

CAS integration tests. Progress report
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 vs. Fricas 1.3.7
via sagemath 9.3

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46 Test file number 81	2214
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48 Test file number 83	2276
49 Test file number 84	2278
50 Test file number 86	2282
51 Test file number 89	2298
52 Test file number 92	2458
53 Test file number 93	2572
54 Test file number 94	2648
55 Test file number 95	2996
56 Test file number 96	3000
57 Test file number 97	3002
58 Test file number 98	3003
59 Test file number 101	3038
60 Test file number 103	3121
61 Test file number 104	3146
62 Test file number 106	3173
63 Test file number 115	3174
64 Test file number 118	3229
65 Test file number 119	3382
66 Test file number 122	3413
67 Test file number 123	3414
68 Test file number 124	3536
69 Test file number 125	3551
70 Test file number 126	3799
71 Test file number 127	3799
72 Test file number 133	3814

73 Test file number 135	3815
74 Test file number 139	3818
75 Test file number 141	3831
76 Test file number 142	3871
77 Test file number 144	3875
78 Test file number 145	3886
79 Test file number 147	3889
80 Test file number 150	3891
81 Test file number 153	3898
82 Test file number 156	3909
83 Test file number 158	3912
84 Test file number 160	3915
85 Test file number 163	3921
86 Test file number 164	3953
87 Test file number 169	3982
88 Test file number 170	4008
89 Test file number 173	4013
90 Test file number 175	4023
91 Test file number 176	4025
92 Test file number 178	4026
93 Test file number 179	4028
94 Test file number 180	4052
95 Test file number 182	4057
96 Test file number 183	4059
97 Test file number 184	4080
98 Test file number 185	4081
99 Test file number 188	4092

100	Test file number 190	4100
101	Test file number 191	4100
102	Test file number 193	4106
103	Test file number 194	4107
104	Test file number 197	4109
105	Test file number 199	4120
106	Test file number 200	4122
107	Test file number 201	4122
108	Test file number 202	4125
109	Test file number 203	4126
110	Test file number 205	4128
111	Test file number 206	4202
112	Test file number 208	4255
113	Test file number 209	4256

1 Table summary of progress report

Table 1: Table summary of progress report

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1	1	41	1 (pass)	0 (not solved)
2	1	175	1 (pass)	0 (not solved)
3	2	25	1 (pass)	0 (not solved)
4	5	174	1 (pass)	-2 (exception) Exception raised: SyntaxError >> Malformed expression
5	10	136	1 (pass)	-1 (time out)
6	10	137	1 (pass)	-1 (time out)
7	10	138	1 (pass)	-1 (time out)
8	10	139	1 (pass)	-1 (time out)
9	10	142	1 (pass)	-1 (time out)
10	10	143	1 (pass)	-1 (time out)
11	10	144	1 (pass)	-1 (time out)
12	10	248	1 (pass)	-1 (time out)
13	10	453	1 (pass)	-1 (time out)
14	11	45	1 (pass)	-1 (time out)
15	11	94	1 (pass)	-1 (time out)
16	14	300	1 (pass)	-1 (time out)
17	14	318	1 (pass)	-1 (time out)
18	14	853	1 (pass)	0 (not solved)
19	14	856	1 (pass)	0 (not solved)
20	14	859	1 (pass)	0 (not solved)
21	14	863	1 (pass)	0 (not solved)
22	14	864	1 (pass)	0 (not solved)
23	14	867	1 (pass)	0 (not solved)
24	14	868	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
25	14	871	1 (pass)	0 (not solved)
26	14	872	1 (pass)	0 (not solved)
27	14	2208	1 (pass)	-1 (time out)
28	14	2209	1 (pass)	-1 (time out)
29	14	2210	1 (pass)	-1 (time out)
30	14	2220	1 (pass)	-1 (time out)
31	14	2221	1 (pass)	-1 (time out)
32	14	2232	1 (pass)	-1 (time out)
33	14	2233	1 (pass)	-1 (time out)
34	14	2261	1 (pass)	-1 (time out)
35	14	2637	1 (pass)	0 (not solved)
36	14	2638	1 (pass)	0 (not solved)
37	14	2639	1 (pass)	0 (not solved)
38	14	2640	1 (pass)	0 (not solved)
39	14	2641	1 (pass)	0 (not solved)
40	14	2642	1 (pass)	0 (not solved)
41	14	2643	1 (pass)	0 (not solved)
42	14	2644	1 (pass)	0 (not solved)
43	14	2645	1 (pass)	0 (not solved)
44	14	2646	1 (pass)	0 (not solved)
45	14	2647	1 (pass)	0 (not solved)
46	14	2648	1 (pass)	0 (not solved)
47	14	2649	1 (pass)	0 (not solved)
48	14	2650	1 (pass)	0 (not solved)
49	14	2651	1 (pass)	0 (not solved)
50	14	2652	1 (pass)	0 (not solved)
51	14	2653	1 (pass)	0 (not solved)
52	14	2654	1 (pass)	0 (not solved)
53	14	2655	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
54	14	2656	1 (pass)	0 (not solved)
55	14	2657	1 (pass)	0 (not solved)
56	14	2658	1 (pass)	0 (not solved)
57	14	2659	1 (pass)	0 (not solved)
58	14	2660	1 (pass)	0 (not solved)
59	14	2661	1 (pass)	0 (not solved)
60	14	2662	1 (pass)	0 (not solved)
61	14	2663	1 (pass)	0 (not solved)
62	14	2664	1 (pass)	0 (not solved)
63	14	2665	1 (pass)	0 (not solved)
64	14	2666	1 (pass)	0 (not solved)
65	14	2667	1 (pass)	0 (not solved)
66	14	2668	1 (pass)	0 (not solved)
67	14	2669	1 (pass)	0 (not solved)
68	14	2670	1 (pass)	0 (not solved)
69	14	2671	1 (pass)	0 (not solved)
70	14	2672	1 (pass)	0 (not solved)
71	14	2673	1 (pass)	0 (not solved)
72	14	2674	1 (pass)	0 (not solved)
73	14	2675	1 (pass)	0 (not solved)
74	14	2676	1 (pass)	0 (not solved)
75	14	2677	1 (pass)	0 (not solved)
76	14	2679	1 (pass)	0 (not solved)
77	14	2680	1 (pass)	0 (not solved)
78	14	2681	1 (pass)	0 (not solved)
79	14	2682	1 (pass)	0 (not solved)
80	14	2683	1 (pass)	0 (not solved)
81	14	2684	1 (pass)	0 (not solved)
82	14	2685	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
83	14	2686	1 (pass)	0 (not solved)
84	14	2687	1 (pass)	0 (not solved)
85	14	2688	1 (pass)	0 (not solved)
86	14	2689	1 (pass)	0 (not solved)
87	14	2690	1 (pass)	0 (not solved)
88	14	2691	1 (pass)	0 (not solved)
89	14	2692	1 (pass)	0 (not solved)
90	14	2693	1 (pass)	0 (not solved)
91	14	2694	1 (pass)	0 (not solved)
92	14	2695	1 (pass)	0 (not solved)
93	14	2696	1 (pass)	0 (not solved)
94	14	2697	1 (pass)	0 (not solved)
95	14	2698	1 (pass)	0 (not solved)
96	14	2699	1 (pass)	0 (not solved)
97	14	2700	1 (pass)	0 (not solved)
98	14	2701	1 (pass)	0 (not solved)
99	14	2702	1 (pass)	0 (not solved)
100	14	2703	1 (pass)	0 (not solved)
101	14	2704	1 (pass)	0 (not solved)
102	14	2705	1 (pass)	0 (not solved)
103	14	2706	1 (pass)	0 (not solved)
104	14	2707	1 (pass)	0 (not solved)
105	14	2708	1 (pass)	0 (not solved)
106	14	2709	1 (pass)	0 (not solved)
107	14	2710	1 (pass)	0 (not solved)
108	14	2711	1 (pass)	0 (not solved)
109	14	2712	1 (pass)	0 (not solved)
110	14	2713	1 (pass)	0 (not solved)
111	14	2714	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
112	14	2715	1 (pass)	0 (not solved)
113	14	2716	1 (pass)	0 (not solved)
114	14	2717	1 (pass)	0 (not solved)
115	14	2718	1 (pass)	0 (not solved)
116	14	2719	1 (pass)	0 (not solved)
117	14	2720	1 (pass)	0 (not solved)
118	14	2721	1 (pass)	0 (not solved)
119	14	2722	1 (pass)	0 (not solved)
120	14	2723	1 (pass)	0 (not solved)
121	14	2724	1 (pass)	0 (not solved)
122	14	2725	1 (pass)	0 (not solved)
123	14	2726	1 (pass)	0 (not solved)
124	14	2727	1 (pass)	0 (not solved)
125	14	2728	1 (pass)	0 (not solved)
126	14	2729	1 (pass)	0 (not solved)
127	14	2730	1 (pass)	0 (not solved)
128	14	2731	1 (pass)	0 (not solved)
129	14	2732	1 (pass)	0 (not solved)
130	14	2733	1 (pass)	0 (not solved)
131	14	2734	1 (pass)	0 (not solved)
132	14	2735	1 (pass)	0 (not solved)
133	14	2736	1 (pass)	0 (not solved)
134	14	2737	1 (pass)	0 (not solved)
135	14	2738	1 (pass)	0 (not solved)
136	14	2739	1 (pass)	0 (not solved)
137	14	2740	1 (pass)	0 (not solved)
138	14	2741	1 (pass)	0 (not solved)
139	14	2742	1 (pass)	0 (not solved)
140	14	2743	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
141	14	2744	1 (pass)	0 (not solved)
142	14	2745	1 (pass)	0 (not solved)
143	14	2746	1 (pass)	0 (not solved)
144	14	2747	1 (pass)	0 (not solved)
145	14	2748	1 (pass)	0 (not solved)
146	14	2749	1 (pass)	0 (not solved)
147	14	2750	1 (pass)	0 (not solved)
148	14	2751	1 (pass)	0 (not solved)
149	14	2752	1 (pass)	0 (not solved)
150	14	2753	1 (pass)	0 (not solved)
151	14	2754	1 (pass)	0 (not solved)
152	14	2755	1 (pass)	0 (not solved)
153	14	2756	1 (pass)	0 (not solved)
154	14	2757	1 (pass)	0 (not solved)
155	14	2758	1 (pass)	0 (not solved)
156	14	2759	1 (pass)	0 (not solved)
157	14	2760	1 (pass)	0 (not solved)
158	14	2761	1 (pass)	0 (not solved)
159	14	2762	1 (pass)	0 (not solved)
160	14	2763	1 (pass)	0 (not solved)
161	14	2764	1 (pass)	0 (not solved)
162	14	2765	1 (pass)	0 (not solved)
163	14	2766	1 (pass)	0 (not solved)
164	14	2767	1 (pass)	0 (not solved)
165	14	2768	1 (pass)	0 (not solved)
166	14	2769	1 (pass)	0 (not solved)
167	14	2770	1 (pass)	0 (not solved)
168	14	2771	1 (pass)	0 (not solved)
169	14	2772	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
170	14	2773	1 (pass)	0 (not solved)
171	14	2774	1 (pass)	0 (not solved)
172	14	2775	1 (pass)	0 (not solved)
173	14	2776	1 (pass)	0 (not solved)
174	14	2777	1 (pass)	0 (not solved)
175	14	2778	1 (pass)	0 (not solved)
176	14	2779	1 (pass)	0 (not solved)
177	14	2780	1 (pass)	0 (not solved)
178	14	2781	1 (pass)	0 (not solved)
179	14	2782	1 (pass)	0 (not solved)
180	14	2783	1 (pass)	0 (not solved)
181	14	2784	1 (pass)	0 (not solved)
182	14	2785	1 (pass)	0 (not solved)
183	14	2786	1 (pass)	0 (not solved)
184	14	2787	1 (pass)	0 (not solved)
185	14	2788	1 (pass)	0 (not solved)
186	14	2789	1 (pass)	0 (not solved)
187	14	2790	1 (pass)	0 (not solved)
188	14	2791	1 (pass)	0 (not solved)
189	14	2792	1 (pass)	0 (not solved)
190	14	2793	1 (pass)	0 (not solved)
191	14	2794	1 (pass)	0 (not solved)
192	14	2795	1 (pass)	0 (not solved)
193	14	2796	1 (pass)	0 (not solved)
194	14	2797	1 (pass)	0 (not solved)
195	14	2798	1 (pass)	0 (not solved)
196	14	2799	1 (pass)	0 (not solved)
197	14	2800	1 (pass)	0 (not solved)
198	14	2801	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
199	14	2802	1 (pass)	0 (not solved)
200	14	2803	1 (pass)	0 (not solved)
201	14	2804	1 (pass)	0 (not solved)
202	14	2805	1 (pass)	0 (not solved)
203	14	2806	1 (pass)	0 (not solved)
204	14	2807	1 (pass)	0 (not solved)
205	14	2808	1 (pass)	0 (not solved)
206	14	2809	1 (pass)	0 (not solved)
207	14	2810	1 (pass)	0 (not solved)
208	14	2811	1 (pass)	0 (not solved)
209	14	2812	1 (pass)	0 (not solved)
210	14	2813	1 (pass)	0 (not solved)
211	14	2814	1 (pass)	0 (not solved)
212	14	2815	1 (pass)	0 (not solved)
213	14	2816	1 (pass)	0 (not solved)
214	14	2817	1 (pass)	0 (not solved)
215	14	2819	1 (pass)	0 (not solved)
216	14	2820	1 (pass)	0 (not solved)
217	14	2821	1 (pass)	0 (not solved)
218	14	2822	1 (pass)	0 (not solved)
219	14	2823	1 (pass)	0 (not solved)
220	14	2824	1 (pass)	0 (not solved)
221	14	2825	1 (pass)	0 (not solved)
222	14	2826	1 (pass)	0 (not solved)
223	14	2827	1 (pass)	0 (not solved)
224	14	2828	1 (pass)	0 (not solved)
225	14	2829	1 (pass)	0 (not solved)
226	14	2830	1 (pass)	0 (not solved)
227	14	2831	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
228	14	2832	1 (pass)	0 (not solved)
229	14	2833	1 (pass)	0 (not solved)
230	14	2834	1 (pass)	0 (not solved)
231	14	2835	1 (pass)	0 (not solved)
232	14	2836	1 (pass)	0 (not solved)
233	14	2837	1 (pass)	0 (not solved)
234	14	2838	1 (pass)	0 (not solved)
235	14	2839	1 (pass)	0 (not solved)
236	14	2840	1 (pass)	0 (not solved)
237	14	2841	1 (pass)	0 (not solved)
238	14	2844	1 (pass)	0 (not solved)
239	14	2846	1 (pass)	0 (not solved)
240	14	2847	1 (pass)	0 (not solved)
241	14	2849	1 (pass)	0 (not solved)
242	14	2852	1 (pass)	0 (not solved)
243	14	2854	1 (pass)	0 (not solved)
244	14	2855	1 (pass)	0 (not solved)
245	14	2857	1 (pass)	0 (not solved)
246	14	2858	1 (pass)	0 (not solved)
247	14	2859	1 (pass)	0 (not solved)
248	14	2860	1 (pass)	0 (not solved)
249	14	2861	1 (pass)	0 (not solved)
250	14	2864	1 (pass)	0 (not solved)
251	14	2865	1 (pass)	0 (not solved)
252	14	2866	1 (pass)	0 (not solved)
253	14	2867	1 (pass)	0 (not solved)
254	14	2868	1 (pass)	0 (not solved)
255	14	2869	1 (pass)	0 (not solved)
256	14	2870	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
257	14	2871	1 (pass)	0 (not solved)
258	14	2872	1 (pass)	0 (not solved)
259	14	2873	1 (pass)	0 (not solved)
260	14	2874	1 (pass)	0 (not solved)
261	14	2875	1 (pass)	0 (not solved)
262	14	2876	1 (pass)	0 (not solved)
263	14	2877	1 (pass)	0 (not solved)
264	14	2878	1 (pass)	0 (not solved)
265	14	2879	1 (pass)	0 (not solved)
266	14	2880	1 (pass)	0 (not solved)
267	14	2881	1 (pass)	0 (not solved)
268	14	2882	1 (pass)	0 (not solved)
269	14	2883	1 (pass)	0 (not solved)
270	14	2884	1 (pass)	0 (not solved)
271	14	2885	1 (pass)	0 (not solved)
272	14	2886	1 (pass)	0 (not solved)
273	14	2887	1 (pass)	0 (not solved)
274	14	2891	1 (pass)	0 (not solved)
275	14	2892	1 (pass)	0 (not solved)
276	14	2893	1 (pass)	0 (not solved)
277	14	2894	1 (pass)	0 (not solved)
278	14	2895	1 (pass)	0 (not solved)
279	14	2896	1 (pass)	0 (not solved)
280	14	2897	1 (pass)	0 (not solved)
281	14	2898	1 (pass)	0 (not solved)
282	14	2899	1 (pass)	0 (not solved)
283	14	2900	1 (pass)	0 (not solved)
284	14	2901	1 (pass)	0 (not solved)
285	14	2902	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
286	14	2903	1 (pass)	0 (not solved)
287	14	2904	1 (pass)	0 (not solved)
288	14	2905	1 (pass)	0 (not solved)
289	14	2906	1 (pass)	0 (not solved)
290	14	2907	1 (pass)	0 (not solved)
291	14	2908	1 (pass)	0 (not solved)
292	14	2909	1 (pass)	0 (not solved)
293	14	2910	1 (pass)	0 (not solved)
294	14	2911	1 (pass)	0 (not solved)
295	14	2912	1 (pass)	0 (not solved)
296	14	2913	1 (pass)	0 (not solved)
297	14	2914	1 (pass)	0 (not solved)
298	14	2915	1 (pass)	0 (not solved)
299	14	2916	1 (pass)	0 (not solved)
300	14	2917	1 (pass)	0 (not solved)
301	14	2918	1 (pass)	0 (not solved)
302	14	2919	1 (pass)	0 (not solved)
303	14	2920	1 (pass)	0 (not solved)
304	14	2921	1 (pass)	0 (not solved)
305	14	2922	1 (pass)	0 (not solved)
306	14	2923	1 (pass)	0 (not solved)
307	14	2924	1 (pass)	0 (not solved)
308	14	2925	1 (pass)	0 (not solved)
309	14	2926	1 (pass)	0 (not solved)
310	14	2927	1 (pass)	0 (not solved)
311	14	2928	1 (pass)	0 (not solved)
312	14	2929	1 (pass)	0 (not solved)
313	14	2930	1 (pass)	0 (not solved)
314	14	2931	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
315	14	2932	1 (pass)	0 (not solved)
316	14	2933	1 (pass)	0 (not solved)
317	14	2934	1 (pass)	0 (not solved)
318	14	2935	1 (pass)	0 (not solved)
319	14	2936	1 (pass)	0 (not solved)
320	14	2937	1 (pass)	0 (not solved)
321	14	2938	1 (pass)	0 (not solved)
322	14	2939	1 (pass)	0 (not solved)
323	14	2940	1 (pass)	0 (not solved)
324	14	2941	1 (pass)	0 (not solved)
325	14	2942	1 (pass)	0 (not solved)
326	14	2943	1 (pass)	0 (not solved)
327	14	2944	1 (pass)	0 (not solved)
328	14	2945	1 (pass)	0 (not solved)
329	14	2946	1 (pass)	0 (not solved)
330	14	2947	1 (pass)	0 (not solved)
331	14	2948	1 (pass)	0 (not solved)
332	14	2949	1 (pass)	0 (not solved)
333	14	2950	1 (pass)	0 (not solved)
334	14	2951	1 (pass)	0 (not solved)
335	14	2952	1 (pass)	0 (not solved)
336	14	2953	1 (pass)	0 (not solved)
337	14	2954	1 (pass)	0 (not solved)
338	14	2955	1 (pass)	0 (not solved)
339	14	2956	1 (pass)	0 (not solved)
340	14	2957	1 (pass)	0 (not solved)
341	14	2958	1 (pass)	0 (not solved)
342	14	2959	1 (pass)	0 (not solved)
343	14	2960	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
344	14	2961	1 (pass)	0 (not solved)
345	14	2962	1 (pass)	0 (not solved)
346	14	2963	1 (pass)	0 (not solved)
347	14	2964	1 (pass)	0 (not solved)
348	14	2965	1 (pass)	0 (not solved)
349	14	2966	1 (pass)	0 (not solved)
350	14	2967	1 (pass)	0 (not solved)
351	14	2968	1 (pass)	0 (not solved)
352	14	2969	1 (pass)	0 (not solved)
353	14	2970	1 (pass)	0 (not solved)
354	14	2971	1 (pass)	0 (not solved)
355	14	2972	1 (pass)	0 (not solved)
356	14	2973	1 (pass)	0 (not solved)
357	14	2974	1 (pass)	0 (not solved)
358	14	2975	1 (pass)	0 (not solved)
359	14	2976	1 (pass)	0 (not solved)
360	14	2977	1 (pass)	0 (not solved)
361	14	2978	1 (pass)	0 (not solved)
362	14	2979	1 (pass)	0 (not solved)
363	14	2980	1 (pass)	0 (not solved)
364	14	2981	1 (pass)	0 (not solved)
365	14	2982	1 (pass)	0 (not solved)
366	14	2983	1 (pass)	0 (not solved)
367	14	2984	1 (pass)	0 (not solved)
368	14	2985	1 (pass)	0 (not solved)
369	14	2986	1 (pass)	0 (not solved)
370	14	2987	1 (pass)	0 (not solved)
371	14	2988	1 (pass)	0 (not solved)
372	14	2989	1 (pass)	0 (not solved)

Continued on next page

Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
373	14	2990	1 (pass)	0 (not solved)
374	14	2991	1 (pass)	0 (not solved)
375	14	2992	1 (pass)	0 (not solved)
376	14	2993	1 (pass)	0 (not solved)
377	14	2994	1 (pass)	0 (not solved)
378	14	2995	1 (pass)	0 (not solved)
379	14	2996	1 (pass)	0 (not solved)
380	14	2997	1 (pass)	0 (not solved)
381	14	2998	1 (pass)	0 (not solved)
382	14	2999	1 (pass)	0 (not solved)
383	14	3000	1 (pass)	0 (not solved)
384	15	4	1 (pass)	-1 (time out)
385	15	33	1 (pass)	0 (not solved)
386	15	34	1 (pass)	0 (not solved)
387	15	35	1 (pass)	0 (not solved)
388	15	36	1 (pass)	0 (not solved)
389	15	37	1 (pass)	0 (not solved)
390	15	38	1 (pass)	0 (not solved)
391	15	44	1 (pass)	0 (not solved)
392	15	45	1 (pass)	0 (not solved)
393	15	46	1 (pass)	0 (not solved)
394	15	47	1 (pass)	0 (not solved)
395	15	51	1 (pass)	0 (not solved)
396	15	52	1 (pass)	0 (not solved)
397	15	53	1 (pass)	0 (not solved)
398	15	60	1 (pass)	0 (not solved)
399	15	61	1 (pass)	0 (not solved)
400	15	62	1 (pass)	0 (not solved)
401	15	68	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
402	15	69	1 (pass)	0 (not solved)
403	17	33	1 (pass)	-1 (time out)
404	17	59	1 (pass)	-1 (time out)
405	17	61	1 (pass)	0 (not solved)
406	17	62	1 (pass)	0 (not solved)
407	17	63	1 (pass)	0 (not solved)
408	17	64	1 (pass)	0 (not solved)
409	17	65	1 (pass)	0 (not solved)
410	17	66	1 (pass)	0 (not solved)
411	17	67	1 (pass)	0 (not solved)
412	17	68	1 (pass)	0 (not solved)
413	17	69	1 (pass)	0 (not solved)
414	17	70	1 (pass)	0 (not solved)
415	17	71	1 (pass)	0 (not solved)
416	17	72	1 (pass)	0 (not solved)
417	17	73	1 (pass)	0 (not solved)
418	17	74	1 (pass)	0 (not solved)
419	17	75	1 (pass)	0 (not solved)
420	17	76	1 (pass)	0 (not solved)
421	17	77	1 (pass)	0 (not solved)
422	17	78	1 (pass)	0 (not solved)
423	18	1	1 (pass)	0 (not solved)
424	18	2	1 (pass)	0 (not solved)
425	18	3	1 (pass)	0 (not solved)
426	18	16	1 (pass)	0 (not solved)
427	18	17	1 (pass)	0 (not solved)
428	18	18	1 (pass)	0 (not solved)
429	18	26	1 (pass)	0 (not solved)
430	18	27	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
431	18	28	1 (pass)	0 (not solved)
432	19	589	1 (pass)	0 (not solved)
433	19	590	1 (pass)	0 (not solved)
434	19	591	1 (pass)	0 (not solved)
435	19	592	1 (pass)	0 (not solved)
436	19	593	1 (pass)	0 (not solved)
437	19	594	1 (pass)	0 (not solved)
438	19	595	1 (pass)	0 (not solved)
439	19	596	1 (pass)	0 (not solved)
440	19	597	1 (pass)	0 (not solved)
441	19	598	1 (pass)	0 (not solved)
442	19	599	1 (pass)	0 (not solved)
443	19	600	1 (pass)	0 (not solved)
444	19	601	1 (pass)	0 (not solved)
445	19	602	1 (pass)	0 (not solved)
446	19	603	1 (pass)	0 (not solved)
447	19	604	1 (pass)	0 (not solved)
448	19	605	1 (pass)	0 (not solved)
449	19	606	1 (pass)	0 (not solved)
450	19	607	1 (pass)	0 (not solved)
451	19	608	1 (pass)	0 (not solved)
452	19	609	1 (pass)	0 (not solved)
453	19	610	1 (pass)	0 (not solved)
454	19	611	1 (pass)	0 (not solved)
455	19	612	1 (pass)	0 (not solved)
456	19	613	1 (pass)	0 (not solved)
457	19	614	1 (pass)	0 (not solved)
458	19	615	1 (pass)	0 (not solved)
459	19	616	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
460	19	617	1 (pass)	0 (not solved)
461	19	618	1 (pass)	0 (not solved)
462	19	619	1 (pass)	0 (not solved)
463	19	620	1 (pass)	0 (not solved)
464	19	621	1 (pass)	0 (not solved)
465	19	622	1 (pass)	0 (not solved)
466	19	623	1 (pass)	0 (not solved)
467	19	624	1 (pass)	0 (not solved)
468	19	625	1 (pass)	0 (not solved)
469	19	626	1 (pass)	0 (not solved)
470	19	627	1 (pass)	0 (not solved)
471	19	628	1 (pass)	0 (not solved)
472	19	629	1 (pass)	0 (not solved)
473	19	630	1 (pass)	0 (not solved)
474	19	631	1 (pass)	0 (not solved)
475	19	632	1 (pass)	0 (not solved)
476	19	633	1 (pass)	0 (not solved)
477	19	634	1 (pass)	0 (not solved)
478	19	635	1 (pass)	0 (not solved)
479	19	636	1 (pass)	0 (not solved)
480	19	637	1 (pass)	0 (not solved)
481	19	638	1 (pass)	0 (not solved)
482	19	639	1 (pass)	0 (not solved)
483	19	640	1 (pass)	0 (not solved)
484	19	641	1 (pass)	0 (not solved)
485	19	642	1 (pass)	0 (not solved)
486	19	643	1 (pass)	0 (not solved)
487	19	644	1 (pass)	0 (not solved)
488	19	645	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
489	19	646	1 (pass)	0 (not solved)
490	19	647	1 (pass)	0 (not solved)
491	19	648	1 (pass)	0 (not solved)
492	19	650	1 (pass)	0 (not solved)
493	20	156	1 (pass)	-1 (time out)
494	20	157	1 (pass)	-1 (time out)
495	20	158	1 (pass)	-1 (time out)
496	20	159	1 (pass)	-1 (time out)
497	20	179	1 (pass)	0 (not solved)
498	20	187	1 (pass)	0 (not solved)
499	20	188	1 (pass)	0 (not solved)
500	20	189	1 (pass)	0 (not solved)
501	20	190	1 (pass)	0 (not solved)
502	20	191	1 (pass)	0 (not solved)
503	20	192	1 (pass)	0 (not solved)
504	20	193	1 (pass)	0 (not solved)
505	20	194	1 (pass)	0 (not solved)
506	20	198	1 (pass)	0 (not solved)
507	20	203	1 (pass)	0 (not solved)
508	20	214	1 (pass)	0 (not solved)
509	20	215	1 (pass)	0 (not solved)
510	20	216	1 (pass)	0 (not solved)
511	20	217	1 (pass)	0 (not solved)
512	20	218	1 (pass)	0 (not solved)
513	20	219	1 (pass)	0 (not solved)
514	20	220	1 (pass)	0 (not solved)
515	20	221	1 (pass)	0 (not solved)
516	20	222	1 (pass)	0 (not solved)
517	20	223	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
518	20	225	1 (pass)	0 (not solved)
519	20	226	1 (pass)	0 (not solved)
520	20	227	1 (pass)	0 (not solved)
521	20	228	1 (pass)	0 (not solved)
522	20	229	1 (pass)	0 (not solved)
523	20	230	1 (pass)	0 (not solved)
524	20	232	1 (pass)	0 (not solved)
525	20	233	1 (pass)	0 (not solved)
526	20	234	1 (pass)	0 (not solved)
527	20	235	1 (pass)	0 (not solved)
528	20	236	1 (pass)	0 (not solved)
529	20	237	1 (pass)	0 (not solved)
530	20	248	1 (pass)	0 (not solved)
531	20	249	1 (pass)	0 (not solved)
532	20	250	1 (pass)	0 (not solved)
533	20	252	1 (pass)	0 (not solved)
534	20	253	1 (pass)	0 (not solved)
535	20	254	1 (pass)	0 (not solved)
536	20	255	1 (pass)	0 (not solved)
537	20	256	1 (pass)	0 (not solved)
538	20	257	1 (pass)	0 (not solved)
539	20	290	1 (pass)	0 (not solved)
540	20	291	1 (pass)	0 (not solved)
541	20	292	1 (pass)	0 (not solved)
542	20	294	1 (pass)	0 (not solved)
543	20	296	1 (pass)	0 (not solved)
544	21	478	1 (pass)	-1 (time out)
545	21	479	1 (pass)	-1 (time out)
546	21	480	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
547	21	482	1 (pass)	-1 (time out)
548	21	484	1 (pass)	-1 (time out)
549	21	491	1 (pass)	-1 (time out)
550	21	493	1 (pass)	-1 (time out)
551	21	785	1 (pass)	0 (not solved)
552	21	786	1 (pass)	0 (not solved)
553	21	787	1 (pass)	0 (not solved)
554	21	788	1 (pass)	0 (not solved)
555	21	789	1 (pass)	0 (not solved)
556	21	790	1 (pass)	0 (not solved)
557	21	791	1 (pass)	0 (not solved)
558	21	792	1 (pass)	0 (not solved)
559	21	793	1 (pass)	0 (not solved)
560	21	794	1 (pass)	0 (not solved)
561	21	795	1 (pass)	0 (not solved)
562	21	796	1 (pass)	0 (not solved)
563	21	797	1 (pass)	0 (not solved)
564	21	798	1 (pass)	0 (not solved)
565	21	799	1 (pass)	0 (not solved)
566	21	800	1 (pass)	0 (not solved)
567	21	801	1 (pass)	0 (not solved)
568	21	802	1 (pass)	0 (not solved)
569	21	803	1 (pass)	0 (not solved)
570	21	804	1 (pass)	0 (not solved)
571	21	805	1 (pass)	0 (not solved)
572	21	806	1 (pass)	0 (not solved)
573	21	807	1 (pass)	0 (not solved)
574	21	808	1 (pass)	0 (not solved)
575	21	809	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
576	21	810	1 (pass)	0 (not solved)
577	21	811	1 (pass)	0 (not solved)
578	21	812	1 (pass)	0 (not solved)
579	21	813	1 (pass)	0 (not solved)
580	21	814	1 (pass)	0 (not solved)
581	21	815	1 (pass)	0 (not solved)
582	21	816	1 (pass)	0 (not solved)
583	21	817	1 (pass)	0 (not solved)
584	21	818	1 (pass)	0 (not solved)
585	21	819	1 (pass)	0 (not solved)
586	21	820	1 (pass)	0 (not solved)
587	21	821	1 (pass)	0 (not solved)
588	21	822	1 (pass)	0 (not solved)
589	21	823	1 (pass)	0 (not solved)
590	21	824	1 (pass)	0 (not solved)
591	21	825	1 (pass)	0 (not solved)
592	21	826	1 (pass)	0 (not solved)
593	21	827	1 (pass)	0 (not solved)
594	21	828	1 (pass)	0 (not solved)
595	21	829	1 (pass)	0 (not solved)
596	21	830	1 (pass)	0 (not solved)
597	21	831	1 (pass)	0 (not solved)
598	21	832	1 (pass)	0 (not solved)
599	21	833	1 (pass)	0 (not solved)
600	21	834	1 (pass)	0 (not solved)
601	21	835	1 (pass)	0 (not solved)
602	21	836	1 (pass)	0 (not solved)
603	21	837	1 (pass)	0 (not solved)
604	21	838	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
605	21	839	1 (pass)	0 (not solved)
606	21	840	1 (pass)	0 (not solved)
607	21	841	1 (pass)	0 (not solved)
608	21	842	1 (pass)	0 (not solved)
609	21	843	1 (pass)	0 (not solved)
610	21	844	1 (pass)	0 (not solved)
611	21	845	1 (pass)	0 (not solved)
612	21	846	1 (pass)	0 (not solved)
613	21	847	1 (pass)	0 (not solved)
614	21	848	1 (pass)	0 (not solved)
615	21	849	1 (pass)	0 (not solved)
616	21	850	1 (pass)	0 (not solved)
617	21	851	1 (pass)	0 (not solved)
618	21	852	1 (pass)	0 (not solved)
619	21	853	1 (pass)	0 (not solved)
620	21	854	1 (pass)	0 (not solved)
621	21	855	1 (pass)	0 (not solved)
622	21	856	1 (pass)	0 (not solved)
623	21	857	1 (pass)	0 (not solved)
624	21	858	1 (pass)	0 (not solved)
625	21	859	1 (pass)	0 (not solved)
626	21	860	1 (pass)	0 (not solved)
627	21	861	1 (pass)	0 (not solved)
628	21	862	1 (pass)	0 (not solved)
629	21	863	1 (pass)	0 (not solved)
630	21	864	1 (pass)	0 (not solved)
631	21	964	1 (pass)	0 (not solved)
632	21	966	1 (pass)	0 (not solved)
633	21	995	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
634	21	997	1 (pass)	0 (not solved)
635	21	999	1 (pass)	0 (not solved)
636	21	1001	1 (pass)	0 (not solved)
637	21	1002	1 (pass)	0 (not solved)
638	21	1003	1 (pass)	0 (not solved)
639	21	1007	1 (pass)	0 (not solved)
640	22	62	1 (pass)	-1 (time out)
641	22	97	1 (pass)	0 (not solved)
642	24	62	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent
643	24	63	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent
644	24	64	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent
645	24	65	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent
646	24	70	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent
647	24	71	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
648	24	72	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent
649	24	73	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent
650	24	78	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent
651	24	79	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent
652	24	80	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent
653	24	81	1 (pass)	-2 (exception) Exception raised: TypeError >> keys do not match self 's parent
654	25	376	1 (pass)	0 (not solved)
655	25	377	1 (pass)	0 (not solved)
656	25	378	1 (pass)	0 (not solved)
657	25	379	1 (pass)	0 (not solved)
658	25	380	1 (pass)	0 (not solved)
659	25	381	1 (pass)	0 (not solved)
660	25	382	1 (pass)	0 (not solved)
661	25	383	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
662	25	384	1 (pass)	0 (not solved)
663	25	385	1 (pass)	0 (not solved)
664	25	386	1 (pass)	0 (not solved)
665	25	394	1 (pass)	0 (not solved)
666	25	395	1 (pass)	0 (not solved)
667	25	396	1 (pass)	0 (not solved)
668	25	397	1 (pass)	0 (not solved)
669	25	398	1 (pass)	0 (not solved)
670	25	399	1 (pass)	0 (not solved)
671	25	400	1 (pass)	0 (not solved)
672	25	401	1 (pass)	0 (not solved)
673	25	402	1 (pass)	0 (not solved)
674	25	403	1 (pass)	0 (not solved)
675	25	411	1 (pass)	0 (not solved)
676	25	412	1 (pass)	0 (not solved)
677	25	413	1 (pass)	0 (not solved)
678	25	414	1 (pass)	0 (not solved)
679	25	415	1 (pass)	0 (not solved)
680	25	416	1 (pass)	0 (not solved)
681	25	417	1 (pass)	0 (not solved)
682	25	418	1 (pass)	0 (not solved)
683	25	419	1 (pass)	0 (not solved)
684	25	420	1 (pass)	0 (not solved)
685	25	428	1 (pass)	0 (not solved)
686	25	429	1 (pass)	0 (not solved)
687	25	430	1 (pass)	0 (not solved)
688	25	431	1 (pass)	0 (not solved)
689	25	432	1 (pass)	0 (not solved)
690	25	433	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
691	25	434	1 (pass)	0 (not solved)
692	25	435	1 (pass)	0 (not solved)
693	25	436	1 (pass)	0 (not solved)
694	25	437	1 (pass)	0 (not solved)
695	25	446	1 (pass)	0 (not solved)
696	25	447	1 (pass)	0 (not solved)
697	25	448	1 (pass)	0 (not solved)
698	25	449	1 (pass)	0 (not solved)
699	25	450	1 (pass)	0 (not solved)
700	25	451	1 (pass)	0 (not solved)
701	25	452	1 (pass)	0 (not solved)
702	25	453	1 (pass)	0 (not solved)
703	25	454	1 (pass)	0 (not solved)
704	25	455	1 (pass)	0 (not solved)
705	25	464	1 (pass)	0 (not solved)
706	25	465	1 (pass)	0 (not solved)
707	25	467	1 (pass)	0 (not solved)
708	25	468	1 (pass)	0 (not solved)
709	25	469	1 (pass)	0 (not solved)
710	25	470	1 (pass)	0 (not solved)
711	25	472	1 (pass)	0 (not solved)
712	25	473	1 (pass)	0 (not solved)
713	25	482	1 (pass)	0 (not solved)
714	25	483	1 (pass)	0 (not solved)
715	25	484	1 (pass)	0 (not solved)
716	25	485	1 (pass)	0 (not solved)
717	25	486	1 (pass)	0 (not solved)
718	25	487	1 (pass)	0 (not solved)
719	25	488	1 (pass)	0 (not solved)

Continued on next page

Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
720	25	489	1 (pass)	0 (not solved)
721	25	490	1 (pass)	0 (not solved)
722	25	491	1 (pass)	0 (not solved)
723	25	500	1 (pass)	0 (not solved)
724	25	501	1 (pass)	0 (not solved)
725	25	503	1 (pass)	0 (not solved)
726	25	504	1 (pass)	0 (not solved)
727	25	505	1 (pass)	0 (not solved)
728	25	506	1 (pass)	0 (not solved)
729	25	508	1 (pass)	0 (not solved)
730	25	509	1 (pass)	0 (not solved)
731	25	518	1 (pass)	-1 (time out)
732	25	776	1 (pass)	0 (not solved)
733	25	777	1 (pass)	0 (not solved)
734	25	778	1 (pass)	0 (not solved)
735	25	779	1 (pass)	0 (not solved)
736	25	796	1 (pass)	0 (not solved)
737	25	797	1 (pass)	0 (not solved)
738	25	799	1 (pass)	0 (not solved)
739	25	803	1 (pass)	0 (not solved)
740	25	804	1 (pass)	0 (not solved)
741	25	809	1 (pass)	0 (not solved)
742	25	810	1 (pass)	0 (not solved)
743	25	821	1 (pass)	0 (not solved)
744	25	822	1 (pass)	0 (not solved)
745	25	823	1 (pass)	0 (not solved)
746	25	824	1 (pass)	0 (not solved)
747	25	825	1 (pass)	0 (not solved)
748	25	841	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
749	25	842	1 (pass)	0 (not solved)
750	25	843	1 (pass)	0 (not solved)
751	25	844	1 (pass)	0 (not solved)
752	25	845	1 (pass)	0 (not solved)
753	25	861	1 (pass)	0 (not solved)
754	25	862	1 (pass)	0 (not solved)
755	25	863	1 (pass)	0 (not solved)
756	25	864	1 (pass)	0 (not solved)
757	25	865	1 (pass)	0 (not solved)
758	25	866	1 (pass)	0 (not solved)
759	25	873	1 (pass)	0 (not solved)
760	25	884	1 (pass)	0 (not solved)
761	25	885	1 (pass)	0 (not solved)
762	25	886	1 (pass)	0 (not solved)
763	25	887	1 (pass)	0 (not solved)
764	25	888	1 (pass)	0 (not solved)
765	25	889	1 (pass)	0 (not solved)
766	25	890	1 (pass)	0 (not solved)
767	25	891	1 (pass)	0 (not solved)
768	25	904	1 (pass)	0 (not solved)
769	25	905	1 (pass)	0 (not solved)
770	25	906	1 (pass)	0 (not solved)
771	25	907	1 (pass)	0 (not solved)
772	25	908	1 (pass)	0 (not solved)
773	25	909	1 (pass)	0 (not solved)
774	25	910	1 (pass)	0 (not solved)
775	25	911	1 (pass)	0 (not solved)
776	25	912	1 (pass)	0 (not solved)
777	25	916	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
778	25	927	1 (pass)	0 (not solved)
779	25	928	1 (pass)	0 (not solved)
780	25	929	1 (pass)	0 (not solved)
781	25	930	1 (pass)	0 (not solved)
782	25	931	1 (pass)	0 (not solved)
783	25	947	1 (pass)	0 (not solved)
784	25	948	1 (pass)	0 (not solved)
785	25	949	1 (pass)	0 (not solved)
786	25	950	1 (pass)	0 (not solved)
787	25	951	1 (pass)	0 (not solved)
788	25	952	1 (pass)	0 (not solved)
789	25	959	1 (pass)	0 (not solved)
790	25	968	1 (pass)	0 (not solved)
791	25	969	1 (pass)	0 (not solved)
792	25	970	1 (pass)	0 (not solved)
793	25	971	1 (pass)	0 (not solved)
794	25	973	1 (pass)	0 (not solved)
795	25	977	1 (pass)	0 (not solved)
796	25	979	1 (pass)	0 (not solved)
797	25	982	1 (pass)	0 (not solved)
798	25	983	1 (pass)	0 (not solved)
799	25	1276	1 (pass)	-1 (time out)
800	25	1396	1 (pass)	0 (not solved)
801	25	1397	1 (pass)	0 (not solved)
802	25	1398	1 (pass)	0 (not solved)
803	25	1402	1 (pass)	0 (not solved)
804	25	1403	1 (pass)	0 (not solved)
805	25	1404	1 (pass)	0 (not solved)
806	25	1420	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
807	25	1421	1 (pass)	0 (not solved)
808	25	1422	1 (pass)	0 (not solved)
809	25	1423	1 (pass)	0 (not solved)
810	25	1428	1 (pass)	0 (not solved)
811	25	1429	1 (pass)	0 (not solved)
812	25	1430	1 (pass)	0 (not solved)
813	25	1431	1 (pass)	0 (not solved)
814	25	1510	1 (pass)	0 (not solved)
815	25	1525	1 (pass)	0 (not solved)
816	25	1527	1 (pass)	0 (not solved)
817	25	1529	1 (pass)	0 (not solved)
818	25	2001	1 (pass)	0 (not solved)
819	25	2002	1 (pass)	0 (not solved)
820	25	2003	1 (pass)	0 (not solved)
821	25	2007	1 (pass)	0 (not solved)
822	25	2008	1 (pass)	0 (not solved)
823	25	2009	1 (pass)	0 (not solved)
824	25	2027	1 (pass)	0 (not solved)
825	25	2028	1 (pass)	0 (not solved)
826	25	2029	1 (pass)	0 (not solved)
827	25	2033	1 (pass)	0 (not solved)
828	25	2034	1 (pass)	0 (not solved)
829	25	2035	1 (pass)	0 (not solved)
830	25	2036	1 (pass)	0 (not solved)
831	25	2047	1 (pass)	0 (not solved)
832	25	2048	1 (pass)	0 (not solved)
833	25	2049	1 (pass)	0 (not solved)
834	25	2053	1 (pass)	0 (not solved)
835	25	2054	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
836	25	2055	1 (pass)	0 (not solved)
837	25	2056	1 (pass)	0 (not solved)
838	25	2062	1 (pass)	0 (not solved)
839	25	2064	1 (pass)	0 (not solved)
840	25	2072	1 (pass)	0 (not solved)
841	25	2080	1 (pass)	0 (not solved)
842	25	2086	1 (pass)	0 (not solved)
843	25	2088	1 (pass)	0 (not solved)
844	25	2094	1 (pass)	0 (not solved)
845	25	2096	1 (pass)	0 (not solved)
846	25	2102	1 (pass)	0 (not solved)
847	25	2104	1 (pass)	0 (not solved)
848	25	2106	1 (pass)	-1 (time out)
849	25	2947	1 (pass)	0 (not solved)
850	25	2948	1 (pass)	0 (not solved)
851	25	2949	1 (pass)	0 (not solved)
852	25	2950	1 (pass)	0 (not solved)
853	25	2951	1 (pass)	0 (not solved)
854	25	2952	1 (pass)	0 (not solved)
855	25	2953	1 (pass)	0 (not solved)
856	25	2954	1 (pass)	0 (not solved)
857	26	26	1 (pass)	-1 (time out)
858	27	185	1 (pass)	0 (not solved)
859	27	186	1 (pass)	0 (not solved)
860	27	187	1 (pass)	0 (not solved)
861	27	188	1 (pass)	0 (not solved)
862	27	189	1 (pass)	0 (not solved)
863	27	190	1 (pass)	0 (not solved)
864	27	191	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
865	27	192	1 (pass)	0 (not solved)
866	27	193	1 (pass)	0 (not solved)
867	27	194	1 (pass)	0 (not solved)
868	27	195	1 (pass)	0 (not solved)
869	27	202	1 (pass)	0 (not solved)
870	27	203	1 (pass)	0 (not solved)
871	27	204	1 (pass)	0 (not solved)
872	27	205	1 (pass)	0 (not solved)
873	27	206	1 (pass)	0 (not solved)
874	27	207	1 (pass)	0 (not solved)
875	27	208	1 (pass)	0 (not solved)
876	27	209	1 (pass)	0 (not solved)
877	27	210	1 (pass)	0 (not solved)
878	27	211	1 (pass)	0 (not solved)
879	27	212	1 (pass)	0 (not solved)
880	27	219	1 (pass)	0 (not solved)
881	27	220	1 (pass)	0 (not solved)
882	27	221	1 (pass)	0 (not solved)
883	27	222	1 (pass)	0 (not solved)
884	27	223	1 (pass)	0 (not solved)
885	27	224	1 (pass)	0 (not solved)
886	27	225	1 (pass)	0 (not solved)
887	27	226	1 (pass)	0 (not solved)
888	27	227	1 (pass)	0 (not solved)
889	27	234	1 (pass)	0 (not solved)
890	27	235	1 (pass)	0 (not solved)
891	27	236	1 (pass)	0 (not solved)
892	27	237	1 (pass)	0 (not solved)
893	27	238	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
894	27	239	1 (pass)	0 (not solved)
895	27	240	1 (pass)	0 (not solved)
896	27	241	1 (pass)	0 (not solved)
897	27	242	1 (pass)	0 (not solved)
898	27	243	1 (pass)	0 (not solved)
899	27	249	1 (pass)	0 (not solved)
900	27	250	1 (pass)	0 (not solved)
901	27	251	1 (pass)	0 (not solved)
902	27	252	1 (pass)	0 (not solved)
903	27	253	1 (pass)	0 (not solved)
904	27	254	1 (pass)	0 (not solved)
905	27	255	1 (pass)	0 (not solved)
906	27	256	1 (pass)	0 (not solved)
907	27	257	1 (pass)	0 (not solved)
908	27	258	1 (pass)	0 (not solved)
909	27	264	1 (pass)	0 (not solved)
910	27	265	1 (pass)	0 (not solved)
911	27	266	1 (pass)	0 (not solved)
912	27	267	1 (pass)	0 (not solved)
913	27	268	1 (pass)	0 (not solved)
914	27	269	1 (pass)	0 (not solved)
915	27	275	1 (pass)	0 (not solved)
916	27	277	1 (pass)	0 (not solved)
917	27	278	1 (pass)	0 (not solved)
918	27	279	1 (pass)	0 (not solved)
919	27	280	1 (pass)	0 (not solved)
920	27	289	1 (pass)	0 (not solved)
921	27	290	1 (pass)	0 (not solved)
922	27	291	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
923	27	292	1 (pass)	0 (not solved)
924	27	293	1 (pass)	0 (not solved)
925	27	294	1 (pass)	0 (not solved)
926	27	302	1 (pass)	0 (not solved)
927	27	303	1 (pass)	0 (not solved)
928	27	304	1 (pass)	0 (not solved)
929	27	306	1 (pass)	0 (not solved)
930	27	307	1 (pass)	0 (not solved)
931	27	315	1 (pass)	0 (not solved)
932	27	316	1 (pass)	0 (not solved)
933	27	318	1 (pass)	0 (not solved)
934	27	319	1 (pass)	0 (not solved)
935	27	320	1 (pass)	0 (not solved)
936	27	321	1 (pass)	0 (not solved)
937	27	322	1 (pass)	0 (not solved)
938	27	323	1 (pass)	0 (not solved)
939	27	324	1 (pass)	0 (not solved)
940	27	332	1 (pass)	0 (not solved)
941	27	333	1 (pass)	0 (not solved)
942	27	334	1 (pass)	0 (not solved)
943	27	335	1 (pass)	0 (not solved)
944	27	336	1 (pass)	0 (not solved)
945	27	337	1 (pass)	0 (not solved)
946	27	338	1 (pass)	0 (not solved)
947	27	339	1 (pass)	0 (not solved)
948	27	340	1 (pass)	0 (not solved)
949	27	341	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
950	27	350	1 (pass)	-2 (exception) Exception raised: TypeError >> Error detected within library code: catdef: division by zero
951	27	351	1 (pass)	-2 (exception) Exception raised: TypeError >> Error detected within library code: catdef: division by zero
952	27	405	1 (pass)	0 (not solved)
953	27	406	1 (pass)	0 (not solved)
954	27	407	1 (pass)	0 (not solved)
955	27	408	1 (pass)	0 (not solved)
956	27	409	1 (pass)	0 (not solved)
957	27	410	1 (pass)	0 (not solved)
958	27	418	1 (pass)	0 (not solved)
959	27	419	1 (pass)	0 (not solved)
960	27	420	1 (pass)	0 (not solved)
961	27	421	1 (pass)	0 (not solved)
962	27	422	1 (pass)	0 (not solved)
963	27	423	1 (pass)	0 (not solved)
964	27	431	1 (pass)	0 (not solved)
965	27	432	1 (pass)	0 (not solved)
966	27	433	1 (pass)	0 (not solved)
967	27	434	1 (pass)	0 (not solved)
968	27	435	1 (pass)	0 (not solved)
969	27	436	1 (pass)	0 (not solved)
970	27	437	1 (pass)	0 (not solved)
971	27	438	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
972	27	439	1 (pass)	0 (not solved)
973	27	440	1 (pass)	0 (not solved)
974	27	441	1 (pass)	0 (not solved)
975	27	449	1 (pass)	0 (not solved)
976	27	450	1 (pass)	0 (not solved)
977	27	451	1 (pass)	0 (not solved)
978	27	452	1 (pass)	0 (not solved)
979	27	453	1 (pass)	0 (not solved)
980	27	454	1 (pass)	0 (not solved)
981	27	455	1 (pass)	0 (not solved)
982	27	456	1 (pass)	0 (not solved)
983	27	457	1 (pass)	0 (not solved)
984	27	458	1 (pass)	0 (not solved)
985	27	459	1 (pass)	0 (not solved)
986	27	526	1 (pass)	0 (not solved)
987	27	550	1 (pass)	0 (not solved)
988	27	555	1 (pass)	0 (not solved)
989	27	556	1 (pass)	0 (not solved)
990	27	558	1 (pass)	0 (not solved)
991	27	560	1 (pass)	0 (not solved)
992	27	561	1 (pass)	0 (not solved)
993	27	563	1 (pass)	0 (not solved)
994	27	564	1 (pass)	0 (not solved)
995	27	566	1 (pass)	0 (not solved)
996	27	577	1 (pass)	-1 (time out)
997	29	59	1 (pass)	0 (not solved)
998	29	60	1 (pass)	0 (not solved)
999	29	61	1 (pass)	0 (not solved)
1000	29	62	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1001	29	63	1 (pass)	0 (not solved)
1002	29	64	1 (pass)	0 (not solved)
1003	29	65	1 (pass)	0 (not solved)
1004	29	66	1 (pass)	0 (not solved)
1005	29	67	1 (pass)	0 (not solved)
1006	29	68	1 (pass)	0 (not solved)
1007	29	69	1 (pass)	0 (not solved)
1008	29	74	1 (pass)	-1 (time out)
1009	29	75	1 (pass)	-1 (time out)
1010	29	79	1 (pass)	0 (not solved)
1011	29	81	1 (pass)	0 (not solved)
1012	29	83	1 (pass)	0 (not solved)
1013	29	84	1 (pass)	0 (not solved)
1014	29	85	1 (pass)	0 (not solved)
1015	29	86	1 (pass)	0 (not solved)
1016	29	87	1 (pass)	0 (not solved)
1017	29	88	1 (pass)	0 (not solved)
1018	29	89	1 (pass)	0 (not solved)
1019	29	90	1 (pass)	0 (not solved)
1020	29	91	1 (pass)	0 (not solved)
1021	29	93	1 (pass)	0 (not solved)
1022	29	95	1 (pass)	0 (not solved)
1023	29	97	1 (pass)	0 (not solved)
1024	29	99	1 (pass)	0 (not solved)
1025	29	100	1 (pass)	0 (not solved)
1026	29	101	1 (pass)	0 (not solved)
1027	29	102	1 (pass)	0 (not solved)
1028	29	103	1 (pass)	0 (not solved)
1029	29	104	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1030	29	105	1 (pass)	0 (not solved)
1031	29	106	1 (pass)	0 (not solved)
1032	29	107	1 (pass)	0 (not solved)
1033	29	108	1 (pass)	0 (not solved)
1034	29	109	1 (pass)	0 (not solved)
1035	29	110	1 (pass)	0 (not solved)
1036	29	111	1 (pass)	0 (not solved)
1037	29	113	1 (pass)	0 (not solved)
1038	29	115	1 (pass)	-1 (time out)
1039	29	116	1 (pass)	-1 (time out)
1040	29	117	1 (pass)	-1 (time out)
1041	29	118	1 (pass)	-1 (time out)
1042	29	119	1 (pass)	-1 (time out)
1043	29	120	1 (pass)	-1 (time out)
1044	29	121	1 (pass)	-1 (time out)
1045	29	122	1 (pass)	-1 (time out)
1046	29	124	1 (pass)	-1 (time out)
1047	29	125	1 (pass)	-1 (time out)
1048	29	126	1 (pass)	-1 (time out)
1049	29	127	1 (pass)	-1 (time out)
1050	29	128	1 (pass)	-1 (time out)
1051	29	129	1 (pass)	-1 (time out)
1052	29	130	1 (pass)	-1 (time out)
1053	29	131	1 (pass)	-1 (time out)
1054	29	132	1 (pass)	-1 (time out)
1055	29	149	1 (pass)	-1 (time out)
1056	29	150	1 (pass)	-1 (time out)
1057	29	151	1 (pass)	-1 (time out)
1058	29	154	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1059	29	157	1 (pass)	-1 (time out)
1060	29	158	1 (pass)	-1 (time out)
1061	29	162	1 (pass)	-1 (time out)
1062	29	165	1 (pass)	-1 (time out)
1063	29	166	1 (pass)	-1 (time out)
1064	29	169	1 (pass)	-1 (time out)
1065	29	170	1 (pass)	-1 (time out)
1066	29	171	1 (pass)	-1 (time out)
1067	29	172	1 (pass)	-1 (time out)
1068	29	173	1 (pass)	-1 (time out)
1069	29	174	1 (pass)	-1 (time out)
1070	29	175	1 (pass)	-1 (time out)
1071	29	176	1 (pass)	-1 (time out)
1072	29	177	1 (pass)	-1 (time out)
1073	29	178	1 (pass)	-1 (time out)
1074	29	192	1 (pass)	-1 (time out)
1075	29	195	1 (pass)	-1 (time out)
1076	29	198	1 (pass)	-1 (time out)
1077	29	210	1 (pass)	0 (not solved)
1078	29	211	1 (pass)	0 (not solved)
1079	29	212	1 (pass)	0 (not solved)
1080	29	213	1 (pass)	0 (not solved)
1081	29	403	1 (pass)	-1 (time out)
1082	29	404	1 (pass)	-1 (time out)
1083	29	405	1 (pass)	-1 (time out)
1084	29	406	1 (pass)	-1 (time out)
1085	29	407	1 (pass)	-1 (time out)
1086	29	408	1 (pass)	-1 (time out)
1087	29	409	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1088	29	410	1 (pass)	-1 (time out)
1089	29	411	1 (pass)	-1 (time out)
1090	29	412	1 (pass)	-1 (time out)
1091	29	413	1 (pass)	-1 (time out)
1092	29	420	1 (pass)	-1 (time out)
1093	29	429	1 (pass)	-1 (time out)
1094	29	430	1 (pass)	0 (not solved)
1095	29	431	1 (pass)	0 (not solved)
1096	29	432	1 (pass)	0 (not solved)
1097	29	433	1 (pass)	0 (not solved)
1098	29	434	1 (pass)	0 (not solved)
1099	29	435	1 (pass)	0 (not solved)
1100	29	436	1 (pass)	0 (not solved)
1101	29	437	1 (pass)	0 (not solved)
1102	29	438	1 (pass)	0 (not solved)
1103	29	439	1 (pass)	0 (not solved)
1104	29	440	1 (pass)	0 (not solved)
1105	29	441	1 (pass)	0 (not solved)
1106	29	442	1 (pass)	0 (not solved)
1107	29	443	1 (pass)	0 (not solved)
1108	29	444	1 (pass)	0 (not solved)
1109	29	445	1 (pass)	0 (not solved)
1110	29	446	1 (pass)	0 (not solved)
1111	29	447	1 (pass)	0 (not solved)
1112	29	448	1 (pass)	0 (not solved)
1113	29	449	1 (pass)	0 (not solved)
1114	29	450	1 (pass)	0 (not solved)
1115	29	451	1 (pass)	0 (not solved)
1116	29	452	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1117	29	453	1 (pass)	0 (not solved)
1118	29	454	1 (pass)	0 (not solved)
1119	29	455	1 (pass)	0 (not solved)
1120	29	456	1 (pass)	0 (not solved)
1121	29	457	1 (pass)	0 (not solved)
1122	29	458	1 (pass)	0 (not solved)
1123	29	459	1 (pass)	0 (not solved)
1124	29	460	1 (pass)	0 (not solved)
1125	29	461	1 (pass)	0 (not solved)
1126	29	462	1 (pass)	0 (not solved)
1127	29	463	1 (pass)	0 (not solved)
1128	29	464	1 (pass)	0 (not solved)
1129	29	465	1 (pass)	0 (not solved)
1130	29	466	1 (pass)	0 (not solved)
1131	29	467	1 (pass)	0 (not solved)
1132	29	468	1 (pass)	0 (not solved)
1133	29	469	1 (pass)	0 (not solved)
1134	29	470	1 (pass)	0 (not solved)
1135	29	471	1 (pass)	0 (not solved)
1136	29	472	1 (pass)	0 (not solved)
1137	29	473	1 (pass)	0 (not solved)
1138	29	485	1 (pass)	-1 (time out)
1139	29	486	1 (pass)	-1 (time out)
1140	29	487	1 (pass)	-1 (time out)
1141	29	488	1 (pass)	-1 (time out)
1142	29	489	1 (pass)	-1 (time out)
1143	29	490	1 (pass)	-1 (time out)
1144	29	491	1 (pass)	-1 (time out)
1145	29	492	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1146	29	493	1 (pass)	-1 (time out)
1147	29	494	1 (pass)	-1 (time out)
1148	30	38	1 (pass)	0 (not solved)
1149	30	39	1 (pass)	0 (not solved)
1150	30	40	1 (pass)	0 (not solved)
1151	30	41	1 (pass)	0 (not solved)
1152	30	42	1 (pass)	0 (not solved)
1153	30	43	1 (pass)	0 (not solved)
1154	30	44	1 (pass)	0 (not solved)
1155	30	45	1 (pass)	0 (not solved)
1156	30	46	1 (pass)	0 (not solved)
1157	30	47	1 (pass)	0 (not solved)
1158	30	48	1 (pass)	0 (not solved)
1159	30	49	1 (pass)	0 (not solved)
1160	30	50	1 (pass)	0 (not solved)
1161	30	51	1 (pass)	0 (not solved)
1162	30	52	1 (pass)	0 (not solved)
1163	30	53	1 (pass)	0 (not solved)
1164	30	54	1 (pass)	0 (not solved)
1165	30	55	1 (pass)	0 (not solved)
1166	30	56	1 (pass)	0 (not solved)
1167	30	57	1 (pass)	0 (not solved)
1168	30	58	1 (pass)	0 (not solved)
1169	30	59	1 (pass)	0 (not solved)
1170	30	60	1 (pass)	0 (not solved)
1171	30	61	1 (pass)	0 (not solved)
1172	30	62	1 (pass)	0 (not solved)
1173	30	63	1 (pass)	0 (not solved)
1174	30	64	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1175	30	65	1 (pass)	0 (not solved)
1176	30	66	1 (pass)	0 (not solved)
1177	30	67	1 (pass)	0 (not solved)
1178	30	68	1 (pass)	0 (not solved)
1179	30	69	1 (pass)	0 (not solved)
1180	30	70	1 (pass)	0 (not solved)
1181	30	71	1 (pass)	0 (not solved)
1182	30	72	1 (pass)	0 (not solved)
1183	30	73	1 (pass)	0 (not solved)
1184	30	74	1 (pass)	0 (not solved)
1185	30	98	1 (pass)	0 (not solved)
1186	30	99	1 (pass)	0 (not solved)
1187	30	102	1 (pass)	0 (not solved)
1188	30	167	1 (pass)	-1 (time out)
1189	30	178	1 (pass)	-1 (time out)
1190	30	179	1 (pass)	-1 (time out)
1191	30	185	1 (pass)	-1 (time out)
1192	30	186	1 (pass)	-1 (time out)
1193	30	187	1 (pass)	-1 (time out)
1194	30	188	1 (pass)	-1 (time out)
1195	30	189	1 (pass)	-1 (time out)
1196	30	194	1 (pass)	-1 (time out)
1197	30	195	1 (pass)	-1 (time out)
1198	30	196	1 (pass)	-1 (time out)
1199	30	197	1 (pass)	-1 (time out)
1200	30	289	1 (pass)	0 (not solved)
1201	30	290	1 (pass)	0 (not solved)
1202	30	291	1 (pass)	0 (not solved)
1203	30	292	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1204	30	293	1 (pass)	0 (not solved)
1205	30	294	1 (pass)	0 (not solved)
1206	30	301	1 (pass)	0 (not solved)
1207	30	302	1 (pass)	0 (not solved)
1208	30	304	1 (pass)	0 (not solved)
1209	30	305	1 (pass)	0 (not solved)
1210	30	307	1 (pass)	0 (not solved)
1211	31	220	1 (pass)	0 (not solved)
1212	31	221	1 (pass)	0 (not solved)
1213	31	222	1 (pass)	0 (not solved)
1214	31	223	1 (pass)	0 (not solved)
1215	31	224	1 (pass)	0 (not solved)
1216	31	225	1 (pass)	0 (not solved)
1217	31	226	1 (pass)	0 (not solved)
1218	31	227	1 (pass)	0 (not solved)
1219	31	228	1 (pass)	0 (not solved)
1220	31	229	1 (pass)	0 (not solved)
1221	31	230	1 (pass)	0 (not solved)
1222	31	231	1 (pass)	0 (not solved)
1223	31	232	1 (pass)	0 (not solved)
1224	31	233	1 (pass)	0 (not solved)
1225	31	234	1 (pass)	0 (not solved)
1226	31	235	1 (pass)	0 (not solved)
1227	31	236	1 (pass)	0 (not solved)
1228	31	237	1 (pass)	0 (not solved)
1229	31	238	1 (pass)	0 (not solved)
1230	31	239	1 (pass)	0 (not solved)
1231	31	240	1 (pass)	0 (not solved)
1232	31	241	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1233	31	242	1 (pass)	0 (not solved)
1234	31	243	1 (pass)	0 (not solved)
1235	31	244	1 (pass)	0 (not solved)
1236	31	245	1 (pass)	0 (not solved)
1237	31	246	1 (pass)	0 (not solved)
1238	31	247	1 (pass)	0 (not solved)
1239	31	248	1 (pass)	0 (not solved)
1240	31	249	1 (pass)	0 (not solved)
1241	31	250	1 (pass)	0 (not solved)
1242	31	251	1 (pass)	0 (not solved)
1243	31	252	1 (pass)	0 (not solved)
1244	31	253	1 (pass)	0 (not solved)
1245	31	254	1 (pass)	0 (not solved)
1246	31	255	1 (pass)	0 (not solved)
1247	31	256	1 (pass)	0 (not solved)
1248	31	257	1 (pass)	0 (not solved)
1249	31	258	1 (pass)	0 (not solved)
1250	31	259	1 (pass)	0 (not solved)
1251	31	260	1 (pass)	0 (not solved)
1252	31	261	1 (pass)	0 (not solved)
1253	31	262	1 (pass)	0 (not solved)
1254	31	263	1 (pass)	0 (not solved)
1255	31	264	1 (pass)	0 (not solved)
1256	31	265	1 (pass)	0 (not solved)
1257	31	266	1 (pass)	0 (not solved)
1258	31	267	1 (pass)	0 (not solved)
1259	31	268	1 (pass)	0 (not solved)
1260	33	386	1 (pass)	0 (not solved)
1261	33	387	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1262	33	388	1 (pass)	0 (not solved)
1263	33	389	1 (pass)	0 (not solved)
1264	33	390	1 (pass)	0 (not solved)
1265	33	391	1 (pass)	0 (not solved)
1266	33	392	1 (pass)	0 (not solved)
1267	33	393	1 (pass)	0 (not solved)
1268	33	394	1 (pass)	0 (not solved)
1269	33	395	1 (pass)	0 (not solved)
1270	33	396	1 (pass)	0 (not solved)
1271	33	397	1 (pass)	0 (not solved)
1272	33	398	1 (pass)	0 (not solved)
1273	33	399	1 (pass)	0 (not solved)
1274	33	400	1 (pass)	0 (not solved)
1275	33	401	1 (pass)	0 (not solved)
1276	33	402	1 (pass)	0 (not solved)
1277	33	403	1 (pass)	0 (not solved)
1278	33	404	1 (pass)	0 (not solved)
1279	33	405	1 (pass)	0 (not solved)
1280	33	406	1 (pass)	0 (not solved)
1281	33	407	1 (pass)	0 (not solved)
1282	33	408	1 (pass)	0 (not solved)
1283	33	409	1 (pass)	0 (not solved)
1284	33	410	1 (pass)	0 (not solved)
1285	33	411	1 (pass)	0 (not solved)
1286	33	412	1 (pass)	0 (not solved)
1287	33	413	1 (pass)	0 (not solved)
1288	33	414	1 (pass)	0 (not solved)
1289	33	415	1 (pass)	0 (not solved)
1290	33	416	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1291	33	417	1 (pass)	0 (not solved)
1292	33	418	1 (pass)	0 (not solved)
1293	33	419	1 (pass)	0 (not solved)
1294	33	420	1 (pass)	0 (not solved)
1295	33	421	1 (pass)	0 (not solved)
1296	33	422	1 (pass)	0 (not solved)
1297	33	423	1 (pass)	0 (not solved)
1298	33	424	1 (pass)	0 (not solved)
1299	33	425	1 (pass)	0 (not solved)
1300	33	426	1 (pass)	0 (not solved)
1301	33	427	1 (pass)	0 (not solved)
1302	33	432	1 (pass)	0 (not solved)
1303	33	433	1 (pass)	0 (not solved)
1304	33	657	1 (pass)	0 (not solved)
1305	33	658	1 (pass)	0 (not solved)
1306	33	659	1 (pass)	0 (not solved)
1307	33	660	1 (pass)	0 (not solved)
1308	33	661	1 (pass)	0 (not solved)
1309	33	662	1 (pass)	0 (not solved)
1310	33	663	1 (pass)	0 (not solved)
1311	33	664	1 (pass)	0 (not solved)
1312	33	665	1 (pass)	0 (not solved)
1313	33	666	1 (pass)	0 (not solved)
1314	33	667	1 (pass)	0 (not solved)
1315	33	668	1 (pass)	0 (not solved)
1316	33	669	1 (pass)	0 (not solved)
1317	33	670	1 (pass)	0 (not solved)
1318	33	671	1 (pass)	0 (not solved)
1319	33	672	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1320	33	673	1 (pass)	0 (not solved)
1321	33	674	1 (pass)	0 (not solved)
1322	33	675	1 (pass)	0 (not solved)
1323	33	676	1 (pass)	0 (not solved)
1324	33	677	1 (pass)	0 (not solved)
1325	33	678	1 (pass)	0 (not solved)
1326	33	679	1 (pass)	0 (not solved)
1327	33	680	1 (pass)	0 (not solved)
1328	33	681	1 (pass)	0 (not solved)
1329	33	682	1 (pass)	0 (not solved)
1330	33	683	1 (pass)	0 (not solved)
1331	33	684	1 (pass)	0 (not solved)
1332	33	685	1 (pass)	0 (not solved)
1333	33	686	1 (pass)	0 (not solved)
1334	33	687	1 (pass)	0 (not solved)
1335	33	688	1 (pass)	0 (not solved)
1336	33	689	1 (pass)	0 (not solved)
1337	33	690	1 (pass)	0 (not solved)
1338	33	691	1 (pass)	0 (not solved)
1339	33	692	1 (pass)	0 (not solved)
1340	33	693	1 (pass)	0 (not solved)
1341	33	694	1 (pass)	0 (not solved)
1342	33	695	1 (pass)	0 (not solved)
1343	33	696	1 (pass)	0 (not solved)
1344	33	697	1 (pass)	0 (not solved)
1345	33	698	1 (pass)	0 (not solved)
1346	33	1327	1 (pass)	0 (not solved)
1347	33	1328	1 (pass)	0 (not solved)
1348	33	1329	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1349	33	1330	1 (pass)	0 (not solved)
1350	33	1331	1 (pass)	0 (not solved)
1351	33	1332	1 (pass)	0 (not solved)
1352	33	1333	1 (pass)	0 (not solved)
1353	33	1334	1 (pass)	0 (not solved)
1354	33	1335	1 (pass)	0 (not solved)
1355	33	1336	1 (pass)	0 (not solved)
1356	33	1337	1 (pass)	0 (not solved)
1357	33	1338	1 (pass)	0 (not solved)
1358	33	1339	1 (pass)	0 (not solved)
1359	33	1340	1 (pass)	0 (not solved)
1360	33	1341	1 (pass)	0 (not solved)
1361	33	1342	1 (pass)	0 (not solved)
1362	33	1343	1 (pass)	0 (not solved)
1363	33	1344	1 (pass)	0 (not solved)
1364	33	1345	1 (pass)	0 (not solved)
1365	33	1346	1 (pass)	0 (not solved)
1366	33	1347	1 (pass)	0 (not solved)
1367	33	1348	1 (pass)	0 (not solved)
1368	33	1349	1 (pass)	0 (not solved)
1369	33	1350	1 (pass)	0 (not solved)
1370	33	1351	1 (pass)	0 (not solved)
1371	33	1352	1 (pass)	0 (not solved)
1372	33	1353	1 (pass)	0 (not solved)
1373	33	1354	1 (pass)	0 (not solved)
1374	33	1355	1 (pass)	0 (not solved)
1375	33	1356	1 (pass)	0 (not solved)
1376	33	1357	1 (pass)	0 (not solved)
1377	33	1358	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1378	33	1359	1 (pass)	0 (not solved)
1379	33	1360	1 (pass)	0 (not solved)
1380	33	1361	1 (pass)	0 (not solved)
1381	33	1362	1 (pass)	0 (not solved)
1382	33	1363	1 (pass)	0 (not solved)
1383	33	1364	1 (pass)	0 (not solved)
1384	33	1365	1 (pass)	0 (not solved)
1385	33	1366	1 (pass)	0 (not solved)
1386	33	1367	1 (pass)	0 (not solved)
1387	33	1368	1 (pass)	0 (not solved)
1388	33	1369	1 (pass)	0 (not solved)
1389	33	1370	1 (pass)	0 (not solved)
1390	33	1371	1 (pass)	0 (not solved)
1391	33	1372	1 (pass)	0 (not solved)
1392	33	1373	1 (pass)	0 (not solved)
1393	33	1375	1 (pass)	0 (not solved)
1394	33	1376	1 (pass)	0 (not solved)
1395	33	1378	1 (pass)	0 (not solved)
1396	33	1379	1 (pass)	0 (not solved)
1397	33	1380	1 (pass)	0 (not solved)
1398	33	1381	1 (pass)	0 (not solved)
1399	33	1382	1 (pass)	0 (not solved)
1400	33	1383	1 (pass)	0 (not solved)
1401	33	1384	1 (pass)	0 (not solved)
1402	33	1385	1 (pass)	0 (not solved)
1403	33	1386	1 (pass)	0 (not solved)
1404	33	1387	1 (pass)	0 (not solved)
1405	33	1388	1 (pass)	0 (not solved)
1406	33	1389	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1407	33	1390	1 (pass)	0 (not solved)
1408	33	1391	1 (pass)	0 (not solved)
1409	33	1392	1 (pass)	0 (not solved)
1410	33	1393	1 (pass)	0 (not solved)
1411	33	1394	1 (pass)	0 (not solved)
1412	33	1395	1 (pass)	0 (not solved)
1413	33	1396	1 (pass)	0 (not solved)
1414	33	1397	1 (pass)	0 (not solved)
1415	33	1398	1 (pass)	0 (not solved)
1416	33	1399	1 (pass)	0 (not solved)
1417	33	1400	1 (pass)	0 (not solved)
1418	33	1401	1 (pass)	0 (not solved)
1419	33	1402	1 (pass)	0 (not solved)
1420	33	1403	1 (pass)	0 (not solved)
1421	33	1404	1 (pass)	0 (not solved)
1422	33	1405	1 (pass)	0 (not solved)
1423	33	1406	1 (pass)	0 (not solved)
1424	33	1407	1 (pass)	0 (not solved)
1425	33	1408	1 (pass)	0 (not solved)
1426	33	1409	1 (pass)	0 (not solved)
1427	33	1410	1 (pass)	0 (not solved)
1428	33	1411	1 (pass)	0 (not solved)
1429	33	2294	1 (pass)	-1 (time out)
1430	33	2299	1 (pass)	-1 (time out)
1431	33	2300	1 (pass)	-1 (time out)
1432	33	2305	1 (pass)	-1 (time out)
1433	33	2440	1 (pass)	0 (not solved)
1434	33	2441	1 (pass)	0 (not solved)
1435	33	2442	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1436	33	2443	1 (pass)	0 (not solved)
1437	33	2444	1 (pass)	0 (not solved)
1438	33	2445	1 (pass)	0 (not solved)
1439	33	2446	1 (pass)	0 (not solved)
1440	33	2447	1 (pass)	0 (not solved)
1441	33	2448	1 (pass)	0 (not solved)
1442	33	2449	1 (pass)	0 (not solved)
1443	33	2450	1 (pass)	0 (not solved)
1444	33	2451	1 (pass)	0 (not solved)
1445	33	2452	1 (pass)	0 (not solved)
1446	33	2453	1 (pass)	0 (not solved)
1447	33	2454	1 (pass)	0 (not solved)
1448	33	2455	1 (pass)	0 (not solved)
1449	33	2456	1 (pass)	0 (not solved)
1450	33	2457	1 (pass)	0 (not solved)
1451	33	2458	1 (pass)	0 (not solved)
1452	33	2459	1 (pass)	0 (not solved)
1453	33	2460	1 (pass)	0 (not solved)
1454	33	2461	1 (pass)	0 (not solved)
1455	33	2462	1 (pass)	0 (not solved)
1456	33	2463	1 (pass)	0 (not solved)
1457	33	2464	1 (pass)	0 (not solved)
1458	33	2465	1 (pass)	0 (not solved)
1459	33	2466	1 (pass)	0 (not solved)
1460	33	2467	1 (pass)	0 (not solved)
1461	33	2468	1 (pass)	0 (not solved)
1462	33	2469	1 (pass)	0 (not solved)
1463	33	2470	1 (pass)	0 (not solved)
1464	33	2471	1 (pass)	0 (not solved)

Continued on next page

Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1465	33	2472	1 (pass)	0 (not solved)
1466	33	2473	1 (pass)	0 (not solved)
1467	33	2474	1 (pass)	0 (not solved)
1468	33	2475	1 (pass)	0 (not solved)
1469	33	2476	1 (pass)	0 (not solved)
1470	33	2477	1 (pass)	0 (not solved)
1471	33	2478	1 (pass)	0 (not solved)
1472	33	2479	1 (pass)	0 (not solved)
1473	33	2480	1 (pass)	0 (not solved)
1474	33	2481	1 (pass)	0 (not solved)
1475	34	433	1 (pass)	0 (not solved)
1476	34	434	1 (pass)	0 (not solved)
1477	34	435	1 (pass)	0 (not solved)
1478	34	436	1 (pass)	0 (not solved)
1479	34	437	1 (pass)	0 (not solved)
1480	34	438	1 (pass)	0 (not solved)
1481	34	439	1 (pass)	0 (not solved)
1482	34	440	1 (pass)	0 (not solved)
1483	34	441	1 (pass)	0 (not solved)
1484	34	442	1 (pass)	0 (not solved)
1485	34	443	1 (pass)	0 (not solved)
1486	34	444	1 (pass)	0 (not solved)
1487	34	445	1 (pass)	0 (not solved)
1488	34	446	1 (pass)	0 (not solved)
1489	34	447	1 (pass)	0 (not solved)
1490	34	448	1 (pass)	0 (not solved)
1491	34	449	1 (pass)	0 (not solved)
1492	34	450	1 (pass)	0 (not solved)
1493	34	451	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1494	34	452	1 (pass)	0 (not solved)
1495	34	453	1 (pass)	0 (not solved)
1496	34	454	1 (pass)	0 (not solved)
1497	34	455	1 (pass)	0 (not solved)
1498	34	456	1 (pass)	0 (not solved)
1499	34	457	1 (pass)	0 (not solved)
1500	34	458	1 (pass)	0 (not solved)
1501	34	459	1 (pass)	0 (not solved)
1502	34	460	1 (pass)	0 (not solved)
1503	34	461	1 (pass)	0 (not solved)
1504	34	462	1 (pass)	0 (not solved)
1505	34	463	1 (pass)	0 (not solved)
1506	34	464	1 (pass)	0 (not solved)
1507	34	465	1 (pass)	0 (not solved)
1508	34	466	1 (pass)	0 (not solved)
1509	34	467	1 (pass)	0 (not solved)
1510	34	468	1 (pass)	0 (not solved)
1511	34	469	1 (pass)	0 (not solved)
1512	34	470	1 (pass)	0 (not solved)
1513	34	471	1 (pass)	0 (not solved)
1514	34	472	1 (pass)	0 (not solved)
1515	34	473	1 (pass)	0 (not solved)
1516	34	474	1 (pass)	0 (not solved)
1517	34	475	1 (pass)	0 (not solved)
1518	34	476	1 (pass)	0 (not solved)
1519	34	477	1 (pass)	0 (not solved)
1520	34	478	1 (pass)	0 (not solved)
1521	34	479	1 (pass)	0 (not solved)
1522	34	480	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1523	34	481	1 (pass)	0 (not solved)
1524	34	482	1 (pass)	0 (not solved)
1525	34	483	1 (pass)	0 (not solved)
1526	34	484	1 (pass)	0 (not solved)
1527	34	1029	1 (pass)	0 (not solved)
1528	34	1030	1 (pass)	0 (not solved)
1529	34	1031	1 (pass)	0 (not solved)
1530	34	1032	1 (pass)	0 (not solved)
1531	34	1033	1 (pass)	0 (not solved)
1532	34	1034	1 (pass)	0 (not solved)
1533	34	1035	1 (pass)	0 (not solved)
1534	34	1036	1 (pass)	0 (not solved)
1535	34	1037	1 (pass)	0 (not solved)
1536	34	1038	1 (pass)	0 (not solved)
1537	34	1039	1 (pass)	0 (not solved)
1538	34	1040	1 (pass)	0 (not solved)
1539	34	1041	1 (pass)	0 (not solved)
1540	34	1042	1 (pass)	0 (not solved)
1541	34	1043	1 (pass)	0 (not solved)
1542	34	1044	1 (pass)	0 (not solved)
1543	34	1045	1 (pass)	0 (not solved)
1544	34	1046	1 (pass)	0 (not solved)
1545	34	1047	1 (pass)	0 (not solved)
1546	34	1048	1 (pass)	0 (not solved)
1547	34	1049	1 (pass)	0 (not solved)
1548	34	1050	1 (pass)	0 (not solved)
1549	34	1051	1 (pass)	0 (not solved)
1550	34	1052	1 (pass)	0 (not solved)
1551	34	1053	1 (pass)	0 (not solved)

Continued on next page

Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1552	34	1054	1 (pass)	0 (not solved)
1553	34	1055	1 (pass)	0 (not solved)
1554	34	1056	1 (pass)	0 (not solved)
1555	34	1057	1 (pass)	0 (not solved)
1556	34	1058	1 (pass)	0 (not solved)
1557	34	1059	1 (pass)	0 (not solved)
1558	34	1060	1 (pass)	0 (not solved)
1559	34	1061	1 (pass)	0 (not solved)
1560	34	1062	1 (pass)	0 (not solved)
1561	34	1063	1 (pass)	0 (not solved)
1562	34	1064	1 (pass)	0 (not solved)
1563	34	1065	1 (pass)	0 (not solved)
1564	34	1066	1 (pass)	0 (not solved)
1565	34	1067	1 (pass)	0 (not solved)
1566	34	1068	1 (pass)	0 (not solved)
1567	34	1069	1 (pass)	0 (not solved)
1568	34	1070	1 (pass)	0 (not solved)
1569	34	1071	1 (pass)	0 (not solved)
1570	34	1072	1 (pass)	0 (not solved)
1571	34	1073	1 (pass)	0 (not solved)
1572	34	1074	1 (pass)	0 (not solved)
1573	34	1075	1 (pass)	0 (not solved)
1574	34	1076	1 (pass)	0 (not solved)
1575	34	1077	1 (pass)	0 (not solved)
1576	34	1078	1 (pass)	0 (not solved)
1577	34	1079	1 (pass)	0 (not solved)
1578	34	1080	1 (pass)	0 (not solved)
1579	34	1081	1 (pass)	0 (not solved)
1580	34	1082	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1581	34	1255	1 (pass)	0 (not solved)
1582	34	1256	1 (pass)	0 (not solved)
1583	34	1257	1 (pass)	0 (not solved)
1584	34	1258	1 (pass)	0 (not solved)
1585	34	1259	1 (pass)	0 (not solved)
1586	34	1260	1 (pass)	0 (not solved)
1587	34	1261	1 (pass)	0 (not solved)
1588	34	1262	1 (pass)	0 (not solved)
1589	34	1263	1 (pass)	0 (not solved)
1590	34	1264	1 (pass)	0 (not solved)
1591	34	1265	1 (pass)	0 (not solved)
1592	34	1266	1 (pass)	0 (not solved)
1593	34	1267	1 (pass)	0 (not solved)
1594	34	1268	1 (pass)	0 (not solved)
1595	34	1269	1 (pass)	0 (not solved)
1596	34	1270	1 (pass)	0 (not solved)
1597	34	1271	1 (pass)	0 (not solved)
1598	34	1272	1 (pass)	0 (not solved)
1599	34	1273	1 (pass)	0 (not solved)
1600	34	1274	1 (pass)	0 (not solved)
1601	34	1275	1 (pass)	0 (not solved)
1602	34	1276	1 (pass)	0 (not solved)
1603	34	1277	1 (pass)	0 (not solved)
1604	34	1278	1 (pass)	0 (not solved)
1605	34	1279	1 (pass)	0 (not solved)
1606	34	1280	1 (pass)	0 (not solved)
1607	34	1281	1 (pass)	0 (not solved)
1608	34	1282	1 (pass)	0 (not solved)
1609	34	1283	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1610	34	1472	1 (pass)	0 (not solved)
1611	34	1473	1 (pass)	0 (not solved)
1612	34	1474	1 (pass)	0 (not solved)
1613	34	1475	1 (pass)	0 (not solved)
1614	34	1476	1 (pass)	0 (not solved)
1615	34	1477	1 (pass)	0 (not solved)
1616	34	1478	1 (pass)	0 (not solved)
1617	34	1479	1 (pass)	0 (not solved)
1618	34	1480	1 (pass)	0 (not solved)
1619	34	1481	1 (pass)	0 (not solved)
1620	34	1482	1 (pass)	0 (not solved)
1621	34	1483	1 (pass)	0 (not solved)
1622	34	1484	1 (pass)	0 (not solved)
1623	34	1485	1 (pass)	0 (not solved)
1624	34	1486	1 (pass)	0 (not solved)
1625	34	1617	1 (pass)	-1 (time out)
1626	34	1622	1 (pass)	-1 (time out)
1627	34	1626	1 (pass)	-1 (time out)
1628	34	1627	1 (pass)	-1 (time out)
1629	34	1628	1 (pass)	0 (not solved)
1630	34	1629	1 (pass)	0 (not solved)
1631	34	1630	1 (pass)	0 (not solved)
1632	34	1631	1 (pass)	0 (not solved)
1633	34	1632	1 (pass)	0 (not solved)
1634	34	1633	1 (pass)	0 (not solved)
1635	34	1634	1 (pass)	0 (not solved)
1636	34	1635	1 (pass)	0 (not solved)
1637	34	1636	1 (pass)	0 (not solved)
1638	34	1637	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1639	34	1638	1 (pass)	0 (not solved)
1640	34	1639	1 (pass)	0 (not solved)
1641	34	1640	1 (pass)	0 (not solved)
1642	34	1641	1 (pass)	0 (not solved)
1643	34	1642	1 (pass)	0 (not solved)
1644	34	1643	1 (pass)	0 (not solved)
1645	34	1644	1 (pass)	0 (not solved)
1646	34	1645	1 (pass)	0 (not solved)
1647	34	1646	1 (pass)	0 (not solved)
1648	34	1647	1 (pass)	0 (not solved)
1649	34	1648	1 (pass)	0 (not solved)
1650	34	1649	1 (pass)	0 (not solved)
1651	34	1650	1 (pass)	0 (not solved)
1652	34	1651	1 (pass)	0 (not solved)
1653	34	1652	1 (pass)	0 (not solved)
1654	34	1653	1 (pass)	0 (not solved)
1655	34	2301	1 (pass)	0 (not solved)
1656	34	2302	1 (pass)	0 (not solved)
1657	34	2303	1 (pass)	0 (not solved)
1658	34	2304	1 (pass)	0 (not solved)
1659	34	2305	1 (pass)	0 (not solved)
1660	34	2487	1 (pass)	-1 (time out)
1661	34	2576	1 (pass)	0 (not solved)
1662	34	2577	1 (pass)	0 (not solved)
1663	34	2578	1 (pass)	0 (not solved)
1664	34	2579	1 (pass)	0 (not solved)
1665	34	2580	1 (pass)	0 (not solved)
1666	34	2581	1 (pass)	0 (not solved)
1667	34	2582	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1668	34	2583	1 (pass)	0 (not solved)
1669	34	2584	1 (pass)	0 (not solved)
1670	34	2585	1 (pass)	0 (not solved)
1671	34	2586	1 (pass)	0 (not solved)
1672	34	2587	1 (pass)	0 (not solved)
1673	34	2588	1 (pass)	0 (not solved)
1674	34	2589	1 (pass)	0 (not solved)
1675	34	2590	1 (pass)	0 (not solved)
1676	34	2591	1 (pass)	0 (not solved)
1677	34	2592	1 (pass)	0 (not solved)
1678	34	2593	1 (pass)	0 (not solved)
1679	34	2594	1 (pass)	0 (not solved)
1680	34	2595	1 (pass)	0 (not solved)
1681	34	2596	1 (pass)	0 (not solved)
1682	34	2597	1 (pass)	0 (not solved)
1683	34	2598	1 (pass)	0 (not solved)
1684	34	2599	1 (pass)	0 (not solved)
1685	34	2600	1 (pass)	0 (not solved)
1686	34	2601	1 (pass)	0 (not solved)
1687	34	2602	1 (pass)	0 (not solved)
1688	34	2603	1 (pass)	0 (not solved)
1689	34	2604	1 (pass)	0 (not solved)
1690	34	2605	1 (pass)	0 (not solved)
1691	34	2606	1 (pass)	0 (not solved)
1692	34	2607	1 (pass)	0 (not solved)
1693	34	2608	1 (pass)	0 (not solved)
1694	34	2609	1 (pass)	0 (not solved)
1695	34	2610	1 (pass)	0 (not solved)
1696	34	2611	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1697	34	2612	1 (pass)	0 (not solved)
1698	34	2613	1 (pass)	0 (not solved)
1699	34	2614	1 (pass)	0 (not solved)
1700	34	2615	1 (pass)	0 (not solved)
1701	34	2616	1 (pass)	0 (not solved)
1702	34	2617	1 (pass)	0 (not solved)
1703	34	2618	1 (pass)	0 (not solved)
1704	34	2619	1 (pass)	0 (not solved)
1705	34	2620	1 (pass)	0 (not solved)
1706	34	2621	1 (pass)	0 (not solved)
1707	34	2622	1 (pass)	0 (not solved)
1708	34	2623	1 (pass)	0 (not solved)
1709	34	2624	1 (pass)	0 (not solved)
1710	34	2625	1 (pass)	0 (not solved)
1711	34	2626	1 (pass)	0 (not solved)
1712	34	2627	1 (pass)	0 (not solved)
1713	34	2628	1 (pass)	0 (not solved)
1714	34	2629	1 (pass)	0 (not solved)
1715	34	2630	1 (pass)	0 (not solved)
1716	34	2631	1 (pass)	0 (not solved)
1717	34	2632	1 (pass)	0 (not solved)
1718	34	2633	1 (pass)	0 (not solved)
1719	34	2634	1 (pass)	0 (not solved)
1720	34	2635	1 (pass)	0 (not solved)
1721	34	2636	1 (pass)	0 (not solved)
1722	34	2637	1 (pass)	0 (not solved)
1723	34	2638	1 (pass)	0 (not solved)
1724	34	2639	1 (pass)	0 (not solved)
1725	35	489	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1726	35	491	1 (pass)	0 (not solved)
1727	35	492	1 (pass)	0 (not solved)
1728	35	494	1 (pass)	0 (not solved)
1729	35	495	1 (pass)	0 (not solved)
1730	35	496	1 (pass)	0 (not solved)
1731	35	498	1 (pass)	0 (not solved)
1732	35	499	1 (pass)	0 (not solved)
1733	35	501	1 (pass)	0 (not solved)
1734	35	502	1 (pass)	0 (not solved)
1735	35	503	1 (pass)	0 (not solved)
1736	35	505	1 (pass)	0 (not solved)
1737	35	506	1 (pass)	0 (not solved)
1738	35	508	1 (pass)	0 (not solved)
1739	35	509	1 (pass)	0 (not solved)
1740	35	510	1 (pass)	0 (not solved)
1741	35	512	1 (pass)	0 (not solved)
1742	35	513	1 (pass)	0 (not solved)
1743	35	515	1 (pass)	0 (not solved)
1744	35	516	1 (pass)	0 (not solved)
1745	35	517	1 (pass)	0 (not solved)
1746	35	519	1 (pass)	0 (not solved)
1747	35	520	1 (pass)	0 (not solved)
1748	35	522	1 (pass)	0 (not solved)
1749	35	523	1 (pass)	0 (not solved)
1750	35	622	1 (pass)	0 (not solved)
1751	35	623	1 (pass)	0 (not solved)
1752	35	624	1 (pass)	0 (not solved)
1753	35	625	1 (pass)	0 (not solved)
1754	35	629	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1755	35	630	1 (pass)	0 (not solved)
1756	35	631	1 (pass)	0 (not solved)
1757	35	632	1 (pass)	0 (not solved)
1758	35	636	1 (pass)	0 (not solved)
1759	35	637	1 (pass)	0 (not solved)
1760	35	638	1 (pass)	0 (not solved)
1761	35	639	1 (pass)	0 (not solved)
1762	35	645	1 (pass)	0 (not solved)
1763	35	646	1 (pass)	0 (not solved)
1764	35	647	1 (pass)	0 (not solved)
1765	35	648	1 (pass)	0 (not solved)
1766	35	656	1 (pass)	0 (not solved)
1767	35	796	1 (pass)	-1 (time out)
1768	35	800	1 (pass)	-1 (time out)
1769	35	886	1 (pass)	0 (not solved)
1770	35	887	1 (pass)	0 (not solved)
1771	35	888	1 (pass)	0 (not solved)
1772	35	889	1 (pass)	0 (not solved)
1773	35	893	1 (pass)	0 (not solved)
1774	35	894	1 (pass)	0 (not solved)
1775	35	895	1 (pass)	0 (not solved)
1776	35	896	1 (pass)	0 (not solved)
1777	35	900	1 (pass)	0 (not solved)
1778	35	901	1 (pass)	0 (not solved)
1779	35	902	1 (pass)	0 (not solved)
1780	35	903	1 (pass)	0 (not solved)
1781	35	909	1 (pass)	0 (not solved)
1782	35	910	1 (pass)	0 (not solved)
1783	35	911	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1784	35	912	1 (pass)	0 (not solved)
1785	37	21	1 (pass)	-1 (time out)
1786	37	54	1 (pass)	-1 (time out)
1787	37	56	1 (pass)	-1 (time out)
1788	37	68	1 (pass)	-1 (time out)
1789	37	71	1 (pass)	-1 (time out)
1790	37	72	1 (pass)	-1 (time out)
1791	37	73	1 (pass)	-1 (time out)
1792	37	74	1 (pass)	-1 (time out)
1793	37	79	1 (pass)	-1 (time out)
1794	37	100	1 (pass)	-1 (time out)
1795	37	103	1 (pass)	-1 (time out)
1796	37	104	1 (pass)	-1 (time out)
1797	37	105	1 (pass)	-1 (time out)
1798	37	106	1 (pass)	-1 (time out)
1799	38	49	1 (pass)	-1 (time out)
1800	38	55	1 (pass)	-1 (time out)
1801	38	61	1 (pass)	-1 (time out)
1802	38	86	1 (pass)	-1 (time out)
1803	38	87	1 (pass)	-1 (time out)
1804	38	105	1 (pass)	-1 (time out)
1805	38	232	1 (pass)	-1 (time out)
1806	38	238	1 (pass)	-1 (time out)
1807	38	239	1 (pass)	-1 (time out)
1808	38	259	1 (pass)	0 (not solved)
1809	38	260	1 (pass)	0 (not solved)
1810	38	261	1 (pass)	0 (not solved)
1811	38	262	1 (pass)	0 (not solved)
1812	38	263	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1813	38	264	1 (pass)	0 (not solved)
1814	38	265	1 (pass)	0 (not solved)
1815	38	266	1 (pass)	0 (not solved)
1816	38	267	1 (pass)	0 (not solved)
1817	38	268	1 (pass)	0 (not solved)
1818	38	269	1 (pass)	0 (not solved)
1819	38	270	1 (pass)	0 (not solved)
1820	38	271	1 (pass)	0 (not solved)
1821	39	16	1 (pass)	0 (not solved)
1822	39	17	1 (pass)	0 (not solved)
1823	39	18	1 (pass)	0 (not solved)
1824	39	19	1 (pass)	0 (not solved)
1825	39	20	1 (pass)	0 (not solved)
1826	39	21	1 (pass)	0 (not solved)
1827	39	22	1 (pass)	0 (not solved)
1828	39	23	1 (pass)	0 (not solved)
1829	39	24	1 (pass)	0 (not solved)
1830	39	25	1 (pass)	0 (not solved)
1831	39	26	1 (pass)	0 (not solved)
1832	39	27	1 (pass)	0 (not solved)
1833	39	28	1 (pass)	0 (not solved)
1834	39	29	1 (pass)	0 (not solved)
1835	39	30	1 (pass)	0 (not solved)
1836	39	31	1 (pass)	0 (not solved)
1837	39	32	1 (pass)	0 (not solved)
1838	39	33	1 (pass)	0 (not solved)
1839	39	34	1 (pass)	0 (not solved)
1840	39	35	1 (pass)	0 (not solved)
1841	39	36	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1842	39	37	1 (pass)	0 (not solved)
1843	39	38	1 (pass)	0 (not solved)
1844	39	39	1 (pass)	0 (not solved)
1845	39	40	1 (pass)	0 (not solved)
1846	39	41	1 (pass)	0 (not solved)
1847	39	66	1 (pass)	0 (not solved)
1848	39	67	1 (pass)	0 (not solved)
1849	39	68	1 (pass)	0 (not solved)
1850	39	69	1 (pass)	0 (not solved)
1851	39	70	1 (pass)	0 (not solved)
1852	39	71	1 (pass)	0 (not solved)
1853	39	72	1 (pass)	0 (not solved)
1854	39	73	1 (pass)	0 (not solved)
1855	39	74	1 (pass)	0 (not solved)
1856	39	75	1 (pass)	0 (not solved)
1857	39	76	1 (pass)	0 (not solved)
1858	39	77	1 (pass)	0 (not solved)
1859	39	78	1 (pass)	0 (not solved)
1860	39	79	1 (pass)	0 (not solved)
1861	39	80	1 (pass)	0 (not solved)
1862	39	81	1 (pass)	0 (not solved)
1863	39	82	1 (pass)	0 (not solved)
1864	39	83	1 (pass)	0 (not solved)
1865	39	84	1 (pass)	0 (not solved)
1866	39	85	1 (pass)	0 (not solved)
1867	39	86	1 (pass)	0 (not solved)
1868	39	87	1 (pass)	0 (not solved)
1869	39	88	1 (pass)	0 (not solved)
1870	39	89	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1871	39	90	1 (pass)	0 (not solved)
1872	39	91	1 (pass)	0 (not solved)
1873	39	92	1 (pass)	0 (not solved)
1874	39	93	1 (pass)	0 (not solved)
1875	39	94	1 (pass)	0 (not solved)
1876	39	98	1 (pass)	0 (not solved)
1877	39	99	1 (pass)	0 (not solved)
1878	39	100	1 (pass)	0 (not solved)
1879	39	101	1 (pass)	0 (not solved)
1880	39	102	1 (pass)	0 (not solved)
1881	39	103	1 (pass)	0 (not solved)
1882	39	104	1 (pass)	0 (not solved)
1883	39	105	1 (pass)	0 (not solved)
1884	39	106	1 (pass)	0 (not solved)
1885	39	107	1 (pass)	0 (not solved)
1886	39	110	1 (pass)	0 (not solved)
1887	39	111	1 (pass)	0 (not solved)
1888	39	112	1 (pass)	0 (not solved)
1889	39	113	1 (pass)	0 (not solved)
1890	39	114	1 (pass)	0 (not solved)
1891	39	115	1 (pass)	0 (not solved)
1892	39	116	1 (pass)	0 (not solved)
1893	39	117	1 (pass)	0 (not solved)
1894	39	118	1 (pass)	0 (not solved)
1895	39	119	1 (pass)	0 (not solved)
1896	39	120	1 (pass)	0 (not solved)
1897	39	121	1 (pass)	0 (not solved)
1898	39	122	1 (pass)	0 (not solved)
1899	39	123	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1900	39	124	1 (pass)	0 (not solved)
1901	39	125	1 (pass)	0 (not solved)
1902	39	126	1 (pass)	0 (not solved)
1903	39	127	1 (pass)	0 (not solved)
1904	39	128	1 (pass)	0 (not solved)
1905	39	129	1 (pass)	0 (not solved)
1906	39	130	1 (pass)	0 (not solved)
1907	39	131	1 (pass)	0 (not solved)
1908	39	132	1 (pass)	0 (not solved)
1909	39	133	1 (pass)	0 (not solved)
1910	39	352	1 (pass)	0 (not solved)
1911	39	353	1 (pass)	0 (not solved)
1912	39	354	1 (pass)	0 (not solved)
1913	39	355	1 (pass)	0 (not solved)
1914	39	356	1 (pass)	0 (not solved)
1915	39	357	1 (pass)	0 (not solved)
1916	39	358	1 (pass)	0 (not solved)
1917	39	359	1 (pass)	0 (not solved)
1918	39	360	1 (pass)	0 (not solved)
1919	39	361	1 (pass)	0 (not solved)
1920	39	362	1 (pass)	0 (not solved)
1921	39	363	1 (pass)	0 (not solved)
1922	39	364	1 (pass)	0 (not solved)
1923	39	365	1 (pass)	0 (not solved)
1924	39	366	1 (pass)	0 (not solved)
1925	39	367	1 (pass)	0 (not solved)
1926	39	368	1 (pass)	0 (not solved)
1927	39	369	1 (pass)	0 (not solved)
1928	39	370	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1929	39	371	1 (pass)	0 (not solved)
1930	39	372	1 (pass)	0 (not solved)
1931	39	373	1 (pass)	0 (not solved)
1932	39	374	1 (pass)	0 (not solved)
1933	39	375	1 (pass)	0 (not solved)
1934	39	376	1 (pass)	0 (not solved)
1935	39	377	1 (pass)	0 (not solved)
1936	39	378	1 (pass)	0 (not solved)
1937	39	379	1 (pass)	0 (not solved)
1938	39	380	1 (pass)	0 (not solved)
1939	39	381	1 (pass)	0 (not solved)
1940	39	382	1 (pass)	0 (not solved)
1941	39	383	1 (pass)	0 (not solved)
1942	39	384	1 (pass)	0 (not solved)
1943	39	385	1 (pass)	0 (not solved)
1944	39	386	1 (pass)	0 (not solved)
1945	39	387	1 (pass)	0 (not solved)
1946	39	388	1 (pass)	0 (not solved)
1947	39	389	1 (pass)	0 (not solved)
1948	39	390	1 (pass)	0 (not solved)
1949	39	391	1 (pass)	0 (not solved)
1950	39	392	1 (pass)	0 (not solved)
1951	39	393	1 (pass)	0 (not solved)
1952	39	394	1 (pass)	0 (not solved)
1953	39	395	1 (pass)	0 (not solved)
1954	39	396	1 (pass)	0 (not solved)
1955	39	397	1 (pass)	0 (not solved)
1956	39	398	1 (pass)	0 (not solved)
1957	39	399	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1958	39	400	1 (pass)	0 (not solved)
1959	39	401	1 (pass)	0 (not solved)
1960	39	402	1 (pass)	0 (not solved)
1961	39	954	1 (pass)	0 (not solved)
1962	39	965	1 (pass)	0 (not solved)
1963	39	978	1 (pass)	0 (not solved)
1964	39	1003	1 (pass)	0 (not solved)
1965	39	1007	1 (pass)	0 (not solved)
1966	39	1011	1 (pass)	0 (not solved)
1967	39	1039	1 (pass)	0 (not solved)
1968	39	1071	1 (pass)	-1 (time out)
1969	39	1078	1 (pass)	-1 (time out)
1970	39	1079	1 (pass)	-1 (time out)
1971	39	1080	1 (pass)	-1 (time out)
1972	39	1082	1 (pass)	-1 (time out)
1973	39	1083	1 (pass)	-1 (time out)
1974	39	1084	1 (pass)	-1 (time out)
1975	39	1086	1 (pass)	-1 (time out)
1976	40	14	1 (pass)	0 (not solved)
1977	40	15	1 (pass)	0 (not solved)
1978	40	22	1 (pass)	0 (not solved)
1979	40	23	1 (pass)	0 (not solved)
1980	40	24	1 (pass)	0 (not solved)
1981	40	25	1 (pass)	0 (not solved)
1982	40	113	1 (pass)	0 (not solved)
1983	40	114	1 (pass)	0 (not solved)
1984	40	115	1 (pass)	0 (not solved)
1985	40	116	1 (pass)	0 (not solved)
1986	40	117	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1987	40	118	1 (pass)	0 (not solved)
1988	40	149	1 (pass)	-1 (time out)
1989	40	235	1 (pass)	0 (not solved)
1990	40	263	1 (pass)	-1 (time out)
1991	40	268	1 (pass)	-1 (time out)
1992	40	303	1 (pass)	0 (not solved)
1993	40	316	1 (pass)	0 (not solved)
1994	40	317	1 (pass)	0 (not solved)
1995	40	318	1 (pass)	0 (not solved)
1996	40	319	1 (pass)	0 (not solved)
1997	40	320	1 (pass)	0 (not solved)
1998	40	324	1 (pass)	0 (not solved)
1999	40	325	1 (pass)	0 (not solved)
2000	40	326	1 (pass)	0 (not solved)
2001	40	327	1 (pass)	0 (not solved)
2002	40	328	1 (pass)	0 (not solved)
2003	40	332	1 (pass)	0 (not solved)
2004	40	333	1 (pass)	0 (not solved)
2005	40	334	1 (pass)	0 (not solved)
2006	40	335	1 (pass)	0 (not solved)
2007	40	339	1 (pass)	0 (not solved)
2008	40	340	1 (pass)	0 (not solved)
2009	40	341	1 (pass)	0 (not solved)
2010	40	367	1 (pass)	0 (not solved)
2011	41	236	1 (pass)	-1 (time out)
2012	41	249	1 (pass)	-1 (time out)
2013	41	250	1 (pass)	-1 (time out)
2014	41	258	1 (pass)	-1 (time out)
2015	41	259	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2016	41	260	1 (pass)	0 (not solved)
2017	41	263	1 (pass)	0 (not solved)
2018	41	264	1 (pass)	0 (not solved)
2019	41	265	1 (pass)	0 (not solved)
2020	41	266	1 (pass)	0 (not solved)
2021	41	296	1 (pass)	-1 (time out)
2022	41	303	1 (pass)	-1 (time out)
2023	41	304	1 (pass)	-1 (time out)
2024	41	307	1 (pass)	-1 (time out)
2025	41	308	1 (pass)	-1 (time out)
2026	41	311	1 (pass)	-1 (time out)
2027	41	314	1 (pass)	-1 (time out)
2028	41	324	1 (pass)	-1 (time out)
2029	41	355	1 (pass)	-1 (time out)
2030	41	358	1 (pass)	-1 (time out)
2031	41	368	1 (pass)	-1 (time out)
2032	41	372	1 (pass)	-1 (time out)
2033	41	386	1 (pass)	-1 (time out)
2034	41	387	1 (pass)	-1 (time out)
2035	41	392	1 (pass)	-1 (time out)
2036	41	393	1 (pass)	-1 (time out)
2037	41	395	1 (pass)	-1 (time out)
2038	41	396	1 (pass)	-1 (time out)
2039	41	397	1 (pass)	-1 (time out)
2040	42	20	1 (pass)	-1 (time out)
2041	42	21	1 (pass)	-1 (time out)
2042	42	64	1 (pass)	-1 (time out)
2043	43	22	1 (pass)	-1 (time out)
2044	43	23	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2045	43	24	1 (pass)	-1 (time out)
2046	43	25	1 (pass)	-1 (time out)
2047	43	68	1 (pass)	-1 (time out)
2048	43	73	1 (pass)	-1 (time out)
2049	43	126	1 (pass)	-1 (time out)
2050	43	127	1 (pass)	-1 (time out)
2051	43	128	1 (pass)	-1 (time out)
2052	43	129	1 (pass)	-1 (time out)
2053	43	130	1 (pass)	-1 (time out)
2054	47	41	1 (pass)	-1 (time out)
2055	48	35	1 (pass)	0 (not solved)
2056	48	36	1 (pass)	0 (not solved)
2057	48	37	1 (pass)	0 (not solved)
2058	48	38	1 (pass)	0 (not solved)
2059	48	39	1 (pass)	0 (not solved)
2060	48	40	1 (pass)	0 (not solved)
2061	48	41	1 (pass)	0 (not solved)
2062	48	42	1 (pass)	0 (not solved)
2063	48	43	1 (pass)	-1 (time out)
2064	48	67	1 (pass)	-1 (time out)
2065	48	70	1 (pass)	-1 (time out)
2066	48	79	1 (pass)	0 (not solved)
2067	48	80	1 (pass)	0 (not solved)
2068	48	81	1 (pass)	0 (not solved)
2069	48	82	1 (pass)	0 (not solved)
2070	50	114	1 (pass)	0 (not solved)
2071	51	61	1 (pass)	-1 (time out)
2072	51	62	1 (pass)	-1 (time out)
2073	51	111	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2074	51	112	1 (pass)	-1 (time out)
2075	51	114	1 (pass)	-1 (time out)
2076	51	115	1 (pass)	-1 (time out)
2077	51	127	1 (pass)	-1 (time out)
2078	51	134	1 (pass)	-1 (time out)
2079	51	138	1 (pass)	-1 (time out)
2080	51	337	1 (pass)	-1 (time out)
2081	51	338	1 (pass)	-1 (time out)
2082	51	342	1 (pass)	-1 (time out)
2083	51	391	1 (pass)	-1 (time out)
2084	51	392	1 (pass)	-1 (time out)
2085	51	394	1 (pass)	-1 (time out)
2086	51	395	1 (pass)	-1 (time out)
2087	51	396	1 (pass)	-1 (time out)
2088	51	398	1 (pass)	-1 (time out)
2089	51	401	1 (pass)	-1 (time out)
2090	51	402	1 (pass)	-1 (time out)
2091	51	403	1 (pass)	-1 (time out)
2092	51	408	1 (pass)	-1 (time out)
2093	51	409	1 (pass)	-1 (time out)
2094	51	410	1 (pass)	-1 (time out)
2095	51	483	1 (pass)	-2 (exception) Exception raised: SyntaxError >> Malformed expression
2096	52	1	1 (pass)	0 (not solved)
2097	52	2	1 (pass)	0 (not solved)
2098	52	9	1 (pass)	0 (not solved)
2099	52	10	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2100	52	11	1 (pass)	0 (not solved)
2101	52	12	1 (pass)	0 (not solved)
2102	52	13	1 (pass)	0 (not solved)
2103	52	52	1 (pass)	0 (not solved)
2104	52	53	1 (pass)	0 (not solved)
2105	52	55	1 (pass)	0 (not solved)
2106	52	56	1 (pass)	-2 (exception) Exception raised: TypeError >> Error detected within library code: catdef: division by zero
2107	52	57	1 (pass)	0 (not solved)
2108	52	58	1 (pass)	0 (not solved)
2109	52	64	1 (pass)	0 (not solved)
2110	52	65	1 (pass)	0 (not solved)
2111	52	66	1 (pass)	0 (not solved)
2112	52	67	1 (pass)	0 (not solved)
2113	52	68	1 (pass)	0 (not solved)
2114	52	73	1 (pass)	0 (not solved)
2115	52	78	1 (pass)	-1 (time out)
2116	52	79	1 (pass)	-1 (time out)
2117	52	80	1 (pass)	-1 (time out)
2118	52	81	1 (pass)	-1 (time out)
2119	52	83	1 (pass)	0 (not solved)
2120	52	84	1 (pass)	0 (not solved)
2121	52	85	1 (pass)	0 (not solved)
2122	52	86	1 (pass)	0 (not solved)
2123	52	91	1 (pass)	0 (not solved)
2124	52	92	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2125	52	93	1 (pass)	0 (not solved)
2126	52	94	1 (pass)	0 (not solved)
2127	52	95	1 (pass)	0 (not solved)
2128	52	100	1 (pass)	0 (not solved)
2129	52	125	1 (pass)	0 (not solved)
2130	52	126	1 (pass)	0 (not solved)
2131	52	127	1 (pass)	0 (not solved)
2132	52	128	1 (pass)	0 (not solved)
2133	52	129	1 (pass)	0 (not solved)
2134	52	130	1 (pass)	0 (not solved)
2135	52	131	1 (pass)	-2 (exception) Exception raised: TypeError >> Error detected within library code: catdef: division by zero
2136	52	133	1 (pass)	0 (not solved)
2137	52	134	1 (pass)	0 (not solved)
2138	52	135	1 (pass)	0 (not solved)
2139	52	136	1 (pass)	0 (not solved)
2140	52	137	1 (pass)	0 (not solved)
2141	52	138	1 (pass)	0 (not solved)
2142	52	139	1 (pass)	0 (not solved)
2143	52	140	1 (pass)	0 (not solved)
2144	52	141	1 (pass)	0 (not solved)
2145	52	142	1 (pass)	0 (not solved)
2146	52	143	1 (pass)	0 (not solved)
2147	52	152	1 (pass)	0 (not solved)
2148	52	153	1 (pass)	0 (not solved)
2149	52	154	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2150	52	155	1 (pass)	0 (not solved)
2151	52	156	1 (pass)	0 (not solved)
2152	52	157	1 (pass)	0 (not solved)
2153	52	158	1 (pass)	0 (not solved)
2154	52	159	1 (pass)	0 (not solved)
2155	52	168	1 (pass)	0 (not solved)
2156	52	169	1 (pass)	0 (not solved)
2157	52	170	1 (pass)	0 (not solved)
2158	52	171	1 (pass)	0 (not solved)
2159	52	208	1 (pass)	0 (not solved)
2160	52	209	1 (pass)	0 (not solved)
2161	52	214	1 (pass)	0 (not solved)
2162	52	215	1 (pass)	0 (not solved)
2163	52	221	1 (pass)	0 (not solved)
2164	52	222	1 (pass)	0 (not solved)
2165	52	260	1 (pass)	0 (not solved)
2166	52	262	1 (pass)	0 (not solved)
2167	52	378	1 (pass)	0 (not solved)
2168	52	380	1 (pass)	0 (not solved)
2169	52	381	1 (pass)	0 (not solved)
2170	52	383	1 (pass)	0 (not solved)
2171	52	387	1 (pass)	0 (not solved)
2172	52	389	1 (pass)	0 (not solved)
2173	52	391	1 (pass)	0 (not solved)
2174	52	392	1 (pass)	0 (not solved)
2175	52	397	1 (pass)	0 (not solved)
2176	52	601	1 (pass)	0 (not solved)
2177	52	764	1 (pass)	0 (not solved)
2178	52	765	1 (pass)	0 (not solved)

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#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2179	52	766	1 (pass)	0 (not solved)
2180	52	767	1 (pass)	0 (not solved)
2181	52	768	1 (pass)	0 (not solved)
2182	52	769	1 (pass)	0 (not solved)
2183	52	770	1 (pass)	0 (not solved)
2184	52	771	1 (pass)	0 (not solved)
2185	52	772	1 (pass)	0 (not solved)
2186	52	773	1 (pass)	0 (not solved)
2187	52	996	1 (pass)	-1 (time out)
2188	61	31	1 (pass)	-1 (time out)
2189	61	106	1 (pass)	0 (not solved)
2190	65	9	1 (pass)	0 (not solved)
2191	65	10	1 (pass)	0 (not solved)
2192	65	11	1 (pass)	0 (not solved)
2193	65	12	1 (pass)	0 (not solved)
2194	65	13	1 (pass)	0 (not solved)
2195	65	14	1 (pass)	0 (not solved)
2196	65	15	1 (pass)	0 (not solved)
2197	65	16	1 (pass)	0 (not solved)
2198	65	17	1 (pass)	0 (not solved)
2199	65	18	1 (pass)	0 (not solved)
2200	65	19	1 (pass)	0 (not solved)
2201	65	20	1 (pass)	0 (not solved)
2202	65	21	1 (pass)	0 (not solved)
2203	65	22	1 (pass)	0 (not solved)
2204	65	23	1 (pass)	0 (not solved)
2205	65	24	1 (pass)	0 (not solved)
2206	65	25	1 (pass)	0 (not solved)
2207	65	26	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2208	65	27	1 (pass)	0 (not solved)
2209	65	28	1 (pass)	0 (not solved)
2210	65	29	1 (pass)	0 (not solved)
2211	65	30	1 (pass)	0 (not solved)
2212	65	31	1 (pass)	0 (not solved)
2213	65	32	1 (pass)	0 (not solved)
2214	65	194	1 (pass)	0 (not solved)
2215	65	195	1 (pass)	0 (not solved)
2216	65	196	1 (pass)	0 (not solved)
2217	65	197	1 (pass)	0 (not solved)
2218	65	198	1 (pass)	0 (not solved)
2219	65	199	1 (pass)	0 (not solved)
2220	65	200	1 (pass)	0 (not solved)
2221	65	201	1 (pass)	0 (not solved)
2222	65	202	1 (pass)	0 (not solved)
2223	65	203	1 (pass)	0 (not solved)
2224	65	211	1 (pass)	0 (not solved)
2225	65	212	1 (pass)	0 (not solved)
2226	65	213	1 (pass)	0 (not solved)
2227	65	214	1 (pass)	0 (not solved)
2228	65	215	1 (pass)	0 (not solved)
2229	65	216	1 (pass)	0 (not solved)
2230	65	217	1 (pass)	0 (not solved)
2231	65	218	1 (pass)	0 (not solved)
2232	65	219	1 (pass)	0 (not solved)
2233	65	220	1 (pass)	0 (not solved)
2234	65	232	1 (pass)	0 (not solved)
2235	65	233	1 (pass)	0 (not solved)
2236	65	234	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2237	65	235	1 (pass)	0 (not solved)
2238	65	236	1 (pass)	0 (not solved)
2239	65	237	1 (pass)	0 (not solved)
2240	65	238	1 (pass)	0 (not solved)
2241	65	239	1 (pass)	0 (not solved)
2242	65	240	1 (pass)	0 (not solved)
2243	65	241	1 (pass)	0 (not solved)
2244	65	269	1 (pass)	0 (not solved)
2245	65	270	1 (pass)	0 (not solved)
2246	65	293	1 (pass)	0 (not solved)
2247	65	294	1 (pass)	0 (not solved)
2248	65	295	1 (pass)	0 (not solved)
2249	65	378	1 (pass)	0 (not solved)
2250	65	379	1 (pass)	0 (not solved)
2251	65	380	1 (pass)	0 (not solved)
2252	65	381	1 (pass)	0 (not solved)
2253	65	382	1 (pass)	0 (not solved)
2254	65	383	1 (pass)	0 (not solved)
2255	65	384	1 (pass)	0 (not solved)
2256	65	391	1 (pass)	0 (not solved)
2257	65	392	1 (pass)	0 (not solved)
2258	65	393	1 (pass)	0 (not solved)
2259	65	394	1 (pass)	0 (not solved)
2260	65	395	1 (pass)	0 (not solved)
2261	65	396	1 (pass)	0 (not solved)
2262	65	404	1 (pass)	0 (not solved)
2263	65	405	1 (pass)	0 (not solved)
2264	65	406	1 (pass)	0 (not solved)
2265	65	407	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2266	65	408	1 (pass)	0 (not solved)
2267	65	409	1 (pass)	0 (not solved)
2268	65	417	1 (pass)	0 (not solved)
2269	65	418	1 (pass)	0 (not solved)
2270	65	419	1 (pass)	0 (not solved)
2271	65	420	1 (pass)	0 (not solved)
2272	65	421	1 (pass)	0 (not solved)
2273	65	422	1 (pass)	0 (not solved)
2274	65	423	1 (pass)	0 (not solved)
2275	65	431	1 (pass)	0 (not solved)
2276	65	432	1 (pass)	0 (not solved)
2277	65	433	1 (pass)	0 (not solved)
2278	65	434	1 (pass)	0 (not solved)
2279	65	435	1 (pass)	0 (not solved)
2280	65	436	1 (pass)	0 (not solved)
2281	65	444	1 (pass)	0 (not solved)
2282	65	445	1 (pass)	0 (not solved)
2283	65	446	1 (pass)	0 (not solved)
2284	65	447	1 (pass)	0 (not solved)
2285	65	448	1 (pass)	0 (not solved)
2286	65	449	1 (pass)	0 (not solved)
2287	65	458	1 (pass)	0 (not solved)
2288	65	459	1 (pass)	0 (not solved)
2289	65	460	1 (pass)	0 (not solved)
2290	65	480	1 (pass)	0 (not solved)
2291	65	481	1 (pass)	0 (not solved)
2292	65	482	1 (pass)	0 (not solved)
2293	65	507	1 (pass)	0 (not solved)
2294	65	508	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2295	65	509	1 (pass)	0 (not solved)
2296	65	510	1 (pass)	0 (not solved)
2297	65	511	1 (pass)	0 (not solved)
2298	65	512	1 (pass)	0 (not solved)
2299	65	513	1 (pass)	0 (not solved)
2300	65	514	1 (pass)	0 (not solved)
2301	65	515	1 (pass)	0 (not solved)
2302	65	516	1 (pass)	0 (not solved)
2303	65	517	1 (pass)	0 (not solved)
2304	65	518	1 (pass)	0 (not solved)
2305	65	519	1 (pass)	0 (not solved)
2306	65	520	1 (pass)	0 (not solved)
2307	65	521	1 (pass)	0 (not solved)
2308	65	522	1 (pass)	0 (not solved)
2309	65	523	1 (pass)	0 (not solved)
2310	65	524	1 (pass)	0 (not solved)
2311	65	525	1 (pass)	0 (not solved)
2312	65	526	1 (pass)	0 (not solved)
2313	65	527	1 (pass)	0 (not solved)
2314	65	528	1 (pass)	0 (not solved)
2315	65	529	1 (pass)	0 (not solved)
2316	65	530	1 (pass)	0 (not solved)
2317	65	531	1 (pass)	0 (not solved)
2318	65	532	1 (pass)	0 (not solved)
2319	65	533	1 (pass)	0 (not solved)
2320	65	534	1 (pass)	0 (not solved)
2321	65	535	1 (pass)	0 (not solved)
2322	65	536	1 (pass)	0 (not solved)
2323	65	537	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2324	67	50	1 (pass)	0 (not solved)
2325	67	51	1 (pass)	0 (not solved)
2326	67	52	1 (pass)	0 (not solved)
2327	67	53	1 (pass)	0 (not solved)
2328	67	54	1 (pass)	0 (not solved)
2329	67	55	1 (pass)	0 (not solved)
2330	67	56	1 (pass)	0 (not solved)
2331	67	57	1 (pass)	0 (not solved)
2332	70	196	1 (pass)	0 (not solved)
2333	70	197	1 (pass)	0 (not solved)
2334	70	198	1 (pass)	0 (not solved)
2335	70	199	1 (pass)	0 (not solved)
2336	70	200	1 (pass)	0 (not solved)
2337	70	201	1 (pass)	0 (not solved)
2338	70	202	1 (pass)	0 (not solved)
2339	70	203	1 (pass)	0 (not solved)
2340	70	204	1 (pass)	0 (not solved)
2341	70	205	1 (pass)	0 (not solved)
2342	70	206	1 (pass)	0 (not solved)
2343	70	207	1 (pass)	0 (not solved)
2344	70	208	1 (pass)	0 (not solved)
2345	70	209	1 (pass)	0 (not solved)
2346	70	210	1 (pass)	0 (not solved)
2347	70	211	1 (pass)	0 (not solved)
2348	70	212	1 (pass)	0 (not solved)
2349	70	213	1 (pass)	0 (not solved)
2350	70	214	1 (pass)	0 (not solved)
2351	70	215	1 (pass)	0 (not solved)
2352	70	216	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2353	70	217	1 (pass)	0 (not solved)
2354	70	218	1 (pass)	0 (not solved)
2355	70	219	1 (pass)	0 (not solved)
2356	70	220	1 (pass)	0 (not solved)
2357	70	221	1 (pass)	0 (not solved)
2358	70	222	1 (pass)	0 (not solved)
2359	70	223	1 (pass)	0 (not solved)
2360	70	224	1 (pass)	0 (not solved)
2361	70	225	1 (pass)	0 (not solved)
2362	70	226	1 (pass)	0 (not solved)
2363	70	227	1 (pass)	0 (not solved)
2364	70	228	1 (pass)	0 (not solved)
2365	70	229	1 (pass)	0 (not solved)
2366	70	230	1 (pass)	0 (not solved)
2367	70	231	1 (pass)	0 (not solved)
2368	70	232	1 (pass)	0 (not solved)
2369	70	233	1 (pass)	0 (not solved)
2370	70	234	1 (pass)	0 (not solved)
2371	70	235	1 (pass)	0 (not solved)
2372	70	236	1 (pass)	0 (not solved)
2373	70	237	1 (pass)	0 (not solved)
2374	70	238	1 (pass)	0 (not solved)
2375	70	239	1 (pass)	0 (not solved)
2376	70	240	1 (pass)	0 (not solved)
2377	70	241	1 (pass)	0 (not solved)
2378	70	242	1 (pass)	0 (not solved)
2379	70	243	1 (pass)	0 (not solved)
2380	70	244	1 (pass)	0 (not solved)
2381	70	245	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2382	70	246	1 (pass)	0 (not solved)
2383	70	247	1 (pass)	0 (not solved)
2384	70	248	1 (pass)	0 (not solved)
2385	70	249	1 (pass)	0 (not solved)
2386	70	250	1 (pass)	0 (not solved)
2387	70	251	1 (pass)	0 (not solved)
2388	70	252	1 (pass)	0 (not solved)
2389	70	253	1 (pass)	0 (not solved)
2390	70	254	1 (pass)	0 (not solved)
2391	70	255	1 (pass)	0 (not solved)
2392	70	256	1 (pass)	0 (not solved)
2393	70	257	1 (pass)	0 (not solved)
2394	70	258	1 (pass)	0 (not solved)
2395	70	259	1 (pass)	0 (not solved)
2396	70	260	1 (pass)	0 (not solved)
2397	70	261	1 (pass)	0 (not solved)
2398	70	262	1 (pass)	0 (not solved)
2399	70	263	1 (pass)	0 (not solved)
2400	70	264	1 (pass)	0 (not solved)
2401	70	265	1 (pass)	0 (not solved)
2402	70	266	1 (pass)	0 (not solved)
2403	70	267	1 (pass)	0 (not solved)
2404	70	268	1 (pass)	0 (not solved)
2405	70	269	1 (pass)	0 (not solved)
2406	70	270	1 (pass)	0 (not solved)
2407	70	271	1 (pass)	0 (not solved)
2408	70	272	1 (pass)	0 (not solved)
2409	70	273	1 (pass)	-1 (time out)
2410	70	274	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2411	70	275	1 (pass)	-1 (time out)
2412	70	280	1 (pass)	-1 (time out)
2413	70	281	1 (pass)	-1 (time out)
2414	70	282	1 (pass)	-1 (time out)
2415	70	283	1 (pass)	-1 (time out)
2416	70	284	1 (pass)	-1 (time out)
2417	70	289	1 (pass)	-1 (time out)
2418	70	290	1 (pass)	-1 (time out)
2419	70	291	1 (pass)	-1 (time out)
2420	70	292	1 (pass)	-1 (time out)
2421	70	293	1 (pass)	-1 (time out)
2422	70	298	1 (pass)	-1 (time out)
2423	70	299	1 (pass)	-1 (time out)
2424	70	300	1 (pass)	-1 (time out)
2425	70	305	1 (pass)	-1 (time out)
2426	70	306	1 (pass)	-1 (time out)
2427	70	307	1 (pass)	-1 (time out)
2428	70	313	1 (pass)	-1 (time out)
2429	70	314	1 (pass)	-1 (time out)
2430	70	315	1 (pass)	-1 (time out)
2431	70	479	1 (pass)	0 (not solved)
2432	70	480	1 (pass)	0 (not solved)
2433	70	481	1 (pass)	0 (not solved)
2434	70	482	1 (pass)	0 (not solved)
2435	70	489	1 (pass)	0 (not solved)
2436	70	490	1 (pass)	0 (not solved)
2437	70	491	1 (pass)	0 (not solved)
2438	70	492	1 (pass)	0 (not solved)
2439	70	493	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2440	70	500	1 (pass)	0 (not solved)
2441	70	501	1 (pass)	0 (not solved)
2442	70	502	1 (pass)	0 (not solved)
2443	70	503	1 (pass)	0 (not solved)
2444	70	504	1 (pass)	0 (not solved)
2445	70	505	1 (pass)	0 (not solved)
2446	70	512	1 (pass)	0 (not solved)
2447	70	513	1 (pass)	0 (not solved)
2448	70	514	1 (pass)	0 (not solved)
2449	70	515	1 (pass)	0 (not solved)
2450	70	522	1 (pass)	0 (not solved)
2451	70	523	1 (pass)	0 (not solved)
2452	70	524	1 (pass)	0 (not solved)
2453	70	525	1 (pass)	0 (not solved)
2454	70	526	1 (pass)	0 (not solved)
2455	70	533	1 (pass)	0 (not solved)
2456	70	534	1 (pass)	0 (not solved)
2457	70	535	1 (pass)	0 (not solved)
2458	70	536	1 (pass)	0 (not solved)
2459	70	537	1 (pass)	0 (not solved)
2460	70	538	1 (pass)	0 (not solved)
2461	70	539	1 (pass)	0 (not solved)
2462	70	540	1 (pass)	0 (not solved)
2463	70	541	1 (pass)	0 (not solved)
2464	70	542	1 (pass)	0 (not solved)
2465	70	543	1 (pass)	0 (not solved)
2466	70	544	1 (pass)	0 (not solved)
2467	70	545	1 (pass)	0 (not solved)
2468	70	546	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2469	70	547	1 (pass)	0 (not solved)
2470	70	548	1 (pass)	0 (not solved)
2471	70	549	1 (pass)	0 (not solved)
2472	70	550	1 (pass)	0 (not solved)
2473	70	551	1 (pass)	0 (not solved)
2474	70	552	1 (pass)	0 (not solved)
2475	70	553	1 (pass)	0 (not solved)
2476	70	554	1 (pass)	0 (not solved)
2477	70	555	1 (pass)	0 (not solved)
2478	70	556	1 (pass)	0 (not solved)
2479	70	557	1 (pass)	0 (not solved)
2480	70	558	1 (pass)	0 (not solved)
2481	70	559	1 (pass)	0 (not solved)
2482	70	560	1 (pass)	0 (not solved)
2483	70	561	1 (pass)	0 (not solved)
2484	70	562	1 (pass)	0 (not solved)
2485	70	563	1 (pass)	0 (not solved)
2486	70	564	1 (pass)	0 (not solved)
2487	70	565	1 (pass)	0 (not solved)
2488	70	566	1 (pass)	0 (not solved)
2489	70	567	1 (pass)	0 (not solved)
2490	70	568	1 (pass)	0 (not solved)
2491	70	569	1 (pass)	0 (not solved)
2492	70	570	1 (pass)	0 (not solved)
2493	70	571	1 (pass)	0 (not solved)
2494	70	572	1 (pass)	0 (not solved)
2495	70	573	1 (pass)	0 (not solved)
2496	73	203	1 (pass)	0 (not solved)
2497	73	204	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2498	73	207	1 (pass)	0 (not solved)
2499	73	208	1 (pass)	0 (not solved)
2500	73	482	1 (pass)	0 (not solved)
2501	73	483	1 (pass)	0 (not solved)
2502	73	484	1 (pass)	0 (not solved)
2503	73	485	1 (pass)	0 (not solved)
2504	73	486	1 (pass)	0 (not solved)
2505	73	487	1 (pass)	0 (not solved)
2506	73	488	1 (pass)	0 (not solved)
2507	73	489	1 (pass)	0 (not solved)
2508	73	490	1 (pass)	0 (not solved)
2509	73	491	1 (pass)	0 (not solved)
2510	73	492	1 (pass)	0 (not solved)
2511	73	493	1 (pass)	0 (not solved)
2512	73	494	1 (pass)	0 (not solved)
2513	73	495	1 (pass)	0 (not solved)
2514	73	496	1 (pass)	0 (not solved)
2515	73	497	1 (pass)	0 (not solved)
2516	73	498	1 (pass)	0 (not solved)
2517	73	499	1 (pass)	0 (not solved)
2518	73	500	1 (pass)	0 (not solved)
2519	73	501	1 (pass)	0 (not solved)
2520	73	502	1 (pass)	0 (not solved)
2521	73	503	1 (pass)	0 (not solved)
2522	73	504	1 (pass)	0 (not solved)
2523	73	505	1 (pass)	0 (not solved)
2524	73	506	1 (pass)	0 (not solved)
2525	73	507	1 (pass)	0 (not solved)
2526	73	508	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2527	73	509	1 (pass)	0 (not solved)
2528	73	510	1 (pass)	0 (not solved)
2529	73	511	1 (pass)	0 (not solved)
2530	73	512	1 (pass)	0 (not solved)
2531	73	513	1 (pass)	0 (not solved)
2532	73	514	1 (pass)	0 (not solved)
2533	73	515	1 (pass)	0 (not solved)
2534	73	516	1 (pass)	0 (not solved)
2535	73	517	1 (pass)	0 (not solved)
2536	73	518	1 (pass)	0 (not solved)
2537	73	519	1 (pass)	0 (not solved)
2538	73	520	1 (pass)	0 (not solved)
2539	73	521	1 (pass)	0 (not solved)
2540	73	704	1 (pass)	-1 (time out)
2541	73	711	1 (pass)	-1 (time out)
2542	73	723	1 (pass)	0 (not solved)
2543	73	724	1 (pass)	0 (not solved)
2544	73	725	1 (pass)	0 (not solved)
2545	73	726	1 (pass)	0 (not solved)
2546	73	727	1 (pass)	0 (not solved)
2547	73	728	1 (pass)	0 (not solved)
2548	73	729	1 (pass)	0 (not solved)
2549	73	730	1 (pass)	0 (not solved)
2550	73	731	1 (pass)	0 (not solved)
2551	73	732	1 (pass)	0 (not solved)
2552	73	733	1 (pass)	0 (not solved)
2553	73	734	1 (pass)	0 (not solved)
2554	73	735	1 (pass)	0 (not solved)
2555	73	736	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2556	73	737	1 (pass)	0 (not solved)
2557	73	738	1 (pass)	0 (not solved)
2558	73	739	1 (pass)	0 (not solved)
2559	73	740	1 (pass)	0 (not solved)
2560	73	741	1 (pass)	0 (not solved)
2561	73	742	1 (pass)	0 (not solved)
2562	73	743	1 (pass)	0 (not solved)
2563	73	744	1 (pass)	0 (not solved)
2564	74	88	1 (pass)	0 (not solved)
2565	74	89	1 (pass)	0 (not solved)
2566	74	90	1 (pass)	0 (not solved)
2567	74	91	1 (pass)	0 (not solved)
2568	74	92	1 (pass)	0 (not solved)
2569	74	93	1 (pass)	0 (not solved)
2570	74	94	1 (pass)	0 (not solved)
2571	74	95	1 (pass)	0 (not solved)
2572	74	96	1 (pass)	0 (not solved)
2573	74	97	1 (pass)	0 (not solved)
2574	74	98	1 (pass)	0 (not solved)
2575	74	99	1 (pass)	0 (not solved)
2576	74	100	1 (pass)	0 (not solved)
2577	74	101	1 (pass)	0 (not solved)
2578	74	102	1 (pass)	0 (not solved)
2579	74	103	1 (pass)	0 (not solved)
2580	74	104	1 (pass)	0 (not solved)
2581	74	105	1 (pass)	0 (not solved)
2582	74	106	1 (pass)	0 (not solved)
2583	74	107	1 (pass)	0 (not solved)
2584	74	108	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2585	74	109	1 (pass)	0 (not solved)
2586	74	110	1 (pass)	0 (not solved)
2587	74	111	1 (pass)	0 (not solved)
2588	74	112	1 (pass)	0 (not solved)
2589	74	113	1 (pass)	0 (not solved)
2590	74	114	1 (pass)	0 (not solved)
2591	74	115	1 (pass)	0 (not solved)
2592	74	116	1 (pass)	0 (not solved)
2593	74	117	1 (pass)	0 (not solved)
2594	74	118	1 (pass)	0 (not solved)
2595	74	119	1 (pass)	0 (not solved)
2596	74	120	1 (pass)	0 (not solved)
2597	74	121	1 (pass)	0 (not solved)
2598	74	122	1 (pass)	0 (not solved)
2599	74	123	1 (pass)	0 (not solved)
2600	74	124	1 (pass)	0 (not solved)
2601	74	125	1 (pass)	0 (not solved)
2602	74	126	1 (pass)	0 (not solved)
2603	74	127	1 (pass)	0 (not solved)
2604	74	128	1 (pass)	0 (not solved)
2605	74	129	1 (pass)	0 (not solved)
2606	74	130	1 (pass)	0 (not solved)
2607	74	131	1 (pass)	0 (not solved)
2608	74	132	1 (pass)	0 (not solved)
2609	74	133	1 (pass)	0 (not solved)
2610	74	134	1 (pass)	0 (not solved)
2611	74	135	1 (pass)	0 (not solved)
2612	74	136	1 (pass)	0 (not solved)
2613	74	137	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2614	74	138	1 (pass)	0 (not solved)
2615	74	139	1 (pass)	0 (not solved)
2616	74	140	1 (pass)	0 (not solved)
2617	74	141	1 (pass)	0 (not solved)
2618	74	142	1 (pass)	0 (not solved)
2619	74	143	1 (pass)	0 (not solved)
2620	74	144	1 (pass)	0 (not solved)
2621	74	145	1 (pass)	0 (not solved)
2622	74	146	1 (pass)	0 (not solved)
2623	74	147	1 (pass)	0 (not solved)
2624	74	148	1 (pass)	0 (not solved)
2625	74	149	1 (pass)	0 (not solved)
2626	74	150	1 (pass)	0 (not solved)
2627	74	1143	1 (pass)	0 (not solved)
2628	74	1144	1 (pass)	0 (not solved)
2629	74	1151	1 (pass)	0 (not solved)
2630	74	1152	1 (pass)	0 (not solved)
2631	74	1160	1 (pass)	0 (not solved)
2632	74	1168	1 (pass)	0 (not solved)
2633	74	1169	1 (pass)	0 (not solved)
2634	74	1170	1 (pass)	0 (not solved)
2635	74	1176	1 (pass)	0 (not solved)
2636	74	1177	1 (pass)	0 (not solved)
2637	74	1178	1 (pass)	0 (not solved)
2638	74	1183	1 (pass)	0 (not solved)
2639	74	1184	1 (pass)	0 (not solved)
2640	74	1185	1 (pass)	0 (not solved)
2641	74	1277	1 (pass)	0 (not solved)
2642	75	39	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2643	78	33	1 (pass)	0 (not solved)
2644	79	7	1 (pass)	0 (not solved)
2645	79	8	1 (pass)	0 (not solved)
2646	79	9	1 (pass)	0 (not solved)
2647	79	10	1 (pass)	0 (not solved)
2648	79	11	1 (pass)	0 (not solved)
2649	79	12	1 (pass)	0 (not solved)
2650	79	150	1 (pass)	0 (not solved)
2651	79	182	1 (pass)	-1 (time out)
2652	79	183	1 (pass)	-1 (time out)
2653	79	184	1 (pass)	-1 (time out)
2654	79	185	1 (pass)	-1 (time out)
2655	79	186	1 (pass)	-1 (time out)
2656	79	187	1 (pass)	-1 (time out)
2657	79	188	1 (pass)	-1 (time out)
2658	79	189	1 (pass)	-1 (time out)
2659	79	190	1 (pass)	-1 (time out)
2660	79	191	1 (pass)	-1 (time out)
2661	79	192	1 (pass)	-1 (time out)
2662	79	193	1 (pass)	-1 (time out)
2663	79	194	1 (pass)	-1 (time out)
2664	79	250	1 (pass)	-1 (time out)
2665	79	251	1 (pass)	-1 (time out)
2666	79	253	1 (pass)	-1 (time out)
2667	79	254	1 (pass)	-1 (time out)
2668	79	351	1 (pass)	0 (not solved)
2669	79	388	1 (pass)	-1 (time out)
2670	79	390	1 (pass)	-1 (time out)
2671	79	391	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2672	79	392	1 (pass)	-1 (time out)
2673	79	393	1 (pass)	-1 (time out)
2674	79	394	1 (pass)	-1 (time out)
2675	79	398	1 (pass)	-1 (time out)
2676	79	400	1 (pass)	-1 (time out)
2677	79	401	1 (pass)	-1 (time out)
2678	79	402	1 (pass)	-1 (time out)
2679	79	403	1 (pass)	-1 (time out)
2680	79	407	1 (pass)	-1 (time out)
2681	79	518	1 (pass)	0 (not solved)
2682	79	591	1 (pass)	0 (not solved)
2683	81	6	1 (pass)	-1 (time out)
2684	81	13	1 (pass)	-1 (time out)
2685	82	9	1 (pass)	0 (not solved)
2686	82	10	1 (pass)	0 (not solved)
2687	82	11	1 (pass)	0 (not solved)
2688	82	12	1 (pass)	0 (not solved)
2689	82	13	1 (pass)	0 (not solved)
2690	82	14	1 (pass)	0 (not solved)
2691	82	15	1 (pass)	0 (not solved)
2692	82	16	1 (pass)	0 (not solved)
2693	82	17	1 (pass)	0 (not solved)
2694	82	18	1 (pass)	0 (not solved)
2695	82	19	1 (pass)	0 (not solved)
2696	82	20	1 (pass)	0 (not solved)
2697	82	21	1 (pass)	0 (not solved)
2698	82	22	1 (pass)	0 (not solved)
2699	82	23	1 (pass)	0 (not solved)
2700	82	24	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2701	82	45	1 (pass)	0 (not solved)
2702	82	46	1 (pass)	0 (not solved)
2703	82	47	1 (pass)	0 (not solved)
2704	82	48	1 (pass)	0 (not solved)
2705	82	49	1 (pass)	0 (not solved)
2706	82	50	1 (pass)	0 (not solved)
2707	82	66	1 (pass)	0 (not solved)
2708	82	67	1 (pass)	0 (not solved)
2709	82	68	1 (pass)	0 (not solved)
2710	82	69	1 (pass)	0 (not solved)
2711	82	70	1 (pass)	0 (not solved)
2712	82	71	1 (pass)	0 (not solved)
2713	82	72	1 (pass)	0 (not solved)
2714	82	73	1 (pass)	0 (not solved)
2715	82	74	1 (pass)	0 (not solved)
2716	82	75	1 (pass)	0 (not solved)
2717	82	76	1 (pass)	0 (not solved)
2718	82	77	1 (pass)	0 (not solved)
2719	82	78	1 (pass)	0 (not solved)
2720	82	79	1 (pass)	0 (not solved)
2721	82	80	1 (pass)	0 (not solved)
2722	82	81	1 (pass)	0 (not solved)
2723	82	82	1 (pass)	0 (not solved)
2724	82	83	1 (pass)	0 (not solved)
2725	82	84	1 (pass)	0 (not solved)
2726	82	85	1 (pass)	0 (not solved)
2727	82	86	1 (pass)	0 (not solved)
2728	82	87	1 (pass)	0 (not solved)
2729	82	88	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2730	82	89	1 (pass)	0 (not solved)
2731	82	90	1 (pass)	0 (not solved)
2732	82	91	1 (pass)	0 (not solved)
2733	82	92	1 (pass)	0 (not solved)
2734	82	93	1 (pass)	0 (not solved)
2735	82	94	1 (pass)	0 (not solved)
2736	82	95	1 (pass)	0 (not solved)
2737	82	96	1 (pass)	0 (not solved)
2738	82	97	1 (pass)	0 (not solved)
2739	82	98	1 (pass)	0 (not solved)
2740	82	99	1 (pass)	0 (not solved)
2741	82	100	1 (pass)	0 (not solved)
2742	82	101	1 (pass)	0 (not solved)
2743	82	102	1 (pass)	0 (not solved)
2744	82	103	1 (pass)	0 (not solved)
2745	82	104	1 (pass)	0 (not solved)
2746	82	105	1 (pass)	0 (not solved)
2747	82	106	1 (pass)	0 (not solved)
2748	82	107	1 (pass)	0 (not solved)
2749	82	108	1 (pass)	0 (not solved)
2750	82	109	1 (pass)	0 (not solved)
2751	82	110	1 (pass)	0 (not solved)
2752	82	111	1 (pass)	0 (not solved)
2753	82	112	1 (pass)	0 (not solved)
2754	82	113	1 (pass)	0 (not solved)
2755	82	114	1 (pass)	0 (not solved)
2756	82	115	1 (pass)	0 (not solved)
2757	82	116	1 (pass)	0 (not solved)
2758	82	117	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2759	82	118	1 (pass)	0 (not solved)
2760	82	119	1 (pass)	0 (not solved)
2761	82	120	1 (pass)	0 (not solved)
2762	82	121	1 (pass)	0 (not solved)
2763	82	122	1 (pass)	0 (not solved)
2764	82	123	1 (pass)	0 (not solved)
2765	82	124	1 (pass)	0 (not solved)
2766	82	125	1 (pass)	0 (not solved)
2767	82	126	1 (pass)	0 (not solved)
2768	82	127	1 (pass)	0 (not solved)
2769	82	128	1 (pass)	0 (not solved)
2770	82	129	1 (pass)	0 (not solved)
2771	82	130	1 (pass)	0 (not solved)
2772	82	131	1 (pass)	0 (not solved)
2773	82	132	1 (pass)	0 (not solved)
2774	82	133	1 (pass)	0 (not solved)
2775	82	134	1 (pass)	0 (not solved)
2776	82	135	1 (pass)	0 (not solved)
2777	82	136	1 (pass)	0 (not solved)
2778	82	137	1 (pass)	0 (not solved)
2779	82	138	1 (pass)	0 (not solved)
2780	82	139	1 (pass)	0 (not solved)
2781	82	263	1 (pass)	0 (not solved)
2782	82	264	1 (pass)	0 (not solved)
2783	82	268	1 (pass)	0 (not solved)
2784	82	269	1 (pass)	0 (not solved)
2785	82	273	1 (pass)	0 (not solved)
2786	82	274	1 (pass)	0 (not solved)
2787	82	277	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2788	82	278	1 (pass)	0 (not solved)
2789	83	76	1 (pass)	0 (not solved)
2790	83	79	1 (pass)	0 (not solved)
2791	83	84	1 (pass)	0 (not solved)
2792	83	87	1 (pass)	0 (not solved)
2793	84	50	1 (pass)	0 (not solved)
2794	84	51	1 (pass)	0 (not solved)
2795	84	52	1 (pass)	0 (not solved)
2796	84	53	1 (pass)	0 (not solved)
2797	84	54	1 (pass)	0 (not solved)
2798	84	55	1 (pass)	0 (not solved)
2799	86	33	1 (pass)	0 (not solved)
2800	86	34	1 (pass)	0 (not solved)
2801	86	35	1 (pass)	0 (not solved)
2802	86	36	1 (pass)	0 (not solved)
2803	86	37	1 (pass)	0 (not solved)
2804	86	38	1 (pass)	0 (not solved)
2805	86	39	1 (pass)	0 (not solved)
2806	86	40	1 (pass)	0 (not solved)
2807	86	41	1 (pass)	0 (not solved)
2808	86	42	1 (pass)	0 (not solved)
2809	86	43	1 (pass)	0 (not solved)
2810	86	44	1 (pass)	0 (not solved)
2811	86	45	1 (pass)	0 (not solved)
2812	86	46	1 (pass)	0 (not solved)
2813	86	47	1 (pass)	0 (not solved)
2814	86	48	1 (pass)	0 (not solved)
2815	86	49	1 (pass)	0 (not solved)
2816	86	50	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2817	86	51	1 (pass)	0 (not solved)
2818	86	52	1 (pass)	0 (not solved)
2819	86	53	1 (pass)	0 (not solved)
2820	86	54	1 (pass)	0 (not solved)
2821	86	55	1 (pass)	0 (not solved)
2822	86	56	1 (pass)	0 (not solved)
2823	86	57	1 (pass)	0 (not solved)
2824	89	146	1 (pass)	0 (not solved)
2825	89	147	1 (pass)	0 (not solved)
2826	89	148	1 (pass)	0 (not solved)
2827	89	149	1 (pass)	0 (not solved)
2828	89	150	1 (pass)	0 (not solved)
2829	89	151	1 (pass)	0 (not solved)
2830	89	152	1 (pass)	0 (not solved)
2831	89	153	1 (pass)	0 (not solved)
2832	89	154	1 (pass)	0 (not solved)
2833	89	155	1 (pass)	0 (not solved)
2834	89	156	1 (pass)	0 (not solved)
2835	89	157	1 (pass)	0 (not solved)
2836	89	158	1 (pass)	0 (not solved)
2837	89	159	1 (pass)	0 (not solved)
2838	89	160	1 (pass)	0 (not solved)
2839	89	161	1 (pass)	0 (not solved)
2840	89	162	1 (pass)	0 (not solved)
2841	89	163	1 (pass)	0 (not solved)
2842	89	164	1 (pass)	0 (not solved)
2843	89	165	1 (pass)	0 (not solved)
2844	89	166	1 (pass)	0 (not solved)
2845	89	167	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2846	89	168	1 (pass)	0 (not solved)
2847	89	169	1 (pass)	0 (not solved)
2848	89	170	1 (pass)	0 (not solved)
2849	89	171	1 (pass)	0 (not solved)
2850	89	172	1 (pass)	0 (not solved)
2851	89	173	1 (pass)	0 (not solved)
2852	89	174	1 (pass)	0 (not solved)
2853	89	175	1 (pass)	0 (not solved)
2854	89	176	1 (pass)	0 (not solved)
2855	89	177	1 (pass)	0 (not solved)
2856	89	178	1 (pass)	0 (not solved)
2857	89	179	1 (pass)	0 (not solved)
2858	89	180	1 (pass)	0 (not solved)
2859	89	181	1 (pass)	0 (not solved)
2860	89	182	1 (pass)	0 (not solved)
2861	89	183	1 (pass)	0 (not solved)
2862	89	184	1 (pass)	0 (not solved)
2863	89	185	1 (pass)	0 (not solved)
2864	89	186	1 (pass)	0 (not solved)
2865	89	187	1 (pass)	0 (not solved)
2866	89	188	1 (pass)	0 (not solved)
2867	89	189	1 (pass)	0 (not solved)
2868	89	190	1 (pass)	0 (not solved)
2869	89	191	1 (pass)	0 (not solved)
2870	89	192	1 (pass)	0 (not solved)
2871	89	193	1 (pass)	0 (not solved)
2872	89	194	1 (pass)	0 (not solved)
2873	89	195	1 (pass)	0 (not solved)
2874	89	196	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2875	89	197	1 (pass)	0 (not solved)
2876	89	291	1 (pass)	0 (not solved)
2877	89	292	1 (pass)	0 (not solved)
2878	89	293	1 (pass)	0 (not solved)
2879	89	294	1 (pass)	0 (not solved)
2880	89	295	1 (pass)	0 (not solved)
2881	89	296	1 (pass)	0 (not solved)
2882	89	297	1 (pass)	0 (not solved)
2883	89	298	1 (pass)	0 (not solved)
2884	89	299	1 (pass)	0 (not solved)
2885	89	300	1 (pass)	0 (not solved)
2886	89	301	1 (pass)	0 (not solved)
2887	89	302	1 (pass)	0 (not solved)
2888	89	303	1 (pass)	0 (not solved)
2889	89	304	1 (pass)	0 (not solved)
2890	89	305	1 (pass)	0 (not solved)
2891	89	306	1 (pass)	0 (not solved)
2892	89	307	1 (pass)	0 (not solved)
2893	89	308	1 (pass)	0 (not solved)
2894	89	309	1 (pass)	0 (not solved)
2895	89	310	1 (pass)	0 (not solved)
2896	89	311	1 (pass)	0 (not solved)
2897	89	312	1 (pass)	0 (not solved)
2898	89	313	1 (pass)	0 (not solved)
2899	89	314	1 (pass)	0 (not solved)
2900	89	315	1 (pass)	0 (not solved)
2901	89	316	1 (pass)	0 (not solved)
2902	89	317	1 (pass)	0 (not solved)
2903	89	318	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2904	89	319	1 (pass)	0 (not solved)
2905	89	320	1 (pass)	0 (not solved)
2906	89	321	1 (pass)	0 (not solved)
2907	89	322	1 (pass)	0 (not solved)
2908	89	323	1 (pass)	0 (not solved)
2909	89	324	1 (pass)	0 (not solved)
2910	89	325	1 (pass)	0 (not solved)
2911	89	326	1 (pass)	0 (not solved)
2912	89	327	1 (pass)	0 (not solved)
2913	89	328	1 (pass)	0 (not solved)
2914	89	329	1 (pass)	0 (not solved)
2915	89	330	1 (pass)	0 (not solved)
2916	89	331	1 (pass)	0 (not solved)
2917	89	332	1 (pass)	0 (not solved)
2918	89	333	1 (pass)	0 (not solved)
2919	89	334	1 (pass)	0 (not solved)
2920	89	335	1 (pass)	0 (not solved)
2921	89	336	1 (pass)	0 (not solved)
2922	89	337	1 (pass)	0 (not solved)
2923	89	338	1 (pass)	0 (not solved)
2924	89	486	1 (pass)	0 (not solved)
2925	89	487	1 (pass)	0 (not solved)
2926	89	488	1 (pass)	0 (not solved)
2927	89	489	1 (pass)	0 (not solved)
2928	89	493	1 (pass)	0 (not solved)
2929	89	494	1 (pass)	0 (not solved)
2930	89	495	1 (pass)	0 (not solved)
2931	89	496	1 (pass)	0 (not solved)
2932	89	500	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2933	89	501	1 (pass)	0 (not solved)
2934	89	502	1 (pass)	0 (not solved)
2935	89	503	1 (pass)	0 (not solved)
2936	89	508	1 (pass)	0 (not solved)
2937	89	509	1 (pass)	0 (not solved)
2938	89	510	1 (pass)	0 (not solved)
2939	89	511	1 (pass)	0 (not solved)
2940	89	512	1 (pass)	0 (not solved)
2941	89	516	1 (pass)	0 (not solved)
2942	89	517	1 (pass)	0 (not solved)
2943	89	518	1 (pass)	0 (not solved)
2944	89	519	1 (pass)	0 (not solved)
2945	89	523	1 (pass)	0 (not solved)
2946	89	524	1 (pass)	0 (not solved)
2947	89	525	1 (pass)	0 (not solved)
2948	89	526	1 (pass)	0 (not solved)
2949	89	530	1 (pass)	0 (not solved)
2950	89	531	1 (pass)	0 (not solved)
2951	89	532	1 (pass)	0 (not solved)
2952	89	533	1 (pass)	0 (not solved)
2953	89	534	1 (pass)	0 (not solved)
2954	89	538	1 (pass)	0 (not solved)
2955	89	539	1 (pass)	0 (not solved)
2956	89	540	1 (pass)	0 (not solved)
2957	89	541	1 (pass)	0 (not solved)
2958	89	542	1 (pass)	0 (not solved)
2959	89	543	1 (pass)	0 (not solved)
2960	89	546	1 (pass)	0 (not solved)
2961	89	547	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2962	89	548	1 (pass)	0 (not solved)
2963	89	549	1 (pass)	0 (not solved)
2964	89	550	1 (pass)	0 (not solved)
2965	89	554	1 (pass)	0 (not solved)
2966	89	555	1 (pass)	0 (not solved)
2967	89	556	1 (pass)	0 (not solved)
2968	89	557	1 (pass)	0 (not solved)
2969	89	561	1 (pass)	0 (not solved)
2970	89	562	1 (pass)	0 (not solved)
2971	89	563	1 (pass)	0 (not solved)
2972	89	564	1 (pass)	0 (not solved)
2973	89	565	1 (pass)	0 (not solved)
2974	89	566	1 (pass)	0 (not solved)
2975	89	567	1 (pass)	0 (not solved)
2976	89	568	1 (pass)	0 (not solved)
2977	89	569	1 (pass)	0 (not solved)
2978	89	570	1 (pass)	0 (not solved)
2979	89	571	1 (pass)	0 (not solved)
2980	89	572	1 (pass)	0 (not solved)
2981	89	573	1 (pass)	0 (not solved)
2982	89	574	1 (pass)	0 (not solved)
2983	89	575	1 (pass)	0 (not solved)
2984	89	576	1 (pass)	0 (not solved)
2985	89	577	1 (pass)	0 (not solved)
2986	89	578	1 (pass)	0 (not solved)
2987	89	579	1 (pass)	0 (not solved)
2988	89	580	1 (pass)	0 (not solved)
2989	89	581	1 (pass)	0 (not solved)
2990	89	690	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
2991	89	691	1 (pass)	0 (not solved)
2992	89	692	1 (pass)	0 (not solved)
2993	89	693	1 (pass)	0 (not solved)
2994	89	694	1 (pass)	0 (not solved)
2995	89	695	1 (pass)	0 (not solved)
2996	89	696	1 (pass)	0 (not solved)
2997	89	697	1 (pass)	0 (not solved)
2998	89	698	1 (pass)	0 (not solved)
2999	89	699	1 (pass)	0 (not solved)
3000	89	700	1 (pass)	0 (not solved)
3001	89	701	1 (pass)	0 (not solved)
3002	89	702	1 (pass)	0 (not solved)
3003	89	703	1 (pass)	0 (not solved)
3004	89	704	1 (pass)	0 (not solved)
3005	89	705	1 (pass)	0 (not solved)
3006	89	706	1 (pass)	0 (not solved)
3007	89	707	1 (pass)	0 (not solved)
3008	89	708	1 (pass)	0 (not solved)
3009	89	709	1 (pass)	0 (not solved)
3010	89	710	1 (pass)	0 (not solved)
3011	89	711	1 (pass)	0 (not solved)
3012	89	794	1 (pass)	0 (not solved)
3013	89	799	1 (pass)	0 (not solved)
3014	89	800	1 (pass)	0 (not solved)
3015	89	801	1 (pass)	0 (not solved)
3016	89	802	1 (pass)	0 (not solved)
3017	89	803	1 (pass)	0 (not solved)
3018	89	804	1 (pass)	0 (not solved)
3019	89	805	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3020	89	806	1 (pass)	0 (not solved)
3021	89	807	1 (pass)	0 (not solved)
3022	89	808	1 (pass)	0 (not solved)
3023	89	809	1 (pass)	0 (not solved)
3024	89	810	1 (pass)	0 (not solved)
3025	89	811	1 (pass)	0 (not solved)
3026	89	812	1 (pass)	0 (not solved)
3027	89	813	1 (pass)	0 (not solved)
3028	89	814	1 (pass)	0 (not solved)
3029	89	815	1 (pass)	0 (not solved)
3030	89	816	1 (pass)	0 (not solved)
3031	89	817	1 (pass)	0 (not solved)
3032	89	818	1 (pass)	0 (not solved)
3033	89	819	1 (pass)	0 (not solved)
3034	89	820	1 (pass)	0 (not solved)
3035	89	821	1 (pass)	0 (not solved)
3036	89	822	1 (pass)	0 (not solved)
3037	89	823	1 (pass)	0 (not solved)
3038	89	824	1 (pass)	0 (not solved)
3039	89	825	1 (pass)	0 (not solved)
3040	89	826	1 (pass)	0 (not solved)
3041	89	827	1 (pass)	0 (not solved)
3042	89	828	1 (pass)	0 (not solved)
3043	89	829	1 (pass)	0 (not solved)
3044	89	830	1 (pass)	0 (not solved)
3045	89	831	1 (pass)	0 (not solved)
3046	89	832	1 (pass)	0 (not solved)
3047	89	833	1 (pass)	0 (not solved)
3048	89	834	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3049	89	835	1 (pass)	0 (not solved)
3050	89	836	1 (pass)	0 (not solved)
3051	89	837	1 (pass)	0 (not solved)
3052	89	838	1 (pass)	0 (not solved)
3053	89	839	1 (pass)	0 (not solved)
3054	89	840	1 (pass)	0 (not solved)
3055	89	841	1 (pass)	0 (not solved)
3056	92	123	1 (pass)	0 (not solved)
3057	92	124	1 (pass)	0 (not solved)
3058	92	125	1 (pass)	0 (not solved)
3059	92	126	1 (pass)	0 (not solved)
3060	92	127	1 (pass)	0 (not solved)
3061	92	128	1 (pass)	0 (not solved)
3062	92	129	1 (pass)	0 (not solved)
3063	92	130	1 (pass)	0 (not solved)
3064	92	131	1 (pass)	0 (not solved)
3065	92	132	1 (pass)	0 (not solved)
3066	92	133	1 (pass)	0 (not solved)
3067	92	134	1 (pass)	0 (not solved)
3068	92	135	1 (pass)	0 (not solved)
3069	92	136	1 (pass)	0 (not solved)
3070	92	137	1 (pass)	0 (not solved)
3071	92	138	1 (pass)	0 (not solved)
3072	92	139	1 (pass)	0 (not solved)
3073	92	140	1 (pass)	0 (not solved)
3074	92	141	1 (pass)	0 (not solved)
3075	92	142	1 (pass)	0 (not solved)
3076	92	143	1 (pass)	0 (not solved)
3077	92	144	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3078	92	145	1 (pass)	0 (not solved)
3079	92	146	1 (pass)	0 (not solved)
3080	92	147	1 (pass)	0 (not solved)
3081	92	148	1 (pass)	0 (not solved)
3082	92	149	1 (pass)	0 (not solved)
3083	92	150	1 (pass)	0 (not solved)
3084	92	151	1 (pass)	0 (not solved)
3085	92	152	1 (pass)	0 (not solved)
3086	92	153	1 (pass)	0 (not solved)
3087	92	154	1 (pass)	0 (not solved)
3088	92	155	1 (pass)	0 (not solved)
3089	92	156	1 (pass)	0 (not solved)
3090	92	157	1 (pass)	0 (not solved)
3091	92	158	1 (pass)	0 (not solved)
3092	92	159	1 (pass)	0 (not solved)
3093	92	160	1 (pass)	0 (not solved)
3094	92	161	1 (pass)	0 (not solved)
3095	92	162	1 (pass)	0 (not solved)
3096	92	163	1 (pass)	0 (not solved)
3097	92	164	1 (pass)	0 (not solved)
3098	92	165	1 (pass)	0 (not solved)
3099	92	279	1 (pass)	-1 (time out)
3100	92	280	1 (pass)	-1 (time out)
3101	92	296	1 (pass)	0 (not solved)
3102	92	297	1 (pass)	0 (not solved)
3103	92	298	1 (pass)	0 (not solved)
3104	92	299	1 (pass)	0 (not solved)
3105	92	304	1 (pass)	0 (not solved)
3106	92	305	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3107	92	306	1 (pass)	0 (not solved)
3108	92	311	1 (pass)	0 (not solved)
3109	92	312	1 (pass)	0 (not solved)
3110	92	313	1 (pass)	0 (not solved)
3111	92	319	1 (pass)	0 (not solved)
3112	92	320	1 (pass)	0 (not solved)
3113	92	321	1 (pass)	0 (not solved)
3114	92	322	1 (pass)	0 (not solved)
3115	92	326	1 (pass)	0 (not solved)
3116	92	327	1 (pass)	0 (not solved)
3117	92	328	1 (pass)	0 (not solved)
3118	92	329	1 (pass)	0 (not solved)
3119	92	333	1 (pass)	0 (not solved)
3120	92	334	1 (pass)	0 (not solved)
3121	92	335	1 (pass)	0 (not solved)
3122	92	336	1 (pass)	0 (not solved)
3123	92	337	1 (pass)	0 (not solved)
3124	92	341	1 (pass)	0 (not solved)
3125	92	343	1 (pass)	0 (not solved)
3126	92	345	1 (pass)	0 (not solved)
3127	92	346	1 (pass)	0 (not solved)
3128	92	347	1 (pass)	0 (not solved)
3129	92	348	1 (pass)	0 (not solved)
3130	92	349	1 (pass)	0 (not solved)
3131	92	350	1 (pass)	0 (not solved)
3132	92	351	1 (pass)	0 (not solved)
3133	92	352	1 (pass)	0 (not solved)
3134	92	353	1 (pass)	0 (not solved)
3135	92	354	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3136	92	355	1 (pass)	0 (not solved)
3137	92	356	1 (pass)	0 (not solved)
3138	92	357	1 (pass)	0 (not solved)
3139	92	358	1 (pass)	0 (not solved)
3140	92	359	1 (pass)	0 (not solved)
3141	92	360	1 (pass)	0 (not solved)
3142	92	361	1 (pass)	0 (not solved)
3143	92	362	1 (pass)	0 (not solved)
3144	92	363	1 (pass)	0 (not solved)
3145	92	364	1 (pass)	0 (not solved)
3146	92	383	1 (pass)	0 (not solved)
3147	92	384	1 (pass)	0 (not solved)
3148	92	385	1 (pass)	0 (not solved)
3149	92	386	1 (pass)	0 (not solved)
3150	92	387	1 (pass)	0 (not solved)
3151	92	388	1 (pass)	0 (not solved)
3152	92	459	1 (pass)	0 (not solved)
3153	92	460	1 (pass)	0 (not solved)
3154	92	461	1 (pass)	0 (not solved)
3155	92	462	1 (pass)	0 (not solved)
3156	92	463	1 (pass)	0 (not solved)
3157	92	464	1 (pass)	0 (not solved)
3158	92	465	1 (pass)	0 (not solved)
3159	92	466	1 (pass)	0 (not solved)
3160	92	467	1 (pass)	0 (not solved)
3161	92	468	1 (pass)	0 (not solved)
3162	92	469	1 (pass)	0 (not solved)
3163	92	470	1 (pass)	0 (not solved)
3164	92	471	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3165	92	472	1 (pass)	0 (not solved)
3166	92	473	1 (pass)	0 (not solved)
3167	92	474	1 (pass)	0 (not solved)
3168	92	475	1 (pass)	0 (not solved)
3169	92	476	1 (pass)	0 (not solved)
3170	92	477	1 (pass)	0 (not solved)
3171	92	478	1 (pass)	0 (not solved)
3172	92	479	1 (pass)	0 (not solved)
3173	92	480	1 (pass)	0 (not solved)
3174	92	481	1 (pass)	0 (not solved)
3175	92	482	1 (pass)	0 (not solved)
3176	92	483	1 (pass)	0 (not solved)
3177	92	484	1 (pass)	0 (not solved)
3178	92	485	1 (pass)	0 (not solved)
3179	92	486	1 (pass)	0 (not solved)
3180	92	487	1 (pass)	0 (not solved)
3181	92	488	1 (pass)	0 (not solved)
3182	92	489	1 (pass)	0 (not solved)
3183	92	490	1 (pass)	0 (not solved)
3184	92	491	1 (pass)	0 (not solved)
3185	92	492	1 (pass)	0 (not solved)
3186	92	550	1 (pass)	0 (not solved)
3187	92	551	1 (pass)	0 (not solved)
3188	92	552	1 (pass)	0 (not solved)
3189	92	553	1 (pass)	0 (not solved)
3190	92	554	1 (pass)	0 (not solved)
3191	92	555	1 (pass)	0 (not solved)
3192	92	556	1 (pass)	0 (not solved)
3193	92	557	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3194	92	558	1 (pass)	0 (not solved)
3195	92	559	1 (pass)	0 (not solved)
3196	92	560	1 (pass)	0 (not solved)
3197	92	561	1 (pass)	0 (not solved)
3198	92	562	1 (pass)	0 (not solved)
3199	92	563	1 (pass)	0 (not solved)
3200	92	564	1 (pass)	0 (not solved)
3201	92	565	1 (pass)	0 (not solved)
3202	92	566	1 (pass)	0 (not solved)
3203	92	584	1 (pass)	0 (not solved)
3204	92	585	1 (pass)	0 (not solved)
3205	92	586	1 (pass)	0 (not solved)
3206	92	587	1 (pass)	0 (not solved)
3207	92	588	1 (pass)	0 (not solved)
3208	92	589	1 (pass)	0 (not solved)
3209	93	16	1 (pass)	0 (not solved)
3210	93	17	1 (pass)	0 (not solved)
3211	93	18	1 (pass)	0 (not solved)
3212	93	19	1 (pass)	0 (not solved)
3213	93	20	1 (pass)	0 (not solved)
3214	93	21	1 (pass)	0 (not solved)
3215	93	22	1 (pass)	0 (not solved)
3216	93	23	1 (pass)	0 (not solved)
3217	93	26	1 (pass)	0 (not solved)
3218	93	27	1 (pass)	0 (not solved)
3219	93	28	1 (pass)	0 (not solved)
3220	93	29	1 (pass)	0 (not solved)
3221	93	30	1 (pass)	0 (not solved)
3222	93	31	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3223	93	32	1 (pass)	0 (not solved)
3224	93	33	1 (pass)	0 (not solved)
3225	93	37	1 (pass)	0 (not solved)
3226	93	38	1 (pass)	0 (not solved)
3227	93	39	1 (pass)	0 (not solved)
3228	93	40	1 (pass)	0 (not solved)
3229	93	41	1 (pass)	0 (not solved)
3230	93	42	1 (pass)	0 (not solved)
3231	93	43	1 (pass)	0 (not solved)
3232	93	44	1 (pass)	0 (not solved)
3233	93	45	1 (pass)	0 (not solved)
3234	93	46	1 (pass)	0 (not solved)
3235	93	47	1 (pass)	0 (not solved)
3236	93	48	1 (pass)	0 (not solved)
3237	93	49	1 (pass)	0 (not solved)
3238	93	50	1 (pass)	0 (not solved)
3239	93	51	1 (pass)	0 (not solved)
3240	93	52	1 (pass)	0 (not solved)
3241	93	53	1 (pass)	0 (not solved)
3242	93	54	1 (pass)	0 (not solved)
3243	93	55	1 (pass)	0 (not solved)
3244	93	56	1 (pass)	0 (not solved)
3245	93	57	1 (pass)	0 (not solved)
3246	93	58	1 (pass)	0 (not solved)
3247	93	59	1 (pass)	0 (not solved)
3248	93	60	1 (pass)	0 (not solved)
3249	93	61	1 (pass)	0 (not solved)
3250	93	62	1 (pass)	0 (not solved)
3251	93	63	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3252	93	64	1 (pass)	0 (not solved)
3253	93	65	1 (pass)	0 (not solved)
3254	93	66	1 (pass)	0 (not solved)
3255	93	67	1 (pass)	0 (not solved)
3256	93	68	1 (pass)	0 (not solved)
3257	93	69	1 (pass)	0 (not solved)
3258	93	70	1 (pass)	0 (not solved)
3259	93	71	1 (pass)	0 (not solved)
3260	93	72	1 (pass)	0 (not solved)
3261	93	73	1 (pass)	0 (not solved)
3262	93	74	1 (pass)	0 (not solved)
3263	93	75	1 (pass)	0 (not solved)
3264	93	76	1 (pass)	0 (not solved)
3265	93	77	1 (pass)	0 (not solved)
3266	93	78	1 (pass)	0 (not solved)
3267	93	79	1 (pass)	0 (not solved)
3268	93	80	1 (pass)	0 (not solved)
3269	93	81	1 (pass)	0 (not solved)
3270	93	82	1 (pass)	0 (not solved)
3271	93	83	1 (pass)	0 (not solved)
3272	93	84	1 (pass)	0 (not solved)
3273	93	85	1 (pass)	0 (not solved)
3274	93	86	1 (pass)	0 (not solved)
3275	93	87	1 (pass)	0 (not solved)
3276	93	88	1 (pass)	0 (not solved)
3277	93	239	1 (pass)	0 (not solved)
3278	93	240	1 (pass)	0 (not solved)
3279	93	241	1 (pass)	0 (not solved)
3280	93	242	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3281	93	243	1 (pass)	0 (not solved)
3282	93	244	1 (pass)	0 (not solved)
3283	93	245	1 (pass)	0 (not solved)
3284	93	246	1 (pass)	0 (not solved)
3285	93	247	1 (pass)	0 (not solved)
3286	93	248	1 (pass)	0 (not solved)
3287	93	249	1 (pass)	0 (not solved)
3288	93	250	1 (pass)	0 (not solved)
3289	93	251	1 (pass)	0 (not solved)
3290	93	252	1 (pass)	0 (not solved)
3291	93	253	1 (pass)	0 (not solved)
3292	93	254	1 (pass)	0 (not solved)
3293	93	255	1 (pass)	0 (not solved)
3294	93	256	1 (pass)	0 (not solved)
3295	93	257	1 (pass)	0 (not solved)
3296	93	258	1 (pass)	0 (not solved)
3297	93	259	1 (pass)	0 (not solved)
3298	93	260	1 (pass)	0 (not solved)
3299	93	261	1 (pass)	0 (not solved)
3300	93	262	1 (pass)	0 (not solved)
3301	93	263	1 (pass)	0 (not solved)
3302	93	264	1 (pass)	0 (not solved)
3303	93	265	1 (pass)	0 (not solved)
3304	93	266	1 (pass)	0 (not solved)
3305	93	267	1 (pass)	0 (not solved)
3306	93	268	1 (pass)	0 (not solved)
3307	93	269	1 (pass)	0 (not solved)
3308	93	270	1 (pass)	0 (not solved)
3309	93	271	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3310	93	272	1 (pass)	0 (not solved)
3311	93	273	1 (pass)	0 (not solved)
3312	93	274	1 (pass)	0 (not solved)
3313	93	275	1 (pass)	0 (not solved)
3314	93	276	1 (pass)	0 (not solved)
3315	93	277	1 (pass)	0 (not solved)
3316	93	278	1 (pass)	0 (not solved)
3317	93	279	1 (pass)	0 (not solved)
3318	93	280	1 (pass)	0 (not solved)
3319	93	281	1 (pass)	0 (not solved)
3320	93	282	1 (pass)	0 (not solved)
3321	93	283	1 (pass)	0 (not solved)
3322	93	284	1 (pass)	0 (not solved)
3323	93	285	1 (pass)	0 (not solved)
3324	93	286	1 (pass)	0 (not solved)
3325	93	287	1 (pass)	0 (not solved)
3326	94	126	1 (pass)	0 (not solved)
3327	94	127	1 (pass)	0 (not solved)
3328	94	128	1 (pass)	0 (not solved)
3329	94	129	1 (pass)	0 (not solved)
3330	94	130	1 (pass)	0 (not solved)
3331	94	131	1 (pass)	0 (not solved)
3332	94	132	1 (pass)	0 (not solved)
3333	94	133	1 (pass)	0 (not solved)
3334	94	134	1 (pass)	0 (not solved)
3335	94	135	1 (pass)	0 (not solved)
3336	94	136	1 (pass)	0 (not solved)
3337	94	137	1 (pass)	0 (not solved)
3338	94	138	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3339	94	139	1 (pass)	0 (not solved)
3340	94	140	1 (pass)	0 (not solved)
3341	94	141	1 (pass)	0 (not solved)
3342	94	142	1 (pass)	0 (not solved)
3343	94	143	1 (pass)	0 (not solved)
3344	94	144	1 (pass)	0 (not solved)
3345	94	145	1 (pass)	0 (not solved)
3346	94	146	1 (pass)	0 (not solved)
3347	94	147	1 (pass)	0 (not solved)
3348	94	148	1 (pass)	0 (not solved)
3349	94	149	1 (pass)	0 (not solved)
3350	94	150	1 (pass)	0 (not solved)
3351	94	151	1 (pass)	0 (not solved)
3352	94	152	1 (pass)	0 (not solved)
3353	94	153	1 (pass)	0 (not solved)
3354	94	154	1 (pass)	0 (not solved)
3355	94	155	1 (pass)	0 (not solved)
3356	94	156	1 (pass)	0 (not solved)
3357	94	157	1 (pass)	0 (not solved)
3358	94	158	1 (pass)	0 (not solved)
3359	94	159	1 (pass)	0 (not solved)
3360	94	160	1 (pass)	0 (not solved)
3361	94	161	1 (pass)	0 (not solved)
3362	94	162	1 (pass)	0 (not solved)
3363	94	163	1 (pass)	0 (not solved)
3364	94	164	1 (pass)	0 (not solved)
3365	94	165	1 (pass)	0 (not solved)
3366	94	166	1 (pass)	0 (not solved)
3367	94	167	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3368	94	168	1 (pass)	0 (not solved)
3369	94	169	1 (pass)	0 (not solved)
3370	94	170	1 (pass)	0 (not solved)
3371	94	283	1 (pass)	0 (not solved)
3372	94	284	1 (pass)	0 (not solved)
3373	94	285	1 (pass)	0 (not solved)
3374	94	286	1 (pass)	0 (not solved)
3375	94	287	1 (pass)	0 (not solved)
3376	94	288	1 (pass)	0 (not solved)
3377	94	289	1 (pass)	0 (not solved)
3378	94	425	1 (pass)	0 (not solved)
3379	94	426	1 (pass)	0 (not solved)
3380	94	427	1 (pass)	0 (not solved)
3381	94	428	1 (pass)	0 (not solved)
3382	94	429	1 (pass)	0 (not solved)
3383	94	430	1 (pass)	0 (not solved)
3384	94	431	1 (pass)	0 (not solved)
3385	94	432	1 (pass)	0 (not solved)
3386	94	433	1 (pass)	0 (not solved)
3387	94	434	1 (pass)	0 (not solved)
3388	94	435	1 (pass)	0 (not solved)
3389	94	436	1 (pass)	0 (not solved)
3390	94	437	1 (pass)	0 (not solved)
3391	94	438	1 (pass)	0 (not solved)
3392	94	439	1 (pass)	0 (not solved)
3393	94	440	1 (pass)	0 (not solved)
3394	94	441	1 (pass)	0 (not solved)
3395	94	442	1 (pass)	0 (not solved)
3396	94	443	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3397	94	444	1 (pass)	0 (not solved)
3398	94	445	1 (pass)	0 (not solved)
3399	94	446	1 (pass)	0 (not solved)
3400	94	447	1 (pass)	0 (not solved)
3401	94	448	1 (pass)	0 (not solved)
3402	94	449	1 (pass)	0 (not solved)
3403	94	450	1 (pass)	0 (not solved)
3404	94	451	1 (pass)	0 (not solved)
3405	94	452	1 (pass)	0 (not solved)
3406	94	453	1 (pass)	0 (not solved)
3407	94	454	1 (pass)	0 (not solved)
3408	94	455	1 (pass)	0 (not solved)
3409	94	456	1 (pass)	0 (not solved)
3410	94	457	1 (pass)	0 (not solved)
3411	94	458	1 (pass)	0 (not solved)
3412	94	459	1 (pass)	0 (not solved)
3413	94	460	1 (pass)	0 (not solved)
3414	94	461	1 (pass)	0 (not solved)
3415	94	462	1 (pass)	0 (not solved)
3416	94	463	1 (pass)	0 (not solved)
3417	94	464	1 (pass)	0 (not solved)
3418	94	465	1 (pass)	0 (not solved)
3419	94	466	1 (pass)	0 (not solved)
3420	94	467	1 (pass)	0 (not solved)
3421	94	468	1 (pass)	0 (not solved)
3422	94	469	1 (pass)	0 (not solved)
3423	94	470	1 (pass)	0 (not solved)
3424	94	471	1 (pass)	0 (not solved)
3425	94	472	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3426	94	473	1 (pass)	0 (not solved)
3427	94	474	1 (pass)	0 (not solved)
3428	94	475	1 (pass)	0 (not solved)
3429	94	623	1 (pass)	0 (not solved)
3430	94	624	1 (pass)	0 (not solved)
3431	94	625	1 (pass)	0 (not solved)
3432	94	630	1 (pass)	0 (not solved)
3433	94	631	1 (pass)	0 (not solved)
3434	94	632	1 (pass)	0 (not solved)
3435	94	638	1 (pass)	0 (not solved)
3436	94	639	1 (pass)	0 (not solved)
3437	94	640	1 (pass)	0 (not solved)
3438	94	646	1 (pass)	0 (not solved)
3439	94	647	1 (pass)	0 (not solved)
3440	94	648	1 (pass)	0 (not solved)
3441	94	649	1 (pass)	0 (not solved)
3442	94	650	1 (pass)	0 (not solved)
3443	94	651	1 (pass)	0 (not solved)
3444	94	656	1 (pass)	0 (not solved)
3445	94	657	1 (pass)	0 (not solved)
3446	94	658	1 (pass)	0 (not solved)
3447	94	659	1 (pass)	0 (not solved)
3448	94	663	1 (pass)	0 (not solved)
3449	94	664	1 (pass)	0 (not solved)
3450	94	665	1 (pass)	0 (not solved)
3451	94	666	1 (pass)	0 (not solved)
3452	94	669	1 (pass)	0 (not solved)
3453	94	670	1 (pass)	0 (not solved)
3454	94	671	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3455	94	672	1 (pass)	0 (not solved)
3456	94	673	1 (pass)	0 (not solved)
3457	94	674	1 (pass)	0 (not solved)
3458	94	675	1 (pass)	0 (not solved)
3459	94	676	1 (pass)	0 (not solved)
3460	94	677	1 (pass)	0 (not solved)
3461	94	678	1 (pass)	0 (not solved)
3462	94	679	1 (pass)	0 (not solved)
3463	94	680	1 (pass)	0 (not solved)
3464	94	681	1 (pass)	0 (not solved)
3465	94	682	1 (pass)	0 (not solved)
3466	94	683	1 (pass)	0 (not solved)
3467	94	684	1 (pass)	0 (not solved)
3468	94	685	1 (pass)	0 (not solved)
3469	94	686	1 (pass)	0 (not solved)
3470	94	687	1 (pass)	0 (not solved)
3471	94	688	1 (pass)	0 (not solved)
3472	94	689	1 (pass)	0 (not solved)
3473	94	690	1 (pass)	0 (not solved)
3474	94	691	1 (pass)	0 (not solved)
3475	94	692	1 (pass)	0 (not solved)
3476	94	693	1 (pass)	0 (not solved)
3477	94	694	1 (pass)	0 (not solved)
3478	94	695	1 (pass)	0 (not solved)
3479	94	696	1 (pass)	0 (not solved)
3480	94	697	1 (pass)	0 (not solved)
3481	94	698	1 (pass)	0 (not solved)
3482	94	699	1 (pass)	0 (not solved)
3483	94	700	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3484	94	701	1 (pass)	0 (not solved)
3485	94	702	1 (pass)	0 (not solved)
3486	94	703	1 (pass)	0 (not solved)
3487	94	814	1 (pass)	0 (not solved)
3488	94	815	1 (pass)	0 (not solved)
3489	94	816	1 (pass)	0 (not solved)
3490	94	820	1 (pass)	0 (not solved)
3491	94	821	1 (pass)	0 (not solved)
3492	94	822	1 (pass)	0 (not solved)
3493	94	827	1 (pass)	0 (not solved)
3494	94	828	1 (pass)	0 (not solved)
3495	94	829	1 (pass)	0 (not solved)
3496	94	835	1 (pass)	0 (not solved)
3497	94	836	1 (pass)	0 (not solved)
3498	94	837	1 (pass)	0 (not solved)
3499	94	841	1 (pass)	0 (not solved)
3500	94	842	1 (pass)	0 (not solved)
3501	94	843	1 (pass)	0 (not solved)
3502	94	844	1 (pass)	0 (not solved)
3503	94	847	1 (pass)	0 (not solved)
3504	94	848	1 (pass)	0 (not solved)
3505	94	849	1 (pass)	0 (not solved)
3506	94	850	1 (pass)	0 (not solved)
3507	94	853	1 (pass)	0 (not solved)
3508	94	854	1 (pass)	0 (not solved)
3509	94	855	1 (pass)	0 (not solved)
3510	94	856	1 (pass)	0 (not solved)
3511	94	857	1 (pass)	0 (not solved)
3512	94	858	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3513	94	859	1 (pass)	0 (not solved)
3514	94	860	1 (pass)	0 (not solved)
3515	94	861	1 (pass)	0 (not solved)
3516	94	862	1 (pass)	0 (not solved)
3517	94	863	1 (pass)	0 (not solved)
3518	94	864	1 (pass)	0 (not solved)
3519	94	865	1 (pass)	0 (not solved)
3520	94	866	1 (pass)	0 (not solved)
3521	94	867	1 (pass)	0 (not solved)
3522	94	868	1 (pass)	0 (not solved)
3523	94	869	1 (pass)	0 (not solved)
3524	94	870	1 (pass)	0 (not solved)
3525	94	871	1 (pass)	0 (not solved)
3526	94	872	1 (pass)	0 (not solved)
3527	94	873	1 (pass)	0 (not solved)
3528	94	874	1 (pass)	0 (not solved)
3529	94	875	1 (pass)	0 (not solved)
3530	94	1000	1 (pass)	-1 (time out)
3531	94	1007	1 (pass)	-1 (time out)
3532	94	1014	1 (pass)	0 (not solved)
3533	94	1015	1 (pass)	0 (not solved)
3534	94	1016	1 (pass)	0 (not solved)
3535	94	1021	1 (pass)	0 (not solved)
3536	94	1022	1 (pass)	0 (not solved)
3537	94	1023	1 (pass)	0 (not solved)
3538	94	1029	1 (pass)	0 (not solved)
3539	94	1030	1 (pass)	0 (not solved)
3540	94	1031	1 (pass)	0 (not solved)
3541	94	1038	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3542	94	1039	1 (pass)	0 (not solved)
3543	94	1040	1 (pass)	0 (not solved)
3544	94	1041	1 (pass)	0 (not solved)
3545	94	1042	1 (pass)	0 (not solved)
3546	94	1047	1 (pass)	0 (not solved)
3547	94	1048	1 (pass)	0 (not solved)
3548	94	1049	1 (pass)	0 (not solved)
3549	94	1053	1 (pass)	0 (not solved)
3550	94	1054	1 (pass)	0 (not solved)
3551	94	1055	1 (pass)	0 (not solved)
3552	94	1056	1 (pass)	0 (not solved)
3553	94	1060	1 (pass)	0 (not solved)
3554	94	1061	1 (pass)	0 (not solved)
3555	94	1062	1 (pass)	0 (not solved)
3556	94	1063	1 (pass)	0 (not solved)
3557	94	1064	1 (pass)	0 (not solved)
3558	94	1065	1 (pass)	0 (not solved)
3559	94	1066	1 (pass)	0 (not solved)
3560	94	1067	1 (pass)	0 (not solved)
3561	94	1068	1 (pass)	0 (not solved)
3562	94	1069	1 (pass)	0 (not solved)
3563	94	1070	1 (pass)	0 (not solved)
3564	94	1071	1 (pass)	0 (not solved)
3565	94	1072	1 (pass)	0 (not solved)
3566	94	1073	1 (pass)	0 (not solved)
3567	94	1074	1 (pass)	0 (not solved)
3568	94	1075	1 (pass)	0 (not solved)
3569	94	1076	1 (pass)	0 (not solved)
3570	94	1077	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3571	94	1078	1 (pass)	0 (not solved)
3572	94	1079	1 (pass)	0 (not solved)
3573	94	1080	1 (pass)	0 (not solved)
3574	94	1081	1 (pass)	0 (not solved)
3575	94	1082	1 (pass)	0 (not solved)
3576	94	1083	1 (pass)	0 (not solved)
3577	94	1084	1 (pass)	0 (not solved)
3578	94	1085	1 (pass)	0 (not solved)
3579	94	1086	1 (pass)	0 (not solved)
3580	94	1087	1 (pass)	0 (not solved)
3581	94	1088	1 (pass)	0 (not solved)
3582	94	1089	1 (pass)	0 (not solved)
3583	94	1090	1 (pass)	0 (not solved)
3584	94	1091	1 (pass)	0 (not solved)
3585	94	1092	1 (pass)	0 (not solved)
3586	94	1093	1 (pass)	0 (not solved)
3587	94	1094	1 (pass)	0 (not solved)
3588	94	1160	1 (pass)	0 (not solved)
3589	94	1161	1 (pass)	0 (not solved)
3590	94	1162	1 (pass)	0 (not solved)
3591	94	1163	1 (pass)	0 (not solved)
3592	94	1164	1 (pass)	0 (not solved)
3593	94	1165	1 (pass)	0 (not solved)
3594	94	1166	1 (pass)	0 (not solved)
3595	94	1167	1 (pass)	0 (not solved)
3596	94	1168	1 (pass)	0 (not solved)
3597	94	1169	1 (pass)	0 (not solved)
3598	94	1170	1 (pass)	0 (not solved)
3599	94	1171	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3600	94	1172	1 (pass)	0 (not solved)
3601	94	1173	1 (pass)	0 (not solved)
3602	94	1174	1 (pass)	0 (not solved)
3603	94	1175	1 (pass)	0 (not solved)
3604	94	1176	1 (pass)	0 (not solved)
3605	94	1177	1 (pass)	0 (not solved)
3606	94	1178	1 (pass)	0 (not solved)
3607	94	1179	1 (pass)	0 (not solved)
3608	94	1180	1 (pass)	0 (not solved)
3609	94	1181	1 (pass)	0 (not solved)
3610	94	1182	1 (pass)	0 (not solved)
3611	94	1183	1 (pass)	0 (not solved)
3612	94	1184	1 (pass)	0 (not solved)
3613	94	1185	1 (pass)	0 (not solved)
3614	94	1186	1 (pass)	0 (not solved)
3615	94	1187	1 (pass)	0 (not solved)
3616	94	1188	1 (pass)	0 (not solved)
3617	94	1189	1 (pass)	0 (not solved)
3618	94	1190	1 (pass)	0 (not solved)
3619	94	1191	1 (pass)	0 (not solved)
3620	94	1192	1 (pass)	0 (not solved)
3621	94	1193	1 (pass)	0 (not solved)
3622	94	1194	1 (pass)	0 (not solved)
3623	94	1195	1 (pass)	0 (not solved)
3624	94	1196	1 (pass)	0 (not solved)
3625	94	1197	1 (pass)	0 (not solved)
3626	94	1198	1 (pass)	0 (not solved)
3627	94	1199	1 (pass)	0 (not solved)
3628	94	1200	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3629	94	1201	1 (pass)	0 (not solved)
3630	94	1202	1 (pass)	0 (not solved)
3631	94	1203	1 (pass)	0 (not solved)
3632	94	1253	1 (pass)	0 (not solved)
3633	94	1254	1 (pass)	0 (not solved)
3634	94	1255	1 (pass)	0 (not solved)
3635	94	1256	1 (pass)	0 (not solved)
3636	94	1257	1 (pass)	0 (not solved)
3637	94	1258	1 (pass)	0 (not solved)
3638	94	1259	1 (pass)	0 (not solved)
3639	94	1260	1 (pass)	0 (not solved)
3640	94	1261	1 (pass)	0 (not solved)
3641	94	1262	1 (pass)	0 (not solved)
3642	94	1263	1 (pass)	0 (not solved)
3643	94	1264	1 (pass)	0 (not solved)
3644	94	1265	1 (pass)	0 (not solved)
3645	94	1266	1 (pass)	0 (not solved)
3646	94	1267	1 (pass)	0 (not solved)
3647	94	1268	1 (pass)	0 (not solved)
3648	94	1269	1 (pass)	0 (not solved)
3649	94	1270	1 (pass)	0 (not solved)
3650	94	1271	1 (pass)	0 (not solved)
3651	94	1272	1 (pass)	0 (not solved)
3652	94	1273	1 (pass)	0 (not solved)
3653	94	1274	1 (pass)	0 (not solved)
3654	94	1275	1 (pass)	0 (not solved)
3655	94	1276	1 (pass)	0 (not solved)
3656	94	1277	1 (pass)	0 (not solved)
3657	94	1278	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3658	94	1279	1 (pass)	0 (not solved)
3659	94	1280	1 (pass)	0 (not solved)
3660	94	1281	1 (pass)	0 (not solved)
3661	94	1282	1 (pass)	0 (not solved)
3662	94	1283	1 (pass)	0 (not solved)
3663	94	1284	1 (pass)	0 (not solved)
3664	94	1285	1 (pass)	0 (not solved)
3665	94	1286	1 (pass)	0 (not solved)
3666	94	1287	1 (pass)	0 (not solved)
3667	94	1288	1 (pass)	0 (not solved)
3668	94	1289	1 (pass)	0 (not solved)
3669	94	1290	1 (pass)	0 (not solved)
3670	94	1291	1 (pass)	0 (not solved)
3671	94	1292	1 (pass)	0 (not solved)
3672	94	1293	1 (pass)	0 (not solved)
3673	94	1294	1 (pass)	0 (not solved)
3674	94	1295	1 (pass)	0 (not solved)
3675	94	1296	1 (pass)	0 (not solved)
3676	94	1297	1 (pass)	0 (not solved)
3677	94	1298	1 (pass)	0 (not solved)
3678	94	1299	1 (pass)	0 (not solved)
3679	94	1300	1 (pass)	0 (not solved)
3680	94	1301	1 (pass)	0 (not solved)
3681	94	1302	1 (pass)	0 (not solved)
3682	94	1303	1 (pass)	0 (not solved)
3683	94	1304	1 (pass)	0 (not solved)
3684	94	1305	1 (pass)	0 (not solved)
3685	94	1306	1 (pass)	0 (not solved)
3686	94	1307	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3687	94	1308	1 (pass)	0 (not solved)
3688	94	1309	1 (pass)	0 (not solved)
3689	94	1359	1 (pass)	-1 (time out)
3690	94	1360	1 (pass)	0 (not solved)
3691	94	1361	1 (pass)	0 (not solved)
3692	94	1362	1 (pass)	0 (not solved)
3693	94	1363	1 (pass)	0 (not solved)
3694	94	1364	1 (pass)	0 (not solved)
3695	94	1365	1 (pass)	0 (not solved)
3696	94	1366	1 (pass)	0 (not solved)
3697	94	1367	1 (pass)	0 (not solved)
3698	94	1368	1 (pass)	0 (not solved)
3699	94	1369	1 (pass)	0 (not solved)
3700	94	1370	1 (pass)	0 (not solved)
3701	94	1371	1 (pass)	0 (not solved)
3702	94	1372	1 (pass)	0 (not solved)
3703	94	1373	1 (pass)	0 (not solved)
3704	94	1374	1 (pass)	0 (not solved)
3705	94	1375	1 (pass)	0 (not solved)
3706	94	1376	1 (pass)	0 (not solved)
3707	94	1377	1 (pass)	0 (not solved)
3708	94	1378	1 (pass)	0 (not solved)
3709	94	1379	1 (pass)	0 (not solved)
3710	94	1380	1 (pass)	0 (not solved)
3711	94	1381	1 (pass)	0 (not solved)
3712	94	1382	1 (pass)	0 (not solved)
3713	94	1383	1 (pass)	0 (not solved)
3714	94	1384	1 (pass)	0 (not solved)
3715	94	1385	1 (pass)	0 (not solved)

Continued on next page

Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3716	94	1386	1 (pass)	0 (not solved)
3717	94	1387	1 (pass)	0 (not solved)
3718	94	1388	1 (pass)	0 (not solved)
3719	94	1389	1 (pass)	0 (not solved)
3720	94	1390	1 (pass)	0 (not solved)
3721	94	1451	1 (pass)	0 (not solved)
3722	94	1452	1 (pass)	0 (not solved)
3723	94	1453	1 (pass)	0 (not solved)
3724	94	1454	1 (pass)	0 (not solved)
3725	94	1455	1 (pass)	0 (not solved)
3726	94	1456	1 (pass)	0 (not solved)
3727	94	1457	1 (pass)	0 (not solved)
3728	94	1458	1 (pass)	0 (not solved)
3729	94	1459	1 (pass)	0 (not solved)
3730	94	1460	1 (pass)	0 (not solved)
3731	94	1461	1 (pass)	0 (not solved)
3732	94	1462	1 (pass)	0 (not solved)
3733	94	1463	1 (pass)	0 (not solved)
3734	94	1464	1 (pass)	0 (not solved)
3735	94	1465	1 (pass)	0 (not solved)
3736	94	1466	1 (pass)	0 (not solved)
3737	94	1467	1 (pass)	0 (not solved)
3738	94	1468	1 (pass)	0 (not solved)
3739	94	1469	1 (pass)	0 (not solved)
3740	94	1470	1 (pass)	0 (not solved)
3741	94	1471	1 (pass)	0 (not solved)
3742	94	1472	1 (pass)	0 (not solved)
3743	94	1473	1 (pass)	0 (not solved)
3744	94	1474	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3745	94	1475	1 (pass)	0 (not solved)
3746	94	1476	1 (pass)	0 (not solved)
3747	94	1477	1 (pass)	0 (not solved)
3748	94	1478	1 (pass)	0 (not solved)
3749	94	1479	1 (pass)	0 (not solved)
3750	94	1480	1 (pass)	0 (not solved)
3751	94	1481	1 (pass)	0 (not solved)
3752	95	61	1 (pass)	0 (not solved)
3753	95	62	1 (pass)	0 (not solved)
3754	95	63	1 (pass)	0 (not solved)
3755	95	75	1 (pass)	-1 (time out)
3756	95	76	1 (pass)	-1 (time out)
3757	95	78	1 (pass)	-1 (time out)
3758	95	79	1 (pass)	-1 (time out)
3759	96	9	1 (pass)	-1 (time out)
3760	96	14	1 (pass)	-1 (time out)
3761	97	8	1 (pass)	-1 (time out)
3762	97	18	1 (pass)	-1 (time out)
3763	98	61	1 (pass)	0 (not solved)
3764	98	62	1 (pass)	0 (not solved)
3765	98	63	1 (pass)	0 (not solved)
3766	98	80	1 (pass)	0 (not solved)
3767	98	81	1 (pass)	0 (not solved)
3768	98	82	1 (pass)	0 (not solved)
3769	98	83	1 (pass)	0 (not solved)
3770	98	101	1 (pass)	0 (not solved)
3771	98	102	1 (pass)	0 (not solved)
3772	98	115	1 (pass)	0 (not solved)
3773	98	117	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3774	98	119	1 (pass)	0 (not solved)
3775	98	120	1 (pass)	0 (not solved)
3776	98	122	1 (pass)	0 (not solved)
3777	98	124	1 (pass)	0 (not solved)
3778	98	126	1 (pass)	0 (not solved)
3779	98	128	1 (pass)	0 (not solved)
3780	98	130	1 (pass)	0 (not solved)
3781	98	132	1 (pass)	0 (not solved)
3782	98	139	1 (pass)	0 (not solved)
3783	98	140	1 (pass)	0 (not solved)
3784	98	141	1 (pass)	0 (not solved)
3785	98	142	1 (pass)	0 (not solved)
3786	98	143	1 (pass)	0 (not solved)
3787	98	144	1 (pass)	0 (not solved)
3788	98	241	1 (pass)	0 (not solved)
3789	98	242	1 (pass)	0 (not solved)
3790	98	243	1 (pass)	0 (not solved)
3791	98	253	1 (pass)	0 (not solved)
3792	98	254	1 (pass)	0 (not solved)
3793	98	255	1 (pass)	0 (not solved)
3794	98	269	1 (pass)	0 (not solved)
3795	98	292	1 (pass)	0 (not solved)
3796	98	294	1 (pass)	0 (not solved)
3797	98	296	1 (pass)	0 (not solved)
3798	98	298	1 (pass)	0 (not solved)
3799	98	299	1 (pass)	0 (not solved)
3800	98	301	1 (pass)	0 (not solved)
3801	98	303	1 (pass)	0 (not solved)
3802	98	305	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3803	98	308	1 (pass)	0 (not solved)
3804	98	310	1 (pass)	0 (not solved)
3805	98	312	1 (pass)	0 (not solved)
3806	98	314	1 (pass)	0 (not solved)
3807	98	316	1 (pass)	0 (not solved)
3808	98	318	1 (pass)	0 (not solved)
3809	98	320	1 (pass)	0 (not solved)
3810	98	323	1 (pass)	0 (not solved)
3811	98	325	1 (pass)	0 (not solved)
3812	98	327	1 (pass)	0 (not solved)
3813	98	329	1 (pass)	0 (not solved)
3814	98	331	1 (pass)	0 (not solved)
3815	98	333	1 (pass)	0 (not solved)
3816	101	185	1 (pass)	0 (not solved)
3817	101	186	1 (pass)	0 (not solved)
3818	101	187	1 (pass)	0 (not solved)
3819	101	188	1 (pass)	0 (not solved)
3820	101	189	1 (pass)	0 (not solved)
3821	101	190	1 (pass)	0 (not solved)
3822	101	191	1 (pass)	0 (not solved)
3823	101	192	1 (pass)	0 (not solved)
3824	101	193	1 (pass)	0 (not solved)
3825	101	194	1 (pass)	0 (not solved)
3826	101	195	1 (pass)	0 (not solved)
3827	101	196	1 (pass)	0 (not solved)
3828	101	197	1 (pass)	0 (not solved)
3829	101	198	1 (pass)	0 (not solved)
3830	101	199	1 (pass)	0 (not solved)
3831	101	200	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3832	101	201	1 (pass)	0 (not solved)
3833	101	202	1 (pass)	0 (not solved)
3834	101	203	1 (pass)	0 (not solved)
3835	101	204	1 (pass)	0 (not solved)
3836	101	205	1 (pass)	0 (not solved)
3837	101	206	1 (pass)	0 (not solved)
3838	101	207	1 (pass)	0 (not solved)
3839	101	208	1 (pass)	0 (not solved)
3840	101	209	1 (pass)	0 (not solved)
3841	101	210	1 (pass)	0 (not solved)
3842	101	211	1 (pass)	0 (not solved)
3843	101	212	1 (pass)	0 (not solved)
3844	101	213	1 (pass)	0 (not solved)
3845	101	214	1 (pass)	0 (not solved)
3846	101	215	1 (pass)	0 (not solved)
3847	101	216	1 (pass)	0 (not solved)
3848	101	217	1 (pass)	0 (not solved)
3849	101	218	1 (pass)	0 (not solved)
3850	101	219	1 (pass)	0 (not solved)
3851	101	220	1 (pass)	0 (not solved)
3852	101	221	1 (pass)	0 (not solved)
3853	101	222	1 (pass)	0 (not solved)
3854	101	223	1 (pass)	0 (not solved)
3855	101	224	1 (pass)	0 (not solved)
3856	101	225	1 (pass)	0 (not solved)
3857	101	226	1 (pass)	0 (not solved)
3858	101	227	1 (pass)	0 (not solved)
3859	101	228	1 (pass)	0 (not solved)
3860	101	229	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3861	101	230	1 (pass)	0 (not solved)
3862	101	231	1 (pass)	0 (not solved)
3863	101	232	1 (pass)	0 (not solved)
3864	101	233	1 (pass)	0 (not solved)
3865	101	234	1 (pass)	0 (not solved)
3866	101	235	1 (pass)	0 (not solved)
3867	101	236	1 (pass)	0 (not solved)
3868	101	237	1 (pass)	0 (not solved)
3869	101	238	1 (pass)	0 (not solved)
3870	101	239	1 (pass)	0 (not solved)
3871	101	240	1 (pass)	0 (not solved)
3872	101	241	1 (pass)	0 (not solved)
3873	101	242	1 (pass)	0 (not solved)
3874	101	243	1 (pass)	0 (not solved)
3875	101	244	1 (pass)	0 (not solved)
3876	101	245	1 (pass)	0 (not solved)
3877	101	246	1 (pass)	0 (not solved)
3878	101	247	1 (pass)	0 (not solved)
3879	101	248	1 (pass)	0 (not solved)
3880	101	249	1 (pass)	0 (not solved)
3881	101	250	1 (pass)	0 (not solved)
3882	101	251	1 (pass)	0 (not solved)
3883	101	252	1 (pass)	0 (not solved)
3884	101	253	1 (pass)	0 (not solved)
3885	101	254	1 (pass)	0 (not solved)
3886	101	255	1 (pass)	0 (not solved)
3887	101	256	1 (pass)	0 (not solved)
3888	101	257	1 (pass)	0 (not solved)
3889	101	258	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3890	101	259	1 (pass)	0 (not solved)
3891	101	260	1 (pass)	0 (not solved)
3892	101	261	1 (pass)	0 (not solved)
3893	101	262	1 (pass)	0 (not solved)
3894	101	578	1 (pass)	0 (not solved)
3895	101	579	1 (pass)	0 (not solved)
3896	101	580	1 (pass)	0 (not solved)
3897	101	581	1 (pass)	0 (not solved)
3898	101	582	1 (pass)	0 (not solved)
3899	101	583	1 (pass)	0 (not solved)
3900	101	584	1 (pass)	0 (not solved)
3901	101	585	1 (pass)	0 (not solved)
3902	101	586	1 (pass)	0 (not solved)
3903	101	587	1 (pass)	0 (not solved)
3904	101	588	1 (pass)	0 (not solved)
3905	101	589	1 (pass)	0 (not solved)
3906	101	590	1 (pass)	0 (not solved)
3907	101	591	1 (pass)	0 (not solved)
3908	101	592	1 (pass)	0 (not solved)
3909	101	593	1 (pass)	0 (not solved)
3910	101	594	1 (pass)	0 (not solved)
3911	101	595	1 (pass)	0 (not solved)
3912	101	596	1 (pass)	0 (not solved)
3913	101	597	1 (pass)	0 (not solved)
3914	101	598	1 (pass)	0 (not solved)
3915	101	599	1 (pass)	0 (not solved)
3916	101	600	1 (pass)	0 (not solved)
3917	101	601	1 (pass)	0 (not solved)
3918	101	602	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3919	101	655	1 (pass)	0 (not solved)
3920	101	656	1 (pass)	0 (not solved)
3921	101	657	1 (pass)	0 (not solved)
3922	101	658	1 (pass)	0 (not solved)
3923	101	659	1 (pass)	0 (not solved)
3924	101	660	1 (pass)	0 (not solved)
3925	101	661	1 (pass)	0 (not solved)
3926	101	662	1 (pass)	0 (not solved)
3927	101	663	1 (pass)	0 (not solved)
3928	101	664	1 (pass)	0 (not solved)
3929	101	665	1 (pass)	0 (not solved)
3930	101	666	1 (pass)	0 (not solved)
3931	101	667	1 (pass)	0 (not solved)
3932	101	668	1 (pass)	0 (not solved)
3933	101	669	1 (pass)	0 (not solved)
3934	101	670	1 (pass)	0 (not solved)
3935	101	671	1 (pass)	0 (not solved)
3936	101	672	1 (pass)	0 (not solved)
3937	103	526	1 (pass)	-1 (time out)
3938	103	552	1 (pass)	-1 (time out)
3939	103	553	1 (pass)	-1 (time out)
3940	103	554	1 (pass)	-1 (time out)
3941	103	597	1 (pass)	-1 (time out)
3942	103	598	1 (pass)	-1 (time out)
3943	103	599	1 (pass)	-1 (time out)
3944	103	600	1 (pass)	-1 (time out)
3945	103	601	1 (pass)	-1 (time out)
3946	103	602	1 (pass)	-1 (time out)
3947	103	603	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3948	103	604	1 (pass)	-1 (time out)
3949	103	605	1 (pass)	-1 (time out)
3950	103	606	1 (pass)	-1 (time out)
3951	103	607	1 (pass)	-1 (time out)
3952	103	671	1 (pass)	-1 (time out)
3953	103	672	1 (pass)	-1 (time out)
3954	103	673	1 (pass)	-1 (time out)
3955	103	674	1 (pass)	-1 (time out)
3956	103	689	1 (pass)	-1 (time out)
3957	103	693	1 (pass)	-1 (time out)
3958	103	1230	1 (pass)	-1 (time out)
3959	103	1232	1 (pass)	-1 (time out)
3960	103	1233	1 (pass)	-1 (time out)
3961	103	1251	1 (pass)	-1 (time out)
3962	103	1252	1 (pass)	-1 (time out)
3963	103	1255	1 (pass)	-1 (time out)
3964	103	1256	1 (pass)	-1 (time out)
3965	103	1257	1 (pass)	-1 (time out)
3966	103	1262	1 (pass)	-1 (time out)
3967	104	321	1 (pass)	-1 (time out)
3968	104	322	1 (pass)	-1 (time out)
3969	104	323	1 (pass)	-1 (time out)
3970	104	324	1 (pass)	-1 (time out)
3971	104	347	1 (pass)	-1 (time out)
3972	104	348	1 (pass)	-1 (time out)
3973	104	349	1 (pass)	-1 (time out)
3974	104	350	1 (pass)	-1 (time out)
3975	104	351	1 (pass)	-1 (time out)
3976	104	352	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
3977	104	353	1 (pass)	-1 (time out)
3978	104	354	1 (pass)	-1 (time out)
3979	104	355	1 (pass)	-1 (time out)
3980	104	356	1 (pass)	-1 (time out)
3981	104	357	1 (pass)	-1 (time out)
3982	104	358	1 (pass)	-1 (time out)
3983	104	359	1 (pass)	-1 (time out)
3984	104	360	1 (pass)	-1 (time out)
3985	104	361	1 (pass)	-1 (time out)
3986	104	385	1 (pass)	-1 (time out)
3987	104	386	1 (pass)	-1 (time out)
3988	104	387	1 (pass)	-1 (time out)
3989	104	388	1 (pass)	-1 (time out)
3990	104	389	1 (pass)	-1 (time out)
3991	104	390	1 (pass)	-1 (time out)
3992	104	391	1 (pass)	-1 (time out)
3993	104	398	1 (pass)	-1 (time out)
3994	104	399	1 (pass)	-1 (time out)
3995	104	400	1 (pass)	-1 (time out)
3996	104	401	1 (pass)	-1 (time out)
3997	104	402	1 (pass)	-1 (time out)
3998	104	403	1 (pass)	-1 (time out)
3999	104	404	1 (pass)	-1 (time out)
4000	104	405	1 (pass)	-1 (time out)
4001	104	406	1 (pass)	-1 (time out)
4002	104	407	1 (pass)	-1 (time out)
4003	104	408	1 (pass)	-1 (time out)
4004	104	474	1 (pass)	-1 (time out)
4005	106	150	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4006	115	9	1 (pass)	0 (not solved)
4007	115	10	1 (pass)	0 (not solved)
4008	115	11	1 (pass)	0 (not solved)
4009	115	12	1 (pass)	0 (not solved)
4010	115	13	1 (pass)	0 (not solved)
4011	115	14	1 (pass)	0 (not solved)
4012	115	15	1 (pass)	0 (not solved)
4013	115	16	1 (pass)	0 (not solved)
4014	115	17	1 (pass)	0 (not solved)
4015	115	18	1 (pass)	0 (not solved)
4016	115	19	1 (pass)	0 (not solved)
4017	115	20	1 (pass)	0 (not solved)
4018	115	21	1 (pass)	0 (not solved)
4019	115	22	1 (pass)	0 (not solved)
4020	115	23	1 (pass)	0 (not solved)
4021	115	24	1 (pass)	0 (not solved)
4022	115	55	1 (pass)	0 (not solved)
4023	115	56	1 (pass)	0 (not solved)
4024	115	57	1 (pass)	0 (not solved)
4025	115	58	1 (pass)	0 (not solved)
4026	115	59	1 (pass)	0 (not solved)
4027	115	60	1 (pass)	0 (not solved)
4028	115	70	1 (pass)	0 (not solved)
4029	115	71	1 (pass)	0 (not solved)
4030	115	72	1 (pass)	0 (not solved)
4031	115	73	1 (pass)	0 (not solved)
4032	115	74	1 (pass)	0 (not solved)
4033	115	75	1 (pass)	0 (not solved)
4034	115	76	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4035	115	77	1 (pass)	0 (not solved)
4036	115	78	1 (pass)	0 (not solved)
4037	115	79	1 (pass)	0 (not solved)
4038	115	80	1 (pass)	0 (not solved)
4039	115	81	1 (pass)	0 (not solved)
4040	115	82	1 (pass)	0 (not solved)
4041	115	83	1 (pass)	0 (not solved)
4042	115	84	1 (pass)	0 (not solved)
4043	115	85	1 (pass)	0 (not solved)
4044	115	86	1 (pass)	0 (not solved)
4045	115	87	1 (pass)	0 (not solved)
4046	115	88	1 (pass)	0 (not solved)
4047	115	89	1 (pass)	0 (not solved)
4048	115	90	1 (pass)	0 (not solved)
4049	115	91	1 (pass)	0 (not solved)
4050	115	92	1 (pass)	0 (not solved)
4051	115	93	1 (pass)	0 (not solved)
4052	115	94	1 (pass)	0 (not solved)
4053	115	95	1 (pass)	0 (not solved)
4054	115	96	1 (pass)	0 (not solved)
4055	115	97	1 (pass)	0 (not solved)
4056	115	98	1 (pass)	0 (not solved)
4057	115	99	1 (pass)	0 (not solved)
4058	115	100	1 (pass)	0 (not solved)
4059	115	101	1 (pass)	0 (not solved)
4060	115	102	1 (pass)	0 (not solved)
4061	115	103	1 (pass)	0 (not solved)
4062	115	104	1 (pass)	0 (not solved)
4063	115	105	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4064	115	106	1 (pass)	0 (not solved)
4065	115	107	1 (pass)	0 (not solved)
4066	115	108	1 (pass)	0 (not solved)
4067	115	109	1 (pass)	0 (not solved)
4068	115	110	1 (pass)	0 (not solved)
4069	115	111	1 (pass)	0 (not solved)
4070	115	112	1 (pass)	0 (not solved)
4071	115	113	1 (pass)	0 (not solved)
4072	115	114	1 (pass)	0 (not solved)
4073	115	115	1 (pass)	0 (not solved)
4074	115	116	1 (pass)	0 (not solved)
4075	115	117	1 (pass)	0 (not solved)
4076	115	118	1 (pass)	0 (not solved)
4077	115	119	1 (pass)	0 (not solved)
4078	115	120	1 (pass)	0 (not solved)
4079	115	121	1 (pass)	0 (not solved)
4080	115	122	1 (pass)	0 (not solved)
4081	115	123	1 (pass)	0 (not solved)
4082	115	124	1 (pass)	0 (not solved)
4083	115	125	1 (pass)	0 (not solved)
4084	115	126	1 (pass)	0 (not solved)
4085	115	127	1 (pass)	0 (not solved)
4086	115	128	1 (pass)	0 (not solved)
4087	115	129	1 (pass)	0 (not solved)
4088	115	130	1 (pass)	0 (not solved)
4089	115	131	1 (pass)	0 (not solved)
4090	115	230	1 (pass)	0 (not solved)
4091	115	232	1 (pass)	0 (not solved)
4092	115	234	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4093	115	246	1 (pass)	0 (not solved)
4094	115	248	1 (pass)	0 (not solved)
4095	115	250	1 (pass)	0 (not solved)
4096	115	252	1 (pass)	0 (not solved)
4097	115	263	1 (pass)	0 (not solved)
4098	115	265	1 (pass)	0 (not solved)
4099	118	165	1 (pass)	0 (not solved)
4100	118	166	1 (pass)	0 (not solved)
4101	118	167	1 (pass)	0 (not solved)
4102	118	168	1 (pass)	0 (not solved)
4103	118	169	1 (pass)	0 (not solved)
4104	118	170	1 (pass)	0 (not solved)
4105	118	171	1 (pass)	0 (not solved)
4106	118	172	1 (pass)	0 (not solved)
4107	118	173	1 (pass)	0 (not solved)
4108	118	174	1 (pass)	0 (not solved)
4109	118	175	1 (pass)	0 (not solved)
4110	118	176	1 (pass)	0 (not solved)
4111	118	177	1 (pass)	0 (not solved)
4112	118	178	1 (pass)	0 (not solved)
4113	118	179	1 (pass)	0 (not solved)
4114	118	180	1 (pass)	0 (not solved)
4115	118	181	1 (pass)	0 (not solved)
4116	118	182	1 (pass)	0 (not solved)
4117	118	183	1 (pass)	0 (not solved)
4118	118	184	1 (pass)	0 (not solved)
4119	118	185	1 (pass)	0 (not solved)
4120	118	186	1 (pass)	0 (not solved)
4121	118	187	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4122	118	188	1 (pass)	0 (not solved)
4123	118	189	1 (pass)	0 (not solved)
4124	118	190	1 (pass)	0 (not solved)
4125	118	191	1 (pass)	0 (not solved)
4126	118	192	1 (pass)	0 (not solved)
4127	118	193	1 (pass)	0 (not solved)
4128	118	194	1 (pass)	0 (not solved)
4129	118	195	1 (pass)	0 (not solved)
4130	118	196	1 (pass)	0 (not solved)
4131	118	197	1 (pass)	0 (not solved)
4132	118	198	1 (pass)	0 (not solved)
4133	118	199	1 (pass)	0 (not solved)
4134	118	200	1 (pass)	0 (not solved)
4135	118	201	1 (pass)	0 (not solved)
4136	118	202	1 (pass)	0 (not solved)
4137	118	203	1 (pass)	0 (not solved)
4138	118	204	1 (pass)	0 (not solved)
4139	118	205	1 (pass)	0 (not solved)
4140	118	206	1 (pass)	0 (not solved)
4141	118	207	1 (pass)	0 (not solved)
4142	118	208	1 (pass)	0 (not solved)
4143	118	209	1 (pass)	0 (not solved)
4144	118	210	1 (pass)	0 (not solved)
4145	118	211	1 (pass)	0 (not solved)
4146	118	212	1 (pass)	0 (not solved)
4147	118	213	1 (pass)	0 (not solved)
4148	118	214	1 (pass)	0 (not solved)
4149	118	215	1 (pass)	0 (not solved)
4150	118	216	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4151	118	217	1 (pass)	0 (not solved)
4152	118	351	1 (pass)	0 (not solved)
4153	118	352	1 (pass)	0 (not solved)
4154	118	353	1 (pass)	0 (not solved)
4155	118	354	1 (pass)	0 (not solved)
4156	118	355	1 (pass)	0 (not solved)
4157	118	356	1 (pass)	0 (not solved)
4158	118	357	1 (pass)	0 (not solved)
4159	118	358	1 (pass)	0 (not solved)
4160	118	359	1 (pass)	0 (not solved)
4161	118	360	1 (pass)	0 (not solved)
4162	118	361	1 (pass)	0 (not solved)
4163	118	362	1 (pass)	0 (not solved)
4164	118	363	1 (pass)	0 (not solved)
4165	118	364	1 (pass)	0 (not solved)
4166	118	365	1 (pass)	0 (not solved)
4167	118	366	1 (pass)	0 (not solved)
4168	118	367	1 (pass)	0 (not solved)
4169	118	368	1 (pass)	0 (not solved)
4170	118	369	1 (pass)	0 (not solved)
4171	118	370	1 (pass)	0 (not solved)
4172	118	371	1 (pass)	0 (not solved)
4173	118	372	1 (pass)	0 (not solved)
4174	118	373	1 (pass)	0 (not solved)
4175	118	374	1 (pass)	0 (not solved)
4176	118	375	1 (pass)	0 (not solved)
4177	118	376	1 (pass)	0 (not solved)
4178	118	377	1 (pass)	0 (not solved)
4179	118	378	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4180	118	379	1 (pass)	0 (not solved)
4181	118	380	1 (pass)	0 (not solved)
4182	118	381	1 (pass)	0 (not solved)
4183	118	382	1 (pass)	0 (not solved)
4184	118	383	1 (pass)	0 (not solved)
4185	118	384	1 (pass)	0 (not solved)
4186	118	385	1 (pass)	0 (not solved)
4187	118	386	1 (pass)	0 (not solved)
4188	118	387	1 (pass)	0 (not solved)
4189	118	388	1 (pass)	0 (not solved)
4190	118	389	1 (pass)	0 (not solved)
4191	118	390	1 (pass)	0 (not solved)
4192	118	391	1 (pass)	0 (not solved)
4193	118	392	1 (pass)	0 (not solved)
4194	118	393	1 (pass)	0 (not solved)
4195	118	394	1 (pass)	0 (not solved)
4196	118	395	1 (pass)	0 (not solved)
4197	118	396	1 (pass)	0 (not solved)
4198	118	397	1 (pass)	0 (not solved)
4199	118	579	1 (pass)	0 (not solved)
4200	118	580	1 (pass)	0 (not solved)
4201	118	581	1 (pass)	0 (not solved)
4202	118	582	1 (pass)	0 (not solved)
4203	118	583	1 (pass)	0 (not solved)
4204	118	584	1 (pass)	0 (not solved)
4205	118	585	1 (pass)	0 (not solved)
4206	118	586	1 (pass)	0 (not solved)
4207	118	587	1 (pass)	0 (not solved)
4208	118	588	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4209	118	589	1 (pass)	0 (not solved)
4210	118	590	1 (pass)	0 (not solved)
4211	118	591	1 (pass)	0 (not solved)
4212	118	592	1 (pass)	0 (not solved)
4213	118	593	1 (pass)	0 (not solved)
4214	118	594	1 (pass)	0 (not solved)
4215	118	595	1 (pass)	0 (not solved)
4216	118	596	1 (pass)	0 (not solved)
4217	118	597	1 (pass)	0 (not solved)
4218	118	598	1 (pass)	0 (not solved)
4219	118	599	1 (pass)	0 (not solved)
4220	118	600	1 (pass)	0 (not solved)
4221	118	601	1 (pass)	0 (not solved)
4222	118	602	1 (pass)	0 (not solved)
4223	118	603	1 (pass)	0 (not solved)
4224	118	604	1 (pass)	0 (not solved)
4225	118	605	1 (pass)	0 (not solved)
4226	118	606	1 (pass)	0 (not solved)
4227	118	607	1 (pass)	0 (not solved)
4228	118	630	1 (pass)	0 (not solved)
4229	118	631	1 (pass)	0 (not solved)
4230	118	632	1 (pass)	0 (not solved)
4231	118	633	1 (pass)	0 (not solved)
4232	118	637	1 (pass)	0 (not solved)
4233	118	638	1 (pass)	0 (not solved)
4234	118	639	1 (pass)	0 (not solved)
4235	118	644	1 (pass)	0 (not solved)
4236	118	645	1 (pass)	0 (not solved)
4237	118	646	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4238	118	650	1 (pass)	0 (not solved)
4239	118	651	1 (pass)	0 (not solved)
4240	118	652	1 (pass)	0 (not solved)
4241	118	653	1 (pass)	0 (not solved)
4242	118	656	1 (pass)	0 (not solved)
4243	118	657	1 (pass)	0 (not solved)
4244	118	658	1 (pass)	0 (not solved)
4245	118	659	1 (pass)	0 (not solved)
4246	118	660	1 (pass)	0 (not solved)
4247	118	663	1 (pass)	0 (not solved)
4248	118	664	1 (pass)	0 (not solved)
4249	118	665	1 (pass)	0 (not solved)
4250	118	666	1 (pass)	0 (not solved)
4251	118	667	1 (pass)	0 (not solved)
4252	118	668	1 (pass)	0 (not solved)
4253	118	669	1 (pass)	0 (not solved)
4254	118	670	1 (pass)	0 (not solved)
4255	118	671	1 (pass)	0 (not solved)
4256	118	672	1 (pass)	0 (not solved)
4257	118	673	1 (pass)	0 (not solved)
4258	118	674	1 (pass)	0 (not solved)
4259	118	675	1 (pass)	0 (not solved)
4260	118	676	1 (pass)	0 (not solved)
4261	118	677	1 (pass)	0 (not solved)
4262	118	678	1 (pass)	0 (not solved)
4263	118	679	1 (pass)	0 (not solved)
4264	118	680	1 (pass)	0 (not solved)
4265	118	681	1 (pass)	0 (not solved)
4266	118	682	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4267	118	683	1 (pass)	0 (not solved)
4268	118	684	1 (pass)	0 (not solved)
4269	118	794	1 (pass)	0 (not solved)
4270	118	795	1 (pass)	0 (not solved)
4271	118	796	1 (pass)	0 (not solved)
4272	118	797	1 (pass)	0 (not solved)
4273	118	798	1 (pass)	0 (not solved)
4274	118	799	1 (pass)	0 (not solved)
4275	118	800	1 (pass)	0 (not solved)
4276	118	801	1 (pass)	0 (not solved)
4277	118	802	1 (pass)	0 (not solved)
4278	118	803	1 (pass)	0 (not solved)
4279	118	804	1 (pass)	0 (not solved)
4280	118	805	1 (pass)	0 (not solved)
4281	118	806	1 (pass)	0 (not solved)
4282	118	807	1 (pass)	0 (not solved)
4283	118	808	1 (pass)	0 (not solved)
4284	118	809	1 (pass)	0 (not solved)
4285	118	810	1 (pass)	0 (not solved)
4286	118	811	1 (pass)	0 (not solved)
4287	118	812	1 (pass)	0 (not solved)
4288	118	813	1 (pass)	0 (not solved)
4289	118	814	1 (pass)	0 (not solved)
4290	118	815	1 (pass)	0 (not solved)
4291	118	816	1 (pass)	0 (not solved)
4292	118	837	1 (pass)	0 (not solved)
4293	118	838	1 (pass)	0 (not solved)
4294	118	839	1 (pass)	0 (not solved)
4295	118	842	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4296	118	843	1 (pass)	0 (not solved)
4297	118	844	1 (pass)	0 (not solved)
4298	118	848	1 (pass)	0 (not solved)
4299	118	849	1 (pass)	0 (not solved)
4300	118	850	1 (pass)	0 (not solved)
4301	118	855	1 (pass)	0 (not solved)
4302	118	856	1 (pass)	0 (not solved)
4303	118	857	1 (pass)	0 (not solved)
4304	118	858	1 (pass)	0 (not solved)
4305	118	862	1 (pass)	0 (not solved)
4306	118	863	1 (pass)	0 (not solved)
4307	118	864	1 (pass)	0 (not solved)
4308	118	865	1 (pass)	0 (not solved)
4309	118	866	1 (pass)	0 (not solved)
4310	118	869	1 (pass)	0 (not solved)
4311	118	870	1 (pass)	0 (not solved)
4312	118	871	1 (pass)	0 (not solved)
4313	118	872	1 (pass)	0 (not solved)
4314	118	873	1 (pass)	0 (not solved)
4315	119	108	1 (pass)	0 (not solved)
4316	119	109	1 (pass)	0 (not solved)
4317	119	110	1 (pass)	0 (not solved)
4318	119	111	1 (pass)	0 (not solved)
4319	119	112	1 (pass)	0 (not solved)
4320	119	113	1 (pass)	0 (not solved)
4321	119	114	1 (pass)	0 (not solved)
4322	119	115	1 (pass)	0 (not solved)
4323	119	116	1 (pass)	0 (not solved)
4324	119	117	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4325	119	118	1 (pass)	0 (not solved)
4326	119	119	1 (pass)	0 (not solved)
4327	119	120	1 (pass)	0 (not solved)
4328	119	121	1 (pass)	0 (not solved)
4329	119	122	1 (pass)	0 (not solved)
4330	119	123	1 (pass)	0 (not solved)
4331	119	124	1 (pass)	0 (not solved)
4332	119	125	1 (pass)	0 (not solved)
4333	119	126	1 (pass)	0 (not solved)
4334	119	127	1 (pass)	0 (not solved)
4335	119	128	1 (pass)	0 (not solved)
4336	119	129	1 (pass)	0 (not solved)
4337	119	130	1 (pass)	0 (not solved)
4338	119	131	1 (pass)	0 (not solved)
4339	119	132	1 (pass)	0 (not solved)
4340	119	133	1 (pass)	0 (not solved)
4341	119	282	1 (pass)	0 (not solved)
4342	119	284	1 (pass)	0 (not solved)
4343	119	286	1 (pass)	0 (not solved)
4344	119	288	1 (pass)	0 (not solved)
4345	119	290	1 (pass)	0 (not solved)
4346	119	292	1 (pass)	0 (not solved)
4347	119	293	1 (pass)	0 (not solved)
4348	119	294	1 (pass)	0 (not solved)
4349	119	295	1 (pass)	0 (not solved)
4350	119	296	1 (pass)	0 (not solved)
4351	119	297	1 (pass)	0 (not solved)
4352	119	298	1 (pass)	0 (not solved)
4353	119	299	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4354	119	300	1 (pass)	0 (not solved)
4355	119	301	1 (pass)	0 (not solved)
4356	119	302	1 (pass)	0 (not solved)
4357	119	303	1 (pass)	0 (not solved)
4358	119	304	1 (pass)	0 (not solved)
4359	119	305	1 (pass)	0 (not solved)
4360	119	306	1 (pass)	0 (not solved)
4361	122	277	1 (pass)	0 (not solved)
4362	123	1	1 (pass)	0 (not solved)
4363	123	2	1 (pass)	0 (not solved)
4364	123	3	1 (pass)	0 (not solved)
4365	123	4	1 (pass)	0 (not solved)
4366	123	5	1 (pass)	0 (not solved)
4367	123	6	1 (pass)	0 (not solved)
4368	123	179	1 (pass)	0 (not solved)
4369	123	180	1 (pass)	0 (not solved)
4370	123	181	1 (pass)	0 (not solved)
4371	123	182	1 (pass)	0 (not solved)
4372	123	183	1 (pass)	0 (not solved)
4373	123	184	1 (pass)	0 (not solved)
4374	123	185	1 (pass)	0 (not solved)
4375	123	186	1 (pass)	0 (not solved)
4376	123	187	1 (pass)	0 (not solved)
4377	123	188	1 (pass)	0 (not solved)
4378	123	189	1 (pass)	0 (not solved)
4379	123	190	1 (pass)	0 (not solved)
4380	123	191	1 (pass)	0 (not solved)
4381	123	192	1 (pass)	0 (not solved)
4382	123	193	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4383	123	194	1 (pass)	0 (not solved)
4384	123	195	1 (pass)	0 (not solved)
4385	123	196	1 (pass)	0 (not solved)
4386	123	197	1 (pass)	0 (not solved)
4387	123	198	1 (pass)	0 (not solved)
4388	123	199	1 (pass)	0 (not solved)
4389	123	200	1 (pass)	0 (not solved)
4390	123	201	1 (pass)	0 (not solved)
4391	123	202	1 (pass)	0 (not solved)
4392	123	203	1 (pass)	0 (not solved)
4393	123	204	1 (pass)	0 (not solved)
4394	123	205	1 (pass)	0 (not solved)
4395	123	206	1 (pass)	0 (not solved)
4396	123	207	1 (pass)	0 (not solved)
4397	123	208	1 (pass)	0 (not solved)
4398	123	209	1 (pass)	0 (not solved)
4399	123	210	1 (pass)	0 (not solved)
4400	123	211	1 (pass)	0 (not solved)
4401	123	212	1 (pass)	0 (not solved)
4402	123	213	1 (pass)	0 (not solved)
4403	123	214	1 (pass)	0 (not solved)
4404	123	215	1 (pass)	0 (not solved)
4405	123	216	1 (pass)	0 (not solved)
4406	123	217	1 (pass)	0 (not solved)
4407	123	218	1 (pass)	0 (not solved)
4408	123	219	1 (pass)	0 (not solved)
4409	123	220	1 (pass)	0 (not solved)
4410	123	221	1 (pass)	0 (not solved)
4411	123	222	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4412	123	223	1 (pass)	0 (not solved)
4413	123	394	1 (pass)	0 (not solved)
4414	123	395	1 (pass)	0 (not solved)
4415	123	396	1 (pass)	0 (not solved)
4416	123	397	1 (pass)	0 (not solved)
4417	123	398	1 (pass)	0 (not solved)
4418	123	399	1 (pass)	0 (not solved)
4419	123	400	1 (pass)	0 (not solved)
4420	123	401	1 (pass)	0 (not solved)
4421	123	402	1 (pass)	0 (not solved)
4422	123	403	1 (pass)	0 (not solved)
4423	123	404	1 (pass)	0 (not solved)
4424	123	405	1 (pass)	0 (not solved)
4425	123	406	1 (pass)	0 (not solved)
4426	123	407	1 (pass)	0 (not solved)
4427	123	408	1 (pass)	0 (not solved)
4428	123	409	1 (pass)	0 (not solved)
4429	123	410	1 (pass)	0 (not solved)
4430	123	411	1 (pass)	0 (not solved)
4431	123	412	1 (pass)	0 (not solved)
4432	123	413	1 (pass)	0 (not solved)
4433	123	414	1 (pass)	0 (not solved)
4434	123	438	1 (pass)	0 (not solved)
4435	123	439	1 (pass)	0 (not solved)
4436	123	440	1 (pass)	0 (not solved)
4437	123	445	1 (pass)	0 (not solved)
4438	123	446	1 (pass)	0 (not solved)
4439	123	447	1 (pass)	0 (not solved)
4440	123	453	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4441	123	454	1 (pass)	0 (not solved)
4442	123	455	1 (pass)	0 (not solved)
4443	123	459	1 (pass)	0 (not solved)
4444	123	460	1 (pass)	0 (not solved)
4445	123	461	1 (pass)	0 (not solved)
4446	123	464	1 (pass)	0 (not solved)
4447	123	465	1 (pass)	0 (not solved)
4448	123	466	1 (pass)	0 (not solved)
4449	123	467	1 (pass)	0 (not solved)
4450	123	469	1 (pass)	0 (not solved)
4451	123	470	1 (pass)	0 (not solved)
4452	123	471	1 (pass)	0 (not solved)
4453	123	472	1 (pass)	0 (not solved)
4454	123	473	1 (pass)	0 (not solved)
4455	123	483	1 (pass)	0 (not solved)
4456	123	484	1 (pass)	0 (not solved)
4457	123	485	1 (pass)	0 (not solved)
4458	123	486	1 (pass)	0 (not solved)
4459	123	487	1 (pass)	0 (not solved)
4460	123	488	1 (pass)	0 (not solved)
4461	123	489	1 (pass)	0 (not solved)
4462	123	490	1 (pass)	0 (not solved)
4463	123	491	1 (pass)	0 (not solved)
4464	123	492	1 (pass)	0 (not solved)
4465	123	493	1 (pass)	0 (not solved)
4466	123	494	1 (pass)	0 (not solved)
4467	123	495	1 (pass)	0 (not solved)
4468	123	496	1 (pass)	0 (not solved)
4469	123	497	1 (pass)	0 (not solved)

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#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4470	123	498	1 (pass)	0 (not solved)
4471	123	499	1 (pass)	0 (not solved)
4472	123	500	1 (pass)	0 (not solved)
4473	123	501	1 (pass)	0 (not solved)
4474	123	502	1 (pass)	0 (not solved)
4475	123	503	1 (pass)	0 (not solved)
4476	123	504	1 (pass)	0 (not solved)
4477	123	505	1 (pass)	0 (not solved)
4478	123	506	1 (pass)	0 (not solved)
4479	123	507	1 (pass)	0 (not solved)
4480	123	508	1 (pass)	0 (not solved)
4481	123	509	1 (pass)	0 (not solved)
4482	123	510	1 (pass)	0 (not solved)
4483	123	511	1 (pass)	0 (not solved)
4484	123	512	1 (pass)	0 (not solved)
4485	123	513	1 (pass)	0 (not solved)
4486	123	514	1 (pass)	0 (not solved)
4487	123	562	1 (pass)	0 (not solved)
4488	123	563	1 (pass)	0 (not solved)
4489	123	564	1 (pass)	0 (not solved)
4490	123	565	1 (pass)	0 (not solved)
4491	123	566	1 (pass)	0 (not solved)
4492	123	567	1 (pass)	0 (not solved)
4493	123	568	1 (pass)	0 (not solved)
4494	123	569	1 (pass)	0 (not solved)
4495	123	570	1 (pass)	0 (not solved)
4496	123	571	1 (pass)	0 (not solved)
4497	123	572	1 (pass)	0 (not solved)
4498	123	573	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4499	123	594	1 (pass)	0 (not solved)
4500	123	595	1 (pass)	0 (not solved)
4501	123	596	1 (pass)	0 (not solved)
4502	123	600	1 (pass)	0 (not solved)
4503	123	601	1 (pass)	0 (not solved)
4504	123	602	1 (pass)	0 (not solved)
4505	123	607	1 (pass)	0 (not solved)
4506	123	608	1 (pass)	0 (not solved)
4507	123	609	1 (pass)	0 (not solved)
4508	123	615	1 (pass)	0 (not solved)
4509	123	616	1 (pass)	0 (not solved)
4510	123	617	1 (pass)	0 (not solved)
4511	123	621	1 (pass)	0 (not solved)
4512	123	622	1 (pass)	0 (not solved)
4513	123	623	1 (pass)	0 (not solved)
4514	123	624	1 (pass)	0 (not solved)
4515	123	628	1 (pass)	0 (not solved)
4516	123	629	1 (pass)	0 (not solved)
4517	123	630	1 (pass)	0 (not solved)
4518	123	631	1 (pass)	0 (not solved)
4519	123	632	1 (pass)	0 (not solved)
4520	124	16	1 (pass)	0 (not solved)
4521	124	17	1 (pass)	0 (not solved)
4522	124	18	1 (pass)	0 (not solved)
4523	124	19	1 (pass)	0 (not solved)
4524	124	20	1 (pass)	0 (not solved)
4525	124	21	1 (pass)	0 (not solved)
4526	124	22	1 (pass)	0 (not solved)
4527	124	23	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4528	124	47	1 (pass)	0 (not solved)
4529	124	48	1 (pass)	0 (not solved)
4530	124	49	1 (pass)	0 (not solved)
4531	124	50	1 (pass)	0 (not solved)
4532	124	51	1 (pass)	0 (not solved)
4533	124	52	1 (pass)	0 (not solved)
4534	124	53	1 (pass)	0 (not solved)
4535	124	65	1 (pass)	0 (not solved)
4536	124	66	1 (pass)	0 (not solved)
4537	124	67	1 (pass)	0 (not solved)
4538	124	68	1 (pass)	0 (not solved)
4539	124	69	1 (pass)	0 (not solved)
4540	124	70	1 (pass)	0 (not solved)
4541	125	207	1 (pass)	0 (not solved)
4542	125	208	1 (pass)	0 (not solved)
4543	125	209	1 (pass)	0 (not solved)
4544	125	210	1 (pass)	0 (not solved)
4545	125	211	1 (pass)	0 (not solved)
4546	125	212	1 (pass)	0 (not solved)
4547	125	213	1 (pass)	0 (not solved)
4548	125	214	1 (pass)	0 (not solved)
4549	125	215	1 (pass)	0 (not solved)
4550	125	216	1 (pass)	0 (not solved)
4551	125	217	1 (pass)	0 (not solved)
4552	125	218	1 (pass)	0 (not solved)
4553	125	219	1 (pass)	0 (not solved)
4554	125	220	1 (pass)	0 (not solved)
4555	125	221	1 (pass)	0 (not solved)
4556	125	222	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4557	125	223	1 (pass)	0 (not solved)
4558	125	224	1 (pass)	0 (not solved)
4559	125	225	1 (pass)	0 (not solved)
4560	125	226	1 (pass)	0 (not solved)
4561	125	227	1 (pass)	0 (not solved)
4562	125	228	1 (pass)	0 (not solved)
4563	125	229	1 (pass)	0 (not solved)
4564	125	230	1 (pass)	0 (not solved)
4565	125	231	1 (pass)	0 (not solved)
4566	125	232	1 (pass)	0 (not solved)
4567	125	233	1 (pass)	0 (not solved)
4568	125	234	1 (pass)	0 (not solved)
4569	125	235	1 (pass)	0 (not solved)
4570	125	236	1 (pass)	0 (not solved)
4571	125	237	1 (pass)	0 (not solved)
4572	125	238	1 (pass)	0 (not solved)
4573	125	239	1 (pass)	0 (not solved)
4574	125	240	1 (pass)	0 (not solved)
4575	125	241	1 (pass)	0 (not solved)
4576	125	242	1 (pass)	0 (not solved)
4577	125	243	1 (pass)	0 (not solved)
4578	125	244	1 (pass)	0 (not solved)
4579	125	245	1 (pass)	0 (not solved)
4580	125	246	1 (pass)	0 (not solved)
4581	125	247	1 (pass)	0 (not solved)
4582	125	248	1 (pass)	0 (not solved)
4583	125	249	1 (pass)	0 (not solved)
4584	125	534	1 (pass)	0 (not solved)
4585	125	535	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4586	125	536	1 (pass)	0 (not solved)
4587	125	537	1 (pass)	0 (not solved)
4588	125	538	1 (pass)	0 (not solved)
4589	125	539	1 (pass)	0 (not solved)
4590	125	540	1 (pass)	0 (not solved)
4591	125	541	1 (pass)	0 (not solved)
4592	125	542	1 (pass)	0 (not solved)
4593	125	543	1 (pass)	0 (not solved)
4594	125	544	1 (pass)	0 (not solved)
4595	125	545	1 (pass)	0 (not solved)
4596	125	546	1 (pass)	0 (not solved)
4597	125	547	1 (pass)	0 (not solved)
4598	125	548	1 (pass)	0 (not solved)
4599	125	549	1 (pass)	0 (not solved)
4600	125	550	1 (pass)	0 (not solved)
4601	125	551	1 (pass)	0 (not solved)
4602	125	552	1 (pass)	0 (not solved)
4603	125	553	1 (pass)	0 (not solved)
4604	125	554	1 (pass)	0 (not solved)
4605	125	555	1 (pass)	0 (not solved)
4606	125	556	1 (pass)	0 (not solved)
4607	125	557	1 (pass)	0 (not solved)
4608	125	558	1 (pass)	0 (not solved)
4609	125	559	1 (pass)	0 (not solved)
4610	125	560	1 (pass)	0 (not solved)
4611	125	561	1 (pass)	0 (not solved)
4612	125	562	1 (pass)	0 (not solved)
4613	125	563	1 (pass)	0 (not solved)
4614	125	564	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4615	125	565	1 (pass)	0 (not solved)
4616	125	566	1 (pass)	0 (not solved)
4617	125	567	1 (pass)	0 (not solved)
4618	125	568	1 (pass)	0 (not solved)
4619	125	569	1 (pass)	0 (not solved)
4620	125	570	1 (pass)	0 (not solved)
4621	125	571	1 (pass)	0 (not solved)
4622	125	572	1 (pass)	0 (not solved)
4623	125	573	1 (pass)	0 (not solved)
4624	125	574	1 (pass)	0 (not solved)
4625	125	575	1 (pass)	0 (not solved)
4626	125	576	1 (pass)	0 (not solved)
4627	125	577	1 (pass)	0 (not solved)
4628	125	981	1 (pass)	0 (not solved)
4629	125	982	1 (pass)	0 (not solved)
4630	125	983	1 (pass)	0 (not solved)
4631	125	984	1 (pass)	0 (not solved)
4632	125	985	1 (pass)	0 (not solved)
4633	125	986	1 (pass)	0 (not solved)
4634	125	987	1 (pass)	0 (not solved)
4635	125	988	1 (pass)	0 (not solved)
4636	125	989	1 (pass)	0 (not solved)
4637	125	990	1 (pass)	0 (not solved)
4638	125	991	1 (pass)	0 (not solved)
4639	125	992	1 (pass)	0 (not solved)
4640	125	993	1 (pass)	0 (not solved)
4641	125	994	1 (pass)	0 (not solved)
4642	125	995	1 (pass)	0 (not solved)
4643	125	996	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4644	125	997	1 (pass)	0 (not solved)
4645	125	998	1 (pass)	0 (not solved)
4646	125	999	1 (pass)	0 (not solved)
4647	125	1000	1 (pass)	0 (not solved)
4648	125	1001	1 (pass)	0 (not solved)
4649	125	1002	1 (pass)	0 (not solved)
4650	125	1003	1 (pass)	0 (not solved)
4651	125	1004	1 (pass)	0 (not solved)
4652	125	1005	1 (pass)	0 (not solved)
4653	125	1006	1 (pass)	0 (not solved)
4654	125	1007	1 (pass)	0 (not solved)
4655	125	1008	1 (pass)	0 (not solved)
4656	125	1009	1 (pass)	0 (not solved)
4657	125	1010	1 (pass)	0 (not solved)
4658	125	1011	1 (pass)	0 (not solved)
4659	125	1035	1 (pass)	0 (not solved)
4660	125	1036	1 (pass)	0 (not solved)
4661	125	1037	1 (pass)	0 (not solved)
4662	125	1043	1 (pass)	0 (not solved)
4663	125	1044	1 (pass)	0 (not solved)
4664	125	1050	1 (pass)	0 (not solved)
4665	125	1051	1 (pass)	0 (not solved)
4666	125	1055	1 (pass)	0 (not solved)
4667	125	1056	1 (pass)	0 (not solved)
4668	125	1057	1 (pass)	0 (not solved)
4669	125	1061	1 (pass)	0 (not solved)
4670	125	1062	1 (pass)	0 (not solved)
4671	125	1063	1 (pass)	0 (not solved)
4672	125	1066	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4673	125	1067	1 (pass)	0 (not solved)
4674	125	1068	1 (pass)	0 (not solved)
4675	125	1069	1 (pass)	0 (not solved)
4676	125	1075	1 (pass)	0 (not solved)
4677	125	1076	1 (pass)	0 (not solved)
4678	125	1077	1 (pass)	0 (not solved)
4679	125	1078	1 (pass)	0 (not solved)
4680	125	1079	1 (pass)	0 (not solved)
4681	125	1080	1 (pass)	0 (not solved)
4682	125	1081	1 (pass)	0 (not solved)
4683	125	1082	1 (pass)	0 (not solved)
4684	125	1083	1 (pass)	0 (not solved)
4685	125	1084	1 (pass)	0 (not solved)
4686	125	1085	1 (pass)	0 (not solved)
4687	125	1086	1 (pass)	0 (not solved)
4688	125	1087	1 (pass)	0 (not solved)
4689	125	1088	1 (pass)	0 (not solved)
4690	125	1089	1 (pass)	0 (not solved)
4691	125	1090	1 (pass)	0 (not solved)
4692	125	1091	1 (pass)	0 (not solved)
4693	125	1092	1 (pass)	0 (not solved)
4694	125	1093	1 (pass)	0 (not solved)
4695	125	1094	1 (pass)	0 (not solved)
4696	125	1095	1 (pass)	0 (not solved)
4697	125	1096	1 (pass)	0 (not solved)
4698	125	1097	1 (pass)	0 (not solved)
4699	125	1098	1 (pass)	0 (not solved)
4700	125	1099	1 (pass)	0 (not solved)
4701	125	1100	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4702	125	1101	1 (pass)	0 (not solved)
4703	125	1102	1 (pass)	0 (not solved)
4704	125	1103	1 (pass)	0 (not solved)
4705	125	1104	1 (pass)	0 (not solved)
4706	125	1105	1 (pass)	0 (not solved)
4707	125	1106	1 (pass)	0 (not solved)
4708	125	1107	1 (pass)	0 (not solved)
4709	125	1108	1 (pass)	0 (not solved)
4710	125	1109	1 (pass)	0 (not solved)
4711	125	1110	1 (pass)	0 (not solved)
4712	125	1111	1 (pass)	0 (not solved)
4713	125	1112	1 (pass)	0 (not solved)
4714	125	1113	1 (pass)	0 (not solved)
4715	125	1114	1 (pass)	0 (not solved)
4716	125	1115	1 (pass)	0 (not solved)
4717	125	1116	1 (pass)	0 (not solved)
4718	125	1117	1 (pass)	0 (not solved)
4719	125	1118	1 (pass)	0 (not solved)
4720	125	1119	1 (pass)	0 (not solved)
4721	125	1120	1 (pass)	0 (not solved)
4722	125	1121	1 (pass)	0 (not solved)
4723	125	1122	1 (pass)	0 (not solved)
4724	125	1123	1 (pass)	0 (not solved)
4725	125	1124	1 (pass)	0 (not solved)
4726	125	1125	1 (pass)	0 (not solved)
4727	125	1126	1 (pass)	0 (not solved)
4728	125	1173	1 (pass)	0 (not solved)
4729	125	1174	1 (pass)	0 (not solved)
4730	125	1175	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4731	125	1176	1 (pass)	0 (not solved)
4732	125	1177	1 (pass)	0 (not solved)
4733	125	1178	1 (pass)	0 (not solved)
4734	125	1179	1 (pass)	0 (not solved)
4735	125	1180	1 (pass)	0 (not solved)
4736	125	1181	1 (pass)	0 (not solved)
4737	125	1182	1 (pass)	0 (not solved)
4738	125	1183	1 (pass)	0 (not solved)
4739	125	1184	1 (pass)	0 (not solved)
4740	125	1185	1 (pass)	0 (not solved)
4741	125	1186	1 (pass)	0 (not solved)
4742	125	1187	1 (pass)	0 (not solved)
4743	125	1188	1 (pass)	0 (not solved)
4744	125	1189	1 (pass)	0 (not solved)
4745	125	1190	1 (pass)	0 (not solved)
4746	125	1191	1 (pass)	0 (not solved)
4747	125	1192	1 (pass)	0 (not solved)
4748	125	1193	1 (pass)	0 (not solved)
4749	125	1194	1 (pass)	0 (not solved)
4750	125	1195	1 (pass)	0 (not solved)
4751	125	1196	1 (pass)	0 (not solved)
4752	125	1197	1 (pass)	0 (not solved)
4753	125	1198	1 (pass)	0 (not solved)
4754	125	1199	1 (pass)	0 (not solved)
4755	125	1200	1 (pass)	0 (not solved)
4756	125	1201	1 (pass)	0 (not solved)
4757	125	1202	1 (pass)	0 (not solved)
4758	125	1203	1 (pass)	0 (not solved)
4759	125	1204	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4760	125	1205	1 (pass)	0 (not solved)
4761	125	1206	1 (pass)	0 (not solved)
4762	125	1207	1 (pass)	0 (not solved)
4763	125	1208	1 (pass)	0 (not solved)
4764	125	1209	1 (pass)	0 (not solved)
4765	125	1210	1 (pass)	0 (not solved)
4766	125	1211	1 (pass)	0 (not solved)
4767	125	1212	1 (pass)	0 (not solved)
4768	125	1213	1 (pass)	0 (not solved)
4769	125	1214	1 (pass)	0 (not solved)
4770	125	1215	1 (pass)	0 (not solved)
4771	125	1216	1 (pass)	0 (not solved)
4772	125	1217	1 (pass)	0 (not solved)
4773	125	1218	1 (pass)	0 (not solved)
4774	125	1219	1 (pass)	0 (not solved)
4775	125	1220	1 (pass)	0 (not solved)
4776	125	1221	1 (pass)	0 (not solved)
4777	125	1222	1 (pass)	0 (not solved)
4778	125	1223	1 (pass)	0 (not solved)
4779	125	1224	1 (pass)	0 (not solved)
4780	125	1225	1 (pass)	0 (not solved)
4781	125	1226	1 (pass)	0 (not solved)
4782	125	1227	1 (pass)	0 (not solved)
4783	125	1228	1 (pass)	0 (not solved)
4784	125	1229	1 (pass)	0 (not solved)
4785	125	1230	1 (pass)	0 (not solved)
4786	125	1231	1 (pass)	0 (not solved)
4787	125	1232	1 (pass)	0 (not solved)
4788	125	1233	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4789	125	1234	1 (pass)	0 (not solved)
4790	125	1235	1 (pass)	0 (not solved)
4791	125	1236	1 (pass)	0 (not solved)
4792	125	1237	1 (pass)	0 (not solved)
4793	125	1238	1 (pass)	0 (not solved)
4794	125	1239	1 (pass)	0 (not solved)
4795	125	1240	1 (pass)	0 (not solved)
4796	125	1241	1 (pass)	0 (not solved)
4797	125	1242	1 (pass)	0 (not solved)
4798	125	1243	1 (pass)	0 (not solved)
4799	125	1244	1 (pass)	0 (not solved)
4800	125	1292	1 (pass)	0 (not solved)
4801	125	1293	1 (pass)	0 (not solved)
4802	125	1294	1 (pass)	0 (not solved)
4803	125	1295	1 (pass)	0 (not solved)
4804	125	1296	1 (pass)	0 (not solved)
4805	125	1297	1 (pass)	0 (not solved)
4806	125	1298	1 (pass)	0 (not solved)
4807	125	1299	1 (pass)	0 (not solved)
4808	125	1300	1 (pass)	0 (not solved)
4809	125	1301	1 (pass)	0 (not solved)
4810	125	1302	1 (pass)	0 (not solved)
4811	125	1303	1 (pass)	0 (not solved)
4812	125	1304	1 (pass)	0 (not solved)
4813	125	1305	1 (pass)	0 (not solved)
4814	125	1306	1 (pass)	0 (not solved)
4815	125	1307	1 (pass)	0 (not solved)
4816	125	1308	1 (pass)	0 (not solved)
4817	125	1309	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4818	125	1310	1 (pass)	0 (not solved)
4819	125	1311	1 (pass)	0 (not solved)
4820	125	1312	1 (pass)	0 (not solved)
4821	125	1313	1 (pass)	0 (not solved)
4822	125	1314	1 (pass)	0 (not solved)
4823	125	1315	1 (pass)	0 (not solved)
4824	125	1316	1 (pass)	0 (not solved)
4825	125	1317	1 (pass)	0 (not solved)
4826	125	1334	1 (pass)	0 (not solved)
4827	125	1335	1 (pass)	0 (not solved)
4828	125	1336	1 (pass)	0 (not solved)
4829	125	1341	1 (pass)	0 (not solved)
4830	125	1342	1 (pass)	0 (not solved)
4831	125	1348	1 (pass)	0 (not solved)
4832	125	1349	1 (pass)	0 (not solved)
4833	125	1356	1 (pass)	0 (not solved)
4834	125	1357	1 (pass)	0 (not solved)
4835	125	1358	1 (pass)	0 (not solved)
4836	125	1363	1 (pass)	0 (not solved)
4837	125	1364	1 (pass)	0 (not solved)
4838	125	1365	1 (pass)	0 (not solved)
4839	125	1368	1 (pass)	0 (not solved)
4840	125	1369	1 (pass)	0 (not solved)
4841	125	1370	1 (pass)	0 (not solved)
4842	125	1371	1 (pass)	0 (not solved)
4843	126	259	1 (pass)	0 (not solved)
4844	127	9	1 (pass)	0 (not solved)
4845	127	10	1 (pass)	0 (not solved)
4846	127	11	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4847	127	12	1 (pass)	0 (not solved)
4848	127	13	1 (pass)	0 (not solved)
4849	127	14	1 (pass)	0 (not solved)
4850	127	15	1 (pass)	0 (not solved)
4851	127	16	1 (pass)	0 (not solved)
4852	127	17	1 (pass)	0 (not solved)
4853	127	18	1 (pass)	0 (not solved)
4854	127	19	1 (pass)	0 (not solved)
4855	127	20	1 (pass)	0 (not solved)
4856	127	21	1 (pass)	0 (not solved)
4857	127	22	1 (pass)	0 (not solved)
4858	127	23	1 (pass)	0 (not solved)
4859	127	24	1 (pass)	0 (not solved)
4860	127	55	1 (pass)	0 (not solved)
4861	127	56	1 (pass)	0 (not solved)
4862	127	57	1 (pass)	0 (not solved)
4863	127	58	1 (pass)	0 (not solved)
4864	127	59	1 (pass)	0 (not solved)
4865	127	60	1 (pass)	0 (not solved)
4866	133	1	1 (pass)	0 (not solved)
4867	135	87	1 (pass)	0 (not solved)
4868	135	110	1 (pass)	0 (not solved)
4869	135	112	1 (pass)	0 (not solved)
4870	135	114	1 (pass)	0 (not solved)
4871	135	175	1 (pass)	0 (not solved)
4872	139	55	1 (pass)	0 (not solved)
4873	139	60	1 (pass)	0 (not solved)
4874	139	64	1 (pass)	0 (not solved)
4875	139	66	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4876	139	68	1 (pass)	0 (not solved)
4877	139	111	1 (pass)	0 (not solved)
4878	139	113	1 (pass)	0 (not solved)
4879	139	115	1 (pass)	0 (not solved)
4880	139	117	1 (pass)	0 (not solved)
4881	139	119	1 (pass)	0 (not solved)
4882	139	121	1 (pass)	0 (not solved)
4883	139	267	1 (pass)	0 (not solved)
4884	139	269	1 (pass)	0 (not solved)
4885	139	271	1 (pass)	0 (not solved)
4886	139	273	1 (pass)	0 (not solved)
4887	139	275	1 (pass)	0 (not solved)
4888	139	277	1 (pass)	0 (not solved)
4889	139	309	1 (pass)	0 (not solved)
4890	139	311	1 (pass)	0 (not solved)
4891	139	313	1 (pass)	0 (not solved)
4892	139	315	1 (pass)	0 (not solved)
4893	139	317	1 (pass)	0 (not solved)
4894	139	319	1 (pass)	0 (not solved)
4895	141	232	1 (pass)	0 (not solved)
4896	141	233	1 (pass)	0 (not solved)
4897	141	234	1 (pass)	0 (not solved)
4898	141	235	1 (pass)	0 (not solved)
4899	141	236	1 (pass)	0 (not solved)
4900	141	237	1 (pass)	0 (not solved)
4901	141	238	1 (pass)	0 (not solved)
4902	141	239	1 (pass)	0 (not solved)
4903	141	240	1 (pass)	0 (not solved)
4904	141	241	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4905	141	242	1 (pass)	0 (not solved)
4906	141	243	1 (pass)	0 (not solved)
4907	141	244	1 (pass)	0 (not solved)
4908	141	245	1 (pass)	0 (not solved)
4909	141	246	1 (pass)	0 (not solved)
4910	141	247	1 (pass)	0 (not solved)
4911	141	403	1 (pass)	0 (not solved)
4912	141	404	1 (pass)	0 (not solved)
4913	141	405	1 (pass)	0 (not solved)
4914	141	406	1 (pass)	0 (not solved)
4915	141	407	1 (pass)	0 (not solved)
4916	141	408	1 (pass)	0 (not solved)
4917	141	409	1 (pass)	0 (not solved)
4918	141	410	1 (pass)	0 (not solved)
4919	141	411	1 (pass)	0 (not solved)
4920	141	412	1 (pass)	0 (not solved)
4921	141	413	1 (pass)	0 (not solved)
4922	141	414	1 (pass)	0 (not solved)
4923	141	415	1 (pass)	0 (not solved)
4924	141	416	1 (pass)	0 (not solved)
4925	141	448	1 (pass)	0 (not solved)
4926	141	449	1 (pass)	0 (not solved)
4927	141	450	1 (pass)	0 (not solved)
4928	141	451	1 (pass)	0 (not solved)
4929	141	452	1 (pass)	0 (not solved)
4930	141	453	1 (pass)	0 (not solved)
4931	141	454	1 (pass)	0 (not solved)
4932	141	455	1 (pass)	0 (not solved)
4933	141	456	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4934	141	457	1 (pass)	0 (not solved)
4935	141	462	1 (pass)	0 (not solved)
4936	141	463	1 (pass)	0 (not solved)
4937	141	464	1 (pass)	0 (not solved)
4938	141	465	1 (pass)	0 (not solved)
4939	141	466	1 (pass)	0 (not solved)
4940	141	467	1 (pass)	0 (not solved)
4941	141	468	1 (pass)	0 (not solved)
4942	141	469	1 (pass)	0 (not solved)
4943	141	470	1 (pass)	0 (not solved)
4944	141	471	1 (pass)	0 (not solved)
4945	141	556	1 (pass)	0 (not solved)
4946	141	557	1 (pass)	0 (not solved)
4947	141	558	1 (pass)	0 (not solved)
4948	141	559	1 (pass)	0 (not solved)
4949	141	560	1 (pass)	0 (not solved)
4950	141	561	1 (pass)	0 (not solved)
4951	141	576	1 (pass)	0 (not solved)
4952	141	938	1 (pass)	0 (not solved)
4953	141	939	1 (pass)	0 (not solved)
4954	142	203	1 (pass)	0 (not solved)
4955	142	204	1 (pass)	0 (not solved)
4956	142	205	1 (pass)	0 (not solved)
4957	142	206	1 (pass)	0 (not solved)
4958	142	207	1 (pass)	0 (not solved)
4959	142	208	1 (pass)	0 (not solved)
4960	144	281	1 (pass)	0 (not solved)
4961	144	282	1 (pass)	0 (not solved)
4962	144	283	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4963	144	284	1 (pass)	0 (not solved)
4964	144	285	1 (pass)	0 (not solved)
4965	144	286	1 (pass)	0 (not solved)
4966	144	287	1 (pass)	0 (not solved)
4967	144	288	1 (pass)	0 (not solved)
4968	144	289	1 (pass)	0 (not solved)
4969	144	290	1 (pass)	0 (not solved)
4970	144	352	1 (pass)	0 (not solved)
4971	144	353	1 (pass)	0 (not solved)
4972	144	354	1 (pass)	0 (not solved)
4973	144	355	1 (pass)	0 (not solved)
4974	144	356	1 (pass)	0 (not solved)
4975	144	393	1 (pass)	0 (not solved)
4976	144	394	1 (pass)	0 (not solved)
4977	144	395	1 (pass)	0 (not solved)
4978	144	396	1 (pass)	0 (not solved)
4979	145	203	1 (pass)	0 (not solved)
4980	145	204	1 (pass)	0 (not solved)
4981	145	205	1 (pass)	0 (not solved)
4982	145	206	1 (pass)	0 (not solved)
4983	145	207	1 (pass)	0 (not solved)
4984	145	208	1 (pass)	0 (not solved)
4985	147	48	1 (pass)	0 (not solved)
4986	147	50	1 (pass)	0 (not solved)
4987	147	52	1 (pass)	0 (not solved)
4988	150	48	1 (pass)	0 (not solved)
4989	150	49	1 (pass)	0 (not solved)
4990	150	50	1 (pass)	0 (not solved)
4991	150	55	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
4992	150	56	1 (pass)	0 (not solved)
4993	150	57	1 (pass)	0 (not solved)
4994	150	63	1 (pass)	0 (not solved)
4995	150	64	1 (pass)	0 (not solved)
4996	150	65	1 (pass)	0 (not solved)
4997	150	99	1 (pass)	0 (not solved)
4998	150	130	1 (pass)	0 (not solved)
4999	153	18	1 (pass)	0 (not solved)
5000	153	19	1 (pass)	0 (not solved)
5001	153	20	1 (pass)	0 (not solved)
5002	153	21	1 (pass)	0 (not solved)
5003	153	22	1 (pass)	0 (not solved)
5004	153	23	1 (pass)	0 (not solved)
5005	153	24	1 (pass)	0 (not solved)
5006	153	25	1 (pass)	0 (not solved)
5007	153	26	1 (pass)	0 (not solved)
5008	153	27	1 (pass)	0 (not solved)
5009	153	28	1 (pass)	0 (not solved)
5010	153	29	1 (pass)	0 (not solved)
5011	156	148	1 (pass)	0 (not solved)
5012	156	170	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5013	156	171	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5014	156	172	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5015	158	154	1 (pass)	0 (not solved)
5016	158	174	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5017	158	175	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5018	158	176	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5019	160	248	1 (pass)	-1 (time out)
5020	160	401	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5021	160	445	1 (pass)	-1 (time out)
5022	160	473	1 (pass)	-1 (time out)
5023	160	486	1 (pass)	-1 (time out)
5024	160	491	1 (pass)	-1 (time out)
5025	160	496	1 (pass)	-1 (time out)
5026	160	500	1 (pass)	-1 (time out)
5027	163	7	1 (pass)	0 (not solved)
5028	163	8	1 (pass)	0 (not solved)
5029	163	9	1 (pass)	0 (not solved)
5030	163	10	1 (pass)	0 (not solved)
5031	163	11	1 (pass)	0 (not solved)
5032	163	12	1 (pass)	0 (not solved)
5033	163	13	1 (pass)	0 (not solved)
5034	163	14	1 (pass)	0 (not solved)
5035	163	15	1 (pass)	0 (not solved)
5036	163	16	1 (pass)	0 (not solved)
5037	163	17	1 (pass)	0 (not solved)
5038	163	18	1 (pass)	0 (not solved)
5039	163	19	1 (pass)	0 (not solved)
5040	163	20	1 (pass)	0 (not solved)
5041	163	21	1 (pass)	0 (not solved)
5042	163	22	1 (pass)	0 (not solved)
5043	163	23	1 (pass)	0 (not solved)
5044	163	24	1 (pass)	0 (not solved)
5045	163	25	1 (pass)	0 (not solved)
5046	163	26	1 (pass)	0 (not solved)
5047	163	27	1 (pass)	0 (not solved)
5048	163	28	1 (pass)	0 (not solved)
5049	163	29	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5050	163	30	1 (pass)	0 (not solved)
5051	163	105	1 (pass)	0 (not solved)
5052	163	106	1 (pass)	0 (not solved)
5053	163	107	1 (pass)	0 (not solved)
5054	163	108	1 (pass)	0 (not solved)
5055	163	109	1 (pass)	0 (not solved)
5056	163	110	1 (pass)	0 (not solved)
5057	163	111	1 (pass)	0 (not solved)
5058	163	126	1 (pass)	0 (not solved)
5059	163	127	1 (pass)	0 (not solved)
5060	163	128	1 (pass)	0 (not solved)
5061	163	137	1 (pass)	0 (not solved)
5062	163	138	1 (pass)	0 (not solved)
5063	163	139	1 (pass)	0 (not solved)
5064	163	146	1 (pass)	0 (not solved)
5065	163	147	1 (pass)	0 (not solved)
5066	163	148	1 (pass)	0 (not solved)
5067	163	149	1 (pass)	0 (not solved)
5068	163	150	1 (pass)	0 (not solved)
5069	163	151	1 (pass)	0 (not solved)
5070	163	279	1 (pass)	0 (not solved)
5071	163	280	1 (pass)	0 (not solved)
5072	163	281	1 (pass)	0 (not solved)
5073	163	282	1 (pass)	0 (not solved)
5074	163	283	1 (pass)	0 (not solved)
5075	163	284	1 (pass)	0 (not solved)
5076	164	58	1 (pass)	-1 (time out)
5077	164	59	1 (pass)	-1 (time out)
5078	164	104	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5079	164	127	1 (pass)	0 (not solved)
5080	164	129	1 (pass)	0 (not solved)
5081	164	171	1 (pass)	-1 (time out)
5082	164	172	1 (pass)	-1 (time out)
5083	164	173	1 (pass)	-1 (time out)
5084	164	174	1 (pass)	-1 (time out)
5085	164	175	1 (pass)	-1 (time out)
5086	164	176	1 (pass)	-1 (time out)
5087	164	177	1 (pass)	-1 (time out)
5088	164	178	1 (pass)	-1 (time out)
5089	164	179	1 (pass)	-1 (time out)
5090	164	180	1 (pass)	-1 (time out)
5091	164	181	1 (pass)	-1 (time out)
5092	164	215	1 (pass)	-1 (time out)
5093	164	216	1 (pass)	-1 (time out)
5094	164	253	1 (pass)	-1 (time out)
5095	164	254	1 (pass)	-1 (time out)
5096	164	255	1 (pass)	-1 (time out)
5097	164	256	1 (pass)	-1 (time out)
5098	164	257	1 (pass)	-1 (time out)
5099	164	258	1 (pass)	-1 (time out)
5100	164	259	1 (pass)	-1 (time out)
5101	164	260	1 (pass)	-1 (time out)
5102	164	261	1 (pass)	-1 (time out)
5103	164	262	1 (pass)	-1 (time out)
5104	164	263	1 (pass)	-1 (time out)
5105	164	264	1 (pass)	-1 (time out)
5106	164	268	1 (pass)	-1 (time out)
5107	164	269	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5108	164	347	1 (pass)	-1 (time out)
5109	164	348	1 (pass)	-1 (time out)
5110	164	379	1 (pass)	0 (not solved)
5111	164	414	1 (pass)	-1 (time out)
5112	164	418	1 (pass)	-1 (time out)
5113	164	487	1 (pass)	0 (not solved)
5114	164	501	1 (pass)	-1 (time out)
5115	164	506	1 (pass)	-1 (time out)
5116	169	7	1 (pass)	0 (not solved)
5117	169	8	1 (pass)	0 (not solved)
5118	169	9	1 (pass)	0 (not solved)
5119	169	10	1 (pass)	0 (not solved)
5120	169	11	1 (pass)	0 (not solved)
5121	169	12	1 (pass)	0 (not solved)
5122	169	13	1 (pass)	0 (not solved)
5123	169	14	1 (pass)	0 (not solved)
5124	169	15	1 (pass)	0 (not solved)
5125	169	16	1 (pass)	0 (not solved)
5126	169	17	1 (pass)	0 (not solved)
5127	169	18	1 (pass)	0 (not solved)
5128	169	19	1 (pass)	0 (not solved)
5129	169	20	1 (pass)	0 (not solved)
5130	169	21	1 (pass)	0 (not solved)
5131	169	22	1 (pass)	0 (not solved)
5132	169	79	1 (pass)	0 (not solved)
5133	169	80	1 (pass)	0 (not solved)
5134	169	81	1 (pass)	0 (not solved)
5135	169	82	1 (pass)	0 (not solved)
5136	169	83	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5137	169	84	1 (pass)	0 (not solved)
5138	169	85	1 (pass)	0 (not solved)
5139	169	86	1 (pass)	0 (not solved)
5140	169	107	1 (pass)	0 (not solved)
5141	169	108	1 (pass)	0 (not solved)
5142	169	109	1 (pass)	0 (not solved)
5143	169	118	1 (pass)	0 (not solved)
5144	169	119	1 (pass)	0 (not solved)
5145	169	120	1 (pass)	0 (not solved)
5146	169	128	1 (pass)	0 (not solved)
5147	169	129	1 (pass)	0 (not solved)
5148	169	130	1 (pass)	0 (not solved)
5149	169	131	1 (pass)	0 (not solved)
5150	169	132	1 (pass)	0 (not solved)
5151	169	133	1 (pass)	0 (not solved)
5152	169	252	1 (pass)	0 (not solved)
5153	169	253	1 (pass)	0 (not solved)
5154	169	254	1 (pass)	0 (not solved)
5155	169	255	1 (pass)	0 (not solved)
5156	169	256	1 (pass)	0 (not solved)
5157	169	257	1 (pass)	0 (not solved)
5158	170	51	1 (pass)	0 (not solved)
5159	170	52	1 (pass)	0 (not solved)
5160	170	55	1 (pass)	0 (not solved)
5161	170	56	1 (pass)	-1 (time out)
5162	170	57	1 (pass)	-1 (time out)
5163	170	65	1 (pass)	-1 (time out)
5164	170	66	1 (pass)	-1 (time out)
5165	170	68	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5166	170	69	1 (pass)	-1 (time out)
5167	173	41	1 (pass)	-1 (time out)
5168	173	42	1 (pass)	-1 (time out)
5169	173	47	1 (pass)	-1 (time out)
5170	173	48	1 (pass)	-1 (time out)
5171	173	65	1 (pass)	-1 (time out)
5172	173	73	1 (pass)	-1 (time out)
5173	173	74	1 (pass)	-1 (time out)
5174	173	76	1 (pass)	-1 (time out)
5175	173	77	1 (pass)	-1 (time out)
5176	173	79	1 (pass)	-1 (time out)
5177	173	200	1 (pass)	-1 (time out)
5178	173	201	1 (pass)	-1 (time out)
5179	173	220	1 (pass)	-1 (time out)
5180	173	246	1 (pass)	-1 (time out)
5181	173	253	1 (pass)	-1 (time out)
5182	173	259	1 (pass)	-1 (time out)
5183	173	263	1 (pass)	-1 (time out)
5184	175	27	1 (pass)	-1 (time out)
5185	175	49	1 (pass)	-1 (time out)
5186	176	8	1 (pass)	-1 (time out)
5187	176	23	1 (pass)	-1 (time out)
5188	176	46	1 (pass)	-1 (time out)
5189	178	84	1 (pass)	-1 (time out)
5190	179	9	1 (pass)	0 (not solved)
5191	179	10	1 (pass)	0 (not solved)
5192	179	11	1 (pass)	0 (not solved)
5193	179	12	1 (pass)	0 (not solved)
5194	179	13	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5195	179	14	1 (pass)	0 (not solved)
5196	179	15	1 (pass)	0 (not solved)
5197	179	16	1 (pass)	0 (not solved)
5198	179	17	1 (pass)	0 (not solved)
5199	179	18	1 (pass)	0 (not solved)
5200	179	19	1 (pass)	0 (not solved)
5201	179	20	1 (pass)	0 (not solved)
5202	179	21	1 (pass)	0 (not solved)
5203	179	22	1 (pass)	0 (not solved)
5204	179	39	1 (pass)	0 (not solved)
5205	179	40	1 (pass)	0 (not solved)
5206	179	41	1 (pass)	0 (not solved)
5207	179	42	1 (pass)	0 (not solved)
5208	179	43	1 (pass)	0 (not solved)
5209	179	44	1 (pass)	0 (not solved)
5210	179	129	1 (pass)	-1 (time out)
5211	179	137	1 (pass)	-1 (time out)
5212	179	145	1 (pass)	-1 (time out)
5213	179	146	1 (pass)	-1 (time out)
5214	179	158	1 (pass)	0 (not solved)
5215	179	162	1 (pass)	0 (not solved)
5216	179	164	1 (pass)	0 (not solved)
5217	179	168	1 (pass)	0 (not solved)
5218	179	170	1 (pass)	0 (not solved)
5219	179	174	1 (pass)	0 (not solved)
5220	179	180	1 (pass)	0 (not solved)
5221	179	196	1 (pass)	0 (not solved)
5222	179	197	1 (pass)	0 (not solved)
5223	179	198	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5224	179	199	1 (pass)	0 (not solved)
5225	179	200	1 (pass)	0 (not solved)
5226	179	201	1 (pass)	0 (not solved)
5227	180	47	1 (pass)	-1 (time out)
5228	180	48	1 (pass)	-1 (time out)
5229	180	168	1 (pass)	-1 (time out)
5230	180	169	1 (pass)	-1 (time out)
5231	180	186	1 (pass)	-1 (time out)
5232	180	193	1 (pass)	-1 (time out)
5233	180	211	1 (pass)	-1 (time out)
5234	180	218	1 (pass)	-1 (time out)
5235	180	220	1 (pass)	-1 (time out)
5236	182	83	1 (pass)	-1 (time out)
5237	183	7	1 (pass)	0 (not solved)
5238	183	8	1 (pass)	0 (not solved)
5239	183	9	1 (pass)	0 (not solved)
5240	183	10	1 (pass)	0 (not solved)
5241	183	11	1 (pass)	0 (not solved)
5242	183	12	1 (pass)	0 (not solved)
5243	183	13	1 (pass)	0 (not solved)
5244	183	14	1 (pass)	0 (not solved)
5245	183	15	1 (pass)	0 (not solved)
5246	183	16	1 (pass)	0 (not solved)
5247	183	17	1 (pass)	0 (not solved)
5248	183	18	1 (pass)	0 (not solved)
5249	183	19	1 (pass)	0 (not solved)
5250	183	20	1 (pass)	0 (not solved)
5251	183	36	1 (pass)	0 (not solved)
5252	183	37	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5253	183	38	1 (pass)	0 (not solved)
5254	183	39	1 (pass)	0 (not solved)
5255	183	40	1 (pass)	0 (not solved)
5256	183	41	1 (pass)	0 (not solved)
5257	183	132	1 (pass)	0 (not solved)
5258	183	136	1 (pass)	0 (not solved)
5259	183	140	1 (pass)	0 (not solved)
5260	183	142	1 (pass)	0 (not solved)
5261	183	144	1 (pass)	0 (not solved)
5262	183	148	1 (pass)	0 (not solved)
5263	183	152	1 (pass)	0 (not solved)
5264	183	154	1 (pass)	0 (not solved)
5265	183	170	1 (pass)	0 (not solved)
5266	183	171	1 (pass)	0 (not solved)
5267	183	172	1 (pass)	0 (not solved)
5268	183	173	1 (pass)	0 (not solved)
5269	183	174	1 (pass)	0 (not solved)
5270	183	175	1 (pass)	0 (not solved)
5271	184	8	1 (pass)	-1 (time out)
5272	184	9	1 (pass)	-1 (time out)
5273	184	15	1 (pass)	-1 (time out)
5274	185	590	1 (pass)	0 (not solved)
5275	185	591	1 (pass)	0 (not solved)
5276	185	592	1 (pass)	0 (not solved)
5277	185	593	1 (pass)	0 (not solved)
5278	185	594	1 (pass)	0 (not solved)
5279	185	595	1 (pass)	0 (not solved)
5280	185	745	1 (pass)	-1 (time out)
5281	185	761	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5282	185	762	1 (pass)	0 (not solved)
5283	185	763	1 (pass)	0 (not solved)
5284	185	764	1 (pass)	0 (not solved)
5285	185	765	1 (pass)	0 (not solved)
5286	185	766	1 (pass)	0 (not solved)
5287	185	767	1 (pass)	0 (not solved)
5288	185	864	1 (pass)	0 (not solved)
5289	188	228	1 (pass)	0 (not solved)
5290	188	229	1 (pass)	0 (not solved)
5291	188	230	1 (pass)	0 (not solved)
5292	188	231	1 (pass)	0 (not solved)
5293	188	232	1 (pass)	0 (not solved)
5294	188	233	1 (pass)	0 (not solved)
5295	188	234	1 (pass)	0 (not solved)
5296	188	235	1 (pass)	0 (not solved)
5297	188	284	1 (pass)	0 (not solved)
5298	188	288	1 (pass)	0 (not solved)
5299	190	141	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5300	191	198	1 (pass)	0 (not solved)
5301	191	199	1 (pass)	0 (not solved)
5302	191	200	1 (pass)	0 (not solved)
5303	191	201	1 (pass)	0 (not solved)
5304	191	202	1 (pass)	0 (not solved)
5305	191	203	1 (pass)	0 (not solved)
5306	191	204	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5307	191	205	1 (pass)	0 (not solved)
5308	193	35	1 (pass)	-1 (time out)
5309	194	119	1 (pass)	0 (not solved)
5310	194	137	1 (pass)	0 (not solved)
5311	194	138	1 (pass)	0 (not solved)
5312	197	16	1 (pass)	0 (not solved)
5313	197	17	1 (pass)	0 (not solved)
5314	197	18	1 (pass)	0 (not solved)
5315	197	19	1 (pass)	0 (not solved)
5316	197	20	1 (pass)	0 (not solved)
5317	197	21	1 (pass)	0 (not solved)
5318	197	22	1 (pass)	0 (not solved)
5319	197	23	1 (pass)	0 (not solved)
5320	197	24	1 (pass)	0 (not solved)
5321	197	25	1 (pass)	0 (not solved)
5322	197	26	1 (pass)	0 (not solved)
5323	197	27	1 (pass)	0 (not solved)
5324	199	359	1 (pass)	0 (not solved)
5325	199	360	1 (pass)	0 (not solved)
5326	199	361	1 (pass)	0 (not solved)
5327	200	166	1 (pass)	0 (not solved)
5328	201	46	1 (pass)	0 (not solved)
5329	201	48	1 (pass)	0 (not solved)
5330	201	50	1 (pass)	0 (not solved)
5331	201	54	1 (pass)	0 (not solved)
5332	202	154	1 (pass)	0 (not solved)
5333	203	40	1 (pass)	0 (not solved)
5334	203	44	1 (pass)	0 (not solved)
5335	205	1	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5336	205	2	1 (pass)	0 (not solved)
5337	205	3	1 (pass)	0 (not solved)
5338	205	4	1 (pass)	0 (not solved)
5339	205	5	1 (pass)	0 (not solved)
5340	205	6	1 (pass)	0 (not solved)
5341	205	7	1 (pass)	0 (not solved)
5342	205	8	1 (pass)	0 (not solved)
5343	205	10	1 (pass)	0 (not solved)
5344	205	11	1 (pass)	0 (not solved)
5345	205	12	1 (pass)	0 (not solved)
5346	205	13	1 (pass)	0 (not solved)
5347	205	14	1 (pass)	0 (not solved)
5348	205	15	1 (pass)	0 (not solved)
5349	205	16	1 (pass)	0 (not solved)
5350	205	17	1 (pass)	0 (not solved)
5351	205	18	1 (pass)	0 (not solved)
5352	205	19	1 (pass)	0 (not solved)
5353	205	20	1 (pass)	0 (not solved)
5354	205	21	1 (pass)	0 (not solved)
5355	205	22	1 (pass)	0 (not solved)
5356	205	25	1 (pass)	0 (not solved)
5357	205	26	1 (pass)	0 (not solved)
5358	205	27	1 (pass)	0 (not solved)
5359	205	28	1 (pass)	0 (not solved)
5360	205	31	1 (pass)	0 (not solved)
5361	205	32	1 (pass)	0 (not solved)
5362	205	34	1 (pass)	0 (not solved)
5363	205	35	1 (pass)	0 (not solved)
5364	205	36	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5365	205	38	1 (pass)	0 (not solved)
5366	205	43	1 (pass)	0 (not solved)
5367	205	47	1 (pass)	0 (not solved)
5368	205	51	1 (pass)	0 (not solved)
5369	205	54	1 (pass)	0 (not solved)
5370	205	55	1 (pass)	0 (not solved)
5371	205	56	1 (pass)	0 (not solved)
5372	205	57	1 (pass)	0 (not solved)
5373	205	58	1 (pass)	0 (not solved)
5374	205	59	1 (pass)	0 (not solved)
5375	205	60	1 (pass)	0 (not solved)
5376	205	65	1 (pass)	0 (not solved)
5377	205	66	1 (pass)	0 (not solved)
5378	205	67	1 (pass)	0 (not solved)
5379	205	68	1 (pass)	0 (not solved)
5380	205	69	1 (pass)	0 (not solved)
5381	205	70	1 (pass)	0 (not solved)
5382	205	71	1 (pass)	0 (not solved)
5383	205	72	1 (pass)	0 (not solved)
5384	205	74	1 (pass)	0 (not solved)
5385	205	75	1 (pass)	0 (not solved)
5386	205	76	1 (pass)	0 (not solved)
5387	205	78	1 (pass)	0 (not solved)
5388	205	79	1 (pass)	0 (not solved)
5389	205	83	1 (pass)	0 (not solved)
5390	205	87	1 (pass)	0 (not solved)
5391	205	92	1 (pass)	0 (not solved)
5392	205	93	1 (pass)	0 (not solved)
5393	205	94	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5394	205	96	1 (pass)	0 (not solved)
5395	205	97	1 (pass)	0 (not solved)
5396	205	98	1 (pass)	0 (not solved)
5397	205	101	1 (pass)	0 (not solved)
5398	205	105	1 (pass)	0 (not solved)
5399	205	109	1 (pass)	0 (not solved)
5400	205	110	1 (pass)	0 (not solved)
5401	205	111	1 (pass)	0 (not solved)
5402	205	112	1 (pass)	0 (not solved)
5403	205	113	1 (pass)	0 (not solved)
5404	205	114	1 (pass)	0 (not solved)
5405	205	115	1 (pass)	0 (not solved)
5406	205	116	1 (pass)	0 (not solved)
5407	205	117	1 (pass)	0 (not solved)
5408	205	119	1 (pass)	0 (not solved)
5409	205	120	1 (pass)	0 (not solved)
5410	205	121	1 (pass)	0 (not solved)
5411	205	122	1 (pass)	0 (not solved)
5412	205	123	1 (pass)	0 (not solved)
5413	205	124	1 (pass)	0 (not solved)
5414	205	125	1 (pass)	0 (not solved)
5415	205	126	1 (pass)	0 (not solved)
5416	205	127	1 (pass)	0 (not solved)
5417	205	128	1 (pass)	0 (not solved)
5418	205	129	1 (pass)	0 (not solved)
5419	205	130	1 (pass)	0 (not solved)
5420	205	131	1 (pass)	0 (not solved)
5421	205	134	1 (pass)	0 (not solved)
5422	205	135	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5423	205	136	1 (pass)	0 (not solved)
5424	205	137	1 (pass)	0 (not solved)
5425	205	140	1 (pass)	0 (not solved)
5426	205	141	1 (pass)	0 (not solved)
5427	205	143	1 (pass)	0 (not solved)
5428	205	144	1 (pass)	0 (not solved)
5429	205	145	1 (pass)	0 (not solved)
5430	205	147	1 (pass)	0 (not solved)
5431	205	152	1 (pass)	0 (not solved)
5432	205	156	1 (pass)	0 (not solved)
5433	205	160	1 (pass)	0 (not solved)
5434	205	163	1 (pass)	0 (not solved)
5435	205	164	1 (pass)	0 (not solved)
5436	205	165	1 (pass)	0 (not solved)
5437	205	166	1 (pass)	0 (not solved)
5438	205	167	1 (pass)	0 (not solved)
5439	205	168	1 (pass)	0 (not solved)
5440	205	169	1 (pass)	0 (not solved)
5441	205	174	1 (pass)	0 (not solved)
5442	205	175	1 (pass)	0 (not solved)
5443	205	176	1 (pass)	0 (not solved)
5444	205	177	1 (pass)	0 (not solved)
5445	205	178	1 (pass)	0 (not solved)
5446	205	179	1 (pass)	0 (not solved)
5447	205	180	1 (pass)	0 (not solved)
5448	205	181	1 (pass)	0 (not solved)
5449	205	183	1 (pass)	0 (not solved)
5450	205	184	1 (pass)	0 (not solved)
5451	205	185	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5452	205	187	1 (pass)	0 (not solved)
5453	205	188	1 (pass)	0 (not solved)
5454	205	192	1 (pass)	0 (not solved)
5455	205	196	1 (pass)	0 (not solved)
5456	205	201	1 (pass)	0 (not solved)
5457	205	202	1 (pass)	0 (not solved)
5458	205	203	1 (pass)	0 (not solved)
5459	205	205	1 (pass)	0 (not solved)
5460	205	206	1 (pass)	0 (not solved)
5461	205	207	1 (pass)	0 (not solved)
5462	205	210	1 (pass)	0 (not solved)
5463	205	214	1 (pass)	0 (not solved)
5464	205	218	1 (pass)	0 (not solved)
5465	206	1	1 (pass)	0 (not solved)
5466	206	2	1 (pass)	0 (not solved)
5467	206	3	1 (pass)	0 (not solved)
5468	206	4	1 (pass)	0 (not solved)
5469	206	5	1 (pass)	0 (not solved)
5470	206	7	1 (pass)	0 (not solved)
5471	206	8	1 (pass)	0 (not solved)
5472	206	10	1 (pass)	0 (not solved)
5473	206	11	1 (pass)	0 (not solved)
5474	206	12	1 (pass)	0 (not solved)
5475	206	13	1 (pass)	0 (not solved)
5476	206	16	1 (pass)	0 (not solved)
5477	206	18	1 (pass)	0 (not solved)
5478	206	19	1 (pass)	0 (not solved)
5479	206	20	1 (pass)	0 (not solved)
5480	206	21	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5481	206	23	1 (pass)	0 (not solved)
5482	206	24	1 (pass)	0 (not solved)
5483	206	26	1 (pass)	0 (not solved)
5484	206	27	1 (pass)	0 (not solved)
5485	206	28	1 (pass)	0 (not solved)
5486	206	32	1 (pass)	0 (not solved)
5487	206	33	1 (pass)	0 (not solved)
5488	206	34	1 (pass)	0 (not solved)
5489	206	35	1 (pass)	0 (not solved)
5490	206	36	1 (pass)	0 (not solved)
5491	206	37	1 (pass)	0 (not solved)
5492	206	38	1 (pass)	0 (not solved)
5493	206	39	1 (pass)	0 (not solved)
5494	206	41	1 (pass)	0 (not solved)
5495	206	42	1 (pass)	0 (not solved)
5496	206	43	1 (pass)	0 (not solved)
5497	206	44	1 (pass)	0 (not solved)
5498	206	45	1 (pass)	0 (not solved)
5499	206	47	1 (pass)	0 (not solved)
5500	206	49	1 (pass)	0 (not solved)
5501	206	50	1 (pass)	0 (not solved)
5502	206	51	1 (pass)	0 (not solved)
5503	206	52	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5504	206	53	1 (pass)	<p>-2 (exception)</p> <p>Exception raised: TypeError >> An error occurred when FriCAS evaluated '(operator(Si)((x*(2)))*(sin((x)*(5)))': There are 1 exposed and 1 unexposed library operations named elt having 1 argument(s) but none was determined to be applicable. Use HyperDoc Browse, or issue</p> <p>)display op elt to learn more about the available operations. Perhaps package-calling the operation or using coercions on the arguments will allow you to apply the operation. Cannot find application of object of type BasicOperator to argument(s) of type(s)</p> <p>Polynomial(Integer)</p>
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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5505	206	54	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred when FriCAS evaluated '(operator(Si)((x*(2)))*(cos((x)*(5)))': There are 1 exposed and 1 unexposed library operations named elt having 1 argument(s) but none was determined to be applicable. Use HyperDoc Browse, or issue)display op elt to learn more about the available operations. Perhaps package-calling the operation or using coercions on the arguments will allow you to apply the operation. Cannot find application of object of type BasicOperator to argument(s) of type(s) Polynomial(Integer)
5506	206	55	1 (pass)	0 (not solved)
5507	206	56	1 (pass)	0 (not solved)
5508	206	57	1 (pass)	0 (not solved)
5509	206	59	1 (pass)	0 (not solved)
5510	206	60	1 (pass)	0 (not solved)
5511	206	61	1 (pass)	0 (not solved)
5512	206	63	1 (pass)	0 (not solved)
5513	206	64	1 (pass)	0 (not solved)
5514	206	66	1 (pass)	0 (not solved)
5515	206	67	1 (pass)	0 (not solved)
5516	206	69	1 (pass)	0 (not solved)
5517	206	70	1 (pass)	0 (not solved)
5518	206	71	1 (pass)	0 (not solved)
5519	206	72	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5520	206	73	1 (pass)	0 (not solved)
5521	206	75	1 (pass)	0 (not solved)
5522	206	76	1 (pass)	0 (not solved)
5523	206	78	1 (pass)	0 (not solved)
5524	206	79	1 (pass)	0 (not solved)
5525	206	81	1 (pass)	0 (not solved)
5526	206	86	1 (pass)	0 (not solved)
5527	206	87	1 (pass)	0 (not solved)
5528	206	88	1 (pass)	0 (not solved)
5529	206	89	1 (pass)	0 (not solved)
5530	206	96	1 (pass)	0 (not solved)
5531	206	100	1 (pass)	0 (not solved)
5532	206	101	1 (pass)	0 (not solved)
5533	206	102	1 (pass)	0 (not solved)
5534	206	103	1 (pass)	0 (not solved)
5535	206	104	1 (pass)	0 (not solved)
5536	206	105	1 (pass)	0 (not solved)
5537	206	106	1 (pass)	0 (not solved)
5538	206	110	1 (pass)	0 (not solved)
5539	206	111	1 (pass)	0 (not solved)
5540	206	112	1 (pass)	0 (not solved)
5541	206	113	1 (pass)	0 (not solved)
5542	206	117	1 (pass)	0 (not solved)
5543	206	118	1 (pass)	0 (not solved)
5544	206	119	1 (pass)	0 (not solved)
5545	206	120	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5546	206	121	1 (pass)	<p>-2 (exception)</p> <p>Exception raised: TypeError >> An error occurred when FriCAS evaluated '(operator(Ci)((x*(2)))*(sin((x)*(5)))': There are 1 exposed and 1 unexposed library operations named elt having 1 argument(s) but none was determined to be applicable. Use HyperDoc Browse, or issue</p> <p>)display op elt to learn more about the available operations. Perhaps package-calling the operation or using coercions on the arguments will allow you to apply the operation. Cannot find application of object of type BasicOperator to argument(s) of type(s)</p> <p>Polynomial(Integer)</p>

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5547	206	122	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred when FriCAS evaluated '(operator(Ci)((x*(2)))*(cos((x)*(5)))': There are 1 exposed and 1 unexposed library operations named elt having 1 argument(s) but none was determined to be applicable. Use HyperDoc Browse, or issue)display op elt to learn more about the available operations. Perhaps package-calling the operation or using coercions on the arguments will allow you to apply the operation. Cannot find application of object of type BasicOperator to argument(s) of type(s) Polynomial(Integer)
5548	206	123	1 (pass)	0 (not solved)
5549	206	124	1 (pass)	0 (not solved)
5550	206	125	1 (pass)	0 (not solved)
5551	206	127	1 (pass)	0 (not solved)
5552	206	128	1 (pass)	0 (not solved)
5553	206	129	1 (pass)	0 (not solved)
5554	206	131	1 (pass)	0 (not solved)
5555	206	132	1 (pass)	0 (not solved)
5556	206	134	1 (pass)	0 (not solved)
5557	206	135	1 (pass)	0 (not solved)
5558	208	145	1 (pass)	0 (not solved)
5559	208	146	1 (pass)	0 (not solved)
5560	209	204	1 (pass)	-1 (time out)
5561	209	587	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5562	209	946	1 (pass)	-1 (time out)
5563	209	1062	1 (pass)	-1 (time out)
5564	209	1063	1 (pass)	-1 (time out)
5565	209	1074	1 (pass)	-1 (time out)
5566	209	1075	1 (pass)	-1 (time out)
5567	209	1087	1 (pass)	-1 (time out)
5568	209	1088	1 (pass)	-1 (time out)
5569	209	1100	1 (pass)	-1 (time out)
5570	209	1101	1 (pass)	-1 (time out)
5571	209	1115	1 (pass)	-1 (time out)
5572	209	1197	1 (pass)	-1 (time out)
5573	209	1198	1 (pass)	-1 (time out)
5574	209	1344	1 (pass)	-1 (time out)
5575	209	1345	1 (pass)	-1 (time out)
5576	209	1366	1 (pass)	-1 (time out)
5577	209	1367	1 (pass)	-1 (time out)
5578	209	1456	1 (pass)	-1 (time out)
5579	209	1457	1 (pass)	-1 (time out)
5580	209	1541	1 (pass)	-1 (time out)
5581	209	1601	1 (pass)	-1 (time out)
5582	209	1868	1 (pass)	-1 (time out)
5583	209	1872	1 (pass)	-1 (time out)
5584	209	1886	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5585	209	1907	1 (pass)	-1 (time out)
5586	209	1908	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5587	209	1926	1 (pass)	-1 (time out)
5588	209	1927	1 (pass)	-1 (time out)
5589	209	1979	1 (pass)	-1 (time out)
5590	209	2026	1 (pass)	-1 (time out)
5591	209	2067	1 (pass)	-1 (time out)
5592	209	2068	1 (pass)	-1 (time out)
5593	209	2141	1 (pass)	-1 (time out)
5594	209	2144	1 (pass)	-1 (time out)
5595	209	2145	1 (pass)	-1 (time out)
5596	209	2279	1 (pass)	-1 (time out)
5597	209	2333	1 (pass)	-1 (time out)
5598	209	2357	1 (pass)	-1 (time out)
5599	209	2377	1 (pass)	-1 (time out)
5600	209	2426	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5601	209	2427	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5602	209	2435	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5603	209	2436	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5604	209	2443	1 (pass)	-1 (time out)
5605	209	2444	1 (pass)	-1 (time out)
5606	209	2446	1 (pass)	-1 (time out)
5607	209	2447	1 (pass)	-1 (time out)
5608	209	2452	1 (pass)	-1 (time out)
5609	209	2453	1 (pass)	-1 (time out)
5610	209	2458	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5611	209	2459	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5612	209	2460	1 (pass)	-1 (time out)
5613	209	2461	1 (pass)	-1 (time out)
5614	209	2464	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5615	209	2465	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5616	209	2486	1 (pass)	-1 (time out)
5617	209	2487	1 (pass)	-1 (time out)
5618	209	2520	1 (pass)	-1 (time out)
5619	209	2521	1 (pass)	-1 (time out)
5620	209	2603	1 (pass)	-1 (time out)
5621	209	2604	1 (pass)	-1 (time out)
5622	209	2646	1 (pass)	-1 (time out)
5623	209	2647	1 (pass)	-1 (time out)
5624	209	2720	1 (pass)	-1 (time out)
5625	209	2799	1 (pass)	-1 (time out)
5626	209	2830	1 (pass)	-1 (time out)
5627	209	2831	1 (pass)	-1 (time out)
5628	209	2876	1 (pass)	-1 (time out)
5629	209	2877	1 (pass)	-1 (time out)
5630	209	2878	1 (pass)	-1 (time out)
5631	209	2891	1 (pass)	-1 (time out)
5632	209	2892	1 (pass)	-1 (time out)
5633	209	2901	1 (pass)	-1 (time out)
5634	209	2902	1 (pass)	-1 (time out)
5635	209	2903	1 (pass)	-1 (time out)
5636	209	2904	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
5637	209	2939	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
5638	209	2946	1 (pass)	-1 (time out)
5639	209	2951	1 (pass)	-1 (time out)
5640	209	3037	1 (pass)	-1 (time out)
5641	209	3038	1 (pass)	-1 (time out)
5642	209	3085	1 (pass)	-1 (time out)
5643	209	3086	1 (pass)	-1 (time out)
5644	209	3099	1 (pass)	-1 (time out)
5645	209	3100	1 (pass)	-1 (time out)
5646	209	3137	1 (pass)	-1 (time out)
5647	209	3153	1 (pass)	-1 (time out)

2 Test file number 1

Test folder name:

test_cases/0_Independent_test_suites/1_Apostol_Problems

2.1 Problem number 41

$$\int \frac{t^3}{\sqrt{4+t^3}} dt$$

Optimal antiderivative

$$\frac{2t\sqrt{t^3+4}}{5} + \frac{8 \cdot 2^{\frac{2}{3}} (2^{\frac{2}{3}} + t) \operatorname{EllipticF}\left(\frac{t+2^{\frac{2}{3}}(1-\sqrt{3})}{t+2^{\frac{2}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{2 \cdot 2^{\frac{1}{3}} - 2^{\frac{2}{3}}t + t^2}{(t+2^{\frac{2}{3}}(1+\sqrt{3}))^2}}}{15\sqrt{t^3+4} \sqrt{\frac{2^{\frac{2}{3}}+t}{(t+2^{\frac{2}{3}}(1+\sqrt{3}))^2}}} \cdot 3^{\frac{3}{4}}$$

command

`integrate(t^3/(t^3+4)^(1/2),t, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{5} \sqrt{t^3+4} t - \frac{16}{5} \operatorname{weierstrassPInverse}(0, -16, t)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{t^3}{\sqrt{t^3+4}}, t\right)$$

2.2 Problem number 175

$$\int \frac{1}{\sqrt{1+t^3}} dt$$

Optimal antiderivative

$$\frac{2(1+t) \operatorname{EllipticF}\left(\frac{1+t-\sqrt{3}}{1+t+\sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{t^2-t+1}{(1+t+\sqrt{3})^2}}}{3\sqrt{t^3+1} \sqrt{\frac{1+t}{(1+t+\sqrt{3})^2}}} \cdot 3^{\frac{3}{4}}$$

command

```
integrate(1/(t^3+1)^(1/2),t, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}(0, -4, t)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{t^3+1}}, t\right)$$

3 Test file number 2

Test folder name:

test_cases/0_Independent_test_suites/2_Bondarenko_Problems

3.1 Problem number 25

$$\int \sqrt{\operatorname{sech}(x) \sinh(2x)} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2}\right) \sqrt{2} \left(\sqrt{\sinh(x)}\right)}{\sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) \sqrt{i \sinh(x)}}$$

command

```
integrate((sinh(2*x)/cosh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2\sqrt{2} \sqrt{\sinh(x)} - 4 \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cosh(x) + \sinh(x)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{\frac{\sinh(2x)}{\cosh(x)}}, x\right)$$

4 Test file number 5

Test folder name:

test_cases/0_Independent_test_suites/5_Hearn_Problems

4.1 Problem number 174

$$\int (2x + \sqrt{2} x^2) dx$$

Optimal antiderivative

$$x^2 + \frac{x^3 \sqrt{2}}{3}$$

command

```
integrate(2*x+x^2*2^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{2} x^3 + x^2$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: SyntaxError

5 Test file number 10

Test folder name:

test_cases/0_Independent_test_suites/10_Timofeev_Problems

5.1 Problem number 136

$$\int \frac{1}{a^5 + x^5} dx$$

Optimal antiderivative

$$\frac{\ln(a+x)}{5a^4} - \frac{\ln\left(a^2 + x^2 - \frac{ax(-\sqrt{5}+1)}{2}\right)(-\sqrt{5}+1)}{20a^4}$$

$$- \frac{\ln\left(a^2 + x^2 - \frac{ax(\sqrt{5}+1)}{2}\right)(\sqrt{5}+1)}{20a^4}$$

$$- \frac{\arctan\left(\frac{(-4x+a(\sqrt{5}+1))\sqrt{50+10\sqrt{5}}}{20a}\right)\sqrt{10-2\sqrt{5}}}{10a^4}$$

$$- \frac{\arctan\left(\frac{-4x+a(-\sqrt{5}+1)}{a\sqrt{10+2\sqrt{5}}}\right)\sqrt{10+2\sqrt{5}}}{10a^4}$$

command

```
integrate(1/(a^5+x^5),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

5.2 Problem number 137

$$\int \frac{x}{a^5 + x^5} dx$$

Optimal antiderivative

$$- \frac{\ln(a+x)}{5a^3} + \frac{\ln\left(a^2 + x^2 - \frac{ax(\sqrt{5}+1)}{2}\right)(-\sqrt{5}+1)}{20a^3}$$

$$+ \frac{\ln\left(a^2 + x^2 - \frac{ax(-\sqrt{5}+1)}{2}\right)(\sqrt{5}+1)}{20a^3} + \frac{\arctan\left(\frac{-4x+a(-\sqrt{5}+1)}{a\sqrt{10+2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}}{10a^3}$$

$$- \frac{\arctan\left(\frac{(-4x+a(\sqrt{5}+1))\sqrt{50+10\sqrt{5}}}{20a}\right)\sqrt{10+2\sqrt{5}}}{10a^3}$$

command

```
integrate(x/(a^5+x^5),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

5.3 Problem number 138

$$\int \frac{x^2}{a^5 + x^5} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\ln(a+x)}{5a^2} - \frac{\ln\left(a^2 + x^2 - \frac{ax(\sqrt{5}+1)}{2}\right)(-\sqrt{5}+1)}{20a^2} \\ & - \frac{\ln\left(a^2 + x^2 - \frac{ax(-\sqrt{5}+1)}{2}\right)(\sqrt{5}+1)}{20a^2} + \frac{\arctan\left(\frac{-4x+a(-\sqrt{5}+1)}{a\sqrt{10+2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}}{10a^2} \\ & - \frac{\arctan\left(\frac{(-4x+a(\sqrt{5}+1))\sqrt{50+10\sqrt{5}}}{20a}\right)\sqrt{10+2\sqrt{5}}}{10a^2} \end{aligned}$$

command

```
integrate(x^2/(a^5+x^5),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

5.4 Problem number 139

$$\int \frac{x^3}{a^5 + x^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\ln(a+x)}{5a} + \frac{\ln\left(a^2+x^2 - \frac{ax(-\sqrt{5}+1)}{2}\right)(-\sqrt{5}+1)}{20a} \\ & + \frac{\ln\left(a^2+x^2 - \frac{ax(\sqrt{5}+1)}{2}\right)(\sqrt{5}+1)}{20a} \\ & - \frac{\arctan\left(\frac{(-4x+a(\sqrt{5}+1))\sqrt{50+10\sqrt{5}}}{20a}\right)\sqrt{10-2\sqrt{5}}}{10a} \\ & - \frac{\arctan\left(\frac{-4x+a(-\sqrt{5}+1)}{a\sqrt{10+2\sqrt{5}}}\right)\sqrt{10+2\sqrt{5}}}{10a} \end{aligned}$$

command

```
integrate(x^3/(a^5+x^5),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

5.5 Problem number 142

$$\int \frac{1}{x^2(a^5+x^5)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{1}{a^5 x} + \frac{\ln(a+x)}{5a^6} - \frac{\ln\left(a^2 + x^2 - \frac{ax(-\sqrt{5}+1)}{2}\right)(-\sqrt{5}+1)}{20a^6} \\
 & - \frac{\ln\left(a^2 + x^2 - \frac{ax(\sqrt{5}+1)}{2}\right)(\sqrt{5}+1)}{20a^6} \\
 & + \frac{\arctan\left(\frac{(-4x+a(\sqrt{5}+1))\sqrt{50+10\sqrt{5}}}{20a}\right)\sqrt{10-2\sqrt{5}}}{10a^6} \\
 & + \frac{\arctan\left(\frac{-4x+a(-\sqrt{5}+1)}{a\sqrt{10+2\sqrt{5}}}\right)\sqrt{10+2\sqrt{5}}}{10a^6}
 \end{aligned}$$

command

`integrate(1/x^2/(a^5+x^5),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

5.6 Problem number 143

$$\int \frac{1}{x^3(a^5+x^5)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{1}{2a^5 x^2} - \frac{\ln(a+x)}{5a^7} + \frac{\ln\left(a^2 + x^2 - \frac{ax(\sqrt{5}+1)}{2}\right)(-\sqrt{5}+1)}{20a^7} \\
 & + \frac{\ln\left(a^2 + x^2 - \frac{ax(-\sqrt{5}+1)}{2}\right)(\sqrt{5}+1)}{20a^7} - \frac{\arctan\left(\frac{-4x+a(-\sqrt{5}+1)}{a\sqrt{10+2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}}{10a^7} \\
 & + \frac{\arctan\left(\frac{(-4x+a(\sqrt{5}+1))\sqrt{50+10\sqrt{5}}}{20a}\right)\sqrt{10+2\sqrt{5}}}{10a^7}
 \end{aligned}$$

command

```
integrate(1/x^3/(a^5+x^5),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

5.7 Problem number 144

$$\int \frac{1}{x^4(a^5+x^5)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1}{3a^5x^3} + \frac{\ln(a+x)}{5a^8} - \frac{\ln\left(a^2+x^2 - \frac{ax(\sqrt{5}+1)}{2}\right)(-\sqrt{5}+1)}{20a^8} \\ & - \frac{\ln\left(a^2+x^2 - \frac{ax(-\sqrt{5}+1)}{2}\right)(\sqrt{5}+1)}{20a^8} - \frac{\arctan\left(\frac{-4x+a(-\sqrt{5}+1)}{a\sqrt{10+2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}}{10a^8} \\ & + \frac{\arctan\left(\frac{(-4x+a(\sqrt{5}+1))\sqrt{50+10\sqrt{5}}}{20a}\right)\sqrt{10+2\sqrt{5}}}{10a^8} \end{aligned}$$

command

```
integrate(1/x^4/(a^5+x^5),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

5.8 Problem number 248

$$\int \frac{B + Ax}{(17 - 18x + 5x^2) \sqrt{13 - 22x + 10x^2}} dx$$

Optimal antiderivative

$$\frac{(2A + B) \arctan\left(\frac{(2-x)\sqrt{35}}{\sqrt{10x^2 - 22x + 13}}\right) \sqrt{35}}{35} - \frac{(A + B) \operatorname{arctanh}\left(\frac{(1-x)\sqrt{35}}{2\sqrt{10x^2 - 22x + 13}}\right) \sqrt{35}}{70}$$

command

```
integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

5.9 Problem number 453

$$\int \frac{\left(5 \cos^2(x) - \sqrt{-1 + 5 \sin^2(x)}\right) \tan(x)}{\sqrt[4]{-1 + 5 \sin^2(x)} \left(2 + \sqrt{-1 + 5 \sin^2(x)}\right)} dx$$

Optimal antiderivative

$$2(-1 + 5(\sin^2(x)))^{\frac{1}{4}} - \frac{3 \arctan\left(\frac{(-1+5(\sin^2(x)))^{\frac{1}{4}} \sqrt{2}}{2}\right) \sqrt{2}}{2} - \frac{\operatorname{arctanh}\left(\frac{(-1+5(\sin^2(x)))^{\frac{1}{4}} \sqrt{2}}{2}\right) \sqrt{2}}{4} - \frac{(-1 + 5(\sin^2(x)))^{\frac{1}{4}}}{2 \left(2 + \sqrt{-1 + 5(\sin^2(x))}\right)}$$

command

```
integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1+5*sin(x)^2)^(1/4)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$70 \left(5 \sqrt{2} \cos(x)^4 - 4 \sqrt{2} \cos(x)^2 \right) \arctan \left(-\frac{2 \left(\left(5 \sqrt{2} \cos(x)^2 - 4 \sqrt{2} \right) \left(-5 \cos(x)^2 + 4 \right)^{\frac{3}{4}} - 2 \sqrt{2} \left(-5 \cos(x)^2 + 4 \right)^{\frac{5}{4}} \right)}{5 \left(5 \cos(x)^4 - 4 \cos(x)^2 \right)} \right) - 5$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

6 Test file number 11

Test folder name:

test_cases/0_Independent_test_suites/11_Welz_Problems

6.1 Problem number 45

$$\int \frac{1 - kx}{(1 + (-2 + k)x)((1 - x)x(1 - kx))^{2/3}} dx$$

Optimal antiderivative

$$\frac{\ln(1 - (2 - k)x) 2^{\frac{1}{3}}}{2(1 - k)^{\frac{1}{3}}} + \frac{\ln(-kx + 1) 2^{\frac{1}{3}}}{4(1 - k)^{\frac{1}{3}}} - \frac{3 \ln\left(-1 + kx + 2^{\frac{2}{3}}(1 - k)^{\frac{1}{3}}((1 - x)x(-kx + 1))^{\frac{1}{3}}\right) 2^{\frac{1}{3}}}{4(1 - k)^{\frac{1}{3}}} - \frac{\arctan\left(\frac{\left(1 + \frac{2^{\frac{1}{3}}(-kx + 1)}{(1 - k)^{\frac{1}{3}}((1 - x)x(-kx + 1))^{\frac{1}{3}}}\right) \sqrt{3}}{3}\right) \sqrt{3} 2^{\frac{1}{3}}}{2(1 - k)^{\frac{1}{3}}}$$

command

`integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{3} 2^{\frac{1}{3}} \arctan \left(\frac{24 \sqrt{3} 2^{\frac{1}{3}} \left((k^5 - 3k^4 - 4k^3 + 22k^2 - 24k + 8)x^4 - 2(k^4 - 10k^3 + 27k^2 - 26k + 8)x^3 - 6(k^3 - 4k^2 + 4k - 1)x^2 - 2(k^2 - 1)x + k - 1 \right) (kx^3 - (k+1)x^2)}{(k-1)^{\frac{1}{3}}} \right)$$

$$2^{\frac{1}{3}} \log \left(\frac{12 \cdot 2^{\frac{2}{3}} (kx^3 - (k+1)x^2 + x)^{\frac{2}{3}} \left((k^3 + k^2 - 4k + 2)x^2 - 2(2k^2 - 3k + 1)x + k - 1 \right) + 6 \left((k^3 + 8k^2 - 8k)x^3 - (11k^2 - 8)x^2 + (11k - 8)x - 1 \right) (kx^3 - (k+1)x^2)}{(k-1)^{\frac{2}{3}} (k^4 - 8k^3 + 24k^2 - 32k + 16)x^4 + 4(k^3 - 6k^2 + 12k - 8)x^3 + 6(k^2 - 4k + 4)x^2 + 2(k - 2)x + 1} \right)$$

$$+ \frac{2^{\frac{1}{3}} \log \left(\frac{6 \cdot 2^{\frac{1}{3}} (kx^3 - (k+1)x^2 + x)^{\frac{1}{3}} (kx - 1) - 2^{\frac{2}{3}} \left((k^2 - 4k + 4)x^2 + 2(k - 2)x + 1 \right) - 12(kx^3 - (k+1)x^2 + x)^{\frac{2}{3}}}{(k-1)^{\frac{1}{3}} (k^2 - 4k + 4)x^2 + 2(k - 2)x + 1} \right)}{6(k-1)^{\frac{1}{3}}} \quad 12(k-1)^{\frac{1}{3}}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

6.2 Problem number 94

$$\int \frac{1}{(1-x^3) \sqrt[3]{a+bx^3}} dx$$

Optimal antiderivative

$$\frac{\ln(-x^3 + 1)}{6(a+b)^{\frac{1}{3}}} - \frac{\ln\left(\left(a+b\right)^{\frac{1}{3}}x - \left(bx^3+a\right)^{\frac{1}{3}}\right)}{2(a+b)^{\frac{1}{3}}} + \frac{\arctan\left(\frac{\left(1+\frac{2(a+b)^{\frac{1}{3}}x}{(bx^3+a)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3(a+b)^{\frac{1}{3}}} \sqrt{3}$$

command

`integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[3 \sqrt{\frac{1}{3}} (a+b) \sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}} \log \left(- \frac{(a^3 - 27a^2b - 108ab^2 - 81b^3)x^9 - 3(10a^3 + 54a^2b + 45ab^2)x^6 - 3(17a^3 + 18a^2b)x^3 - a^3 + 9((2a^2 + 3a + b)x^2 + 2(a+b)x + a)}{(a+b)^{\frac{1}{3}} (bx^3 + a)^{\frac{1}{3}}} \right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

7 Test file number 14

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/14_1.1.1.3-a+b_x-
^m-c+d_x-^n-e+f_x-^p

7.1 Problem number 300

$$\int \frac{1}{x^3(a+bx)^2(c+dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1}{2a^2c^3x^2} + \frac{3ad+2bc}{a^3c^4x} + \frac{b^5}{a^3(-ad+bc)^3(bx+a)} + \frac{d^4}{2c^3(-ad+bc)^2(dx+c)^2} \\ & + \frac{d^4(-3ad+5bc)}{c^4(-ad+bc)^3(dx+c)} + \frac{3(2a^2d^2+2abcd+b^2c^2)\ln(x)}{a^4c^5} \\ & - \frac{3b^5(-2ad+bc)\ln(bx+a)}{a^4(-ad+bc)^4} - \frac{3d^4(2a^2d^2-6abcd+5b^2c^2)\ln(dx+c)}{c^5(-ad+bc)^4} \end{aligned}$$

command

`integrate(1/x^3/(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{a^3b^4c^8 - 4a^4b^3c^7d + 6a^5b^2c^6d^2 - 4a^6bc^5d^3 + a^7c^4d^4 - 6(ab^6c^6d^2 - 2a^2b^5c^5d^3 + 5a^4b^3c^3d^5 - 6a^5b^2c^2d^6 + 2a^6b^2c^2d^7)}{a^4c^5(-ad+bc)^4}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

7.2 Problem number 318

$$\int \frac{1}{x^2(a+bx)^3(c+dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1}{a^3 c^3 x} - \frac{b^4}{2a^2(-ad+bc)^3(bx+a)^2} - \frac{b^4(-5ad+2bc)}{a^3(-ad+bc)^4(bx+a)} \\ & + \frac{d^4}{2c^2(-ad+bc)^3(dx+c)^2} + \frac{d^4(-2ad+5bc)}{c^3(-ad+bc)^4(dx+c)} - \frac{3(ad+bc)\ln(x)}{a^4 c^4} \\ & + \frac{3b^4(5a^2d^2-4abcd+b^2c^2)\ln(bx+a)}{a^4(-ad+bc)^5} - \frac{3d^4(a^2d^2-4abcd+5b^2c^2)\ln(dx+c)}{c^4(-ad+bc)^5} \end{aligned}$$

command

```
integrate(1/x^2/(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2a^3b^5c^8 - 10a^4b^4c^7d + 20a^5b^3c^6d^2 - 20a^6b^2c^5d^3 + 10a^7bc^4d^4 - 2a^8c^3d^5 + 6(ab^7c^6d^2 - 4a^2b^6c^5d^3 + 5a^3b^5c^4d^4)}{a^4(-ad+bc)^5}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

7.3 Problem number 853

$$\int \frac{1}{\sqrt{-x}\sqrt{a-bx}\sqrt{a+bx}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{b}\sqrt{-x}}{\sqrt{a}}, i\right) \sqrt{a}\sqrt{1-\frac{bx}{a}}\sqrt{1+\frac{bx}{a}}}{\sqrt{b}\sqrt{-bx+a}\sqrt{bx+a}}$$

command

```
integrate(1/(-x)^(1/2)/(-b*x+a)^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassPInverse}\left(\frac{4a^2}{b^2}, 0, x\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx+a}\sqrt{-bx+a}\sqrt{-x}}{b^2x^3-a^2x}, x\right)$$

7.4 Problem number 856

$$\int \frac{1}{\sqrt{-x} \sqrt{2-bx} \sqrt{2+bx}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{\sqrt{b} \sqrt{-x} \sqrt{2}}{2}, i\right) \sqrt{2}}{\sqrt{b}}$$

command

`integrate(1/(-x)^(1/2)/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \text{weierstrassPInverse}\left(\frac{16}{b^2}, 0, x\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx+2} \sqrt{-bx+2} \sqrt{-x}}{b^2 x^3 - 4x}, x\right)$$

7.5 Problem number 859

$$\int \frac{1}{\sqrt{2-3x} \sqrt{-x} \sqrt{2+3x}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{\sqrt{6} \sqrt{-x}}{2}, i\right) \sqrt{6}}{3}$$

command

`integrate(1/(2-3*x)^(1/2)/(-x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{3} \text{weierstrassPInverse}\left(\frac{16}{9}, 0, x\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x} \sqrt{3x+2} \sqrt{-3x+2}}{9x^3 - 4x}, x\right)$$

7.6 Problem number 863

$$\int \frac{1}{\sqrt{bx} \sqrt{1-cx} \sqrt{1+cx}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{bx}}{\sqrt{b}}, i\right)}{\sqrt{b} \sqrt{c}}$$

command

```
integrate(1/(b*x)^(1/2)/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2 \sqrt{-bc^2} \operatorname{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right)}{bc^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{bx} \sqrt{cx+1} \sqrt{-cx+1}}{bc^2x^3 - bx}, x\right)$$

7.7 Problem number 864

$$\int \frac{1}{\sqrt{bx} \sqrt{1-cx} \sqrt{1+dx}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{bx}}{\sqrt{b}}, \sqrt{-\frac{d}{c}}\right)}{\sqrt{b} \sqrt{c}}$$

command

```
integrate(1/(b*x)^(1/2)/(-c*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2 \sqrt{-bcd} \operatorname{weierstrassPInverse}\left(\frac{4(c^2+cd+d^2)}{3c^2d^2}, -\frac{4(2c^3+3c^2d-3cd^2-2d^3)}{27c^3d^3}, \frac{3cdx+c-d}{3cd}\right)}{bcd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{bx} \sqrt{-cx+1} \sqrt{dx+1}}{bcdx^3 + (bc-bd)x^2 - bx}, x\right)$$

7.8 Problem number 867

$$\int \frac{\sqrt{1+cx}}{\sqrt{bx}\sqrt{1-cx}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{bx}}{\sqrt{b}}, i\right)}{\sqrt{b}\sqrt{c}}$$

command

```
integrate((c*x+1)^(1/2)/(b*x)^(1/2)/(-c*x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{-bc^2} \operatorname{cweierstrassZeta}\left(\frac{4}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right)\right) - \sqrt{-bc^2} \operatorname{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right)\right)}{bc^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{bx}\sqrt{cx+1}\sqrt{-cx+1}}{bcx^2-bx}, x\right)$$

7.9 Problem number 868

$$\int \frac{\sqrt{1+cx}}{\sqrt{bx}\sqrt{1-dx}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{d}\sqrt{bx}}{\sqrt{b}}, \sqrt{-\frac{c}{d}}\right)}{\sqrt{b}\sqrt{d}}$$

command

```
integrate((c*x+1)^(1/2)/(b*x)^(1/2)/(-d*x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3\sqrt{-bcd} \operatorname{cdweierstrassZeta}\left(\frac{4(c^2+cd+d^2)}{3c^2d^2}, \frac{4(2c^3+3c^2d-3cd^2-2d^3)}{27c^3d^3}, \operatorname{weierstrassPInverse}\left(\frac{4(c^2+cd+d^2)}{3c^2d^2}, \frac{4(2c^3+3c^2d-3cd^2-2d^3)}{27c^3d^3}\right)\right)\right)}{3bcd^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{bx}\sqrt{cx+1}\sqrt{-dx+1}}{bdx^2-bx}, x\right)$$

7.10 Problem number 871

$$\int \frac{\sqrt{1-cx}}{\sqrt{bx}\sqrt{1+cx}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{bx}}{\sqrt{-b}}, i\right)}{\sqrt{-b}\sqrt{c}}$$

command

```
integrate((-c*x+1)^(1/2)/(b*x)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\left(\sqrt{-bc^2} \operatorname{cweierstrassZeta}\left(\frac{4}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right)\right) + \sqrt{-bc^2} \operatorname{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right)\right)}{bc^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx}\sqrt{cx+1}\sqrt{-cx+1}}{bcx^2+bx}, x\right)$$

7.11 Problem number 872

$$\int \frac{\sqrt{1-cx}}{\sqrt{bx}\sqrt{1+dx}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{d}\sqrt{bx}}{\sqrt{-b}}, \sqrt{-\frac{c}{d}}\right)}{\sqrt{-b}\sqrt{d}}$$

command

```
integrate((-c*x+1)^(1/2)/(b*x)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\left(3\sqrt{-bcd} \operatorname{cdweierstrassZeta}\left(\frac{4(c^2+cd+d^2)}{3c^2d^2}, -\frac{4(2c^3+3c^2d-3cd^2-2d^3)}{27c^3d^3}, \operatorname{weierstrassPInverse}\left(\frac{4(c^2+cd+d^2)}{3c^2d^2}, -\frac{4(2c^3+3c^2d-3cd^2-2d^3)}{27c^3d^3}, x\right)\right)\right)}{3c^2d^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx}\sqrt{-cx+1}\sqrt{dx+1}}{bdx^2+bx}, x\right)$$

7.15 Problem number 2220

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{(d+ex)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-Ae+Bd)(bx+a)^{\frac{5}{2}}}{11e(-ae+bd)(ex+d)^{\frac{11}{2}}} + \frac{2(6Abe-11Bae+5Bbd)(bx+a)^{\frac{5}{2}}}{99e(-ae+bd)^2(ex+d)^{\frac{9}{2}}} \\ & + \frac{8b(6Abe-11Bae+5Bbd)(bx+a)^{\frac{5}{2}}}{693e(-ae+bd)^3(ex+d)^{\frac{7}{2}}} + \frac{16b^2(6Abe-11Bae+5Bbd)(bx+a)^{\frac{5}{2}}}{3465e(-ae+bd)^4(ex+d)^{\frac{5}{2}}} \end{aligned}$$

command

```
integrate((b*x+a)^(3/2)*(B*x+A)/(e*x+d)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(495Bb^5d^3x^3 + 99(8Bab^4 + 7Ab^5)d^3x^2 + 99(Ba^2b^3 + 14Aab^4)d^3x - 99(2Ba^3b^2 - 7Aa^2b^3)d^3 - (315Aa^5 + 8A^2b^2d^2)x^4 + 3465(b^4d^{10} + a^4x^6e^{10} - 2(2a^3bdx^6 - 3a^4dx^5)e^9 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^8 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^7 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^6 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^5 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^4 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^3 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^2 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))}{3465(b^4d^{10} + a^4x^6e^{10} - 2(2a^3bdx^6 - 3a^4dx^5)e^9 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^8 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^7 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^6 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^5 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^4 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^3 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e^2 + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))e + 3(2a^2bd^2x^5 - 3a^3bdx^4 - 3a^4dx^3 + 3a^5d^2x^2 - 3a^6dx + 3a^7d^2))}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

7.16 Problem number 2221

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{(d+ex)^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-Ae+Bd)(bx+a)^{\frac{5}{2}}}{13e(-ae+bd)(ex+d)^{\frac{13}{2}}} + \frac{2(8Abe-13Bae+5Bbd)(bx+a)^{\frac{5}{2}}}{143e(-ae+bd)^2(ex+d)^{\frac{11}{2}}} \\ & + \frac{4b(8Abe-13Bae+5Bbd)(bx+a)^{\frac{5}{2}}}{429e(-ae+bd)^3(ex+d)^{\frac{9}{2}}} + \frac{16b^2(8Abe-13Bae+5Bbd)(bx+a)^{\frac{5}{2}}}{3003e(-ae+bd)^4(ex+d)^{\frac{7}{2}}} \\ & + \frac{32b^3(8Abe-13Bae+5Bbd)(bx+a)^{\frac{5}{2}}}{15015e(-ae+bd)^5(ex+d)^{\frac{5}{2}}} \end{aligned}$$

command

```
integrate((b*x+a)^(3/2)*(B*x+A)/(e*x+d)^(15/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2145 Bb^6 d^4 x^3 + 429(8 Bab^5 + 7 Ab^6) d^4 x^2 + 429(Ba^2 b^4 + 14 Aab^5) d^4 x - 429(2 Ba^3 b^3 - 7 Aa^2 b^4) d^4 + (1155 A - 15015(b^5 d^{12} - a^5 x^7 e^{12} + (5 a^4 b d x^7 - 7 a^5 d x^7))}{15015(b^5 d^{12} - a^5 x^7 e^{12} + (5 a^4 b d x^7 - 7 a^5 d x^7)}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

7.17 Problem number 2232

$$\int \frac{(a + bx)^{5/2}(A + Bx)}{(d + ex)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-Ae + Bd)(bx + a)^{\frac{7}{2}}}{11e(-ae + bd)(ex + d)^{\frac{11}{2}}} + \frac{2(4Abe - 11Bae + 7Bbd)(bx + a)^{\frac{7}{2}}}{99e(-ae + bd)^2(ex + d)^{\frac{9}{2}}} \\ & + \frac{4b(4Abe - 11Bae + 7Bbd)(bx + a)^{\frac{7}{2}}}{693e(-ae + bd)^3(ex + d)^{\frac{7}{2}}} \end{aligned}$$

command

```
integrate((b*x+a)^(5/2)*(B*x+A)/(e*x+d)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(77 Bb^5 d^2 x^4 + 11(19 Bab^4 + 9 Ab^5) d^2 x^3 + 33(5 Ba^2 b^3 + 9 Aab^4) d^2 x^2 + 11(Ba^3 b^2 + 27 Aa^2 b^3) d^2 x - 11(2 Ba^4 b - 15015(b^3 d^9 - a^3 x^6 e^9 + 3(a^2 b d x^6 - 2 a^3 d x^5) e^8 - 3(ab^2 d^2 x^6 - 6 a^2 b d x^6))}{693(b^3 d^9 - a^3 x^6 e^9 + 3(a^2 b d x^6 - 2 a^3 d x^5) e^8 - 3(ab^2 d^2 x^6 - 6 a^2 b d x^6)}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

7.18 Problem number 2233

$$\int \frac{(a + bx)^{5/2}(A + Bx)}{(d + ex)^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-Ae + Bd)(bx + a)^{\frac{7}{2}}}{13e(-ae + bd)(ex + d)^{\frac{13}{2}}} + \frac{2(6Abe - 13Bae + 7Bbd)(bx + a)^{\frac{7}{2}}}{143e(-ae + bd)^2(ex + d)^{\frac{11}{2}}} \\ & + \frac{8b(6Abe - 13Bae + 7Bbd)(bx + a)^{\frac{7}{2}}}{1287e(-ae + bd)^3(ex + d)^{\frac{9}{2}}} + \frac{16b^2(6Abe - 13Bae + 7Bbd)(bx + a)^{\frac{7}{2}}}{9009e(-ae + bd)^4(ex + d)^{\frac{7}{2}}} \end{aligned}$$

command

```
integrate((b*x+a)^(5/2)*(B*x+A)/(e*x+d)^(15/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(1001 B b^6 d^3 x^4 + 143(19 B a b^5 + 9 A b^6) d^3 x^3 + 429(5 B a^2 b^4 + 9 A a b^5) d^3 x^2 + 143(B a^3 b^3 + 27 A a^2 b^4) d^3 x - 143 A a^2 b^3}{9009(b^4 d^{11} + a^4 x^7 e^{11} - (4 a^5 d^2 + 2 a^4 x^7 e^{11}) b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

7.19 Problem number 2261

$$\int \frac{A + Bx}{(a + bx)^{5/2}(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(Ab - Ba)}{3b(-ae + bd)(bx + a)^{\frac{3}{2}}(ex + d)^{\frac{7}{2}}} - \frac{2(-10Abe + 7Bae + 3Bbd)}{3b(-ae + bd)^2(ex + d)^{\frac{7}{2}}\sqrt{bx + a}} \\ & - \frac{16e(-10Abe + 7Bae + 3Bbd)\sqrt{bx + a}}{21b(-ae + bd)^3(ex + d)^{\frac{7}{2}}} - \frac{32e(-10Abe + 7Bae + 3Bbd)\sqrt{bx + a}}{35(-ae + bd)^4(ex + d)^{\frac{5}{2}}} \\ & - \frac{128be(-10Abe + 7Bae + 3Bbd)\sqrt{bx + a}}{105(-ae + bd)^5(ex + d)^{\frac{3}{2}}} - \frac{256b^2e(-10Abe + 7Bae + 3Bbd)\sqrt{bx + a}}{105(-ae + bd)^6\sqrt{ex + d}} \end{aligned}$$

command

```
integrate((B*x+A)/(b*x+a)^(5/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(105 B b^5 d^5 x + 35(2 B a b^4 + A b^5) d^5 + (15 A a^5 + 128(7 B a b^4 - 10 A b^5) x^5 + 192(7 B a^4 d - 10 A a^4 b) x^4 + 128 A a^4 b^2) d^4 - 2(3 a^5 b^3 d x^6 + 4 a^6 b^2 d x^5 - a^7 b d x^4 - 2 a^8 d x^3)}{105(b^8 d^{10} x^2 + 2 a b^7 d^{10} x + a^2 b^6 d^{10} + (a^6 b^2 x^6 + 2 a^7 b x^5 + a^8 x^4) e^{10} - 2(3 a^5 b^3 d x^6 + 4 a^6 b^2 d x^5 - a^7 b d x^4 - 2 a^8 d x^3) b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

7.20 Problem number 2637

$$\int \frac{1}{\sqrt{a+bx} \sqrt{c + \frac{b(-1+c)x}{a}} \sqrt{e + \frac{b(-1+e)x}{a}}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{1-c} \sqrt{bx+a}}{\sqrt{a}}, \sqrt{\frac{1-e}{1-c}}\right) \sqrt{a}}{b\sqrt{1-c}}$$

command

`integrate(1/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, algorithm="fricas"`
 Fracas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2a^2 \sqrt{-\frac{b^3c - b^3 - (b^3c - b^3)e}{a^2}} \operatorname{weierstrassPInverse}\left(\frac{4(a^2c^2 - a^2c + a^2e^2 + a^2 - (a^2c + a^2)e)}{3(b^2c^2 - 2b^2c + b^2 + (b^2c^2 - 2b^2c + b^2)e^2 - 2(b^2c^2 - 2b^2c + b^2)e)}, \frac{27(b^3c^3 - 3b^3c^2 - b^3c - b^3 - (b^3c^3 - 3b^3c^2 - b^3c - b^3)e)}{b^3c - b^3 - (b^3c - b^3)e}\right)}{b^3c - b^3 - (b^3c - b^3)e}$$

Fracas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx+a} a^2 \sqrt{\frac{ac + (bc-b)x}{a}} \sqrt{\frac{ae + (be-b)x}{a}}}{a^3ce - (b^3c - b^3 - (b^3c - b^3)e)x^3 - (2ab^2c - ab^2 - (3ab^2c - 2ab^2)e)x^2 - (a^2bc - (3a^2bc - a^2b)e)x}, x\right)$$

7.21 Problem number 2638

$$\int \frac{1}{\sqrt{a+bx} \sqrt{c+dx} \sqrt{e + \frac{b(-1+e)x}{a}}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{1-e} \sqrt{bx+a}}{\sqrt{a}}, \sqrt{-\frac{ad}{(-ad+bc)(1-e)}}\right) \sqrt{a} \sqrt{\frac{b(dx+c)}{-ad+bc}}}{b\sqrt{1-e} \sqrt{dx+c}}$$

command

`integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2a \sqrt{\frac{b^2de - b^2d}{a}} \text{weierstrassPInverse} \left(\frac{4(b^2c^2 - abcd + a^2d^2 + (b^2c^2 - 2abcd + a^2d^2)e^2 - (2b^2c^2 - 3abcd + a^2d^2)e)}{3(b^2d^2e^2 - 2b^2d^2e + b^2d^2)}, \frac{4(2b^3c^3 - 3ab^2c^2d - 3a^2c^2d^2 - 3ab^2cd^2 - 3a^2bd^2e)}{27b^3d^3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{bx+a} \sqrt{dx+c} a \sqrt{\frac{ae + (be-b)x}{a}}}{a^2ce + (b^2de - b^2d)x^3 - (b^2c + abd - (b^2c + 2abd)e)x^2 - (abc - (2abc + a^2d)e)x}, x \right)$$

7.22 Problem number 2639

$$\int \frac{1}{\sqrt{a+bx} \sqrt{c+dx} \sqrt{e+fx}} dx$$

Optimal antiderivative

$$\frac{2 \text{EllipticF} \left(\frac{\sqrt{d} \sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{\frac{(-ad+bc)f}{d(-af+be)}} \right) \sqrt{ad-bc} \sqrt{\frac{b(dx+c)}{-ad+bc}} \sqrt{\frac{b(fx+e)}{-af+be}}}{b\sqrt{d} \sqrt{dx+c} \sqrt{fx+e}}$$

command

```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{bdf} \text{weierstrassPInverse} \left(\frac{4(b^2d^2e^2 + (b^2c^2 - abcd + a^2d^2)f^2 - (b^2cd + abd^2)fe)}{3b^2d^2f^2}, -\frac{4(2b^3d^3e^3 + (2b^3c^3 - 3ab^2c^2d - 3a^2bcd^2 + 2a^3d^3)f^3 - 3ab^2cd^2e - 3a^2bd^2e^2 - 3a^2cd^2e^2)}{27b^3d^3} \right)$$

bdf

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e}}{bdfx^3 + ace + (bde + (bc+ad)f)x^2 + (acf + (bc+ad)e)x}, x \right)$$

7.26 Problem number 2643

$$\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{d}\sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{\frac{(-ad+bc)f}{d(-af+be)}}\right) \sqrt{ad-bc} \sqrt{\frac{b(dx+c)}{-ad+bc}} \sqrt{fx+e}}{b\sqrt{d}\sqrt{dx+c} \sqrt{\frac{b(fx+e)}{-af+be}}}$$

command

`integrate((f*x+e)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3\sqrt{bdf}\operatorname{bdw} \operatorname{weierstrassZeta}\left(\frac{4(b^2d^2e^2+(b^2c^2-abcd+a^2d^2)f^2-(b^2cd+abd^2)fe)}{3b^2d^2f^2}, -\frac{4(2b^3d^3e^3+(2b^3c^3-3ab^2c^2d-3a^2bcd^2+2a^3d^3)}{3b^2d^2f^2}\right)\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}{bdx^2+ac+(bc+ad)x}, x\right)$$

7.27 Problem number 2644

$$\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}\sqrt{e+fx}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{dx+c}\sqrt{2}}{2\sqrt{c}}, \sqrt{-\frac{2cf}{-cf+de}}\right) \sqrt{c} \sqrt{\frac{-dx+c}{c}} \sqrt{\frac{d(fx+e)}{-cf+de}}}{d\sqrt{dx-c}\sqrt{fx+e}}$$

command

`integrate(1/(d*x-c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{d^2f}\operatorname{weierstrassPInverse}\left(\frac{4(3c^2f^2+d^2e^2)}{3d^2f^2}, \frac{8(9c^2f^2e-d^2e^3)}{27d^2f^3}, \frac{3fx+e}{3f}\right)}{d^2f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx+c}\sqrt{dx-c}\sqrt{fx+e}}{d^2fx^3+d^2ex^2-c^2fx-c^2e}, x\right)$$

7.28 Problem number 2645

$$\int \sqrt{1-2x} (2+3x)^{5/2} \sqrt{3+5x} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2911577 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1771875} \\ & - \frac{175111 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3543750} \\ & - \frac{23(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{1575} + \frac{2(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{45} \\ & - \frac{1244(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{13125} - \frac{175111\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{236250} \end{aligned}$$

command

`integrate((2+3*x)^(5/2)*(1-2*x)^(1/2)*(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{236250} (472500x^3 + 861750x^2 + 410490x - 136987) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((9x^2 + 12x + 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}, x\right)$$

7.29 Problem number 2646

$$\int \sqrt{1-2x} (2+3x)^{3/2} \sqrt{3+5x} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{55019 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{78750} \\ & - \frac{823 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{39375} + \frac{2(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{35} \\ & - \frac{27(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{875} - \frac{823\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{2625} \end{aligned}$$

command

```
integrate((2+3*x)^(3/2)*(1-2*x)^(1/2)*(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2625} (2250 x^2 + 2445 x - 166) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{5x+3} (3x+2)^{\frac{3}{2}} \sqrt{-2x+1}, x\right)$$

7.30 Problem number 2647

$$\int \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1159 \text{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3375} \\ & - \frac{31 \text{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3375} \\ & + \frac{2(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{25} - \frac{31 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{225} \end{aligned}$$

command

```
integrate((1-2*x)^(1/2)*(2+3*x)^(1/2)*(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{225} (90x+23) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}, x\right)$$

7.31 Problem number 2648

$$\int \frac{\sqrt{1-2x} \sqrt{3+5x}}{\sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{37 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{135} \\ & + \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{135} + \frac{2\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{9} \end{aligned}$$

command

`integrate((1-2*x)^(1/2)*(3+5*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{9} \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3} \sqrt{-2x+1}}{\sqrt{3x+2}}, x\right)$$

7.32 Problem number 2649

$$\int \frac{\sqrt{1-2x} \sqrt{3+5x}}{(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9} \\ & - \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9} - \frac{2\sqrt{1-2x} \sqrt{3+5x}}{3\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(1/2)*(3+5*x)^(1/2)/(2+3*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\sqrt{5x+3}\sqrt{-2x+1}}{3\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9x^2+12x+4}, x\right)$$

7.33 Problem number 2650

$$\int \frac{\sqrt{1-2x}\sqrt{3+5x}}{(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{74 \text{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{189} \\ & + \frac{4 \text{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{189} \\ & - \frac{2\sqrt{1-2x}\sqrt{3+5x}}{9(2+3x)^{3/2}} + \frac{74\sqrt{1-2x}\sqrt{3+5x}}{63\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(1/2)*(3+5*x)^(1/2)/(2+3*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(111x+67)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{63(9x^2+12x+4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{27x^3+54x^2+36x+8}, x\right)$$

7.34 Problem number 2651

$$\int \frac{\sqrt{1-2x} \sqrt{3+5x}}{(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4636 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{6615} \\ & - \frac{124 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{6615} \\ & - \frac{2\sqrt{1-2x} \sqrt{3+5x}}{15(2+3x)^{\frac{5}{2}}} + \frac{74\sqrt{1-2x} \sqrt{3+5x}}{315(2+3x)^{\frac{3}{2}}} + \frac{4636\sqrt{1-2x} \sqrt{3+5x}}{2205\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(1/2)*(3+5*x)^(1/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(20862x^2 + 28593x + 9643)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2205(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{81x^4 + 216x^3 + 216x^2 + 96x + 16}, x\right)$$

7.35 Problem number 2652

$$\int \frac{\sqrt{1-2x} \sqrt{3+5x}}{(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{220076 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{108045} \\ & - \frac{6584 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{108045} - \frac{2\sqrt{1-2x} \sqrt{3+5x}}{21(2+3x)^{\frac{7}{2}}} \\ & + \frac{74\sqrt{1-2x} \sqrt{3+5x}}{735(2+3x)^{\frac{5}{2}}} + \frac{3184\sqrt{1-2x} \sqrt{3+5x}}{5145(2+3x)^{\frac{3}{2}}} + \frac{220076\sqrt{1-2x} \sqrt{3+5x}}{36015\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(1/2)*(3+5*x)^(1/2)/(2+3*x)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2971026x^3 + 6042348x^2 + 4100535x + 926791)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{36015(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32}, x\right)$$

7.36 Problem number 2653

$$\int \sqrt{1-2x} (2+3x)^{5/2} (3+5x)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1508889271 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{233887500} \\ & -\frac{11346991 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{58471875} - \frac{23(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{2475} \\ & + \frac{2(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{55} - \frac{342971(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{866250} \\ & - \frac{543(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{9625} - \frac{11346991\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{3898125} \end{aligned}$$

command

`integrate((2+3*x)^(5/2)*(3+5*x)^(3/2)*(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{7796250} (63787500x^4 + 156161250x^3 + 132234750x^2 + 29706255x - 27010769)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((45x^3 + 87x^2 + 56x + 12)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}, x\right)$$

7.37 Problem number 2654

$$\int \sqrt{1-2x} (2+3x)^{3/2} (3+5x)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5327983 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2126250} \\ & - \frac{160297 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2126250} \\ & + \frac{2(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{45} - \frac{1208(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{7875} \\ & - \frac{3(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{175} - \frac{160297\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{141750} \end{aligned}$$

command

`integrate((2+3*x)^(3/2)*(3+5*x)^(3/2)*(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{141750} (472500x^3 + 821250x^2 + 366480x - 133999) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(15x^2 + 19x + 6\right)\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}, x\right)$$

7.38 Problem number 2655

$$\int \sqrt{1-2x} \sqrt{2+3x} (3+5x)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{148831 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{141750} \\ & - \frac{2252 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{70875} - \frac{31(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{525} \\ & + \frac{2(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{35} - \frac{2252\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{4725} \end{aligned}$$

command

```
integrate((3+5*x)^(3/2)*(1-2*x)^(1/2)*(2+3*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{4725} (6750x^2 + 6705x - 659) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(5x+3)^{\frac{3}{2}} \sqrt{3x+2} \sqrt{-2x+1}}{\sqrt{2+3x}}, x\right)$$

7.39 Problem number 2656

$$\int \frac{\sqrt{1-2x} (3+5x)^{3/2}}{\sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{974 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2025} \\ & - \frac{41 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2025} \\ & + \frac{2(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{15} - \frac{41 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{135} \end{aligned}$$

command

```
integrate((3+5*x)^(3/2)*(1-2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{135} (90x + 13) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(5x+3)^{\frac{3}{2}} \sqrt{-2x+1}}{\sqrt{3x+2}}, x\right)$$

7.40 Problem number 2657

$$\int \frac{\sqrt{1-2x} (3+5x)^{3/2}}{(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{49 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{81} \\ & + \frac{8 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{81} \\ & - \frac{2(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{3\sqrt{2+3x}} + \frac{40\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{27} \end{aligned}$$

command

`integrate((3+5*x)^(3/2)*(1-2*x)^(1/2)/(2+3*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(15x+13)\sqrt{5x+3}\sqrt{-2x+1}}{27\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(5x+3)^{\frac{3}{2}}\sqrt{3x+2}\sqrt{-2x+1}}{9x^2+12x+4}, x\right)$$

7.41 Problem number 2658

$$\int \frac{\sqrt{1-2x} (3+5x)^{3/2}}{(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{494 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{567} \\ & - \frac{214 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{567} \\ & - \frac{2(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{9(2+3x)^{\frac{3}{2}}} - \frac{214\sqrt{1-2x} \sqrt{3+5x}}{189\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(3/2)*(1-2*x)^(1/2)/(2+3*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(426x + 277)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{189(9x^2 + 12x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(5x + 3)^{\frac{3}{2}}\sqrt{3x + 2}\sqrt{-2x + 1}}{27x^3 + 54x^2 + 36x + 8}, x\right)$$

7.42 Problem number 2659

$$\int \frac{\sqrt{1 - 2x} (3 + 5x)^{3/2}}{(2 + 3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{8314 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{19845} \\ & + \frac{824 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{19845} - \frac{2(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{15(2+3x)^{\frac{5}{2}}} \\ & - \frac{214\sqrt{1-2x}\sqrt{3+5x}}{945(2+3x)^{\frac{3}{2}}} + \frac{8314\sqrt{1-2x}\sqrt{3+5x}}{6615\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(3/2)*(1-2*x)^(1/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(37413x^2 + 45432x + 13807)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{6615(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(5x + 3)^{\frac{3}{2}}\sqrt{3x + 2}\sqrt{-2x + 1}}{81x^4 + 216x^3 + 216x^2 + 96x + 16}, x\right)$$

7.43 Problem number 2660

$$\int \frac{\sqrt{1-2x} (3+5x)^{3/2}}{(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{475592 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{972405} \\ & - \frac{10628 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{972405} - \frac{2(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{21(2+3x)^{\frac{7}{2}}} \\ & - \frac{214\sqrt{1-2x} \sqrt{3+5x}}{2205(2+3x)^{\frac{5}{2}}} + \frac{8578\sqrt{1-2x} \sqrt{3+5x}}{46305(2+3x)^{\frac{3}{2}}} + \frac{475592\sqrt{1-2x} \sqrt{3+5x}}{324135\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(3/2)*(1-2*x)^(1/2)/(2+3*x)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(6420492x^3 + 13111191x^2 + 8796570x + 1944697)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{324135(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(5x+3)^{\frac{3}{2}}\sqrt{3x+2}\sqrt{-2x+1}}{243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32}, x\right)$$

7.44 Problem number 2661

$$\int \frac{\sqrt{1-2x} (3+5x)^{3/2}}{(2+3x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{22738708 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{20420505} \\ & - \frac{673072 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{20420505} \\ & - \frac{2(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{27(2+3x)^{\frac{9}{2}}} - \frac{214\sqrt{1-2x} \sqrt{3+5x}}{3969(2+3x)^{\frac{7}{2}}} + \frac{8842\sqrt{1-2x} \sqrt{3+5x}}{138915(2+3x)^{\frac{5}{2}}} \\ & + \frac{332372\sqrt{1-2x} \sqrt{3+5x}}{972405(2+3x)^{\frac{3}{2}}} + \frac{22738708\sqrt{1-2x} \sqrt{3+5x}}{6806835\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(3/2)*(1-2*x)^(1/2)/(2+3*x)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(920917674x^4 + 2487189618x^3 + 2520548433x^2 + 1134125364x + 190959271)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{6806835(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(5x+3)^{\frac{3}{2}}\sqrt{3x+2}\sqrt{-2x+1}}{729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64}, x\right)$$

7.45 Problem number 2662

$$\int \sqrt{1-2x} (2+3x)^{5/2} (3+5x)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16416987253 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{608107500} \\ & - \frac{493825477 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{608107500} \\ & - \frac{23(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{7}{2}}\sqrt{1-2x}}{3575} + \frac{2(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{7}{2}}\sqrt{1-2x}}{65} \\ & - \frac{1865989(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{1126125} - \frac{564731(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{2252250} \\ & - \frac{2014(3+5x)^{\frac{7}{2}}\sqrt{1-2x}\sqrt{2+3x}}{53625} - \frac{493825477\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{40540500} \end{aligned}$$

command

```
integrate((2+3*x)^(5/2)*(3+5*x)^(5/2)*(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{40540500} (1403325000x^5 + 4299277500x^4 + 5075689500x^3 + 2626854750x^2 + 139824180x - 707313559)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(225x^4 + 570x^3 + 541x^2 + 228x + 36)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{40540500}, x\right)$$

7.46 Problem number 2663

$$\int \sqrt{1-2x} (2+3x)^{3/2} (3+5x)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{30926081 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3118500} \\ & - \frac{465127 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1559250} + \frac{2(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{7}{2}}\sqrt{1-2x}}{55} \\ & - \frac{7031(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{11550} - \frac{177(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{1925} \\ & - \frac{3(3+5x)^{\frac{7}{2}}\sqrt{1-2x}\sqrt{2+3x}}{275} - \frac{465127\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{103950} \end{aligned}$$

command

`integrate((2+3*x)^(3/2)*(3+5*x)^(5/2)*(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{103950} (1417500x^4 + 3354750x^3 + 2737800x^2 + 570555x - 567484) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((75x^3 + 140x^2 + 87x + 18) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}, x\right)$$

7.47 Problem number 2664

$$\int \sqrt{1-2x} \sqrt{2+3x} (3+5x)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{488149 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{127575} \\ & - \frac{29357 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{255150} \\ & - \frac{223(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{945} - \frac{31(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{945} \\ & + \frac{2(3+5x)^{\frac{7}{2}}\sqrt{1-2x}\sqrt{2+3x}}{45} - \frac{29357\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{17010} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)*(1-2*x)^(1/2)*(2+3*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{17010} (94500x^3 + 156150x^2 + 65250x - 26009) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{\sqrt{3x+2}}, x\right)$$

7.48 Problem number 2665

$$\int \frac{\sqrt{1-2x} (3+5x)^{5/2}}{\sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9013 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{5670} \\ & - \frac{131 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2835} - \frac{(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{7} \\ & + \frac{2(3+5x)^{\frac{5}{2}} \sqrt{1-2x} \sqrt{2+3x}}{21} - \frac{131\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{189} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)*(1-2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5}{189} (90x^2 + 81x - 10) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{-2x+1}}{\sqrt{3x+2}}, x\right)$$

7.49 Problem number 2666

$$\int \frac{\sqrt{1-2x} (3+5x)^{5/2}}{(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{15} - \frac{3 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{5} - \frac{2(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{3\sqrt{2+3x}} + \frac{4(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{3} - \sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}$$

command

`integrate((3+5*x)^(5/2)*(1-2*x)^(1/2)/(2+3*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(10x^2 + 7x)\sqrt{5x+3}\sqrt{-2x+1}}{3\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9x^2 + 12x + 4}, x\right)$$

7.50 Problem number 2667

$$\int \frac{\sqrt{1-2x} (3+5x)^{5/2}}{(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2209 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1701} + \frac{494 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1701} - \frac{2(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{9(2+3x)^{\frac{3}{2}}} - \frac{118(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{63\sqrt{2+3x}} + \frac{2470\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{567}$$

command

`integrate((3+5*x)^(5/2)*(1-2*x)^(1/2)/(2+3*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 (1575 x^2 + 2841 x + 1187) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}}{567 (9 x^2 + 12 x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(25 x^2 + 30 x + 9) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}}{27 x^3 + 54 x^2 + 36 x + 8}, x \right)$$

7.51 Problem number 2668

$$\int \frac{\sqrt{1-2x} (3+5x)^{5/2}}{(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{31588 \text{EllipticE} \left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33} \right) \sqrt{33}}{19845} \\ & - \frac{12758 \text{EllipticF} \left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33} \right) \sqrt{33}}{19845} \\ & - \frac{118(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{315 (2+3x)^{\frac{3}{2}}} - \frac{2(3+5x)^{\frac{5}{2}} \sqrt{1-2x}}{15 (2+3x)^{\frac{5}{2}}} - \frac{12758 \sqrt{1-2x} \sqrt{3+5x}}{6615 \sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(5/2)*(1-2*x)^(1/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2 (87021 x^2 + 113319 x + 36919) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}}{6615 (27 x^3 + 54 x^2 + 36 x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(25 x^2 + 30 x + 9) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}}{81 x^4 + 216 x^3 + 216 x^2 + 96 x + 16}, x \right)$$

7.52 Problem number 2669

$$\int \frac{\sqrt{1-2x} (3+5x)^{5/2}}{(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{173482 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{324135} \\ & + \frac{23612 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{324135} - \frac{118(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{735(2+3x)^{\frac{5}{2}}} \\ & - \frac{2(3+5x)^{\frac{5}{2}} \sqrt{1-2x}}{21(2+3x)^{\frac{7}{2}}} - \frac{4282 \sqrt{1-2x} \sqrt{3+5x}}{15435(2+3x)^{\frac{3}{2}}} + \frac{173482 \sqrt{1-2x} \sqrt{3+5x}}{108045 \sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(5/2)*(1-2*x)^(1/2)/(2+3*x)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2342007x^3 + 4290411x^2 + 2623695x + 535637)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{108045(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32}, x\right)$$

7.53 Problem number 2670

$$\int \frac{\sqrt{1-2x} (3+5x)^{5/2}}{(2+3x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{27198452 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{61261515} \\ & - \frac{442868 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{61261515} \\ & - \frac{118(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{1323(2+3x)^{\frac{7}{2}}} - \frac{2(3+5x)^{\frac{5}{2}} \sqrt{1-2x}}{27(2+3x)^{\frac{9}{2}}} - \frac{12934 \sqrt{1-2x} \sqrt{3+5x}}{138915(2+3x)^{\frac{5}{2}}} \\ & + \frac{568318 \sqrt{1-2x} \sqrt{3+5x}}{2917215(2+3x)^{\frac{3}{2}}} + \frac{27198452 \sqrt{1-2x} \sqrt{3+5x}}{20420505 \sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)*(1-2*x)^(1/2)/(2+3*x)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(1101537306x^4 + 2991138867x^3 + 3003721227x^2 + 1325733891x + 217427099)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{20420505(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64}, x\right)$$

7.54 Problem number 2671

$$\int \frac{\sqrt{1-2x}(3+5x)^{5/2}}{(2+3x)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1305025844 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1572378885} \\ & - \frac{37904696 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1572378885} - \frac{118(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{2079(2+3x)^{\frac{9}{2}}} \\ & - \frac{2(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{33(2+3x)^{\frac{11}{2}}} - \frac{13022\sqrt{1-2x}\sqrt{3+5x}}{305613(2+3x)^{\frac{7}{2}}} + \frac{627806\sqrt{1-2x}\sqrt{3+5x}}{10696455(2+3x)^{\frac{5}{2}}} \\ & + \frac{19417096\sqrt{1-2x}\sqrt{3+5x}}{74875185(2+3x)^{\frac{3}{2}}} + \frac{1305025844\sqrt{1-2x}\sqrt{3+5x}}{524126295\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)*(1-2*x)^(1/2)/(2+3*x)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(158560640046x^5 + 534040213536x^4 + 719808574005x^3 + 484598540169x^2 + 162787885893x + 21813966691)}{524126295(729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 + 6048x^2 + 1344x + 128}, x\right)$$

7.55 Problem number 2672

$$\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{d}\sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{\frac{(-ad+bc)f}{d(-af+be)}}\right) \sqrt{ad-bc} \sqrt{\frac{b(dx+c)}{-ad+bc}} \sqrt{fx+e}}{b\sqrt{d}\sqrt{dx+c} \sqrt{\frac{b(fx+e)}{-af+be}}}$$

command

`integrate((f*x+e)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3\sqrt{bdf} \operatorname{bdw} \operatorname{weierstrassZeta}\left(\frac{4(b^2d^2e^2+(b^2c^2-abcd+a^2d^2)f^2-(b^2cd+abd^2)fe)}{3b^2d^2f^2}\right), -\frac{4(2b^3d^3e^3+(2b^3c^3-3ab^2c^2d-3a^2bcd^2+2a^3d^3)}{3b^2d^2f^2}\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}{bdx^2+ac+(bc+ad)x}, x\right)$$

7.56 Problem number 2673

$$\int \frac{\sqrt{e+fx}}{(a+bx)^{3/2}\sqrt{c+dx}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{dx+c}\sqrt{fx+e}}{(-ad+bc)\sqrt{bx+a}} + \frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{f}\sqrt{bx+a}}{\sqrt{af-be}}, \sqrt{\frac{d(-af+be)}{(-ad+bc)f}}\right) \sqrt{f}\sqrt{af-be}\sqrt{dx+c}\sqrt{\frac{b(fx+e)}{-af+be}}}{b(-ad+bc)\sqrt{\frac{b(dx+c)}{-ad+bc}}\sqrt{fx+e}}$$

command

`integrate((f*x+e)^(1/2)/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e} b^2 df - \sqrt{bdf} \left((2b^2c - abd)fx + (2abc - a^2d)f - (b^2dx + abd)e \right) \right) \text{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e}}{b^2 dx^3 + a^2 c + (b^2 c + 2abd)x^2 + (2abc + a^2 d)x}, x \right)$$

7.57 Problem number 2674

$$\int \frac{\sqrt{1-2x}}{\sqrt{-3-5x} \sqrt{2+3x}} dx$$

Optimal antiderivative

$$\frac{2 \text{EllipticE} \left(\sqrt{5} \sqrt{2+3x}, \frac{\sqrt{70}}{35} \right) \sqrt{35}}{15}$$

command

`integrate((1-2*x)^(1/2)/(-3-5*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{68}{675} \sqrt{30} \text{weierstrassPInverse} \left(\frac{1159}{675}, \frac{38998}{91125}, x + \frac{23}{90} \right) + \frac{2}{15} \sqrt{30} \text{weierstrassZeta} \left(\frac{1159}{675}, \frac{38998}{91125}, \text{weierstrassPInverse} \left(\frac{1159}{675}, \frac{38998}{91125}, x + \frac{23}{90} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{3x+2} \sqrt{-2x+1} \sqrt{-5x-3}}{15x^2 + 19x + 6}, x \right)$$

7.58 Problem number 2675

$$\int \frac{\sqrt{1-2x} (2+3x)^{5/2}}{\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{61151 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{131250} \\ & - \frac{314 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{21875} - \frac{23(2+3x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{3+5x}}{875} \\ & + \frac{2(2+3x)^{\frac{5}{2}} \sqrt{1-2x} \sqrt{3+5x}}{35} - \frac{859 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{4375} \end{aligned}$$

command

`integrate((2+3*x)^(5/2)*(1-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{4375} (2250x^2 + 2655x - 89) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(9x^2 + 12x + 4) \sqrt{3x+2} \sqrt{-2x+1}}{\sqrt{5x+3}}, x\right)$$

7.59 Problem number 2676

$$\int \frac{\sqrt{1-2x} (2+3x)^{3/2}}{\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{17 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1875} \\ & - \frac{146 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{625} \\ & + \frac{2(2+3x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{3+5x}}{25} - \frac{9 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{125} \end{aligned}$$

command

```
integrate((2+3*x)^(3/2)*(1-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{125} (30x + 11) \sqrt{5x + 3} \sqrt{3x + 2} \sqrt{-2x + 1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(3x + 2)^{\frac{3}{2}} \sqrt{-2x + 1}}{\sqrt{5x + 3}}, x\right)$$

7.60 Problem number 2677

$$\int \frac{\sqrt{1 - 2x} \sqrt{2 + 3x}}{\sqrt{3 + 5x}} dx$$

Optimal antiderivative

$$\frac{31 \text{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1 - 2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{225} - \frac{4 \text{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1 - 2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{225} + \frac{2\sqrt{1 - 2x} \sqrt{2 + 3x} \sqrt{3 + 5x}}{15}$$

command

```
integrate((1-2*x)^(1/2)*(2+3*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{15} \sqrt{5x + 3} \sqrt{3x + 2} \sqrt{-2x + 1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{3x + 2} \sqrt{-2x + 1}}{\sqrt{5x + 3}}, x\right)$$

7.61 Problem number 2679

$$\int \frac{\sqrt{1-2x}}{(2+3x)^{3/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3} + \frac{2\sqrt{1-2x} \sqrt{3+5x}}{\sqrt{2+3x}}$$

command

`integrate((1-2*x)^(1/2)/(2+3*x)^(3/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{5x+3} \sqrt{-2x+1}}{\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{45x^3 + 87x^2 + 56x + 12}, x\right)$$

7.62 Problem number 2680

$$\int \frac{\sqrt{1-2x}}{(2+3x)^{5/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$-\frac{136 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{63} - \frac{4 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{63} + \frac{2\sqrt{1-2x} \sqrt{3+5x}}{3(2+3x)^{\frac{3}{2}}} + \frac{136\sqrt{1-2x} \sqrt{3+5x}}{21\sqrt{2+3x}}$$

command

`integrate((1-2*x)^(1/2)/(2+3*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(204x + 143)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{21(9x^2 + 12x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{135x^4 + 351x^3 + 342x^2 + 148x + 24}, x\right)$$

7.63 Problem number 2681

$$\int \frac{\sqrt{1 - 2x}}{(2 + 3x)^{7/2}\sqrt{3 + 5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6388 \text{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1 - 2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{735} \\ & - \frac{64 \text{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1 - 2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{245} + \frac{2\sqrt{1 - 2x}\sqrt{3 + 5x}}{5(2 + 3x)^{\frac{5}{2}}} \\ & + \frac{92\sqrt{1 - 2x}\sqrt{3 + 5x}}{35(2 + 3x)^{\frac{3}{2}}} + \frac{6388\sqrt{1 - 2x}\sqrt{3 + 5x}}{245\sqrt{2 + 3x}} \end{aligned}$$

command

`integrate((1-2*x)^(1/2)/(2+3*x)^(7/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(28746x^2 + 39294x + 13469)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{245(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{405x^5 + 1323x^4 + 1728x^3 + 1128x^2 + 368x + 48}, x\right)$$

7.64 Problem number 2682

$$\int \frac{\sqrt{1-2x} (2+3x)^{7/2}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{203179 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{656250} \\ & - \frac{38723 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3609375} \\ & - \frac{2(2+3x)^{7/2} \sqrt{1-2x}}{5\sqrt{3+5x}} + \frac{183(2+3x)^{3/2} \sqrt{1-2x} \sqrt{3+5x}}{4375} \\ & + \frac{48(2+3x)^{5/2} \sqrt{1-2x} \sqrt{3+5x}}{175} - \frac{2486\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{21875} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)*(1-2*x)^(1/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(33750x^3 + 63225x^2 + 25955x + 32)\sqrt{3x+2}\sqrt{-2x+1}}{21875\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(27x^3 + 54x^2 + 36x + 8)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{25x^2 + 30x + 9}, x\right)$$

7.65 Problem number 2683

$$\int \frac{\sqrt{1-2x} (2+3x)^{5/2}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1409 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9375} \\ & - \frac{1091 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{103125} - \frac{2(2+3x)^{5/2} \sqrt{1-2x}}{5\sqrt{3+5x}} \\ & + \frac{36(2+3x)^{3/2} \sqrt{1-2x} \sqrt{3+5x}}{125} + \frac{13\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{625} \end{aligned}$$

command

```
integrate((2+3*x)^(5/2)*(1-2*x)^(1/2)/(3+5*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(450x^2 + 485x + 119)\sqrt{3x+2}\sqrt{-2x+1}}{625\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(9x^2 + 12x + 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{25x^2 + 30x + 9}, x\right)$$

7.66 Problem number 2684

$$\int \frac{\sqrt{1-2x}(2+3x)^{3/2}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{19 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{375} \\ & -\frac{106 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{4125} \\ & -\frac{2(2+3x)^{\frac{3}{2}}\sqrt{1-2x}}{5\sqrt{3+5x}} + \frac{8\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{25} \end{aligned}$$

command

```
integrate((2+3*x)^(3/2)*(1-2*x)^(1/2)/(3+5*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(5x+2)\sqrt{3x+2}\sqrt{-2x+1}}{25\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}(3x+2)^{\frac{3}{2}}\sqrt{-2x+1}}{25x^2 + 30x + 9}, x\right)$$

7.67 Problem number 2685

$$\int \frac{\sqrt{1-2x} \sqrt{2+3x}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{62 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{825} + \frac{4 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{25} - \frac{2\sqrt{1-2x} \sqrt{2+3x}}{5\sqrt{3+5x}}$$

command

`integrate((1-2*x)^(1/2)*(2+3*x)^(1/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\sqrt{3x+2}\sqrt{-2x+1}}{5\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{25x^2+30x+9}, x\right)$$

7.68 Problem number 2686

$$\int \frac{\sqrt{1-2x}}{\sqrt{2+3x}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{55} \sqrt{1-2x}}{11}, \frac{\sqrt{1155}}{35}\right) \sqrt{35}}{5} - \frac{2\sqrt{1-2x} \sqrt{2+3x}}{\sqrt{3+5x}}$$

command

`integrate((1-2*x)^(1/2)/(3+5*x)^(3/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\sqrt{3x+2}\sqrt{-2x+1}}{\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{75x^3+140x^2+87x+18}, x\right)$$

7.69 Problem number 2687

$$\int \frac{\sqrt{1-2x}}{(2+3x)^{3/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{4 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{33} + 4 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}$$

$$+ \frac{2\sqrt{1-2x}}{\sqrt{2+3x} \sqrt{3+5x}} - \frac{20\sqrt{1-2x} \sqrt{2+3x}}{\sqrt{3+5x}}$$

command

`integrate((1-2*x)^(1/2)/(2+3*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(30x+19)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{15x^2+19x+6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{225x^4+570x^3+541x^2+228x+36}, x\right)$$

7.70 Problem number 2688

$$\int \frac{\sqrt{1-2x}}{(2+3x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{556 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{21}$$

$$+ \frac{184 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{231}$$

$$+ \frac{2\sqrt{1-2x}}{3(2+3x)^{\frac{3}{2}} \sqrt{3+5x}} + \frac{92\sqrt{1-2x}}{7\sqrt{2+3x} \sqrt{3+5x}} - \frac{2780\sqrt{1-2x} \sqrt{2+3x}}{21\sqrt{3+5x}}$$

command

`integrate((1-2*x)^(1/2)/(2+3*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(4170x^2 + 5422x + 1759)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{7(45x^3 + 87x^2 + 56x + 12)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{675x^5 + 2160x^4 + 2763x^3 + 1766x^2 + 564x + 72}, x\right)$$

7.71 Problem number 2689

$$\int \frac{\sqrt{1-2x}}{(2+3x)^{7/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{116464 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{735} \\ & + \frac{38536 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{8085} + \frac{2\sqrt{1-2x}}{5(2+3x)^{5/2}\sqrt{3+5x}} \\ & + \frac{416\sqrt{1-2x}}{105(2+3x)^{3/2}\sqrt{3+5x}} + \frac{19268\sqrt{1-2x}}{245\sqrt{2+3x}\sqrt{3+5x}} - \frac{116464\sqrt{1-2x}\sqrt{2+3x}}{147\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(1/2)/(2+3*x)^(7/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(2620440x^3 + 5154174x^2 + 3376856x + 736871)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{245(135x^4 + 351x^3 + 342x^2 + 148x + 24)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2025x^6 + 7830x^5 + 12609x^4 + 10824x^3 + 5224x^2 + 1344x + 144}, x\right)$$

7.72 Problem number 2690

$$\int \frac{\sqrt{1-2x} (2+3x)^{9/2}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1473539 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{7218750} \\ & - \frac{31288 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3609375} - \frac{2(2+3x)^{9/2} \sqrt{1-2x}}{15(3+5x)^{3/2}} \\ & - \frac{118(2+3x)^{7/2} \sqrt{1-2x}}{165\sqrt{3+5x}} + \frac{5153(2+3x)^{3/2} \sqrt{1-2x} \sqrt{3+5x}}{48125} \\ & + \frac{958(2+3x)^{5/2} \sqrt{1-2x} \sqrt{3+5x}}{1925} - \frac{12601\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{240625} \end{aligned}$$

command

`integrate((2+3*x)^(9/2)*(1-2*x)^(1/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(3341250x^4 + 8575875x^3 + 6882975x^2 + 1854575x + 54083)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{721875(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(81x^4 + 216x^3 + 216x^2 + 96x + 16)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{125x^3 + 225x^2 + 135x + 27}, x\right)$$

7.73 Problem number 2691

$$\int \frac{\sqrt{1-2x} (2+3x)^{7/2}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{523 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{46875} \\ & - \frac{47342 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{515625} \\ & - \frac{2(2+3x)^{7/2} \sqrt{1-2x}}{15(3+5x)^{3/2}} - \frac{458(2+3x)^{5/2} \sqrt{1-2x}}{825\sqrt{3+5x}} \\ & + \frac{2818(2+3x)^{3/2} \sqrt{1-2x} \sqrt{3+5x}}{6875} + \frac{2719\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{34375} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)*(1-2*x)^(1/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(222750 x^3 + 398475 x^2 + 221200 x + 37273) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{103125 (25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(27x^3 + 54x^2 + 36x + 8) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{125x^3 + 225x^2 + 135x + 27}, x\right)$$

7.74 Problem number 2692

$$\int \frac{\sqrt{1-2x} (2+3x)^{5/2}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{169 \text{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{20625} \\ & - \frac{496 \text{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{20625} - \frac{2(2+3x)^{5/2} \sqrt{1-2x}}{15(3+5x)^{3/2}} \\ & - \frac{326(2+3x)^{3/2} \sqrt{1-2x}}{825\sqrt{3+5x}} + \frac{458\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{1375} \end{aligned}$$

command

`integrate((2+3*x)^(5/2)*(1-2*x)^(1/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2475x^2 + 1825x + 193) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{4125(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(9x^2 + 12x + 4) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{125x^3 + 225x^2 + 135x + 27}, x\right)$$

7.75 Problem number 2693

$$\int \frac{\sqrt{1-2x} (2+3x)^{3/2}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{458 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{4125} - \frac{178 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{4125} - \frac{2(2+3x)^{\frac{3}{2}} \sqrt{1-2x}}{15(3+5x)^{\frac{3}{2}}} - \frac{194\sqrt{1-2x} \sqrt{2+3x}}{825\sqrt{3+5x}}$$

command

`integrate((2+3*x)^(3/2)*(1-2*x)^(1/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(650x+401)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{825(25x^2+30x+9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}(3x+2)^{\frac{3}{2}}\sqrt{-2x+1}}{125x^3+225x^2+135x+27}, x\right)$$

7.76 Problem number 2694

$$\int \frac{\sqrt{1-2x} \sqrt{2+3x}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{62 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{825} + \frac{8 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{825} - \frac{2\sqrt{1-2x} \sqrt{2+3x}}{15(3+5x)^{\frac{3}{2}}} - \frac{62\sqrt{1-2x} \sqrt{2+3x}}{165\sqrt{3+5x}}$$

command

`integrate((1-2*x)^(1/2)*(2+3*x)^(1/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(155x + 104)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{165(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{125x^3 + 225x^2 + 135x + 27}, x\right)$$

7.77 Problem number 2695

$$\int \frac{\sqrt{1 - 2x}}{\sqrt{2 + 3x}(3 + 5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{136 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{165} \\ & - \frac{4 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{165} \\ & - \frac{2\sqrt{1-2x}\sqrt{2+3x}}{3(3+5x)^{\frac{3}{2}}} + \frac{136\sqrt{1-2x}\sqrt{2+3x}}{33\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(1/2)/(3+5*x)^(5/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(340x + 193)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{33(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{375x^4 + 925x^3 + 855x^2 + 351x + 54}, x\right)$$

7.78 Problem number 2696

$$\int \frac{\sqrt{1-2x}}{(2+3x)^{3/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{532 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{33} \\ & - \frac{16 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{33} + \frac{2\sqrt{1-2x}}{(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & - \frac{40\sqrt{1-2x}\sqrt{2+3x}}{3(3+5x)^{\frac{3}{2}}} + \frac{2660\sqrt{1-2x}\sqrt{2+3x}}{33\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(1/2)/(2+3*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(19950x^2 + 24610x + 7573)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{33(75x^3 + 140x^2 + 87x + 18)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1125x^5 + 3525x^4 + 4415x^3 + 2763x^2 + 864x + 108}, x\right)$$

7.79 Problem number 2697

$$\int \frac{\sqrt{1-2x}}{(2+3x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{36968 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{231} \\ & - \frac{1112 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{231} + \frac{2\sqrt{1-2x}}{3(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} \\ & + \frac{416\sqrt{1-2x}}{21(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} - \frac{2780\sqrt{1-2x}\sqrt{2+3x}}{21(3+5x)^{\frac{3}{2}}} + \frac{184840\sqrt{1-2x}\sqrt{2+3x}}{231\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(1/2)/(2+3*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(4158900x^3 + 7902930x^2 + 4998904x + 1052533)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{231(225x^4 + 570x^3 + 541x^2 + 228x + 36)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3375x^6 + 12825x^5 + 20295x^4 + 17119x^3 + 8118x^2 + 2052x + 216}, x\right)$$

7.80 Problem number 2698

$$\int \frac{\sqrt{1-2x}}{(2+3x)^{7/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10312712 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{8085} \\ & - \frac{310208 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{8085} \\ & + \frac{2\sqrt{1-2x}}{5(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}} + \frac{556\sqrt{1-2x}}{105(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} + \frac{116044\sqrt{1-2x}}{735(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & - \frac{155104\sqrt{1-2x}\sqrt{2+3x}}{147(3+5x)^{\frac{3}{2}}} + \frac{10312712\sqrt{1-2x}\sqrt{2+3x}}{1617\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(1/2)/(2+3*x)^(7/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3480540300x^4 + 8934240060x^3 + 8592783498x^2 + 3669873602x + 587237237)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8085(675x^5 + 2160x^4 + 2763x^3 + 1766x^2 + 564x + 72)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{10125x^7 + 45225x^6 + 86535x^5 + 91947x^4 + 58592x^3 + 22392x^2 + 4752x + 432}, x\right)$$

7.81 Problem number 2699

$$\int (1-2x)^{3/2}(2+3x)^{5/2}\sqrt{3+5x} \, dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{55} - \frac{604915631 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{584718750} \\ & - \frac{18177329 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{584718750} \\ & + \frac{1103(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{259875} + \frac{178(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{7425} \\ & - \frac{124891(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{2165625} - \frac{18177329\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{38981250} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(2+3*x)^(5/2)*(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{38981250} (127575000x^4 + 140805000x^3 - 48345750x^2 - 89595360x - 4295257)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(- (18x^3 + 15x^2 - 4x - 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}, x\right)$$

7.82 Problem number 2700

$$\int (1-2x)^{3/2}(2+3x)^{3/2}\sqrt{3+5x} \, dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{45} - \frac{5684677 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{10631250} \\ & - \frac{84134 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{5315625} + \frac{62(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{1575} \\ & - \frac{347(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{39375} - \frac{84134\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{354375} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(2+3*x)^(3/2)*(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{354375} (472500 x^3 + 153000 x^2 - 359685 x - 84697) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(- (6x^2 + x - 2) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}, x\right)$$

7.83 Problem number 2701

$$\int (1-2x)^{3/2} \sqrt{2+3x} \sqrt{3+5x} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{118898 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{354375} \\ & -\frac{2657 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{354375} + \frac{2(1-2x)^{\frac{3}{2}} (3+5x)^{\frac{3}{2}} \sqrt{2+3x}}{35} \\ & + \frac{194(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{2625} - \frac{2657 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{23625} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(2+3*x)^(1/2)*(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{23625} (13500 x^2 - 7380 x - 6631) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{5x+3} \sqrt{3x+2} (-2x+1)^{\frac{3}{2}}, x\right)$$

7.84 Problem number 2702

$$\int \frac{(1-2x)^{3/2} \sqrt{3+5x}}{\sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4157 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{10125} \\ & + \frac{412 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{10125} \\ & + \frac{2(1-2x)^{\frac{3}{2}} \sqrt{2+3x} \sqrt{3+5x}}{15} + \frac{214 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{675} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{4}{675} (45x - 76) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3} (-2x+1)^{\frac{3}{2}}}{\sqrt{3x+2}}, x\right)$$

7.85 Problem number 2703

$$\int \frac{(1-2x)^{3/2} \sqrt{3+5x}}{(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{494 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{405} \\ & - \frac{214 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{405} \\ & - \frac{2(1-2x)^{\frac{3}{2}} \sqrt{3+5x}}{3\sqrt{2+3x}} - \frac{16 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{27} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(3+5*x)^(1/2)/(2+3*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(6x+25)\sqrt{5x+3}\sqrt{-2x+1}}{27\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{9x^2+12x+4}, x\right)$$

7.86 Problem number 2704

$$\int \frac{(1-2x)^{3/2}\sqrt{3+5x}}{(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{98 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{81} \\ & + \frac{16 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{81} \\ & - \frac{2(1-2x)^{\frac{3}{2}}\sqrt{3+5x}}{9(2+3x)^{\frac{3}{2}}} + \frac{82\sqrt{1-2x}\sqrt{3+5x}}{27\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(3+5*x)^(1/2)/(2+3*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(129x+79)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{27(9x^2+12x+4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{27x^3+54x^2+36x+8}, x\right)$$

7.87 Problem number 2705

$$\int \frac{(1-2x)^{3/2} \sqrt{3+5x}}{(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3896 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2835} \\ & - \frac{164 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2835} \\ & - \frac{2(1-2x)^{\frac{3}{2}} \sqrt{3+5x}}{15(2+3x)^{\frac{5}{2}}} + \frac{82\sqrt{1-2x} \sqrt{3+5x}}{135(2+3x)^{\frac{3}{2}}} + \frac{3896\sqrt{1-2x} \sqrt{3+5x}}{945\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(1/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(17532x^2 + 24363x + 8303)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{945(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{81x^4 + 216x^3 + 216x^2 + 96x + 16}, x\right)$$

7.88 Problem number 2706

$$\int \frac{(1-2x)^{3/2} \sqrt{3+5x}}{(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{595324 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{138915} \\ & - \frac{18016 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{138915} - \frac{2(1-2x)^{\frac{3}{2}} \sqrt{3+5x}}{21(2+3x)^{\frac{7}{2}}} \\ & + \frac{82\sqrt{1-2x} \sqrt{3+5x}}{315(2+3x)^{\frac{5}{2}}} + \frac{8516\sqrt{1-2x} \sqrt{3+5x}}{6615(2+3x)^{\frac{3}{2}}} + \frac{595324\sqrt{1-2x} \sqrt{3+5x}}{46305\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(3+5*x)^(1/2)/(2+3*x)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(8036874x^3 + 16342002x^2 + 11095995x + 2510369)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{46305(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32}, x\right)$$

7.89 Problem number 2707

$$\int \frac{(1-2x)^{3/2}\sqrt{3+5x}}{(2+3x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{42623864 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2917215} \\ & - \frac{1282376 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2917215} \\ & - \frac{2(1-2x)^{\frac{3}{2}}\sqrt{3+5x}}{27(2+3x)^{\frac{9}{2}}} + \frac{82\sqrt{1-2x}\sqrt{3+5x}}{567(2+3x)^{\frac{7}{2}}} + \frac{13136\sqrt{1-2x}\sqrt{3+5x}}{19845(2+3x)^{\frac{5}{2}}} \\ & + \frac{613276\sqrt{1-2x}\sqrt{3+5x}}{138915(2+3x)^{\frac{3}{2}}} + \frac{42623864\sqrt{1-2x}\sqrt{3+5x}}{972405\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(3+5*x)^(1/2)/(2+3*x)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(1726266492x^4 + 4661331894x^3 + 4722182964x^2 + 2127363207x + 359554583)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{972405(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64}, x\right)$$

7.90 Problem number 2708

$$\int (1-2x)^{3/2}(2+3x)^{5/2}(3+5x)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{65} \\ & - \frac{51601293223 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15202687500} \\ & - \frac{776112041 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{7601343750} \\ & + \frac{601(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{160875} + \frac{178(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{10725} \\ & - \frac{11725073(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{56306250} - \frac{18034(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{625625} \\ & - \frac{776112041\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{506756250} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(2+3*x)^(5/2)*(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{506756250} (7016625000 x^5 + 12374775000 x^4 + 3047388750 x^3 - 5775295500 x^2 - 3548873565 x + 325972172) \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(- (90 x^4 + 129 x^3 + 25 x^2 - 32 x - 12) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}, x\right)$$

7.91 Problem number 2709

$$\int (1-2x)^{3/2}(2+3x)^{3/2}(3+5x)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{55} - \frac{90397364 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{58471875} \\ & - \frac{5442127 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{116943750} \\ & + \frac{62(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{2475} - \frac{40703(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{433125} \\ & - \frac{23(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{9625} - \frac{5442127\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{7796250} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(2+3*x)^(3/2)*(3+5*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{7796250} (42525000x^4 + 43470000x^3 - 17237250x^2 - 27227430x - 810641) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(- (30x^3 + 23x^2 - 7x - 6) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}, x\right)$$

7.92 Problem number 2710

$$\int (1-2x)^{3/2} \sqrt{2+3x} (3+5x)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4971289 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{6378750} \\ & - \frac{76163 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3189375} \\ & + \frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}\sqrt{2+3x}}{45} - \frac{839(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{23625} \\ & + \frac{194(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{4725} - \frac{76163\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{212625} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(3/2)*(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{212625} (472500x^3 + 112500x^2 - 337545x - 64804) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(10x^2 + x - 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{\sqrt{3x+2}}, x\right)$$

7.93 Problem number 2711

$$\int \frac{(1-2x)^{3/2}(3+5x)^{3/2}}{\sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{29933 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{70875} \\ & -\frac{1847 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{70875} + \frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}\sqrt{2+3x}}{21} \\ & + \frac{74(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{525} - \frac{1847\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{4725} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(3/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{4725} (4500x^2 - 2880x - 1501) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(10x^2 + x - 3)\sqrt{5x+3}\sqrt{-2x+1}}{\sqrt{3x+2}}, x\right)$$

7.94 Problem number 2712

$$\int \frac{(1-2x)^{3/2}(3+5x)^{3/2}}{(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2209 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2025} \\ & + \frac{494 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2025} - \frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{3\sqrt{2+3x}} \\ & - \frac{8(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{15} + \frac{494\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{135} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(3/2)/(2+3*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(90x^2 - 102x - 143)\sqrt{5x+3}\sqrt{-2x+1}}{135\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(10x^2+x-3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9x^2+12x+4}, x\right)$$

7.95 Problem number 2713

$$\int \frac{(1-2x)^{3/2}(3+5x)^{3/2}}{(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{9(2+3x)^{\frac{3}{2}}} + \frac{592 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{243} \\ & - \frac{230 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{243} \\ & + \frac{74(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{9\sqrt{2+3x}} - \frac{1150\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{81} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(3/2)/(2+3*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(90x^2 + 564x + 329)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{81(9x^2 + 12x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(10x^2 + x - 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{27x^3 + 54x^2 + 36x + 8}, x\right)$$

7.96 Problem number 2714

$$\int \frac{(1-2x)^{3/2}(3+5x)^{3/2}}{(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{15(2+3x)^{\frac{5}{2}}} - \frac{4418 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2835} \\ & + \frac{988 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2835} \\ & + \frac{74(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{45(2+3x)^{\frac{3}{2}}} + \frac{988\sqrt{1-2x}\sqrt{3+5x}}{945\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(3/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(16731x^2 + 20754x + 6449)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{945(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(10x^2 + x - 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{81x^4 + 216x^3 + 216x^2 + 96x + 16}, x\right)$$

7.97 Problem number 2715

$$\int \frac{(1-2x)^{3/2}(3+5x)^{3/2}}{(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{21(2+3x)^{\frac{7}{2}}} - \frac{119732 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{138915} \\ & - \frac{7388 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{138915} + \frac{74(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{105(2+3x)^{\frac{5}{2}}} \\ & - \frac{3632\sqrt{1-2x}\sqrt{3+5x}}{6615(2+3x)^{\frac{3}{2}}} + \frac{119732\sqrt{1-2x}\sqrt{3+5x}}{46305\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(3/2)/(2+3*x)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(1616382x^3 + 3385161x^2 + 2314860x + 519367)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{46305(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(10x^2+x-3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{243x^5+810x^4+1080x^3+720x^2+240x+32}, x\right)$$

7.98 Problem number 2716

$$\int \frac{(1-2x)^{3/2}(3+5x)^{3/2}}{(2+3x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{27(2+3x)^{\frac{9}{2}}} - \frac{19885156 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{8751645} \\ & - \frac{609304 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{8751645} + \frac{74(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{189(2+3x)^{\frac{7}{2}}} \\ & - \frac{8252\sqrt{1-2x}\sqrt{3+5x}}{19845(2+3x)^{\frac{5}{2}}} + \frac{280904\sqrt{1-2x}\sqrt{3+5x}}{416745(2+3x)^{\frac{3}{2}}} + \frac{19885156\sqrt{1-2x}\sqrt{3+5x}}{2917215\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(3+5*x)^(3/2)/(2+3*x)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(805348818x^4 + 2174142276x^3 + 2204875881x^2 + 993561978x + 167622907)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2917215(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(10x^2 + x - 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64}, x\right)$$

7.99 Problem number 2717

$$\int \frac{(1-2x)^{3/2}(3+5x)^{3/2}}{(2+3x)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{33(2+3x)^{\frac{11}{2}}} - \frac{1446357824 \text{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{224625555} \\ & - \frac{43537016 \text{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{224625555} + \frac{74(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{297(2+3x)^{\frac{9}{2}}} \\ & - \frac{12872\sqrt{1-2x}\sqrt{3+5x}}{43659(2+3x)^{\frac{7}{2}}} + \frac{442076\sqrt{1-2x}\sqrt{3+5x}}{1528065(2+3x)^{\frac{5}{2}}} \\ & + \frac{20799916\sqrt{1-2x}\sqrt{3+5x}}{10696455(2+3x)^{\frac{3}{2}}} + \frac{1446357824\sqrt{1-2x}\sqrt{3+5x}}{74875185\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(3+5*x)^(3/2)/(2+3*x)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(175732475616x^5 + 591671694906x^4 + 797050394730x^3 + 537061687749x^2 + 180988667568x + 24398176891)}{74875185(729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(10x^2 + x - 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 + 6048x^2 + 1344x + 128}, x\right)$$

7.100 Problem number 2718

$$\int (1-2x)^{3/2}(2+3x)^{5/2}(3+5x)^{5/2} dx$$

Optimal antiderivative

$$\frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{7}{2}}}{75} - \frac{836091184171 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{68412093750} - \frac{50299451003 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{136824187500} + \frac{2503(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{7}{2}} \sqrt{1-2x}}{804375} + \frac{178(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{7}{2}} \sqrt{1-2x}}{14625} - \frac{380132617(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{506756250} - \frac{57509209(3+5x)^{\frac{5}{2}} \sqrt{1-2x} \sqrt{2+3x}}{506756250} - \frac{199721(3+5x)^{\frac{7}{2}} \sqrt{1-2x} \sqrt{2+3x}}{12065625} - \frac{50299451003 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{9121612500}$$

command

```
integrate((1-2*x)^(3/2)*(2+3*x)^(5/2)*(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{9121612500} (547296750000 x^6 + 1316318850000 x^5 + 888419542500 x^4 - 227285730000 x^3 - 522917547750 x^2 -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(450 x^5 + 915 x^4 + 512 x^3 - 85 x^2 - 156 x - 36\right) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}, x\right)$$

7.101 Problem number 2719

$$\int (1-2x)^{3/2}(2+3x)^{3/2}(3+5x)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{7}{2}}}{65} - \frac{9380126059 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1824322500} \\ & - \frac{70536439 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{456080625} + \frac{62(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{7}{2}}\sqrt{1-2x}}{3575} \\ & - \frac{2133359(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{6756750} - \frac{160084(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{3378375} \\ & - \frac{67(3+5x)^{\frac{7}{2}}\sqrt{1-2x}\sqrt{2+3x}}{160875} - \frac{70536439\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{30405375} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(2+3*x)^(3/2)*(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{60810750} (1403325000 x^5 + 2364390000 x^4 + 496455750 x^3 - 1110242250 x^2 - 638983395 x + 67302101) \sqrt{5x+3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(- (150x^4 + 205x^3 + 34x^2 - 51x - 18) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}, x\right)$$

7.102 Problem number 2720

$$\int (1-2x)^{3/2} \sqrt{2+3x} (3+5x)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{97540001 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{42099750} \\ & - \frac{2930159 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{42099750} + \frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{7}{2}}\sqrt{2+3x}}{55} \\ & - \frac{22576(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{155925} - \frac{2377(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{155925} \\ & + \frac{194(3+5x)^{\frac{7}{2}}\sqrt{1-2x}\sqrt{2+3x}}{7425} - \frac{2930159\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{2806650} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(5/2)*(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2806650} (25515000 x^4 + 24003000 x^3 - 10837350 x^2 - 14851260 x - 201247) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(50x^3 + 35x^2 - 12x - 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{\sqrt{3x+2}}, x\right)$$

7.103 Problem number 2721

$$\int \frac{(1-2x)^{3/2}(3+5x)^{5/2}}{\sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{886499 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{765450} \\ & -\frac{11908 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{382725} \\ & + \frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}\sqrt{2+3x}}{27} - \frac{499(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{2835} \\ & + \frac{46(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{567} - \frac{11908\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{25515} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(5/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{25515} (94500 x^3 + 14400 x^2 - 62325 x - 10259) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(50x^3 + 35x^2 - 12x - 9)\sqrt{5x+3}\sqrt{-2x+1}}{\sqrt{3x+2}}, x\right)$$

7.104 Problem number 2722

$$\int \frac{(1-2x)^{3/2}(3+5x)^{5/2}}{(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2894 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{8505} \\ & - \frac{1061 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{8505} \\ & - \frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{3\sqrt{2+3x}} + \frac{202(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{63} \\ & - \frac{32(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{63} - \frac{1061\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{567} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(5/2)/(2+3*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2700x^3 - 180x^2 - 1767x - 200)\sqrt{5x+3}\sqrt{-2x+1}}{567\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(50x^3 + 35x^2 - 12x - 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9x^2 + 12x + 4}, x\right)$$

7.105 Problem number 2723

$$\int \frac{(1-2x)^{3/2}(3+5x)^{5/2}}{(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{9(2+3x)^{\frac{3}{2}}} - \frac{9587 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3645} \\ & + \frac{2632 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3645} + \frac{362(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{27\sqrt{2+3x}} \\ & - \frac{614(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{27} + \frac{2632\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{243} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(3+5*x)^(5/2)/(2+3*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(810x^3 - 468x^2 - 2463x - 1187)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{243(9x^2 + 12x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(50x^3 + 35x^2 - 12x - 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{27x^3 + 54x^2 + 36x + 8}, x\right)$$

7.106 Problem number 2724

$$\int \frac{(1-2x)^{3/2}(3+5x)^{5/2}}{(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{15(2+3x)^{\frac{5}{2}}} + \frac{116854 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{25515} \\ & -\frac{43214 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{25515} + \frac{362(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{135(2+3x)^{\frac{3}{2}}} \\ & + \frac{9808(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{945\sqrt{2+3x}} - \frac{43214\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{1701} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(3+5*x)^(5/2)/(2+3*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(47250x^3 + 377793x^2 + 432387x + 134497)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8505(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(50x^3 + 35x^2 - 12x - 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{81x^4 + 216x^3 + 216x^2 + 96x + 16}, x\right)$$

7.107 Problem number 2725

$$\int \frac{(1-2x)^{3/2}(3+5x)^{5/2}}{(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{21(2+3x)^{\frac{7}{2}}} - \frac{962678 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{416745} \\ & + \frac{249448 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{416745} + \frac{2108(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{6615(2+3x)^{\frac{3}{2}}} \\ & + \frac{362(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{315(2+3x)^{\frac{5}{2}}} + \frac{249448\sqrt{1-2x}\sqrt{3+5x}}{138915\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(5/2)/(2+3*x)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(10680903x^3 + 20067219x^2 + 12594615x + 2640643)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{138915(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(50x^3 + 35x^2 - 12x - 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32}, x\right)$$

7.108 Problem number 2726

$$\int \frac{(1-2x)^{3/2}(3+5x)^{5/2}}{(2+3x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{27(2+3x)^{\frac{9}{2}}} - \frac{17830424 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{26254935} \\ & - \frac{1717916 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{26254935} - \frac{1864(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{6615(2+3x)^{\frac{5}{2}}} \\ & + \frac{362(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{567(2+3x)^{\frac{7}{2}}} - \frac{558524\sqrt{1-2x}\sqrt{3+5x}}{1250235(2+3x)^{\frac{3}{2}}} + \frac{17830424\sqrt{1-2x}\sqrt{3+5x}}{8751645\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(3+5*x)^(5/2)/(2+3*x)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(722132172x^4 + 2043155529x^3 + 2115318249x^2 + 955601637x + 159578303)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8751645(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(50x^3 + 35x^2 - 12x - 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64}, x\right)$$

7.109 Problem number 2727

$$\int \frac{(1-2x)^{3/2}(3+5x)^{5/2}}{(2+3x)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{33(2+3x)^{\frac{11}{2}}} - \frac{3316711588 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2021629995} \\ & - \frac{103970992 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2021629995} \\ & - \frac{13292(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{43659(2+3x)^{\frac{7}{2}}} + \frac{362(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{891(2+3x)^{\frac{9}{2}}} - \frac{1366496\sqrt{1-2x}\sqrt{3+5x}}{4584195(2+3x)^{\frac{5}{2}}} \\ & + \frac{45748292\sqrt{1-2x}\sqrt{3+5x}}{96268095(2+3x)^{\frac{3}{2}}} + \frac{3316711588\sqrt{1-2x}\sqrt{3+5x}}{673876665\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(3+5*x)^(5/2)/(2+3*x)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(402980457942x^5 + 1356237833922x^4 + 1829570010885x^3 + 1234133449713x^2 + 415681177941x + 558751077)}{673876665(729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(50x^3 + 35x^2 - 12x - 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 + 6048x^2 + 1344x + 128}, x\right)$$

7.110 Problem number 2728

$$\int \frac{(1-2x)^{3/2}(3+5x)^{5/2}}{(2+3x)^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{39(2+3x)^{\frac{13}{2}}} - \frac{245282464136 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{61322776515} \\ & - \frac{7391549624 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{61322776515} - \frac{20992(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{81081(2+3x)^{\frac{9}{2}}} \\ & + \frac{362(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{1287(2+3x)^{\frac{11}{2}}} - \frac{2174468\sqrt{1-2x}\sqrt{3+5x}}{11918907(2+3x)^{\frac{7}{2}}} + \frac{73596464\sqrt{1-2x}\sqrt{3+5x}}{417161745(2+3x)^{\frac{5}{2}}} \\ & + \frac{3523482724\sqrt{1-2x}\sqrt{3+5x}}{2920132215(2+3x)^{\frac{3}{2}}} + \frac{245282464136\sqrt{1-2x}\sqrt{3+5x}}{20440925505\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(3+5*x)^(5/2)/(2+3*x)^(15/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(89405458177572x^6 + 360618554767050x^5 + 606171513555828x^4 + 543590753927373x^3 + 274263621177573x^2 + 20440925505(2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3))}{20440925505(2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(50x^3 + 35x^2 - 12x - 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{6561x^8 + 34992x^7 + 81648x^6 + 108864x^5 + 90720x^4 + 48384x^3 + 16128x^2 + 3072x + 256}, x\right)$$

7.111 Problem number 2729

$$\int \frac{(1-2x)^{3/2}(2+3x)^{5/2}}{\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6515539 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{17718750} \\ & - \frac{104663 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{8859375} \\ & + \frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}} \sqrt{3+5x}}{45} + \frac{403(2+3x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{3+5x}}{118125} \\ & + \frac{178(2+3x)^{\frac{5}{2}} \sqrt{1-2x} \sqrt{3+5x}}{4725} - \frac{87476 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{590625} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(2+3*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{590625} (472500x^3 + 193500x^2 - 378045x - 110554) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(18x^3 + 15x^2 - 4x - 4) \sqrt{3x+2} \sqrt{-2x+1}}{\sqrt{5x+3}}, x\right)$$

7.112 Problem number 2730

$$\int \frac{(1-2x)^{3/2}(2+3x)^{3/2}}{\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{46159 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{196875} \\ & - \frac{2281 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{196875} + \frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}} \sqrt{3+5x}}{35} \\ & + \frac{62(2+3x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{3+5x}}{875} - \frac{487 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{13125} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(2+3*x)^(3/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{13125} (4500x^2 - 2040x - 2873) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(6x^2 + x - 2)\sqrt{3x+2}\sqrt{-2x+1}}{\sqrt{5x+3}}, x\right)$$

7.113 Problem number 2731

$$\int \frac{(1-2x)^{3/2} \sqrt{2+3x}}{\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2797 \text{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{16875} \\ & - \frac{598 \text{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{16875} \\ & + \frac{2(1-2x)^{\frac{3}{2}} \sqrt{2+3x} \sqrt{3+5x}}{25} + \frac{194 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{1125} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(2+3*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{4}{1125} (45x - 71) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{\sqrt{5x+3}}, x\right)$$

7.114 Problem number 2732

$$\int \frac{(1-2x)^{3/2}}{\sqrt{2+3x}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\frac{272 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{675} - \frac{202 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{675} - \frac{4\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{45}$$

command

`integrate((1-2*x)^(3/2)/(2+3*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{4}{45} \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{15x^2+19x+6}, x\right)$$

7.115 Problem number 2733

$$\int \frac{(1-2x)^{3/2}}{(2+3x)^{3/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\frac{74 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{45} + \frac{4 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{45} + \frac{14\sqrt{1-2x}\sqrt{3+5x}}{3\sqrt{2+3x}}$$

command

`integrate((1-2*x)^(3/2)/(2+3*x)^(3/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{14 \sqrt{5x+3} \sqrt{-2x+1}}{3 \sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3} \sqrt{3x+2} (-2x+1)^{\frac{3}{2}}}{45x^3 + 87x^2 + 56x + 12}, x\right)$$

7.116 Problem number 2734

$$\int \frac{(1-2x)^{3/2}}{(2+3x)^{5/2} \sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{124 \text{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{27} \\ & - \frac{4 \text{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{27} \\ & + \frac{14 \sqrt{1-2x} \sqrt{3+5x}}{9(2+3x)^{\frac{3}{2}}} + \frac{124 \sqrt{1-2x} \sqrt{3+5x}}{9 \sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)/(2+3*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(186x+131)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9(9x^2+12x+4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3} \sqrt{3x+2} (-2x+1)^{\frac{3}{2}}}{135x^4 + 351x^3 + 342x^2 + 148x + 24}, x\right)$$

7.117 Problem number 2735

$$\int \frac{(1-2x)^{3/2}}{(2+3x)^{7/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{17804 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{945} \\ & - \frac{536 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{945} + \frac{14\sqrt{1-2x}\sqrt{3+5x}}{15(2+3x)^{\frac{5}{2}}} \\ & + \frac{256\sqrt{1-2x}\sqrt{3+5x}}{45(2+3x)^{\frac{3}{2}}} + \frac{17804\sqrt{1-2x}\sqrt{3+5x}}{315\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)/(2+3*x)^(7/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(80118x^2 + 109512x + 37547)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{315(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{405x^5 + 1323x^4 + 1728x^3 + 1128x^2 + 368x + 48}, x\right)$$

7.118 Problem number 2736

$$\int \frac{(1-2x)^{3/2}}{(2+3x)^{9/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{1255552 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15435} \\ & - \frac{37768 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15435} + \frac{2\sqrt{1-2x}\sqrt{3+5x}}{3(2+3x)^{\frac{7}{2}}} \\ & + \frac{388\sqrt{1-2x}\sqrt{3+5x}}{105(2+3x)^{\frac{5}{2}}} + \frac{18068\sqrt{1-2x}\sqrt{3+5x}}{735(2+3x)^{\frac{3}{2}}} + \frac{1255552\sqrt{1-2x}\sqrt{3+5x}}{5145\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)/(2+3*x)^(9/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(16949952x^3 + 34469046x^2 + 23387310x + 5295887)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{5145(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{1215x^6 + 4779x^5 + 7830x^4 + 6840x^3 + 3360x^2 + 880x + 96}, x\right)$$

7.119 Problem number 2737

$$\int \frac{(1-2x)^{3/2}(2+3x)^{7/2}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1473539 \text{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{5906250} \\ & - \frac{31288 \text{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2953125} - \frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{7}{2}}}{5\sqrt{3+5x}} \\ & + \frac{5153(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{39375} + \frac{958(2+3x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{3+5x}}{1575} \\ & - \frac{8(2+3x)^{\frac{7}{2}}\sqrt{1-2x}\sqrt{3+5x}}{45} - \frac{12601\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{196875} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(2+3*x)^(7/2)/(3+5*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(472500x^4 + 517500x^3 - 252225x^2 - 377530x - 83787)\sqrt{3x+2}\sqrt{-2x+1}}{196875\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(54x^4 + 81x^3 + 18x^2 - 20x - 8)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{25x^2 + 30x + 9}, x\right)$$

7.120 Problem number 2738

$$\int \frac{(1-2x)^{3/2}(2+3x)^{5/2}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{47342 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{328125} \\ & - \frac{5753 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{328125} \\ & - \frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}}{5\sqrt{3+5x}} + \frac{2818(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{4375} \\ & - \frac{32(2+3x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{3+5x}}{175} + \frac{2719\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{21875} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(2+3*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(22500x^3 + 5400x^2 - 22305x - 9697)\sqrt{3x+2}\sqrt{-2x+1}}{21875\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(18x^3 + 15x^2 - 4x - 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{25x^2 + 30x + 9}, x\right)$$

7.121 Problem number 2739

$$\int \frac{(1-2x)^{3/2}(2+3x)^{3/2}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{169 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9375} \\ & - \frac{496 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9375} - \frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}}{5\sqrt{3+5x}} \\ & - \frac{24(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{125} + \frac{458\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{625} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(2+3*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(150x^2 - 130x - 77)\sqrt{3x+2}\sqrt{-2x+1}}{625\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(6x^2 + x - 2)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{25x^2 + 30x + 9}, x\right)$$

7.122 Problem number 2740

$$\int \frac{(1-2x)^{3/2}\sqrt{2+3x}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{458 \text{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1125} \\ & - \frac{178 \text{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1125} \\ & - \frac{2(1-2x)^{\frac{3}{2}}\sqrt{2+3x}}{5\sqrt{3+5x}} - \frac{16\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{75} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(2+3*x)^(1/2)/(3+5*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(10x + 39)\sqrt{3x+2}\sqrt{-2x+1}}{75\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{25x^2 + 30x + 9}, x\right)$$

7.123 Problem number 2741

$$\int \frac{(1-2x)^{3/2}}{\sqrt{2+3x} (3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{62 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{75} + \frac{8 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{75} - \frac{22\sqrt{1-2x} \sqrt{2+3x}}{5\sqrt{3+5x}}$$

command

`integrate((1-2*x)^(3/2)/(3+5*x)^(3/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{22 \sqrt{3x+2} \sqrt{-2x+1}}{5 \sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3} \sqrt{3x+2} (-2x+1)^{\frac{3}{2}}}{75x^3 + 140x^2 + 87x + 18}, x\right)$$

7.124 Problem number 2742

$$\int \frac{(1-2x)^{3/2}}{(2+3x)^{3/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{136 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15} + \frac{4 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15} + \frac{14\sqrt{1-2x}}{3\sqrt{2+3x} \sqrt{3+5x}} - \frac{136\sqrt{1-2x} \sqrt{2+3x}}{3\sqrt{3+5x}}$$

command

`integrate((1-2*x)^(3/2)/(2+3*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(68x+43)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{15x^2+19x+6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{225x^4+570x^3+541x^2+228x+36}, x\right)$$

7.125 Problem number 2743

$$\int \frac{(1-2x)^{3/2}}{(2+3x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{532 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{9} \\ & + \frac{16 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{9} \\ & + \frac{14\sqrt{1-2x}}{9(2+3x)^{\frac{3}{2}}\sqrt{3+5x}} + \frac{88\sqrt{1-2x}}{3\sqrt{2+3x}\sqrt{3+5x}} - \frac{2660\sqrt{1-2x}\sqrt{2+3x}}{9\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)/(2+3*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(3990x^2+5188x+1683)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3(45x^3+87x^2+56x+12)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{675x^5+2160x^4+2763x^3+1766x^2+564x+72}, x\right)$$

7.126 Problem number 2744

$$\int \frac{(1-2x)^{3/2}}{(2+3x)^{7/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{36968 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{105} \\ & + \frac{1112 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{105} + \frac{14\sqrt{1-2x}}{15(2+3x)^{5/2}\sqrt{3+5x}} \\ & + \frac{44\sqrt{1-2x}}{5(2+3x)^{3/2}\sqrt{3+5x}} + \frac{6116\sqrt{1-2x}}{35\sqrt{2+3x}\sqrt{3+5x}} - \frac{36968\sqrt{1-2x}\sqrt{2+3x}}{21\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)/(2+3*x)^(7/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(831780x^3 + 1636038x^2 + 1071882x + 233897)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{35(135x^4 + 351x^3 + 342x^2 + 148x + 24)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{3/2}}{2025x^6 + 7830x^5 + 12609x^4 + 10824x^3 + 5224x^2 + 1344x + 144}, x\right)$$

7.127 Problem number 2745

$$\int \frac{(1-2x)^{3/2}}{(2+3x)^{9/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10312712 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{5145} \\ & + \frac{310208 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{5145} \\ & + \frac{2\sqrt{1-2x}}{3(2+3x)^{7/2}\sqrt{3+5x}} + \frac{176\sqrt{1-2x}}{35(2+3x)^{5/2}\sqrt{3+5x}} + \frac{12276\sqrt{1-2x}}{245(2+3x)^{3/2}\sqrt{3+5x}} \\ & + \frac{1706144\sqrt{1-2x}}{1715\sqrt{2+3x}\sqrt{3+5x}} - \frac{10312712\sqrt{1-2x}\sqrt{2+3x}}{1029\sqrt{3+5x}} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)/(2+3*x)^(9/2)/(3+5*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(696108060x^4 + 1833255216x^3 + 1809835578x^2 + 793777840x + 130497191)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1715(405x^5 + 1323x^4 + 1728x^3 + 1128x^2 + 368x + 48)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{6075x^7 + 27540x^6 + 53487x^5 + 57690x^4 + 37320x^3 + 14480x^2 + 3120x + 288}, x\right)$$

7.128 Problem number 2746

$$\int \frac{(1-2x)^{3/2}(2+3x)^{7/2}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{7}{2}}}{15(3+5x)^{\frac{3}{2}}} - \frac{24369 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1203125} \\ & - \frac{25643 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{328125} \\ & - \frac{6(2+3x)^{\frac{7}{2}}\sqrt{1-2x}}{\sqrt{3+5x}} + \frac{3872(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{4375} \\ & + \frac{622(2+3x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{3+5x}}{175} + \frac{4801\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{21875} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(2+3*x)^(7/2)/(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(202500x^4 + 189000x^3 - 174525x^2 - 216050x - 52067)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{65625(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(54x^4 + 81x^3 + 18x^2 - 20x - 8)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{125x^3 + 225x^2 + 135x + 27}, x\right)$$

7.129 Problem number 2747

$$\int \frac{(1-2x)^{3/2}(2+3x)^{5/2}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}}{15(3+5x)^{\frac{3}{2}}} - \frac{8366 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{171875} \\ & + \frac{1973 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{46875} - \frac{106(2+3x)^{\frac{5}{2}} \sqrt{1-2x}}{25\sqrt{3+5x}} \\ & + \frac{1558(2+3x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{3+5x}}{625} + \frac{2264\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{3125} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)*(2+3*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(6750x^3 - 1050x^2 - 2975x + 106)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9375(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(18x^3 + 15x^2 - 4x - 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{125x^3 + 225x^2 + 135x + 27}, x\right)$$

7.130 Problem number 2748

$$\int \frac{(1-2x)^{3/2}(2+3x)^{3/2}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}}{15(3+5x)^{\frac{3}{2}}} - \frac{582 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{6875} \\ & + \frac{496 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1875} \\ & - \frac{62(2+3x)^{\frac{3}{2}} \sqrt{1-2x}}{25\sqrt{3+5x}} + \frac{178\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{125} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(2+3*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(150x^2 + 800x + 437)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{375(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(6x^2 + x - 2)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{125x^3 + 225x^2 + 135x + 27}, x\right)$$

7.131 Problem number 2749

$$\int \frac{(1-2x)^{3/2}\sqrt{2+3x}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{38 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{375} \\ & + \frac{212 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{4125} \\ & - \frac{2(1-2x)^{\frac{3}{2}}\sqrt{2+3x}}{15(3+5x)^{\frac{3}{2}}} - \frac{18\sqrt{1-2x}\sqrt{2+3x}}{25\sqrt{3+5x}} \end{aligned}$$

command

```
integrate((1-2*x)^(3/2)*(2+3*x)^(1/2)/(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(125x + 86)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{75(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{125x^3 + 225x^2 + 135x + 27}, x\right)$$

7.132 Problem number 2750

$$\int \frac{(1-2x)^{3/2}}{\sqrt{2+3x} (3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{148 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{75} - \frac{52 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{825} - \frac{22\sqrt{1-2x}\sqrt{2+3x}}{15(3+5x)^{3/2}} + \frac{148\sqrt{1-2x}\sqrt{2+3x}}{15\sqrt{3+5x}}$$

command

`integrate((1-2*x)^(3/2)/(3+5*x)^(5/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(370x+211)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{15(25x^2+30x+9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{3/2}}{375x^4+925x^3+855x^2+351x+54}, x\right)$$

7.133 Problem number 2751

$$\int \frac{(1-2x)^{3/2}}{(2+3x)^{3/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{556 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15} - \frac{184 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{165} + \frac{14\sqrt{1-2x}}{3(3+5x)^{3/2}\sqrt{2+3x}} - \frac{92\sqrt{1-2x}\sqrt{2+3x}}{3(3+5x)^{3/2}} + \frac{556\sqrt{1-2x}\sqrt{2+3x}}{3\sqrt{3+5x}}$$

command

`integrate((1-2*x)^(3/2)/(2+3*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(4170x^2 + 5144x + 1583)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3(75x^3 + 140x^2 + 87x + 18)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{1125x^5 + 3525x^4 + 4415x^3 + 2763x^2 + 864x + 108}, x\right)$$

7.134 Problem number 2752

$$\int \frac{(1-2x)^{3/2}}{(2+3x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{120 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{11} \\ & -\frac{1088 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{3} + \frac{14\sqrt{1-2x}}{9(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} \\ & + \frac{404\sqrt{1-2x}}{9(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} - \frac{300\sqrt{1-2x}\sqrt{2+3x}}{(3+5x)^{\frac{3}{2}}} + \frac{5440\sqrt{1-2x}\sqrt{2+3x}}{3\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)/(2+3*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(122400x^3 + 232590x^2 + 147122x + 30977)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3(225x^4 + 570x^3 + 541x^2 + 228x + 36)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{\frac{3}{2}}}{3375x^6 + 12825x^5 + 20295x^4 + 17119x^3 + 8118x^2 + 2052x + 216}, x\right)$$

7.135 Problem number 2753

$$\int \frac{(1-2x)^{3/2}}{(2+3x)^{7/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{33232 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{385} \\ & - \frac{301304 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{105} \\ & + \frac{14\sqrt{1-2x}}{15(2+3x)^{5/2}(3+5x)^{3/2}} + \frac{536\sqrt{1-2x}}{45(2+3x)^{3/2}(3+5x)^{3/2}} + \frac{111884\sqrt{1-2x}}{315(3+5x)^{3/2}\sqrt{2+3x}} \\ & - \frac{16616\sqrt{1-2x}\sqrt{2+3x}}{7(3+5x)^{3/2}} + \frac{301304\sqrt{1-2x}\sqrt{2+3x}}{21\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(3/2)/(2+3*x)^(7/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(101690100x^4 + 261029520x^3 + 251053266x^2 + 107221804x + 17157169)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{105(675x^5 + 2160x^4 + 2763x^3 + 1766x^2 + 564x + 72)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}(-2x+1)^{3/2}}{10125x^7 + 45225x^6 + 86535x^5 + 91947x^4 + 58592x^3 + 22392x^2 + 4752x + 432}, x\right)$$

7.136 Problem number 2754

$$\int (1-2x)^{5/2}(2+3x)^{5/2}\sqrt{3+5x} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{62(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{2145} + \frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{65} \\ & - \frac{1163388067 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1266890625} \\ & - \frac{69808931 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2533781250} \\ & + \frac{32717(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{1126125} + \frac{34(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{2475} \\ & - \frac{445024(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{9384375} - \frac{69808931 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{168918750} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(2+3*x)^(5/2)*(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{168918750} (935550000 x^5 + 433755000 x^4 - 936022500 x^3 - 309143250 x^2 + 380959290 x + 84411073) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(36x^4 + 12x^3 - 23x^2 - 4x + 4\right) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}, x\right)$$

7.137 Problem number 2755

$$\int (1-2x)^{5/2} (2+3x)^{3/2} \sqrt{3+5x} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{106(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{2475} + \frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{55} \\ & - \frac{326256461 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{584718750} \\ & - \frac{4738087 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{292359375} \\ & + \frac{2866(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{86625} + \frac{38729(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{2165625} \\ & - \frac{4738087 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{19490625} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(2+3*x)^(3/2)*(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{19490625} (42525000 x^4 - 13702500 x^3 - 35750250 x^2 + 16294455 x + 9437696) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(12x^3 - 4x^2 - 5x + 2\right)\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}, x\right)$$

7.138 Problem number 2756

$$\int (1-2x)^{5/2} \sqrt{2+3x} \sqrt{3+5x} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6799613 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15946875} \\ & - \frac{110717 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15946875} \\ & + \frac{326(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}\sqrt{2+3x}}{4725} + \frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}\sqrt{2+3x}}{45} \\ & + \frac{10214(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{118125} - \frac{110717\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{1063125} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(2+3*x)^(1/2)*(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{1063125} (945000 x^3 - 1111500 x^2 + 55530 x + 526861) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(4x^2 - 4x + 1\right)\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}, x\right)$$

7.139 Problem number 2757

$$\int \frac{(1-2x)^{5/2} \sqrt{3+5x}}{\sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{86741 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{118125} \\ & + \frac{11806 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{118125} + \frac{118(1-2x)^{\frac{3}{2}} \sqrt{2+3x} \sqrt{3+5x}}{525} \\ & + \frac{2(1-2x)^{\frac{5}{2}} \sqrt{2+3x} \sqrt{3+5x}}{21} + \frac{4282 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{7875} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{7875} (1500x^2 - 3270x + 3401) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2 - 4x + 1) \sqrt{5x+3} \sqrt{-2x+1}}{\sqrt{3x+2}}, x\right)$$

7.140 Problem number 2758

$$\int \frac{(1-2x)^{5/2} \sqrt{3+5x}}{(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{31588 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{10125} \\ & - \frac{12758 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{10125} - \frac{2(1-2x)^{\frac{5}{2}} \sqrt{3+5x}}{3\sqrt{2+3x}} \\ & - \frac{8(1-2x)^{\frac{3}{2}} \sqrt{2+3x} \sqrt{3+5x}}{15} - \frac{1076 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{675} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(3+5*x)^(1/2)/(2+3*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(180x^2 - 534x - 1661)\sqrt{5x+3}\sqrt{-2x+1}}{675\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9x^2 + 12x + 4}, x\right)$$

7.141 Problem number 2759

$$\int \frac{(1-2x)^{5/2}\sqrt{3+5x}}{(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4418 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1215} \\ & + \frac{988 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1215} - \frac{2(1-2x)^{5/2}\sqrt{3+5x}}{9(2+3x)^{3/2}} \\ & + \frac{10(1-2x)^{3/2}\sqrt{3+5x}}{3\sqrt{2+3x}} + \frac{196\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{81} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(3+5*x)^(1/2)/(2+3*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(36x^2 + 1077x + 653)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{81(9x^2 + 12x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{27x^3 + 54x^2 + 36x + 8}, x\right)$$

7.142 Problem number 2760

$$\int \frac{(1-2x)^{5/2} \sqrt{3+5x}}{(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15} \\ & - \frac{12 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15} \\ & - \frac{2(1-2x)^{5/2} \sqrt{3+5x}}{15(2+3x)^{5/2}} + \frac{2(1-2x)^{3/2} \sqrt{3+5x}}{3(2+3x)^{3/2}} + \frac{8\sqrt{1-2x} \sqrt{3+5x}}{\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(1/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(506x^2 + 719x + 249)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{15(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{81x^4 + 216x^3 + 216x^2 + 96x + 16}, x\right)$$

7.143 Problem number 2761

$$\int \frac{(1-2x)^{5/2} \sqrt{3+5x}}{(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{36052 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3969} \\ & - \frac{1048 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3969} - \frac{2(1-2x)^{5/2} \sqrt{3+5x}}{21(2+3x)^{7/2}} \\ & + \frac{2(1-2x)^{3/2} \sqrt{3+5x}}{7(2+3x)^{5/2}} + \frac{524\sqrt{1-2x} \sqrt{3+5x}}{189(2+3x)^{3/2}} + \frac{36052\sqrt{1-2x} \sqrt{3+5x}}{1323\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(1/2)/(2+3*x)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(486702x^3 + 988524x^2 + 671007x + 151859)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1323(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32}, x\right)$$

7.144 Problem number 2762

$$\int \frac{(1-2x)^{5/2}\sqrt{3+5x}}{(2+3x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7810384 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{250047} \\ & - \frac{234856 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{250047} \\ & - \frac{2(1-2x)^{5/2}\sqrt{3+5x}}{27(2+3x)^{9/2}} + \frac{10(1-2x)^{3/2}\sqrt{3+5x}}{63(2+3x)^{7/2}} + \frac{832\sqrt{1-2x}\sqrt{3+5x}}{567(2+3x)^{5/2}} \\ & + \frac{112436\sqrt{1-2x}\sqrt{3+5x}}{11907(2+3x)^{3/2}} + \frac{7810384\sqrt{1-2x}\sqrt{3+5x}}{83349\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(1/2)/(2+3*x)^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(316320552x^4 + 854146674x^3 + 865270206x^2 + 389804925x + 65886031)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{83349(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64}, x\right)$$

7.145 Problem number 2763

$$\int \frac{(1-2x)^{5/2} \sqrt{3+5x}}{(2+3x)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{247408648 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2139291} \\ & - \frac{7442032 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2139291} - \frac{2(1-2x)^{5/2} \sqrt{3+5x}}{33(2+3x)^{11/2}} \\ & + \frac{10(1-2x)^{3/2} \sqrt{3+5x}}{99(2+3x)^{9/2}} + \frac{1900\sqrt{1-2x} \sqrt{3+5x}}{2079(2+3x)^{7/2}} + \frac{76492\sqrt{1-2x} \sqrt{3+5x}}{14553(2+3x)^{5/2}} \\ & + \frac{3560432\sqrt{1-2x} \sqrt{3+5x}}{101871(2+3x)^{3/2}} + \frac{247408648\sqrt{1-2x} \sqrt{3+5x}}{713097\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(1/2)/(2+3*x)^(13/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(30060150732x^5 + 101209884912x^4 + 136342955970x^3 + 91862628912x^2 + 30956769477x + 4174268813)\sqrt{5x}}{713097(729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 + 6048x^2 + 1344x + 128}, x\right)$$

7.146 Problem number 2764

$$\int (1-2x)^{5/2}(2+3x)^{5/2}(3+5x)^{3/2} dx$$

Optimal antiderivative

$$\frac{62(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{2925} + \frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{75}$$

$$\frac{1764163292393 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{684120937500}$$

$$\frac{13267820528 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{171030234375}$$

$$+ \frac{142391(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{7239375} + \frac{3698(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{482625}$$

$$\frac{400516993(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{2533781250} - \frac{569519(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{28153125}$$

$$\frac{13267820528\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{11402015625}$$

command

`integrate((1-2*x)^(5/2)*(2+3*x)^(5/2)*(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{22804031250} (547296750000 x^6 + 621672975000 x^5 - 336683182500 x^4 - 528977216250 x^3 + 48836706750 x^2 + 1725000000 x - 125000000)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(180x^5 + 168x^4 - 79x^3 - 89x^2 + 8x + 12\right)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}, x\right)$$

7.147 Problem number 2765

$$\int (1-2x)^{5/2}(2+3x)^{3/2}(3+5x)^{3/2} dx$$

Optimal antiderivative

$$\frac{106(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{3575} + \frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{65}$$

$$\frac{30660308017 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{22804031250}$$

$$\frac{923943703 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{22804031250}$$

$$+ \frac{8318(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{482625} - \frac{6794792(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{84459375}$$

$$+ \frac{25603(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{1876875} - \frac{923943703\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{1520268750}$$

command

`integrate((1-2*x)^(5/2)*(2+3*x)^(3/2)*(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{1520268750} (14033250000 x^5 + 5400675000 x^4 - 13684072500 x^3 - 3707642250 x^2 + 5290733520 x + 1020785999)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((60 x^4 + 16 x^3 - 37 x^2 - 5 x + 6) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}, x\right)$$

7.148 Problem number 2766

$$\int (1 - 2x)^{5/2} \sqrt{2 + 3x} (3 + 5x)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{829177897 \text{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1052493750} \\ & - \frac{12996374 \text{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{526246875} \\ & + \frac{326(1-2x)^{\frac{3}{2}} (3+5x)^{\frac{5}{2}} \sqrt{2+3x}}{7425} + \frac{2(1-2x)^{\frac{5}{2}} (3+5x)^{\frac{5}{2}} \sqrt{2+3x}}{55} \\ & - \frac{78797(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{3898125} + \frac{30362(3+5x)^{\frac{5}{2}} \sqrt{1-2x} \sqrt{2+3x}}{779625} \\ & - \frac{12996374 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{35083125} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(3/2)*(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{35083125} (127575000 x^4 - 51502500 x^3 - 95024250 x^2 + 48272535 x + 22517617) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((20 x^3 - 8 x^2 - 7 x + 3) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}, x\right)$$

7.149 Problem number 2767

$$\int \frac{(1-2x)^{5/2}(3+5x)^{3/2}}{\sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4457606 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9568125} \\ & - \frac{429479 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9568125} \\ & + \frac{362(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}} \sqrt{2+3x}}{2835} + \frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}} \sqrt{2+3x}}{27} \\ & + \frac{14318(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{70875} - \frac{429479 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{637875} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(3/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{637875} (945000x^3 - 1192500x^2 + 232110x + 343207) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(20x^3 - 8x^2 - 7x + 3) \sqrt{5x+3} \sqrt{-2x+1}}{\sqrt{3x+2}}, x\right)$$

7.150 Problem number 2768

$$\int \frac{(1-2x)^{5/2}(3+5x)^{3/2}}{(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{481339 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{212625} \\ & + \frac{124724 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{212625} \\ & - \frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{3\sqrt{2+3x}} - \frac{32(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}} \sqrt{2+3x}}{63} \\ & - \frac{2108(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{1575} + \frac{124724 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{14175} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(3+5*x)^(3/2)/(2+3*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(13500x^3 - 21690x^2 + 14727x + 32033)\sqrt{5x+3}\sqrt{-2x+1}}{14175\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(20x^3 - 8x^2 - 7x + 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9x^2 + 12x + 4}, x\right)$$

7.151 Problem number 2769

$$\int \frac{(1-2x)^{5/2}(3+5x)^{3/2}}{(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{9(2+3x)^{\frac{3}{2}}} + \frac{116854 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{18225} \\ & - \frac{43214 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{18225} + \frac{230(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{27\sqrt{2+3x}} \\ & + \frac{788(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{135} - \frac{43214\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{1215} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(3+5*x)^(3/2)/(2+3*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(1620x^3 - 3906x^2 - 23538x - 13231)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1215(9x^2 + 12x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(20x^3 - 8x^2 - 7x + 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{27x^3 + 54x^2 + 36x + 8}, x\right)$$

7.152 Problem number 2770

$$\int \frac{(1-2x)^{5/2}(3+5x)^{3/2}}{(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{15(2+3x)^{\frac{5}{2}}} + \frac{46(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{27(2+3x)^{\frac{3}{2}}} \\ & - \frac{19174 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{3645} \\ & + \frac{5264 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{3645} \\ & - \frac{316(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{27\sqrt{2+3x}} + \frac{5264\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{243} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(3/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2700x^3 + 68913x^2 + 83412x + 25927)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1215(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(20x^3 - 8x^2 - 7x + 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{81x^4 + 216x^3 + 216x^2 + 96x + 16}, x\right)$$

7.153 Problem number 2771

$$\int \frac{(1-2x)^{5/2}(3+5x)^{3/2}}{(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{21(2+3x)^{\frac{7}{2}}} + \frac{46(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{63(2+3x)^{\frac{5}{2}}} \\ & - \frac{11576 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \frac{\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{11907} \\ & - \frac{4244 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \frac{\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{11907} \\ & + \frac{608(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{189(2+3x)^{\frac{3}{2}}} - \frac{4244 \sqrt{1-2x} \sqrt{3+5x}}{3969 \sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(3/2)/(2+3*x)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(182736x^3 + 409005x^2 + 292578x + 67759)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3969(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(20x^3 - 8x^2 - 7x + 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32}, x\right)$$

7.154 Problem number 2772

$$\int \frac{(1-2x)^{5/2}(3+5x)^{3/2}}{(2+3x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{27(2+3x)^{\frac{9}{2}}} + \frac{230(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{567(2+3x)^{\frac{7}{2}}} \\ & - \frac{3545996 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \frac{\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{750141} \\ & - \frac{95264 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \frac{\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{750141} + \frac{1532(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{567(2+3x)^{\frac{5}{2}}} \\ & - \frac{104036 \sqrt{1-2x} \sqrt{3+5x}}{35721(2+3x)^{\frac{3}{2}}} + \frac{3545996 \sqrt{1-2x} \sqrt{3+5x}}{250047 \sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(3+5*x)^(3/2)/(2+3*x)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(143612838x^4 + 386630766x^3 + 391601529x^2 + 176436240x + 29785139)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{250047(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(20x^3 - 8x^2 - 7x + 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64}, x\right)$$

7.155 Problem number 2773

$$\int \frac{(1-2x)^{5/2}(3+5x)^{3/2}}{(2+3x)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{33(2+3x)^{\frac{11}{2}}} + \frac{230(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{891(2+3x)^{\frac{9}{2}}} \\ & - \frac{780320008 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{57760857} \\ & - \frac{23441272 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{57760857} \\ & + \frac{12280(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{6237(2+3x)^{\frac{7}{2}}} - \frac{325796\sqrt{1-2x}\sqrt{3+5x}}{130977(2+3x)^{\frac{5}{2}}} \\ & + \frac{11243972\sqrt{1-2x}\sqrt{3+5x}}{2750517(2+3x)^{\frac{3}{2}}} + \frac{780320008\sqrt{1-2x}\sqrt{3+5x}}{19253619\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(3+5*x)^(3/2)/(2+3*x)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(94808880972x^5 + 319217269302x^4 + 429993423180x^3 + 289719086787x^2 + 97637232762x + 13163824553)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{19253619(729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(20x^3 - 8x^2 - 7x + 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 + 6048x^2 + 1344x + 128}, x\right)$$

7.156 Problem number 2774

$$\int \frac{(1-2x)^{5/2}(3+5x)^{3/2}}{(2+3x)^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{39(2+3x)^{\frac{13}{2}}} + \frac{230(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{1287(2+3x)^{\frac{11}{2}}} \\ & -\frac{75041008472 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1752079329} \\ & -\frac{2257166048 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1752079329} + \frac{1300(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{891(2+3x)^{\frac{9}{2}}} \\ & -\frac{3347620\sqrt{1-2x}\sqrt{3+5x}}{1702701(2+3x)^{\frac{7}{2}}} + \frac{23210828\sqrt{1-2x}\sqrt{3+5x}}{11918907(2+3x)^{\frac{5}{2}}} \\ & +\frac{1079936248\sqrt{1-2x}\sqrt{3+5x}}{83432349(2+3x)^{\frac{3}{2}}} + \frac{75041008472\sqrt{1-2x}\sqrt{3+5x}}{584026443\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(3+5*x)^(3/2)/(2+3*x)^(15/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(27352447588044x^6 + 110328276131100x^5 + 185457331738206x^4 + 166295375376786x^3 + 83893544414217x^2 + 584026443(2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(20x^3 - 8x^2 - 7x + 3)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{6561x^8 + 34992x^7 + 81648x^6 + 108864x^5 + 90720x^4 + 48384x^3 + 16128x^2 + 3072x + 256}, x\right)$$

7.157 Problem number 2775

$$\int (1-2x)^{5/2}(2+3x)^{3/2}(3+5x)^{5/2} dx$$

Optimal antiderivative

$$\frac{106(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{7}{2}}}{4875} + \frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{7}{2}}}{75}$$

$$\frac{1580201444291 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{410472562500}$$

$$\frac{23763809947 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{205236281250}$$

$$+ \frac{8038(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{7}{2}}\sqrt{1-2x}}{804375} - \frac{359748241(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{1520268750}$$

$$- \frac{26534891(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{760134375} + \frac{364267(3+5x)^{\frac{7}{2}}\sqrt{1-2x}\sqrt{2+3x}}{36196875}$$

$$- \frac{23763809947\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{13682418750}$$

command

`integrate((1-2*x)^(5/2)*(2+3*x)^(3/2)*(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{13682418750} (547296750000 x^6 + 579573225000 x^5 - 352885207500 x^4 - 487924998750 x^3 + 59959633500 x^2 + 1500000000 x - 150000000)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(300x^5 + 260x^4 - 137x^3 - 136x^2 + 15x + 18\right)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}, x\right)$$

7.158 Problem number 2776

$$\int (1-2x)^{5/2} \sqrt{2+3x} (3+5x)^{5/2} dx$$

Optimal antiderivative

$$\frac{8120161139 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{4104725625}$$

$$\frac{486785077 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{8209451250}$$

$$+ \frac{326(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{7}{2}}\sqrt{2+3x}}{10725} + \frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{7}{2}}\sqrt{2+3x}}{65}$$

$$- \frac{3872003(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{30405375} - \frac{121031(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{30405375}$$

$$+ \frac{2314(3+5x)^{\frac{7}{2}}\sqrt{1-2x}\sqrt{2+3x}}{111375} - \frac{486785077\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{547296750}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(5/2)*(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{547296750} (8419950000 x^5 + 2577015000 x^4 - 7942630500 x^3 - 1730459250 x^2 + 2923422930 x + 495379991) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(100x^4 + 20x^3 - 59x^2 - 6x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{\sqrt{2+3x}}, x\right)$$

7.159 Problem number 2777

$$\int \frac{(1-2x)^{5/2}(3+5x)^{5/2}}{\sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{146222113 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{126299250} \\ & - \frac{1654421 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{63149625} + \frac{74(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}\sqrt{2+3x}}{891} \\ & + \frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}\sqrt{2+3x}}{33} - \frac{146963(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{467775} \\ & + \frac{9698(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{93555} - \frac{1654421\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{4209975} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(5/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{4209975} (25515000 x^4 - 12379500 x^3 - 16381350 x^2 + 9143865 x + 3748468) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(100x^4 + 20x^3 - 59x^2 - 6x + 9)\sqrt{5x+3}\sqrt{-2x+1}}{\sqrt{3x+2}}, x\right)$$

7.160 Problem number 2778

$$\int \frac{(1-2x)^{5/2}(3+5x)^{5/2}}{(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{25111 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1148175} \\ & - \frac{310399 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1148175} - \frac{2(1-2x)^{5/2}(3+5x)^{5/2}}{3\sqrt{2+3x}} \\ & - \frac{40(1-2x)^{3/2}(3+5x)^{5/2}\sqrt{2+3x}}{81} + \frac{64628(3+5x)^{3/2}\sqrt{1-2x}\sqrt{2+3x}}{8505} \\ & - \frac{2108(3+5x)^{5/2}\sqrt{1-2x}\sqrt{2+3x}}{1701} - \frac{310399\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{76545} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(5/2)/(2+3*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(567000x^4 - 386100x^3 - 259650x^2 + 245751x + 21964)\sqrt{5x+3}\sqrt{-2x+1}}{76545\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(100x^4 + 20x^3 - 59x^2 - 6x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9x^2 + 12x + 4}, x\right)$$

7.161 Problem number 2779

$$\int \frac{(1-2x)^{5/2}(3+5x)^{5/2}}{(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(1-2x)^{5/2}(3+5x)^{5/2}}{9(2+3x)^{3/2}} - \frac{452399 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{76545} \\ & + \frac{135334 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{76545} \\ & + \frac{370(1-2x)^{3/2}(3+5x)^{5/2}}{27\sqrt{2+3x}} - \frac{31298(3+5x)^{3/2}\sqrt{1-2x}\sqrt{2+3x}}{567} \\ & + \frac{5260(3+5x)^{5/2}\sqrt{1-2x}\sqrt{2+3x}}{567} + \frac{135334\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{5103} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(5/2)/(2+3*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(24300x^4 - 25110x^3 + 5949x^2 + 108285x + 56963)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{5103(9x^2 + 12x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(100x^4 + 20x^3 - 59x^2 - 6x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{27x^3 + 54x^2 + 36x + 8}, x\right)$$

7.162 Problem number 2780

$$\int \frac{(1-2x)^{5/2}(3+5x)^{5/2}}{(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{15(2+3x)^{\frac{5}{2}}} + \frac{74(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{27(2+3x)^{\frac{3}{2}}} \\ & + \frac{136028 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{10935} \\ & - \frac{48478 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{10935} - \frac{6464(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{81\sqrt{2+3x}} \\ & + \frac{11036(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{81} - \frac{48478\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{729} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(5/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(24300x^4 - 45090x^3 - 461043x^2 - 517257x - 158237)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3645(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(100x^4 + 20x^3 - 59x^2 - 6x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{81x^4 + 216x^3 + 216x^2 + 96x + 16}, x\right)$$

7.163 Problem number 2781

$$\int \frac{(1-2x)^{5/2}(3+5x)^{5/2}}{(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{21(2+3x)^{\frac{7}{2}}} + \frac{74(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{63(2+3x)^{\frac{5}{2}}} \\ & - \frac{904798 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{107163} \\ & + \frac{270668 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{107163} - \frac{1844(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{567(2+3x)^{\frac{3}{2}}} \\ & - \frac{62596(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{3969\sqrt{2+3x}} + \frac{1353340\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{35721} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(5/2)/(2+3*x)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(396900x^4 + 9846603x^3 + 17788023x^2 + 11107911x + 2337569)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{35721(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(100x^4 + 20x^3 - 59x^2 - 6x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32}, x\right)$$

7.164 Problem number 2782

$$\int \frac{(1-2x)^{5/2}(3+5x)^{5/2}}{(2+3x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{27(2+3x)^{\frac{9}{2}}} + \frac{370(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{567(2+3x)^{\frac{7}{2}}} \\
& - \frac{100444 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2250423} \\
& - \frac{1241596 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2250423} - \frac{13316(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{35721(2+3x)^{\frac{3}{2}}} \\
& + \frac{2776(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{1701(2+3x)^{\frac{5}{2}}} - \frac{1241596\sqrt{1-2x}\sqrt{3+5x}}{750141\sqrt{2+3x}}
\end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(3+5*x)^(5/2)/(2+3*x)^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(29072682x^4 + 115002639x^3 + 142557831x^2 + 71920155x + 12903031)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{750141(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(100x^4 + 20x^3 - 59x^2 - 6x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64}, x\right)$$

7.165 Problem number 2783

$$\int \frac{(1-2x)^{5/2}(3+5x)^{5/2}}{(2+3x)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{33(2+3x)^{\frac{11}{2}}} + \frac{370(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{891(2+3x)^{\frac{9}{2}}} \\
& - \frac{584888452 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{173282571} \\
& - \frac{13235368 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{173282571} - \frac{55772(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{43659(2+3x)^{\frac{5}{2}}} \\
& + \frac{36980(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{18711(2+3x)^{\frac{7}{2}}} - \frac{17089252\sqrt{1-2x}\sqrt{3+5x}}{8251551(2+3x)^{\frac{3}{2}}} + \frac{584888452\sqrt{1-2x}\sqrt{3+5x}}{57760857\sqrt{2+3x}}
\end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(3+5*x)^(5/2)/(2+3*x)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(71063946918x^5 + 237923150688x^4 + 320012032635x^3 + 215597947743x^2 + 72620507583x + 9770732477)\sqrt{5}}{57760857(729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(100x^4 + 20x^3 - 59x^2 - 6x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 + 6048x^2 + 1344x + 128}, x\right)$$

7.166 Problem number 2784

$$\int \frac{(1-2x)^{5/2}(3+5x)^{5/2}}{(2+3x)^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{39(2+3x)^{\frac{13}{2}}} + \frac{370(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{1287(2+3x)^{\frac{11}{2}}} \\ & - \frac{129922578224 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{15768713961} \\ & - \frac{3894280616 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{15768713961} - \frac{2622980(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{1702701(2+3x)^{\frac{7}{2}}} \\ & + \frac{60080(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{34749(2+3x)^{\frac{9}{2}}} - \frac{54281308\sqrt{1-2x}\sqrt{3+5x}}{35756721(2+3x)^{\frac{5}{2}}} \\ & + \frac{1876198516\sqrt{1-2x}\sqrt{3+5x}}{750891141(2+3x)^{\frac{3}{2}}} + \frac{129922578224\sqrt{1-2x}\sqrt{3+5x}}{5256237987\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(3+5*x)^(5/2)/(2+3*x)^(15/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(47356779762648x^6 + 191022825888450x^5 + 321056742490902x^4 + 287874442427697x^3 + 145238558453649x^2 + 47356779762648x + 191022825888450)}{5256237987(2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 + 6048x^2 + 1344x + 128)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(100x^4 + 20x^3 - 59x^2 - 6x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{6561x^8 + 34992x^7 + 81648x^6 + 108864x^5 + 90720x^4 + 48384x^3 + 16128x^2 + 3072x + 256}, x\right)$$

7.167 Problem number 2785

$$\int \frac{(1-2x)^{5/2}(3+5x)^{5/2}}{(2+3x)^{17/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{45(2+3x)^{\frac{15}{2}}} + \frac{74(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{351(2+3x)^{\frac{13}{2}}} \\ & -\frac{12641611554328 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{551904988635} \\ & -\frac{380220959152 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{551904988635} \\ & -\frac{1085156(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{729729(2+3x)^{\frac{9}{2}}} + \frac{16636(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{11583(2+3x)^{\frac{11}{2}}} \\ & -\frac{112817764\sqrt{1-2x}\sqrt{3+5x}}{107270163(2+3x)^{\frac{7}{2}}} + \frac{3914701972\sqrt{1-2x}\sqrt{3+5x}}{3754455705(2+3x)^{\frac{5}{2}}} \\ & +\frac{181941877952\sqrt{1-2x}\sqrt{3+5x}}{26281189935(2+3x)^{\frac{3}{2}}} + \frac{12641611554328\sqrt{1-2x}\sqrt{3+5x}}{183968329545\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(3+5*x)^(5/2)/(2+3*x)^(17/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(13823602234657668x^7 + 64974368463330312x^6 + 130900492508039982x^5 + 146528498784887100x^4 + 984274183968329545x^3 + 34992x^7 + 81648x^6 + 1088x^5)}{183968329545(6561x^8 + 34992x^7 + 81648x^6 + 1088x^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(100x^4 + 20x^3 - 59x^2 - 6x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{19683x^9 + 118098x^8 + 314928x^7 + 489888x^6 + 489888x^5 + 326592x^4 + 145152x^3 + 41472x^2 + 6912x + 1088}\right)$$

7.168 Problem number 2786

$$\int \frac{(1-2x)^{5/2}(2+3x)^{5/2}}{\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{231061879 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{584718750} \\ & - \frac{3963068 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{292359375} + \frac{62(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}} \sqrt{3+5x}}{1485} \\ & + \frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{5}{2}} \sqrt{3+5x}}{55} + \frac{181333(2+3x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{3+5x}}{3898125} \\ & + \frac{4258(2+3x)^{\frac{5}{2}} \sqrt{1-2x} \sqrt{3+5x}}{155925} - \frac{2865161 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{19490625} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(2+3*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{19490625} (25515000 x^4 - 6142500 x^3 - 23717250 x^2 + 9526995 x + 7167169) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(36x^4 + 12x^3 - 23x^2 - 4x + 4) \sqrt{3x+2} \sqrt{-2x+1}}{\sqrt{5x+3}}, x\right)$$

7.169 Problem number 2787

$$\int \frac{(1-2x)^{5/2}(2+3x)^{3/2}}{\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8024546 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{26578125} \\ & - \frac{509189 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{26578125} \\ & + \frac{106(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}} \sqrt{3+5x}}{1575} + \frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{3}{2}} \sqrt{3+5x}}{45} \\ & + \frac{8878(2+3x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{3+5x}}{118125} + \frac{21547 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{1771875} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(2+3*x)^(3/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{1771875} (945000x^3 - 1030500x^2 - 113490x + 683887) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(12x^3 - 4x^2 - 5x + 2)\sqrt{3x+2}\sqrt{-2x+1}}{\sqrt{5x+3}}, x\right)$$

7.170 Problem number 2788

$$\int \frac{(1-2x)^{5/2} \sqrt{2+3x}}{\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{408311 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1771875} \\ & - \frac{132824 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1771875} + \frac{326(1-2x)^{\frac{3}{2}} \sqrt{2+3x} \sqrt{3+5x}}{2625} \\ & + \frac{2(1-2x)^{\frac{5}{2}} \sqrt{2+3x} \sqrt{3+5x}}{35} + \frac{30922 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{118125} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(2+3*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{118125} (13500x^2 - 28170x + 26171) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{3x+2}\sqrt{-2x+1}}{\sqrt{5x+3}}, x\right)$$

7.171 Problem number 2789

$$\int \frac{(1-2x)^{5/2}}{\sqrt{2+3x}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\frac{53194 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{50625} - \frac{34154 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{50625} - \frac{4(1-2x)^{\frac{3}{2}}\sqrt{2+3x}\sqrt{3+5x}}{75} - \frac{1088\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{3375}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4}{3375} (90x - 317)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{15x^2 + 19x + 6}, x\right)$$

7.172 Problem number 2790

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{3/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\frac{8314 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2025} + \frac{824 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2025} + \frac{14(1-2x)^{\frac{3}{2}}\sqrt{3+5x}}{3\sqrt{2+3x}} + \frac{428\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{135}$$

command

```
integrate((1-2*x)^(5/2)/(2+3*x)^(3/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(12x + 743)\sqrt{5x + 3}\sqrt{-2x + 1}}{135\sqrt{3x + 2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{45x^3 + 87x^2 + 56x + 12}, x\right)$$

7.173 Problem number 2791

$$\int \frac{(1 - 2x)^{5/2}}{(2 + 3x)^{5/2}\sqrt{3 + 5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3896 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{405} \\ & - \frac{164 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{405} \\ & + \frac{14(1-2x)^{\frac{3}{2}}\sqrt{3+5x}}{9(2+3x)^{\frac{3}{2}}} + \frac{812\sqrt{1-2x}\sqrt{3+5x}}{27\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)/(2+3*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{98(24x + 17)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{27(9x^2 + 12x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{135x^4 + 351x^3 + 342x^2 + 148x + 24}, x\right)$$

7.174 Problem number 2792

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{7/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16564 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{405} \\ & - \frac{496 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{405} + \frac{14(1-2x)^{\frac{3}{2}} \sqrt{3+5x}}{15(2+3x)^{\frac{5}{2}}} \\ & + \frac{1736 \sqrt{1-2x} \sqrt{3+5x}}{135(2+3x)^{\frac{3}{2}}} + \frac{16564 \sqrt{1-2x} \sqrt{3+5x}}{135 \sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(7/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(74538x^2 + 101862x + 34927)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{135(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{405x^5 + 1323x^4 + 1728x^3 + 1128x^2 + 368x + 48}, x\right)$$

7.175 Problem number 2793

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{9/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{703480 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3969} \\ & - \frac{21160 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3969} + \frac{2(1-2x)^{\frac{3}{2}} \sqrt{3+5x}}{3(2+3x)^{\frac{7}{2}}} \\ & + \frac{76 \sqrt{1-2x} \sqrt{3+5x}}{9(2+3x)^{\frac{5}{2}}} + \frac{10124 \sqrt{1-2x} \sqrt{3+5x}}{189(2+3x)^{\frac{3}{2}}} + \frac{703480 \sqrt{1-2x} \sqrt{3+5x}}{1323 \sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(9/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(9496980x^3 + 19312866x^2 + 13103724x + 2967269)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1323(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1215x^6 + 4779x^5 + 7830x^4 + 6840x^3 + 3360x^2 + 880x + 96}, x\right)$$

7.176 Problem number 2794

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{11/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{66055016 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{83349} \\ & - \frac{1986944 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{83349} \\ & + \frac{14(1-2x)^{\frac{3}{2}}\sqrt{3+5x}}{27(2+3x)^{\frac{9}{2}}} + \frac{512\sqrt{1-2x}\sqrt{3+5x}}{81(2+3x)^{\frac{7}{2}}} + \frac{20420\sqrt{1-2x}\sqrt{3+5x}}{567(2+3x)^{\frac{5}{2}}} \\ & + \frac{950584\sqrt{1-2x}\sqrt{3+5x}}{3969(2+3x)^{\frac{3}{2}}} + \frac{66055016\sqrt{1-2x}\sqrt{3+5x}}{27783\sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(11/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2675228148x^4 + 7223771916x^3 + 7318104714x^2 + 3296666850x + 557240459)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{27783(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3645x^7 + 16767x^6 + 33048x^5 + 36180x^4 + 23760x^3 + 9360x^2 + 2048x + 192}, x\right)$$

7.177 Problem number 2795

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{13/2} \sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{23204503328 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{6417873} \\ & - \frac{697995152 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{6417873} + \frac{14(1-2x)^{\frac{3}{2}} \sqrt{3+5x}}{33(2+3x)^{\frac{11}{2}}} \\ & + \frac{4508 \sqrt{1-2x} \sqrt{3+5x}}{891(2+3x)^{\frac{9}{2}}} + \frac{171004 \sqrt{1-2x} \sqrt{3+5x}}{6237(2+3x)^{\frac{7}{2}}} + \frac{7173272 \sqrt{1-2x} \sqrt{3+5x}}{43659(2+3x)^{\frac{5}{2}}} \\ & + \frac{333930952 \sqrt{1-2x} \sqrt{3+5x}}{305613(2+3x)^{\frac{3}{2}}} + \frac{23204503328 \sqrt{1-2x} \sqrt{3+5x}}{2139291 \sqrt{2+3x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(13/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2819347154352x^5 + 9492493272732x^4 + 12787628716260x^3 + 8615827181322x^2 + 2903435279352x + 391506)}{2139291(729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{10935x^8 + 57591x^7 + 132678x^6 + 174636x^5 + 143640x^4 + 75600x^3 + 24864x^2 + 4672x + 384}, x\right)$$

7.178 Problem number 2796

$$\int \frac{(1-2x)^{5/2}(2+3x)^{7/2}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{264260033 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{974531250} \\ & - \frac{7261561 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{487265625} \\ & - \frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{7}{2}}}{5\sqrt{3+5x}} - \frac{48(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{7}{2}}\sqrt{3+5x}}{275} \\ & + \frac{2020841(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{6496875} + \frac{346636(2+3x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{3+5x}}{259875} \\ & - \frac{2972(2+3x)^{\frac{7}{2}}\sqrt{1-2x}\sqrt{3+5x}}{7425} - \frac{703672\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{32484375} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(2+3*x)^(7/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(127575000x^5 + 56227500x^4 - 141221250x^3 - 32807925x^2 + 71568535x + 26378214)\sqrt{3x+2}\sqrt{-2x+1}}{32484375\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(108x^5 + 108x^4 - 45x^3 - 58x^2 + 4x + 8)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{25x^2 + 30x + 9}, x\right)$$

7.179 Problem number 2797

$$\int \frac{(1-2x)^{5/2}(2+3x)^{5/2}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1509007 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{8859375} \\ & - \frac{299863 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{8859375} - \frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{5}{2}}}{5\sqrt{3+5x}} \\ & - \frac{8(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}\sqrt{3+5x}}{45} + \frac{167228(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{118125} \\ & - \frac{1972(2+3x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{3+5x}}{4725} + \frac{196499\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{590625} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(2+3*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(945000x^4 - 382500x^3 - 844650x^2 + 650155x + 443337)\sqrt{3x+2}\sqrt{-2x+1}}{590625\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(36x^4 + 12x^3 - 23x^2 - 4x + 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{25x^2 + 30x + 9}, x\right)$$

7.180 Problem number 2798

$$\int \frac{(1-2x)^{5/2}(2+3x)^{3/2}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{53279 \text{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{984375} \\ & - \frac{110014 \text{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{984375} \\ & - \frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{3}{2}}}{5\sqrt{3+5x}} - \frac{32(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}\sqrt{3+5x}}{175} \\ & - \frac{1972(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{4375} + \frac{106772\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{65625} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(2+3*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(22500x^3 - 31350x^2 + 9545x + 9168)\sqrt{3x+2}\sqrt{-2x+1}}{65625\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(12x^3 - 4x^2 - 5x + 2)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{25x^2 + 30x + 9}, x\right)$$

7.181 Problem number 2799

$$\int \frac{(1-2x)^{5/2} \sqrt{2+3x}}{(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{81164 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{84375} \\ & - \frac{28174 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{84375} - \frac{2(1-2x)^{5/2} \sqrt{2+3x}}{5\sqrt{3+5x}} \\ & - \frac{24(1-2x)^{3/2} \sqrt{2+3x} \sqrt{3+5x}}{125} - \frac{3028\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{5625} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(2+3*x)^(1/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(900x^2 - 2530x - 7287)\sqrt{3x+2}\sqrt{-2x+1}}{5625\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{25x^2 + 30x + 9}, x\right)$$

7.182 Problem number 2800

$$\int \frac{(1-2x)^{5/2}}{\sqrt{2+3x} (3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5594 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3375} \\ & + \frac{1196 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3375} \\ & - \frac{22(1-2x)^{3/2} \sqrt{2+3x}}{5\sqrt{3+5x}} - \frac{388\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{225} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)/(3+5*x)^(3/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(20x - 1077)\sqrt{3x+2}\sqrt{-2x+1}}{225\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{75x^3 + 140x^2 + 87x + 18}, x\right)$$

7.183 Problem number 2801

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{3/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4636 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{225} \\ & + \frac{124 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{225} \\ & + \frac{14(1-2x)^{\frac{3}{2}}}{3\sqrt{2+3x}\sqrt{3+5x}} - \frac{1496\sqrt{1-2x}\sqrt{2+3x}}{15\sqrt{3+5x}} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)/(2+3*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(2314x + 1461)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{15(15x^2 + 19x + 6)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{225x^4 + 570x^3 + 541x^2 + 228x + 36}, x\right)$$

7.184 Problem number 2802

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{17804 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{135} \\ & + \frac{536 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{135} + \frac{14(1-2x)^{\frac{3}{2}}}{9(2+3x)^{\frac{3}{2}}\sqrt{3+5x}} \\ & + \frac{1792\sqrt{1-2x}}{27\sqrt{2+3x}\sqrt{3+5x}} - \frac{17804\sqrt{1-2x}\sqrt{2+3x}}{27\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(26706x^2 + 34726x + 11265)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9(45x^3 + 87x^2 + 56x + 12)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{675x^5 + 2160x^4 + 2763x^3 + 1766x^2 + 564x + 72}, x\right)$$

7.185 Problem number 2803

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{7/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{105584 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{135} \\ & + \frac{3176 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{135} + \frac{14(1-2x)^{\frac{3}{2}}}{15(2+3x)^{\frac{5}{2}}\sqrt{3+5x}} \\ & + \frac{2716\sqrt{1-2x}}{135(2+3x)^{\frac{3}{2}}\sqrt{3+5x}} + \frac{17468\sqrt{1-2x}}{45\sqrt{2+3x}\sqrt{3+5x}} - \frac{105584\sqrt{1-2x}\sqrt{2+3x}}{27\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(7/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2375640x^3 + 4672674x^2 + 3061396x + 668031)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{45(135x^4 + 351x^3 + 342x^2 + 148x + 24)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2025x^6 + 7830x^5 + 12609x^4 + 10824x^3 + 5224x^2 + 1344x + 144}, x\right)$$

7.186 Problem number 2804

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{9/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1959032 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{441} \\ & + \frac{58928 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{441} \\ & + \frac{2(1-2x)^{\frac{3}{2}}}{3(2+3x)^{\frac{7}{2}}\sqrt{3+5x}} + \frac{104\sqrt{1-2x}}{9(2+3x)^{\frac{5}{2}}\sqrt{3+5x}} + \frac{2332\sqrt{1-2x}}{21(2+3x)^{\frac{3}{2}}\sqrt{3+5x}} \\ & + \frac{324104\sqrt{1-2x}}{147\sqrt{2+3x}\sqrt{3+5x}} - \frac{9795160\sqrt{1-2x}\sqrt{2+3x}}{441\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(9/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(132234660x^4 + 348250356x^3 + 343801494x^2 + 150788294x + 24789615)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{147(405x^5 + 1323x^4 + 1728x^3 + 1128x^2 + 368x + 48)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{6075x^7 + 27540x^6 + 53487x^5 + 57690x^4 + 37320x^3 + 14480x^2 + 3120x + 288}, x\right)$$

7.187 Problem number 2805

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{11/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{683150096 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{27783} \\ & + \frac{20549264 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{27783} + \frac{14(1-2x)^{3/2}}{27(2+3x)^{9/2}\sqrt{3+5x}} \\ & + \frac{652\sqrt{1-2x}}{81(2+3x)^{7/2}\sqrt{3+5x}} + \frac{11660\sqrt{1-2x}}{189(2+3x)^{5/2}\sqrt{3+5x}} + \frac{813208\sqrt{1-2x}}{1323(2+3x)^{3/2}\sqrt{3+5x}} \\ & + \frac{113020952\sqrt{1-2x}}{9261\sqrt{2+3x}\sqrt{3+5x}} - \frac{3415750480\sqrt{1-2x}\sqrt{2+3x}}{27783\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(11/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(138337894440x^5 + 456548966244x^4 + 602551975428x^3 + 397527527442x^2 + 131099014240x + 172891788270)}{9261(1215x^6 + 4779x^5 + 7830x^4 + 6840x^3 + 3360x^2 + 880x + 96)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{18225x^8 + 94770x^7 + 215541x^6 + 280044x^5 + 227340x^4 + 118080x^3 + 38320x^2 + 7104x + 576}, x\right)$$

7.188 Problem number 2806

$$\int \frac{(1-2x)^{5/2}(2+3x)^{7/2}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(1-2x)^{5/2}(2+3x)^{7/2}}{15(3+5x)^{3/2}} - \frac{1065118 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{14765625} \\ & - \frac{595387 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{14765625} - \frac{442(1-2x)^{3/2}(2+3x)^{7/2}}{75\sqrt{3+5x}} \\ & + \frac{373022(2+3x)^{3/2}\sqrt{1-2x}\sqrt{3+5x}}{196875} + \frac{59662(2+3x)^{5/2}\sqrt{1-2x}\sqrt{3+5x}}{7875} \\ & - \frac{524(2+3x)^{7/2}\sqrt{1-2x}\sqrt{3+5x}}{225} + \frac{500501\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{984375} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(2+3*x)^(7/2)/(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(4725000 x^5 + 1327500 x^4 - 5654250 x^3 + 470675 x^2 + 4026600 x + 1215489) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{984375 (25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(108 x^5 + 108 x^4 - 45 x^3 - 58 x^2 + 4 x + 8) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{125 x^3 + 225 x^2 + 135 x + 27}, x\right)$$

7.189 Problem number 2807

$$\int \frac{(1-2x)^{5/2}(2+3x)^{5/2}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{5}{2}}}{15(3+5x)^{\frac{3}{2}}} + \frac{49321 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{328125} \\ & -\frac{32836 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{328125} \\ & -\frac{62(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}}{15\sqrt{3+5x}} + \frac{22866(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{4375} \\ & -\frac{284(2+3x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{3+5x}}{175} + \frac{33778\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{21875} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(2+3*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(67500 x^4 - 47250 x^3 - 41025 x^2 - 23425 x - 19087) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{65625 (25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(36 x^4 + 12 x^3 - 23 x^2 - 4 x + 4) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{125 x^3 + 225 x^2 + 135 x + 27}, x\right)$$

7.190 Problem number 2808

$$\int \frac{(1-2x)^{5/2}(2+3x)^{3/2}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(1-2x)^{\frac{5}{2}}(2+3x)^{\frac{3}{2}}}{15(3+5x)^{\frac{3}{2}}} - \frac{7738 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{46875} \\ & + \frac{9206 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{15625} - \frac{178(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}}{75\sqrt{3+5x}} \\ & - \frac{572(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{625} + \frac{8874\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{3125} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)*(2+3*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(4500x^3 - 9450x^2 - 48650x - 25421)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9375(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(12x^3 - 4x^2 - 5x + 2)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{125x^3 + 225x^2 + 135x + 27}, x\right)$$

7.191 Problem number 2809

$$\int \frac{(1-2x)^{5/2}\sqrt{2+3x}}{(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{338 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{5625} \\ & + \frac{992 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{5625} - \frac{2(1-2x)^{\frac{5}{2}}\sqrt{2+3x}}{15(3+5x)^{\frac{3}{2}}} \\ & - \frac{46(1-2x)^{\frac{3}{2}}\sqrt{2+3x}}{75\sqrt{3+5x}} - \frac{76\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{375} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)*(2+3*x)^(1/2)/(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(100x^2 - 925x - 712)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{375(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{125x^3 + 225x^2 + 135x + 27}, x\right)$$

7.192 Problem number 2810

$$\int \frac{(1-2x)^{5/2}}{\sqrt{2+3x}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{68 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{375} \\ & - \frac{584 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{125} \\ & - \frac{22(1-2x)^{\frac{3}{2}}\sqrt{2+3x}}{15(3+5x)^{\frac{3}{2}}} + \frac{572\sqrt{1-2x}\sqrt{2+3x}}{25\sqrt{3+5x}} \end{aligned}$$

command

```
integrate((1-2*x)^(5/2)/(3+5*x)^(5/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{22(400x + 229)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{75(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{375x^4 + 925x^3 + 855x^2 + 351x + 54}, x\right)$$

7.193 Problem number 2811

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{3/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6388 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{75} \\ & - \frac{64 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{25} + \frac{14(1-2x)^{\frac{3}{2}}}{3(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & - \frac{1012\sqrt{1-2x}\sqrt{2+3x}}{15(3+5x)^{\frac{3}{2}}} + \frac{6388\sqrt{1-2x}\sqrt{2+3x}}{15\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(47910x^2 + 59098x + 18187)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{15(75x^3 + 140x^2 + 87x + 18)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1125x^5 + 3525x^4 + 4415x^3 + 2763x^2 + 864x + 108}, x\right)$$

7.194 Problem number 2812

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14(1-2x)^{\frac{3}{2}}}{9(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} - \frac{36968 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{45} \\ & - \frac{1112 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{45} + \frac{308\sqrt{1-2x}}{3(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & - \frac{6116\sqrt{1-2x}\sqrt{2+3x}}{9(3+5x)^{\frac{3}{2}}} + \frac{36968\sqrt{1-2x}\sqrt{2+3x}}{9\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(277260x^3 + 526862x^2 + 333260x + 70169)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3(225x^4 + 570x^3 + 541x^2 + 228x + 36)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3375x^6 + 12825x^5 + 20295x^4 + 17119x^3 + 8118x^2 + 2052x + 216}, x\right)$$

7.195 Problem number 2813

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{7/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14(1-2x)^{\frac{3}{2}}}{15(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}} - \frac{96808 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{15} \\ & - \frac{2912 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{15} + \frac{1232\sqrt{1-2x}}{45(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} \\ & + \frac{35948\sqrt{1-2x}}{45(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} - \frac{16016\sqrt{1-2x}\sqrt{2+3x}}{3(3+5x)^{\frac{3}{2}}} + \frac{96808\sqrt{1-2x}\sqrt{2+3x}}{3\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(7/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(32672700x^4 + 83867940x^3 + 80662602x^2 + 34450018x + 5512543)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{15(675x^5 + 2160x^4 + 2763x^3 + 1766x^2 + 564x + 72)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{10125x^7 + 45225x^6 + 86535x^5 + 91947x^4 + 58592x^3 + 22392x^2 + 4752x + 432}, x\right)$$

7.196 Problem number 2814

$$\int \frac{(1-2x)^{5/2}}{(2+3x)^{9/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(1-2x)^{\frac{3}{2}}}{3(2+3x)^{\frac{7}{2}}(3+5x)^{\frac{3}{2}}} - \frac{201616 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{147} \\ & - \frac{2234208 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{49} \\ & + \frac{44\sqrt{1-2x}}{3(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}} + \frac{11924\sqrt{1-2x}}{63(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} + \frac{2488904\sqrt{1-2x}}{441(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & - \frac{5544440\sqrt{1-2x}\sqrt{2+3x}}{147(3+5x)^{\frac{3}{2}}} + \frac{11171040\sqrt{1-2x}\sqrt{2+3x}}{49\sqrt{3+5x}} \end{aligned}$$

command

`integrate((1-2*x)^(5/2)/(2+3*x)^(9/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(6786406800x^5 + 21944379060x^4 + 28367736228x^3 + 18325125498x^2 + 5915384456x + 763335749)\sqrt{5x+3}}{147(2025x^6 + 7830x^5 + 12609x^4 + 10824x^3 + 5224x^2 + 1344x + 144)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2 - 4x + 1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{30375x^8 + 155925x^7 + 350055x^6 + 448911x^5 + 359670x^4 + 184360x^3 + 59040x^2 + 10800x + 864}, x\right)$$

7.197 Problem number 2815

$$\int \frac{(2+3x)^{5/2}\sqrt{3+5x}}{\sqrt{1-2x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5057 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{26250} \\ & - \frac{56041 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{8750} - \frac{104(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{175} \\ & - \frac{(2+3x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{3+5x}}{7} - \frac{4839\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{1750} \end{aligned}$$

command

```
integrate((2+3*x)^(5/2)*(3+5*x)^(1/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{1750} (2250x^2 + 6120x + 7919) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(9x^2 + 12x + 4) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{2x-1}, x\right)$$

7.198 Problem number 2816

$$\int \frac{(2+3x)^{3/2} \sqrt{3+5x}}{\sqrt{1-2x}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1597 \text{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{750} \\ & -\frac{8 \text{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{125} \\ & -\frac{(2+3x)^{3/2} \sqrt{1-2x} \sqrt{3+5x}}{5} - \frac{23 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{25} \end{aligned}$$

command

```
integrate((2+3*x)^(3/2)*(3+5*x)^(1/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{3}{25} (5x + 11) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3} (3x+2)^{3/2} \sqrt{-2x+1}}{2x-1}, x\right)$$

7.199 Problem number 2817

$$\int \frac{\sqrt{2+3x} \sqrt{3+5x}}{\sqrt{1-2x}} dx$$

Optimal antiderivative

$$\frac{34 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{45} - \frac{\operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{45} - \frac{\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{3}$$

command

`integrate((2+3*x)^(1/2)*(3+5*x)^(1/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{3} \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{2x-1}, x\right)$$

7.200 Problem number 2819

$$\int \frac{\sqrt{3+5x}}{\sqrt{1-2x} (2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE}\left(\sqrt{5} \sqrt{2+3x}, \frac{\sqrt{70}}{35}\right) \sqrt{35} \sqrt{-3-5x}}{21 \sqrt{3+5x}} - \frac{2 \sqrt{1-2x} \sqrt{3+5x}}{7 \sqrt{2+3x}}$$

command

`integrate((3+5*x)^(1/2)/(2+3*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2 \sqrt{5x+3} \sqrt{-2x+1}}{7 \sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{18x^3+15x^2-4x-4}, x\right)$$

7.201 Problem number 2820

$$\int \frac{\sqrt{3+5x}}{\sqrt{1-2x} (2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{62 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{441} - \frac{8 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{441} - \frac{2\sqrt{1-2x} \sqrt{3+5x}}{21(2+3x)^{\frac{3}{2}}} + \frac{62\sqrt{1-2x} \sqrt{3+5x}}{147\sqrt{2+3x}}$$

command

`integrate((3+5*x)^(1/2)/(2+3*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(93x+55)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{147(9x^2+12x+4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{54x^4+81x^3+18x^2-20x-8}, x\right)$$

7.202 Problem number 2821

$$\int \frac{\sqrt{3+5x}}{\sqrt{1-2x} (2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\frac{68 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{5145} - \frac{584 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1715} - \frac{2\sqrt{1-2x} \sqrt{3+5x}}{35(2+3x)^{\frac{5}{2}}} + \frac{18\sqrt{1-2x} \sqrt{3+5x}}{245(2+3x)^{\frac{3}{2}}} + \frac{1752\sqrt{1-2x} \sqrt{3+5x}}{1715\sqrt{2+3x}}$$

command

```
integrate((3+5*x)^(1/2)/(2+3*x)^(7/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(7884x^2 + 10701x + 3581)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1715(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{162x^5 + 351x^4 + 216x^3 - 24x^2 - 64x - 16}, x\right)$$

7.203 Problem number 2822

$$\int \frac{(2+3x)^{5/2}(3+5x)^{3/2}}{\sqrt{1-2x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{44109377 \text{EllipticE}\left(\frac{\sqrt{21}}{7}, \frac{\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1417500} \\ & - \frac{663409 \text{EllipticF}\left(\frac{\sqrt{21}}{7}, \frac{\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{708750} \\ & - \frac{137(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{315} - \frac{(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{9} \\ & - \frac{9547(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{5250} - \frac{663409\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{47250} \end{aligned}$$

command

```
integrate((2+3*x)^(5/2)*(3+5*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{47250}(236250x^3 + 765000x^2 + 1114065x + 1107478)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(45x^3 + 87x^2 + 56x + 12)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2x-1}, x\right)$$

7.204 Problem number 2823

$$\int \frac{(2+3x)^{3/2}(3+5x)^{3/2}}{\sqrt{1-2x}} dx$$

Optimal antiderivative

$$\frac{78472 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{4721 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}} - \frac{(2+3x)^{3/2}(3+5x)^{3/2}\sqrt{1-2x}}{15750} - \frac{102(3+5x)^{3/2}\sqrt{1-2x}\sqrt{2+3x}}{175} - \frac{4721\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{1050}$$

command

`integrate((2+3*x)^(3/2)*(3+5*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{1050} (2250x^2 + 5910x + 7457) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(15x^2 + 19x + 6) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{2x-1}, x\right)$$

7.205 Problem number 2824

$$\int \frac{\sqrt{2+3x}(3+5x)^{3/2}}{\sqrt{1-2x}} dx$$

Optimal antiderivative

$$\frac{4451 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{67 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}} - \frac{(3+5x)^{3/2}\sqrt{1-2x}\sqrt{2+3x}}{5} - \frac{67\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{45}$$

command

`integrate((3+5*x)^(3/2)*(2+3*x)^(1/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{45} (45x + 94) \sqrt{5x + 3} \sqrt{3x + 2} \sqrt{-2x + 1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(5x + 3)^{\frac{3}{2}} \sqrt{3x + 2} \sqrt{-2x + 1}}{2x - 1}, x\right)$$

7.206 Problem number 2825

$$\int \frac{(3 + 5x)^{3/2}}{\sqrt{1 - 2x} \sqrt{2 + 3x}} dx$$

Optimal antiderivative

$$\frac{31 \text{EllipticE}\left(\frac{\sqrt{21} \sqrt{1 - 2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{\text{EllipticF}\left(\frac{\sqrt{21} \sqrt{1 - 2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33} - \frac{5\sqrt{1 - 2x} \sqrt{2 + 3x} \sqrt{3 + 5x}}{9}}$$

command

`integrate((3+5*x)^(3/2)/(1-2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{5}{9} \sqrt{5x + 3} \sqrt{3x + 2} \sqrt{-2x + 1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(5x + 3)^{\frac{3}{2}} \sqrt{3x + 2} \sqrt{-2x + 1}}{6x^2 + x - 2}, x\right)$$

7.207 Problem number 2826

$$\int \frac{(3+5x)^{3/2}}{\sqrt{1-2x} (2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{37 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{63} + \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{63} + \frac{2\sqrt{1-2x} \sqrt{3+5x}}{21\sqrt{2+3x}}$$

command

`integrate((3+5*x)^(3/2)/(2+3*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{5x+3}\sqrt{-2x+1}}{21\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x+3)^{\frac{3}{2}}\sqrt{3x+2}\sqrt{-2x+1}}{18x^3+15x^2-4x-4}, x\right)$$

7.208 Problem number 2827

$$\int \frac{(3+5x)^{3/2}}{\sqrt{1-2x} (2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{272 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1323} - \frac{202 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1323} + \frac{2\sqrt{1-2x} \sqrt{3+5x}}{63(2+3x)^{\frac{3}{2}}} - \frac{272\sqrt{1-2x} \sqrt{3+5x}}{441\sqrt{2+3x}}$$

command

```
integrate((3+5*x)^(3/2)/(2+3*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(408x + 265)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{441(9x^2 + 12x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(5x + 3)^{\frac{3}{2}}\sqrt{3x + 2}\sqrt{-2x + 1}}{54x^4 + 81x^3 + 18x^2 - 20x - 8}, x\right)$$

7.209 Problem number 2828

$$\int \frac{(3 + 5x)^{3/2}}{\sqrt{1 - 2x}(2 + 3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5594 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{46305} \\ & -\frac{1196 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{46305} \\ & + \frac{2\sqrt{1-2x}\sqrt{3+5x}}{105(2+3x)^{\frac{5}{2}}} - \frac{404\sqrt{1-2x}\sqrt{3+5x}}{2205(2+3x)^{\frac{3}{2}}} + \frac{5594\sqrt{1-2x}\sqrt{3+5x}}{15435\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(3/2)/(2+3*x)^(7/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(25173x^2 + 29322x + 8507)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{15435(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(5x + 3)^{\frac{3}{2}}\sqrt{3x + 2}\sqrt{-2x + 1}}{162x^5 + 351x^4 + 216x^3 - 24x^2 - 64x - 16}, x\right)$$

7.210 Problem number 2829

$$\int \frac{(3+5x)^{3/2}}{\sqrt{1-2x}(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{184636 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{756315} \\ & - \frac{9124 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{756315} + \frac{2\sqrt{1-2x} \sqrt{3+5x}}{147(2+3x)^{7/2}} \\ & - \frac{536\sqrt{1-2x} \sqrt{3+5x}}{5145(2+3x)^{5/2}} + \frac{974\sqrt{1-2x} \sqrt{3+5x}}{36015(2+3x)^{3/2}} + \frac{184636\sqrt{1-2x} \sqrt{3+5x}}{252105\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(3/2)/(2+3*x)^(9/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2492586x^3 + 5015853x^2 + 3324960x + 727631)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{252105(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x+3)^{3/2}\sqrt{3x+2}\sqrt{-2x+1}}{486x^6 + 1377x^5 + 1350x^4 + 360x^3 - 240x^2 - 176x - 32}, x\right)$$

7.211 Problem number 2830

$$\int \frac{(2+3x)^{7/2}(3+5x)^{5/2}}{\sqrt{1-2x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{610627101631 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1216215000} \\ & - \frac{2295970088 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{152026875} \\ & - \frac{14303(2+3x)^{3/2}(3+5x)^{5/2}\sqrt{1-2x}}{12870} - \frac{41(2+3x)^{5/2}(3+5x)^{5/2}\sqrt{1-2x}}{143} \\ & - \frac{(2+3x)^{7/2}(3+5x)^{5/2}\sqrt{1-2x}}{13} - \frac{138809831(3+5x)^{3/2}\sqrt{1-2x}\sqrt{2+3x}}{4504500} \\ & - \frac{221673(3+5x)^{5/2}\sqrt{1-2x}\sqrt{2+3x}}{50050} - \frac{2295970088\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{10135125} \end{aligned}$$

command

```
integrate((2+3*x)^(7/2)*(3+5*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{40540500} (2104987500 x^5 + 9351247500 x^4 + 18620894250 x^3 + 22592085750 x^2 + 19961825445 x + 1600170005)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(675 x^5 + 2160 x^4 + 2763 x^3 + 1766 x^2 + 564 x + 72) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}}{2 x - 1}, x\right)$$

7.212 Problem number 2831

$$\int \frac{(2+3x)^{5/2}(3+5x)^{5/2}}{\sqrt{1-2x}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{725140729 \text{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{4677750} \\ & -\frac{43624697 \text{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9355500} - \frac{34(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}} \sqrt{1-2x}}{99} \\ & - \frac{(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}} \sqrt{1-2x}}{329683(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}} \\ & - \frac{11}{1053(3+5x)^{\frac{5}{2}} \sqrt{1-2x} \sqrt{2+3x}} - \frac{34650}{43624697 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}} \\ & - \frac{11}{770} - \frac{34650}{623700} \end{aligned}$$

command

```
integrate((2+3*x)^(5/2)*(3+5*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{623700} (12757500 x^4 + 48384000 x^3 + 81985950 x^2 + 86822370 x + 75000749) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(225 x^4 + 570 x^3 + 541 x^2 + 228 x + 36) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}}{2 x - 1}, x\right)$$

7.213 Problem number 2832

$$\int \frac{(2+3x)^{3/2}(3+5x)^{5/2}}{\sqrt{1-2x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8256877 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{170100} \\ & - \frac{62092 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{42525} \\ & - \frac{(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}} \sqrt{1-2x}}{9} - \frac{1877(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{630} \\ & - \frac{3(3+5x)^{\frac{5}{2}} \sqrt{1-2x} \sqrt{2+3x}}{7} - \frac{62092 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{2835} \end{aligned}$$

command

`integrate((2+3*x)^(3/2)*(3+5*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{5670} (47250x^3 + 148950x^2 + 212175x + 208073) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(75x^3 + 140x^2 + 87x + 18) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{2x-1}, x\right)$$

7.214 Problem number 2833

$$\int \frac{\sqrt{2+3x} (3+5x)^{5/2}}{\sqrt{1-2x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{17587 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1134} \\ & - \frac{529 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1134} - \frac{20(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{21} \\ & - \frac{(3+5x)^{\frac{5}{2}} \sqrt{1-2x} \sqrt{2+3x}}{7} - \frac{2645 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{378} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)*(2+3*x)^(1/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{378} (1350x^2 + 3420x + 4211) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(25x^2 + 30x + 9) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{2x-1}, x\right)$$

7.215 Problem number 2834

$$\int \frac{(3+5x)^{5/2}}{\sqrt{1-2x} \sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4141 \text{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{810} \\ & - \frac{62 \text{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{405} \\ & - \frac{(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}}{3} - \frac{62 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{27} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)/(1-2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{27} (45x + 89) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(25x^2 + 30x + 9) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{6x^2 + x - 2}, x\right)$$

7.216 Problem number 2835

$$\int \frac{(3+5x)^{5/2}}{\sqrt{1-2x}(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{974 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{567} \\ & - \frac{41 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{567} \\ & + \frac{2(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{21\sqrt{2+3x}} - \frac{205\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{189} \end{aligned}$$

command

`integrate((3+5*x)^(5/2)/(2+3*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(525x+356)\sqrt{5x+3}\sqrt{-2x+1}}{189\sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(25x^2+30x+9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{18x^3+15x^2-4x-4}, x\right)$$

7.217 Problem number 2836

$$\int \frac{(3+5x)^{5/2}}{\sqrt{1-2x}(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4157 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3969} \\ & + \frac{412 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3969} \\ & + \frac{2(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{63(2+3x)^{\frac{3}{2}}} + \frac{412\sqrt{1-2x} \sqrt{3+5x}}{1323\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)/(2+3*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(723x + 475)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{1323(9x^2 + 12x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(25x^2 + 30x + 9)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{54x^4 + 81x^3 + 18x^2 - 20x - 8}, x\right)$$

7.218 Problem number 2837

$$\int \frac{(3 + 5x)^{5/2}}{\sqrt{1 - 2x} (2 + 3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{53194 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{138915} \\ & - \frac{34154 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{138915} \\ & + \frac{2(3 + 5x)^{\frac{3}{2}} \sqrt{1 - 2x}}{105(2 + 3x)^{\frac{5}{2}}} + \frac{544\sqrt{1 - 2x} \sqrt{3 + 5x}}{6615(2 + 3x)^{\frac{3}{2}}} - \frac{53194\sqrt{1 - 2x} \sqrt{3 + 5x}}{46305\sqrt{2 + 3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)/(2+3*x)^(7/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(239373x^2 + 311247x + 101257)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{46305(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(25x^2 + 30x + 9)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{162x^5 + 351x^4 + 216x^3 - 24x^2 - 64x - 16}, x\right)$$

7.219 Problem number 2838

$$\int \frac{(3+5x)^{5/2}}{\sqrt{1-2x}(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{816622 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{6806835} \\ & - \frac{265648 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{6806835} + \frac{2(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{147(2+3x)^{\frac{7}{2}}} \\ & + \frac{676\sqrt{1-2x} \sqrt{3+5x}}{15435(2+3x)^{\frac{5}{2}}} - \frac{101902\sqrt{1-2x} \sqrt{3+5x}}{324135(2+3x)^{\frac{3}{2}}} + \frac{816622\sqrt{1-2x} \sqrt{3+5x}}{2268945\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(5/2)/(2+3*x)^(9/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(11024397x^3 + 18838881x^2 + 10645545x + 1985537)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2268945(81x^4 + 216x^3 + 216x^2 + 96x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{486x^6 + 1377x^5 + 1350x^4 + 360x^3 - 240x^2 - 176x - 32}, x\right)$$

7.220 Problem number 2839

$$\int \frac{(3+5x)^{5/2}}{\sqrt{1-2x}(2+3x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32098184 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{142943535} \\ & - \frac{2036756 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{142943535} \\ & + \frac{2(3+5x)^{\frac{3}{2}} \sqrt{1-2x}}{189(2+3x)^{\frac{9}{2}}} + \frac{808\sqrt{1-2x} \sqrt{3+5x}}{27783(2+3x)^{\frac{7}{2}}} - \frac{168034\sqrt{1-2x} \sqrt{3+5x}}{972405(2+3x)^{\frac{5}{2}}} \\ & - \frac{43094\sqrt{1-2x} \sqrt{3+5x}}{6806835(2+3x)^{\frac{3}{2}}} + \frac{32098184\sqrt{1-2x} \sqrt{3+5x}}{47647845\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)/(2+3*x)^(11/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(1299976452x^4 + 3462531489x^3 + 3421407609x^2 + 1489220097x + 241253543)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{47647845(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1458x^7 + 5103x^6 + 6804x^5 + 3780x^4 - 1008x^2 - 448x - 64}, x\right)$$

7.221 Problem number 2840

$$\int \frac{(3+5x)^{5/2}}{\sqrt{1-2x}(2+3x)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{924247516 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2201330439} \\ & -\frac{31704544 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{2201330439} + \frac{2(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{231(2+3x)^{\frac{11}{2}}} \\ & + \frac{940\sqrt{1-2x}\sqrt{3+5x}}{43659(2+3x)^{\frac{9}{2}}} - \frac{251590\sqrt{1-2x}\sqrt{3+5x}}{2139291(2+3x)^{\frac{7}{2}}} - \frac{362666\sqrt{1-2x}\sqrt{3+5x}}{14975037(2+3x)^{\frac{5}{2}}} \\ & + \frac{11460644\sqrt{1-2x}\sqrt{3+5x}}{104825259(2+3x)^{\frac{3}{2}}} + \frac{924247516\sqrt{1-2x}\sqrt{3+5x}}{733776813\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)/(2+3*x)^(13/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(112296073194x^5 + 377569336554x^4 + 507518001945x^3 + 340525216341x^2 + 113962415157x + 15211411193)}{733776813(729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4374x^8 + 18225x^7 + 30618x^6 + 24948x^5 + 7560x^4 - 3024x^3 - 3360x^2 - 1088x - 128}, x\right)$$

7.222 Problem number 2841

$$\int \frac{1}{\sqrt{1+x} \sqrt{2+x} \sqrt{3+x}} dx$$

Optimal antiderivative

$$-2 \operatorname{EllipticF}\left(\frac{1}{\sqrt{3+x}}, \sqrt{2}\right)$$

command

```
integrate(1/(1+x)^(1/2)/(2+x)^(1/2)/(3+x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}(4, 0, x + 2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x+3} \sqrt{x+2} \sqrt{x+1}}{x^3 + 6x^2 + 11x + 6}, x\right)$$

7.223 Problem number 2844

$$\int \frac{1}{\sqrt{2-x} \sqrt{3-x} \sqrt{1+x}} dx$$

Optimal antiderivative

$$\operatorname{EllipticF}\left(\frac{\sqrt{1+x} \sqrt{3}}{3}, \frac{\sqrt{3}}{2}\right)$$

command

```
integrate(1/(2-x)^(1/2)/(3-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}\left(\frac{52}{3}, -\frac{280}{27}, x - \frac{4}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x+1} \sqrt{-x+3} \sqrt{-x+2}}{x^3 - 4x^2 + x + 6}, x\right)$$

7.224 Problem number 2846

$$\int \frac{1}{\sqrt{1-x} \sqrt{3-x} \sqrt{2+x}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{2+x} \sqrt{3}}{3}, \frac{\sqrt{15}}{5}\right) \sqrt{5}}{5}$$

command

`integrate(1/(1-x)^(1/2)/(3-x)^(1/2)/(2+x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}\left(\frac{76}{3}, -\frac{224}{27}, x - \frac{2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x+2} \sqrt{-x+3} \sqrt{-x+1}}{x^3 - 2x^2 - 5x + 6}, x\right)$$

7.225 Problem number 2847

$$\int \frac{1}{\sqrt{1-x} \sqrt{2-x} \sqrt{3+x}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{3+x}}{2}, \frac{2\sqrt{5}}{5}\right) \sqrt{5}}{5}$$

command

`integrate(1/(1-x)^(1/2)/(2-x)^(1/2)/(3+x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}(28, -24, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x+3} \sqrt{-x+2} \sqrt{-x+1}}{x^3 - 7x + 6}, x\right)$$

7.226 Problem number 2849

$$\int \frac{1}{\sqrt{-3+x} \sqrt{-2+x} \sqrt{-1+x}} dx$$

Optimal antiderivative

$$-2 \operatorname{EllipticF}\left(\frac{1}{\sqrt{-1+x}}, \sqrt{2}\right)$$

command

`integrate(1/(-3+x)^(1/2)/(-2+x)^(1/2)/(-1+x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}(4, 0, x - 2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x-1} \sqrt{x-2} \sqrt{x-3}}{x^3 - 6x^2 + 11x - 6}, x\right)$$

7.227 Problem number 2852

$$\int \frac{1}{\sqrt{-3-x} \sqrt{-2-x} \sqrt{-1+x}} dx$$

Optimal antiderivative

$$-\frac{2i \operatorname{EllipticK}(4) \sqrt{2+x}}{\sqrt{-2-x}} + \frac{\operatorname{EllipticK}\left(\frac{3}{4}\right) \sqrt{3+x}}{\sqrt{-3-x}} - \frac{\operatorname{EllipticF}\left(\frac{2}{\sqrt{3+x}}, \frac{1}{2}\right) \sqrt{2+x} \sqrt{3+x}}{\sqrt{-3-x} \sqrt{-2-x}}$$

command

`integrate(1/(-3-x)^(1/2)/(-2-x)^(1/2)/(-1+x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}\left(\frac{52}{3}, \frac{280}{27}, x + \frac{4}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x-1} \sqrt{-x-2} \sqrt{-x-3}}{x^3 + 4x^2 + x - 6}, x\right)$$

7.228 Problem number 2854

$$\int \frac{1}{\sqrt{-3-x} \sqrt{-1-x} \sqrt{-2+x}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{EllipticF}\left(\frac{5}{\sqrt{15+5x}}, \frac{\sqrt{10}}{5}\right) \sqrt{1+x} \sqrt{3+x} \sqrt{5}}{5\sqrt{-3-x} \sqrt{-1-x}}$$

command

`integrate(1/(-3-x)^(1/2)/(-1-x)^(1/2)/(-2+x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}\left(\frac{76}{3}, \frac{224}{27}, x + \frac{2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x-2} \sqrt{-x-1} \sqrt{-x-3}}{x^3 + 2x^2 - 5x - 6}, x\right)$$

7.229 Problem number 2855

$$\int \frac{1}{\sqrt{-2-x} \sqrt{-1-x} \sqrt{-3+x}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{EllipticF}\left(\frac{5}{\sqrt{10+5x}}, \frac{\sqrt{5}}{5}\right) \sqrt{1+x} \sqrt{2+x} \sqrt{5}}{5\sqrt{-2-x} \sqrt{-1-x}}$$

command

`integrate(1/(-2-x)^(1/2)/(-1-x)^(1/2)/(-3+x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}(28, 24, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x-3} \sqrt{-x-1} \sqrt{-x-2}}{x^3 - 7x - 6}, x\right)$$

7.230 Problem number 2857

$$\int \frac{1}{(a+bx)^{3/2} \sqrt{c+dx} \sqrt{e+fx}} dx$$

Optimal antiderivative

$$\frac{2b\sqrt{dx+c} \sqrt{fx+e}}{(-ad+bc)(-af+be)\sqrt{bx+a}} + \frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{f} \sqrt{dx+c}}{\sqrt{cf-de}}, \sqrt{-\frac{b(-cf+de)}{(-ad+bc)f}}\right) \sqrt{f} \sqrt{cf-de} \sqrt{bx+a} \sqrt{\frac{d(fx+e)}{-cf+de}}}{(-ad+bc)(-af+be) \sqrt{-\frac{d(bx+a)}{-ad+bc}} \sqrt{fx+e}}$$

command

`integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e} b^2 df + \sqrt{bdf} \left((b^2c - 2abd)fx + (abc - 2a^2d)f + (b^2dx + abd)e \right) \right) \operatorname{weierstrassPI}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e}}{b^2dfx^4 + a^2ce + (b^2de + (b^2c + 2abd)f)x^3 + ((b^2c + 2abd)e + (2abc + a^2d)f)x^2 + (a^2cf + (2abc + a^2d))x + a^2e}\right)$$

7.231 Problem number 2858

$$\int \frac{1}{(a+bx)^{5/2} \sqrt{c+dx} \sqrt{e+fx}} dx$$

Optimal antiderivative

$$\frac{2b\sqrt{dx+c} \sqrt{fx+e}}{3(-ad+bc)(-af+be)(bx+a)^{\frac{3}{2}}} + \frac{4b(-2adf+bcf+bde) \sqrt{dx+c} \sqrt{fx+e}}{3(-ad+bc)^2(-af+be)^2 \sqrt{bx+a}} + \frac{4(-2adf+bcf+bde) \operatorname{EllipticE}\left(\frac{\sqrt{d} \sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{\frac{(-ad+bc)f}{d(-af+be)}}\right) \sqrt{d} \sqrt{\frac{b(dx+c)}{-ad+bc}} \sqrt{fx+e}}{3(ad-bc)^{\frac{3}{2}}(-af+be)^2 \sqrt{dx+c} \sqrt{\frac{b(fx+e)}{-af+be}}} + \frac{2(-3adf+bcf+2bde) \operatorname{EllipticF}\left(\frac{\sqrt{d} \sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{\frac{(-ad+bc)f}{d(-af+be)}}\right) \sqrt{d} \sqrt{\frac{b(dx+c)}{-ad+bc}} \sqrt{\frac{b(fx+e)}{-af+be}}}{3b(ad-bc)^{\frac{3}{2}}(-af+be) \sqrt{dx+c} \sqrt{fx+e}}$$

command

```
integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \left(2 \left(b^4 c d - 2 a b^3 d^2 \right) f^2 x + \left(3 a b^3 c d - 5 a^2 b^2 d^2 \right) f^2 + \left(2 b^4 d^2 f x - \left(b^4 c d - 3 a b^3 d^2 \right) f \right) e \right) \sqrt{b x + a} \sqrt{d x + c} \sqrt{f x + e} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b x + a} \sqrt{d x + c} \sqrt{f x + e}}{b^3 d f x^5 + a^3 c e + (b^3 d e + (b^3 c + 3 a b^2 d) f) x^4 + ((b^3 c + 3 a b^2 d) e + 3 (a b^2 c + a^2 b d) f) x^3 + (3 (a b^2 c + a^2 b d) e} \right)$$

7.232 Problem number 2859

$$\int \frac{(2+3x)^{7/2}}{\sqrt{1-2x} \sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{270248 \text{EllipticE} \left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33} \right) \sqrt{33}}{65625} \\ & - \frac{178879 \text{EllipticF} \left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33} \right) \sqrt{33}}{1443750} - \frac{333(2+3x)^{3/2} \sqrt{1-2x} \sqrt{3+5x}}{875} \\ & - \frac{3(2+3x)^{5/2} \sqrt{1-2x} \sqrt{3+5x}}{35} - \frac{15553 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{8750} \end{aligned}$$

command

```
integrate((2+3*x)^(7/2)/(1-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{8750} (6750 x^2 + 18990 x + 25213) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{(27 x^3 + 54 x^2 + 36 x + 8) \sqrt{5 x + 3} \sqrt{3 x + 2} \sqrt{-2 x + 1}}{10 x^2 + x - 3}, x \right)$$

7.233 Problem number 2860

$$\int \frac{(2+3x)^{5/2}}{\sqrt{1-2x}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\frac{5161 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3750} - \frac{857 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{20625} - \frac{3(2+3x)^{3/2} \sqrt{1-2x} \sqrt{3+5x}}{25} - \frac{74\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{125}$$

command

`integrate((2+3*x)^(5/2)/(1-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{125} (45x + 104) \sqrt{5x + 3} \sqrt{3x + 2} \sqrt{-2x + 1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(9x^2 + 12x + 4) \sqrt{5x + 3} \sqrt{3x + 2} \sqrt{-2x + 1}}{10x^2 + x - 3}, x\right)$$

7.234 Problem number 2861

$$\int \frac{(2+3x)^{3/2}}{\sqrt{1-2x}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\frac{37 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{75} - \frac{13 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{825} - \frac{\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{5}$$

command

`integrate((2+3*x)^(3/2)/(1-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{5} \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3} (3x+2)^{\frac{3}{2}} \sqrt{-2x+1}}{10x^2+x-3}, x\right)$$

7.235 Problem number 2864

$$\int \frac{1}{\sqrt{1-2x} (2+3x)^{3/2} \sqrt{3+5x}} dx$$

Optimal antiderivative

$$-\frac{2 \text{EllipticE}\left(\frac{\sqrt{55} \sqrt{1-2x}}{11}, \frac{\sqrt{1155}}{35}\right) \sqrt{35}}{7} + \frac{6\sqrt{1-2x} \sqrt{3+5x}}{7\sqrt{2+3x}}$$

command

`integrate(1/(2+3*x)^(3/2)/(1-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \sqrt{5x+3} \sqrt{-2x+1}}{7 \sqrt{3x+2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{90x^4+129x^3+25x^2-32x-12}, x\right)$$

7.236 Problem number 2865

$$\int \frac{1}{\sqrt{1-2x} (2+3x)^{5/2} \sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{148 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{147} \\ & - \frac{52 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1617} \\ & + \frac{2\sqrt{1-2x} \sqrt{3+5x}}{7(2+3x)^{\frac{3}{2}}} + \frac{148\sqrt{1-2x} \sqrt{3+5x}}{49\sqrt{2+3x}} \end{aligned}$$

command

`integrate(1/(2+3*x)^(5/2)/(1-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(222x + 155)\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{49(9x^2 + 12x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{270x^5 + 567x^4 + 333x^3 - 46x^2 - 100x - 24}, x\right)$$

7.237 Problem number 2866

$$\int \frac{1}{\sqrt{1-2x} (2+3x)^{7/2} \sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{20644 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{5145} \\ & - \frac{6856 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{56595} + \frac{6\sqrt{1-2x} \sqrt{3+5x}}{35(2+3x)^{\frac{5}{2}}} \\ & + \frac{296\sqrt{1-2x} \sqrt{3+5x}}{245(2+3x)^{\frac{3}{2}}} + \frac{20644\sqrt{1-2x} \sqrt{3+5x}}{1715\sqrt{2+3x}} \end{aligned}$$

command

`integrate(1/(2+3*x)^(7/2)/(1-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(92898x^2 + 126972x + 43507)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1715(27x^3 + 54x^2 + 36x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{810x^6 + 2241x^5 + 2133x^4 + 528x^3 - 392x^2 - 272x - 48}, x\right)$$

7.238 Problem number 2867

$$\int \frac{(2+3x)^{7/2}}{\sqrt{1-2x}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{61151 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{68750} \\ & - \frac{942 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{34375} - \frac{2(2+3x)^{5/2}\sqrt{1-2x}}{55\sqrt{3+5x}} \\ & - \frac{69(2+3x)^{3/2}\sqrt{1-2x}\sqrt{3+5x}}{1375} - \frac{2577\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{6875} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)/(3+5*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(7425x^2 + 22440x + 10801)\sqrt{3x+2}\sqrt{-2x+1}}{6875\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(27x^3 + 54x^2 + 36x + 8)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{50x^3 + 35x^2 - 12x - 9}, x\right)$$

7.239 Problem number 2868

$$\int \frac{(2+3x)^{5/2}}{\sqrt{1-2x} (3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{438 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1375} - \frac{17 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1375} - \frac{2(2+3x)^{\frac{3}{2}} \sqrt{1-2x}}{55\sqrt{3+5x}} - \frac{27\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{275}$$

command

`integrate((2+3*x)^(5/2)/(3+5*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(165x+101)\sqrt{3x+2}\sqrt{-2x+1}}{275\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(9x^2+12x+4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{50x^3+35x^2-12x-9}, x\right)$$

7.240 Problem number 2869

$$\int \frac{(2+3x)^{3/2}}{\sqrt{1-2x} (3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{31 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{275} - \frac{4 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{275} - \frac{2\sqrt{1-2x} \sqrt{2+3x}}{55\sqrt{3+5x}}$$

command

```
integrate((2+3*x)^(3/2)/(3+5*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\sqrt{3x+2}\sqrt{-2x+1}}{55\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}(3x+2)^{\frac{3}{2}}\sqrt{-2x+1}}{50x^3+35x^2-12x-9}, x\right)$$

7.241 Problem number 2870

$$\int \frac{\sqrt{2+3x}}{\sqrt{1-2x}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2\text{EllipticE}\left(\sqrt{5}\sqrt{2+3x}, \frac{\sqrt{70}}{35}\right)\sqrt{35}\sqrt{-3-5x}}{55\sqrt{3+5x}} - \frac{2\sqrt{1-2x}\sqrt{2+3x}}{11\sqrt{3+5x}}$$

command

```
integrate((2+3*x)^(1/2)/(3+5*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\sqrt{3x+2}\sqrt{-2x+1}}{11\sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{50x^3+35x^2-12x-9}, x\right)$$

7.242 Problem number 2871

$$\int \frac{1}{\sqrt{1-2x} \sqrt{2+3x} (3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{11} - \frac{10\sqrt{1-2x} \sqrt{2+3x}}{11\sqrt{3+5x}}$$

command

`integrate(1/(3+5*x)^(3/2)/(1-2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{10 \sqrt{3x+2} \sqrt{-2x+1}}{11 \sqrt{5x+3}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{150x^4 + 205x^3 + 34x^2 - 51x - 18}, x\right)$$

7.243 Problem number 2872

$$\int \frac{1}{\sqrt{1-2x} (2+3x)^{3/2} (3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{136 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{77} \\ & + \frac{4 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{77} \\ & + \frac{6\sqrt{1-2x}}{7\sqrt{2+3x} \sqrt{3+5x}} - \frac{680\sqrt{1-2x} \sqrt{2+3x}}{77\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(2+3*x)^(3/2)/(3+5*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(1020x + 647)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{77(15x^2 + 19x + 6)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{450x^5 + 915x^4 + 512x^3 - 85x^2 - 156x - 36}, x\right)$$

7.244 Problem number 2873

$$\int \frac{1}{\sqrt{1-2x}(2+3x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6388 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{539} \\ & + \frac{192 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{539} + \frac{2\sqrt{1-2x}}{7(2+3x)^{\frac{3}{2}}\sqrt{3+5x}} \\ & + \frac{288\sqrt{1-2x}}{49\sqrt{2+3x}\sqrt{3+5x}} - \frac{31940\sqrt{1-2x}\sqrt{2+3x}}{539\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(2+3*x)^(5/2)/(3+5*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(143730x^2 + 186888x + 60635)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{539(45x^3 + 87x^2 + 56x + 12)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1350x^6 + 3645x^5 + 3366x^4 + 769x^3 - 638x^2 - 420x - 72}, x\right)$$

7.245 Problem number 2874

$$\int \frac{1}{\sqrt{1-2x} (2+3x)^{7/2} (3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1344984 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{18865} \\ & + \frac{40456 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{18865} + \frac{6\sqrt{1-2x}}{35(2+3x)^{5/2}\sqrt{3+5x}} \\ & + \frac{436\sqrt{1-2x}}{245(2+3x)^{3/2}\sqrt{3+5x}} + \frac{60684\sqrt{1-2x}}{1715\sqrt{2+3x}\sqrt{3+5x}} - \frac{1344984\sqrt{1-2x}\sqrt{2+3x}}{3773\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(2+3*x)^(7/2)/(3+5*x)^(3/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(90786420x^3 + 178568982x^2 + 116993058x + 25529443)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{18865(135x^4 + 351x^3 + 342x^2 + 148x + 24)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4050x^7 + 13635x^6 + 17388x^5 + 9039x^4 - 376x^3 - 2536x^2 - 1056x - 144}, x\right)$$

7.246 Problem number 2875

$$\int \frac{(2+3x)^{9/2}}{\sqrt{1-2x} (3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6515539 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{11343750} \\ & -\frac{104663 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{5671875} \\ & -\frac{2(2+3x)^{7/2}\sqrt{1-2x}}{165(3+5x)^{3/2}} - \frac{668(2+3x)^{5/2}\sqrt{1-2x}}{9075\sqrt{3+5x}} \\ & + \frac{403(2+3x)^{3/2}\sqrt{1-2x}\sqrt{3+5x}}{75625} - \frac{87476\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{378125} \end{aligned}$$

command

`integrate((2+3*x)^(9/2)/(3+5*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(3675375x^3 + 13721400x^2 + 12517925x + 3365042)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1134375(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(81x^4 + 216x^3 + 216x^2 + 96x + 16)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{250x^4 + 325x^3 + 45x^2 - 81x - 27}, x\right)$$

7.247 Problem number 2876

$$\int \frac{(2+3x)^{7/2}}{\sqrt{1-2x}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{46159 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{226875} \\ & -\frac{2281 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{226875} - \frac{2(2+3x)^{5/2}\sqrt{1-2x}}{165(3+5x)^{3/2}} \\ & -\frac{536(2+3x)^{3/2}\sqrt{1-2x}}{9075\sqrt{3+5x}} - \frac{487\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{15125} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)/(3+5*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(81675x^2 + 101350x + 31429)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{45375(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(27x^3 + 54x^2 + 36x + 8)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{250x^4 + 325x^3 + 45x^2 - 81x - 27}, x\right)$$

7.248 Problem number 2877

$$\int \frac{(2+3x)^{5/2}}{\sqrt{1-2x} (3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2797 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{45375} - \frac{598 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{45375} - \frac{2(2+3x)^{\frac{3}{2}} \sqrt{1-2x}}{165 (3+5x)^{\frac{3}{2}}} - \frac{404 \sqrt{1-2x} \sqrt{2+3x}}{9075 \sqrt{3+5x}}$$

command

`integrate((2+3*x)^(5/2)/(3+5*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(1175x+716)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9075(25x^2+30x+9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(9x^2+12x+4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{250x^4+325x^3+45x^2-81x-27}, x\right)$$

7.249 Problem number 2878

$$\int \frac{(2+3x)^{3/2}}{\sqrt{1-2x} (3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{272 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9075} - \frac{202 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9075} - \frac{2\sqrt{1-2x} \sqrt{2+3x}}{165 (3+5x)^{\frac{3}{2}}} - \frac{272\sqrt{1-2x} \sqrt{2+3x}}{1815 \sqrt{3+5x}}$$

command

`integrate((2+3*x)^(3/2)/(3+5*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(680x + 419)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{1815(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x + 3}(3x + 2)^{\frac{3}{2}}\sqrt{-2x + 1}}{250x^4 + 325x^3 + 45x^2 - 81x - 27}, x\right)$$

7.250 Problem number 2879

$$\int \frac{\sqrt{2 + 3x}}{\sqrt{1 - 2x}(3 + 5x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{74 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33} - 4 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1815} - \frac{2\sqrt{1-2x}\sqrt{2+3x}}{33(3+5x)^{\frac{3}{2}}} - \frac{74\sqrt{1-2x}\sqrt{2+3x}}{363\sqrt{3+5x}}$$

command

`integrate((2+3*x)^(1/2)/(3+5*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(185x + 122)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{363(25x^2 + 30x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{250x^4 + 325x^3 + 45x^2 - 81x - 27}, x\right)$$

7.251 Problem number 2880

$$\int \frac{1}{\sqrt{1-2x} \sqrt{2+3x} (3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{124 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{363} \\ & - \frac{4 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{363} \\ & - \frac{10\sqrt{1-2x} \sqrt{2+3x}}{33(3+5x)^{\frac{3}{2}}} + \frac{620\sqrt{1-2x} \sqrt{2+3x}}{363\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(3+5*x)^(5/2)/(1-2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{50(62x+35)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{363(25x^2+30x+9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{750x^5+1475x^4+785x^3-153x^2-243x-54}, x\right)$$

7.252 Problem number 2881

$$\int \frac{1}{\sqrt{1-2x} (2+3x)^{3/2} (3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{17804 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2541} \\ & - \frac{536 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2541} + \frac{6\sqrt{1-2x}}{7(3+5x)^{\frac{3}{2}} \sqrt{2+3x}} \\ & - \frac{1340\sqrt{1-2x} \sqrt{2+3x}}{231(3+5x)^{\frac{3}{2}}} + \frac{89020\sqrt{1-2x} \sqrt{2+3x}}{2541\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(2+3*x)^(3/2)/(3+5*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(667650x^2 + 823580x + 253409)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2541(75x^3 + 140x^2 + 87x + 18)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2250x^6 + 5925x^5 + 5305x^4 + 1111x^3 - 1035x^2 - 648x - 108}, x\right)$$

7.253 Problem number 2882

$$\int \frac{1}{\sqrt{1-2x}(2+3x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1255552 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{17787} \\ & - \frac{37768 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{17787} + \frac{2\sqrt{1-2x}}{7(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} \\ & + \frac{428\sqrt{1-2x}}{49(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} - \frac{94420\sqrt{1-2x}\sqrt{2+3x}}{1617(3+5x)^{\frac{3}{2}}} + \frac{6277760\sqrt{1-2x}\sqrt{2+3x}}{17787\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(2+3*x)^(5/2)/(3+5*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(141249600x^3 + 268408770x^2 + 169778606x + 35747225)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{17787(225x^4 + 570x^3 + 541x^2 + 228x + 36)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{6750x^7 + 22275x^6 + 27765x^5 + 13943x^4 - 883x^3 - 4014x^2 - 1620x - 216}, x\right)$$

7.254 Problem number 2883

$$\int \frac{1}{\sqrt{1-2x} (2+3x)^{7/2} (3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{352875016 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{622545} \\ & - \frac{10614544 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{622545} \\ & + \frac{6\sqrt{1-2x}}{35(2+3x)^{5/2}(3+5x)^{3/2}} + \frac{576\sqrt{1-2x}}{245(2+3x)^{3/2}(3+5x)^{3/2}} + \frac{120324\sqrt{1-2x}}{1715(3+5x)^{3/2}\sqrt{2+3x}} \\ & - \frac{5307272\sqrt{1-2x}\sqrt{2+3x}}{11319(3+5x)^{3/2}} + \frac{352875016\sqrt{1-2x}\sqrt{2+3x}}{124509\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(2+3*x)^(7/2)/(3+5*x)^(5/2)/(1-2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(119095317900x^4 + 305707177080x^3 + 294023389014x^2 + 125573817736x + 20093773321)\sqrt{5x+3}\sqrt{3x+2}}{622545(675x^5 + 2160x^4 + 2763x^3 + 1766x^2 + 564x + 72)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{20250x^8 + 80325x^7 + 127845x^6 + 97359x^5 + 25237x^4 - 13808x^3 - 12888x^2 - 3888x - 432}, x\right)$$

7.255 Problem number 2884

$$\int \frac{\sqrt{x}}{\sqrt{a+2x}\sqrt{c+2x}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticE}\left(\frac{\sqrt{a+2x}}{\sqrt{a-c}}, \sqrt{1-\frac{c}{a}}\right) \sqrt{a-c} \sqrt{x} \sqrt{\frac{-c-2x}{a-c}} \sqrt{2}}{2\sqrt{-\frac{x}{a}} \sqrt{c+2x}}$$

command

```
integrate(x^(1/2)/(a+2*x)^(1/2)/(c+2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6}(a+c)\text{weierstrassPInverse}\left(\frac{1}{3}a^2-\frac{1}{3}ac+\frac{1}{3}c^2,-\frac{1}{27}a^3+\frac{1}{18}a^2c+\frac{1}{18}ac^2-\frac{1}{27}c^3,\frac{1}{6}a+\frac{1}{6}c+x\right)-\text{weierstrassZeta}\left(\frac{1}{3}a^2-\frac{1}{3}ac+\frac{1}{3}c^2,-\frac{1}{27}a^3+\frac{1}{18}a^2c+\frac{1}{18}ac^2-\frac{1}{27}c^3,\text{weierstrassPInverse}\left(\frac{1}{3}a^2-\frac{1}{3}ac+\frac{1}{3}c^2,-\frac{1}{27}a^3+\frac{1}{18}a^2c+\frac{1}{18}ac^2-\frac{1}{27}c^3,\frac{1}{6}a+\frac{1}{6}c+x\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{a+2x}\sqrt{c+2x}\sqrt{x}}{ac+2(a+c)x+4x^2},x\right)$$

7.256 Problem number 2885

$$\int \frac{1}{\sqrt{4-x}\sqrt{5-x}\sqrt{-3+x}} dx$$

Optimal antiderivative

$$\text{EllipticF}\left(\sqrt{-3+x},\frac{\sqrt{2}}{2}\right)\sqrt{2}$$

command

```
integrate(1/(4-x)^(1/2)/(5-x)^(1/2)/(-3+x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\text{weierstrassPInverse}(4,0,x-4)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x-3}\sqrt{-x+5}\sqrt{-x+4}}{x^3-12x^2+47x-60},x\right)$$

7.257 Problem number 2886

$$\int \frac{1}{\sqrt{4-x} \sqrt{(5-x)(-3+x)}} dx$$

Optimal antiderivative

$$-2 \operatorname{EllipticF}(\sqrt{4-x}, i)$$

command

```
integrate(1/(4-x)^(1/2)/((5-x)*(-3+x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}(4, 0, x - 4)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^2 + 8x - 15} \sqrt{-x + 4}}{x^3 - 12x^2 + 47x - 60}, x\right)$$

7.258 Problem number 2887

$$\int \frac{1}{\sqrt{4-x} \sqrt{-15 + 8x - x^2}} dx$$

Optimal antiderivative

$$-2 \operatorname{EllipticF}(\sqrt{4-x}, i)$$

command

```
integrate(1/(4-x)^(1/2)/(-x^2+8*x-15)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}(4, 0, x - 4)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^2 + 8x - 15} \sqrt{-x + 4}}{x^3 - 12x^2 + 47x - 60}, x\right)$$

7.259 Problem number 2891

$$\int \frac{(2+3x)^{7/2} \sqrt{3+5x}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4071079 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{52500} \\ & + \frac{673523 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{288750} \\ & + \frac{(2+3x)^{7/2} \sqrt{3+5x}}{\sqrt{1-2x}} + \frac{2517(2+3x)^{3/2} \sqrt{1-2x} \sqrt{3+5x}}{350} \\ & + \frac{12(2+3x)^{5/2} \sqrt{1-2x} \sqrt{3+5x}}{7} + \frac{29293 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{875} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)*(3+5*x)^(1/2)/(1-2*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(6750x^3 + 26010x^2 + 54757x - 109756) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{1750(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(27x^3 + 54x^2 + 36x + 8) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{4x^2 - 4x + 1}, x\right)$$

7.260 Problem number 2892

$$\int \frac{(2+3x)^{5/2} \sqrt{3+5x}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7279 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{375} \\ & + \frac{4817 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{8250} + \frac{(2+3x)^{5/2} \sqrt{3+5x}}{\sqrt{1-2x}} \\ & + \frac{9(2+3x)^{3/2} \sqrt{1-2x} \sqrt{3+5x}}{5} + \frac{419 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{50} \end{aligned}$$

command

```
integrate((2+3*x)^(5/2)*(3+5*x)^(1/2)/(1-2*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(90x^2 + 328x - 799)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{50(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(9x^2 + 12x + 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4x^2 - 4x + 1}, x\right)$$

7.261 Problem number 2893

$$\int \frac{(2+3x)^{3/2}\sqrt{3+5x}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{139 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{30} \\ & + \frac{23 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{165} \\ & + \frac{(2+3x)^{3/2}\sqrt{3+5x}}{\sqrt{1-2x}} + 2\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x} \end{aligned}$$

command

```
integrate((2+3*x)^(3/2)*(3+5*x)^(1/2)/(1-2*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{5x+3}\sqrt{3x+2}(x-4)\sqrt{-2x+1}}{2x-1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}(3x+2)^{3/2}\sqrt{-2x+1}}{4x^2 - 4x + 1}, x\right)$$

7.262 Problem number 2894

$$\int \frac{\sqrt{2+3x} \sqrt{3+5x}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{33} + \text{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33} + \frac{\sqrt{2+3x} \sqrt{3+5x}}{\sqrt{1-2x}}$$

command

`integrate((2+3*x)^(1/2)*(3+5*x)^(1/2)/(1-2*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{2x-1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{4x^2-4x+1}, x\right)$$

7.263 Problem number 2895

$$\int \frac{\sqrt{3+5x}}{(1-2x)^{3/2} \sqrt{2+3x}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(\frac{\sqrt{55} \sqrt{1-2x}}{11}, \frac{\sqrt{1155}}{35}\right) \sqrt{35}}{7} + \frac{2\sqrt{2+3x} \sqrt{3+5x}}{7\sqrt{1-2x}}$$

command

`integrate((3+5*x)^(1/2)/(1-2*x)^(3/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{7(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{12x^3-4x^2-5x+2}, x\right)$$

7.264 Problem number 2896

$$\int \frac{\sqrt{3+5x}}{(1-2x)^{3/2}(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{62 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1617} \\ & + \frac{4 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{49} \\ & + \frac{2\sqrt{3+5x}}{7\sqrt{1-2x}\sqrt{2+3x}} - \frac{12\sqrt{1-2x}\sqrt{3+5x}}{49\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(1/2)/(1-2*x)^(3/2)/(2+3*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(12x+1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{49(6x^2+x-2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{36x^4+12x^3-23x^2-4x+4}, x\right)$$

7.265 Problem number 2897

$$\int \frac{\sqrt{3+5x}}{(1-2x)^{3/2}(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{38 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1029} \\ & - \frac{212 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{11319} \\ & + \frac{2\sqrt{3+5x}}{7(2+3x)^{\frac{3}{2}}\sqrt{1-2x}} - \frac{8\sqrt{1-2x}\sqrt{3+5x}}{49(2+3x)^{\frac{3}{2}}} + \frac{38\sqrt{1-2x}\sqrt{3+5x}}{343\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(1/2)/(1-2*x)^(3/2)/(2+3*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(114x^2 - 37x - 59)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{343(18x^3 + 15x^2 - 4x - 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{108x^5 + 108x^4 - 45x^3 - 58x^2 + 4x + 8}, x\right)$$

7.266 Problem number 2898

$$\int \frac{\sqrt{3+5x}}{(1-2x)^{3/2}(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5636 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{36015} \\ & -\frac{4364 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{396165} + \frac{2\sqrt{3+5x}}{7(2+3x)^{\frac{5}{2}}\sqrt{1-2x}} \\ & -\frac{36\sqrt{1-2x}\sqrt{3+5x}}{245(2+3x)^{\frac{5}{2}}} - \frac{26\sqrt{1-2x}\sqrt{3+5x}}{1715(2+3x)^{\frac{3}{2}}} + \frac{5636\sqrt{1-2x}\sqrt{3+5x}}{12005\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(1/2)/(1-2*x)^(3/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(50724x^3 + 41724x^2 - 13127x - 11923)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{12005(54x^4 + 81x^3 + 18x^2 - 20x - 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{324x^6 + 540x^5 + 81x^4 - 264x^3 - 104x^2 + 32x + 16}, x\right)$$

7.267 Problem number 2899

$$\int \frac{(2+3x)^{7/2}(3+5x)^{3/2}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{112543103 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{236250} \\ & + \frac{6770629 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{472500} + \frac{(2+3x)^{7/2}(3+5x)^{3/2}}{\sqrt{1-2x}} \\ & + \frac{1397(2+3x)^{3/2}(3+5x)^{3/2}\sqrt{1-2x}}{210} + \frac{5(2+3x)^{5/2}(3+5x)^{3/2}\sqrt{1-2x}}{3} \\ & + \frac{24358(3+5x)^{3/2}\sqrt{1-2x}\sqrt{2+3x}}{875} + \frac{6770629\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{31500} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)*(3+5*x)^(3/2)/(1-2*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(472500x^4 + 2002500x^3 + 4128030x^2 + 6609296x - 12044593)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{31500(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(135x^4 + 351x^3 + 342x^2 + 148x + 24)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4x^2 - 4x + 1}, x\right)$$

7.268 Problem number 2900

$$\int \frac{(2+3x)^{5/2}(3+5x)^{3/2}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1289089 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{10500} \\ & + \frac{9694 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2625} \\ & + \frac{(2+3x)^{5/2}(3+5x)^{3/2}}{\sqrt{1-2x}} + \frac{12(2+3x)^{3/2}(3+5x)^{3/2}\sqrt{1-2x}}{7} \\ & + \frac{2511(3+5x)^{3/2}\sqrt{1-2x}\sqrt{2+3x}}{350} + \frac{9694\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{175} \end{aligned}$$

command

```
integrate((2+3*x)^(5/2)*(3+5*x)^(3/2)/(1-2*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2250x^3 + 8460x^2 + 17487x - 34721)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{350(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(45x^3 + 87x^2 + 56x + 12)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4x^2 - 4x + 1}, x\right)$$

7.269 Problem number 2901

$$\int \frac{(2+3x)^{3/2}(3+5x)^{3/2}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4621 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{150} \\ & + \frac{139 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{150} + \frac{(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{\sqrt{1-2x}} \\ & + \frac{9(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{5} + \frac{139\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{10} \end{aligned}$$

command

```
integrate((2+3*x)^(3/2)*(3+5*x)^(3/2)/(1-2*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(30x^2 + 106x - 253)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{10(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(15x^2 + 19x + 6)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4x^2 - 4x + 1}, x\right)$$

7.270 Problem number 2902

$$\int \frac{\sqrt{2+3x} (3+5x)^{3/2}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{133 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{18} + \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9} + \frac{(3+5x)^{3/2} \sqrt{2+3x}}{\sqrt{1-2x}} + \frac{10\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{3}$$

command

`integrate((3+5*x)^(3/2)*(2+3*x)^(1/2)/(1-2*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{5x+3} (5x-19) \sqrt{3x+2} \sqrt{-2x+1}}{3(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(5x+3)^{3/2} \sqrt{3x+2} \sqrt{-2x+1}}{4x^2-4x+1}, x\right)$$

7.271 Problem number 2903

$$\int \frac{(3+5x)^{3/2}}{(1-2x)^{3/2} \sqrt{2+3x}} dx$$

Optimal antiderivative

$$\frac{34 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{21} + \frac{\operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{21} + \frac{11\sqrt{2+3x} \sqrt{3+5x}}{7\sqrt{1-2x}}$$

command

`integrate((3+5*x)^(3/2)/(1-2*x)^(3/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{11\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{7(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(5x+3)^{\frac{3}{2}}\sqrt{3x+2}\sqrt{-2x+1}}{12x^3-4x^2-5x+2}, x\right)$$

7.272 Problem number 2904

$$\int \frac{(3+5x)^{3/2}}{(1-2x)^{3/2}(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{31 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{147} + \frac{4 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{147} \\ + \frac{11\sqrt{3+5x}}{7\sqrt{1-2x}\sqrt{2+3x}} - \frac{31\sqrt{1-2x}\sqrt{3+5x}}{49\sqrt{2+3x}}$$

command

`integrate((3+5*x)^(3/2)/(1-2*x)^(3/2)/(2+3*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(31x+23)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{49(6x^2+x-2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(5x+3)^{\frac{3}{2}}\sqrt{3x+2}\sqrt{-2x+1}}{36x^4+12x^3-23x^2-4x+4}, x\right)$$

7.273 Problem number 2905

$$\int \frac{(3+5x)^{3/2}}{(1-2x)^{3/2}(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{458 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3087} \\ & - \frac{178 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3087} \\ & + \frac{11\sqrt{3+5x}}{7(2+3x)^{\frac{3}{2}} \sqrt{1-2x}} - \frac{97\sqrt{1-2x} \sqrt{3+5x}}{147(2+3x)^{\frac{3}{2}}} - \frac{458\sqrt{1-2x} \sqrt{3+5x}}{1029\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(3/2)/(1-2*x)^(3/2)/(2+3*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(1374x^2 + 908x + 11)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1029(18x^3 + 15x^2 - 4x - 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(5x+3)^{\frac{3}{2}}\sqrt{3x+2}\sqrt{-2x+1}}{108x^5 + 108x^4 - 45x^3 - 58x^2 + 4x + 8}, x\right)$$

7.274 Problem number 2906

$$\int \frac{(3+5x)^{3/2}}{(1-2x)^{3/2}(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{338 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{36015} \\ & - \frac{992 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{36015} + \frac{11\sqrt{3+5x}}{7(2+3x)^{\frac{5}{2}} \sqrt{1-2x}} \\ & - \frac{163\sqrt{1-2x} \sqrt{3+5x}}{245(2+3x)^{\frac{5}{2}}} - \frac{458\sqrt{1-2x} \sqrt{3+5x}}{1715(2+3x)^{\frac{3}{2}}} + \frac{338\sqrt{1-2x} \sqrt{3+5x}}{12005\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(3/2)/(1-2*x)^(3/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3042x^3 - 7083x^2 - 10266x - 2909)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{12005(54x^4 + 81x^3 + 18x^2 - 20x - 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(5x+3)^{\frac{3}{2}}\sqrt{3x+2}\sqrt{-2x+1}}{324x^6 + 540x^5 + 81x^4 - 264x^3 - 104x^2 + 32x + 16}, x\right)$$

7.275 Problem number 2907

$$\int \frac{(3+5x)^{3/2}}{(1-2x)^{3/2}(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{189368 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1764735} \\ & - \frac{23012 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1764735} \\ & + \frac{11\sqrt{3+5x}}{7(2+3x)^{\frac{7}{2}}\sqrt{1-2x}} - \frac{229\sqrt{1-2x}\sqrt{3+5x}}{343(2+3x)^{\frac{7}{2}}} - \frac{2818\sqrt{1-2x}\sqrt{3+5x}}{12005(2+3x)^{\frac{5}{2}}} \\ & - \frac{5438\sqrt{1-2x}\sqrt{3+5x}}{84035(2+3x)^{\frac{3}{2}}} + \frac{189368\sqrt{1-2x}\sqrt{3+5x}}{588245\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(3/2)/(1-2*x)^(3/2)/(2+3*x)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(5112936x^4 + 7326810x^3 + 1004571x^2 - 2279324x - 809083)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{588245(162x^5 + 351x^4 + 216x^3 - 24x^2 - 64x - 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(5x+3)^{\frac{3}{2}}\sqrt{3x+2}\sqrt{-2x+1}}{972x^7 + 2268x^6 + 1323x^5 - 630x^4 - 840x^3 - 112x^2 + 112x + 32}, x\right)$$

7.276 Problem number 2908

$$\int \frac{(2+3x)^{7/2}(3+5x)^{5/2}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{17888580643 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{6237000} \\ & + \frac{269045681 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3118500} \\ & + \frac{(2+3x)^{7/2}(3+5x)^{5/2}}{\sqrt{1-2x}} + \frac{419(2+3x)^{3/2}(3+5x)^{5/2}\sqrt{1-2x}}{66} \\ & + \frac{18(2+3x)^{5/2}(3+5x)^{5/2}\sqrt{1-2x}}{11} + \frac{4066493(3+5x)^{3/2}\sqrt{1-2x}\sqrt{2+3x}}{23100} \\ & + \frac{9741(3+5x)^{5/2}\sqrt{1-2x}\sqrt{2+3x}}{385} + \frac{269045681\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{207900} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)*(3+5*x)^(5/2)/(1-2*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(12757500x^5 + 60196500x^4 + 133330950x^3 + 198895770x^2 + 273928969x - 477155552)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{207900(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(675x^5 + 2160x^4 + 2763x^3 + 1766x^2 + 564x + 72)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4x^2 - 4x + 1}, x\right)$$

7.277 Problem number 2909

$$\int \frac{(2+3x)^{5/2}(3+5x)^{5/2}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{42696881 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{56700} \\ & + \frac{1284329 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{56700} + \frac{(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{\sqrt{1-2x}} \\ & + \frac{5(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{3} + \frac{4853(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{105} \\ & + \frac{93(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{14} + \frac{1284329\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{3780} \end{aligned}$$

command

`integrate((2+3*x)^(5/2)*(3+5*x)^(5/2)/(1-2*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(94500x^4 + 392400x^3 + 795150x^2 + 1258906x - 2283923)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3780(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(225x^4 + 570x^3 + 541x^2 + 228x + 36)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4x^2 - 4x + 1}, x\right)$$

7.278 Problem number 2910

$$\int \frac{(2+3x)^{3/2}(3+5x)^{5/2}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{244879 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1260} \\ & + \frac{3683 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{630} \\ & + \frac{(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{\sqrt{1-2x}} + \frac{167(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{14} \\ & + \frac{12(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{7} + \frac{3683\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{42} \end{aligned}$$

command

```
integrate((2+3*x)^(3/2)*(3+5*x)^(5/2)/(1-2*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(450x^3 + 1650x^2 + 3349x - 6590)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{42(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(75x^3 + 140x^2 + 87x + 18)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4x^2 - 4x + 1}, x\right)$$

7.279 Problem number 2911

$$\int \frac{\sqrt{2+3x}(3+5x)^{5/2}}{(1-2x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6599 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{135} \\ & + \frac{397 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{270} + \frac{(3+5x)^{5/2}\sqrt{2+3x}}{\sqrt{1-2x}} \\ & + 3(3+5x)^{3/2}\sqrt{1-2x}\sqrt{2+3x} + \frac{397\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{18} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)*(2+3*x)^(1/2)/(1-2*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(90x^2 + 308x - 721)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{18(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4x^2 - 4x + 1}, x\right)$$

7.280 Problem number 2912

$$\int \frac{(3+5x)^{5/2}}{(1-2x)^{3/2}\sqrt{2+3x}} dx$$

Optimal antiderivative

$$\frac{4451 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{378} + \frac{67 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{189} + \frac{11(3+5x)^{\frac{3}{2}} \sqrt{2+3x}}{7\sqrt{1-2x}} + \frac{335\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{63}$$

command

`integrate((3+5*x)^(5/2)/(1-2*x)^(3/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(175x - 632)\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{63(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{12x^3 - 4x^2 - 5x + 2}, x\right)$$

7.281 Problem number 2913

$$\int \frac{(3+5x)^{5/2}}{(1-2x)^{3/2}(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1159 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{441} + \frac{31 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{441} + \frac{11(3+5x)^{\frac{3}{2}}}{7\sqrt{1-2x} \sqrt{2+3x}} + \frac{31\sqrt{1-2x} \sqrt{3+5x}}{147\sqrt{2+3x}}$$

command

```
integrate((3+5*x)^(5/2)/(1-2*x)^(3/2)/(2+3*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(1093x + 724)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{147(6x^2 + x - 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{36x^4 + 12x^3 - 23x^2 - 4x + 4}, x\right)$$

7.282 Problem number 2914

$$\int \frac{(3 + 5x)^{5/2}}{(1 - 2x)^{3/2}(2 + 3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2797 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9261} \\ & + \frac{598 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9261} \\ & + \frac{11(3+5x)^{\frac{3}{2}}}{7(2+3x)^{\frac{3}{2}}\sqrt{1-2x}} + \frac{97\sqrt{1-2x}\sqrt{3+5x}}{441(2+3x)^{\frac{3}{2}}} - \frac{2797\sqrt{1-2x}\sqrt{3+5x}}{3087\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)/(1-2*x)^(3/2)/(2+3*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(8391x^2 + 12847x + 4819)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{3087(18x^3 + 15x^2 - 4x - 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{108x^5 + 108x^4 - 45x^3 - 58x^2 + 4x + 8}, x\right)$$

7.283 Problem number 2915

$$\int \frac{(3+5x)^{5/2}}{(1-2x)^{3/2}(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\frac{81164 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{324135} - \frac{28174 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{324135} + \frac{11(3+5x)^{\frac{3}{2}}}{7(2+3x)^{\frac{5}{2}}\sqrt{1-2x}} + \frac{163\sqrt{1-2x}\sqrt{3+5x}}{735(2+3x)^{\frac{5}{2}}} - \frac{15601\sqrt{1-2x}\sqrt{3+5x}}{15435(2+3x)^{\frac{3}{2}}} - \frac{81164\sqrt{1-2x}\sqrt{3+5x}}{108045\sqrt{2+3x}}$$

command

`integrate((3+5*x)^(5/2)/(1-2*x)^(3/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(730476x^3 + 936351x^2 + 292777x - 4877)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{108045(54x^4 + 81x^3 + 18x^2 - 20x - 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{324x^6 + 540x^5 + 81x^4 - 264x^3 - 104x^2 + 32x + 16}, x\right)$$

7.284 Problem number 2916

$$\int \frac{(3+5x)^{5/2}}{(1-2x)^{3/2}(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\frac{106558 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{5294205} - \frac{220028 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{5294205} + \frac{11(3+5x)^{\frac{3}{2}}}{7(2+3x)^{\frac{7}{2}}\sqrt{1-2x}} + \frac{229\sqrt{1-2x}\sqrt{3+5x}}{1029(2+3x)^{\frac{7}{2}}} - \frac{37117\sqrt{1-2x}\sqrt{3+5x}}{36015(2+3x)^{\frac{5}{2}}} - \frac{106772\sqrt{1-2x}\sqrt{3+5x}}{252105(2+3x)^{\frac{3}{2}}} - \frac{106558\sqrt{1-2x}\sqrt{3+5x}}{1764735\sqrt{2+3x}}$$

command

```
integrate((3+5*x)^(5/2)/(1-2*x)^(3/2)/(2+3*x)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2877066x^4 + 11042235x^3 + 12020751x^2 + 4889131x + 616327)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1764735(162x^5 + 351x^4 + 216x^3 - 24x^2 - 64x - 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{972x^7 + 2268x^6 + 1323x^5 - 630x^4 - 840x^3 - 112x^2 + 112x + 32}, x\right)$$

7.285 Problem number 2917

$$\int \frac{(3+5x)^{5/2}}{(1-2x)^{3/2}(2+3x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6036028 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{66706983} \\ & - \frac{1199452 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{66706983} + \frac{11(3+5x)^{\frac{3}{2}}}{7(2+3x)^{\frac{9}{2}}\sqrt{1-2x}} \\ & + \frac{295\sqrt{1-2x}\sqrt{3+5x}}{1323(2+3x)^{\frac{9}{2}}} - \frac{67345\sqrt{1-2x}\sqrt{3+5x}}{64827(2+3x)^{\frac{7}{2}}} - \frac{167228\sqrt{1-2x}\sqrt{3+5x}}{453789(2+3x)^{\frac{5}{2}}} \\ & - \frac{392998\sqrt{1-2x}\sqrt{3+5x}}{3176523(2+3x)^{\frac{3}{2}}} + \frac{6036028\sqrt{1-2x}\sqrt{3+5x}}{22235661\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)/(1-2*x)^(3/2)/(2+3*x)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(488918268x^5 + 985046292x^4 + 466728543x^3 - 227945505x^2 - 243200677x - 52688263)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{22235661(486x^6 + 1377x^5 + 1350x^4 + 360x^3 - 240x^2 - 176x - 32)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2916x^8 + 8748x^7 + 8505x^6 + 756x^5 - 3780x^4 - 2016x^3 + 112x^2 + 320x + 64}, x\right)$$

7.286 Problem number 2918

$$\int \frac{(2+3x)^{7/2}}{(1-2x)^{3/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{168123 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{13750} \\ & + \frac{5057 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{13750} + \frac{7(2+3x)^{\frac{5}{2}} \sqrt{3+5x}}{11\sqrt{1-2x}} \\ & + \frac{312(2+3x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{3+5x}}{275} + \frac{14517\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{2750} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)/(1-2*x)^(3/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2970x^2 + 11154x - 27757)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2750(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(27x^3 + 54x^2 + 36x + 8)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{20x^3 - 8x^2 - 7x + 3}, x\right)$$

7.287 Problem number 2919

$$\int \frac{(2+3x)^{5/2}}{(1-2x)^{3/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1597 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{550} \\ & + \frac{24 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{275} \\ & + \frac{7(2+3x)^{\frac{3}{2}} \sqrt{3+5x}}{11\sqrt{1-2x}} + \frac{69\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{55} \end{aligned}$$

command

```
integrate((2+3*x)^(5/2)/(1-2*x)^(3/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(33x - 139)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{55(2x - 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(9x^2 + 12x + 4)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{20x^3 - 8x^2 - 7x + 3}, x\right)$$

7.288 Problem number 2920

$$\int \frac{(2 + 3x)^{3/2}}{(1 - 2x)^{3/2}\sqrt{3 + 5x}} dx$$

Optimal antiderivative

$$\frac{34 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{55} + \frac{\text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{55} + \frac{7\sqrt{2+3x}\sqrt{3+5x}}{11\sqrt{1-2x}}$$

command

```
integrate((2+3*x)^(3/2)/(1-2*x)^(3/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{7\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{11(2x - 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x + 3}(3x + 2)^{\frac{3}{2}}\sqrt{-2x + 1}}{20x^3 - 8x^2 - 7x + 3}, x\right)$$

7.289 Problem number 2921

$$\int \frac{\sqrt{2+3x}}{(1-2x)^{3/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{11} + \frac{2\sqrt{2+3x}\sqrt{3+5x}}{11\sqrt{1-2x}}$$

command

`integrate((2+3*x)^(1/2)/(1-2*x)^(3/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{11(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{20x^3-8x^2-7x+3}, x\right)$$

7.290 Problem number 2922

$$\int \frac{1}{(1-2x)^{3/2}\sqrt{2+3x}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\frac{2\text{EllipticE}\left(\sqrt{5}\sqrt{2+3x}, \frac{\sqrt{70}}{35}\right)\sqrt{35}\sqrt{-3-5x}}{77\sqrt{3+5x}} + \frac{4\sqrt{2+3x}\sqrt{3+5x}}{77\sqrt{1-2x}}$$

command

`integrate(1/(1-2*x)^(3/2)/(2+3*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{4\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{77(2x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{60x^4+16x^3-37x^2-5x+6}, x\right)$$

7.291 Problem number 2923

$$\int \frac{1}{(1-2x)^{3/2}(2+3x)^{3/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{62 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{539} \\ & - \frac{8 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{539} \\ & + \frac{4\sqrt{3+5x}}{77\sqrt{1-2x}\sqrt{2+3x}} + \frac{186\sqrt{1-2x}\sqrt{3+5x}}{539\sqrt{2+3x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(3/2)/(2+3*x)^(3/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(186x - 107)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{539(6x^2 + x - 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{180x^5 + 168x^4 - 79x^3 - 89x^2 + 8x + 12}, x\right)$$

7.292 Problem number 2924

$$\int \frac{1}{(1-2x)^{3/2}(2+3x)^{5/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1752 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3773} \\ & - \frac{68 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3773} + \frac{4\sqrt{3+5x}}{77(2+3x)^{\frac{3}{2}}\sqrt{1-2x}} \\ & + \frac{54\sqrt{1-2x}\sqrt{3+5x}}{539(2+3x)^{\frac{3}{2}}} + \frac{5256\sqrt{1-2x}\sqrt{3+5x}}{3773\sqrt{2+3x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(3/2)/(2+3*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(15768x^2 + 3006x - 5543)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3773(18x^3 + 15x^2 - 4x - 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{540x^6 + 864x^5 + 99x^4 - 425x^3 - 154x^2 + 52x + 24}, x\right)$$

7.293 Problem number 2925

$$\int \frac{1}{(1-2x)^{3/2}(2+3x)^{7/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{244604 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{132055} \\ & - \frac{7536 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{132055} + \frac{4\sqrt{3+5x}}{77(2+3x)^{\frac{5}{2}}\sqrt{1-2x}} \\ & + \frac{138\sqrt{1-2x}\sqrt{3+5x}}{2695(2+3x)^{\frac{5}{2}}} + \frac{10308\sqrt{1-2x}\sqrt{3+5x}}{18865(2+3x)^{\frac{3}{2}}} + \frac{733812\sqrt{1-2x}\sqrt{3+5x}}{132055\sqrt{2+3x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(3/2)/(2+3*x)^(7/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(6604308x^3 + 5720058x^2 - 1424784x - 1546591)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{132055(54x^4 + 81x^3 + 18x^2 - 20x - 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1620x^7 + 3672x^6 + 2025x^5 - 1077x^4 - 1312x^3 - 152x^2 + 176x + 48}, x\right)$$

7.294 Problem number 2926

$$\int \frac{(2+3x)^{9/2}}{(1-2x)^{3/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2911577 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{378125} \\ & + \frac{175111 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{756250} \\ & + \frac{7(2+3x)^{7/2}}{11\sqrt{1-2x}\sqrt{3+5x}} - \frac{37(2+3x)^{5/2}\sqrt{1-2x}}{605\sqrt{3+5x}} \\ & + \frac{10851(2+3x)^{3/2}\sqrt{1-2x}\sqrt{3+5x}}{15125} + \frac{502941\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{151250} \end{aligned}$$

command

`integrate((2+3*x)^(9/2)/(1-2*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(490050x^3 + 2188890x^2 - 3684629x - 2892883)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{151250(10x^2 + x - 3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(81x^4 + 216x^3 + 216x^2 + 96x + 16)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{100x^4 + 20x^3 - 59x^2 - 6x + 9}, x\right)$$

7.295 Problem number 2927

$$\int \frac{(2+3x)^{7/2}}{(1-2x)^{3/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{55019 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{30250} \\ & + \frac{823 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15125} + \frac{7(2+3x)^{5/2}}{11\sqrt{1-2x}\sqrt{3+5x}} \\ & - \frac{37(2+3x)^{3/2}\sqrt{1-2x}}{605\sqrt{3+5x}} + \frac{2388\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{3025} \end{aligned}$$

command

```
integrate((2+3*x)^(7/2)/(1-2*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(5445x^2 - 20897x - 14494)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3025(10x^2+x-3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(27x^3 + 54x^2 + 36x + 8)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{100x^4 + 20x^3 - 59x^2 - 6x + 9}, x\right)$$

7.296 Problem number 2928

$$\int \frac{(2+3x)^{5/2}}{(1-2x)^{3/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1159 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{3025} \\ & + \frac{31 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{3025} \\ & + \frac{7(2+3x)^{\frac{3}{2}}}{11\sqrt{1-2x}\sqrt{3+5x}} - \frac{37\sqrt{1-2x}\sqrt{2+3x}}{605\sqrt{3+5x}} \end{aligned}$$

command

```
integrate((2+3*x)^(5/2)/(1-2*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(1229x + 733)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{605(10x^2+x-3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(9x^2 + 12x + 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{100x^4 + 20x^3 - 59x^2 - 6x + 9}, x\right)$$

7.297 Problem number 2929

$$\int \frac{(2+3x)^{3/2}}{(1-2x)^{3/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{37 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{605} - \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{605}$$

$$+ \frac{7\sqrt{2+3x}}{11\sqrt{1-2x}\sqrt{3+5x}} - \frac{37\sqrt{1-2x}\sqrt{2+3x}}{121\sqrt{3+5x}}$$

command

`integrate((2+3*x)^(3/2)/(1-2*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(37x+20)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{121(10x^2+x-3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}(3x+2)^{\frac{3}{2}}\sqrt{-2x+1}}{100x^4+20x^3-59x^2-6x+9}, x\right)$$

7.298 Problem number 2930

$$\int \frac{\sqrt{2+3x}}{(1-2x)^{3/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{4 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{121} - \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{121}$$

$$+ \frac{2\sqrt{2+3x}}{11\sqrt{1-2x}\sqrt{3+5x}} - \frac{20\sqrt{1-2x}\sqrt{2+3x}}{121\sqrt{3+5x}}$$

command

`integrate((2+3*x)^(1/2)/(1-2*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(20x+1)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{121(10x^2+x-3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{100x^4+20x^3-59x^2-6x+9}, x\right)$$

7.299 Problem number 2931

$$\int \frac{1}{(1-2x)^{3/2}\sqrt{2+3x}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{74 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33} - 4 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{847} + \frac{4\sqrt{2+3x}}{77\sqrt{1-2x}\sqrt{3+5x}} - \frac{370\sqrt{1-2x}\sqrt{2+3x}}{847\sqrt{3+5x}}$$

command

`integrate(1/(1-2*x)^(3/2)/(3+5*x)^(3/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(370x-163)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{847(10x^2+x-3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{300x^5+260x^4-137x^3-136x^2+15x+18}, x\right)$$

7.300 Problem number 2932

$$\int \frac{1}{(1-2x)^{3/2}(2+3x)^{3/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{4636 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{5929} + \frac{124 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{5929} + \frac{4}{77\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}} + \frac{186\sqrt{1-2x}}{539\sqrt{2+3x}\sqrt{3+5x}} - \frac{23180\sqrt{1-2x}\sqrt{2+3x}}{5929\sqrt{3+5x}}$$

command

`integrate(1/(1-2*x)^(3/2)/(2+3*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(69540x^2 + 9544x - 22003)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{5929(30x^3 + 23x^2 - 7x - 6)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{900x^6 + 1380x^5 + 109x^4 - 682x^3 - 227x^2 + 84x + 36}, x\right)$$

7.301 Problem number 2933

$$\int \frac{1}{(1-2x)^{3/2}(2+3x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{220076 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{41503} + \frac{6584 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{41503} + \frac{4}{77(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}} + \frac{54\sqrt{1-2x}}{539(2+3x)^{\frac{3}{2}}\sqrt{3+5x}} + \frac{9876\sqrt{1-2x}}{3773\sqrt{2+3x}\sqrt{3+5x}} - \frac{1100380\sqrt{1-2x}\sqrt{2+3x}}{41503\sqrt{3+5x}}$$

command

`integrate(1/(1-2*x)^(3/2)/(2+3*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(9903420x^3 + 7926942x^2 - 2259236x - 2088967)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{41503(90x^4 + 129x^3 + 25x^2 - 32x - 12)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2700x^7 + 5940x^6 + 3087x^5 - 1828x^4 - 2045x^3 - 202x^2 + 276x + 72}, x\right)$$

7.302 Problem number 2934

$$\int \frac{1}{(1-2x)^{3/2}(2+3x)^{7/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{46585232 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1452605} \\ & + \frac{1400888 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1452605} + \frac{4}{77(2+3x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{3+5x}} \\ & + \frac{138\sqrt{1-2x}}{2695(2+3x)^{\frac{5}{2}}\sqrt{3+5x}} + \frac{14928\sqrt{1-2x}}{18865(2+3x)^{\frac{3}{2}}\sqrt{3+5x}} \\ & + \frac{2101332\sqrt{1-2x}}{132055\sqrt{2+3x}\sqrt{3+5x}} - \frac{46585232\sqrt{1-2x}\sqrt{2+3x}}{290521\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(3/2)/(2+3*x)^(7/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(6289006320x^4 + 9225477612x^3 + 1919527182x^2 - 2283681406x - 884250959)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1452605(270x^5 + 567x^4 + 333x^3 - 46x^2 - 100x - 24)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8100x^8 + 23220x^7 + 21141x^6 + 690x^5 - 9791x^4 - 4696x^3 + 424x^2 + 768x + 144}, x\right)$$

7.303 Problem number 2935

$$\int \frac{(2+3x)^{11/2}}{(1-2x)^{3/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{604915631 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{124781250} \\ & + \frac{18177329 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{124781250} \\ & + \frac{7(2+3x)^{9/2}}{11(3+5x)^{3/2}\sqrt{1-2x}} - \frac{107(2+3x)^{7/2}\sqrt{1-2x}}{1815(3+5x)^{3/2}} - \frac{4553(2+3x)^{5/2}\sqrt{1-2x}}{99825\sqrt{3+5x}} \\ & + \frac{380188(2+3x)^{3/2}\sqrt{1-2x}\sqrt{3+5x}}{831875} + \frac{17427983\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{831875} \end{aligned}$$

command

`integrate((2+3*x)^(11/2)/(1-2*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(242574750x^4 + 1255998150x^3 - 1267558775x^2 - 2667846028x - 904528061)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{24956250(50x^3 + 35x^2 - 12x - 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{500x^5 + 400x^4 - 235x^3 - 207x^2 + 27x + 27}, x\right)$$

7.304 Problem number 2936

$$\int \frac{(2+3x)^{9/2}}{(1-2x)^{3/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5684677 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{4991250} \\ & + \frac{84134 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2495625} + \frac{7(2+3x)^{7/2}}{11(3+5x)^{3/2}\sqrt{1-2x}} \\ & - \frac{107(2+3x)^{5/2}\sqrt{1-2x}}{1815(3+5x)^{3/2}} - \frac{4421(2+3x)^{3/2}\sqrt{1-2x}}{99825\sqrt{3+5x}} + \frac{83093\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{166375} \end{aligned}$$

command

`integrate((2+3*x)^(9/2)/(1-2*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2695275 x^3 - 9376775 x^2 - 14153413 x - 4534181) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{499125 (50x^3 + 35x^2 - 12x - 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(81x^4 + 216x^3 + 216x^2 + 96x + 16) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{500x^5 + 400x^4 - 235x^3 - 207x^2 + 27x + 27}, x \right)$$

7.305 Problem number 2937

$$\int \frac{(2+3x)^{7/2}}{(1-2x)^{3/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{118898 \text{EllipticE} \left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33} \right) \sqrt{33}}{499125} \\ & + \frac{2657 \text{EllipticF} \left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33} \right) \sqrt{33}}{499125} + \frac{7(2+3x)^{5/2}}{11(3+5x)^{3/2} \sqrt{1-2x}} \\ & - \frac{107(2+3x)^{3/2} \sqrt{1-2x}}{1815(3+5x)^{3/2}} - \frac{4289 \sqrt{1-2x} \sqrt{2+3x}}{99825 \sqrt{3+5x}} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)/(1-2*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(649925 x^2 + 772474 x + 229463) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{99825 (50x^3 + 35x^2 - 12x - 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(27x^3 + 54x^2 + 36x + 8) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{500x^5 + 400x^4 - 235x^3 - 207x^2 + 27x + 27}, x \right)$$

7.306 Problem number 2938

$$\int \frac{(2+3x)^{5/2}}{(1-2x)^{3/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4157 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{99825} \\ & - \frac{412 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{99825} \\ & + \frac{7(2+3x)^{\frac{3}{2}}}{11(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} - \frac{107\sqrt{1-2x}\sqrt{2+3x}}{1815(3+5x)^{\frac{3}{2}}} - \frac{4157\sqrt{1-2x}\sqrt{2+3x}}{19965\sqrt{3+5x}} \end{aligned}$$

command

`integrate((2+3*x)^(5/2)/(1-2*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(20785x^2 + 22313x + 5881)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{19965(50x^3 + 35x^2 - 12x - 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(9x^2 + 12x + 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{500x^5 + 400x^4 - 235x^3 - 207x^2 + 27x + 27}, x\right)$$

7.307 Problem number 2939

$$\int \frac{(2+3x)^{3/2}}{(1-2x)^{3/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{494 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{19965} \\ & - \frac{214 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{19965} \\ & + \frac{7\sqrt{2+3x}}{11(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} - \frac{107\sqrt{1-2x}\sqrt{2+3x}}{363(3+5x)^{\frac{3}{2}}} - \frac{494\sqrt{1-2x}\sqrt{2+3x}}{3993\sqrt{3+5x}} \end{aligned}$$

command

`integrate((2+3*x)^(3/2)/(1-2*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(2470x^2 + 1424x - 59)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3993(50x^3 + 35x^2 - 12x - 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}(3x+2)^{\frac{3}{2}}\sqrt{-2x+1}}{500x^5 + 400x^4 - 235x^3 - 207x^2 + 27x + 27}, x\right)$$

7.308 Problem number 2940

$$\int \frac{\sqrt{2+3x}}{(1-2x)^{3/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{98 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{3993} \\ & - \frac{16 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{3993} + \frac{2\sqrt{2+3x}}{11(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} \\ & - \frac{40\sqrt{1-2x}\sqrt{2+3x}}{363(3+5x)^{\frac{3}{2}}} - \frac{490\sqrt{1-2x}\sqrt{2+3x}}{3993\sqrt{3+5x}} \end{aligned}$$

command

`integrate((2+3*x)^(1/2)/(1-2*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(2450x^2 + 685x - 592)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3993(50x^3 + 35x^2 - 12x - 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{500x^5 + 400x^4 - 235x^3 - 207x^2 + 27x + 27}, x\right)$$

7.309 Problem number 2941

$$\int \frac{1}{(1-2x)^{3/2} \sqrt{2+3x} (3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3896 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{27951} \\ & - \frac{164 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{27951} + \frac{4\sqrt{2+3x}}{77(3+5x)^{\frac{3}{2}} \sqrt{1-2x}} \\ & - \frac{410\sqrt{1-2x} \sqrt{2+3x}}{2541(3+5x)^{\frac{3}{2}}} + \frac{19480\sqrt{1-2x} \sqrt{2+3x}}{27951\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(3/2)/(3+5*x)^(5/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(97400x^2 + 5230x - 27691)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{27951(50x^3 + 35x^2 - 12x - 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1500x^6 + 2200x^5 + 95x^4 - 1091x^3 - 333x^2 + 135x + 54}, x\right)$$

7.310 Problem number 2942

$$\int \frac{1}{(1-2x)^{3/2}(2+3x)^{3/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{595324 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{195657} \\ & - \frac{18016 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{195657} + \frac{4}{77(3+5x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{2+3x}} \\ & + \frac{186\sqrt{1-2x}}{539(3+5x)^{\frac{3}{2}} \sqrt{2+3x}} - \frac{45040\sqrt{1-2x} \sqrt{2+3x}}{17787(3+5x)^{\frac{3}{2}}} + \frac{2976620\sqrt{1-2x} \sqrt{2+3x}}{195657\sqrt{3+5x}} \end{aligned}$$

command

```
integrate(1/(1-2*x)^(3/2)/(2+3*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(44649300x^3 + 32744810x^2 - 10598372x - 8473261)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{195657(150x^4 + 205x^3 + 34x^2 - 51x - 18)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4500x^7 + 9600x^6 + 4685x^5 - 3083x^4 - 3181x^3 - 261x^2 + 432x + 108}, x\right)$$

7.311 Problem number 2943

$$\int \frac{1}{(1-2x)^{3/2}(2+3x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{42623864 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1369599} \\ & - \frac{1282376 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1369599} \\ & + \frac{4}{77(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} + \frac{54\sqrt{1-2x}}{539(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} + \frac{14496\sqrt{1-2x}}{3773(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & - \frac{3205940\sqrt{1-2x}\sqrt{2+3x}}{124509(3+5x)^{\frac{3}{2}}} + \frac{213119320\sqrt{1-2x}\sqrt{2+3x}}{1369599\sqrt{3+5x}} \end{aligned}$$

command

```
integrate(1/(1-2*x)^(3/2)/(2+3*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(9590369400x^4 + 13428808080x^3 + 2415287594x^2 - 3336610202x - 1213551469)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1369599(450x^5 + 915x^4 + 512x^3 - 85x^2 - 156x - 36)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{13500x^8 + 37800x^7 + 33255x^6 + 121x^5 - 15709x^4 - 7145x^3 + 774x^2 + 1188x + 216}, x\right)$$

7.312 Problem number 2944

$$\int \frac{1}{(1-2x)^{3/2}(2+3x)^{7/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{12071114168 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \frac{\sqrt{1-2x}}{\sqrt{33}}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{47935965} \\ & - \frac{363103712 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \frac{\sqrt{1-2x}}{\sqrt{33}}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{47935965} \\ & + \frac{4}{77(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} + \frac{138\sqrt{1-2x}}{2695(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}} \\ & + \frac{19548\sqrt{1-2x}}{18865(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} + \frac{4115652\sqrt{1-2x}}{132055(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & - \frac{181551856\sqrt{1-2x}\sqrt{2+3x}}{871563(3+5x)^{\frac{3}{2}}} + \frac{12071114168\sqrt{1-2x}\sqrt{2+3x}}{9587193\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(3/2)/(2+3*x)^(7/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(8148002063400x^5 + 16841199826980x^4 + 9658241620704x^3 - 1466692421066x^2 - 2920885694212x - 687365)}{47935965(1350x^6 + 3645x^5 + 3366x^4 + 769x^3 - 638x^2 - 420x - 72)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{40500x^9 + 140400x^8 + 175365x^7 + 66873x^6 - 46885x^5 - 52853x^4 - 11968x^3 + 5112x^2 + 3024x + 43}\right)$$

7.313 Problem number 2945

$$\int \frac{(2+3x)^{9/2}\sqrt{3+5x}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{112543103 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{288750} \\ & - \frac{6770629 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{577500} + \frac{(2+3x)^{\frac{9}{2}} \sqrt{3+5x}}{3(1-2x)^{\frac{3}{2}}} \\ & - \frac{166(2+3x)^{\frac{7}{2}} \sqrt{3+5x}}{33\sqrt{1-2x}} - \frac{139163(2+3x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{3+5x}}{3850} \\ & - \frac{1327(2+3x)^{\frac{5}{2}} \sqrt{1-2x} \sqrt{3+5x}}{154} - \frac{6478333 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{38500} \end{aligned}$$

command

`integrate((2+3*x)^(9/2)*(3+5*x)^(1/2)/(1-2*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(1336500x^4 + 6664680x^3 + 19375686x^2 - 94671446x + 35797779)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{115500(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(81x^4 + 216x^3 + 216x^2 + 96x + 16)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.314 Problem number 2946

$$\int \frac{(2+3x)^{7/2} \sqrt{3+5x}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1289089 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{16500} \\ & - \frac{9694 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{4125} \\ & + \frac{(2+3x)^{\frac{7}{2}} \sqrt{3+5x}}{3(1-2x)^{\frac{3}{2}}} - \frac{133(2+3x)^{\frac{5}{2}} \sqrt{3+5x}}{33\sqrt{1-2x}} \\ & - \frac{797(2+3x)^{\frac{3}{2}} \sqrt{1-2x} \sqrt{3+5x}}{110} - \frac{18551 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{550} \end{aligned}$$

command

```
integrate((2+3*x)^(7/2)*(3+5*x)^(1/2)/(1-2*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(8910x^3 + 45342x^2 - 275587x + 101763)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1650(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(27x^3 + 54x^2 + 36x + 8)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.315 Problem number 2947

$$\int \frac{(2+3x)^{5/2}\sqrt{3+5x}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4621 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{330} \\ & -\frac{139 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{330} + \frac{(2+3x)^{5/2}\sqrt{3+5x}}{3(1-2x)^{3/2}} \\ & -\frac{100(2+3x)^{3/2}\sqrt{3+5x}}{33\sqrt{1-2x}} - \frac{133\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{22} \end{aligned}$$

command

```
integrate((2+3*x)^(5/2)*(3+5*x)^(1/2)/(1-2*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(198x^2 - 2060x + 711)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{66(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(9x^2 + 12x + 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.316 Problem number 2948

$$\int \frac{(2+3x)^{3/2} \sqrt{3+5x}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{133 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{66} - \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{33} + \frac{(2+3x)^{\frac{3}{2}} \sqrt{3+5x}}{3(1-2x)^{\frac{3}{2}}} - \frac{67\sqrt{2+3x} \sqrt{3+5x}}{33\sqrt{1-2x}}$$

command

```
integrate((2+3*x)^(3/2)*(3+5*x)^(1/2)/(1-2*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(167x - 45)\sqrt{5x + 3} \sqrt{3x + 2} \sqrt{-2x + 1}}{33(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}(3x+2)^{\frac{3}{2}}\sqrt{-2x+1}}{8x^3-12x^2+6x-1}, x\right)$$

7.317 Problem number 2949

$$\int \frac{\sqrt{2+3x} \sqrt{3+5x}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\frac{34 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{231} - \frac{\operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{231} + \frac{\sqrt{2+3x} \sqrt{3+5x}}{3(1-2x)^{\frac{3}{2}}} - \frac{68\sqrt{2+3x} \sqrt{3+5x}}{231\sqrt{1-2x}}$$

command

```
integrate((2+3*x)^(1/2)*(3+5*x)^(1/2)/(1-2*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(136x + 9)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{231(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.318 Problem number 2950

$$\int \frac{\sqrt{3 + 5x}}{(1 - 2x)^{5/2}\sqrt{2 + 3x}} dx$$

Optimal antiderivative

$$\frac{31 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1617} + \frac{4 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1617}$$

$$+ \frac{2\sqrt{2+3x}\sqrt{3+5x}}{21(1-2x)^{\frac{3}{2}}} + \frac{62\sqrt{2+3x}\sqrt{3+5x}}{1617\sqrt{1-2x}}$$

command

`integrate((3+5*x)^(1/2)/(1-2*x)^(5/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{4(31x - 54)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{1617(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{24x^4 - 20x^3 - 6x^2 + 9x - 2}, x\right)$$

7.319 Problem number 2951

$$\int \frac{\sqrt{3+5x}}{(1-2x)^{5/2}(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{458 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{11319} \\ & - \frac{178 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{11319} \\ & + \frac{2\sqrt{3+5x}}{21(1-2x)^{\frac{3}{2}}\sqrt{2+3x}} + \frac{194\sqrt{3+5x}}{1617\sqrt{1-2x}\sqrt{2+3x}} - \frac{458\sqrt{1-2x}\sqrt{3+5x}}{3773\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(1/2)/(1-2*x)^(5/2)/(2+3*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2748x^2 - 1390x - 531)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{11319(12x^3 - 4x^2 - 5x + 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{72x^5 - 12x^4 - 58x^3 + 15x^2 + 12x - 4}, x\right)$$

7.320 Problem number 2952

$$\int \frac{\sqrt{3+5x}}{(1-2x)^{5/2}(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{338 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{79233} \\ & - \frac{992 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{79233} + \frac{2\sqrt{3+5x}}{21(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}} \\ & + \frac{326\sqrt{3+5x}}{1617(2+3x)^{\frac{3}{2}}\sqrt{1-2x}} - \frac{458\sqrt{1-2x}\sqrt{3+5x}}{3773(2+3x)^{\frac{3}{2}}} + \frac{338\sqrt{1-2x}\sqrt{3+5x}}{26411\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(1/2)/(1-2*x)^(5/2)/(2+3*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(6084x^3 - 21264x^2 + 727x + 7965)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{79233(36x^4 + 12x^3 - 23x^2 - 4x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{216x^6 + 108x^5 - 198x^4 - 71x^3 + 66x^2 + 12x - 8}, x\right)$$

7.321 Problem number 2953

$$\int \frac{\sqrt{3+5x}}{(1-2x)^{5/2}(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2092 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{252105} \\ & - \frac{189368 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2773155} \\ & + \frac{2\sqrt{3+5x}}{21(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}} + \frac{458\sqrt{3+5x}}{1617(2+3x)^{\frac{5}{2}}\sqrt{1-2x}} - \frac{2818\sqrt{1-2x}\sqrt{3+5x}}{18865(2+3x)^{\frac{5}{2}}} \\ & - \frac{5438\sqrt{1-2x}\sqrt{3+5x}}{132055(2+3x)^{\frac{3}{2}}} + \frac{189368\sqrt{1-2x}\sqrt{3+5x}}{924385\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(1/2)/(1-2*x)^(5/2)/(2+3*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(10225872x^4 + 2723436x^3 - 7133292x^2 - 807691x + 1339677)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2773155(108x^5 + 108x^4 - 45x^3 - 58x^2 + 4x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{648x^7 + 756x^6 - 378x^5 - 609x^4 + 56x^3 + 168x^2 - 16}, x\right)$$

7.322 Problem number 2954

$$\int \frac{(2+3x)^{7/2}(3+5x)^{3/2}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2+3x)^{\frac{7}{2}}(3+5x)^{\frac{3}{2}}}{3(1-2x)^{\frac{3}{2}}} - \frac{722133 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{38500} \\ & - \frac{6547351 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{10500} \\ & - \frac{56(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{11\sqrt{1-2x}} - \frac{1341(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}}{154} \\ & - \frac{140289(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{3850} - \frac{2166399\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{7700} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)*(3+5*x)^(3/2)/(1-2*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(40500x^4 + 198180x^3 + 567906x^2 - 2751916x + 1041609)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2100(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(135x^4 + 351x^3 + 342x^2 + 148x + 24)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.323 Problem number 2955

$$\int \frac{(2+3x)^{5/2}(3+5x)^{3/2}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}}{3(1-2x)^{\frac{3}{2}}} - \frac{2077 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{550} \\ & - \frac{37663 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{300} - \frac{45(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{11\sqrt{1-2x}} \\ & - \frac{807(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{110} - \frac{6231\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{110} \end{aligned}$$

command

```
integrate((2+3*x)^(5/2)*(3+5*x)^(3/2)/(1-2*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(270x^3 + 1344x^2 - 8039x + 2976)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{30(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(45x^3 + 87x^2 + 56x + 12)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.324 Problem number 2956

$$\int \frac{(2+3x)^{3/2}(3+5x)^{3/2}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}}{3(1-2x)^{\frac{3}{2}}} - \frac{15 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{22} \\ & - \frac{68 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{3} \\ & - \frac{34(3+5x)^{\frac{3}{2}}\sqrt{2+3x}}{11\sqrt{1-2x}} - \frac{225\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{22} \end{aligned}$$

command

```
integrate((2+3*x)^(3/2)*(3+5*x)^(3/2)/(1-2*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(30x^2 - 302x + 105)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{6(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(15x^2 + 19x + 6)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.325 Problem number 2957

$$\int \frac{\sqrt{2+3x} (3+5x)^{3/2}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{139 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{42} \\ & - \frac{23 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{231} \\ & + \frac{(3+5x)^{\frac{3}{2}} \sqrt{2+3x}}{3(1-2x)^{\frac{3}{2}}} - \frac{23\sqrt{2+3x} \sqrt{3+5x}}{7\sqrt{1-2x}} \end{aligned}$$

command

`integrate((3+5*x)^(3/2)*(2+3*x)^(1/2)/(1-2*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(173x - 48)\sqrt{5x + 3} \sqrt{3x + 2} \sqrt{-2x + 1}}{21(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x + 3)^{\frac{3}{2}} \sqrt{3x + 2} \sqrt{-2x + 1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.326 Problem number 2958

$$\int \frac{(3+5x)^{3/2}}{(1-2x)^{5/2} \sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{37 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{147} \\ & - \frac{13 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1617} \\ & + \frac{11\sqrt{2+3x} \sqrt{3+5x}}{21(1-2x)^{\frac{3}{2}}} - \frac{74\sqrt{2+3x} \sqrt{3+5x}}{147\sqrt{1-2x}} \end{aligned}$$

command

```
integrate((3+5*x)^(3/2)/(1-2*x)^(5/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(148x + 3)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{147(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(5x + 3)^{\frac{3}{2}}\sqrt{3x + 2}\sqrt{-2x + 1}}{24x^4 - 20x^3 - 6x^2 + 9x - 2}, x\right)$$

7.327 Problem number 2959

$$\int \frac{(3 + 5x)^{3/2}}{(1 - 2x)^{5/2}(2 + 3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{19 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{1029} \\ & + \frac{106 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{11319} \\ & + \frac{11\sqrt{3+5x}}{21(1-2x)^{\frac{3}{2}}\sqrt{2+3x}} - \frac{8\sqrt{3+5x}}{147\sqrt{1-2x}\sqrt{2+3x}} - \frac{19\sqrt{1-2x}\sqrt{3+5x}}{343\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(3/2)/(1-2*x)^(5/2)/(2+3*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(114x^2 - 170x - 213)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{1029(12x^3 - 4x^2 - 5x + 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(5x + 3)^{\frac{3}{2}}\sqrt{3x + 2}\sqrt{-2x + 1}}{72x^5 - 12x^4 - 58x^3 + 15x^2 + 12x - 4}, x\right)$$

7.328 Problem number 2960

$$\int \frac{(3+5x)^{3/2}}{(1-2x)^{5/2}(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{582 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{26411} \\ & + \frac{496 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{7203} + \frac{11\sqrt{3+5x}}{21(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}} \\ & + \frac{58\sqrt{3+5x}}{147(2+3x)^{\frac{3}{2}}\sqrt{1-2x}} - \frac{89\sqrt{1-2x}\sqrt{3+5x}}{343(2+3x)^{\frac{3}{2}}} - \frac{496\sqrt{1-2x}\sqrt{3+5x}}{2401\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(3/2)/(1-2*x)^(5/2)/(2+3*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(8928x^3 + 762x^2 - 4616x - 885)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{7203(36x^4 + 12x^3 - 23x^2 - 4x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x+3)^{\frac{3}{2}}\sqrt{3x+2}\sqrt{-2x+1}}{216x^6 + 108x^5 - 198x^4 - 71x^3 + 66x^2 + 12x - 8}, x\right)$$

7.329 Problem number 2961

$$\int \frac{(3+5x)^{3/2}}{(1-2x)^{5/2}(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16732 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{924385} \\ & + \frac{3946 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{252105} \\ & + \frac{11\sqrt{3+5x}}{21(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}} + \frac{124\sqrt{3+5x}}{147(2+3x)^{\frac{5}{2}}\sqrt{1-2x}} - \frac{779\sqrt{1-2x}\sqrt{3+5x}}{1715(2+3x)^{\frac{5}{2}}} \\ & - \frac{2264\sqrt{1-2x}\sqrt{3+5x}}{12005(2+3x)^{\frac{3}{2}}} - \frac{3946\sqrt{1-2x}\sqrt{3+5x}}{84035\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(3/2)/(1-2*x)^(5/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(213084x^4 + 356292x^3 - 2199x^2 - 158902x - 43881)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{252105(108x^5 + 108x^4 - 45x^3 - 58x^2 + 4x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(5x+3)^{\frac{3}{2}}\sqrt{3x+2}\sqrt{-2x+1}}{648x^7 + 756x^6 - 378x^5 - 609x^4 + 56x^3 + 168x^2 - 16}, x\right)$$

7.330 Problem number 2962

$$\int \frac{(2+3x)^{7/2}(3+5x)^{5/2}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2+3x)^{\frac{7}{2}}(3+5x)^{\frac{5}{2}}}{3(1-2x)^{\frac{3}{2}}} - \frac{174654791 \text{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{37800} \\ & - \frac{1313411 \text{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{9450} - \frac{203(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{33\sqrt{1-2x}} \\ & - \frac{225(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}\sqrt{1-2x}}{22} - \frac{1310203(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{4620} \\ & - \frac{6277(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{154} - \frac{1313411\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{630} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)*(3+5*x)^(5/2)/(1-2*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(94500x^5 + 486900x^4 + 1279350x^3 + 2783146x^2 - 12151171x + 4641769)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1260(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(675x^5 + 2160x^4 + 2763x^3 + 1766x^2 + 564x + 72)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.331 Problem number 2963

$$\int \frac{(2+3x)^{5/2}(3+5x)^{5/2}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{5}{2}}}{3(1-2x)^{\frac{3}{2}}} - \frac{12601 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{420} \\ & - \frac{69819 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{70} \\ & - \frac{170(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{33\sqrt{1-2x}} - \frac{28283(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{462} \\ & - \frac{1355(3+5x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{2+3x}}{154} - \frac{12601\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{28} \end{aligned}$$

command

`integrate((2+3*x)^(5/2)*(3+5*x)^(5/2)/(1-2*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(2700x^4 + 12960x^3 + 36606x^2 - 175958x + 66663)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{84(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(225x^4 + 570x^3 + 541x^2 + 228x + 36)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.332 Problem number 2964

$$\int \frac{(2+3x)^{3/2}(3+5x)^{5/2}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{5}{2}}}{3(1-2x)^{\frac{3}{2}}} - \frac{12101 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{60} \\ & - \frac{91 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15} - \frac{137(3+5x)^{\frac{5}{2}}\sqrt{2+3x}}{33\sqrt{1-2x}} \\ & - \frac{817(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}}{66} - 91\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x} \end{aligned}$$

command

```
integrate((2+3*x)^(3/2)*(3+5*x)^(5/2)/(1-2*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(90x^3 + 438x^2 - 2579x + 957)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{6(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(75x^3 + 140x^2 + 87x + 18)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.333 Problem number 2965

$$\int \frac{\sqrt{2+3x}(3+5x)^{5/2}}{(1-2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4621 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{126} \\ & -\frac{139 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{126} + \frac{(3+5x)^{5/2}\sqrt{2+3x}}{3(1-2x)^{3/2}} \\ & -\frac{104(3+5x)^{3/2}\sqrt{2+3x}}{21\sqrt{1-2x}} - \frac{695\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{42} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)*(2+3*x)^(1/2)/(1-2*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(350x^2 - 3408x + 1193)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{42(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{8x^3 - 12x^2 + 6x - 1}, x\right)$$

7.334 Problem number 2966

$$\int \frac{(3+5x)^{5/2}}{(1-2x)^{5/2}\sqrt{2+3x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1597 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{294} \\ & - \frac{8 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{49} \\ & + \frac{11(3+5x)^{\frac{3}{2}} \sqrt{2+3x}}{21(1-2x)^{\frac{3}{2}}} - \frac{264\sqrt{2+3x} \sqrt{3+5x}}{49\sqrt{1-2x}} \end{aligned}$$

command

`integrate((3+5*x)^(5/2)/(1-2*x)^(5/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{11(179x-51)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{147(4x^2-4x+1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(25x^2+30x+9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{24x^4-20x^3-6x^2+9x-2}, x\right)$$

7.335 Problem number 2967

$$\int \frac{(3+5x)^{5/2}}{(1-2x)^{5/2}(2+3x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{17 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1029} \\ & - \frac{146 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{343} \\ & + \frac{11(3+5x)^{\frac{3}{2}}}{21(1-2x)^{\frac{3}{2}}\sqrt{2+3x}} - \frac{143\sqrt{3+5x}}{49\sqrt{1-2x}\sqrt{2+3x}} + \frac{438\sqrt{1-2x}\sqrt{3+5x}}{343\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(5/2)/(1-2*x)^(5/2)/(2+3*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(5256x^2 + 3445x - 72)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1029(12x^3 - 4x^2 - 5x + 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{72x^5 - 12x^4 - 58x^3 + 15x^2 + 12x - 4}, x\right)$$

7.336 Problem number 2968

$$\int \frac{(3+5x)^{5/2}}{(1-2x)^{5/2}(2+3x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{11(3+5x)^{\frac{3}{2}}}{21(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}} + \frac{169 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{21609} \\ & + \frac{496 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{21609} - \frac{22\sqrt{3+5x}}{49(2+3x)^{\frac{3}{2}}\sqrt{1-2x}} \\ & + \frac{229\sqrt{1-2x}\sqrt{3+5x}}{1029(2+3x)^{\frac{3}{2}}} - \frac{169\sqrt{1-2x}\sqrt{3+5x}}{7203\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(5/2)/(1-2*x)^(5/2)/(2+3*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(1014x^3 - 3544x^2 - 9883x - 4675)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{7203(36x^4 + 12x^3 - 23x^2 - 4x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{216x^6 + 108x^5 - 198x^4 - 71x^3 + 66x^2 + 12x - 8}, x\right)$$

7.337 Problem number 2969

$$\int \frac{(3+5x)^{5/2}}{(1-2x)^{5/2}(2+3x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{11(3+5x)^{\frac{3}{2}}}{21(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}} - \frac{7738 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{252105} \\ & + \frac{9206 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{84035} + \frac{99\sqrt{3+5x}}{49(2+3x)^{\frac{5}{2}}\sqrt{1-2x}} \\ & - \frac{1432\sqrt{1-2x}\sqrt{3+5x}}{1715(2+3x)^{\frac{5}{2}}} - \frac{4437\sqrt{1-2x}\sqrt{3+5x}}{12005(2+3x)^{\frac{3}{2}}} - \frac{27618\sqrt{1-2x}\sqrt{3+5x}}{84035\sqrt{2+3x}} \end{aligned}$$

command

`integrate((3+5*x)^(5/2)/(1-2*x)^(5/2)/(2+3*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(1491372x^4 + 1056186x^3 - 718167x^2 - 640441x - 88623)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{252105(108x^5 + 108x^4 - 45x^3 - 58x^2 + 4x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{648x^7 + 756x^6 - 378x^5 - 609x^4 + 56x^3 + 168x^2 - 16}, x\right)$$

7.338 Problem number 2970

$$\int \frac{(3+5x)^{5/2}}{(1-2x)^{5/2}(2+3x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{11(3+5x)^{\frac{3}{2}}}{21(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{7}{2}}} + \frac{98642 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2470629} \\ & - \frac{65672 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2470629} + \frac{220\sqrt{3+5x}}{49(2+3x)^{\frac{7}{2}}\sqrt{1-2x}} \\ & - \frac{4545\sqrt{1-2x}\sqrt{3+5x}}{2401(2+3x)^{\frac{7}{2}}} - \frac{11433\sqrt{1-2x}\sqrt{3+5x}}{16807(2+3x)^{\frac{5}{2}}} \\ & - \frac{33778\sqrt{1-2x}\sqrt{3+5x}}{117649(2+3x)^{\frac{3}{2}}} - \frac{98642\sqrt{1-2x}\sqrt{3+5x}}{823543\sqrt{2+3x}} \end{aligned}$$

command

```
integrate((3+5*x)^(5/2)/(1-2*x)^(5/2)/(2+3*x)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(15980004x^5 + 28748088x^4 + 7681599x^3 - 10746933x^2 - 6524789x - 866085)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2470629(324x^6 + 540x^5 + 81x^4 - 264x^3 - 104x^2 + 32x + 16)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(25x^2 + 30x + 9)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1944x^8 + 3564x^7 + 378x^6 - 2583x^5 - 1050x^4 + 616x^3 + 336x^2 - 48x - 32}, x\right)$$

7.339 Problem number 2971

$$\int \frac{(2+3x)^{9/2}}{(1-2x)^{5/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{44109377 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{907500} \\ & -\frac{663409 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{453750} \\ & + \frac{7(2+3x)^{7/2}\sqrt{3+5x}}{33(1-2x)^{3/2}} - \frac{910(2+3x)^{5/2}\sqrt{3+5x}}{363\sqrt{1-2x}} \\ & - \frac{27271(2+3x)^{3/2}\sqrt{1-2x}\sqrt{3+5x}}{6050} - \frac{317384\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{15125} \end{aligned}$$

command

```
integrate((2+3*x)^(9/2)/(1-2*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(294030x^3 + 1528956x^2 - 9445541x + 3478434)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{90750(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(81x^4 + 216x^3 + 216x^2 + 96x + 16)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{40x^4 - 36x^3 - 6x^2 + 13x - 3}, x\right)$$

7.340 Problem number 2972

$$\int \frac{(2+3x)^{7/2}}{(1-2x)^{5/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{78472 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9075} \\ & - \frac{4721 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{18150} + \frac{7(2+3x)^{5/2} \sqrt{3+5x}}{33(1-2x)^{3/2}} \\ & - \frac{679(2+3x)^{3/2} \sqrt{3+5x}}{363\sqrt{1-2x}} - \frac{4517\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}}{1210} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)/(1-2*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(6534x^2 - 70234x + 24051)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3630(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(27x^3 + 54x^2 + 36x + 8)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{40x^4 - 36x^3 - 6x^2 + 13x - 3}, x\right)$$

7.341 Problem number 2973

$$\int \frac{(2+3x)^{5/2}}{(1-2x)^{5/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4451 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3630} \\ & - \frac{67 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1815} \\ & + \frac{7(2+3x)^{3/2} \sqrt{3+5x}}{33(1-2x)^{3/2}} - \frac{448\sqrt{2+3x} \sqrt{3+5x}}{363\sqrt{1-2x}} \end{aligned}$$

command

`integrate((2+3*x)^(5/2)/(1-2*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{49(23x - 6)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{363(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(9x^2 + 12x + 4)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{40x^4 - 36x^3 - 6x^2 + 13x - 3}, x\right)$$

7.342 Problem number 2974

$$\int \frac{(2 + 3x)^{3/2}}{(1 - 2x)^{5/2}\sqrt{3 + 5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{31 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{363} \\ & -\frac{\text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{363} + \frac{7\sqrt{2+3x}\sqrt{3+5x}}{33(1-2x)^{\frac{3}{2}}} - \frac{62\sqrt{2+3x}\sqrt{3+5x}}{363\sqrt{1-2x}} \end{aligned}$$

command

`integrate((2+3*x)^(3/2)/(1-2*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(124x + 15)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{363(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x + 3}(3x + 2)^{\frac{3}{2}}\sqrt{-2x + 1}}{40x^4 - 36x^3 - 6x^2 + 13x - 3}, x\right)$$

7.343 Problem number 2975

$$\int \frac{\sqrt{2+3x}}{(1-2x)^{5/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\frac{37 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2541} - \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2541} + \frac{2\sqrt{2+3x}\sqrt{3+5x}}{33(1-2x)^{\frac{3}{2}}} + \frac{74\sqrt{2+3x}\sqrt{3+5x}}{2541\sqrt{1-2x}}$$

command

`integrate((2+3*x)^(1/2)/(1-2*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{4(37x-57)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2541(4x^2-4x+1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{40x^4-36x^3-6x^2+13x-3}, x\right)$$

7.344 Problem number 2976

$$\int \frac{1}{(1-2x)^{5/2}\sqrt{2+3x}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\frac{272 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{17787} - \frac{202 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{17787} + \frac{4\sqrt{2+3x}\sqrt{3+5x}}{231(1-2x)^{\frac{3}{2}}} + \frac{544\sqrt{2+3x}\sqrt{3+5x}}{17787\sqrt{1-2x}}$$

command

`integrate(1/(1-2*x)^(5/2)/(2+3*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{4(272x - 213)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{17787(4x^2 - 4x + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{120x^5 - 28x^4 - 90x^3 + 27x^2 + 17x - 6}, x\right)$$

7.345 Problem number 2977

$$\int \frac{1}{(1 - 2x)^{5/2}(2 + 3x)^{3/2}\sqrt{3 + 5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5594 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{124509} \\ & - \frac{1196 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{124509} + \frac{4\sqrt{3+5x}}{231(1-2x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & + \frac{808\sqrt{3+5x}}{17787\sqrt{1-2x}\sqrt{2+3x}} + \frac{5594\sqrt{1-2x}\sqrt{3+5x}}{41503\sqrt{2+3x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(5/2)/(2+3*x)^(3/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(33564x^2 - 39220x + 12297)\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{124509(12x^3 - 4x^2 - 5x + 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x + 3}\sqrt{3x + 2}\sqrt{-2x + 1}}{360x^6 + 156x^5 - 326x^4 - 99x^3 + 105x^2 + 16x - 12}, x\right)$$

7.346 Problem number 2978

$$\int \frac{1}{(1-2x)^{5/2}(2+3x)^{5/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{184636 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{871563} \\ & - \frac{9124 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{871563} + \frac{4\sqrt{3+5x}}{231(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}} \\ & + \frac{1072\sqrt{3+5x}}{17787(2+3x)^{\frac{3}{2}}\sqrt{1-2x}} + \frac{974\sqrt{1-2x}\sqrt{3+5x}}{41503(2+3x)^{\frac{3}{2}}} + \frac{184636\sqrt{1-2x}\sqrt{3+5x}}{290521\sqrt{2+3x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(5/2)/(2+3*x)^(5/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3323448x^3 - 1066908x^2 - 1478206x + 597945)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{871563(36x^4 + 12x^3 - 23x^2 - 4x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1080x^7 + 1188x^6 - 666x^5 - 949x^4 + 117x^3 + 258x^2 - 4x - 24}, x\right)$$

7.347 Problem number 2979

$$\int \frac{1}{(1-2x)^{5/2}(2+3x)^{7/2}\sqrt{3+5x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{26062156 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{30504705} \\ & - \frac{837304 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{30504705} + \frac{4\sqrt{3+5x}}{231(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}} \\ & + \frac{1336\sqrt{3+5x}}{17787(2+3x)^{\frac{5}{2}}\sqrt{1-2x}} - \frac{806\sqrt{1-2x}\sqrt{3+5x}}{207515(2+3x)^{\frac{5}{2}}} \\ & + \frac{349904\sqrt{1-2x}\sqrt{3+5x}}{1452605(2+3x)^{\frac{3}{2}}} + \frac{26062156\sqrt{1-2x}\sqrt{3+5x}}{10168235\sqrt{2+3x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(5/2)/(2+3*x)^(7/2)/(3+5*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(1407356424x^4 + 513206712x^3 - 914077314x^2 - 176797172x + 165071409)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{30504705(108x^5 + 108x^4 - 45x^3 - 58x^2 + 4x + 8)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3240x^8 + 5724x^7 + 378x^6 - 4179x^5 - 1547x^4 + 1008x^3 + 504x^2 - 80x - 48}, x\right)$$

7.348 Problem number 2980

$$\int \frac{(2+3x)^{11/2}}{(1-2x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1508889271 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{49912500} \\ & - \frac{11346991 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{12478125} \\ & + \frac{7(2+3x)^{\frac{9}{2}}}{33(1-2x)^{\frac{3}{2}}\sqrt{3+5x}} - \frac{896(2+3x)^{\frac{7}{2}}}{363\sqrt{1-2x}\sqrt{3+5x}} + \frac{4439(2+3x)^{\frac{5}{2}}\sqrt{1-2x}}{19965\sqrt{3+5x}} \\ & - \frac{932783(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{332750} - \frac{21713939\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{1663750} \end{aligned}$$

command

`integrate((2+3*x)^(11/2)/(1-2*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(48514950x^4 + 286777260x^3 - 1463754851x^2 - 376752444x + 356556921)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{4991250(20x^3 - 8x^2 - 7x + 3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{200x^5 - 60x^4 - 138x^3 + 47x^2 + 24x - 9}, x\right)$$

7.349 Problem number 2981

$$\int \frac{(2+3x)^{9/2}}{(1-2x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5327983 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{998250} \\ & - \frac{160297 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{998250} + \frac{7(2+3x)^{7/2}}{33(1-2x)^{3/2}\sqrt{3+5x}} \\ & - \frac{665(2+3x)^{5/2}}{363\sqrt{1-2x}\sqrt{3+5x}} + \frac{3284(2+3x)^{3/2}\sqrt{1-2x}}{19965\sqrt{3+5x}} - \frac{153319\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{66550} \end{aligned}$$

command

`integrate((2+3*x)^(9/2)/(1-2*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(1078110x^3 - 11321446x^2 - 3117099x + 2438391)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{199650(20x^3 - 8x^2 - 7x + 3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(81x^4 + 216x^3 + 216x^2 + 96x + 16)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{200x^5 - 60x^4 - 138x^3 + 47x^2 + 24x - 9}, x\right)$$

7.350 Problem number 2982

$$\int \frac{(2+3x)^{7/2}}{(1-2x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{148831 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{199650} \\ & - \frac{2252 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{99825} + \frac{7(2+3x)^{5/2}}{33(1-2x)^{3/2}\sqrt{3+5x}} \\ & - \frac{434(2+3x)^{3/2}}{363\sqrt{1-2x}\sqrt{3+5x}} + \frac{2129\sqrt{1-2x}\sqrt{2+3x}}{19965\sqrt{3+5x}} \end{aligned}$$

command

`integrate((2+3*x)^(7/2)/(1-2*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(189851x^2 + 66174x - 28671)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{19965(20x^3 - 8x^2 - 7x + 3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(27x^3 + 54x^2 + 36x + 8)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{200x^5 - 60x^4 - 138x^3 + 47x^2 + 24x - 9}, x\right)$$

7.351 Problem number 2983

$$\int \frac{(2+3x)^{5/2}}{(1-2x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{974 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{19965} \\ & -\frac{41 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{19965} + \frac{7(2+3x)^{\frac{3}{2}}}{33(1-2x)^{\frac{3}{2}}\sqrt{3+5x}} \\ & -\frac{203\sqrt{2+3x}}{363\sqrt{1-2x}\sqrt{3+5x}} + \frac{974\sqrt{1-2x}\sqrt{2+3x}}{3993\sqrt{3+5x}} \end{aligned}$$

command

`integrate((2+3*x)^(5/2)/(1-2*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(3896x^2 + 3111x + 435)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3993(20x^3 - 8x^2 - 7x + 3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(9x^2 + 12x + 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{200x^5 - 60x^4 - 138x^3 + 47x^2 + 24x - 9}, x\right)$$

7.352 Problem number 2984

$$\int \frac{(2+3x)^{3/2}}{(1-2x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{49 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3993} - \frac{8 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3993} + \frac{7\sqrt{2+3x}}{33(1-2x)^{3/2}\sqrt{3+5x}} + \frac{8\sqrt{2+3x}}{363\sqrt{1-2x}\sqrt{3+5x}} - \frac{245\sqrt{1-2x}\sqrt{2+3x}}{3993\sqrt{3+5x}}$$

command

`integrate((2+3*x)^(3/2)/(1-2*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(490x^2 - 402x - 345)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3993(20x^3 - 8x^2 - 7x + 3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}(3x+2)^{3/2}\sqrt{-2x+1}}{200x^5 - 60x^4 - 138x^3 + 47x^2 + 24x - 9}, x\right)$$

7.353 Problem number 2985

$$\int \frac{\sqrt{2+3x}}{(1-2x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{494 \operatorname{EllipticE}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{27951} - \frac{214 \operatorname{EllipticF}\left(\frac{\sqrt{21}}{7} \sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{27951} + \frac{2\sqrt{2+3x}}{33(1-2x)^{3/2}\sqrt{3+5x}} + \frac{214\sqrt{2+3x}}{2541\sqrt{1-2x}\sqrt{3+5x}} - \frac{2470\sqrt{1-2x}\sqrt{2+3x}}{27951\sqrt{3+5x}}$$

command

`integrate((2+3*x)^(1/2)/(1-2*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(4940x^2 - 2586x - 789)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{27951(20x^3 - 8x^2 - 7x + 3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{200x^5 - 60x^4 - 138x^3 + 47x^2 + 24x - 9}, x\right)$$

7.354 Problem number 2986

$$\int \frac{1}{(1-2x)^{5/2}\sqrt{2+3x}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8314 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{195657} \\ & - \frac{824 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{195657} + \frac{4\sqrt{2+3x}}{231(1-2x)^{\frac{3}{2}}\sqrt{3+5x}} \\ & + \frac{824\sqrt{2+3x}}{17787\sqrt{1-2x}\sqrt{3+5x}} - \frac{41570\sqrt{1-2x}\sqrt{2+3x}}{195657\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(5/2)/(3+5*x)^(3/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(83140x^2 - 74076x + 14559)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{195657(20x^3 - 8x^2 - 7x + 3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{600x^6 + 220x^5 - 534x^4 - 135x^3 + 166x^2 + 21x - 18}, x\right)$$

7.355 Problem number 2987

$$\int \frac{1}{(1-2x)^{5/2}(2+3x)^{3/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{475592 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1369599} \\ & + \frac{10628 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1369599} \\ & + \frac{4}{231(1-2x)^{\frac{3}{2}} \sqrt{2+3x} \sqrt{3+5x}} + \frac{1088}{17787\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}} \\ & + \frac{5314\sqrt{1-2x}}{41503\sqrt{2+3x} \sqrt{3+5x}} - \frac{2377960\sqrt{1-2x} \sqrt{2+3x}}{1369599\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(5/2)/(2+3*x)^(3/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(14267760x^3 - 5106644x^2 - 5510400x + 2236533)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1369599(60x^4 + 16x^3 - 37x^2 - 5x + 6)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1800x^7 + 1860x^6 - 1162x^5 - 1473x^4 + 228x^3 + 395x^2 - 12x - 36}, x\right)$$

7.356 Problem number 2988

$$\int \frac{1}{(1-2x)^{5/2}(2+3x)^{5/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{22738708 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9587193} \\ & + \frac{673072 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{9587193} + \frac{4}{231(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}\sqrt{3+5x}} \\ & + \frac{1352}{17787(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}} + \frac{694\sqrt{1-2x}}{41503(2+3x)^{\frac{3}{2}}\sqrt{3+5x}} \\ & + \frac{336536\sqrt{1-2x}}{290521\sqrt{2+3x}\sqrt{3+5x}} - \frac{113693540\sqrt{1-2x}\sqrt{2+3x}}{9587193\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(5/2)/(2+3*x)^(5/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2046483720x^4 + 615527112x^3 - 1285584962x^2 - 198573504x + 215753865)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9587193(180x^5 + 168x^4 - 79x^3 - 89x^2 + 8x + 12)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{5400x^8 + 9180x^7 + 234x^6 - 6743x^5 - 2262x^4 + 1641x^3 + 754x^2 - 132x - 72}, x\right)$$

7.357 Problem number 2989

$$\int \frac{1}{(1-2x)^{5/2}(2+3x)^{7/2}(3+5x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4839325048 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{335551755} \\ & + \frac{145418632 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{335551755} \\ & + \frac{4}{231(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}\sqrt{3+5x}} + \frac{1616}{17787(2+3x)^{\frac{5}{2}}\sqrt{1-2x}\sqrt{3+5x}} \\ & - \frac{2206\sqrt{1-2x}}{207515(2+3x)^{\frac{5}{2}}\sqrt{3+5x}} + \frac{499564\sqrt{1-2x}}{1452605(2+3x)^{\frac{3}{2}}\sqrt{3+5x}} \\ & + \frac{72709316\sqrt{1-2x}}{10168235\sqrt{2+3x}\sqrt{3+5x}} - \frac{4839325048\sqrt{1-2x}\sqrt{2+3x}}{67110351\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(5/2)/(2+3*x)^(7/2)/(3+5*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(1306617762960x^5 + 1263428429256x^4 - 559512908172x^3 - 673871013766x^2 + 53503915182x + 9185592224)}{335551755(540x^6 + 864x^5 + 99x^4 - 425x^3 - 154x^2 + 52x + 24)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{16200x^9 + 38340x^8 + 19062x^7 - 19761x^6 - 20272x^5 + 399x^4 + 5544x^3 + 1112x^2 - 480x - 144}, x\right)$$

7.358 Problem number 2990

$$\int \frac{(2+3x)^{13/2}}{(1-2x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(2+3x)^{\frac{11}{2}}}{33(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} - \frac{51601293223 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2745187500} \\ & - \frac{776112041 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{1372593750} - \frac{294(2+3x)^{\frac{9}{2}}}{121(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} \\ & + \frac{4373(2+3x)^{\frac{7}{2}}\sqrt{1-2x}}{19965(3+5x)^{\frac{3}{2}}} + \frac{150812(2+3x)^{\frac{5}{2}}\sqrt{1-2x}}{1098075\sqrt{3+5x}} \\ & - \frac{31887029(2+3x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{3+5x}}{18301250} - \frac{371279941\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{45753125} \end{aligned}$$

command

`integrate((2+3*x)^(13/2)/(1-2*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(8004966750x^5 + 53010668700x^4 - 222254370925x^3 - 215557803774x^2 + 21979664649x + 36533948644)\sqrt{5x}}{274518750(100x^4 + 20x^3 - 59x^2 - 6x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1000x^6 + 300x^5 - 870x^4 - 179x^3 + 261x^2 + 27x - 27}, x\right)$$

7.359 Problem number 2991

$$\int \frac{(2+3x)^{11/2}}{(1-2x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(2+3x)^{\frac{9}{2}}}{33(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} - \frac{90397364 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{27451875} \\ & - \frac{5442127 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{54903750} \\ & - \frac{217(2+3x)^{\frac{7}{2}}}{121(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} + \frac{3218(2+3x)^{\frac{5}{2}}\sqrt{1-2x}}{19965(3+5x)^{\frac{3}{2}}} \\ & + \frac{110519(2+3x)^{\frac{3}{2}}\sqrt{1-2x}}{1098075\sqrt{3+5x}} - \frac{5199979\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x}}{3660250} \end{aligned}$$

command

`integrate((2+3*x)^(11/2)/(1-2*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(177888150x^4 - 1825153850x^3 - 1696384053x^2 + 89252928x + 246962693)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{10980750(100x^4 + 20x^3 - 59x^2 - 6x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1000x^6 + 300x^5 - 870x^4 - 179x^3 + 261x^2 + 27x - 27}, x\right)$$

7.360 Problem number 2992

$$\int \frac{(2+3x)^{9/2}}{(1-2x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(2+3x)^{\frac{7}{2}}}{33(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} - \frac{4971289 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{10980750} \\ & - \frac{76163 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{5490375} - \frac{140(2+3x)^{\frac{5}{2}}}{121(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} \\ & + \frac{2063(2+3x)^{\frac{3}{2}}\sqrt{1-2x}}{19965(3+5x)^{\frac{3}{2}}} + \frac{70226\sqrt{1-2x}\sqrt{2+3x}}{1098075\sqrt{3+5x}} \end{aligned}$$

command

```
integrate((2+3*x)^(9/2)/(1-2*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(31924075 x^3 + 30619782 x^2 + 2244393 x - 2780992) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{1098075 (100 x^4 + 20 x^3 - 59 x^2 - 6 x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{(81 x^4 + 216 x^3 + 216 x^2 + 96 x + 16) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{1000 x^6 + 300 x^5 - 870 x^4 - 179 x^3 + 261 x^2 + 27 x - 27}, x \right)$$

7.361 Problem number 2993

$$\int \frac{(2+3x)^{7/2}}{(1-2x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(2+3x)^{\frac{5}{2}}}{33(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} - \frac{29933 \text{EllipticE} \left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33} \right) \sqrt{33}}{1098075} \\ & - \frac{1847 \text{EllipticF} \left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33} \right) \sqrt{33}}{1098075} - \frac{63(2+3x)^{\frac{3}{2}}}{121(3+5x)^{\frac{3}{2}} \sqrt{1-2x}} \\ & + \frac{908 \sqrt{1-2x} \sqrt{2+3x}}{19965(3+5x)^{\frac{3}{2}}} + \frac{29933 \sqrt{1-2x} \sqrt{2+3x}}{219615 \sqrt{3+5x}} \end{aligned}$$

command

```
integrate((2+3*x)^(7/2)/(1-2*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(598660 x^3 + 905823 x^2 + 423882 x + 57437) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{219615 (100 x^4 + 20 x^3 - 59 x^2 - 6 x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{(27 x^3 + 54 x^2 + 36 x + 8) \sqrt{5x+3} \sqrt{3x+2} \sqrt{-2x+1}}{1000 x^6 + 300 x^5 - 870 x^4 - 179 x^3 + 261 x^2 + 27 x - 27}, x \right)$$

7.362 Problem number 2994

$$\int \frac{(2+3x)^{5/2}}{(1-2x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(2+3x)^{\frac{3}{2}}}{33(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} + \frac{2209 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{219615} \\ & - \frac{494 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{219615} + \frac{14\sqrt{2+3x}}{121(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} \\ & - \frac{247\sqrt{1-2x}\sqrt{2+3x}}{3993(3+5x)^{\frac{3}{2}}} - \frac{2209\sqrt{1-2x}\sqrt{2+3x}}{43923\sqrt{3+5x}} \end{aligned}$$

command

`integrate((2+3*x)^(5/2)/(1-2*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(22090x^3 - 3402x^2 - 22059x - 7186)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{43923(100x^4 + 20x^3 - 59x^2 - 6x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(9x^2 + 12x + 4)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1000x^6 + 300x^5 - 870x^4 - 179x^3 + 261x^2 + 27x - 27}, x\right)$$

7.363 Problem number 2995

$$\int \frac{(2+3x)^{3/2}}{(1-2x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{592 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{43923} \\ & - \frac{230 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{43923} + \frac{7\sqrt{2+3x}}{33(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} \\ & + \frac{26\sqrt{2+3x}}{121(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} - \frac{575\sqrt{1-2x}\sqrt{2+3x}}{3993(3+5x)^{\frac{3}{2}}} - \frac{2960\sqrt{1-2x}\sqrt{2+3x}}{43923\sqrt{3+5x}} \end{aligned}$$

command

`integrate((2+3*x)^(3/2)/(1-2*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(29600x^3 + 810x^2 - 13572x - 1775)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{43923(100x^4 + 20x^3 - 59x^2 - 6x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}(3x+2)^{\frac{3}{2}}\sqrt{-2x+1}}{1000x^6 + 300x^5 - 870x^4 - 179x^3 + 261x^2 + 27x - 27}, x\right)$$

7.364 Problem number 2996

$$\int \frac{\sqrt{2+3x}}{(1-2x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4418 \text{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{307461} \\ & - \frac{988 \text{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{307461} + \frac{2\sqrt{2+3x}}{33(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} \\ & + \frac{118\sqrt{2+3x}}{847(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} - \frac{2470\sqrt{1-2x}\sqrt{2+3x}}{27951(3+5x)^{\frac{3}{2}}} - \frac{22090\sqrt{1-2x}\sqrt{2+3x}}{307461\sqrt{3+5x}} \end{aligned}$$

command

`integrate((2+3*x)^(1/2)/(1-2*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2(220900x^3 - 34020x^2 - 88821x + 15986)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{307461(100x^4 + 20x^3 - 59x^2 - 6x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{1000x^6 + 300x^5 - 870x^4 - 179x^3 + 261x^2 + 27x - 27}, x\right)$$

7.365 Problem number 2997

$$\int \frac{1}{(1-2x)^{5/2} \sqrt{2+3x} (3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{119732 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2152227} \\ & - \frac{7388 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{2152227} + \frac{4\sqrt{2+3x}}{231(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} \\ & + \frac{368\sqrt{2+3x}}{5929(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} - \frac{18470\sqrt{1-2x}\sqrt{2+3x}}{195657(3+5x)^{\frac{3}{2}}} + \frac{598660\sqrt{1-2x}\sqrt{2+3x}}{2152227\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(5/2)/(3+5*x)^(5/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(5986600x^3 - 2800980x^2 - 1822554x + 881831)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{2152227(100x^4 + 20x^3 - 59x^2 - 6x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{3000x^7 + 2900x^6 - 2010x^5 - 2277x^4 + 425x^3 + 603x^2 - 27x - 54}, x\right)$$

7.366 Problem number 2998

$$\int \frac{1}{(1-2x)^{5/2}(2+3x)^{3/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{19885156 \operatorname{EllipticE}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15065589} \\ & - \frac{609304 \operatorname{EllipticF}\left(\frac{\sqrt{21} \sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{15065589} + \frac{4}{231(1-2x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & + \frac{456}{5929(3+5x)^{\frac{3}{2}}\sqrt{1-2x}\sqrt{2+3x}} + \frac{5034\sqrt{1-2x}}{41503(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & - \frac{1523260\sqrt{1-2x}\sqrt{2+3x}}{1369599(3+5x)^{\frac{3}{2}}} + \frac{99425780\sqrt{1-2x}\sqrt{2+3x}}{15065589\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(5/2)/(2+3*x)^(3/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2982773400x^4 + 694871080x^3 - 1802210526x^2 - 211488180x + 283144937)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{15065589(300x^5 + 260x^4 - 137x^3 - 136x^2 + 15x + 18)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{9000x^8 + 14700x^7 - 230x^6 - 10851x^5 - 3279x^4 + 2659x^3 + 1125x^2 - 216x - 108}, x\right)$$

7.367 Problem number 2999

$$\int \frac{1}{(1-2x)^{5/2}(2+3x)^{5/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4}{231(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} \\ & - \frac{1446357824 \text{EllipticE}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{105459123} \\ & - \frac{43537016 \text{EllipticF}\left(\frac{\sqrt{21}}{7}\sqrt{1-2x}, \frac{\sqrt{1155}}{33}\right)\sqrt{33}}{105459123} \\ & + \frac{544}{5929(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} \\ & + \frac{414\sqrt{1-2x}}{41503(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} + \frac{488436\sqrt{1-2x}}{290521(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & - \frac{108842540\sqrt{1-2x}\sqrt{2+3x}}{9587193(3+5x)^{\frac{3}{2}}} + \frac{7231789120\sqrt{1-2x}\sqrt{2+3x}}{105459123\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(5/2)/(2+3*x)^(5/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(650861020800x^5 + 585919463160x^4 - 291775464272x^3 - 308398535118x^2 + 30866656614x + 41179778225)\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{105459123(900x^6 + 1380x^5 + 109x^4 - 682x^3 - 227x^2 + 84x + 36)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{27000x^9 + 62100x^8 + 28710x^7 - 33013x^6 - 31539x^5 + 1419x^4 + 8693x^3 + 1602x^2 - 756x - 216}, x\right)$$

7.368 Problem number 3000

$$\int \frac{1}{(1-2x)^{5/2}(2+3x)^{7/2}(3+5x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4}{231(1-2x)^{\frac{3}{2}}(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}} \\ & - \frac{412810345784 \operatorname{EllipticE}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3691069305} \\ & - \frac{12417792656 \operatorname{EllipticF}\left(\frac{\sqrt{21}\sqrt{1-2x}}{7}, \frac{\sqrt{1155}}{33}\right) \sqrt{33}}{3691069305} \\ & + \frac{632}{5929(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}\sqrt{1-2x}} - \frac{3606\sqrt{1-2x}}{207515(2+3x)^{\frac{5}{2}}(3+5x)^{\frac{3}{2}}} \\ & + \frac{649224\sqrt{1-2x}}{1452605(2+3x)^{\frac{3}{2}}(3+5x)^{\frac{3}{2}}} + \frac{140700876\sqrt{1-2x}}{10168235(3+5x)^{\frac{3}{2}}\sqrt{2+3x}} \\ & - \frac{6208896328\sqrt{1-2x}\sqrt{2+3x}}{67110351(3+5x)^{\frac{3}{2}}} + \frac{412810345784\sqrt{1-2x}\sqrt{2+3x}}{738213861\sqrt{3+5x}} \end{aligned}$$

command

`integrate(1/(1-2*x)^(5/2)/(2+3*x)^(7/2)/(3+5*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(557293966808400x^6 + 873229924799280x^5 + 84649478011164x^4 - 430611138612568x^3 - 149619576926754x^2 - 3691069305(2700x^7 + 5940x^6 + 3087x^5 - 1828x^4 - 2045x^3 -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{5x+3}\sqrt{3x+2}\sqrt{-2x+1}}{81000x^{10} + 240300x^9 + 210330x^8 - 41619x^7 - 160643x^6 - 58821x^5 + 28917x^4 + 22192x^3 + 936x^2}\right)$$

8 Test file number 15

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/15_1.1.1.4-a+b_x-
 $\hat{m}-c+d_x-\hat{n}-e+f_x-\hat{p}-g+h_x-\hat{q}$

8.1 Problem number 4

$$\int \frac{a + bx}{(c + dx)(e + fx)(g + hx)} dx$$

Optimal antiderivative

$$-\frac{(-ad + bc) \ln(dx + c)}{(-cf + de)(-ch + dg)} + \frac{(-af + be) \ln(fx + e)}{(-cf + de)(-eh + fg)} - \frac{(-ah + bg) \ln(hx + g)}{(-ch + dg)(-eh + fg)}$$

command

`integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{((bc - ad)fg - (bc - ad)he) \log(dx + c) + (adfg - acfh - (bdg - bch)e) \log(fx + e) - (bcfg - acfh - (bdg - adf)g^2 - c^2f^2gh + (d^2gh - cdh^2)e^2 - (d^2fg^2 - c^2fh^2)e}{(bc - ad)fg - (bc - ad)he}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

8.2 Problem number 33

$$\int \frac{A + Bx}{\sqrt{a + bx} \sqrt{c + \frac{b(-1 + c)x}{a}} \sqrt{e + \frac{b(-1 + e)x}{a}}} dx$$

Optimal antiderivative

$$-\frac{2a^{\frac{3}{2}} B \operatorname{EllipticE} \left(\frac{\sqrt{1-c} \sqrt{bx+a}}{\sqrt{a}}, \sqrt{\frac{1-e}{1-c}} \right)}{b^2 (1-e) \sqrt{1-c}} + \frac{2(aBe + A(-be + b)) \operatorname{EllipticF} \left(\frac{\sqrt{1-c} \sqrt{bx+a}}{\sqrt{a}}, \sqrt{\frac{1-e}{1-c}} \right) \sqrt{a}}{b^2 (1-e) \sqrt{1-c}}$$

command

`integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((Ba^3 - 3Aa^2b - (2Ba^3 - 3Aa^2b)c - (2Ba^3 - 3Aa^2b - 3(Ba^3 - Aa^2b)c)e) \sqrt{-\frac{b^3c - b^3 - (b^3c - b^3)e}{a^2}} \right) \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Ba^2x + Aa^2) \sqrt{bx+a} \sqrt{\frac{ac + (bc-b)x}{a}} \sqrt{\frac{ae + (be-b)x}{a}}}{a^3ce - (b^3c - b^3 - (b^3c - b^3)e)x^3 - (2ab^2c - ab^2 - (3ab^2c - 2ab^2)e)x^2 - (a^2bc - (3a^2bc - a^2b)e)x}, x \right)$$

8.3 Problem number 34

$$\int \frac{A + Bx}{\sqrt{a+bx} \sqrt{c+dx} \sqrt{e + \frac{b(-1+e)x}{a}}} dx$$

Optimal antiderivative

$$\frac{2(aBe + A(-be + b)) \text{EllipticF} \left(\frac{\sqrt{1-e} \sqrt{bx+a}}{\sqrt{a}}, \sqrt{-\frac{ad}{(-ad+bc)(1-e)}} \right) \sqrt{a} \sqrt{\frac{b(dx+c)}{-ad+bc}}}{b^2(1-e)^{\frac{3}{2}} \sqrt{dx+c}} + \frac{2aB \text{EllipticE} \left(\frac{\sqrt{d} \sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{-\frac{(-ad+bc)(1-e)}{ad}} \right) \sqrt{ad-bc} \sqrt{\frac{b(dx+c)}{-ad+bc}}}{b^2(1-e) \sqrt{d} \sqrt{dx+c}}$$

command

`integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((Babc + (Ba^2 - 3Aab)d - (Babc + (2Ba^2 - 3Aab)d)e) \sqrt{\frac{b^2de - b^2d}{a}} \text{weierstrassPInverse} \left(\frac{4(b^2c^2 - abcd + a^2d^2 + \dots)}{a} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bax + Aa) \sqrt{bx+a} \sqrt{dx+c} \sqrt{\frac{ae + (be-b)x}{a}}}{a^2ce + (b^2de - b^2d)x^3 - (b^2c + abd - (b^2c + 2abd)e)x^2 - (abc - (2abc + a^2d)e)x}, x \right)$$

8.4 Problem number 35

$$\int \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x} (7+5x)^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{522167393 \operatorname{EllipticF}\left(\frac{\sqrt{33} \sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{139968 \sqrt{-5+2x}} \\ & - \frac{6489123157 \operatorname{EllipticE}\left(\frac{2\sqrt{2-3x} \sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{699840 \sqrt{5-2x}} \\ & - \frac{1182926269 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{1603800} \\ & - \frac{12243139(7+5x) \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{356400} \\ & - \frac{17561(7+5x)^2 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{8910} \\ & - \frac{427(7+5x)^3 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{2970} \\ & + \frac{2(7+5x)^4 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{55} \end{aligned}$$

command

```
integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{641520} (14580000 x^4 + 70119000 x^3 + 91429200 x^2 - 106456131 x - 665014315) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((125 x^3 + 525 x^2 + 735 x + 343) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}, x\right)$$

8.5 Problem number 36

$$\int \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x} (7+5x)^2 dx$$

Optimal antiderivative

$$\frac{5592499 \operatorname{EllipticF}\left(\frac{\sqrt{33}\sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{23328\sqrt{-5+2x}} - \frac{17746949 \operatorname{EllipticE}\left(\frac{2\sqrt{2-3x}\sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{29160\sqrt{5-2x}} - \frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141(7+5x)\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2700} - \frac{61(7+5x)^2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{270} + \frac{2(7+5x)^3\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{45}$$

command

```
integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{19440} (108000x^3 + 343800x^2 + 34524x - 1380515) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((25x^2 + 70x + 49) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}, x\right)$$

8.6 Problem number 37

$$\int \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x} (7+5x) dx$$

Optimal antiderivative

$$\frac{5(-5+2x)^{\frac{3}{2}}(1+4x)^{\frac{3}{2}}\sqrt{2-3x}}{28} + \frac{72479 \operatorname{EllipticF}\left(\frac{\sqrt{33}\sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{4536\sqrt{-5+2x}} + \frac{136(1+4x)^{\frac{3}{2}}\sqrt{2-3x}\sqrt{-5+2x}}{105} - \frac{954811 \operatorname{EllipticE}\left(\frac{2\sqrt{2-3x}\sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{22680\sqrt{5-2x}} - \frac{20911\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3780}$$

command

`integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{1890} (2700x^2 + 3717x - 9695) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((5x+7)\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}, x\right)$$

8.7 Problem number 38

$$\int \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{121 \text{EllipticF}\left(\frac{\sqrt{33}\sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{108\sqrt{-5+2x}} + \frac{(1+4x)^{\frac{3}{2}} \sqrt{2-3x} \sqrt{-5+2x}}{10} \\ & - \frac{847 \text{EllipticE}\left(\frac{2\sqrt{2-3x}\sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{270\sqrt{5-2x}} \\ & - \frac{22\sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{45} \end{aligned}$$

command

`integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{90} (36x - 35) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}, x\right)$$

8.8 Problem number 44

$$\int \frac{\sqrt{2-3x} \sqrt{1+4x} (7+5x)^3}{\sqrt{-5+2x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2161804579 \operatorname{EllipticF}\left(\frac{\sqrt{33} \sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{326592 \sqrt{-5+2x}} \\ & + \frac{2629157597 \operatorname{EllipticE}\left(\frac{2\sqrt{2-3x} \sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{163296 \sqrt{5-2x}} \\ & + \frac{46134551 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{38880} \\ & + \frac{26291(7+5x) \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{540} \\ & + \frac{1679(7+5x)^2 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{756} \\ & + \frac{(7+5x)^3 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{9} \end{aligned}$$

command

```
integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{54432} (756000 x^3 + 6197400 x^2 + 26158104 x + 91137277) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(125 x^3 + 525 x^2 + 735 x + 343) \sqrt{4x+1} \sqrt{-3x+2}}{\sqrt{2x-5}}, x\right)$$

8.9 Problem number 45

$$\int \frac{\sqrt{2-3x} \sqrt{1+4x} (7+5x)^2}{\sqrt{-5+2x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1679161 \operatorname{EllipticF}\left(\frac{\sqrt{33}\sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{4536\sqrt{-5+2x}} \\ & + \frac{8198333 \operatorname{EllipticE}\left(\frac{2\sqrt{2-3x}\sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{9072\sqrt{5-2x}} \\ & + \frac{73207\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1080} + \frac{173(7+5x)\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{60} \\ & + \frac{(7+5x)^2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7} \end{aligned}$$

command

```
integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{1512} (5400x^2 + 36918x + 143591) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(25x^2 + 70x + 49)\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}, x\right)$$

8.10 Problem number 46

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4543 \operatorname{EllipticF}\left(\frac{\sqrt{33}\sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{216\sqrt{-5+2x}} + \frac{(1+4x)^{\frac{3}{2}} \sqrt{2-3x} \sqrt{-5+2x}}{4} \\ & + \frac{1397 \operatorname{EllipticE}\left(\frac{2\sqrt{2-3x}\sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{27\sqrt{5-2x}} \\ & + \frac{95\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{18} \end{aligned}$$

command

```
integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{36} (36x + 199) \sqrt{4x + 1} \sqrt{2x - 5} \sqrt{-3x + 2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(5x + 7) \sqrt{4x + 1} \sqrt{-3x + 2}}{\sqrt{2x - 5}}, x \right)$$

8.11 Problem number 47

$$\int \frac{\sqrt{2 - 3x} \sqrt{1 + 4x}}{\sqrt{-5 + 2x}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{11 \text{EllipticF} \left(\frac{\sqrt{33} \sqrt{1 + 4x}}{11}, \frac{\sqrt{3}}{3} \right) \sqrt{66} \sqrt{5 - 2x}}{9\sqrt{-5 + 2x}} \\ & + \frac{55 \text{EllipticE} \left(\frac{2\sqrt{2 - 3x} \sqrt{11}}{11}, \frac{i\sqrt{2}}{2} \right) \sqrt{11} \sqrt{-5 + 2x}}{18\sqrt{5 - 2x}} + \frac{\sqrt{2 - 3x} \sqrt{-5 + 2x} \sqrt{1 + 4x}}{3} \end{aligned}$$

command

`integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{4x + 1} \sqrt{2x - 5} \sqrt{-3x + 2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{4x + 1} \sqrt{-3x + 2}}{\sqrt{2x - 5}}, x \right)$$

8.12 Problem number 51

$$\int \frac{\sqrt{2-3x} (7+5x)^3}{\sqrt{-5+2x} \sqrt{1+4x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{25260049 \operatorname{EllipticF}\left(\frac{\sqrt{33} \sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{36288 \sqrt{-5+2x}} \\ & + \frac{15629623 \operatorname{EllipticE}\left(\frac{2\sqrt{2-3x} \sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{9072 \sqrt{5-2x}} \\ & + \frac{110743 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{864} + \frac{121(7+5x) \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{24} \\ & + \frac{5(7+5x)^2 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{28} \end{aligned}$$

command

`integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5}{6048} (5400x^2 + 45612x + 208313) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(125x^3 + 525x^2 + 735x + 343) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}}{8x^2 - 18x - 5}, x\right)$$

8.13 Problem number 52

$$\int \frac{\sqrt{2-3x} (7+5x)^2}{\sqrt{-5+2x} \sqrt{1+4x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{17533 \operatorname{EllipticF}\left(\frac{\sqrt{33} \sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{432 \sqrt{-5+2x}} \\ & + \frac{44569 \operatorname{EllipticE}\left(\frac{2\sqrt{2-3x} \sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{432 \sqrt{5-2x}} \\ & + \frac{68 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{9} + \frac{(7+5x) \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{4} \end{aligned}$$

command

```
integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5}{36} (9x + 67) \sqrt{4x + 1} \sqrt{2x - 5} \sqrt{-3x + 2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(25x^2 + 70x + 49)\sqrt{4x + 1}\sqrt{2x - 5}\sqrt{-3x + 2}}{8x^2 - 18x - 5}, x\right)$$

8.14 Problem number 53

$$\int \frac{\sqrt{2 - 3x} (7 + 5x)}{\sqrt{-5 + 2x} \sqrt{1 + 4x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{179 \operatorname{EllipticF}\left(\frac{\sqrt{33}\sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{72\sqrt{-5+2x}} \\ & + \frac{241 \operatorname{EllipticE}\left(\frac{2\sqrt{2-3x}\sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{36\sqrt{5-2x}} + \frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{12} \end{aligned}$$

command

```
integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5}{12} \sqrt{4x + 1} \sqrt{2x - 5} \sqrt{-3x + 2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(5x + 7)\sqrt{4x + 1}\sqrt{2x - 5}\sqrt{-3x + 2}}{8x^2 - 18x - 5}, x\right)$$

8.15 Problem number 60

$$\int \frac{(7+5x)^4}{\sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}} dx$$

Optimal antiderivative

$$\frac{392989907 \operatorname{EllipticF}\left(\frac{\sqrt{33} \sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{133056 \sqrt{-5+2x}} - \frac{5109835 \operatorname{EllipticE}\left(\frac{2\sqrt{2-3x} \sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{756 \sqrt{5-2x}} - \frac{120355 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{288} - \frac{305(7+5x) \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{24} - \frac{25(7+5x)^2 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{84}$$

command

`integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{25}{2016} (600x^2 + 6804x + 42049) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(625x^4 + 3500x^3 + 7350x^2 + 6860x + 2401) \sqrt{4x+1} \sqrt{2x-5} \sqrt{-3x+2}}{24x^3 - 70x^2 + 21x + 10}, x\right)$$

8.16 Problem number 61

$$\int \frac{(7+5x)^3}{\sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}} dx$$

Optimal antiderivative

$$\frac{2474201 \operatorname{EllipticF}\left(\frac{\sqrt{33} \sqrt{1+4x}}{11}, \frac{\sqrt{3}}{3}\right) \sqrt{66} \sqrt{5-2x}}{14256 \sqrt{-5+2x}} - \frac{487585 \operatorname{EllipticE}\left(\frac{2\sqrt{2-3x} \sqrt{11}}{11}, \frac{i\sqrt{2}}{2}\right) \sqrt{11} \sqrt{-5+2x}}{1296 \sqrt{5-2x}} - \frac{2135 \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{108} - \frac{5(7+5x) \sqrt{2-3x} \sqrt{-5+2x} \sqrt{1+4x}}{12}$$

command

```
integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{25}{108} (9x + 98) \sqrt{4x + 1} \sqrt{2x - 5} \sqrt{-3x + 2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{(125x^3 + 525x^2 + 735x + 343) \sqrt{4x + 1} \sqrt{2x - 5} \sqrt{-3x + 2}}{24x^3 - 70x^2 + 21x + 10}, x \right)$$

8.17 Problem number 62

$$\int \frac{(7 + 5x)^2}{\sqrt{2 - 3x} \sqrt{-5 + 2x} \sqrt{1 + 4x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{24353 \text{EllipticF} \left(\frac{\sqrt{33} \sqrt{1 + 4x}}{11}, \frac{\sqrt{3}}{3} \right) \sqrt{66} \sqrt{5 - 2x}}{2376 \sqrt{-5 + 2x}} \\ & - \frac{2135 \text{EllipticE} \left(\frac{2\sqrt{2 - 3x} \sqrt{11}}{11}, \frac{i\sqrt{2}}{2} \right) \sqrt{11} \sqrt{-5 + 2x}}{108 \sqrt{5 - 2x}} \\ & - \frac{25 \sqrt{2 - 3x} \sqrt{-5 + 2x} \sqrt{1 + 4x}}{36} \end{aligned}$$

command

```
integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{25}{36} \sqrt{4x + 1} \sqrt{2x - 5} \sqrt{-3x + 2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{(25x^2 + 70x + 49) \sqrt{4x + 1} \sqrt{2x - 5} \sqrt{-3x + 2}}{24x^3 - 70x^2 + 21x + 10}, x \right)$$

8.18 Problem number 68

$$\int \frac{ci + dix}{\sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal antiderivative

$$\frac{2i \operatorname{EllipticE} \left(\frac{\sqrt{h} \sqrt{fx + e}}{\sqrt{eh - fg}}, \sqrt{-\frac{d(-eh + fg)}{(-cf + de)h}} \right) \sqrt{eh - fg} \sqrt{dx + c} \sqrt{\frac{f(hx + g)}{-eh + fg}}}{f \sqrt{h} \sqrt{-\frac{f(dx + c)}{-cf + de}} \sqrt{hx + g}}$$

command

`integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3i \sqrt{dfh} \operatorname{dfrhweierstrassZeta} \left(\frac{4(d^2 f^2 g^2 - cdf^2 gh + c^2 f^2 h^2 + d^2 h^2 e^2 - (d^2 fgh + cdfh^2)e)}{3d^2 f^2 h^2}, -\frac{4(2d^3 f^3 g^3 - 3cd^2 f^3 g^2 h - 3c^2 df^3 gh^2 + 2c^3 f^3 h^3)}{3d^2 f^2 h^2} \right) \right)}{f \sqrt{h} \sqrt{-\frac{f(dx + c)}{-cf + de}} \sqrt{hx + g}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g} i}{fhx^2 + eg + (fg + eh)x}, x \right)$$

8.19 Problem number 69

$$\int \frac{a + bx}{\sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal antiderivative

$$\frac{2b \operatorname{EllipticE} \left(\frac{\sqrt{f} \sqrt{dx + c}}{\sqrt{cf - de}}, \sqrt{\frac{(-cf + de)h}{f(-ch + dg)}} \right) \sqrt{cf - de} \sqrt{\frac{d(fx + e)}{-cf + de}} \sqrt{hx + g}}{dh \sqrt{f} \sqrt{fx + e} \sqrt{\frac{d(hx + g)}{-ch + dg}}}$$

$$\frac{2(-ah + bg) \operatorname{EllipticF} \left(\frac{\sqrt{f} \sqrt{dx + c}}{\sqrt{cf - de}}, \sqrt{\frac{(-cf + de)h}{f(-ch + dg)}} \right) \sqrt{cf - de} \sqrt{\frac{d(fx + e)}{-cf + de}} \sqrt{\frac{d(hx + g)}{-ch + dg}}}{dh \sqrt{f} \sqrt{fx + e} \sqrt{hx + g}}$$

command

```
integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{dfh} \operatorname{bd}fh\operatorname{weierstrassZeta} \left(\frac{4(d^2 f^2 g^2 - cdf^2 gh + c^2 f^2 h^2 + d^2 h^2 e^2 - (d^2 fgh + cdfh^2)e)}{3d^2 f^2 h^2}, -\frac{4(2d^3 f^3 g^3 - 3cd^2 f^3 g^2 h - 3c^2 df^3 gh^2 + 2c^3 d^2 f^2 h^2)}{3d^2 f^2 h^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{dfhx^3 + ceg + (dfg + (de+cf)h)x^2 + (ceh + (de+cf)g)x}, x \right)$$

9 Test file number 17

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/17_1.1.1.6_P-x-a+b_x-^m-c+d_x-ⁿ-e+f_x-^p

9.1 Problem number 33

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3} dx$$

Optimal antiderivative

$$\frac{f \left(A + \frac{e(-Bf+Ce)}{f^2} \right) (-b^2 x^2 + a^2)}{2(-a^2 f^2 + b^2 e^2)(fx+e)^2 \sqrt{bx+a}\sqrt{-bcx+ac}} + \frac{(2a^2 f^2(-Bf+2Ce) - b^2 e(Ce^2 + f(-3Af+Be))) (-b^2 x^2 + a^2)}{2f(-a^2 f^2 + b^2 e^2)^2 (fx+e)\sqrt{bx+a}\sqrt{-bcx+ac}} + \frac{(A(a^2 b^2 f^2 + 2b^4 e^2) + a^2(2a^2 C f^2 + b^2 e(-3Bf+Ce))) \arctan \left(\frac{(b^2 ex+a^2 f)\sqrt{c}}{\sqrt{-a^2 f^2 + b^2 e^2} \sqrt{-b^2 c x^2 + a^2 c}} \right) \sqrt{-b^2 c x^2}}{2(-a^2 f^2 + b^2 e^2)^{\frac{5}{2}} \sqrt{c} \sqrt{bx+a}\sqrt{-bcx+ac}}$$

command

```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{((2Ca^4 + Aa^2b^2)f^4x^2 + (Ca^2b^2 + 2Ab^4)e^4 - (3Ba^2b^2f - 2(Ca^2b^2 + 2Ab^4)fx)e^3 - (6Ba^2b^2f^2x - (Ca^2b^2 + 2Ab^4)e^2)ef - (3Aa^2b^2f^3 + (Ca^2b^2 + 2Ab^4)e^2)ef^2 - (3Aa^2b^2f^3 + (Ca^2b^2 + 2Ab^4)e^2)ef^2 - (3Aa^2b^2f^3 + (Ca^2b^2 + 2Ab^4)e^2)ef^2 - (3Aa^2b^2f^3 + (Ca^2b^2 + 2Ab^4)e^2)ef^2)}{2(-a^2 f^2 + b^2 e^2)^{\frac{5}{2}} \sqrt{c} \sqrt{bx+a}\sqrt{-bcx+ac}} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

9.2 Problem number 59

$$\int \frac{A + Bx + Cx^2}{(a + bx)^3 \sqrt{c + dx} \sqrt{e + fx}} dx$$

Optimal antiderivative

$$\frac{(b^2(3Ad^2e^2 - 2cde(-Af + 2Be)) + c^2(3Af^2 - 4Bef + 8Ce^2)) + ab(d^2e(-8Af + Be) - c^2f(-Bf + 8Ce) - 2c^2d^2e^2) + 4(-ad + bc)^2(-af + be)(bx + a)^2}{(Ab^2 - a(bB - aC)) \sqrt{dx + c} \sqrt{fx + e} + \frac{(2a^3Cdf + ab^2(-6Adf + Bcf + Bde + 8cCe) - b^3(4Bce - 3A(cf + de)) + a^2b(2Bdf - 5C(cf + de))) \sqrt{dx + c}}{4b(-ad + bc)^2(-af + be)^2(bx + a)}}$$

command

```
integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

9.3 Problem number 61

$$\int \sqrt{a + bx} \sqrt{c + dx} \sqrt{e + fx} (A + Bx + Cx^2) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2C(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}(fx+e)^{\frac{3}{2}}}{9bdf} \\
& - \frac{2(2aCdf - b(3Bdf - 2C(cf+de)))(dx+c)^{\frac{3}{2}}(fx+e)^{\frac{3}{2}}\sqrt{bx+a}}{21bd^2f^2} \\
& - \frac{2(7bdf(-3Abdf + acCf + aCde + bcCe) + (adf - 4b(cf+de))(2aCdf - b(3Bdf - 2C(cf+de))))(fx+e)^{\frac{3}{2}}\sqrt{bx+a}}{105b^2d^2f^3} \\
& + \frac{2(8a^3Cd^3f^3 + 3a^2bd^2f^2(-4Bdf - cCf + Cde) - 3ab^2df^2((-7Ad^2 + c^2C)f + Bd(-2cf + de)) - b^3(C(-8c^3f^2 + 3c^2d^2f + 3cd^2e - 3d^3e^2)))}{315b^3} \\
& - \frac{2(16a^4Cd^4f^4 - 8a^3bd^3f^3(3Bdf + cCf + Cde) + 3a^2b^2d^2f^2(df(14Adf + 5Bcf + 5Bde) - 2C(c^2f^2 - cdef + d^2e^2)))}{315b^3} \\
& - \frac{2(-af + be)(-cf + de)(8a^3Cd^3f^3 + 3a^2bd^2f^2(-4Bdf - cCf + Cde) - 3ab^2df^2((-7Ad^2 + c^2C)f + Bd(-2cf + de)))}{315b^3}
\end{aligned}$$

command

```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}{\sqrt{a+bx}}, x\right)$$

9.4 Problem number 62

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}} dx$$

Optimal antiderivative

$$\frac{2C(dx+c)^{\frac{3}{2}}(fx+e)^{\frac{3}{2}}\sqrt{bx+a}}{7bdf}$$

$$-\frac{2(6aCdf-b(7Bdf-4C(cf+de)))(fx+e)^{\frac{3}{2}}\sqrt{bx+a}\sqrt{dx+c}}{35b^2df^2}$$

$$-\frac{2(5bdf(3aC(cf+de)+b(-7Adf+cCe))-(4adf-bcf+2bde)(6aCdf-b(7Bdf-4C(cf+de))))\sqrt{bx+a}\sqrt{dx+c}}{105b^3d^2f^2}$$

$$2(3bdf(5bcf(3aC(cf+de)+b(-7Adf+cCe))-(3acf+ade+bce)(6aCdf-b(7Bdf-4C(cf+de))))+2\left(\frac{bd}{2}\right. \\ \left.-\frac{2(-af+be)(-cf+de)(24a^2Cd^2f^2+abdf(-28Bdf-5Ccf+13Cde))-b^2(7df(-5Adf-Bcf+2Bde))-C(-}{105b^4d^{\frac{5}{2}}f^3\sqrt{dx-c}}\right)$$

command

```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(3\left(15Cb^4d^4f^4x^2-4Cb^4d^4f^2e^2+3\left(Cb^4cd^3-(6Cab^3-7Bb^4)d^4\right)f^4x-(4Cb^4c^2d^2+(5Cab^3-7Bb^4)cd^3-\right.\right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{\sqrt{bx+a}},x\right)$$

9.5 Problem number 63

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(Ab^2 - a(bB - aC))(dx + c)^{\frac{3}{2}}(fx + e)^{\frac{3}{2}}}{b(-ad + bc)(-af + be)\sqrt{bx + a}} \\
& + \frac{2(6a^2Cdf + b^2(5Adf + cCe) - ab(5Bdf + cCf + Cde))(fx + e)^{\frac{3}{2}}\sqrt{bx + a}\sqrt{dx + c}}{5b^2(-ad + bc)f(-af + be)} \\
& + \frac{2(24a^2Cd f^2 - abf(20Bdf + cCf + 7Cde) + b^2(5df(3Af + Be) - Ce(-cf + 2de)))\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{15b^3df(-af + be)} \\
& + \frac{2(48a^2C d^2 f^2 - 8abdf(5Bdf + cCf + Cde) + b^2(5df(6Adf + Bcf + Bde) - 2C(c^2 f^2 - cdef + d^2 e^2))) \text{EllipticE}}{15b^4 d^{\frac{3}{2}} f^2 \sqrt{dx + c} \sqrt{\frac{b(fx + e)}{-af + be}}} \\
& - \frac{2(-cf + de)(24a^2Cd f^2 - abf(20Bdf + cCf + 7Cde) + b^2(5df(3Af + Be) - Ce(-cf + 2de))) \text{EllipticF}\left(\frac{\sqrt{d}}{\sqrt{a}}\right)}{15b^4 d^{\frac{3}{2}} f^2 \sqrt{dx + c} \sqrt{fx + e}}
\end{aligned}$$

command

```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3\left(3Cb^4d^3f^3x^2 + (Cb^4cd^2 - (6Cab^3 - 5Bb^4)d^3\right)f^3x + (Cab^3cd^2 - (24Ca^2b^2 - 20Bab^3 + 15Ab^4)d^3\right)f^3 + (C\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^2x^2 + 2abx + a^2}, x\right)$$

9.6 Problem number 64

$$\int \frac{\sqrt{c + dx}\sqrt{e + fx}(A + Bx + Cx^2)}{(a + bx)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(Ab^2 - a(bB - aC))(dx + c)^{\frac{3}{2}}(fx + e)^{\frac{3}{2}}}{3b(-ad + bc)(-af + be)(bx + a)^{\frac{3}{2}}} - \frac{2(bB - 2aC)(fx + e)^{\frac{3}{2}}\sqrt{dx + c}}{b^2(-af + be)\sqrt{bx + a}} \\
& + \frac{2(8a^2Cdf + b^2(Adf + 3Bcf + cCe) - ab(4Bdf + 7cCf + Cde))\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{3b^3(-ad + bc)(-af + be)} \\
& + \frac{2(16a^3C d^2 f^2 - 8a^2 bdf(Bdf + 2C(cf + de)) - b^3(c^2 Cef + A d^2 ef + cd(A f^2 + 6Bef + C e^2)) + a b^2(df(2Adf + \\
& \hspace{20em} 3b^4 f(-af + be)\sqrt{d}\sqrt{ad} \\
& 2(-cf + de)(8a^2 Cdf + b^2(Adf + 3Bcf + cCe) - ab(4Bdf + 7cCf + Cde))\text{EllipticF}\left(\frac{\sqrt{d}\sqrt{bx + a}}{\sqrt{ad - bc}}, \sqrt{\frac{-ad +}{d(-af}}\right)}{3b^4 f\sqrt{d}\sqrt{ad - bc}\sqrt{dx + c}\sqrt{fx + e}}
\end{aligned}$$

command

```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3}, x\right)$$

9.7 Problem number 65

$$\int \frac{\sqrt{c + dx}\sqrt{e + fx}(A + Bx + Cx^2)}{(a + bx)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{2(Ab^2 - a(bB - aC))(dx + c)^{\frac{3}{2}}(fx + e)^{\frac{3}{2}}}{5b(-ad + bc)(-af + be)(bx + a)^{\frac{5}{2}}} \\
 & + \frac{2(6a^3Cdf + ab^2(-4Adf + 3Bcf + 3Bde + 10cCe) - b^3(5Bce - 2A(cf + de)) - a^2b(Bdf + 8C(cf + de)))(fx + e)}{15b^2(-ad + bc)(-af + be)^2(bx + a)^{\frac{3}{2}}} \\
 & + \frac{2(24a^3C d^2 f - a^2bd(4Bdf + 41cCf + 23Cde) - b^3(15c^2Ce - 2A d^2e + cd(Af + 5Be)) + ab^2(15c^2Cf + d^2(-Af + e))}{15b^3(-ad + bc)^2(-af + be)\sqrt{bx + a}} \\
 & + \frac{2(48a^4C d^2 f^2 - 8a^3bdf(Bdf + 11C(cf + de)) - b^4(2A d^2e^2 - cde(2Af + 5Be) - c^2(-2A f^2 + 5Bef + 30C e^2))}{15b^4(ad - bc)^{\frac{3}{2}}(-af + be)}
 \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^4x^4 + 4ab^3x^3 + 6a^2b^2x^2 + 4a^3bx + a^4}, x\right)$$

9.8 Problem number 66

$$\int \frac{\sqrt{c + dx}\sqrt{e + fx}(A + Bx + Cx^2)}{(a + bx)^{9/2}} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^5x^5 + 5ab^4x^4 + 10a^2b^3x^3 + 10a^3b^2x^2 + 5a^4bx + a^5}, x\right)$$

9.9 Problem number 67

$$\int \frac{(a + bx)^{3/2}\sqrt{c + dx}(A + Bx + Cx^2)}{\sqrt{e + fx}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(4aCdf + b(-9Bdf + 6cCf + 8Cde))(bx + a)^{\frac{3}{2}}(dx + c)^{\frac{3}{2}}\sqrt{fx + e}}{63bd^2f^2} \\ + & \frac{2C(bx + a)^{\frac{5}{2}}(dx + c)^{\frac{3}{2}}\sqrt{fx + e}}{9bdf} \\ & \frac{2(7bdf(-9Abdf + acCf + 3aCde + 5bcCe) - (-3adf + 4bcf + 6bde)(4aCdf + b(-9Bdf + 6cCf + 8Cde)))(dx + c)^{\frac{3}{2}}\sqrt{fx + e}}{315bd^3f^3} \\ & \frac{2(5bdf(7adf(-9Abdf + acCf + 3aCde + 5bcCe) - (acf + 3ade + 3bce)(4aCdf + b(-9Bdf + 6cCf + 8Cde))) - (-3adf + 4bcf + 6bde)(4aCdf + b(-9Bdf + 6cCf + 8Cde)))(dx + c)^{\frac{3}{2}}\sqrt{fx + e}}{315bd^3f^3} \\ + & \frac{2(8a^4C d^4 f^4 + a^3 b d^3 f^3(-18Bdf - 7cCf + 11Cde) - 3a^2 b^2 d^2 f^2(3df(-7Adf - 3Bcf + 4Bde) - C(-3c^2 f^2 - 5cde)) - 3a b^2 d f^2(3df(-7Adf - 3Bcf + 4Bde) - C(-3c^2 f^2 - 5cde)))(dx + c)^{\frac{3}{2}}\sqrt{fx + e}}{315bd^3f^3} \\ + & \frac{2(-af + be)(-cf + de)(4a^3 C d^3 f^3 + 3a^2 b d^2 f^2(-3Bdf - cCf + 3Cde) - 3a b^2 df(3df(-21Adf + 3Bcf + 16Bde) - C(-3c^2 f^2 - 5cde)))(dx + c)^{\frac{3}{2}}\sqrt{fx + e}}{315bd^3f^3} \end{aligned}$$

command

```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cb^3x^3 + (Ca + Bb)x^2 + Aa + (Ba + Ab)x)\sqrt{bx + a}\sqrt{dx + c}}{\sqrt{fx + e}}, x\right)$$

9.10 Problem number 68

$$\int \frac{\sqrt{a+bx} \sqrt{c+dx} (A+Bx+Cx^2)}{\sqrt{e+fx}} dx$$

Optimal antiderivative

$$\frac{2C(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\sqrt{fx+e}}{7bdf}$$

$$- \frac{2(4aCdf + b(-7Bdf + 4Cf + 6Cde))(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\sqrt{fx+e}}{35bd^2f^2}$$

$$- \frac{2(5bdf(-7Abdf + acCf + 3aCde + 3bcCe) + (adf - 2b(cf + 2de))(4aCdf + b(-7Bdf + 4Cf + 6Cde)))\sqrt{bx+a}}{105b^2d^2f^3}$$

$$- \frac{2(3bdf(5adf(-7Abdf + acCf + 3aCde + 3bcCe) - (acf + 3ade + bce)(4aCdf + b(-7Bdf + 4Cf + 6Cde))) +$$

$$+ \frac{2(-af + be)(-cf + de)(4a^2C d^2 f^2 + abdf(-7Bdf - 2cCf + 8Cde) - b^2(7df(-10Adf + Bcf + 8Bde) - 4C(c^2 + dx + e)))}{105b^3d^{\frac{5}{2}}f^4\sqrt{dx+e}}$$

command

```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(15Cb^4d^4f^4x^2 + 24Cb^4d^4f^2e^2 + 3(Cb^4cd^3 + (Cab^3 + 7Bb^4)d^4)f^4x - (4Cb^4c^2d^2 - (2Cab^3 + 7Bb^4)cd^3 +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}}, x\right)$$

9.11 Problem number 69

$$\int \frac{\sqrt{c+dx} (A+Bx+Cx^2)}{\sqrt{a+bx} \sqrt{e+fx}} dx$$

Optimal antiderivative

$$\frac{2C(dx+c)^{\frac{3}{2}} \sqrt{bx+a} \sqrt{fx+e}}{5bdf}$$

$$- \frac{2(4aCdf + b(-5Bdf + 2cCf + 4Cde)) \sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e}}{15b^2d f^2}$$

$$2(3bdf(-5Abdf + acCf + 3aCde + bcCe) - (2adf - bcf + 2bde) (4aCdf + b(-5Bdf + 2cCf + 4Cde))) \text{EllipticF}$$

$$\frac{15b^3d^{\frac{3}{2}}f^3\sqrt{dx+c} \sqrt{\frac{b(fx+e)}{-af+be}}}{15b^3d^{\frac{3}{2}}f^3\sqrt{dx+c} \sqrt{fx+e}}$$

$$2(-cf + de) (4a^2Cdf^2 + abf(-5Bdf - cCf + 3Cde) - b^2(5df(-3Af + 2Be) - Ce(cf + 8de))) \text{EllipticF} \left(\frac{\sqrt{d}}{\sqrt{e}} \right)$$

$$\frac{15b^3d^{\frac{3}{2}}f^3\sqrt{dx+c} \sqrt{fx+e}}{15b^3d^{\frac{3}{2}}f^3\sqrt{dx+c} \sqrt{fx+e}}$$

command

```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(3Cb^3d^3f^3x - 4Cb^3d^3f^2e + (Cb^3cd^2 - (4Cab^2 - 5Bb^3)d^3)f^3) \sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e} - (8Cb^3d^3e^3 - \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cx^2 + Bx + A) \sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e}}{bfx^2 + ae + (be + af)x}, x \right)$$

9.12 Problem number 70

$$\int \frac{\sqrt{c+dx} (A+Bx+Cx^2)}{(a+bx)^{3/2} \sqrt{e+fx}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Ab^2 - a(bB - aC))(dx + c)^{\frac{3}{2}} \sqrt{fx + e}}{b(-ad + bc)(-af + be)\sqrt{bx + a}} \\
& + \frac{2(4a^2Cdf + b^2(3Adf + cCe) - ab(3Bdf + cCf + Cde))\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{3b^2(-ad + bc)f(-af + be)} \\
& + \frac{2(8a^2Cd f^2 - abf(6Bdf + cCf + 3Cde) + b^2(3df(Af + Be) - Ce(-cf + 2de))) \operatorname{EllipticE}\left(\frac{\sqrt{d}\sqrt{bx + a}}{\sqrt{ad - bc}}, \sqrt{\frac{(-ad + bc)f}{d(-af + be)}}\right)}{3b^3 f^2 (-af + be) \sqrt{d} \sqrt{dx + c} \sqrt{\frac{b(fx + e)}{-af + be}}} \\
& + \frac{2(-cf + de)(-3Bbf + 4Caf + 2Cbe) \operatorname{EllipticF}\left(\frac{\sqrt{d}\sqrt{bx + a}}{\sqrt{ad - bc}}, \sqrt{\frac{(-ad + bc)f}{d(-af + be)}}\right) \sqrt{ad - bc} \sqrt{\frac{b(dx + c)}{-ad + bc}}}{3b^3 f^2 \sqrt{d} \sqrt{dx + c} \sqrt{fx + e}}
\end{aligned}$$

command

```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3(Cab^3d^2f^3x + (4Ca^2b^2 - 3Bab^3 + 3Ab^4)d^2f^3 - (Cb^4d^2f^2x + Cab^3d^2f^2)e)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e} - \dots\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^2fx^3 + a^2e + (b^2e + 2abf)x^2 + (2abe + a^2f)x}, x\right)$$

9.13 Problem number 71

$$\int \frac{\sqrt{c + dx}(A + Bx + Cx^2)}{(a + bx)^{5/2}\sqrt{e + fx}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Ab^2 - a(bB - aC))(dx + c)^{\frac{3}{2}} \sqrt{fx + e}}{3b(-ad + bc)(-af + be)(bx + a)^{\frac{3}{2}}} \\
& - \frac{2(4a^2Cf + b^2(-2Af + 3Be) - ab(Bf + 6Ce)) \sqrt{dx + c} \sqrt{fx + e}}{3b^2(-af + be)^2 \sqrt{bx + a}} \\
& + \frac{2(8a^3Cd f^2 - a^2bf(2Bdf + 7Cf + 13Cde) + a b^2(3Ce(4cf + de) + f(-Adf + Bcf + 4Bde)) - b^3(Adef + c(-2 \\
& \hspace{20em} 3b^3 f(-af + be)^2 \sqrt{ad - bc} \sqrt{da \\
& + \frac{2(-cf + de)(4a^2Cdf + b^2(Adf + 3cCe) - ab(Bdf + 3C(cf + de))) \operatorname{EllipticF}\left(\frac{\sqrt{d} \sqrt{bx + a}}{\sqrt{ad - bc}}, \sqrt{\frac{(-ad + bc) f}{d(-af + be)}}\right)}{3b^3 f(-af + be) \sqrt{d} \sqrt{ad - bc} \sqrt{dx + c} \sqrt{fx + e}}
\end{aligned}$$

command

```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx + a} \sqrt{dx + c} \sqrt{fx + e}}{b^3fx^4 + a^3e + (b^3e + 3ab^2f)x^3 + 3(ab^2e + a^2bf)x^2 + (3a^2be + a^3f)x}, x\right)$$

9.14 Problem number 72

$$\int \frac{\sqrt{c + dx} (A + Bx + Cx^2)}{(a + bx)^{7/2} \sqrt{e + fx}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Ab^2 - a(bB - aC))(dx + c)^{\frac{3}{2}} \sqrt{fx + e}}{5b(-ad + bc)(-af + be)(bx + a)^{\frac{5}{2}}} \\
& + \frac{2(4a^3Cdf - b^3(-4Acf - 2Ade + 5Bce) + ab^2(-6Adf + Bcf + 3Bde + 10cCe) - a^2b(-Bdf + 6cCf + 8Cde)) \sqrt{fx + e}}{15b^2(-ad + bc)(-af + be)^2(bx + a)^{\frac{3}{2}}} \\
& - \frac{2(8a^4C d^2 f^2 - a^3 bdf(-2Bdf + 13cCf + 23Cde) - b^4(2A d^2 e^2 - cde(-3Af + 5Be) - c^2(8A f^2 - 10Bef + 15Cde))) \sqrt{fx + e}}{15b^3(ad - bc)^{\frac{3}{2}}(-af + be)^2 \sqrt{dx + c}} \\
& + \frac{2(8a^4C d^2 f^2 - a^3 bdf(-2Bdf + 13cCf + 23Cde) - b^4(2A d^2 e^2 - cde(-3Af + 5Be) - c^2(8A f^2 - 10Bef + 15Cde))) \sqrt{fx + e}}{15b^3(ad - bc)^{\frac{3}{2}}(-af + be)^2 \sqrt{dx + c}} \\
& + \frac{2(-cf + de)(4a^3Cdf - b^3(-4Acf - 2Ade + 5Bce) + ab^2(-6Adf + Bcf + 3Bde + 10cCe) - a^2b(-Bdf + 6cCf + 8Cde)) \sqrt{fx + e}}{15b^3(ad - bc)^{\frac{3}{2}}(-af + be)^2 \sqrt{dx + c}}
\end{aligned}$$

command

```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cx^2 + Bx + A) \sqrt{bx + a} \sqrt{dx + c} \sqrt{fx + e}}{b^4fx^5 + a^4e + (b^4e + 4ab^3f)x^4 + 2(2ab^3e + 3a^2b^2f)x^3 + 2(3a^2b^2e + 2a^3bf)x^2 + (4a^3be + a^4f)x}, x \right)$$

9.15 Problem number 73

$$\int \frac{(a + bx)^{3/2} (A + Bx + Cx^2)}{\sqrt{c + dx} \sqrt{e + fx}} dx$$

Optimal antiderivative

$$\frac{2(2aCdf - b(7Bdf - 6C(cf + de))) (bx + a)^{\frac{3}{2}} \sqrt{dx + c} \sqrt{fx + e}}{35b d^2 f^2}$$

$$+ \frac{2C(bx + a)^{\frac{5}{2}} \sqrt{dx + c} \sqrt{fx + e}}{7bdf}$$

$$\frac{2(5bdf(-7Abdf + acCf + aCde + 5bcCe) + (3adf - 4b(cf + de))(2aCdf - b(7Bdf - 6C(cf + de)))) \sqrt{bx + a}}{105b d^3 f^3}$$

$$2 \left(3bdf(5adf(-7Abdf + acCf + aCde + 5bcCe) - (acf + ade + 3bce)(2aCdf - b(7Bdf - 6C(cf + de)))) + 2 \left(\frac{a}{b} \right) \right)$$

$$2(-af + be) (3a^2 C d^2 f^2 (-cf + de) - 3abdf(7df(-5Adf + 2Bcf + 3Bde) - C(11c^2 f^2 + 8cdef + 16d^2 e^2)) - b^2)$$

command

```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(15Cb^4d^4f^4x^2 + 24Cb^4d^4f^2e^2 - 3(6Cb^4cd^3 - (8Cab^3 + 7Bb^4)d^4)f^4x + (24Cb^4c^2d^2 - (33Cab^3 + 28Bb^4) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cb^3x^3 + (Ca + Bb)x^2 + Aa + (Ba + Ab)x) \sqrt{bx + a} \sqrt{dx + c} \sqrt{fx + e}}{dfx^2 + ce + (de + cf)x}, x \right)$$

9.16 Problem number 74

$$\int \frac{\sqrt{a + bx} (A + Bx + Cx^2)}{\sqrt{c + dx} \sqrt{e + fx}} dx$$

Optimal antiderivative

$$\frac{2C(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}}{5bdf} - \frac{2(2aCdf - b(5Bdf - 4C(cf+de)))\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}{15bd^2f^2}$$

$$- \frac{2(3bdf(-5Abdf + acCf + aCde + 3bcCe) + (adf - 2b(cf+de))(2aCdf - b(5Bdf - 4C(cf+de))))\text{EllipticE}\left(\frac{\sqrt{d}\sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{\frac{(-ad+bc)f}{d(-af+be)}}\right)\sqrt{ad-bc}\sqrt{\frac{b(dx+c)}{-ad+bc}}\sqrt{fx+e}}{15b^2d^{\frac{5}{2}}f^3\sqrt{dx+c}\sqrt{fx+e}}$$

$$+ \frac{2(-af+be)(aCdf(-cf+de) - b(5df(-3Adf + Bcf + 2Bde) - C(4c^2f^2 + 3cdef + 8d^2e^2)))\text{EllipticF}\left(\frac{\sqrt{d}\sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{\frac{(-ad+bc)f}{d(-af+be)}}\right)\sqrt{ad-bc}}{15b^2d^{\frac{5}{2}}f^3\sqrt{dx+c}\sqrt{fx+e}}$$

command

`integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(3(3Cb^3d^3f^3x - 4Cb^3d^3f^2e - (4Cb^3cd^2 - (Cab^2 + 5Bb^3)d^3)f^3)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e} - (8Cb^3d^3e^3 + \dots)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}{dfx^2 + ce + (de + cf)x}, x\right)$$

9.17 Problem number 75

$$\int \frac{A + Bx + Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}} dx$$

Optimal antiderivative

$$\frac{2C\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}{3bdf} - \frac{2(2aCdf - b(3Bdf - 2C(cf+de)))\text{EllipticE}\left(\frac{\sqrt{d}\sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{\frac{(-ad+bc)f}{d(-af+be)}}\right)\sqrt{ad-bc}\sqrt{\frac{b(dx+c)}{-ad+bc}}\sqrt{fx+e}}{3b^2d^{\frac{3}{2}}f^2\sqrt{dx+c}\sqrt{\frac{b(fx+e)}{-af+be}}}$$

$$+ \frac{2(aCf(-cf+de) - b(3df(-Af + Be) - Ce(cf + 2de)))\text{EllipticF}\left(\frac{\sqrt{d}\sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{\frac{(-ad+bc)f}{d(-af+be)}}\right)\sqrt{ad-bc}}{3b^2d^{\frac{3}{2}}f^2\sqrt{dx+c}\sqrt{fx+e}}$$

command

```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e} Cb^2d^2f^2 + (2Cb^2d^2e^2 + (2Cb^2c^2 + (Cab - 3Bb^2)cd + (2Ca^2 - 3Bab + 9Ab^2)) \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cx^2 + Bx + A) \sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e}}{bdfx^3 + ace + (bde + (bc + ad)f)x^2 + (acf + (bc + ad)e)x}, x \right)$$

9.18 Problem number 76

$$\int \frac{A + Bx + Cx^2}{(a + bx)^{3/2} \sqrt{c + dx} \sqrt{e + fx}} dx$$

Optimal antiderivative

$$\frac{2(Ab^2 - a(bB - aC)) \sqrt{dx+c} \sqrt{fx+e}}{b(-ad+bc)(-af+be)\sqrt{bx+a}}$$

$$2(2a^2Cdf + b^2(Adf + cCe) - ab(Bdf + cCf + Cde)) \text{EllipticE} \left(\frac{\sqrt{d} \sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{\frac{(-ad+bc)f}{d(-af+be)}} \right) \sqrt{\frac{b(dx+c)}{-ad+bc}}$$

$$b^2f(-af+be) \sqrt{d} \sqrt{ad-bc} \sqrt{dx+c} \sqrt{\frac{b(fx+e)}{-af+be}}$$

$$2(aC(-cf+de) - b(Adf - Bcf + cCe)) \text{EllipticF} \left(\frac{\sqrt{d} \sqrt{bx+a}}{\sqrt{ad-bc}}, \sqrt{\frac{(-ad+bc)f}{d(-af+be)}} \right) \sqrt{\frac{b(dx+c)}{-ad+bc}} \sqrt{\frac{b(fx+e)}{-af+be}}$$

$$b^2f\sqrt{d} \sqrt{ad-bc} \sqrt{dx+c} \sqrt{fx+e}$$

command

```
integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 (Ca^2b^2 - Bab^3 + Ab^4) \sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e} d^2f^2 - ((Cab^3c^2 + (2Ca^2b^2 - 2Bab^3 - Ab^4)cd - (2Ca^3) \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cx^2 + Bx + A) \sqrt{bx+a} \sqrt{dx+c} \sqrt{fx+e}}{b^2dfx^4 + a^2ce + (b^2de + (b^2c + 2abd)f)x^3 + ((b^2c + 2abd)e + (2abc + a^2d)f)x^2 + (a^2cf + (2abc + a^2d))} \right)$$

9.19 Problem number 77

$$\int \frac{A + Bx + Cx^2}{(a + bx)^{5/2} \sqrt{c + dx} \sqrt{e + fx}} dx$$

Optimal antiderivative

$$\frac{2(Ab^2 - a(bB - aC)) \sqrt{dx + c} \sqrt{fx + e}}{3b(-ad + bc)(-af + be)(bx + a)^{\frac{3}{2}}} + \frac{2(2a^3Cdf + ab^2(-4Adf + Bcf + Bde + 6cCe) - b^3(3Bce - 2A(cf + de)) + a^2b(Bdf - 4C(cf + de))) \sqrt{dx + c}}{3b(-ad + bc)^2(-af + be)^2 \sqrt{bx + a}}$$

$$\frac{2(2a^3Cdf + ab^2(-4Adf + Bcf + Bde + 6cCe) - b^3(3Bce - 2A(cf + de)) + a^2b(Bdf - 4C(cf + de))) \text{EllipticE}}{3b^2(ad - bc)^{\frac{3}{2}}(-af + be)^2 \sqrt{dx + c} \sqrt{\frac{b(fx + e)}{-af + be}}}$$

$$\frac{2(a^2Cd(-cf + de) - b^2(Acdf + 2Ad^2e - 3Bcde + 3c^2Ce) + ab(3(Ad^2 + c^2C)f - Bd(2cf + de))) \text{EllipticF}}{3b^2(ad - bc)^{\frac{3}{2}}(-af + be) \sqrt{d} \sqrt{dx + c} \sqrt{fx + e}}$$

command

```
integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx + a} \sqrt{dx + c} \sqrt{fx + e}}{b^3dfx^5 + a^3ce + (b^3de + (b^3c + 3ab^2d)f)x^4 + ((b^3c + 3ab^2d)e + 3(ab^2c + a^2bd)f)x^3 + (3(ab^2c + a^2bd)e + (b^3d + 3ab^2c)f)x^2 + (3ab^2de + (b^3c + 3ab^2d)e + 3(ab^2c + a^2bd)f)x + 3ab^2de}, x\right)$$

9.20 Problem number 78

$$\int \frac{A + Bx + Cx^2}{(a + bx)^{7/2} \sqrt{c + dx} \sqrt{e + fx}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{2(Ab^2 - a(bB - aC)) \sqrt{dx + c} \sqrt{fx + e}}{5b(-ad + bc)(-af + be)(bx + a)^{\frac{5}{2}}} \\
 + & \frac{2(2a^3Cdf + ab^2(-8Adf + Bcf + Bde + 10cCe) - b^3(5Bce - 4A(cf + de)) + 3a^2b(Bdf - 2C(cf + de))) \sqrt{dx + c}}{15b(-ad + bc)^2(-af + be)^2(bx + a)^{\frac{3}{2}}} \\
 + & \frac{2(2a^4C d^2 f^2 + a^3 bdf(3Bdf - 7C(cf + de)) - b^4(8A d^2 e^2 - cde(-7Af + 10Be) + c^2(8A f^2 - 10Bef + 15C e^2))}{2(2a^4C d^2 f^2 + a^3 bdf(3Bdf - 7C(cf + de)) - b^4(8A d^2 e^2 - cde(-7Af + 10Be) + c^2(8A f^2 - 10Bef + 15C e^2))} \\
 + & \frac{2(a^3Cdf(-cf + de) + b^3(8A d^2 e^2 - cde(-3Af + 10Be) + c^2(4A f^2 - 5Bef + 15C e^2)) + ab^2(d^2e(-19Af + 2B))}{\dots}
 \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cx^2 + Bx + A) \sqrt{bx}}{b^4 d f x^6 + a^4 c e + (b^4 d e + (b^4 c + 4 a b^3 d) f) x^5 + ((b^4 c + 4 a b^3 d) e + 2 (2 a b^3 c + 3 a^2 b^2 d) f) x^4 + 2 ((2 a b^3 c + 3 a^2 b^2 d) e + 2 a b^3 c f) x^3 + (2 a b^3 c e + 2 a^2 b^2 d f) x^2 + (2 a b^3 c f + 2 a^2 b^2 d e) x + 2 a b^3 c e} \right)$$

10 Test file number 18

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/18_1.1.1.7_P-x-a+b_x-^m-c+d_x-^n-e+f_x-^p-g+h_x-^q

10.1 Problem number 1

$$\int \frac{(a + bx)^2(A + Bx)}{\sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal antiderivative

$$\frac{2b(7aBdfh + b(5Adfh - 4B(cf h + deh + df g))) \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}}{15d^2 f^2 h^2} + \frac{2bB(bx + a) \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}}{5dfh}$$

$$+ \frac{2(15a^2 B d^2 f^2 h^2 + 10abdfh(3Adfh - 2B(cf h + deh + df g)) - b^2(10Adfh(cf h + deh + df g) - B(8c^2 f^2 h^2 + 7cdf$$

$$15d^3 f^{\frac{5}{2}} h^3$$

$$- \frac{2(15a^2 d^2 f^2 h^2 (-Ah + Bg) + 10abdfh(3Adfgh - B(ch(-eh + fg) + dg(eh + 2fg))) - b^2(5Adfh(ch(-eh + fg) -$$

command

```
integrate((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(3Bb^2d^3f^3h^3x - 4Bb^2d^3f^3gh^2 - 4Bb^2d^3f^2h^3e - (4Bb^2cd^2 - 5(2Bab + Ab^2)d^3)f^3h^3) \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bb^2x^3 + Aa^2 + (2Bab + Ab^2)x^2 + (Ba^2 + 2Aab)x) \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}}{dfhx^3 + ceg + (dfg + (de + cf)h)x^2 + (ceh + (de + cf)g)x}, x \right)$$

10.2 Problem number 2

$$\int \frac{(a + bx)(A + Bx)}{\sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal antiderivative

$$\frac{2bB\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{3dfh} + \frac{2(3aBdfh + b(3Adfh - 2B(cf h + deh + df g))) \operatorname{EllipticE}\left(\frac{\sqrt{f}\sqrt{dx+c}}{\sqrt{cf-de}}, \sqrt{\frac{(-cf+de)h}{f(-ch+dg)}}\right) \sqrt{cf-de} \sqrt{\frac{d(fx+e)}{-cf+de}}}{3d^2 f^{\frac{3}{2}} h^2 \sqrt{fx+e} \sqrt{\frac{d(hx+g)}{-ch+dg}}} - \frac{2(3adfh(-Ah+Bg) + b(3Adfgh - B(ch(-eh+fg) + dg(eh+2fg)))) \operatorname{EllipticF}\left(\frac{\sqrt{f}\sqrt{dx+c}}{\sqrt{cf-de}}, \sqrt{\frac{(-cf+de)h}{f(-ch+dg)}}\right) \sqrt{cf-de} \sqrt{\frac{d(hx+g)}{-ch+dg}}}{3d^2 f^{\frac{3}{2}} h^2 \sqrt{fx+e} \sqrt{hx+g}}$$

command

```
integrate((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{dx+c} \sqrt{fx+e} \sqrt{hx+g} Bbd^2 f^2 h^2 + (2 Bbd^2 f^2 g^2 + 2 Bbd^2 h^2 e^2 + (Bbcd - 3(Ba + Ab)d^2) f^2 gh + (2 Bb$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bbx^2 + Aa + (Ba + Ab)x)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{dfhx^3 + ceg + (dfg + (de + cf)h)x^2 + (ceh + (de + cf)g)x}, x\right)$$

10.3 Problem number 3

$$\int \frac{A + Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}} dx$$

Optimal antiderivative

$$\frac{2B \operatorname{EllipticE}\left(\frac{\sqrt{f}\sqrt{dx+c}}{\sqrt{cf-de}}, \sqrt{\frac{(-cf+de)h}{f(-ch+dg)}}\right) \sqrt{cf-de} \sqrt{\frac{d(fx+e)}{-cf+de}} \sqrt{hx+g}}{dh\sqrt{f}\sqrt{fx+e}\sqrt{\frac{d(hx+g)}{-ch+dg}}} - \frac{2(-Ah+Bg) \operatorname{EllipticF}\left(\frac{\sqrt{f}\sqrt{dx+c}}{\sqrt{cf-de}}, \sqrt{\frac{(-cf+de)h}{f(-ch+dg)}}\right) \sqrt{cf-de} \sqrt{\frac{d(fx+e)}{-cf+de}} \sqrt{\frac{d(hx+g)}{-ch+dg}}}{dh\sqrt{f}\sqrt{fx+e}\sqrt{hx+g}}$$

command

`integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{dfh} Bdfh \text{weierstrassZeta} \left(\frac{4(d^2 f^2 g^2 - cdf^2 gh + c^2 f^2 h^2 + d^2 h^2 e^2 - (d^2 fgh + cdfh^2)e)}{3d^2 f^2 h^2}, -\frac{4(2d^3 f^3 g^3 - 3cd^2 f^3 g^2 h - 3c^2 df^3 gh^2 + 2c^3 d^2 f^3 h^2)}{3d^2 f^2 h^2} \right), \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bx + A) \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}}{dfhx^3 + ceg + (dfg + (de + cf)h)x^2 + (ceh + (de + cf)g)x}, x \right)$$

10.4 Problem number 16

$$\int \frac{(a + bx)(abB - a^2C + b^2Bx + b^2Cx^2)}{\sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal antiderivative

$$\frac{2b^2(5bBdfh + 2C(adfh - 2b(cf h + deh + dfg))) \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}}{15d^2 f^2 h^2}$$

$$+ \frac{2b^2C(bx + a) \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}}{5dfh}$$

$$2b(15a^2C d^2 f^2 h^2 - 10abdfh(3Bdfh - C(cf h + deh + dfg))) + b^2(10Bdfh(cf h + deh + dfg) - C(8c^2 f^2 h^2 + 7cd^2 f^2 h^2))$$

$$15d^3 f^{\frac{5}{2}} h^{\frac{5}{2}}$$

$$2(15a^3C d^2 f^2 h^3 - 15a^2b d^2 f^2 h^2(Bh + Cg) + 5a b^2 dfh(6Bdfgh - C(ch(-eh + fg) + dg(eh + 2fg)))) - b^3(5Bdfh^2 + 5Cdfh^2))$$

command

`integrate((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(3Cb^3d^3f^3h^3x - 4Cb^3d^3f^3gh^2 - 4Cb^3d^3f^2h^3e - (4Cb^3cd^2 - 5(Cab^2 + Bb^3)d^3)f^3h^3) \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g} \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cb^3x^3 - Ca^3 + Ba^2b + (Cab^2 + Bb^3)x^2 - (Ca^2b - 2Bab^2)x) \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}}{dfhx^3 + ceg + (dfg + (de + cf)h)x^2 + (ceh + (de + cf)g)x}, x \right)$$

10.5 Problem number 17

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}} dx$$

Optimal antiderivative

$$\frac{2b^2C\sqrt{dx+c} \sqrt{fx+e} \sqrt{hx+g}}{3dfh} + \frac{2b^2(3Bdfh - 2C(cf h + deh + df g)) \operatorname{EllipticE}\left(\frac{\sqrt{f} \sqrt{dx+c}}{\sqrt{cf-de}}, \sqrt{\frac{(-cf+de)h}{f(-ch+dg)}}\right) \sqrt{cf-de} \sqrt{\frac{d(fx+e)}{-cf+de}} \sqrt{hx+g}}{3d^2 f^{\frac{3}{2}} h^2 \sqrt{fx+e} \sqrt{\frac{d(hx+g)}{-ch+dg}}} + \frac{2(3abBdf h^2 - 3a^2Cdf h^2 - b^2(3Bdfgh - C(ch(-eh+fg) + dg(eh+2fg)))) \operatorname{EllipticF}\left(\frac{\sqrt{f} \sqrt{dx+c}}{\sqrt{cf-de}}, \sqrt{\frac{(-cf+de)h}{f(-ch+dg)}}\right) \sqrt{cf-de} \sqrt{\frac{d(fx+e)}{-cf+de}} \sqrt{hx+g}}{3d^2 f^{\frac{3}{2}} h^2 \sqrt{fx+e} \sqrt{hx+g}}$$

command

`integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm='Fricas')`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3\sqrt{dx+c} \sqrt{fx+e} \sqrt{hx+g} Cb^2d^2f^2h^2 + (2Cb^2d^2f^2g^2 + 2Cb^2d^2h^2e^2 + (Cb^2cd - 3Bb^2d^2)f^2gh + (2Cb^2c^2 - 2Bb^2cd + B^2c^2)g^2)\sqrt{fx+e}\sqrt{hx+g} + (2Cb^2cd - 3Bb^2d^2)f^2gh + (2Cb^2c^2 - 2Bb^2cd + B^2c^2)g^2\right)}{3d^2 f^{\frac{3}{2}} h^2 \sqrt{fx+e} \sqrt{hx+g}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Cb^2x^2 + Bb^2x - Ca^2 + Bab)\sqrt{dx+c} \sqrt{fx+e} \sqrt{hx+g}}{dfhx^3 + ceg + (dfg + (de+cf)h)x^2 + (ceh + (de+cf)g)x}, x\right)$$

10.6 Problem number 18

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}} dx$$

Optimal antiderivative

$$\frac{2bC \operatorname{EllipticE}\left(\frac{\sqrt{f} \sqrt{dx+c}}{\sqrt{cf-de}}, \sqrt{\frac{(-cf+de)h}{f(-ch+dg)}}\right) \sqrt{cf-de} \sqrt{\frac{d(fx+e)}{-cf+de}} \sqrt{hx+g}}{dh \sqrt{f} \sqrt{fx+e} \sqrt{\frac{d(hx+g)}{-ch+dg}}} - \frac{2(-bBh + aCh + bCg) \operatorname{EllipticF}\left(\frac{\sqrt{f} \sqrt{dx+c}}{\sqrt{cf-de}}, \sqrt{\frac{(-cf+de)h}{f(-ch+dg)}}\right) \sqrt{cf-de} \sqrt{\frac{d(fx+e)}{-cf+de}} \sqrt{\frac{d(hx+g)}{-ch+dg}}}{dh \sqrt{f} \sqrt{fx+e} \sqrt{hx+g}}$$

command

`integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{dfh} \operatorname{CbdfhweierstrassZeta} \left(\frac{4(d^2 f^2 g^2 - cdf^2 gh + c^2 f^2 h^2 + d^2 h^2 e^2 - (d^2 fgh + cdfh^2)e)}{3d^2 f^2 h^2}, -\frac{4(2d^3 f^3 g^3 - 3cd^2 f^3 g^2 h - 3c^2 df^3 gh^2 + \dots}{\dots} \right), \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Cbx - Ca + Bb)\sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}}{dfhx^3 + ceg + (dfg + (de + cf)h)x^2 + (ceh + (de + cf)g)x}, x \right)$$

10.7 Problem number 26

$$\int \frac{(a + bx)^2 (A + Cx^2)}{\sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal antiderivative

$$\frac{2(4C(2adfh - 3b(cf h + deh + df g)) (adfh - 2b(cf h + deh + df g)) + 5bdfh(7Abdfh - C(5b(ceh + cf g + deg) + 2 \dots))}{105d^3 f^3 h^3}$$

$$+ \frac{4C(2adfh - 3b(cf h + deh + df g)) (bx + a) \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}}{35d^2 f^2 h^2}$$

$$+ \frac{2C(bx + a)^2 \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}}{7dfh}$$

$$4(35a^2 C d^2 f^2 h^2 (cf h + deh + df g) - 7abdfh(15A d^2 f^2 h^2 + C(8c^2 f^2 h^2 + 7cdfh(eh + fg) + d^2(8e^2 h^2 + 7efgh + \dots)))$$

$$+ \frac{2(35a^2 d^2 f^2 h^2 (3Adf h^2 + C(ch(-eh + fg) + dg(eh + 2fg))) - 14abdfh(15A d^2 f^2 g h^2 + C(4c^2 f h^2(-eh + fg) + \dots))}{\dots}$$

command

`integrate((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas"`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 (15 C b^2 d^4 f^4 h^4 x^2 + 24 C b^2 d^4 f^4 g^2 h^2 + 24 C b^2 d^4 f^2 h^4 e^2 + (23 C b^2 c d^3 - 56 C a b d^4) f^4 g h^3 + (24 C b^2 c^2 d^2 - 56 C \dots) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cb^2x^4 + 2Cabx^3 + 2Aabx + Aa^2 + (Ca^2 + Ab^2)x^2)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{dfhx^3 + ceg + (dfg + (de + cf)h)x^2 + (ceh + (de + cf)g)x}, x\right)$$

10.8 Problem number 27

$$\int \frac{(a + bx)(A + Cx^2)}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4C(adfh - 2b(cf h + deh + df g))\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{15d^2f^2h^2} \\ & + \frac{2C(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{5dfh} \\ & \frac{2(10aCdfh(cf h + deh + df g) - b(15A d^2f^2h^2 + C(8c^2f^2h^2 + 7cdfh(eh + fg) + d^2(8e^2h^2 + 7efgh + 8f^2g^2))))}{15d^3f^{\frac{5}{2}}h^3\sqrt{fx+e}\sqrt{\frac{d(hx+g)}{-ch+dg}}} \\ & + \frac{2(5adfh(3Adf h^2 + C(ch(-eh + fg) + dg(eh + 2fg))) - b(15A d^2f^2gh^2 + C(4c^2fh^2(-eh + fg) + cdh(-4e^2h^2 + 7efgh + 8f^2g^2))))}{15} \end{aligned}$$

command

```
integrate((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\frac{3(3Cbd^3f^3h^3x - 4Cbd^3f^3gh^2 - 4Cbd^3f^2h^3e - (4Cbcd^2 - 5Cad^3)f^3h^3)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g} - (8Cbd^3f^3h^3x - 4Cbd^3f^3gh^2 - 4Cbd^3f^2h^3e - (4Cbcd^2 - 5Cad^3)f^3h^3)}{15d^3f^{\frac{5}{2}}h^3\sqrt{fx+e}\sqrt{\frac{d(hx+g)}{-ch+dg}}}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cb^3x^3 + Cax^2 + Abx + Aa)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{dfhx^3 + ceg + (dfg + (de + cf)h)x^2 + (ceh + (de + cf)g)x}, x\right)$$

10.9 Problem number 28

$$\int \frac{A + Cx^2}{\sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal antiderivative

$$\frac{2C\sqrt{dx+c} \sqrt{fx+e} \sqrt{hx+g}}{3dfh}$$

$$- \frac{4C(cf h + deh + dfg) \operatorname{EllipticE}\left(\frac{\sqrt{f} \sqrt{dx+c}}{\sqrt{cf-de}}, \sqrt{\frac{(-cf+de)h}{f(-ch+dg)}}\right) \sqrt{cf-de} \sqrt{\frac{d(fx+e)}{-cf+de}} \sqrt{hx+g}}{3d^2 f^{\frac{3}{2}} h^2 \sqrt{fx+e} \sqrt{\frac{d(hx+g)}{-ch+dg}}}$$

$$+ \frac{2(3Adf h^2 + C(ch(-eh+fg) + dg(eh+2fg))) \operatorname{EllipticF}\left(\frac{\sqrt{f} \sqrt{dx+c}}{\sqrt{cf-de}}, \sqrt{\frac{(-cf+de)h}{f(-ch+dg)}}\right) \sqrt{cf-de} \sqrt{\frac{d(fx+e)}{-cf+de}}}{3d^2 f^{\frac{3}{2}} h^2 \sqrt{fx+e} \sqrt{hx+g}}$$

command

```
integrate((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{dx+c} \sqrt{fx+e} \sqrt{hx+g} Cd^2 f^2 h^2 + (2Cd^2 f^2 g^2 + Ccdf^2 gh + 2Cd^2 h^2 e^2 + (2Cc^2 + 9Ad^2) f^2 h^2 + (Cd^2 f^2 g^2 + Ccdf^2 gh + 2Cd^2 h^2 e^2) \sqrt{dx+c} \sqrt{fx+e} \sqrt{hx+g} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Cx^2 + A)\sqrt{dx+c} \sqrt{fx+e} \sqrt{hx+g}}{dfhx^3 + ceg + (dfg + (de+cf)h)x^2 + (ceh + (de+cf)g)x}, x\right)$$

11 Test file number 19

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.2_Quadratic/19_1.1.2.2-c_x~m-a+b_x^2~p

11.1 Problem number 589

$$\int (cx)^{7/2} \sqrt{a+bx^2} \, dx$$

Optimal antiderivative

$$\frac{4ac(cx)^{\frac{5}{2}} \sqrt{bx^2+a}}{77b} + \frac{2(cx)^{\frac{9}{2}} \sqrt{bx^2+a}}{11c} - \frac{20a^2c^3 \sqrt{cx} \sqrt{bx^2+a}}{231b^2}$$

$$+ \frac{10a^{\frac{11}{4}} c^{\frac{7}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{1}{(\sqrt{a} + x\sqrt{b})^2}}}{231 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right) b^{\frac{9}{4}} \sqrt{bx^2+a}}$$

command

```
integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10 \sqrt{bc} a^3 c^3 \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (21 b^3 c^3 x^4 + 6 ab^2 c^3 x^2 - 10 a^2 bc^3) \sqrt{bx^2+a} \sqrt{cx} \right)}{231 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{bx^2+a} \sqrt{cx} c^3 x^3, x\right)$$

11.2 Problem number 590

$$\int (cx)^{5/2} \sqrt{a+bx^2} \, dx$$

Optimal antiderivative

$$\frac{4ac(cx)^{\frac{3}{2}} \sqrt{bx^2 + a}}{45b} + \frac{2(cx)^{\frac{7}{2}} \sqrt{bx^2 + a}}{9c} - \frac{4a^2c^2\sqrt{cx} \sqrt{bx^2 + a}}{15b^{\frac{3}{2}} (\sqrt{a} + x\sqrt{b})}$$

$$+ \frac{4a^{\frac{9}{4}}c^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{b}{(\sqrt{a}})}}}{15 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) b^{\frac{7}{4}} \sqrt{bx^2 + a}}$$

$$- \frac{2a^{\frac{9}{4}}c^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{b}{(\sqrt{a}})}}}{15 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) b^{\frac{7}{4}} \sqrt{bx^2 + a}}$$

command

`integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 \sqrt{bc} a^2 c^2 \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (5b^2c^2x^3 + 2abc^2x) \sqrt{bx^2 + a} \sqrt{cx} \right)}{45b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{bx^2 + a} \sqrt{cx} c^2 x^2, x\right)$$

11.3 Problem number 591

$$\int (cx)^{3/2} \sqrt{a + bx^2} dx$$

Optimal antiderivative

$$\frac{2(cx)^{\frac{5}{2}} \sqrt{bx^2 + a}}{7c} + \frac{4ac\sqrt{cx} \sqrt{bx^2 + a}}{21b}$$

$$+ \frac{2a^{\frac{7}{4}}c^{\frac{3}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{b}{(\sqrt{a}})}}}{21 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) b^{\frac{5}{4}} \sqrt{bx^2 + a}}$$

command

```
integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{bc} a^2 \text{cweierstrassPInverse} \left(-\frac{4a}{b}, 0, x \right) - (3b^2cx^2 + 2abc) \sqrt{bx^2 + a} \sqrt{cx} \right)}{21b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{bx^2 + a} \sqrt{cx} cx, x \right)$$

11.4 Problem number 592

$$\int \sqrt{cx} \sqrt{a + bx^2} dx$$

Optimal antiderivative

$$\frac{2(cx)^{\frac{3}{2}} \sqrt{bx^2 + a}}{5c} + \frac{4a\sqrt{cx} \sqrt{bx^2 + a}}{5\sqrt{b} (\sqrt{a} + x\sqrt{b})}$$

$$4a^{\frac{5}{4}} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{a} + x\sqrt{b}) \sqrt{c} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right)}{2}}$$

$$5 \cos \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right) b^{\frac{3}{4}} \sqrt{bx^2 + a}$$

$$2a^{\frac{5}{4}} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{a} + x\sqrt{b}) \sqrt{c} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right)}{2}}$$

+

$$5 \cos \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right) b^{\frac{3}{4}} \sqrt{bx^2 + a}$$

command

```
integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{bx^2 + a} \sqrt{cx} bx - 2 \sqrt{bc} a \text{weierstrassZeta} \left(-\frac{4a}{b}, 0, \text{weierstrassPInverse} \left(-\frac{4a}{b}, 0, x \right) \right) \right)}{5b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{bx^2 + a} \sqrt{cx}, x \right)$$

11.5 Problem number 593

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{cx} \sqrt{bx^2+a}}{3c} + \frac{2a^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2}{(\sqrt{a} + x\sqrt{b})^2}}}{3 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) b^{\frac{1}{4}}\sqrt{c} \sqrt{bx^2+a}}$$

command

```
integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{bc} \operatorname{aweierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + \sqrt{bx^2+a} \sqrt{cx} b \right)}{3bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a} \sqrt{cx}}{cx}, x\right)$$

11.6 Problem number 594

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2\sqrt{bx^2+a}}{c\sqrt{cx}} + \frac{4\sqrt{b}\sqrt{cx}\sqrt{bx^2+a}}{c^2(\sqrt{a}+x\sqrt{b})} \\
 & + \frac{4a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{b}{(\sqrt{a}+x\sqrt{b})^2}}}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)c^{\frac{3}{2}}\sqrt{bx^2+a}} \\
 & + \frac{2a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)c^{\frac{3}{2}}\sqrt{bx^2+a}}
 \end{aligned}$$

command

`integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{bc}x\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + \sqrt{bx^2+a}\sqrt{cx}\right)}{c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{c^2x^2}, x\right)$$

11.7 Problem number 595

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2\sqrt{bx^2+a}}{3c(cx)^{\frac{3}{2}}} \\
 & + \frac{2b^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{3\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{1}{4}}c^{\frac{5}{2}}\sqrt{bx^2+a}}
 \end{aligned}$$

command

```
integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{bc} x^2 \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - \sqrt{bx^2 + a} \sqrt{cx} \right)}{3 c^3 x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^2 + a} \sqrt{cx}}{c^3 x^3}, x\right)$$

11.8 Problem number 596

$$\int \frac{\sqrt{a + bx^2}}{(cx)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{bx^2 + a}}{5c(cx)^{\frac{5}{2}}} - \frac{4b\sqrt{bx^2 + a}}{5ac^3\sqrt{cx}} + \frac{4b^{\frac{3}{2}}\sqrt{cx}\sqrt{bx^2 + a}}{5ac^4(\sqrt{a} + x\sqrt{b})} \\ & \frac{4b^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{\frac{bx^2}{(\sqrt{a} + x\sqrt{b})^2}}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{3}{4}}c^{\frac{7}{2}}\sqrt{bx^2 + a}} \\ & + \frac{2b^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{\frac{bx^2}{(\sqrt{a} + x\sqrt{b})^2}}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{3}{4}}c^{\frac{7}{2}}\sqrt{bx^2 + a}} \end{aligned}$$

command

```
integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{bc} bx^3 \text{weierstrassZeta}\left(-\frac{4a}{b}, 0, \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (2bx^2 + a) \sqrt{bx^2 + a} \sqrt{cx} \right)}{5ac^4x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^2 + a} \sqrt{cx}}{c^4 x^4}, x\right)$$

11.9 Problem number 597

$$\int (cx)^{7/2} (a + bx^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{2(cx)^{\frac{9}{2}} (bx^2 + a)^{\frac{3}{2}}}{15c} + \frac{8a^2c(cx)^{\frac{5}{2}} \sqrt{bx^2 + a}}{385b} + \frac{4a(cx)^{\frac{9}{2}} \sqrt{bx^2 + a}}{55c} - \frac{8a^3c^3 \sqrt{cx} \sqrt{bx^2 + a}}{231b^2}$$

$$+ \frac{4a^{\frac{15}{4}} c^{\frac{7}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{b}{(\sqrt{a}})}}}{231 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right) b^{\frac{9}{4}} \sqrt{bx^2 + a}}$$

command

```
integrate((c*x)^(7/2)*(b*x^2+a)^(3/2), x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(20 \sqrt{bc} a^4 c^3 \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (77 b^4 c^3 x^6 + 119 a b^3 c^3 x^4 + 12 a^2 b^2 c^3 x^2 - 20 a^3 b c^3) \sqrt{bx^2 + a} \sqrt{cx} \right)}{1155 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bc^3x^5 + ac^3x^3) \sqrt{bx^2 + a} \sqrt{cx}, x\right)$$

11.10 Problem number 598

$$\int (cx)^{5/2} (a + bx^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{2(cx)^{\frac{7}{2}}(bx^2+a)^{\frac{3}{2}}}{13c} + \frac{8a^2c(cx)^{\frac{3}{2}}\sqrt{bx^2+a}}{195b} + \frac{4a(cx)^{\frac{7}{2}}\sqrt{bx^2+a}}{39c} - \frac{8a^3c^2\sqrt{cx}\sqrt{bx^2+a}}{65b^{\frac{3}{2}}(\sqrt{a}+x\sqrt{b})}$$

$$+ \frac{8a^{\frac{13}{4}}c^{\frac{5}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{b}{(\sqrt{a}+x\sqrt{b})^2}}}{65\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^2+a}}$$

$$- \frac{4a^{\frac{13}{4}}c^{\frac{5}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{b}{(\sqrt{a}+x\sqrt{b})^2}}}{65\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^2+a}}$$

command

```
integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12\sqrt{bc}a^3c^2\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (15b^3c^2x^5 + 25ab^2c^2x^3 + 4a^2bc^2x)\sqrt{bx^2+a}\right)}{195b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bc^2x^4 + ac^2x^2)\sqrt{bx^2+a}\sqrt{cx}, x\right)$$

11.11 Problem number 599

$$\int (cx)^{3/2} (a + bx^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{2(cx)^{\frac{5}{2}}(bx^2+a)^{\frac{3}{2}}}{11c} + \frac{12a(cx)^{\frac{5}{2}}\sqrt{bx^2+a}}{77c} + \frac{8a^2c\sqrt{cx}\sqrt{bx^2+a}}{77b}$$

$$+ \frac{4a^{\frac{11}{4}}c^{\frac{3}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{b}{(\sqrt{a}+x\sqrt{b})^2}}}{77\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)b^{\frac{5}{4}}\sqrt{bx^2+a}}$$

command

`integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 \sqrt{bc} a^3 \text{weierstrassPInverse} \left(-\frac{4a}{b}, 0, x \right) - (7b^3cx^4 + 13ab^2cx^2 + 4a^2bc) \sqrt{bx^2 + a} \sqrt{cx} \right)}{77b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((bcx^3 + acx) \sqrt{bx^2 + a} \sqrt{cx}, x \right)$$

11.12 Problem number 600

$$\int \sqrt{cx} (a + bx^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{2(cx)^{\frac{3}{2}} (bx^2 + a)^{\frac{3}{2}}}{9c} + \frac{4a(cx)^{\frac{3}{2}} \sqrt{bx^2 + a}}{15c} + \frac{8a^2 \sqrt{cx} \sqrt{bx^2 + a}}{15\sqrt{b} (\sqrt{a} + x\sqrt{b})}$$

$$8a^{\frac{9}{4}} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{a} + x\sqrt{b}) \sqrt{c} \sqrt{\frac{bx^2 + a}{bx^2 + a}}$$

$$15 \cos \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right) b^{\frac{3}{4}} \sqrt{bx^2 + a}$$

$$4a^{\frac{9}{4}} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{a} + x\sqrt{b}) \sqrt{c} \sqrt{\frac{bx^2 + a}{bx^2 + a}}$$

$$15 \cos \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}} \right) \right) b^{\frac{3}{4}} \sqrt{bx^2 + a}$$

command

`integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 \sqrt{bc} a^2 \text{weierstrassZeta} \left(-\frac{4a}{b}, 0, \text{weierstrassPInverse} \left(-\frac{4a}{b}, 0, x \right) \right) - (5b^2x^3 + 11abx) \sqrt{bx^2 + a} \sqrt{cx} \right)}{45b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((bx^2 + a)^{\frac{3}{2}} \sqrt{cx}, x \right)$$

11.13 Problem number 601

$$\int \frac{(a + bx^2)^{3/2}}{\sqrt{cx}} dx$$

Optimal antiderivative

$$\frac{2(bx^2 + a)^{\frac{3}{2}} \sqrt{cx}}{7c} + \frac{4a\sqrt{cx} \sqrt{bx^2 + a}}{7c} + \frac{4a^{\frac{7}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{7 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right) b^{\frac{1}{4}} \sqrt{c} \sqrt{bx^2 + a}}$$

command

```
integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 \sqrt{bc} a^2 \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (b^2 x^2 + 3ab) \sqrt{bx^2 + a} \sqrt{cx} \right)}{7bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bx^2 + a)^{\frac{3}{2}} \sqrt{cx}}{cx}, x\right)$$

11.14 Problem number 602

$$\int \frac{(a + bx^2)^{3/2}}{(cx)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(bx^2+a)^{\frac{3}{2}}}{c\sqrt{cx}} + \frac{12b(cx)^{\frac{3}{2}}\sqrt{bx^2+a}}{5c^3} + \frac{24a\sqrt{b}\sqrt{cx}\sqrt{bx^2+a}}{5c^2(\sqrt{a}+x\sqrt{b})} \\
 & \frac{24a^{\frac{5}{4}}b^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{b}{(\sqrt{a}+x\sqrt{b})^2}}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)c^{\frac{3}{2}}\sqrt{bx^2+a}} \\
 & + \frac{12a^{\frac{5}{4}}b^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{b}{(\sqrt{a}+x\sqrt{b})^2}}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)c^{\frac{3}{2}}\sqrt{bx^2+a}}
 \end{aligned}$$

command

`integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12\sqrt{bc}\operatorname{axweierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) - \sqrt{bx^2+a}(bx^2-5a)\sqrt{cx}\right)}{5c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bx^2+a)^{\frac{3}{2}}\sqrt{cx}}{c^2x^2}, x\right)$$

11.15 Problem number 603

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(bx^2+a)^{\frac{3}{2}}}{3c(cx)^{\frac{3}{2}}} + \frac{4b\sqrt{cx}\sqrt{bx^2+a}}{3c^3} \\
 & \frac{4a^{\frac{3}{4}}b^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{b}{(\sqrt{a}+x\sqrt{b})^2}}}{3\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)c^{\frac{5}{2}}\sqrt{bx^2+a}}
 \end{aligned}$$

command

```
integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 \sqrt{bc} ax^2 \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + \sqrt{bx^2 + a} (bx^2 - a) \sqrt{cx} \right)}{3 c^3 x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{3}{2}} \sqrt{cx}}{c^3 x^3}, x\right)$$

11.16 Problem number 604

$$\int \frac{(a + bx^2)^{3/2}}{(cx)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(bx^2 + a)^{\frac{3}{2}}}{5c(cx)^{\frac{5}{2}}} - \frac{12b\sqrt{bx^2 + a}}{5c^3\sqrt{cx}} + \frac{24b^{\frac{3}{2}}\sqrt{cx}\sqrt{bx^2 + a}}{5c^4(\sqrt{a} + x\sqrt{b})} \\ & \frac{24a^{\frac{1}{4}}b^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{\frac{b}{(\sqrt{a} + x\sqrt{b})^2}}}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)c^{\frac{7}{2}}\sqrt{bx^2 + a}} \\ & \frac{12a^{\frac{1}{4}}b^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{\frac{b}{(\sqrt{a} + x\sqrt{b})^2}}}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)c^{\frac{7}{2}}\sqrt{bx^2 + a}} \end{aligned}$$

command

```
integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 \sqrt{bc} bx^3 \text{weierstrassZeta}\left(-\frac{4a}{b}, 0, \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (7bx^2 + a) \sqrt{bx^2 + a} \sqrt{cx} \right)}{5 c^4 x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{3}{2}}\sqrt{cx}}{c^4x^4}, x\right)$$

11.17 Problem number 605

$$\int \frac{(a + bx^2)^{3/2}}{(cx)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(bx^2 + a)^{\frac{3}{2}}}{7c(cx)^{\frac{7}{2}}} - \frac{4b\sqrt{bx^2 + a}}{7c^3(cx)^{\frac{3}{2}}} \\ & + \frac{4b^{\frac{7}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{7\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{1}{4}}c^{\frac{9}{2}}\sqrt{bx^2 + a}} \end{aligned}$$

command

`integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4\sqrt{bc}bx^4\text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (3bx^2 + a)\sqrt{bx^2 + a}\sqrt{cx}\right)}{7c^5x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{3}{2}}\sqrt{cx}}{c^5x^5}, x\right)$$

11.18 Problem number 606

$$\int \frac{(a + bx^2)^{3/2}}{(cx)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(bx^2 + a)^{\frac{3}{2}}}{9c(cx)^{\frac{9}{2}}} - \frac{4b\sqrt{bx^2 + a}}{15c^3(cx)^{\frac{5}{2}}} - \frac{8b^2\sqrt{bx^2 + a}}{15ac^5\sqrt{cx}} + \frac{8b^{\frac{5}{2}}\sqrt{cx}\sqrt{bx^2 + a}}{15ac^6(\sqrt{a} + x\sqrt{b})} \\ & - \frac{8b^{\frac{9}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{15 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{3}{4}} c^{\frac{11}{2}} \sqrt{bx^2 + a}} \\ & + \frac{4b^{\frac{9}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{15 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{3}{4}} c^{\frac{11}{2}} \sqrt{bx^2 + a}} \end{aligned}$$

command

`integrate((b*x^2+a)^(3/2)/(c*x)^(11/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 \sqrt{bc} b^2 x^5 \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (12 b^2 x^4 + 11 abx^2 + 5 a^2) \sqrt{bx^2 + a} \right)}{45 ac^6 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bx^2 + a)^{\frac{3}{2}} \sqrt{cx}}{c^6 x^6}, x\right)$$

11.19 Problem number 607

$$\int (cx)^{5/2} \sqrt{3a - 2ax^2} dx$$

Optimal antiderivative

$$\frac{3 \cdot 6^{\frac{1}{4}} a c^2 \operatorname{EllipticE}\left(\frac{\sqrt{3-x}\sqrt{6}}{6}, \sqrt{2}\right) \sqrt{cx} \sqrt{-2x^2+3}}{5\sqrt{x} \sqrt{-2ax^2+3a}} - \frac{2c(cx)^{\frac{3}{2}} \sqrt{-2ax^2+3a}}{15} + \frac{2(cx)^{\frac{7}{2}} \sqrt{-2ax^2+3a}}{9c}$$

command

`integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3}{5} \sqrt{2} \sqrt{-ac} c^2 \operatorname{weierstrassZeta}(6, 0, \operatorname{weierstrassPInverse}(6, 0, x)) + \frac{2}{45} (5c^2x^3 - 3c^2x) \sqrt{-2ax^2+3a} \sqrt{cx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{-2ax^2+3a} \sqrt{cx} c^2x^2, x\right)$$

11.20 Problem number 608

$$\int (cx)^{3/2} \sqrt{3a - 2ax^2} dx$$

Optimal antiderivative

$$\frac{6^{\frac{3}{4}} a c^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}, i\right) \sqrt{-2x^2+3}}{7\sqrt{a}(-2x^2+3)} + \frac{2(cx)^{\frac{5}{2}} \sqrt{-2ax^2+3a}}{7c} - \frac{2c\sqrt{cx} \sqrt{-2ax^2+3a}}{7}$$

command

`integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{3}{7} \sqrt{2} \sqrt{-ac} c \operatorname{weierstrassPInverse}(6, 0, x) + \frac{2}{7} \sqrt{-2ax^2+3a} (cx^2 - c) \sqrt{cx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{-2ax^2+3a} \sqrt{cx} cx, x\right)$$

11.21 Problem number 609

$$\int \sqrt{cx} \sqrt{3a - 2ax^2} dx$$

Optimal antiderivative

$$\frac{6 \cdot 6^{\frac{1}{4}} a \operatorname{EllipticE}\left(\frac{\sqrt{3-x}\sqrt{6}}{6}, \sqrt{2}\right) \sqrt{cx} \sqrt{-2x^2+3}}{5\sqrt{x} \sqrt{-2ax^2+3a}} + \frac{2(cx)^{\frac{3}{2}} \sqrt{-2ax^2+3a}}{5c}$$

command

```
integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{5} \sqrt{-2ax^2+3a} \sqrt{cx} x + \frac{6}{5} \sqrt{2} \sqrt{-ac} \operatorname{weierstrassZeta}(6, 0, \operatorname{weierstrassPInverse}(6, 0, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{-2ax^2+3a} \sqrt{cx}, x\right)$$

11.22 Problem number 610

$$\int \frac{\sqrt{3a - 2ax^2}}{\sqrt{cx}} dx$$

Optimal antiderivative

$$\frac{2 \cdot 2^{\frac{3}{4}} a \operatorname{EllipticF}\left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}, i\right) \sqrt{-2x^2+3} \cdot 3^{\frac{3}{4}}}{3\sqrt{c} \sqrt{a(-2x^2+3)}} + \frac{2\sqrt{cx} \sqrt{-2ax^2+3a}}{3c}$$

command

```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3\sqrt{2} \sqrt{-ac} \operatorname{weierstrassPInverse}(6, 0, x) - \sqrt{-2ax^2+3a} \sqrt{cx}\right)}{3c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-2ax^2+3a} \sqrt{cx}}{cx}, x\right)$$

11.23 Problem number 611

$$\int \frac{\sqrt{3a - 2ax^2}}{(cx)^{3/2}} dx$$

Optimal antiderivative

$$\frac{46^{\frac{1}{4}} a \operatorname{EllipticE}\left(\frac{\sqrt{3 - x\sqrt{6}} \sqrt{6}}{6}, \sqrt{2}\right) \sqrt{cx} \sqrt{-2x^2 + 3}}{c^2 \sqrt{x} \sqrt{-2ax^2 + 3a}} - \frac{2\sqrt{-2ax^2 + 3a}}{c\sqrt{cx}}$$

command

`integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{2}\sqrt{-ac}x\operatorname{weierstrassZeta}(6,0,\operatorname{weierstrassPInverse}(6,0,x)) + \sqrt{-2ax^2 + 3a}\sqrt{cx}\right)}{c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-2ax^2 + 3a}\sqrt{cx}}{c^2x^2}, x\right)$$

11.24 Problem number 612

$$\int \frac{\sqrt{3a - 2ax^2}}{(cx)^{5/2}} dx$$

Optimal antiderivative

$$\frac{42^{\frac{3}{4}} a \operatorname{EllipticF}\left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}, i\right) \sqrt{-2x^2 + 3} 3^{\frac{3}{4}}}{9c^{\frac{5}{2}} \sqrt{a(-2x^2 + 3)}} - \frac{2\sqrt{-2ax^2 + 3a}}{3c(cx)^{\frac{3}{2}}}$$

command

`integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{2}\sqrt{-ac}x^2\operatorname{weierstrassPInverse}(6,0,x) - \sqrt{-2ax^2 + 3a}\sqrt{cx}\right)}{3c^3x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-2ax^2 + 3a}\sqrt{cx}}{c^3x^3}, x\right)$$

11.25 Problem number 613

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}} dx$$

Optimal antiderivative

$$\frac{2c(cx)^{\frac{5}{2}} \sqrt{bx^2+a}}{7b} - \frac{10ac^3 \sqrt{cx} \sqrt{bx^2+a}}{21b^2} + \frac{5a^{\frac{7}{4}} c^{\frac{7}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right)}{2}} + \frac{1}{2}}{21 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right) b^{\frac{9}{4}} \sqrt{bx^2+a}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2+a}{(\sqrt{a} + x\sqrt{b})^2}}$$

command

```
integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(5\sqrt{bc}a^2c^3\operatorname{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(3b^2c^3x^2-5abc^3\right)\sqrt{bx^2+a}\sqrt{cx}\right)}{21b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx}c^3x^3}{\sqrt{bx^2+a}},x\right)$$

11.26 Problem number 614

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}} dx$$

Optimal antiderivative

$$\frac{2c(cx)^{\frac{3}{2}} \sqrt{bx^2 + a}}{5b} - \frac{6ac^2 \sqrt{cx} \sqrt{bx^2 + a}}{5b^{\frac{3}{2}} (\sqrt{a} + x\sqrt{b})}$$

$$+ \frac{6a^{\frac{5}{4}} c^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right) b^{\frac{7}{4}} \sqrt{bx^2 + a}}$$

$$- \frac{3a^{\frac{5}{4}} c^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right) b^{\frac{7}{4}} \sqrt{bx^2 + a}}$$

command

`integrate((c*x)^(5/2)/(b*x^2+a)^(1/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{bx^2 + a} \sqrt{cx} bc^2 x + 3 \sqrt{bc} ac^2 \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) \right)}{5b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx} c^2 x^2}{\sqrt{bx^2 + a}}, x\right)$$

11.27 Problem number 615

$$\int \frac{(cx)^{3/2}}{\sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\frac{2c\sqrt{cx} \sqrt{bx^2 + a}}{3b}$$

$$+ \frac{a^{\frac{3}{4}} c^{\frac{3}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{3 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{cx}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right) b^{\frac{5}{4}} \sqrt{bx^2 + a}}$$

command

`integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{bc} \operatorname{acweierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - \sqrt{bx^2 + a} \sqrt{cx} bc \right)}{3b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx} cx}{\sqrt{bx^2 + a}}, x\right)$$

11.28 Problem number 616

$$\int \frac{\sqrt{cx}}{\sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{cx} \sqrt{bx^2 + a}}{\sqrt{b} (\sqrt{a} + x\sqrt{b})}$$

$$\frac{2a^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{c} \sqrt{\frac{b}{(\sqrt{a} + x\sqrt{b})^2}}}{\cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) b^{\frac{3}{4}} \sqrt{bx^2 + a}}$$

$$+ \frac{a^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{c} \sqrt{\frac{b}{(\sqrt{a} + x\sqrt{b})^2}}}{\cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) b^{\frac{3}{4}} \sqrt{bx^2 + a}}$$

command

`integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{bc} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx}}{\sqrt{bx^2 + a}}, x\right)$$

11.29 Problem number 617

$$\int \frac{1}{\sqrt{cx} \sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})}}}{\cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{c} \sqrt{bx^2 + a}}$$

command

```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{bc} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)}{bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2 + a} \sqrt{cx}}{bcx^3 + acx}, x\right)$$

11.30 Problem number 618

$$\int \frac{1}{(cx)^{3/2} \sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^2+a}}{ac\sqrt{cx}} + \frac{2\sqrt{b}\sqrt{cx}\sqrt{bx^2+a}}{ac^2(\sqrt{a}+x\sqrt{b})}$$

$$2b^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b}) \sqrt{\frac{bx^2}{(\sqrt{a}+x\sqrt{b})}}$$

$$\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{3}{4}} c^{\frac{3}{2}} \sqrt{bx^2+a}$$

$$b^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b}) \sqrt{\frac{bx^2}{(\sqrt{a}+x\sqrt{b})}}$$

$$+ \frac{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{3}{4}} c^{\frac{3}{2}} \sqrt{bx^2+a}}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{3}{4}} c^{\frac{3}{2}} \sqrt{bx^2+a}}$$

command

```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{bc} x \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + \sqrt{bx^2+a} \sqrt{cx}\right)}{ac^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{bc^2x^4+ac^2x^2}, x\right)$$

11.31 Problem number 619

$$\int \frac{1}{(cx)^{5/2} \sqrt{a+bx^2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^2+a}}{3ac(cx)^{\frac{3}{2}}}$$

$$b^{\frac{3}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b}) \sqrt{\frac{bx^2}{(\sqrt{a}+x\sqrt{b})}}$$

$$3 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{5}{4}} c^{\frac{5}{2}} \sqrt{bx^2+a}$$

command

`integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{bc} x^2 \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + \sqrt{bx^2 + a} \sqrt{cx} \right)}{3ac^3x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^2 + a} \sqrt{cx}}{bc^3x^5 + ac^3x^3}, x\right)$$

11.32 Problem number 620

$$\int \frac{1}{(cx)^{7/2} \sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{bx^2 + a}}{5ac(cx)^{\frac{5}{2}}} + \frac{6b\sqrt{bx^2 + a}}{5a^2c^3\sqrt{cx}} - \frac{6b^{\frac{3}{2}}\sqrt{cx}\sqrt{bx^2 + a}}{5a^2c^4(\sqrt{a} + x\sqrt{b})} \\ & + \frac{6b^{\frac{5}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2}{(\sqrt{a} + x\sqrt{b})^2}}}{5 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{7}{4}}c^{\frac{7}{2}}\sqrt{bx^2 + a}} \\ & - \frac{3b^{\frac{5}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2}{(\sqrt{a} + x\sqrt{b})^2}}}{5 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{7}{4}}c^{\frac{7}{2}}\sqrt{bx^2 + a}} \end{aligned}$$

command

`integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \sqrt{bc} bx^3 \text{weierstrassZeta}\left(-\frac{4a}{b}, 0, \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (3bx^2 - a) \sqrt{bx^2 + a} \sqrt{cx} \right)}{5a^2c^4x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^2 + a} \sqrt{cx}}{bc^4x^6 + ac^4x^4}, x\right)$$

11.33 Problem number 621

$$\int \frac{(cx)^{7/2}}{(a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{c(cx)^{\frac{5}{2}}}{b\sqrt{bx^2+a}} + \frac{5c^3\sqrt{cx}\sqrt{bx^2+a}}{3b^2}$$

$$5a^{\frac{3}{4}}c^{\frac{7}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{\frac{b}{(\sqrt{a} + x\sqrt{b})^2}}$$

$$6\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)b^{\frac{9}{4}}\sqrt{bx^2+a}$$

command

```
integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(abc^3x^2 + a^2c^3)\sqrt{bc}\operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (2b^2c^3x^2 + 5abc^3)\sqrt{bx^2+a}\sqrt{cx}}{3(b^4x^2 + ab^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}c^3x^3}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

11.34 Problem number 622

$$\int \frac{(cx)^{5/2}}{(a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{c(cx)^{\frac{3}{2}}}{b\sqrt{bx^2+a}} + \frac{3c^2\sqrt{cx}\sqrt{bx^2+a}}{b^{\frac{3}{2}}(\sqrt{a}+x\sqrt{b})} \\
 & + \frac{3a^{\frac{1}{4}}c^{\frac{5}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^2+a}} \\
 & + \frac{3a^{\frac{1}{4}}c^{\frac{5}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^2+a}}
 \end{aligned}$$

command

```
integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{bx^2+a}\sqrt{cx}bc^2x+3(bc^2x^2+ac^2)\sqrt{bc}\operatorname{weierstrassZeta}\left(-\frac{4a}{b},0,\operatorname{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{b^3x^2+ab^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}c^2x^2}{b^2x^4+2abx^2+a^2},x\right)$$

11.35 Problem number 623

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{c\sqrt{cx}}{b\sqrt{bx^2+a}} \\
 & + \frac{c^{\frac{3}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{1}{4}}b^{\frac{5}{4}}\sqrt{bx^2+a}}
 \end{aligned}$$

command

```
integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{bx^2+a} \sqrt{cx} bc - (bcx^2 + ac) \sqrt{bc} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)}{b^3x^2 + ab^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a} \sqrt{cx} cx}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

11.36 Problem number 624

$$\int \frac{\sqrt{cx}}{(a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(cx)^{\frac{3}{2}}}{ac\sqrt{bx^2+a}} - \frac{\sqrt{cx} \sqrt{bx^2+a}}{a\sqrt{b}(\sqrt{a} + x\sqrt{b})}$$

$$+ \frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{c} \sqrt{\frac{bx^2}{(\sqrt{a} + x\sqrt{b})^2}}}{\cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{3}{4}} b^{\frac{3}{4}} \sqrt{bx^2+a}}$$

$$- \frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{c} \sqrt{\frac{bx^2}{(\sqrt{a} + x\sqrt{b})^2}}}{2 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{3}{4}} b^{\frac{3}{4}} \sqrt{bx^2+a}}$$

command

```
integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{bx^2+a} \sqrt{cx} bx + (bx^2 + a) \sqrt{bc} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right)}{ab^2x^2 + a^2b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a} \sqrt{cx}}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

11.37 Problem number 625

$$\int \frac{1}{\sqrt{cx} (a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{cx}}{ac\sqrt{bx^2+a}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{1/4}\sqrt{cx}}{a^{1/4}\sqrt{c}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{1/4}\sqrt{cx}}{a^{1/4}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2+a}{(\sqrt{a} + x\sqrt{b})^2}} + \frac{2\cos\left(2\arctan\left(\frac{b^{1/4}\sqrt{cx}}{a^{1/4}\sqrt{c}}\right)\right) a^{5/4} b^{1/4} \sqrt{c} \sqrt{bx^2+a}}{2}$$

command

```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(bx^2 + a)\sqrt{bc} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + \sqrt{bx^2 + a} \sqrt{cx} b}{ab^2cx^2 + a^2bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a} \sqrt{cx}}{b^2cx^5 + 2abcx^3 + a^2cx}, x\right)$$

11.38 Problem number 626

$$\int \frac{1}{(cx)^{3/2} (a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{ac\sqrt{cx} \sqrt{bx^2+a}} - \frac{3\sqrt{bx^2+a}}{a^2c\sqrt{cx}} + \frac{3\sqrt{b} \sqrt{cx} \sqrt{bx^2+a}}{a^2c^2(\sqrt{a}+x\sqrt{b})}$$

$$3b^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b}) \sqrt{\frac{bx^2}{(\sqrt{a}+x\sqrt{b})}}$$

$$\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{7}{4}}c^{\frac{3}{2}}\sqrt{bx^2+a}$$

$$3b^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b}) \sqrt{\frac{bx^2}{(\sqrt{a}+x\sqrt{b})}}$$

$$+ \frac{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{7}{4}}c^{\frac{3}{2}}\sqrt{bx^2+a}}$$

command

```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2), x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(bx^3+ax)\sqrt{bc} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (3bx^2+2a)\sqrt{bx^2+a}\sqrt{cx}}{a^2bc^2x^3+a^3c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^2c^2x^6+2abc^2x^4+a^2c^2x^2}, x\right)$$

11.39 Problem number 627

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{ac(cx)^{\frac{3}{2}}\sqrt{bx^2+a}} - \frac{5\sqrt{bx^2+a}}{3a^2c(cx)^{\frac{3}{2}}}$$

$$5b^{\frac{3}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b}) \sqrt{\frac{bx^2}{(\sqrt{a}+x\sqrt{b})}}$$

$$6\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{9}{4}}c^{\frac{5}{2}}\sqrt{bx^2+a}$$

command

`integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 (bx^4 + ax^2) \sqrt{bc} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (5bx^2 + 2a) \sqrt{bx^2 + a} \sqrt{cx}}{3(a^2bc^3x^4 + a^3c^3x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2 + a} \sqrt{cx}}{b^2c^3x^7 + 2abc^3x^5 + a^2c^3x^3}, x\right)$$

11.40 Problem number 628

$$\int \frac{1}{(cx)^{7/2} (a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1}{ac (cx)^{\frac{5}{2}} \sqrt{bx^2 + a}} - \frac{7\sqrt{bx^2 + a}}{5a^2c (cx)^{\frac{5}{2}}} + \frac{21b\sqrt{bx^2 + a}}{5a^3c^3\sqrt{cx}} - \frac{21b^{\frac{3}{2}}\sqrt{cx} \sqrt{bx^2 + a}}{5a^3c^4 (\sqrt{a} + x\sqrt{b})} \\ & + \frac{21b^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2}{(\sqrt{a} + x\sqrt{b})}}}{5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{11}{4}} c^{\frac{7}{2}} \sqrt{bx^2 + a}} \\ & - \frac{21b^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2}{(\sqrt{a} + x\sqrt{b})}}}{10 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{11}{4}} c^{\frac{7}{2}} \sqrt{bx^2 + a}} \end{aligned}$$

command

`integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21 (b^2x^5 + abx^3) \sqrt{bc} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (21b^2x^4 + 14abx^2 - 2a^2) \sqrt{bx^2 + a}}{5(a^3bc^4x^5 + a^4c^4x^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2 + a} \sqrt{cx}}{b^2c^4x^8 + 2abc^4x^6 + a^2c^4x^4}, x\right)$$

11.41 Problem number 629

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{c(cx)^{\frac{5}{2}}}{3b(bx^2+a)^{\frac{3}{2}}} - \frac{5c^3\sqrt{cx}}{6b^2\sqrt{bx^2+a}} + \frac{5c^{\frac{7}{2}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{12 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{1}{4}} b^{\frac{9}{4}} \sqrt{bx^2+a}}$$

command

`integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(b^2c^3x^4 + 2abc^3x^2 + a^2c^3)\sqrt{bc} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (7b^2c^3x^2 + 5abc^3)\sqrt{bx^2+a}\sqrt{cx}}{6(b^5x^4 + 2ab^4x^2 + a^2b^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}c^3x^3}{b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3}, x\right)$$

11.42 Problem number 630

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{c(cx)^{\frac{3}{2}}}{3b(bx^2+a)^{\frac{3}{2}}} + \frac{c(cx)^{\frac{3}{2}}}{2ab\sqrt{bx^2+a}} - \frac{c^2\sqrt{cx}\sqrt{bx^2+a}}{2ab^{\frac{3}{2}}(\sqrt{a}+x\sqrt{b})} \\
 & + \frac{c^{\frac{5}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{3}{4}}b^{\frac{7}{4}}\sqrt{bx^2+a}} \\
 & - \frac{c^{\frac{5}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{4\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{3}{4}}b^{\frac{7}{4}}\sqrt{bx^2+a}}
 \end{aligned}$$

command

`integrate((c*x)^(5/2)/(b*x^2+a)^(5/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(b^2c^2x^4 + 2abc^2x^2 + a^2c^2)\sqrt{bc}\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (3b^2c^2x^3 + abc^2x)\sqrt{bc}}{6(ab^4x^4 + 2a^2b^3x^2 + a^3b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}c^2x^2}{b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3}, x\right)$$

11.43 Problem number 631

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{c\sqrt{cx}}{3b(bx^2+a)^{\frac{3}{2}}} + \frac{c\sqrt{cx}}{6ab\sqrt{bx^2+a}} \\
 & + \frac{c^{\frac{3}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{12\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{5}{4}}b^{\frac{5}{4}}\sqrt{bx^2+a}}
 \end{aligned}$$

command

```
integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(b^2cx^4 + 2abcx^2 + a^2c)\sqrt{bc} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (b^2cx^2 - abc)\sqrt{bx^2 + a}\sqrt{cx}}{6(ab^4x^4 + 2a^2b^3x^2 + a^3b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2 + a}\sqrt{cx}cx}{b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3}, x\right)$$

11.44 Problem number 632

$$\int \frac{\sqrt{cx}}{(a + bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(cx)^{\frac{3}{2}}}{3ac(bx^2 + a)^{\frac{3}{2}}} + \frac{(cx)^{\frac{3}{2}}}{2a^2c\sqrt{bx^2 + a}} - \frac{\sqrt{cx}\sqrt{bx^2 + a}}{2a^2\sqrt{b}(\sqrt{a} + x\sqrt{b})} \\ & + \frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{c}\sqrt{\frac{bx^2}{(\sqrt{a} + x\sqrt{b})^2}}}}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{7}{4}}b^{\frac{3}{4}}\sqrt{bx^2 + a}} \\ & - \frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{c}\sqrt{\frac{bx^2}{(\sqrt{a} + x\sqrt{b})^2}}}}{4\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{7}{4}}b^{\frac{3}{4}}\sqrt{bx^2 + a}} \end{aligned}$$

command

```
integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(b^2x^4 + 2abx^2 + a^2)\sqrt{bc} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (3b^2x^3 + 5abx)\sqrt{bx^2 + a}}{6(a^2b^3x^4 + 2a^3b^2x^2 + a^4b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2 + a}\sqrt{cx}}{b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3}, x\right)$$

11.45 Problem number 633

$$\int \frac{1}{\sqrt{cx} (a + bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{cx}}{3ac(bx^2 + a)^{\frac{3}{2}}} + \frac{5\sqrt{cx}}{6a^2c\sqrt{bx^2 + a}}$$

$$+ \frac{5\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{12\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right) a^{\frac{9}{4}} b^{\frac{1}{4}} \sqrt{c} \sqrt{bx^2 + a}}$$

command

`integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(b^2x^4 + 2abx^2 + a^2)\sqrt{bc} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (5b^2x^2 + 7ab)\sqrt{bx^2 + a}\sqrt{cx}}{6(a^2b^3cx^4 + 2a^3b^2cx^2 + a^4bc)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2 + a}\sqrt{cx}}{b^3cx^7 + 3ab^2cx^5 + 3a^2bcx^3 + a^3cx}, x\right)$$

11.46 Problem number 634

$$\int \frac{1}{(cx)^{3/2} (a + bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{1}{3ac(bx^2+a)^{\frac{3}{2}}\sqrt{cx}} + \frac{7}{6a^2c\sqrt{cx}\sqrt{bx^2+a}} - \frac{7\sqrt{bx^2+a}}{2a^3c\sqrt{cx}} + \frac{7\sqrt{b}\sqrt{cx}\sqrt{bx^2+a}}{2a^3c^2(\sqrt{a}+x\sqrt{b})}$$

$$\frac{7b^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{bx^2}{(\sqrt{a}+x\sqrt{b})^2}}}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{11}{4}}c^{\frac{3}{2}}\sqrt{bx^2+a}}$$

$$+ \frac{7b^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{bx^2}{(\sqrt{a}+x\sqrt{b})^2}}}{4\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{11}{4}}c^{\frac{3}{2}}\sqrt{bx^2+a}}$$

command

`integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21(b^2x^5 + 2abx^3 + a^2x)\sqrt{bc}\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (21b^2x^4 + 35abx^2 + 12a^2x)\sqrt{bc}}{6(a^3b^2c^2x^5 + 2a^4bc^2x^3 + a^5c^2x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^3c^2x^8 + 3ab^2c^2x^6 + 3a^2bc^2x^4 + a^3c^2x^2}, x\right)$$

11.47 Problem number 635

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{1}{3ac(cx)^{\frac{3}{2}}(bx^2+a)^{\frac{3}{2}}} + \frac{3}{2a^2c(cx)^{\frac{3}{2}}\sqrt{bx^2+a}} - \frac{5\sqrt{bx^2+a}}{2a^3c(cx)^{\frac{3}{2}}}$$

$$\frac{5b^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{\frac{bx^2}{(\sqrt{a}+x\sqrt{b})^2}}}{4\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{cx}}{a^{\frac{1}{4}}\sqrt{c}}\right)\right)a^{\frac{13}{4}}c^{\frac{5}{2}}\sqrt{bx^2+a}}$$

command

```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 (b^2 x^6 + 2 a b x^4 + a^2 x^2) \sqrt{b c} \operatorname{weierstrassPInverse}\left(-\frac{4 a}{b}, 0, x\right) + (15 b^2 x^4 + 21 a b x^2 + 4 a^2) \sqrt{b x^2 + a} \sqrt{c x}}{6 (a^3 b^2 c^3 x^6 + 2 a^4 b c^3 x^4 + a^5 c^3 x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b x^2 + a} \sqrt{c x}}{b^3 c^3 x^9 + 3 a b^2 c^3 x^7 + 3 a^2 b c^3 x^5 + a^3 c^3 x^3}, x\right)$$

11.48 Problem number 636

$$\int \frac{1}{(c x)^{7/2} (a + b x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1}{3 a c (c x)^{\frac{5}{2}} (b x^2 + a)^{\frac{3}{2}}} + \frac{11}{6 a^2 c (c x)^{\frac{5}{2}} \sqrt{b x^2 + a}} \\ & - \frac{77 \sqrt{b x^2 + a}}{30 a^3 c (c x)^{\frac{5}{2}}} + \frac{77 b \sqrt{b x^2 + a}}{10 a^4 c^3 \sqrt{c x}} - \frac{77 b^{\frac{3}{2}} \sqrt{c x} \sqrt{b x^2 + a}}{10 a^4 c^4 (\sqrt{a} + x \sqrt{b})} \\ & + \frac{77 b^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{c x}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{c x}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x \sqrt{b}) \sqrt{\frac{b x^2}{(\sqrt{a} + x \sqrt{b})}}}{10 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{c x}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right) a^{\frac{15}{4}} c^{\frac{7}{2}} \sqrt{b x^2 + a}} \\ & - \frac{77 b^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{c x}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{c x}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x \sqrt{b}) \sqrt{\frac{b x^2}{(\sqrt{a} + x \sqrt{b})}}}{20 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{c x}}{a^{\frac{1}{4}} \sqrt{c}}\right)\right) a^{\frac{15}{4}} c^{\frac{7}{2}} \sqrt{b x^2 + a}} \end{aligned}$$

command

```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{231 (b^3 x^7 + 2 ab^2 x^5 + a^2 b x^3) \sqrt{bc} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (231 b^3 x^6 + 385 ab^2 x^5)}{30 (a^4 b^2 c^4 x^7 + 2 a^5 b c^4 x^5 + a^6 c^4 x^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2 + a} \sqrt{cx}}{b^3 c^4 x^{10} + 3 ab^2 c^4 x^8 + 3 a^2 b c^4 x^6 + a^3 c^4 x^4}, x\right)$$

11.49 Problem number 637

$$\int \frac{(cx)^{5/2}}{\sqrt{3a - 2ax^2}} dx$$

Optimal antiderivative

$$\frac{9 \cdot 3^{1/4} c^2 \operatorname{EllipticE}\left(\frac{\sqrt{3 - x\sqrt{6}} \sqrt{6}}{6}, \sqrt{2}\right) \sqrt{cx} \sqrt{-2x^2 + 3} \cdot 2^{1/4}}{10 \sqrt{x} \sqrt{-2ax^2 + 3a}} - \frac{c(cx)^{3/2} \sqrt{-2ax^2 + 3a}}{5a}$$

command

`integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{-2ax^2 + 3a} \sqrt{cx} c^2 x - 9 \sqrt{2} \sqrt{-ac} c^2 \operatorname{weierstrassZeta}(6, 0, \operatorname{weierstrassPInverse}(6, 0, x))}{10a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2ax^2 + 3a} \sqrt{cx} c^2 x^2}{2ax^2 - 3a}, x\right)$$

11.50 Problem number 638

$$\int \frac{(cx)^{3/2}}{\sqrt{3a - 2ax^2}} dx$$

Optimal antiderivative

$$\frac{c^{3/2} \operatorname{EllipticF}\left(\frac{2^{1/4} 3^{3/4} \sqrt{cx}}{3\sqrt{c}}, i\right) \sqrt{-2x^2 + 3} \cdot 6^{3/4}}{6 \sqrt{a(-2x^2 + 3)}} - \frac{c \sqrt{cx} \sqrt{-2ax^2 + 3a}}{3a}$$

command

```
integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{2}\sqrt{-ac}\operatorname{cweierstrassPInverse}(6,0,x)+2\sqrt{-2ax^2+3a}\sqrt{cx}c}{6a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2ax^2+3a}\sqrt{cx}cx}{2ax^2-3a},x\right)$$

11.51 Problem number 639

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}} dx$$

Optimal antiderivative

$$\frac{6^{\frac{1}{4}}\operatorname{EllipticE}\left(\frac{\sqrt{3-x\sqrt{6}}\sqrt{6}}{6},\sqrt{2}\right)\sqrt{cx}\sqrt{-2x^2+3}}{\sqrt{x}\sqrt{-2ax^2+3a}}$$

command

```
integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}\sqrt{-ac}\operatorname{weierstrassZeta}(6,0,\operatorname{weierstrassPInverse}(6,0,x))}{a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2ax^2+3a}\sqrt{cx}}{2ax^2-3a},x\right)$$

11.52 Problem number 640

$$\int \frac{1}{\sqrt{cx} \sqrt{3a - 2ax^2}} dx$$

Optimal antiderivative

$$\frac{2^{\frac{3}{4}} \text{EllipticF}\left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}, i\right) \sqrt{-2x^2 + 3} 3^{\frac{3}{4}}}{3\sqrt{c} \sqrt{a(-2x^2 + 3)}}$$

command

`integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{2} \sqrt{-ac} \text{weierstrassPInverse}(6, 0, x)}{ac}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2ax^2 + 3a} \sqrt{cx}}{2acx^3 - 3acx}, x\right)$$

11.53 Problem number 641

$$\int \frac{1}{(cx)^{3/2} \sqrt{3a - 2ax^2}} dx$$

Optimal antiderivative

$$\frac{2 \cdot 2^{\frac{1}{4}} \text{EllipticE}\left(\frac{\sqrt{3-x}\sqrt{6}}{6} \sqrt{6}, \sqrt{2}\right) \sqrt{cx} \sqrt{-2x^2 + 3} 3^{\frac{1}{4}}}{3c^2 \sqrt{x} \sqrt{-2ax^2 + 3a}} - \frac{2\sqrt{-2ax^2 + 3a}}{3ac\sqrt{cx}}$$

command

`integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\left(\sqrt{2} \sqrt{-ac} x \text{weierstrassZeta}(6, 0, \text{weierstrassPInverse}(6, 0, x)) + \sqrt{-2ax^2 + 3a} \sqrt{cx}\right)}{3ac^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2ax^2 + 3a} \sqrt{cx}}{2ac^2x^4 - 3ac^2x^2}, x\right)$$

11.54 Problem number 642

$$\int \frac{1}{(cx)^{5/2} \sqrt{3a - 2ax^2}} dx$$

Optimal antiderivative

$$\frac{2 \cdot 2^{3/4} \operatorname{EllipticF}\left(\frac{2^{1/4} 3^{3/4} \sqrt{cx}}{3\sqrt{c}}, i\right) \sqrt{-2x^2 + 3} \cdot 3^{3/4}}{27c^{5/2} \sqrt{a(-2x^2 + 3)}} - \frac{2\sqrt{-2ax^2 + 3a}}{9ac(cx)^{3/2}}$$

command

`integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\left(\sqrt{2}\sqrt{-ac}x^2\operatorname{weierstrassPInverse}(6,0,x) + \sqrt{-2ax^2 + 3a}\sqrt{cx}\right)}{9ac^3x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2ax^2 + 3a}\sqrt{cx}}{2ac^3x^5 - 3ac^3x^3}, x\right)$$

11.55 Problem number 643

$$\int \frac{(cx)^{5/2}}{(3a - 2ax^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{c(cx)^{3/2}}{2a\sqrt{-2ax^2 + 3a}} + \frac{3 \cdot 3^{1/4} c^2 \operatorname{EllipticE}\left(\frac{\sqrt{3-x\sqrt{6}}\sqrt{6}}{6}, \sqrt{2}\right) \sqrt{cx} \sqrt{-2x^2 + 3} \cdot 2^{1/4}}{4a\sqrt{x} \sqrt{-2ax^2 + 3a}}$$

command

`integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{-2ax^2 + 3a}\sqrt{cx}c^2x + 3\sqrt{2}(2c^2x^2 - 3c^2)\sqrt{-ac}\operatorname{weierstrassZeta}(6,0,\operatorname{weierstrassPInverse}(6,0,x))}{4(2a^2x^2 - 3a^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-2ax^2 + 3a}\sqrt{cx}c^2x^2}{4a^2x^4 - 12a^2x^2 + 9a^2}, x\right)$$

11.56 Problem number 644

$$\int \frac{(cx)^{3/2}}{(3a - 2ax^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{c^{3/2} \operatorname{EllipticF}\left(\frac{2^{1/4} 3^{3/4} \sqrt{cx}}{3\sqrt{c}}, i\right) \sqrt{-2x^2 + 3} 6^{3/4}}{12a\sqrt{a(-2x^2 + 3)}} + \frac{c\sqrt{cx}}{2a\sqrt{-2ax^2 + 3a}}$$

command

```
integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (2cx^2 - 3c) \sqrt{-ac} \operatorname{weierstrassPInverse}(6, 0, x) - 2 \sqrt{-2ax^2 + 3a} \sqrt{cx} c}{4(2a^2x^2 - 3a^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-2ax^2 + 3a} \sqrt{cx} cx}{4a^2x^4 - 12a^2x^2 + 9a^2}, x\right)$$

11.57 Problem number 645

$$\int \frac{\sqrt{cx}}{(3a - 2ax^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(cx)^{3/2}}{3ac\sqrt{-2ax^2 + 3a}} + \frac{\operatorname{EllipticE}\left(\frac{\sqrt{3 - x\sqrt{6}} \sqrt{6}}{6}, \sqrt{2}\right) \sqrt{cx} \sqrt{-2x^2 + 3} 6^{1/4}}{6a\sqrt{x} \sqrt{-2ax^2 + 3a}}$$

command

```
integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-ac} (2x^2 - 3) \operatorname{weierstrassZeta}(6, 0, \operatorname{weierstrassPInverse}(6, 0, x)) + 2 \sqrt{-2ax^2 + 3a} \sqrt{cx} x}{6(2a^2x^2 - 3a^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-2ax^2 + 3a} \sqrt{cx}}{4a^2x^4 - 12a^2x^2 + 9a^2}, x\right)$$

11.58 Problem number 646

$$\int \frac{1}{\sqrt{cx} (3a - 2ax^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{2^{1/4} 3^{3/4} \sqrt{cx}}{3\sqrt{c}}, i\right) \sqrt{-2x^2 + 3} 6^{3/4}}{18a\sqrt{c} \sqrt{a(-2x^2 + 3)}} + \frac{\sqrt{cx}}{3ac\sqrt{-2ax^2 + 3a}}$$

command

`integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{2} \sqrt{-ac} (2x^2 - 3) \text{weierstrassPInverse}(6, 0, x) + 2 \sqrt{-2ax^2 + 3a} \sqrt{cx}}{6(2a^2cx^2 - 3a^2c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-2ax^2 + 3a} \sqrt{cx}}{4a^2cx^5 - 12a^2cx^3 + 9a^2cx}, x\right)$$

11.59 Problem number 647

$$\int \frac{1}{(cx)^{3/2} (3a - 2ax^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{3ac\sqrt{cx} \sqrt{-2ax^2 + 3a}} + \frac{2^{1/4} \text{EllipticE}\left(\frac{\sqrt{3-x}\sqrt{6}}{6} \sqrt{6}, \sqrt{2}\right) \sqrt{cx} \sqrt{-2x^2 + 3} 3^{1/4}}{3a c^2 \sqrt{x} \sqrt{-2ax^2 + 3a}} - \frac{\sqrt{-2ax^2 + 3a}}{3a^2 c \sqrt{cx}}$$

command

`integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{2} (2x^3 - 3x) \sqrt{-ac} \text{weierstrassZeta}(6, 0, \text{weierstrassPInverse}(6, 0, x)) + 2 \sqrt{-2ax^2 + 3a} \sqrt{cx} (x^2 - 1)}{3(2a^2c^2x^3 - 3a^2c^2x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-2ax^2 + 3a} \sqrt{cx}}{4a^2c^2x^6 - 12a^2c^2x^4 + 9a^2c^2x^2}, x\right)$$

11.60 Problem number 648

$$\int \frac{1}{(cx)^{5/2} (3a - 2ax^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{5 \cdot 2^{\frac{3}{4}} \operatorname{EllipticF}\left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}, i\right) \sqrt{-2x^2 + 3} \cdot 3^{\frac{3}{4}}}{81a c^{\frac{5}{2}} \sqrt{a(-2x^2 + 3)}} + \frac{1}{3ac (cx)^{\frac{3}{2}} \sqrt{-2ax^2 + 3a}} - \frac{5\sqrt{-2ax^2 + 3a}}{27a^2 c (cx)^{\frac{3}{2}}}$$

command

`integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(2x^4 - 3x^2)\sqrt{-ac} \operatorname{weierstrassPInverse}(6, 0, x) + 2\sqrt{-2ax^2 + 3a}\sqrt{cx}(5x^2 - 3)}{27(2a^2c^3x^4 - 3a^2c^3x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-2ax^2 + 3a}\sqrt{cx}}{4a^2c^3x^7 - 12a^2c^3x^5 + 9a^2c^3x^3}, x\right)$$

11.61 Problem number 650

$$\int \frac{1}{\sqrt{x} \sqrt{1 + ax^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(a^{\frac{1}{4}}\sqrt{x}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(a^{\frac{1}{4}}\sqrt{x}\right)\right), \frac{\sqrt{2}}{2}\right) (1 + x\sqrt{a}) \sqrt{\frac{ax^2 + 1}{(1 + x\sqrt{a})^2}}}{\cos\left(2 \arctan\left(a^{\frac{1}{4}}\sqrt{x}\right)\right) a^{\frac{1}{4}} \sqrt{ax^2 + 1}}$$

command

`integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassPInverse}\left(-\frac{4}{a}, 0, x\right)}{\sqrt{a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{ax^2 + 1}\sqrt{x}}{ax^3 + x}, x\right)$$

12 Test file number 20

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.2_Quadratic/20_1.1.2.3-a+b_x^2-^p-c+d_x^2-^q

12.1 Problem number 156

$$\int \frac{1}{\sqrt[3]{2+3x^2} (6d+dx^2)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{6}}\left(2^{\frac{1}{3}}-(3x^2+2)^{\frac{1}{3}}\right)}{x}\right) 2^{\frac{1}{6}}}{8d} \\ & + \frac{\operatorname{arctan}\left(\frac{\left(2^{\frac{1}{3}}-(3x^2+2)^{\frac{1}{3}}\right)^2 2^{\frac{5}{6}}\sqrt{3}}{18x}\right) 2^{\frac{1}{6}}\sqrt{3}}{24d} + \frac{\operatorname{arctan}\left(\frac{x\sqrt{6}}{6}\right) 2^{\frac{1}{6}}\sqrt{3}}{24d} \end{aligned}$$

command

```
integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

12.2 Problem number 157

$$\int \frac{1}{\sqrt[3]{2-3x^2} (-6d+dx^2)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{arctan}\left(\frac{2^{\frac{1}{6}}\left(2^{\frac{1}{3}}-(-3x^2+2)^{\frac{1}{3}}\right)}{x}\right) 2^{\frac{1}{6}}}{8d} \\ & + \frac{\operatorname{arctanh}\left(\frac{\left(2^{\frac{1}{3}}-(-3x^2+2)^{\frac{1}{3}}\right)^2 2^{\frac{5}{6}}\sqrt{3}}{18x}\right) 2^{\frac{1}{6}}\sqrt{3}}{24d} - \frac{\operatorname{arctanh}\left(\frac{x\sqrt{6}}{6}\right) 2^{\frac{1}{6}}\sqrt{3}}{24d} \end{aligned}$$

command

```
integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

12.3 Problem number 158

$$\int \frac{1}{\sqrt[3]{-2+3x^2}(-6d+dx^2)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2^{\frac{1}{6}}\left(2^{\frac{1}{3}}+(3x^2-2)^{\frac{1}{3}}\right)}{x}\right)2^{\frac{1}{6}}}{8d} - \frac{\operatorname{arctanh}\left(\frac{\left(2^{\frac{1}{3}}+(3x^2-2)^{\frac{1}{3}}\right)^2 2^{\frac{5}{6}}\sqrt{3}}{18x}\right)2^{\frac{1}{6}}\sqrt{3}}{24d} + \frac{\operatorname{arctanh}\left(\frac{x\sqrt{6}}{6}\right)2^{\frac{1}{6}}\sqrt{3}}{24d}$$

command

```
integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

12.4 Problem number 159

$$\int \frac{1}{\sqrt[3]{-2-3x^2}(6d+dx^2)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{6}}\left(2^{\frac{1}{3}}+(-3x^2-2)^{\frac{1}{3}}\right)}{x}\right)2^{\frac{1}{6}}}{8d} - \frac{\operatorname{arctan}\left(\frac{\left(2^{\frac{1}{3}}+(-3x^2-2)^{\frac{1}{3}}\right)^2 2^{\frac{5}{6}}\sqrt{3}}{18x}\right)2^{\frac{1}{6}}\sqrt{3}}{24d} - \frac{\operatorname{arctan}\left(\frac{x\sqrt{6}}{6}\right)2^{\frac{1}{6}}\sqrt{3}}{24d}$$

command

```
integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

12.5 Problem number 179

$$\int \sqrt{3-6x^2} \sqrt{2+4x^2} dx$$

Optimal antiderivative

$$\frac{2\operatorname{EllipticF}\left(x\sqrt{2}, i\right)\sqrt{3}}{3} + \frac{x\sqrt{6}\sqrt{-4x^2+1}}{3}$$

command

```
integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3}\sqrt{4x^2+2}\sqrt{-6x^2+3}x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{4x^2+2}\sqrt{-6x^2+3}, x\right)$$

12.6 Problem number 187

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}} dx$$

Optimal antiderivative

$$\text{EllipticE}(x, i)$$

command

```
integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{x^2+1}\sqrt{-x^2+1}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-x^2+1}}{x^2-1}, x\right)$$

12.7 Problem number 188

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(\frac{x\sqrt{6}}{2}, \frac{i\sqrt{6}}{3}\right)\sqrt{3}}{3}$$

command

```
integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{x^2+1}\sqrt{-3x^2+2}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-3x^2+2}}{3x^2-2}, x\right)$$

12.8 Problem number 189

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE}\left(\frac{x\sqrt{6}}{2}, \frac{i\sqrt{6}}{6}\right) \sqrt{3}}{3}$$

command

```
integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{x^2+4} \sqrt{-3x^2+2}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{x^2+4} \sqrt{-3x^2+2}}{3x^2-2}, x\right)$$

12.9 Problem number 190

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticE}\left(\frac{x\sqrt{6}}{2}, \frac{2i\sqrt{6}}{3}\right) \sqrt{3}}{3}$$

command

```
integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{4x^2+1} \sqrt{-3x^2+2}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{4x^2+1} \sqrt{-3x^2+2}}{3x^2-2}, x\right)$$

12.10 Problem number 191

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}} dx$$

Optimal antiderivative

$$- \text{EllipticE}(x, i) + 2 \text{EllipticF}(x, i)$$

command

```
integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{x^2+1} \sqrt{-x^2+1}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}, x\right)$$

12.11 Problem number 192

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}} dx$$

Optimal antiderivative

$$\frac{5 \text{EllipticF}\left(x, \frac{i\sqrt{6}}{2}\right) \sqrt{2}}{6} - \frac{\text{EllipticE}\left(x, \frac{i\sqrt{6}}{2}\right) \sqrt{2}}{3}$$

command

```
integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{3x^2+2} \sqrt{-x^2+1}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^2+1}}{\sqrt{3x^2+2}}, x\right)$$

12.12 Problem number 193

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{EllipticE}\left(\frac{x}{2}, i\sqrt{6}\right) \sqrt{2}}{3} + \frac{7 \operatorname{EllipticF}\left(\frac{x}{2}, i\sqrt{6}\right) \sqrt{2}}{3}$$

command

```
integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{3x^2+2} \sqrt{-x^2+4}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^2+4}}{\sqrt{3x^2+2}}, x\right)$$

12.13 Problem number 194

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}} dx$$

Optimal antiderivative

$$\frac{11 \operatorname{EllipticF}\left(2x, \frac{i\sqrt{6}}{4}\right) \sqrt{2}}{12} - \frac{2 \operatorname{EllipticE}\left(2x, \frac{i\sqrt{6}}{4}\right) \sqrt{2}}{3}$$

command

```
integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{3x^2+2} \sqrt{-4x^2+1}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-4x^2+1}}{\sqrt{3x^2+2}}, x\right)$$

12.14 Problem number 198

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(x\sqrt{2}, \frac{\sqrt{2}}{2}\right) \sqrt{-2x^2+1} \sqrt{2}}{2\sqrt{2x^2-1}}$$

command

```
integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2x^2-1} \sqrt{-x^2+1}}{2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^2+1}}{\sqrt{2x^2-1}}, x\right)$$

12.15 Problem number 203

$$\int \frac{1}{\sqrt{a+bx^2} \sqrt{c+dx^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{1+\frac{dx^2}{c}}} \sqrt{1+\frac{dx^2}{c}} \text{EllipticF}\left(\frac{x\sqrt{d}}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}, \sqrt{1-\frac{bc}{ad}}\right) \sqrt{c} \sqrt{bx^2+a}}{a\sqrt{d} \sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}} \sqrt{dx^2+c}}$$

command

```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{ac} \sqrt{-\frac{b}{a}} \operatorname{ellipticF}\left(x \sqrt{-\frac{b}{a}}, \frac{ad}{bc}\right)}{bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a} \sqrt{dx^2+c}}{bdx^4+(bc+ad)x^2+ac}, x\right)$$

12.16 Problem number 214

$$\int \frac{1}{\sqrt{a+bx^2} \sqrt{c+dx^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{1+\frac{dx^2}{c}}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticF}\left(\frac{x\sqrt{d}}{\sqrt{c} \sqrt{1+\frac{dx^2}{c}}}, \sqrt{1-\frac{bc}{ad}}\right) \sqrt{c} \sqrt{bx^2+a}}{a\sqrt{d} \sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}} \sqrt{dx^2+c}}$$

command

```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{ac} \sqrt{-\frac{b}{a}} \operatorname{ellipticF}\left(x \sqrt{-\frac{b}{a}}, \frac{ad}{bc}\right)}{bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a} \sqrt{dx^2+c}}{bdx^4+(bc+ad)x^2+ac}, x\right)$$

12.17 Problem number 215

$$\int \frac{1}{\sqrt{a - bx^2} \sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{b}}{\sqrt{a}}, \sqrt{-\frac{ad}{bc}}\right) \sqrt{a} \sqrt{1 - \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}}}{\sqrt{b} \sqrt{-bx^2 + a} \sqrt{dx^2 + c}}$$

command

`integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{ac} \sqrt{\frac{b}{a}} \text{ellipticF}\left(x \sqrt{\frac{b}{a}}, -\frac{ad}{bc}\right)}{bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-bx^2 + a} \sqrt{dx^2 + c}}{bdx^4 + (bc - ad)x^2 - ac}, x\right)$$

12.18 Problem number 216

$$\int \frac{1}{\sqrt{a + bx^2} \sqrt{c - dx^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{d}}{\sqrt{c}}, \sqrt{-\frac{bc}{ad}}\right) \sqrt{c} \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 - \frac{dx^2}{c}}}{\sqrt{d} \sqrt{bx^2 + a} \sqrt{-dx^2 + c}}$$

command

`integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{ac} \sqrt{\frac{d}{c}} \text{ellipticF}\left(x \sqrt{\frac{d}{c}}, -\frac{bc}{ad}\right)}{ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{bx^2 + a} \sqrt{-dx^2 + c}}{bdx^4 - (bc - ad)x^2 - ac}, x\right)$$

12.19 Problem number 217

$$\int \frac{1}{\sqrt{a-bx^2} \sqrt{c-dx^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{d}}{\sqrt{c}}, \sqrt{\frac{bc}{ad}}\right) \sqrt{c} \sqrt{1-\frac{bx^2}{a}} \sqrt{1-\frac{dx^2}{c}}}{\sqrt{d} \sqrt{-bx^2+a} \sqrt{-dx^2+c}}$$

command

```
integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{ac} \sqrt{\frac{d}{c}} \text{ellipticF}\left(x \sqrt{\frac{d}{c}}, \frac{bc}{ad}\right)}{ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-bx^2+a} \sqrt{-dx^2+c}}{bdx^4 - (bc+ad)x^2 + ac}, x\right)$$

12.20 Problem number 218

$$\int \frac{1}{\sqrt{1-x^2} \sqrt{2+5x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x, \frac{i\sqrt{10}}{2}\right) \sqrt{2}}{2}$$

command

```
integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}\left(x, -\frac{5}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x^2+2} \sqrt{-x^2+1}}{5x^4 - 3x^2 - 2}, x\right)$$

12.21 Problem number 219

$$\int \frac{1}{\sqrt{1-x^2} \sqrt{2+4x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x, i\sqrt{2}\right) \sqrt{2}}{2}$$

command

```
integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}(x, -2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{4x^2+2} \sqrt{-x^2+1}}{2(2x^4-x^2-1)}, x\right)$$

12.22 Problem number 220

$$\int \frac{1}{\sqrt{1-x^2} \sqrt{2+3x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x, \frac{i\sqrt{6}}{2}\right) \sqrt{2}}{2}$$

command

```
integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}\left(x, -\frac{3}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2+2} \sqrt{-x^2+1}}{3x^4-x^2-2}, x\right)$$

12.23 Problem number 221

$$\int \frac{1}{\sqrt{1-x^2} \sqrt{2+2x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}(x, i) \sqrt{2}}{2}$$

command

```
integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}(x, -1)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{2x^2+2} \sqrt{-x^2+1}}{2(x^4-1)}, x\right)$$

12.24 Problem number 222

$$\int \frac{1}{\sqrt{1-x^2} \sqrt{2+x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x, \frac{i\sqrt{2}}{2}\right) \sqrt{2}}{2}$$

command

```
integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}\left(x, -\frac{1}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{x^2+2} \sqrt{-x^2+1}}{x^4+x^2-2}, x\right)$$

12.25 Problem number 223

$$\int \frac{1}{\sqrt{1-x^2} \sqrt{2-x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x, \frac{\sqrt{2}}{2}\right) \sqrt{2}}{2}$$

command

```
integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}\left(x, \frac{1}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^2+2} \sqrt{-x^2+1}}{x^4-3x^2+2}, x\right)$$

12.26 Problem number 225

$$\int \frac{1}{\sqrt{2-3x^2} \sqrt{1-x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x, \frac{\sqrt{6}}{2}\right) \sqrt{2}}{2}$$

command

```
integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}\left(x, \frac{3}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^2+1} \sqrt{-3x^2+2}}{3x^4-5x^2+2}, x\right)$$

12.27 Problem number 226

$$\int \frac{1}{\sqrt{2-4x^2} \sqrt{1-x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x, \sqrt{2}\right) \sqrt{2}}{2}$$

command

```
integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}(x, 2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^2+1} \sqrt{-4x^2+2}}{2(2x^4-3x^2+1)}, x\right)$$

12.28 Problem number 227

$$\int \frac{1}{\sqrt{2-5x^2} \sqrt{1-x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x, \frac{\sqrt{10}}{2}\right) \sqrt{2}}{2}$$

command

```
integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}\left(x, \frac{5}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^2+1} \sqrt{-5x^2+2}}{5x^4-7x^2+2}, x\right)$$

12.29 Problem number 228

$$\int \frac{1}{\sqrt{1+x^2} \sqrt{2+5x^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{x^2+1}} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2+1}}, \frac{i\sqrt{6}}{2}\right) \sqrt{5x^2+2} \sqrt{2}}{2\sqrt{\frac{5x^2+2}{x^2+1}}}$$

command

```
integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2}i\sqrt{2} \operatorname{ellipticF}\left(ix, \frac{5}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{5x^2+2} \sqrt{x^2+1}}{5x^4+7x^2+2}, x\right)$$

12.30 Problem number 229

$$\int \frac{1}{\sqrt{1+x^2} \sqrt{2+4x^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{x^2+1}} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2+1}}, i\right) \sqrt{2x^2+1} \sqrt{2}}{2\sqrt{\frac{2x^2+1}{x^2+1}}}$$

command

```
integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2}i\sqrt{2} \operatorname{ellipticF}(ix, 2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{4x^2+2} \sqrt{x^2+1}}{2(2x^4+3x^2+1)}, x\right)$$

12.31 Problem number 230

$$\int \frac{1}{\sqrt{1+x^2} \sqrt{2+3x^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{x^2+1}} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2+1}}, \frac{i\sqrt{2}}{2}\right) \sqrt{3x^2+2} \sqrt{2}}{2\sqrt{\frac{3x^2+2}{x^2+1}}}$$

command

`integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2}i\sqrt{2} \operatorname{ellipticF}\left(ix, \frac{3}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{3x^2+2} \sqrt{x^2+1}}{3x^4+5x^2+2}, x\right)$$

12.32 Problem number 232

$$\int \frac{1}{\sqrt{1+x^2} \sqrt{2+x^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{x^2+1}} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2+1}}, \frac{\sqrt{2}}{2}\right) \sqrt{x^2+2} \sqrt{2}}{2\sqrt{\frac{x^2+2}{x^2+1}}}$$

command

`integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \operatorname{ellipticF}\left(\frac{1}{2}i\sqrt{2}x, 2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^2+2} \sqrt{x^2+1}}{x^4+3x^2+2}, x\right)$$

12.33 Problem number 233

$$\int \frac{1}{\sqrt{2-x^2} \sqrt{1+x^2}} dx$$

Optimal antiderivative

$$\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)$$

command

```
integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\text{ellipticF}\left(\frac{1}{2}\sqrt{2}x, -2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-x^2+2}}{x^4-x^2-2}, x\right)$$

12.34 Problem number 234

$$\int \frac{1}{\sqrt{2-2x^2} \sqrt{1+x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}(x, i)\sqrt{2}}{2}$$

command

```
integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2}\sqrt{2}\text{ellipticF}(x, -1)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-2x^2+2}}{2(x^4-1)}, x\right)$$

12.35 Problem number 235

$$\int \frac{1}{\sqrt{2-3x^2} \sqrt{1+x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{6}}{2}, \frac{i\sqrt{6}}{3}\right) \sqrt{3}}{3}$$

command

```
integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{3} \text{ellipticF}\left(\frac{1}{2} \sqrt{3} \sqrt{2} x, -\frac{2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{x^2+1} \sqrt{-3x^2+2}}{3x^4+x^2-2}, x\right)$$

12.36 Problem number 236

$$\int \frac{1}{\sqrt{2-4x^2} \sqrt{1+x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x\sqrt{2}, \frac{i\sqrt{2}}{2}\right)}{2}$$

command

```
integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \text{ellipticF}\left(\sqrt{2} x, -\frac{1}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{x^2+1} \sqrt{-4x^2+2}}{2(2x^4+x^2-1)}, x\right)$$

12.37 Problem number 237

$$\int \frac{1}{\sqrt{2-5x^2} \sqrt{1+x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{10}}{2}, \frac{i\sqrt{10}}{5}\right) \sqrt{5}}{5}$$

command

```
integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{5} \sqrt{5} \text{ellipticF}\left(\frac{1}{2} \sqrt{5} \sqrt{2} x, -\frac{2}{5}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{x^2+1} \sqrt{-5x^2+2}}{5x^4+3x^2-2}, x\right)$$

12.38 Problem number 248

$$\int \frac{1}{\sqrt{-1-x^2} \sqrt{2+5x^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{x^2+1}} \sqrt{x^2+1} \text{EllipticF}\left(\frac{x}{\sqrt{x^2+1}}, \frac{i\sqrt{6}}{2}\right) \sqrt{5x^2+2} \sqrt{2}}{2\sqrt{-x^2-1} \sqrt{\frac{5x^2+2}{x^2+1}}}$$

command

```
integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} i \sqrt{-2} \text{ellipticF}\left(ix, \frac{5}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{5x^2+2} \sqrt{-x^2-1}}{5x^4+7x^2+2}, x\right)$$

12.39 Problem number 249

$$\int \frac{1}{\sqrt{-1-x^2} \sqrt{2+4x^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{x^2+1}} \sqrt{x^2+1} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2+1}}, i\right) \sqrt{2x^2+1} \sqrt{2}}{2\sqrt{-x^2-1} \sqrt{\frac{2x^2+1}{x^2+1}}}$$

command

```
integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} i \sqrt{-2} \operatorname{ellipticF}(i x, 2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{4x^2+2} \sqrt{-x^2-1}}{2(2x^4+3x^2+1)}, x\right)$$

12.40 Problem number 250

$$\int \frac{1}{\sqrt{-1-x^2} \sqrt{2+3x^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{x^2+1}} \sqrt{x^2+1} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2+1}}, \frac{i\sqrt{2}}{2}\right) \sqrt{3x^2+2} \sqrt{2}}{2\sqrt{-x^2-1} \sqrt{\frac{3x^2+2}{x^2+1}}}$$

command

```
integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} i \sqrt{-2} \operatorname{ellipticF}\left(i x, \frac{3}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+2} \sqrt{-x^2-1}}{3x^4+5x^2+2}, x\right)$$

12.41 Problem number 252

$$\int \frac{1}{\sqrt{-1-x^2} \sqrt{2+x^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{x^2+1}} \sqrt{x^2+1} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2+1}}, \frac{\sqrt{2}}{2}\right) \sqrt{x^2+2} \sqrt{2}}{2\sqrt{-x^2-1} \sqrt{\frac{x^2+2}{x^2+1}}}$$

command

`integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2}i \sqrt{2} \sqrt{-2} \operatorname{ellipticF}\left(\frac{1}{2}i \sqrt{2} x, 2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{x^2+2} \sqrt{-x^2-1}}{x^4+3x^2+2}, x\right)$$

12.42 Problem number 253

$$\int \frac{1}{\sqrt{-1-x^2} \sqrt{2-x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \sqrt{x^2+1}}{\sqrt{-x^2-1}}$$

command

`integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2} \sqrt{2} \sqrt{-2} \operatorname{ellipticF}\left(\frac{1}{2} \sqrt{2} x, -2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^2+2} \sqrt{-x^2-1}}{x^4-x^2-2}, x\right)$$

12.43 Problem number 254

$$\int \frac{1}{\sqrt{2-2x^2} \sqrt{-1-x^2}} dx$$

Optimal antiderivative

$$-\frac{x^2 \operatorname{EllipticF}\left(\frac{1}{x}, i\right) \sqrt{1-\frac{1}{x^4}}}{\sqrt{-2x^2+2} \sqrt{-x^2-1}}$$

command

```
integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2} \sqrt{-2} \operatorname{ellipticF}(x, -1)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^2-1} \sqrt{-2x^2+2}}{2(x^4-1)}, x\right)$$

12.44 Problem number 255

$$\int \frac{1}{\sqrt{2-3x^2} \sqrt{-1-x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticF}\left(\frac{x\sqrt{6}}{2}, \frac{i\sqrt{6}}{3}\right) \sqrt{x^2+1} \sqrt{3}}{3\sqrt{-x^2-1}}$$

command

```
integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{3} \sqrt{2} \sqrt{-2} \operatorname{ellipticF}\left(\frac{1}{2} \sqrt{3} \sqrt{2} x, -\frac{2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^2-1} \sqrt{-3x^2+2}}{3x^4+x^2-2}, x\right)$$

12.45 Problem number 256

$$\int \frac{1}{\sqrt{2-4x^2} \sqrt{-1-x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x\sqrt{2}, \frac{i\sqrt{2}}{2}\right) \sqrt{x^2+1}}{2\sqrt{-x^2-1}}$$

command

`integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{4} \sqrt{2} \sqrt{-2} \text{ellipticF}\left(\sqrt{2} x, -\frac{1}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^2-1} \sqrt{-4x^2+2}}{2(2x^4+x^2-1)}, x\right)$$

12.46 Problem number 257

$$\int \frac{1}{\sqrt{2-5x^2} \sqrt{-1-x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{10}}{2}, \frac{i\sqrt{10}}{5}\right) \sqrt{x^2+1} \sqrt{5}}{5\sqrt{-x^2-1}}$$

command

`integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{10} \sqrt{5} \sqrt{2} \sqrt{-2} \text{ellipticF}\left(\frac{1}{2} \sqrt{5} \sqrt{2} x, -\frac{2}{5}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^2-1} \sqrt{-5x^2+2}}{5x^4+3x^2-2}, x\right)$$

12.47 Problem number 290

$$\int \frac{1}{\sqrt{2+bx^2} \sqrt{3+dx^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{3dx^2+9}} \sqrt{3dx^2+9} \operatorname{EllipticF}\left(\frac{x\sqrt{d}\sqrt{3}}{\sqrt{3dx^2+9}}, \sqrt{\frac{4-\frac{6b}{d}}{2}}\right) \sqrt{2} \sqrt{bx^2+2}}{2\sqrt{d} \sqrt{\frac{bx^2+2}{dx^2+3}} \sqrt{dx^2+3}}$$

command

`integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{6} \sqrt{2} \sqrt{-b} \operatorname{ellipticF}\left(\frac{1}{2} \sqrt{2} \sqrt{-b} x, \frac{2d}{3b}\right)}{6b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+2} \sqrt{dx^2+3}}{bdx^4+(3b+2d)x^2+6}, x\right)$$

12.48 Problem number 291

$$\int \frac{1}{\sqrt{4-x^2} \sqrt{c+dx^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticF}\left(\frac{x}{2}, 2\sqrt{-\frac{d}{c}}\right) \sqrt{1+\frac{dx^2}{c}}}{\sqrt{dx^2+c}}$$

command

`integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\operatorname{ellipticF}\left(\frac{1}{2} x, -\frac{4d}{c}\right)}{\sqrt{c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^2+c} \sqrt{-x^2+4}}{dx^4+(c-4d)x^2-4c}, x\right)$$

12.49 Problem number 292

$$\int \frac{1}{\sqrt{4+x^2} \sqrt{c+dx^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{x^2+4}} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2+4}}, \sqrt{1-\frac{4d}{c}}\right) \sqrt{dx^2+c}}{c \sqrt{\frac{dx^2+c}{c(x^2+4)}}$$

command

```
integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{i \operatorname{ellipticF}\left(\frac{1}{2}ix, \frac{4d}{c}\right)}{\sqrt{c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^2+c} \sqrt{x^2+4}}{dx^4+(c+4d)x^2+4c}, x\right)$$

12.50 Problem number 294

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{EllipticE}(cx, i)}{c} + \frac{2 \operatorname{EllipticF}(cx, i)}{c}$$

command

```
integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{c^2x^2+1} \sqrt{-c^2x^2+1}}{c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}, x\right)$$

12.51 Problem number 296

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{x^2} \operatorname{EllipticE}\left(\frac{\sqrt{-6x^2+4}}{2}, \sqrt{2}\right) \sqrt{3}}{3x}$$

command

```
integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{3x^2-1} \sqrt{-3x^2+2}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2-1} \sqrt{-3x^2+2}}{3x^2-2}, x\right)$$

13 Test file number 21

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.2_Quadratic/21_1.1.2.4-
e_x~m-a+b_x^2~p-c+d_x^2~q

13.1 Problem number 478

$$\int \frac{1}{x^{5/2} (a+bx^2)(c+dx^2)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{7ad - 4bc}{6a^2 c^2 (-ad + bc) x^{\frac{3}{2}}} - \frac{d}{2c(-ad + bc) x^{\frac{3}{2}} (dx^2 + c)} + \frac{b^{\frac{11}{4}} \arctan\left(1 - \frac{b^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{2a^{\frac{7}{4}} (-ad + bc)^2} \\
& - \frac{b^{\frac{11}{4}} \arctan\left(1 + \frac{b^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{2a^{\frac{7}{4}} (-ad + bc)^2} - \frac{d^{\frac{7}{4}} (-7ad + 11bc) \arctan\left(1 - \frac{d^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{8c^{\frac{11}{4}} (-ad + bc)^2} \\
& + \frac{d^{\frac{7}{4}} (-7ad + 11bc) \arctan\left(1 + \frac{d^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{8c^{\frac{11}{4}} (-ad + bc)^2} \\
& + \frac{b^{\frac{11}{4}} \ln\left(\sqrt{a} + x\sqrt{b} - a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4a^{\frac{7}{4}} (-ad + bc)^2} \\
& - \frac{b^{\frac{11}{4}} \ln\left(\sqrt{a} + x\sqrt{b} + a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4a^{\frac{7}{4}} (-ad + bc)^2} \\
& - \frac{d^{\frac{7}{4}} (-7ad + 11bc) \ln\left(\sqrt{c} + x\sqrt{d} - c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{16c^{\frac{11}{4}} (-ad + bc)^2} \\
& + \frac{d^{\frac{7}{4}} (-7ad + 11bc) \ln\left(\sqrt{c} + x\sqrt{d} + c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{16c^{\frac{11}{4}} (-ad + bc)^2}
\end{aligned}$$

command

```
integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

13.2 Problem number 479

$$\int \frac{1}{x^{7/2} (a + bx^2) (c + dx^2)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{9ad - 4bc}{10a^2 c^2 (-ad + bc) x^{\frac{5}{2}}} - \frac{d}{2c(-ad + bc) x^{\frac{5}{2}} (dx^2 + c)} - \frac{b^{\frac{13}{4}} \arctan\left(1 - \frac{b^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{2a^{\frac{9}{4}} (-ad + bc)^2} \\
& + \frac{b^{\frac{13}{4}} \arctan\left(1 + \frac{b^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{2a^{\frac{9}{4}} (-ad + bc)^2} + \frac{d^{\frac{9}{4}} (-9ad + 13bc) \arctan\left(1 - \frac{d^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{8c^{\frac{13}{4}} (-ad + bc)^2} \\
& - \frac{d^{\frac{9}{4}} (-9ad + 13bc) \arctan\left(1 + \frac{d^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{8c^{\frac{13}{4}} (-ad + bc)^2} \\
& + \frac{b^{\frac{13}{4}} \ln\left(\sqrt{a} + x\sqrt{b} - a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4a^{\frac{9}{4}} (-ad + bc)^2} \\
& - \frac{b^{\frac{13}{4}} \ln\left(\sqrt{a} + x\sqrt{b} + a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4a^{\frac{9}{4}} (-ad + bc)^2} \\
& - \frac{d^{\frac{9}{4}} (-9ad + 13bc) \ln\left(\sqrt{c} + x\sqrt{d} - c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{16c^{\frac{13}{4}} (-ad + bc)^2} \\
& + \frac{d^{\frac{9}{4}} (-9ad + 13bc) \ln\left(\sqrt{c} + x\sqrt{d} + c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{16c^{\frac{13}{4}} (-ad + bc)^2} + \frac{-9a^2 d^2 + 4abcd + 4b^2 c^2}{2a^2 c^3 (-ad + bc) \sqrt{x}}
\end{aligned}$$

command

```
integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

13.3 Problem number 480

$$\int \frac{x^{7/2}}{(a + bx^2)(c + dx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{a^{\frac{5}{4}} b^{\frac{3}{4}} \arctan\left(1 - \frac{b^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{2(-ad + bc)^3} + \frac{a^{\frac{5}{4}} b^{\frac{3}{4}} \arctan\left(1 + \frac{b^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{2(-ad + bc)^3} \\
& - \frac{(-5a^2 d^2 - 30abcd + 3b^2 c^2) \arctan\left(1 - \frac{d^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{64c^{\frac{3}{4}} d^{\frac{5}{4}} (-ad + bc)^3} \\
& + \frac{(-5a^2 d^2 - 30abcd + 3b^2 c^2) \arctan\left(1 + \frac{d^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{64c^{\frac{3}{4}} d^{\frac{5}{4}} (-ad + bc)^3} \\
& - \frac{a^{\frac{5}{4}} b^{\frac{3}{4}} \ln\left(\sqrt{a} + x\sqrt{b} - a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4(-ad + bc)^3} \\
& + \frac{a^{\frac{5}{4}} b^{\frac{3}{4}} \ln\left(\sqrt{a} + x\sqrt{b} + a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4(-ad + bc)^3} \\
& - \frac{(-5a^2 d^2 - 30abcd + 3b^2 c^2) \ln\left(\sqrt{c} + x\sqrt{d} - c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{128c^{\frac{3}{4}} d^{\frac{5}{4}} (-ad + bc)^3} \\
& + \frac{(-5a^2 d^2 - 30abcd + 3b^2 c^2) \ln\left(\sqrt{c} + x\sqrt{d} + c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{128c^{\frac{3}{4}} d^{\frac{5}{4}} (-ad + bc)^3} \\
& - \frac{c\sqrt{x}}{4d(-ad + bc)(dx^2 + c)^2} + \frac{(-9ad + bc)\sqrt{x}}{16d(-ad + bc)^2(dx^2 + c)}
\end{aligned}$$

command

```
integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

13.4 Problem number 482

$$\int \frac{x^{3/2}}{(a + bx^2)(c + dx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{a^{\frac{1}{4}} b^{\frac{7}{4}} \arctan\left(1 - \frac{b^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{2(-ad + bc)^3} - \frac{a^{\frac{1}{4}} b^{\frac{7}{4}} \arctan\left(1 + \frac{b^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{2(-ad + bc)^3} \\
& - \frac{(-3a^2 d^2 + 14abcd + 21b^2 c^2) \arctan\left(1 - \frac{d^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{64c^{\frac{7}{4}} d^{\frac{1}{4}} (-ad + bc)^3} \\
& + \frac{(-3a^2 d^2 + 14abcd + 21b^2 c^2) \arctan\left(1 + \frac{d^{\frac{1}{4}} \sqrt{2} \sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{64c^{\frac{7}{4}} d^{\frac{1}{4}} (-ad + bc)^3} \\
& + \frac{a^{\frac{1}{4}} b^{\frac{7}{4}} \ln\left(\sqrt{a} + x\sqrt{b} - a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4(-ad + bc)^3} \\
& - \frac{a^{\frac{1}{4}} b^{\frac{7}{4}} \ln\left(\sqrt{a} + x\sqrt{b} + a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4(-ad + bc)^3} \\
& - \frac{(-3a^2 d^2 + 14abcd + 21b^2 c^2) \ln\left(\sqrt{c} + x\sqrt{d} - c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{128c^{\frac{7}{4}} d^{\frac{1}{4}} (-ad + bc)^3} \\
& + \frac{(-3a^2 d^2 + 14abcd + 21b^2 c^2) \ln\left(\sqrt{c} + x\sqrt{d} + c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{128c^{\frac{7}{4}} d^{\frac{1}{4}} (-ad + bc)^3} \\
& + \frac{\sqrt{x}}{4(-ad + bc)(dx^2 + c)^2} + \frac{(ad + 7bc)\sqrt{x}}{16c(-ad + bc)^2(dx^2 + c)}
\end{aligned}$$

command

```
integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

13.5 Problem number 484

$$\int \frac{1}{\sqrt{x} (a + bx^2) (c + dx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{b^{\frac{11}{4}} \arctan\left(1 - \frac{b^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{2a^{\frac{3}{4}}(-ad+bc)^3} + \frac{b^{\frac{11}{4}} \arctan\left(1 + \frac{b^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{2a^{\frac{3}{4}}(-ad+bc)^3} \\
& + \frac{d^{\frac{3}{4}}(21a^2d^2 - 66abcd + 77b^2c^2) \arctan\left(1 - \frac{d^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{64c^{\frac{11}{4}}(-ad+bc)^3} \\
& - \frac{d^{\frac{3}{4}}(21a^2d^2 - 66abcd + 77b^2c^2) \arctan\left(1 + \frac{d^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{64c^{\frac{11}{4}}(-ad+bc)^3} \\
& - \frac{b^{\frac{11}{4}} \ln\left(\sqrt{a} + x\sqrt{b} - a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{4a^{\frac{3}{4}}(-ad+bc)^3} \\
& + \frac{b^{\frac{11}{4}} \ln\left(\sqrt{a} + x\sqrt{b} + a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{4a^{\frac{3}{4}}(-ad+bc)^3} \\
& + \frac{d^{\frac{3}{4}}(21a^2d^2 - 66abcd + 77b^2c^2) \ln\left(\sqrt{c} + x\sqrt{d} - c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{128c^{\frac{11}{4}}(-ad+bc)^3} \\
& - \frac{d^{\frac{3}{4}}(21a^2d^2 - 66abcd + 77b^2c^2) \ln\left(\sqrt{c} + x\sqrt{d} + c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{128c^{\frac{11}{4}}(-ad+bc)^3} \\
& - \frac{d\sqrt{x}}{4c(-ad+bc)(dx^2+c)^2} - \frac{d(-7ad+15bc)\sqrt{x}}{16c^2(-ad+bc)^2(dx^2+c)}
\end{aligned}$$

command

```
integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

13.6 Problem number 491

$$\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d(ad+bc)x^{\frac{3}{2}}}{2ac(-ad+bc)^2(dx^2+c)} + \frac{bx^{\frac{3}{2}}}{2a(-ad+bc)(bx^2+a)(dx^2+c)} \\ & - \frac{b^{\frac{5}{4}}(-9ad+bc) \arctan\left(1 - \frac{b^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{8a^{\frac{5}{4}}(-ad+bc)^3} \\ & + \frac{b^{\frac{5}{4}}(-9ad+bc) \arctan\left(1 + \frac{b^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{8a^{\frac{5}{4}}(-ad+bc)^3} \\ & - \frac{d^{\frac{5}{4}}(-ad+9bc) \arctan\left(1 - \frac{d^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{8c^{\frac{5}{4}}(-ad+bc)^3} \\ & + \frac{d^{\frac{5}{4}}(-ad+9bc) \arctan\left(1 + \frac{d^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{8c^{\frac{5}{4}}(-ad+bc)^3} \\ & + \frac{b^{\frac{5}{4}}(-9ad+bc) \ln\left(\sqrt{a} + x\sqrt{b} - a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{16a^{\frac{5}{4}}(-ad+bc)^3} \\ & - \frac{b^{\frac{5}{4}}(-9ad+bc) \ln\left(\sqrt{a} + x\sqrt{b} + a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{16a^{\frac{5}{4}}(-ad+bc)^3} \\ & + \frac{d^{\frac{5}{4}}(-ad+9bc) \ln\left(\sqrt{c} + x\sqrt{d} - c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{16c^{\frac{5}{4}}(-ad+bc)^3} \\ & - \frac{d^{\frac{5}{4}}(-ad+9bc) \ln\left(\sqrt{c} + x\sqrt{d} + c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{16c^{\frac{5}{4}}(-ad+bc)^3} \end{aligned}$$

command

```
integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

13.7 Problem number 493

$$\int \frac{1}{x^{3/2} (a + bx^2)^2 (c + dx^2)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^{\frac{9}{4}}(-13ad + 5bc) \arctan\left(1 - \frac{b^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{8a^{\frac{9}{4}}(-ad + bc)^3} \\ & - \frac{b^{\frac{9}{4}}(-13ad + 5bc) \arctan\left(1 + \frac{b^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{8a^{\frac{9}{4}}(-ad + bc)^3} \\ & + \frac{d^{\frac{9}{4}}(-5ad + 13bc) \arctan\left(1 - \frac{d^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{8c^{\frac{9}{4}}(-ad + bc)^3} \\ & - \frac{d^{\frac{9}{4}}(-5ad + 13bc) \arctan\left(1 + \frac{d^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{c^{\frac{1}{4}}}\right) \sqrt{2}}{8c^{\frac{9}{4}}(-ad + bc)^3} \\ & - \frac{b^{\frac{9}{4}}(-13ad + 5bc) \ln\left(\sqrt{a} + x\sqrt{b} - a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{16a^{\frac{9}{4}}(-ad + bc)^3} \\ & + \frac{b^{\frac{9}{4}}(-13ad + 5bc) \ln\left(\sqrt{a} + x\sqrt{b} + a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{16a^{\frac{9}{4}}(-ad + bc)^3} \\ & - \frac{d^{\frac{9}{4}}(-5ad + 13bc) \ln\left(\sqrt{c} + x\sqrt{d} - c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{16c^{\frac{9}{4}}(-ad + bc)^3} \\ & + \frac{d^{\frac{9}{4}}(-5ad + 13bc) \ln\left(\sqrt{c} + x\sqrt{d} + c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}\sqrt{x}\right) \sqrt{2}}{16c^{\frac{9}{4}}(-ad + bc)^3} + \frac{-5a^2d^2 + 8abcd - 5b^2c^2}{2a^2c^2(-ad + bc)^2\sqrt{x}} \\ & + \frac{d(ad + bc)}{2ac(-ad + bc)^2(dx^2 + c)\sqrt{x}} + \frac{b}{2a(-ad + bc)(bx^2 + a)(dx^2 + c)\sqrt{x}} \end{aligned}$$

command

```
integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

13.8 Problem number 785

$$\int (ex)^{3/2} \sqrt{a+bx^2} (A+Bx^2) dx$$

Optimal antiderivative

$$\frac{2B(ex)^{\frac{5}{2}} (bx^2+a)^{\frac{3}{2}}}{11be} + \frac{2(11Ab-5Ba)(ex)^{\frac{5}{2}} \sqrt{bx^2+a}}{77be} + \frac{4a(11Ab-5Ba)e\sqrt{ex} \sqrt{bx^2+a}}{231b^2}$$

$$2a^{\frac{7}{4}}(11Ab-5Ba)e^{\frac{3}{2}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x$$

$$231 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) b^{\frac{9}{4}} \sqrt{bx^2+a}$$

command

```
integrate((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\left(5Ba^3-11Aa^2b\right)\sqrt{b}e^{\frac{3}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(21Bb^3x^4-10Ba^2b+22Aab^2+3\left(2Bab^2+11Aa^2b\right)\right)\sqrt{ex}\sqrt{bx^2+a}\right)}{231b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bex^3+Aex\right)\sqrt{bx^2+a}\sqrt{ex},x\right)$$

13.9 Problem number 786

$$\int \sqrt{ex} \sqrt{a+bx^2} (A+Bx^2) dx$$

Optimal antiderivative

$$\frac{2B(ex)^{\frac{3}{2}}(bx^2+a)^{\frac{3}{2}}}{9be} + \frac{2(3Ab-Ba)(ex)^{\frac{3}{2}}\sqrt{bx^2+a}}{15be} + \frac{4a(3Ab-Ba)\sqrt{ex}\sqrt{bx^2+a}}{15b^{\frac{3}{2}}(\sqrt{a}+x\sqrt{b})}$$

$$4a^{\frac{5}{4}}(3Ab-Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})$$

$$15\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^2+a}$$

$$2a^{\frac{5}{4}}(3Ab-Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})$$

$$15\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^2+a}$$

command

`integrate((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6(Ba^2-3Aab)\sqrt{b}e^{\frac{1}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{b},0,\operatorname{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+(5Bb^2x^3+(2Bab+9Ab^2)x)\right)}{45b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((Bx^2+A)\sqrt{bx^2+a}\sqrt{ex},x\right)$$

13.10 Problem number 787

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{\sqrt{ex}} dx$$

Optimal antiderivative

$$\frac{2B(bx^2+a)^{\frac{3}{2}}\sqrt{ex}}{7be} + \frac{2(7Ab-Ba)\sqrt{ex}\sqrt{bx^2+a}}{21be}$$

$$2a^{\frac{3}{4}}(7Ab-Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})$$

$$21\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)b^{\frac{5}{4}}\sqrt{e}\sqrt{bx^2+a}$$

command

`integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 (Ba^2 - 7Aab) \sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (3Bb^2x^2 + 2Bab + 7Ab^2) \sqrt{bx^2 + a} \sqrt{x} \right) e^{-\frac{1}{2}}}{21b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A) \sqrt{bx^2 + a} \sqrt{ex}}{ex}, x\right)$$

13.11 Problem number 788

$$\int \frac{\sqrt{a + bx^2} (A + Bx^2)}{(ex)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2A(bx^2 + a)^{\frac{3}{2}}}{ae\sqrt{ex}} + \frac{2(5Ab + Ba)(ex)^{\frac{3}{2}} \sqrt{bx^2 + a}}{5ae^3} + \frac{4(5Ab + Ba) \sqrt{ex} \sqrt{bx^2 + a}}{5e^2 \sqrt{b} (\sqrt{a} + x\sqrt{b})}$$

$$4a^{\frac{1}{4}}(5Ab + Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})$$

$$5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right) b^{\frac{3}{4}} e^{\frac{3}{2}} \sqrt{bx^2 + a}$$

$$2a^{\frac{1}{4}}(5Ab + Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})$$

+

$$5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right) b^{\frac{3}{4}} e^{\frac{3}{2}} \sqrt{bx^2 + a}$$

command

`integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 (Ba + 5Ab) \sqrt{b} x \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) - (Bbx^2 - 5Ab) \sqrt{bx^2 + a} \sqrt{x} \right)}{5bx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A) \sqrt{bx^2 + a} \sqrt{ex}}{e^2 x^2}, x\right)$$

13.12 Problem number 789

$$\int \frac{\sqrt{a+bx^2} (A+Bx^2)}{(ex)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2A(bx^2+a)^{\frac{3}{2}}}{3ae(ex)^{\frac{3}{2}}} + \frac{2(Ab+Ba)\sqrt{ex}\sqrt{bx^2+a}}{3ae^3}$$

$$+ \frac{2(Ab+Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})}{3\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) a^{\frac{1}{4}}b^{\frac{1}{4}}e^{\frac{5}{2}}\sqrt{bx^2+a}}$$

command

```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(Ba+Ab)\sqrt{b}x^2\operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (Bbx^2 - Ab)\sqrt{bx^2+a}\sqrt{x}\right)e^{(-\frac{5}{2})}}{3bx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2+A)\sqrt{bx^2+a}\sqrt{ex}}{e^3x^3}, x\right)$$

13.13 Problem number 790

$$\int \frac{\sqrt{a+bx^2} (A+Bx^2)}{(ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2A(bx^2 + a)^{\frac{3}{2}}}{5ae(ex)^{\frac{5}{2}}} - \frac{2(Ab + 5Ba)\sqrt{bx^2 + a}}{5ae^3\sqrt{ex}} + \frac{4(Ab + 5Ba)\sqrt{b}\sqrt{ex}\sqrt{bx^2 + a}}{5ae^4(\sqrt{a} + x\sqrt{b})}$$

$$4b^{\frac{1}{4}}(Ab + 5Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})$$

$$5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{bx^2 + a}$$

$$2b^{\frac{1}{4}}(Ab + 5Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})$$

$$5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{bx^2 + a}$$

command

```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(5Ba + Ab)\sqrt{b}x^3\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (5Ba + 2Ab)x^2 + Aa\right)\sqrt{bx^2 + a}}{5ax^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}}{e^4x^4}, x\right)$$

13.14 Problem number 791

$$\int \frac{\sqrt{a + bx^2}(A + Bx^2)}{x^{9/2}} dx$$

Optimal antiderivative

$$\frac{2A(bx^2 + a)^{\frac{3}{2}}}{7ax^{\frac{7}{2}}} + \frac{2(Ab - 7Ba)\sqrt{bx^2 + a}}{21ax^{\frac{3}{2}}}$$

$$2b^{\frac{3}{4}}(Ab - 7Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})$$

$$21\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{5}{4}}\sqrt{bx^2 + a}$$

command

```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 (7 B a - A b) \sqrt{b} x^4 \text{weierstrassPInverse} \left(-\frac{4a}{b}, 0, x \right) - ((7 B a + 2 A b) x^2 + 3 A a) \sqrt{b x^2 + a} \sqrt{x} \right)}{21 a x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B x^2 + A) \sqrt{b x^2 + a}}{x^{\frac{9}{2}}}, x \right)$$

13.15 Problem number 792

$$\int \frac{\sqrt{a + b x^2} (A + B x^2)}{x^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2A(bx^2+a)^{\frac{3}{2}}}{9ax^{\frac{9}{2}}} + \frac{2(Ab-3Ba)\sqrt{bx^2+a}}{15ax^{\frac{5}{2}}} \\ & + \frac{4b(Ab-3Ba)\sqrt{bx^2+a}}{15a^2\sqrt{x}} - \frac{4b^{\frac{3}{2}}(Ab-3Ba)\sqrt{x}\sqrt{bx^2+a}}{15a^2(\sqrt{a}+x\sqrt{b})} \\ & + \frac{4b^{\frac{5}{4}}(Ab-3Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})}{15\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}\sqrt{bx^2+a}} \\ & - \frac{2b^{\frac{5}{4}}(Ab-3Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})}{15\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}\sqrt{bx^2+a}} \end{aligned}$$

command

```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 (3 Bab - Ab^2) \sqrt{b} x^5 \text{weierstrassZeta} \left(-\frac{4a}{b}, 0, \text{weierstrassPInverse} \left(-\frac{4a}{b}, 0, x \right) \right) + (6 (3 Bab - Ab^2) x^4 + 5 Aa^2) \right)}{45 a^2 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(\frac{(Bx^2 + A) \sqrt{bx^2 + a}}{x^{\frac{11}{2}}}, x \right) \right)$$

13.16 Problem number 793

$$\int \frac{\sqrt{a + bx^2} (A + Bx^2)}{x^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2A(bx^2 + a)^{\frac{3}{2}}}{11ax^{\frac{11}{2}}} + \frac{2(5Ab - 11Ba) \sqrt{bx^2 + a}}{77ax^{\frac{7}{2}}} + \frac{4b(5Ab - 11Ba) \sqrt{bx^2 + a}}{231a^2x^{\frac{3}{2}}} \\ & + \frac{2b^{\frac{7}{4}}(5Ab - 11Ba) \sqrt{\frac{\cos \left(4 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{a} + x\sqrt{b})}{231 \cos \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}} \right) \right) a^{\frac{9}{4}} \sqrt{bx^2 + a}} \end{aligned}$$

command

```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 (11 Bab - 5 Ab^2) \sqrt{b} x^6 \text{weierstrassPInverse} \left(-\frac{4a}{b}, 0, x \right) + (2 (11 Bab - 5 Ab^2) x^4 + 21 Aa^2 + 3 (11 Ba^2 + 2 Aa^2) \right)}{231 a^2 x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(\frac{(Bx^2 + A) \sqrt{bx^2 + a}}{x^{\frac{13}{2}}}, x \right) \right)$$

13.17 Problem number 794

$$\int (ex)^{3/2} (a + bx^2)^{3/2} (A + Bx^2) dx$$

Optimal antiderivative

$$\frac{2(3Ab - Ba)(ex)^{\frac{5}{2}}(bx^2 + a)^{\frac{3}{2}}}{33be} + \frac{2B(ex)^{\frac{5}{2}}(bx^2 + a)^{\frac{5}{2}}}{15be}$$

$$+ \frac{4a(3Ab - Ba)(ex)^{\frac{5}{2}}\sqrt{bx^2 + a}}{77be} + \frac{8a^2(3Ab - Ba)e\sqrt{ex}\sqrt{bx^2 + a}}{231b^2}$$

$$4a^{\frac{11}{4}}(3Ab - Ba)e^{\frac{3}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + xv$$

$$231 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) b^{\frac{9}{4}}\sqrt{bx^2 + a}$$

command

```
integrate((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(20(Ba^4 - 3Aa^3b)\sqrt{b}e^{\frac{3}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (77Bb^4x^6 - 20Ba^3b + 60Aa^2b^2 + 7(17Bab^3 + 15Aa^2b^2))\sqrt{bx^2 + a}\sqrt{ex}\right)}{1155b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bbex^5 + (Ba + Ab)ex^3 + Aaex\right)\sqrt{bx^2 + a}\sqrt{ex}, x\right)$$

13.18 Problem number 795

$$\int \sqrt{ex} (a + bx^2)^{3/2} (A + Bx^2) dx$$

Optimal antiderivative

$$\frac{2(13Ab - 3Ba)(ex)^{\frac{3}{2}}(bx^2 + a)^{\frac{3}{2}}}{117be} + \frac{2B(ex)^{\frac{3}{2}}(bx^2 + a)^{\frac{5}{2}}}{13be}$$

$$+ \frac{4a(13Ab - 3Ba)(ex)^{\frac{3}{2}}\sqrt{bx^2 + a}}{195be} + \frac{8a^2(13Ab - 3Ba)\sqrt{ex}\sqrt{bx^2 + a}}{195b^{\frac{3}{2}}(\sqrt{a} + x\sqrt{b})}$$

$$8a^{\frac{9}{4}}(13Ab - 3Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})$$

$$195\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^2 + a}$$

$$4a^{\frac{9}{4}}(13Ab - 3Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})$$

$$195\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^2 + a}$$

command

`integrate((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12(3Ba^3 - 13Aa^2b)\sqrt{b}e^{\frac{1}{2}}\text{weierstrassZeta}\left(-\frac{4a}{b}, 0, \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (45Bb^3x^5 + 5(15Bab^2\right)}{585b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bbx^4 + (Ba + Ab)x^2 + Aa\right)\sqrt{bx^2 + a}\sqrt{ex}, x\right)$$

13.19 Problem number 796

$$\int \frac{(a + bx^2)^{3/2}(A + Bx^2)}{\sqrt{ex}} dx$$

Optimal antiderivative

$$\frac{2(11Ab - Ba)(bx^2 + a)^{\frac{3}{2}}\sqrt{ex}}{77be} + \frac{2B(bx^2 + a)^{\frac{5}{2}}\sqrt{ex}}{11be} + \frac{4a(11Ab - Ba)\sqrt{ex}\sqrt{bx^2 + a}}{77be}$$

$$4a^{\frac{7}{4}}(11Ab - Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})$$

$$77\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)b^{\frac{5}{4}}\sqrt{e}\sqrt{bx^2 + a}$$

command

```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 (Ba^3 - 11 Aa^2b) \sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (7 Bb^3x^4 + 4 Ba^2b + 33 Aab^2 + (13 Bab^2 + 11 Ab^3)x^3) \right)}{77b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bbx^4 + (Ba + Ab)x^2 + Aa) \sqrt{bx^2 + a} \sqrt{ex}}{ex}, x\right)$$

13.20 Problem number 797

$$\int \frac{(a + bx^2)^{3/2} (A + Bx^2)}{(ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9Ab + Ba) (ex)^{\frac{3}{2}} (bx^2 + a)^{\frac{3}{2}}}{9ae^3} - \frac{2A(bx^2 + a)^{\frac{5}{2}}}{ae\sqrt{ex}} \\ & + \frac{4(9Ab + Ba) (ex)^{\frac{3}{2}} \sqrt{bx^2 + a}}{15e^3} + \frac{8a(9Ab + Ba) \sqrt{ex} \sqrt{bx^2 + a}}{15e^2\sqrt{b} (\sqrt{a} + x\sqrt{b})} \\ & + \frac{8a^{\frac{5}{4}}(9Ab + Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})}{15 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) b^{\frac{3}{4}} e^{\frac{3}{2}} \sqrt{bx^2 + a}} \\ & + \frac{4a^{\frac{5}{4}}(9Ab + Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})}{15 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) b^{\frac{3}{4}} e^{\frac{3}{2}} \sqrt{bx^2 + a}} \end{aligned}$$

command

```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 (Ba^2 + 9 Aab) \sqrt{b} x \operatorname{weierstrassZeta} \left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse} \left(-\frac{4a}{b}, 0, x \right) \right) - (5 Bb^2 x^4 - 45 Aab + (11 B \right)}{45 bx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Bbx^4 + (Ba + Ab)x^2 + Aa) \sqrt{bx^2 + a} \sqrt{ex}}{e^2 x^2}, x \right)$$

13.21 Problem number 798

$$\int \frac{(a + bx^2)^{3/2} (A + Bx^2)}{(ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2A(bx^2 + a)^{\frac{5}{2}}}{3ae(ex)^{\frac{3}{2}}} + \frac{2(7Ab + 3Ba)(bx^2 + a)^{\frac{3}{2}} \sqrt{ex}}{21ae^3} + \frac{4(7Ab + 3Ba) \sqrt{ex} \sqrt{bx^2 + a}}{21e^3} \\ & + \frac{4a^{\frac{3}{4}}(7Ab + 3Ba) \sqrt{\frac{\cos \left(4 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{a} + x\sqrt{b}}{21 \cos \left(2 \arctan \left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}} \right) \right) b^{\frac{1}{4}} e^{\frac{5}{2}} \sqrt{bx^2 + a}} \end{aligned}$$

command

```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 (3 Ba^2 + 7 Aab) \sqrt{b} x^2 \operatorname{weierstrassPInverse} \left(-\frac{4a}{b}, 0, x \right) + (3 Bb^2 x^4 - 7 Aab + (9 Bab + 7 Ab^2) x^2) \sqrt{bx^2 + a} \sqrt{e} \right)}{21 bx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Bbx^4 + (Ba + Ab)x^2 + Aa) \sqrt{bx^2 + a} \sqrt{ex}}{e^3 x^3}, x \right)$$

13.22 Problem number 799

$$\int \frac{(a + bx^2)^{3/2} (A + Bx^2)}{(ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2A(bx^2 + a)^{\frac{5}{2}}}{5ae(ex)^{\frac{5}{2}}} - \frac{2(Ab + Ba)(bx^2 + a)^{\frac{3}{2}}}{ae^3\sqrt{ex}} \\ & + \frac{12b(Ab + Ba)(ex)^{\frac{3}{2}}\sqrt{bx^2 + a}}{5ae^5} + \frac{24(Ab + Ba)\sqrt{b}\sqrt{ex}\sqrt{bx^2 + a}}{5e^4(\sqrt{a} + x\sqrt{b})} \\ & \frac{24a^{\frac{1}{4}}b^{\frac{1}{4}}(Ab + Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)e^{\frac{7}{2}}\sqrt{bx^2 + a}} \\ & \frac{12a^{\frac{1}{4}}b^{\frac{1}{4}}(Ab + Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)e^{\frac{7}{2}}\sqrt{bx^2 + a}} \end{aligned}$$

command

```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12(Ba + Ab)\sqrt{b}x^3\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) - (Bbx^4 - (5Ba + 7Ab)x^2 - A)\sqrt{bx^2 + a}\sqrt{ex}\right)}{5x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bbx^4 + (Ba + Ab)x^2 + Aa)\sqrt{bx^2 + a}\sqrt{ex}}{e^4x^4}, x\right)$$

13.23 Problem number 800

$$\int \frac{(ex)^{5/2} (A + Bx^2)}{\sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\frac{2(9Ab - 7Ba) e(ex)^{\frac{3}{2}} \sqrt{bx^2 + a}}{45b^2} + \frac{2B(ex)^{\frac{7}{2}} \sqrt{bx^2 + a}}{9be} - \frac{2a(9Ab - 7Ba) e^2 \sqrt{ex} \sqrt{bx^2 + a}}{15b^{\frac{5}{2}} (\sqrt{a} + x\sqrt{b})}$$

$$+ \frac{2a^{\frac{5}{4}}(9Ab - 7Ba) e^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})}{15 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) b^{\frac{11}{4}} \sqrt{bx^2 + a}}$$

$$+ \frac{a^{\frac{5}{4}}(9Ab - 7Ba) e^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})}{15 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) b^{\frac{11}{4}} \sqrt{bx^2 + a}}$$

command

```
integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 (7 Ba^2 - 9 Aab) \sqrt{b} e^{\frac{5}{2}} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) - (5 Bb^2x^3 - (7 Bab - 9 Ab^2) \sqrt{bx^2 + a}) \right)}{45 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^2x^4 + Ae^2x^2)\sqrt{ex}}{\sqrt{bx^2 + a}}, x\right)$$

13.24 Problem number 801

$$\int \frac{(ex)^{3/2} (A + Bx^2)}{\sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\frac{2B(ex)^{\frac{5}{2}} \sqrt{bx^2 + a}}{7be} + \frac{2(7Ab - 5Ba) e \sqrt{ex} \sqrt{bx^2 + a}}{21b^2}$$

$$a^{\frac{3}{4}} (7Ab - 5Ba) e^{\frac{3}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x \sqrt{bx^2 + a})}$$

$$21 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right) b^{\frac{9}{4}} \sqrt{bx^2 + a}$$

command

```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((5Ba^2 - 7Aab) \sqrt{b} e^{\frac{3}{2}} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (3Bb^2x^2 - 5Bab + 7Ab^2) \sqrt{bx^2 + a} \sqrt{x} e^{\frac{3}{2}} \right)}{21b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bex^3 + Aex)\sqrt{ex}}{\sqrt{bx^2 + a}}, x\right)$$

13.25 Problem number 802

$$\int \frac{\sqrt{ex} (A + Bx^2)}{\sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\frac{2B(ex)^{\frac{3}{2}} \sqrt{bx^2 + a}}{5be} + \frac{2(5Ab - 3Ba) \sqrt{ex} \sqrt{bx^2 + a}}{5b^{\frac{3}{2}} (\sqrt{a} + x\sqrt{b})}$$

$$2a^{\frac{1}{4}}(5Ab - 3Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})$$

$$5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) b^{\frac{7}{4}} \sqrt{bx^2 + a}$$

$$a^{\frac{1}{4}}(5Ab - 3Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})$$

$$5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) b^{\frac{7}{4}} \sqrt{bx^2 + a}$$

command

```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{bx^2 + a} Bbx^{\frac{3}{2}}e^{\frac{1}{2}} + (3Ba - 5Ab)\sqrt{b}e^{\frac{1}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right)\right)}{5b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A)\sqrt{ex}}{\sqrt{bx^2 + a}}, x\right)$$

13.26 Problem number 803

$$\int \frac{A + Bx^2}{\sqrt{ex} \sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\frac{2B\sqrt{ex} \sqrt{bx^2 + a}}{3be}$$

$$(3Ab - Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\dots}$$

$$3 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) a^{\frac{1}{4}}b^{\frac{5}{4}}\sqrt{e} \sqrt{bx^2 + a}$$

command

`integrate((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{bx^2 + a} Bb\sqrt{x} - (Ba - 3Ab)\sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) \right) e^{(-\frac{1}{2})}}{3b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}}{bex^3 + aex}, x\right)$$

13.27 Problem number 804

$$\int \frac{A + Bx^2}{(ex)^{3/2} \sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2A\sqrt{bx^2 + a}}{ae\sqrt{ex}} + \frac{2(Ab + Ba)\sqrt{ex}\sqrt{bx^2 + a}}{ae^2\sqrt{b}(\sqrt{a} + x\sqrt{b})} \\ & + \frac{2(Ab + Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{3}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{bx^2 + a}}} \\ & + \frac{(Ab + Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{3}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{bx^2 + a}}} \end{aligned}$$

command

`integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((Ba + Ab)\sqrt{b} x \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + \sqrt{bx^2 + a} Ab\sqrt{x} \right) e^{(-\frac{3}{2})}}{abx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}}{be^2x^4 + ae^2x^2}, x\right)$$

13.28 Problem number 805

$$\int \frac{A + Bx^2}{(ex)^{5/2} \sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\frac{2A\sqrt{bx^2 + a}}{3ae(ex)^{\frac{3}{2}}} + (Ab - 3Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{\frac{3 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) a^{\frac{5}{4}} b^{\frac{1}{4}} e^{\frac{5}{2}} \sqrt{bx^2 + a}}{3 abx^2}}$$

command

```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((3Ba - Ab)\sqrt{b} x^2 \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - \sqrt{bx^2 + a} Ab\sqrt{x} \right) e^{(-\frac{5}{2})}}{3 abx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}}{be^3x^5 + ae^3x^3}, x\right)$$

13.29 Problem number 806

$$\int \frac{A + Bx^2}{(ex)^{7/2} \sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\frac{2A\sqrt{bx^2+a}}{5ae(ex)^{\frac{5}{2}}} + \frac{2(3Ab-5Ba)\sqrt{bx^2+a}}{5a^2e^3\sqrt{ex}} - \frac{2(3Ab-5Ba)\sqrt{b}\sqrt{ex}\sqrt{bx^2+a}}{5a^2e^4(\sqrt{a}+x\sqrt{b})}$$

$$+ \frac{2b^{\frac{1}{4}}(3Ab-5Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{7}{4}}e^{\frac{7}{2}}\sqrt{bx^2+a}}$$

$$+ \frac{b^{\frac{1}{4}}(3Ab-5Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{7}{4}}e^{\frac{7}{2}}\sqrt{bx^2+a}}$$

command

`integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((5Ba-3Ab)\sqrt{b}x^3\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + ((5Ba-3Ab)x^2 + Aa)\sqrt{bx^2+a}\right)}{5a^2x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2+A)\sqrt{bx^2+a}\sqrt{ex}}{be^4x^6+ae^4x^4}, x\right)$$

13.30 Problem number 807

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(7Ab-9Ba)e(ex)^{\frac{5}{2}}}{7b^2\sqrt{bx^2+a}} + \frac{2B(ex)^{\frac{9}{2}}}{7be\sqrt{bx^2+a}} + \frac{5(7Ab-9Ba)e^3\sqrt{ex}\sqrt{bx^2+a}}{21b^3}$$

$$+ \frac{5a^{\frac{3}{4}}(7Ab-9Ba)e^{\frac{7}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})}{42\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)b^{\frac{13}{4}}\sqrt{bx^2+a}}$$

command

`integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(9Ba^3 - 7Aa^2b + (9Ba^2b - 7Aab^2)x^2)\sqrt{b}e^{\frac{7}{2}}\text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (6Bb^3x^4 - 45Ba^2b + 35Aab^2 - 5a^3)}{21(b^5x^2 + ab^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Be^3x^5 + Ae^3x^3)\sqrt{bx^2 + a}\sqrt{ex}}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

13.31 Problem number 808

$$\int \frac{(ex)^{5/2}(A + Bx^2)}{(a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(5Ab - 7Ba)e(ex)^{\frac{3}{2}}}{5b^2\sqrt{bx^2 + a}} + \frac{2B(ex)^{\frac{7}{2}}}{5be\sqrt{bx^2 + a}} + \frac{3(5Ab - 7Ba)e^2\sqrt{ex}\sqrt{bx^2 + a}}{5b^{\frac{5}{2}}(\sqrt{a} + x\sqrt{b})} \\ & + 3a^{\frac{1}{4}}(5Ab - 7Ba)e^{\frac{5}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})} \\ & - \frac{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)b^{\frac{11}{4}}\sqrt{bx^2 + a}}{10\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)b^{\frac{11}{4}}\sqrt{bx^2 + a}} \\ & + 3a^{\frac{1}{4}}(5Ab - 7Ba)e^{\frac{5}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})} \\ & + \frac{10\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)b^{\frac{11}{4}}\sqrt{bx^2 + a}}{10\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)b^{\frac{11}{4}}\sqrt{bx^2 + a}} \end{aligned}$$

command

`integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(7Ba^2 - 5Aab + (7Bab - 5Ab^2)x^2)\sqrt{b}e^{\frac{5}{2}}\text{weierstrassZeta}\left(-\frac{4a}{b}, 0, \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (2Bb^2x^3 - 3a^2)}{5(b^4x^2 + ab^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Be^2x^4 + Ae^2x^2)\sqrt{bx^2 + a}\sqrt{ex}}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

13.32 Problem number 809

$$\int \frac{(ex)^{3/2} (A + Bx^2)}{(a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2B(ex)^{\frac{5}{2}}}{3be\sqrt{bx^2+a}} - \frac{(3Ab-5Ba)e\sqrt{ex}}{3b^2\sqrt{bx^2+a}}$$

$$+ \frac{(3Ab-5Ba)e^{\frac{3}{2}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})}{6 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) a^{\frac{1}{4}} b^{\frac{9}{4}} \sqrt{bx^2+a}}$$

command

```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(5Ba^2 - 3Aab + (5Bab - 3Ab^2)x^2)\sqrt{b}e^{\frac{3}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (2Bb^2x^2 + 5Bab - 3Ab^2)\sqrt{bx^2+a}}{3(b^4x^2 + ab^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bex^3 + Aex)\sqrt{bx^2+a}\sqrt{ex}}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

13.33 Problem number 810

$$\int \frac{\sqrt{ex} (A + Bx^2)}{(a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(Ab - Ba)(ex)^{\frac{3}{2}}}{abe\sqrt{bx^2 + a}} - \frac{(Ab - 3Ba)\sqrt{ex}\sqrt{bx^2 + a}}{ab^{\frac{3}{2}}(\sqrt{a} + x\sqrt{b})}$$

$$+ \frac{(Ab - 3Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{e}}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{3}{4}}b^{\frac{7}{4}}\sqrt{bx^2 + a}}$$

$$+ \frac{(Ab - 3Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{e}}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{3}{4}}b^{\frac{7}{4}}\sqrt{bx^2 + a}}$$

command

```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(Bab - Ab^2)\sqrt{bx^2 + a}x^{\frac{3}{2}}e^{\frac{1}{2}} + (3Ba^2 - Aab + (3Bab - Ab^2)x^2)\sqrt{b}e^{\frac{1}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPI}\right)}{ab^3x^2 + a^2b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

13.34 Problem number 811

$$\int \frac{A + Bx^2}{\sqrt{ex}(a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(Ab - Ba)\sqrt{ex}}{abe\sqrt{bx^2 + a}}$$

$$+ \frac{(Ab + Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{e}}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{5}{4}}b^{\frac{5}{4}}\sqrt{e}\sqrt{bx^2 + a}}$$

command

`integrate((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left((Ba^2 + Aab + (Bab + Ab^2)x^2)\sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (Bab - Ab^2)\sqrt{bx^2 + a} \sqrt{x} \right) e^{(-\frac{1}{2})}}{ab^3x^2 + a^2b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a} \sqrt{ex}}{b^2ex^5 + 2abex^3 + a^2ex}, x\right)$$

13.35 Problem number 812

$$\int \frac{A + Bx^2}{(ex)^{3/2} (a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3Ab - Ba)(ex)^{\frac{3}{2}}}{a^2e^3\sqrt{bx^2 + a}} - \frac{2A}{ae\sqrt{ex}\sqrt{bx^2 + a}} + \frac{(3Ab - Ba)\sqrt{ex}\sqrt{bx^2 + a}}{a^2e^2\sqrt{b}(\sqrt{a} + x\sqrt{b})} \\ & \frac{(3Ab - Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{7}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{bx^2 + a}} \\ & + \frac{(3Ab - Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{7}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{bx^2 + a}} \end{aligned}$$

command

`integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(((Bab - 3Ab^2)x^3 + (Ba^2 - 3Aab)x)\sqrt{b} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) - (2Aab - (Bab - 3Ab^2)x)\sqrt{bx^2 + a} \right) e^{(-\frac{1}{2})}}{a^2b^2x^3 + a^3bx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a} \sqrt{ex}}{b^2e^2x^6 + 2abe^2x^4 + a^2e^2x^2}, x\right)$$

13.36 Problem number 813

$$\int \frac{A + Bx^2}{(ex)^{5/2} (a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2A}{3ae (ex)^{\frac{3}{2}} \sqrt{bx^2 + a}} - \frac{(5Ab - 3Ba) \sqrt{ex}}{3a^2 e^3 \sqrt{bx^2 + a}}$$

$$(5Ab - 3Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})$$

$$6 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right) a^{\frac{9}{4}} b^{\frac{1}{4}} e^{\frac{5}{2}} \sqrt{bx^2 + a}$$

command

```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\left(3 Bab - 5 Ab^2\right)x^4 + \left(3 Ba^2 - 5 Aab\right)x^2\right) \sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - \left(2 Aab - \left(3 Bab - 5 Ab^2\right)x^2\right) \sqrt{b}}{3\left(a^2 b^2 x^4 + a^3 b x^2\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A) \sqrt{bx^2 + a} \sqrt{ex}}{b^2 e^3 x^7 + 2 a b e^3 x^5 + a^2 e^3 x^3}, x\right)$$

13.37 Problem number 814

$$\int \frac{A + Bx^2}{(ex)^{7/2} (a + bx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2A}{5ae(ex)^{\frac{5}{2}}\sqrt{bx^2+a}} + \frac{-7Ab+5Ba}{5a^2e^3\sqrt{ex}\sqrt{bx^2+a}} \\
 & + \frac{3(7Ab-5Ba)\sqrt{bx^2+a}}{5a^3e^3\sqrt{ex}} - \frac{3(7Ab-5Ba)\sqrt{b}\sqrt{ex}\sqrt{bx^2+a}}{5a^3e^4(\sqrt{a}+x\sqrt{b})} \\
 & + \frac{3b^{\frac{1}{4}}(7Ab-5Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{11}{4}}e^{\frac{7}{2}}\sqrt{bx^2+a}} \\
 & + \frac{3b^{\frac{1}{4}}(7Ab-5Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})}{10\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{11}{4}}e^{\frac{7}{2}}\sqrt{bx^2+a}}
 \end{aligned}$$

command

```
integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(3\left((5Bab-7Ab^2)x^5+(5Ba^2-7Aab)x^3\right)\sqrt{b}\operatorname{weierstrassZeta}\left(-\frac{4a}{b},0,\operatorname{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+3\left(\dots\right)\right)}{5(a^3bx^5+a^4x^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2+A)\sqrt{bx^2+a}\sqrt{ex}}{b^2e^4x^8+2abe^4x^6+a^2e^4x^4},x\right)$$

13.38 Problem number 815

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(Ab - 3Ba) e(ex)^{\frac{5}{2}}}{3b^2 (bx^2 + a)^{\frac{3}{2}}} + \frac{2B(ex)^{\frac{9}{2}}}{3be (bx^2 + a)^{\frac{3}{2}}} - \frac{5(Ab - 3Ba) e^3 \sqrt{ex}}{6b^3 \sqrt{bx^2 + a}}$$

$$+ \frac{5(Ab - 3Ba) e^{\frac{7}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})}{12 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right) a^{\frac{1}{4}} b^{\frac{13}{4}} \sqrt{bx^2 + a}}$$

command

`integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \left((3Bab^2 - Ab^3)x^4 + 3Ba^3 - Aa^2b + 2(3Ba^2b - Aab^2)x^2 \right) \sqrt{b} e^{\frac{7}{2}} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (4Bb^3x^4 + 6(b^6x^4 + 2ab^5x^2 + a^2b^4))}{6(b^6x^4 + 2ab^5x^2 + a^2b^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^3x^5 + Ae^3x^3) \sqrt{bx^2 + a} \sqrt{ex}}{b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3}, x\right)$$

13.39 Problem number 816

$$\int \frac{(ex)^{5/2} (A + Bx^2)}{(a + bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(Ab - Ba) (ex)^{\frac{7}{2}}}{3abe (bx^2 + a)^{\frac{3}{2}}} + \frac{(Ab - 7Ba) e(ex)^{\frac{3}{2}}}{6ab^2 \sqrt{bx^2 + a}} - \frac{(Ab - 7Ba) e^2 \sqrt{ex} \sqrt{bx^2 + a}}{2ab^{\frac{5}{2}} (\sqrt{a} + x\sqrt{b})}$$

$$+ \frac{(Ab - 7Ba) e^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})}{2 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right) a^{\frac{3}{4}} b^{\frac{11}{4}} \sqrt{bx^2 + a}}$$

$$+ \frac{(Ab - 7Ba) e^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})}{4 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right) a^{\frac{3}{4}} b^{\frac{11}{4}} \sqrt{bx^2 + a}}$$

command

```
integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3((7Bab^2 - Ab^3)x^4 + 7Ba^3 - Aa^2b + 2(7Ba^2b - Aab^2)x^2)\sqrt{b}e^{\frac{5}{2}}\text{weierstrassZeta}\left(-\frac{4a}{b}, 0, \text{weierstrassPInverse}\right)}{6(ab^5x^4 + 2a^2b^4x^2 + a^3b^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Be^2x^4 + Ae^2x^2)\sqrt{bx^2 + a}\sqrt{ex}}{b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3}, x\right)$$

13.40 Problem number 817

$$\int \frac{(ex)^{3/2}(A + Bx^2)}{(a + bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(Ab - Ba)(ex)^{\frac{5}{2}}}{3abe(bx^2 + a)^{\frac{3}{2}}} - \frac{(Ab + 5Ba)e\sqrt{ex}}{6ab^2\sqrt{bx^2 + a}}$$

$$+ \frac{(Ab + 5Ba)e^{\frac{3}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}}{\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})}}{12\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{5}{4}}b^{\frac{9}{4}}\sqrt{bx^2 + a}}$$

command

```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{((5Bab^2 + Ab^3)x^4 + 5Ba^3 + Aa^2b + 2(5Ba^2b + Aab^2)x^2)\sqrt{b}e^{\frac{3}{2}}\text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (5Ba^2b + Aa^2b^2)}{6(ab^5x^4 + 2a^2b^4x^2 + a^3b^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bex^3 + Aex)\sqrt{bx^2 + a}\sqrt{ex}}{b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3}, x\right)$$

13.41 Problem number 818

$$\int \frac{\sqrt{ex} (A + Bx^2)}{(a + bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(Ab - Ba)(ex)^{\frac{3}{2}}}{3abe(bx^2 + a)^{\frac{3}{2}}} + \frac{(Ab + Ba)(ex)^{\frac{3}{2}}}{2a^2be\sqrt{bx^2 + a}} - \frac{(Ab + Ba)\sqrt{ex}\sqrt{bx^2 + a}}{2a^2b^{\frac{3}{2}}(\sqrt{a} + x\sqrt{b})}$$

$$+ \frac{(Ab + Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{e}}{2 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) a^{\frac{7}{4}} b^{\frac{7}{4}} \sqrt{bx^2 + a}}$$

$$- \frac{(Ab + Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{e}}{4 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right) a^{\frac{7}{4}} b^{\frac{7}{4}} \sqrt{bx^2 + a}}$$

command

```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \left((Bab^2 + Ab^3)x^4 + Ba^3 + Aa^2b + 2(Ba^2b + Aab^2)x^2 \right) \sqrt{b} e^{\frac{1}{2}} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}\right)\right)}{6(a^2b^4x^4 + 2a^3b^3x^2 + a^4b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}}{b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3}, x\right)$$

13.42 Problem number 819

$$\int \frac{A + Bx^2}{\sqrt{ex} (a + bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(Ab - Ba) \sqrt{ex}}{3abe (bx^2 + a)^{\frac{3}{2}}} + \frac{(5Ab + Ba) \sqrt{ex}}{6a^2be \sqrt{bx^2 + a}}$$

$$+ \frac{(5Ab + Ba) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b})}{12 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{ex}}{a^{\frac{1}{4}} \sqrt{e}}\right)\right) a^{\frac{9}{4}} b^{\frac{5}{4}} \sqrt{e} \sqrt{bx^2 + a}}$$

command

`integrate((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\left(\left(Bab^2 + 5Ab^3\right)x^4 + Ba^3 + 5Aa^2b + 2\left(Ba^2b + 5Aab^2\right)x^2\right)\sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - \left(Ba^2b - 7Aab^2\right)\sqrt{bx^2 + a}\right)}{6\left(a^2b^4x^4 + 2a^3b^3x^2 + a^4b^2\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A) \sqrt{bx^2 + a} \sqrt{ex}}{b^3ex^7 + 3ab^2ex^5 + 3a^2bex^3 + a^3ex}, x\right)$$

13.43 Problem number 820

$$\int \frac{A + Bx^2}{(ex)^{3/2} (a + bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(7Ab - Ba)(ex)^{\frac{3}{2}}}{3a^2e^3(bx^2 + a)^{\frac{3}{2}}} - \frac{2A}{ae(bx^2 + a)^{\frac{3}{2}}\sqrt{ex}} - \frac{(7Ab - Ba)(ex)^{\frac{3}{2}}}{2a^3e^3\sqrt{bx^2 + a}} + \frac{(7Ab - Ba)\sqrt{ex}\sqrt{bx^2 + a}}{2a^3e^2\sqrt{b}(\sqrt{a} + x\sqrt{b})}$$

$$(7Ab - Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{\frac{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{11}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{bx^2 + a}}{2}}$$

$$(7Ab - Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{\frac{4\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{11}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{bx^2 + a}}{4}}$$

command

`integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(3\left((Bab^2 - 7Ab^3)x^5 + 2(Ba^2b - 7Aab^2)x^3 + (Ba^3 - 7Aa^2b)x\right)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4a}{b}, 0, \text{weierstrassPInverse}\right)\right)}{6(a^3b^3x^5 + 2a^4b^2x^3 + a^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}}{b^3e^2x^8 + 3ab^2e^2x^6 + 3a^2be^2x^4 + a^3e^2x^2}, x\right)$$

13.44 Problem number 821

$$\int \frac{A + Bx^2}{(ex)^{5/2}(a + bx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2A}{3ae(ex)^{\frac{3}{2}}(bx^2 + a)^{\frac{3}{2}}} - \frac{(3Ab - Ba)\sqrt{ex}}{3a^2e^3(bx^2 + a)^{\frac{3}{2}}} - \frac{5(3Ab - Ba)\sqrt{ex}}{6a^3e^3\sqrt{bx^2 + a}}$$

$$5(3Ab - Ba)\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{\frac{12\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{ex}}{a^{\frac{1}{4}}\sqrt{e}}\right)\right)a^{\frac{13}{4}}b^{\frac{1}{4}}e^{\frac{5}{2}}\sqrt{bx^2 + a}}{12}}$$

command

```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(5 \left((Bab^2 - 3Ab^3)x^6 + 2(Ba^2b - 3Aab^2)x^4 + (Ba^3 - 3Aa^2b)x^2\right)\sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + 5(Bab^2)\right)}{6(a^3b^3x^6 + 2a^4b^2x^4 + a^5bx^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}}{b^3e^3x^9 + 3ab^2e^3x^7 + 3a^2be^3x^5 + a^3e^3x^3}, x\right)$$

13.45 Problem number 822

$$\int (ex)^{3/2} (a + bx^2)^2 \sqrt{c + dx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(-10ad + 3bc)(ex)^{\frac{5}{2}}(dx^2 + c)^{\frac{3}{2}}}{55d^2e} + \frac{2b^2(ex)^{\frac{9}{2}}(dx^2 + c)^{\frac{3}{2}}}{15de^3} \\ & + \frac{2(11a^2d^2 + bc(-10ad + 3bc))(ex)^{\frac{5}{2}}\sqrt{dx^2 + c}}{77d^2e} \\ & + \frac{4c(11a^2d^2 + bc(-10ad + 3bc))e\sqrt{ex}\sqrt{dx^2 + c}}{231d^3} \\ & + \frac{2c^{\frac{7}{4}}(11a^2d^2 + bc(-10ad + 3bc))e^{\frac{3}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)\right)}{231\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{13}{4}}\sqrt{dx^2 + c}} \end{aligned}$$

command

```
integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(10(3b^2c^4 - 10abc^3d + 11a^2c^2d^2)\sqrt{d}e^{\frac{3}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) - (77b^2d^4x^6 + 30b^2c^3d - 100abc^2d^2)\right)}{1155d^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((b^2ex^5 + 2abex^3 + a^2ex)\sqrt{dx^2 + c}\sqrt{ex}, x\right)$$

13.46 Problem number 823

$$\int \sqrt{ex} (a + bx^2)^2 \sqrt{c + dx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(-26ad + 7bc)(ex)^{\frac{3}{2}}(dx^2 + c)^{\frac{3}{2}}}{117d^2e} + \frac{2b^2(ex)^{\frac{7}{2}}(dx^2 + c)^{\frac{3}{2}}}{13de^3} \\ & + \frac{2(39a^2d^2 + bc(-26ad + 7bc))(ex)^{\frac{3}{2}}\sqrt{dx^2 + c}}{195d^2e} \\ & + \frac{4c(39a^2d^2 + bc(-26ad + 7bc))\sqrt{ex}\sqrt{dx^2 + c}}{195d^{\frac{5}{2}}(\sqrt{c} + x\sqrt{d})} \\ & - \frac{4c^{\frac{5}{4}}(39a^2d^2 + bc(-26ad + 7bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}}{195\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{11}{4}}\sqrt{dx^2 + c}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) \\ & + \frac{2c^{\frac{5}{4}}(39a^2d^2 + bc(-26ad + 7bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}}{195\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{11}{4}}\sqrt{dx^2 + c}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) \end{aligned}$$

command

```
integrate((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6(7b^2c^3 - 26abc^2d + 39a^2cd^2)\sqrt{d}e^{\frac{1}{2}}\operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) - (45b^2d^3x^5 + \dots)\right)}{585d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((b^2x^4 + 2abx^2 + a^2)\sqrt{dx^2 + c}\sqrt{ex}, x\right)$$

13.47 Problem number 824

$$\int \frac{(a + bx^2)^2 \sqrt{c + dx^2}}{\sqrt{ex}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(ex)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{11de^3} - \frac{2b(-22ad+5bc)(dx^2+c)^{\frac{3}{2}}\sqrt{ex}}{77d^2e} \\ & + \frac{2(77a^2d^2-22abcd+5b^2c^2)\sqrt{ex}\sqrt{dx^2+c}}{231d^2e} \\ & + \frac{2c^{\frac{3}{4}}(77a^2d^2-22abcd+5b^2c^2)\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right),\frac{\sqrt{2}}{2}\right)}}{231\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{9}{4}}\sqrt{e}\sqrt{dx^2+c}} \end{aligned}$$

command

`integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\left(5b^2c^3-22abc^2d+77a^2cd^2\right)\sqrt{d}\operatorname{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)+\left(21b^2d^3x^4-10b^2c^2d+44abcd^2+77a^2d^3\right)\sqrt{ex}\sqrt{dx^2+c}\right)}{231d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4+2abx^2+a^2)\sqrt{dx^2+c}\sqrt{ex}}{ex},x\right)$$

13.48 Problem number 825

$$\int \frac{(a + bx^2)^2 \sqrt{c + dx^2}}{(ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b^2(ex)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}{9de^3} - \frac{2a^2(dx^2+c)^{\frac{3}{2}}}{ce\sqrt{ex}} - \frac{2(b^2c^2-3ad(5ad+2bc))(ex)^{\frac{3}{2}}\sqrt{dx^2+c}}{15cde^3}$$

$$- \frac{4(b^2c^2-3ad(5ad+2bc))\sqrt{ex}\sqrt{dx^2+c}}{15d^{\frac{3}{2}}e^2(\sqrt{c}+x\sqrt{d})}$$

$$+ \frac{4c^{\frac{1}{4}}(b^2c^2-3ad(5ad+2bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{15\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{dx^2+c}}$$

$$- \frac{2c^{\frac{1}{4}}(b^2c^2-3ad(5ad+2bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{15\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{dx^2+c}}$$

command

```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6(b^2c^2-6abcd-15a^2d^2)\sqrt{d}x\operatorname{weierstrassZeta}\left(-\frac{4c}{d},0,\operatorname{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)+(5b^2d^2x^4-45a^2d^2)\right)}{45d^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4+2abx^2+a^2)\sqrt{dx^2+c}\sqrt{ex}}{e^2x^2},x\right)$$

13.49 Problem number 826

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2a^2(dx^2+c)^{\frac{3}{2}}}{3ce(ex)^{\frac{3}{2}}} + \frac{2b^2(dx^2+c)^{\frac{3}{2}}\sqrt{ex}}{7de^3} - \frac{2(b^2c^2-7ad(ad+2bc))\sqrt{ex}\sqrt{dx^2+c}}{21cde^3} \\
& \frac{2(b^2c^2-7ad(ad+2bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{c}}{21\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)c^{\frac{1}{4}}d^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{dx^2+c}}
\end{aligned}$$

command

```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(b^2c^2-14abcd-7a^2d^2)\sqrt{d}x^2\operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) - (3b^2d^2x^4 - 7a^2d^2 + 2(b^2cd + 7abd^2)x^2)\sqrt{ex}\right)}{21d^2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{dx^2+c}\sqrt{ex}}{e^3x^3}, x\right)$$

13.50 Problem number 827

$$\int \frac{(a + bx^2)^2 \sqrt{c + dx^2}}{(ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^2(dx^2 + c)^{\frac{3}{2}}}{5ce(ex)^{\frac{5}{2}}} - \frac{2a(ad + 10bc)(dx^2 + c)^{\frac{3}{2}}}{5c^2e^3\sqrt{ex}} \\ & + \frac{2(b^2c^2 + ad(ad + 10bc))(ex)^{\frac{3}{2}}\sqrt{dx^2 + c}}{5c^2e^5} + \frac{4(b^2c^2 + ad(ad + 10bc))\sqrt{ex}\sqrt{dx^2 + c}}{5ce^4\sqrt{d}(\sqrt{c} + x\sqrt{d})} \\ & - \frac{4(b^2c^2 + ad(ad + 10bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{c}}{5\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)c^{\frac{3}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{dx^2 + c}} \\ & + \frac{2(b^2c^2 + ad(ad + 10bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{c}}{5\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)c^{\frac{3}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{dx^2 + c}} \end{aligned}$$

command

```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(b^2c^2 + 10abcd + a^2d^2)\sqrt{d}x^3\operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) - (b^2cdx^4 - a^2cd - 2\right)}{5cdx^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{dx^2 + c}\sqrt{ex}}{e^4x^4}, x\right)$$

13.51 Problem number 828

$$\int \frac{(a + bx^2)^2 \sqrt{c + dx^2}}{x^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^2(dx^2 + c)^{\frac{3}{2}}}{7cx^{\frac{7}{2}}} - \frac{2a(-ad + 14bc)(dx^2 + c)^{\frac{3}{2}}}{21c^2x^{\frac{3}{2}}} + \frac{2(7b^2c^2 + ad(-ad + 14bc))\sqrt{x}\sqrt{dx^2 + c}}{21c^2} \\ & + \frac{2(7b^2c^2 + ad(-ad + 14bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{c}}{21\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right)c^{\frac{5}{4}}d^{\frac{1}{4}}\sqrt{dx^2 + c}} \end{aligned}$$

command

`integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(7b^2c^2 + 14abcd - a^2d^2)\sqrt{d}x^4\operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) + (7b^2cdx^4 - 3a^2cd - 2(7abcd + a^2d^2)x^2)\sqrt{c}\right)}{21cdx^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{dx^2 + c}}{x^{\frac{9}{2}}}, x\right)$$

13.52 Problem number 829

$$\int \frac{(a + bx^2)^2 \sqrt{c + dx^2}}{x^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{2a^2(d^2x^2 + c)^{\frac{3}{2}}}{9cx^{\frac{9}{2}}} - \frac{2a(-ad + 6bc)(d^2x^2 + c)^{\frac{3}{2}}}{15c^2x^{\frac{5}{2}}} - \frac{2(15b^2c^2 + ad(-ad + 6bc))\sqrt{dx^2 + c}}{15c^2\sqrt{x}} \\
 & + \frac{4(15b^2c^2 + ad(-ad + 6bc))\sqrt{d}\sqrt{x}\sqrt{dx^2 + c}}{15c^2(\sqrt{c} + x\sqrt{d})} \\
 & - \frac{4d^{\frac{1}{4}}(15b^2c^2 + ad(-ad + 6bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)}{15\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{dx^2 + c}} \\
 & + \frac{2d^{\frac{1}{4}}(15b^2c^2 + ad(-ad + 6bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)}{15\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{dx^2 + c}}
 \end{aligned}$$

command

```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6(15b^2c^2 + 6abcd - a^2d^2)\sqrt{d}x^5\operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) + (3(15b^2c^2 + 12abcd - a^2d^2)\sqrt{d}x^5 + 6abcd)\sqrt{dx^2 + c}\right)}{45c^2x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{dx^2 + c}}{x^{\frac{11}{2}}}, x\right)$$

13.53 Problem number 830

$$\int \frac{(a + bx^2)^2 \sqrt{c + dx^2}}{x^{13/2}} dx$$

Optimal antiderivative

$$\frac{2a^2(dx^2+c)^{\frac{3}{2}}}{11cx^{\frac{11}{2}}} - \frac{2a(-5ad+22bc)(dx^2+c)^{\frac{3}{2}}}{77c^2x^{\frac{7}{2}}} - \frac{2(5a^2d^2-22abcd+77b^2c^2)\sqrt{dx^2+c}}{231c^2x^{\frac{3}{2}}}$$

$$+ \frac{2d^{\frac{3}{4}}(5a^2d^2-22abcd+77b^2c^2)\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)}{231\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right)c^{\frac{9}{4}}\sqrt{dx^2+c}}$$

command

```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(77b^2c^2-22abcd+5a^2d^2)\sqrt{d}x^6\operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) - ((77b^2c^2+44abcd-10a^2d^2)x^4+21a^2c)\right)}{231c^2x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4+2abx^2+a^2)\sqrt{dx^2+c}}{x^{\frac{13}{2}}}, x\right)$$

13.54 Problem number 831

$$\int \frac{(a + bx^2)^2 \sqrt{c + dx^2}}{x^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(d x^2 + c)^{\frac{3}{2}}}{13c x^{\frac{13}{2}}} - \frac{2a(-7ad + 26bc)(d x^2 + c)^{\frac{3}{2}}}{117c^2 x^{\frac{9}{2}}} - \frac{2(7a^2d^2 - 26abcd + 39b^2c^2) \sqrt{d x^2 + c}}{195c^2 x^{\frac{5}{2}}} \\ & - \frac{4d(7a^2d^2 - 26abcd + 39b^2c^2) \sqrt{d x^2 + c}}{195c^3 \sqrt{x}} + \frac{4d^{\frac{3}{2}}(7a^2d^2 - 26abcd + 39b^2c^2) \sqrt{x} \sqrt{d x^2 + c}}{195c^3 (\sqrt{c} + x\sqrt{d})} \\ & - \frac{4d^{\frac{5}{4}}(7a^2d^2 - 26abcd + 39b^2c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)}{195 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right) c^{\frac{11}{4}} \sqrt{d x^2 + c}} \\ & + \frac{2d^{\frac{5}{4}}(7a^2d^2 - 26abcd + 39b^2c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)}{195 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{x}}{c^{\frac{1}{4}}}\right)\right) c^{\frac{11}{4}} \sqrt{d x^2 + c}} \end{aligned}$$

command

```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 (39 b^2 c^2 d - 26 a b c d^2 + 7 a^2 d^3) \sqrt{d} x^7 \operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) + 6 (39 b^2 c^2 d \right)}{x^{15/2}}$$

585

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2 x^4 + 2 a b x^2 + a^2) \sqrt{d x^2 + c}}{x^{\frac{15}{2}}}, x\right)$$

13.55 Problem number 832

$$\int (ex)^{5/2} (a + bx^2)^2 (c + dx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(51a^2d^2 + bc(-42ad + 11bc)) (ex)^{7/2} (dx^2 + c)^{3/2}}{663d^2e} - \frac{2b(-42ad + 11bc) (ex)^{7/2} (dx^2 + c)^{5/2}}{357d^2e} \\ & + \frac{2b^2(ex)^{11/2} (dx^2 + c)^{5/2}}{21de^3} + \frac{8c^2(51a^2d^2 + bc(-42ad + 11bc)) e(ex)^{3/2} \sqrt{dx^2 + c}}{9945d^3} \\ & + \frac{4c(51a^2d^2 + bc(-42ad + 11bc)) (ex)^{7/2} \sqrt{dx^2 + c}}{1989d^2e} \\ & - \frac{8c^3(51a^2d^2 + bc(-42ad + 11bc)) e^2 \sqrt{ex} \sqrt{dx^2 + c}}{3315d^{7/2} (\sqrt{c} + x\sqrt{d})} \\ & + \frac{8c^{13/4} (51a^2d^2 + bc(-42ad + 11bc)) e^{5/2} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{1/4}\sqrt{ex}}{c^{1/4}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{d^{1/4}\sqrt{ex}}{c^{1/4}\sqrt{e}}\right)\right)\right)}{3315 \cos\left(2 \arctan\left(\frac{d^{1/4}\sqrt{ex}}{c^{1/4}\sqrt{e}}\right)\right) d^{15/4} \sqrt{dx^2 + c}} \\ & - \frac{4c^{13/4} (51a^2d^2 + bc(-42ad + 11bc)) e^{5/2} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{1/4}\sqrt{ex}}{c^{1/4}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{1/4}\sqrt{ex}}{c^{1/4}\sqrt{e}}\right)\right)\right)}{3315 \cos\left(2 \arctan\left(\frac{d^{1/4}\sqrt{ex}}{c^{1/4}\sqrt{e}}\right)\right) d^{15/4} \sqrt{dx^2 + c}} \end{aligned}$$

command

```
integrate((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(84 (11 b^2 c^5 - 42 abc^4 d + 51 a^2 c^3 d^2) \sqrt{d} e^{5/2} \operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) + (3315 b^2 d^5) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((b^2 de^2 x^8 + (b^2 c + 2 abd) e^2 x^6 + a^2 ce^2 x^2 + (2 abc + a^2 d) e^2 x^4) \sqrt{dx^2 + c} \sqrt{ex}, x\right)$$

13.56 Problem number 833

$$\int (ex)^{3/2} (a + bx^2)^2 (c + dx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(57a^2d^2 + bc(-38ad + 9bc)) (ex)^{\frac{5}{2}} (dx^2 + c)^{\frac{3}{2}}}{627d^2e} - \frac{2b(-38ad + 9bc) (ex)^{\frac{5}{2}} (dx^2 + c)^{\frac{5}{2}}}{285d^2e} \\ & + \frac{2b^2(ex)^{\frac{9}{2}} (dx^2 + c)^{\frac{5}{2}}}{19de^3} + \frac{4c(57a^2d^2 + bc(-38ad + 9bc)) (ex)^{\frac{5}{2}} \sqrt{dx^2 + c}}{1463d^2e} \\ & + \frac{8c^2(57a^2d^2 + bc(-38ad + 9bc)) e\sqrt{ex} \sqrt{dx^2 + c}}{4389d^3} \\ & + \frac{4c^{\frac{11}{4}} (57a^2d^2 + bc(-38ad + 9bc)) e^{\frac{3}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), 1\right)}{4389 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) d^{\frac{13}{4}} \sqrt{dx^2 + c}} \end{aligned}$$

command

```
integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(20\left(9b^2c^5 - 38abc^4d + 57a^2c^3d^2\right)\sqrt{d}e^{\frac{3}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) - \left(1155b^2d^5x^8 + 180b^2c^4d - 760abc^5\right)\right)}{4389 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) d^{\frac{13}{4}} \sqrt{dx^2 + c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2dex^7 + (b^2c + 2abd)ex^5 + a^2cex + (2abc + a^2d)ex^3\right)\sqrt{dx^2 + c} \sqrt{ex}, x\right)$$

13.57 Problem number 834

$$\int \sqrt{ex} (a + bx^2)^2 (c + dx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{2(221a^2d^2 + 3bc(-34ad + 7bc)) (ex)^{\frac{3}{2}} (dx^2 + c)^{\frac{3}{2}}}{1989d^2e} - \frac{2b(-34ad + 7bc) (ex)^{\frac{3}{2}} (dx^2 + c)^{\frac{5}{2}}}{221d^2e} \\
 & + \frac{2b^2(ex)^{\frac{7}{2}} (dx^2 + c)^{\frac{5}{2}}}{17de^3} + \frac{4c(221a^2d^2 + 3bc(-34ad + 7bc)) (ex)^{\frac{3}{2}} \sqrt{dx^2 + c}}{3315d^2e} \\
 & + \frac{8c^2(221a^2d^2 + 3bc(-34ad + 7bc)) \sqrt{ex} \sqrt{dx^2 + c}}{3315d^{\frac{5}{2}} (\sqrt{c} + x\sqrt{d})} \\
 & - \frac{8c^{\frac{9}{4}}(221a^2d^2 + 3bc(-34ad + 7bc)) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}}{\text{EllipticE}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)\right), \sqrt{dx^2 + c}}}{3315 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) d^{\frac{11}{4}} \sqrt{dx^2 + c}} \\
 & + \frac{4c^{\frac{9}{4}}(221a^2d^2 + 3bc(-34ad + 7bc)) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}}{\text{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)\right), \sqrt{dx^2 + c}}}{3315 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) d^{\frac{11}{4}} \sqrt{dx^2 + c}}
 \end{aligned}$$

command

```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 (21 b^2 c^4 - 102 abc^3 d + 221 a^2 c^2 d^2) \sqrt{d} e^{\frac{1}{2}} \text{weierstrassZeta}\left(-\frac{4c}{d}, 0, \text{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) - (585 b^2 c^2 d^2 + 3315 a^2 c d^2) \sqrt{dx^2 + c} \sqrt{ex} \right)}{3315 d^{\frac{11}{4}} \sqrt{dx^2 + c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 dx^6 + (b^2 c + 2 abd)x^4 + a^2 c + (2 abc + a^2 d)x^2\right) \sqrt{dx^2 + c} \sqrt{ex}, x\right)$$

13.58 Problem number 835

$$\int \frac{(a + bx^2)^2 (c + dx^2)^{3/2}}{\sqrt{ex}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(ex)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}{15de^3} + \frac{2(33a^2d^2+bc(-6ad+bc))(dx^2+c)^{\frac{3}{2}}\sqrt{ex}}{231d^2e} \\ & - \frac{2b(-6ad+bc)(dx^2+c)^{\frac{5}{2}}\sqrt{ex}}{33d^2e} + \frac{4c(33a^2d^2+bc(-6ad+bc))\sqrt{ex}\sqrt{dx^2+c}}{231d^2e} \\ & + \frac{4c^{\frac{7}{4}}(33a^2d^2+bc(-6ad+bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{231\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{9}{4}}\sqrt{e}\sqrt{dx^2+c}} \end{aligned}$$

command

```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(20(b^2c^4-6abc^3d+33a^2c^2d^2)\sqrt{d}\operatorname{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)+(77b^2d^4x^6-20b^2c^3d+120abc^2d^2+495a^2cd^2)\sqrt{dx^2+c}\sqrt{ex}\right)}{1155d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2dx^6+(b^2c+2abd)x^4+a^2c+(2abc+a^2d)x^2)\sqrt{dx^2+c}\sqrt{ex}}{ex},x\right)$$

13.59 Problem number 836

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(3b^2c^2 - 13ad(9ad + 2bc))(ex)^{\frac{3}{2}}(dx^2 + c)^{\frac{3}{2}}}{117cd e^3} + \frac{2b^2(ex)^{\frac{3}{2}}(dx^2 + c)^{\frac{5}{2}}}{13d e^3} - \frac{2a^2(dx^2 + c)^{\frac{5}{2}}}{ce\sqrt{ex}} \\
 & -\frac{4(3b^2c^2 - 13ad(9ad + 2bc))(ex)^{\frac{3}{2}}\sqrt{dx^2 + c}}{195d e^3} - \frac{8c(3b^2c^2 - 13ad(9ad + 2bc))\sqrt{ex}\sqrt{dx^2 + c}}{195d^{\frac{3}{2}}e^2(\sqrt{c} + x\sqrt{d})} \\
 & + \frac{8c^{\frac{5}{4}}(3b^2c^2 - 13ad(9ad + 2bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{195\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{dx^2 + c}} \\
 & - \frac{4c^{\frac{5}{4}}(3b^2c^2 - 13ad(9ad + 2bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{195\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{dx^2 + c}}
 \end{aligned}$$

command

```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12(3b^2c^3 - 26abc^2d - 117a^2cd^2)\sqrt{d}x\operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) + (45b^2d^3x^6 - 585d^2x\right)}{585d^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2dx^6 + (b^2c + 2abd)x^4 + a^2c + (2abc + a^2d)x^2)\sqrt{dx^2 + c}\sqrt{ex}}{e^2x^2}, x\right)$$

13.60 Problem number 837

$$\int \frac{(a + bx^2)^2(c + dx^2)^{3/2}}{(ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a^2(dx^2+c)^{\frac{5}{2}}}{3ce(ex)^{\frac{3}{2}}} - \frac{2(3b^2c^2-11ad(7ad+6bc))(dx^2+c)^{\frac{3}{2}}\sqrt{ex}}{231cde^3}$$

$$+ \frac{2b^2(dx^2+c)^{\frac{5}{2}}\sqrt{ex}}{11de^3} - \frac{4(3b^2c^2-11ad(7ad+6bc))\sqrt{ex}\sqrt{dx^2+c}}{231de^3}$$

$$4c^{\frac{3}{4}}(3b^2c^2-11ad(7ad+6bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)$$

$$231\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{dx^2+c}$$

command

```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4(3b^2c^3-66abc^2d-77a^2cd^2)\sqrt{d}x^2\operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) - (21b^2d^3x^6 - 77a^2cd^2 + 3(13b^2cd^2 + \dots)\right)}{231d^2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2dx^6 + (b^2c + 2abd)x^4 + a^2c + (2abc + a^2d)x^2)\sqrt{dx^2+c}\sqrt{ex}}{e^3x^3}, x\right)$$

13.61 Problem number 838

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(b^2c^2 + 9ad(ad + 2bc)) (ex)^{\frac{3}{2}} (dx^2 + c)^{\frac{3}{2}}}{9c^2e^5} - \frac{2a^2(dx^2 + c)^{\frac{5}{2}}}{5ce(ex)^{\frac{5}{2}}} - \frac{2a(ad + 2bc)(dx^2 + c)^{\frac{5}{2}}}{c^2e^3\sqrt{ex}} \\
& + \frac{4(b^2c^2 + 9ad(ad + 2bc)) (ex)^{\frac{3}{2}} \sqrt{dx^2 + c}}{15ce^5} + \frac{8(b^2c^2 + 9ad(ad + 2bc)) \sqrt{ex} \sqrt{dx^2 + c}}{15e^4\sqrt{d} (\sqrt{c} + x\sqrt{d})} \\
& - \frac{8c^{\frac{1}{4}}(b^2c^2 + 9ad(ad + 2bc)) \sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{dx^2 + c})}{15 \cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{dx^2 + c}} \\
& + \frac{4c^{\frac{1}{4}}(b^2c^2 + 9ad(ad + 2bc)) \sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{dx^2 + c})}{15 \cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{dx^2 + c}}
\end{aligned}$$

command

```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 (b^2c^2 + 18abcd + 9a^2d^2) \sqrt{d} x^3 \operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) - (5b^2d^2x^6 + (11b^2cd + 9ad^2)x^3 + 3c^2) \sqrt{dx^2 + c} \sqrt{ex} \right)}{45 dx^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2dx^6 + (b^2c + 2abd)x^4 + a^2c + (2abc + a^2d)x^2) \sqrt{dx^2 + c} \sqrt{ex}}{e^4x^4}, x\right)$$

13.62 Problem number 839

$$\int \frac{(ex)^{5/2} (a + bx^2)^2}{\sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(117a^2d^2 + 7bc(-26ad + 11bc)) e(ex)^{\frac{3}{2}} \sqrt{dx^2 + c}}{585d^3} - \frac{2b(-26ad + 11bc) (ex)^{\frac{7}{2}} \sqrt{dx^2 + c}}{117d^2e} \\ & + \frac{2b^2(ex)^{\frac{11}{2}} \sqrt{dx^2 + c}}{13de^3} - \frac{2c(117a^2d^2 + 7bc(-26ad + 11bc)) e^2 \sqrt{ex} \sqrt{dx^2 + c}}{195d^{\frac{7}{2}} (\sqrt{c} + x\sqrt{d})} \\ & + \frac{2c^{\frac{5}{4}} (117a^2d^2 + 7bc(-26ad + 11bc)) e^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)\right)}{195 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right) d^{\frac{15}{4}} \sqrt{dx^2 + c}} \\ & + \frac{c^{\frac{5}{4}} (117a^2d^2 + 7bc(-26ad + 11bc)) e^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)\right)}{195 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right) d^{\frac{15}{4}} \sqrt{dx^2 + c}} \end{aligned}$$

command

```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 (77 b^2 c^3 - 182 abc^2 d + 117 a^2 cd^2) \sqrt{d} e^{\frac{5}{2}} \operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) + (45 b^2 d^3 x^5 \right)}{585 d^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2 e^2 x^6 + 2 abe^2 x^4 + a^2 e^2 x^2) \sqrt{ex}}{\sqrt{dx^2 + c}}, x\right)$$

13.63 Problem number 840

$$\int \frac{(ex)^{3/2} (a + bx^2)^2}{\sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2b(-22ad + 9bc)(ex)^{\frac{5}{2}}\sqrt{dx^2 + c}}{77d^2e} + \frac{2b^2(ex)^{\frac{9}{2}}\sqrt{dx^2 + c}}{11de^3} \\
 & + \frac{2(77a^2d^2 + 5bc(-22ad + 9bc))e\sqrt{ex}\sqrt{dx^2 + c}}{231d^3} \\
 & + \frac{c^{\frac{3}{4}}(77a^2d^2 + 5bc(-22ad + 9bc))e^{\frac{3}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}}{\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)\right), \sqrt{dx^2 + c}}}{231\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{13}{4}}\sqrt{dx^2 + c}}
 \end{aligned}$$

command

```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((45b^2c^3 - 110abc^2d + 77a^2cd^2)\sqrt{d}e^{\frac{3}{2}}\text{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) - (21b^2d^3x^4 + 45b^2c^2d - 110abcd^2 + \dots)\right)}{231d^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(b^2ex^5 + 2abex^3 + a^2ex)\sqrt{ex}}{\sqrt{dx^2 + c}}, x\right)$$

13.64 Problem number 841

$$\int \frac{\sqrt{ex}(a + bx^2)^2}{\sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2b(-18ad + 7bc)(ex)^{\frac{3}{2}}\sqrt{dx^2 + c}}{45d^2e} + \frac{2b^2(ex)^{\frac{7}{2}}\sqrt{dx^2 + c}}{9de^3} \\
 & + \frac{2(15a^2d^2 + bc(-18ad + 7bc))\sqrt{ex}\sqrt{dx^2 + c}}{15d^{\frac{5}{2}}(\sqrt{c} + x\sqrt{d})} \\
 & + \frac{2c^{\frac{1}{4}}(15a^2d^2 + bc(-18ad + 7bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{15\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{11}{4}}\sqrt{dx^2 + c}} \\
 & + \frac{c^{\frac{1}{4}}(15a^2d^2 + bc(-18ad + 7bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{15\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)d^{\frac{11}{4}}\sqrt{dx^2 + c}}
 \end{aligned}$$

command

```
integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3(7b^2c^2 - 18abcd + 15a^2d^2)\sqrt{d}e^{\frac{1}{2}}\operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) - (5b^2d^2x^3 - (7b^2d^2x^3 - 7b^2d^2x^3))\right)}{45d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{ex}}{\sqrt{dx^2 + c}}, x\right)$$

13.65 Problem number 842

$$\int \frac{(a + bx^2)^2}{\sqrt{ex}\sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\frac{2b^2(ex)^{\frac{5}{2}} \sqrt{dx^2+c}}{7de^3} - \frac{2b(-14ad+5bc) \sqrt{ex} \sqrt{dx^2+c}}{21d^2e}$$

$$+ \frac{(21a^2d^2 - 14abcd + 5b^2c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c}}{21 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{1}{4}}d^{\frac{9}{4}}\sqrt{e} \sqrt{dx^2+c}}$$

command

`integrate((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((5b^2c^2 - 14abcd + 21a^2d^2) \sqrt{d} \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) + (3b^2d^2x^2 - 5b^2cd + 14abd^2) \sqrt{dx^2+c} \sqrt{x} \right)}{21d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2) \sqrt{dx^2+c} \sqrt{ex}}{dex^3 + cex}, x\right)$$

13.66 Problem number 843

$$\int \frac{(a + bx^2)^2}{(ex)^{3/2} \sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\frac{2b^2(ex)^{\frac{3}{2}} \sqrt{dx^2+c}}{5de^3} - \frac{2a^2 \sqrt{dx^2+c}}{ce\sqrt{ex}} - \frac{2(3b^2c^2 - 5ad(ad+2bc)) \sqrt{ex} \sqrt{dx^2+c}}{5cd^{\frac{3}{2}}e^2(\sqrt{c} + x\sqrt{d})}$$

$$+ \frac{2(3b^2c^2 - 5ad(ad+2bc)) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c}}{5 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{3}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}} \sqrt{dx^2+c}}$$

$$+ \frac{(3b^2c^2 - 5ad(ad+2bc)) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c}}{5 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{3}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}} \sqrt{dx^2+c}}$$

command

```
integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((3b^2c^2 - 10abcd - 5a^2d^2) \sqrt{d} x \operatorname{weierstrassZeta} \left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse} \left(-\frac{4c}{d}, 0, x \right) \right) + (b^2cdx^2 - 5a^2d^2) \sqrt{d} \right)}{5cd^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(b^2x^4 + 2abx^2 + a^2) \sqrt{dx^2 + c} \sqrt{ex}}{de^2x^4 + ce^2x^2}, x \right)$$

13.67 Problem number 844

$$\int \frac{(a + bx^2)^2}{(ex)^{5/2} \sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sqrt{dx^2 + c}}{3ce (ex)^{\frac{3}{2}}} + \frac{2b^2 \sqrt{ex} \sqrt{dx^2 + c}}{3de^3} + \frac{(a^2d^2 - 6abcd + b^2c^2) \sqrt{\frac{\cos \left(4 \arctan \left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}} \right) \right)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{c} + \sqrt{dx^2 + c})}{3 \cos \left(2 \arctan \left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}} \right) \right) c^{\frac{5}{4}} d^{\frac{5}{4}} e^{\frac{5}{2}} \sqrt{dx^2 + c}}$$

command

```
integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((b^2c^2 - 6abcd + a^2d^2) \sqrt{d} x^2 \operatorname{weierstrassPInverse} \left(-\frac{4c}{d}, 0, x \right) - (b^2cdx^2 - a^2d^2) \sqrt{dx^2 + c} \sqrt{x} \right) e^{-\frac{5}{2}}}{3cd^2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(b^2x^4 + 2abx^2 + a^2) \sqrt{dx^2 + c} \sqrt{ex}}{de^3x^5 + ce^3x^3}, x \right)$$

13.68 Problem number 845

$$\int \frac{(a + bx^2)^2}{(ex)^{7/2} \sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^2 \sqrt{dx^2 + c}}{5ce (ex)^{\frac{5}{2}}} - \frac{2a(-3ad + 10bc) \sqrt{dx^2 + c}}{5c^2 e^3 \sqrt{ex}} + \frac{2(-3a^2 d^2 + 10abcd + 5b^2 c^2) \sqrt{ex} \sqrt{dx^2 + c}}{5c^2 e^4 \sqrt{d} (\sqrt{c} + x\sqrt{d})} \\ & - \frac{2(-3a^2 d^2 + 10abcd + 5b^2 c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{5 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right) c^{\frac{7}{4}} d^{\frac{3}{4}} e^{\frac{7}{2}} \sqrt{dx^2 + c}} \\ & + \frac{(-3a^2 d^2 + 10abcd + 5b^2 c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{5 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right) c^{\frac{7}{4}} d^{\frac{3}{4}} e^{\frac{7}{2}} \sqrt{dx^2 + c}} \end{aligned}$$

command

`integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((5b^2c^2 + 10abcd - 3a^2d^2) \sqrt{d} x^3 \operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) + (a^2cd + (10abcd) \right)}{5c^2dx^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2) \sqrt{dx^2 + c} \sqrt{ex}}{de^4x^6 + ce^4x^4}, x\right)$$

13.69 Problem number 846

$$\int \frac{(a + bx^2)^2}{(ex)^{9/2} \sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sqrt{dx^2 + c}}{7ce (ex)^{\frac{7}{2}}} - \frac{2a(-5ad + 14bc) \sqrt{dx^2 + c}}{21c^2 e^3 (ex)^{\frac{3}{2}}}$$

$$+ \frac{(5a^2 d^2 - 14abcd + 21b^2 c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{dx^2 + c}}{21 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right) c^{\frac{9}{4}} d^{\frac{1}{4}} e^{\frac{9}{2}} \sqrt{dx^2 + c}}$$

command

`integrate((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((21 b^2 c^2 - 14 abcd + 5 a^2 d^2) \sqrt{d} x^4 \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) - (3 a^2 cd + (14 abcd - 5 a^2 d^2) x^2) \sqrt{dx^2 + c} \right)}{21 c^2 dx^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2 x^4 + 2 abx^2 + a^2) \sqrt{dx^2 + c} \sqrt{ex}}{de^5 x^7 + ce^5 x^5}, x\right)$$

13.70 Problem number 847

$$\int \frac{(a + bx^2)^2}{(ex)^{11/2} \sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \sqrt{dx^2 + c}}{9ce (ex)^{\frac{9}{2}}} - \frac{2a(-7ad + 18bc) \sqrt{dx^2 + c}}{45c^2e^3 (ex)^{\frac{5}{2}}} - \frac{2(7a^2d^2 - 18abcd + 15b^2c^2) \sqrt{dx^2 + c}}{15c^3e^5 \sqrt{ex}} \\ & + \frac{2(7a^2d^2 - 18abcd + 15b^2c^2) \sqrt{d} \sqrt{ex} \sqrt{dx^2 + c}}{15c^3e^6 (\sqrt{c} + x\sqrt{d})} \\ & \frac{2d^{\frac{1}{4}}(7a^2d^2 - 18abcd + 15b^2c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{15 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{11}{4}} e^{\frac{11}{2}} \sqrt{dx^2 + c}} \\ & \frac{d^{\frac{1}{4}}(7a^2d^2 - 18abcd + 15b^2c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{15 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{11}{4}} e^{\frac{11}{2}} \sqrt{dx^2 + c}} \end{aligned}$$

command

```
integrate((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 (15 b^2 c^2 - 18 a b c d + 7 a^2 d^2) \sqrt{d} x^5 \operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right)\right) + (3 (15 b^2 c^2 - 18 a b c d + 7 a^2 d^2) \sqrt{d} x^5 + 3 (15 b^2 c^2 - 18 a b c d + 7 a^2 d^2) \sqrt{d} x^4 + \dots \right)}{45 c^3 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2) \sqrt{dx^2 + c} \sqrt{ex}}{de^6x^8 + ce^6x^6}, x\right)$$

13.71 Problem number 848

$$\int \frac{(a + bx^2)^2}{(ex)^{13/2} \sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sqrt{dx^2 + c}}{11ce (ex)^{\frac{11}{2}}} - \frac{2a(-9ad + 22bc) \sqrt{dx^2 + c}}{77c^2 e^3 (ex)^{\frac{7}{2}}} - \frac{2(77b^2c^2 - 5ad(-9ad + 22bc)) \sqrt{dx^2 + c}}{231c^3 e^5 (ex)^{\frac{3}{2}}}$$

$$d^{\frac{3}{4}}(77b^2c^2 - 5ad(-9ad + 22bc)) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)$$

$$231 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{13}{4}} e^{\frac{13}{2}} \sqrt{dx^2 + c}$$

command

```
integrate((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((77b^2c^2 - 110abcd + 45a^2d^2)\sqrt{d}x^6 \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) + ((77b^2c^2 - 110abcd + 45a^2d^2)x^4 + 21\right)}{231c^3x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{dx^2 + c}\sqrt{ex}}{de^7x^9 + ce^7x^7}, x\right)$$

13.72 Problem number 849

$$\int \frac{(ex)^{7/2} (a + bx^2)^2}{(c + dx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-ad + bc)^2 (ex)^{\frac{9}{2}}}{cd^2e\sqrt{dx^2 + c}} - \frac{(77a^2d^2 - 198abcd + 117b^2c^2) e(ex)^{\frac{5}{2}} \sqrt{dx^2 + c}}{77cd^3}$$

$$+ \frac{2b^2(ex)^{\frac{9}{2}} \sqrt{dx^2 + c}}{11d^2e} + \frac{5(77a^2d^2 - 198abcd + 117b^2c^2) e^3 \sqrt{ex} \sqrt{dx^2 + c}}{231d^4}$$

$$5c^{\frac{3}{4}}(77a^2d^2 - 198abcd + 117b^2c^2) e^{\frac{7}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)$$

$$462 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) d^{\frac{17}{4}} \sqrt{dx^2 + c}$$

command

```
integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \left(117 b^2 c^4 - 198 a b c^3 d + 77 a^2 c^2 d^2 + (117 b^2 c^3 d - 198 a b c^2 d^2 + 77 a^2 c d^3) x^2 \right) \sqrt{d} e^{\frac{7}{2}} \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, \dots\right)}{d^2 x^4 + 2 c d x^2 + c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2 e^3 x^7 + 2 a b e^3 x^5 + a^2 e^3 x^3) \sqrt{d x^2 + c} \sqrt{e x}}{d^2 x^4 + 2 c d x^2 + c^2}, x\right)$$

13.73 Problem number 850

$$\int \frac{(e x)^{5/2} (a + b x^2)^2}{(c + d x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ad + bc)^2 (ex)^{\frac{7}{2}}}{cd^2 e \sqrt{dx^2 + c}} - \frac{(45a^2 d^2 - 126abcd + 77b^2 c^2) e (ex)^{\frac{3}{2}} \sqrt{dx^2 + c}}{45c d^3} \\ & + \frac{2b^2 (ex)^{\frac{7}{2}} \sqrt{dx^2 + c}}{9d^2 e} + \frac{(45a^2 d^2 - 126abcd + 77b^2 c^2) e^2 \sqrt{ex} \sqrt{dx^2 + c}}{15d^{\frac{7}{2}} (\sqrt{c} + x\sqrt{d})} \\ & - \frac{c^{\frac{1}{4}} (45a^2 d^2 - 126abcd + 77b^2 c^2) e^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2}}}{15 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right) d^{\frac{15}{4}} \sqrt{dx^2 + c}} \\ & + \frac{c^{\frac{1}{4}} (45a^2 d^2 - 126abcd + 77b^2 c^2) e^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2}}}{30 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right) d^{\frac{15}{4}} \sqrt{dx^2 + c}} \end{aligned}$$

command

```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(77b^2c^3 - 126abc^2d + 45a^2cd^2 + (77b^2c^2d - 126abcd^2 + 45a^2d^3)x^2)\sqrt{d}e^{\frac{5}{2}}\text{weierstrassZeta}\left(-\frac{4c}{d}, 0, \text{weierstrass}\right)}{45(d^5x^2 + cd^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(b^2e^2x^6 + 2abe^2x^4 + a^2e^2x^2)\sqrt{dx^2 + c}\sqrt{ex}}{d^2x^4 + 2cdx^2 + c^2}, x\right)$$

13.74 Problem number 851

$$\int \frac{(ex)^{3/2} (a + bx^2)^2}{(c + dx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ad + bc)^2 (ex)^{\frac{5}{2}}}{cd^2e\sqrt{dx^2 + c}} + \frac{2b^2(ex)^{\frac{5}{2}}\sqrt{dx^2 + c}}{7d^2e} - \frac{(21a^2d^2 - 70abcd + 45b^2c^2)e\sqrt{ex}\sqrt{dx^2 + c}}{21cd^3} \\ & (21a^2d^2 - 70abcd + 45b^2c^2)e^{\frac{3}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)} \\ & + \frac{42\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)c^{\frac{1}{4}}d^{\frac{13}{4}}\sqrt{dx^2 + c}}{21(d^5x^2 + cd^4)} \end{aligned}$$

command

```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(45b^2c^3 - 70abc^2d + 21a^2cd^2 + (45b^2c^2d - 70abcd^2 + 21a^2d^3)x^2)\sqrt{d}e^{\frac{3}{2}}\text{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) + (6b^2d^2)}{21(d^5x^2 + cd^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(b^2ex^5 + 2abex^3 + a^2ex)\sqrt{dx^2 + c}\sqrt{ex}}{d^2x^4 + 2cdx^2 + c^2}, x\right)$$

13.75 Problem number 852

$$\int \frac{\sqrt{ex} (a + bx^2)^2}{(c + dx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-ad + bc)^2 (ex)^{\frac{3}{2}}}{cd^2 e \sqrt{dx^2 + c}} + \frac{2b^2 (ex)^{\frac{3}{2}} \sqrt{dx^2 + c}}{5d^2 e} - \frac{(5a^2 d^2 - 30abcd + 21b^2 c^2) \sqrt{ex} \sqrt{dx^2 + c}}{5c d^{\frac{5}{2}} (\sqrt{c} + x\sqrt{d})}$$

$$(5a^2 d^2 - 30abcd + 21b^2 c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{\dots})$$

$$+ \frac{5 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right) c^{\frac{3}{4}} d^{\frac{11}{4}} \sqrt{dx^2 + c}}{5c d^{\frac{5}{2}} (\sqrt{c} + x\sqrt{d})}$$

$$(5a^2 d^2 - 30abcd + 21b^2 c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{\dots})$$

$$- \frac{10 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right) c^{\frac{3}{4}} d^{\frac{11}{4}} \sqrt{dx^2 + c}}{5c d^{\frac{5}{2}} (\sqrt{c} + x\sqrt{d})}$$

command

```
integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(21 b^2 c^3 - 30 abc^2 d + 5 a^2 cd^2 + (21 b^2 c^2 d - 30 abcd^2 + 5 a^2 d^3) x^2) \sqrt{d} e^{\frac{1}{2}} \operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInverse}\right)}{5 (cd^4 x^2 + c^2 d^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2 x^4 + 2 abx^2 + a^2) \sqrt{dx^2 + c} \sqrt{ex}}{d^2 x^4 + 2 cdx^2 + c^2}, x\right)$$

13.76 Problem number 853

$$\int \frac{(a + bx^2)^2}{\sqrt{ex} (c + dx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-ad + bc)^2 \sqrt{ex}}{cd^2e\sqrt{dx^2 + c}} + \frac{2b^2 \sqrt{ex} \sqrt{dx^2 + c}}{3d^2e}$$

$$(-3a^2d^2 - 6abcd + 5b^2c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{1/4}\sqrt{ex}}{c^{1/4}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{1/4}\sqrt{ex}}{c^{1/4}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) \left(\sqrt{\frac{6 \cos\left(2 \arctan\left(\frac{d^{1/4}\sqrt{ex}}{c^{1/4}\sqrt{e}}\right)\right) c^{5/4} d^{9/4} \sqrt{e} \sqrt{dx^2 + c}}{3(cd^4x^2 + c^2d^3)}}\right)$$

command

`integrate((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left((5b^2c^3 - 6abc^2d - 3a^2cd^2 + (5b^2c^2d - 6abcd^2 - 3a^2d^3)x^2)\sqrt{d} \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) - (2b^2cd^2x^2 + \dots)\right)}{3(cd^4x^2 + c^2d^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{dx^2 + c}\sqrt{ex}}{d^2ex^5 + 2cdex^3 + c^2ex}, x\right)$$

13.77 Problem number 854

$$\int \frac{(a + bx^2)^2}{(ex)^{3/2} (c + dx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(3a^2d^2 - 2abcd + b^2c^2)(ex)^{\frac{3}{2}}}{c^2de^3\sqrt{dx^2+c}} - \frac{2a^2}{ce\sqrt{ex}\sqrt{dx^2+c}} + \frac{(3a^2d^2 - 2abcd + 3b^2c^2)\sqrt{ex}\sqrt{dx^2+c}}{c^2d^{\frac{3}{2}}e^2(\sqrt{c} + x\sqrt{d})}$$

$$(3a^2d^2 - 2abcd + 3b^2c^2) \sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c} + x\sqrt{d})$$

$$\frac{\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{7}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{dx^2+c}}{(3a^2d^2 - 2abcd + 3b^2c^2) \sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c} + x\sqrt{d})}$$

$$+ \frac{2\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{7}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{dx^2+c}}{2\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{7}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{dx^2+c}}$$

command

`integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\left((3b^2c^2d - 2abcd^2 + 3a^2d^3)x^3 + (3b^2c^3 - 2abc^2d + 3a^2cd^2)x\right)\sqrt{d}\operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPInvers}\right)\right)}{c^2d^3x^3 + c^3d^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{dx^2+c}\sqrt{ex}}{d^2e^2x^6 + 2cde^2x^4 + c^2e^2x^2}, x\right)$$

13.78 Problem number 855

$$\int \frac{(a + bx^2)^2}{(ex)^{5/2}(c + dx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2a^2}{3ce(ex)^{\frac{3}{2}}\sqrt{dx^2+c}} - \frac{(5a^2d^2 - 6abcd + 3b^2c^2)\sqrt{ex}}{3c^2de^3\sqrt{dx^2+c}}$$

$$(3b^2c^2 + ad(-5ad + 6bc)) \sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c} + x\sqrt{d})$$

$$+ \frac{6\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{9}{4}}d^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{dx^2+c}}{6\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{9}{4}}d^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{dx^2+c}}$$

command

`integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\left(3b^2c^2d + 6abcd^2 - 5a^2d^3\right)x^4 + \left(3b^2c^3 + 6abc^2d - 5a^2cd^2\right)x^2\right)\sqrt{d} \operatorname{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) - \left(2a^2cd^2 + \dots\right)}{3\left(c^2d^3x^4 + c^3d^2x^2\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{dx^2 + c}\sqrt{ex}}{d^2e^3x^7 + 2cde^3x^5 + c^2e^3x^3}, x\right)$$

13.79 Problem number 856

$$\int \frac{(a + bx^2)^2}{(ex)^{7/2}(c + dx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^2}{5ce(ex)^{\frac{5}{2}}\sqrt{dx^2+c}} + \frac{(5b^2c^2 - 3ad(-7ad + 10bc))(ex)^{\frac{3}{2}}}{5c^3e^5\sqrt{dx^2+c}} \\ & -\frac{2a(-7ad + 10bc)}{5c^2e^3\sqrt{ex}\sqrt{dx^2+c}} - \frac{(5b^2c^2 - 3ad(-7ad + 10bc))\sqrt{ex}\sqrt{dx^2+c}}{5c^3e^4\sqrt{d}\left(\sqrt{c} + x\sqrt{d}\right)} \\ & + \frac{(5b^2c^2 - 3ad(-7ad + 10bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{5\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)c^{\frac{11}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{dx^2+c}} \\ & - \frac{(5b^2c^2 - 3ad(-7ad + 10bc))\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{10\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)c^{\frac{11}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{dx^2+c}} \end{aligned}$$

command

`integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left((5b^2c^2d - 30abcd^2 + 21a^2d^3)x^5 + (5b^2c^3 - 30abc^2d + 21a^2cd^2)x^3 \right) \sqrt{d} \operatorname{weierstrassZeta}\left(-\frac{4c}{d}, 0, \operatorname{weierstrassPI}\right)}{5(c^3d^2x^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{dx^2 + c}\sqrt{ex}}{d^2e^4x^8 + 2cde^4x^6 + c^2e^4x^4}, x\right)$$

13.80 Problem number 857

$$\int \frac{(ex)^{7/2} (a + bx^2)^2}{(c + dx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ad + bc)^2 (ex)^{\frac{9}{2}}}{3cd^2e(dx^2 + c)^{\frac{3}{2}}} + \frac{(7a^2d^2 - 42abcd + 39b^2c^2)e(ex)^{\frac{5}{2}}}{14cd^3\sqrt{dx^2 + c}} \\ & + \frac{2b^2(ex)^{\frac{9}{2}}}{7d^2e\sqrt{dx^2 + c}} - \frac{5(7a^2d^2 - 42abcd + 39b^2c^2)e^3\sqrt{ex}\sqrt{dx^2 + c}}{42cd^4} \\ & + \frac{5(7a^2d^2 - 42abcd + 39b^2c^2)e^{\frac{7}{2}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}}{84\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)c^{\frac{1}{4}}d^{\frac{17}{4}}\sqrt{dx^2 + c}} \end{aligned}$$

command

`integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(39b^2c^4 - 42abc^3d + 7a^2c^2d^2 + (39b^2c^2d^2 - 42abcd^3 + 7a^2d^4)x^4 + 2(39b^2c^3d - 42abc^2d^2 + 7a^2cd^3)x^2)\sqrt{d}e^{\frac{7}{2}}}{5(c^3d^2x^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2e^3x^7 + 2abe^3x^5 + a^2e^3x^3)\sqrt{dx^2 + c}\sqrt{ex}}{d^3x^6 + 3cd^2x^4 + 3c^2dx^2 + c^3}, x\right)$$

13.81 Problem number 858

$$\int \frac{(ex)^{5/2} (a + bx^2)^2}{(c + dx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ad + bc)^2 (ex)^{\frac{7}{2}}}{3cd^2e(dx^2 + c)^{\frac{3}{2}}} + \frac{(5a^2d^2 - 70abcd + 77b^2c^2) e(ex)^{\frac{3}{2}}}{30cd^3\sqrt{dx^2 + c}} \\ & + \frac{2b^2(ex)^{\frac{7}{2}}}{5d^2e\sqrt{dx^2 + c}} - \frac{(5a^2d^2 - 70abcd + 77b^2c^2) e^2\sqrt{ex} \sqrt{dx^2 + c}}{10cd^{\frac{7}{2}}(\sqrt{c} + x\sqrt{d})} \\ & (5a^2d^2 - 70abcd + 77b^2c^2) e^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) \\ & + \frac{10 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{3}{4}} d^{\frac{15}{4}} \sqrt{dx^2 + c}}{10cd^{\frac{7}{2}}(\sqrt{c} + x\sqrt{d})} \\ & (5a^2d^2 - 70abcd + 77b^2c^2) e^{\frac{5}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) \\ & - \frac{20 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{3}{4}} d^{\frac{15}{4}} \sqrt{dx^2 + c}}{10cd^{\frac{7}{2}}(\sqrt{c} + x\sqrt{d})} \end{aligned}$$

command

```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3(77b^2c^4 - 70abc^3d + 5a^2c^2d^2 + (77b^2c^2d^2 - 70abcd^3 + 5a^2d^4)x^4 + 2(77b^2c^3d - 70abc^2d^2 + 5a^2cd^3)x^2)\sqrt{d}e^{\frac{5}{2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2e^2x^6 + 2abe^2x^4 + a^2e^2x^2)\sqrt{dx^2 + c}\sqrt{ex}}{d^3x^6 + 3cd^2x^4 + 3c^2dx^2 + c^3}, x\right)$$

13.82 Problem number 859

$$\int \frac{(ex)^{3/2} (a + bx^2)^2}{(c + dx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(-ad + bc)^2 (ex)^{\frac{5}{2}}}{3cd^2e(dx^2 + c)^{\frac{3}{2}}} + \frac{2b^2(ex)^{\frac{5}{2}}}{3d^2e\sqrt{dx^2 + c}} + \frac{(-a^2d^2 - 10abcd + 15b^2c^2)e\sqrt{ex}}{6cd^3\sqrt{dx^2 + c}}$$

$$(-a^2d^2 - 10abcd + 15b^2c^2)e^{\frac{3}{2}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}$$

$$12 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{5}{4}} d^{\frac{13}{4}} \sqrt{dx^2 + c}$$

command

```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(15b^2c^4 - 10abc^3d - a^2c^2d^2 + (15b^2c^2d^2 - 10abcd^3 - a^2d^4)x^4 + 2(15b^2c^3d - 10abc^2d^2 - a^2cd^3)x^2)\sqrt{d}e^{\frac{3}{2}}}{6(cd^6x^2 + \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2ex^5 + 2abex^3 + a^2ex)\sqrt{dx^2 + c}\sqrt{ex}}{d^3x^6 + 3cd^2x^4 + 3c^2dx^2 + c^3}, x\right)$$

13.83 Problem number 860

$$\int \frac{\sqrt{ex} (a + bx^2)^2}{(c + dx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(-ad + bc)^2 (ex)^{\frac{3}{2}}}{3cd^2e(dx^2 + c)^{\frac{3}{2}}} - \frac{(-ad + bc)(ad + 3bc)(ex)^{\frac{3}{2}}}{2c^2d^2e\sqrt{dx^2 + c}} + \frac{(-a^2d^2 - 2abcd + 7b^2c^2)\sqrt{ex}\sqrt{dx^2 + c}}{2c^2d^{\frac{5}{2}}(\sqrt{c} + x\sqrt{d})}$$

$$(-a^2d^2 - 2abcd + 7b^2c^2) \sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c} + x\sqrt{d})$$

$$\frac{2\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)c^{\frac{7}{4}}d^{\frac{11}{4}}\sqrt{dx^2 + c}}{(-a^2d^2 - 2abcd + 7b^2c^2) \sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c} + x\sqrt{d})}$$

$$+ \frac{4\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)c^{\frac{7}{4}}d^{\frac{11}{4}}\sqrt{dx^2 + c}}{(-a^2d^2 - 2abcd + 7b^2c^2) \sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c} + x\sqrt{d})}$$

command

`integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(7b^2c^4 - 2abc^3d - a^2c^2d^2 + (7b^2c^2d^2 - 2abcd^3 - a^2d^4)x^4 + 2(7b^2c^3d - 2abc^2d^2 - a^2cd^3)x^2)\sqrt{d}e^{\frac{1}{2}}\operatorname{weierstrass}\operatorname{sn}\left(\frac{2\sqrt{c}\sqrt{dx^2 + c}}{\sqrt{d}}, \frac{\sqrt{2}}{2}\right)}{6(d^{\frac{11}{4}}c^{\frac{7}{4}}\sqrt{dx^2 + c})}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{dx^2 + c}\sqrt{ex}}{d^3x^6 + 3cd^2x^4 + 3c^2dx^2 + c^3}, x\right)$$

13.84 Problem number 861

$$\int \frac{(a + bx^2)^2}{\sqrt{ex}(c + dx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(-ad + bc)^2\sqrt{ex}}{3cd^2e(dx^2 + c)^{\frac{3}{2}}} - \frac{(-ad + bc)(5ad + 7bc)\sqrt{ex}}{6c^2d^2e\sqrt{dx^2 + c}}$$

$$(5a^2d^2 + 2abcd + 5b^2c^2) \sqrt{\frac{\cos\left(4\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c} + x\sqrt{d})$$

$$+ \frac{12\cos\left(2\arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)c^{\frac{9}{4}}d^{\frac{9}{4}}\sqrt{e}\sqrt{dx^2 + c}}{(-ad + bc)(5ad + 7bc)\sqrt{ex}}$$

command

```
integrate((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left((5b^2c^4 + 2abc^3d + 5a^2c^2d^2 + (5b^2c^2d^2 + 2abcd^3 + 5a^2d^4)x^4 + 2(5b^2c^3d + 2abc^2d^2 + 5a^2cd^3)x^2) \sqrt{d} \operatorname{weierstrass} \right)}{6(c^2d^5x^4 + 2c^3d^4x^2 + c^4d^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(b^2x^4 + 2abx^2 + a^2) \sqrt{dx^2 + c} \sqrt{ex}}{d^3ex^7 + 3cd^2ex^5 + 3c^2dex^3 + c^3ex}, x \right)$$

13.85 Problem number 862

$$\int \frac{(a + bx^2)^2}{(ex)^{3/2} (c + dx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(7a^2d^2 - 2abcd + b^2c^2)(ex)^{\frac{3}{2}}}{3c^2de^3(dx^2 + c)^{\frac{3}{2}}} - \frac{2a^2}{ce(dx^2 + c)^{\frac{3}{2}}\sqrt{ex}} \\ & + \frac{(b^2c^2 + ad(-7ad + 2bc))(ex)^{\frac{3}{2}}}{2c^3de^3\sqrt{dx^2 + c}} - \frac{(b^2c^2 + ad(-7ad + 2bc))\sqrt{ex}\sqrt{dx^2 + c}}{2c^3d^{\frac{3}{2}}e^2(\sqrt{c} + x\sqrt{d})} \\ & + \frac{(b^2c^2 + ad(-7ad + 2bc)) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c} + x\sqrt{d})}{2 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{11}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{dx^2 + c}} \\ & - \frac{(b^2c^2 + ad(-7ad + 2bc)) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{c} + x\sqrt{d})}{4 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}}\sqrt{ex}}{c^{\frac{1}{4}}\sqrt{e}}\right)\right) c^{\frac{11}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{dx^2 + c}} \end{aligned}$$

command

```
integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$\left(3 \left((b^2 c^2 d^2 + 2 abcd^3 - 7 a^2 d^4) x^5 + 2 (b^2 c^3 d + 2 abc^2 d^2 - 7 a^2 cd^3) x^3 + (b^2 c^4 + 2 abc^3 d - 7 a^2 c^2 d^2) x \right) \sqrt{d} \text{ weierstra}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b^2 x^4 + 2 abx^2 + a^2) \sqrt{dx^2 + c} \sqrt{ex}}{d^3 e^2 x^8 + 3 cd^2 e^2 x^6 + 3 c^2 de^2 x^4 + c^3 e^2 x^2}, x \right)$$

13.86 Problem number 863

$$\int \frac{(a + bx^2)^2}{(ex)^{5/2} (c + dx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^2}{3ce (ex)^{\frac{3}{2}} (dx^2 + c)^{\frac{3}{2}}} - \frac{(3a^2 d^2 - 2abcd + b^2 c^2) \sqrt{ex}}{3c^2 d e^3 (dx^2 + c)^{\frac{3}{2}}} + \frac{(b^2 c^2 + 5ad(-3ad + 2bc)) \sqrt{ex}}{6c^3 d e^3 \sqrt{dx^2 + c}} \\ & (b^2 c^2 + 5ad(-3ad + 2bc)) \sqrt{\frac{\cos \left(4 \arctan \left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(2 \arctan \left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{c} \\ & + \frac{12 \cos \left(2 \arctan \left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}} \right) \right) c^{\frac{13}{4}} d^{\frac{5}{4}} e^{\frac{5}{2}} \sqrt{dx^2 + c}}{12 \cos \left(2 \arctan \left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}} \right) \right) c^{\frac{13}{4}} d^{\frac{5}{4}} e^{\frac{5}{2}} \sqrt{dx^2 + c}} \end{aligned}$$

command

`integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$\left(((b^2 c^2 d^2 + 10 abcd^3 - 15 a^2 d^4) x^6 + 2 (b^2 c^3 d + 10 abc^2 d^2 - 15 a^2 cd^3) x^4 + (b^2 c^4 + 10 abc^3 d - 15 a^2 c^2 d^2) x^2 \right) \sqrt{d} \text{ we}$

6 (c^3 d

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b^2 x^4 + 2 abx^2 + a^2) \sqrt{dx^2 + c} \sqrt{ex}}{d^3 e^3 x^9 + 3 cd^2 e^3 x^7 + 3 c^2 de^3 x^5 + c^3 e^3 x^3}, x \right)$$

13.87 Problem number 864

$$\int \frac{(a + bx^2)^2}{(ex)^{7/2} (c + dx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^2}{5ce (ex)^{\frac{5}{2}} (dx^2 + c)^{\frac{3}{2}}} + \frac{(77a^2d^2 - 70abcd + 5b^2c^2) (ex)^{\frac{3}{2}}}{15c^3e^5 (dx^2 + c)^{\frac{3}{2}}} - \frac{2a(-11ad + 10bc)}{5c^2e^3 (dx^2 + c)^{\frac{3}{2}} \sqrt{ex}} \\ & + \frac{(77a^2d^2 - 70abcd + 5b^2c^2) (ex)^{\frac{3}{2}}}{10c^4e^5 \sqrt{dx^2 + c}} - \frac{(77a^2d^2 - 70abcd + 5b^2c^2) \sqrt{ex} \sqrt{dx^2 + c}}{10c^4e^4 \sqrt{d} (\sqrt{c} + x\sqrt{d})} \\ & (77a^2d^2 - 70abcd + 5b^2c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{ex} \sqrt{dx^2 + c}) \\ & + \frac{10 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right) c^{\frac{15}{4}} d^{\frac{3}{4}} e^{\frac{7}{2}} \sqrt{dx^2 + c}}{(77a^2d^2 - 70abcd + 5b^2c^2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{ex} \sqrt{dx^2 + c}) \\ & - \frac{20 \cos\left(2 \arctan\left(\frac{d^{\frac{1}{4}} \sqrt{ex}}{c^{\frac{1}{4}} \sqrt{e}}\right)\right) c^{\frac{15}{4}} d^{\frac{3}{4}} e^{\frac{7}{2}} \sqrt{dx^2 + c}}{\end{aligned}$$

command

```
integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(3 \left((5b^2c^2d^2 - 70abcd^3 + 77a^2d^4)x^7 + 2(5b^2c^3d - 70abc^2d^2 + 77a^2cd^3)x^5 + (5b^2c^4 - 70abc^3d + 77a^2c^2d^2)x^3\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2x^4 + 2abx^2 + a^2) \sqrt{dx^2 + c} \sqrt{ex}}{d^3e^4x^{10} + 3cd^2e^4x^8 + 3c^2de^4x^6 + c^3e^4x^4}, x\right)$$

13.88 Problem number 964

$$\int \frac{x^4 \sqrt{-1 + 3x^2}}{\sqrt{2 - 3x^2}} dx$$

Optimal antiderivative

$$\frac{8\sqrt{x^2} \operatorname{EllipticE}\left(\frac{\sqrt{-6x^2+4}}{2}, \sqrt{2}\right) \sqrt{3}}{135x} - \frac{2\sqrt{x^2} \operatorname{EllipticF}\left(\frac{\sqrt{-6x^2+4}}{2}, \sqrt{2}\right) \sqrt{3}}{81x} - \frac{7x\sqrt{-3x^2+2} \sqrt{3x^2-1}}{135} - \frac{x^3\sqrt{-3x^2+2} \sqrt{3x^2-1}}{15}$$

command

```
integrate(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(9x^4 + 7x^2 + 8) \sqrt{3x^2 - 1} \sqrt{-3x^2 + 2}}{135x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2-1} \sqrt{-3x^2+2} x^4}{3x^2-2}, x\right)$$

13.89 Problem number 966

$$\int \frac{x^2 \sqrt{-1 + 3x^2}}{\sqrt{2 - 3x^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{x^2} \operatorname{EllipticE}\left(\frac{\sqrt{-6x^2+4}}{2}, \sqrt{2}\right) \sqrt{3}}{9x} - \frac{\sqrt{x^2} \operatorname{EllipticF}\left(\frac{\sqrt{-6x^2+4}}{2}, \sqrt{2}\right) \sqrt{3}}{27x} - \frac{x\sqrt{-3x^2+2} \sqrt{3x^2-1}}{9}$$

command

```
integrate(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{3x^2-1}(x^2+1)\sqrt{-3x^2+2}}{9x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2-1}\sqrt{-3x^2+2}x^2}{3x^2-2}, x\right)$$

13.90 Problem number 995

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(x, \frac{i\sqrt{6}}{2}\right)\sqrt{2}}{3} - \frac{\text{EllipticF}\left(x, \frac{i\sqrt{6}}{2}\right)\sqrt{2}}{3}$$

command

```
integrate(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{3x^2+2}\sqrt{-x^2+1}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2+2}\sqrt{-x^2+1}x^2}{3x^4-x^2-2}, x\right)$$

13.91 Problem number 997

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(\frac{x}{2}, i\sqrt{6}\right)\sqrt{2}}{3} - \frac{\text{EllipticF}\left(\frac{x}{2}, i\sqrt{6}\right)\sqrt{2}}{3}$$

command

```
integrate(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{3x^2+2}\sqrt{-x^2+4}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2+2}\sqrt{-x^2+4}x^2}{3x^4-10x^2-8},x\right)$$

13.92 Problem number 999

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(2x, \frac{i\sqrt{6}}{4}\right)\sqrt{2}}{6} - \frac{\text{EllipticF}\left(2x, \frac{i\sqrt{6}}{4}\right)\sqrt{2}}{6}$$

command

```
integrate(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{3x^2+2}\sqrt{-4x^2+1}}{12x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2+2}\sqrt{-4x^2+1}x^2}{12x^4+5x^2-2},x\right)$$

13.93 Problem number 1001

$$\int \frac{x^2}{\sqrt{2-3x^2} \sqrt{1+x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(\frac{x\sqrt{6}}{2}, \frac{i\sqrt{6}}{3}\right) \sqrt{3}}{3} - \frac{\text{EllipticF}\left(\frac{x\sqrt{6}}{2}, \frac{i\sqrt{6}}{3}\right) \sqrt{3}}{3}$$

command

`integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{x^2+1} \sqrt{-3x^2+2}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{x^2+1} \sqrt{-3x^2+2} x^2}{3x^4+x^2-2}, x\right)$$

13.94 Problem number 1002

$$\int \frac{x^2}{\sqrt{2-3x^2} \sqrt{4+x^2}} dx$$

Optimal antiderivative

$$\frac{2 \text{EllipticE}\left(\frac{x\sqrt{6}}{2}, \frac{i\sqrt{6}}{6}\right) \sqrt{3}}{3} - \frac{2 \text{EllipticF}\left(\frac{x\sqrt{6}}{2}, \frac{i\sqrt{6}}{6}\right) \sqrt{3}}{3}$$

command

`integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{x^2+4} \sqrt{-3x^2+2}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{x^2+4} \sqrt{-3x^2+2} x^2}{3x^4+10x^2-8}, x\right)$$

13.95 Problem number 1003

$$\int \frac{x^2}{\sqrt{2-3x^2} \sqrt{1+4x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(\frac{x\sqrt{6}}{2}, \frac{2i\sqrt{6}}{3}\right) \sqrt{3}}{12} - \frac{\text{EllipticF}\left(\frac{x\sqrt{6}}{2}, \frac{2i\sqrt{6}}{3}\right) \sqrt{3}}{12}$$

command

```
integrate(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{4x^2+1} \sqrt{-3x^2+2}}{12x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{4x^2+1} \sqrt{-3x^2+2} x^2}{12x^4-5x^2-2}, x\right)$$

13.96 Problem number 1007

$$\int \frac{x^2}{\sqrt{1-x^2} \sqrt{-1+2x^2}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{x^2} \text{EllipticE}\left(\sqrt{-x^2+1}, \sqrt{2}\right)}{2x} - \frac{\sqrt{x^2} \text{EllipticF}\left(\sqrt{-x^2+1}, \sqrt{2}\right)}{2x}$$

command

```
integrate(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{2x^2-1} \sqrt{-x^2+1}}{2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{2x^2-1} \sqrt{-x^2+1} x^2}{2x^4-3x^2+1}, x\right)$$

14 Test file number 22

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.2_Quadratic/22_1.1.2.5-a+b_x^2-^p-c+d_x^2-^q-e+f_x^2-^r

14.1 Problem number 62

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}} dx$$

Optimal antiderivative

$$\frac{b \arctan\left(\frac{x\sqrt{-af+be}}{\sqrt{a}\sqrt{fx^2+e}}\right)}{(-ad+bc)\sqrt{a}\sqrt{-af+be}} - \frac{d \arctan\left(\frac{x\sqrt{-cf+de}}{\sqrt{c}\sqrt{fx^2+e}}\right)}{(-ad+bc)\sqrt{c}\sqrt{-cf+de}}$$

command

`integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{(bc^2f - bcde)\sqrt{a^2f - abe} \log\left(\frac{8a^2f^2x^4 - 4(2afx^3 - (bx^3 - ax)e)\sqrt{a^2f - abe}\sqrt{fx^2 + e} + (b^2x^4 - 6abx^2 + a^2)e^2 - 8(abfx^4 - b^2x^4 + 2abx^2 + a^2)}{b^2x^4 + 2abx^2 + a^2}\right)}{4((a^2bc^3 - a^3c^2d)f^2 - (ab^2c^3 - a^3cd^2)fe - (a^2df - abde)\sqrt{c^2f - cde})} \right. \\ \left. - \frac{2(bc^2f - bcde)\sqrt{-a^2f + abe} \arctan\left(\frac{(2afx^2 - (bx^2 - a)e)\sqrt{-a^2f + abe}\sqrt{fx^2 + e}}{2(a^2f^2x^3 - abxe^2 - (abfx^3 - a^2fx)e)}\right)}{4((a^2bc^3 - a^3c^2d)f^2 - (ab^2c^3 - a^3cd^2)fe - (a^2df - abde)\sqrt{c^2f - cde})} \right. \\ \left. - \frac{(bc^2f - bcde)\sqrt{-a^2f + abe} \arctan\left(\frac{(2afx^2 - (bx^2 - a)e)\sqrt{-a^2f + abe}\sqrt{fx^2 + e}}{2(a^2f^2x^3 - abxe^2 - (abfx^3 - a^2fx)e)}\right)}{2((a^2bc^3 - a^3c^2d)f^2 - (ab^2c^3 - a^3cd^2)fe + (ab^2c^2d - a^2bcd^2))} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

14.2 Problem number 97

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}} dx$$

Optimal antiderivative

$$-\frac{\text{EllipticF}\left(x, \sqrt{-\frac{b}{a}}\right) \sqrt{1 + \frac{bx^2}{a}}}{\sqrt{bx^2 + a}}$$

command

```
integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\text{ellipticF}\left(x, -\frac{b}{a}\right)}{\sqrt{a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^2 + a} \sqrt{-x^2 + 1}}{bx^4 + (a - b)x^2 - a}, x\right)$$

15 Test file number 24

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.2_Quadratic/24_1.1.2.8_P-x-c_x^-m-a+b_x^2-^p

15.1 Problem number 62

$$\int x^3(a + bx^2)(A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$\frac{aAx^4}{4} + \frac{aBx^5}{5} + \frac{(Ab + aC)x^6}{6} + \frac{(bB + aD)x^7}{7} + \frac{bCx^8}{8} + \frac{bDx^9}{9}$$

command

```
integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{9}Dbx^9 + \frac{1}{8}Cbx^8 + \frac{1}{7}(Da + Bb)x^7 + \frac{1}{5}Bax^5 + \frac{1}{6}(Ca + Ab)x^6 + \frac{1}{4}Aax^4$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

15.2 Problem number 63

$$\int x^2(a + bx^2)(A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$\frac{aAx^3}{3} + \frac{aBx^4}{4} + \frac{(Ab + aC)x^5}{5} + \frac{(bB + aD)x^6}{6} + \frac{bCx^7}{7} + \frac{bDx^8}{8}$$

command

```
integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{8}Dbx^8 + \frac{1}{7}Cbx^7 + \frac{1}{6}(Da + Bb)x^6 + \frac{1}{4}Bax^4 + \frac{1}{5}(Ca + Ab)x^5 + \frac{1}{3}Aax^3$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

15.3 Problem number 64

$$\int x(a + bx^2)(A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$\frac{aAx^2}{2} + \frac{aBx^3}{3} + \frac{(Ab + aC)x^4}{4} + \frac{(bB + aD)x^5}{5} + \frac{bCx^6}{6} + \frac{bDx^7}{7}$$

command

```
integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{7}Dbx^7 + \frac{1}{6}Cbx^6 + \frac{1}{5}(Da + Bb)x^5 + \frac{1}{3}Bax^3 + \frac{1}{4}(Ca + Ab)x^4 + \frac{1}{2}Aax^2$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

15.4 Problem number 65

$$\int (a + bx^2) (A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$aAx + \frac{aBx^2}{2} + \frac{(Ab + aC)x^3}{3} + \frac{(bB + aD)x^4}{4} + \frac{bCx^5}{5} + \frac{bDx^6}{6}$$

command

```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6}Dbx^6 + \frac{1}{5}Cbx^5 + \frac{1}{4}(Da + Bb)x^4 + \frac{1}{2}Bax^2 + \frac{1}{3}(Ca + Ab)x^3 + Aax$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

15.5 Problem number 70

$$\int x^3 (a + bx^2)^2 (A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2Ax^4}{4} + \frac{a^2Bx^5}{5} + \frac{a(2Ab + aC)x^6}{6} + \frac{a(2bB + aD)x^7}{7} \\ & + \frac{b(Ab + 2aC)x^8}{8} + \frac{b(bB + 2aD)x^9}{9} + \frac{b^2Cx^{10}}{10} + \frac{b^2Dx^{11}}{11} \end{aligned}$$

command

```
integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{11}Db^2x^{11} + \frac{1}{10}Cb^2x^{10} + \frac{1}{9}(2Dab + Bb^2)x^9 + \frac{1}{8}(2Cab + Ab^2)x^8 \\ & + \frac{1}{5}Ba^2x^5 + \frac{1}{7}(Da^2 + 2Bab)x^7 + \frac{1}{4}Aa^2x^4 + \frac{1}{6}(Ca^2 + 2Aab)x^6 \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

15.6 Problem number 71

$$\int x^2 (a + bx^2)^2 (A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$\frac{a^2 A x^3}{3} + \frac{a^2 B x^4}{4} + \frac{a(2Ab + aC) x^5}{5} + \frac{a(2bB + aD) x^6}{6} \\ + \frac{b(Ab + 2aC) x^7}{7} + \frac{b(bB + 2aD) x^8}{8} + \frac{b^2 C x^9}{9} + \frac{b^2 D x^{10}}{10}$$

command

```
integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{10} D b^2 x^{10} + \frac{1}{9} C b^2 x^9 + \frac{1}{8} (2 D a b + B b^2) x^8 + \frac{1}{7} (2 C a b + A b^2) x^7 \\ + \frac{1}{4} B a^2 x^4 + \frac{1}{6} (D a^2 + 2 B a b) x^6 + \frac{1}{3} A a^2 x^3 + \frac{1}{5} (C a^2 + 2 A a b) x^5$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

15.7 Problem number 72

$$\int x (a + bx^2)^2 (A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$\frac{a^2 B x^3}{3} + \frac{a^2 C x^4}{4} + \frac{a(2bB + aD) x^5}{5} + \frac{abC x^6}{3} + \frac{b(bB + 2aD) x^7}{7} + \frac{b^2 C x^8}{8} + \frac{b^2 D x^9}{9} + \frac{A(bx^2 + a)^3}{6b}$$

command

```
integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{9} D b^2 x^9 + \frac{1}{8} C b^2 x^8 + \frac{1}{7} (2 D a b + B b^2) x^7 + \frac{1}{6} (2 C a b + A b^2) x^6 \\ + \frac{1}{3} B a^2 x^3 + \frac{1}{5} (D a^2 + 2 B a b) x^5 + \frac{1}{2} A a^2 x^2 + \frac{1}{4} (C a^2 + 2 A a b) x^4$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

15.8 Problem number 73

$$\int (a + bx^2)^2 (A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$a^2Ax + \frac{a(2Ab + aC)x^3}{3} + \frac{a^2Dx^4}{4} + \frac{b(Ab + 2aC)x^5}{5} + \frac{abDx^6}{3} + \frac{b^2Cx^7}{7} + \frac{b^2Dx^8}{8} + \frac{B(bx^2 + a)^3}{6b}$$

command

```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{8}Db^2x^8 + \frac{1}{7}Cb^2x^7 + \frac{1}{6}(2Dab + Bb^2)x^6 + \frac{1}{5}(2Cab + Ab^2)x^5 + \frac{1}{2}Ba^2x^2 + \frac{1}{4}(Da^2 + 2Bab)x^4 + Aa^2x + \frac{1}{3}(Ca^2 + 2Aab)x^3$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

15.9 Problem number 78

$$\int x^3(a + bx^2)^3 (A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$\frac{a^3Ax^4}{4} + \frac{a^3Bx^5}{5} + \frac{a^2(3Ab + aC)x^6}{6} + \frac{a^2(3bB + aD)x^7}{7} + \frac{3ab(Ab + aC)x^8}{8} + \frac{ab(bB + aD)x^9}{3} + \frac{b^2(Ab + 3aC)x^{10}}{10} + \frac{b^2(bB + 3aD)x^{11}}{11} + \frac{b^3Cx^{12}}{12} + \frac{b^3Dx^{13}}{13}$$

command

```
integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{13}Db^3x^{13} + \frac{1}{12}Cb^3x^{12} + \frac{1}{11}(3Dab^2 + Bb^3)x^{11} + \frac{1}{10}(3Cab^2 + Ab^3)x^{10} + \frac{1}{3}(Da^2b + Bab^2)x^9 + \frac{1}{5}Ba^3x^5 + \frac{3}{8}(Ca^2b + Aab^2)x^8 + \frac{1}{4}Aa^3x^4 + \frac{1}{7}(Da^3 + 3Ba^2b)x^7 + \frac{1}{6}(Ca^3 + 3Aa^2b)x^6$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

15.10 Problem number 79

$$\int x^2(a + bx^2)^3 (A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$\frac{a^3 A x^3}{3} + \frac{a^3 B x^4}{4} + \frac{a^2(3Ab + aC) x^5}{5} + \frac{a^2(3bB + aD) x^6}{6} + \frac{3ab(Ab + aC) x^7}{7} + \frac{3ab(bB + aD) x^8}{8} + \frac{b^2(Ab + 3aC) x^9}{9} + \frac{b^2(bB + 3aD) x^{10}}{10} + \frac{b^3 C x^{11}}{11} + \frac{b^3 D x^{12}}{12}$$

command

```
integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{12} D b^3 x^{12} + \frac{1}{11} C b^3 x^{11} + \frac{1}{10} (3 D a b^2 + B b^3) x^{10} + \frac{1}{9} (3 C a b^2 + A b^3) x^9 + \frac{3}{8} (D a^2 b + B a b^2) x^8 + \frac{1}{4} B a^3 x^4 + \frac{3}{7} (C a^2 b + A a b^2) x^7 + \frac{1}{3} A a^3 x^3 + \frac{1}{6} (D a^3 + 3 B a^2 b) x^6 + \frac{1}{5} (C a^3 + 3 A a^2 b) x^5$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

15.11 Problem number 80

$$\int x(a + bx^2)^3 (A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$\frac{a^3 B x^3}{3} + \frac{a^3 C x^4}{4} + \frac{a^2(3bB + aD) x^5}{5} + \frac{a^2 b C x^6}{2} + \frac{3ab(bB + aD) x^7}{7} + \frac{3a b^2 C x^8}{8} + \frac{b^2(bB + 3aD) x^9}{9} + \frac{b^3 C x^{10}}{10} + \frac{b^3 D x^{11}}{11} + \frac{A(bx^2 + a)^4}{8b}$$

command

```
integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{11} D b^3 x^{11} + \frac{1}{10} C b^3 x^{10} + \frac{1}{9} (3 D a b^2 + B b^3) x^9 + \frac{1}{8} (3 C a b^2 + A b^3) x^8 + \frac{3}{7} (D a^2 b + B a b^2) x^7 + \frac{1}{3} B a^3 x^3 + \frac{1}{2} (C a^2 b + A a b^2) x^6 + \frac{1}{2} A a^3 x^2 + \frac{1}{5} (D a^3 + 3 B a^2 b) x^5 + \frac{1}{4} (C a^3 + 3 A a^2 b) x^4$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

15.12 Problem number 81

$$\int (a + bx^2)^3 (A + Bx + Cx^2 + Dx^3) dx$$

Optimal antiderivative

$$\begin{aligned} & a^3 Ax + \frac{a^2(3Ab + aC)x^3}{3} + \frac{a^3 Dx^4}{4} + \frac{3ab(Ab + aC)x^5}{5} + \frac{a^2 b Dx^6}{2} \\ & + \frac{b^2(Ab + 3aC)x^7}{7} + \frac{3a b^2 Dx^8}{8} + \frac{b^3 C x^9}{9} + \frac{b^3 Dx^{10}}{10} + \frac{B(bx^2 + a)^4}{8b} \end{aligned}$$

command

```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{10} Db^3 x^{10} + \frac{1}{9} Cb^3 x^9 + \frac{1}{8} (3Dab^2 + Bb^3)x^8 + \frac{1}{7} (3Cab^2 + Ab^3)x^7 + \frac{1}{2} (Da^2b + Bab^2)x^6 \\ & + \frac{1}{2} Ba^3 x^2 + \frac{3}{5} (Ca^2b + Aab^2)x^5 + Aa^3 x + \frac{1}{4} (Da^3 + 3Ba^2b)x^4 + \frac{1}{3} (Ca^3 + 3Aa^2b)x^3 \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

16 Test file number 25

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.3_General/25_1.1.3.2-c_x-
^m-a+b_x^n-p

16.1 Problem number 376

$$\int x^6 \sqrt{a + bx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{48a^2 x \sqrt{bx^3 + a}}{935b^2} + \frac{6a x^4 \sqrt{bx^3 + a}}{187b} + \frac{2x^7 \sqrt{bx^3 + a}}{17} \\ & + \frac{32 \cdot 3^{\frac{3}{4}} a^3 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{935 b^{\frac{7}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

```
integrate(x^6*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(48 a^3 \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (55 b^3 x^7 + 15 a b^2 x^4 - 24 a^2 b x) \sqrt{b x^3 + a} \right)}{935 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b x^3 + a} x^6, x\right)$$

16.2 Problem number 377

$$\int x^3 \sqrt{a + b x^3} dx$$

Optimal antiderivative

$$\frac{6 a x \sqrt{b x^3 + a}}{55 b} + \frac{2 x^4 \sqrt{b x^3 + a}}{11} + \frac{4 \cdot 3^{\frac{3}{4}} a^2 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i \sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{55 b^{\frac{4}{3}} \sqrt{b x^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}$$

command

```
integrate(x^3*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 a^2 \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - (5 b^2 x^4 + 3 a b x) \sqrt{b x^3 + a} \right)}{55 b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b x^3 + a} x^3, x\right)$$

16.3 Problem number 378

$$\int \sqrt{a + bx^3} dx$$

Optimal antiderivative

$$\frac{2x\sqrt{bx^3+a}}{5} + \frac{23^{\frac{3}{4}}a\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{5b^{\frac{1}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

`integrate((b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{bx^3+a}bx+3a\sqrt{b}\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)}{5b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{bx^3+a},x\right)$$

16.4 Problem number 379

$$\int \frac{\sqrt{a + bx^3}}{x^3} dx$$

Optimal antiderivative

$$-\frac{\sqrt{bx^3+a}}{2x^2} + \frac{3^{\frac{3}{4}}b^{\frac{2}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{2\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

```
integrate((b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{b}x^2 \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - \sqrt{bx^3 + a}}{2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}}{x^3}, x\right)$$

16.5 Problem number 380

$$\int \frac{\sqrt{a + bx^3}}{x^6} dx$$

Optimal antiderivative

$$\frac{\frac{\sqrt{bx^3 + a}}{5x^5} - \frac{3b\sqrt{bx^3 + a}}{20ax^2} + 3^{\frac{3}{4}}b^{\frac{5}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{20a\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((b*x^3+a)^(1/2)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3b^{\frac{3}{2}}x^5 \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (3bx^3 + 4a)\sqrt{bx^3 + a}}{20ax^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}}{x^6}, x\right)$$

16.6 Problem number 381

$$\int \frac{\sqrt{a + bx^3}}{x^9} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{bx^3 + a}}{8x^8} - \frac{3b\sqrt{bx^3 + a}}{80ax^5} + \frac{21b^2\sqrt{bx^3 + a}}{320a^2x^2} \\ & + \frac{7 \cdot 3^{\frac{3}{4}} b^{\frac{8}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{320a^2\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(1/2)/x^9,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21 b^{\frac{5}{2}} x^8 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (21 b^2 x^6 - 12 a b x^3 - 40 a^2) \sqrt{bx^3 + a}}{320 a^2 x^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + a}}{x^9}, x\right)$$

16.7 Problem number 382

$$\int x^7 \sqrt{a + bx^3} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{60a^2x^2\sqrt{bx^3+a}}{1729b^2} + \frac{6ax^5\sqrt{bx^3+a}}{247b} + \frac{2x^8\sqrt{bx^3+a}}{19} + \frac{240a^3\sqrt{bx^3+a}}{1729b^{\frac{8}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
 & + \frac{80\cdot 3^{\frac{3}{4}}a^{\frac{10}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{1729b^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
 & + \frac{120\cdot 3^{\frac{1}{4}}a^{\frac{10}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{1729b^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
 \end{aligned}$$

command

```
integrate(x^7*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(120a^3\sqrt{b}\operatorname{weierstrassZeta}\left(0,-\frac{4a}{b},\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-(91b^3x^8+21ab^2x^5-30a^2bx^2)\sqrt{bx^3+a}\right)}{1729b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{bx^3+a}x^7,x\right)$$

16.8 Problem number 383

$$\int x^4\sqrt{a+bx^3}dx$$

Optimal antiderivative

$$\frac{6a x^2 \sqrt{bx^3 + a}}{91b} + \frac{2x^5 \sqrt{bx^3 + a}}{13} - \frac{24a^2 \sqrt{bx^3 + a}}{91b^{\frac{5}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)}$$

$$+ \frac{8 \cdot 3^{\frac{3}{4}} a^{\frac{7}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{91b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

$$+ \frac{12 \cdot 3^{\frac{1}{4}} a^{\frac{7}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{91b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate(x^4*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 a^2 \sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + (7b^2x^5 + 3abx^2) \sqrt{bx^3 + a}\right)}{91b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{bx^3 + a} x^4, x\right)$$

16.9 Problem number 384

$$\int x \sqrt{a + bx^3} dx$$

Optimal antiderivative

$$\frac{2x^2\sqrt{bx^3+a}}{7} + \frac{6a\sqrt{bx^3+a}}{7b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)}$$

$$+ \frac{23^{\frac{3}{4}}a^{\frac{4}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{7b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

$$+ \frac{33^{\frac{1}{4}}a^{\frac{4}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{7b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

```
integrate(x*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{bx^3+a}bx^2-3a\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{7b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{bx^3+a}x,x\right)$$

16.10 Problem number 385

$$\int \frac{\sqrt{a+bx^3}}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{\sqrt{bx^3+a}}{x} + \frac{3b^{\frac{1}{3}}\sqrt{bx^3+a}}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})} \\
 & + \frac{3^{\frac{3}{4}}a^{\frac{1}{3}}b^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
 & - \frac{3^{\frac{3}{4}}a^{\frac{1}{3}}b^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{2\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
 \end{aligned}$$

command

`integrate((b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{b}x\operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + \sqrt{bx^3+a}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}}{x^2}, x\right)$$

16.11 Problem number 386

$$\int \frac{\sqrt{a+bx^3}}{x^5} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{\sqrt{bx^3+a}}{4x^4} - \frac{3b\sqrt{bx^3+a}}{8ax} + \frac{3b^{\frac{4}{3}}\sqrt{bx^3+a}}{8a\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
 & + \frac{3^{\frac{3}{4}}b^{\frac{4}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}\sqrt{2}}{8a^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
 & - \frac{3^{\frac{3}{4}}b^{\frac{4}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{16a^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
 \end{aligned}$$

command

```
integrate((b*x^3+a)^(1/2)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3b^{\frac{3}{2}}x^4\text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + (3bx^3 + 2a)\sqrt{bx^3+a}}{8ax^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{x^5}, x\right)$$

16.12 Problem number 394

$$\int x^6(a+bx^3)^{3/2} dx$$

Optimal antiderivative

$$\frac{2x^7(bx^3+a)^{\frac{3}{2}}}{23} - \frac{432a^3x\sqrt{bx^3+a}}{21505b^2} + \frac{54a^2x^4\sqrt{bx^3+a}}{4301b} + \frac{18ax^7\sqrt{bx^3+a}}{391}$$

$$+ \frac{288 \cdot 3^{\frac{3}{4}} a^4 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{21505b^{\frac{7}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

`integrate(x^6*(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(432a^4\sqrt{b}\operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (935b^4x^{10} + 1430ab^3x^7 + 135a^2b^2x^4 - 216a^3bx)\sqrt{bx^3+a}\right)}{21505b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bx^9 + ax^6)\sqrt{bx^3 + a}, x\right)$$

16.13 Problem number 395

$$\int x^3(a + bx^3)^{3/2} dx$$

Optimal antiderivative

$$\frac{2x^4(bx^3+a)^{\frac{3}{2}}}{17} + \frac{54a^2x\sqrt{bx^3+a}}{935b} + \frac{18ax^4\sqrt{bx^3+a}}{187}$$

$$- \frac{36 \cdot 3^{\frac{3}{4}} a^3 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{935b^{\frac{4}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

`integrate(x^3*(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(54 a^3 \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - (55 b^3 x^7 + 100 a b^2 x^4 + 27 a^2 b x) \sqrt{b x^3 + a} \right)}{935 b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((b x^6 + a x^3) \sqrt{b x^3 + a}, x\right)$$

16.14 Problem number 396

$$\int (a + b x^3)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x(bx^3 + a)^{\frac{3}{2}}}{11} + \frac{18ax\sqrt{bx^3 + a}}{55} \\ & + \frac{18 \cdot 3^{\frac{3}{4}} a^2 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}{55 b^{\frac{1}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate((b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(27 a^2 \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (5 b^2 x^4 + 14 a b x) \sqrt{b x^3 + a} \right)}{55 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((b x^3 + a)^{\frac{3}{2}}, x\right)$$

16.15 Problem number 397

$$\int \frac{(a + bx^3)^{3/2}}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(bx^3 + a)^{\frac{3}{2}}}{2x^2} + \frac{9bx\sqrt{bx^3 + a}}{10} \\ & + \frac{9 \cdot 3^{\frac{3}{4}} a b^{\frac{2}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}{10\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate((b*x^3+a)^(3/2)/x^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{27 a \sqrt{b} x^2 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (4bx^3 - 5a) \sqrt{bx^3 + a}}{10x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bx^3 + a)^{\frac{3}{2}}}{x^3}, x\right)$$

16.16 Problem number 398

$$\int \frac{(a + bx^3)^{3/2}}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(bx^3 + a)^{\frac{3}{2}}}{5x^5} - \frac{9b\sqrt{bx^3 + a}}{20x^2} \\ & + \frac{9 \cdot 3^{\frac{3}{4}} b^{\frac{5}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}{20\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{27 b^{\frac{3}{2}} x^5 \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - (13 b x^3 + 4 a) \sqrt{b x^3 + a}}{20 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(b x^3 + a)^{\frac{3}{2}}}{x^6}, x\right)$$

16.17 Problem number 399

$$\int x^7 (a + b x^3)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^8(bx^3+a)^{\frac{3}{2}}}{25} - \frac{108a^3x^2\sqrt{bx^3+a}}{8645b^2} + \frac{54a^2x^5\sqrt{bx^3+a}}{6175b} \\ & + \frac{18ax^8\sqrt{bx^3+a}}{475} + \frac{432a^4\sqrt{bx^3+a}}{8645b^{\frac{8}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & + \frac{144\sqrt[3]{4}a^{\frac{13}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{8645b^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\ & + \frac{216\sqrt[3]{4}a^{\frac{13}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{8645b^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate(x^7*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(1080 a^4 \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) - (1729 b^4 x^{11} + 2548 ab^3 x^8 + 189 a^2 b^2 x^5) \right)}{43225 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left((bx^{10} + ax^7) \sqrt{bx^3 + a}, x \right)$$

16.18 Problem number 400

$$\int x^4 (a + bx^3)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^5 (bx^3 + a)^{\frac{3}{2}}}{19} + \frac{54a^2 x^2 \sqrt{bx^3 + a}}{1729b} + \frac{18a x^5 \sqrt{bx^3 + a}}{247} - \frac{216a^3 \sqrt{bx^3 + a}}{1729b^{\frac{5}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\ & \frac{72 \cdot 3^{\frac{3}{4}} a^{\frac{10}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{1729b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \\ & + \frac{108 \cdot 3^{\frac{1}{4}} a^{\frac{10}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \operatorname{EllipticE} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{1729b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

`integrate(x^4*(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(108 a^3 \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) + (91 b^3 x^8 + 154 ab^2 x^5 + 27 a^2 b x^2) \sqrt{bx^3 + a} \right)}{1729 b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left((bx^7 + ax^4) \sqrt{bx^3 + a}, x \right)$$

16.19 Problem number 401

$$\int x(a + bx^3)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^2(bx^3 + a)^{\frac{3}{2}}}{13} + \frac{18ax^2\sqrt{bx^3 + a}}{91} + \frac{54a^2\sqrt{bx^3 + a}}{91b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{183^{\frac{3}{4}}a^{\frac{7}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{91b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{273^{\frac{1}{4}}a^{\frac{7}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{91b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate(x*(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(27a^2\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - (7b^2x^5 + 16abx^2)\sqrt{bx^3 + a}\right)}{91b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bx^4 + ax)\sqrt{bx^3 + a}, x\right)$$

16.20 Problem number 402

$$\int \frac{(a + bx^3)^{3/2}}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(bx^3 + a)^{\frac{3}{2}}}{x} + \frac{9bx^2\sqrt{bx^3 + a}}{7} + \frac{27ab^{\frac{1}{3}}\sqrt{bx^3 + a}}{7\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{93^{\frac{3}{4}}a^{\frac{4}{3}}b^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{7\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{273^{\frac{1}{4}}a^{\frac{4}{3}}b^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{14\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{27a\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - (2bx^3 - 7a)\sqrt{bx^3 + a}}{7x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bx^3 + a)^{\frac{3}{2}}}{x^2}, x\right)$$

16.21 Problem number 403

$$\int \frac{(a + bx^3)^{3/2}}{x^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(bx^3 + a)^{\frac{3}{2}}}{4x^4} - \frac{9b\sqrt{bx^3 + a}}{8x} + \frac{27b^{\frac{4}{3}}\sqrt{bx^3 + a}}{8\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{93^{\frac{3}{4}}a^{\frac{1}{3}}b^{\frac{4}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \sqrt{2}}{8\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{273^{\frac{1}{4}}a^{\frac{1}{3}}b^{\frac{4}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{16\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{27b^{\frac{3}{2}}x^4 \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + (11bx^3 + 2a)\sqrt{bx^3 + a}}{8x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bx^3 + a)^{\frac{3}{2}}}{x^5}, x\right)$$

16.22 Problem number 411

$$\int \frac{x^6}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{16ax\sqrt{bx^3+a}}{55b^2} + \frac{2x^4\sqrt{bx^3+a}}{11b} \\ & + \frac{32a^2\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{165b^{\frac{7}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} 3^{\frac{3}{4}} \end{aligned}$$

command

`integrate(x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(16a^2\sqrt{b}\operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (5b^2x^4 - 8abx)\sqrt{bx^3+a}\right)}{55b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^6}{\sqrt{bx^3+a}}, x\right)$$

16.23 Problem number 412

$$\int \frac{x^3}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x\sqrt{bx^3+a}}{5b} \\ & + \frac{4a\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{15b^{\frac{4}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} 3^{\frac{3}{4}} \end{aligned}$$

command

```
integrate(x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{bx^3 + a} bx - 2a\sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) \right)}{5b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^3}{\sqrt{bx^3 + a}}, x\right)$$

16.24 Problem number 413

$$\int \frac{1}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} 3^{\frac{3}{4}}}{3b^{\frac{1}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}$$

command

```
integrate(1/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)}{\sqrt{b}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{bx^3 + a}}, x\right)$$

16.25 Problem number 414

$$\int \frac{1}{x^3 \sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{\sqrt{bx^3 + a}}{2ax^2} - \frac{b^{\frac{2}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{3^{\frac{3}{4}}} - \frac{6a \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{1}$$

command

`integrate(1/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{b} x^2 \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) + \sqrt{bx^3 + a}}{2ax^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{bx^3 + a}}{bx^6 + ax^3}, x \right)$$

16.26 Problem number 415

$$\int \frac{1}{x^6 \sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{\sqrt{bx^3 + a}}{5ax^5} + \frac{7b\sqrt{bx^3 + a}}{20a^2x^2} - \frac{7b^{\frac{5}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{3^{\frac{3}{4}}} + \frac{60a^2 \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{1}$$

command

```
integrate(1/x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{7b^{\frac{3}{2}}x^5 \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (7bx^3 - 4a)\sqrt{bx^3 + a}}{20a^2x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}}{bx^9 + ax^6}, x\right)$$

16.27 Problem number 416

$$\int \frac{x^7}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{20ax^2\sqrt{bx^3+a}}{91b^2} + \frac{2x^5\sqrt{bx^3+a}}{13b} + \frac{80a^2\sqrt{bx^3+a}}{91b^{\frac{8}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & + \frac{80a^{\frac{7}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{3}{4}}} \\ & + \frac{273b^{\frac{8}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{91b^{\frac{8}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\ & + \frac{40\cdot 3^{\frac{1}{4}}a^{\frac{7}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{91b^{\frac{8}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate(x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(40 a^2 \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) - (7 b^2 x^5 - 10 a b x^2) \sqrt{b x^3 + a} \right)}{91 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{x^7}{\sqrt{b x^3 + a}}, x \right)$$

16.28 Problem number 417

$$\int \frac{x^4}{\sqrt{a + b x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^2 \sqrt{b x^3 + a}}{7b} - \frac{8a \sqrt{b x^3 + a}}{7b^{\frac{5}{3}} \left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\ & 8a^{\frac{4}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} 3^{\frac{3}{4}} \\ & - \frac{21b^{\frac{5}{3}} \sqrt{b x^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{43^{\frac{1}{4}} a^{\frac{4}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticE} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} \\ & + \frac{7b^{\frac{5}{3}} \sqrt{b x^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} \end{aligned}$$

command

`integrate(x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{b x^3 + a} b x^2 + 4 a \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) \right)}{7 b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{x^4}{\sqrt{b x^3 + a}}, x \right)$$

16.29 Problem number 418

$$\int \frac{x}{\sqrt{a+bx^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^3+a}}{b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} + \frac{2a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} 3^{\frac{3}{4}}}{3b^{\frac{2}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} + \frac{3^{\frac{1}{4}}a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{b^{\frac{2}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

```
integrate(x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right)}{\sqrt{b}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x}{\sqrt{bx^3+a}}, x\right)$$

16.30 Problem number 419

$$\int \frac{1}{x^2 \sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{bx^3 + a}}{ax} + \frac{b^{\frac{1}{3}} \sqrt{bx^3 + a}}{a \left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\ & + \frac{b^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} 3^{\frac{3}{4}}}{3a^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \\ & - \frac{3^{\frac{1}{4}} b^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticE} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{2a^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

`integrate(1/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{b} x \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) + \sqrt{bx^3 + a}}{ax}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{bx^3 + a}}{bx^5 + ax^2}, x \right)$$

16.31 Problem number 420

$$\int \frac{1}{x^5 \sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{bx^3+a}}{4ax^4} + \frac{5b\sqrt{bx^3+a}}{8a^2x} - \frac{5b^{\frac{4}{3}}\sqrt{bx^3+a}}{8a^2\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & - \frac{5b^{\frac{4}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & - \frac{24a^{\frac{5}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{53^{\frac{1}{4}}b^{\frac{4}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\ & + \frac{16a^{\frac{5}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \end{aligned}$$

command

`integrate(1/x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5b^{\frac{3}{2}}x^4 \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + (5bx^3 - 2a)\sqrt{bx^3+a}}{8a^2x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}}{bx^8+ax^5}, x\right)$$

16.32 Problem number 428

$$\int \frac{x^6}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x^4}{3b\sqrt{bx^3 + a}} + \frac{16x\sqrt{bx^3 + a}}{15b^2} + \frac{32a\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{45b^{\frac{7}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} 3^{\frac{3}{4}}$$

command

`integrate(x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(16(abx^3 + a^2)\sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - (3b^2x^4 + 8abx)\sqrt{bx^3 + a}\right)}{15(b^4x^3 + ab^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + a} x^6}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

16.33 Problem number 429

$$\int \frac{x^3}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x}{3b\sqrt{bx^3 + a}} + \frac{4\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{9b^{\frac{4}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} 3^{\frac{3}{4}}$$

command

```
integrate(x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\left(\sqrt{bx^3+a}bx-2(bx^3+a)\sqrt{b}\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)}{3(b^3x^3+ab^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}x^3}{b^2x^6+2abx^3+a^2},x\right)$$

16.34 Problem number 430

$$\int \frac{1}{(a+bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x}{3a\sqrt{bx^3+a}} + \frac{2\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})},i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{9ab^{\frac{1}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}} 3^{\frac{3}{4}}$$

command

```
integrate(1/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{bx^3+a}bx+(bx^3+a)\sqrt{b}\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)}{3(ab^2x^3+a^2b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}}{b^2x^6+2abx^3+a^2},x\right)$$

16.35 Problem number 431

$$\int \frac{1}{x^3 (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2}{3ax^2\sqrt{bx^3+a}} - \frac{7\sqrt{bx^3+a}}{6a^2x^2} + \frac{7b^{\frac{2}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{18a^2\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \quad 3^{\frac{3}{4}}$$

command

`integrate(1/x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{7(bx^5 + ax^2)\sqrt{b} \operatorname{weierstrassPInverse}(0, -\frac{4a}{b}, x) + (7bx^3 + 3a)\sqrt{bx^3 + a}}{6(a^2bx^5 + a^3x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}}{b^2x^9 + 2abx^6 + a^2x^3}, x\right)$$

16.36 Problem number 432

$$\int \frac{1}{x^6 (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2}{3ax^5\sqrt{bx^3+a}} - \frac{13\sqrt{bx^3+a}}{15a^2x^5} + \frac{91b\sqrt{bx^3+a}}{60a^3x^2} + \frac{91b^{\frac{5}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{180a^3\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \quad 3^{\frac{3}{4}}$$

command

```
integrate(1/x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{91 (b^2 x^8 + a b x^5) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (91 b^2 x^6 + 39 a b x^3 - 12 a^2) \sqrt{b x^3 + a}}{60 (a^3 b x^8 + a^4 x^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b x^3 + a}}{b^2 x^{12} + 2 a b x^9 + a^2 x^6}, x\right)$$

16.37 Problem number 433

$$\int \frac{x^7}{(a + b x^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2x^5}{3b\sqrt{bx^3+a}} + \frac{20x^2\sqrt{bx^3+a}}{21b^2} - \frac{80a\sqrt{bx^3+a}}{21b^{\frac{8}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & 80a^{\frac{4}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}3^{\frac{3}{4}} \\ & -\frac{63b^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{40a^{\frac{4}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}3^{\frac{1}{4}} \\ & +\frac{21b^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \end{aligned}$$

command

```
integrate(x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(40 (abx^3 + a^2) \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) + (3b^2x^5 + 10abx^2) \sqrt{bx^3 + a} \right)}{21 (b^4x^3 + ab^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{bx^3 + a} x^7}{b^2x^6 + 2abx^3 + a^2}, x \right)$$

16.38 Problem number 434

$$\int \frac{x^4}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2x^2}{3b\sqrt{bx^3 + a}} + \frac{8\sqrt{bx^3 + a}}{3b^{5/3} \left(b^{1/3}x + a^{1/3} (1 + \sqrt{3}) \right)} \\ & + \frac{8a^{1/3} \left(a^{1/3} + b^{1/3}x \right) \operatorname{EllipticF} \left(\frac{b^{1/3}x + a^{1/3} (1 - \sqrt{3})}{b^{1/3}x + a^{1/3} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{\left(b^{1/3}x + a^{1/3} (1 + \sqrt{3}) \right)^2}} 3^{3/4}}{9b^{5/3} \sqrt{bx^3 + a} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3}x \right)}{\left(b^{1/3}x + a^{1/3} (1 + \sqrt{3}) \right)^2}}} \\ & + \frac{4a^{1/3} \left(a^{1/3} + b^{1/3}x \right) \operatorname{EllipticE} \left(\frac{b^{1/3}x + a^{1/3} (1 - \sqrt{3})}{b^{1/3}x + a^{1/3} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{\left(b^{1/3}x + a^{1/3} (1 + \sqrt{3}) \right)^2}} 3^{1/4}}{3b^{5/3} \sqrt{bx^3 + a} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3}x \right)}{\left(b^{1/3}x + a^{1/3} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

`integrate(x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{bx^3 + a} bx^2 + 4 (bx^3 + a) \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) \right)}{3 (b^3x^3 + ab^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{bx^3 + a} x^4}{b^2x^6 + 2abx^3 + a^2}, x \right)$$

16.39 Problem number 435

$$\int \frac{x}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2x^2}{3a\sqrt{bx^3+a}} - \frac{2\sqrt{bx^3+a}}{3ab^{2/3}\left(b^{1/3}x+a^{1/3}(1+\sqrt{3})\right)}}{2\left(a^{1/3}+b^{1/3}x\right)\operatorname{EllipticF}\left(\frac{b^{1/3}x+a^{1/3}(1-\sqrt{3})}{b^{1/3}x+a^{1/3}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(b^{1/3}x+a^{1/3}(1+\sqrt{3})\right)^2}}3^{3/4}}$$

$$- \frac{9a^{2/3}b^{2/3}\sqrt{bx^3+a}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(b^{1/3}x+a^{1/3}(1+\sqrt{3})\right)^2}}}{\left(a^{1/3}+b^{1/3}x\right)\operatorname{EllipticE}\left(\frac{b^{1/3}x+a^{1/3}(1-\sqrt{3})}{b^{1/3}x+a^{1/3}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(b^{1/3}x+a^{1/3}(1+\sqrt{3})\right)^2}}3^{1/4}}$$

$$+ \frac{3a^{2/3}b^{2/3}\sqrt{bx^3+a}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(b^{1/3}x+a^{1/3}(1+\sqrt{3})\right)^2}}$$

command

`integrate(x/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{bx^3+a}bx^2+(bx^3+a)\sqrt{b}\operatorname{weierstrassZeta}\left(0,-\frac{4a}{b},\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{3(ab^2x^3+a^2b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}x}{b^2x^6+2abx^3+a^2},x\right)$$

16.40 Problem number 436

$$\int \frac{1}{x^2 (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2}{3ax\sqrt{bx^3+a}} - \frac{5\sqrt{bx^3+a}}{3a^2x} + \frac{5b^{\frac{1}{3}}\sqrt{bx^3+a}}{3a^2\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & + \frac{5b^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{3}{4}}} \\ & + \frac{9a^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{5b^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{1}{4}}} \\ & - \frac{6a^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{\end{aligned}$$

command

```
integrate(1/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(bx^4+ax)\sqrt{b}\operatorname{weierstrassZeta}\left(0,-\frac{4a}{b},\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(5bx^3+3a)\sqrt{bx^3+a}}{3(a^2bx^4+a^3x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}}{b^2x^8+2abx^5+a^2x^2},x\right)$$

16.41 Problem number 437

$$\int \frac{1}{x^5 (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2}{3ax^4\sqrt{bx^3+a}} - \frac{11\sqrt{bx^3+a}}{12a^2x^4} + \frac{55b\sqrt{bx^3+a}}{24a^3x} - \frac{55b^{\frac{4}{3}}\sqrt{bx^3+a}}{24a^3\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & \frac{55b^{\frac{4}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & - \frac{72a^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{55b^{\frac{4}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{1}{4}}} \\ & + \frac{48a^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \end{aligned}$$

command

```
integrate(1/x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{55(b^2x^7+abx^4)\sqrt{b}\operatorname{weierstrassZeta}\left(0,-\frac{4a}{b},\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(55b^2x^6+33abx^3-6a^2)\sqrt{bx^3+a}}{24(a^3bx^7+a^4x^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}}{b^2x^{11}+2abx^8+a^2x^5},x\right)$$

16.42 Problem number 446

$$\int \frac{x^6}{\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{16x\sqrt{x^3+1}}{55} + \frac{2x^4\sqrt{x^3+1}}{11} \\ & + \frac{32(1+x)\operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{165\sqrt{x^3+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate(x^6/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{55}(5x^4-8x)\sqrt{x^3+1} + \frac{32}{55}\operatorname{weierstrassPInverse}(0,-4,x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^6}{\sqrt{x^3+1}}, x\right)$$

16.43 Problem number 447

$$\int \frac{x^3}{\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x\sqrt{x^3+1}}{5} - \frac{4(1+x)\operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{15\sqrt{x^3+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

`integrate(x^3/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{5} \sqrt{x^3 + 1} x - \frac{4}{5} \text{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{x^3}{\sqrt{x^3 + 1}}, x\right)$$

16.44 Problem number 448

$$\int \frac{1}{\sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2(1+x) \text{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(1/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \text{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{1}{\sqrt{x^3 + 1}}, x\right)$$

16.45 Problem number 449

$$\int \frac{1}{x^3 \sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\frac{\frac{\sqrt{x^3+1}}{2x^2} (1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{6\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate(1/x^3/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{x^2 \operatorname{weierstrassPInverse}(0, -4, x) + \sqrt{x^3+1}}{2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^3+1}}{x^6+x^3}, x\right)$$

16.46 Problem number 450

$$\int \frac{1}{x^6 \sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\frac{-\frac{\sqrt{x^3+1}}{5x^5} + \frac{7\sqrt{x^3+1}}{20x^2} + 7(1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{60\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(1/x^6/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{7x^5 \text{weierstrassPInverse}(0, -4, x) + (7x^3 - 4)\sqrt{x^3 + 1}}{20x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3 + 1}}{x^9 + x^6}, x\right)$$

16.47 Problem number 451

$$\int \frac{x^7}{\sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{20x^2\sqrt{x^3+1}}{91} + \frac{2x^5\sqrt{x^3+1}}{13} + \frac{80\sqrt{x^3+1}}{91(1+x+\sqrt{3})} \\ & + \frac{80(1+x)\text{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}3^{\frac{3}{4}}}{273\sqrt{x^3+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \\ & - \frac{403^{\frac{1}{4}}(1+x)\text{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{91\sqrt{x^3+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

`integrate(x^7/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{91}(7x^5 - 10x^2)\sqrt{x^3 + 1} - \frac{80}{91}\text{weierstrassZeta}(0, -4, \text{weierstrassPInverse}(0, -4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{x^7}{\sqrt{x^3 + 1}}, x\right)$$

16.48 Problem number 452

$$\int \frac{x^4}{\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^2\sqrt{x^3+1}}{7} - \frac{8\sqrt{x^3+1}}{7(1+x+\sqrt{3})} \\ & - \frac{8(1+x)\operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{21\sqrt{x^3+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \\ & + \frac{43^{\frac{1}{4}}(1+x)\operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{7\sqrt{x^3+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

`integrate(x^4/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{7}\sqrt{x^3+1}x^2 + \frac{8}{7}\operatorname{weierstrassZeta}(0,-4,\operatorname{weierstrassPInverse}(0,-4,x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^4}{\sqrt{x^3+1}},x\right)$$

16.49 Problem number 453

$$\int \frac{x}{\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{x^3+1}}{1+x+\sqrt{3}} + \frac{2(1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} - \frac{3^{\frac{1}{4}}(1+x) \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(x/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2 \operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x}{\sqrt{x^3+1}}, x\right)$$

16.50 Problem number 454

$$\int \frac{1}{x^2 \sqrt{1+x^3}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{x^3+1}}{x} + \frac{\sqrt{x^3+1}}{1+x+\sqrt{3}} + \frac{(1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} + \frac{3^{\frac{1}{4}}(1+x) \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{2\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate(1/x^2/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-\text{weierstrassZeta}(0, -4, \text{weierstrassPInverse}(0, -4, x)) + \sqrt{x^3 + 1}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3 + 1}}{x^5 + x^2}, x\right)$$

16.51 Problem number 455

$$\int \frac{1}{x^5 \sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{x^3 + 1}}{4x^4} + \frac{5\sqrt{x^3 + 1}}{8x} - \frac{5\sqrt{x^3 + 1}}{8(1 + x + \sqrt{3})} \\ & - \frac{5(1 + x) \text{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{x^2 - x + 1}{(1 + x + \sqrt{3})^2}} 3^{\frac{3}{4}}}{24\sqrt{x^3 + 1} \sqrt{\frac{1 + x}{(1 + x + \sqrt{3})^2}}} \\ & + \frac{5 \cdot 3^{\frac{1}{4}} (1 + x) \text{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1 + x + \sqrt{3})^2}}}{16\sqrt{x^3 + 1} \sqrt{\frac{1 + x}{(1 + x + \sqrt{3})^2}}} \end{aligned}$$

command

```
integrate(1/x^5/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5x^4 \text{weierstrassZeta}(0, -4, \text{weierstrassPInverse}(0, -4, x)) + (5x^3 - 2)\sqrt{x^3 + 1}}{8x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3 + 1}}{x^8 + x^5}, x\right)$$

16.52 Problem number 464

$$\int \frac{x^6}{\sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\frac{-\frac{16x\sqrt{-x^3+1}}{55} - \frac{2x^4\sqrt{-x^3+1}}{11} + 32(1-x)\operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{165\sqrt{-x^3+1}\sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}$$

command

`integrate(x^6/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{55}(5x^4+8x)\sqrt{-x^3+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3+1}x^6}{x^3-1}, x\right)$$

16.53 Problem number 465

$$\int \frac{x^3}{\sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\frac{\frac{2x\sqrt{-x^3+1}}{5} + 4(1-x)\operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{15\sqrt{-x^3+1}\sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}$$

command

`integrate(x^3/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{5} \sqrt{-x^3 + 1} x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^3 + 1} x^3}{x^3 - 1}, x\right)$$

16.54 Problem number 467

$$\int \frac{1}{x^3 \sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\frac{\sqrt{-x^3 + 1}}{2x^2} - \frac{(1-x) \text{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 + x + 1}{(1-x + \sqrt{3})^2}} 3^{\frac{3}{4}}}{6\sqrt{-x^3 + 1} \sqrt{\frac{1-x}{(1-x + \sqrt{3})^2}}}$$

command

`integrate(1/x^3/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{-x^3 + 1}}{2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^3 + 1}}{x^6 - x^3}, x\right)$$

16.55 Problem number 468

$$\int \frac{1}{x^6 \sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\frac{-\frac{\sqrt{-x^3+1}}{5x^5} - \frac{7\sqrt{-x^3+1}}{20x^2} + 7(1-x) \operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{60\sqrt{-x^3+1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}$$

command

`integrate(1/x^6/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(7x^3+4)\sqrt{-x^3+1}}{20x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3+1}}{x^9-x^6}, x\right)$$

16.56 Problem number 469

$$\int \frac{x^7}{\sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{20x^2\sqrt{-x^3+1}}{91} - \frac{2x^5\sqrt{-x^3+1}}{13} + \frac{80\sqrt{-x^3+1}}{91(1-x+\sqrt{3})} \\
& + \frac{80(1-x)\operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}}3^{\frac{3}{4}}}{273\sqrt{-x^3+1}\sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}} \\
& - \frac{40\cdot 3^{\frac{1}{4}}(1-x)\operatorname{EllipticE}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}}}{91\sqrt{-x^3+1}\sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}
\end{aligned}$$

command

```
integrate(x^7/(-x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{91}(7x^5+10x^2)\sqrt{-x^3+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3+1}x^7}{x^3-1}, x\right)$$

16.57 Problem number 470

$$\int \frac{x^4}{\sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2x^2\sqrt{-x^3+1}}{7} + \frac{8\sqrt{-x^3+1}}{7(1-x+\sqrt{3})} \\
& + \frac{8(1-x)\operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}}}{21\sqrt{-x^3+1}\sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}} 3^{\frac{3}{4}} \\
& - \frac{4\cdot 3^{\frac{1}{4}}(1-x)\operatorname{EllipticE}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}}}{7\sqrt{-x^3+1}\sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}
\end{aligned}$$

command

```
integrate(x^4/(-x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{7}\sqrt{-x^3+1}x^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3+1}x^4}{x^3-1}, x\right)$$

16.58 Problem number 472

$$\int \frac{1}{x^2\sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{\sqrt{-x^3+1}}{x} - \frac{\sqrt{-x^3+1}}{1-x+\sqrt{3}} \\
& \frac{(1-x)\operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}}3^{\frac{3}{4}}}{3\sqrt{-x^3+1}\sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}} \\
& + \frac{3^{\frac{1}{4}}(1-x)\operatorname{EllipticE}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}}}{2\sqrt{-x^3+1}\sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}
\end{aligned}$$

command

```
integrate(1/x^2/(-x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{-x^3+1}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3+1}}{x^5-x^2}, x\right)$$

16.59 Problem number 473

$$\int \frac{1}{x^5\sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{\sqrt{-x^3+1}}{4x^4} - \frac{5\sqrt{-x^3+1}}{8x} - \frac{5\sqrt{-x^3+1}}{8(1-x+\sqrt{3})} \\
 & - \frac{5(1-x) \operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{24\sqrt{-x^3+1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}} \\
 & + \frac{5 \cdot 3^{\frac{1}{4}}(1-x) \operatorname{EllipticE}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}}}{16\sqrt{-x^3+1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}
 \end{aligned}$$

command

`integrate(1/x^5/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(5x^3+2)\sqrt{-x^3+1}}{8x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3+1}}{x^8-x^5}, x\right)$$

16.60 Problem number 482

$$\int \frac{x^6}{\sqrt{-1+x^3}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{16x\sqrt{x^3-1}}{55} + \frac{2x^4\sqrt{x^3-1}}{11} \\
 & - \frac{32(1-x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{165\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}
 \end{aligned}$$

command

```
integrate(x^6/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{55} (5x^4 + 8x) \sqrt{x^3 - 1} + \frac{32}{55} \text{weierstrassPInverse}(0, 4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{x^6}{\sqrt{x^3 - 1}}, x\right)$$

16.61 Problem number 483

$$\int \frac{x^3}{\sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2x\sqrt{x^3-1}}{5} - \frac{4(1-x)\text{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{15\sqrt{x^3-1}\sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

command

```
integrate(x^3/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{5} \sqrt{x^3 - 1} x + \frac{4}{5} \text{weierstrassPInverse}(0, 4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{x^3}{\sqrt{x^3 - 1}}, x\right)$$

16.62 Problem number 484

$$\int \frac{1}{\sqrt{-1+x^3}} dx$$

Optimal antiderivative

$$\frac{2(1-x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

command

```
integrate(1/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}(0, 4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{x^3-1}}, x\right)$$

16.63 Problem number 485

$$\int \frac{1}{x^3 \sqrt{-1+x^3}} dx$$

Optimal antiderivative

$$\frac{\sqrt{x^3-1}}{2x^2} - \frac{(1-x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{6\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

command

```
integrate(1/x^3/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{x^2 \text{weierstrassPInverse}(0, 4, x) + \sqrt{x^3 - 1}}{2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3 - 1}}{x^6 - x^3}, x\right)$$

16.64 Problem number 486

$$\int \frac{1}{x^6 \sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{\frac{\sqrt{x^3 - 1}}{5x^5} + \frac{7\sqrt{x^3 - 1}}{20x^2} + 7(1-x) \text{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i - i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 + x + 1}{(1-x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{60\sqrt{x^3 - 1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

command

```
integrate(1/x^6/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{7x^5 \text{weierstrassPInverse}(0, 4, x) + (7x^3 + 4)\sqrt{x^3 - 1}}{20x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3 - 1}}{x^9 - x^6}, x\right)$$

16.65 Problem number 487

$$\int \frac{x^7}{\sqrt{-1+x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{20x^2\sqrt{x^3-1}}{91} + \frac{2x^5\sqrt{x^3-1}}{13} - \frac{80\sqrt{x^3-1}}{91(1-x-\sqrt{3})} \\ & - \frac{80(1-x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) \sqrt{2} \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{273\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}} \\ & + \frac{40 \cdot 3^{\frac{1}{4}}(1-x) \operatorname{EllipticE}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{91\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}} \end{aligned}$$

command

`integrate(x^7/(x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{91} (7x^5 + 10x^2) \sqrt{x^3-1} - \frac{80}{91} \operatorname{weierstrassZeta}(0, 4, \operatorname{weierstrassPInverse}(0, 4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^7}{\sqrt{x^3-1}}, x\right)$$

16.66 Problem number 488

$$\int \frac{x^4}{\sqrt{-1+x^3}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2x^2 \sqrt{x^3 - 1}}{7} - \frac{8 \sqrt{x^3 - 1}}{7(1 - x - \sqrt{3})} \\
& - \frac{8(1 - x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i - i\sqrt{3}\right) \sqrt{2} \sqrt{\frac{x^2 + x + 1}{(1 - x - \sqrt{3})^2}} 3^{\frac{3}{4}}}{21 \sqrt{x^3 - 1} \sqrt{\frac{-1 + x}{(1 - x - \sqrt{3})^2}}} \\
& + \frac{43^{\frac{1}{4}}(1 - x) \operatorname{EllipticE}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i - i\sqrt{3}\right) \sqrt{\frac{x^2 + x + 1}{(1 - x - \sqrt{3})^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{7 \sqrt{x^3 - 1} \sqrt{\frac{-1 + x}{(1 - x - \sqrt{3})^2}}}
\end{aligned}$$

command

```
integrate(x^4/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{7} \sqrt{x^3 - 1} x^2 - \frac{8}{7} \operatorname{weierstrassZeta}(0, 4, \operatorname{weierstrassPInverse}(0, 4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^4}{\sqrt{x^3 - 1}}, x\right)$$

16.67 Problem number 489

$$\int \frac{x}{\sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{\frac{2\sqrt{x^3-1}}{1-x-\sqrt{3}} - \frac{2(1-x)\operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right)\sqrt{2}\sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}}3^{\frac{3}{4}}}{3\sqrt{x^3-1}\sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}} + \frac{3^{\frac{1}{4}}(1-x)\operatorname{EllipticE}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right)\sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-1}\sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}{\sqrt{x^3-1}\sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

command

`integrate(x/(x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2 \operatorname{weierstrassZeta}(0, 4, \operatorname{weierstrassPInverse}(0, 4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x}{\sqrt{x^3-1}}, x\right)$$

16.68 Problem number 490

$$\int \frac{1}{x^2\sqrt{-1+x^3}} dx$$

Optimal antiderivative

$$\frac{\frac{\sqrt{x^3-1}}{x} + \frac{\sqrt{x^3-1}}{1-x-\sqrt{3}}}{\frac{(1-x)\operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right)\sqrt{2}\sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}}3^{\frac{3}{4}}}{3\sqrt{x^3-1}\sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}} + \frac{3^{\frac{1}{4}}(1-x)\operatorname{EllipticE}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right)\sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{2\sqrt{x^3-1}\sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}}$$

command

```
integrate(1/x^2/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{x\text{weierstrassZeta}(0, 4, \text{weierstrassPInverse}(0, 4, x)) + \sqrt{x^3 - 1}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3 - 1}}{x^5 - x^2}, x\right)$$

16.69 Problem number 491

$$\int \frac{1}{x^5 \sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{x^3 - 1}}{4x^4} + \frac{5\sqrt{x^3 - 1}}{8x} + \frac{5\sqrt{x^3 - 1}}{8(1 - x - \sqrt{3})} \\ & + \frac{5(1 - x) \text{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i - i\sqrt{3}\right) \sqrt{2} \sqrt{\frac{x^2 + x + 1}{(1 - x - \sqrt{3})^2}} 3^{\frac{3}{4}}}{24\sqrt{x^3 - 1} \sqrt{\frac{-1 + x}{(1 - x - \sqrt{3})^2}}} \\ & - \frac{5 \cdot 3^{\frac{1}{4}}(1 - x) \text{EllipticE}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i - i\sqrt{3}\right) \sqrt{\frac{x^2 + x + 1}{(1 - x - \sqrt{3})^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{16\sqrt{x^3 - 1} \sqrt{\frac{-1 + x}{(1 - x - \sqrt{3})^2}}} \end{aligned}$$

command

```
integrate(1/x^5/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5x^4\text{weierstrassZeta}(0, 4, \text{weierstrassPInverse}(0, 4, x)) + (5x^3 + 2)\sqrt{x^3 - 1}}{8x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3 - 1}}{x^8 - x^5}, x\right)$$

16.70 Problem number 500

$$\int \frac{x^6}{\sqrt{-1-x^3}} dx$$

Optimal antiderivative

$$\frac{16x\sqrt{-x^3-1}}{55} - \frac{2x^4\sqrt{-x^3-1}}{11} + \frac{32(1+x)\operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}}3^{\frac{3}{4}}}{165\sqrt{-x^3-1}\sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

command

`integrate(x^6/(-x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{55}(5x^4-8x)\sqrt{-x^3-1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3-1}x^6}{x^3+1}, x\right)$$

16.71 Problem number 501

$$\int \frac{x^3}{\sqrt{-1-x^3}} dx$$

Optimal antiderivative

$$\frac{2x\sqrt{-x^3-1}}{5} - \frac{4(1+x)\operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}}3^{\frac{3}{4}}}{15\sqrt{-x^3-1}\sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

command

`integrate(x^3/(-x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{5} \sqrt{-x^3 - 1} x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^3 - 1} x^3}{x^3 + 1}, x\right)$$

16.72 Problem number 503

$$\int \frac{1}{x^3 \sqrt{-1 - x^3}} dx$$

Optimal antiderivative

$$\frac{\sqrt{-x^3 - 1}}{2x^2} - \frac{(1+x) \text{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i - i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{6\sqrt{-x^3 - 1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

command

`integrate(1/x^3/(-x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^3 - 1}}{2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^3 - 1}}{x^6 + x^3}, x\right)$$

16.73 Problem number 504

$$\int \frac{1}{x^6 \sqrt{-1-x^3}} dx$$

Optimal antiderivative

$$\frac{\sqrt{-x^3-1}}{5x^5} - \frac{7\sqrt{-x^3-1}}{20x^2} + \frac{7(1+x) \operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{60\sqrt{-x^3-1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

command

`integrate(1/x^6/(-x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(7x^3-4)\sqrt{-x^3-1}}{20x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3-1}}{x^9+x^6}, x\right)$$

16.74 Problem number 505

$$\int \frac{x^7}{\sqrt{-1-x^3}} dx$$

Optimal antiderivative

$$\frac{20x^2\sqrt{-x^3-1}}{91} - \frac{2x^5\sqrt{-x^3-1}}{13} - \frac{80\sqrt{-x^3-1}}{91(1+x-\sqrt{3})}$$

$$- \frac{80(1+x)\operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right)\sqrt{2}\sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}}3^{\frac{3}{4}}}{273\sqrt{-x^3-1}\sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

$$+ \frac{40\cdot 3^{\frac{1}{4}}(1+x)\operatorname{EllipticE}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right)\sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{91\sqrt{-x^3-1}\sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

command

`integrate(x^7/(-x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{91}(7x^5-10x^2)\sqrt{-x^3-1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3-1}x^7}{x^3+1}, x\right)$$

16.75 Problem number 506

$$\int \frac{x^4}{\sqrt{-1-x^3}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2x^2\sqrt{-x^3-1}}{7} + \frac{8\sqrt{-x^3-1}}{7(1+x-\sqrt{3})} \\
& + \frac{8(1+x)\operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right)\sqrt{2}\sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}}3^{\frac{3}{4}}}{21\sqrt{-x^3-1}\sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}} \\
& - \frac{4\cdot 3^{\frac{1}{4}}(1+x)\operatorname{EllipticE}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right)\sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{7\sqrt{-x^3-1}\sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}
\end{aligned}$$

command

```
integrate(x^4/(-x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{7}\sqrt{-x^3-1}x^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3-1}x^4}{x^3+1}, x\right)$$

16.76 Problem number 508

$$\int \frac{1}{x^2\sqrt{-1-x^3}} dx$$

Optimal antiderivative

$$\frac{\sqrt{-x^3-1}}{x} - \frac{\sqrt{-x^3-1}}{1+x-\sqrt{3}}$$

$$- \frac{(1+x) \operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right) \sqrt{2} \sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{-x^3-1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

$$+ \frac{3^{\frac{1}{4}}(1+x) \operatorname{EllipticE}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right) \sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{2\sqrt{-x^3-1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

command

```
integrate(1/x^2/(-x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^3-1}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3-1}}{x^5+x^2}, x\right)$$

16.77 Problem number 509

$$\int \frac{1}{x^5 \sqrt{-1-x^3}} dx$$

Optimal antiderivative

$$\frac{\sqrt{-x^3-1}}{4x^4} - \frac{5\sqrt{-x^3-1}}{8x} + \frac{5\sqrt{-x^3-1}}{8(1+x-\sqrt{3})}$$

$$+ \frac{5(1+x) \operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right) \sqrt{2} \sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{24\sqrt{-x^3-1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

$$- \frac{5 \cdot 3^{\frac{1}{4}}(1+x) \operatorname{EllipticE}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right) \sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{16\sqrt{-x^3-1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

command

`integrate(1/x^5/(-x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(5x^3-2)\sqrt{-x^3-1}}{8x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3-1}}{x^8+x^5}, x\right)$$

16.78 Problem number 518

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2} dx$$

Optimal antiderivative

$$-\frac{(bx^3+a)^{\frac{1}{3}}}{x} - \frac{b^{\frac{1}{3}} \ln\left(b^{\frac{1}{3}}x - (bx^3+a)^{\frac{1}{3}}\right)}{2} - \frac{b^{\frac{1}{3}} \arctan\left(\frac{\left(1 + \frac{2b^{\frac{1}{3}}x}{(bx^3+a)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right) \sqrt{3}}{3}$$

command

```
integrate((b*x^3+a)^(1/3)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{3}(-b)^{\frac{1}{3}}x \arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2+2\sqrt{3}(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{1}{3}}x+\sqrt{3}a}{3(2bx^3+a)}\right) + (-b)^{\frac{1}{3}}x \log\left(-(-b)^{\frac{1}{3}}bx^2 + (bx^3+a)\right)}{6x}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

16.79 Problem number 776

$$\int x^4 \sqrt{a + cx^4} dx$$

Optimal antiderivative

$$\frac{2ax\sqrt{cx^4+a}}{21c} + \frac{x^5\sqrt{cx^4+a}}{7} + \frac{a^{\frac{7}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x^2\sqrt{c})\sqrt{\frac{cx^4+a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{21\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)c^{\frac{5}{4}}\sqrt{cx^4+a}}$$

command

```
integrate(x^4*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2a\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}, -1\right) - (3cx^5 + 2ax)\sqrt{cx^4+a}}{21c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^4+a}x^4, x\right)$$

16.80 Problem number 777

$$\int \sqrt{a + cx^4} dx$$

Optimal antiderivative

$$\frac{x\sqrt{cx^4+a}}{3} + \frac{a^{\frac{3}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4+a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{3 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) c^{\frac{1}{4}} \sqrt{cx^4+a}}$$

command

`integrate((c*x^4+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{3} \sqrt{c} \left(-\frac{a}{c}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}, -1\right) + \frac{1}{3} \sqrt{cx^4+a} x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^4+a}, x\right)$$

16.81 Problem number 778

$$\int \frac{\sqrt{a + cx^4}}{x^4} dx$$

Optimal antiderivative

$$-\frac{\sqrt{cx^4+a}}{3x^3} + \frac{c^{\frac{3}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4+a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{3 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} \sqrt{cx^4+a}}$$

command

```
integrate((c*x^4+a)^(1/2)/x^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{a}x^3\left(-\frac{c}{a}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}},-1\right)+\sqrt{cx^4+a}}{3x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+a}}{x^4},x\right)$$

16.82 Problem number 779

$$\int \frac{\sqrt{a+cx^4}}{x^8} dx$$

Optimal antiderivative

$$\frac{\frac{\sqrt{cx^4+a}}{7x^7} - \frac{2c\sqrt{cx^4+a}}{21ax^3} + \frac{c^{\frac{7}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x^2\sqrt{c})\sqrt{\frac{cx^4+a}{(\sqrt{a}+x^2\sqrt{c})^2}}}{21\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)a^{\frac{5}{4}}\sqrt{cx^4+a}}$$

command

```
integrate((c*x^4+a)^(1/2)/x^8,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{a}cx^7\left(-\frac{c}{a}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}},-1\right)-(2cx^4+3a)\sqrt{cx^4+a}}{21ax^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+a}}{x^8},x\right)$$

16.83 Problem number 796

$$\int x^4 (a + cx^4)^{3/2} dx$$

Optimal antiderivative

$$\frac{x^5 (cx^4 + a)^{\frac{3}{2}}}{11} + \frac{4a^2 x \sqrt{cx^4 + a}}{77c} + \frac{6a x^5 \sqrt{cx^4 + a}}{77}$$

$$2a^{\frac{11}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2 \sqrt{c}) \sqrt{\frac{cx^4 + a}{(\sqrt{a} + x^2 \sqrt{c})}}$$

$$77 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} x}{a^{\frac{1}{4}}}\right)\right) c^{\frac{5}{4}} \sqrt{cx^4 + a}$$

command

```
integrate(x^4*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4a^2 \sqrt{c} \left(-\frac{a}{c}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}, -1\right) - (7c^2 x^9 + 13acx^5 + 4a^2 x) \sqrt{cx^4 + a}}{77c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((cx^8 + ax^4) \sqrt{cx^4 + a}, x\right)$$

16.84 Problem number 797

$$\int (a + cx^4)^{3/2} dx$$

Optimal antiderivative

$$\frac{x (cx^4 + a)^{\frac{3}{2}}}{7} + \frac{2ax \sqrt{cx^4 + a}}{7}$$

$$2a^{\frac{7}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2 \sqrt{c}) \sqrt{\frac{cx^4 + a}{(\sqrt{a} + x^2 \sqrt{c})^2}}$$

$$+ \frac{7 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} x}{a^{\frac{1}{4}}}\right)\right) c^{\frac{1}{4}} \sqrt{cx^4 + a}}{77}$$

command

```
integrate((c*x^4+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4}{7} a \sqrt{c} \left(-\frac{a}{c}\right)^{\frac{3}{4}} \text{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}, -1\right) + \frac{1}{7} (cx^5 + 3ax) \sqrt{cx^4 + a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(cx^4 + a\right)^{\frac{3}{2}}, x\right)$$

16.85 Problem number 799

$$\int \frac{(a + cx^4)^{3/2}}{x^8} dx$$

Optimal antiderivative

$$\frac{(cx^4 + a)^{\frac{3}{2}}}{7x^7} - \frac{2c\sqrt{cx^4 + a}}{7x^3} + \frac{2c^{\frac{7}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4 + a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{7 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} \sqrt{cx^4 + a}}$$

command

```
integrate((c*x^4+a)^(3/2)/x^8,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{4\sqrt{a}cx^7\left(-\frac{c}{a}\right)^{\frac{3}{4}} \text{ellipticF}\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}, -1\right) + (3cx^4 + a)\sqrt{cx^4 + a}}{7x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(cx^4 + a)^{\frac{3}{2}}}{x^8}, x\right)$$

16.86 Problem number 803

$$\int (1+x^4)^{3/2} dx$$

Optimal antiderivative

$$\frac{x(x^4+1)^{\frac{3}{2}}}{7} + \frac{2x\sqrt{x^4+1}}{7} + \frac{2(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4+1}{(x^2+1)^2}}}{7\cos(2\arctan(x))\sqrt{x^4+1}}$$

command

```
integrate((x^4+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{7}(x^5+3x)\sqrt{x^4+1} + \frac{4}{7}i\sqrt{i}\operatorname{ellipticF}\left(\frac{\sqrt{i}}{x}, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((x^4+1)^{\frac{3}{2}}, x\right)$$

16.87 Problem number 804

$$\int (1-x^4)^{3/2} dx$$

Optimal antiderivative

$$\frac{x(-x^4+1)^{\frac{3}{2}}}{7} + \frac{4\operatorname{EllipticF}(x, i)}{7} + \frac{2x\sqrt{-x^4+1}}{7}$$

command

```
integrate((-x^4+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{7}(x^5-3x)\sqrt{-x^4+1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((-x^4+1)^{\frac{3}{2}}, x\right)$$

16.88 Problem number 809

$$\int \sqrt{1+x^4} dx$$

Optimal antiderivative

$$\frac{x\sqrt{x^4+1}}{3} + \frac{(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^4+1}{(x^2+1)^2}}}{3\cos(2\arctan(x))\sqrt{x^4+1}}$$

command

```
integrate((x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3}\sqrt{x^4+1}x + \frac{2}{3}i\sqrt{i}\operatorname{ellipticF}\left(\frac{\sqrt{i}}{x}, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{x^4+1}, x\right)$$

16.89 Problem number 810

$$\int \sqrt{1-x^4} dx$$

Optimal antiderivative

$$\frac{2\operatorname{EllipticF}(x, i)}{3} + \frac{x\sqrt{-x^4+1}}{3}$$

command

```
integrate((-x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3}\sqrt{-x^4+1}x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{-x^4+1}, x\right)$$

16.90 Problem number 821

$$\int \frac{x^8}{\sqrt{a+bx^4}} dx$$

Optimal antiderivative

$$\frac{-\frac{5ax\sqrt{bx^4+a}}{21b^2} + \frac{x^5\sqrt{bx^4+a}}{7b}}{5a^{\frac{7}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4+a}{(\sqrt{a} + x^2\sqrt{b})^2}}}$$

$$+ \frac{42 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) b^{\frac{9}{4}} \sqrt{bx^4+a}}{42 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) b^{\frac{9}{4}} \sqrt{bx^4+a}}$$

command

`integrate(x^8/(b*x^4+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5a\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}, -1\right) + (3bx^5 - 5ax)\sqrt{bx^4+a}}{21b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^8}{\sqrt{bx^4+a}}, x\right)$$

16.91 Problem number 822

$$\int \frac{x^4}{\sqrt{a+bx^4}} dx$$

Optimal antiderivative

$$\frac{\frac{x\sqrt{bx^4+a}}{3b}}{a^{\frac{3}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4+a}{(\sqrt{a} + x^2\sqrt{b})^2}}}$$

$$- \frac{6 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) b^{\frac{5}{4}} \sqrt{bx^4+a}}{6 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) b^{\frac{5}{4}} \sqrt{bx^4+a}}$$

command

```
integrate(x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{b} \left(-\frac{a}{b}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}, -1\right) - \sqrt{bx^4 + a} x}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^4}{\sqrt{bx^4 + a}}, x\right)$$

16.92 Problem number 823

$$\int \frac{1}{\sqrt{a + bx^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \left(\sqrt{a} + x^2\sqrt{b}\right) \sqrt{\frac{bx^4 + a}{\left(\sqrt{a} + x^2\sqrt{b}\right)^2}}}{2 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{bx^4 + a}}$$

command

```
integrate(1/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{a} \left(-\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}, -1\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{bx^4 + a}}, x\right)$$

16.93 Problem number 824

$$\int \frac{1}{x^4 \sqrt{a + bx^4}} dx$$

Optimal antiderivative

$$\frac{-\frac{\sqrt{bx^4 + a}}{3ax^3} + b^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4 + a}{(\sqrt{a} + x^2\sqrt{b})^2}}}{6 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{5}{4}} \sqrt{bx^4 + a}}$$

command

`integrate(1/x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{a} x^3 \left(-\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x \left(-\frac{b}{a}\right)^{\frac{1}{4}}, -1\right) - \sqrt{bx^4 + a}}{3ax^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^4 + a}}{bx^8 + ax^4}, x\right)$$

16.94 Problem number 825

$$\int \frac{1}{x^8 \sqrt{a + bx^4}} dx$$

Optimal antiderivative

$$\frac{-\frac{\sqrt{bx^4 + a}}{7ax^7} + \frac{5b\sqrt{bx^4 + a}}{21a^2x^3} + 5b^{\frac{7}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4 + a}{(\sqrt{a} + x^2\sqrt{b})^2}}}{42 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{9}{4}} \sqrt{bx^4 + a}}$$

command

```
integrate(1/x^8/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{a}bx^7\left(-\frac{b}{a}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}},-1\right)-(5bx^4-3a)\sqrt{bx^4+a}}{21a^2x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^4+a}}{bx^{12}+ax^8},x\right)$$

16.95 Problem number 841

$$\int \frac{x^8}{\sqrt{a-bx^4}} dx$$

Optimal antiderivative

$$-\frac{5ax\sqrt{-bx^4+a}}{21b^2}-\frac{x^5\sqrt{-bx^4+a}}{7b}+\frac{5a^{\frac{9}{4}}\operatorname{EllipticF}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}},i\right)\sqrt{1-\frac{bx^4}{a}}}{21b^{\frac{9}{4}}\sqrt{-bx^4+a}}$$

command

```
integrate(x^8/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5a\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x},-1\right)-(3bx^5+5ax)\sqrt{-bx^4+a}}{21b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-bx^4+a}x^8}{bx^4-a},x\right)$$

16.96 Problem number 842

$$\int \frac{x^4}{\sqrt{a - bx^4}} dx$$

Optimal antiderivative

$$-\frac{x\sqrt{-bx^4+a}}{3b} + \frac{a^{\frac{5}{4}} \operatorname{EllipticF}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}, i\right) \sqrt{1 - \frac{bx^4}{a}}}{3b^{\frac{5}{4}} \sqrt{-bx^4+a}}$$

command

`integrate(x^4/(-b*x^4+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-b} \left(\frac{a}{b}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\left(\frac{a}{b}\right)^{\frac{1}{4}} x, -1\right) - \sqrt{-bx^4+a} x}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-bx^4+a} x^4}{bx^4-a}, x\right)$$

16.97 Problem number 843

$$\int \frac{1}{\sqrt{a - bx^4}} dx$$

Optimal antiderivative

$$\frac{a^{\frac{1}{4}} \operatorname{EllipticF}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}, i\right) \sqrt{1 - \frac{bx^4}{a}}}{b^{\frac{1}{4}} \sqrt{-bx^4+a}}$$

command

`integrate(1/(-b*x^4+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{a} \left(\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}, -1\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-bx^4+a}}{bx^4-a}, x\right)$$

16.98 Problem number 844

$$\int \frac{1}{x^4 \sqrt{a - bx^4}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{-bx^4 + a}}{3ax^3} + \frac{b^{\frac{3}{4}} \operatorname{EllipticF}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}, i\right) \sqrt{1 - \frac{bx^4}{a}}}{3a^{\frac{3}{4}} \sqrt{-bx^4 + a}}$$

command

```
integrate(1/x^4/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{a} x^3 \left(\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x \left(\frac{b}{a}\right)^{\frac{1}{4}}, -1\right) - \sqrt{-bx^4 + a}}{3ax^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-bx^4 + a}}{bx^8 - ax^4}, x\right)$$

16.99 Problem number 845

$$\int \frac{1}{x^8 \sqrt{a - bx^4}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{-bx^4 + a}}{7ax^7} - \frac{5b\sqrt{-bx^4 + a}}{21a^2x^3} + \frac{5b^{\frac{7}{4}} \operatorname{EllipticF}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}, i\right) \sqrt{1 - \frac{bx^4}{a}}}{21a^{\frac{7}{4}} \sqrt{-bx^4 + a}}$$

command

```
integrate(1/x^8/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{a} bx^7 \left(\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x \left(\frac{b}{a}\right)^{\frac{1}{4}}, -1\right) - (5bx^4 + 3a) \sqrt{-bx^4 + a}}{21a^2x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-bx^4 + a}}{bx^{12} - ax^8}, x\right)$$

16.100 Problem number 861

$$\int \frac{x^{12}}{(a + bx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x^9}{2b\sqrt{bx^4+a}} - \frac{15ax\sqrt{bx^4+a}}{14b^3} + \frac{9x^5\sqrt{bx^4+a}}{14b^2} \\ & + \frac{15a^{7/4} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{1/4}x}{a^{1/4}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{1/4}x}{a^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4+a}{(\sqrt{a} + x^2\sqrt{b})}}}{28 \cos\left(2\arctan\left(\frac{b^{1/4}x}{a^{1/4}}\right)\right) b^{13/4} \sqrt{bx^4+a}} \end{aligned}$$

command

`integrate(x^12/(b*x^4+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15(abx^4 + a^2)\sqrt{b}\left(-\frac{a}{b}\right)^{3/4} \operatorname{ellipticF}\left(\frac{\left(-\frac{a}{b}\right)^{1/4}}{x}, -1\right) + (2b^2x^9 - 6abx^5 - 15a^2x)\sqrt{bx^4+a}}{14(b^4x^4 + ab^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^4+a}x^{12}}{b^2x^8 + 2abx^4 + a^2}, x\right)$$

16.101 Problem number 862

$$\int \frac{x^8}{(a + bx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x^5}{2b\sqrt{bx^4+a}} + \frac{5x\sqrt{bx^4+a}}{6b^2} \\ & - \frac{5a^{3/4} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{1/4}x}{a^{1/4}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{1/4}x}{a^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4+a}{(\sqrt{a} + x^2\sqrt{b})}}}{12 \cos\left(2\arctan\left(\frac{b^{1/4}x}{a^{1/4}}\right)\right) b^{9/4} \sqrt{bx^4+a}} \end{aligned}$$

command

```
integrate(x^8/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 (bx^4 + a) \sqrt{b} \left(-\frac{a}{b}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}, -1\right) - (2bx^5 + 5ax) \sqrt{bx^4 + a}}{6 (b^3x^4 + ab^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^4 + a} x^8}{b^2x^8 + 2abx^4 + a^2}, x\right)$$

16.102 Problem number 863

$$\int \frac{x^4}{(a + bx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{x}{2b\sqrt{bx^4 + a}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4 + a}{(\sqrt{a} + x^2\sqrt{b})^2}}}{4 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} b^{\frac{5}{4}} \sqrt{bx^4 + a}}$$

command

```
integrate(x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(bx^4 + a) \sqrt{a} \left(-\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x \left(-\frac{b}{a}\right)^{\frac{1}{4}}, -1\right) + \sqrt{bx^4 + a} bx}{2 (b^3x^4 + ab^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^4 + a} x^4}{b^2x^8 + 2abx^4 + a^2}, x\right)$$

16.103 Problem number 864

$$\int \frac{1}{(a + bx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x}{2a\sqrt{bx^4+a}} + \frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4+a}{(\sqrt{a} + x^2\sqrt{b})^2}}}{4\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{5}{4}} b^{\frac{1}{4}} \sqrt{bx^4+a}}$$

command

`integrate(1/(b*x^4+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(bx^4 + a)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}, -1\right) - \sqrt{bx^4+a} bx}{2(ab^2x^4 + a^2b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^4+a}}{b^2x^8 + 2abx^4 + a^2}, x\right)$$

16.104 Problem number 865

$$\int \frac{1}{x^4(a + bx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{2ax^3\sqrt{bx^4+a}} - \frac{5\sqrt{bx^4+a}}{6a^2x^3} + \frac{5b^{\frac{3}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4+a}{(\sqrt{a} + x^2\sqrt{b})^2}}}{12\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{9}{4}} \sqrt{bx^4+a}}$$

command

```
integrate(1/x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 (bx^7 + ax^3) \sqrt{a} \left(-\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x \left(-\frac{b}{a}\right)^{\frac{1}{4}}, -1\right) - (5bx^4 + 2a) \sqrt{bx^4 + a}}{6 (a^2bx^7 + a^3x^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^4 + a}}{b^2x^{12} + 2abx^8 + a^2x^4}, x\right)$$

16.105 Problem number 866

$$\int \frac{1}{x^8 (a + bx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{2ax^7 \sqrt{bx^4 + a}} - \frac{9\sqrt{bx^4 + a}}{14a^2x^7} + \frac{15b\sqrt{bx^4 + a}}{14a^3x^3} + \frac{15b^{\frac{7}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4 + a}{(\sqrt{a} + x^2\sqrt{b})}}}{28 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{13}{4}} \sqrt{bx^4 + a}}$$

command

```
integrate(1/x^8/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 (b^2x^{11} + abx^7) \sqrt{a} \left(-\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x \left(-\frac{b}{a}\right)^{\frac{1}{4}}, -1\right) - (15b^2x^8 + 6abx^4 - 2a^2) \sqrt{bx^4 + a}}{14 (a^3bx^{11} + a^4x^7)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^4 + a}}{b^2x^{16} + 2abx^{12} + a^2x^8}, x\right)$$

16.106 Problem number 873

$$\int \frac{1}{(a + bx^4)^{5/2}} dx$$

Optimal antiderivative

$$\frac{x}{6a(bx^4 + a)^{3/2}} + \frac{5x}{12a^2\sqrt{bx^4 + a}}$$

$$+ \frac{5 \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{1/4}x}{a^{1/4}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{1/4}x}{a^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4 + a}{(\sqrt{a} + x^2\sqrt{b})^2}}}{24 \cos\left(2 \arctan\left(\frac{b^{1/4}x}{a^{1/4}}\right)\right) a^{9/4} b^{1/4} \sqrt{bx^4 + a}}$$

command

```
integrate(1/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(b^2x^8 + 2abx^4 + a^2)\sqrt{a}\left(-\frac{b}{a}\right)^{3/4} \operatorname{ellipticF}\left(x\left(-\frac{b}{a}\right)^{1/4}, -1\right) - (5b^2x^5 + 7abx)\sqrt{bx^4 + a}}{12(a^2b^3x^8 + 2a^3b^2x^4 + a^4b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^4 + a}}{b^3x^{12} + 3ab^2x^8 + 3a^2bx^4 + a^3}, x\right)$$

16.107 Problem number 884

$$\int \frac{x^8}{\sqrt{1-x^4}} dx$$

Optimal antiderivative

$$\frac{5 \operatorname{EllipticF}(x, i)}{21} - \frac{5x\sqrt{-x^4 + 1}}{21} - \frac{x^5\sqrt{-x^4 + 1}}{7}$$

command

```
integrate(x^8/(-x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{21} (3x^5 + 5x) \sqrt{-x^4 + 1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4 + 1} x^8}{x^4 - 1}, x\right)$$

16.108 Problem number 885

$$\int \frac{x^4}{\sqrt{1 - x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}(x, i)}{3} - \frac{x \sqrt{-x^4 + 1}}{3}$$

command

`integrate(x^4/(-x^4+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{3} \sqrt{-x^4 + 1} x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4 + 1} x^4}{x^4 - 1}, x\right)$$

16.109 Problem number 886

$$\int \frac{1}{\sqrt{1 - x^4}} dx$$

Optimal antiderivative

$$\text{EllipticF}(x, i)$$

command

`integrate(1/(-x^4+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\text{ellipticF}(x, -1)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}}{x^4-1}, x\right)$$

16.110 Problem number 887

$$\int \frac{1}{x^4 \sqrt{1-x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}(x, i)}{3} - \frac{\sqrt{-x^4+1}}{3x^3}$$

command

`integrate(1/x^4/(-x^4+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{x^3 \text{ellipticF}(x, -1) - \sqrt{-x^4+1}}{3x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}}{x^8-x^4}, x\right)$$

16.111 Problem number 888

$$\int \frac{1}{x^8 \sqrt{1-x^4}} dx$$

Optimal antiderivative

$$\frac{5 \text{EllipticF}(x, i)}{21} - \frac{\sqrt{-x^4+1}}{7x^7} - \frac{5\sqrt{-x^4+1}}{21x^3}$$

command

`integrate(1/x^8/(-x^4+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5x^7 \operatorname{ellipticF}(x, -1) - (5x^4 + 3)\sqrt{-x^4 + 1}}{21x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^4 + 1}}{x^{12} - x^8}, x\right)$$

16.112 Problem number 889

$$\int \frac{x^{10}}{\sqrt{1-x^4}} dx$$

Optimal antiderivative

$$\frac{7 \operatorname{EllipticE}(x, i)}{15} - \frac{7 \operatorname{EllipticF}(x, i)}{15} - \frac{7x^3 \sqrt{-x^4 + 1}}{45} - \frac{x^7 \sqrt{-x^4 + 1}}{9}$$

command

`integrate(x^10/(-x^4+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(5x^8 + 7x^4 + 21)\sqrt{-x^4 + 1}}{45x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^4 + 1} x^{10}}{x^4 - 1}, x\right)$$

16.113 Problem number 890

$$\int \frac{x^6}{\sqrt{1-x^4}} dx$$

Optimal antiderivative

$$\frac{3 \operatorname{EllipticE}(x, i)}{5} - \frac{3 \operatorname{EllipticF}(x, i)}{5} - \frac{x^3 \sqrt{-x^4 + 1}}{5}$$

command

```
integrate(x^6/(-x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(x^4 + 3)\sqrt{-x^4 + 1}}{5x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4 + 1} x^6}{x^4 - 1}, x\right)$$

16.114 Problem number 891

$$\int \frac{x^2}{\sqrt{1-x^4}} dx$$

Optimal antiderivative

$$\text{EllipticE}(x, i) - \text{EllipticF}(x, i)$$

command

```
integrate(x^2/(-x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{-x^4 + 1}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4 + 1} x^2}{x^4 - 1}, x\right)$$

16.115 Problem number 904

$$\int \frac{x^{12}}{(1-x^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{15 \text{EllipticF}(x, i)}{14} + \frac{x^9}{2\sqrt{-x^4 + 1}} + \frac{15x\sqrt{-x^4 + 1}}{14} + \frac{9x^5\sqrt{-x^4 + 1}}{14}$$

command

```
integrate(x^12/(-x^4+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2x^9 + 6x^5 - 15x)\sqrt{-x^4 + 1}}{14(x^4 - 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^4 + 1} x^{12}}{x^8 - 2x^4 + 1}, x\right)$$

16.116 Problem number 905

$$\int \frac{x^8}{(1-x^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{5 \operatorname{EllipticF}(x, i)}{6} + \frac{x^5}{2\sqrt{-x^4 + 1}} + \frac{5x\sqrt{-x^4 + 1}}{6}$$

command

```
integrate(x^8/(-x^4+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2x^5 - 5x)\sqrt{-x^4 + 1}}{6(x^4 - 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^4 + 1} x^8}{x^8 - 2x^4 + 1}, x\right)$$

16.117 Problem number 906

$$\int \frac{x^4}{(1-x^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\text{EllipticF}(x, i)}{2} + \frac{x}{2\sqrt{-x^4+1}}$$

command

```
integrate(x^4/(-x^4+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(x^4-1)\text{ellipticF}(x, -1) + \sqrt{-x^4+1} x}{2(x^4-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^4+1} x^4}{x^8-2x^4+1}, x\right)$$

16.118 Problem number 907

$$\int \frac{1}{(1-x^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}(x, i)}{2} + \frac{x}{2\sqrt{-x^4+1}}$$

command

```
integrate(1/(-x^4+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(x^4-1)\text{ellipticF}(x, -1) - \sqrt{-x^4+1} x}{2(x^4-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^4+1}}{x^8-2x^4+1}, x\right)$$

16.119 Problem number 908

$$\int \frac{1}{x^4 (1-x^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{5 \operatorname{EllipticF}(x, i)}{6} + \frac{1}{2x^3 \sqrt{-x^4 + 1}} - \frac{5\sqrt{-x^4 + 1}}{6x^3}$$

command

`integrate(1/x^4/(-x^4+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(x^7 - x^3)\operatorname{ellipticF}(x, -1) - (5x^4 - 2)\sqrt{-x^4 + 1}}{6(x^7 - x^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^4 + 1}}{x^{12} - 2x^8 + x^4}, x\right)$$

16.120 Problem number 909

$$\int \frac{1}{x^8 (1-x^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{15 \operatorname{EllipticF}(x, i)}{14} + \frac{1}{2x^7 \sqrt{-x^4 + 1}} - \frac{9\sqrt{-x^4 + 1}}{14x^7} - \frac{15\sqrt{-x^4 + 1}}{14x^3}$$

command

`integrate(1/x^8/(-x^4+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15(x^{11} - x^7)\operatorname{ellipticF}(x, -1) - (15x^8 - 6x^4 - 2)\sqrt{-x^4 + 1}}{14(x^{11} - x^7)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^4 + 1}}{x^{16} - 2x^{12} + x^8}, x\right)$$

16.121 Problem number 910

$$\int \frac{x^{14}}{(1-x^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{77 \operatorname{EllipticE}(x, i)}{30} + \frac{77 \operatorname{EllipticF}(x, i)}{30} + \frac{x^{11}}{2\sqrt{-x^4+1}} + \frac{77x^3\sqrt{-x^4+1}}{90} + \frac{11x^7\sqrt{-x^4+1}}{18}$$

command

```
integrate(x^14/(-x^4+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(10x^{12} + 22x^8 + 154x^4 - 231)\sqrt{-x^4+1}}{90(x^5-x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^4+1}x^{14}}{x^8-2x^4+1}, x\right)$$

16.122 Problem number 911

$$\int \frac{x^{10}}{(1-x^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{21 \operatorname{EllipticE}(x, i)}{10} + \frac{21 \operatorname{EllipticF}(x, i)}{10} + \frac{x^7}{2\sqrt{-x^4+1}} + \frac{7x^3\sqrt{-x^4+1}}{10}$$

command

```
integrate(x^10/(-x^4+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2x^8 + 14x^4 - 21)\sqrt{-x^4+1}}{10(x^5-x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^4+1}x^{10}}{x^8-2x^4+1}, x\right)$$

16.123 Problem number 912

$$\int \frac{x^6}{(1-x^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{3 \operatorname{EllipticE}(x, i)}{2} + \frac{3 \operatorname{EllipticF}(x, i)}{2} + \frac{x^3}{2\sqrt{-x^4+1}}$$

command

`integrate(x^6/(-x^4+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2x^4 - 3)\sqrt{-x^4 + 1}}{2(x^5 - x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^4+1} x^6}{x^8 - 2x^4 + 1}, x\right)$$

16.124 Problem number 916

$$\int \frac{1}{(1-x^4)^{5/2}} dx$$

Optimal antiderivative

$$\frac{x}{6(-x^4+1)^{3/2}} + \frac{5 \operatorname{EllipticF}(x, i)}{12} + \frac{5x}{12\sqrt{-x^4+1}}$$

command

`integrate(1/(-x^4+1)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(x^8 - 2x^4 + 1)\operatorname{ellipticF}(x, -1) - (5x^5 - 7x)\sqrt{-x^4 + 1}}{12(x^8 - 2x^4 + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^4+1}}{x^{12} - 3x^8 + 3x^4 - 1}, x\right)$$

16.125 Problem number 927

$$\int \frac{x^8}{\sqrt{1+x^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5x\sqrt{x^4+1}}{21} + \frac{x^5\sqrt{x^4+1}}{7} \\ & + \frac{5(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4+1}{(x^2+1)^2}}}{42\cos(2\arctan(x))\sqrt{x^4+1}} \end{aligned}$$

command

`integrate(x^8/(x^4+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{21} (3x^5 - 5x)\sqrt{x^4+1} + \frac{5}{21}i\sqrt{i} \operatorname{ellipticF}\left(\frac{\sqrt{i}}{x}, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^8}{\sqrt{x^4+1}}, x\right)$$

16.126 Problem number 928

$$\int \frac{x^4}{\sqrt{1+x^4}} dx$$

Optimal antiderivative

$$\frac{x\sqrt{x^4+1}}{3} - \frac{(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4+1}{(x^2+1)^2}}}{6\cos(2\arctan(x))\sqrt{x^4+1}}$$

command

`integrate(x^4/(x^4+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{x^4 + 1} x - \frac{1}{3} i \sqrt{i} \operatorname{ellipticF}\left(\frac{\sqrt{i}}{x}, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^4}{\sqrt{x^4 + 1}}, x\right)$$

16.127 Problem number 929

$$\int \frac{1}{\sqrt{1 + x^4}} dx$$

Optimal antiderivative

$$\frac{(x^2 + 1) \sqrt{\frac{\cos(4 \arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4 + 1}{(x^2 + 1)^2}}}{2 \cos(2 \arctan(x)) \sqrt{x^4 + 1}}$$

command

`integrate(1/(x^4+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{i} \operatorname{ellipticF}\left(\sqrt{i} x, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{x^4 + 1}}, x\right)$$

16.128 Problem number 930

$$\int \frac{1}{x^4 \sqrt{1 + x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{x^4 + 1}}{3x^3} - \frac{(x^2 + 1) \sqrt{\frac{\cos(4 \arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4 + 1}{(x^2 + 1)^2}}}{6 \cos(2 \arctan(x)) \sqrt{x^4 + 1}}$$

command

```
integrate(1/x^4/(x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{i} x^3 \operatorname{ellipticF}\left(\sqrt{i} x, -1\right) - \sqrt{x^4 + 1}}{3 x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^4 + 1}}{x^8 + x^4}, x\right)$$

16.129 Problem number 931

$$\int \frac{1}{x^8 \sqrt{1 + x^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{x^4 + 1}}{7x^7} + \frac{5\sqrt{x^4 + 1}}{21x^3} \\ & + \frac{5(x^2 + 1) \sqrt{\frac{\cos(4 \arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4 + 1}{(x^2 + 1)^2}}}{42 \cos(2 \arctan(x)) \sqrt{x^4 + 1}} \end{aligned}$$

command

```
integrate(1/x^8/(x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{i} x^7 \operatorname{ellipticF}\left(\sqrt{i} x, -1\right) + (5x^4 - 3) \sqrt{x^4 + 1}}{21 x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^4 + 1}}{x^{12} + x^8}, x\right)$$

16.130 Problem number 947

$$\int \frac{x^{12}}{(1+x^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x^9}{2\sqrt{x^4+1}} - \frac{15x\sqrt{x^4+1}}{14} + \frac{9x^5\sqrt{x^4+1}}{14} \\ & + \frac{15(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4+1}{(x^2+1)^2}}}{28\cos(2\arctan(x))\sqrt{x^4+1}} \end{aligned}$$

command

`integrate(x^12/(x^4+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{15\sqrt{i}(-ix^4-i)\operatorname{ellipticF}\left(\frac{\sqrt{i}}{x}, -1\right) - (2x^9 - 6x^5 - 15x)\sqrt{x^4+1}}{14(x^4+1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^4+1}x^{12}}{x^8+2x^4+1}, x\right)$$

16.131 Problem number 948

$$\int \frac{x^8}{(1+x^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x^5}{2\sqrt{x^4+1}} + \frac{5x\sqrt{x^4+1}}{6} \\ & - \frac{5(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4+1}{(x^2+1)^2}}}{12\cos(2\arctan(x))\sqrt{x^4+1}} \end{aligned}$$

command

`integrate(x^8/(x^4+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{5\sqrt{i}(ix^4+i)\operatorname{ellipticF}\left(\frac{\sqrt{i}}{x},-1\right)-(2x^5+5x)\sqrt{x^4+1}}{6(x^4+1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^4+1}x^8}{x^8+2x^4+1},x\right)$$

16.132 Problem number 949

$$\int \frac{x^4}{(1+x^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{x}{2\sqrt{x^4+1}} + \frac{(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4+1}{(x^2+1)^2}}}{4\cos(2\arctan(x))\sqrt{x^4+1}}$$

command

`integrate(x^4/(x^4+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{i}(-ix^4-i)\operatorname{ellipticF}\left(\sqrt{i}x,-1\right)-\sqrt{x^4+1}x}{2(x^4+1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^4+1}x^4}{x^8+2x^4+1},x\right)$$

16.133 Problem number 950

$$\int \frac{1}{(1+x^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x}{2\sqrt{x^4+1}} + \frac{(x^2+1) \sqrt{\frac{\cos(4 \arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4+1}{(x^2+1)^2}}}{4 \cos(2 \arctan(x)) \sqrt{x^4+1}}$$

command

```
integrate(1/(x^4+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{i}(-ix^4 - i) \operatorname{ellipticF}(\sqrt{i}x, -1) + \sqrt{x^4+1}x}{2(x^4+1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^4+1}}{x^8+2x^4+1}, x\right)$$

16.134 Problem number 951

$$\int \frac{1}{x^4(1+x^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{2x^3\sqrt{x^4+1}} - \frac{5\sqrt{x^4+1}}{6x^3} - \frac{5(x^2+1) \sqrt{\frac{\cos(4 \arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4+1}{(x^2+1)^2}}}{12 \cos(2 \arctan(x)) \sqrt{x^4+1}}$$

command

```
integrate(1/x^4/(x^4+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{i}(-ix^7 - ix^3)\text{ellipticF}(\sqrt{i}x, -1) + (5x^4 + 2)\sqrt{x^4 + 1}}{6(x^7 + x^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^4 + 1}}{x^{12} + 2x^8 + x^4}, x\right)$$

16.135 Problem number 952

$$\int \frac{1}{x^8(1+x^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{2x^7\sqrt{x^4 + 1}} - \frac{9\sqrt{x^4 + 1}}{14x^7} + \frac{15\sqrt{x^4 + 1}}{14x^3} + \frac{15(x^2 + 1)\sqrt{\frac{\cos(4\arctan(x))}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^4 + 1}{(x^2 + 1)^2}}}{28\cos(2\arctan(x))\sqrt{x^4 + 1}}$$

command

`integrate(1/x^8/(x^4+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15\sqrt{i}(ix^{11} + ix^7)\text{ellipticF}(\sqrt{i}x, -1) - (15x^8 + 6x^4 - 2)\sqrt{x^4 + 1}}{14(x^{11} + x^7)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^4 + 1}}{x^{16} + 2x^{12} + x^8}, x\right)$$

16.136 Problem number 959

$$\int \frac{1}{(1+x^4)^{5/2}} dx$$

Optimal antiderivative

$$\frac{x}{6(x^4+1)^{\frac{3}{2}}} + \frac{5x}{12\sqrt{x^4+1}} + \frac{5(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4+1}{(x^2+1)^2}}}{24\cos(2\arctan(x))\sqrt{x^4+1}}$$

command

```
integrate(1/(x^4+1)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{i}(ix^8+2ix^4+i)\operatorname{ellipticF}(\sqrt{i}x,-1) - (5x^5+7x)\sqrt{x^4+1}}{12(x^8+2x^4+1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^4+1}}{x^{12}+3x^8+3x^4+1}, x\right)$$

16.137 Problem number 968

$$\int \frac{x^6}{\sqrt{16-x^4}} dx$$

Optimal antiderivative

$$\frac{96\operatorname{EllipticE}\left(\frac{x}{2}, i\right)}{5} - \frac{96\operatorname{EllipticF}\left(\frac{x}{2}, i\right)}{5} - \frac{x^3\sqrt{-x^4+16}}{5}$$

command

```
integrate(x^6/(-x^4+16)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(x^4+48)\sqrt{-x^4+16}}{5x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^4+16}x^6}{x^4-16}, x\right)$$

16.138 Problem number 969

$$\int \frac{x^4}{\sqrt{16-x^4}} dx$$

Optimal antiderivative

$$\frac{8 \operatorname{EllipticF}\left(\frac{x}{2}, i\right)}{3} - \frac{x \sqrt{-x^4 + 16}}{3}$$

command

```
integrate(x^4/(-x^4+16)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{3} \sqrt{-x^4 + 16} x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^4 + 16} x^4}{x^4 - 16}, x\right)$$

16.139 Problem number 970

$$\int \frac{x^2}{\sqrt{16-x^4}} dx$$

Optimal antiderivative

$$2 \operatorname{EllipticE}\left(\frac{x}{2}, i\right) - 2 \operatorname{EllipticF}\left(\frac{x}{2}, i\right)$$

command

```
integrate(x^2/(-x^4+16)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{-x^4 + 16}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^4 + 16} x^2}{x^4 - 16}, x\right)$$

16.140 Problem number 971

$$\int \frac{1}{\sqrt{16-x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x}{2}, i\right)}{2}$$

command

`integrate(1/(-x^4+16)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \text{ellipticF}\left(\frac{1}{2}x, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4+16}}{x^4-16}, x\right)$$

16.141 Problem number 973

$$\int \frac{1}{x^4 \sqrt{16-x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x}{2}, i\right)}{96} - \frac{\sqrt{-x^4+16}}{48x^3}$$

command

`integrate(1/x^4/(-x^4+16)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{x^3 \text{ellipticF}\left(\frac{1}{2}x, -1\right) - 2\sqrt{-x^4+16}}{96x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4+16}}{x^8-16x^4}, x\right)$$

16.142 Problem number 977

$$\int \frac{x^4}{\sqrt{-1+x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}, \frac{\sqrt{2}}{2}\right) \sqrt{x^2-1} \sqrt{x^2+1} \sqrt{2}}{6\sqrt{x^4-1}} + \frac{x\sqrt{x^4-1}}{3}$$

command

`integrate(x^4/(x^4-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{x^4-1} x - \frac{1}{3} \text{ellipticF}\left(\frac{1}{x}, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{x^4}{\sqrt{x^4-1}}, x\right)$$

16.143 Problem number 979

$$\int \frac{1}{x^4 \sqrt{-1+x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}, \frac{\sqrt{2}}{2}\right) \sqrt{x^2-1} \sqrt{x^2+1} \sqrt{2}}{6\sqrt{x^4-1}} + \frac{\sqrt{x^4-1}}{3x^3}$$

command

`integrate(1/x^4/(x^4-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{x^4-1}}{3x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^4-1}}{x^8-x^4}, x\right)$$

16.144 Problem number 982

$$\int \frac{1}{x^2 \sqrt{-1+x^4}} dx$$

Optimal antiderivative

$$\frac{x(x^2+1)}{\sqrt{x^4-1}} - \frac{\text{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}, \frac{\sqrt{2}}{2}\right) \sqrt{x^2-1} \sqrt{x^2+1} \sqrt{2}}{2\sqrt{x^4-1}} \\ + \frac{\text{EllipticE}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}, \frac{\sqrt{2}}{2}\right) \sqrt{2} \sqrt{x^2-1} \sqrt{x^2+1}}{\sqrt{x^4-1}} + \frac{\sqrt{x^4-1}}{x}$$

command

```
integrate(1/x^2/(x^4-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{x^4-1}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^4-1}}{x^6-x^2}, x\right)$$

16.145 Problem number 983

$$\int \frac{x^2}{\sqrt{3-2x^4}} dx$$

Optimal antiderivative

$$\frac{3^{\frac{1}{4}} \text{EllipticE}\left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} x}{3}, i\right) 2^{\frac{1}{4}}}{2} - \frac{3^{\frac{1}{4}} \text{EllipticF}\left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} x}{3}, i\right) 2^{\frac{1}{4}}}{2}$$

command

```
integrate(x^2/(-2*x^4+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{-2x^4+3}}{2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4+3} x^2}{2x^4-3}, x\right)$$

16.146 Problem number 1276

$$\int \frac{1}{a + bx^5} dx$$

Optimal antiderivative

$$\frac{\ln\left(a^{\frac{1}{5}} + b^{\frac{1}{5}}x\right) - \frac{\ln\left(2a^{\frac{2}{5}} + 2b^{\frac{2}{5}}x^2 - a^{\frac{1}{5}}b^{\frac{1}{5}}x(-\sqrt{5} + 1)\right)(-\sqrt{5} + 1)}{5a^{\frac{4}{5}}b^{\frac{1}{5}}}}{\frac{\ln\left(2a^{\frac{2}{5}} + 2b^{\frac{2}{5}}x^2 - a^{\frac{1}{5}}b^{\frac{1}{5}}x(\sqrt{5} + 1)\right)(\sqrt{5} + 1)}{20a^{\frac{4}{5}}b^{\frac{1}{5}}}} - \frac{\arctan\left(\frac{-4b^{\frac{1}{5}}x + a^{\frac{1}{5}}(\sqrt{5} + 1)}{a^{\frac{1}{5}}\sqrt{10 - 2\sqrt{5}}}\right)\sqrt{10 - 2\sqrt{5}}}{10a^{\frac{4}{5}}b^{\frac{1}{5}}}}{\frac{\arctan\left(\frac{-4b^{\frac{1}{5}}x + a^{\frac{1}{5}}(-\sqrt{5} + 1)}{a^{\frac{1}{5}}\sqrt{10 + 2\sqrt{5}}}\right)\sqrt{10 + 2\sqrt{5}}}{10a^{\frac{4}{5}}b^{\frac{1}{5}}}}$$

command

`integrate(1/(b*x^5+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

16.147 Problem number 1396

$$\int \frac{x^7}{\sqrt{2 + x^6}} dx$$

Optimal antiderivative

$$\frac{x^2\sqrt{x^6 + 2}}{5} + \frac{2 \cdot 2^{\frac{5}{6}} \left(2^{\frac{1}{3}} + x^2\right) \operatorname{EllipticF}\left(\frac{x^2 + 2^{\frac{1}{3}}(1 - \sqrt{3})}{x^2 + 2^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}} - 2^{\frac{1}{3}}x^2 + x^4}{\left(x^2 + 2^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{3^{\frac{3}{4}}} - \frac{15\sqrt{x^6 + 2} \sqrt{\frac{2^{\frac{1}{3}} + x^2}{\left(x^2 + 2^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{3^{\frac{3}{4}}}$$

command

```
integrate(x^7/(x^6+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{5} \sqrt{x^6 + 2} x^2 - \frac{4}{5} \text{weierstrassPInverse}(0, -8, x^2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{x^7}{\sqrt{x^6 + 2}}, x\right)$$

16.148 Problem number 1397

$$\int \frac{x}{\sqrt{2 + x^6}} dx$$

Optimal antiderivative

$$\frac{2^{\frac{5}{6}} \left(2^{\frac{1}{3}} + x^2\right) \text{EllipticF}\left(\frac{x^2 + 2^{\frac{1}{3}}(1 - \sqrt{3})}{x^2 + 2^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}} - 2^{\frac{1}{3}}x^2 + x^4}{\left(x^2 + 2^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}}{6\sqrt{x^6 + 2} \sqrt{\frac{2^{\frac{1}{3}} + x^2}{\left(x^2 + 2^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate(x/(x^6+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\text{weierstrassPInverse}(0, -8, x^2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{x}{\sqrt{x^6 + 2}}, x\right)$$

16.149 Problem number 1398

$$\int \frac{1}{x^5 \sqrt{2+x^6}} dx$$

Optimal antiderivative

$$\frac{\sqrt{x^6+2}}{8x^4} - \frac{2^{\frac{5}{6}} \left(2^{\frac{1}{3}} + x^2\right) \operatorname{EllipticF}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}} - 2^{\frac{1}{3}}x^2 + x^4}{\left(x^2 + 2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{48\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}} + x^2}{\left(x^2 + 2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} 3^{\frac{3}{4}}$$

command

`integrate(1/x^5/(x^6+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{x^4 \operatorname{weierstrassPInverse}(0, -8, x^2) + \sqrt{x^6+2}}{8x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^6+2}}{x^{11} + 2x^5}, x\right)$$

16.150 Problem number 1402

$$\int \frac{x^9}{\sqrt{2+x^6}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{x^4 \sqrt{x^6 + 2}}{7} - \frac{8 \sqrt{x^6 + 2}}{7 \left(x^2 + 2^{\frac{1}{3}} (1 + \sqrt{3})\right)} \\
& \frac{8 \cdot 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + x^2\right) \operatorname{EllipticF}\left(\frac{x^2 + 2^{\frac{1}{3}} (1 - \sqrt{3})}{x^2 + 2^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{2^{\frac{2}{3}} - 2^{\frac{1}{3}} x^2 + x^4}{\left(x^2 + 2^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}} \cdot 3^{\frac{3}{4}}}{21 \sqrt{x^6 + 2} \sqrt{\frac{2^{\frac{1}{3}} + x^2}{\left(x^2 + 2^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}} \\
& \frac{4 \cdot 2^{\frac{1}{6}} \cdot 3^{\frac{1}{4}} \left(2^{\frac{1}{3}} + x^2\right) \operatorname{EllipticE}\left(\frac{x^2 + 2^{\frac{1}{3}} (1 - \sqrt{3})}{x^2 + 2^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}} - 2^{\frac{1}{3}} x^2 + x^4}{\left(x^2 + 2^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}{7 \sqrt{x^6 + 2} \sqrt{\frac{2^{\frac{1}{3}} + x^2}{\left(x^2 + 2^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}
\end{aligned}$$

command

```
integrate(x^9/(x^6+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{7} \sqrt{x^6 + 2} x^4 + \frac{8}{7} \operatorname{weierstrassZeta}(0, -8, \operatorname{weierstrassPInverse}(0, -8, x^2))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^9}{\sqrt{x^6 + 2}}, x\right)$$

16.151 Problem number 1403

$$\int \frac{x^3}{\sqrt{2 + x^6}} dx$$

Optimal antiderivative

$$\frac{\sqrt{x^6+2}}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})} + \frac{2^{\frac{2}{3}}(2^{\frac{1}{3}}+x^2) \operatorname{EllipticF}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{\frac{2^{\frac{2}{3}}-2^{\frac{1}{3}}x^2+x^4}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}} 3^{\frac{3}{4}}}{3\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}} - \frac{2^{\frac{1}{6}}3^{\frac{1}{4}}(2^{\frac{1}{3}}+x^2) \operatorname{EllipticE}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}}-2^{\frac{1}{3}}x^2+x^4}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{2\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}}$$

command

```
integrate(x^3/(x^6+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\operatorname{weierstrassZeta}(0, -8, \operatorname{weierstrassPInverse}(0, -8, x^2))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^3}{\sqrt{x^6+2}}, x\right)$$

16.152 Problem number 1404

$$\int \frac{1}{x^3 \sqrt{2+x^6}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{\sqrt{x^6+2}}{4x^2} + \frac{\sqrt{x^6+2}}{4x^2 + 4 \cdot 2^{\frac{1}{3}}(1+\sqrt{3})} \\
 & + \frac{2^{\frac{2}{3}}(2^{\frac{1}{3}}+x^2) \operatorname{EllipticF}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{\frac{2^{\frac{2}{3}}-2^{\frac{1}{3}}x^2+x^4}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{12\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \\
 & - \frac{2^{\frac{1}{6}}3^{\frac{1}{4}}(2^{\frac{1}{3}}+x^2) \operatorname{EllipticE}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}}-2^{\frac{1}{3}}x^2+x^4}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{8\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}}
 \end{aligned}$$

command

```
integrate(1/x^3/(x^6+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{x^2 \operatorname{weierstrassZeta}(0, -8, \operatorname{weierstrassPInverse}(0, -8, x^2)) + \sqrt{x^6+2}}{4x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^6+2}}{x^9+2x^3}, x\right)$$

16.153 Problem number 1420

$$\int \frac{x^{13}}{(2+x^6)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x^8}{3\sqrt{x^6+2}} + \frac{8x^2\sqrt{x^6+2}}{15} + \frac{16 \cdot 2^{\frac{5}{6}} (2^{\frac{1}{3}} + x^2) \operatorname{EllipticF}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}} - 2^{\frac{1}{3}}x^2 + x^4}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{3}{4}}}$$

$$45\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}$$

command

`integrate(x^13/(x^6+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{32(x^6+2)\operatorname{weierstrassPInverse}(0,-8,x^2) - (3x^8+16x^2)\sqrt{x^6+2}}{15(x^6+2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^6+2}x^{13}}{x^{12}+4x^6+4}, x\right)$$

16.154 Problem number 1421

$$\int \frac{x^7}{(2+x^6)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x^2}{3\sqrt{x^6+2}} + \frac{2^{\frac{5}{6}} (2^{\frac{1}{3}} + x^2) \operatorname{EllipticF}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}} - 2^{\frac{1}{3}}x^2 + x^4}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{3}{4}}}$$

$$9\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}$$

command

`integrate(x^7/(x^6+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{x^6+2} x^2 - 2(x^6+2)\text{weierstrassPInverse}(0, -8, x^2)}{3(x^6+2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^6+2} x^7}{x^{12}+4x^6+4}, x\right)$$

16.155 Problem number 1422

$$\int \frac{x}{(2+x^6)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x^2}{6\sqrt{x^6+2}} + \frac{2^{\frac{5}{6}}(2^{\frac{1}{3}}+x^2)\text{EllipticF}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{2^{\frac{2}{3}}-2^{\frac{1}{3}}x^2+x^4}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{36\sqrt{x^6+2}\sqrt{\frac{2^{\frac{1}{3}}+x^2}{(x^2+2^{\frac{1}{3}}(1+\sqrt{3}))^2}}}}{3^{\frac{3}{4}}}$$

command

`integrate(x/(x^6+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{x^6+2} x^2 + (x^6+2)\text{weierstrassPInverse}(0, -8, x^2)}{6(x^6+2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^6+2} x}{x^{12}+4x^6+4}, x\right)$$

16.156 Problem number 1423

$$\int \frac{1}{x^5 (2 + x^6)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{6x^4 \sqrt{x^6 + 2}} - \frac{7\sqrt{x^6 + 2}}{48x^4} - \frac{7 \cdot 2^{\frac{5}{6}} \left(2^{\frac{1}{3}} + x^2\right) \operatorname{EllipticF}\left(\frac{x^2 + 2^{\frac{1}{3}}(1 - \sqrt{3})}{x^2 + 2^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}} - 2^{\frac{1}{3}}x^2 + x^4}{\left(x^2 + 2^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{288\sqrt{x^6 + 2} \sqrt{\frac{2^{\frac{1}{3}} + x^2}{\left(x^2 + 2^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \cdot 3^{\frac{3}{4}}$$

command

```
integrate(1/x^5/(x^6+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{7(x^{10} + 2x^4)\operatorname{weierstrassPInverse}(0, -8, x^2) + (7x^6 + 6)\sqrt{x^6 + 2}}{48(x^{10} + 2x^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^6 + 2}}{x^{17} + 4x^{11} + 4x^5}, x\right)$$

16.157 Problem number 1428

$$\int \frac{x^{15}}{(2 + x^6)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{x^{10}}{3\sqrt{x^6+2}} + \frac{10x^4\sqrt{x^6+2}}{21} - \frac{80\sqrt{x^6+2}}{21\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
& \quad 80 \cdot 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + x^2\right) \operatorname{EllipticF}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{2^{\frac{2}{3}} - 2^{\frac{1}{3}}x^2 + x^4}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \cdot 3^{\frac{3}{4}} \\
& \quad \frac{63\sqrt{x^6+2}}{\sqrt{\frac{2^{\frac{1}{3}} + x^2}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
& \quad 40 \cdot 2^{\frac{1}{6}} \cdot 3^{\frac{1}{4}} \left(2^{\frac{1}{3}} + x^2\right) \operatorname{EllipticE}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}} - 2^{\frac{1}{3}}x^2 + x^4}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \\
& + \frac{21\sqrt{x^6+2}}{\sqrt{\frac{2^{\frac{1}{3}} + x^2}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
\end{aligned}$$

command

```
integrate(x^15/(x^6+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{80(x^6+2)\operatorname{weierstrassZeta}(0,-8,\operatorname{weierstrassPInverse}(0,-8,x^2)) + (3x^{10} + 20x^4)\sqrt{x^6+2}}{21(x^6+2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^6+2}x^{15}}{x^{12}+4x^6+4},x\right)$$

16.158 Problem number 1429

$$\int \frac{x^9}{(2+x^6)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{x^4}{3\sqrt{x^6+2}} + \frac{4\sqrt{x^6+2}}{3\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
& + \frac{4 \cdot 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + x^2\right) \operatorname{EllipticF}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{\frac{2^{\frac{2}{3}}-2^{\frac{1}{3}}x^2+x^4}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \cdot 3^{\frac{3}{4}}}{9\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
& + \frac{2 \cdot 2^{\frac{1}{6}} \cdot 3^{\frac{1}{4}} \left(2^{\frac{1}{3}} + x^2\right) \operatorname{EllipticE}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}}-2^{\frac{1}{3}}x^2+x^4}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
\end{aligned}$$

command

```
integrate(x^9/(x^6+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{x^6+2} x^4 + 4(x^6+2) \operatorname{weierstrassZeta}(0, -8, \operatorname{weierstrassPInverse}(0, -8, x^2))}{3(x^6+2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^6+2} x^9}{x^{12}+4x^6+4}, x\right)$$

16.159 Problem number 1430

$$\int \frac{x^3}{(2+x^6)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{x^4}{6\sqrt{x^6+2}} - \frac{\sqrt{x^6+2}}{6\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
& \frac{2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+x^2\right) \operatorname{EllipticF}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{\frac{2^{\frac{2}{3}}-2^{\frac{1}{3}}x^2+x^4}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{3}{4}}} \\
& - \frac{18\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{2^{\frac{1}{6}}3^{\frac{1}{4}}\left(2^{\frac{1}{3}}+x^2\right) \operatorname{EllipticE}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}}-2^{\frac{1}{3}}x^2+x^4}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
& + \frac{12\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{12\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
\end{aligned}$$

command

```
integrate(x^3/(x^6+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{x^6+2} x^4 + (x^6+2) \operatorname{weierstrassZeta}(0, -8, \operatorname{weierstrassPInverse}(0, -8, x^2))}{6(x^6+2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^6+2} x^3}{x^{12}+4x^6+4}, x\right)$$

16.160 Problem number 1431

$$\int \frac{1}{x^3(2+x^6)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{6x^2\sqrt{x^6+2}} - \frac{5\sqrt{x^6+2}}{24x^2} + \frac{5\sqrt{x^6+2}}{24\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)}$$

$$+ \frac{5 \cdot 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + x^2\right) \operatorname{EllipticF}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{\frac{2^{\frac{2}{3}}-2^{\frac{1}{3}}x^2+x^4}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \cdot 3^{\frac{3}{4}}}{72\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

$$- \frac{5 \cdot 2^{\frac{1}{6}} \cdot 3^{\frac{1}{4}} \left(2^{\frac{1}{3}} + x^2\right) \operatorname{EllipticE}\left(\frac{x^2+2^{\frac{1}{3}}(1-\sqrt{3})}{x^2+2^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{2^{\frac{2}{3}}-2^{\frac{1}{3}}x^2+x^4}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{48\sqrt{x^6+2} \sqrt{\frac{2^{\frac{1}{3}}+x^2}{\left(x^2+2^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

`integrate(1/x^3/(x^6+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(x^8+2x^2)\operatorname{weierstrassZeta}(0,-8,\operatorname{weierstrassPInverse}(0,-8,x^2))+(5x^6+6)\sqrt{x^6+2}}{24(x^8+2x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^6+2}}{x^{15}+4x^9+4x^3},x\right)$$

16.161 Problem number 1510

$$\int x\sqrt{1+x^8} dx$$

Optimal antiderivative

$$\frac{x^2\sqrt{x^8+1}}{6} + \frac{(x^4+1)\sqrt{\frac{\cos(4\arctan(x^2))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin(2\arctan(x^2)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^8+1}{(x^4+1)^2}}}{6\cos(2\arctan(x^2))\sqrt{x^8+1}}$$

command

```
integrate(x*(x^8+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{x^8 + 1} x^2 + \frac{1}{3} i \sqrt{i} \operatorname{ellipticF}\left(\frac{\sqrt{i}}{x^2}, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{x^8 + 1} x, x\right)$$

16.162 Problem number 1525

$$\int \frac{x^9}{\sqrt{1 + x^8}} dx$$

Optimal antiderivative

$$\frac{x^2 \sqrt{x^8 + 1}}{6} - \frac{(x^4 + 1) \sqrt{\frac{\cos(4 \arctan(x^2))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \arctan(x^2)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^8 + 1}{(x^4 + 1)^2}}}{12 \cos(2 \arctan(x^2)) \sqrt{x^8 + 1}}$$

command

```
integrate(x^9/(x^8+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{x^8 + 1} x^2 - \frac{1}{6} i \sqrt{i} \operatorname{ellipticF}\left(\frac{\sqrt{i}}{x^2}, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^9}{\sqrt{x^8 + 1}}, x\right)$$

16.163 Problem number 1527

$$\int \frac{x}{\sqrt{1+x^8}} dx$$

Optimal antiderivative

$$\frac{(x^4 + 1) \sqrt{\frac{\cos(4 \arctan(x^2))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \arctan(x^2)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^8 + 1}{(x^4 + 1)^2}}}{4 \cos(2 \arctan(x^2)) \sqrt{x^8 + 1}}$$

command

`integrate(x/(x^8+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2}i \sqrt{i} \operatorname{ellipticF}\left(\sqrt{i} x^2, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x}{\sqrt{x^8 + 1}}, x\right)$$

16.164 Problem number 1529

$$\int \frac{1}{x^7 \sqrt{1+x^8}} dx$$

Optimal antiderivative

$$\frac{\sqrt{x^8 + 1}}{6x^6} - \frac{(x^4 + 1) \sqrt{\frac{\cos(4 \arctan(x^2))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \arctan(x^2)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^8 + 1}{(x^4 + 1)^2}}}{12 \cos(2 \arctan(x^2)) \sqrt{x^8 + 1}}$$

command

`integrate(1/x^7/(x^8+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{i} x^6 \operatorname{ellipticF}\left(\sqrt{i} x^2, -1\right) - \sqrt{x^8 + 1}}{6x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^8 + 1}}{x^{15} + x^7}, x\right)$$

16.165 Problem number 2001

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^2} dx$$

Optimal antiderivative

$$\frac{2\sqrt{a + \frac{b}{x^3}}}{5x} - \frac{23^{\frac{3}{4}} a \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \text{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{5b^{\frac{1}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

`integrate((a+b/x^3)^(1/2)/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3a\sqrt{b} \text{xweierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right) + b\sqrt{\frac{ax^3 + b}{x^3}} \right)}{5bx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{x^2}, x\right)$$

16.166 Problem number 2002

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^5} dx$$

Optimal antiderivative

$$\frac{2\sqrt{a + \frac{b}{x^3}}}{11x^4} - \frac{6a\sqrt{a + \frac{b}{x^3}}}{55bx} + \frac{43^{\frac{3}{4}}a^2\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{55b^{\frac{4}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((a+b/x^3)^(1/2)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 a^2 \sqrt{b} x^4 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right) - (3 abx^3 + 5 b^2) \sqrt{\frac{ax^3 + b}{x^3}} \right)}{55 b^2 x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{x^5}, x\right)$$

16.167 Problem number 2003

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^8} dx$$

Optimal antiderivative

$$\frac{2\sqrt{a + \frac{b}{x^3}}}{17x^7} - \frac{6a\sqrt{a + \frac{b}{x^3}}}{187bx^4} + \frac{48a^2\sqrt{a + \frac{b}{x^3}}}{935b^2x} + \frac{32 \cdot 3^{\frac{3}{4}} a^3 \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{935b^{\frac{7}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

command

`integrate((a+b/x^3)^(1/2)/x^8,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(48 a^3 \sqrt{b} x^7 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right) - (24 a^2 b x^6 - 15 a b^2 x^3 - 55 b^3) \sqrt{\frac{a x^3 + b}{x^3}} \right)}{935 b^3 x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{a x^3 + b}{x^3}}}{x^8}, x\right)$$

16.168 Problem number 2007

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^3} dx$$

Optimal antiderivative

$$\frac{2\sqrt{a + \frac{b}{x^3}}}{7x^2} - \frac{6a\sqrt{a + \frac{b}{x^3}}}{7b^{\frac{2}{3}}\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)}$$

$$- \frac{23^{\frac{3}{4}}a^{\frac{4}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{7b^{\frac{2}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

$$+ \frac{33^{\frac{1}{4}}a^{\frac{4}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticE}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{7b^{\frac{2}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

`integrate((a+b/x^3)^(1/2)/x^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3a\sqrt{b}x^2\operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right) - b\sqrt{\frac{ax^3 + b}{x^3}}\right)}{7bx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{x^3}, x\right)$$

16.169 Problem number 2008

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{a + \frac{b}{x^3}}}{13x^5} - \frac{6a\sqrt{a + \frac{b}{x^3}}}{91bx^2} + \frac{24a^2\sqrt{a + \frac{b}{x^3}}}{91b^{\frac{5}{3}}\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{83^{\frac{3}{4}}a^{\frac{7}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{91b^{\frac{5}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{123^{\frac{1}{4}}a^{\frac{7}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticE}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{91b^{\frac{5}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate((a+b/x^3)^(1/2)/x^6,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 a^2 \sqrt{b} x^5 \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right) + (3 abx^3 + 7 b^2) \sqrt{\frac{ax^3 + b}{x^3}} \right)}{91 b^2 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{ax^3 + b}}{x^6}, x\right)$$

16.170 Problem number 2009

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^9} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{a + \frac{b}{x^3}}}{19x^8} - \frac{6a\sqrt{a + \frac{b}{x^3}}}{247bx^5} + \frac{60a^2\sqrt{a + \frac{b}{x^3}}}{1729b^2x^2} - \frac{240a^3\sqrt{a + \frac{b}{x^3}}}{1729b^{\frac{8}{3}}\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{80 \cdot 3^{\frac{3}{4}} a^{\frac{10}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{1729b^{\frac{8}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{120 \cdot 3^{\frac{1}{4}} a^{\frac{10}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticE}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{1729b^{\frac{8}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate((a+b/x^3)^(1/2)/x^9,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(120 a^3 \sqrt{b} x^8 \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right) + (30 a^2 b x^6 - 21 a b^2 x^3 - 91 b^3) \sqrt{\frac{a x^3 + b}{x^3}} \right)}{1729 b^3 x^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a x^3 + b}}{x^9}, x\right)$$

16.171 Problem number 2027

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^2} dx$$

Optimal antiderivative

$$\frac{2 \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right) \operatorname{EllipticF} \left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} 3^{\frac{3}{4}}}{3b^{\frac{1}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}$$

command

```
integrate(1/x^2/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)}{\sqrt{b}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{x \sqrt{\frac{ax^3 + b}{x^3}}}{ax^3 + b}, x \right)$$

16.172 Problem number 2028

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^5} dx$$

Optimal antiderivative

$$\frac{2\sqrt{a + \frac{b}{x^3}}}{5bx} + \frac{4a\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2} 3^{\frac{3}{4}} + \frac{15b^{\frac{4}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}$$

command

```
integrate(1/x^5/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2a\sqrt{b} \operatorname{xweierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right) - b\sqrt{\frac{ax^3 + b}{x^3}}\right)}{5b^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{ax^5 + bx^2}, x\right)$$

16.173 Problem number 2029

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^8} dx$$

Optimal antiderivative

$$\frac{2\sqrt{a + \frac{b}{x^3}}}{11bx^4} + \frac{16a\sqrt{a + \frac{b}{x^3}}}{55b^2x} + \frac{32a^2\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2} 3^{\frac{3}{4}}$$

$$165b^{\frac{7}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

command

```
integrate(1/x^8/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(16a^2\sqrt{b}x^4\operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right) - (8abx^3 - 5b^2)\sqrt{\frac{ax^3 + b}{x^3}}\right)}{55b^3x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{ax^8 + bx^5}, x\right)$$

16.174 Problem number 2033

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2\sqrt{a + \frac{b}{x^3}}}{b^{\frac{2}{3}} \left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\
& 2a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right) \text{EllipticF} \left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} 3^{\frac{3}{4}} \\
& \frac{3b^{\frac{2}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{3^{\frac{1}{4}} a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right) \text{EllipticE} \left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} \\
& + \frac{b^{\frac{2}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}
\end{aligned}$$

command

```
integrate(1/x^3/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right)}{\sqrt{b}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{ax^3 + b}, x \right)$$

16.175 Problem number 2034

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{a + \frac{b}{x^3}}}{7bx^2} + \frac{8a\sqrt{a + \frac{b}{x^3}}}{7b^{\frac{5}{3}} \left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\ & + \frac{8a^{\frac{4}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right) \operatorname{EllipticF} \left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} 3^{\frac{3}{4}}}{21b^{\frac{5}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \\ & - \frac{43^{\frac{1}{4}} a^{\frac{4}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right) \operatorname{EllipticE} \left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{7b^{\frac{5}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

`integrate(1/x^6/(a+b/x^3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4a\sqrt{b} x^2 \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, \frac{1}{x} \right) \right) + b \sqrt{\frac{ax^3 + b}{x^3}} \right)}{7b^2 x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{ax^6 + bx^3}, x \right)$$

16.176 Problem number 2035

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^9} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{a + \frac{b}{x^3}}}{13bx^5} + \frac{20a\sqrt{a + \frac{b}{x^3}}}{91b^2x^2} - \frac{80a^2\sqrt{a + \frac{b}{x^3}}}{91b^{\frac{8}{3}} \left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\ & \frac{80a^{\frac{7}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right) \text{EllipticF} \left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} 3^{\frac{3}{4}}}{\sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \\ & \frac{273b^{\frac{8}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{\sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \\ & + \frac{40 \cdot 3^{\frac{1}{4}} a^{\frac{7}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right) \text{EllipticE} \left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{\sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \\ & + \frac{91b^{\frac{8}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{\sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x} \right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

```
integrate(1/x^9/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(40 a^2 \sqrt{b} x^5 \text{weierstrassZeta} \left(0, -\frac{4a}{b}, \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, \frac{1}{x} \right) \right) + (10 abx^3 - 7b^2) \sqrt{\frac{ax^3 + b}{x^3}} \right)}{91 b^3 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{ax^9 + bx^6}, x \right)$$

16.177 Problem number 2036

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^{12}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{a + \frac{b}{x^3}}}{19bx^8} + \frac{32a\sqrt{a + \frac{b}{x^3}}}{247b^2x^5} - \frac{320a^2\sqrt{a + \frac{b}{x^3}}}{1729b^3x^2} + \frac{1280a^3\sqrt{a + \frac{b}{x^3}}}{1729b^{\frac{11}{3}} \left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{1280a^{\frac{10}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \text{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}}{5187b^{\frac{11}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & - \frac{640 \cdot 3^{\frac{1}{4}} a^{\frac{10}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \text{EllipticE}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{1729b^{\frac{11}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate(1/x^12/(a+b/x^3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(640 a^3 \sqrt{b} x^8 \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right) + (160 a^2 b x^6 - 112 a b^2 x^3 + 91 b^3) \sqrt{\frac{ax^3 + b}{ax^{12} + bx^9}} \right)}{1729 b^4 x^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{ax^{12} + bx^9}, x\right)$$

16.178 Problem number 2047

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^2} dx$$

Optimal antiderivative

$$\frac{2}{3ax\sqrt{a + \frac{b}{x^3}}} \frac{2\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{9ab^{\frac{1}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} 3^{\frac{3}{4}}$$

command

`integrate(1/(a+b/x^3)^(3/2)/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(bx^2\sqrt{\frac{ax^3+b}{x^3}} + (ax^3+b)\sqrt{b}\operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right)}{3(a^2bx^3 + ab^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^4\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6 + 2abx^3 + b^2}, x\right)$$

16.179 Problem number 2048

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^5} dx$$

Optimal antiderivative

$$\frac{2}{3bx\sqrt{a + \frac{b}{x^3}}} \frac{4\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{9b^{\frac{4}{3}}\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} 3^{\frac{3}{4}}$$

command

`integrate(1/(a+b/x^3)^(3/2)/x^5,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(bx^2\sqrt{\frac{ax^3 + b}{x^3}} - 2(ax^3 + b)\sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right)}{3(ab^2x^3 + b^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x\sqrt{\frac{ax^3 + b}{x^3}}}{a^2x^6 + 2abx^3 + b^2}, x\right)$$

16.180 Problem number 2049

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^8} dx$$

Optimal antiderivative

$$\frac{2}{3bx^4\sqrt{a+\frac{b}{x^3}}} - \frac{16\sqrt{a+\frac{b}{x^3}}}{15b^2x}$$

$$+ \frac{32a\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1-\sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{45b^{\frac{7}{3}}\sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} 3^{\frac{3}{4}}$$

command

```
integrate(1/(a+b/x^3)^(3/2)/x^8,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(16(a^2x^4 + abx)\sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right) - (8abx^3 + 3b^2)\sqrt{\frac{ax^3 + b}{x^3}}\right)}{15(ab^3x^4 + b^4x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{a^2x^8 + 2abx^5 + b^2x^2}, x\right)$$

16.181 Problem number 2053

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^3} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2}{3ax^2\sqrt{a+\frac{b}{x^3}}} + \frac{2\sqrt{a+\frac{b}{x^3}}}{3ab^{\frac{2}{3}}\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
 & + \frac{2\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1-\sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2} 3^{\frac{3}{4}} \\
 & + \frac{9a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticE}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1-\sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} 3^{\frac{1}{4}} \\
 & - \frac{3a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}
 \end{aligned}$$

command

```
integrate(1/(a+b/x^3)^(3/2)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(bx\sqrt{\frac{ax^3+b}{x^3}} + (ax^3+b)\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right)\right)}{3(a^2bx^3 + ab^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^3\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6 + 2abx^3 + b^2}, x\right)$$

16.182 Problem number 2054

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^6} dx$$

Optimal antiderivative

$$\frac{\frac{2}{3bx^2\sqrt{a+\frac{b}{x^3}}} - \frac{8\sqrt{a+\frac{b}{x^3}}}{3b^{\frac{5}{3}}\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)}}{8a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1-\sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} 3^{\frac{3}{4}}}}{+ \frac{9b^{\frac{5}{3}}\sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}{4a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \operatorname{EllipticE}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1-\sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} 3^{\frac{1}{4}}}}{+ \frac{3b^{\frac{5}{3}}\sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}$$

command

```
integrate(1/(a+b/x^3)^(3/2)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(bx\sqrt{\frac{ax^3+b}{x^3}} + 4(ax^3+b)\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right)\right)}{3(ab^2x^3 + b^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6 + 2abx^3 + b^2}, x\right)$$

16.183 Problem number 2055

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^9} dx$$

Optimal antiderivative

$$\frac{2}{3b x^5 \sqrt{a + \frac{b}{x^3}}} - \frac{20 \sqrt{a + \frac{b}{x^3}}}{21b^2 x^2} + \frac{80a \sqrt{a + \frac{b}{x^3}}}{21b^{\frac{8}{3}} \left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})\right)}$$

$$+ \frac{80a^{\frac{4}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \text{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}{3^{\frac{3}{4}}}$$

$$+ \frac{63b^{\frac{8}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}{40a^{\frac{4}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \text{EllipticE}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}{3^{\frac{1}{4}}}$$

$$+ \frac{21b^{\frac{8}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}$$

command

```
integrate(1/(a+b/x^3)^(3/2)/x^9,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(40 (a^2 x^5 + abx^2) \sqrt{b} \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right) + (10 abx^3 + 3b^2) \sqrt{\frac{ax^3 + b}{x^3}} \right)}{21 (ab^3 x^5 + b^4 x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{a^2 x^9 + 2 abx^6 + b^2 x^3}, x\right)$$

16.184 Problem number 2056

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^{12}} dx$$

Optimal antiderivative

$$\frac{2}{3b x^8 \sqrt{a + \frac{b}{x^3}}} - \frac{32 \sqrt{a + \frac{b}{x^3}}}{39b^2 x^5} + \frac{320a \sqrt{a + \frac{b}{x^3}}}{273b^3 x^2} - \frac{1280a^2 \sqrt{a + \frac{b}{x^3}}}{273b^{\frac{11}{3}} \left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})\right)}$$

$$- \frac{1280a^{\frac{7}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \text{EllipticF}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}}{819b^{\frac{11}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}$$

$$+ \frac{640a^{\frac{7}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right) \text{EllipticE}\left(\frac{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 - \sqrt{3})}{\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + \frac{b^{\frac{2}{3}}}{x^2} - \frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{x}}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}} 3^{\frac{1}{4}}}{273b^{\frac{11}{3}} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + \frac{b^{\frac{1}{3}}}{x}\right)}{\left(\frac{b^{\frac{1}{3}}}{x} + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}$$

command

```
integrate(1/(a+b/x^3)^(3/2)/x^12,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(640 (a^3 x^8 + a^2 b x^5) \sqrt{b} \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right) + (160 a^2 b x^6 + 48 a b^2 x^3 - 21 b^3) \right)}{273 (ab^4 x^8 + b^5 x^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3 + b}{x^3}}}{a^2 x^{12} + 2 ab x^9 + b^2 x^6}, x\right)$$

16.185 Problem number 2062

$$\int \sqrt{a + \frac{b}{x^4}} x^2 dx$$

Optimal antiderivative

$$\frac{x^3 \sqrt{a + \frac{b}{x^4}}}{3} + \frac{b^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}} x}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}} x}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}}{3 \cos\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}} x}{b^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} \sqrt{a + \frac{b}{x^4}}}$$

command

`integrate(x^2*(a+b/x^4)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} x^3 \sqrt{\frac{ax^4 + b}{x^4}} + \frac{2}{3} \sqrt{a} \left(-\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x^2 \sqrt{\frac{ax^4 + b}{x^4}}, x\right)$$

16.186 Problem number 2064

$$\int \frac{\sqrt{a + \frac{b}{x^4}}}{x^2} dx$$

Optimal antiderivative

$$\frac{\sqrt{a + \frac{b}{x^4}}}{3x} - \frac{a^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}}{3 \cos\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right) b^{\frac{1}{4}} \sqrt{a + \frac{b}{x^4}}}$$

command

`integrate((a+b/x^4)^(1/2)/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}, -1\right) + \sqrt{\frac{ax^4 + b}{x^4}}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{ax^4 + b}{x^4}}}{x^2}, x\right)$$

16.187 Problem number 2072

$$\int \frac{\left(a + \frac{b}{x^4}\right)^{3/2}}{x^2} dx$$

Optimal antiderivative

$$\frac{\left(a + \frac{b}{x^4}\right)^{\frac{3}{2}}}{7x} - \frac{2a\sqrt{a + \frac{b}{x^4}}}{7x} - \frac{2a^{\frac{7}{4}} \sqrt{\frac{\cos\left(4 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}}{7 \cos\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right) b^{\frac{1}{4}} \sqrt{a + \frac{b}{x^4}}}$$

command

```
integrate((a+b/x^4)^(3/2)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4a\sqrt{b}x^5\left(-\frac{a}{b}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}},-1\right)+(3ax^4+b)\sqrt{\frac{ax^4+b}{x^4}}}{7x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(ax^4+b)\sqrt{\frac{ax^4+b}{x^4}}}{x^6},x\right)$$

16.188 Problem number 2080

$$\int \frac{\left(a + \frac{b}{x^4}\right)^{5/2}}{x^2} dx$$

Optimal antiderivative

$$\frac{\frac{10a\left(a + \frac{b}{x^4}\right)^{\frac{3}{2}}}{77x} - \frac{\left(a + \frac{b}{x^4}\right)^{\frac{5}{2}}}{11x} - \frac{20a^2\sqrt{a + \frac{b}{x^4}}}{77x}}{20a^{\frac{11}{4}}\sqrt{\frac{\cos\left(4\operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(2\operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)\sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}}$$

$$77\cos\left(2\operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right)b^{\frac{1}{4}}\sqrt{a + \frac{b}{x^4}}$$

command

```
integrate((a+b/x^4)^(5/2)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{40a^2\sqrt{b}x^9\left(-\frac{a}{b}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}},-1\right)+(37a^2x^8+24abx^4+7b^2)\sqrt{\frac{ax^4+b}{x^4}}}{77x^9}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(a^2x^8+2abx^4+b^2)\sqrt{\frac{ax^4+b}{x^4}}}{x^{10}},x\right)$$

16.189 Problem number 2086

$$\int \frac{x^2}{\sqrt{a + \frac{b}{x^4}}} dx$$

Optimal antiderivative

$$\frac{x^3 \sqrt{a + \frac{b}{x^4}}}{3a} + \frac{b^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}}{6 \cos\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right) a^{\frac{5}{4}} \sqrt{a + \frac{b}{x^4}}}$$

command

```
integrate(x^2/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{x^3 \sqrt{\frac{ax^4 + b}{x^4}} - \sqrt{a} \left(-\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}, -1\right)}{3a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^6 \sqrt{\frac{ax^4 + b}{x^4}}}{ax^4 + b}, x\right)$$

16.190 Problem number 2088

$$\int \frac{1}{\sqrt{a + \frac{b}{x^4}} x^2} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}}{2 \cos\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{a + \frac{b}{x^4}}}$$

command

`integrate(1/x^2/(a+b/x^4)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{b} \left(-\frac{a}{b}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x \left(-\frac{a}{b}\right)^{\frac{1}{4}}, -1\right)}{a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^2 \sqrt{\frac{ax^4 + b}{x^4}}}{ax^4 + b}, x\right)$$

16.191 Problem number 2094

$$\int \frac{x^2}{\left(a + \frac{b}{x^4}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{-\frac{x^3}{2a \sqrt{a + \frac{b}{x^4}}} + \frac{5x^3 \sqrt{a + \frac{b}{x^4}}}{6a^2}}{5b^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}}} + \frac{12 \cos\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right) a^{\frac{9}{4}} \sqrt{a + \frac{b}{x^4}}}{}$$

command

`integrate(x^2/(a+b/x^4)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(ax^4 + b)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}, -1\right) - (2ax^7 + 5bx^3)\sqrt{\frac{ax^4 + b}{x^4}}}{6(a^3x^4 + a^2b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^{10}\sqrt{\frac{ax^4 + b}{x^4}}}{a^2x^8 + 2abx^4 + b^2}, x\right)$$

16.192 Problem number 2096

$$\int \frac{1}{\left(a + \frac{b}{x^4}\right)^{3/2} x^2} dx$$

Optimal antiderivative

$$\frac{1}{2ax\sqrt{a + \frac{b}{x^4}} \sqrt{\frac{\cos\left(4 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}}{4 \cos\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right) a^{\frac{5}{4}} b^{\frac{1}{4}} \sqrt{a + \frac{b}{x^4}}}$$

command

`integrate(1/(a+b/x^4)^(3/2)/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{ax^3\sqrt{\frac{ax^4 + b}{x^4}} + (ax^4 + b)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}, -1\right)}{2(a^3x^4 + a^2b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^6\sqrt{\frac{ax^4 + b}{x^4}}}{a^2x^8 + 2abx^4 + b^2}, x\right)$$

16.193 Problem number 2102

$$\int \frac{x^2}{\left(a + \frac{b}{x^4}\right)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x^3}{6a\left(a + \frac{b}{x^4}\right)^{\frac{3}{2}}} - \frac{3x^3}{4a^2\sqrt{a + \frac{b}{x^4}}} + \frac{5x^3\sqrt{a + \frac{b}{x^4}}}{4a^3} \\ & + \frac{5b^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)\sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}}{8\cos\left(2\operatorname{arccot}\left(\frac{a^{\frac{1}{4}}x}{b^{\frac{1}{4}}}\right)\right)a^{\frac{13}{4}}\sqrt{a + \frac{b}{x^4}}} \end{aligned}$$

command

`integrate(x^2/(a+b/x^4)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15\left(a^2x^8 + 2abx^4 + b^2\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}, -1\right) - \left(4a^2x^{11} + 21abx^7 + 15b^2x^3\right)\sqrt{\frac{ax^4 + b}{x^4}}}{12\left(a^5x^8 + 2a^4bx^4 + a^3b^2\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^{14}\sqrt{\frac{ax^4 + b}{x^4}}}{a^3x^{12} + 3a^2bx^8 + 3ab^2x^4 + b^3}, x\right)$$

16.194 Problem number 2104

$$\int \frac{1}{\left(a + \frac{b}{x^4}\right)^{5/2}x^2} dx$$

Optimal antiderivative

$$\frac{1}{6a \left(a + \frac{b}{x^4}\right)^{\frac{3}{2}} x} - \frac{5}{12a^2 x \sqrt{a + \frac{b}{x^4}}} + \frac{5 \sqrt{\frac{\cos\left(4 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}} x}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}} x}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}}{24 \cos\left(2 \operatorname{arccot}\left(\frac{a^{\frac{1}{4}} x}{b^{\frac{1}{4}}}\right)\right) a^{\frac{9}{4}} b^{\frac{1}{4}} \sqrt{a + \frac{b}{x^4}}}$$

command

```
integrate(1/(a+b/x^4)^(5/2)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(a^2 x^8 + 2abx^4 + b^2) \sqrt{b} \left(-\frac{a}{b}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x \left(-\frac{a}{b}\right)^{\frac{1}{4}}, -1\right) + (7a^2 x^7 + 5abx^3) \sqrt{\frac{ax^4 + b}{x^4}}}{12(a^5 x^8 + 2a^4 bx^4 + a^3 b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^{10} \sqrt{\frac{ax^4 + b}{x^4}}}{a^3 x^{12} + 3a^2 bx^8 + 3ab^2 x^4 + b^3}, x\right)$$

16.195 Problem number 2106

$$\int \frac{1}{a + \frac{b}{x^5}} dx$$

Optimal antiderivative

$$\frac{x}{a} - \frac{b^{\frac{1}{5}} \ln\left(b^{\frac{1}{5}} + a^{\frac{1}{5}}x\right)}{5a^{\frac{6}{5}}} + \frac{b^{\frac{1}{5}} \ln\left(b^{\frac{2}{5}} + a^{\frac{2}{5}}x^2 - \frac{a^{\frac{1}{5}}b^{\frac{1}{5}}x(-\sqrt{5}+1)}{2}\right)}{20a^{\frac{6}{5}}}(-\sqrt{5}+1)$$

$$+ \frac{b^{\frac{1}{5}} \ln\left(b^{\frac{2}{5}} + a^{\frac{2}{5}}x^2 - \frac{a^{\frac{1}{5}}b^{\frac{1}{5}}x(\sqrt{5}+1)}{2}\right)}{20a^{\frac{6}{5}}}(\sqrt{5}+1)$$

$$- \frac{b^{\frac{1}{5}} \arctan\left(\frac{a^{\frac{1}{5}}x\sqrt{50+10\sqrt{5}}}{5b^{\frac{1}{5}}} - \frac{\sqrt{25+10\sqrt{5}}}{5}\right)\sqrt{10-2\sqrt{5}}}{10a^{\frac{6}{5}}}$$

$$+ \frac{b^{\frac{1}{5}} \arctan\left(\frac{\sqrt{25-10\sqrt{5}}}{5} + \frac{2a^{\frac{1}{5}}x\sqrt{2}}{\sqrt{5+\sqrt{5}}b^{\frac{1}{5}}}\right)\sqrt{10+2\sqrt{5}}}{10a^{\frac{6}{5}}}$$

command

```
integrate(1/(a+b/x^5),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

16.196 Problem number 2947

$$\int x^3 \sqrt{a + b(cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x^4 \sqrt{a + b(cx^2)^{3/2}}}{11} + \frac{6a\sqrt{cx^2} \sqrt{a + b(cx^2)^{3/2}}}{55b^2 c^2}$$

$$+ 4 \cdot 3^{\frac{3}{4}} a^2 \operatorname{EllipticF}\left(\frac{a^{\frac{1}{3}}(1-\sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}}{a^{\frac{1}{3}}(1+\sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}}, i\sqrt{3} + 2i\right) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\sqrt{cx^2}\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + b^{\frac{2}{3}}cx^2 - a^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{cx^2}}{\left(a^{\frac{1}{3}}(1+\sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}\right)^2}}$$

$$- \frac{55b^{\frac{4}{3}}c^2 \sqrt{a + b(cx^2)^{3/2}}}{\sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}\sqrt{cx^2})}{\left(a^{\frac{1}{3}}(1+\sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}\right)^2}}}$$

command

```
integrate(x^3*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 \sqrt{\frac{\sqrt{cx^2} bc}{x}} a^2 \text{weierstrassPInverse} \left(0, -\frac{4\sqrt{cx^2} a}{bc^2 x}, x \right) - (5b^2c^3x^4 + 3\sqrt{cx^2} abc) \sqrt{\sqrt{cx^2} bcx^2 + a} \right)}{55b^2c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{\sqrt{cx^2} bcx^2 + a} x^3, x \right)$$

16.197 Problem number 2948

$$\int \sqrt{a + b(cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x \sqrt{a + b(cx^2)^{3/2}}}{5}$$

$$+ \frac{2 \cdot 3^{3/4} a x \text{EllipticF} \left(\frac{a^{1/3} (1 - \sqrt{3}) + b^{1/3} \sqrt{cx^2}}{a^{1/3} (1 + \sqrt{3}) + b^{1/3} \sqrt{cx^2}}, i\sqrt{3} + 2i \right) (a^{1/3} + b^{1/3} \sqrt{cx^2}) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} cx^2 - a^{1/3} b^{1/3} \sqrt{cx^2}}{(a^{1/3} (1 + \sqrt{3}) + b^{1/3} \sqrt{cx^2})^2}}}{5b^{1/3} \sqrt{cx^2} \sqrt{a + b(cx^2)^{3/2}} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} \sqrt{cx^2})}{(a^{1/3} (1 + \sqrt{3}) + b^{1/3} \sqrt{cx^2})^2}}}$$

command

```
integrate((a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{\sqrt{cx^2} bcx^2 + a} bc^2 x^2 + 3\sqrt{cx^2} \sqrt{\frac{\sqrt{cx^2} bc}{x}} a \text{weierstrassPInverse} \left(0, -\frac{4\sqrt{cx^2} a}{bc^2 x}, x \right) \right)}{5bc^2 x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{\sqrt{cx^2} bcx^2 + a}, x \right)$$

16.198 Problem number 2949

$$\int \frac{\sqrt{a + b(cx^2)^{3/2}}}{x^3} dx$$

Optimal antiderivative

$$\frac{\sqrt{a + b(cx^2)^{3/2}}}{2x^2} + \frac{3^{3/4} b^{2/3} c \operatorname{EllipticF}\left(\frac{a^{1/3}(1-\sqrt{3}) + b^{1/3}\sqrt{cx^2}}{a^{1/3}(1+\sqrt{3}) + b^{1/3}\sqrt{cx^2}}, i\sqrt{3} + 2i\right) \left(a^{1/3} + b^{1/3}\sqrt{cx^2}\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{2/3} + b^{2/3}cx^2 - a^{1/3}b^{1/3}\sqrt{cx^2}}{\left(a^{1/3}(1+\sqrt{3}) + b^{1/3}\sqrt{cx^2}\right)^2}}}{2\sqrt{a + b(cx^2)^{3/2}} \sqrt{\frac{a^{1/3}\left(a^{1/3} + b^{1/3}\sqrt{cx^2}\right)}{\left(a^{1/3}(1+\sqrt{3}) + b^{1/3}\sqrt{cx^2}\right)^2}}}$$

command

```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \sqrt{\frac{\sqrt{cx^2} bc}{x}} x^2 \operatorname{weierstrassPInverse}\left(0, -\frac{4\sqrt{cx^2} a}{bc^2 x}, x\right) - \sqrt{\sqrt{cx^2} bcx^2 + a}}{2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sqrt{cx^2} bcx^2 + a}}{x^3}, x\right)$$

16.199 Problem number 2950

$$\int \frac{\sqrt{a + b(cx^2)^{3/2}}}{x^6} dx$$

Optimal antiderivative

$$\frac{\sqrt{a + b(cx^2)^{\frac{3}{2}}}}{5x^5} - \frac{3b(cx^2)^{\frac{5}{2}} \sqrt{a + b(cx^2)^{\frac{3}{2}}}}{20acx^7} + 3^{\frac{3}{4}} b^{\frac{5}{3}} (cx^2)^{\frac{5}{2}} \operatorname{EllipticF} \left(\frac{a^{\frac{1}{3}}(1-\sqrt{3}) + b^{\frac{1}{3}} \sqrt{cx^2}}{a^{\frac{1}{3}}(1+\sqrt{3}) + b^{\frac{1}{3}} \sqrt{cx^2}}, i\sqrt{3} + 2i \right) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} \sqrt{cx^2} \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} + b^{\frac{2}{3}} cx^2 -}{(a^{\frac{1}{3}}(1+\sqrt{3}))^2}}$$

$$20ax^5 \sqrt{a + b(cx^2)^{\frac{3}{2}}} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}} \sqrt{cx^2})}{(a^{\frac{1}{3}}(1+\sqrt{3}) + b^{\frac{1}{3}} \sqrt{cx^2})^2}}$$

command

```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \sqrt{cx^2} \sqrt{\frac{\sqrt{cx^2} bc}{x}} b c x^4 \operatorname{weierstrassPInverse} \left(0, -\frac{4 \sqrt{cx^2} a}{bc^2 x}, x \right) + (3 \sqrt{cx^2} b c x^2 + 4 a) \sqrt{\sqrt{cx^2} b c x^2 + a}}{20 a x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{\sqrt{cx^2} b c x^2 + a}}{x^6}, x \right)$$

16.200 Problem number 2951

$$\int x^4 \sqrt{a + b(cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x^5 \sqrt{a + b(cx^2)^{\frac{3}{2}}}}{13} + \frac{6acx^7 \sqrt{a + b(cx^2)^{\frac{3}{2}}}}{91b(cx^2)^{\frac{5}{2}}} - \frac{24a^2x^5 \sqrt{a + b(cx^2)^{\frac{3}{2}}}}{91b^{\frac{5}{3}}(cx^2)^{\frac{5}{2}} \left(a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2} \right)}$$

$$+ \frac{8 \cdot 3^{\frac{3}{4}} a^{\frac{7}{3}} x^5 \operatorname{EllipticF} \left(\frac{a^{\frac{1}{3}}(1 - \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}}{a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}}, i\sqrt{3} + 2i \right) \sqrt{2} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\sqrt{cx^2} \right) \sqrt{\frac{a^{\frac{2}{3}} + b^{\frac{2}{3}}cx^2 - a^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{cx^2}}{\left(a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2} \right)^2}}}{91b^{\frac{5}{3}}(cx^2)^{\frac{5}{2}} \sqrt{a + b(cx^2)^{\frac{3}{2}}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\sqrt{cx^2} \right)}}{\left(a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2} \right)^2}}$$

$$+ \frac{12 \cdot 3^{\frac{1}{4}} a^{\frac{7}{3}} x^5 \operatorname{EllipticE} \left(\frac{a^{\frac{1}{3}}(1 - \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}}{a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}}, i\sqrt{3} + 2i \right) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\sqrt{cx^2} \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} + b^{\frac{2}{3}}cx^2 - a^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{cx^2}}{\left(a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2} \right)^2}}}{91b^{\frac{5}{3}}(cx^2)^{\frac{5}{2}} \sqrt{a + b(cx^2)^{\frac{3}{2}}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\sqrt{cx^2} \right)}}{\left(a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2} \right)^2}}$$

command

```
integrate(x^4*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 \sqrt{\frac{\sqrt{cx^2} bc}{x}} a^2 \operatorname{weierstrassZeta} \left(0, -\frac{4\sqrt{cx^2} a}{bc^2 x}, \operatorname{weierstrassPInverse} \left(0, -\frac{4\sqrt{cx^2} a}{bc^2 x}, x \right) \right) + (7b^2c^3x^5 + 3\sqrt{cx^2}) \right)}{91b^2c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{\sqrt{cx^2} bcx^2 + a} x^4, x \right)$$

16.201 Problem number 2952

$$\int x \sqrt{a + b(cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x^2 \sqrt{a + b(cx^2)^{\frac{3}{2}}}}{7} + \frac{6a \sqrt{a + b(cx^2)^{\frac{3}{2}}}}{7b^{\frac{2}{3}}c \left(a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2} \right)}$$

$$+ \frac{2 \cdot 3^{\frac{3}{4}} a^{\frac{4}{3}} \operatorname{EllipticF} \left(\frac{a^{\frac{1}{3}}(1 - \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}}{a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}}, i\sqrt{3} + 2i \right) \sqrt{2} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\sqrt{cx^2} \right) \sqrt{\frac{a^{\frac{2}{3}} + b^{\frac{2}{3}}cx^2 - a^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{cx^2}}{\left(a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2} \right)^2}}}{7b^{\frac{2}{3}}c \sqrt{a + b(cx^2)^{\frac{3}{2}}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\sqrt{cx^2} \right)}{\left(a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2} \right)^2}}$$

$$+ \frac{3 \cdot 3^{\frac{1}{4}} a^{\frac{4}{3}} \operatorname{EllipticE} \left(\frac{a^{\frac{1}{3}}(1 - \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}}{a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2}}, i\sqrt{3} + 2i \right) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\sqrt{cx^2} \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} + b^{\frac{2}{3}}cx^2 - a^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{cx^2}}{\left(a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2} \right)^2}}}{7b^{\frac{2}{3}}c \sqrt{a + b(cx^2)^{\frac{3}{2}}} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\sqrt{cx^2} \right)}{\left(a^{\frac{1}{3}}(1 + \sqrt{3}) + b^{\frac{1}{3}}\sqrt{cx^2} \right)^2}}$$

command

```
integrate(x*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{\sqrt{cx^2} bcx^2 + a} bc^2 x^3 - 3 \sqrt{cx^2} \sqrt{\frac{\sqrt{cx^2} bc}{x}} \operatorname{awierstrassZeta} \left(0, -\frac{4\sqrt{cx^2} a}{bc^2 x}, \operatorname{weierstrassPInverse} \left(0, -\frac{4\sqrt{cx^2}}{bc^2 x} \right) \right)}{7bc^2 x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{\sqrt{cx^2} bcx^2 + a} x, x \right)$$

16.202 Problem number 2953

$$\int \frac{\sqrt{a + b(cx^2)^{3/2}}}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{\sqrt{a+b(cx^2)^{\frac{3}{2}}}}{x} + \frac{3b^{\frac{1}{3}}\sqrt{cx^2}\sqrt{a+b(cx^2)^{\frac{3}{2}}}}{x\left(a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}\right)} \\
 & + \frac{3^{\frac{3}{4}}a^{\frac{1}{3}}b^{\frac{1}{3}}\operatorname{EllipticF}\left(\frac{a^{\frac{1}{3}}(1-\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}}{a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{cx^2}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\sqrt{cx^2}\right)\sqrt{\frac{a^{\frac{2}{3}}+b^{\frac{2}{3}}cx^2-a^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{cx^2}}{\left(a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}\right)^2}}}{x\sqrt{a+b(cx^2)^{\frac{3}{2}}}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\sqrt{cx^2}\right)}{\left(a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}\right)^2}}} \\
 & + \frac{3^{\frac{3}{4}}a^{\frac{1}{3}}b^{\frac{1}{3}}\operatorname{EllipticE}\left(\frac{a^{\frac{1}{3}}(1-\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}}{a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}}, i\sqrt{3}+2i\right)\sqrt{cx^2}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\sqrt{cx^2}\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}+b^{\frac{2}{3}}cx^2}{\left(a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}\right)^2}}}{2x\sqrt{a+b(cx^2)^{\frac{3}{2}}}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\sqrt{cx^2}\right)}{\left(a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}\right)^2}}}
 \end{aligned}$$

command

`integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{\frac{\sqrt{cx^2}bc}{x}}x\operatorname{weierstrassZeta}\left(0, -\frac{4\sqrt{cx^2}a}{bc^2x}, \operatorname{weierstrassPInverse}\left(0, -\frac{4\sqrt{cx^2}a}{bc^2x}, x\right)\right) + \sqrt{\sqrt{cx^2}bcx^2+a}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sqrt{cx^2}bcx^2+a}}{x^2}, x\right)$$

16.203 Problem number 2954

$$\int \frac{\sqrt{a+b(cx^2)^{3/2}}}{x^5} dx$$

Optimal antiderivative

$$\frac{\sqrt{a+b(cx^2)^{\frac{3}{2}}}}{4x^4} - \frac{3bc^2\sqrt{a+b(cx^2)^{\frac{3}{2}}}}{8a\sqrt{cx^2}} + \frac{3b^{\frac{4}{3}}c^2\sqrt{a+b(cx^2)^{\frac{3}{2}}}}{8a\left(a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}\right)}$$

$$+ \frac{3^{\frac{3}{4}}b^{\frac{4}{3}}c^2 \operatorname{EllipticF}\left(\frac{a^{\frac{1}{3}}(1-\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}}{a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}}, i\sqrt{3}+2i\right)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\sqrt{cx^2}\right)\sqrt{\frac{a^{\frac{2}{3}}+b^{\frac{2}{3}}cx^2-a^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{cx^2}}{\left(a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}\right)^2}}\sqrt{2}}{8a^{\frac{2}{3}}\sqrt{a+b(cx^2)^{\frac{3}{2}}}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\sqrt{cx^2}\right)}{\left(a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}\right)^2}}}$$

$$- \frac{3^{\frac{3}{4}}b^{\frac{4}{3}}c^2 \operatorname{EllipticE}\left(\frac{a^{\frac{1}{3}}(1-\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}}{a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}}, i\sqrt{3}+2i\right)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\sqrt{cx^2}\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}+b^{\frac{2}{3}}cx^2-a^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{cx^2}}{\left(a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}\right)^2}}}{16a^{\frac{2}{3}}\sqrt{a+b(cx^2)^{\frac{3}{2}}}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\sqrt{cx^2}\right)}{\left(a^{\frac{1}{3}}(1+\sqrt{3})+b^{\frac{1}{3}}\sqrt{cx^2}\right)^2}}}$$

command

```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{cx^2}\sqrt{\frac{\sqrt{cx^2}bc}{x}}bcx^3\operatorname{weierstrassZeta}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},\operatorname{weierstrassPInverse}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},x\right)\right)+\left(3\sqrt{cx^2}bcx^2\right)}{8ax^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sqrt{cx^2}bcx^2+a}}{x^5},x\right)$$

17 Test file number 26

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.3_General/26_1.1.3.3-a+b_x^n-
~p-c+d_x^n-~q

17.1 Problem number 26

$$\int \frac{1}{(a + bx^3)^2 (c + dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d(ad + bc)x}{3ac(-ad + bc)^2(dx^3 + c)} + \frac{bx}{3a(-ad + bc)(bx^3 + a)(dx^3 + c)} \\ & + \frac{2b^{\frac{5}{3}}(-4ad + bc)\ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{9a^{\frac{5}{3}}(-ad + bc)^3} + \frac{2d^{\frac{5}{3}}(-ad + 4bc)\ln\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right)}{9c^{\frac{5}{3}}(-ad + bc)^3} \\ & - \frac{b^{\frac{5}{3}}(-4ad + bc)\ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{9a^{\frac{5}{3}}(-ad + bc)^3} - \frac{d^{\frac{5}{3}}(-ad + 4bc)\ln\left(c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2\right)}{9c^{\frac{5}{3}}(-ad + bc)^3} \\ & - \frac{2b^{\frac{5}{3}}(-4ad + bc)\arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{9a^{\frac{5}{3}}(-ad + bc)^3} \\ & - \frac{2d^{\frac{5}{3}}(-ad + 4bc)\arctan\left(\frac{\left(c^{\frac{1}{3}} - 2d^{\frac{1}{3}}x\right)\sqrt{3}}{3c^{\frac{1}{3}}}\right)\sqrt{3}}{9c^{\frac{5}{3}}(-ad + bc)^3} \end{aligned}$$

command

```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3(b^3c^2d - a^2bd^3)x^4 + 2\sqrt{3}\left((b^3c^2d - 4ab^2cd^2)x^6 + ab^2c^3 - 4a^2bc^2d + (b^3c^3 - 3ab^2c^2d - 4a^2bcd^2)x^3\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \arctan$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

18 Test file number 27

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.3_General/27_1.1.3.4-e_x-
 $\hat{m}-a+b_x\hat{n}-\hat{p}-c+d_x\hat{n}-\hat{q}$

18.1 Problem number 185

$$\int x^3 \sqrt{a + bx^3} (A + Bx^3) dx$$

Optimal antiderivative

$$\frac{2Bx^4(bx^3 + a)^{\frac{3}{2}}}{17b} + \frac{6a(17Ab - 8Ba)x\sqrt{bx^3 + a}}{935b^2} + \frac{2(17Ab - 8Ba)x^4\sqrt{bx^3 + a}}{187b}$$

$$43^{\frac{3}{4}}a^2(17Ab - 8Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

$$935b^{\frac{7}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

command

```
integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6\left(8Ba^3 - 17Aa^2b\right)\sqrt{b} \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + \left(55Bb^3x^7 + 5\left(3Bab^2 + 17Ab^3\right)x^4 - 3\left(8Ba^2b - 17Aa^2b\right)\right)}{935b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bx^6 + Ax^3\right)\sqrt{bx^3 + a}, x\right)$$

18.2 Problem number 186

$$\int \sqrt{a + bx^3} (A + Bx^3) dx$$

Optimal antiderivative

$$\frac{2Bx(bx^3 + a)^{\frac{3}{2}}}{11b} + \frac{2(11Ab - 2Ba)x\sqrt{bx^3 + a}}{55b}$$

$$+ \frac{2 \cdot 3^{\frac{3}{4}} a (11Ab - 2Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}{55b^{\frac{4}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \left(2 Ba^2 - 11 Aab \right) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - \left(5 Bb^2 x^4 + \left(3 Bab + 11 Ab^2 \right) x \right) \sqrt{bx^3 + a} \right)}{55 b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bx^3 + A\right) \sqrt{bx^3 + a}, x\right)$$

18.3 Problem number 187

$$\int \frac{\sqrt{a + bx^3} (A + Bx^3)}{x^3} dx$$

Optimal antiderivative

$$-\frac{A(bx^3 + a)^{\frac{3}{2}}}{2ax^2} + \frac{(5Ab + 4Ba)x\sqrt{bx^3 + a}}{10a}$$

$$+ \frac{3^{\frac{3}{4}} (5Ab + 4Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}{10b^{\frac{1}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(4Ba + 5Ab)\sqrt{b}x^2 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (4Bbx^3 - 5Ab)\sqrt{bx^3 + a}}{10bx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^3}, x\right)$$

18.4 Problem number 188

$$\int \frac{\sqrt{a + bx^3}(A + Bx^3)}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A(bx^3 + a)^{\frac{3}{2}}}{5ax^5} + \frac{(Ab - 10Ba)\sqrt{bx^3 + a}}{20ax^2} \\ & \frac{3^{\frac{3}{4}}b^{\frac{2}{3}}(Ab - 10Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)}}}{20a\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(10Ba - Ab)\sqrt{b}x^5 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - ((10Ba + 3Ab)x^3 + 4Aa)\sqrt{bx^3 + a}}{20ax^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^6}, x\right)$$

18.5 Problem number 189

$$\int \frac{\sqrt{a + bx^3} (A + Bx^3)}{x^9} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A(bx^3 + a)^{\frac{3}{2}}}{8ax^8} + \frac{(7Ab - 16Ba)\sqrt{bx^3 + a}}{80ax^5} + \frac{3b(7Ab - 16Ba)\sqrt{bx^3 + a}}{320a^2x^2} \\ & + \frac{3^{\frac{3}{4}}b^{\frac{5}{3}}(7Ab - 16Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{320a^2\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(16Bab - 7Ab^2)\sqrt{b}x^8 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (3(16Bab - 7Ab^2)x^6 + 4(16Ba^2 + 3Aab)x^3 + 40Aa^2)}{320a^2x^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^9}, x\right)$$

18.6 Problem number 190

$$\int x^4 \sqrt{a + bx^3} (A + Bx^3) dx$$

Optimal antiderivative

$$\frac{2Bx^5(bx^3+a)^{\frac{3}{2}}}{19b} + \frac{6a(19Ab-10Ba)x^2\sqrt{bx^3+a}}{1729b^2}$$

$$+ \frac{2(19Ab-10Ba)x^5\sqrt{bx^3+a}}{247b} - \frac{24a^2(19Ab-10Ba)\sqrt{bx^3+a}}{1729b^{\frac{8}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)}$$

$$- \frac{8 \cdot 3^{\frac{3}{4}} a^{\frac{7}{3}} (19Ab-10Ba) \left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{1729b^{\frac{8}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

$$+ \frac{12 \cdot 3^{\frac{1}{4}} a^{\frac{7}{3}} (19Ab-10Ba) \left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{1729b^{\frac{8}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

```
integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 (10 Ba^3 - 19 Aa^2b) \sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - (91 Bb^3x^8 + 7 (3 Bab^2 - \dots) \right)}{1729b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bx^7 + Ax^4\right)\sqrt{bx^3+a}, x\right)$$

18.7 Problem number 191

$$\int x \sqrt{a + bx^3} (A + Bx^3) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Bx^2(bx^3+a)^{\frac{3}{2}}}{13b} + \frac{2(13Ab-4Ba)x^2\sqrt{bx^3+a}}{91b} + \frac{6a(13Ab-4Ba)\sqrt{bx^3+a}}{91b^{\frac{5}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & + \frac{2 \cdot 3^{\frac{3}{4}} a^{\frac{4}{3}} (13Ab-4Ba) \left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{91b^{\frac{5}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\ & + \frac{3 \cdot 3^{\frac{1}{4}} a^{\frac{4}{3}} (13Ab-4Ba) \left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{91b^{\frac{5}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \left(4Ba^2 - 13Aab \right) \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) + \left(7Bb^2x^5 + \left(3Bab + 13Ab^2 \right) \right)}{91b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left((Bx^4 + Ax) \sqrt{bx^3 + a}, x \right)$$

18.8 Problem number 192

$$\int \frac{\sqrt{a+bx^3} (A+Bx^3)}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{A(bx^3+a)^{\frac{3}{2}}}{ax} + \frac{(7Ab+2Ba)x^2\sqrt{bx^3+a}}{7a} + \frac{3(7Ab+2Ba)\sqrt{bx^3+a}}{7b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
 & + \frac{3^{\frac{3}{4}}a^{\frac{1}{3}}(7Ab+2Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{7b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
 & + \frac{3\cdot 3^{\frac{1}{4}}a^{\frac{1}{3}}(7Ab+2Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{14b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
 \end{aligned}$$

command

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(2Ba+7Ab)\sqrt{b}\text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - (2Bbx^3-7Ab)\sqrt{bx^3+a}}{7bx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}}{x^2}, x\right)$$

18.9 Problem number 193

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{A(bx^3+a)^{\frac{3}{2}}}{4ax^4} - \frac{(Ab+8Ba)\sqrt{bx^3+a}}{8ax} + \frac{3b^{\frac{1}{3}}(Ab+8Ba)\sqrt{bx^3+a}}{8a\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
 & + \frac{3^{\frac{3}{4}}b^{\frac{1}{3}}(Ab+8Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}\sqrt{2}}{8a^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
 & + \frac{3\cdot 3^{\frac{1}{4}}b^{\frac{1}{3}}(Ab+8Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{16a^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
 \end{aligned}$$

command

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(8Ba+Ab)\sqrt{b}x^4\text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + ((8Ba+3Ab)x^3+2Aa)\sqrt{bx^3+a}}{8ax^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}}{x^5}, x\right)$$

18.10 Problem number 194

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{A(bx^3+a)^{\frac{3}{2}}}{7ax^7} + \frac{(5Ab-14Ba)\sqrt{bx^3+a}}{56a^4} \\
 & + \frac{3b(5Ab-14Ba)\sqrt{bx^3+a}}{112a^2x} - \frac{3b^{\frac{4}{3}}(5Ab-14Ba)\sqrt{bx^3+a}}{112a^2\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
 & - \frac{3^{\frac{3}{4}}b^{\frac{4}{3}}(5Ab-14Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}\sqrt{2}}{112a^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
 & + \frac{3^{\frac{3}{4}}b^{\frac{4}{3}}(5Ab-14Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{224a^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
 \end{aligned}$$

command

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(14Bab-5Ab^2)\sqrt{b}x^7\text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + (3(14Bab-5Ab^2)x^6 + 2}{112a^2x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}}{x^8}, x\right)$$

18.11 Problem number 195

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{A(bx^3+a)^{\frac{3}{2}}}{10ax^{10}} + \frac{(11Ab-20Ba)\sqrt{bx^3+a}}{140ax^7} + \frac{3b(11Ab-20Ba)\sqrt{bx^3+a}}{1120a^2x^4} \\
 & -\frac{3b^2(11Ab-20Ba)\sqrt{bx^3+a}}{448a^3x} + \frac{3b^{\frac{7}{3}}(11Ab-20Ba)\sqrt{bx^3+a}}{448a^3\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
 & + \frac{3^{\frac{3}{4}}b^{\frac{7}{3}}(11Ab-20Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}\sqrt{2}}{448a^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
 & + \frac{3^{\frac{3}{4}}b^{\frac{7}{3}}(11Ab-20Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{896a^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
 \end{aligned}$$

command

```
integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15(20Bab^2-11Ab^3)\sqrt{b}x^{10}\text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + (15(20Bab^2-11Ab^3)x^9)}{2240a^3x^{10}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{11}}, x\right)$$

18.12 Problem number 202

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3) dx$$

Optimal antiderivative

$$\frac{2(23Ab - 8Ba)x^4(bx^3 + a)^{\frac{3}{2}}}{391b} + \frac{2Bx^4(bx^3 + a)^{\frac{5}{2}}}{23b} + \frac{54a^2(23Ab - 8Ba)x\sqrt{bx^3 + a}}{21505b^2} + \frac{18a(23Ab - 8Ba)x^4\sqrt{bx^3 + a}}{4301b}$$

$$+ \frac{36 \cdot 3^{\frac{3}{4}} a^3 (23Ab - 8Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{21505b^{\frac{7}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(54(8Ba^4 - 23Aa^3b)\sqrt{b}\text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (935Bb^4x^{10} + 55(26Bab^3 + 23Ab^4)x^7 + 5(27Ba^2b^3 - 23Aa^3b)\sqrt{b}\right)}{21505b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bbx^9 + (Ba + Ab)x^6 + Aax^3\right)\sqrt{bx^3 + a}, x\right)$$

18.13 Problem number 203

$$\int (a + bx^3)^{3/2} (A + Bx^3) dx$$

Optimal antiderivative

$$\frac{2(17Ab - 2Ba)x(bx^3 + a)^{\frac{3}{2}}}{187b} + \frac{2Bx(bx^3 + a)^{\frac{5}{2}}}{17b} + \frac{18a(17Ab - 2Ba)x\sqrt{bx^3 + a}}{935b}$$

$$+ \frac{18 \cdot 3^{\frac{3}{4}} a^2 (17Ab - 2Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{935b^{\frac{4}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(27 (2 B a^3 - 17 A a^2 b) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - (55 B b^3 x^7 + 5 (20 B a b^2 + 17 A b^3) x^4 + (27 B a^2 b + \dots \right)}{935 b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B b x^6 + (B a + A b) x^3 + A a) \sqrt{b x^3 + a}}{x^3}, x\right)$$

18.14 Problem number 204

$$\int \frac{(a + b x^3)^{3/2} (A + B x^3)}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(11 A b + 4 B a) x (b x^3 + a)^{\frac{3}{2}}}{22 a} - \frac{A (b x^3 + a)^{\frac{5}{2}}}{2 a x^2} + \frac{9 (11 A b + 4 B a) x \sqrt{b x^3 + a}}{110} \\ & + \frac{9 \cdot 3^{\frac{3}{4}} a (11 A b + 4 B a) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i \sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}}{110 b^{\frac{1}{3}} \sqrt{b x^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{27 (4 B a^2 + 11 A a b) \sqrt{b} x^2 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (20 B b^2 x^6 + 4 (14 B a b + 11 A b^2) x^3 - 55 A a b) \sqrt{b x^3 + a}}{110 b x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B b x^6 + (B a + A b) x^3 + A a) \sqrt{b x^3 + a}}{x^3}, x\right)$$

18.15 Problem number 205

$$\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(Ab + 2Ba)(bx^3 + a)^{\frac{3}{2}}}{4ax^2} - \frac{A(bx^3 + a)^{\frac{5}{2}}}{5a^2x^5} + \frac{9b(Ab + 2Ba)x\sqrt{bx^3 + a}}{20a} \\ & + \frac{9 \cdot 3^{\frac{3}{4}} b^{\frac{2}{3}} (Ab + 2Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{20\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{27(2Ba + Ab)\sqrt{b}x^5 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (8Bbx^6 - (10Ba + 13Ab)x^3 - 4Aa)\sqrt{bx^3 + a}}{20x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bbx^6 + (Ba + Ab)x^3 + Aa)\sqrt{bx^3 + a}}{x^6}, x\right)$$

18.16 Problem number 206

$$\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x^9} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(Ab - 16Ba)(bx^3 + a)^{\frac{3}{2}}}{80a^2x^5} - \frac{A(bx^3 + a)^{\frac{5}{2}}}{8a^2x^8} + \frac{9b(Ab - 16Ba)\sqrt{bx^3 + a}}{320a^2x^2} \\ & + \frac{9 \cdot 3^{\frac{3}{4}} b^{\frac{5}{3}} (Ab - 16Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{320a\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{27 (16 Bab - Ab^2) \sqrt{b} x^8 \text{weierstrassPInverse}(0, -\frac{4a}{b}, x) - ((208 Bab + 27 Ab^2)x^6 + 4(16 Ba^2 + 19 Aab)x^3 + 40 A)}{320 ax^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bbx^6 + (Ba + Ab)x^3 + Aa)\sqrt{bx^3 + a}}{x^9}, x\right)$$

18.17 Problem number 207

$$\int x^4(a + bx^3)^{3/2} (A + Bx^3) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5Ab - 2Ba)x^5(bx^3 + a)^{\frac{3}{2}}}{95b} + \frac{2Bx^5(bx^3 + a)^{\frac{5}{2}}}{25b} + \frac{54a^2(5Ab - 2Ba)x^2\sqrt{bx^3 + a}}{8645b^2} \\ & + \frac{18a(5Ab - 2Ba)x^5\sqrt{bx^3 + a}}{1235b} - \frac{216a^3(5Ab - 2Ba)\sqrt{bx^3 + a}}{8645b^{\frac{8}{3}}(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & \frac{723^{\frac{3}{4}}a^{\frac{10}{3}}(5Ab - 2Ba)(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \text{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{8645b^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & \frac{1083^{\frac{1}{4}}a^{\frac{10}{3}}(5Ab - 2Ba)(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \text{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{8645b^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(540 (2 B a^4 - 5 A a^3 b) \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) - (1729 B b^4 x^{11} + 91 (28 B a^4 - 5 A a^3 b) x^9) \right)}{43225 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left((B b x^{10} + (B a + A b) x^7 + A a x^4) \sqrt{b x^3 + a}, x \right)$$

18.18 Problem number 208

$$\int x (a + b x^3)^{3/2} (A + B x^3) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(19Ab - 4Ba) x^2 (b x^3 + a)^{\frac{3}{2}}}{247b} + \frac{2B x^2 (b x^3 + a)^{\frac{5}{2}}}{19b} \\ & + \frac{18a(19Ab - 4Ba) x^2 \sqrt{b x^3 + a}}{1729b} + \frac{54a^2(19Ab - 4Ba) \sqrt{b x^3 + a}}{1729b^{\frac{5}{3}} \left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\ & + \frac{18 \cdot 3^{\frac{3}{4}} a^{\frac{7}{3}} (19Ab - 4Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{1729b^{\frac{5}{3}} \sqrt{b x^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \\ & + \frac{27 \cdot 3^{\frac{1}{4}} a^{\frac{7}{3}} (19Ab - 4Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticE} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{1729b^{\frac{5}{3}} \sqrt{b x^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

`integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(27 (4 B a^3 - 19 A a^2 b) \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) + (91 B b^3 x^8 + 7 (22 B a b^2 - 19 A a^2 b) x^6) \right)}{1729 b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left((B b x^7 + (B a + A b) x^4 + A a x) \sqrt{b x^3 + a}, x \right)$$

18.19 Problem number 209

$$\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(13Ab + 2Ba) x^2 (bx^3 + a)^{\frac{3}{2}}}{13a} - \frac{A(bx^3 + a)^{\frac{5}{2}}}{ax} \\ & + \frac{9(13Ab + 2Ba) x^2 \sqrt{bx^3 + a}}{91} + \frac{27a(13Ab + 2Ba) \sqrt{bx^3 + a}}{91b^{\frac{2}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\ & + \frac{93^{\frac{3}{4}} a^{\frac{4}{3}} (13Ab + 2Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{91b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \\ & + \frac{273^{\frac{1}{4}} a^{\frac{4}{3}} (13Ab + 2Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \operatorname{EllipticE} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{182b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{27(2Ba^2 + 13Aab)\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - (14Bb^2x^6 + 2(16Bab + 13Aa^2))\sqrt{bx^3 + a}}{91bx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bbx^6 + (Ba + Ab)x^3 + Aa)\sqrt{bx^3 + a}}{x^2}, x\right)$$

18.20 Problem number 210

$$\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(7Ab + 8Ba)(bx^3 + a)^{\frac{3}{2}}}{8ax} - \frac{A(bx^3 + a)^{\frac{5}{2}}}{4ax^4} \\ & + \frac{9b(7Ab + 8Ba)x^2\sqrt{bx^3 + a}}{56a} + \frac{27b^{\frac{1}{3}}(7Ab + 8Ba)\sqrt{bx^3 + a}}{56\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{93^{\frac{3}{4}}a^{\frac{1}{3}}b^{\frac{1}{3}}(7Ab + 8Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \sqrt{2}}{56\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{273^{\frac{1}{4}}a^{\frac{1}{3}}b^{\frac{1}{3}}(7Ab + 8Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{112\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{27(8Ba + 7Ab)\sqrt{b}x^4 \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - (16Bbx^6 - 7(8Ba + 11Ab)x^5)}{56x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bbx^6 + (Ba + Ab)x^3 + Aa)\sqrt{bx^3 + a}}{x^5}, x\right)$$

18.21 Problem number 211

$$\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x^8} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(Ab + 14Ba)(bx^3 + a)^{\frac{3}{2}}}{56a^4} - \frac{A(bx^3 + a)^{\frac{5}{2}}}{7ax^7} \\ & - \frac{9b(Ab + 14Ba)\sqrt{bx^3 + a}}{112ax} + \frac{27b^{\frac{4}{3}}(Ab + 14Ba)\sqrt{bx^3 + a}}{112a\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{9 \cdot 3^{\frac{3}{4}} b^{\frac{4}{3}} (Ab + 14Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \sqrt{2}}{112a^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & - \frac{27 \cdot 3^{\frac{1}{4}} b^{\frac{4}{3}} (Ab + 14Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{224a^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{27(14Bab + Ab^2)\sqrt{b}x^7 \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + ((154Bab + 27Ab^2)x^6 + 2(14Bab + Ab^2)\sqrt{b}x^3 + 2Aa)}{112ax^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bbx^6 + (Ba + Ab)x^3 + Aa)\sqrt{bx^3 + a}}{x^8}, x\right)$$

18.22 Problem number 212

$$\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x^{11}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(Ab - 4Ba)(bx^3 + a)^{\frac{3}{2}}}{28a x^7} - \frac{A(bx^3 + a)^{\frac{5}{2}}}{10a x^{10}} + \frac{9b(Ab - 4Ba)\sqrt{bx^3 + a}}{224a x^4} \\ & + \frac{27b^2(Ab - 4Ba)\sqrt{bx^3 + a}}{448a^2 x} - \frac{27b^{\frac{7}{3}}(Ab - 4Ba)\sqrt{bx^3 + a}}{448a^2 \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{9 \cdot 3^{\frac{3}{4}} b^{\frac{7}{3}} (Ab - 4Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \sqrt{2}}{448a^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{27 \cdot 3^{\frac{1}{4}} b^{\frac{7}{3}} (Ab - 4Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{896a^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{135(4Bab^2 - Ab^3)\sqrt{b}x^{10}\operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + (135(4Bab^2 - Ab^3)x^9 + 2240a^2x^{10}}{2240a^2x^{10}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bbx^6 + (Ba + Ab)x^3 + Aa)\sqrt{bx^3 + a}}{x^{11}}, x\right)$$

18.23 Problem number 219

$$\int \frac{x^3(A + Bx^3)}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2(11Ab - 8Ba)x\sqrt{bx^3 + a}}{55b^2} + \frac{2Bx^4\sqrt{bx^3 + a}}{11b}$$

$$+ \frac{4a(11Ab - 8Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{165b^{\frac{7}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(8Ba^2 - 11Aab)\sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (5Bb^2x^4 - (8Bab - 11Ab^2)x)\sqrt{bx^3 + a}\right)}{55b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bx^6 + Ax^3}{\sqrt{bx^3 + a}}, x\right)$$

18.24 Problem number 220

$$\int \frac{A + Bx^3}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2Bx\sqrt{bx^3 + a}}{5b}$$

$$+ \frac{2(5Ab - 2Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{15b^{\frac{4}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{bx^3 + a} Bbx - (2Ba - 5Ab)\sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) \right)}{5b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bx^3 + A}{\sqrt{bx^3 + a}}, x\right)$$

18.25 Problem number 221

$$\int \frac{A + Bx^3}{x^3 \sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{\frac{A\sqrt{bx^3 + a}}{2ax^2} (Ab - 4Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{6ab^{\frac{1}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(4Ba - Ab)\sqrt{b}x^2 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - \sqrt{bx^3 + a} Ab}{2abx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{bx^6 + ax^3}, x\right)$$

18.26 Problem number 222

$$\int \frac{A + Bx^3}{x^6 \sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A\sqrt{bx^3+a}}{5ax^5} + \frac{(7Ab-10Ba)\sqrt{bx^3+a}}{20a^2x^2} \\ & + \frac{b^{\frac{2}{3}}(7Ab-10Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{60a^2\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(10Ba-7Ab)\sqrt{b}x^5 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + ((10Ba-7Ab)x^3 + 4Aa)\sqrt{bx^3+a}}{20a^2x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}}{bx^9+ax^6}, x\right)$$

18.27 Problem number 223

$$\int \frac{x^4(A + Bx^3)}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2(13Ab - 10Ba)x^2\sqrt{bx^3 + a}}{91b^2} + \frac{2Bx^5\sqrt{bx^3 + a}}{13b} - \frac{8a(13Ab - 10Ba)\sqrt{bx^3 + a}}{91b^{\frac{8}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)}$$

$$+ \frac{8a^{\frac{4}{3}}(13Ab - 10Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{3^{\frac{3}{4}}}$$

$$+ \frac{273b^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{43^{\frac{1}{4}}a^{\frac{4}{3}}(13Ab - 10Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

$$+ \frac{91b^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

command

```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4(10Ba^2 - 13Aab)\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - (7Bb^2x^5 - (10Bab - 13Aa^2))\sqrt{bx^3 + a}\right)}{91b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bx^7 + Ax^4}{\sqrt{bx^3 + a}}, x\right)$$

18.28 Problem number 224

$$\int \frac{x(A + Bx^3)}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2Bx^2\sqrt{bx^3+a}}{7b} + \frac{2(7Ab-4Ba)\sqrt{bx^3+a}}{7b^{\frac{5}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)}$$

$$+ \frac{2a^{\frac{1}{3}}(7Ab-4Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{3}{4}}}$$

$$+ \frac{21b^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{1}{4}}a^{\frac{1}{3}}(7Ab-4Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

$$- \frac{7b^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}{}$$

command

```
integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{bx^3+a}Bbx^2+(4Ba-7Ab)\sqrt{b}\operatorname{weierstrassZeta}\left(0,-\frac{4a}{b},\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{7b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bx^4+Ax}{\sqrt{bx^3+a}},x\right)$$

18.29 Problem number 225

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{A\sqrt{bx^3+a}}{ax} + \frac{(Ab+2Ba)\sqrt{bx^3+a}}{ab^{\frac{2}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
 & + \frac{(Ab+2Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
 & + \frac{3^{\frac{1}{4}}(Ab+2Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{2a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
 \end{aligned}$$

command

`integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2Ba+Ab)\sqrt{b}x\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\sqrt{bx^3+a}Ab}{abx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}}{bx^5+ax^2},x\right)$$

18.30 Problem number 226

$$\int \frac{A+Bx^3}{x^5\sqrt{a+Bx^3}}dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{A\sqrt{bx^3+a}}{4ax^4} + \frac{(5Ab-8Ba)\sqrt{bx^3+a}}{8a^2x} - \frac{b^{\frac{1}{3}}(5Ab-8Ba)\sqrt{bx^3+a}}{8a^2\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
 & + \frac{b^{\frac{1}{3}}(5Ab-8Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\
 & + \frac{24a^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{1}{4}}b^{\frac{1}{3}}(5Ab-8Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
 & + \frac{16a^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{8a^2x^4}
 \end{aligned}$$

command

```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(8Ba-5Ab)\sqrt{b}x^4\text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + ((8Ba-5Ab)x^3 + 2Aa)\sqrt{bx^3+a}}{8a^2x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}}{bx^8+ax^5}, x\right)$$

18.31 Problem number 227

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{A\sqrt{bx^3+a}}{7ax^7} + \frac{(11Ab-14Ba)\sqrt{bx^3+a}}{56a^2x^4} \\
 & -\frac{5b(11Ab-14Ba)\sqrt{bx^3+a}}{112a^3x} + \frac{5b^{\frac{4}{3}}(11Ab-14Ba)\sqrt{bx^3+a}}{112a^3\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
 & + \frac{5b^{\frac{4}{3}}(11Ab-14Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}3^{\frac{3}{4}}\sqrt{2}}{336a^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
 & - \frac{5\cdot 3^{\frac{1}{4}}b^{\frac{4}{3}}(11Ab-14Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{224a^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
 \end{aligned}$$

command

```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(14Bab-11Ab^2)\sqrt{b}x^7\operatorname{weierstrassZeta}\left(0,-\frac{4a}{b},\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(5(14Bab-11Ab^2)x^6-2}{112a^3x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}}{bx^{11}+ax^8},x\right)$$

18.32 Problem number 234

$$\int \frac{x^6(A+Bx^3)}{(a+Bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(11Ab - 14Ba)x^4}{33b^2\sqrt{bx^3 + a}} + \frac{2Bx^7}{11b\sqrt{bx^3 + a}} + \frac{16(11Ab - 14Ba)x\sqrt{bx^3 + a}}{165b^3}$$

$$+ \frac{32a(11Ab - 14Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{495b^{\frac{10}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

`integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(16(14Ba^3 - 11Aa^2b + (14Ba^2b - 11Aab^2)x^3)\sqrt{b}\operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (15Bb^3x^7 - 3(14Bab^2 - 11Aa^2b)x^4)\sqrt{b}\right)}{165(b^5x^3 + ab^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^9 + Ax^6)\sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

18.33 Problem number 235

$$\int \frac{x^3(A + Bx^3)}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(5Ab - 8Ba)x}{15b^2\sqrt{bx^3 + a}} + \frac{2Bx^4}{5b\sqrt{bx^3 + a}}$$

$$+ \frac{4(5Ab - 8Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{45b^{\frac{7}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \left((8 Bab - 5 Ab^2)x^3 + 8 Ba^2 - 5 Aab \right) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - \left(3 Bb^2x^4 + (8 Bab - 5 Ab^2)x \right) \sqrt{bx^3 + a} \right)}{15 (b^4x^3 + ab^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^6 + Ax^3)\sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

18.34 Problem number 236

$$\int \frac{A + Bx^3}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba)x}{3ab\sqrt{bx^3 + a}}$$

$$+ \frac{2(Ab + 2Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{9ab^{\frac{4}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{bx^3 + a} (Bab - Ab^2)x - \left((2 Bab + Ab^2)x^3 + 2 Ba^2 + Aab \right) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) \right)}{3 (ab^3x^3 + a^2b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

18.35 Problem number 237

$$\int \frac{A + Bx^3}{x^3 (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A}{2a x^2 \sqrt{bx^3 + a}} - \frac{(7Ab - 4Ba)x}{6a^2 \sqrt{bx^3 + a}} \\ & (7Ab - 4Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \text{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}} \\ & - \frac{18a^2 b^{\frac{1}{3}} \sqrt{bx^3 + a}}{\sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

`integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left((4Bab - 7Ab^2)x^5 + (4Ba^2 - 7Aab)x^2 \right) \sqrt{b} \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + \left((4Bab - 7Ab^2)x^3 - 3Aab \right) \sqrt{bx^3 + a}}{6(a^2b^2x^5 + a^3bx^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bx^3 + A) \sqrt{bx^3 + a}}{b^2x^9 + 2abx^6 + a^2x^3}, x \right)$$

18.36 Problem number 238

$$\int \frac{A + Bx^3}{x^6 (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A}{5a x^5 \sqrt{bx^3 + a}} + \frac{-13Ab + 10Ba}{15a^2 x^2 \sqrt{bx^3 + a}} + \frac{7(13Ab - 10Ba) \sqrt{bx^3 + a}}{60a^3 x^2} \\ & 7b^{\frac{2}{3}}(13Ab - 10Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \text{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}} \\ & + \frac{180a^3 \sqrt{bx^3 + a}}{\sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{7 \left((10 Bab - 13 Ab^2)x^8 + (10 Ba^2 - 13 Aab)x^5 \right) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (7 (10 Bab - 13 Ab^2)x^6 + \dots}{60 (a^3bx^8 + a^4x^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{b^2x^{12} + 2abx^9 + a^2x^6}, x\right)$$

18.37 Problem number 239

$$\int \frac{x^4(A + Bx^3)}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(7Ab - 10Ba)x^2}{21b^2\sqrt{bx^3 + a}} + \frac{2Bx^5}{7b\sqrt{bx^3 + a}} + \frac{8(7Ab - 10Ba)\sqrt{bx^3 + a}}{21b^{\frac{8}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{8a^{\frac{1}{3}}(7Ab - 10Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}}{63b^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{4a^{\frac{1}{3}}(7Ab - 10Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{21b^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 \left((10 Bab - 7 Ab^2)x^3 + 10 Ba^2 - 7 Aab \right) \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) + (3 Bb^2) \right)}{21 (b^4 x^3 + ab^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Bx^7 + Ax^4) \sqrt{bx^3 + a}}{b^2 x^6 + 2 abx^3 + a^2}, x \right)$$

18.38 Problem number 240

$$\int \frac{x(A + Bx^3)}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba)x^2}{3ab\sqrt{bx^3 + a}} - \frac{2(Ab - 4Ba)\sqrt{bx^3 + a}}{3ab^{\frac{5}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)}$$

$$+ \frac{2(Ab - 4Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}} 3^{\frac{3}{4}}}{9a^{\frac{2}{3}}b^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}}$$

$$+ \frac{(Ab - 4Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \operatorname{EllipticE} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}} 3^{\frac{3}{4}}}{3a^{\frac{2}{3}}b^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}}$$

command

`integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{bx^3 + a} (Bab - Ab^2)x^2 + \left((4 Bab - Ab^2)x^3 + 4 Ba^2 - Aab \right) \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) \right)}{3 (ab^3 x^3 + a^2 b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Bx^4 + Ax) \sqrt{bx^3 + a}}{b^2 x^6 + 2 abx^3 + a^2}, x \right)$$

18.39 Problem number 241

$$\int \frac{A + Bx^3}{x^2 (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A}{ax\sqrt{bx^3+a}} - \frac{(5Ab-2Ba)x^2}{3a^2\sqrt{bx^3+a}} + \frac{(5Ab-2Ba)\sqrt{bx^3+a}}{3a^2b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & (5Ab-2Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}3^{\frac{3}{4}} \\ & + \frac{9a^{\frac{5}{3}}b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{(5Ab-2Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\ & - \frac{6a^{\frac{5}{3}}b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3(a^2b^2x^4+a^3bx)} \end{aligned}$$

command

```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left((2 Bab - 5 Ab^2)x^4 + (2 Ba^2 - 5 Aab)x\right)\sqrt{b} \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + \left((2 Bab - 5 Ab^2)x^4 + (2 Ba^2 - 5 Aab)x\right)\sqrt{b}}{3(a^2b^2x^4 + a^3bx)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{b^2x^8 + 2abx^5 + a^2x^2}, x\right)$$

18.40 Problem number 242

$$\int \frac{A + Bx^3}{x^5 (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A}{4a x^4 \sqrt{bx^3 + a}} + \frac{-11Ab + 8Ba}{12a^2 x \sqrt{bx^3 + a}} + \frac{5(11Ab - 8Ba) \sqrt{bx^3 + a}}{24a^3 x} - \frac{5b^{\frac{1}{3}}(11Ab - 8Ba) \sqrt{bx^3 + a}}{24a^3 \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{5b^{\frac{1}{3}}(11Ab - 8Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & + \frac{72a^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{5b^{\frac{1}{3}}(11Ab - 8Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & + \frac{48a^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \end{aligned}$$

command

```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \left((8 Bab - 11 Ab^2)x^7 + (8 Ba^2 - 11 Aab)x^4 \right) \sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + (5}{24(a^3bx^7 + a^4x^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{b^2x^{11} + 2abx^8 + a^2x^5}, x\right)$$

18.41 Problem number 243

$$\int \frac{A + Bx^3}{x^8 (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A}{7a x^7 \sqrt{bx^3 + a}} + \frac{-17Ab + 14Ba}{21a^2 x^4 \sqrt{bx^3 + a}} + \frac{11(17Ab - 14Ba) \sqrt{bx^3 + a}}{168a^3 x^4} \\ & - \frac{55b(17Ab - 14Ba) \sqrt{bx^3 + a}}{336a^4 x} + \frac{55b^{\frac{4}{3}}(17Ab - 14Ba) \sqrt{bx^3 + a}}{336a^4 \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{55b^{\frac{4}{3}}(17Ab - 14Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}} \sqrt{2}}{1008a^{\frac{11}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & - \frac{55b^{\frac{4}{3}}(17Ab - 14Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{672a^{\frac{11}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{55 \left((14 Bab^2 - 17 Ab^3) x^{10} + (14 Ba^2 b - 17 Aab^2) x^7 \right) \sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right)}{336 (a^4 b x^{10} + \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3 + A) \sqrt{bx^3 + a}}{b^2 x^{14} + 2 abx^{11} + a^2 x^8}, x\right)$$

18.42 Problem number 249

$$\int \frac{x^6 (A + Bx^3)}{(a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(5Ab - 14Ba)x^4}{45b^2(bx^3 + a)^{\frac{3}{2}}} + \frac{2Bx^7}{5b(bx^3 + a)^{\frac{3}{2}}} - \frac{16(5Ab - 14Ba)x}{135b^3\sqrt{bx^3 + a}} \\ & + \frac{32(5Ab - 14Ba)(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{405b^{\frac{10}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

`integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(16 \left((14 Bab^2 - 5 Ab^3)x^6 + 14 Ba^3 - 5 Aa^2b + 2 (14 Ba^2b - 5 Aab^2)x^3 \right) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - \right)}{135 (b^6x^6 + 2ab^5x^3 + a^2b^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^9 + Ax^6)\sqrt{bx^3 + a}}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

18.43 Problem number 250

$$\int \frac{x^3 (A + Bx^3)}{(a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab - Ba)x^4}{9ab(bx^3 + a)^{\frac{3}{2}}} - \frac{2(Ab + 8Ba)x}{27ab^2\sqrt{bx^3 + a}} \\ & + \frac{4(Ab + 8Ba)(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{81ab^{\frac{7}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((8 Bab^2 + Ab^3)x^6 + 8 Ba^3 + Aa^2b + 2 (8 Ba^2b + Aab^2)x^3 \right) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - ((11 Bab^2 - 27(ab^5x^6 + 2a^2b^4x^3 + a^3b^3))}{27(ab^5x^6 + 2a^2b^4x^3 + a^3b^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^6 + Ax^3)\sqrt{bx^3 + a}}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

18.44 Problem number 251

$$\int \frac{A + Bx^3}{(a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba)x}{9ab(bx^3 + a)^{\frac{3}{2}}} + \frac{2(7Ab + 2Ba)x}{27a^2b\sqrt{bx^3 + a}}$$

$$+ \frac{2(7Ab + 2Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{81a^2b^{\frac{4}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((2 Bab^2 + 7 Ab^3)x^6 + 2 Ba^3 + 7 Aa^2b + 2 (2 Ba^2b + 7 Aab^2)x^3 \right) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + ((2 Bab^2 - 27(a^2b^4x^6 + 2a^3b^3x^3 + a^4b^2))}{27(a^2b^4x^6 + 2a^3b^3x^3 + a^4b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

18.45 Problem number 252

$$\int \frac{A + Bx^3}{x^3 (a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A}{2a x^2 (b x^3 + a)^{\frac{3}{2}}} - \frac{(13Ab - 4Ba) x}{18a^2 (b x^3 + a)^{\frac{3}{2}}} - \frac{7(13Ab - 4Ba) x}{54a^3 \sqrt{b x^3 + a}} \\ & + \frac{7(13Ab - 4Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{162a^3 b^{\frac{1}{3}} \sqrt{b x^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

`integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{7 \left((4 Bab^2 - 13 Ab^3) x^8 + 2 (4 Ba^2 b - 13 Aab^2) x^5 + (4 Ba^3 - 13 Aa^2 b) x^2 \right) \sqrt{b} \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) + (7)}{54 (a^3 b^3 x^8 + 2 a^4 b^2 x^5 + a^5 b x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Bx^3 + A) \sqrt{bx^3 + a}}{b^3 x^{12} + 3 ab^2 x^9 + 3 a^2 b x^6 + a^3 x^3}, x \right)$$

18.46 Problem number 253

$$\int \frac{A + Bx^3}{x^6 (a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A}{5a x^5 (b x^3 + a)^{\frac{3}{2}}} + \frac{-19Ab + 10Ba}{45a^2 x^2 (b x^3 + a)^{\frac{3}{2}}} - \frac{13(19Ab - 10Ba)}{135a^3 x^2 \sqrt{b x^3 + a}} + \frac{91(19Ab - 10Ba) \sqrt{b x^3 + a}}{540a^4 x^2} \\ & + \frac{91b^{\frac{2}{3}} (19Ab - 10Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{1620a^4 \sqrt{b x^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

```
integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{91 \left((10 Bab^2 - 19 Ab^3)x^{11} + 2(10 Ba^2b - 19 Aab^2)x^8 + (10 Ba^3 - 19 Aa^2b)x^5 \right) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, 540(a^4b^2x^{11} + 2\right)}{540(a^4b^2x^{11} + 2\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{b^3x^{15} + 3ab^2x^{12} + 3a^2bx^9 + a^3x^6}, x\right)$$

18.47 Problem number 254

$$\int \frac{x^7(A + Bx^3)}{(a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(7Ab - 16Ba)x^5}{63b^2(bx^3 + a)^{\frac{3}{2}}} + \frac{2Bx^8}{7b(bx^3 + a)^{\frac{3}{2}}} - \frac{20(7Ab - 16Ba)x^2}{189b^3\sqrt{bx^3 + a}} + \frac{80(7Ab - 16Ba)\sqrt{bx^3 + a}}{189b^{\frac{11}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{80a^{\frac{1}{3}}(7Ab - 16Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}}{567b^{\frac{11}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{40a^{\frac{1}{3}}(7Ab - 16Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{189b^{\frac{11}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(40 \left((16 Bab^2 - 7 Ab^3)x^6 + 16 Ba^3 - 7 Aa^2b + 2 (16 Ba^2b - 7 Aab^2)x^3 \right) \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassP} \right) \right)}{189 (b^6 x^6 + 2 ab^5 x^3 - \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Bx^{10} + Ax^7) \sqrt{bx^3 + a}}{b^3 x^9 + 3 ab^2 x^6 + 3 a^2 b x^3 + a^3}, x \right)$$

18.48 Problem number 255

$$\int \frac{x^4 (A + Bx^3)}{(a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab - Ba)x^5}{9ab(bx^3 + a)^{\frac{3}{2}}} + \frac{2(Ab - 10Ba)x^2}{27ab^2\sqrt{bx^3 + a}} - \frac{8(Ab - 10Ba)\sqrt{bx^3 + a}}{27ab^{\frac{8}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & 8(Ab - 10Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}} \\ & \frac{81a^{\frac{2}{3}}b^{\frac{8}{3}}\sqrt{bx^3 + a}}{\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & 4(Ab - 10Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & + \frac{27a^{\frac{2}{3}}b^{\frac{8}{3}}\sqrt{bx^3 + a}}{\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 \left((10 Bab^2 - Ab^3)x^6 + 10 Ba^3 - Aa^2b + 2 (10 Ba^2b - Aab^2)x^3 \right) \sqrt{b} \operatorname{weierstrassZeta} \left(0, -\frac{4a}{b}, \operatorname{weierstrassP} \right) \right)}{27 (ab^5 x^6 + 2 a^2 b^4 x^3 + a^3 b^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Bx^7 + Ax^4) \sqrt{bx^3 + a}}{b^3 x^9 + 3 ab^2 x^6 + 3 a^2 b x^3 + a^3}, x \right)$$

18.49 Problem number 256

$$\int \frac{x(A + Bx^3)}{(a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba)x^2}{9ab(bx^3 + a)^{3/2}} + \frac{2(5Ab + 4Ba)x^2}{27a^2b\sqrt{bx^3 + a}} - \frac{2(5Ab + 4Ba)\sqrt{bx^3 + a}}{27a^2b^{5/3}\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)}$$

$$+ \frac{2(5Ab + 4Ba)\left(a^{1/3} + b^{1/3}x\right) \operatorname{EllipticF}\left(\frac{b^{1/3}x + a^{1/3}(1 - \sqrt{3})}{b^{1/3}x + a^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}} 3^{3/4}}{81a^{5/3}b^{5/3}\sqrt{bx^3 + a} \sqrt{\frac{a^{1/3}(a^{1/3} + b^{1/3}x)}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}}$$

$$+ \frac{(5Ab + 4Ba)\left(a^{1/3} + b^{1/3}x\right) \operatorname{EllipticE}\left(\frac{b^{1/3}x + a^{1/3}(1 - \sqrt{3})}{b^{1/3}x + a^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}} 3^{3/4}}{27a^{5/3}b^{5/3}\sqrt{bx^3 + a} \sqrt{\frac{a^{1/3}(a^{1/3} + b^{1/3}x)}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\left((4Bab^2 + 5Ab^3)x^6 + 4Ba^3 + 5Aa^2b + 2(4Ba^2b + 5Aab^2)x^3\right)\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(27(a^2b^4x^6 + 2a^3b^3x^3 + a^4b^2)\right)\right)\right)}{27(a^2b^4x^6 + 2a^3b^3x^3 + a^4b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^4 + Ax)\sqrt{bx^3 + a}}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

18.50 Problem number 257

$$\int \frac{A + Bx^3}{x^2 (a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{A}{ax(bx^3+a)^{\frac{3}{2}}} - \frac{(11Ab-2Ba)x^2}{9a^2(bx^3+a)^{\frac{3}{2}}} - \frac{5(11Ab-2Ba)x^2}{27a^3\sqrt{bx^3+a}} + \frac{5(11Ab-2Ba)\sqrt{bx^3+a}}{27a^3b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & + \frac{5(11Ab-2Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{81a^{\frac{8}{3}}b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\ & + \frac{5(11Ab-2Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{54a^{\frac{8}{3}}b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\left(\left(2Bab^2-11Ab^3\right)x^7+2\left(2Ba^2b-11Aab^2\right)x^4+\left(2Ba^3-11Aa^2b\right)x\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassP}\right)}{27\left(a^3b^3x^7+2a^4b^2x^4+a^5\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}}{b^3x^{11}+3ab^2x^8+3a^2bx^5+a^3x^2},x\right)$$

18.51 Problem number 258

$$\int \frac{A + Bx^3}{x^5 (a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A}{4a x^4 (bx^3 + a)^{\frac{3}{2}}} + \frac{-17Ab + 8Ba}{36a^2 x (bx^3 + a)^{\frac{3}{2}}} - \frac{11(17Ab - 8Ba)}{108a^3 x \sqrt{bx^3 + a}} \\ & + \frac{55(17Ab - 8Ba) \sqrt{bx^3 + a}}{216a^4 x} - \frac{55b^{\frac{1}{3}}(17Ab - 8Ba) \sqrt{bx^3 + a}}{216a^4 \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & 55b^{\frac{1}{3}}(17Ab - 8Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}\sqrt{2} \\ & - \frac{648a^{\frac{11}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{55b^{\frac{1}{3}}(17Ab - 8Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticE} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & + \frac{432a^{\frac{11}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \end{aligned}$$

command

```
integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{55 \left((8 Bab^2 - 17 Ab^3)x^{10} + 2(8 Ba^2b - 17 Aab^2)x^7 + (8 Ba^3 - 17 Aa^2b)x^4 \right) \sqrt{b} \text{weierstrassZeta} \left(0, -\frac{4a}{b} \right) \text{weierstrassZeta} \left(0, -\frac{4a}{b} \right)}{216 (a^4 b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bx^3 + A) \sqrt{bx^3 + a}}{b^3 x^{14} + 3 ab^2 x^{11} + 3 a^2 b x^8 + a^3 x^5}, x \right)$$

18.52 Problem number 264

$$\int \frac{x^4 \sqrt{c + dx^3}}{4c + dx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{22^{\frac{1}{3}} c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} - 2^{\frac{1}{3}} d^{\frac{1}{3}} x)}{\sqrt{dx^3 + c}}\right)}{d^{\frac{5}{3}}} + \frac{22^{\frac{1}{3}} c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{\sqrt{c}}\right)}{3d^{\frac{5}{3}}} \\ & - \frac{22^{\frac{1}{3}} c^{\frac{7}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + 2^{\frac{1}{3}} d^{\frac{1}{3}} x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{3d^{\frac{5}{3}}} + \frac{22^{\frac{1}{3}} c^{\frac{7}{6}} \operatorname{arctan}\left(\frac{\sqrt{dx^3 + c}\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{3d^{\frac{5}{3}}} \\ & + \frac{2x^2 \sqrt{dx^3 + c}}{7d} - \frac{50c \sqrt{dx^3 + c}}{7d^{\frac{5}{3}} (d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3}))} \\ & - \frac{50c^{\frac{4}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}} x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 - \sqrt{3})}{d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}} d^{\frac{1}{3}} x + d^{\frac{2}{3}} x^2}{(d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}} \\ & - \frac{21d^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}} x)}{(d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3}))^2}}}{25 \cdot 3^{\frac{1}{4}} c^{\frac{4}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}} x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 - \sqrt{3})}{d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}} d^{\frac{1}{3}} x + d^{\frac{2}{3}} x^2}{(d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3}))^2}}} \\ & + \frac{7d^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}} x)}{(d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3}))^2}}}{\end{aligned}$$

command

```
integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c} x^4}{dx^3 + 4c}, x\right)$$

18.53 Problem number 265

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}}-2^{\frac{1}{3}}d^{\frac{1}{3}}x)}{\sqrt{dx^3+c}}\right) 2^{\frac{1}{3}}}{2d^{\frac{2}{3}}} - \frac{c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right) 2^{\frac{1}{3}}}{6d^{\frac{2}{3}}} \\ & + \frac{c^{\frac{1}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}}+2^{\frac{1}{3}}d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3+c}}\right) 2^{\frac{1}{3}}\sqrt{3}}{6d^{\frac{2}{3}}} \\ & - \frac{c^{\frac{1}{6}} \operatorname{arctan}\left(\frac{\sqrt{dx^3+c}\sqrt{3}}{3\sqrt{c}}\right) 2^{\frac{1}{3}}\sqrt{3}}{6d^{\frac{2}{3}}} + \frac{2\sqrt{dx^3+c}}{d^{\frac{2}{3}}(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))} \\ & + \frac{2c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{3}{4}}} \\ & + \frac{3d^{\frac{2}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{1}{4}}c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \\ & - \frac{d^{\frac{2}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{d^{\frac{2}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3+c}x}{dx^3+4c}, x\right)$$

18.54 Problem number 266

$$\int \frac{\sqrt{c + dx^3}}{x^2(4c + dx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} - 2^{\frac{1}{3}}d^{\frac{1}{3}}x)}{\sqrt{dx^3 + c}}\right) 2^{\frac{1}{3}}}{8c^{\frac{5}{6}}} + \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{\sqrt{c}}\right) 2^{\frac{1}{3}}}{24c^{\frac{5}{6}}} \\ & - \frac{d^{\frac{1}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + 2^{\frac{1}{3}}d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) 2^{\frac{1}{3}}\sqrt{3}}{24c^{\frac{5}{6}}} + \frac{d^{\frac{1}{3}} \operatorname{arctan}\left(\frac{\sqrt{dx^3 + c}\sqrt{3}}{3\sqrt{c}}\right) 2^{\frac{1}{3}}\sqrt{3}}{24c^{\frac{5}{6}}} \\ & - \frac{\sqrt{dx^3 + c}}{4cx} + \frac{d^{\frac{1}{3}}\sqrt{dx^3 + c}}{4c(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{3}{4}}\sqrt{2}}{12c^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{3^{\frac{1}{4}}d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{8c^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c}}{dx^5 + 4cx^2}, x\right)$$

18.55 Problem number 267

$$\int \frac{x^3 \sqrt{c + dx^3}}{4c + dx^3} dx$$

Optimal antiderivative

$$\frac{x^4 F_1\left(\frac{4}{3}, -\frac{1}{2}, 1, \frac{7}{3}, -\frac{dx^3}{c}, -\frac{dx^3}{4c}\right) \sqrt{dx^3 + c}}{16c \sqrt{1 + \frac{dx^3}{c}}}$$

command

```
integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3 + c} x^3}{dx^3 + 4c}, x\right)$$

18.56 Problem number 268

$$\int \frac{\sqrt{c + dx^3}}{4c + dx^3} dx$$

Optimal antiderivative

$$\frac{x F_1\left(\frac{1}{3}, -\frac{1}{2}, 1, \frac{4}{3}, -\frac{dx^3}{c}, -\frac{dx^3}{4c}\right) \sqrt{dx^3 + c}}{4c \sqrt{1 + \frac{dx^3}{c}}}$$

command

```
integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}}{dx^3 + 4c}, x\right)$$

18.57 Problem number 269

$$\int \frac{\sqrt{c + dx^3}}{x^3(4c + dx^3)} dx$$

Optimal antiderivative

$$-\frac{F_1\left(-\frac{2}{3}, -\frac{1}{2}, 1, \frac{1}{3}, -\frac{dx^3}{c}, -\frac{dx^3}{4c}\right) \sqrt{dx^3 + c}}{8cx^2 \sqrt{1 + \frac{dx^3}{c}}}$$

command

```
integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}}{dx^6 + 4cx^3}, x\right)$$

18.58 Problem number 275

$$\int \frac{x^4}{\sqrt{c+dx^3} (4c+dx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{22^{\frac{1}{3}} c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}}-2^{\frac{1}{3}}d^{\frac{1}{3}}x)}{\sqrt{dx^3+c}}\right)}{3d^{\frac{5}{3}}} - \frac{22^{\frac{1}{3}} c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{9d^{\frac{5}{3}}} \\ & + \frac{22^{\frac{1}{3}} c^{\frac{1}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}}+2^{\frac{1}{3}}d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3+c}}\right)\sqrt{3}}{9d^{\frac{5}{3}}} \\ & - \frac{22^{\frac{1}{3}} c^{\frac{1}{6}} \operatorname{arctan}\left(\frac{\sqrt{dx^3+c}\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d^{\frac{5}{3}}} + \frac{2\sqrt{dx^3+c}}{d^{\frac{5}{3}}(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))} \\ & + \frac{2c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{3}{4}}} \\ & + \frac{3d^{\frac{5}{3}}\sqrt{dx^3+c}\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{1}{4}}c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \\ & - \frac{d^{\frac{5}{3}}\sqrt{dx^3+c}\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{\end{aligned}$$

command

```
integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3+c}x^4}{d^2x^6+5cdx^3+4c^2},x\right)$$

18.59 Problem number 277

$$\int \frac{1}{x^2 \sqrt{c + dx^3} (4c + dx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{1}{3}} \operatorname{arctanh} \left(\frac{c^{\frac{1}{6}} (c^{\frac{1}{3}} - 2^{\frac{1}{3}} d^{\frac{1}{3}} x)}{\sqrt{dx^3 + c}} \right) 2^{\frac{1}{3}}}{24c^{\frac{11}{6}}} - \frac{d^{\frac{1}{3}} \operatorname{arctanh} \left(\frac{\sqrt{dx^3 + c}}{\sqrt{c}} \right) 2^{\frac{1}{3}}}{72c^{\frac{11}{6}}} \\ & + \frac{d^{\frac{1}{3}} \operatorname{arctan} \left(\frac{c^{\frac{1}{6}} (c^{\frac{1}{3}} + 2^{\frac{1}{3}} d^{\frac{1}{3}} x) \sqrt{3}}{\sqrt{dx^3 + c}} \right) 2^{\frac{1}{3}} \sqrt{3}}{72c^{\frac{11}{6}}} - \frac{d^{\frac{1}{3}} \operatorname{arctan} \left(\frac{\sqrt{dx^3 + c} \sqrt{3}}{3\sqrt{c}} \right) 2^{\frac{1}{3}} \sqrt{3}}{72c^{\frac{11}{6}}} \\ & - \frac{\sqrt{dx^3 + c}}{4c^2 x} + \frac{d^{\frac{1}{3}} \sqrt{dx^3 + c}}{4c^2 (d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3}))} \\ & + \frac{d^{\frac{1}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}} x) \operatorname{EllipticF} \left(\frac{d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 - \sqrt{3})}{d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}} d^{\frac{1}{3}} x + d^{\frac{2}{3}} x^2}{(d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3}))^2}} 3^{\frac{3}{4}} \sqrt{2}}{12c^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}} x)}{(d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3}))^2}}} \\ & - \frac{3^{\frac{1}{4}} d^{\frac{1}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}} x) \operatorname{EllipticE} \left(\frac{d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 - \sqrt{3})}{d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}} d^{\frac{1}{3}} x + d^{\frac{2}{3}} x^2}{(d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3}))^2}}}{8c^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}} x)}{(d^{\frac{1}{3}} x + c^{\frac{1}{3}} (1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{dx^3 + c}}{d^2 x^8 + 5cdx^5 + 4c^2 x^2}, x \right)$$

18.60 Problem number 278

$$\int \frac{x^3}{\sqrt{c+dx^3} (4c+dx^3)} dx$$

Optimal antiderivative

$$\frac{x^4 F_1\left(\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{dx^3}{c}, -\frac{dx^3}{4c}\right) \sqrt{1 + \frac{dx^3}{c}}}{16c\sqrt{dx^3+c}}$$

command

`integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3+c} x^3}{d^2x^6+5cdx^3+4c^2}, x\right)$$

18.61 Problem number 279

$$\int \frac{1}{\sqrt{c+dx^3} (4c+dx^3)} dx$$

Optimal antiderivative

$$\frac{x F_1\left(\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{dx^3}{c}, -\frac{dx^3}{4c}\right) \sqrt{1 + \frac{dx^3}{c}}}{4c\sqrt{dx^3+c}}$$

command

`integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^2x^6+5cdx^3+4c^2}, x\right)$$

18.62 Problem number 280

$$\int \frac{1}{x^3 \sqrt{c + dx^3} (4c + dx^3)} dx$$

Optimal antiderivative

$$-\frac{F_1\left(-\frac{2}{3}, \frac{1}{2}, 1, \frac{1}{3}, -\frac{dx^3}{c}, -\frac{dx^3}{4c}\right) \sqrt{1 + \frac{dx^3}{c}}}{8cx^2 \sqrt{dx^3 + c}}$$

command

```
integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}}{d^2x^9 + 5cdx^6 + 4c^2x^3}, x\right)$$

18.63 Problem number 289

$$\int \frac{x^7 \sqrt{c + dx^3}}{8c - dx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32c^{\frac{13}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{d^{\frac{8}{3}}} - \frac{32c^{\frac{13}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{d^{\frac{8}{3}}} \\ & - \frac{32c^{\frac{13}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{d^{\frac{8}{3}}} - \frac{214cx^2 \sqrt{dx^3 + c}}{91d^2} \\ & - \frac{2x^5 \sqrt{dx^3 + c}}{13d} - \frac{12248c^2 \sqrt{dx^3 + c}}{91d^{\frac{8}{3}} (d^{\frac{1}{3}}x + c^{\frac{1}{3}} (1 + \sqrt{3}))} \\ & - \frac{12248c^{\frac{7}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}} \\ & + \frac{273d^{\frac{8}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{61243^{\frac{1}{4}}c^{\frac{7}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & + \frac{91d^{\frac{8}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} \end{aligned}$$

command

```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c} x^7}{dx^3 - 8c}, x\right)$$

18.64 Problem number 290

$$\int \frac{x^4 \sqrt{c + dx^3}}{8c - dx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{d^{\frac{5}{3}}} - \frac{4c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{d^{\frac{5}{3}}} \\ & - \frac{4c^{\frac{7}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{d^{\frac{5}{3}}} - \frac{2x^2 \sqrt{dx^3 + c}}{7d} - \frac{118c \sqrt{dx^3 + c}}{7d^{\frac{5}{3}} (d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & - \frac{118c^{\frac{4}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}} \\ & - \frac{21d^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{59 \cdot 3^{\frac{1}{4}} c^{\frac{4}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & + \frac{7d^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{\end{aligned}$$

command

```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c} x^4}{dx^3 - 8c}, x\right)$$

18.65 Problem number 291

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3} dx$$

Optimal antiderivative

$$\frac{c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3+c}}\right)}{2d^{\frac{2}{3}}} - \frac{c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{2d^{\frac{2}{3}}}$$

$$- \frac{c^{\frac{1}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3+c}}\right)\sqrt{3}}{2d^{\frac{2}{3}}} - \frac{2\sqrt{dx^3+c}}{d^{\frac{2}{3}}(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))}$$

$$- \frac{2c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{3}{4}}}$$

$$- \frac{3d^{\frac{2}{3}}\sqrt{dx^3+c}\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{3}{4}}c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}$$

$$+ \frac{d^{\frac{2}{3}}\sqrt{dx^3+c}\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{d^{\frac{2}{3}}\sqrt{dx^3+c}}$$

command

```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3+c}x}{dx^3-8c}, x\right)$$

18.66 Problem number 292

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3+c}}\right)}{16c^{\frac{5}{6}}} - \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{16c^{\frac{5}{6}}} \\ & - \frac{d^{\frac{1}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3+c}}\right)\sqrt{3}}{16c^{\frac{5}{6}}} - \frac{\sqrt{dx^3+c}}{8cx} + \frac{d^{\frac{1}{3}}\sqrt{dx^3+c}}{8c(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))} \\ & + \frac{d^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & + \frac{24c^{\frac{2}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{16c^{\frac{2}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \\ & + \frac{3^{\frac{1}{4}}d^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{16c^{\frac{2}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3+c}}{dx^5-8cx^2}, x\right)$$

18.67 Problem number 293

$$\int \frac{\sqrt{c + dx^3}}{x^5 (8c - dx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{128c^{\frac{11}{6}}} - \frac{d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{128c^{\frac{11}{6}}} - \frac{d^{\frac{4}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{128c^{\frac{11}{6}}} \\ & - \frac{\sqrt{dx^3 + c}}{32cx^4} - \frac{d\sqrt{dx^3 + c}}{16c^2x} + \frac{d^{\frac{4}{3}}\sqrt{dx^3 + c}}{16c^2(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{3}{4}}\sqrt{2}}{48c^{\frac{5}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & + \frac{3^{\frac{1}{4}}d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{32c^{\frac{5}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c}}{dx^8 - 8cx^5}, x\right)$$

18.68 Problem number 294

$$\int \frac{\sqrt{c + dx^3}}{x^8(8c - dx^3)} dx$$

Optimal antiderivative

$$\frac{d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right) - d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right) - d^{\frac{7}{3}} \arctan\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{1024c^{\frac{17}{6}} - \frac{\sqrt{dx^3 + c}}{56cx^7} - \frac{19d\sqrt{dx^3 + c}}{1792c^2x^4} + \frac{d^2\sqrt{dx^3 + c}}{112c^3x} - \frac{d^{\frac{7}{3}}\sqrt{dx^3 + c}}{112c^3(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))}}$$

$$- \frac{d^{\frac{7}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}}$$

$$+ \frac{336c^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{1}{4}}d^{\frac{7}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}$$

$$+ \frac{224c^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{}$$

command

```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c}}{dx^{11} - 8cx^8}, x\right)$$

18.69 Problem number 302

$$\int \frac{x^7 (c + dx^3)^{3/2}}{8c - dx^3} dx$$

Optimal antiderivative

$$\frac{288c^{\frac{19}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{d^{\frac{8}{3}}} - \frac{288c^{\frac{19}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{d^{\frac{8}{3}}}$$

$$- \frac{288c^{\frac{19}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{d^{\frac{8}{3}}} - \frac{36534c^2 x^2 \sqrt{dx^3 + c}}{1729d^2}$$

$$- \frac{348c x^5 \sqrt{dx^3 + c}}{247d} - \frac{2x^8 \sqrt{dx^3 + c}}{19} - \frac{2094648c^3 \sqrt{dx^3 + c}}{1729d^{\frac{8}{3}} (d^{\frac{1}{3}}x + c^{\frac{1}{3}} (1 + \sqrt{3}))}$$

$$- \frac{698216 3^{\frac{3}{4}} c^{\frac{10}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{1729d^{\frac{8}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}$$

$$+ \frac{1047324 3^{\frac{1}{4}} c^{\frac{10}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{1729d^{\frac{8}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}$$

command

```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(dx^{10} + cx^7) \sqrt{dx^3 + c}}{dx^3 - 8c}, x\right)$$

18.70 Problem number 303

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{36c^{\frac{13}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3+c}}\right)}{d^{\frac{5}{3}}} - \frac{36c^{\frac{13}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{d^{\frac{5}{3}}} \\ & - \frac{36c^{\frac{13}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3+c}}\right)\sqrt{3}}{d^{\frac{5}{3}}} - \frac{240cx^2\sqrt{dx^3+c}}{91d} \\ & - \frac{2x^5\sqrt{dx^3+c}}{13} - \frac{13782c^2\sqrt{dx^3+c}}{91d^{\frac{5}{3}}(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))} \\ & - \frac{45943^{\frac{3}{4}}c^{\frac{7}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2} \sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{91d^{\frac{5}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \\ & + \frac{68913^{\frac{1}{4}}c^{\frac{7}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{91d^{\frac{5}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(dx^7+cx^4)\sqrt{dx^3+c}}{dx^3-8c}, x\right)$$

18.71 Problem number 304

$$\int \frac{x(c + dx^3)^{3/2}}{8c - dx^3} dx$$

Optimal antiderivative

$$\frac{9c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{2d^{\frac{2}{3}}} - \frac{9c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{2d^{\frac{2}{3}}}$$

$$- \frac{9c^{\frac{7}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{2d^{\frac{2}{3}}} - \frac{2x^2\sqrt{dx^3 + c}}{7} - \frac{132c\sqrt{dx^3 + c}}{7d^{\frac{2}{3}}(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))}$$

$$- \frac{443^{\frac{3}{4}}c^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right)\sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{7d^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}$$

$$+ \frac{663^{\frac{1}{4}}c^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{7d^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}$$

command

```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(dx^4 + cx)\sqrt{dx^3 + c}}{dx^3 - 8c}, x\right)$$

18.72 Problem number 306

$$\int \frac{(c + dx^3)^{3/2}}{x^5 (8c - dx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{128c^{\frac{5}{6}}} - \frac{9d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{128c^{\frac{5}{6}}} \\ & - \frac{9d^{\frac{4}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{128c^{\frac{5}{6}}} - \frac{\sqrt{dx^3 + c}}{32x^4} - \frac{3d\sqrt{dx^3 + c}}{16cx} + \frac{3d^{\frac{4}{3}}\sqrt{dx^3 + c}}{16c(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{3^{\frac{3}{4}}d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} \sqrt{2}}{16c^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{33^{\frac{1}{4}}d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{32c^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(dx^3 + c)^{\frac{3}{2}}}{dx^8 - 8cx^5}, x\right)$$

18.73 Problem number 307

$$\int \frac{(c + dx^3)^{3/2}}{x^8 (8c - dx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9d^{7/3} \operatorname{arctanh}\left(\frac{(c^{1/3} + d^{1/3}x)^2}{3c^{1/6} \sqrt{dx^3 + c}}\right)}{1024c^{11/6}} - \frac{9d^{7/3} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{1024c^{11/6}} \\ & - \frac{9d^{7/3} \operatorname{arctan}\left(\frac{c^{1/6}(c^{1/3} + d^{1/3}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{1024c^{11/6}} - \frac{\sqrt{dx^3 + c}}{56x^7} \\ & - \frac{75d\sqrt{dx^3 + c}}{1792cx^4} - \frac{3d^2\sqrt{dx^3 + c}}{56c^2x} + \frac{3d^{7/3}\sqrt{dx^3 + c}}{56c^2(d^{1/3}x + c^{1/3}(1 + \sqrt{3}))} \\ & + \frac{3^{3/4}d^{7/3}(c^{1/3} + d^{1/3}x) \operatorname{EllipticF}\left(\frac{d^{1/3}x + c^{1/3}(1 - \sqrt{3})}{d^{1/3}x + c^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{(d^{1/3}x + c^{1/3}(1 + \sqrt{3}))^2}} \sqrt{2}}{56c^{5/3}\sqrt{dx^3 + c} \sqrt{\frac{c^{1/3}(c^{1/3} + d^{1/3}x)}{(d^{1/3}x + c^{1/3}(1 + \sqrt{3}))^2}}} \\ & - \frac{3 \cdot 3^{1/4} d^{7/3} (c^{1/3} + d^{1/3}x) \operatorname{EllipticE}\left(\frac{d^{1/3}x + c^{1/3}(1 - \sqrt{3})}{d^{1/3}x + c^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{(d^{1/3}x + c^{1/3}(1 + \sqrt{3}))^2}}}{112c^{5/3}\sqrt{dx^3 + c} \sqrt{\frac{c^{1/3}(c^{1/3} + d^{1/3}x)}{(d^{1/3}x + c^{1/3}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(dx^3 + c)^{3/2}}{dx^{11} - 8cx^8}, x\right)$$

18.74 Problem number 315

$$\int \frac{x^7}{(8c - dx^3) \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\frac{32c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{9d^{\frac{8}{3}}} - \frac{32c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{9d^{\frac{8}{3}}}$$

$$- \frac{32c^{\frac{7}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{9d^{\frac{8}{3}}} - \frac{2x^2 \sqrt{dx^3 + c}}{7d^2} - \frac{104c \sqrt{dx^3 + c}}{7d^{\frac{8}{3}} (d^{\frac{1}{3}}x + c^{\frac{1}{3}} (1 + \sqrt{3}))}$$

$$- \frac{104c^{\frac{4}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}}$$

$$- \frac{21d^{\frac{8}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{523^{\frac{1}{4}} c^{\frac{4}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}$$

$$+ \frac{7d^{\frac{8}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{}$$

command

```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c} x^7}{d^2 x^6 - 7cdx^3 - 8c^2}, x\right)$$

18.75 Problem number 316

$$\int \frac{x^4}{(8c - dx^3) \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{9d^{\frac{5}{3}}} - \frac{4c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{9d^{\frac{5}{3}}} \\ & - \frac{4c^{\frac{1}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{9d^{\frac{5}{3}}} - \frac{2\sqrt{dx^3 + c}}{d^{\frac{5}{3}}(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & - \frac{2c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}} \\ & + \frac{3d^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{1}{4}} c^{\frac{1}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & + \frac{d^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{\end{aligned}$$

command

`integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c} x^4}{d^2 x^6 - 7cdx^3 - 8c^2}, x\right)$$

18.76 Problem number 318

$$\int \frac{1}{x^2 (8c - dx^3) \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{144c^{\frac{11}{6}}} - \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{144c^{\frac{11}{6}}} \\ & - \frac{d^{\frac{1}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{144c^{\frac{11}{6}}} - \frac{\sqrt{dx^3 + c}}{8c^2x} + \frac{d^{\frac{1}{3}} \sqrt{dx^3 + c}}{8c^2(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & + \frac{24c^{\frac{5}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{1}{4}}d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{16c^{\frac{5}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{16c^{\frac{5}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

`integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^2x^8 - 7cdx^5 - 8c^2x^2}, x\right)$$

18.77 Problem number 319

$$\int \frac{1}{x^5 (8c - dx^3) \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\frac{d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right) - d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right) - d^{\frac{4}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{1152c^{\frac{17}{6}} \sqrt{dx^3 + c} - \frac{1152c^{\frac{17}{6}}}{32c^2x^4} + \frac{d\sqrt{dx^3 + c}}{16c^3x} - \frac{1152c^{\frac{17}{6}}}{16c^3(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))}}$$

$$- \frac{d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}}$$

$$+ \frac{48c^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{32c^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}$$

$$+ \frac{3^{\frac{1}{4}}d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{32c^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}$$

command

```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^2x^{11} - 7cdx^8 - 8c^2x^5}, x\right)$$

18.78 Problem number 320

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right) - d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right) - d^{\frac{7}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{9216c^{\frac{23}{6}}} \\ & - \frac{\sqrt{dx^3 + c}}{56c^2x^7} + \frac{37d\sqrt{dx^3 + c}}{1792c^3x^4} - \frac{3d^2\sqrt{dx^3 + c}}{56c^4x} + \frac{3d^{\frac{7}{3}}\sqrt{dx^3 + c}}{56c^4(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{3^{\frac{3}{4}}d^{\frac{7}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} \sqrt{2}}{56c^{\frac{11}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{3^{\frac{3}{4}}d^{\frac{7}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{112c^{\frac{11}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^2x^{14} - 7cdx^{11} - 8c^2x^8}, x\right)$$

18.79 Problem number 321

$$\int \frac{x^3}{(8c - dx^3) \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\frac{x^4 F_1\left(\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{32c \sqrt{dx^3 + c}}$$

command

```
integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{dx^3 + c} x^3}{d^2 x^6 - 7 c dx^3 - 8 c^2}, x\right)$$

18.80 Problem number 322

$$\int \frac{1}{(8c - dx^3) \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\frac{x F_1\left(\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{8c \sqrt{dx^3 + c}}$$

command

```
integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^2 x^6 - 7 c dx^3 - 8 c^2}, x\right)$$

18.81 Problem number 323

$$\int \frac{1}{x^3 (8c - dx^3) \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$-\frac{F_1\left(-\frac{2}{3}, \frac{1}{2}, 1, \frac{1}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{16c x^2 \sqrt{dx^3 + c}}$$

command

```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^2x^9 - 7cdx^6 - 8c^2x^3}, x\right)$$

18.82 Problem number 324

$$\int \frac{1}{x^6 (8c - dx^3) \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$-\frac{F_1\left(-\frac{5}{3}, \frac{1}{2}, 1, -\frac{2}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{40c x^5 \sqrt{dx^3 + c}}$$

command

```
integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^2x^{12} - 7cdx^9 - 8c^2x^6}, x\right)$$

18.83 Problem number 332

$$\int \frac{x^7}{(8c - dx^3)(c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{81d^{\frac{8}{3}}} - \frac{32c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{81d^{\frac{8}{3}}} \\ & - \frac{32c^{\frac{1}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{81d^{\frac{8}{3}}} + \frac{2x^2}{27d^2 \sqrt{dx^3 + c}} - \frac{56\sqrt{dx^3 + c}}{27d^{\frac{8}{3}}(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & - \frac{56c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{81d^{\frac{8}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & + \frac{28c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{27d^{\frac{8}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} 3^{\frac{3}{4}} \end{aligned}$$

command

```
integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c} x^7}{d^3 x^9 - 6cd^2 x^6 - 15c^2 dx^3 - 8c^3}, x\right)$$

18.84 Problem number 333

$$\int \frac{x^4}{(8c - dx^3)(c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4 \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{81c^{\frac{5}{6}}d^{\frac{5}{3}}} - \frac{4 \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{81c^{\frac{5}{6}}d^{\frac{5}{3}}} \\ & - \frac{4 \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{81c^{\frac{5}{6}}d^{\frac{5}{3}}} - \frac{2x^2}{27cd\sqrt{dx^3 + c}} + \frac{2\sqrt{dx^3 + c}}{27cd^{\frac{5}{3}}(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{2(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{3}{4}}}{81c^{\frac{2}{3}}d^{\frac{5}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{1}{4}}}{27c^{\frac{2}{3}}d^{\frac{5}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

`integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c} x^4}{d^3x^9 - 6cd^2x^6 - 15c^2dx^3 - 8c^3}, x\right)$$

18.85 Problem number 334

$$\int \frac{x}{(8c - dx^3)(c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{162c^{\frac{11}{6}}d^{\frac{2}{3}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{162c^{\frac{11}{6}}d^{\frac{2}{3}}}$$

$$- \frac{\operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{162c^{\frac{11}{6}}d^{\frac{2}{3}}} + \frac{2x^2}{27c^2\sqrt{dx^3 + c}} - \frac{2\sqrt{dx^3 + c}}{27c^2d^{\frac{2}{3}}(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))}$$

$$- \frac{2(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{3}{4}}}{81c^{\frac{5}{3}}d^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}$$

$$+ \frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{1}{4}}}{27c^{\frac{5}{3}}d^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}$$

command

```
integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c} x}{d^3x^9 - 6cd^2x^6 - 15c^2dx^3 - 8c^3}, x\right)$$

18.86 Problem number 335

$$\int \frac{1}{x^2 (8c - dx^3) (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{1296c^{\frac{17}{6}}} - \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{1296c^{\frac{17}{6}}} - \frac{d^{\frac{1}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{1296c^{\frac{17}{6}}} \\ & + \frac{2}{27c^2x\sqrt{dx^3 + c}} - \frac{43\sqrt{dx^3 + c}}{216c^3x} + \frac{43d^{\frac{1}{3}}\sqrt{dx^3 + c}}{216c^3(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{43d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{3}{4}}\sqrt{2}}{648c^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{43d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{1}{4}}}{432c^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^3x^{11} - 6cd^2x^8 - 15c^2dx^5 - 8c^3x^2}, x\right)$$

18.87 Problem number 336

$$\int \frac{1}{x^5 (8c - dx^3) (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3+c}}\right)}{10368c^{\frac{23}{6}}} - \frac{d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{10368c^{\frac{23}{6}}} - \frac{d^{\frac{4}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3+c}}\right)\sqrt{3}}{10368c^{\frac{23}{6}}} \\ & + \frac{2}{27c^2x^4\sqrt{dx^3+c}} - \frac{91\sqrt{dx^3+c}}{864c^3x^4} + \frac{113d\sqrt{dx^3+c}}{432c^4x} - \frac{113d^{\frac{4}{3}}\sqrt{dx^3+c}}{432c^4\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & \frac{113d^{\frac{4}{3}}\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} 3^{\frac{3}{4}}\sqrt{2}}{1296c^{\frac{11}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\ & + \frac{113d^{\frac{4}{3}}\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} 3^{\frac{1}{4}}}{864c^{\frac{11}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3+c}}{d^3x^{14} - 6cd^2x^{11} - 15c^2dx^8 - 8c^3x^5}, x\right)$$

18.88 Problem number 337

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{82944c^{\frac{29}{6}}} - \frac{d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{82944c^{\frac{29}{6}}} \\ & - \frac{d^{\frac{7}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{82944c^{\frac{29}{6}}} + \frac{2}{27c^2x^7\sqrt{dx^3 + c}} - \frac{139\sqrt{dx^3 + c}}{1512c^3x^7} \\ & + \frac{6095d\sqrt{dx^3 + c}}{48384c^4x^4} - \frac{953d^2\sqrt{dx^3 + c}}{3024c^5x} + \frac{953d^{\frac{7}{3}}\sqrt{dx^3 + c}}{3024c^5(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{953d^{\frac{7}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & + \frac{9072c^{\frac{14}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{9072c^{\frac{14}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{953d^{\frac{7}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{1}{4}}} \\ & - \frac{6048c^{\frac{14}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{6048c^{\frac{14}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^3x^{17} - 6cd^2x^{14} - 15c^2dx^{11} - 8c^3x^8}, x\right)$$

18.89 Problem number 338

$$\int \frac{x^3}{(8c - dx^3)(c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x^4 F_1\left(\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{32c^2 \sqrt{dx^3 + c}}$$

command

`integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{dx^3 + c} x^3}{d^3 x^9 - 6 cd^2 x^6 - 15 c^2 dx^3 - 8 c^3}, x\right)$$

18.90 Problem number 339

$$\int \frac{1}{(8c - dx^3)(c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x F_1\left(\frac{1}{3}, \frac{3}{2}, 1, \frac{4}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{8c^2 \sqrt{dx^3 + c}}$$

command

`integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^3 x^9 - 6 cd^2 x^6 - 15 c^2 dx^3 - 8 c^3}, x\right)$$

18.91 Problem number 340

$$\int \frac{1}{x^3 (8c - dx^3) (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{F_1\left(-\frac{2}{3}, \frac{3}{2}, 1, \frac{1}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{16c^2 x^2 \sqrt{dx^3 + c}}$$

command

```
integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^3 x^{12} - 6cd^2 x^9 - 15c^2 dx^6 - 8c^3 x^3}, x\right)$$

18.92 Problem number 341

$$\int \frac{1}{x^6 (8c - dx^3) (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{F_1\left(-\frac{5}{3}, \frac{3}{2}, 1, -\frac{2}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{40c^2 x^5 \sqrt{dx^3 + c}}$$

command

```
integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^3 x^{15} - 6cd^2 x^{12} - 15c^2 dx^9 - 8c^3 x^6}, x\right)$$

18.93 Problem number 350

$$\int \frac{x}{\sqrt{a+bx^3} \left(2(5+3\sqrt{3})a+bx^3\right)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{3^{\frac{1}{4}}a^{\frac{1}{6}}(a^{\frac{1}{3}}+b^{\frac{1}{3}}x)(1+\sqrt{3})\sqrt{2}}{2\sqrt{bx^3+a}}\right)(2-\sqrt{3})3^{\frac{1}{4}}\sqrt{2}}{12a^{\frac{5}{6}}b^{\frac{2}{3}}}$$

$$\frac{\arctan\left(\frac{((1-\sqrt{3})\sqrt{bx^3+a})^{\frac{1}{3}}3^{\frac{1}{4}}\sqrt{2}}{6\sqrt{a}}\right)(2-\sqrt{3})3^{\frac{1}{4}}\sqrt{2}}{18a^{\frac{5}{6}}b^{\frac{2}{3}}}$$

$$\frac{\operatorname{arctanh}\left(\frac{3^{\frac{1}{4}}a^{\frac{1}{6}}(a^{\frac{1}{3}}+b^{\frac{1}{3}}x)(1-\sqrt{3})\sqrt{2}}{2\sqrt{bx^3+a}}\right)(2-\sqrt{3})3^{\frac{3}{4}}\sqrt{2}}{36a^{\frac{5}{6}}b^{\frac{2}{3}}}$$

$$\frac{\operatorname{arctanh}\left(\frac{3^{\frac{1}{4}}a^{\frac{1}{6}}(-2b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3}))\sqrt{2}}{2\sqrt{bx^3+a}}\right)(2-\sqrt{3})3^{\frac{3}{4}}\sqrt{2}}{18a^{\frac{5}{6}}b^{\frac{2}{3}}}$$

command

```
integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

18.94 Problem number 351

$$\int \frac{x}{\sqrt{a-bx^3} \left(2(5+3\sqrt{3})a-bx^3\right)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{3^{\frac{1}{4}}a^{\frac{1}{6}}(a^{\frac{1}{3}}-b^{\frac{1}{3}}x)(1+\sqrt{3})\sqrt{2}}{2\sqrt{-bx^3+a}}\right)(2-\sqrt{3})3^{\frac{1}{4}}\sqrt{2}}{12a^{\frac{5}{6}}b^{\frac{2}{3}}}$$

$$\frac{\arctan\left(\frac{(1-\sqrt{3})\sqrt{-bx^3+a}3^{\frac{1}{4}}\sqrt{2}}{6\sqrt{a}}\right)(2-\sqrt{3})3^{\frac{1}{4}}\sqrt{2}}{18a^{\frac{5}{6}}b^{\frac{2}{3}}}$$

$$\frac{\operatorname{arctanh}\left(\frac{3^{\frac{1}{4}}a^{\frac{1}{6}}(a^{\frac{1}{3}}-b^{\frac{1}{3}}x)(1-\sqrt{3})\sqrt{2}}{2\sqrt{-bx^3+a}}\right)(2-\sqrt{3})3^{\frac{3}{4}}\sqrt{2}}{36a^{\frac{5}{6}}b^{\frac{2}{3}}}$$

$$\frac{\operatorname{arctanh}\left(\frac{3^{\frac{1}{4}}a^{\frac{1}{6}}(2b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3}))\sqrt{2}}{2\sqrt{-bx^3+a}}\right)(2-\sqrt{3})3^{\frac{3}{4}}\sqrt{2}}{18a^{\frac{5}{6}}b^{\frac{2}{3}}}$$

command

```
integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

18.95 Problem number 405

$$\int \frac{x^7 \sqrt{c+dx^3}}{(8c-dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{76c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3+c}}\right)}{9d^{\frac{8}{3}}} + \frac{76c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9d^{\frac{8}{3}}} \\
& + \frac{76c^{\frac{7}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3+c}}\right)\sqrt{3}}{9d^{\frac{8}{3}}} + \frac{13x^2\sqrt{dx^3+c}}{21d^2} \\
& + \frac{x^5\sqrt{dx^3+c}}{3d(-dx^3+8c)} + \frac{746c\sqrt{dx^3+c}}{21d^{\frac{8}{3}}(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))} \\
& + \frac{746c^{\frac{4}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2} \sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{3}{4}}} \\
& + \frac{63d^{\frac{8}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{373\cdot 3^{\frac{1}{4}}c^{\frac{4}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \\
& - \frac{21d^{\frac{8}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{21d^{\frac{8}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}
\end{aligned}$$

command

```
integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3+c} x^7}{d^2x^6-16cdx^3+64c^2}, x\right)$$

18.96 Problem number 406

$$\int \frac{x^4 \sqrt{c + dx^3}}{(8c - dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{9d^{\frac{5}{3}}} + \frac{5c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{9d^{\frac{5}{3}}} \\ & + \frac{5c^{\frac{1}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{9d^{\frac{5}{3}}} + \frac{x^2 \sqrt{dx^3 + c}}{3d(-dx^3 + 8c)} + \frac{7\sqrt{dx^3 + c}}{3d^{\frac{5}{3}}(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{7c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{9d^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \quad 3^{\frac{3}{4}} \\ & + \frac{7 \cdot 3^{\frac{1}{4}} c^{\frac{1}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{6d^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c} x^4}{d^2x^6 - 16cdx^3 + 64c^2}, x\right)$$

18.97 Problem number 407

$$\int \frac{x \sqrt{c + dx^3}}{(8c - dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3+c}}\right)}{144c^{\frac{5}{6}}d^{\frac{2}{3}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{144c^{\frac{5}{6}}d^{\frac{2}{3}}} \\ & + \frac{\operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3+c}}\right)\sqrt{3}}{144c^{\frac{5}{6}}d^{\frac{2}{3}}} + \frac{x^2\sqrt{dx^3+c}}{24c(-dx^3+8c)} + \frac{\sqrt{dx^3+c}}{24cd^{\frac{2}{3}}(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))} \\ & + \frac{(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)\operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & + \frac{72c^{\frac{2}{3}}d^{\frac{2}{3}}\sqrt{dx^3+c}\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)\operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{1}{4}}} \\ & - \frac{48c^{\frac{2}{3}}d^{\frac{2}{3}}\sqrt{dx^3+c}\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}} \end{aligned}$$

command

```
integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3+c}x}{d^2x^6-16cdx^3+64c^2},x\right)$$

18.98 Problem number 408

$$\int \frac{\sqrt{c + dx^3}}{x^2 (8c - dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{144c^{\frac{11}{6}}} - \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{144c^{\frac{11}{6}}} - \frac{d^{\frac{1}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{144c^{\frac{11}{6}}} \\ & - \frac{\sqrt{dx^3 + c}}{48c^2x} + \frac{\sqrt{dx^3 + c}}{24cx(-dx^3 + 8c)} + \frac{d^{\frac{1}{3}}\sqrt{dx^3 + c}}{48c^2(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{144c^{\frac{5}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & + \frac{3^{\frac{1}{4}}d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{96c^{\frac{5}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c}}{d^2x^8 - 16cdx^5 + 64c^2x^2}, x\right)$$

18.99 Problem number 409

$$\int \frac{\sqrt{c + dx^3}}{x^5 (8c - dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{17d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{9216c^{\frac{17}{6}}} - \frac{17d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{9216c^{\frac{17}{6}}} \\ & - \frac{17d^{\frac{4}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{9216c^{\frac{17}{6}}} - \frac{7\sqrt{dx^3 + c}}{768c^2x^4} \\ & - \frac{d\sqrt{dx^3 + c}}{96c^3x} + \frac{\sqrt{dx^3 + c}}{24cx^4(-dx^3 + 8c)} + \frac{d^{\frac{4}{3}}\sqrt{dx^3 + c}}{96c^3(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & + \frac{288c^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{1}{4}}d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{192c^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{\end{aligned}$$

command

```
integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c}}{d^2x^{11} - 16cdx^8 + 64c^2x^5}, x\right)$$

18.100 Problem number 410

$$\int \frac{\sqrt{c + dx^3}}{x^8 (8c - dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{13d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{36864c^{\frac{23}{6}}} - \frac{13d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{36864c^{\frac{23}{6}}} \\ & - \frac{13d^{\frac{7}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{36864c^{\frac{23}{6}}} - \frac{5\sqrt{dx^3 + c}}{672c^2x^7} - \frac{53d\sqrt{dx^3 + c}}{21504c^3x^4} \\ & - \frac{d^2\sqrt{dx^3 + c}}{5376c^4x} + \frac{\sqrt{dx^3 + c}}{24cx^7(-dx^3 + 8c)} + \frac{d^{\frac{7}{3}}\sqrt{dx^3 + c}}{5376c^4(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{3^{\frac{3}{4}}d^{\frac{7}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} \sqrt{2}}{16128c^{\frac{11}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{3^{\frac{1}{4}}d^{\frac{7}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{10752c^{\frac{11}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c}}{d^2x^{14} - 16cdx^{11} + 64c^2x^8}, x\right)$$

18.101 Problem number 418

$$\int \frac{x^7 (c + dx^3)^{3/2}}{(8c - dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^5 (dx^3 + c)^{\frac{3}{2}}}{3d(-dx^3 + 8c)} - \frac{108c^{\frac{13}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{d^{\frac{8}{3}}} \\ & + \frac{108c^{\frac{13}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{d^{\frac{8}{3}}} + \frac{108c^{\frac{13}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{d^{\frac{8}{3}}} \\ & + \frac{103cx^2\sqrt{dx^3 + c}}{13d^2} + \frac{19x^5\sqrt{dx^3 + c}}{39d} + \frac{5906c^2\sqrt{dx^3 + c}}{13d^{\frac{8}{3}}\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{5906c^{\frac{7}{3}}\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right)\sqrt{2}\sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{39d^{\frac{8}{3}}\sqrt{dx^3 + c}\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & - \frac{2953\frac{1}{4}c^{\frac{7}{3}}\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right)\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right)\sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{13d^{\frac{8}{3}}\sqrt{dx^3 + c}\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(dx^{10} + cx^7)\sqrt{dx^3 + c}}{d^2x^6 - 16cdx^3 + 64c^2}, x\right)$$

18.102 Problem number 419

$$\int \frac{x^4 (c + dx^3)^{3/2}}{(8c - dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^2 (dx^3 + c)^{\frac{3}{2}}}{3d(-dx^3 + 8c)} - \frac{9c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{d^{\frac{5}{3}}} + \frac{9c^{\frac{7}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{d^{\frac{5}{3}}} \\ & + \frac{9c^{\frac{7}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{d^{\frac{5}{3}}} + \frac{13x^2 \sqrt{dx^3 + c}}{21d} + \frac{265c \sqrt{dx^3 + c}}{7d^{\frac{5}{3}} (d^{\frac{1}{3}}x + c^{\frac{1}{3}} (1 + \sqrt{3}))} \\ & + \frac{265c^{\frac{4}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{21d^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} 3^{\frac{3}{4}} \\ & - \frac{265 \cdot 3^{\frac{1}{4}} c^{\frac{4}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{14d^{\frac{5}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(dx^7 + cx^4) \sqrt{dx^3 + c}}{d^2x^6 - 16cdx^3 + 64c^2}, x\right)$$

18.103 Problem number 420

$$\int \frac{x(c + dx^3)^{3/2}}{(8c - dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{9c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{16d^{\frac{2}{3}}} + \frac{9c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{16d^{\frac{2}{3}}} \\ & + \frac{9c^{\frac{1}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{16d^{\frac{2}{3}}} + \frac{3x^2\sqrt{dx^3 + c}}{8(-dx^3 + 8c)} + \frac{19\sqrt{dx^3 + c}}{8d^{\frac{2}{3}}(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{19c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right)\sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}} \\ & + \frac{24d^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{193^{\frac{1}{4}}c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{16d^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{16d^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(dx^4 + cx)\sqrt{dx^3 + c}}{d^2x^6 - 16cdx^3 + 64c^2}, x\right)$$

18.104 Problem number 421

$$\int \frac{(c + dx^3)^{3/2}}{x^2 (8c - dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{dx^3 + c}}{16cx} + \frac{3\sqrt{dx^3 + c}}{8x(-dx^3 + 8c)} + \frac{d^{\frac{1}{3}}\sqrt{dx^3 + c}}{16c\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{d^{\frac{1}{3}}\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}\sqrt{2}}{48c^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right)}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{3^{\frac{1}{4}}d^{\frac{1}{3}}\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{32c^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right)}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(dx^4 - 8cx)\sqrt{d} \operatorname{weierstrassZeta}\left(0, -\frac{4c}{d}, \operatorname{weierstrassPInverse}\left(0, -\frac{4c}{d}, x\right)\right) + \sqrt{dx^3 + c}(dx^3 - 2c)}{16(cd^2x^4 - 8c^2x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(dx^3 + c)^{\frac{3}{2}}}{d^2x^8 - 16cdx^5 + 64c^2x^2}, x\right)$$

18.105 Problem number 422

$$\int \frac{(c + dx^3)^{3/2}}{x^5 (8c - dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3+c}}\right)}{1024c^{\frac{11}{6}}} - \frac{9d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{1024c^{\frac{11}{6}}} \\ & - \frac{9d^{\frac{4}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3+c}}\right)\sqrt{3}}{1024c^{\frac{11}{6}}} - \frac{13\sqrt{dx^3+c}}{256c^4} \\ & - \frac{d\sqrt{dx^3+c}}{32c^2x} + \frac{3\sqrt{dx^3+c}}{8x^4(-dx^3+8c)} + \frac{d^{\frac{4}{3}}\sqrt{dx^3+c}}{32c^2(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))} \\ & + \frac{d^{\frac{4}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & + \frac{96c^{\frac{5}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{3^{\frac{1}{4}}d^{\frac{4}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \\ & - \frac{64c^{\frac{5}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{\end{aligned}$$

command

```
integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(dx^3+c)^{\frac{3}{2}}}{d^2x^{11}-16cdx^8+64c^2x^5}, x\right)$$

18.106 Problem number 423

$$\int \frac{(c + dx^3)^{3/2}}{x^8 (8c - dx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9d^{7/3} \operatorname{arctanh}\left(\frac{(c^{1/3} + d^{1/3}x)^2}{3c^{1/6} \sqrt{dx^3 + c}}\right)}{4096c^{17/6}} - \frac{9d^{7/3} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{4096c^{17/6}} \\ & - \frac{9d^{7/3} \operatorname{arctan}\left(\frac{c^{1/6}(c^{1/3} + d^{1/3}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{4096c^{17/6}} - \frac{11\sqrt{dx^3 + c}}{224c^7} - \frac{83d\sqrt{dx^3 + c}}{7168c^2x^4} \\ & - \frac{19d^2\sqrt{dx^3 + c}}{1792c^3x} + \frac{3\sqrt{dx^3 + c}}{8x^7(-dx^3 + 8c)} + \frac{19d^{7/3}\sqrt{dx^3 + c}}{1792c^3(d^{1/3}x + c^{1/3}(1 + \sqrt{3}))} \\ & + \frac{19d^{7/3}(c^{1/3} + d^{1/3}x) \operatorname{EllipticF}\left(\frac{d^{1/3}x + c^{1/3}(1 - \sqrt{3})}{d^{1/3}x + c^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{(d^{1/3}x + c^{1/3}(1 + \sqrt{3}))^2}}}{3^{3/4}\sqrt{2}} \\ & + \frac{5376c^{8/3}\sqrt{dx^3 + c} \sqrt{\frac{c^{1/3}(c^{1/3} + d^{1/3}x)}{(d^{1/3}x + c^{1/3}(1 + \sqrt{3}))^2}}}{193^{1/4}d^{7/3}(c^{1/3} + d^{1/3}x) \operatorname{EllipticE}\left(\frac{d^{1/3}x + c^{1/3}(1 - \sqrt{3})}{d^{1/3}x + c^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{(d^{1/3}x + c^{1/3}(1 + \sqrt{3}))^2}}} \\ & - \frac{3584c^{8/3}\sqrt{dx^3 + c} \sqrt{\frac{c^{1/3}(c^{1/3} + d^{1/3}x)}{(d^{1/3}x + c^{1/3}(1 + \sqrt{3}))^2}}}{\end{aligned}$$

command

```
integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(dx^3 + c)^{3/2}}{d^2x^{14} - 16cdx^{11} + 64c^2x^8}, x\right)$$

18.107 Problem number 431

$$\int \frac{x^7}{(8c - dx^3)^2 \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{44c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{81d^{\frac{8}{3}}} + \frac{44c^{\frac{1}{6}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{81d^{\frac{8}{3}}} \\ & + \frac{44c^{\frac{1}{6}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{81d^{\frac{8}{3}}} + \frac{8x^2 \sqrt{dx^3 + c}}{27d^2(-dx^3 + 8c)} + \frac{62\sqrt{dx^3 + c}}{27d^{\frac{8}{3}}(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{62c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{81d^{\frac{8}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & + \frac{31c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{27d^{\frac{8}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c} x^7}{d^3x^9 - 15cd^2x^6 + 48c^2dx^3 + 64c^3}, x\right)$$

18.108 Problem number 432

$$\int \frac{x^4}{(8c - dx^3)^2 \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3+c}}\right)}{81c^{\frac{5}{6}}d^{\frac{5}{3}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{81c^{\frac{5}{6}}d^{\frac{5}{3}}} \\ & + \frac{\operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3+c}}\right)\sqrt{3}}{81c^{\frac{5}{6}}d^{\frac{5}{3}}} + \frac{x^2\sqrt{dx^3+c}}{27cd(-dx^3+8c)} + \frac{\sqrt{dx^3+c}}{27cd^{\frac{5}{3}}\left(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & + \frac{(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)\operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{81c^{\frac{2}{3}}d^{\frac{5}{3}}\sqrt{dx^3+c}\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{\left(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} 3^{\frac{3}{4}} \\ & + \frac{(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)\operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{c^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{1}{3}}x+d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{54c^{\frac{2}{3}}d^{\frac{5}{3}}\sqrt{dx^3+c}\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}}+d^{\frac{1}{3}}x)}{\left(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} 3^{\frac{1}{4}} \end{aligned}$$

command

```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3+c}x^4}{d^3x^9-15cd^2x^6+48c^2dx^3+64c^3},x\right)$$

18.109 Problem number 433

$$\int \frac{x}{(8c - dx^3)^2 \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7 \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{1296c^{\frac{11}{6}} d^{\frac{2}{3}}} - \frac{7 \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{1296c^{\frac{11}{6}} d^{\frac{2}{3}}} \\ & - \frac{7 \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{1296c^{\frac{11}{6}} d^{\frac{2}{3}}} + \frac{x^2 \sqrt{dx^3 + c}}{216c^2 (-dx^3 + 8c)} + \frac{\sqrt{dx^3 + c}}{216c^2 d^{\frac{2}{3}} (d^{\frac{1}{3}}x + c^{\frac{1}{3}} (1 + \sqrt{3}))} \\ & + \frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{3}{4}}}{648c^{\frac{5}{3}} d^{\frac{2}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{1}{4}}}{432c^{\frac{5}{3}} d^{\frac{2}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c} x}{d^3x^9 - 15cd^2x^6 + 48c^2dx^3 + 64c^3}, x\right)$$

18.110 Problem number 434

$$\int \frac{1}{x^2 (8c - dx^3)^2 \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{648c^{\frac{17}{6}}} - \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{648c^{\frac{17}{6}}} - \frac{d^{\frac{1}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{648c^{\frac{17}{6}}} \\ & - \frac{7\sqrt{dx^3 + c}}{432c^3x} + \frac{\sqrt{dx^3 + c}}{216c^2x(-dx^3 + 8c)} + \frac{7d^{\frac{1}{3}}\sqrt{dx^3 + c}}{432c^3(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{7d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{3}{4}}\sqrt{2}}{1296c^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{7d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}} 3^{\frac{1}{4}}}{864c^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c}}{d^3x^{11} - 15cd^2x^8 + 48c^2dx^5 + 64c^3x^2}, x\right)$$

18.111 Problem number 435

$$\int \frac{1}{x^5 (8c - dx^3)^2 \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{25d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{82944c^{\frac{23}{6}}} - \frac{25d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{82944c^{\frac{23}{6}}} \\ & - \frac{25d^{\frac{4}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{82944c^{\frac{23}{6}}} - \frac{31\sqrt{dx^3 + c}}{6912c^3x^4} \\ & + \frac{5d\sqrt{dx^3 + c}}{864c^4x} + \frac{\sqrt{dx^3 + c}}{216c^2x^4(-dx^3 + 8c)} - \frac{5d^{\frac{4}{3}}\sqrt{dx^3 + c}}{864c^4(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & - \frac{5d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & - \frac{2592c^{\frac{11}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{5d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{1}{4}}} \\ & + \frac{1728c^{\frac{11}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{\end{aligned}$$

command

`integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c}}{d^3x^{14} - 15cd^2x^{11} + 48c^2dx^8 + 64c^3x^5}, x\right)$$

18.112 Problem number 436

$$\int \frac{1}{x^8 (8c - dx^3)^2 \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{17d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{331776c^{\frac{29}{6}}} - \frac{17d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{331776c^{\frac{29}{6}}} \\ & - \frac{17d^{\frac{7}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{331776c^{\frac{29}{6}}} - \frac{17\sqrt{dx^3 + c}}{6048c^3x^7} + \frac{391d\sqrt{dx^3 + c}}{193536c^4x^4} \\ & - \frac{289d^2\sqrt{dx^3 + c}}{48384c^5x} + \frac{\sqrt{dx^3 + c}}{216c^2x^7(-dx^3 + 8c)} + \frac{289d^{\frac{7}{3}}\sqrt{dx^3 + c}}{48384c^5(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{289d^{\frac{7}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & + \frac{145152c^{\frac{14}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & - \frac{289d^{\frac{7}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{1}{4}}} \\ & - \frac{96768c^{\frac{14}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{\sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c}}{d^3x^{17} - 15cd^2x^{14} + 48c^2dx^{11} + 64c^3x^8}, x\right)$$

18.113 Problem number 437

$$\int \frac{x^6}{(8c - dx^3)^2 \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\frac{x^7 F_1\left(\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{448c^2 \sqrt{dx^3 + c}}$$

command

`integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3 + c} x^6}{d^3 x^9 - 15 cd^2 x^6 + 48 c^2 dx^3 + 64 c^3}, x\right)$$

18.114 Problem number 438

$$\int \frac{x^3}{(8c - dx^3)^2 \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\frac{x^4 F_1\left(\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{256c^2 \sqrt{dx^3 + c}}$$

command

`integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3 + c} x^3}{d^3 x^9 - 15 cd^2 x^6 + 48 c^2 dx^3 + 64 c^3}, x\right)$$

18.115 Problem number 439

$$\int \frac{1}{(8c - dx^3)^2 \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\frac{{}_x F_1\left(\frac{1}{3}, \frac{1}{2}, 2, \frac{4}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{64c^2 \sqrt{dx^3 + c}}$$

command

`integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}}{d^3x^9 - 15cd^2x^6 + 48c^2dx^3 + 64c^3}, x\right)$$

18.116 Problem number 440

$$\int \frac{1}{x^3 (8c - dx^3)^2 \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$-\frac{F_1\left(-\frac{2}{3}, \frac{1}{2}, 2, \frac{1}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{128c^2x^2 \sqrt{dx^3 + c}}$$

command

`integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}}{d^3x^{12} - 15cd^2x^9 + 48c^2dx^6 + 64c^3x^3}, x\right)$$

18.117 Problem number 441

$$\int \frac{1}{x^6 (8c - dx^3)^2 \sqrt{c + dx^3}} dx$$

Optimal antiderivative

$$\frac{F_1\left(-\frac{5}{3}, \frac{1}{2}, 2, -\frac{2}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{320c^2x^5 \sqrt{dx^3 + c}}$$

command

```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}}{d^3x^{15} - 15cd^2x^{12} + 48c^2dx^9 + 64c^3x^6}, x\right)$$

18.118 Problem number 449

$$\int \frac{x^7}{(8c - dx^3)^2 (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4 \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{243c^{\frac{5}{6}}d^{\frac{8}{3}}} + \frac{4 \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{243c^{\frac{5}{6}}d^{\frac{8}{3}}} + \frac{4 \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{243c^{\frac{5}{6}}d^{\frac{8}{3}}} \\ & -\frac{2x^2}{81cd^2\sqrt{dx^3 + c}} + \frac{8x^2}{27d^2(-dx^3 + 8c)\sqrt{dx^3 + c}} + \frac{2\sqrt{dx^3 + c}}{81cd^{\frac{8}{3}}\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{2\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}}{243c^{\frac{2}{3}}d^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{1}{4}}}{81c^{\frac{2}{3}}d^{\frac{8}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c} x^7}{d^4 x^{12} - 14cd^3 x^9 + 33c^2 d^2 x^6 + 112c^3 dx^3 + 64c^4}, x\right)$$

18.119 Problem number 450

$$\int \frac{x^4}{(8c - dx^3)^2 (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3+c}}\right)}{243c^{\frac{11}{6}}d^{\frac{5}{3}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{243c^{\frac{11}{6}}d^{\frac{5}{3}}} - \frac{\operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3+c}}\right)\sqrt{3}}{243c^{\frac{11}{6}}d^{\frac{5}{3}}} \\ & - \frac{x^2}{81c^2d\sqrt{dx^3+c}} + \frac{x^2}{27cd(-dx^3+8c)\sqrt{dx^3+c}} + \frac{\sqrt{dx^3+c}}{81c^2d^{\frac{5}{3}}\left(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & + \frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{243c^{\frac{5}{3}}d^{\frac{5}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \\ & + \frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1-\sqrt{3})}{d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{162c^{\frac{5}{3}}d^{\frac{5}{3}}\sqrt{dx^3+c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x+c^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3+c}x^4}{d^4x^{12}-14cd^3x^9+33c^2d^2x^6+112c^3dx^3+64c^4},x\right)$$

18.120 Problem number 451

$$\int \frac{x}{(8c - dx^3)^2 (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5 \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{3888c^{\frac{17}{6}}d^{\frac{2}{3}}} - \frac{5 \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{3888c^{\frac{17}{6}}d^{\frac{2}{3}}} - \frac{5 \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{3888c^{\frac{17}{6}}d^{\frac{2}{3}}} \\ & + \frac{5x^2}{648c^3\sqrt{dx^3 + c}} + \frac{x^2}{216c^2(-dx^3 + 8c)\sqrt{dx^3 + c}} - \frac{5\sqrt{dx^3 + c}}{648c^3d^{\frac{2}{3}}\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{5\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}\sqrt{2}}{\dots} \\ & - \frac{1944c^{\frac{8}{3}}d^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{\dots} \\ & + \frac{5\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{1}{4}}}{\dots} \\ & + \frac{1296c^{\frac{8}{3}}d^{\frac{2}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{\dots} \end{aligned}$$

command

`integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c} x}{d^4x^{12} - 14cd^3x^9 + 33c^2d^2x^6 + 112c^3dx^3 + 64c^4}, x\right)$$

18.121 Problem number 452

$$\int \frac{1}{x^2 (8c - dx^3)^2 (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{3888c^{\frac{23}{6}}} - \frac{d^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{3888c^{\frac{23}{6}}} \\ & - \frac{d^{\frac{1}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{3888c^{\frac{23}{6}}} + \frac{5}{648c^3x\sqrt{dx^3 + c}} \\ & + \frac{1}{216c^2x(-dx^3 + 8c)\sqrt{dx^3 + c}} - \frac{31\sqrt{dx^3 + c}}{1296c^4x} + \frac{31d^{\frac{1}{3}}\sqrt{dx^3 + c}}{1296c^4(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{31d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}} \\ & + \frac{3888c^{\frac{11}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{31d^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{1}{4}}} \\ & - \frac{2592c^{\frac{11}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{2592c^{\frac{11}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c}}{d^4x^{14} - 14cd^3x^{11} + 33c^2d^2x^8 + 112c^3dx^5 + 64c^4x^2}, x\right)$$

18.122 Problem number 453

$$\int \frac{1}{x^5 (8c - dx^3)^2 (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{11d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}}\sqrt{dx^3 + c}}\right)}{248832c^{\frac{29}{6}}} - \frac{11d^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{248832c^{\frac{29}{6}}} \\ & - \frac{11d^{\frac{4}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right)\sqrt{3}}{248832c^{\frac{29}{6}}} + \frac{5}{648c^3x^4\sqrt{dx^3 + c}} + \frac{1}{216c^2x^4(-dx^3 + 8c)\sqrt{dx^3 + c}} \\ & - \frac{253\sqrt{dx^3 + c}}{20736c^4x^4} + \frac{77d\sqrt{dx^3 + c}}{2592c^5x} - \frac{77d^{\frac{4}{3}}\sqrt{dx^3 + c}}{2592c^5(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & - \frac{77d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}}\sqrt{2}}}{\dots} \\ & + \frac{7776c^{\frac{14}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{\dots} \\ & + \frac{77d^{\frac{4}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{1}{4}}}}{\dots} \\ & + \frac{5184c^{\frac{14}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{\dots} \end{aligned}$$

command

`integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c}}{d^4x^{17} - 14cd^3x^{14} + 33c^2d^2x^{11} + 112c^3dx^8 + 64c^4x^5}, x\right)$$

18.123 Problem number 454

$$\int \frac{1}{x^8 (8c - dx^3)^2 (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)^2}{3c^{\frac{1}{6}} \sqrt{dx^3 + c}}\right)}{995328c^{\frac{35}{6}}} - \frac{7d^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{c}}\right)}{995328c^{\frac{35}{6}}} \\ & - \frac{7d^{\frac{7}{3}} \operatorname{arctan}\left(\frac{c^{\frac{1}{6}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{dx^3 + c}}\right) \sqrt{3}}{995328c^{\frac{35}{6}}} + \frac{5}{648c^3 x^7 \sqrt{dx^3 + c}} + \frac{1}{216c^2 x^7 (-dx^3 + 8c) \sqrt{dx^3 + c}} \\ & - \frac{191 \sqrt{dx^3 + c}}{18144c^4 x^7} + \frac{8257d \sqrt{dx^3 + c}}{580608c^5 x^4} - \frac{5179d^2 \sqrt{dx^3 + c}}{145152c^6 x} + \frac{5179d^{\frac{7}{3}} \sqrt{dx^3 + c}}{145152c^6 (d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{5179d^{\frac{7}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{3}{4}} \sqrt{2}} \\ & + \frac{435456c^{\frac{17}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{5179d^{\frac{7}{3}} (c^{\frac{1}{3}} + d^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3^{\frac{1}{4}}} \\ & - \frac{290304c^{\frac{17}{3}} \sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{\end{aligned}$$

command

```
integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx^3 + c}}{d^4 x^{20} - 14cd^3 x^{17} + 33c^2 d^2 x^{14} + 112c^3 dx^{11} + 64c^4 x^8}, x\right)$$

18.124 Problem number 455

$$\int \frac{x^6}{(8c - dx^3)^2 (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x(dx^3 + 4c)}{81cd^2(-dx^3 + 8c)\sqrt{dx^3 + c}}$$

$$2\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right) \text{EllipticF}\left(\frac{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 - \sqrt{3})}{d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}$$

$$243cd^{\frac{7}{3}}\sqrt{dx^3 + c} \sqrt{\frac{c^{\frac{1}{3}}(c^{\frac{1}{3}} + d^{\frac{1}{3}}x)}{\left(d^{\frac{1}{3}}x + c^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

command

`integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((d^2x^6 - 7cdx^3 - 8c^2)\sqrt{d} \text{weierstrassPInverse}\left(0, -\frac{4c}{d}, x\right) + (d^2x^4 + 4cdx)\sqrt{dx^3 + c}\right)}{81(cd^5x^6 - 7c^2d^4x^3 - 8c^3d^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3 + c} x^6}{d^4x^{12} - 14cd^3x^9 + 33c^2d^2x^6 + 112c^3dx^3 + 64c^4}, x\right)$$

18.125 Problem number 456

$$\int \frac{x^3}{(8c - dx^3)^2 (c + dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x^4 F_1\left(\frac{4}{3}, \frac{3}{2}, 2, \frac{7}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1 + \frac{dx^3}{c}}}{256c^3 \sqrt{dx^3 + c}}$$

command

`integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3+c} x^3}{d^4x^{12}-14cd^3x^9+33c^2d^2x^6+112c^3dx^3+64c^4}, x\right)$$

18.126 Problem number 457

$$\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{{}_x F_1\left(\frac{1}{3}, \frac{3}{2}, 2, \frac{4}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1+\frac{dx^3}{c}}}{64c^3 \sqrt{dx^3+c}}$$

command

`integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{12}-14cd^3x^9+33c^2d^2x^6+112c^3dx^3+64c^4}, x\right)$$

18.127 Problem number 458

$$\int \frac{1}{x^3(8c-dx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{{}_x F_1\left(-\frac{2}{3}, \frac{3}{2}, 2, \frac{1}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1+\frac{dx^3}{c}}}{128c^3x^2 \sqrt{dx^3+c}}$$

command

```
integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{15}-14cd^3x^{12}+33c^2d^2x^9+112c^3dx^6+64c^4x^3},x\right)$$

18.128 Problem number 459

$$\int \frac{1}{x^6(8c-dx^3)^2(c+dx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{F_1\left(-\frac{5}{3}, \frac{3}{2}, 2, -\frac{2}{3}, -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) \sqrt{1+\frac{dx^3}{c}}}{320c^3x^5\sqrt{dx^3+c}}$$

command

```
integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{18}-14cd^3x^{15}+33c^2d^2x^{12}+112c^3dx^9+64c^4x^6},x\right)$$

18.129 Problem number 526

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}} dx$$

Optimal antiderivative

$$\frac{-\frac{2A(bx^3+a)^{\frac{3}{2}}}{11ax^{\frac{11}{2}}} + \frac{2(2Ab-11Ba)\sqrt{bx^3+a}}{55ax^{\frac{5}{2}}}{3^{\frac{3}{4}}b(2Ab-11Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)^2}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)\text{EllipticF}\left(\sqrt{1-\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}}\right)}{55\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)a^{\frac{4}{3}}\sqrt{bx^3+a}\sqrt{\frac{b^{\frac{1}{3}}x\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}}$$

command

`integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3\left(11Bab-2Ab^2\right)\sqrt{a}x^6\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)+\left(11Ba^2+3Aab\right)x^3+5Aa^2\right)\sqrt{bx^3+a}\sqrt{x}}{55a^2x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{13}{2}}},x\right)$$

18.130 Problem number 550

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}} dx$$

Optimal antiderivative

$$\frac{\frac{2A\sqrt{bx^3+a}}{5ae(ex)^{\frac{5}{2}}}}{(2Ab-5Ba)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)^2}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)\text{EllipticF}\left(\sqrt{1-\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}}\right)}{15\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)a^{\frac{4}{3}}e^4\sqrt{bx^3+a}\sqrt{\frac{b^{\frac{1}{3}}x\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}}$$

command

```
integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((5Ba - 2Ab) \sqrt{a} x^3 \text{weierstrassPInverse}\left(0, -\frac{4b}{a}, \frac{1}{x}\right) + \sqrt{bx^3 + a} Aa \sqrt{x} \right) e^{(-\frac{7}{2})}}{5a^2x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3 + A) \sqrt{bx^3 + a} \sqrt{ex}}{be^4x^7 + ae^4x^4}, x\right)$$

18.131 Problem number 555

$$\int \frac{A + Bx^3}{\sqrt{ex} (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba) \sqrt{ex}}{3abe \sqrt{bx^3 + a}}$$

$$+ \frac{(2Ab + Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right) \text{EllipticF}\left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}}\right)}{9 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right) a^{\frac{4}{3}} be \sqrt{bx^3 + a} \sqrt{\frac{b^{\frac{1}{3}}x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(((Bab + 2Ab^2)x^3 + Ba^2 + 2Aab) \sqrt{a} \text{weierstrassPInverse}\left(0, -\frac{4b}{a}, \frac{1}{x}\right) + \sqrt{bx^3 + a} (Ba^2 - Aab) \sqrt{x} \right) e^{(-\frac{1}{2})}}{3(a^2b^2x^3 + a^3b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3 + A) \sqrt{bx^3 + a} \sqrt{ex}}{b^2ex^7 + 2abex^4 + a^2ex}, x\right)$$

18.132 Problem number 556

$$\int \frac{A + Bx^3}{(ex)^{3/2} (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(4Ab - Ba)(ex)^{\frac{5}{2}}}{3a^2e^4\sqrt{bx^3 + a}} - \frac{2A}{ae\sqrt{ex}\sqrt{bx^3 + a}} + \frac{2(4Ab - Ba)(1 + \sqrt{3})\sqrt{ex}\sqrt{bx^3 + a}}{3a^2b^{\frac{2}{3}}e^2\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)}$$

$$\frac{2(4Ab - Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right) \text{EllipticE}\left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}}\right)}{3\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right) a^{\frac{5}{3}}b^{\frac{2}{3}}e^2\sqrt{bx^3 + a} \sqrt{\frac{b^{\frac{1}{3}}x\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}}}$$

$$\frac{(4Ab - Ba)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right) \text{EllipticF}\left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}}\right)}{9\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right) a^{\frac{5}{3}}b^{\frac{2}{3}}e^2\sqrt{bx^3 + a} \sqrt{\frac{b^{\frac{1}{3}}x\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\left((Bab - 4Ab^2)x^4 + (Ba^2 - 4Aab)x\right)\sqrt{a}\text{weierstrassZeta}\left(0, -\frac{4b}{a}, \text{weierstrassPInverse}\left(0, -\frac{4b}{a}, \frac{1}{x}\right)\right) + \sqrt{bx^3 + a}\right)}{3(a^2b^2x^4 + a^3bx)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}\sqrt{ex}}{b^2e^2x^8 + 2abe^2x^5 + a^2e^2x^2}, x\right)$$

18.133 Problem number 558

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2A}{5ae (ex)^{\frac{5}{2}} \sqrt{bx^3 + a}} - \frac{2(8Ab - 5Ba) \sqrt{ex}}{15a^2 e^4 \sqrt{bx^3 + a}}$$

$$+ \frac{2(8Ab - 5Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right) \text{EllipticF}\left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}}\right)}{45 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right) a^{\frac{7}{3}} e^4 \sqrt{bx^3 + a} \sqrt{\frac{b^{\frac{1}{3}}x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}}}$$

command

`integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((5 Bab - 8 Ab^2)x^6 + (5 Ba^2 - 8 Aab)x^3 \right) \sqrt{a} \text{weierstrassPInverse}\left(0, -\frac{4b}{a}, \frac{1}{x}\right) - ((5 Ba^2 - 8 Aab)x^3 - 3 Aa^2)}{15 (a^3 bx^6 + a^4 x^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3 + A) \sqrt{bx^3 + a} \sqrt{ex}}{b^2 e^4 x^{10} + 2 a b e^4 x^7 + a^2 e^4 x^4}, x\right)$$

18.134 Problem number 560

$$\int \frac{(ex)^{5/2} (A + Bx^3)}{(a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba) (ex)^{\frac{7}{2}}}{9abe (bx^3 + a)^{\frac{3}{2}}} - \frac{2(2Ab + 7Ba) e^2 \sqrt{ex}}{27a b^2 \sqrt{bx^3 + a}}$$

$$+ \frac{(2Ab + 7Ba) e^2 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right) \text{EllipticF} \left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \right)}{81 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right) a^{\frac{4}{3}} b^2 \sqrt{bx^3 + a} \sqrt{\frac{b^{\frac{1}{3}} x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}}$$

command

```
integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((7 Bab^2 + 2 Ab^3) x^6 + 7 Ba^3 + 2 Aa^2 b + 2 (7 Ba^2 b + 2 Aab^2) x^3 \right) \sqrt{a} e^{\frac{5}{2}} \text{weierstrassPInverse} \left(0, -\frac{4b}{a}, \frac{1}{x} \right) + (7 B a^3 + 2 A a^2 b)}{27 (a^2 b^4 x^6 + 2 a^3 b^3 x^3 + a^4 b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Be^2 x^5 + Ae^2 x^2) \sqrt{bx^3 + a} \sqrt{ex}}{b^3 x^9 + 3 ab^2 x^6 + 3 a^2 b x^3 + a^3}, x \right)$$

18.135 Problem number 561

$$\int \frac{(ex)^{3/2} (A + Bx^3)}{(a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba)(ex)^{\frac{5}{2}}}{9abe(bx^3 + a)^{\frac{3}{2}}} + \frac{2(4Ab + 5Ba)(ex)^{\frac{5}{2}}}{27a^2be\sqrt{bx^3 + a}} - \frac{2(4Ab + 5Ba)e(1 + \sqrt{3})\sqrt{ex}\sqrt{bx^3 + a}}{27a^2b^{\frac{5}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))}$$

$$+ \frac{2(4Ab + 5Ba)e(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)\sqrt{\frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))^2}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))\text{EllipticE}\left(\sqrt{1 - \frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))^2}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}\right)}{27(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))a^{\frac{5}{3}}b^{\frac{5}{3}}\sqrt{bx^3 + a}\sqrt{\frac{b^{\frac{1}{3}}x(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}}}$$

$$+ \frac{(4Ab + 5Ba)e(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)\sqrt{\frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))^2}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))\text{EllipticF}\left(\sqrt{1 - \frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))^2}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}\right)}{81(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))a^{\frac{5}{3}}b^{\frac{5}{3}}\sqrt{bx^3 + a}\sqrt{\frac{b^{\frac{1}{3}}x(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}}}$$

command

```
integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\left((5Bab^2 + 4Ab^3)x^7 + 2(5Ba^2b + 4Aab^2)x^4 + (5Ba^3 + 4Aa^2b)x\right)\sqrt{a}e^{\frac{3}{2}}\text{weierstrassZeta}\left(0, -\frac{4b}{a}, \text{weierstrass}\right)\right)}{27(a^2b^4x^7 + 2a^3b^3x^4 + a^4b^2x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bex^4 + Aex)\sqrt{bx^3 + a}\sqrt{ex}}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

18.136 Problem number 563

$$\int \frac{A + Bx^3}{\sqrt{ex}(a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba) \sqrt{ex}}{9abe (bx^3 + a)^{\frac{3}{2}}} + \frac{2(8Ab + Ba) \sqrt{ex}}{27a^2be \sqrt{bx^3 + a}}$$

$$+ \frac{2(8Ab + Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right) \text{EllipticF} \left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}}\right)}{81 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right) a^{\frac{7}{3}}be \sqrt{bx^3 + a} \sqrt{\frac{b^{\frac{1}{3}}x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}}$$

command

```
integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \left((Bab^2 + 8Ab^3)x^6 + Ba^3 + 8Aa^2b + 2(Ba^2b + 8Aab^2)x^3 \right) \sqrt{a} \text{weierstrassPInverse} \left(0, -\frac{4b}{a}, \frac{1}{x} \right) + (2Ba^3 - \dots \right)}{27(a^3b^3x^6 + 2a^4b^2x^3 + a^5b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bx^3 + A) \sqrt{bx^3 + a} \sqrt{ex}}{b^3ex^{10} + 3ab^2ex^7 + 3a^2bex^4 + a^3ex}, x \right)$$

18.137 Problem number 564

$$\int \frac{A + Bx^3}{(ex)^{3/2} (a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2(10Ab - Ba)(ex)^{\frac{5}{2}}}{9a^2e^4(bx^3 + a)^{\frac{3}{2}}} - \frac{2A}{ae(bx^3 + a)^{\frac{3}{2}}\sqrt{ex}} \\
& -\frac{8(10Ab - Ba)(ex)^{\frac{5}{2}}}{27a^3e^4\sqrt{bx^3 + a}} + \frac{8(10Ab - Ba)(1 + \sqrt{3})\sqrt{ex}\sqrt{bx^3 + a}}{27a^3b^{\frac{2}{3}}e^2(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))} \\
& -\frac{8(10Ab - Ba)(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)\sqrt{\frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))^2}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))\text{EllipticE}\left(\sqrt{1 - \frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))^2}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}\right)}{27(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))a^{\frac{8}{3}}b^{\frac{2}{3}}e^2\sqrt{bx^3 + a}\sqrt{\frac{b^{\frac{1}{3}}x(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}}} \\
& -\frac{4(10Ab - Ba)(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)\sqrt{\frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))^2}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))\text{EllipticF}\left(\sqrt{1 - \frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))^2}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}\right)}{81(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3}))a^{\frac{8}{3}}b^{\frac{2}{3}}e^2\sqrt{bx^3 + a}\sqrt{\frac{b^{\frac{1}{3}}x}{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3}))^2}}}
\end{aligned}$$

command

```
integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4((Bab^2 - 10Ab^3)x^7 + 2(Ba^2b - 10Aab^2)x^4 + (Ba^3 - 10Aa^2b)x)\sqrt{a}\text{weierstrassZeta}\left(0, -\frac{4b}{a}, \text{weierstrassP}\right)\right)}{27(a^3b^3x^7 + 2a^4b^2x^4 + a^5b)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}\sqrt{ex}}{b^3e^2x^{11} + 3ab^2e^2x^8 + 3a^2be^2x^5 + a^3e^2x^2}, x\right)$$

18.138 Problem number 566

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2A}{5ae (ex)^{\frac{5}{2}} (bx^3 + a)^{\frac{3}{2}}} - \frac{2(14Ab - 5Ba) \sqrt{ex}}{45a^2e^4 (bx^3 + a)^{\frac{3}{2}}} - \frac{16(14Ab - 5Ba) \sqrt{ex}}{135a^3e^4 \sqrt{bx^3 + a}}$$

$$- \frac{16(14Ab - 5Ba) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}}}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)} \text{EllipticF} \left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 + \sqrt{3})\right)^2}} \right)}{405 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x(1 - \sqrt{3})\right) a^{\frac{10}{3}} e^4 \sqrt{bx^3 + a} \sqrt{\frac{b^{\frac{1}{3}}x \left(a^{\frac{1}{3}}\right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}}$$

command

`integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(16 \left((5 Bab^2 - 14 Ab^3)x^9 + 2 (5 Ba^2b - 14 Aab^2)x^6 + (5 Ba^3 - 14 Aa^2b)x^3 \right) \sqrt{a} \text{weierstrassPInverse} \left(0, -\frac{4b}{a}, \frac{1}{x} \right) \right)}{135 (a^4b^2x^9 + 2a^5bx^6 + a^6)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bx^3 + A) \sqrt{bx^3 + a} \sqrt{ex}}{b^3e^4x^{13} + 3ab^2e^4x^{10} + 3a^2be^4x^7 + a^3e^4x^4}, x \right)$$

18.139 Problem number 577

$$\int \frac{\sqrt[3]{a + bx^3}}{x^2(ad - bdx^3)} dx$$

Optimal antiderivative

$$-\frac{(bx^3 + a)^{\frac{1}{3}}}{adx} + \frac{b^{\frac{1}{3}} \ln(-bdx^3 + ad) 2^{\frac{1}{3}}}{6ad} - \frac{b^{\frac{1}{3}} \ln\left(2^{\frac{1}{3}}b^{\frac{1}{3}}x - (bx^3 + a)^{\frac{1}{3}}\right) 2^{\frac{1}{3}}}{2ad}$$

$$- \frac{2^{\frac{1}{3}}b^{\frac{1}{3}} \arctan\left(\frac{\left(1 + \frac{22^{\frac{1}{3}}b^{\frac{1}{3}}x}{(bx^3 + a)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right) \sqrt{3}}{3ad}$$

command

```
integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\sqrt{3}2^{\frac{1}{3}}(-b)^{\frac{1}{3}}x \arctan\left(\frac{6\sqrt{3}2^{\frac{2}{3}}(19b^2x^8+16abx^5+a^2x^2)(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}+6\sqrt{3}2^{\frac{1}{3}}(5b^2x^7-4abx^4-a^2x)(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{1}{3}}+\sqrt{3}(7}{3(109b^3x^9+105ab^2x^6+3a^2bx^3-a^3)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19 Test file number 29

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.3_General/29_1.1.3.8_P-x-c_x^-m-a+b_x^n-p

19.1 Problem number 59

$$\int (a + bx^3)^{3/2} (ac + adx + bcx^3 + bdx^4) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{30a(187dx^2 + 247cx)(bx^3 + a)^{\frac{3}{2}}}{46189} + \frac{2(17dx^2 + 19cx)(bx^3 + a)^{\frac{5}{2}}}{323} \\ & + \frac{54a^2(935dx^2 + 1729cx)\sqrt{bx^3 + a}}{323323} + \frac{810a^3d\sqrt{bx^3 + a}}{1729b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & 4053^{\frac{1}{4}}a^{\frac{10}{3}}d\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & \frac{1729b^{\frac{2}{3}}\sqrt{bx^3 + a}}{\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & 543^{\frac{3}{4}}a^3\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(1729b^{\frac{1}{3}}c - 935a^{\frac{1}{3}}d(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & \frac{323323b^{\frac{2}{3}}\sqrt{bx^3 + a}}{\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(140049 a^3 \sqrt{b} \operatorname{cweierstrassPInverse}(0, -\frac{4a}{b}, x) - 75735 a^3 \sqrt{b} \operatorname{dweierstrassZeta}(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}(0, -\frac{4a}{b}, x)) \right)$$

3233

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((b^2 dx^7 + b^2 cx^6 + 2 abdx^4 + 2 abcx^3 + a^2 dx + a^2 c) \sqrt{bx^3 + a}, x\right)$$

19.2 Problem number 60

$$\int \sqrt{a + bx^3} (ac + adx + bcx^3 + bdx^4) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(11dx^2 + 13cx)(bx^3 + a)^{\frac{3}{2}}}{143} + \frac{18a(55dx^2 + 91cx)\sqrt{bx^3 + a}}{5005} + \frac{54a^2 d \sqrt{bx^3 + a}}{91b^{\frac{2}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + 27 \cdot 3^{\frac{1}{4}} a^{\frac{7}{3}} d \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & - \frac{91b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{91b^{\frac{2}{3}} \sqrt{bx^3 + a}} \\ & + 18 \cdot 3^{\frac{3}{4}} a^2 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(91b^{\frac{1}{3}}c - 55a^{\frac{1}{3}}d(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & + \frac{5005b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{5005b^{\frac{2}{3}} \sqrt{bx^3 + a}} \end{aligned}$$

command

`integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2457 a^2 \sqrt{b} \operatorname{cweierstrassPInverse}(0, -\frac{4a}{b}, x) - 1485 a^2 \sqrt{b} \operatorname{dweierstrassZeta}(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}(0, -\frac{4a}{b}, x)) \right)$$

5005 b

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bdx^4 + bcx^3 + adx + ac) \sqrt{bx^3 + a}, x\right)$$

19.3 Problem number 61

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5dx^2 + 7cx) \sqrt{bx^3 + a}}{35} + \frac{6ad\sqrt{bx^3 + a}}{7b^{\frac{2}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & \frac{33^{\frac{1}{4}}a^{\frac{4}{3}}d \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{7b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{23^{\frac{3}{4}}a \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(7b^{\frac{1}{3}}c - 5a^{\frac{1}{3}}d(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{35b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21 a \sqrt{b} \operatorname{cweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - 15 a \sqrt{b} d \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) \right)}{35 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{bx^3 + a} (dx + c), x\right)$$

19.4 Problem number 62

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2d\sqrt{bx^3+a}}{b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} + \frac{3^{\frac{1}{4}}a^{\frac{1}{3}}d\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} + \frac{2\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(b^{\frac{1}{3}}c-a^{\frac{1}{3}}d(1-\sqrt{3})\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{b}\operatorname{cweierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}\operatorname{dweierstrassZeta}\left(0,-\frac{4a}{b},\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{dx+c}{\sqrt{bx^3+a}},x\right)$$

19.5 Problem number 63

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2x(dx + c)}{3a\sqrt{bx^3 + a}} - \frac{2d\sqrt{bx^3 + a}}{3ab^{\frac{2}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)}$$

$$+ \frac{d\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{3a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

$$+ \frac{2\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(b^{\frac{1}{3}}c + a^{\frac{1}{3}}d(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{9ab^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((bcx^3 + ac)\sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (bdx^3 + ad)\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right)\right)}{3(ab^2x^3 + a^2b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + a}(dx + c)}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

19.6 Problem number 64

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x(dx + c)}{9a(bx^3 + a)^{\frac{3}{2}}} + \frac{2x(5dx + 7c)}{27a^2\sqrt{bx^3 + a}} - \frac{10d\sqrt{bx^3 + a}}{27a^2b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{5d\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{27a^{\frac{5}{3}}b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \cdot 3^{\frac{1}{4}} \\ & + \frac{2\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(7b^{\frac{1}{3}}c + 5a^{\frac{1}{3}}d(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{81a^2b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(7\left(b^2cx^6 + 2abcx^3 + a^2c\right)\sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 5\left(b^2dx^6 + 2abdx^3 + a^2d\right)\sqrt{b} \operatorname{weierstrassZeta}\left(0, \frac{4a}{b}\right)\right)}{27\left(a^2b^3x^6 + 2a^3b^2x^3 + a^4\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + a}(dx + c)}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

19.7 Problem number 65

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x(dx + c)}{15a(bx^3 + a)^{\frac{5}{2}}} + \frac{2x(11dx + 13c)}{135a^2(bx^3 + a)^{\frac{3}{2}}} + \frac{2x(55dx + 91c)}{405a^3\sqrt{bx^3 + a}} - \frac{22d\sqrt{bx^3 + a}}{81a^3b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{11d\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{81a^{\frac{8}{3}}b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} 3^{\frac{1}{4}} \\ & + \frac{2\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(91b^{\frac{1}{3}}c + 55a^{\frac{1}{3}}d(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{1215a^3b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(91 (b^3cx^9 + 3ab^2cx^6 + 3a^2bcx^3 + a^3c) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 55 (b^3dx^9 + 3ab^2dx^6 + 3a^2bdx^3 + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + a} (dx + c)}{b^4x^{12} + 4ab^3x^9 + 6a^2b^2x^6 + 4a^3bx^3 + a^4}, x\right)$$

19.8 Problem number 66

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e\sqrt{bx^3+a}}{3b} + \frac{2fx\sqrt{bx^3+a}}{5b} + \frac{2gx^2\sqrt{bx^3+a}}{7b} + \frac{2(-4ag+7bd)\sqrt{bx^3+a}}{7b^{\frac{5}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & 3^{\frac{1}{4}}a^{\frac{1}{3}}(-4ag+7bd)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \\ & + \frac{7b^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}{2\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(7b^{\frac{1}{3}}(-2af+5bc)-5a^{\frac{1}{3}}(-4ag+7bd)(1-\sqrt{3})\right)\left(\frac{\sqrt{6}}{2}\right)} \\ & + \frac{105b^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(21(5bc-2af)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-15(7bd-4ag)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{105b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^3+a}},x\right)$$

19.9 Problem number 67

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x(bc - af + (-ag + bd)x + be x^2)}{3ab\sqrt{bx^3 + a}} - \frac{2e\sqrt{bx^3 + a}}{3ab} - \frac{2(-4ag + bd)\sqrt{bx^3 + a}}{3ab^{5/3}\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)} \\ & (-4ag + bd)\left(a^{1/3} + b^{1/3}x\right) \text{EllipticE}\left(\frac{b^{1/3}x + a^{1/3}(1 - \sqrt{3})}{b^{1/3}x + a^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}} \\ & + \frac{3a^{2/3}b^{5/3}\sqrt{bx^3 + a} \sqrt{\frac{a^{1/3}\left(a^{1/3} + b^{1/3}x\right)}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}}{2\left(a^{1/3} + b^{1/3}x\right) \text{EllipticF}\left(\frac{b^{1/3}x + a^{1/3}(1 - \sqrt{3})}{b^{1/3}x + a^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(b^{1/3}(2af + bc) + a^{1/3}(-4ag + bd)(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)} \\ & + \frac{9ab^{5/3}\sqrt{bx^3 + a} \sqrt{\frac{a^{1/3}\left(a^{1/3} + b^{1/3}x\right)}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}}{3(ab^3x^3 + a^2)} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\left((b^2c + 2abf)x^3 + abc + 2a^2f\right)\sqrt{b} \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + \left((b^2d - 4abg)x^3 + abd - 4a^2g\right)\sqrt{b} \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right)}{3(ab^3x^3 + a^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

19.10 Problem number 68

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^3)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x(bc - af + (-ag + bd)x + be x^2)}{9ab(bx^3 + a)^{\frac{3}{2}}} \\ & - \frac{2(3ae - x(7bc + 2af + (4ag + 5bd)x))}{27a^2b\sqrt{bx^3 + a}} - \frac{2(4ag + 5bd)\sqrt{bx^3 + a}}{27a^2b^{\frac{5}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{(4ag + 5bd)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{27a^{\frac{5}{3}}b^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{2\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(b^{\frac{1}{3}}(2af + 7bc) + a^{\frac{1}{3}}(4ag + 5bd)(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{81a^2b^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(((7b^3c + 2ab^2f)x^6 + 7a^2bc + 2a^3f + 2(7ab^2c + 2a^2bf)x^3) \sqrt{b} \operatorname{weierstrassPInverse}(0, -\frac{4a}{b}, x) + ((5b^3d + 4a$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

19.11 Problem number 69

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^3)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x(bc - af + (-ag + bd)x + be x^2)}{15ab(bx^3 + a)^{\frac{5}{2}}} - \frac{2(9ae - x(13bc + 2af + (4ag + 11bd)x))}{135a^2b(bx^3 + a)^{\frac{3}{2}}} \\ & + \frac{2x(14af + 91bc + 5(4ag + 11bd)x)}{405a^3b\sqrt{bx^3 + a}} - \frac{2(4ag + 11bd)\sqrt{bx^3 + a}}{81a^3b^{\frac{5}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{(4ag + 11bd)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{81a^{\frac{8}{3}}b^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{2\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(7b^{\frac{1}{3}}(2af + 13bc) + 5a^{\frac{1}{3}}(4ag + 11bd)(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right)}{1215a^3b^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(7 \left((13b^4c + 2ab^3f)x^9 + 3(13ab^3c + 2a^2b^2f)x^6 + 13a^3bc + 2a^4f + 3(13a^2b^2c + 2a^3bf)x^3 \right) \sqrt{b} \operatorname{weierstrassPI} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{b^4x^{12} + 4ab^3x^9 + 6a^2b^2x^6 + 4a^3bx^3 + a^4}, x\right)$$

19.12 Problem number 74

$$\int \frac{(a + bx + cx^2)^3}{d + ex^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-6abce - b^3e + c^3d)x}{e^2} + \frac{3c(ac + b^2)x^2}{2e} + \frac{bc^2x^3}{e} + \frac{c^3x^4}{4e} \\ & + \frac{\left(c^3d^2 - 6abcde - e(-a^3e + b^3d) + 3d^{\frac{1}{3}}e^{\frac{2}{3}}(-a^2be + ac^2d + b^2cd)\right) \ln\left(d^{\frac{1}{3}} + e^{\frac{1}{3}}x\right)}{3d^{\frac{2}{3}}e^{\frac{7}{3}}} \\ & - \frac{\left(c^3d^2 - 6abcde - e(-a^3e + b^3d) + 3d^{\frac{1}{3}}e^{\frac{2}{3}}(-a^2be + ac^2d + b^2cd)\right) \ln\left(d^{\frac{2}{3}} - d^{\frac{1}{3}}e^{\frac{1}{3}}x + e^{\frac{2}{3}}x^2\right)}{6d^{\frac{2}{3}}e^{\frac{7}{3}}} \\ & - \frac{(-a^2ce - ab^2e + bc^2d) \ln(ex^3 + d)}{e^2} \\ & - \frac{\left(c^3d^2 - 3b^2cd^{\frac{4}{3}}e^{\frac{2}{3}} - 3ac^2d^{\frac{4}{3}}e^{\frac{2}{3}} - b^3de - 6abcde + 3a^2bd^{\frac{1}{3}}e^{\frac{5}{3}} + a^3e^2\right) \arctan\left(\frac{(d^{\frac{1}{3}} - 2e^{\frac{1}{3}}x)\sqrt{3}}{3d^{\frac{1}{3}}}\right) \sqrt{3}}{3d^{\frac{2}{3}}e^{\frac{7}{3}}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.13 Problem number 75

$$\int \frac{(a + bx + cx^2)^4}{d + ex^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(-6a^2bce - 2ab^3e + 2ac^3d + 3b^2c^2d)x}{e^2} - \frac{(-6a^2c^2e - 12ab^2ce - b^4e + 4bc^3d)x^2}{2e^2} \\
& - \frac{c(-12abce - 4b^3e + c^3d)x^3}{3e^2} + \frac{c^2(2ac + 3b^2)x^4}{2e} + \frac{4bc^3x^5}{5e} + \frac{c^4x^6}{6e} \\
& + \frac{\left(e^{\frac{1}{3}}(a^4e^2 - 12a^2bcde - 4ab^3de + 4ac^3d^2 + 6b^2c^2d^2) + d^{\frac{1}{3}}(b^4de + 12ab^2cde + 6a^2c^2de - 4b(a^3e^2 + c^3d^2))\right) \ln(d)}{3d^{\frac{2}{3}}e^{\frac{8}{3}}} \\
& - \frac{\left(e^{\frac{1}{3}}(a^4e^2 - 12a^2bcde - 4ab^3de + 4ac^3d^2 + 6b^2c^2d^2) + d^{\frac{1}{3}}(b^4de + 12ab^2cde + 6a^2c^2de - 4b(a^3e^2 + c^3d^2))\right) \ln(a)}{6d^{\frac{2}{3}}e^{\frac{8}{3}}} \\
& + \frac{(c^4d^2 - 12abc^2de + 6a^2b^2e^2 - 4ce(-a^3e + b^3d)) \ln(ex^3 + d)}{3e^3} \\
& - \frac{\left(bd^{\frac{1}{3}} + ae^{\frac{1}{3}}\right) \left(4c^3d^2 + 6c^2\left(bd^{\frac{5}{3}}e^{\frac{1}{3}} - ad^{\frac{4}{3}}e^{\frac{2}{3}}\right) - 12abcde - e\left(b^3d + 3ab^2d^{\frac{2}{3}}e^{\frac{1}{3}} - 3a^2bd^{\frac{1}{3}}e^{\frac{2}{3}} - a^3e\right)\right) \arctan\left(\frac{d^{\frac{1}{3}}}{e^{\frac{1}{3}}}\right)}{3d^{\frac{2}{3}}e^{\frac{8}{3}}}
\end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.14 Problem number 79

$$\int \frac{1 + \sqrt{3} + x}{\sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{x^3+1}}{1+x+\sqrt{3}} - \frac{3^{\frac{1}{4}}(1+x) \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} + \frac{4 \cdot 3^{\frac{1}{4}}(1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(\sqrt{3}+1) \operatorname{weierstrassPInverse}(0, -4, x) - 2 \operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}, x\right)$$

19.15 Problem number 81

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{x^3-1}}{1-x-\sqrt{3}} - \frac{3^{\frac{1}{4}}(1-x) \operatorname{EllipticE}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

command

```
integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{3} + 1 \right) \text{weierstrassPInverse}(0, 4, x) + 2 \text{weierstrassZeta}(0, 4, \text{weierstrassPInverse}(0, 4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{x - \sqrt{3} - 1}{\sqrt{x^3 - 1}}, x \right)$$

19.16 Problem number 83

$$\int \frac{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^3+a}}{b^{\frac{1}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3}) \right)} \\ 3^{\frac{1}{4}} a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \text{EllipticE} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3}) \right)^2}} \\ - \frac{b^{\frac{1}{3}} \sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3}) \right)^2}}}{b^{\frac{1}{3}} \sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3}) \right)^2}}} \\ 4 \cdot 3^{\frac{1}{4}} a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \text{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3}) \right)^2}} \\ + \frac{b^{\frac{1}{3}} \sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3}) \right)^2}}}{b^{\frac{1}{3}} \sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3}) \right)^2}}}$$

command

```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(a^{\frac{1}{3}} \sqrt{b} \left(\sqrt{3} + 1 \right) \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) - b^{\frac{5}{6}} \text{weierstrassZeta} \left(0, -\frac{4a}{b}, \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(\sqrt{3} + 1 \right)}{\sqrt{bx^3 + a}}, x \right)$$

19.17 Problem number 84

$$\int \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{b} x}{\sqrt{a - bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{-bx^3+a}}{b^{\frac{1}{3}}\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & + \frac{3^{\frac{1}{4}}a^{\frac{1}{3}}\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{b^{\frac{1}{3}}\sqrt{-bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\ & + \frac{43^{\frac{1}{4}}a^{\frac{1}{3}}\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{b^{\frac{1}{3}}\sqrt{-bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(a^{\frac{1}{3}} \sqrt{-b} \left(\sqrt{3} + 1 \right) \text{weierstrassPInverse} \left(0, \frac{4a}{b}, x \right) + \sqrt{-b} b^{\frac{1}{3}} \text{weierstrassZeta} \left(0, \frac{4a}{b}, \text{weierstrassPInverse} \left(0, \frac{4a}{b}, x \right) \right) \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{-bx^3+a} b^{\frac{1}{3}} x - \sqrt{-bx^3+a} a^{\frac{1}{3}} \left(\sqrt{3} + 1 \right)}{bx^3 - a}, x \right)$$

19.18 Problem number 85

$$\int \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{b} x}{\sqrt{-a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^3 - a}}{b^{\frac{1}{3}} \left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)} 3^{\frac{1}{4}} a^{\frac{1}{3}} \left(a^{\frac{1}{3}} - b^{\frac{1}{3}}x\right) \text{EllipticE} \left(\frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}, 2i - i\sqrt{3} \right) \sqrt{\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)$$

$$b^{\frac{1}{3}} \sqrt{bx^3 - a} \sqrt{-\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} - b^{\frac{1}{3}}x)}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)^2}}$$

command

`integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(a^{\frac{1}{3}} \sqrt{b} \left(\sqrt{3} + 1 \right) \text{weierstrassPInverse} \left(0, \frac{4a}{b}, x \right) + b^{\frac{5}{6}} \text{weierstrassZeta} \left(0, \frac{4a}{b}, \text{weierstrassPInverse} \left(0, \frac{4a}{b}, x \right) \right) \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{bx^3 - a}}, x \right)$$

19.19 Problem number 86

$$\int \frac{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{-a - bx^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{-bx^3-a}}{b^{\frac{1}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})\right)} + \frac{3^{\frac{1}{4}}a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}, 2i-i\sqrt{3}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})\right)^2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{b^{\frac{1}{3}}\sqrt{-bx^3-a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})\right)^2}}}$$

command

```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}\left(\sqrt{3}+1\right)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,\right.\right.\right.}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-bx^3-a}b^{\frac{1}{3}}x+\sqrt{-bx^3-a}a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{bx^3+a},x\right)$$

19.20 Problem number 87

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}} dx$$

Optimal antiderivative

$$\frac{2\left(\frac{b}{a}\right)^{\frac{1}{3}} \sqrt{bx^3 + a}}{b^{\frac{2}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} - \frac{3^{\frac{1}{4}} a^{\frac{1}{3}} \left(\frac{b}{a}\right)^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} + \frac{2\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(-a^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}(1 - \sqrt{3}) + b^{\frac{1}{3}}(1 + \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{3b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{b}\left(\sqrt{3} + 1\right) \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - \sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}} \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{bx^3 + a}}, x\right)$$

19.21 Problem number 88

$$\int \frac{1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\sqrt{a - bx^3}} dx$$

Optimal antiderivative

$$\frac{2\left(\frac{b}{a}\right)^{\frac{1}{3}}\sqrt{-bx^3+a}}{b^{\frac{2}{3}}\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} + \frac{3^{\frac{1}{4}}a^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})},i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{b^{\frac{2}{3}}\sqrt{-bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} + \frac{2\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})},i\sqrt{3}+2i\right)\left(-a^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}(1-\sqrt{3})+b^{\frac{1}{3}}(1+\sqrt{3})\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{3b^{\frac{2}{3}}\sqrt{-bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{-b}\left(\sqrt{3}+1\right)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},\right.\right.\right.}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-bx^3+a}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{-bx^3+a}\left(\sqrt{3}+1\right)}{bx^3-a},x\right)$$

19.22 Problem number 89

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}dx$$

Optimal antiderivative

$$\frac{2\left(\frac{b}{a}\right)^{\frac{2}{3}} \sqrt{bx^3 - a}}{b\left(1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x - \sqrt{3}\right)} + \frac{3^{\frac{1}{4}} \left(1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x\right) \text{EllipticE}\left(\frac{1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \sqrt{3}}{1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x - \sqrt{3}}, 2i - i\sqrt{3}\right) \sqrt{\frac{1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \left(\frac{b}{a}\right)^{\frac{2}{3}} x^2}{\left(1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x - \sqrt{3}\right)^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{\left(\frac{b}{a}\right)^{\frac{1}{3}} \sqrt{bx^3 - a} \sqrt{\frac{-1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x - \sqrt{3}\right)^2}}}$$

command

`integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{b}\left(\sqrt{3} + 1\right) \text{weierstrassPInverse}\left(0, \frac{4a}{b}, x\right) + \sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}} \text{weierstrassZeta}\left(0, \frac{4a}{b}, \text{weierstrassPInverse}\left(0, \frac{4a}{b}, x\right)\right)\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a}}, x\right)$$

19.23 Problem number 90

$$\int \frac{1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}} x}{\sqrt{-a - bx^3}} dx$$

Optimal antiderivative

$$\frac{2\left(\frac{b}{a}\right)^{\frac{2}{3}} \sqrt{-bx^3 - a}}{b\left(1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x - \sqrt{3}\right)} + \frac{3^{\frac{1}{4}} \left(1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x\right) \text{EllipticE}\left(\frac{1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \sqrt{3}}{1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x - \sqrt{3}}, 2i - i\sqrt{3}\right) \sqrt{\frac{1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \left(\frac{b}{a}\right)^{\frac{2}{3}} x^2}{\left(1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x - \sqrt{3}\right)^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{\left(\frac{b}{a}\right)^{\frac{1}{3}} \sqrt{-bx^3 - a} \sqrt{\frac{-1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x - \sqrt{3}\right)^2}}}$$

command

```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{-b} \left(\sqrt{3} + 1 \right) \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) - \sqrt{-b} \left(\frac{b}{a} \right)^{\frac{1}{3}} \text{weierstrassZeta} \left(0, -\frac{4a}{b}, \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{-bx^3 - a} x \left(\frac{b}{a} \right)^{\frac{1}{3}} + \sqrt{-bx^3 - a} \left(\sqrt{3} + 1 \right)}{bx^3 + a}, x \right)$$

19.24 Problem number 91

$$\int \frac{1 - \sqrt{3} + x}{\sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{x^3+1}}{1+x+\sqrt{3}} - \frac{3^{\frac{1}{4}}(1+x) \text{EllipticE} \left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -2 \left(\sqrt{3} - 1 \right) \text{weierstrassPInverse} \left(0, -4, x \right) \\ & - 2 \text{weierstrassZeta} \left(0, -4, \text{weierstrassPInverse} \left(0, -4, x \right) \right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{x - \sqrt{3} + 1}{\sqrt{x^3 + 1}}, x \right)$$

19.25 Problem number 93

$$\int \frac{1 - \sqrt{3} - x}{\sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{x^3 - 1}}{1 - x - \sqrt{3}} + \frac{43^{\frac{1}{4}}(1 - x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i - i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 + x + 1}{(1 - x - \sqrt{3})^2}}}{\sqrt{x^3 - 1} \sqrt{\frac{-1 + x}{(1 - x - \sqrt{3})^2}}} - \frac{3^{\frac{1}{4}}(1 - x) \operatorname{EllipticE}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i - i\sqrt{3}\right) \sqrt{\frac{x^2 + x + 1}{(1 - x - \sqrt{3})^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{\sqrt{x^3 - 1} \sqrt{\frac{-1 + x}{(1 - x - \sqrt{3})^2}}}$$

command

`integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2 \left(\sqrt{3} - 1 \right) \operatorname{weierstrassPInverse}(0, 4, x) + 2 \operatorname{weierstrassZeta}(0, 4, \operatorname{weierstrassPInverse}(0, 4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{x + \sqrt{3} - 1}{\sqrt{x^3 - 1}}, x\right)$$

19.26 Problem number 95

$$\int \frac{-1 + \sqrt{3} - x}{\sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{x^3 + 1}}{1 + x + \sqrt{3}} + \frac{3^{\frac{1}{4}}(1 + x) \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1 + x + \sqrt{3})^2}}}{\sqrt{x^3 + 1} \sqrt{\frac{1 + x}{(1 + x + \sqrt{3})^2}}}$$

command

```
integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,-4,x)+2\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}},x\right)$$

19.27 Problem number 97

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{x^3-1}}{1-x-\sqrt{3}} \\ & \frac{43^{\frac{1}{4}}(1-x)\text{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}},2i-i\sqrt{3}\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}}}{\sqrt{x^3-1}\sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}} \\ & +\frac{3^{\frac{1}{4}}(1-x)\text{EllipticE}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}},2i-i\sqrt{3}\right)\sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-1}\sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,4,x)-2\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}},x\right)$$

19.28 Problem number 99

$$\int \frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^3 + a}}{b^{\frac{1}{3}} \left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\ 3^{\frac{1}{4}} a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \text{EllipticE} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} \\ \frac{b^{\frac{1}{3}} \sqrt{bx^3 + a}}{\sqrt{\frac{a^{\frac{1}{3}} (a^{\frac{1}{3}} + b^{\frac{1}{3}} x)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}$$

command

`integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(a^{\frac{1}{3}} \sqrt{b} \left(\sqrt{3} - 1 \right) \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) + b^{\frac{5}{6}} \text{weierstrassZeta} \left(0, -\frac{4a}{b}, \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}} (\sqrt{3} - 1)}{\sqrt{bx^3 + a}}, x \right)$$

19.29 Problem number 100

$$\int \frac{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{b} x}{\sqrt{a - bx^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{-bx^3+a}}{b^{\frac{1}{3}}\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} + \frac{3^{\frac{1}{4}}a^{\frac{1}{3}}\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{b^{\frac{1}{3}}\sqrt{-bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}\left(\sqrt{3}-1\right)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-bx^3+a}b^{\frac{1}{3}}x+\sqrt{-bx^3+a}a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{bx^3-a},x\right)$$

19.30 Problem number 101

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^3 - a}}{b^{\frac{1}{3}} \left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)} + \frac{43^{\frac{1}{4}}a^{\frac{1}{3}} \left(a^{\frac{1}{3}} - b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}, 2i - i\sqrt{3}\right) \sqrt{\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)^2}} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right)}{b^{\frac{1}{3}} \sqrt{bx^3 - a} \sqrt{-\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} - b^{\frac{1}{3}}x)}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)^2}}} + \frac{3^{\frac{1}{4}}a^{\frac{1}{3}} \left(a^{\frac{1}{3}} - b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}, 2i - i\sqrt{3}\right) \sqrt{\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{b^{\frac{1}{3}} \sqrt{bx^3 - a} \sqrt{-\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} - b^{\frac{1}{3}}x)}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)^2}}}$$

command

```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(a^{\frac{1}{3}} \sqrt{b} \left(\sqrt{3} - 1\right) \operatorname{weierstrassPInverse}\left(0, \frac{4a}{b}, x\right) - b^{\frac{5}{6}} \operatorname{weierstrassZeta}\left(0, \frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, \frac{4a}{b}, x\right)\right)\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} - 1)}{\sqrt{bx^3 - a}}, x\right)$$

19.31 Problem number 102

$$\int \frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{-a - bx^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{-bx^3-a}}{b^{\frac{1}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})\right)} + \frac{4\sqrt[4]{3}a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}, 2i-i\sqrt{3}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})\right)^2}}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{b^{\frac{1}{3}}\sqrt{-bx^3-a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})\right)^2}}} + \frac{3^{\frac{1}{4}}a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}, 2i-i\sqrt{3}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})\right)^2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{b^{\frac{1}{3}}\sqrt{-bx^3-a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})\right)^2}}}$$

command

```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}\left(\sqrt{3}-1\right)\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{-b}b^{\frac{1}{3}}\operatorname{weierstrassZeta}\left(0,-\frac{4a}{b},\operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-bx^3-a}b^{\frac{1}{3}}x-\sqrt{-bx^3-a}a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{bx^3+a},x\right)$$

19.32 Problem number 103

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}} dx$$

Optimal antiderivative

$$\frac{2\left(\frac{b}{a}\right)^{\frac{2}{3}} \sqrt{bx^3 + a}}{b\left(1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \sqrt{3}\right)} + \frac{3^{\frac{1}{4}} \left(1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x\right) \operatorname{EllipticE}\left(\frac{1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x - \sqrt{3}}{1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \left(\frac{b}{a}\right)^{\frac{2}{3}} x^2}{\left(1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \sqrt{3}\right)^2}}}{\left(\frac{b}{a}\right)^{\frac{1}{3}} \sqrt{bx^3 + a} \sqrt{\frac{1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \sqrt{3}\right)^2}}}$$

command

`integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{b}\left(\sqrt{3}-1\right) \operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right) + \sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}} \operatorname{weierstrassZeta}\left(0,-\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0,-\frac{4a}{b}\right)\right)\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a}}, x\right)$$

19.33 Problem number 104

$$\int \frac{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}} x}{\sqrt{a - bx^3}} dx$$

Optimal antiderivative

$$\frac{2\left(\frac{b}{a}\right)^{\frac{2}{3}} \sqrt{-bx^3 + a}}{b\left(1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \sqrt{3}\right)} + \frac{3^{\frac{1}{4}} \left(1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x\right) \operatorname{EllipticE}\left(\frac{1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x - \sqrt{3}}{1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{1 + \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \left(\frac{b}{a}\right)^{\frac{2}{3}} x^2}{\left(1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \sqrt{3}\right)^2}}}{\left(\frac{b}{a}\right)^{\frac{1}{3}} \sqrt{-bx^3 + a} \sqrt{\frac{1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1 - \left(\frac{b}{a}\right)^{\frac{1}{3}} x + \sqrt{3}\right)^2}}} +$$

command

```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{-b} \left(\sqrt{3} - 1 \right) \text{weierstrassPInverse} \left(0, \frac{4a}{b}, x \right) - \sqrt{-b} \left(\frac{b}{a} \right)^{\frac{1}{3}} \text{weierstrassZeta} \left(0, \frac{4a}{b}, \text{weierstrassPInverse} \left(0, \frac{4a}{b}, x \right) \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{-bx^3 + a} x \left(\frac{b}{a} \right)^{\frac{1}{3}} + \sqrt{-bx^3 + a} \left(\sqrt{3} - 1 \right)}{bx^3 - a}, x \right)$$

19.34 Problem number 105

$$\int \frac{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}} x}{\sqrt{-a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2 \left(\frac{b}{a} \right)^{\frac{1}{3}} \sqrt{bx^3 - a}}{b^{\frac{2}{3}} \left(-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right) \right)}$$

$$2 \left(a^{\frac{1}{3}} - b^{\frac{1}{3}} x \right) \text{EllipticF} \left(\frac{-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 + \sqrt{3} \right)}{-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right)}, 2i - i\sqrt{3} \right) \left(b^{\frac{1}{3}} \left(1 - \sqrt{3} \right) - a^{\frac{1}{3}} \left(\frac{b}{a} \right)^{\frac{1}{3}} \left(1 + \sqrt{3} \right) \right) \sqrt{\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}} b^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right) \right)^2}}$$

$$3b^{\frac{2}{3}} \sqrt{bx^3 - a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} - b^{\frac{1}{3}} x \right)}{\left(-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right) \right)^2}}$$

$$3^{\frac{1}{4}} a^{\frac{1}{3}} \left(\frac{b}{a} \right)^{\frac{1}{3}} \left(a^{\frac{1}{3}} - b^{\frac{1}{3}} x \right) \text{EllipticE} \left(\frac{-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 + \sqrt{3} \right)}{-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right)}, 2i - i\sqrt{3} \right) \sqrt{\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right) \right)^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right)$$

$$b^{\frac{2}{3}} \sqrt{bx^3 - a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} - b^{\frac{1}{3}} x \right)}{\left(-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right) \right)^2}}$$

command

```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{b} \left(\sqrt{3} - 1 \right) \text{weierstrassPInverse} \left(0, \frac{4a}{b}, x \right) - \sqrt{b} \left(\frac{b}{a} \right)^{\frac{1}{3}} \text{weierstrassZeta} \left(0, \frac{4a}{b}, \text{weierstrassPInverse} \left(0, \frac{4a}{b}, x \right) \right) \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{x \left(\frac{b}{a} \right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{bx^3 - a}}, x \right)$$

19.35 Problem number 106

$$\int \frac{1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}} x}{\sqrt{-a - bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \left(\frac{b}{a} \right)^{\frac{1}{3}} \sqrt{-bx^3 - a}}{b^{\frac{2}{3}} \left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right) \right)} \\ & + \frac{2 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \text{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 + \sqrt{3} \right)}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right)}, 2i - i\sqrt{3} \right) \left(b^{\frac{1}{3}} \left(1 - \sqrt{3} \right) - a^{\frac{1}{3}} \left(\frac{b}{a} \right)^{\frac{1}{3}} \left(1 + \sqrt{3} \right) \right)}{\sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right) \right)^2}}} \\ & + \frac{3b^{\frac{2}{3}} \sqrt{-bx^3 - a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right) \right)^2}}}{3^{\frac{3}{4}} a^{\frac{1}{3}} \left(\frac{b}{a} \right)^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \text{EllipticE} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 + \sqrt{3} \right)}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right)}, 2i - i\sqrt{3} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right) \right)^2}}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \\ & + \frac{b^{\frac{2}{3}} \sqrt{-bx^3 - a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right) \right)^2}}}{b^{\frac{2}{3}} \sqrt{-bx^3 - a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right) \right)^2}}} \end{aligned}$$

command

```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{-b} \left(\sqrt{3} - 1 \right) \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) + \sqrt{-b} \left(\frac{b}{a} \right)^{\frac{1}{3}} \text{weierstrassZeta} \left(0, -\frac{4a}{b}, \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{-bx^3 - a} x \left(\frac{b}{a} \right)^{\frac{1}{3}} - \sqrt{-bx^3 - a} \left(\sqrt{3} - 1 \right)}{bx^3 + a}, x \right)$$

19.36 Problem number 107

$$\int \frac{c + dx}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2d\sqrt{bx^3 + a}}{b^{\frac{2}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} \left(1 + \sqrt{3} \right) \right)} + \frac{3^{\frac{1}{4}} a^{\frac{1}{3}} d \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \text{EllipticE} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right)}{b^{\frac{1}{3}}x + a^{\frac{1}{3}} \left(1 + \sqrt{3} \right)}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} \left(1 + \sqrt{3} \right) \right)^2}}}{b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} \left(1 + \sqrt{3} \right) \right)^2}}} + \frac{2 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \text{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}} \left(1 - \sqrt{3} \right)}{b^{\frac{1}{3}}x + a^{\frac{1}{3}} \left(1 + \sqrt{3} \right)}, i\sqrt{3} + 2i \right) \left(b^{\frac{1}{3}}c - a^{\frac{1}{3}}d \left(1 - \sqrt{3} \right) \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} \left(1 + \sqrt{3} \right) \right)^2}}}{3b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} \left(1 + \sqrt{3} \right) \right)^2}}}$$

command

`integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{b} c \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) - \sqrt{b} d \text{weierstrassZeta} \left(0, -\frac{4a}{b}, \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{dx + c}{\sqrt{bx^3 + a}}, x \right)$$

19.37 Problem number 108

$$\int \frac{c + dx}{\sqrt{a - bx^3}} dx$$

Optimal antiderivative

$$\frac{2d\sqrt{-bx^3+a}}{b^{\frac{2}{3}}\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} + \frac{3^{\frac{1}{4}}a^{\frac{1}{3}}d\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{b^{\frac{2}{3}}\sqrt{-bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} + \frac{2\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(b^{\frac{1}{3}}c+a^{\frac{1}{3}}d(1-\sqrt{3})\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3b^{\frac{2}{3}}\sqrt{-bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

```
integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{-b}\text{cweierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}\text{dweierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-bx^3+a}(dx+c)}{bx^3-a},x\right)$$

19.38 Problem number 109

$$\int \frac{c + dx}{\sqrt{-a + bx^3}} dx$$

Optimal antiderivative

$$\frac{\frac{2d\sqrt{bx^3 - a}}{b^{\frac{2}{3}}(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))} + 2(a^{\frac{1}{3}} - b^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}, 2i - i\sqrt{3}\right) (b^{\frac{1}{3}}c + a^{\frac{1}{3}}d(1 + \sqrt{3})) \sqrt{\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))^2}}}{3b^{\frac{2}{3}}\sqrt{bx^3 - a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} - b^{\frac{1}{3}}x)}{(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))^2}} + 3^{\frac{1}{4}}a^{\frac{1}{3}}d(a^{\frac{1}{3}} - b^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}, 2i - i\sqrt{3}\right) \sqrt{\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) + \frac{b^{\frac{2}{3}}\sqrt{bx^3 - a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} - b^{\frac{1}{3}}x)}{(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))^2}}}{b}}$$

command

```
integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{b} \operatorname{cweierstrassPInverse}\left(0, \frac{4a}{b}, x\right) - \sqrt{b} \operatorname{dweierstrassZeta}\left(0, \frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, \frac{4a}{b}, x\right)\right)\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{dx + c}{\sqrt{bx^3 - a}}, x\right)$$

19.39 Problem number 110

$$\int \frac{c + dx}{\sqrt{-a - bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2d\sqrt{-bx^3 - a}}{b^{\frac{2}{3}}(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))} \\ & + \frac{2(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}, 2i - i\sqrt{3}\right) (b^{\frac{1}{3}}c - a^{\frac{1}{3}}d(1 + \sqrt{3})) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))^2}} \left(\frac{\sqrt{6}}{2}\right)}{3b^{\frac{2}{3}}\sqrt{-bx^3 - a} \sqrt{-\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))^2}}} \\ & + \frac{3^{\frac{1}{4}}a^{\frac{1}{3}}d(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}, 2i - i\sqrt{3}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))^2}} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{b^{\frac{2}{3}}\sqrt{-bx^3 - a} \sqrt{-\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))^2}}} \end{aligned}$$

command

`integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{-b} \operatorname{cweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - \sqrt{-b} \operatorname{dweierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right)\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-bx^3 - a}(dx + c)}{bx^3 + a}, x\right)$$

19.40 Problem number 111

$$\int \frac{c + dx}{\sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2d\sqrt{x^3+1}}{1+x+\sqrt{3}} - \frac{3^{\frac{1}{4}}d(1+x)\operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{\sqrt{x^3+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

$$+ \frac{2(1+x)\operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)(c-d(1-\sqrt{3}))\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}3^{\frac{3}{4}}}{3\sqrt{x^3+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\operatorname{cweierstrassPInverse}(0,-4,x) - 2d\operatorname{weierstrassZeta}(0,-4,\operatorname{weierstrassPInverse}(0,-4,x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{dx+c}{\sqrt{x^3+1}},x\right)$$

19.41 Problem number 113

$$\int \frac{c + dx}{\sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2d\sqrt{x^3-1}}{1-x-\sqrt{3}} - \frac{2(1-x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) (c+d+d\sqrt{3}) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}}}{3\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}} + \frac{3^{\frac{1}{4}}d(1-x) \operatorname{EllipticE}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}} \left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

command

```
integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{cweierstrassPInverse}(0, 4, x) - 2 \operatorname{dweierstrassZeta}(0, 4, \operatorname{weierstrassPInverse}(0, 4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{dx+c}{\sqrt{x^3-1}}, x\right)$$

19.42 Problem number 115

$$\int \frac{c+dx}{a-bx^4} dx$$

Optimal antiderivative

$$\frac{c \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)}{2a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{c \operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)}{2a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

command

```
integrate((d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.43 Problem number 116

$$\int \frac{c + dx}{a + bx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c \arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{4a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{c \arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{4a^{\frac{3}{4}}b^{\frac{1}{4}}} \\ & - \frac{c \ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \sqrt{2}}{8a^{\frac{3}{4}}b^{\frac{1}{4}}} \\ & + \frac{c \ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \sqrt{2}}{8a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} \end{aligned}$$

command

```
integrate((d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.44 Problem number 117

$$\int \frac{c + dx}{(a - bx^4)^2} dx$$

Optimal antiderivative

$$\frac{x(dx + c)}{4a(-bx^4 + a)} + \frac{3c \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)}{8a^{\frac{7}{4}}b^{\frac{1}{4}}} + \frac{3c \operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)}{8a^{\frac{7}{4}}b^{\frac{1}{4}}} + \frac{d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}}$$

command

```
integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.45 Problem number 118

$$\int \frac{c + dx}{(a + bx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(dx + c)}{4a(bx^4 + a)} + \frac{3c \arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{1}{4}}} \\ & + \frac{3c \arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{1}{4}}} - \frac{3c \ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{1}{4}}} \\ & + \frac{3c \ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{1}{4}}} + \frac{d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}} \end{aligned}$$

command

```
integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.46 Problem number 119

$$\int \frac{c + dx}{(a - bx^4)^3} dx$$

Optimal antiderivative

$$\frac{x(dx + c)}{8a(-bx^4 + a)^2} + \frac{x(6dx + 7c)}{32a^2(-bx^4 + a)} + \frac{21c \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)}{64a^{\frac{11}{4}}b^{\frac{1}{4}}} + \frac{21c \operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)}{64a^{\frac{11}{4}}b^{\frac{1}{4}}} + \frac{3d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}}$$

command

```
integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.47 Problem number 120

$$\int \frac{c + dx}{(a + bx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(dx + c)}{8a(bx^4 + a)^2} + \frac{x(6dx + 7c)}{32a^2(bx^4 + a)} + \frac{21c \arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{1}{4}}} \\ & + \frac{21c \arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{1}{4}}} - \frac{21c \ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{1}{4}}} \\ & + \frac{21c \ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{1}{4}}} + \frac{3d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}} \end{aligned}$$

command

```
integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.48 Problem number 121

$$\int \frac{c + dx}{(a - bx^4)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(dx + c)}{12a(-bx^4 + a)^3} + \frac{x(10dx + 11c)}{96a^2(-bx^4 + a)^2} + \frac{x(60dx + 77c)}{384a^3(-bx^4 + a)} \\ & + \frac{77c \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)}{256a^{\frac{15}{4}}b^{\frac{1}{4}}} + \frac{77c \operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)}{256a^{\frac{15}{4}}b^{\frac{1}{4}}} + \frac{5d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}} \end{aligned}$$

command

```
integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.49 Problem number 122

$$\int \frac{c + dx}{(a + bx^4)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(dx + c)}{12a(bx^4 + a)^3} + \frac{x(10dx + 11c)}{96a^2(bx^4 + a)^2} + \frac{x(60dx + 77c)}{384a^3(bx^4 + a)} + \frac{77c \arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{512a^{\frac{15}{4}}b^{\frac{1}{4}}} \\ & + \frac{77c \arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{512a^{\frac{15}{4}}b^{\frac{1}{4}}} - \frac{77c \ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \sqrt{2}}{1024a^{\frac{15}{4}}b^{\frac{1}{4}}} \\ & + \frac{77c \ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \sqrt{2}}{1024a^{\frac{15}{4}}b^{\frac{1}{4}}} + \frac{5d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}} \end{aligned}$$

command

```
integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.50 Problem number 124

$$\int \frac{c + dx}{1 + x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d \arctan(x^2)}{2} + \frac{c \arctan(-1 + x\sqrt{2}) \sqrt{2}}{4} + \frac{c \arctan(1 + x\sqrt{2}) \sqrt{2}}{4} \\ & - \frac{c \ln(1 + x^2 - x\sqrt{2}) \sqrt{2}}{8} + \frac{c \ln(1 + x^2 + x\sqrt{2}) \sqrt{2}}{8} \end{aligned}$$

command

```
integrate((d*x+c)/(x^4+1),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.51 Problem number 125

$$\int \frac{c + dx + ex^2}{a - bx^4} dx$$

Optimal antiderivative

$$\frac{d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(-e\sqrt{a} + c\sqrt{b})}{2a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(e\sqrt{a} + c\sqrt{b})}{2a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

command

```
integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.52 Problem number 126

$$\int \frac{c + dx + ex^2}{a + bx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d \operatorname{arctan}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)(-e\sqrt{a} + c\sqrt{b})\sqrt{2}}{8a^{\frac{3}{4}}b^{\frac{3}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)(-e\sqrt{a} + c\sqrt{b})\sqrt{2}}{8a^{\frac{3}{4}}b^{\frac{3}{4}}} \\ & + \frac{\operatorname{arctan}\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)(e\sqrt{a} + c\sqrt{b})\sqrt{2}}{4a^{\frac{3}{4}}b^{\frac{3}{4}}} \\ & + \frac{\operatorname{arctan}\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)(e\sqrt{a} + c\sqrt{b})\sqrt{2}}{4a^{\frac{3}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.53 Problem number 127

$$\int \frac{c + dx + ex^2}{(a - bx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex^2 + dx + c)}{4a(-bx^4 + a)} + \frac{d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}} \\ & + \frac{\operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(-e\sqrt{a} + 3c\sqrt{b})}{8a^{\frac{7}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(e\sqrt{a} + 3c\sqrt{b})}{8a^{\frac{7}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.54 Problem number 128

$$\int \frac{c + dx + ex^2}{(a + bx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex^2 + dx + c)}{4a(bx^4 + a)} + \frac{d \operatorname{arctan}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)(-e\sqrt{a} + 3c\sqrt{b})\sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{3}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)(-e\sqrt{a} + 3c\sqrt{b})\sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{3}{4}}} \\ & + \frac{\operatorname{arctan}\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)(e\sqrt{a} + 3c\sqrt{b})\sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{3}{4}}} \\ & + \frac{\operatorname{arctan}\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)(e\sqrt{a} + 3c\sqrt{b})\sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.55 Problem number 129

$$\int \frac{c + dx + ex^2}{(a - bx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex^2 + dx + c)}{8a(-bx^4 + a)^2} + \frac{x(5ex^2 + 6dx + 7c)}{32a^2(-bx^4 + a)} + \frac{3d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}} \\ & + \frac{\operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(-5e\sqrt{a} + 21c\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(5e\sqrt{a} + 21c\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.56 Problem number 130

$$\int \frac{c + dx + ex^2}{(a + bx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{x(e x^2 + dx + c)}{8a(b x^4 + a)^2} + \frac{x(5e x^2 + 6dx + 7c)}{32a^2(b x^4 + a)} + \frac{3d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}} \\
& - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{3}{4}}} \\
& + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{3}{4}}} \\
& + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{3}{4}}} \\
& + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{3}{4}}}
\end{aligned}$$

command

```
integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.57 Problem number 131

$$\int \frac{c + dx + ex^2}{(a - bx^4)^4} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{x(e x^2 + dx + c)}{12a(-b x^4 + a)^3} + \frac{x(9e x^2 + 10dx + 11c)}{96a^2(-b x^4 + a)^2} + \frac{x(45e x^2 + 60dx + 77c)}{384a^3(-b x^4 + a)} + \frac{5d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}} \\
& + \frac{\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\left(-15e\sqrt{a} + 77c\sqrt{b}\right)}{256a^{\frac{15}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\left(15e\sqrt{a} + 77c\sqrt{b}\right)}{256a^{\frac{15}{4}}b^{\frac{3}{4}}}
\end{aligned}$$

command

```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.58 Problem number 132

$$\int \frac{c + dx + ex^2}{(a + bx^4)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex^2 + dx + c)}{12a(bx^4 + a)^3} + \frac{x(9ex^2 + 10dx + 11c)}{96a^2(bx^4 + a)^2} + \frac{x(45ex^2 + 60dx + 77c)}{384a^3(bx^4 + a)} \\ & + \frac{5d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}} - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-15e\sqrt{a} + 77c\sqrt{b}\right)\sqrt{2}}{1024a^{\frac{15}{4}}b^{\frac{3}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-15e\sqrt{a} + 77c\sqrt{b}\right)\sqrt{2}}{1024a^{\frac{15}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(15e\sqrt{a} + 77c\sqrt{b}\right)\sqrt{2}}{512a^{\frac{15}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(15e\sqrt{a} + 77c\sqrt{b}\right)\sqrt{2}}{512a^{\frac{15}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.59 Problem number 149

$$\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{af + bx(ex^2 + dx + c)}{4ab(-bx^4 + a)} + \frac{d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}} \\ & + \frac{\operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(-e\sqrt{a} + 3c\sqrt{b})}{8a^{\frac{7}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(e\sqrt{a} + 3c\sqrt{b})}{8a^{\frac{7}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.60 Problem number 150

$$\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(5ex^2 + 6dx + 7c)}{32a^2(-bx^4 + a)} + \frac{af + bx(ex^2 + dx + c)}{8ab(-bx^4 + a)^2} + \frac{3d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}} \\ & + \frac{\operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(-5e\sqrt{a} + 21c\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(5e\sqrt{a} + 21c\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.61 Problem number 151

$$\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(9ex^2 + 10dx + 11c)}{96a^2(-bx^4 + a)^2} + \frac{x(45ex^2 + 60dx + 77c)}{384a^3(-bx^4 + a)} \\ & + \frac{af + bx(ex^2 + dx + c)}{12ab(-bx^4 + a)^3} + \frac{5d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}} \\ & + \frac{\operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(-15e\sqrt{a} + 77c\sqrt{b})}{256a^{\frac{15}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(15e\sqrt{a} + 77c\sqrt{b})}{256a^{\frac{15}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.62 Problem number 154

$$\int \frac{a + bx}{2 + 3x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a \operatorname{arctan}\left(-1 + 6^{\frac{1}{4}}x\right) 6^{\frac{3}{4}}}{24} + \frac{a \operatorname{arctan}\left(1 + 6^{\frac{1}{4}}x\right) 6^{\frac{3}{4}}}{24} - \frac{a \ln\left(-6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) 6^{\frac{3}{4}}}{48} \\ & + \frac{a \ln\left(6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) 6^{\frac{3}{4}}}{48} + \frac{b \operatorname{arctan}\left(\frac{x^2\sqrt{6}}{2}\right) \sqrt{6}}{12} \end{aligned}$$

command

```
integrate((b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.63 Problem number 157

$$\int \frac{bx + cx^2}{2 + 3x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c \arctan\left(-1 + 6^{\frac{1}{4}}x\right) 6^{\frac{1}{4}}}{12} + \frac{c \arctan\left(1 + 6^{\frac{1}{4}}x\right) 6^{\frac{1}{4}}}{12} + \frac{c \ln\left(-6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) 6^{\frac{1}{4}}}{24} \\ & - \frac{c \ln\left(6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) 6^{\frac{1}{4}}}{24} + \frac{b \arctan\left(\frac{x^2\sqrt{6}}{2}\right) \sqrt{6}}{12} \end{aligned}$$

command

```
integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.64 Problem number 158

$$\int \frac{a + bx + cx^2}{2 + 3x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b \arctan\left(\frac{x^2\sqrt{6}}{2}\right) \sqrt{6}}{12} - \frac{\ln\left(-6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) (-2c + a\sqrt{6}) 6^{\frac{1}{4}}}{48} \\ & + \frac{\ln\left(6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) (-2c + a\sqrt{6}) 6^{\frac{1}{4}}}{48} \\ & + \frac{\arctan\left(-1 + 6^{\frac{1}{4}}x\right) (2c + a\sqrt{6}) 6^{\frac{1}{4}}}{24} + \frac{\arctan\left(1 + 6^{\frac{1}{4}}x\right) (2c + a\sqrt{6}) 6^{\frac{1}{4}}}{24} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.65 Problem number 162

$$\int \frac{a + bx + dx^3}{2 + 3x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a \arctan\left(-1 + 6^{\frac{1}{4}}x\right) 6^{\frac{3}{4}}}{24} + \frac{a \arctan\left(1 + 6^{\frac{1}{4}}x\right) 6^{\frac{3}{4}}}{24} + \frac{d \ln(3x^4 + 2)}{12} \\ & - \frac{a \ln\left(-6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) 6^{\frac{3}{4}}}{48} + \frac{a \ln\left(6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) 6^{\frac{3}{4}}}{48} + \frac{b \arctan\left(\frac{x^2\sqrt{6}}{2}\right) \sqrt{6}}{12} \end{aligned}$$

command

```
integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.66 Problem number 165

$$\int \frac{bx + cx^2 + dx^3}{2 + 3x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c \arctan\left(-1 + 6^{\frac{1}{4}}x\right) 6^{\frac{1}{4}}}{12} + \frac{c \arctan\left(1 + 6^{\frac{1}{4}}x\right) 6^{\frac{1}{4}}}{12} + \frac{d \ln(3x^4 + 2)}{12} \\ & + \frac{c \ln\left(-6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) 6^{\frac{1}{4}}}{24} - \frac{c \ln\left(6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) 6^{\frac{1}{4}}}{24} + \frac{b \arctan\left(\frac{x^2\sqrt{6}}{2}\right) \sqrt{6}}{12} \end{aligned}$$

command

```
integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.67 Problem number 166

$$\int \frac{a + bx + cx^2 + dx^3}{2 + 3x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d \ln(3x^4 + 2)}{12} + \frac{b \arctan\left(\frac{x^2\sqrt{6}}{2}\right) \sqrt{6}}{12} - \frac{\ln\left(-6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) (-2c + a\sqrt{6}) 6^{\frac{1}{4}}}{48} \\ & + \frac{\ln\left(6^{\frac{3}{4}}x + 3x^2 + \sqrt{6}\right) (-2c + a\sqrt{6}) 6^{\frac{1}{4}}}{48} \\ & + \frac{\arctan\left(-1 + 6^{\frac{1}{4}}x\right) (2c + a\sqrt{6}) 6^{\frac{1}{4}}}{24} + \frac{\arctan\left(1 + 6^{\frac{1}{4}}x\right) (2c + a\sqrt{6}) 6^{\frac{1}{4}}}{24} \end{aligned}$$

command

```
integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.68 Problem number 169

$$\int \frac{1 + x + x^2 + x^3}{a - bx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\ln(-bx^4 + a)}{4b} - \frac{\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (\sqrt{a} - \sqrt{b})}{2a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (\sqrt{a} + \sqrt{b})}{2a^{\frac{3}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.69 Problem number 170

$$\int \frac{1+x+x^2+x^3}{a+bx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\ln(bx^4+a)}{4b} + \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) (\sqrt{a} - \sqrt{b}) \sqrt{2}}{8a^{\frac{3}{4}}b^{\frac{3}{4}}} \\ & - \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) (\sqrt{a} - \sqrt{b}) \sqrt{2}}{8a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{\arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (\sqrt{a} + \sqrt{b}) \sqrt{2}}{4a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (\sqrt{a} + \sqrt{b}) \sqrt{2}}{4a^{\frac{3}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.70 Problem number 171

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{gx}{b} - \frac{f \ln(-bx^4+a)}{4b} + \frac{d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} \\ & + \frac{\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (bc+ag-e\sqrt{a}\sqrt{b})}{2a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (bc+ag+e\sqrt{a}\sqrt{b})}{2a^{\frac{3}{4}}b^{\frac{5}{4}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.71 Problem number 172

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a - bx^4)^2} dx$$

Optimal antiderivative

$$\frac{x(bf x^3 + be x^2 + bdx + ag + bc)}{4ab(-bx^4 + a)} + \frac{d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}}$$

$$+ \frac{\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(3bc - ag - e\sqrt{a}\sqrt{b})}{8a^{\frac{7}{4}}b^{\frac{5}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(3bc - ag + e\sqrt{a}\sqrt{b})}{8a^{\frac{7}{4}}b^{\frac{5}{4}}}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.72 Problem number 173

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a - bx^4)^3} dx$$

Optimal antiderivative

$$\frac{x(bf x^3 + be x^2 + bdx + ag + bc)}{8ab(-bx^4 + a)^2} + \frac{4af + x(5be x^2 + 6bdx - ag + 7bc)}{32a^2b(-bx^4 + a)} + \frac{3d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}}$$

$$+ \frac{\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(21bc - 3ag - 5e\sqrt{a}\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{5}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)(21bc - 3ag + 5e\sqrt{a}\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{5}{4}}}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.73 Problem number 174

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a - bx^4)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(bfx^3 + bex^2 + bdx + ag + bc)}{12ab(-bx^4 + a)^3} + \frac{x(45bex^2 + 60bdx - 7ag + 77bc)}{384a^3b(-bx^4 + a)} \\ & + \frac{8af + x(9bex^2 + 10bdx - ag + 11bc)}{96a^2b(-bx^4 + a)^2} + \frac{5d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}} \\ & + \frac{\operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (77bc - 7ag - 15e\sqrt{a}\sqrt{b})}{256a^{\frac{15}{4}}b^{\frac{5}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (77bc - 7ag + 15e\sqrt{a}\sqrt{b})}{256a^{\frac{15}{4}}b^{\frac{5}{4}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.74 Problem number 175

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{a + bx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{gx}{b} + \frac{f \ln(bx^4 + a)}{4b} + \frac{d \operatorname{arctan}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) (bc - ag - e\sqrt{a}\sqrt{b}) \sqrt{2}}{8a^{\frac{3}{4}}b^{\frac{5}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) (bc - ag - e\sqrt{a}\sqrt{b}) \sqrt{2}}{8a^{\frac{3}{4}}b^{\frac{5}{4}}} \\ & + \frac{\operatorname{arctan}\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (bc - ag + e\sqrt{a}\sqrt{b}) \sqrt{2}}{4a^{\frac{3}{4}}b^{\frac{5}{4}}} \\ & + \frac{\operatorname{arctan}\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (bc - ag + e\sqrt{a}\sqrt{b}) \sqrt{2}}{4a^{\frac{3}{4}}b^{\frac{5}{4}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.75 Problem number 176

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(bf x^3 + be x^2 + bdx - ag + bc)}{4ab(bx^4 + a)} + \frac{d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(3bc + ag - e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{5}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(3bc + ag - e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{5}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(3bc + ag + e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{5}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(3bc + ag + e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{5}{4}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.76 Problem number 177

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(bf x^3 + be x^2 + bdx - ag + bc)}{8ab(bx^4 + a)^2} + \frac{-4af + x(5be x^2 + 6bdx + ag + 7bc)}{32a^2b(bx^4 + a)} \\ & + \frac{3d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}} - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(21bc + 3ag - 5e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{5}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(21bc + 3ag - 5e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{5}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(21bc + 3ag + 5e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{5}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(21bc + 3ag + 5e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{5}{4}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.77 Problem number 178

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^4} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{x(bf x^3 + be x^2 + bdx - ag + bc)}{12ab(bx^4 + a)^3} + \frac{x(45be x^2 + 60bdx + 7ag + 77bc)}{384a^3b(bx^4 + a)} \\
 & + \frac{-8af + x(9be x^2 + 10bdx + ag + 11bc)}{96a^2b(bx^4 + a)^2} + \frac{5d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}} \\
 & - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) (77bc + 7ag - 15e\sqrt{a}\sqrt{b}) \sqrt{2}}{1024a^{\frac{15}{4}}b^{\frac{5}{4}}} \\
 & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) (77bc + 7ag - 15e\sqrt{a}\sqrt{b}) \sqrt{2}}{1024a^{\frac{15}{4}}b^{\frac{5}{4}}} \\
 & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (77bc + 7ag + 15e\sqrt{a}\sqrt{b}) \sqrt{2}}{512a^{\frac{15}{4}}b^{\frac{5}{4}}} \\
 & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (77bc + 7ag + 15e\sqrt{a}\sqrt{b}) \sqrt{2}}{512a^{\frac{15}{4}}b^{\frac{5}{4}}}
 \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.78 Problem number 192

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a - bx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{x(bc + ag + (ah + bd)x + be x^2 + bf x^3)}{4ab(-bx^4 + a)} + \frac{(-ah + bd) \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}b^{\frac{3}{2}}} \\
 & + \frac{\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (3bc - ag - e\sqrt{a}\sqrt{b})}{8a^{\frac{7}{4}}b^{\frac{5}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (3bc - ag + e\sqrt{a}\sqrt{b})}{8a^{\frac{7}{4}}b^{\frac{5}{4}}}
 \end{aligned}$$

command

```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.79 Problem number 195

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(bc - ag + (-ah + bd)x + be x^2 + bf x^3)}{4ab(bx^4 + a)} + \frac{(ah + bd) \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}b^{\frac{3}{2}}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(3bc + ag - e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{5}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(3bc + ag - e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{5}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(3bc + ag + e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{5}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(3bc + ag + e\sqrt{a}\sqrt{b}\right) \sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{5}{4}}} \end{aligned}$$

command

```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.80 Problem number 198

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a - bx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(bc + ag + (ah + bd)x + be x^2 + bf x^3)}{8ab(-bx^4 + a)^2} + \frac{4af + x(7bc - ag + 2(-ah + 3bd)x + 5be x^2)}{32a^2b(-bx^4 + a)} \\ & + \frac{(-ah + 3bd) \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}b^{\frac{3}{2}}} + \frac{\operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (21bc - 3ag - 5e\sqrt{a}\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{5}{4}}} \\ & + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (21bc - 3ag + 5e\sqrt{a}\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{5}{4}}} \end{aligned}$$

command

```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.81 Problem number 210

$$\int \frac{c + dx}{\sqrt{a + bx^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{bx^4 + a}}\right)}{2\sqrt{b}} \\ & + \frac{c \sqrt{\frac{\cos\left(4 \operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{b}) \sqrt{\frac{bx^4 + a}{(\sqrt{a} + x^2\sqrt{b})^2}}}{2 \cos\left(2 \operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{bx^4 + a}} \end{aligned}$$

command

`integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 b^{\frac{3}{2}} c \left(-\frac{a}{b}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}, -1\right) + a \sqrt{b} d \log\left(-2 b x^4 - 2 \sqrt{b x^4 + a} \sqrt{b} x^2 - a\right)}{4 a b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{dx + c}{\sqrt{b x^4 + a}}, x\right)$$

19.82 Problem number 211

$$\int \frac{c + dx}{\sqrt{a - b x^4}} dx$$

Optimal antiderivative

$$\frac{d \arctan\left(\frac{x^2 \sqrt{b}}{\sqrt{-b x^4 + a}}\right)}{2 \sqrt{b}} + \frac{a^{\frac{1}{4}} c \operatorname{EllipticF}\left(\frac{b^{\frac{1}{4}} x}{a^{\frac{1}{4}}}, i\right) \sqrt{1 - \frac{b x^4}{a}}}{b^{\frac{1}{4}} \sqrt{-b x^4 + a}}$$

command

`integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \sqrt{-b} b c \left(\frac{a}{b}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}, -1\right) - a \sqrt{-b} d \log\left(2 b x^4 - 2 \sqrt{-b x^4 + a} \sqrt{-b} x^2 - a\right)}{4 a b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-b x^4 + a} (dx + c)}{b x^4 - a}, x\right)$$

19.83 Problem number 212

$$\int \frac{c + dx}{\sqrt{-a + bx^4}} dx$$

Optimal antiderivative

$$\frac{d \operatorname{arctanh} \left(\frac{x^2 \sqrt{b}}{\sqrt{bx^4 - a}} \right)}{2\sqrt{b}} + \frac{a^{\frac{1}{4}} c \operatorname{EllipticF} \left(\frac{b^{\frac{1}{4}} x}{a^{\frac{1}{4}}}, i \right) \sqrt{1 - \frac{bx^4}{a}}}{b^{\frac{1}{4}} \sqrt{bx^4 - a}}$$

command

`integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4b^{\frac{3}{2}}c\left(\frac{a}{b}\right)^{\frac{3}{4}} \operatorname{ellipticF} \left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}, -1 \right) - a\sqrt{b} d \log \left(2bx^4 + 2\sqrt{bx^4 - a} \sqrt{b} x^2 - a \right)}{4ab}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{dx + c}{\sqrt{bx^4 - a}}, x \right)$$

19.84 Problem number 213

$$\int \frac{c + dx}{\sqrt{-a - bx^4}} dx$$

Optimal antiderivative

$$\frac{d \operatorname{arctan} \left(\frac{x^2 \sqrt{b}}{\sqrt{-bx^4 - a}} \right)}{2\sqrt{b}} + \frac{c \sqrt{\frac{\cos \left(4 \operatorname{arctan} \left(\frac{b^{\frac{1}{4}} x}{a^{\frac{1}{4}} \right) \right)}{2}} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(2 \operatorname{arctan} \left(\frac{b^{\frac{1}{4}} x}{a^{\frac{1}{4}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{a} + x^2 \sqrt{b}) \sqrt{\frac{bx^4 + a}{(\sqrt{a} + x^2 \sqrt{b})^2}}}{2 \cos \left(2 \operatorname{arctan} \left(\frac{b^{\frac{1}{4}} x}{a^{\frac{1}{4}} \right) \right) a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{-bx^4 - a}}$$

command

`integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4\sqrt{-b}bc\left(-\frac{a}{b}\right)^{\frac{3}{4}}\text{ellipticF}\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}, -1\right) + a\sqrt{-b}d\log\left(-2bx^4 + 2\sqrt{-bx^4 - a}\sqrt{-b}x^2 - a\right)}{4ab}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-bx^4 - a}(dx + c)}{bx^4 + a}, x\right)$$

19.85 Problem number 403

$$\int \frac{x^4(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a(-ah + be)x}{b^3} + \frac{(-af + bc)x^2}{2b^2} + \frac{(-ag + bd)x^3}{3b^2} + \frac{(-ah + be)x^4}{4b^2} \\ & + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{a^{\frac{2}{3}}\left(b^{\frac{2}{3}}(-af + bc) + a^{\frac{2}{3}}(-ah + be)\right)\ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3b^{\frac{10}{3}}} \\ & - \frac{a^{\frac{2}{3}}\left(b^{\frac{2}{3}}(-af + bc) + a^{\frac{2}{3}}(-ah + be)\right)\ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6b^{\frac{10}{3}}} \\ & - \frac{a(-ag + bd)\ln(bx^3 + a)}{3b^3} \\ & + \frac{a^{\frac{2}{3}}\left(b^{\frac{5}{3}}c - a^{\frac{2}{3}}be - ab^{\frac{2}{3}}f + a^{\frac{5}{3}}h\right)\arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{3b^{\frac{10}{3}}} \end{aligned}$$

command

`integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.86 Problem number 404

$$\int \frac{x^3(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-af + bc)x}{b^2} + \frac{(-ag + bd)x^2}{2b^2} + \frac{(-ah + be)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} \\ & + \frac{hx^6}{6b} - \frac{a^{\frac{1}{3}}(b^{\frac{1}{3}}(-af + bc) - a^{\frac{1}{3}}(-ag + bd)) \ln(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{3b^{\frac{8}{3}}} \\ & + \frac{a^{\frac{1}{3}}(b^{\frac{1}{3}}(-af + bc) - a^{\frac{1}{3}}(-ag + bd)) \ln(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2)}{6b^{\frac{8}{3}}} - \frac{a(-ah + be) \ln(bx^3 + a)}{3b^3} \\ & + \frac{a^{\frac{1}{3}}(b^{\frac{4}{3}}c + a^{\frac{1}{3}}bd - ab^{\frac{1}{3}}f - a^{\frac{4}{3}}g) \arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{3b^{\frac{8}{3}}} \end{aligned}$$

command

```
integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.87 Problem number 405

$$\int \frac{x^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ag + bd)x}{b^2} + \frac{(-ah + be)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} \\ & - \frac{a^{\frac{1}{3}}(b^{\frac{1}{3}}(-ag + bd) - a^{\frac{1}{3}}(-ah + be)) \ln(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{3b^{\frac{8}{3}}} \\ & + \frac{a^{\frac{1}{3}}(b^{\frac{1}{3}}(-ag + bd) - a^{\frac{1}{3}}(-ah + be)) \ln(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2)}{6b^{\frac{8}{3}}} + \frac{(-af + bc) \ln(bx^3 + a)}{3b^2} \\ & + \frac{a^{\frac{1}{3}}(b^{\frac{4}{3}}d + a^{\frac{1}{3}}be - ab^{\frac{1}{3}}g - a^{\frac{4}{3}}h) \arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{3b^{\frac{8}{3}}} \end{aligned}$$

command

```
integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.88 Problem number 406

$$\int \frac{x(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ah + be)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\left(b^{\frac{2}{3}}(-af + bc) + a^{\frac{2}{3}}(-ah + be)\right) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3a^{\frac{1}{3}}b^{\frac{7}{3}}} \\ & + \frac{\left(b^{\frac{2}{3}}(-af + bc) + a^{\frac{2}{3}}(-ah + be)\right) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6a^{\frac{1}{3}}b^{\frac{7}{3}}} + \frac{(-ag + bd) \ln(bx^3 + a)}{3b^2} \\ & - \frac{\left(b^{\frac{5}{3}}c - a^{\frac{2}{3}}be - ab^{\frac{2}{3}}f + a^{\frac{5}{3}}h\right) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{3a^{\frac{1}{3}}b^{\frac{7}{3}}} \end{aligned}$$

command

```
integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.89 Problem number 407

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{a + bx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} + \frac{\left(b^{\frac{1}{3}}(-af + bc) - a^{\frac{1}{3}}(-ag + bd)\right) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3a^{\frac{2}{3}}b^{\frac{5}{3}}} \\ & - \frac{\left(b^{\frac{1}{3}}(-af + bc) - a^{\frac{1}{3}}(-ag + bd)\right) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{5}{3}}} + \frac{(-ah + be) \ln(bx^3 + a)}{3b^2} \\ & - \frac{\left(b^{\frac{4}{3}}c + a^{\frac{1}{3}}bd - ab^{\frac{1}{3}}f - a^{\frac{4}{3}}g\right) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{3a^{\frac{2}{3}}b^{\frac{5}{3}}} \end{aligned}$$

command

```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.90 Problem number 408

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x(a + bx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{gx}{b} + \frac{hx^2}{2b} + \frac{c \ln(x)}{a} + \frac{\left(b^{\frac{1}{3}}(-ag + bd) - a^{\frac{1}{3}}(-ah + be)\right) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3a^{\frac{2}{3}}b^{\frac{5}{3}}} \\ & - \frac{\left(b^{\frac{1}{3}}(-ag + bd) - a^{\frac{1}{3}}(-ah + be)\right) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{5}{3}}} - \frac{(-af + bc) \ln(bx^3 + a)}{3ab} \\ & - \frac{\left(b^{\frac{4}{3}}d + a^{\frac{1}{3}}be - ab^{\frac{1}{3}}g - a^{\frac{4}{3}}h\right) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{3a^{\frac{2}{3}}b^{\frac{5}{3}}} \end{aligned}$$

command

`integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.91 Problem number 409

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^2(a + bx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c}{ax} + \frac{hx}{b} + \frac{d \ln(x)}{a} + \frac{\left(b^{\frac{2}{3}}(-af + bc) + a^{\frac{2}{3}}(-ah + be)\right) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3a^{\frac{4}{3}}b^{\frac{4}{3}}} \\ & - \frac{\left(b^{\frac{2}{3}}(-af + bc) + a^{\frac{2}{3}}(-ah + be)\right) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6a^{\frac{4}{3}}b^{\frac{4}{3}}} - \frac{(-ag + bd) \ln(bx^3 + a)}{3ab} \\ & + \frac{\left(b^{\frac{5}{3}}c - a^{\frac{2}{3}}be - ab^{\frac{2}{3}}f + a^{\frac{5}{3}}h\right) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{3a^{\frac{4}{3}}b^{\frac{4}{3}}} \end{aligned}$$

command

`integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.92 Problem number 410

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^3(a + bx^3)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e \ln(x)}{a} - \frac{\left(b^{\frac{1}{3}}(-af+bc) - a^{\frac{1}{3}}(-ag+bd)\right) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3a^{\frac{5}{3}}b^{\frac{2}{3}}} \\
 & + \frac{\left(b^{\frac{1}{3}}(-af+bc) - a^{\frac{1}{3}}(-ag+bd)\right) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6a^{\frac{5}{3}}b^{\frac{2}{3}}} - \frac{(-ah+be) \ln(bx^3+a)}{3ab} \\
 & + \frac{\left(b^{\frac{4}{3}}c + a^{\frac{1}{3}}bd - ab^{\frac{1}{3}}f - a^{\frac{4}{3}}g\right) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{3a^{\frac{5}{3}}b^{\frac{2}{3}}}
 \end{aligned}$$

command

```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.93 Problem number 411

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^4(a + bx^3)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} - \frac{(-af+bc) \ln(x)}{a^2} - \frac{\left(b^{\frac{1}{3}}(-ag+bd) - a^{\frac{1}{3}}(-ah+be)\right) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3a^{\frac{5}{3}}b^{\frac{2}{3}}} \\
 & + \frac{\left(b^{\frac{1}{3}}(-ag+bd) - a^{\frac{1}{3}}(-ah+be)\right) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6a^{\frac{5}{3}}b^{\frac{2}{3}}} + \frac{(-af+bc) \ln(bx^3+a)}{3a^2} \\
 & + \frac{\left(b^{\frac{4}{3}}d + a^{\frac{1}{3}}be - ab^{\frac{1}{3}}g - a^{\frac{4}{3}}h\right) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{3a^{\frac{5}{3}}b^{\frac{2}{3}}}
 \end{aligned}$$

command

```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.94 Problem number 412

$$\int \frac{x^4(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-2ah + be)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(-ah + be) - b(-af + bc)x - b(-ag + bd)x^2)}{3b^3(bx^3 + a)} \\ & - \frac{\left(b^{\frac{2}{3}}(-5af + 2bc) + a^{\frac{2}{3}}(-7ah + 4be)\right) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{9a^{\frac{1}{3}}b^{\frac{10}{3}}} \\ & + \frac{\left(b^{\frac{2}{3}}(-5af + 2bc) + a^{\frac{2}{3}}(-7ah + 4be)\right) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{18a^{\frac{1}{3}}b^{\frac{10}{3}}} \\ & + \frac{(-2ag + bd) \ln(bx^3 + a)}{3b^3} \\ & - \frac{\left(2b^{\frac{5}{3}}c - 4a^{\frac{2}{3}}be - 5ab^{\frac{2}{3}}f + 7a^{\frac{5}{3}}h\right) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{9a^{\frac{1}{3}}b^{\frac{10}{3}}} \end{aligned}$$

command

```
integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.95 Problem number 413

$$\int \frac{x^3(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc - af + (-ag + bd)x + (-ah + be)x^2)}{3b^2(bx^3 + a)} \\ & + \frac{\left(b^{\frac{1}{3}}(-4af + bc) - a^{\frac{1}{3}}(-5ag + 2bd)\right) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{9a^{\frac{2}{3}}b^{\frac{8}{3}}} \\ & - \frac{\left(b^{\frac{1}{3}}(-4af + bc) - a^{\frac{1}{3}}(-5ag + 2bd)\right) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{18a^{\frac{2}{3}}b^{\frac{8}{3}}} + \frac{(-2ah + be) \ln(bx^3 + a)}{3b^3} \\ & - \frac{\left(b^{\frac{4}{3}}c + 2a^{\frac{1}{3}}bd - 4ab^{\frac{1}{3}}f - 5a^{\frac{4}{3}}g\right) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{9a^{\frac{2}{3}}b^{\frac{8}{3}}} \end{aligned}$$

command

```
integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.96 Problem number 420

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^4 (a + bx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x(bd - ag + (-ah + be)x - b(\frac{bc}{a} - f)x^2)}{3a^2(bx^3 + a)} \\ & - \frac{(-af + 2bc) \ln(x)}{a^3} - \frac{(b^{\frac{1}{3}}(-2ag + 5bd) - a^{\frac{1}{3}}(-ah + 4be)) \ln(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{9a^{\frac{8}{3}}b^{\frac{2}{3}}} \\ & + \frac{(b^{\frac{1}{3}}(-2ag + 5bd) - a^{\frac{1}{3}}(-ah + 4be)) \ln(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2)}{18a^{\frac{8}{3}}b^{\frac{2}{3}}} + \frac{(-af + 2bc) \ln(bx^3 + a)}{3a^3} \\ & + \frac{(5b^{\frac{4}{3}}d + 4a^{\frac{1}{3}}be - 2ab^{\frac{1}{3}}g - a^{\frac{4}{3}}h) \arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{9a^{\frac{8}{3}}b^{\frac{2}{3}}} \end{aligned}$$

command

```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.97 Problem number 429

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^4 (a + bx^3)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x(bd - ag + (-ah + be)x - b(\frac{bc}{a} - f)x^2)}{6a^2(bx^3 + a)^2} \\ & - \frac{x(11bd - 5ag + 2(-2ah + 5be)x - 3b(\frac{5bc}{a} - 3f)x^2)}{18a^3(bx^3 + a)} - \frac{(-af + 3bc)\ln(x)}{a^4} \\ & - \frac{\left(5b^{\frac{1}{3}}(-ag + 4bd) - 2a^{\frac{1}{3}}(-ah + 7be)\right)\ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{27a^{\frac{11}{3}}b^{\frac{2}{3}}} \\ & + \frac{\left(5b^{\frac{1}{3}}(-ag + 4bd) - 2a^{\frac{1}{3}}(-ah + 7be)\right)\ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{54a^{\frac{11}{3}}b^{\frac{2}{3}}} \\ & + \frac{(-af + 3bc)\ln(bx^3 + a)}{3a^4} \\ & + \frac{\left(20b^{\frac{4}{3}}d + 14a^{\frac{1}{3}}be - 5ab^{\frac{1}{3}}g - 2a^{\frac{4}{3}}h\right)\arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{27a^{\frac{11}{3}}b^{\frac{2}{3}}} \end{aligned}$$

command

```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.98 Problem number 430

$$\int \frac{x^3(c + dx + ex^2)}{\sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{4ae\sqrt{bx^3+a}}{9b^2} + \frac{2cx\sqrt{bx^3+a}}{5b} + \frac{2dx^2\sqrt{bx^3+a}}{7b} \\
 & + \frac{2ex^3\sqrt{bx^3+a}}{9b} - \frac{8ad\sqrt{bx^3+a}}{7b^{\frac{5}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
 & + \frac{43^{\frac{1}{4}}a^{\frac{4}{3}}d\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{7b^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
 & - \frac{4a\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(7b^{\frac{1}{3}}c-10a^{\frac{1}{3}}d(1-\sqrt{3})\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{105b^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
 \end{aligned}$$

command

```
integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(126a\sqrt{b}\text{cweierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-180a\sqrt{b}\text{dweierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{315b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{ex^5+dx^4+cx^3}{\sqrt{bx^3+a}},x\right)$$

19.99 Problem number 431

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}} dx$$

Optimal antiderivative

$$\frac{2c\sqrt{bx^3+a}}{3b} + \frac{2dx\sqrt{bx^3+a}}{5b} + \frac{2ex^2\sqrt{bx^3+a}}{7b} - \frac{8ae\sqrt{bx^3+a}}{7b^{\frac{5}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)}$$

$$+ \frac{43^{\frac{1}{4}}a^{\frac{4}{3}}e\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{7b^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

$$- \frac{4a\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(7b^{\frac{1}{3}}d-10a^{\frac{1}{3}}e(1-\sqrt{3})\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{105b^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

`integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(42a\sqrt{b}d\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-60a\sqrt{b}e\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{105b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{ex^4+dx^3+cx^2}{\sqrt{bx^3+a}},x\right)$$

19.100 Problem number 432

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}} dx$$

Optimal antiderivative

$$\frac{2d\sqrt{bx^3+a}}{3b} + \frac{2ex\sqrt{bx^3+a}}{5b} + \frac{2c\sqrt{bx^3+a}}{b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)}$$

$$3^{\frac{1}{4}}a^{\frac{1}{3}}c\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})},i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}$$

$$b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}$$

$$2\cdot 3^{\frac{3}{4}}a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})},i\sqrt{3}+2i\right)\left(2a^{\frac{2}{3}}e+5b^{\frac{2}{3}}c(1-\sqrt{3})\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}$$

$$15b^{\frac{4}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}$$

command

```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6a\sqrt{b}\text{eweierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+15b^{\frac{3}{2}}c\text{eweierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}\right)}{15b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{ex^3+dx^2+cx}{\sqrt{bx^3+a}},x\right)$$

19.101 Problem number 433

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}} dx$$

Optimal antiderivative

$$\frac{2e\sqrt{bx^3+a}}{3b} + \frac{2d\sqrt{bx^3+a}}{b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)}$$

$$- \frac{3^{\frac{1}{4}}a^{\frac{1}{3}}d\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

$$+ \frac{2\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(b^{\frac{1}{3}}c-a^{\frac{1}{3}}d(1-\sqrt{3})\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})\right)^2}}}{3b^{\frac{2}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

`integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3\sqrt{b}\text{cweierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-3\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\sqrt{bx^3+a}\right)}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{ex^2+dx+c}{\sqrt{bx^3+a}},x\right)$$

19.102 Problem number 434

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2c \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{2e\sqrt{bx^3+a}}{b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
& - \frac{3^{\frac{1}{4}}a^{\frac{1}{3}}e\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{b^{\frac{2}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
& + \frac{2\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(b^{\frac{1}{3}}d - a^{\frac{1}{3}}e(1-\sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3b^{\frac{2}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
\end{aligned}$$

command

```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{\sqrt{a} bc \log\left(-\frac{b^2x^6+8abx^3-4(bx^3+2a)\sqrt{bx^3+a}\sqrt{a}+8a^2}{x^6}\right) + 12a\sqrt{b} d \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - 12a\sqrt{b} e w}{6ab} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}(ex^2+dx+c)}{bx^4+ax}, x\right)$$

19.103 Problem number 435

$$\int \frac{c + dx + ex^2}{x^2 \sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2d \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{c\sqrt{bx^3 + a}}{ax} + \frac{b^{\frac{1}{3}}c\sqrt{bx^3 + a}}{a\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)}$$

$$- \frac{3^{\frac{1}{4}}b^{\frac{1}{3}}c\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{2a^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

$$+ \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(-2a^{\frac{2}{3}}e + b^{\frac{2}{3}}c(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{3a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{\sqrt{a} b dx \log\left(\frac{b^2 x^6 + 8 ab x^3 - 4(bx^3 + 2a)\sqrt{bx^3 + a}\sqrt{a} + 8a^2}{x^6}\right) + 12 a \sqrt{b} \operatorname{reweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - 6 b^{\frac{3}{2}} c x \operatorname{reweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)}{6 abx} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + a}(ex^2 + dx + c)}{bx^5 + ax^2}, x\right)$$

19.104 Problem number 436

$$\int \frac{c + dx + ex^2}{x^3 \sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2e \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{c\sqrt{bx^3+a}}{2ax^2} - \frac{d\sqrt{bx^3+a}}{ax} + \frac{b^{\frac{1}{3}}d\sqrt{bx^3+a}}{a\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)}$$

$$+ \frac{3^{\frac{3}{4}}b^{\frac{1}{3}}d\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{2a^{\frac{2}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

$$+ \frac{3^{\frac{3}{4}}b^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(b^{\frac{1}{3}}c + 2a^{\frac{1}{3}}d(1-\sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{6a\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}$$

command

`integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{\sqrt{a} x^2 e \log\left(\frac{b^2 x^6 + 8 a b x^3 - 4 (b x^3 + 2 a) \sqrt{b x^3 + a} \sqrt{a} + 8 a^2}{x^6}\right) - 3 \sqrt{b} c x^2 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - 6 \sqrt{b} d x^2}{6 a x^2} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}(ex^2+dx+c)}{bx^6+ax^3}, x\right)$$

19.105 Problem number 437

$$\int \frac{x^5(c + dx + ex^2)}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x(-bcx^2 + aex + ad)}{3b^2\sqrt{bx^3 + a}} + \frac{4c\sqrt{bx^3 + a}}{3b^2} + \frac{2dx\sqrt{bx^3 + a}}{5b^2} \\ & + \frac{2ex^2\sqrt{bx^3 + a}}{7b^2} - \frac{80ae\sqrt{bx^3 + a}}{21b^{\frac{8}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{40a^{\frac{4}{3}}e\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{21b^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} 3^{\frac{1}{4}} \\ & + \frac{16a\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(14b^{\frac{1}{3}}d - 25a^{\frac{1}{3}}e(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{315b^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(200\left(abx^3 + a^2\right)\sqrt{b} \operatorname{ewierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - 112\left(abdx^3 + a^2d\right)\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, x\right)\right)}{105\left(b^4x^3 + ab^2x + a^3\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(ex^7 + dx^6 + cx^5\right)\sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

19.106 Problem number 438

$$\int \frac{x^4(c + dx + ex^2)}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x(-bdx^2 - bcx + ae)}{3b^2\sqrt{bx^3 + a}} + \frac{4d\sqrt{bx^3 + a}}{3b^2} + \frac{2ex\sqrt{bx^3 + a}}{5b^2} + \frac{8c\sqrt{bx^3 + a}}{3b^{\frac{5}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)}$$

$$4a^{\frac{1}{3}}c\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{1}{4}}$$

$$3b^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

$$8a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(4a^{\frac{2}{3}}e + 5b^{\frac{2}{3}}c(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

$$45b^{\frac{7}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

command

```
integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(16(abx^3 + a^2)\sqrt{b} \text{eweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 20(b^2cx^3 + abc)\sqrt{b} \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrass}\right)\right)}{15(b^4x^3 + ab^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(ex^6 + dx^5 + cx^4)\sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

19.107 Problem number 439

$$\int \frac{x^3(c + dx + ex^2)}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x(ex^2 + dx + c)}{3b\sqrt{bx^3 + a}} + \frac{4e\sqrt{bx^3 + a}}{3b^2} + \frac{8d\sqrt{bx^3 + a}}{3b^{5/3}\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)}$$

$$+ \frac{4a^{1/3}d\left(a^{1/3} + b^{1/3}x\right) \operatorname{EllipticE}\left(\frac{b^{1/3}x + a^{1/3}(1 - \sqrt{3})}{b^{1/3}x + a^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}}{3^{1/4}}$$

$$+ \frac{3b^{5/3}\sqrt{bx^3 + a} \sqrt{\frac{a^{1/3}\left(a^{1/3} + b^{1/3}x\right)}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}}{4\left(a^{1/3} + b^{1/3}x\right) \operatorname{EllipticF}\left(\frac{b^{1/3}x + a^{1/3}(1 - \sqrt{3})}{b^{1/3}x + a^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(b^{1/3}c - 2a^{1/3}d(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}$$

$$+ \frac{9b^{5/3}\sqrt{bx^3 + a} \sqrt{\frac{a^{1/3}\left(a^{1/3} + b^{1/3}x\right)}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}$$

command

`integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(bc x^3 + ac)\sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - 4(bdx^3 + ad)\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\right)\right)}{3(b^3x^3 + ab^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(ex^5 + dx^4 + cx^3)\sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

19.108 Problem number 440

$$\int \frac{x^2(c + dx + ex^2)}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(e x^2 + dx + c)}{3b\sqrt{bx^3 + a}} + \frac{8e\sqrt{bx^3 + a}}{3b^{5/3}\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)}$$

$$4a^{1/3}e\left(a^{1/3} + b^{1/3}x\right) \text{EllipticE}\left(\frac{b^{1/3}x + a^{1/3}(1 - \sqrt{3})}{b^{1/3}x + a^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}} 3^{1/4}$$

$$3b^{5/3}\sqrt{bx^3 + a} \sqrt{\frac{a^{1/3}\left(a^{1/3} + b^{1/3}x\right)}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}$$

$$4\left(a^{1/3} + b^{1/3}x\right) \text{EllipticF}\left(\frac{b^{1/3}x + a^{1/3}(1 - \sqrt{3})}{b^{1/3}x + a^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(b^{1/3}d - 2a^{1/3}e(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}$$

$$9b^{5/3}\sqrt{bx^3 + a} \sqrt{\frac{a^{1/3}\left(a^{1/3} + b^{1/3}x\right)}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}$$

command

```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4\left(bx^3 + a\right)\sqrt{b} \text{ewierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - 2\left(bdx^3 + ad\right)\sqrt{b} \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right)}{3\left(b^3x^3 + ab^2\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(ex^4 + dx^3 + cx^2\right)\sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

19.109 Problem number 441

$$\int \frac{x(c + dx + ex^2)}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x(-bdx^2 - bcx + ae)}{3ab\sqrt{bx^3 + a}} - \frac{2d\sqrt{bx^3 + a}}{3ab} - \frac{2c\sqrt{bx^3 + a}}{3ab^{\frac{2}{3}}(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & + \frac{c(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{3a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} 3^{\frac{1}{4}} \\ & + \frac{2(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) (2a^{\frac{2}{3}}e + b^{\frac{2}{3}}(c - c\sqrt{3})) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{9a^{\frac{2}{3}}b^{\frac{4}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

`integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 (abx^3 + a^2) \sqrt{b} \operatorname{eweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (b^2cx^3 + abc) \sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) \right)}{3(ab^3x^3 + a^2b^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + a} (ex^3 + dx^2 + cx)}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

19.110 Problem number 442

$$\int \frac{c + dx + ex^2}{(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(ae - bx(dx + c))}{3ab\sqrt{bx^3 + a}} - \frac{2d\sqrt{bx^3 + a}}{3ab^{2/3}\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)}$$

$$+ \frac{d\left(a^{1/3} + b^{1/3}x\right) \operatorname{EllipticE}\left(\frac{b^{1/3}x + a^{1/3}(1 - \sqrt{3})}{b^{1/3}x + a^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}}{3a^{2/3}b^{2/3}\sqrt{bx^3 + a} \sqrt{\frac{a^{1/3}\left(a^{1/3} + b^{1/3}x\right)}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}} 3^{1/4}$$

$$+ \frac{2\left(a^{1/3} + b^{1/3}x\right) \operatorname{EllipticF}\left(\frac{b^{1/3}x + a^{1/3}(1 - \sqrt{3})}{b^{1/3}x + a^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(b^{1/3}c + a^{1/3}d(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}}{9ab^{2/3}\sqrt{bx^3 + a} \sqrt{\frac{a^{1/3}\left(a^{1/3} + b^{1/3}x\right)}{\left(b^{1/3}x + a^{1/3}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((bcx^3 + ac)\sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (bdx^3 + ad)\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right)\right)}{3(ab^2x^3 + a^2b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + a}(ex^2 + dx + c)}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

19.111 Problem number 443

$$\int \frac{c + dx + ex^2}{x(a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2c \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3a^{3/2}} + \frac{2x(-bcx^2 + aex + ad)}{3a^2 \sqrt{bx^3+a}} \\ & + \frac{2c\sqrt{bx^3+a}}{3a^2} - \frac{2e\sqrt{bx^3+a}}{3ab^{2/3}\left(b^{1/3}x + a^{1/3}(1+\sqrt{3})\right)} \\ & + \frac{e\left(a^{1/3} + b^{1/3}x\right) \operatorname{EllipticE}\left(\frac{b^{1/3}x + a^{1/3}(1-\sqrt{3})}{b^{1/3}x + a^{1/3}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{\left(b^{1/3}x + a^{1/3}(1+\sqrt{3})\right)^2}}}{3a^{2/3}b^{2/3}\sqrt{bx^3+a} \sqrt{\frac{a^{1/3}\left(a^{1/3} + b^{1/3}x\right)}{\left(b^{1/3}x + a^{1/3}(1+\sqrt{3})\right)^2}}} 3^{1/4} \\ & + \frac{2\left(a^{1/3} + b^{1/3}x\right) \operatorname{EllipticF}\left(\frac{b^{1/3}x + a^{1/3}(1-\sqrt{3})}{b^{1/3}x + a^{1/3}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(b^{1/3}d + a^{1/3}e(1-\sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x}{\left(b^{1/3}x + a^{1/3}(1+\sqrt{3})\right)^2}}}{9ab^{2/3}\sqrt{bx^3+a} \sqrt{\frac{a^{1/3}\left(a^{1/3} + b^{1/3}x\right)}{\left(b^{1/3}x + a^{1/3}(1+\sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{4(abx^3 + a^2)\sqrt{b} \operatorname{eweierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + (b^2cx^3 + abc)\sqrt{a} \log\left(\frac{b^2x^6 + 8abx^3 - \dots}{\dots}\right)}{6(a \dots)} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}(ex^2+dx+c)}{b^2x^7+2abx^4+a^2x}, x\right)$$

19.112 Problem number 444

$$\int \frac{c + dx + ex^2}{x^2 (a + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2d \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}} + \frac{2x(-bdx^2 - bcx + ae)}{3a^2 \sqrt{bx^3+a}} \\ & + \frac{2d\sqrt{bx^3+a}}{3a^2} - \frac{c\sqrt{bx^3+a}}{a^2x} + \frac{5b^{\frac{1}{3}}c\sqrt{bx^3+a}}{3a^2\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & - \frac{5b^{\frac{1}{3}}c\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{3^{\frac{1}{4}}} \\ & - \frac{6a^{\frac{5}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(-2a^{\frac{2}{3}}e + 5b^{\frac{2}{3}}c(1-\sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \\ & - \frac{9a^{\frac{5}{3}}b^{\frac{1}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \end{aligned}$$

command

`integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{4(abx^4 + a^2x)\sqrt{b} \operatorname{eweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + (b^2dx^4 + abdx)\sqrt{a} \log\left(\frac{b^2x^6 + 8abx^3 - 4(bx^3 + 2a)\sqrt{bx^3+a}}{x^6}\right)}{\dots} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}(ex^2+dx+c)}{b^2x^8+2abx^5+a^2x^2}, x\right)$$

19.113 Problem number 445

$$\int x^3 \sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4a^2e\sqrt{bx^3+a}}{45b^2} + \frac{6a(-8af+17bc)x\sqrt{bx^3+a}}{935b^2} + \frac{6a(-10ag+19bd)x^2\sqrt{bx^3+a}}{1729b^2} \\ & + \frac{2aex^3\sqrt{bx^3+a}}{45b} + \frac{6afx^4\sqrt{bx^3+a}}{187b} + \frac{6agx^5\sqrt{bx^3+a}}{247b} \\ & + \frac{2x^3(36465gx^5+40755fx^4+46189ex^3+53295dx^2+62985cx)\sqrt{bx^3+a}}{692835} \\ & - \frac{24a^2(-10ag+19bd)\sqrt{bx^3+a}}{1729b^{\frac{8}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & + \frac{123^{\frac{1}{4}}a^{\frac{7}{3}}(-10ag+19bd)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{1729b^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\ & + \frac{43^{\frac{3}{4}}a^2\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(1729b^{\frac{1}{3}}(-8af+17bc)-1870a^{\frac{1}{3}}(-10ag+19bd)\right)}{1616615b^{\frac{8}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(93366(17a^2bc-8a^3f)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-100980(19a^2bd-10a^3g)\sqrt{b}\text{weierstrassZeta}\left(0,\right.\right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(gx^7+fx^6+ex^5+dx^4+cx^3\right)\sqrt{bx^3+a},x\right)$$

19.114 Problem number 446

$$\int x^2 \sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(-2af + 5bc) \sqrt{bx^3 + a}}{45b^2} + \frac{6a(-8ag + 17bd) x \sqrt{bx^3 + a}}{935b^2} \\ & + \frac{6ae x^2 \sqrt{bx^3 + a}}{91b} + \frac{2af x^3 \sqrt{bx^3 + a}}{45b} + \frac{6ag x^4 \sqrt{bx^3 + a}}{187b} \\ & + \frac{2x^2(6435g x^5 + 7293f x^4 + 8415e x^3 + 9945d x^2 + 12155cx) \sqrt{bx^3 + a}}{109395} \\ & - \frac{24a^2 e \sqrt{bx^3 + a}}{91b^{\frac{5}{3}} \left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\ & + \frac{123^{\frac{1}{4}} a^{\frac{7}{3}} e \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \text{EllipticE} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{91b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \\ & + \frac{43^{\frac{3}{4}} a^2 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \text{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(1547bd - 728ag - 1870a^{\frac{1}{3}} b^{\frac{2}{3}} e (1 - \sqrt{3}) \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right)}{85085b^{\frac{7}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

`integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(100980 a^2 b^{\frac{3}{2}} \text{weierstrassZeta} \left(0, -\frac{4a}{b}, \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) - 4914 (17 a^2 b d - 8 a^3 g) \sqrt{b} \text{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((gx^6 + fx^5 + ex^4 + dx^3 + cx^2) \sqrt{bx^3 + a}, x \right)$$

19.115 Problem number 447

$$\int x \sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(-2ag + 5bd) \sqrt{bx^3 + a}}{45b^2} + \frac{6aex \sqrt{bx^3 + a}}{55b} + \frac{6afx^2 \sqrt{bx^3 + a}}{91b} + \frac{2agx^3 \sqrt{bx^3 + a}}{45b} \\ & + \frac{2x(3003gx^5 + 3465fx^4 + 4095ex^3 + 5005dx^2 + 6435cx) \sqrt{bx^3 + a}}{45045} \\ & + \frac{6a(-4af + 13bc) \sqrt{bx^3 + a}}{91b^{\frac{5}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)} \end{aligned}$$

$$33^{\frac{1}{4}} a^{\frac{4}{3}} (-4af + 13bc) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \text{EllipticE} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}$$

$$91b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}$$

$$23^{\frac{3}{4}} a^{\frac{4}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \text{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(182a^{\frac{2}{3}}b^{\frac{1}{3}}e + 55(-4af + 13bc)(1 - \sqrt{3}) \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right)$$

$$5005b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}$$

command

```
integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(4914 a^2 \sqrt{b} \text{ewierstrassPInverse}(0, -\frac{4a}{b}, x) + 1485 (13 abc - 4 a^2 f) \sqrt{b} \text{weierstrassZeta}(0, -\frac{4a}{b}, \text{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((gx^5 + fx^4 + ex^3 + dx^2 + cx) \sqrt{bx^3 + a}, x \right)$$

19.116 Problem number 448

$$\int \sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ae\sqrt{bx^3+a}}{9b} + \frac{6afx\sqrt{bx^3+a}}{55b} + \frac{6agx^2\sqrt{bx^3+a}}{91b} \\ & + \frac{2(3465gx^5 + 4095fx^4 + 5005ex^3 + 6435dx^2 + 9009cx)\sqrt{bx^3+a}}{45045} \\ & + \frac{6a(-4ag + 13bd)\sqrt{bx^3+a}}{91b^{\frac{5}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{33^{\frac{1}{4}}a^{\frac{4}{3}}(-4ag + 13bd)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)\text{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right)\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{91b^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{23^{\frac{3}{4}}a\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)\text{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right)\left(91b^{\frac{1}{3}}(-2af + 11bc) - 55a^{\frac{1}{3}}(-4ag + 13bd)\left(1 - \sqrt{3}\right)\right)}{5005b^{\frac{5}{3}}\sqrt{bx^3+a}\sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(2457(11abc - 2a^2f)\sqrt{b}\text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - 1485(13abd - 4a^2g)\sqrt{b}\text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{we}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}, x\right)$$

19.117 Problem number 449

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2c \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right) \sqrt{a}}{3} + \frac{2af\sqrt{bx^3+a}}{9b} + \frac{6agx\sqrt{bx^3+a}}{55b} \\ & + \frac{2(315gx^5+385fx^4+495ex^3+693dx^2+1155cx)\sqrt{bx^3+a}}{3465x} + \frac{6ae\sqrt{bx^3+a}}{7b^{\frac{2}{3}}(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3}))} \\ & - \frac{33^{\frac{1}{4}}a^{\frac{4}{3}}e(a^{\frac{1}{3}}+b^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3}))^2}}}{7b^{\frac{2}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}}+b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \\ & + \frac{23^{\frac{3}{4}}a(a^{\frac{1}{3}}+b^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) (77bd-14ag-55a^{\frac{1}{3}}b^{\frac{2}{3}}e(1-\sqrt{3})) \left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{385b^{\frac{4}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}}+b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1155 \sqrt{a} b^2 c \log\left(-\frac{b^2 x^6 + 8 a b x^3 - 4 (b x^3 + 2 a) \sqrt{b x^3 + a} \sqrt{a} + 8 a^2}{x^6}\right) - 5940 a b^{\frac{3}{2}} e \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPI}\right)}{\dots} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x}, x\right)$$

19.118 Problem number 450

$$\int \frac{\sqrt{a+bx^3} (c+dx+ex^2+fx^3+gx^4)}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2d \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right) \sqrt{a}}{3} + \frac{2ag\sqrt{bx^3+a}}{9b} - \frac{3c\sqrt{bx^3+a}}{x} \\ & + \frac{2(35gx^5+45fx^4+63ex^3+105dx^2+315cx)\sqrt{bx^3+a}}{315x^2} + \frac{3(2af+7bc)\sqrt{bx^3+a}}{7b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\ & - \frac{33^{\frac{1}{4}}a^{\frac{1}{3}}(2af+7bc)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)}}}{14b^{\frac{2}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\ & + \frac{33^{\frac{1}{4}}a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(14a^{\frac{2}{3}}b^{\frac{1}{3}}e-5(2af+7bc)(1-\sqrt{3})\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{35b^{\frac{2}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{105\sqrt{a} bdx \log\left(-\frac{b^2x^6+8abx^3-4(bx^3+2a)\sqrt{bx^3+a}\sqrt{a}+8a^2}{x^6}\right) + 756a\sqrt{b} x \operatorname{eWeierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - 270}{1} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^2}, x\right)$$

19.119 Problem number 451

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right) \sqrt{a}}{3} + \frac{3c\sqrt{bx^3+a}}{2x^2} - \frac{3d\sqrt{bx^3+a}}{x} \\ & - \frac{2(-15gx^5 - 21fx^4 - 35ex^3 - 105dx^2 + 105cx) \sqrt{bx^3+a}}{105x^3} + \frac{3(2ag + 7bd) \sqrt{bx^3+a}}{7b^{\frac{2}{3}}(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & - \frac{33^{\frac{1}{4}}a^{\frac{1}{3}}(2ag + 7bd) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{14b^{\frac{2}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & + \frac{3^{\frac{3}{4}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) (7b^{\frac{1}{3}}(4af + 5bc) - 10a^{\frac{1}{3}}(2ag + 7bd)(1 - \sqrt{3})) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right)}{70b^{\frac{2}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{35 \sqrt{a} b x^2 e \log\left(-\frac{b^2 x^6 + 8 a b x^3 - 4 (b x^3 + 2 a) \sqrt{b x^3 + a} \sqrt{a} + 8 a^2}{x^6}\right) + 63 (5 b c + 4 a f) \sqrt{b} x^2 \operatorname{weierstrassPInverse}\left(0, -\frac{4 a}{b}\right)}{\right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(g x^4 + f x^3 + e x^2 + d x + c) \sqrt{b x^3 + a}}{x^3}, x\right)$$

19.120 Problem number 452

$$\int \frac{\sqrt{a+bx^3} (c+dx+ex^2+fx^3+gx^4)}{x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(2af+bc) \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{c\sqrt{bx^3+a}}{3x^3} + \frac{3d\sqrt{bx^3+a}}{2x^2} - \frac{3e\sqrt{bx^3+a}}{x} \\ & -\frac{2(-3gx^5-5fx^4-15ex^3+15dx^2+5cx)\sqrt{bx^3+a}}{15x^4} + \frac{3b^{\frac{1}{3}}e\sqrt{bx^3+a}}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})} \\ & -\frac{3^{\frac{3}{4}}a^{\frac{1}{3}}b^{\frac{1}{3}}e\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{2\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\ & +\frac{3^{\frac{3}{4}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(5bd+4ag-10a^{\frac{1}{3}}b^{\frac{2}{3}}e(1-\sqrt{3})\right) \left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{10b^{\frac{1}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{180ab^{\frac{3}{2}}x^3 \operatorname{eweierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - 5(b^2c+2abf)\sqrt{a}x^3 \log\left(-\frac{b^2x^6+8abx^3-}{bx^3+a}\right)}{90ab^{\frac{3}{2}}x^3 \operatorname{eweierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - 5(b^2c+2abf)\sqrt{-a}x^3 \arctan\left(\frac{2\sqrt{bx^3+a}}{bx^3+a}\right)} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^4}, x\right)$$

19.121 Problem number 453

$$\int \frac{\sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4)}{x^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(2ag + bd) \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{3c\sqrt{bx^3 + a}}{20x^4} \\ & + \frac{d\sqrt{bx^3 + a}}{3x^3} + \frac{3e\sqrt{bx^3 + a}}{2x^2} - \frac{3(8af + bc)\sqrt{bx^3 + a}}{8ax} \\ & - \frac{2(-5gx^5 - 15fx^4 + 15ex^3 + 5dx^2 + 3cx)\sqrt{bx^3 + a}}{15x^5} + \frac{3b^{\frac{1}{3}}(8af + bc)\sqrt{bx^3 + a}}{8a\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{33^{\frac{1}{4}}b^{\frac{1}{3}}(8af + bc)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{16a^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{3^{\frac{3}{4}}b^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(4a^{\frac{2}{3}}b^{\frac{1}{3}}e - (8af + bc)(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{8a^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{36 a \sqrt{b} x^4 \operatorname{eweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 2 (bd + 2 ag) \sqrt{a} x^4 \log\left(-\frac{b^2 x^6 + 8 abx^3 - 4 (bx^3 + 2a) \sqrt{bx^3 + a} \sqrt{a} + 8 a^2}{x^6}\right)}{\dots} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^5}, x\right)$$

19.122 Problem number 454

$$\int \frac{\sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4)}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{be \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{\left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x}\right)\sqrt{bx^3 + a}}{60} \\ & - \frac{3bc\sqrt{bx^3 + a}}{20ax^2} - \frac{3bd\sqrt{bx^3 + a}}{8ax} + \frac{3b^{\frac{1}{3}}(8ag + bd)\sqrt{bx^3 + a}}{8a\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + 3^{\frac{3}{4}}b^{\frac{1}{3}}(8ag + bd)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & + 16a^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & + 3^{\frac{3}{4}}b^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(2b^{\frac{1}{3}}(-10af + bc) + 5a^{\frac{1}{3}}(8ag + bd)(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2}\right) \\ & + 40a\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[10\sqrt{a}bx^5e \log\left(-\frac{b^2x^6 + 8abx^3 - 4(bx^3 + 2a)\sqrt{bx^3 + a}\sqrt{a} + 8a^2}{x^6}\right) - 18(bc - 10af)\sqrt{b}x^5 \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}\right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^6}, x\right)$$

19.123 Problem number 455

$$\int \frac{\sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4)}{x^7} dx$$

Optimal antiderivative

$$\frac{b(-4af + bc) \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right) - \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2}\right) \sqrt{bx^3 + a}}{12a^{\frac{3}{2}}} - \frac{bc\sqrt{bx^3 + a}}{12ax^3} - \frac{3bd\sqrt{bx^3 + a}}{20ax^2} - \frac{3be\sqrt{bx^3 + a}}{8ax} + \frac{3b^{\frac{4}{3}}e\sqrt{bx^3 + a}}{8a\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)}$$

$$- \frac{3^{\frac{3}{4}}b^{\frac{4}{3}}e\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{16a^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

$$- \frac{3^{\frac{3}{4}}b^{\frac{2}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(2bd - 20ag + 5a^{\frac{1}{3}}b^{\frac{2}{3}}e(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{40a\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{90 ab^{\frac{3}{2}} x^6 \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + 5 (b^2 c - 4 abf) \sqrt{a} x^6 \log\left(\frac{b^2 x^6 + 8 abx^3 - 4 (b^2 c - 4 abf) x^3}{b^2 x^6 + 8 abx^3 - 4 (b^2 c - 4 abf) x^3}\right)}{45 ab^{\frac{3}{2}} x^6 \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + 5 (b^2 c - 4 abf) \sqrt{-a} x^6 \arctan\left(\frac{(bx^3+2a)\sqrt{a}}{2(a-bx^3)}\right)} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^7}, x\right)$$

19.124 Problem number 456

$$\int \frac{\sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4)}{x^8} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b(-4ag + bd) \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right) - \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3}\right) \sqrt{bx^3 + a}}{12a^{\frac{3}{2}}} - \frac{\left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3}\right) \sqrt{bx^3 + a}}{420} \\ & - \frac{3bc\sqrt{bx^3 + a}}{56ax^4} - \frac{bd\sqrt{bx^3 + a}}{12ax^3} - \frac{3be\sqrt{bx^3 + a}}{20ax^2} \\ & + \frac{3b(-14af + 5bc)\sqrt{bx^3 + a}}{112a^2x} - \frac{3b^{\frac{4}{3}}(-14af + 5bc)\sqrt{bx^3 + a}}{112a^2\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{3^{\frac{3}{4}}b^{\frac{4}{3}}(-14af + 5bc)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{224a^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{3^{\frac{3}{4}}b^{\frac{4}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(28a^{\frac{2}{3}}b^{\frac{1}{3}}e - 5(-14af + 5bc)(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{560a^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{252 ab^{\frac{3}{2}} x^7 \text{eweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 35 (b^2 d - 4 abg) \sqrt{a} x^7 \log\left(\frac{b^2 x^6 + 8 abx^3 - 4 (bx^3 + 2a) \sqrt{bx^3 + a} \sqrt{a} + 8}{x^6}\right)}{\dots} \right]$$

$$\frac{252 ab^{\frac{3}{2}} x^7 \text{eweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 70 (b^2 d - 4 abg) \sqrt{-a} x^7 \arctan\left(\frac{(bx^3 + 2a) \sqrt{bx^3 + a} \sqrt{-a}}{2(abx^3 + a^2)}\right) - 45 (\dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c) \sqrt{bx^3 + a}}{x^8}, x\right)$$

19.125 Problem number 457

$$\int \frac{\sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4)}{x^9} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b^2 e \operatorname{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right) - \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4}\right) \sqrt{bx^3+a}}{12a^{\frac{3}{2}}} - \frac{840}{840} \\
& - \frac{3bc\sqrt{bx^3+a}}{80ax^5} - \frac{3bd\sqrt{bx^3+a}}{56ax^4} - \frac{be\sqrt{bx^3+a}}{12ax^3} + \frac{3b(-16af+7bc)\sqrt{bx^3+a}}{320a^2x^2} \\
& + \frac{3b(-14ag+5bd)\sqrt{bx^3+a}}{112a^2x} - \frac{3b^{\frac{4}{3}}(-14ag+5bd)\sqrt{bx^3+a}}{112a^2\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)} \\
& + 3^{\frac{1}{4}}b^{\frac{4}{3}}(-14ag+5bd)\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}} \\
& + \frac{224a^{\frac{5}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{224a^{\frac{5}{3}}\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}} \\
& + 3^{\frac{3}{4}}b^{\frac{4}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1-\sqrt{3})}{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(7b^{\frac{1}{3}}(-16af+7bc)+20a^{\frac{1}{3}}(-14ag+5bd)(1-\sqrt{3})\right) \\
& + \frac{2240a^2\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}{2240a^2\sqrt{bx^3+a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})\right)^2}}}
\end{aligned}$$

command

```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{140\sqrt{a}b^2x^8e \log\left(\frac{b^2x^6+8abx^3+4(bx^3+2a)\sqrt{bx^3+a}\sqrt{a+8a^2}}{x^6}\right) + 63(7b^2c-16abf)\sqrt{b}x^8\operatorname{weierstrassPInverse}(0, \right.$$

$$\left. 280\sqrt{-a}b^2x^8 \arctan\left(\frac{(bx^3+2a)\sqrt{bx^3+a}\sqrt{-a}}{2(abx^3+a^2)}\right) e - 63(7b^2c-16abf)\sqrt{b}x^8\operatorname{weierstrassPInverse}(0, -\frac{4a}{b}, x) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^9}, x\right)$$

19.126 Problem number 458

$$\int x^3 (a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^3(bx^3 + a)^{\frac{3}{2}} (156009gx^5 + 169575fx^4 + 185725ex^3 + 205275dx^2 + 229425cx)}{3900225} \\ & - \frac{4a^3e\sqrt{bx^3 + a}}{105b^2} + \frac{54a^2(-8af + 23bc)x\sqrt{bx^3 + a}}{21505b^2} + \frac{54a^2(-2ag + 5bd)x^2\sqrt{bx^3 + a}}{8645b^2} \\ & + \frac{2a^2ex^3\sqrt{bx^3 + a}}{105b} + \frac{54a^2fx^4\sqrt{bx^3 + a}}{4301b} + \frac{54a^2gx^5\sqrt{bx^3 + a}}{6175b} \\ & + \frac{2ax^3(3522519gx^5 + 4279275fx^4 + 5311735ex^3 + 6774075dx^2 + 8947575cx)\sqrt{bx^3 + a}}{185910725} \\ & - \frac{216a^3(-2ag + 5bd)\sqrt{bx^3 + a}}{8645b^{\frac{8}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{108 \cdot 3^{\frac{1}{4}} a^{\frac{10}{3}} (-2ag + 5bd) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + \dots}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{8645b^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{36 \cdot 3^{\frac{3}{4}} a^3 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(1729b^{\frac{1}{3}}(-8af + 23bc) - 8602a^{\frac{1}{3}}(-2ag + 5bd)\right) \left(1 - \dots\right)}{37182145b^{\frac{8}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(1400490 (23 a^3 bc - 8 a^4 f) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - 6967620 (5 a^3 bd - 2 a^4 g) \sqrt{b} \operatorname{weierstrassZeta}\left(\dots\right)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bgx^{10} + bfx^9 + bex^8 + (bd + ag)x^7 + aex^5 + (bc + af)x^6 + adx^4 + acx^3) \sqrt{bx^3 + a}, x\right)$$

19.127 Problem number 459

$$\int x^2 (a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^2(bx^3 + a)^{\frac{3}{2}} (33915gx^5 + 37145fx^4 + 41055ex^3 + 45885dx^2 + 52003cx)}{780045} \\ & + \frac{2a^2(-2af + 7bc) \sqrt{bx^3 + a}}{105b^2} + \frac{54a^2(-8ag + 23bd) x \sqrt{bx^3 + a}}{21505b^2} \\ & + \frac{54a^2e x^2 \sqrt{bx^3 + a}}{1729b} + \frac{2a^2f x^3 \sqrt{bx^3 + a}}{105b} + \frac{54a^2g x^4 \sqrt{bx^3 + a}}{4301b} \\ & + \frac{2a x^2 (2567565g x^5 + 3187041f x^4 + 4064445e x^3 + 5368545d x^2 + 7436429cx) \sqrt{bx^3 + a}}{111546435} \\ & - \frac{216a^3e \sqrt{bx^3 + a}}{1729b^{\frac{5}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\ & + \frac{108 \cdot 3^{\frac{1}{4}} a^{\frac{10}{3}} e \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \text{EllipticE} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{1729b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \\ & + \frac{36 \cdot 3^{\frac{3}{4}} a^3 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \text{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(13832ag - 39767bd + 43010a^{\frac{1}{3}}b^{\frac{2}{3}}e(1 - \sqrt{3}) \right) \left(\sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}} \right)}{37182145b^{\frac{7}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

```
integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6967620 a^3 b^{\frac{3}{2}} \text{ewierstrassZeta} \left(0, -\frac{4a}{b}, \text{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right) \right) - 280098 (23 a^3 b d - 8 a^4 g) \sqrt{b} \text{weierstra} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((bgx^9 + bfx^8 + bex^7 + (bd + ag)x^6 + aex^4 + (bc + af)x^5 + adx^3 + acx^2) \sqrt{bx^3 + a}, x \right)$$

19.128 Problem number 460

$$\int x(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x(bx^3 + a)^{\frac{3}{2}} (20995gx^5 + 23205fx^4 + 25935ex^3 + 29393dx^2 + 33915cx)}{440895} \\ & + \frac{2a^2(-2ag + 7bd) \sqrt{bx^3 + a}}{105b^2} + \frac{54a^2ex \sqrt{bx^3 + a}}{935b} + \frac{54a^2fx^2 \sqrt{bx^3 + a}}{1729b} + \frac{2a^2gx^3 \sqrt{bx^3 + a}}{105b} \\ & + \frac{2ax(138567gx^5 + 176715fx^4 + 233415ex^3 + 323323dx^2 + 479655cx) \sqrt{bx^3 + a}}{4849845} \\ & + \frac{54a^2(-4af + 19bc) \sqrt{bx^3 + a}}{1729b^{\frac{5}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{273^{\frac{1}{4}}a^{\frac{7}{3}}(-4af + 19bc) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + a}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{1729b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & - \frac{183^{\frac{3}{4}}a^{\frac{7}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(3458a^{\frac{2}{3}}b^{\frac{1}{3}}e + 935(-4af + 19bc)(1 - \sqrt{3})\right) \left(\frac{\sqrt{2}}{2}\right)}{1616615b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(280098 a^3 \sqrt{b} \operatorname{eweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 75735 (19 a^2 bc - 4 a^3 f) \sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassP}\left(0, -\frac{4a}{b}, x\right)\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bgx^8 + bfx^7 + bex^6 + (bd + ag)x^5 + aex^3 + (bc + af)x^4 + adx^2 + acx) \sqrt{bx^3 + a}, x\right)$$

19.129 Problem number 461

$$\int (a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bx^3 + a)^{\frac{3}{2}} (36465g x^5 + 40755f x^4 + 46189e x^3 + 53295d x^2 + 62985cx)}{692835} \\ & + \frac{2a^2 e \sqrt{bx^3 + a}}{15b} + \frac{54a^2 f x \sqrt{bx^3 + a}}{935b} + \frac{54a^2 g x^2 \sqrt{bx^3 + a}}{1729b} \\ & + \frac{2a(176715g x^5 + 233415f x^4 + 323323e x^3 + 479655d x^2 + 793611cx) \sqrt{bx^3 + a}}{4849845} \\ & + \frac{54a^2(-4ag + 19bd) \sqrt{bx^3 + a}}{1729b^{\frac{5}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{273^{\frac{1}{4}} a^{\frac{7}{3}} (-4ag + 19bd) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + a^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{1729b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{183^{\frac{3}{4}} a^2 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(1729b^{\frac{1}{3}}(-2af + 17bc) - 935a^{\frac{1}{3}}(-4ag + 19bd)\right) \left(1 - \frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}\right)}{1616615b^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(140049 (17 a^2 bc - 2 a^3 f) \sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - 75735 (19 a^2 bd - 4 a^3 g) \sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, x\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(bg x^7 + b f x^6 + b e x^5 + (b d + a g) x^4 + a e x^2 + (b c + a f) x^3 + a d x + a c\right) \sqrt{b x^3 + a}, x\right)$$

19.130 Problem number 462

$$\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bx^3 + a)^{\frac{3}{2}} (6435gx^5 + 7293fx^4 + 8415ex^3 + 9945dx^2 + 12155cx)}{109395x} \\ & - \frac{2a^{\frac{3}{2}}c \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right)}{3} + \frac{2a^2f\sqrt{bx^3 + a}}{15b} + \frac{54a^2gx\sqrt{bx^3 + a}}{935b} \\ & + \frac{2a(12285gx^5 + 17017fx^4 + 25245ex^3 + 41769dx^2 + 85085cx)\sqrt{bx^3 + a}}{255255x} \\ & + \frac{54a^2e\sqrt{bx^3 + a}}{91b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{273^{\frac{1}{4}}a^{\frac{7}{3}}e\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{91b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{183^{\frac{3}{4}}a^2\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(1547bd - 182ag - 935a^{\frac{1}{3}}b^{\frac{2}{3}}e(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{85085b^{\frac{4}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{255255 a^{\frac{3}{2}} b^2 c \log\left(-\frac{b^2 x^6 + 8 a b x^3 - 4 (b x^3 + 2 a) \sqrt{b x^3 + a} \sqrt{a} + 8 a^2}{x^6}\right) - 908820 a^2 b^{\frac{3}{2}} e \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrass}\right)}{\dots} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x}, x\right)$$

19.131 Problem number 463

$$\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bx^3 + a)^{\frac{3}{2}} (3003gx^5 + 3465fx^4 + 4095ex^3 + 5005dx^2 + 6435cx)}{45045x^2} \\ & - \frac{2a^{\frac{3}{2}} d \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right)}{3} + \frac{2a^2 g \sqrt{bx^3 + a}}{15b} - \frac{27ac \sqrt{bx^3 + a}}{7x} \\ & + \frac{2a(1001gx^5 + 1485fx^4 + 2457ex^3 + 5005dx^2 + 19305cx) \sqrt{bx^3 + a}}{15015x^2} \\ & + \frac{27a(2af + 13bc) \sqrt{bx^3 + a}}{91b^{\frac{2}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{273^{\frac{1}{4}} a^{\frac{4}{3}} (2af + 13bc) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{182b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{93^{\frac{3}{4}} a^{\frac{4}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(182a^{\frac{2}{3}}b^{\frac{1}{3}}e - 55(2af + 13bc)(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{5005b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{15015 a^{\frac{3}{2}} b d x \log\left(-\frac{b^2 x^6 + 8 a b x^3 - 4 (b x^3 + 2 a) \sqrt{b x^3 + a} \sqrt{a} + 8 a^2}{x^6}\right) + 88452 a^2 \sqrt{b} x \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)}{\right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac) \sqrt{bx^3 + a}}{x^2}, x\right)$$

19.132 Problem number 464

$$\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bx^3 + a)^{\frac{3}{2}} (3465gx^5 + 4095fx^4 + 5005ex^3 + 6435dx^2 + 9009cx)}{45045x^3} \\ & - \frac{2a^{\frac{3}{2}} e \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right)}{3} + \frac{27ac\sqrt{bx^3 + a}}{10x^2} - \frac{27ad\sqrt{bx^3 + a}}{7x} \\ & - \frac{2a(-1485gx^5 - 2457fx^4 - 5005ex^3 - 19305dx^2 + 27027cx)\sqrt{bx^3 + a}}{15015x^3} \\ & + \frac{27a(2ag + 13bd)\sqrt{bx^3 + a}}{91b^{\frac{2}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{273^{\frac{1}{4}}a^{\frac{4}{3}}(2ag + 13bd)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{182b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{93^{\frac{3}{4}}a\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(91b^{\frac{1}{3}}(4af + 11bc) - 110a^{\frac{1}{3}}(2ag + 13bd)(1 - \sqrt{3})\right)}{10010b^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{15015 a^{\frac{3}{2}} b x^2 e \log\left(-\frac{b^2 x^6 + 8 a b x^3 - 4 (b x^3 + 2 a) \sqrt{b x^3 + a} \sqrt{a} + 8 a^2}{x^6}\right) + 22113 (11 a b c + 4 a^2 f) \sqrt{b} x^2 \operatorname{weierstrassPInverse}\left(\frac{b x^3 + a}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}\right)}{\right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^3}, x\right)$$

19.133 Problem number 465

$$\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bx^3 + a)^{\frac{3}{2}} (315gx^5 + 385fx^4 + 495ex^3 + 693dx^2 + 1155cx)}{3465x^4} \\ & - \frac{(2af + 3bc) \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right) \sqrt{a}}{3} + \frac{ac\sqrt{bx^3 + a}}{x^3} + \frac{27ad\sqrt{bx^3 + a}}{10x^2} - \frac{27ae\sqrt{bx^3 + a}}{7x} \\ & - \frac{2a(-189gx^5 - 385fx^4 - 1485ex^3 + 2079dx^2 + 1155cx) \sqrt{bx^3 + a}}{1155x^4} + \frac{27ab^{\frac{1}{3}}e\sqrt{bx^3 + a}}{7(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))} \\ & - \frac{273^{\frac{1}{4}}a^{\frac{4}{3}}b^{\frac{1}{3}}e(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{14\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \\ & + \frac{93^{\frac{3}{4}}a(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) (77bd + 28ag - 110a^{\frac{1}{3}}b^{\frac{2}{3}}e(1 - \sqrt{3})) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{770b^{\frac{1}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{53460 ab^{\frac{3}{2}} x^3 \operatorname{eweierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - 1155 (3b^2c + 2abf) \sqrt{a} x^3 \log\left(-\frac{b^2x^6}{\dots}\right)}{26730 ab^{\frac{3}{2}} x^3 \operatorname{eweierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - 1155 (3b^2c + 2abf) \sqrt{-a} x^3 \arctan\left(\frac{2\sqrt{bx^3 + a}}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}\right)} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^4}, x\right)$$

19.134 Problem number 466

$$\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^5} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bx^3 + a)^{\frac{3}{2}} (35gx^5 + 45fx^4 + 63ex^3 + 105dx^2 + 315cx)}{315x^5} \\ & - \frac{(2ag + 3bd) \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right) \sqrt{a}}{3} + \frac{27ac\sqrt{bx^3 + a}}{20x^4} \\ & + \frac{ad\sqrt{bx^3 + a}}{x^3} + \frac{27ae\sqrt{bx^3 + a}}{10x^2} - \frac{27(8af + 7bc)\sqrt{bx^3 + a}}{56x} \\ & - \frac{2a(-35gx^5 - 135fx^4 + 189ex^3 + 105dx^2 + 189cx)\sqrt{bx^3 + a}}{105x^5} + \frac{27b^{\frac{1}{3}}(8af + 7bc)\sqrt{bx^3 + a}}{56\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{27 \cdot 3^{\frac{1}{4}} a^{\frac{1}{3}} b^{\frac{1}{3}} (8af + 7bc) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{112\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & + \frac{9 \cdot 3^{\frac{3}{4}} a^{\frac{1}{3}} b^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(28a^{\frac{2}{3}}b^{\frac{1}{3}}e - 5(8af + 7bc)(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{280\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{6804 a \sqrt{b} x^4 \text{eweiterstrassPInverse}(0, -\frac{4a}{b}, x) + 210 (3bd + 2ag) \sqrt{a} x^4 \log \left(-\frac{b^2 x^6 + 8abx^3 - 4(bx^3 + 2a) \sqrt{bx^3 + a} \sqrt{a}}{x^6} \right)}{\dots} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac) \sqrt{bx^3 + a}}{x^5}, x \right)$$

19.135 Problem number 467

$$\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) (bx^3 + a)^{\frac{3}{2}}}{60} \\ & - be \operatorname{arctanh} \left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}} \right) \sqrt{a} + \frac{27bc \sqrt{bx^3 + a}}{20x^2} - \frac{27bd \sqrt{bx^3 + a}}{8x} \\ & - \frac{b(-180gx^5 - 126fx^4 - 140ex^3 - 315dx^2 + 252cx) \sqrt{bx^3 + a}}{140x^3} + \frac{27b^{\frac{1}{3}}(8ag + 7bd) \sqrt{bx^3 + a}}{56 \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)} \\ & 273^{\frac{1}{4}} a^{\frac{1}{3}} b^{\frac{1}{3}} (8ag + 7bd) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \operatorname{EllipticE} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + \dots}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}} \\ & \frac{112 \sqrt{bx^3 + a}}{\sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}} \\ & 93^{\frac{3}{4}} b^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(14b^{\frac{1}{3}}(2af + bc) - 5a^{\frac{1}{3}}(8ag + 7bd)(1 - \sqrt{3}) \right) \left(\frac{\sqrt{2}}{2} \right) \\ & + \frac{280 \sqrt{bx^3 + a}}{\sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

`integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{210 \sqrt{a} b x^5 e \log \left(-\frac{b^2 x^6 + 8 a b x^3 - 4 (b x^3 + 2 a) \sqrt{b x^3 + a} \sqrt{a + 8 a^2}}{x^6} \right) + 1134 (b c + 2 a f) \sqrt{b} x^5 \text{weierstrassPInverse}(0, -}{\right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b g x^7 + b f x^6 + b e x^5 + (b d + a g) x^4 + a e x^2 + (b c + a f) x^3 + a d x + a c) \sqrt{b x^3 + a}}{x^6}, x \right)$$

19.136 Problem number 468

$$\int \frac{(a + b x^3)^{3/2} (c + d x + e x^2 + f x^3 + g x^4)}{x^7} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) (b x^3 + a)^{\frac{3}{2}}}{60} - \frac{b(4af + bc) \operatorname{arctanh} \left(\frac{\sqrt{b x^3 + a}}{\sqrt{a}} \right)}{4\sqrt{a}} \\ & + \frac{bc \sqrt{b x^3 + a}}{4x^3} + \frac{27bd \sqrt{b x^3 + a}}{20x^2} - \frac{27be \sqrt{b x^3 + a}}{8x} \\ & - \frac{b(-18g x^5 - 20f x^4 - 45e x^3 + 36d x^2 + 10cx) \sqrt{b x^3 + a}}{20x^4} + \frac{27b^{\frac{4}{3}} e \sqrt{b x^3 + a}}{8 \left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)} \\ & - \frac{27 \cdot 3^{\frac{1}{4}} a^{\frac{1}{3}} b^{\frac{4}{3}} e \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticE} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{16 \sqrt{b x^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \\ & + \frac{9 \cdot 3^{\frac{3}{4}} b^{\frac{2}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(2bd + 4ag - 5a^{\frac{1}{3}} b^{\frac{2}{3}} e (1 - \sqrt{3}) \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{40 \sqrt{b x^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{810 ab^{\frac{3}{2}} x^6 \text{e} \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - 15 (b^2 c + 4 abf) \sqrt{a} x^6 \log\left(-\frac{b^2 x^6 + 8 abx^3}{b^3}\right)}{405 ab^{\frac{3}{2}} x^6 \text{e} \text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) - 15 (b^2 c + 4 abf) \sqrt{-a} x^6 \arctan\left(\frac{2 \sqrt{bx^3 + a}}{bx^3}\right)} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac) \sqrt{bx^3 + a}}{x^7}, x\right)$$

19.137 Problem number 469

$$\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^8} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{\left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3}\right) (bx^3 + a)^{\frac{3}{2}}}{420} - \frac{b(4ag + bd) \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right)}{4\sqrt{a}} \\
& + \frac{27bc\sqrt{bx^3 + a}}{280x^4} + \frac{bd\sqrt{bx^3 + a}}{4x^3} + \frac{27be\sqrt{bx^3 + a}}{20x^2} - \frac{27b(14af + bc)\sqrt{bx^3 + a}}{112ax} \\
& - \frac{b(-140gx^5 - 315fx^4 + 252ex^3 + 70dx^2 + 36cx)\sqrt{bx^3 + a}}{140x^5} + \frac{27b^{\frac{4}{3}}(14af + bc)\sqrt{bx^3 + a}}{112a\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\
& - \frac{273^{\frac{1}{4}}b^{\frac{4}{3}}(14af + bc)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{224a^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\
& + \frac{93^{\frac{3}{4}}b^{\frac{4}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(28a^{\frac{2}{3}}b^{\frac{1}{3}}e - 5(14af + bc)(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{560a^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}
\end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{2268 ab^{\frac{3}{2}} x^7 \operatorname{eweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 105 (b^2d + 4abg) \sqrt{a} x^7 \log\left(-\frac{b^2x^6 + 8abx^3 - 4(bx^3 + 2a)\sqrt{bx^3 + a}\sqrt{a}}{x^6}\right)}{\dots} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^8}, x\right)$$

19.138 Problem number 470

$$\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^9} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4}\right) (bx^3 + a)^{\frac{3}{2}}}{840} - \frac{b^2 e \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right)}{4\sqrt{a}} \\ & - \frac{b\left(\frac{63c}{x^5} + \frac{90d}{x^4} + \frac{140e}{x^3} + \frac{252f}{x^2} + \frac{630g}{x}\right) \sqrt{bx^3 + a}}{560} - \frac{27b^2 c \sqrt{bx^3 + a}}{320a x^2} \\ & - \frac{27b^2 d \sqrt{bx^3 + a}}{112ax} + \frac{27b^{\frac{4}{3}}(14ag + bd) \sqrt{bx^3 + a}}{112a \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + 27 \cdot 3^{\frac{1}{4}} b^{\frac{4}{3}} (14ag + bd) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & + 224a^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & + 9 \cdot 3^{\frac{3}{4}} b^{\frac{4}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(7b^{\frac{1}{3}}(-16af + bc) + 20a^{\frac{1}{3}}(14ag + bd)(1 - \sqrt{3})\right) \\ & + 2240a \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{420 \sqrt{a} b^2 x^8 e \log\left(-\frac{b^2 x^6 + 8 a b x^3 - 4 (b x^3 + 2 a) \sqrt{b x^3 + a} \sqrt{a} + 8 a^2}{x^6}\right) - 567 (b^2 c - 16 a b f) \sqrt{b} x^8 \operatorname{weierstrassPInverse}(0)}{\dots} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac) \sqrt{bx^3 + a}}{x^9}, x\right)$$

19.139 Problem number 471

$$\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^{10}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{280c}{x^9} + \frac{315d}{x^8} + \frac{360e}{x^7} + \frac{420f}{x^6} + \frac{504g}{x^5}\right) (bx^3 + a)^{\frac{3}{2}}}{2520} + \frac{b^2(-6af + bc) \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right)}{24a^{\frac{3}{2}}} \\ & - \frac{b\left(\frac{140c}{x^6} + \frac{189d}{x^5} + \frac{270e}{x^4} + \frac{420f}{x^3} + \frac{756g}{x^2}\right) \sqrt{bx^3 + a}}{1680} - \frac{b^2c\sqrt{bx^3 + a}}{24ax^3} \\ & - \frac{27b^2d\sqrt{bx^3 + a}}{320ax^2} - \frac{27b^2e\sqrt{bx^3 + a}}{112ax} + \frac{27b^{\frac{7}{3}}e\sqrt{bx^3 + a}}{112a\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & - \frac{273^{\frac{1}{4}}b^{\frac{7}{3}}e\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{224a^{\frac{2}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & - \frac{93^{\frac{3}{4}}b^{\frac{5}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(7bd - 112ag + 20a^{\frac{1}{3}}b^{\frac{2}{3}}e(1 - \sqrt{3})\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{2240a\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{4860 ab^{\frac{5}{2}} x^9 \operatorname{eweierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + 210 (b^3c - 6ab^2f) \sqrt{a} x^9 \log\left(\frac{b^2x^6 + 8abx^3 + a^2}{(bx^3 + a)^2}\right)}{4860 ab^{\frac{5}{2}} x^9 \operatorname{eweierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + 420 (b^3c - 6ab^2f) \sqrt{-a} x^9 \arctan\left(\frac{bx^3 + a}{\sqrt{-a}}\right)} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^{10}}, x\right)$$

19.140 Problem number 472

$$\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^{11}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7} + \frac{420g}{x^6}\right)(bx^3 + a)^{\frac{3}{2}}}{2520} + \frac{b^2(-6ag + bd) \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right)}{24a^{\frac{3}{2}}} \\ & -\frac{b\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right)\sqrt{bx^3 + a}}{1680} - \frac{27b^2c\sqrt{bx^3 + a}}{1120ax^4} - \frac{b^2d\sqrt{bx^3 + a}}{24ax^3} \\ & -\frac{27b^2e\sqrt{bx^3 + a}}{320ax^2} + \frac{27b^2(-4af + bc)\sqrt{bx^3 + a}}{448a^2x} - \frac{27b^{\frac{7}{3}}(-4af + bc)\sqrt{bx^3 + a}}{448a^2\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\ & + \frac{273^{\frac{1}{4}}b^{\frac{7}{3}}(-4af + bc)\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right)\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{896a^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\ & -\frac{93^{\frac{3}{4}}b^{\frac{7}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right)\left(7a^{\frac{2}{3}}b^{\frac{1}{3}}e - 5(-4af + bc)(1 - \sqrt{3})\right)\left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)}{2240a^{\frac{5}{3}}\sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1701 ab^{\frac{5}{2}} x^{10} \text{eweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 210 (b^3 d - 6 ab^2 g) \sqrt{a} x^{10} \log\left(\frac{b^2 x^6 + 8 abx^3 - 4 (bx^3 + 2a) \sqrt{bx^3 + a}}{x^6}\right)}{1701 ab^{\frac{5}{2}} x^{10} \text{eweierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + 420 (b^3 d - 6 ab^2 g) \sqrt{-a} x^{10} \arctan\left(\frac{(bx^3 + 2a) \sqrt{bx^3 + a} \sqrt{-a}}{2(abx^3 + a^2)}\right)} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac) \sqrt{bx^3 + a}}{x^{11}}, x\right)$$

19.141 Problem number 473

$$\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^{12}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{\left(\frac{2520c}{x^{11}} + \frac{2772d}{x^{10}} + \frac{3080e}{x^9} + \frac{3465f}{x^8} + \frac{3960g}{x^7}\right) (bx^3 + a)^{\frac{3}{2}}}{27720} + \frac{b^3 e \operatorname{arctanh}\left(\frac{\sqrt{bx^3 + a}}{\sqrt{a}}\right)}{24a^{\frac{3}{2}}} \\
& - \frac{b\left(\frac{945c}{x^8} + \frac{1188d}{x^7} + \frac{1540e}{x^6} + \frac{2079f}{x^5} + \frac{2970g}{x^4}\right) \sqrt{bx^3 + a}}{18480} - \frac{27b^2 c \sqrt{bx^3 + a}}{1760a x^5} \\
& - \frac{27b^2 d \sqrt{bx^3 + a}}{1120a x^4} - \frac{b^2 e \sqrt{bx^3 + a}}{24a x^3} + \frac{27b^2(-22af + 7bc) \sqrt{bx^3 + a}}{7040a^2 x^2} \\
& + \frac{27b^2(-4ag + bd) \sqrt{bx^3 + a}}{448a^2 x} - \frac{27b^{\frac{7}{3}}(-4ag + bd) \sqrt{bx^3 + a}}{448a^2 \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)} \\
& + \frac{27 \cdot 3^{\frac{1}{4}} b^{\frac{7}{3}}(-4ag + bd) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{896a^{\frac{5}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \\
& + \frac{93^{\frac{3}{4}} b^{\frac{7}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(7b^{\frac{1}{3}}(-22af + 7bc) + 110a^{\frac{1}{3}}(-4ag + bd)(1 - \sqrt{3})\right)}{49280a^2 \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}
\end{aligned}$$

command

```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
& \left[\frac{4620 \sqrt{a} b^3 x^{11} e \log\left(\frac{b^2 x^6 + 8 a b x^3 + 4 (b x^3 + 2 a) \sqrt{b x^3 + a} \sqrt{a} + 8 a^2}{x^6}\right) + 1701 (7 b^3 c - 22 a b^2 f) \sqrt{b} x^{11} \operatorname{weierstrassPInverse}\left(\frac{b x^3 + a}{a}, \sqrt{b}\right)}{9240 \sqrt{-a} b^3 x^{11} \arctan\left(\frac{(b x^3 + 2 a) \sqrt{b x^3 + a} \sqrt{-a}}{2 (a b x^3 + a^2)}\right) e - 1701 (7 b^3 c - 22 a b^2 f) \sqrt{b} x^{11} \operatorname{weierstrassPInverse}\left(0, -\frac{4}{7}\right)} \right]
\end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac) \sqrt{bx^3 + a}}{x^{12}}, x \right)$$

19.142 Problem number 485

$$\int \frac{c + dx + ex^2 + fx^3}{a - bx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{f \ln(-bx^4 + a)}{4b} + \frac{d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} \\ & + \frac{\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (-e\sqrt{a} + c\sqrt{b})}{2a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (e\sqrt{a} + c\sqrt{b})}{2a^{\frac{3}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.143 Problem number 486

$$\int \frac{x^3(c + dx + ex^2 + fx^3)}{a - bx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{dx}{b} - \frac{ex^2}{2b} - \frac{fx^3}{3b} - \frac{c \ln(-bx^4 + a)}{4b} + \frac{e \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right) \sqrt{a}}{2b^{\frac{3}{2}}} \\ & + \frac{a^{\frac{1}{4}} \arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (-f\sqrt{a} + d\sqrt{b})}{2b^{\frac{7}{4}}} + \frac{a^{\frac{1}{4}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (f\sqrt{a} + d\sqrt{b})}{2b^{\frac{7}{4}}} \end{aligned}$$

command

```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.144 Problem number 487

$$\int \frac{c + dx + ex^2 + fx^3}{a + bx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{f \ln(bx^4 + a)}{4b} + \frac{d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-e\sqrt{a} + c\sqrt{b}\right) \sqrt{2}}{8a^{\frac{3}{4}}b^{\frac{3}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-e\sqrt{a} + c\sqrt{b}\right) \sqrt{2}}{8a^{\frac{3}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(e\sqrt{a} + c\sqrt{b}\right) \sqrt{2}}{4a^{\frac{3}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(e\sqrt{a} + c\sqrt{b}\right) \sqrt{2}}{4a^{\frac{3}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.145 Problem number 488

$$\int \frac{x^3(c + dx + ex^2 + fx^3)}{a + bx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{dx}{b} + \frac{e x^2}{2b} + \frac{f x^3}{3b} + \frac{c \ln(bx^4 + a)}{4b} - \frac{e \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right) \sqrt{a}}{2b^{\frac{3}{2}}} \\ & + \frac{a^{\frac{1}{4}} \ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-f\sqrt{a} + d\sqrt{b}\right) \sqrt{2}}{8b^{\frac{7}{4}}} \\ & - \frac{a^{\frac{1}{4}} \ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-f\sqrt{a} + d\sqrt{b}\right) \sqrt{2}}{8b^{\frac{7}{4}}} \\ & - \frac{a^{\frac{1}{4}} \arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(f\sqrt{a} + d\sqrt{b}\right) \sqrt{2}}{4b^{\frac{7}{4}}} \\ & - \frac{a^{\frac{1}{4}} \arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(f\sqrt{a} + d\sqrt{b}\right) \sqrt{2}}{4b^{\frac{7}{4}}} \end{aligned}$$

command

```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.146 Problem number 489

$$\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{-af + bx(ex^2 + dx + c)}{4ab(bx^4 + a)} + \frac{d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{b}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-e\sqrt{a} + 3c\sqrt{b}\right) \sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{3}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-e\sqrt{a} + 3c\sqrt{b}\right) \sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(e\sqrt{a} + 3c\sqrt{b}\right) \sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(e\sqrt{a} + 3c\sqrt{b}\right) \sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.147 Problem number 490

$$\int \frac{x^3(c + dx + ex^2 + fx^3)}{(a + bx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{-fx^3 - ex^2 - dx - c}{4b(bx^4 + a)} + \frac{e \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}\sqrt{a}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-3f\sqrt{a} + d\sqrt{b}\right)\sqrt{2}}{32a^{\frac{3}{4}}b^{\frac{7}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-3f\sqrt{a} + d\sqrt{b}\right)\sqrt{2}}{32a^{\frac{3}{4}}b^{\frac{7}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(3f\sqrt{a} + d\sqrt{b}\right)\sqrt{2}}{16a^{\frac{3}{4}}b^{\frac{7}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(3f\sqrt{a} + d\sqrt{b}\right)\sqrt{2}}{16a^{\frac{3}{4}}b^{\frac{7}{4}}} \end{aligned}$$

command

```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.148 Problem number 491

$$\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(5ex^2 + 6dx + 7c)}{32a^2(bx^4 + a)} + \frac{-af + bx(ex^2 + dx + c)}{8ab(bx^4 + a)^2} + \frac{3d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{3}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.149 Problem number 492

$$\int \frac{x^3(c + dx + ex^2 + fx^3)}{(a + bx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{-fx^3 - ex^2 - dx - c}{8b(bx^4 + a)^2} + \frac{x(3fx^2 + 2ex + d)}{32ab(bx^4 + a)} + \frac{e \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}b^{\frac{3}{2}}} \\
& - \frac{3 \ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-f\sqrt{a} + d\sqrt{b}\right) \sqrt{2}}{256a^{\frac{7}{4}}b^{\frac{7}{4}}} \\
& + \frac{3 \ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-f\sqrt{a} + d\sqrt{b}\right) \sqrt{2}}{256a^{\frac{7}{4}}b^{\frac{7}{4}}} \\
& + \frac{3 \arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(f\sqrt{a} + d\sqrt{b}\right) \sqrt{2}}{128a^{\frac{7}{4}}b^{\frac{7}{4}}} \\
& + \frac{3 \arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(f\sqrt{a} + d\sqrt{b}\right) \sqrt{2}}{128a^{\frac{7}{4}}b^{\frac{7}{4}}}
\end{aligned}$$

command

```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.150 Problem number 493

$$\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^4} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{x(9ex^2 + 10dx + 11c)}{96a^2(bx^4 + a)^2} + \frac{x(45ex^2 + 60dx + 77c)}{384a^3(bx^4 + a)} + \frac{-af + bx(ex^2 + dx + c)}{12ab(bx^4 + a)^3} \\
& + \frac{5d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}} - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-15e\sqrt{a} + 77c\sqrt{b}\right) \sqrt{2}}{1024a^{\frac{15}{4}}b^{\frac{3}{4}}} \\
& + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-15e\sqrt{a} + 77c\sqrt{b}\right) \sqrt{2}}{1024a^{\frac{15}{4}}b^{\frac{3}{4}}} \\
& + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(15e\sqrt{a} + 77c\sqrt{b}\right) \sqrt{2}}{512a^{\frac{15}{4}}b^{\frac{3}{4}}} \\
& + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(15e\sqrt{a} + 77c\sqrt{b}\right) \sqrt{2}}{512a^{\frac{15}{4}}b^{\frac{3}{4}}}
\end{aligned}$$

command

```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

19.151 Problem number 494

$$\int \frac{x^3(c + dx + ex^2 + fx^3)}{(a + bx^4)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{-fx^3 - ex^2 - dx - c}{12b(bx^4 + a)^3} + \frac{x(3fx^2 + 2ex + d)}{96ab(bx^4 + a)^2} + \frac{x(15fx^2 + 12ex + 7d)}{384a^2b(bx^4 + a)} \\ & + \frac{e \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{32a^{\frac{5}{2}}b^{\frac{3}{2}}} - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-5f\sqrt{a} + 7d\sqrt{b}\right)\sqrt{2}}{1024a^{\frac{11}{4}}b^{\frac{7}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-5f\sqrt{a} + 7d\sqrt{b}\right)\sqrt{2}}{1024a^{\frac{11}{4}}b^{\frac{7}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(5f\sqrt{a} + 7d\sqrt{b}\right)\sqrt{2}}{512a^{\frac{11}{4}}b^{\frac{7}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(5f\sqrt{a} + 7d\sqrt{b}\right)\sqrt{2}}{512a^{\frac{11}{4}}b^{\frac{7}{4}}} \end{aligned}$$

command

```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20 Test file number 30

Test folder name:

test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.4_Improper/30_1.1.4.2-c_x-
 $\hat{m}-a_x^j+b_x^n-\hat{p}$

20.1 Problem number 38

$$\int x^3 \sqrt{ax + bx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{20a^2 \sqrt{bx^3 + ax}}{231b^2} + \frac{4ax^2 \sqrt{bx^3 + ax}}{77b} + \frac{2x^4 \sqrt{bx^3 + ax}}{11} \\ & + 10a^{\frac{11}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{\sqrt{a}}{\sqrt{a+b^3x^3}}} \\ & + \frac{231 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{9}{4}} \sqrt{bx^3 + ax}}{231 b^3} \end{aligned}$$

command

```
integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10 a^3 \sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (21 b^3 x^4 + 6 a b^2 x^2 - 10 a^2 b) \sqrt{bx^3 + ax} \right)}{231 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{bx^3 + ax} x^3, x\right)$$

20.2 Problem number 39

$$\int x^2 \sqrt{ax + bx^3} dx$$

Optimal antiderivative

$$\frac{4a^2x(bx^2+a)}{15b^{\frac{3}{2}}(\sqrt{a}+x\sqrt{b})\sqrt{bx^3+ax}} + \frac{4ax\sqrt{bx^3+ax}}{45b} + \frac{2x^3\sqrt{bx^3+ax}}{9}$$

$$+ \frac{4a^{\frac{9}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}\sqrt{\frac{b}{(\sqrt{a}})}}}{15\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^3+ax}}$$

$$- \frac{2a^{\frac{9}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}\sqrt{\frac{b}{(\sqrt{a}})}}}{15\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^3+ax}}$$

command

`integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6a^2\sqrt{b}\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (5b^2x^3 + 2abx)\sqrt{bx^3+ax}\right)}{45b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{bx^3+ax}x^2, x\right)$$

20.3 Problem number 40

$$\int x\sqrt{ax+bx^3}dx$$

Optimal antiderivative

$$\frac{4a\sqrt{bx^3+ax}}{21b} + \frac{2x^2\sqrt{bx^3+ax}}{7}$$

$$+ \frac{2a^{\frac{7}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}\sqrt{\frac{b}{(\sqrt{a}})}}}{21\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)b^{\frac{5}{4}}\sqrt{bx^3+ax}}$$

command

```
integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 a^2 \sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (3 b^2 x^2 + 2 a b) \sqrt{b x^3 + a x} \right)}{21 b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b x^3 + a x} x, x\right)$$

20.4 Problem number 41

$$\int \sqrt{a x + b x^3} dx$$

Optimal antiderivative

$$\frac{4 a x (b x^2 + a)}{5 \sqrt{b} (\sqrt{a} + x \sqrt{b}) \sqrt{b x^3 + a x}} + \frac{2 x \sqrt{b x^3 + a x}}{5}$$

$$+ \frac{4 a^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x \sqrt{b}) \sqrt{x} \sqrt{\frac{b}{(\sqrt{a}})}}}{5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{b x^3 + a x}}$$

$$+ \frac{2 a^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x \sqrt{b}) \sqrt{x} \sqrt{\frac{b}{(\sqrt{a}})}}}{5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{b x^3 + a x}}$$

command

```
integrate((b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{b x^3 + a x} b x - 2 a \sqrt{b} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) \right)}{5 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b x^3 + a x}, x\right)$$

20.5 Problem number 42

$$\int \frac{\sqrt{ax + bx^3}}{x} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^3 + ax}}{3} + \frac{2a^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{bx^3 + ax}{a}}}{3 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{1}{4}} \sqrt{bx^3 + ax}}$$

command

```
integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 a \sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + \sqrt{bx^3 + ax} b \right)}{3 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax}}{x}, x\right)$$

20.6 Problem number 43

$$\int \frac{\sqrt{ax + bx^3}}{x^2} dx$$

Optimal antiderivative

$$\frac{4x(bx^2 + a)\sqrt{b}}{(\sqrt{a} + x\sqrt{b})\sqrt{bx^3 + ax}} - \frac{2\sqrt{bx^3 + ax}}{x}$$

$$+ \frac{4a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)\sqrt{bx^3 + ax}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{x}\sqrt{\frac{bx^3 + ax}{(\sqrt{a} + x\sqrt{b})^2}}$$

$$+ \frac{2a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)\sqrt{bx^3 + ax}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{x}\sqrt{\frac{bx^3 + ax}{(\sqrt{a} + x\sqrt{b})^2}}$$

command

```
integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{b}x\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + \sqrt{bx^3 + ax}\right)}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax}}{x^2}, x\right)$$

20.7 Problem number 44

$$\int \frac{\sqrt{ax + bx^3}}{x^3} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^3 + ax}}{3x^2}$$

$$+ \frac{2b^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{3\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{1}{4}}\sqrt{bx^3 + ax}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{x}\sqrt{\frac{bx^3 + ax}{(\sqrt{a} + x\sqrt{b})^2}}$$

command

```
integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{b} x^2 \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - \sqrt{bx^3 + ax} \right)}{3x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{x^3}, x\right)$$

20.8 Problem number 45

$$\int \frac{\sqrt{ax + bx^3}}{x^4} dx$$

Optimal antiderivative

$$\frac{4b^{\frac{3}{2}}x(bx^2 + a)}{5a(\sqrt{a} + x\sqrt{b})\sqrt{bx^3 + ax}} - \frac{2\sqrt{bx^3 + ax}}{5x^3} - \frac{4b\sqrt{bx^3 + ax}}{5ax}$$

$$4b^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{b}{(\sqrt{a}})}}$$

$$5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{3}{4}} \sqrt{bx^3 + ax}$$

$$2b^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{bx^3 + ax}{(\sqrt{a}})}}$$

$$+ \frac{5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{3}{4}} \sqrt{bx^3 + ax}}{5ax^3}$$

command

```
integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 b^{\frac{3}{2}} x^3 \text{weierstrassZeta}\left(-\frac{4a}{b}, 0, \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + \sqrt{bx^3 + ax} (2bx^2 + a) \right)}{5ax^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{x^4}, x\right)$$

20.9 Problem number 46

$$\int x^2(ax + bx^3)^{3/2} dx$$

Optimal antiderivative

$$\frac{2x^3(bx^3 + ax)^{\frac{3}{2}}}{15} - \frac{8a^3\sqrt{bx^3 + ax}}{231b^2} + \frac{8a^2x^2\sqrt{bx^3 + ax}}{385b} + \frac{4ax^4\sqrt{bx^3 + ax}}{55}$$

$$+ \frac{4a^{\frac{15}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{b}{(\sqrt{a}})}}}{231 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{9}{4}} \sqrt{bx^3 + ax}}$$

command

```
integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(20 a^4 \sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (77 b^4 x^6 + 119 a b^3 x^4 + 12 a^2 b^2 x^2 - 20 a^3 b) \sqrt{bx^3 + ax} \right)}{1155 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(bx^5 + ax^3\right) \sqrt{bx^3 + ax}, x\right)$$

20.10 Problem number 47

$$\int x(ax + bx^3)^{3/2} dx$$

Optimal antiderivative

$$\frac{2x^2(bx^3+ax)^{\frac{3}{2}}}{13} - \frac{8a^3x(bx^2+a)}{65b^{\frac{3}{2}}(\sqrt{a}+x\sqrt{b})\sqrt{bx^3+ax}} + \frac{8a^2x\sqrt{bx^3+ax}}{195b} + \frac{4ax^3\sqrt{bx^3+ax}}{39}$$

$$+ \frac{8a^{\frac{13}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b})\sqrt{x} \sqrt{\frac{b}{(\sqrt{a}+x\sqrt{b})^2}}}{65 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{7}{4}}\sqrt{bx^3+ax}}$$

$$- \frac{4a^{\frac{13}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b})\sqrt{x} \sqrt{\frac{b}{(\sqrt{a}+x\sqrt{b})^2}}}{65 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{7}{4}}\sqrt{bx^3+ax}}$$

command

`integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12a^3\sqrt{b}\operatorname{weierstrassZeta}\left(-\frac{4a}{b},0,\operatorname{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right) + (15b^3x^5 + 25ab^2x^3 + 4a^2bx)\sqrt{bx^3+ax}\right)}{195b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bx^4+ax^2)\sqrt{bx^3+ax},x\right)$$

20.11 Problem number 48

$$\int (ax+bx^3)^{3/2} dx$$

Optimal antiderivative

$$\frac{2x(bx^3+ax)^{\frac{3}{2}}}{11} + \frac{8a^2\sqrt{bx^3+ax}}{77b} + \frac{12ax^2\sqrt{bx^3+ax}}{77}$$

$$+ \frac{4a^{\frac{11}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b})\sqrt{x} \sqrt{\frac{b}{(\sqrt{a}+x\sqrt{b})^2}}}{77 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{5}{4}}\sqrt{bx^3+ax}}$$

command

`integrate((b*x^3+a*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 a^3 \sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (7 b^3 x^4 + 13 a b^2 x^2 + 4 a^2 b) \sqrt{b x^3 + a x} \right)}{77 b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((b x^3 + a x)^{\frac{3}{2}}, x\right)$$

20.12 Problem number 49

$$\int \frac{(a x + b x^3)^{3/2}}{x} dx$$

Optimal antiderivative

$$\frac{2(b x^3 + a x)^{\frac{3}{2}}}{9} + \frac{8 a^2 x (b x^2 + a)}{15 \sqrt{b} (\sqrt{a} + x \sqrt{b}) \sqrt{b x^3 + a x}} + \frac{4 a x \sqrt{b x^3 + a x}}{15}$$

$$\frac{8 a^{\frac{9}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x \sqrt{b}) \sqrt{x} \sqrt{\frac{b}{(\sqrt{a})^2}}}}{15 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{b x^3 + a x}}$$

$$+ \frac{4 a^{\frac{9}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x \sqrt{b}) \sqrt{x} \sqrt{\frac{b}{(\sqrt{a})^2}}}}{15 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{b x^3 + a x}}$$

command

`integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 a^2 \sqrt{b} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) - (5 b^2 x^3 + 11 a b x) \sqrt{b x^3 + a x} \right)}{45 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b x^3 + a x} (b x^2 + a), x\right)$$

20.13 Problem number 50

$$\int \frac{(ax + bx^3)^{3/2}}{x^2} dx$$

Optimal antiderivative

$$\frac{2(bx^3 + ax)^{\frac{3}{2}}}{7x} + \frac{4a\sqrt{bx^3 + ax}}{7} + \frac{4a^{\frac{7}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{bx^3 + ax}{(\sqrt{a} + x\sqrt{b})^2}}}{7 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{1}{4}} \sqrt{bx^3 + ax}}$$

command

```
integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4a^2\sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (b^2x^2 + 3ab)\sqrt{bx^3 + ax}\right)}{7b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax}(bx^2 + a)}{x}, x\right)$$

20.14 Problem number 51

$$\int \frac{(ax + bx^3)^{3/2}}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(bx^3+ax)^{\frac{3}{2}}}{x^2} + \frac{24ax(bx^2+a)\sqrt{b}}{5(\sqrt{a}+x\sqrt{b})\sqrt{bx^3+ax}} + \frac{12bx\sqrt{bx^3+ax}}{5} \\
 & \frac{24a^{\frac{5}{4}}b^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)\sqrt{bx^3+ax}} \\
 & + \frac{12a^{\frac{5}{4}}b^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)\sqrt{bx^3+ax}}
 \end{aligned}$$

command

`integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12a\sqrt{b}x\operatorname{weierstrassZeta}\left(-\frac{4a}{b},0,\operatorname{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)-\sqrt{bx^3+ax}(bx^2-5a)\right)}{5x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+ax}(bx^2+a)}{x^2},x\right)$$

20.15 Problem number 52

$$\int \frac{(ax+bx^3)^{3/2}}{x^4} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(bx^3+ax)^{\frac{3}{2}}}{3x^3} + \frac{4b\sqrt{bx^3+ax}}{3} \\
 & \frac{4a^{\frac{3}{4}}b^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}}{3\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)\sqrt{bx^3+ax}}
 \end{aligned}$$

command

```
integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 a \sqrt{b} x^2 \text{weierstrassPInverse} \left(-\frac{4a}{b}, 0, x \right) + \sqrt{bx^3 + ax} (bx^2 - a) \right)}{3 x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{bx^3 + ax} (bx^2 + a)}{x^3}, x \right)$$

20.16 Problem number 53

$$\int \frac{(ax + bx^3)^{3/2}}{x^5} dx$$

Optimal antiderivative

$$-\frac{2(bx^3 + ax)^{\frac{3}{2}}}{5x^4} + \frac{24b^{\frac{3}{2}}x(bx^2 + a)}{5(\sqrt{a} + x\sqrt{b})\sqrt{bx^3 + ax}} - \frac{12b\sqrt{bx^3 + ax}}{5x}$$

$$- \frac{24a^{\frac{1}{4}}b^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x}}{5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) \sqrt{bx^3 + ax}}$$

$$+ \frac{12a^{\frac{1}{4}}b^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x}}{5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) \sqrt{bx^3 + ax}}$$

command

```
integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 b^{\frac{3}{2}} x^3 \text{weierstrassZeta} \left(-\frac{4a}{b}, 0, \text{weierstrassPInverse} \left(-\frac{4a}{b}, 0, x \right) \right) + \sqrt{bx^3 + ax} (7bx^2 + a) \right)}{5 x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{bx^3 + ax} (bx^2 + a)}{x^4}, x \right)$$

20.17 Problem number 54

$$\int \frac{(ax + bx^3)^{3/2}}{x^6} dx$$

Optimal antiderivative

$$\frac{2(bx^3 + ax)^{\frac{3}{2}}}{7x^5} - \frac{4b\sqrt{bx^3 + ax}}{7x^2} + \frac{4b^{\frac{7}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{bx^3 + ax}{(\sqrt{a})^2}}}{7 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} \sqrt{bx^3 + ax}}$$

command

```
integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 b^{\frac{3}{2}} x^4 \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - \sqrt{bx^3 + ax} (3bx^2 + a) \right)}{7x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax} (bx^2 + a)}{x^5}, x\right)$$

20.18 Problem number 55

$$\int \frac{(ax + bx^3)^{3/2}}{x^7} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(bx^3+ax)^{\frac{3}{2}}}{9x^6} + \frac{8b^{\frac{5}{2}}x(bx^2+a)}{15a(\sqrt{a}+x\sqrt{b})\sqrt{bx^3+ax}} - \frac{4b\sqrt{bx^3+ax}}{15x^3} - \frac{8b^2\sqrt{bx^3+ax}}{15ax} \\
 & - \frac{8b^{\frac{9}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b})\sqrt{x} \sqrt{\frac{b}{(\sqrt{a}})}}{15 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{3}{4}}\sqrt{bx^3+ax}} \\
 & + \frac{4b^{\frac{9}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b})\sqrt{x} \sqrt{\frac{bx}{(\sqrt{a}})}}{15 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{3}{4}}\sqrt{bx^3+ax}}
 \end{aligned}$$

command

`integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12b^{\frac{5}{2}}x^5\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (12b^2x^4 + 11abx^2 + 5a^2)\sqrt{bx^3+ax}\right)}{45ax^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+ax}(bx^2+a)}{x^6}, x\right)$$

20.19 Problem number 56

$$\int \frac{(ax+bx^3)^{3/2}}{x^8} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(bx^3+ax)^{\frac{3}{2}}}{11x^7} - \frac{12b\sqrt{bx^3+ax}}{77x^4} - \frac{8b^2\sqrt{bx^3+ax}}{77ax^2} \\
 & - \frac{4b^{\frac{11}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b})\sqrt{x} \sqrt{\frac{bx}{(\sqrt{a}})}}{77 \cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{5}{4}}\sqrt{bx^3+ax}}
 \end{aligned}$$

command

```
integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 b^{\frac{5}{2}} x^6 \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (4 b^2 x^4 + 13 a b x^2 + 7 a^2) \sqrt{b x^3 + a x} \right)}{77 a x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b x^3 + a x} (b x^2 + a)}{x^7}, x\right)$$

20.20 Problem number 57

$$\int \frac{x^4}{\sqrt{a x + b x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{10 a \sqrt{b x^3 + a x}}{21 b^2} + \frac{2 x^2 \sqrt{b x^3 + a x}}{7 b} \\ & + \frac{5 a^{\frac{7}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x \sqrt{b}) \sqrt{x} \sqrt{\frac{b x^3 + a x}{\sqrt{a}}}}{21 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{9}{4}} \sqrt{b x^3 + a x}} \end{aligned}$$

command

```
integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5 a^2 \sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (3 b^2 x^2 - 5 a b) \sqrt{b x^3 + a x} \right)}{21 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b x^3 + a x} x^3}{b x^2 + a}, x\right)$$

20.21 Problem number 58

$$\int \frac{x^3}{\sqrt{ax + bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6ax(bx^2 + a)}{5b^{\frac{3}{2}}(\sqrt{a} + x\sqrt{b})\sqrt{bx^3 + ax}} + \frac{2x\sqrt{bx^3 + ax}}{5b} \\ & + \frac{6a^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{x}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^3 + ax}} \sqrt{\frac{b}{(\sqrt{a}}}} \\ & + \frac{3a^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{x}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^3 + ax}} \sqrt{\frac{b}{(\sqrt{a}}}} \end{aligned}$$

command

```
integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{bx^3 + ax}bx + 3a\sqrt{b}\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right)\right)}{5b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax}x^2}{bx^2 + a}, x\right)$$

20.22 Problem number 59

$$\int \frac{x^2}{\sqrt{ax + bx^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^3 + ax}}{3b} a^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{bx^3 + ax}{(\sqrt{a} + x\sqrt{b})^2}} - \frac{3 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{5}{4}} \sqrt{bx^3 + ax}}{3b^2}$$

command

```
integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(a\sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - \sqrt{bx^3 + ax} b \right)}{3b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax} x}{bx^2 + a}, x\right)$$

20.23 Problem number 60

$$\int \frac{x}{\sqrt{ax + bx^3}} dx$$

Optimal antiderivative

$$\frac{2x(bx^2 + a)}{\sqrt{b}(\sqrt{a} + x\sqrt{b})\sqrt{bx^3 + ax}}$$

$$2a^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}$$

$$\cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{bx^3 + ax}$$

$$+ \frac{a^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{\cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{bx^3 + ax}}$$

command

```
integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right)}{\sqrt{b}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax}}{bx^2 + a}, x\right)$$

20.24 Problem number 61

$$\int \frac{1}{\sqrt{ax + bx^3}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{\cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{bx^3 + ax}}$$

command

`integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)}{\sqrt{b}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{bx^3+ax}}, x\right)$$

20.25 Problem number 62

$$\int \frac{1}{x \sqrt{ax+bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x(bx^2+a)\sqrt{b}}{a(\sqrt{a}+x\sqrt{b})\sqrt{bx^3+ax}} - \frac{2\sqrt{bx^3+ax}}{ax} \\ & \frac{2b^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b})\sqrt{x} \sqrt{\frac{bx^2}{(\sqrt{a}+x\sqrt{b})^2}}}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{3}{4}}\sqrt{bx^3+ax}} \\ & + \frac{b^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b})\sqrt{x} \sqrt{\frac{bx^2}{(\sqrt{a}+x\sqrt{b})^2}}}{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{3}{4}}\sqrt{bx^3+ax}} \end{aligned}$$

command

`integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{b}x\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + \sqrt{bx^3+ax}\right)}{ax}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+ax}}{bx^4+ax^2}, x\right)$$

20.26 Problem number 63

$$\int \frac{1}{x^2 \sqrt{ax + bx^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^3 + ax}}{3ax^2} + b^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{bx^3 + ax}{(\sqrt{a} - x\sqrt{b})^2}} - \frac{3 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{5}{4}} \sqrt{bx^3 + ax}}{3ax^2}$$

command

```
integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{b}x^2 \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + \sqrt{bx^3 + ax}\right)}{3ax^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax}}{bx^5 + ax^3}, x\right)$$

20.27 Problem number 64

$$\int \frac{1}{x^3 \sqrt{ax + bx^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6b^{\frac{3}{2}}x(bx^2+a)}{5a^2(\sqrt{a}+x\sqrt{b})\sqrt{bx^3+ax}} - \frac{2\sqrt{bx^3+ax}}{5ax^3} + \frac{6b\sqrt{bx^3+ax}}{5a^2x} \\ & + \frac{6b^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}\sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}\sqrt{bx^3+ax}} \\ & + \frac{3b^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}\sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}\sqrt{bx^3+ax}} \end{aligned}$$

command

`integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3b^{\frac{3}{2}}x^3\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + \sqrt{bx^3+ax}(3bx^2-a)\right)}{5a^2x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+ax}}{bx^6+ax^4}, x\right)$$

20.28 Problem number 65

$$\int \frac{x^7}{(ax+bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x^5}{b\sqrt{bx^3+ax}} - \frac{15a\sqrt{bx^3+ax}}{7b^3} + \frac{9x^2\sqrt{bx^3+ax}}{7b^2} \\ & + \frac{15a^{\frac{7}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}\sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}}{14\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)b^{\frac{13}{4}}\sqrt{bx^3+ax}} \end{aligned}$$

command

```
integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 (a^2 b x^2 + a^3) \sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + (2 b^3 x^4 - 6 a b^2 x^2 - 15 a^2 b) \sqrt{b x^3 + a x}}{7 (b^5 x^2 + a b^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b x^3 + a x} x^5}{b^2 x^4 + 2 a b x^2 + a^2}, x\right)$$

20.29 Problem number 66

$$\int \frac{x^6}{(a x + b x^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x^4}{b \sqrt{b x^3 + a x}} - \frac{21 a x (b x^2 + a)}{5 b^{\frac{5}{2}} (\sqrt{a} + x \sqrt{b}) \sqrt{b x^3 + a x}} + \frac{7 x \sqrt{b x^3 + a x}}{5 b^2} \\ & + \frac{21 a^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x \sqrt{b}) \sqrt{x} \sqrt{\frac{b}{(\sqrt{a} + x \sqrt{b})^2}}}}{5 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{11}{4}} \sqrt{b x^3 + a x}} \\ & + \frac{21 a^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x \sqrt{b}) \sqrt{x} \sqrt{\frac{b}{(\sqrt{a} + x \sqrt{b})^2}}}}{10 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) b^{\frac{11}{4}} \sqrt{b x^3 + a x}} \end{aligned}$$

command

```
integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21 (a b x^2 + a^2) \sqrt{b} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (2 b^2 x^3 + 7 a b x) \sqrt{b x^3 + a x}}{5 (b^4 x^2 + a b^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b x^3 + a x} x^4}{b^2 x^4 + 2 a b x^2 + a^2}, x\right)$$

20.30 Problem number 67

$$\int \frac{x^5}{(ax + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x^3}{b\sqrt{bx^3 + ax}} + \frac{5\sqrt{bx^3 + ax}}{3b^2} - \frac{5a^{3/4} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{b}{(\sqrt{a}})}}}{6 \cos\left(2 \arctan\left(\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right)\right) b^{9/4} \sqrt{bx^3 + ax}}$$

command

```
integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(abx^2 + a^2)\sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - (2b^2x^2 + 5ab)\sqrt{bx^3 + ax}}{3(b^4x^2 + ab^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax} x^3}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

20.31 Problem number 68

$$\int \frac{x^4}{(ax + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{x^2}{b\sqrt{bx^3+ax}} + \frac{3x(bx^2+a)}{b^{\frac{3}{2}}(\sqrt{a}+x\sqrt{b})\sqrt{bx^3+ax}} \\
 & + \frac{3a^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^3+ax}} \sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}} \\
 & + \frac{3a^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)b^{\frac{7}{4}}\sqrt{bx^3+ax}} \sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}
 \end{aligned}$$

command

`integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{bx^3+ax}bx + 3(bx^2+a)\sqrt{b}\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right)}{b^3x^2+ab^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+ax}x^2}{b^2x^4+2abx^2+a^2}, x\right)$$

20.32 Problem number 69

$$\int \frac{x^3}{(ax+bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{x}{b\sqrt{bx^3+ax}} \\
 & + \frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{b})\sqrt{x}}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{1}{4}}b^{\frac{5}{4}}\sqrt{bx^3+ax}} \sqrt{\frac{bx^2+a}{(\sqrt{a}+x\sqrt{b})^2}}
 \end{aligned}$$

command

```
integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(bx^2 + a)\sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) - \sqrt{bx^3 + ax} b}{b^3x^2 + ab^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax} x}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

20.33 Problem number 70

$$\int \frac{x^2}{(ax + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x^2}{a\sqrt{bx^3 + ax}} - \frac{x(bx^2 + a)}{a\sqrt{b}(\sqrt{a} + x\sqrt{b})\sqrt{bx^3 + ax}}$$

$$+ \frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{x} \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{\cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{3}{4}} b^{\frac{3}{4}} \sqrt{bx^3 + ax}}$$

$$- \frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{x} \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{2 \cos\left(2 \arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{3}{4}} b^{\frac{3}{4}} \sqrt{bx^3 + ax}}$$

command

```
integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{bx^3 + ax} bx + (bx^2 + a)\sqrt{b} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right)}{ab^2x^2 + a^2b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax}}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

20.34 Problem number 71

$$\int \frac{x}{(ax + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x}{a\sqrt{bx^3 + ax}} + \frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{b}) \sqrt{x} \sqrt{\frac{bx^2 + a}{(\sqrt{a} + x\sqrt{b})^2}}}{2\cos\left(2\arctan\left(\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right)\right) a^{5/4} b^{1/4} \sqrt{bx^3 + ax}}$$

command

`integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(bx^2 + a)\sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + \sqrt{bx^3 + ax} b}{ab^2x^2 + a^2b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax}}{b^2x^5 + 2abx^3 + a^2x}, x\right)$$

20.35 Problem number 72

$$\int \frac{1}{(ax + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{a\sqrt{bx^3+ax}} + \frac{3x(bx^2+a)\sqrt{b}}{a^2(\sqrt{a}+x\sqrt{b})\sqrt{bx^3+ax}} - \frac{3\sqrt{bx^3+ax}}{a^2x}$$

$$3b^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b})\sqrt{x} \sqrt{\frac{b}{(\sqrt{a}})}$$

$$\frac{\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{7}{4}}\sqrt{bx^3+ax}}{3b^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b})\sqrt{x} \sqrt{\frac{b}{(\sqrt{a}}}}$$

$$+ \frac{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{7}{4}}\sqrt{bx^3+ax}}{2\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{7}{4}}\sqrt{bx^3+ax}}$$

command

```
integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(bx^3+ax)\sqrt{b}\operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + \sqrt{bx^3+ax}(3bx^2+2a)}{a^2bx^3+a^3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+ax}}{b^2x^6+2abx^4+a^2x^2}, x\right)$$

20.36 Problem number 73

$$\int \frac{1}{x(ax+bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{ax\sqrt{bx^3+ax}} - \frac{5\sqrt{bx^3+ax}}{3a^2x^2}$$

$$5b^{\frac{3}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{b})\sqrt{x} \sqrt{\frac{b}{(\sqrt{a}}}}$$

$$\frac{6\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{9}{4}}\sqrt{bx^3+ax}}{6\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{9}{4}}\sqrt{bx^3+ax}}$$

command

`integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(bx^4 + ax^2)\sqrt{b} \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right) + \sqrt{bx^3 + ax}(5bx^2 + 2a)}{3(a^2bx^4 + a^3x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax}}{b^2x^7 + 2abx^5 + a^2x^3}, x\right)$$

20.37 Problem number 74

$$\int \frac{1}{x^2(ax + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1}{ax^2\sqrt{bx^3 + ax}} - \frac{21b^{\frac{3}{2}}x(bx^2 + a)}{5a^3(\sqrt{a} + x\sqrt{b})\sqrt{bx^3 + ax}} - \frac{7\sqrt{bx^3 + ax}}{5a^2x^3} + \frac{21b\sqrt{bx^3 + ax}}{5a^3x} \\ & + \frac{21b^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{x}\sqrt{\frac{b}{(\sqrt{a}})}}}{5\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{11}{4}}\sqrt{bx^3 + ax}} \\ & - \frac{21b^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{b})\sqrt{x}\sqrt{\frac{b}{(\sqrt{a}})}}}{10\cos\left(2\arctan\left(\frac{b^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{11}{4}}\sqrt{bx^3 + ax}} \end{aligned}$$

command

`integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21(b^2x^5 + abx^3)\sqrt{b} \operatorname{weierstrassZeta}\left(-\frac{4a}{b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (21b^2x^4 + 14abx^2 - 2a^2)\sqrt{bx^3 + ax}}{5(a^3bx^5 + a^4x^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^3 + ax}}{b^2x^8 + 2abx^6 + a^2x^4}, x\right)$$

20.38 Problem number 98

$$\int \frac{1}{\sqrt{ax + bx^4}} dx$$

Optimal antiderivative

$$\frac{x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right) \operatorname{EllipticF} \left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}}, \frac{\sqrt{6}}{4} \right)}{3 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right) a^{\frac{1}{3}} \sqrt{bx^4 + ax} \sqrt{\frac{b^{\frac{1}{3}} x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}}$$

command

`integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassPInverse}\left(0, -\frac{4b}{a}, \frac{1}{x}\right)}{\sqrt{a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{bx^4 + ax}}, x\right)$$

20.39 Problem number 99

$$\int \frac{1}{x^3 \sqrt{ax + bx^4}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{bx^4 + ax}}{5a x^3} - \frac{2bx \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right) \operatorname{EllipticF} \left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \right)}{15 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right) a^{\frac{4}{3}} \sqrt{bx^4 + ax} \sqrt{\frac{b^{\frac{1}{3}} x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}}$$

command

```
integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{a} b x^3 \text{weierstrassPInverse}\left(0, -\frac{4b}{a}, \frac{1}{x}\right) - \sqrt{b x^4 + a x} a \right)}{5 a^2 x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b x^4 + a x}}{b x^7 + a x^4}, x\right)$$

20.40 Problem number 102

$$\int \frac{1}{x \sqrt{a x + b x^4}} dx$$

Optimal antiderivative

$$\frac{2 b^{\frac{1}{3}} x (b x^3 + a) (1 + \sqrt{3})}{a \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right) \sqrt{b x^4 + a x}} - \frac{2 \sqrt{b x^4 + a x}}{a x}$$

$$- \frac{2 \cdot 3^{\frac{1}{4}} b^{\frac{1}{3}} x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right) \text{EllipticE} \left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right) a^{\frac{2}{3}} \sqrt{b x^4 + a x} \sqrt{\frac{b^{\frac{1}{3}} x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}}$$

$$- \frac{b^{\frac{1}{3}} x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right) \text{EllipticF} \left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right) a^{\frac{2}{3}} \sqrt{b x^4 + a x} \sqrt{\frac{b^{\frac{1}{3}} x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}}$$

command

```
integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassZeta}\left(0, -\frac{4b}{a}, \operatorname{weierstrassPInverse}\left(0, -\frac{4b}{a}, \frac{1}{x}\right)\right)}{\sqrt{a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^4 + ax}}{bx^5 + ax^2}, x\right)$$

20.41 Problem number 167

$$\int x^3 \sqrt{bx^{2/3} + ax} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{524288b^9 \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{4345965a^{10}} + \frac{8388608b^{12} \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{152108775a^{13}x} - \frac{4194304b^{11} \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{50702925a^{12}x^{\frac{2}{3}}} \\ & + \frac{1048576b^{10} \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{10140585a^{11}x^{\frac{1}{3}}} + \frac{65536b^8 x^{\frac{1}{3}} \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{482885a^9} \\ & - \frac{360448b^7 x^{\frac{2}{3}} \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{2414425a^8} + \frac{90112b^6 x \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{557175a^7} \\ & - \frac{45056b^5 x^{\frac{4}{3}} \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{260015a^6} + \frac{2816b^4 x^{\frac{5}{3}} \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{15295a^5} - \frac{1408b^3 x^2 \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{7245a^4} \\ & + \frac{352b^2 x^{\frac{7}{3}} \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{1725a^3} - \frac{16b x^{\frac{8}{3}} \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{75a^2} + \frac{2x^3 \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}}}{9a} \end{aligned}$$

command

```
integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$(211106232532992 b^{19} + 43980465111040 b^{18} + 206158430208 (64 a^3 - 3) b^{16} - 4123168604160 b^{17} - 1073741824$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.42 Problem number 178

$$\int (bx^{2/3} + ax)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bx^{2/3} + ax)^{5/2}}{5a} - \frac{512b^5(bx^{2/3} + ax)^{5/2}}{15015a^6x^{5/3}} + \frac{256b^4(bx^{2/3} + ax)^{5/2}}{3003a^5x^{4/3}} \\ & - \frac{64b^3(bx^{2/3} + ax)^{5/2}}{429a^4x} + \frac{32b^2(bx^{2/3} + ax)^{5/2}}{143a^3x^{2/3}} - \frac{4b(bx^{2/3} + ax)^{5/2}}{13a^2x^{1/3}} \end{aligned}$$

command

```
integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(4 (805306368 b^{13} + 167772160 b^{12} + 786432 (64 a^3 - 3) b^{10} - 15728640 b^{11} - 4096 (11264 a^3 - 53) b^9 + 4372368 \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.43 Problem number 179

$$\int \frac{(bx^{2/3} + ax)^{3/2}}{x} dx$$

Optimal antiderivative

$$\frac{16b^2(bx^{2/3} + ax)^{5/2}}{105a^3x^{5/3}} - \frac{8b(bx^{2/3} + ax)^{5/2}}{21a^2x^{4/3}} + \frac{2(bx^{2/3} + ax)^{5/2}}{3ax}$$

command

```
integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$(201326592 b^{10} + 41943040 b^9 + 196608 (6784 a^3 - 3) b^7 - 3932160 b^8 + 1024 (257536 a^3 + 53) b^6 - 407680 a^6 - 3 \dots)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.44 Problem number 185

$$\int \frac{x^4}{\sqrt{bx^{2/3} + ax}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8388608b^{12}\sqrt{bx^{\frac{2}{3}} + ax}}{11700675a^{13}} - \frac{16777216b^{13}\sqrt{bx^{\frac{2}{3}} + ax}}{11700675a^{14}x^{\frac{1}{3}}} - \frac{2097152b^{11}x^{\frac{1}{3}}\sqrt{bx^{\frac{2}{3}} + ax}}{3900225a^{12}} \\ & + \frac{1048576b^{10}x^{\frac{2}{3}}\sqrt{bx^{\frac{2}{3}} + ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{\frac{2}{3}} + ax}}{334305a^{10}} + \frac{65536b^8x^{\frac{4}{3}}\sqrt{bx^{\frac{2}{3}} + ax}}{185725a^9} \\ & - \frac{180224b^7x^{\frac{5}{3}}\sqrt{bx^{\frac{2}{3}} + ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{\frac{2}{3}} + ax}}{3900225a^7} \\ & - \frac{73216b^5x^{\frac{7}{3}}\sqrt{bx^{\frac{2}{3}} + ax}}{260015a^6} + \frac{36608b^4x^{\frac{8}{3}}\sqrt{bx^{\frac{2}{3}} + ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{\frac{2}{3}} + ax}}{36225a^4} \\ & + \frac{416b^2x^{\frac{10}{3}}\sqrt{bx^{\frac{2}{3}} + ax}}{1725a^3} - \frac{52bx^{\frac{11}{3}}\sqrt{bx^{\frac{2}{3}} + ax}}{225a^2} + \frac{2x^4\sqrt{bx^{\frac{2}{3}} + ax}}{9a} \end{aligned}$$

command

```
integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(211106232532992b^{19} + 43980465111040b^{18} + 206158430208(64a^3 - 3)b^{16} - 4123168604160b^{17} - 1073741824($$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.45 Problem number 186

$$\int \frac{x^3}{\sqrt{bx^{2/3} + ax}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{262144b^9\sqrt{bx^{\frac{2}{3}}+ax}}{323323a^{10}} + \frac{524288b^{10}\sqrt{bx^{\frac{2}{3}}+ax}}{323323a^{11}x^{\frac{1}{3}}} + \frac{196608b^8x^{\frac{1}{3}}\sqrt{bx^{\frac{2}{3}}+ax}}{323323a^9} \\ & - \frac{163840b^7x^{\frac{2}{3}}\sqrt{bx^{\frac{2}{3}}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{\frac{2}{3}}+ax}}{46189a^7} \\ & - \frac{18432b^5x^{\frac{4}{3}}\sqrt{bx^{\frac{2}{3}}+ax}}{46189a^6} + \frac{1536b^4x^{\frac{5}{3}}\sqrt{bx^{\frac{2}{3}}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{\frac{2}{3}}+ax}}{2261a^4} \\ & + \frac{720b^2x^{\frac{7}{3}}\sqrt{bx^{\frac{2}{3}}+ax}}{2261a^3} - \frac{40bx^{\frac{8}{3}}\sqrt{bx^{\frac{2}{3}}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{\frac{2}{3}}+ax}}{7a} \end{aligned}$$

command

```
integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((3298534883328 b^{16} + 687194767360 b^{15} + 3221225472 (64 a^3 - 3) b^{13} - 64424509440 b^{14} - 16777216 (11264 a^3 - 53) b^{12} + 16777216 (11264 a^3 - 53) b^{11} - 16777216 (11264 a^3 - 53) b^{10} + 16777216 (11264 a^3 - 53) b^9 + 16777216 (11264 a^3 - 53) b^8 + 16777216 (11264 a^3 - 53) b^7 + 16777216 (11264 a^3 - 53) b^6 + 16777216 (11264 a^3 - 53) b^5 + 16777216 (11264 a^3 - 53) b^4 + 16777216 (11264 a^3 - 53) b^3 + 16777216 (11264 a^3 - 53) b^2 + 16777216 (11264 a^3 - 53) b + 16777216 (11264 a^3 - 53)) \sqrt{bx^{\frac{2}{3}}+ax} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.46 Problem number 187

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2048b^6\sqrt{bx^{\frac{2}{3}}+ax}}{2145a^7} - \frac{4096b^7\sqrt{bx^{\frac{2}{3}}+ax}}{2145a^8x^{\frac{1}{3}}} - \frac{512b^5x^{\frac{1}{3}}\sqrt{bx^{\frac{2}{3}}+ax}}{715a^6} + \frac{256b^4x^{\frac{2}{3}}\sqrt{bx^{\frac{2}{3}}+ax}}{429a^5} \\ & - \frac{224b^3x\sqrt{bx^{\frac{2}{3}}+ax}}{429a^4} + \frac{336b^2x^{\frac{4}{3}}\sqrt{bx^{\frac{2}{3}}+ax}}{715a^3} - \frac{28bx^{\frac{5}{3}}\sqrt{bx^{\frac{2}{3}}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{\frac{2}{3}}+ax}}{5a} \end{aligned}$$

command

```
integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$(51539607552 b^{13} + 10737418240 b^{12} + 50331648 (64 a^3 - 3) b^{10} - 1006632960 b^{11} - 262144 (11264 a^3 - 53) b^9 + 49152000 (11264 a^3 - 53) b^8 - 262144 (11264 a^3 - 53) b^7 + 49152000 (11264 a^3 - 53) b^6 - 262144 (11264 a^3 - 53) b^5 + 49152000 (11264 a^3 - 53) b^4 - 262144 (11264 a^3 - 53) b^3 + 49152000 (11264 a^3 - 53) b^2 - 262144 (11264 a^3 - 53) b + 49152000 (11264 a^3 - 53)) \sqrt{bx^{\frac{2}{3}}+ax}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.47 Problem number 188

$$\int \frac{x}{\sqrt{bx^{2/3} + ax}} dx$$

Optimal antiderivative

$$-\frac{128b^3 \sqrt{bx^{\frac{2}{3}} + ax}}{105a^4} + \frac{256b^4 \sqrt{bx^{\frac{2}{3}} + ax}}{105a^5 x^{\frac{1}{3}}} + \frac{32b^2 x^{\frac{1}{3}} \sqrt{bx^{\frac{2}{3}} + ax}}{35a^3} \\ - \frac{16bx^{\frac{2}{3}} \sqrt{bx^{\frac{2}{3}} + ax}}{21a^2} + \frac{2x \sqrt{bx^{\frac{2}{3}} + ax}}{3a}$$

command

```
integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 (805306368 b^{10} + 167772160 b^9 + 786432 (64 a^3 - 3) b^7 - 15728640 b^8 - 4096 (11264 a^3 - 53) b^6 - 101920 a^6 \right.$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.48 Problem number 189

$$\int \frac{1}{\sqrt{bx^{2/3} + ax}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^{\frac{2}{3}} + ax}}{a} - \frac{4b\sqrt{bx^{\frac{2}{3}} + ax}}{a^2 x^{\frac{1}{3}}}$$

command

```
integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$(50331648 b^7 + 10485760 b^6 + 49152 (512 a^3 - 3) b^4 - 983040 b^5 + 256 (24576 a^3 + 53) b^3 + 11648 a^3 - 96 (2048 a^3 -$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.49 Problem number 194

$$\int \frac{x^4}{(bx^{2/3} + ax)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} - \frac{524288b^9\sqrt{bx^{2/3} + ax}}{29393a^{11}} + \frac{1048576b^{10}\sqrt{bx^{2/3} + ax}}{29393a^{12}x^{1/3}} \\ & + \frac{393216b^8x^{1/3}\sqrt{bx^{2/3} + ax}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3} + ax}}{29393a^9} + \frac{40960b^6x\sqrt{bx^{2/3} + ax}}{4199a^8} \\ & - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3} + ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^5} \\ & + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^4} - \frac{880bx^{8/3}\sqrt{bx^{2/3} + ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3} + ax}}{7a^2} \end{aligned}$$

command

```
integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.50 Problem number 195

$$\int \frac{x^3}{(bx^{2/3} + ax)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6x^3}{a\sqrt{bx^{2/3} + ax}} + \frac{32768b^6\sqrt{bx^{2/3} + ax}}{2145a^8} - \frac{65536b^7\sqrt{bx^{2/3} + ax}}{2145a^9x^{1/3}} \\ & - \frac{8192b^5x^{1/3}\sqrt{bx^{2/3} + ax}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3} + ax}}{429a^6} - \frac{3584b^3x\sqrt{bx^{2/3} + ax}}{429a^5} \\ & + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3} + ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3} + ax}}{5a^2} \end{aligned}$$

command

```
integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.51 Problem number 196

$$\int \frac{x^2}{(bx^{2/3} + ax)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6x^2}{a\sqrt{bx^{\frac{2}{3}} + ax}} - \frac{256b^3\sqrt{bx^{\frac{2}{3}} + ax}}{21a^5} + \frac{512b^4\sqrt{bx^{\frac{2}{3}} + ax}}{21a^6x^{\frac{1}{3}}} \\ & + \frac{64b^2x^{\frac{1}{3}}\sqrt{bx^{\frac{2}{3}} + ax}}{7a^4} - \frac{160bx^{\frac{2}{3}}\sqrt{bx^{\frac{2}{3}} + ax}}{21a^3} + \frac{20x\sqrt{bx^{\frac{2}{3}} + ax}}{3a^2} \end{aligned}$$

command

```
integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.52 Problem number 197

$$\int \frac{x}{(bx^{2/3} + ax)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{6x}{a\sqrt{bx^{\frac{2}{3}} + ax}} + \frac{8\sqrt{bx^{\frac{2}{3}} + ax}}{a^2} - \frac{16b\sqrt{bx^{\frac{2}{3}} + ax}}{a^3x^{\frac{1}{3}}}$$

command

```
integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$(98304 a^3 b^{10} + 81920 a^3 b^9 - 30720 a^3 b^8 + 1456 a^9 + 6144 (16 a^6 - 3 a^3) b^7 + 6784 (8 a^6 + a^3) b^6 - 192 (236 a^6 + a^3) b^5 - 192 (236 a^6 + a^3) b^4 - 192 (236 a^6 + a^3) b^3 - 192 (236 a^6 + a^3) b^2 - 192 (236 a^6 + a^3) b - 192 (236 a^6 + a^3)) \sqrt{bx^5 + ax^2}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

20.53 Problem number 289

$$\int \frac{x^4}{\sqrt{ax^2 + bx^5}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^5 + ax^2}}{5b} + \frac{4ax \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{15b^{\frac{4}{3}} \sqrt{bx^5 + ax^2} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} 3^{\frac{3}{4}}$$

command

```
integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2a\sqrt{b} \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - \sqrt{bx^5 + ax^2} b \right)}{5b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^5 + ax^2} x^2}{bx^3 + a}, x\right)$$

20.54 Problem number 290

$$\int \frac{x}{\sqrt{ax^2 + bx^5}} dx$$

Optimal antiderivative

$$\frac{2x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{3b^{\frac{1}{3}} \sqrt{bx^5 + ax^2} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} 3^{\frac{3}{4}}$$

command

`integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassPInverse} \left(0, -\frac{4a}{b}, x \right)}{\sqrt{b}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{bx^5 + ax^2}}{bx^4 + ax}, x \right)$$

20.55 Problem number 291

$$\int \frac{1}{x^2 \sqrt{ax^2 + bx^5}} dx$$

Optimal antiderivative

$$\frac{\frac{\sqrt{bx^5 + ax^2}}{2ax^3} + b^{\frac{2}{3}} x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{6a \sqrt{bx^5 + ax^2} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} 3^{\frac{3}{4}}$$

command

```
integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{b} x^3 \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) + \sqrt{bx^5 + ax^2}}{2ax^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}}{bx^7 + ax^4}, x\right)$$

20.56 Problem number 292

$$\int \frac{x^5}{\sqrt{ax^2 + bx^5}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{8ax(bx^3 + a)}{7b^{\frac{5}{3}}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)\sqrt{bx^5 + ax^2}} + \frac{2x\sqrt{bx^5 + ax^2}}{7b} \\ & 8a^{\frac{4}{3}}x\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}} \\ & - \frac{21b^{\frac{5}{3}}\sqrt{bx^5 + ax^2} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{43^{\frac{1}{4}}a^{\frac{4}{3}}x\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \text{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \\ & + \frac{7b^{\frac{5}{3}}\sqrt{bx^5 + ax^2} \sqrt{\frac{a^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} \end{aligned}$$

command

```
integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{bx^5 + ax^2} bx + 4a\sqrt{b} \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) \right)}{7b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^5 + ax^2} x^3}{bx^3 + a}, x\right)$$

20.57 Problem number 293

$$\int \frac{x^2}{\sqrt{ax^2 + bx^5}} dx$$

Optimal antiderivative

$$\frac{2x(bx^3 + a)}{b^{\frac{2}{3}} \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right) \sqrt{bx^5 + ax^2}} + \frac{2a^{\frac{1}{3}}x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}} 3^{\frac{3}{4}}}{3b^{\frac{2}{3}} \sqrt{bx^5 + ax^2} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}} + \frac{3^{\frac{1}{4}} a^{\frac{1}{3}} x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}}{b^{\frac{2}{3}} \sqrt{bx^5 + ax^2} \sqrt{\frac{a^{\frac{1}{3}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3}) \right)^2}}}$$

command

`integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right)}{\sqrt{b}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^5 + ax^2}}{bx^3 + a}, x\right)$$

20.58 Problem number 294

$$\int \frac{1}{x \sqrt{ax^2 + bx^5}} dx$$

Optimal antiderivative

$$\frac{b^{\frac{1}{3}}x(bx^3 + a)}{a \left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right) \sqrt{bx^5 + ax^2}} - \frac{\sqrt{bx^5 + ax^2}}{ax^2}$$

$$+ \frac{b^{\frac{1}{3}}x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}}{3a^{\frac{2}{3}} \sqrt{bx^5 + ax^2} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

$$- \frac{3^{\frac{1}{4}}b^{\frac{1}{3}}x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticE}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}{2a^{\frac{2}{3}} \sqrt{bx^5 + ax^2} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

`integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{b} x^2 \operatorname{weierstrassZeta}\left(0, -\frac{4a}{b}, \operatorname{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right)\right) + \sqrt{bx^5 + ax^2}}{ax^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^5 + ax^2}}{bx^6 + ax^3}, x\right)$$

20.59 Problem number 301

$$\int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^5}} dx$$

Optimal antiderivative

$$\frac{x^{\frac{3}{2}} \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \sqrt{\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \operatorname{EllipticF} \left(\sqrt{1 - \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}}, \sqrt{\frac{b^{\frac{1}{3}} x \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \right)}{3 \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right) a^{\frac{1}{3}} \sqrt{bx^5 + ax^2}}$$

command

```
integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassPInverse} \left(0, -\frac{4b}{a}, \frac{1}{x} \right)}{\sqrt{a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{x}}{\sqrt{bx^5 + ax^2}}, x \right)$$

20.60 Problem number 302

$$\int \frac{1}{\sqrt{x} \sqrt{ax^2 + bx^5}} dx$$

Optimal antiderivative

$$\frac{2b^{\frac{1}{3}}x^{\frac{3}{2}}(bx^3+a)(1+\sqrt{3})}{a\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)\sqrt{bx^5+ax^2}} - \frac{2\sqrt{bx^5+ax^2}}{ax^{\frac{3}{2}}}$$

$$23^{\frac{1}{4}}b^{\frac{1}{3}}x^{\frac{3}{2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)^2}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)\text{EllipticE}\left(\sqrt{1-\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)^2}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}\right)$$

$$\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)a^{\frac{2}{3}}\sqrt{bx^5+ax^2}}{\sqrt{\frac{b^{\frac{1}{3}}x\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}}$$

$$b^{\frac{1}{3}}x^{\frac{3}{2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)^2}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)\text{EllipticF}\left(\sqrt{1-\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)^2}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}\right)$$

$$\frac{3\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)a^{\frac{2}{3}}\sqrt{bx^5+ax^2}}{\sqrt{\frac{b^{\frac{1}{3}}x\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}}$$

command

```
integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\text{weierstrassZeta}\left(0,-\frac{4b}{a},\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)}{\sqrt{a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\sqrt{x}}{bx^6+ax^3},x\right)$$

20.61 Problem number 304

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^5+ax^2}}{5ax^{\frac{7}{2}}}$$

$$2bx^{\frac{3}{2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\left(1-\sqrt{3}\right)\right)^2}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\left(1+\sqrt{3}\right)\right)^2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\left(1+\sqrt{3}\right)\right)\text{EllipticF}\left(\sqrt{1-\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\left(1-\sqrt{3}\right)\right)^2}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\left(1+\sqrt{3}\right)\right)^2}}\right)$$

$$15\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\left(1-\sqrt{3}\right)\right)a^{\frac{4}{3}}\sqrt{bx^5+ax^2}\sqrt{\frac{b^{\frac{1}{3}}x\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\left(1+\sqrt{3}\right)\right)^2}}$$

command

```
integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{a}bx^4\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^5+ax^2}a\sqrt{x}\right)}{5a^2x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\sqrt{x}}{bx^8+ax^5},x\right)$$

20.62 Problem number 305

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}} dx$$

Optimal antiderivative

$$\frac{8b^{\frac{4}{3}}x^{\frac{3}{2}}(bx^3+a)(1+\sqrt{3})}{7a^2\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)\sqrt{bx^5+ax^2}} - \frac{2\sqrt{bx^5+ax^2}}{7ax^{\frac{9}{2}}} + \frac{8b\sqrt{bx^5+ax^2}}{7a^2x^{\frac{3}{2}}}$$

$$+ \frac{83^{\frac{1}{4}}b^{\frac{4}{3}}x^{\frac{3}{2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)^2}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)\text{EllipticE}\left(\sqrt{1-\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)}}\right)}{7\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)a^{\frac{5}{3}}\sqrt{bx^5+ax^2}\sqrt{\frac{b^{\frac{1}{3}}x\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)}}}$$

$$+ \frac{4b^{\frac{4}{3}}x^{\frac{3}{2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)^2}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)\text{EllipticF}\left(\sqrt{1-\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)}}\right)}{21\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)a^{\frac{5}{3}}\sqrt{bx^5+ax^2}\sqrt{\frac{b^{\frac{1}{3}}x\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)}}}$$

command

```
integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4\sqrt{a}bx^5\text{weierstrassZeta}\left(0,-\frac{4b}{a},\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)+\sqrt{bx^5+ax^2}a\sqrt{x}\right)}{7a^2x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\sqrt{x}}{bx^9+ax^6},x\right)$$

20.63 Problem number 307

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2\sqrt{bx^5+ax^2}}{11ax^{\frac{13}{2}}} + \frac{16b\sqrt{bx^5+ax^2}}{55a^2x^{\frac{7}{2}}} \\
 & + \frac{16b^2x^{\frac{3}{2}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)^2}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)^2}}}{165\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)a^{\frac{7}{3}}\sqrt{bx^5+ax^2}} \operatorname{EllipticF}\left(\sqrt{1-\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1-\sqrt{3})\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)}}, \sqrt{\frac{b^{\frac{1}{3}}x\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x(1+\sqrt{3})\right)}}\right)
 \end{aligned}$$

command

```
integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(16\sqrt{a}b^2x^7\operatorname{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^5+ax^2}\left(8abx^3-5a^2\right)\sqrt{x}\right)}{55a^3x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{bx^5+ax^2}\sqrt{x}}{bx^{11}+ax^8},x\right)$$

21 Test file number 31

Test folder name:

```
test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.4_Improper/31_1.1.4.3-e_x-
^m-a_x^j+b_x^k-^p-c+d_x^n-^q
```

21.1 Problem number 220

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}dx$$

Optimal antiderivative

$$\frac{2Bx^{\frac{3}{2}}(cx^4 + bx^2)^{\frac{3}{2}}}{15c} - \frac{4b(-5Ac + 3bB)x^{\frac{3}{2}}\sqrt{cx^4 + bx^2}}{385c^2}$$

$$- \frac{2(-5Ac + 3bB)x^{\frac{7}{2}}\sqrt{cx^4 + bx^2}}{55c} + \frac{4b^2(-5Ac + 3bB)\sqrt{cx^4 + bx^2}}{231c^3\sqrt{x}}$$

$$- \frac{2b^{\frac{11}{4}}(-5Ac + 3bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{cx^4 + bx^2})}{231\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{13}{4}}\sqrt{cx^4 + bx^2}}$$

command

```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(10(3Bb^4 - 5Ab^3c)\sqrt{c}x\operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - (77Bc^4x^6 + 30Bb^3c - 50Ab^2c^2 + 7(2Bbc^3 + 15A))\sqrt{cx^4 + bx^2}\right)}{1155c^4x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((Bx^4 + Ax^2)\sqrt{cx^4 + bx^2}\sqrt{x}, x\right)$$

21.2 Problem number 221

$$\int x^{3/2}(A + Bx^2)\sqrt{bx^2 + cx^4} dx$$

Optimal antiderivative

$$\frac{2B(c^2x^4 + bx^2)^{\frac{3}{2}}\sqrt{x}}{13c} + \frac{4b^2(-13Ac + 7bB)x^{\frac{3}{2}}(cx^2 + b)}{195c^{\frac{5}{2}}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}}$$

$$- \frac{2(-13Ac + 7bB)x^{\frac{5}{2}}\sqrt{cx^4 + bx^2}}{117c} - \frac{4b(-13Ac + 7bB)\sqrt{x}\sqrt{cx^4 + bx^2}}{585c^2}$$

$$+ \frac{4b^{\frac{9}{4}}(-13Ac + 7bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{195\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4 + bx^2}}$$

$$+ \frac{2b^{\frac{9}{4}}(-13Ac + 7bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{195\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4 + bx^2}}$$

command

`integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6(7Bb^3 - 13Ab^2c)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) - (45Bc^3x^4 - 14Bb^2c + 26b^2)\sqrt{cx^4 + bx^2}\right)}{585c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^4 + bx^2}(Bx^3 + Ax)\sqrt{x}, x\right)$$

21.3 Problem number 222

$$\int \sqrt{x}(A + Bx^2)\sqrt{bx^2 + cx^4} dx$$

Optimal antiderivative

$$\frac{2B(c^2x^4 + bx^2)^{\frac{3}{2}}\sqrt{x}}{11c\sqrt{x}} - \frac{2(-11Ac + 5bB)x^{\frac{3}{2}}\sqrt{cx^4 + bx^2}}{77c} - \frac{4b(-11Ac + 5bB)\sqrt{cx^4 + bx^2}}{231c^2\sqrt{x}}$$

$$+ \frac{2b^{\frac{7}{4}}(-11Ac + 5bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{231\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{9}{4}}\sqrt{cx^4 + bx^2}}$$

command

`integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 (5 B b^3 - 11 A b^2 c) \sqrt{c} \operatorname{xweierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (21 B c^3 x^4 - 10 B b^2 c + 22 A b c^2 + 3 (2 B b c^2 + 11 A c^3) \sqrt{c} x\right)}{231 c^3 x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c x^4 + b x^2} (B x^2 + A) \sqrt{x}}{\sqrt{x}}, x\right)$$

21.4 Problem number 223

$$\int \frac{(A + B x^2) \sqrt{b x^2 + c x^4}}{\sqrt{x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(c x^4 + b x^2)^{\frac{3}{2}}}{9c x^{\frac{3}{2}}} - \frac{4b(-3Ac + bB) x^{\frac{3}{2}}(c x^2 + b)}{15c^{\frac{3}{2}}(\sqrt{b} + x\sqrt{c})\sqrt{c x^4 + b x^2}} - \frac{2(-3Ac + bB) \sqrt{x} \sqrt{c x^4 + b x^2}}{15c} \\ & + \frac{4b^{\frac{5}{4}}(-3Ac + bB) x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{c x^4 + b x^2}} \\ & + \frac{2b^{\frac{5}{4}}(-3Ac + bB) x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{c x^4 + b x^2}} \end{aligned}$$

command

`integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 (B b^2 - 3 A b c) \sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (5 B c^2 x^2 + 2 B b c + 9 A c^2) \sqrt{c x^4 + b x^2} \right)}{45 c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c x^4 + b x^2} (B x^2 + A)}{\sqrt{x}}, x\right)$$

21.5 Problem number 224

$$\int \frac{(A + Bx^2) \sqrt{bx^2 + cx^4}}{x^{3/2}} dx$$

Optimal antiderivative

$$\frac{2B(cx^4 + bx^2)^{\frac{3}{2}}}{7cx^{\frac{5}{2}}} - \frac{2(-7Ac + bB) \sqrt{cx^4 + bx^2}}{21c\sqrt{x}}$$

$$2b^{\frac{3}{4}}(-7Ac + bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})$$

$$21 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{5}{4}} \sqrt{cx^4 + bx^2}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(Bb^2 - 7Abc)\sqrt{c} \operatorname{xweierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - (3Bc^2x^2 + 2Bbc + 7Ac^2)\sqrt{cx^4 + bx^2}\sqrt{x}\right)}{21c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)}{x^{\frac{3}{2}}}, x\right)$$

21.6 Problem number 225

$$\int \frac{(A + Bx^2) \sqrt{bx^2 + cx^4}}{x^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2A(cx^4 + bx^2)^{\frac{3}{2}}}{bx^{\frac{7}{2}}} + \frac{4(5Ac + bB)x^{\frac{3}{2}}(cx^2 + b)}{5\sqrt{c}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} + \frac{2(5Ac + bB)\sqrt{x}\sqrt{cx^4 + bx^2}}{5b} \\
 & 4b^{\frac{1}{4}}(5Ac + bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c}) \\
 & - \frac{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{cx^4 + bx^2}}{2b^{\frac{1}{4}}(5Ac + bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})} \\
 & + \frac{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{cx^4 + bx^2}}{2b^{\frac{1}{4}}(5Ac + bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}
 \end{aligned}$$

command

`integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(Bb + 5Ac)\sqrt{c}x^2\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) - \sqrt{cx^4 + bx^2}(Bcx^2 - 5Ac)\sqrt{x}\right)}{5cx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)}{x^{\frac{5}{2}}}, x\right)$$

21.7 Problem number 226

$$\int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2A(cx^4 + bx^2)^{\frac{3}{2}}}{3bx^{\frac{9}{2}}} + \frac{2(Ac + bB)\sqrt{cx^4 + bx^2}}{3b\sqrt{x}} \\
 & 2(Ac + bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} \\
 & + \frac{3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{cx^4 + bx^2}}{2(Ac + bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}
 \end{aligned}$$

command

`integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 (Bb + Ac) \sqrt{c} x^3 \text{weierstrassPInverse} \left(-\frac{4b}{c}, 0, x \right) + \sqrt{cx^4 + bx^2} (Bcx^2 - Ac) \sqrt{x} \right)}{3cx^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^4 + bx^2} (Bx^2 + A)}{x^{\frac{7}{2}}}, x \right)$$

21.8 Problem number 227

$$\int \frac{(A + Bx^2) \sqrt{bx^2 + cx^4}}{x^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2A(cx^4 + bx^2)^{\frac{3}{2}}}{5bx^{\frac{11}{2}}} + \frac{4(Ac + 5bB)x^{\frac{3}{2}}(cx^2 + b)\sqrt{c}}{5b(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{2(Ac + 5bB)\sqrt{cx^4 + bx^2}}{5bx^{\frac{3}{2}}} \\ & + \frac{4c^{\frac{1}{4}}(Ac + 5bB)x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{5 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{cx^4 + bx^2}} \\ & + \frac{2c^{\frac{1}{4}}(Ac + 5bB)x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{5 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{cx^4 + bx^2}} \end{aligned}$$

command

`integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 (5Bb + Ac) \sqrt{c} x^4 \text{weierstrassZeta} \left(-\frac{4b}{c}, 0, \text{weierstrassPInverse} \left(-\frac{4b}{c}, 0, x \right) \right) + \sqrt{cx^4 + bx^2} ((5Bb + 2Ac)x^2 \right)}{5bx^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^4 + bx^2} (Bx^2 + A)}{x^{\frac{9}{2}}}, x \right)$$

21.9 Problem number 228

$$\int \frac{(A + Bx^2) \sqrt{bx^2 + cx^4}}{x^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2A(cx^4 + bx^2)^{\frac{3}{2}}}{7bx^{\frac{13}{2}}} - \frac{2(-Ac + 7bB) \sqrt{cx^4 + bx^2}}{21bx^{\frac{5}{2}}} \\ & + \frac{2c^{\frac{3}{4}}(-Ac + 7bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})}{21 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{5}{4}} \sqrt{cx^4 + bx^2}} \end{aligned}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2(7Bb - Ac)\sqrt{c} x^5 \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - \sqrt{cx^4 + bx^2} ((7Bb + 2Ac)x^2 + 3Ab)\sqrt{x} \right)}{21bx^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} (Bx^2 + A)}{x^{\frac{11}{2}}}, x\right)$$

21.10 Problem number 229

$$\int \frac{(A + Bx^2) \sqrt{bx^2 + cx^4}}{x^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2A(cx^4 + bx^2)^{\frac{3}{2}}}{9bx^{\frac{15}{2}}} + \frac{4c^{\frac{3}{2}}(-Ac + 3bB)x^{\frac{3}{2}}(cx^2 + b)}{15b^2(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} \\
 & -\frac{2(-Ac + 3bB)\sqrt{cx^4 + bx^2}}{15bx^{\frac{7}{2}}} - \frac{4c(-Ac + 3bB)\sqrt{cx^4 + bx^2}}{15b^2x^{\frac{3}{2}}} \\
 & -\frac{4c^{\frac{5}{4}}(-Ac + 3bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{7}{4}}\sqrt{cx^4 + bx^2}} \\
 & +\frac{2c^{\frac{5}{4}}(-Ac + 3bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{7}{4}}\sqrt{cx^4 + bx^2}}
 \end{aligned}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6(3Bbc - Ac^2)\sqrt{c}x^6\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (6(3Bbc - Ac^2)x^4 + 5Ab^2)\right)}{45b^2x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)}{x^{\frac{13}{2}}}, x\right)$$

21.11 Problem number 230

$$\int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^{15/2}} dx$$

Optimal antiderivative

$$\frac{2A(cx^4 + bx^2)^{\frac{3}{2}}}{11bx^{\frac{17}{2}}} - \frac{2(-5Ac + 11bB)\sqrt{cx^4 + bx^2}}{77bx^{\frac{9}{2}}} - \frac{4c(-5Ac + 11bB)\sqrt{cx^4 + bx^2}}{231b^2x^{\frac{5}{2}}}$$

$$+ \frac{2c^{\frac{7}{4}}(-5Ac + 11bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}}}{231 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{9}{4}} \sqrt{cx^4 + bx^2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(11Bbc - 5Ac^2)\sqrt{c}x^7 \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (2(11Bbc - 5Ac^2)x^4 + 21Ab^2 + 3(11Bb^2 + 2Ab^2))\sqrt{cx^4 + bx^2}\right)}{231b^2x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)}{x^{\frac{15}{2}}}, x\right)$$

21.12 Problem number 231

$$\int x^{7/2}(A+Bx^2)(bx^2+cx^4)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-5Ac+3bB)x^{\frac{9}{2}}(cx^4+bx^2)^{\frac{3}{2}}}{105c} + \frac{2Bx^{\frac{5}{2}}(cx^4+bx^2)^{\frac{5}{2}}}{25c} + \frac{88b^5(-5Ac+3bB)x^{\frac{3}{2}}(cx^2+b)}{16575c^{\frac{9}{2}}(\sqrt{b}+x\sqrt{c})\sqrt{cx^4+bx^2}} \\ & + \frac{88b^3(-5Ac+3bB)x^{\frac{5}{2}}\sqrt{cx^4+bx^2}}{69615c^3} - \frac{8b^2(-5Ac+3bB)x^{\frac{9}{2}}\sqrt{cx^4+bx^2}}{7735c^2} \\ & - \frac{4b(-5Ac+3bB)x^{\frac{13}{2}}\sqrt{cx^4+bx^2}}{595c} - \frac{88b^4(-5Ac+3bB)\sqrt{x}\sqrt{cx^4+bx^2}}{49725c^4} \\ & - \frac{88b^{\frac{21}{4}}(-5Ac+3bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})}{16575\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{19}{4}}\sqrt{cx^4+bx^2}} \\ & + \frac{44b^{\frac{21}{4}}(-5Ac+3bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})}{16575\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{19}{4}}\sqrt{cx^4+bx^2}} \end{aligned}$$

command

```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(924(3Bb^6-5Ab^5c)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c},0,\operatorname{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)-(13923Bc^6x^{10}+663(27A^2c^3+13B^2c^2)x^7+13923Bc^6x^{10}+663(27A^2c^3+13B^2c^2)x^7)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((Bcx^9+(Bb+Ac)x^7+Abx^5)\sqrt{cx^4+bx^2}\sqrt{x},x\right)$$

21.13 Problem number 232

$$\int x^{5/2}(A + Bx^2)(bx^2 + cx^4)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-23Ac + 13bB)x^{\frac{7}{2}}(cx^4 + bx^2)^{\frac{3}{2}}}{437c} + \frac{2Bx^{\frac{3}{2}}(cx^4 + bx^2)^{\frac{5}{2}}}{23c} \\ & + \frac{72b^3(-23Ac + 13bB)x^{\frac{3}{2}}\sqrt{cx^4 + bx^2}}{168245c^3} - \frac{8b^2(-23Ac + 13bB)x^{\frac{7}{2}}\sqrt{cx^4 + bx^2}}{24035c^2} \\ & - \frac{4b(-23Ac + 13bB)x^{\frac{11}{2}}\sqrt{cx^4 + bx^2}}{2185c} - \frac{24b^4(-23Ac + 13bB)\sqrt{cx^4 + bx^2}}{33649c^4\sqrt{x}} \\ & + \frac{12b^{\frac{19}{4}}(-23Ac + 13bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x)}{33649\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{17}{4}}\sqrt{cx^4 + bx^2}} \end{aligned}$$

command

```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(60(13Bb^6 - 23Ab^5c)\sqrt{c}x\operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (7315Bc^6x^{10} + 385(25Bbc^5 + 23Ac^6)x^8 - 780B\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((Bcx^8 + (Bb + Ac)x^6 + Abx^4)\sqrt{cx^4 + bx^2}\sqrt{x}, x\right)$$

21.14 Problem number 233

$$\int x^{3/2}(A + Bx^2)(bx^2 + cx^4)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2(-21Ac + 11bB)x^{\frac{5}{2}}(cx^4 + bx^2)^{\frac{3}{2}}}{357c} + \frac{2B(cx^4 + bx^2)^{\frac{5}{2}}\sqrt{x}}{21c} \\
& -\frac{8b^4(-21Ac + 11bB)x^{\frac{3}{2}}(cx^2 + b)}{3315c^{\frac{7}{2}}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{8b^2(-21Ac + 11bB)x^{\frac{5}{2}}\sqrt{cx^4 + bx^2}}{13923c^2} \\
& -\frac{4b(-21Ac + 11bB)x^{\frac{9}{2}}\sqrt{cx^4 + bx^2}}{1547c} + \frac{8b^3(-21Ac + 11bB)\sqrt{x}\sqrt{cx^4 + bx^2}}{9945c^3} \\
& + \frac{8b^{\frac{17}{4}}(-21Ac + 11bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x}}{3315\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{15}{4}}\sqrt{cx^4 + bx^2}} \\
& + \frac{4b^{\frac{17}{4}}(-21Ac + 11bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x}}{3315\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{15}{4}}\sqrt{cx^4 + bx^2}}
\end{aligned}$$

command

```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(84(11Bb^5 - 21Ab^4c)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (3315Bc^5x^8 + 195(23Bb^5 - 21Ab^4c)c^2x^5 + 195(11Bb^5 - 21Ab^4c)c^2x^2 + 195(11Bb^5 - 21Ab^4c)c^2)\sqrt{cx^4 + bx^2}\sqrt{x}\right)$$

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Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((Bcx^7 + (Bb + Ac)x^5 + Abx^3)\sqrt{cx^4 + bx^2}\sqrt{x}, x\right)$$

21.15 Problem number 234

$$\int \sqrt{x}(A + Bx^2)(bx^2 + cx^4)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-19Ac + 9bB)x^{\frac{3}{2}}(cx^4 + bx^2)^{\frac{3}{2}}}{285c} + \frac{2B(cx^4 + bx^2)^{\frac{5}{2}}}{19c\sqrt{x}} - \frac{8b^2(-19Ac + 9bB)x^{\frac{3}{2}}\sqrt{cx^4 + bx^2}}{7315c^2} \\ & -\frac{4b(-19Ac + 9bB)x^{\frac{7}{2}}\sqrt{cx^4 + bx^2}}{1045c} + \frac{8b^3(-19Ac + 9bB)\sqrt{cx^4 + bx^2}}{4389c^3\sqrt{x}} \\ & -\frac{4b^{\frac{15}{4}}(-19Ac + 9bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{cx^4 + bx^2})}{4389\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{13}{4}}\sqrt{cx^4 + bx^2}} \end{aligned}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(20(9Bb^5 - 19Ab^4c)\sqrt{c}x\operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - (1155Bc^5x^8 + 77(21Bbc^4 + 19Ac^5)x^6 + 180Bb^2c^3x^4 + 180Ab^2c^2x^2 + 180A^2c)x^2\right)}{21945c^4x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bcx^6 + (Bb + Ac)x^4 + Abx^2\right)\sqrt{cx^4 + bx^2}\sqrt{x}, x\right)$$

21.16 Problem number 235

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{\sqrt{x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(cx^4 + bx^2)^{\frac{5}{2}}}{17cx^{\frac{3}{2}}} - \frac{2(-17Ac + 7bB)(cx^4 + bx^2)^{\frac{3}{2}}\sqrt{x}}{221c} + \frac{8b^3(-17Ac + 7bB)x^{\frac{3}{2}}(cx^2 + b)}{1105c^{\frac{5}{2}}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} \\ & - \frac{4b(-17Ac + 7bB)x^{\frac{5}{2}}\sqrt{cx^4 + bx^2}}{663c} - \frac{8b^2(-17Ac + 7bB)\sqrt{x}\sqrt{cx^4 + bx^2}}{3315c^2} \\ & + \frac{8b^{\frac{13}{4}}(-17Ac + 7bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{1105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4 + bx^2}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c}) \\ & - \frac{4b^{\frac{13}{4}}(-17Ac + 7bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{1105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4 + bx^2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c}) \\ & + \frac{4b^{\frac{13}{4}}(-17Ac + 7bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{1105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4 + bx^2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c}) \end{aligned}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12(7Bb^4 - 17Ab^3c)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) - (195Bc^4x^6 - 28Bb^3c + 6B^2b^2)\sqrt{cx^4 + bx^2}\sqrt{x}\right)}{3315c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((Bcx^5 + (Bb + Ac)x^3 + Abx)\sqrt{cx^4 + bx^2}\sqrt{x}, x\right)$$

21.17 Problem number 236

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(cx^4 + bx^2)^{\frac{5}{2}}}{15cx^{\frac{5}{2}}} - \frac{2(-3Ac + bB)(cx^4 + bx^2)^{\frac{3}{2}}}{33c\sqrt{x}} \\ & - \frac{4b(-3Ac + bB)x^{\frac{3}{2}}\sqrt{cx^4 + bx^2}}{77c} - \frac{8b^2(-3Ac + bB)\sqrt{cx^4 + bx^2}}{231c^2\sqrt{x}} \\ & + \frac{4b^{\frac{11}{4}}(-3Ac + bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c}}{231\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{9}{4}}\sqrt{cx^4 + bx^2}} \end{aligned}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(20(Bb^4 - 3Ab^3c)\sqrt{c}x\operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (77Bc^4x^6 - 20Bb^3c + 60Ab^2c^2 + 7(17Bbc^3 + 15Ac^4)\right)}{1155c^3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((Bcx^4 + (Bb + Ac)x^2 + Ab)\sqrt{cx^4 + bx^2}\sqrt{x}, x\right)$$

21.18 Problem number 237

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2(-13Ac + 3bB)(cx^4 + bx^2)^{\frac{3}{2}}}{117cx^{\frac{3}{2}}} + \frac{2B(cx^4 + bx^2)^{\frac{5}{2}}}{13cx^{\frac{7}{2}}} \\
& -\frac{8b^2(-13Ac + 3bB)x^{\frac{3}{2}}(cx^2 + b)}{195c^{\frac{3}{2}}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{4b(-13Ac + 3bB)\sqrt{x}\sqrt{cx^4 + bx^2}}{195c} \\
& + \frac{8b^{\frac{9}{4}}(-13Ac + 3bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{195\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{cx^4 + bx^2}}} \\
& + \frac{4b^{\frac{9}{4}}(-13Ac + 3bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{195\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{cx^4 + bx^2}}
\end{aligned}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12(3Bb^3 - 13Ab^2c)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (45Bc^3x^4 + 12Bb^2c + 143Bc^2)\sqrt{x}\sqrt{cx^4 + bx^2}\right)}{585c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^4 + (Bb + Ac)x^2 + Ab)\sqrt{cx^4 + bx^2}}{\sqrt{x}}, x\right)$$

21.19 Problem number 238

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(-11Ac + bB)(cx^4 + bx^2)^{\frac{3}{2}}}{77cx^{\frac{5}{2}}} + \frac{2B(cx^4 + bx^2)^{\frac{5}{2}}}{11cx^{\frac{9}{2}}} - \frac{4b(-11Ac + bB)\sqrt{cx^4 + bx^2}}{77c\sqrt{x}}$$

$$4b^{\frac{7}{4}}(-11Ac + bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})$$

$$77\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{5}{4}}\sqrt{cx^4 + bx^2}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4(Bb^3 - 11Ab^2c)\sqrt{c}x\operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - (7Bc^3x^4 + 4Bb^2c + 33Abc^2 + (13Bbc^2 + 11Ac^3)x^2)\sqrt{cx^4 + bx^2}\right)}{77c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^4 + (Bb + Ac)x^2 + Ab)\sqrt{cx^4 + bx^2}}{x^{\frac{3}{2}}}, x\right)$$

21.20 Problem number 239

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9Ac + bB)(cx^4 + bx^2)^{\frac{3}{2}}}{9bx^{\frac{3}{2}}} - \frac{2A(cx^4 + bx^2)^{\frac{5}{2}}}{bx^{\frac{11}{2}}} \\ & + \frac{8b(9Ac + bB)x^{\frac{3}{2}}(cx^2 + b)}{15\sqrt{c}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} + \frac{4(9Ac + bB)\sqrt{x}\sqrt{cx^4 + bx^2}}{15} \\ & 8b^{\frac{5}{4}}(9Ac + bB)x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c}) \\ & - \frac{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{cx^4 + bx^2}}{4b^{\frac{5}{4}}(9Ac + bB)x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})} \\ & + \frac{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{cx^4 + bx^2}}{15} \end{aligned}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12(Bb^2 + 9Abc)\sqrt{c}x^2\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) - (5Bc^2x^4 - 45Abc + (11B\right)}{45cx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^4 + (Bb + Ac)x^2 + Ab)\sqrt{cx^4 + bx^2}}{x^{\frac{5}{2}}}, x\right)$$

21.21 Problem number 240

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{11/2}} dx$$

Optimal antiderivative

$$\frac{2(7Ac + 3bB)(cx^4 + bx^2)^{\frac{3}{2}}}{21bx^{\frac{5}{2}}} - \frac{2A(cx^4 + bx^2)^{\frac{5}{2}}}{3bx^{\frac{13}{2}}} + \frac{4(7Ac + 3bB)\sqrt{cx^4 + bx^2}}{21\sqrt{x}}$$

$$+ \frac{4b^{\frac{3}{4}}(7Ac + 3bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})}{21 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{1}{4}} \sqrt{cx^4 + bx^2}}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 (3 B b^2 + 7 A b c) \sqrt{c} x^3 \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (3 B c^2 x^4 - 7 A b c + (9 B b c + 7 A c^2) x^2) \sqrt{c x^4 + b x^2} \right)}{21 c x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B c x^4 + (B b + A c) x^2 + A b) \sqrt{c x^4 + b x^2}}{x^{\frac{7}{2}}}, x\right)$$

21.22 Problem number 241

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Ac + bB)(cx^4 + bx^2)^{\frac{3}{2}}}{bx^{\frac{7}{2}}} - \frac{2A(cx^4 + bx^2)^{\frac{5}{2}}}{5bx^{\frac{15}{2}}} \\
& + \frac{24(Ac + bB)x^{\frac{3}{2}}(cx^2 + b)\sqrt{c}}{5(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} + \frac{12c(Ac + bB)\sqrt{x}\sqrt{cx^4 + bx^2}}{5b} \\
& - \frac{24b^{\frac{1}{4}}c^{\frac{1}{4}}(Ac + bB)x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)\sqrt{cx^4 + bx^2}} \\
& + \frac{12b^{\frac{1}{4}}c^{\frac{1}{4}}(Ac + bB)x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)\sqrt{cx^4 + bx^2}}
\end{aligned}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12(Bb + Ac)\sqrt{c}x^4\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) - (Bcx^4 - (5Bb + 7Ac)x^2 - Ab)\sqrt{cx^4 + bx^2}\right)}{5x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^4 + (Bb + Ac)x^2 + Ab)\sqrt{cx^4 + bx^2}}{x^{\frac{9}{2}}}, x\right)$$

21.23 Problem number 242

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(3Ac + 7bB)(cx^4 + bx^2)^{\frac{3}{2}}}{21bx^{\frac{9}{2}}} - \frac{2A(cx^4 + bx^2)^{\frac{5}{2}}}{7bx^{\frac{17}{2}}} + \frac{4c(3Ac + 7bB)\sqrt{cx^4 + bx^2}}{21b\sqrt{x}} \\ & + \frac{4c^{\frac{3}{4}}(3Ac + 7bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{21\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{1}{4}}\sqrt{cx^4 + bx^2}} \end{aligned}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4(7Bb + 3Ac)\sqrt{c}x^5\operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (7Bcx^4 - (7Bb + 9Ac)x^2 - 3Ab)\sqrt{cx^4 + bx^2}\sqrt{x}\right)}{21x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^4 + (Bb + Ac)x^2 + Ab)\sqrt{cx^4 + bx^2}}{x^{\frac{11}{2}}}, x\right)$$

21.24 Problem number 243

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{17/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(Ac + 9bB)(cx^4 + bx^2)^{\frac{3}{2}}}{45bx^{\frac{11}{2}}} - \frac{2A(cx^4 + bx^2)^{\frac{5}{2}}}{9bx^{\frac{19}{2}}} \\ & + \frac{8c^{\frac{3}{2}}(Ac + 9bB)x^{\frac{3}{2}}(cx^2 + b)}{15b(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{4c(Ac + 9bB)\sqrt{cx^4 + bx^2}}{15bx^{\frac{3}{2}}} \\ & - \frac{8c^{\frac{5}{4}}(Ac + 9bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{cx^4 + bx^2}} \\ & + \frac{4c^{\frac{5}{4}}(Ac + 9bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{cx^4 + bx^2}} \end{aligned}$$

command

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 (9 Bbc + Ac^2) \sqrt{c} x^6 \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (3 (21 Bbc + 4 Ac^2) x^4 + 5 \dots \right)}{45 bx^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^4 + (Bb + Ac)x^2 + Ab)\sqrt{cx^4 + bx^2}}{x^{\frac{13}{2}}}, x\right)$$

21.25 Problem number 244

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6b(-15Ac+13bB)x^{\frac{3}{2}}\sqrt{cx^4+bx^2}}{385c^3} - \frac{2(-15Ac+13bB)x^{\frac{7}{2}}\sqrt{cx^4+bx^2}}{165c^2} \\ & + \frac{2Bx^{\frac{11}{2}}\sqrt{cx^4+bx^2}}{15c} - \frac{2b^2(-15Ac+13bB)\sqrt{cx^4+bx^2}}{77c^4\sqrt{x}} \\ & + \frac{b^{\frac{11}{4}}(-15Ac+13bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})}{77\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{17}{4}}\sqrt{cx^4+bx^2}} \end{aligned}$$

command

```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(15(13Bb^4-15Ab^3c)\sqrt{c}x\operatorname{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+(77Bc^4x^6-195Bb^3c+225Ab^2c^2-7(13Bbc^3-1155c^5x)\right)}{1155c^5x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^6+Ax^4)\sqrt{cx^4+bx^2}\sqrt{x}}{cx^2+b},x\right)$$

21.26 Problem number 245

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{14b^2(-13Ac + 11bB)x^{\frac{3}{2}}(cx^2 + b)}{195c^{\frac{7}{2}}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{2(-13Ac + 11bB)x^{\frac{5}{2}}\sqrt{cx^4 + bx^2}}{117c^2} \\ & + \frac{2Bx^{\frac{9}{2}}\sqrt{cx^4 + bx^2}}{13c} + \frac{14b(-13Ac + 11bB)\sqrt{x}\sqrt{cx^4 + bx^2}}{585c^3} \\ & + \frac{14b^{\frac{9}{4}}(-13Ac + 11bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x}}{195\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{15}{4}}\sqrt{cx^4 + bx^2}} \\ & + \frac{7b^{\frac{9}{4}}(-13Ac + 11bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x}}{195\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{15}{4}}\sqrt{cx^4 + bx^2}} \end{aligned}$$

command

```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(21(11Bb^3 - 13Ab^2c)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (45Bc^3x^4 + 77Bb^2c - 91c^2)\sqrt{cx^4 + bx^2}\right)}{585c^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^5 + Ax^3)\sqrt{cx^4 + bx^2}\sqrt{x}}{cx^2 + b}, x\right)$$

21.27 Problem number 246

$$\int \frac{x^{9/2}(A + Bx^2)}{\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2(-11Ac + 9bB)x^{\frac{3}{2}}\sqrt{cx^4 + bx^2}}{77c^2} + \frac{2Bx^{\frac{7}{2}}\sqrt{cx^4 + bx^2}}{11c} + \frac{10b(-11Ac + 9bB)\sqrt{cx^4 + bx^2}}{231c^3\sqrt{x}}$$

$$\frac{5b^{\frac{7}{4}}(-11Ac + 9bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{231\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{13}{4}}\sqrt{cx^4 + bx^2}}$$

command

`integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(5(9Bb^3 - 11Ab^2c)\sqrt{c}x\operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - (21Bc^3x^4 + 45Bb^2c - 55Abc^2 - 3(9Bbc^2 - 11Ac^2))\sqrt{cx^4 + bx^2}\right)}{231c^4x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^4 + Ax^2)\sqrt{cx^4 + bx^2}\sqrt{x}}{cx^2 + b}, x\right)$$

21.28 Problem number 247

$$\int \frac{x^{7/2}(A + Bx^2)}{\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2b(-9Ac + 7bB)x^{\frac{3}{2}}(cx^2 + b)}{15c^{\frac{5}{2}}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} + \frac{2Bx^{\frac{5}{2}}\sqrt{cx^4 + bx^2}}{9c} - \frac{2(-9Ac + 7bB)\sqrt{x}\sqrt{cx^4 + bx^2}}{45c^2}$$

$$\frac{2b^{\frac{5}{4}}(-9Ac + 7bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4 + bx^2}}$$

$$\frac{b^{\frac{5}{4}}(-9Ac + 7bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4 + bx^2}}$$

command

```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 (7 B b^2 - 9 A b c) \sqrt{c} \operatorname{weierstrassZeta} \left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse} \left(-\frac{4b}{c}, 0, x \right) \right) - (5 B c^2 x^2 - 7 B b c + 9 A c^2) \sqrt{x} \right)}{45 c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{c x^4 + b x^2} (B x^3 + A x) \sqrt{x}}{c x^2 + b}, x \right)$$

21.29 Problem number 248

$$\int \frac{x^{5/2} (A + B x^2)}{\sqrt{b x^2 + c x^4}} dx$$

Optimal antiderivative

$$\frac{2 B x^{\frac{3}{2}} \sqrt{c x^4 + b x^2}}{7 c} - \frac{2 (-7 A c + 5 b B) \sqrt{c x^4 + b x^2}}{21 c^2 \sqrt{x}}$$

$$+ \frac{b^{\frac{3}{4}} (-7 A c + 5 b B) x \sqrt{\frac{\cos \left(4 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{b} + x \sqrt{c})}{21 \cos \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right) c^{\frac{9}{4}} \sqrt{c x^4 + b x^2}}$$

command

```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((5 B b^2 - 7 A b c) \sqrt{c} x \operatorname{weierstrassPInverse} \left(-\frac{4b}{c}, 0, x \right) + (3 B c^2 x^2 - 5 B b c + 7 A c^2) \sqrt{c x^4 + b x^2} \sqrt{x} \right)}{21 c^3 x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{c x^4 + b x^2} (B x^2 + A) \sqrt{x}}{c x^2 + b}, x \right)$$

21.30 Problem number 249

$$\int \frac{x^{3/2}(A + Bx^2)}{\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-5Ac + 3bB)x^{\frac{3}{2}}(cx^2 + b)}{5c^{\frac{3}{2}}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} + \frac{2B\sqrt{x}\sqrt{cx^4 + bx^2}}{5c} \\ & + \frac{2b^{\frac{1}{4}}(-5Ac + 3bB)x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{cx^4 + bx^2}} \\ & + \frac{b^{\frac{1}{4}}(-5Ac + 3bB)x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{cx^4 + bx^2}} \end{aligned}$$

command

```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{cx^4 + bx^2} Bc\sqrt{x} + (3Bb - 5Ac)\sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right)\right)}{5c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)\sqrt{x}}{cx^3 + bx}, x\right)$$

21.31 Problem number 250

$$\int \frac{\sqrt{x} (A + Bx^2)}{\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2B\sqrt{cx^4 + bx^2}}{3c\sqrt{x}}$$

$$\frac{(-3Ac + bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})}{3 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{1}{4}} c^{\frac{5}{4}} \sqrt{cx^4 + bx^2}}$$

command

```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((Bb - 3Ac)\sqrt{c} \operatorname{xweierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - \sqrt{cx^4 + bx^2} Bc\sqrt{x} \right)}{3c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx^2 + A)\sqrt{x}}{\sqrt{cx^4 + bx^2}}, x\right)$$

21.32 Problem number 251

$$\int \frac{A + Bx^2}{\sqrt{x} \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2(Ac + bB)x^{\frac{3}{2}}(cx^2 + b)}{b\sqrt{c}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{2A\sqrt{cx^4 + bx^2}}{bx^{\frac{3}{2}}}$$

$$+ \frac{2(Ac + bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c}) \sqrt{\frac{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right)}{2} + \frac{1}{2}}}{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right) b^{\frac{3}{4}} c^{\frac{3}{4}} \sqrt{cx^4 + bx^2}}$$

$$+ \frac{(Ac + bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c}) \sqrt{\frac{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right)}{2} + \frac{1}{2}}}{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right) b^{\frac{3}{4}} c^{\frac{3}{4}} \sqrt{cx^4 + bx^2}}$$

command

`integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((Bb + Ac)\sqrt{c}x^2 \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{cx^4 + bx^2} Ac\sqrt{x}\right)}{bcx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)\sqrt{x}}{cx^5 + bx^3}, x\right)$$

21.33 Problem number 252

$$\int \frac{A + Bx^2}{x^{3/2}\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2A\sqrt{cx^4 + bx^2}}{3bx^{\frac{5}{2}}}$$

$$+ \frac{(-Ac + 3bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c}) \sqrt{\frac{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right)}{2} + \frac{1}{2}}}{3 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}\right)}{b^{\frac{1}{4}}}\right) b^{\frac{5}{4}} c^{\frac{1}{4}} \sqrt{cx^4 + bx^2}}$$

command

`integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((3Bb - Ac) \sqrt{c} x^3 \text{weierstrassPInverse} \left(-\frac{4b}{c}, 0, x \right) - \sqrt{cx^4 + bx^2} Ac \sqrt{x} \right)}{3bcx^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^4 + bx^2} (Bx^2 + A) \sqrt{x}}{cx^6 + bx^4}, x \right)$$

21.34 Problem number 253

$$\int \frac{A + Bx^2}{x^{5/2} \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2(-3Ac + 5bB) x^{\frac{3}{2}} (cx^2 + b) \sqrt{c}}{5b^2 (\sqrt{b} + x\sqrt{c}) \sqrt{cx^4 + bx^2}} - \frac{2A \sqrt{cx^4 + bx^2}}{5bx^{\frac{7}{2}}} - \frac{2(-3Ac + 5bB) \sqrt{cx^4 + bx^2}}{5b^2 x^{\frac{3}{2}}}$$

$$2c^{\frac{1}{4}}(-3Ac + 5bB) x \sqrt{\frac{\cos \left(4 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{b} + x\sqrt{c})$$

$$5 \cos \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right) b^{\frac{7}{4}} \sqrt{cx^4 + bx^2}$$

$$c^{\frac{1}{4}}(-3Ac + 5bB) x \sqrt{\frac{\cos \left(4 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{b} + x\sqrt{c})$$

$$+ \frac{5 \cos \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right) b^{\frac{7}{4}} \sqrt{cx^4 + bx^2}}{5b^2 x^4}$$

command

`integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((5Bb - 3Ac) \sqrt{c} x^4 \text{weierstrassZeta} \left(-\frac{4b}{c}, 0, \text{weierstrassPInverse} \left(-\frac{4b}{c}, 0, x \right) \right) + \sqrt{cx^4 + bx^2} ((5Bb - 3Ac)x^2 \right)}{5b^2 x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^4 + bx^2} (Bx^2 + A) \sqrt{x}}{cx^7 + bx^5}, x \right)$$

21.35 Problem number 254

$$\int \frac{A + Bx^2}{x^{7/2} \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2A\sqrt{cx^4 + bx^2}}{7bx^{\frac{9}{2}}} - \frac{2(-5Ac + 7bB)\sqrt{cx^4 + bx^2}}{21b^2x^{\frac{5}{2}}} + \frac{c^{\frac{3}{4}}(-5Ac + 7bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}}}{21 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{9}{4}} \sqrt{cx^4 + bx^2}} \text{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})$$

command

```
integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((7Bb - 5Ac)\sqrt{c}x^5 \text{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + \sqrt{cx^4 + bx^2}((7Bb - 5Ac)x^2 + 3Ab)\sqrt{x}\right)}{21b^2x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)\sqrt{x}}{cx^8 + bx^6}, x\right)$$

21.36 Problem number 255

$$\int \frac{A + Bx^2}{x^{9/2} \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2c^{\frac{3}{2}}(-7Ac + 9bB)x^{\frac{3}{2}}(cx^2 + b)}{15b^3(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{2A\sqrt{cx^4 + bx^2}}{9bx^{\frac{11}{2}}} \\ & - \frac{2(-7Ac + 9bB)\sqrt{cx^4 + bx^2}}{45b^2x^{\frac{7}{2}}} + \frac{2c(-7Ac + 9bB)\sqrt{cx^4 + bx^2}}{15b^3x^{\frac{3}{2}}} \\ & + \frac{2c^{\frac{5}{4}}(-7Ac + 9bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{11}{4}}\sqrt{cx^4 + bx^2}} \\ & + \frac{c^{\frac{5}{4}}(-7Ac + 9bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{11}{4}}\sqrt{cx^4 + bx^2}} \end{aligned}$$

command

```
integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3(9Bbc - 7Ac^2)\sqrt{c}x^6\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (3(9Bbc - 7Ac^2)x^4 - 5Ab^2)\sqrt{cx^4 + bx^2}\right)}{45b^3x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)\sqrt{x}}{cx^9 + bx^7}, x\right)$$

21.37 Problem number 256

$$\int \frac{A + Bx^2}{x^{11/2}\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2A\sqrt{cx^4+bx^2}}{11bx^{\frac{13}{2}}} - \frac{2(-9Ac+11bB)\sqrt{cx^4+bx^2}}{77b^2x^{\frac{9}{2}}} + \frac{10c(-9Ac+11bB)\sqrt{cx^4+bx^2}}{231b^3x^{\frac{5}{2}}}$$

$$+ \frac{5c^{\frac{7}{4}}(-9Ac+11bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})}{231\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{13}{4}}\sqrt{cx^4+bx^2}}$$

command

`integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(5(11Bbc-9Ac^2)\sqrt{c}x^7\operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (5(11Bbc-9Ac^2)x^4 - 21Ab^2 - 3(11Bb^2-9Abc))\right)}{231b^3x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}(Bx^2+A)\sqrt{x}}{cx^{10}+bx^8}, x\right)$$

21.38 Problem number 257

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-Ac+bB)x^{\frac{15}{2}}}{bc\sqrt{cx^4+bx^2}} - \frac{9(-11Ac+13bB)x^{\frac{3}{2}}\sqrt{cx^4+bx^2}}{77c^3}$$

$$+ \frac{(-11Ac+13bB)x^{\frac{7}{2}}\sqrt{cx^4+bx^2}}{11bc^2} + \frac{15b(-11Ac+13bB)\sqrt{cx^4+bx^2}}{77c^4\sqrt{x}}$$

$$+ \frac{15b^{\frac{7}{4}}(-11Ac+13bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})}{154\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{17}{4}}\sqrt{cx^4+bx^2}}$$

command

```
integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \left((13 B b^3 c - 11 A b^2 c^2) x^3 + (13 B b^4 - 11 A b^3 c) x \right) \sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - (14 B c^4 x^6 + 195 B b^3 c^2 x^3 + 145 b^4 c^2 x) \sqrt{c x^4 + b x^2}}{77 (c^6 x^3 + b c^5 x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B x^6 + A x^4) \sqrt{c x^4 + b x^2} \sqrt{x}}{c^2 x^4 + 2 b c x^2 + b^2}, x\right)$$

21.39 Problem number 258

$$\int \frac{x^{15/2} (A + B x^2)}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-Ac + bB) x^{\frac{13}{2}}}{bc \sqrt{c x^4 + b x^2}} + \frac{7b(-9Ac + 11bB) x^{\frac{3}{2}} (c x^2 + b)}{15c^{\frac{7}{2}} (\sqrt{b} + x\sqrt{c}) \sqrt{c x^4 + b x^2}} \\ & + \frac{(-9Ac + 11bB) x^{\frac{5}{2}} \sqrt{c x^4 + b x^2}}{9b c^2} - \frac{7(-9Ac + 11bB) \sqrt{x} \sqrt{c x^4 + b x^2}}{45c^3} \\ & - \frac{7b^{\frac{5}{4}} (-9Ac + 11bB) x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{15}{4}} \sqrt{c x^4 + b x^2}} \\ & + \frac{7b^{\frac{5}{4}} (-9Ac + 11bB) x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})}{30 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{15}{4}} \sqrt{c x^4 + b x^2}} \end{aligned}$$

command

```
integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21 \left((11 B b^3 - 9 A b^2 c + (11 B b^2 c - 9 A b c^2) x^2) \sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) - (10 B^2 c^2 - 15 A b c^2) \sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) \right)}{45 (c^5 x^2 + b c^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B x^5 + A x^3) \sqrt{c x^4 + b x^2} \sqrt{x}}{c^2 x^4 + 2 b c x^2 + b^2}, x\right)$$

21.40 Problem number 259

$$\int \frac{x^{13/2} (A + B x^2)}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-Ac + bB) x^{\frac{11}{2}}}{bc \sqrt{c x^4 + b x^2}} + \frac{(-7Ac + 9bB) x^{\frac{3}{2}} \sqrt{c x^4 + b x^2}}{7b c^2} - \frac{5(-7Ac + 9bB) \sqrt{c x^4 + b x^2}}{21c^3 \sqrt{x}} \\ & + \frac{5b^{\frac{3}{4}} (-7Ac + 9bB) x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c})}{42 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{13}{4}} \sqrt{c x^4 + b x^2}} \end{aligned}$$

command

`integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \left((9 B b^2 c - 7 A b c^2) x^3 + (9 B b^3 - 7 A b^2 c) x \right) \sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (6 B c^3 x^4 - 45 B b^2 c + 35 A b c^2 - 10 A^2 c) \sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)}{21 (c^5 x^3 + b c^4 x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B x^4 + A x^2) \sqrt{c x^4 + b x^2} \sqrt{x}}{c^2 x^4 + 2 b c x^2 + b^2}, x\right)$$

21.41 Problem number 260

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-Ac+bB)x^{\frac{9}{2}}}{bc\sqrt{cx^4+bx^2}} - \frac{3(-5Ac+7bB)x^{\frac{3}{2}}(cx^2+b)}{5c^{\frac{5}{2}}(\sqrt{b}+x\sqrt{c})\sqrt{cx^4+bx^2}} + \frac{(-5Ac+7bB)\sqrt{x}\sqrt{cx^4+bx^2}}{5bc^2} \\ & + \frac{3b^{\frac{1}{4}}(-5Ac+7bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4+bx^2}} \\ & + \frac{3b^{\frac{1}{4}}(-5Ac+7bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})}{10\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4+bx^2}} \end{aligned}$$

command

```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(7Bb^2-5Abc+(7Bbc-5Ac^2)x^2)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c},0,\operatorname{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+(2Bc^2x^2+7Ac^2)}{5(c^4x^2+bc^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}(Bx^3+Ax)\sqrt{x}}{c^2x^4+2bcx^2+b^2},x\right)$$

21.42 Problem number 261

$$\int \frac{x^{9/2}(A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-Ac + bB)x^{\frac{7}{2}}}{bc\sqrt{cx^4 + bx^2}} + \frac{(-3Ac + 5bB)\sqrt{cx^4 + bx^2}}{3bc^2\sqrt{x}} \\ & (-3Ac + 5bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \\ & \frac{6 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{1}{4}} c^{\frac{9}{4}} \sqrt{cx^4 + bx^2}}{6 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{1}{4}} c^{\frac{9}{4}} \sqrt{cx^4 + bx^2}} \end{aligned}$$

command

```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{((5Bbc - 3Ac^2)x^3 + (5Bb^2 - 3Abc)x)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - (2Bc^2x^2 + 5Bbc - 3Ac^2)\sqrt{cx^4 + bx^2}}{3(c^4x^3 + bc^3x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)\sqrt{x}}{c^2x^4 + 2bcx^2 + b^2}, x\right)$$

21.43 Problem number 262

$$\int \frac{x^{7/2}(A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-Ac + bB)x^{\frac{5}{2}}}{bc\sqrt{cx^4 + bx^2}} + \frac{(-Ac + 3bB)x^{\frac{3}{2}}(cx^2 + b)}{bc^{\frac{3}{2}}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} \\ & (-Ac + 3bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} b^{\frac{3}{4}}c^{\frac{7}{4}}\sqrt{cx^4 + bx^2}} \\ & (-Ac + 3bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} b^{\frac{3}{4}}c^{\frac{7}{4}}\sqrt{cx^4 + bx^2}} \\ & + \frac{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} b^{\frac{3}{4}}c^{\frac{7}{4}}\sqrt{cx^4 + bx^2} \end{aligned}$$

command

`integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(3Bb^2 - Abc + (3Bbc - Ac^2)x^2)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{cx^4 + bx^2}(B - bc^3x^2 + b^2c^2)}{bc^3x^2 + b^2c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)\sqrt{x}}{c^2x^5 + 2bcx^3 + b^2x}, x\right)$$

21.44 Problem number 263

$$\int \frac{x^{5/2}(A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-Ac + bB)x^{\frac{3}{2}}}{bc\sqrt{cx^4 + bx^2}} \\ & (Ac + bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} b^{\frac{5}{4}}c^{\frac{5}{4}}\sqrt{cx^4 + bx^2}} \\ & + \frac{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} b^{\frac{5}{4}}c^{\frac{5}{4}}\sqrt{cx^4 + bx^2} \end{aligned}$$

command

`integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{((Bbc + Ac^2)x^3 + (Bb^2 + Abc)x)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - \sqrt{cx^4 + bx^2} (Bbc - Ac^2)\sqrt{x}}{bc^3x^3 + b^2c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} (Bx^2 + A)\sqrt{x}}{c^2x^6 + 2bcx^4 + b^2x^2}, x\right)$$

21.45 Problem number 264

$$\int \frac{x^{3/2}(A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-3Ac + bB)x^{\frac{5}{2}}}{b^2\sqrt{cx^4 + bx^2}} - \frac{(-3Ac + bB)x^{\frac{3}{2}}(cx^2 + b)}{b^2\sqrt{c}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{2A\sqrt{x}}{b\sqrt{cx^4 + bx^2}}$$

$$+ \frac{(-3Ac + bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})}{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{7}{4}}c^{\frac{3}{4}}\sqrt{cx^4 + bx^2}}$$

$$+ \frac{(-3Ac + bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})}{2 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{7}{4}}c^{\frac{3}{4}}\sqrt{cx^4 + bx^2}}$$

command

`integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{((Bbc - 3Ac^2)x^4 + (Bb^2 - 3Abc)x^2)\sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) - \sqrt{cx^4 + bx^2}}{b^2c^2x^4 + b^3cx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} (Bx^2 + A)\sqrt{x}}{c^2x^7 + 2bcx^5 + b^2x^3}, x\right)$$

21.46 Problem number 265

$$\int \frac{\sqrt{x} (A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-5Ac + 3bB)x^{\frac{3}{2}}}{3b^2\sqrt{cx^4 + bx^2}} - \frac{2A}{3b\sqrt{x}\sqrt{cx^4 + bx^2}}$$

$$+ \frac{(-5Ac + 3bB)x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c})}{6 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{9}{4}} c^{\frac{1}{4}} \sqrt{cx^4 + bx^2}}$$

command

```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{((3Bbc - 5Ac^2)x^5 + (3Bb^2 - 5Abc)x^3)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - \sqrt{cx^4 + bx^2} (2Abc - (3Bbc - 5Ac^2)x^2)}{3(b^2c^2x^5 + b^3cx^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} (Bx^2 + A)\sqrt{x}}{c^2x^8 + 2bcx^6 + b^2x^4}, x\right)$$

21.47 Problem number 266

$$\int \frac{A + Bx^2}{\sqrt{x} (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2A}{5bx^{\frac{3}{2}}\sqrt{cx^4+bx^2}} + \frac{3(-7Ac+5bB)x^{\frac{3}{2}}(cx^2+b)\sqrt{c}}{5b^3(\sqrt{b}+x\sqrt{c})\sqrt{cx^4+bx^2}} \\
 & + \frac{(-7Ac+5bB)\sqrt{x}}{5b^2\sqrt{cx^4+bx^2}} - \frac{3(-7Ac+5bB)\sqrt{cx^4+bx^2}}{5b^3x^{\frac{3}{2}}} \\
 & \frac{3c^{\frac{1}{4}}(-7Ac+5bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{11}{4}}\sqrt{cx^4+bx^2}} \\
 & \frac{3c^{\frac{1}{4}}(-7Ac+5bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})}{10\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{11}{4}}\sqrt{cx^4+bx^2}}
 \end{aligned}$$

command

`integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3((5Bbc-7Ac^2)x^6+(5Bb^2-7Abc)x^4)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c},0,\operatorname{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+(3(5Bbc-7Ac^2)x^6+(5Bb^2-7Abc)x^4)\sqrt{c}}{5(b^3cx^6+b^4x^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}(Bx^2+A)\sqrt{x}}{c^2x^9+2bcx^7+b^2x^5},x\right)$$

21.48 Problem number 267

$$\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2A}{7bx^{\frac{5}{2}}\sqrt{cx^4+bx^2}} + \frac{-9Ac+7bB}{7b^2\sqrt{x}\sqrt{cx^4+bx^2}} - \frac{5(-9Ac+7bB)\sqrt{cx^4+bx^2}}{21b^3x^{\frac{5}{2}}} \\
 & 5c^{\frac{3}{4}}(-9Ac+7bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c}} \\
 & \frac{42\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{13}{4}}\sqrt{cx^4+bx^2}}{42\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{13}{4}}\sqrt{cx^4+bx^2}}
 \end{aligned}$$

command

```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5((7Bbc-9Ac^2)x^7+(7Bb^2-9Abc)x^5)\sqrt{c}\operatorname{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+(5(7Bbc-9Ac^2)x^4+6Ab^2+21(b^3cx^7+b^4x^5))}{21(b^3cx^7+b^4x^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}(Bx^2+A)\sqrt{x}}{c^2x^{10}+2bcx^8+b^2x^6},x\right)$$

21.49 Problem number 268

$$\int \frac{A + Bx^2}{x^{5/2} (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2A}{9bx^{\frac{7}{2}}\sqrt{cx^4+bx^2}} + \frac{-11Ac+9bB}{9b^2x^{\frac{3}{2}}\sqrt{cx^4+bx^2}} - \frac{7c^{\frac{3}{2}}(-11Ac+9bB)x^{\frac{3}{2}}(cx^2+b)}{15b^4(\sqrt{b}+x\sqrt{c})\sqrt{cx^4+bx^2}} \\ & - \frac{7(-11Ac+9bB)\sqrt{cx^4+bx^2}}{45b^3x^{\frac{7}{2}}} + \frac{7c(-11Ac+9bB)\sqrt{cx^4+bx^2}}{15b^4x^{\frac{3}{2}}} \\ & + \frac{7c^{\frac{5}{4}}(-11Ac+9bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{15}{4}}\sqrt{cx^4+bx^2}} \\ & + \frac{7c^{\frac{5}{4}}(-11Ac+9bB)x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})}{30\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{15}{4}}\sqrt{cx^4+bx^2}} \end{aligned}$$

command

```
integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21((9Bbc^2 - 11Ac^3)x^8 + (9Bb^2c - 11Abc^2)x^6)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (21Bc^2x^7 + 11Ac^3x^5)\sqrt{c}}{45(b^4cx^8 + b^5x^6)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}(Bx^2+A)\sqrt{x}}{c^2x^{11}+2bcx^9+b^2x^7}, x\right)$$

22 Test file number 33

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/33_1.2.1.2-d+e_x^-m-a+b_x+c_x^2-^p

22.1 Problem number 386

$$\int (d + ex)^{3/2} \sqrt{bx + cx^2} dx$$

Optimal antiderivative

$$\frac{2e(cx^2 + bx)^{\frac{3}{2}} \sqrt{ex + d}}{7c}$$

$$\frac{2(-be + 2cd) (8b^2e^2 - 3bcde + 3c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{105c^{\frac{5}{2}}e^2 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}$$

$$+ \frac{4d(-be + cd) (2b^2e^2 - 3bcde + 3c^2d^2) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{105c^{\frac{5}{2}}e^2 \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

$$+ \frac{2(3c^2d^2 + 9bcde - 4b^2e^2 + 12ce(-be + 2cd)x) \sqrt{ex + d} \sqrt{cx^2 + bx}}{105c^2e}$$

command

```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((6c^4d^4 - 12bc^3d^3e - 17b^2c^2d^2e^2 + 23b^3cde^3 - 8b^4e^4) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2 - bcde + b^2e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d^2 - bcde + b^2e^2)}{3c^2}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^2 + bx} (ex + d)^{\frac{3}{2}}, x\right)$$

22.2 Problem number 387

$$\int \sqrt{d+ex} \sqrt{bx+cx^2} dx$$

Optimal antiderivative

$$\frac{4(b^2e^2 - bcde + c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1+\frac{cx}{b}} \sqrt{ex+d}}{15c^{\frac{3}{2}}e^2 \sqrt{1+\frac{ex}{d}} \sqrt{cx^2+bx}} + \frac{2d(-be+cd)(-be+2cd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1+\frac{cx}{b}} \sqrt{1+\frac{ex}{d}}}{15c^{\frac{3}{2}}e^2 \sqrt{ex+d} \sqrt{cx^2+bx}} + \frac{2(ex+d)^{\frac{3}{2}} \sqrt{cx^2+bx}}{5e} - \frac{2(-be+2cd) \sqrt{ex+d} \sqrt{cx^2+bx}}{15ce}$$

command

`integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^3d^3 - 3bc^2d^2e - 3b^2cde^2 + 2b^3e^3) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2 - bcde + b^2e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d^3 - 3bc^2d^2e - 3b^2cde^2 + 2b^3e^3)}{27c^3}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^2+bx} \sqrt{ex+d}, x\right)$$

22.3 Problem number 388

$$\int \frac{\sqrt{bx+cx^2}}{\sqrt{d+ex}} dx$$

Optimal antiderivative

$$\frac{2(-be+2cd) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1+\frac{cx}{b}} \sqrt{ex+d}}{3e^2 \sqrt{c} \sqrt{1+\frac{ex}{d}} \sqrt{cx^2+bx}} + \frac{4d(-be+cd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1+\frac{cx}{b}} \sqrt{1+\frac{ex}{d}}}{3e^2 \sqrt{c} \sqrt{ex+d} \sqrt{cx^2+bx}} + \frac{2\sqrt{ex+d} \sqrt{cx^2+bx}}{3e}$$

command

```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cx^2 + bx} \sqrt{xe + d} c^2 e^2 + (2c^2 d^2 - 2bcde - b^2 e^2) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4(c^2 d^2 - bcde + b^2 e^2) e^{(-2)}}{3c^2}, -\frac{4(2c^3 d^2 - 3bc^2 d e + 3b^2 c d e^2 + 2b^3 e^3)}{27c^3} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx}}{\sqrt{ex + d}}, x \right)$$

22.4 Problem number 389

$$\int \frac{\sqrt{bx + cx^2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{4 \text{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{e^2 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} - \frac{2(-be + 2cd) \text{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{e^2 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}} - \frac{2\sqrt{cx^2 + bx}}{e\sqrt{ex + d}}$$

command

```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2cd^2 - bxe^2 + (2cdx - bd)e) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4(c^2 d^2 - bcde + b^2 e^2) e^{(-2)}}{3c^2}, -\frac{4(2c^3 d^3 - 3bc^2 d^2 e - 3b^2 c d e^2 + 2b^3 e^3)}{27c^3} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx} \sqrt{ex + d}}{e^2 x^2 + 2dex + d^2}, x \right)$$

22.5 Problem number 390

$$\int \frac{\sqrt{bx + cx^2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(-be + 2cd) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{3de^2(-be + cd) \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} + \frac{4 \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3e^2 \sqrt{ex + d} \sqrt{cx^2 + bx}} - \frac{2\sqrt{cx^2 + bx}}{3e(ex + d)^{3/2}} + \frac{2(-be + 2cd) \sqrt{cx^2 + bx}}{3de(-be + cd) \sqrt{ex + d}}$$

command

```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^2d^4 - b^2x^2e^4 - 2(bcdx^2 + b^2dx)e^3 + (2c^2d^2x^2 - 4bcd^2x - b^2d^2)e^2 + 2(2c^2d^3x - bcd^3)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrass} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx} \sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

22.6 Problem number 391

$$\int \frac{\sqrt{bx + cx^2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{4(b^2e^2 - bcde + c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{15d^2e^2(-be + cd)^2 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} + \frac{2(-be + 2cd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{15de^2(-be + cd) \sqrt{ex + d} \sqrt{cx^2 + bx}} - \frac{2\sqrt{cx^2 + bx}}{5e(ex + d)^{\frac{5}{2}}} + \frac{2(-be + 2cd) \sqrt{cx^2 + bx}}{15de(-be + cd)(ex + d)^{\frac{3}{2}}} + \frac{4(b^2e^2 - bcde + c^2d^2) \sqrt{cx^2 + bx}}{15d^2e(-be + cd)^2 \sqrt{ex + d}}$$

command

`integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^3d^6 + 2b^3x^3e^6 - 3(b^2cdx^3 - 2b^3dx^2)e^5 - 3(bc^2d^2x^3 + 3b^2cd^2x^2 - 2b^3d^2x)e^4 + (2c^3d^3x^3 - 9bc^2d^3x^2 - 9b$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx} \sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

22.7 Problem number 392

$$\int (d + ex)^{3/2} (bx + cx^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{2(c^2d^2 + 13bcde - 6b^2e^2 + 14ce(-be + 2cd)x)(cx^2 + bx)^{\frac{3}{2}} \sqrt{ex + d}}{231c^2e} + \frac{2e(cx^2 + bx)^{\frac{5}{2}} \sqrt{ex + d}}{11c} - \frac{16(-2be + cd)(-be + 2cd)(be + cd)(b^2e^2 - bcde + c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{1155c^{\frac{7}{2}}e^4 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} + \frac{2d(-be + cd)(-8b^4e^4 + 13b^3cde^3 + 3b^2c^2d^2e^2 - 32b^3c^3d^3e + 16c^4d^4) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{1155c^{\frac{7}{2}}e^4 \sqrt{ex + d} \sqrt{cx^2 + bx}} + \frac{2(8c^4d^4 - 19bc^3d^3e + 6b^2c^2d^2e^2 - 19b^3cde^3 + 8b^4e^4 - 3ce(-be + 2cd)(8b^2e^2 - bcde + c^2d^2)x) \sqrt{ex + d} \sqrt{cx^2 + bx}}{1155c^3e^3}$$

command

```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^6d^6 - 48bc^5d^5e + 33b^2c^4d^4e^2 + 14b^3c^3d^3e^3 + 33b^4c^2d^2e^4 - 48b^5cde^5 + 16b^6e^6) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((cex^3 + bdx + (cd + be)x^2) \sqrt{cx^2 + bx} \sqrt{ex + d}, x \right)$$

22.8 Problem number 393

$$\int \sqrt{d + ex} (bx + cx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ex + d)^{\frac{3}{2}} (cx^2 + bx)^{\frac{3}{2}}}{9e} - \frac{2(-be + 2cd) (cx^2 + bx)^{\frac{3}{2}} \sqrt{ex + d}}{21ce} \\ & - \frac{2(-8b^4e^4 + 7b^3cde^3 + 9b^2c^2d^2e^2 - 32b^3c^3d^3e + 16c^4d^4) \text{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{315c^{\frac{5}{2}}e^4 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\ & + \frac{8d(-be + cd) (-be + 2cd) (-b^2e^2 - 2bcde + 2c^2d^2) \text{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{315c^{\frac{5}{2}}e^4 \sqrt{ex + d} \sqrt{cx^2 + bx}} \\ & + \frac{2(8c^3d^3 - 15bc^2d^2e + 3b^2cde^2 - 4b^3e^3 - 6ce(2b^2e^2 - bcde + c^2d^2) x) \sqrt{ex + d} \sqrt{cx^2 + bx}}{315c^2e^3} \end{aligned}$$

command

```
integrate((c*x^2+b*x)^(3/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^5d^5 - 40bc^4d^4e + 22b^2c^3d^3e^2 + 7b^3c^2d^2e^3 + 11b^4cde^4 - 8b^5e^5) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + b^2e^2)}{3c^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((cx^2 + bx)^{\frac{3}{2}} \sqrt{ex + d}, x \right)$$

22.9 Problem number 394

$$\int \frac{(bx + cx^2)^{3/2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(cx^2 + bx)^{\frac{3}{2}} \sqrt{ex + d}}{7e} \\ & - \frac{4(-be + 2cd) (-b^2e^2 - 4bcde + 4c^2d^2) \operatorname{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{35c^{\frac{3}{2}} e^4 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\ & + \frac{2d(-be + cd) (-b^2e^2 - 16bcde + 16c^2d^2) \operatorname{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{35c^{\frac{3}{2}} e^4 \sqrt{ex + d} \sqrt{cx^2 + bx}} \\ & + \frac{2(8c^2d^2 - 11bcde + b^2e^2 - 3ce(-be + 2cd)x) \sqrt{ex + d} \sqrt{cx^2 + bx}}{35ce^3} \end{aligned}$$

command

`integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^4d^4 - 32bc^3d^3e + 13b^2c^2d^2e^2 + 3b^3cde^3 + 2b^4e^4) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + b^2e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3}{3} \right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(cx^2 + bx)^{\frac{3}{2}}}{\sqrt{ex + d}}, x \right)$$

22.10 Problem number 395

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(cx^2 + bx)^{\frac{3}{2}}}{e\sqrt{ex + d}} \\
& + \frac{2(b^2e^2 - 16bcde + 16c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{5e^4\sqrt{c} \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\
& - \frac{16d(-be + cd)(-be + 2cd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{5e^4\sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}} \\
& - \frac{2(-6cex - 7be + 8cd) \sqrt{ex + d} \sqrt{cx^2 + bx}}{5e^3}
\end{aligned}$$

command

```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((16c^3d^4 + b^3xe^4 + (6b^2cdx + b^3d)e^3 - 6(4bc^2d^2x - b^2cd^2)e^2 + 8(2c^3d^3x - 3bc^2d^3)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInv} \right)}{5e^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + bx)^{\frac{3}{2}} \sqrt{ex + d}}{e^2x^2 + 2dex + d^2}, x\right)$$

22.11 Problem number 396

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2(cx^2 + bx)^{\frac{3}{2}}}{3e(ex + d)^{\frac{3}{2}}} \\
& - \frac{16(-be + 2cd) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{3e^4 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\
& + \frac{2(-3be + 4cd)(-be + 4cd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3e^4 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}} \\
& + \frac{2(2cex - 3be + 8cd) \sqrt{cx^2 + bx}}{3e^3 \sqrt{ex + d}}
\end{aligned}$$

command

```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^2d^4 + b^2x^2e^4 - 2(8bcdx^2 - b^2dx)e^3 + (16c^2d^2x^2 - 32bcd^2x + b^2d^2)e^2 + 16(2c^2d^3x - bcd^3)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weier} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + bx)^{\frac{3}{2}} \sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

22.12 Problem number 397

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(cx^2 + bx)^{\frac{3}{2}}}{5e(ex + d)^{\frac{5}{2}}} \\
& + \frac{2(b^2e^2 - 16bcde + 16c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{5de^4(-be + cd) \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\
& - \frac{16(-be + 2cd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{5e^4 \sqrt{ex + d} \sqrt{cx^2 + bx}} \\
& - \frac{2(cd^2(-7be + 8cd) + e(b^2e^2 - 10bcde + 10c^2d^2)x) \sqrt{cx^2 + bx}}{5de^3(-be + cd)(ex + d)^{\frac{3}{2}}}
\end{aligned}$$

command

```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((16c^3d^6 + b^3x^3e^6 + 3(2b^2cdx^3 + b^3dx^2))e^5 - 3(8bc^2d^2x^3 - 6b^2cd^2x^2 - b^3d^2x)e^4 + (16c^3d^3x^3 - 72bc^2d^3x^2 + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + bx)^{\frac{3}{2}} \sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

22.13 Problem number 398

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(cx^2 + bx)^{\frac{3}{2}}}{7e(ex + d)^{\frac{7}{2}}} \\
& - \frac{4(-be + 2cd)(-b^2e^2 - 4bcde + 4c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{35d^2e^4(-be + cd)^2 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\
& + \frac{2(-b^2e^2 - 16bcde + 16c^2d^2) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{35de^4(-be + cd) \sqrt{ex + d} \sqrt{cx^2 + bx}} \\
& - \frac{2(d(-2b^2e^2 - 5bcde + 8c^2d^2) + e(b^2e^2 - 14bcde + 14c^2d^2)x) \sqrt{cx^2 + bx}}{35de^3(-be + cd)(ex + d)^{\frac{5}{2}}} \\
& + \frac{4(-be + 2cd)(-b^2e^2 - 4bcde + 4c^2d^2) \sqrt{cx^2 + bx}}{35d^2e^3(-be + cd)^2 \sqrt{ex + d}}
\end{aligned}$$

command

```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^4d^8 + 2b^4x^4e^8 + (3b^3cdx^4 + 8b^4dx^3)e^7 + (13b^2c^2d^2x^4 + 12b^3cd^2x^3 + 12b^4d^2x^2)e^6 - 2(16bc^3d^3x^4 - 26b^2c^2d^2x^3 + 12b^3cd^2x^2 - 12b^4d^2x)e^5 + (16c^4d^8 + 2b^4x^4e^8) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + bx)^{\frac{3}{2}} \sqrt{ex + d}}{e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5}, x\right)$$

22.14 Problem number 399

$$\int \sqrt{d + ex} (bx + cx^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(ex+d)^{\frac{3}{2}}(cx^2+bx)^{\frac{5}{2}}}{13e} \\
& + \frac{10(16c^3d^3 - 31bc^2d^2e + 9b^2cd^2e^2 - 18b^3e^3 - 14ce(3b^2e^2 - bcde + c^2d^2)x)(cx^2+bx)^{\frac{3}{2}}\sqrt{ex+d}}{9009c^2e^3} \\
& - \frac{10(-be+2cd)(cx^2+bx)^{\frac{5}{2}}\sqrt{ex+d}}{143ce} \\
& - \frac{4(24b^6e^6 - 20b^5cde^5 - 21b^4c^2d^2e^4 - 46b^3c^3d^3e^3 + 343b^2c^4d^4e^2 - 384bc^5d^5e + 128c^6d^6) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right)}{9009c^{\frac{7}{2}}e^6\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}} \\
& + \frac{2d(-be+cd)(-be+2cd)(24b^4e^4 + 49b^3cde^3 + 79b^2c^2d^2e^2 - 256bc^3d^3e + 128c^4d^4) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right)}{9009c^{\frac{7}{2}}e^6\sqrt{ex+d}\sqrt{cx^2+bx}} \\
& + \frac{2(128c^5d^5 - 368bc^4d^4e + 303b^2c^3d^3e^2 - 22b^3c^2d^2e^3 - 17b^4cde^4 + 24b^5e^5 - 3ce(-24b^4e^4 + 11b^3cde^3 + 21b^2c^2d^2e^2))}{9009c^3e^5}
\end{aligned}$$

command

```
integrate((c*x^2+b*x)^(5/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((256c^7d^7 - 896bc^6d^6e + 1022b^2c^5d^5e^2 - 315b^3c^4d^4e^3 - 68b^4c^3d^3e^4 - 31b^5c^2d^2e^5 - 64b^6cde^6 + 48b^7e^7)\sqrt{c}e^{\frac{1}{2}}\right)}{9009c^3e^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(c^2x^4 + 2bcx^3 + b^2x^2\right)\sqrt{cx^2+bx}\sqrt{ex+d}, x\right)$$

22.15 Problem number 400

$$\int \frac{(bx+cx^2)^{5/2}}{\sqrt{d+ex}} dx$$

Optimal antiderivative

$$\frac{10(16c^2d^2 - 23bcde + 3b^2e^2 - 7ce(-be + 2cd)x)(cx^2 + bx)^{\frac{3}{2}}\sqrt{ex + d}}{693ce^3} + \frac{2(cx^2 + bx)^{\frac{5}{2}}\sqrt{ex + d}}{11e}$$

$$- \frac{2(-be + 2cd)(8b^4e^4 + 29b^3cde^3 + 99b^2c^2d^2e^2 - 256b^3c^3d^3e + 128c^4d^4)\text{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right)\sqrt{-b}\sqrt{x}}{693c^{\frac{5}{2}}e^6\sqrt{1 + \frac{ex}{d}}\sqrt{cx^2 + bx}}$$

$$+ \frac{4d(-be + cd)(2b^4e^4 + 5b^3cde^3 + 123b^2c^2d^2e^2 - 256b^3c^3d^3e + 128c^4d^4)\text{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right)\sqrt{-b}\sqrt{x}}{693c^{\frac{5}{2}}e^6\sqrt{ex + d}\sqrt{cx^2 + bx}}$$

$$+ \frac{2(128c^4d^4 - 304b^3c^3d^3e + 195b^2c^2d^2e^2 - 7b^3cde^3 - 4b^4e^4 - 12ce(-be + 2cd)(-b^2e^2 - 4bcde + 4c^2d^2)x)\sqrt{ex + d}}{693c^2e^5}$$

command

`integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left((256c^6d^6 - 768bc^5d^5e + 726b^2c^4d^4e^2 - 172b^3c^3d^3e^3 - 33b^4c^2d^2e^4 - 9b^5cde^5 - 8b^6e^6)\sqrt{c}e^{\frac{1}{2}}\text{weierstrassPInver}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(c^2x^4 + 2bcx^3 + b^2x^2)\sqrt{cx^2 + bx}}{\sqrt{ex + d}}, x\right)$$

22.16 Problem number 401

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(cx^2 + bx)^{\frac{5}{2}}}{e\sqrt{ex + d}} - \frac{10(-14ce x - 15be + 16cd)(cx^2 + bx)^{\frac{3}{2}}\sqrt{ex + d}}{63e^3}$$

$$+ \frac{4(-b^4e^4 - 7b^3cde^3 + 135b^2c^2d^2e^2 - 256b^3c^3d^3e + 128c^4d^4)\text{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right)\sqrt{-b}\sqrt{x}\sqrt{1 + \frac{cx}{b}}}{63c^{\frac{3}{2}}e^6\sqrt{1 + \frac{ex}{d}}\sqrt{cx^2 + bx}}$$

$$- \frac{2d(-be + cd)(-be + 2cd)(-b^2e^2 - 128bcde + 128c^2d^2)\text{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right)\sqrt{-b}\sqrt{x}\sqrt{1 + \frac{cx}{b}}}{63c^{\frac{3}{2}}e^6\sqrt{ex + d}\sqrt{cx^2 + bx}}$$

$$- \frac{2(128c^3d^3 - 240b^2c^2d^2e + 111b^2cde^2 - b^3e^3 - 3ce(b^2e^2 - 32bcde + 32c^2d^2)x)\sqrt{ex + d}\sqrt{cx^2 + bx}}{63ce^5}$$

command

```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((256 c^5 d^6 - 2 b^5 x e^6 - (13 b^4 c d x + 2 b^5 d) e^5 - (77 b^3 c^2 d^2 x + 13 b^4 c d^2) e^4 + (478 b^2 c^3 d^3 x - 77 b^3 c^2 d^3) e^3 - 2 (32$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(c^2 x^4 + 2 b c x^3 + b^2 x^2) \sqrt{c x^2 + b x} \sqrt{e x + d}}{e^2 x^2 + 2 d e x + d^2}, x \right)$$

22.17 Problem number 402

$$\int \frac{(b x + c x^2)^{5/2}}{(d + e x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(c x^2 + b x)^{\frac{5}{2}}}{3e (e x + d)^{\frac{3}{2}}} + \frac{10(2cex - 7be + 16cd) (c x^2 + b x)^{\frac{3}{2}}}{21e^3 \sqrt{e x + d}} \\ & - \frac{2(-be + 2cd) (3b^2 e^2 - 128bcde + 128c^2 d^2) \text{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{e x + d}}{21e^6 \sqrt{c} \sqrt{1 + \frac{ex}{d}} \sqrt{c x^2 + b x}} \\ & + \frac{4d(-be + cd) (27b^2 e^2 - 128bcde + 128c^2 d^2) \text{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{21e^6 \sqrt{c} \sqrt{e x + d} \sqrt{c x^2 + b x}} \\ & + \frac{2(128c^2 d^2 - 176bcde + 51b^2 e^2 - 48ce(-be + 2cd) x) \sqrt{e x + d} \sqrt{c x^2 + b x}}{21e^5} \end{aligned}$$

command

```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((256 c^4 d^6 - 3 b^4 x^2 e^6 - 2 (11 b^3 c d x^2 + 3 b^4 d x) e^5 + (278 b^2 c^2 d^2 x^2 - 44 b^3 c d^2 x - 3 b^4 d^2) e^4 - 2 (256 b c^3 d^3 x^2 - 278$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(c^2 x^4 + 2 b c x^3 + b^2 x^2) \sqrt{c x^2 + b x} \sqrt{e x + d}}{e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3}, x \right)$$

22.18 Problem number 403

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(6cex - 5be + 16cd)(cx^2 + bx)^{\frac{3}{2}}}{15e^3(ex + d)^{\frac{3}{2}}} - \frac{2(cx^2 + bx)^{\frac{5}{2}}}{5e(ex + d)^{\frac{5}{2}}}$$

$$+ \frac{4(23b^2e^2 - 128bcde + 128c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{15e^6 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}$$

$$- \frac{2(-be + 2cd)(15b^2e^2 - 128bcde + 128c^2d^2) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{15e^6 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

$$- \frac{2(128c^2d^2 - 112bcde + 15b^2e^2 + 16ce(-be + 2cd)x) \sqrt{cx^2 + bx}}{15e^5 \sqrt{ex + d}}$$

command

```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((256c^3d^6 + b^3x^3e^6 + 3(42b^2cdx^3 + b^3dx^2))e^5 - 3(128bc^2d^2x^3 - 126b^2cd^2x^2 - b^3d^2x)e^4 + (256c^3d^3x^3 - 115$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + b^2x^2) \sqrt{cx^2 + bx} \sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

22.19 Problem number 404

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(c d^2(-13be + 16cd) + e(3b^2e^2 - 22bcde + 22c^2d^2)x)(cx^2 + bx)^{\frac{3}{2}}}{21de^3(-be + cd)(ex + d)^{\frac{5}{2}}} - \frac{2(cx^2 + bx)^{\frac{5}{2}}}{7e(ex + d)^{\frac{7}{2}}} \\
& - \frac{2(-be + 2cd)(3b^2e^2 - 128bcde + 128c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{21de^6(-be + cd) \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\
& + \frac{4(27b^2e^2 - 128bcde + 128c^2d^2) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{21e^6 \sqrt{ex + d} \sqrt{cx^2 + bx}} \\
& + \frac{2c(d(51b^2e^2 - 176bcde + 128c^2d^2) + e(3b^2e^2 - 32bcde + 32c^2d^2)x) \sqrt{cx^2 + bx}}{21de^5(-be + cd) \sqrt{ex + d}}
\end{aligned}$$

command

```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((256c^4d^8 - 3b^4x^4e^8 - 2(11b^3cdx^4 + 6b^4dx^3))e^7 + 2(139b^2c^2d^2x^4 - 44b^3cd^2x^3 - 9b^4d^2x^2)e^6 - 4(128bc^3d^3x^4 \right)}{21de^5(-be + cd) \sqrt{ex + d}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + b^2x^2) \sqrt{cx^2 + bx} \sqrt{ex + d}}{e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5}, x\right)$$

22.20 Problem number 405

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^{11/2}} dx$$

Optimal antiderivative

$$\frac{2(d(-2b^2e^2 - 11bcde + 16c^2d^2) + e(3b^2e^2 - 26bcde + 26c^2d^2)x)(cx^2 + bx)^{\frac{3}{2}}}{63de^3(-be + cd)(ex + d)^{\frac{7}{2}}} - \frac{2(cx^2 + bx)^{\frac{5}{2}}}{9e(ex + d)^{\frac{9}{2}}}$$

$$+ \frac{4(-b^4e^4 - 7b^3cde^3 + 135b^2c^2d^2e^2 - 256b^3c^3d^3e + 128c^4d^4) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}}}{63d^2e^6(-be + cd)^2 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}$$

$$- \frac{2(-be + 2cd)(-b^2e^2 - 128bcde + 128c^2d^2) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{63de^6(-be + cd) \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

$$- \frac{2(cd^2(-b^3e^3 + 111b^2cde^2 - 240b^2c^2d^2e + 128c^3d^3) + e(-2b^4e^4 - 11b^3cde^3 + 171b^2c^2d^2e^2 - 320b^3c^3d^3e + 160c^4d^4))}{63d^2e^5(-be + cd)^2(ex + d)^{\frac{3}{2}}}$$

command

```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((256c^5d^{10} - 2b^5x^5e^{10} - (13b^4cdx^5 + 10b^5dx^4)e^9 - (77b^3c^2d^2x^5 + 65b^4cd^2x^4 + 20b^5d^2x^3)e^8 + (478b^2c^3d^3x^5\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + b^2x^2)\sqrt{cx^2 + bx}\sqrt{ex + d}}{e^6x^6 + 6de^5x^5 + 15d^2e^4x^4 + 20d^3e^3x^3 + 15d^4e^2x^2 + 6d^5ex + d^6}, x\right)$$

22.21 Problem number 406

$$\int \frac{(d + ex)^{7/2}}{\sqrt{bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{16(-be + 2cd)(6b^2e^2 - 11bcde + 11c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{105c^{\frac{7}{2}} \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}$$

$$- \frac{2d(-be + cd)(24b^2e^2 - 71bcde + 71c^2d^2) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{105c^{\frac{7}{2}} \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

$$+ \frac{12e(-be + 2cd)(ex + d)^{\frac{3}{2}} \sqrt{cx^2 + bx}}{35c^2} + \frac{2e(ex + d)^{\frac{5}{2}} \sqrt{cx^2 + bx}}{7c}$$

$$+ \frac{2e(24b^2e^2 - 71bcde + 71c^2d^2) \sqrt{ex + d} \sqrt{cx^2 + bx}}{105c^3}$$

command

`integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((139c^4d^4 - 278bc^3d^3e + 347b^2c^2d^2e^2 - 208b^3cde^3 + 48b^4e^4) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2 - bcde + b^2e^2)e^{(-2)}}{3c^2}\right), x \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{ex + d}}{\sqrt{cx^2 + bx}}, x\right)$$

22.22 Problem number 407

$$\int \frac{(d + ex)^{5/2}}{\sqrt{bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(8b^2e^2 - 23bcde + 23c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{15c^{\frac{5}{2}} \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}$$

$$- \frac{8d(-be + cd)(-be + 2cd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{15c^{\frac{5}{2}} \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

$$+ \frac{2e(ex + d)^{\frac{3}{2}} \sqrt{cx^2 + bx}}{5c} + \frac{8e(-be + 2cd) \sqrt{ex + d} \sqrt{cx^2 + bx}}{15c^2}$$

command

```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((22 c^3 d^3 - 33 b c^2 d^2 e + 27 b^2 c d e^2 - 8 b^3 e^3) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4 (c^2 d^2 - b c d e + b^2 e^2) e^{(-2)}}{3 c^2}, -\frac{4 (2 c^3 d^3 - 3 b c^2 d^2 e - 3 b^2 c d e^2 - 8 b^3 e^3)}{27 c^3} \right), x \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(e^2 x^2 + 2 d e x + d^2) \sqrt{e x + d}}{\sqrt{c x^2 + b x}}, x \right)$$

22.23 Problem number 408

$$\int \frac{(d + e x)^{3/2}}{\sqrt{b x + c x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(-be + 2cd) \text{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{3c^{\frac{3}{2}} \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\ & - \frac{2d(-be + cd) \text{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3c^{\frac{3}{2}} \sqrt{ex + d} \sqrt{cx^2 + bx}} \\ & + \frac{2e \sqrt{ex + d} \sqrt{cx^2 + bx}}{3c} \end{aligned}$$

command

```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cx^2 + bx} \sqrt{ex + d} c^2 e^2 + (5 c^2 d^2 - 5 b c d e + 2 b^2 e^2) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4 (c^2 d^2 - b c d e + b^2 e^2) e^{(-2)}}{3 c^2}, -\frac{4 (2 c^3 d^3 - 3 b c^2 d^2 e - 3 b^2 c d e^2 - 8 b^3 e^3)}{27 c^3} \right), x \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(e x + d)^{\frac{3}{2}}}{\sqrt{c x^2 + b x}}, x \right)$$

22.24 Problem number 409

$$\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b}\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{ex+d}}{\sqrt{c}\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}}$$

command

```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2cd - be)\sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2 - bcde + b^2e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d^3 - 3bc^2d^2e - 3b^2cde^2 + 2b^3e^3)e^{(-3)}}{27c^3}, \frac{(cd + (3cx + b)e)e^{(-1)}}{3c}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{ex+d}}{\sqrt{cx^2+bx}}, x\right)$$

22.25 Problem number 410

$$\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b}\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}}{\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+bx}}$$

command

```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 e^{(-\frac{1}{2})} \operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2 - bcde + b^2e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d^3 - 3bc^2d^2e - 3b^2cde^2 + 2b^3e^3)e^{(-3)}}{27c^3}, \frac{(cd + (3cx + b)e)e^{(-1)}}{3c}\right)}{\sqrt{c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{(cx^3+bdx+(cd+be)x^2), x}\right)$$

22.26 Problem number 411

$$\int \frac{1}{(d+ex)^{3/2} \sqrt{bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1+\frac{cx}{b}} \sqrt{ex+d}}{d(-be+cd) \sqrt{1+\frac{ex}{d}} \sqrt{cx^2+bx}} - \frac{2e\sqrt{cx^2+bx}}{d(-be+cd) \sqrt{ex+d}}$$

command

`integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2cd^2 - bxe^2 + (2cdx - bd)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2 - bcde + b^2e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d^3 - 3bc^2d^2e - 3b^2cde^2 + 2b^3e^3)}{27c^3}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+bx} \sqrt{ex+d}}{ce^2x^4 + bd^2x + (2cde + be^2)x^3 + (cd^2 + 2bde)x^2}, x\right)$$

22.27 Problem number 412

$$\int \frac{1}{(d+ex)^{5/2} \sqrt{bx+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(-be+2cd) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1+\frac{cx}{b}} \sqrt{ex+d}}{3d^2(-be+cd)^2 \sqrt{1+\frac{ex}{d}} \sqrt{cx^2+bx}} \\ & - \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1+\frac{cx}{b}} \sqrt{1+\frac{ex}{d}}}{3d(-be+cd) \sqrt{ex+d} \sqrt{cx^2+bx}} \\ & - \frac{2e\sqrt{cx^2+bx}}{3d(-be+cd)(ex+d)^{\frac{3}{2}}} - \frac{4e(-be+2cd) \sqrt{cx^2+bx}}{3d^2(-be+cd)^2 \sqrt{ex+d}} \end{aligned}$$

command

`integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((5c^2d^4 + 2b^2x^2e^4 - (5bcdx^2 - 4b^2dx)e^3 + (5c^2d^2x^2 - 10bcd^2x + 2b^2d^2)e^2 + 5(2c^2d^3x - bcd^3)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx} \sqrt{ex + d}}{ce^3x^5 + bd^3x + (3cde^2 + be^3)x^4 + 3(cd^2e + bde^2)x^3 + (cd^3 + 3bd^2e)x^2}, x \right)$$

22.28 Problem number 413

$$\int \frac{1}{(d + ex)^{7/2} \sqrt{bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(8b^2e^2 - 23bcde + 23c^2d^2) \operatorname{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{15d^3 (-be + cd)^3 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} - \frac{8(-be + 2cd) \operatorname{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{15d^2 (-be + cd)^2 \sqrt{ex + d} \sqrt{cx^2 + bx}} - \frac{2e \sqrt{cx^2 + bx}}{5d (-be + cd) (ex + d)^{\frac{5}{2}}} - \frac{8e(-be + 2cd) \sqrt{cx^2 + bx}}{15d^2 (-be + cd)^2 (ex + d)^{\frac{3}{2}}} - \frac{2e(8b^2e^2 - 23bcde + 23c^2d^2) \sqrt{cx^2 + bx}}{15d^3 (-be + cd)^3 \sqrt{ex + d}}$$

command

`integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((22c^3d^6 - 8b^3x^3e^6 + 3(9b^2cdx^3 - 8b^3dx^2)e^5 - 3(11bc^2d^2x^3 - 27b^2cd^2x^2 + 8b^3d^2x)e^4 + (22c^3d^3x^3 - 99bc^2d$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx} \sqrt{ex + d}}{ce^4x^6 + bd^4x + (4cde^3 + be^4)x^5 + 2(3cd^2e^2 + 2bde^3)x^4 + 2(2cd^3e + 3bd^2e^2)x^3 + (cd^4 + 4bd^3e)x^2}, x \right)$$

22.29 Problem number 414

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ex+d)^{\frac{5}{2}}(bd+(-be+2cd)x)}{b^2\sqrt{cx^2+bx}} \\ & + \frac{2(-be+2cd)(8b^2e^2-3bcde+3c^2d^2)\operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{ex+d}}{3(-b)^{\frac{3}{2}}c^{\frac{5}{2}}\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}} \\ & - \frac{4d(-be+cd)(2b^2e^2-3bcde+3c^2d^2)\operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}}{3(-b)^{\frac{3}{2}}c^{\frac{5}{2}}\sqrt{ex+d}\sqrt{cx^2+bx}} \\ & + \frac{2e(-be+2cd)(ex+d)^{\frac{3}{2}}\sqrt{cx^2+bx}}{b^2c} + \frac{4e(2b^2e^2-3bcde+3c^2d^2)\sqrt{ex+d}\sqrt{cx^2+bx}}{3b^2c^2} \end{aligned}$$

command

```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((6c^5d^4x^2+6bc^4d^4x-8(b^4cx^2+b^5x)e^4+23(b^3c^2dx^2+b^4cdx)e^3-17(b^2c^3d^2x^2+b^3c^2d^2x)e^2-12(bc^4d^3x^2\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^3x^3+3de^2x^2+3d^2ex+d^3)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^2x^4+2bcx^3+b^2x^2},x\right)$$

22.30 Problem number 415

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ex+d)^{\frac{3}{2}}(bd+(-be+2cd)x)}{b^2\sqrt{cx^2+bx}} \\ & + \frac{4(b^2e^2-bcde+c^2d^2)\operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{ex+d}}{(-b)^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}} \\ & - \frac{2d(-be+cd)(-be+2cd)\operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}}{(-b)^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{ex+d}\sqrt{cx^2+bx}} \\ & + \frac{2e(-be+2cd)\sqrt{ex+d}\sqrt{cx^2+bx}}{b^2c} \end{aligned}$$

command

```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((2c^4d^3x^2+2bc^3d^3x+2(b^3cx^2+b^4x)e^3-3(b^2c^2dx^2+b^3cdx)e^2-3(bc^3d^2x^2+b^2c^2d^2x)e)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrass}\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^2x^2+2dex+d^2)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^2x^4+2bcx^3+b^2x^2},x\right)$$

22.31 Problem number 416

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bd+(-be+2cd)x)\sqrt{ex+d}}{b^2\sqrt{cx^2+bx}} \\ & + \frac{2(-be+2cd)\operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{ex+d}}{(-b)^{\frac{3}{2}}\sqrt{c}\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}} \\ & - \frac{4d(-be+cd)\operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}}{(-b)^{\frac{3}{2}}\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+bx}} \end{aligned}$$

command

```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((2c^3d^2x^2 + 2bc^2d^2x - (b^2cx^2 + b^3x)e^2 - 2(bc^2dx^2 + b^2cdx)e) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + b^2e^2)e(-}{3c^2} \right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx} (ex + d)^{\frac{3}{2}}}{c^2x^4 + 2bcx^3 + b^2x^2}, x \right)$$

22.32 Problem number 417

$$\int \frac{\sqrt{d + ex}}{(bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(2cx + b) \sqrt{ex + d}}{b^2 \sqrt{cx^2 + bx}} + \frac{4 \text{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{(-b)^{\frac{3}{2}} \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}$$

$$- \frac{2(-be + 2cd) \text{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{(-b)^{\frac{3}{2}} \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

command

```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((2c^2dx^2 + 2bcdx - (bcx^2 + b^2x)e) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + b^2e^2)e(-2)}{3c^2}, -\frac{4(2c^3d^3 - 3bc^2d^2e - 3b^2cde^2 -}{27c^3} \right. \right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx} \sqrt{ex + d}}{c^2x^4 + 2bcx^3 + b^2x^2}, x \right)$$

22.33 Problem number 418

$$\int \frac{1}{\sqrt{d+ex} (bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(b(-be+cd) + c(-be+2cd)x) \sqrt{ex+d}}{b^2 d (-be+cd) \sqrt{cx^2+bx}} \\ & + \frac{2(-be+2cd) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x} \sqrt{1+\frac{cx}{b}} \sqrt{ex+d}}{(-b)^{\frac{3}{2}} d (-be+cd) \sqrt{1+\frac{ex}{d}} \sqrt{cx^2+bx}} \\ & - \frac{4 \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x} \sqrt{1+\frac{cx}{b}} \sqrt{1+\frac{ex}{d}}}{(-b)^{\frac{3}{2}} \sqrt{ex+d} \sqrt{cx^2+bx}} \end{aligned}$$

command

`integrate(1/(c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^3d^2x^2 + 2bc^2d^2x - (b^2cx^2 + b^3x)e^2 - 2(bc^2dx^2 + b^2cdx)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2 - bcde + b^2e^2)e^{(-2)}}{3c^2}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+bx} \sqrt{ex+d}}{c^2ex^5 + b^2dx^2 + (c^2d + 2bce)x^4 + (2bcd + b^2e)x^3}, x\right)$$

22.34 Problem number 419

$$\int \frac{1}{(d+ex)^{3/2} (bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(b(-be + cd) + c(-be + 2cd)x)}{b^2d(-be + cd)\sqrt{ex + d}\sqrt{cx^2 + bx}} \\
& + \frac{4(b^2e^2 - bcde + c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x}\sqrt{1 + \frac{cx}{b}}\sqrt{ex + d}}{(-b)^{\frac{3}{2}}d^2(-be + cd)^2\sqrt{1 + \frac{ex}{d}}\sqrt{cx^2 + bx}} \\
& - \frac{2(-be + 2cd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x}\sqrt{1 + \frac{cx}{b}}\sqrt{1 + \frac{ex}{d}}}{(-b)^{\frac{3}{2}}d(-be + cd)\sqrt{ex + d}\sqrt{cx^2 + bx}} \\
& - \frac{4e(b^2e^2 - bcde + c^2d^2)\sqrt{cx^2 + bx}}{b^2d^2(-be + cd)^2\sqrt{ex + d}}
\end{aligned}$$

command

```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((2c^4d^4x^2 + 2bc^3d^4x + 2(b^3cx^3 + b^4x^2)e^4 - (3b^2c^2dx^3 + b^3cdx^2 - 2b^4dx)e^3 - 3(bc^3d^2x^3 + 2b^2c^2d^2x^2 + b^3cd\right)}{$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx}\sqrt{ex + d}}{c^2e^2x^6 + b^2d^2x^2 + 2(c^2de + bce^2)x^5 + (c^2d^2 + 4bcde + b^2e^2)x^4 + 2(bcd^2 + b^2de)x^3}, x\right)$$

22.35 Problem number 420

$$\int \frac{1}{(d + ex)^{5/2}(bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(b(-be + cd) + c(-be + 2cd)x)}{b^2 d(-be + cd)(ex + d)^{\frac{3}{2}} \sqrt{cx^2 + bx}} \\
& + \frac{2(-be + 2cd)(8b^2e^2 - 3bcde + 3c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x}\sqrt{1 + \frac{cx}{b}}\sqrt{ex + d}}{3(-b)^{\frac{3}{2}}d^3(-be + cd)^3\sqrt{1 + \frac{ex}{d}}\sqrt{cx^2 + bx}} \\
& - \frac{4(2b^2e^2 - 3bcde + 3c^2d^2) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x}\sqrt{1 + \frac{cx}{b}}\sqrt{1 + \frac{ex}{d}}}{3(-b)^{\frac{3}{2}}d^2(-be + cd)^2\sqrt{ex + d}\sqrt{cx^2 + bx}} \\
& - \frac{4e(2b^2e^2 - 3bcde + 3c^2d^2)\sqrt{cx^2 + bx}}{3b^2d^2(-be + cd)^2(ex + d)^{\frac{3}{2}}} - \frac{2e(-be + 2cd)(8b^2e^2 - 3bcde + 3c^2d^2)\sqrt{cx^2 + bx}}{3b^2d^3(-be + cd)^3\sqrt{ex + d}}
\end{aligned}$$

command

```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((6c^5d^6x^2 + 6bc^4d^6x - 8(b^4cx^4 + b^5x^3))e^6 + (23b^3c^2dx^4 + 7b^4cdx^3 - 16b^5dx^2)e^5 - (17b^2c^3d^2x^4 - 29b^3c^2d^2x^3\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx}\sqrt{ex + d}}{c^2e^3x^7 + b^2d^3x^2 + (3c^2de^2 + 2bce^3)x^6 + (3c^2d^2e + 6bcde^2 + b^2e^3)x^5 + (c^2d^3 + 6bcd^2e + 3b^2de^2)x^4 + (2\right)$$

22.36 Problem number 421

$$\int \frac{(d + ex)^{9/2}}{(bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ex+d)^{\frac{7}{2}}(bd+(-be+2cd)x)}{3b^2(cx^2+bx)^{\frac{3}{2}}} \\ & + \frac{2(ex+d)^{\frac{3}{2}}(bcd^2(-11be+8cd)+(-be+2cd)(-3b^2e^2-8bcde+8c^2d^2)x)}{3b^4c\sqrt{cx^2+bx}} \\ & - \frac{2(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4)\operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{ex+d}}{3(-b)^{\frac{7}{2}}c^{\frac{5}{2}}\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}} \\ & + \frac{8d(-be+cd)(-be+2cd)(-b^2e^2-2bcde+2c^2d^2)\operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}}{3(-b)^{\frac{7}{2}}c^{\frac{5}{2}}\sqrt{ex+d}\sqrt{cx^2+bx}} \\ & - \frac{8e(b^3e^3-6bc^2d^2e+4c^3d^3)\sqrt{ex+d}\sqrt{cx^2+bx}}{3b^4c^2} \end{aligned}$$

command

`integrate((e*x+d)^(9/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^7d^5x^4 + 32bc^6d^5x^3 + 16b^2c^5d^5x^2 - 8(b^5c^2x^4 + 2b^6cx^3 + b^7x^2)e^5 + 11(b^4c^3dx^4 + 2b^5c^2dx^3 + b^6cdx^2)e^4 + \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}, x\right)$$

22.37 Problem number 422

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(ex+d)^{\frac{5}{2}}(bd+(-be+2cd)x)}{3b^2(cx^2+bx)^{\frac{3}{2}}} \\
+ & \frac{2(bcd^2(-9be+8cd)+(-be+2cd)(-b^2e^2-8bcde+8c^2d^2)x)\sqrt{ex+d}}{3b^4c\sqrt{cx^2+bx}} \\
& - \frac{4(-be+2cd)(-b^2e^2-4bcde+4c^2d^2)\operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{ex+d}}{3(-b)^{\frac{7}{2}}c^{\frac{3}{2}}\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}} \\
+ & \frac{2d(-be+cd)(-b^2e^2-16bcde+16c^2d^2)\operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}}{3(-b)^{\frac{7}{2}}c^{\frac{3}{2}}\sqrt{ex+d}\sqrt{cx^2+bx}}
\end{aligned}$$

command

```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^6d^4x^4 + 32bc^5d^4x^3 + 16b^2c^4d^4x^2 + 2(b^4c^2x^4 + 2b^5cx^3 + b^6x^2))e^4 + 3(b^3c^3dx^4 + 2b^4c^2dx^3 + b^5cdx^2)e^3 + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}, x\right)$$

22.38 Problem number 423

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(ex+d)^{\frac{3}{2}}(bd+(-be+2cd)x)}{3b^2(cx^2+bx)^{\frac{3}{2}}} \\
& + \frac{2(bd(-7be+8cd)+(b^2e^2-16bcde+16c^2d^2)x)\sqrt{ex+d}}{3b^4\sqrt{cx^2+bx}} \\
& - \frac{2(b^2e^2-16bcde+16c^2d^2)\operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{ex+d}}{3(-b)^{\frac{7}{2}}\sqrt{c}\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}} \\
& + \frac{16d(-be+cd)(-be+2cd)\operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}}{3(-b)^{\frac{7}{2}}\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+bx}}
\end{aligned}$$

command

```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^5d^3x^4 + 32bc^4d^3x^3 + 16b^2c^3d^3x^2 + (b^3c^2x^4 + 2b^4cx^3 + b^5x^2)e^3 + 6(b^2c^3dx^4 + 2b^3c^2dx^3 + b^4cdx^2)e^2 - 24$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^2x^2+2dex+d^2)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3x^6+3bc^2x^5+3b^2cx^4+b^3x^3},x\right)$$

22.39 Problem number 424

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(bd + (-be + 2cd)x) \sqrt{ex + d}}{3b^2 (cx^2 + bx)^{\frac{3}{2}}} \\
& + \frac{2(b(-5be + 8cd)(-be + cd) + 8c(-be + cd)(-be + 2cd)x) \sqrt{ex + d}}{3b^4 (-be + cd) \sqrt{cx^2 + bx}} \\
& - \frac{16(-be + 2cd) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{3(-b)^{\frac{7}{2}} \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\
& + \frac{2(-3be + 4cd)(-be + 4cd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3(-b)^{\frac{7}{2}} \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}}
\end{aligned}$$

command

```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((16c^4d^2x^4 + 32bc^3d^2x^3 + 16b^2c^2d^2x^2 + (b^2c^2x^4 + 2b^3cx^3 + b^4x^2)e^2 - 16(bc^3dx^4 + 2b^2c^2dx^3 + b^3cdx^2)e) \sqrt{c} e \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx} (ex + d)^{\frac{3}{2}}}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}, x\right)$$

22.40 Problem number 425

$$\int \frac{\sqrt{d + ex}}{(bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(2cx + b) \sqrt{ex + d}}{3b^2 (cx^2 + bx)^{\frac{3}{2}}} \\
& + \frac{2(b(-be + cd)(-be + 8cd) + c(b^2e^2 - 16bcde + 16c^2d^2)x) \sqrt{ex + d}}{3b^4d(-be + cd) \sqrt{cx^2 + bx}} \\
& - \frac{2(b^2e^2 - 16bcde + 16c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{3(-b)^{\frac{7}{2}} d(-be + cd) \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\
& + \frac{16(-be + 2cd) \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3(-b)^{\frac{7}{2}} \sqrt{ex + d} \sqrt{cx^2 + bx}}
\end{aligned}$$

command

```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((16c^5d^3x^4 + 32bc^4d^3x^3 + 16b^2c^3d^3x^2 + (b^3c^2x^4 + 2b^4cx^3 + b^5x^2)e^3 + 6(b^2c^3dx^4 + 2b^3c^2dx^3 + b^4cdx^2)e^2 - \dots \right)}{
}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx} \sqrt{ex + d}}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}, x\right)$$

22.41 Problem number 426

$$\int \frac{1}{\sqrt{d + ex} (bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(b(-be + cd) + c(-be + 2cd)x) \sqrt{ex + d}}{3b^2d(-be + cd)(cx^2 + bx)^{\frac{3}{2}}} \\
+ & \frac{2(b(-be + cd)(-2b^2e^2 - 5bcde + 8c^2d^2) + 2c(-be + 2cd)(-b^2e^2 - 4bcde + 4c^2d^2)x) \sqrt{ex + d}}{3b^4d^2(-be + cd)^2 \sqrt{cx^2 + bx}} \\
& \frac{4(-be + 2cd)(-b^2e^2 - 4bcde + 4c^2d^2) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x}\sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{3(-b)^{\frac{7}{2}}d^2(-be + cd)^2 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\
+ & \frac{2(-b^2e^2 - 16bcde + 16c^2d^2) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x}\sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3(-b)^{\frac{7}{2}}d(-be + cd) \sqrt{ex + d} \sqrt{cx^2 + bx}}
\end{aligned}$$

command

```
integrate(1/(c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^6d^4x^4 + 32bc^5d^4x^3 + 16b^2c^4d^4x^2 + 2(b^4c^2x^4 + 2b^5cx^3 + b^6x^2)e^4 + 3(b^3c^3dx^4 + 2b^4c^2dx^3 + b^5cdx^2)e^3 + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx} \sqrt{ex + d}}{c^3ex^7 + b^3dx^3 + (c^3d + 3bc^2e)x^6 + 3(bc^2d + b^2ce)x^5 + (3b^2cd + b^3e)x^4}, x\right)$$

22.42 Problem number 427

$$\int \frac{1}{(d + ex)^{3/2} (bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(b(-be+cd)+c(-be+2cd)x)}{3b^2d(-be+cd)(cx^2+bx)^{\frac{3}{2}}\sqrt{ex+d}} + \frac{2b(-be+cd)(-4b^2e^2-3bcde+8c^2d^2)}{3} + \frac{8c(b^3e^3-6bc^2d^2e+4c^3d^3)x}{3}}{b^4d^2(-be+cd)^2\sqrt{ex+d}\sqrt{cx^2+bx}}$$

$$2(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4)\text{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{c}\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{ex+d}$$

$$3(-b)^{\frac{7}{2}}d^3(-be+cd)^3\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}$$

$$8(-be+2cd)(-b^2e^2-2bcde+2c^2d^2)\text{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{c}\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}$$

$$3(-b)^{\frac{7}{2}}d^2(-be+cd)^2\sqrt{ex+d}\sqrt{cx^2+bx}$$

$$+ \frac{2e(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4)\sqrt{cx^2+bx}}{3b^4d^3(-be+cd)^3\sqrt{ex+d}}$$

command

```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left((16c^7d^6x^4+32bc^6d^6x^3+16b^2c^5d^6x^2-8(b^5c^2x^5+2b^6cx^4+b^7x^3)e^6+(11b^4c^3dx^5+14b^5c^2dx^4-5b^6cdx^3\right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3e^2x^8+b^3d^2x^3+(2c^3de+3bc^2e^2)x^7+(c^3d^2+6bc^2de+3b^2ce^2)x^6+(3bc^2d^2+6b^2cde+b^3e^2)x^5+(3\right.$$

22.43 Problem number 432

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}} dx$$

Optimal antiderivative

$$-2\text{EllipticE}(\sqrt{-x}, i)$$

command

```
integrate((1-x)^(1/2)/(-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\text{weierstrassPInverse}(4, 0, x) + 2\text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x}\sqrt{x+1}\sqrt{-x+1}}{x^2+x}, x\right)$$

22.44 Problem number 433

$$\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}} dx$$

Optimal antiderivative

$$-2 \operatorname{EllipticE}(\sqrt{-x}, i)$$

command

`integrate((1-x)^(1/2)/(-x^2-x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \operatorname{weierstrassPInverse}(4, 0, x) + 2 \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^2-x}\sqrt{-x+1}}{x^2+x}, x\right)$$

22.45 Problem number 657

$$\int (d+ex)^{3/2} \sqrt{a+cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e(cx^2+a)^{\frac{3}{2}}\sqrt{ex+d}}{7c} + \frac{2(24cdex-5ae^2+3cd^2)\sqrt{ex+d}\sqrt{cx^2+a}}{105ce} \\ & + \frac{4d(-29ae^2+3cd^2)\operatorname{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}}}{105e^2\sqrt{c}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} \\ & + \frac{4(-5ae^2+3cd^2)(ae^2+cd^2)\operatorname{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{105c^{\frac{3}{2}}e^2\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

`integrate((e*x+d)^(3/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 (3 c^2 d^4 + 52 a c d^2 e^2 - 15 a^2 e^4) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4 (c d^2 - 3 a e^2) e^{(-2)}}{3 c}, -\frac{8 (c d^3 + 9 a d e^2) e^{(-3)}}{27 c}, \frac{1}{3} (3 x e + d) e^{(-1)} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{c x^2 + a} (e x + d)^{\frac{3}{2}}, x \right)$$

22.46 Problem number 658

$$\int \sqrt{d + e x} \sqrt{a + c x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(e x + d)^{\frac{3}{2}} \sqrt{c x^2 + a}}{5 e} - \frac{4 d \sqrt{e x + d} \sqrt{c x^2 + a}}{15 e} \\ & + \frac{4(-3 a e^2 + c d^2) \text{EllipticE} \left(\frac{\sqrt{1 - \frac{x \sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2 a e}{-a e + d \sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{e x + d} \sqrt{1 + \frac{c x^2}{a}}}{15 e^2 \sqrt{c} \sqrt{c x^2 + a} \sqrt{\frac{(e x + d) \sqrt{c}}{e \sqrt{-a} + d \sqrt{c}}}} \\ & + \frac{4 d (a e^2 + c d^2) \text{EllipticF} \left(\frac{\sqrt{1 - \frac{x \sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2 a e}{-a e + d \sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{c x^2}{a}} \sqrt{\frac{(e x + d) \sqrt{c}}{e \sqrt{-a} + d \sqrt{c}}}}{15 e^2 \sqrt{c} \sqrt{e x + d} \sqrt{c x^2 + a}} \end{aligned}$$

command

`integrate((e*x+d)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 (c d^3 + 9 a d e^2) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4 (c d^2 - 3 a e^2) e^{(-2)}}{3 c}, -\frac{8 (c d^3 + 9 a d e^2) e^{(-3)}}{27 c}, \frac{1}{3} (3 x e + d) e^{(-1)} \right) + 6 (c d^2 e - 3 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{c x^2 + a} \sqrt{e x + d}, x \right)$$

22.47 Problem number 659

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{d+ex}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{ex+d}\sqrt{cx^2+a}}{3e} + \frac{4d \operatorname{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{c}\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}}}{3e^2\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} + \frac{4(ae^2+cd^2) \operatorname{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{3e^2\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+a}}$$

command

`integrate((c*x^2+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(6c^{\frac{3}{2}}de^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c}\right), \operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}}{\sqrt{ex+d}}, x\right)$$

22.48 Problem number 660

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{cx^2+a}}{e\sqrt{ex+d}} - \frac{4 \operatorname{EllipticE}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}, \sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{c}\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}}}{e^2\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} + \frac{4d \operatorname{EllipticF}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}, \sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{c}\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{e^2\sqrt{ex+d}\sqrt{cx^2+a}}$$

command

```
integrate((c*x^2+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(dx+e)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{-2}}{3c}, -\frac{8(cd^3+9ade^2)e^{-3}}{27c}, \frac{1}{3}(3xe+d)e^{-1}\right) + 6(xe^2+de)\right)}{e^2\sqrt{ex+d}\sqrt{cx^2+a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{e^2x^2+2dex+d^2}, x\right)$$

22.49 Problem number 661

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{cx^2+a}}{3e(ex+d)^{\frac{3}{2}}} + \frac{4cd\sqrt{cx^2+a}}{3e(ae^2+cd^2)\sqrt{ex+d}} \\ & + \frac{4c^{\frac{3}{2}}d \operatorname{EllipticE}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}, \sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{ex+d} \sqrt{1+\frac{cx^2}{a}}}{3e^2(ae^2+cd^2)\sqrt{cx^2+a} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} \\ & + \frac{4 \operatorname{EllipticF}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}, \sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{c} \sqrt{1+\frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{3e^2\sqrt{ex+d} \sqrt{cx^2+a}} \end{aligned}$$

command

`integrate((c*x^2+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2(2cd^3xe + cd^4 + 3ax^2e^4 + 6adx e^3 + (cd^2x^2 + 3ad^2)e^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ae^2)}{3c}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a} \sqrt{ex+d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

22.50 Problem number 662

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{cx^2+a}}{5e(ex+d)^{5/2}} + \frac{4cd\sqrt{cx^2+a}}{15e(ae^2+cd^2)(ex+d)^{3/2}} + \frac{4c(-3ae^2+cd^2)\sqrt{cx^2+a}}{15e(ae^2+cd^2)^2\sqrt{ex+d}} \\ & + \frac{4c^{3/2}(-3ae^2+cd^2)\text{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}}}{15e^2(ae^2+cd^2)^2\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} \\ & + \frac{4c^{3/2}d\text{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{15e^2(ae^2+cd^2)\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((c*x^2+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(2(3c^2d^5xe+c^2d^6+9acdx^3e^5+27acd^2x^2e^4+(c^2d^3x^3+27acd^3x)e^3+3(c^2d^4x^2+3acd^4)e^2)\sqrt{c}e^{\frac{1}{2}}\text{weierstrass}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{e^4x^4+4de^3x^3+6d^2e^2x^2+4d^3ex+d^4}, x\right)$$

22.51 Problem number 663

$$\int (d + ex)^{3/2} (a + cx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(28cdex - 3ae^2 + cd^2)(cx^2 + a)^{\frac{3}{2}}\sqrt{ex+d}}{231ce} + \frac{2e(cx^2 + a)^{\frac{5}{2}}\sqrt{ex+d}}{11c} \\ & + \frac{4(4c^2d^4 + 21acd^2e^2 - 15a^2e^4 - 3cde(-31ae^2 + cd^2)x)\sqrt{ex+d}\sqrt{cx^2+a}}{1155ce^3} \\ & + \frac{32d(-3ae^2 + cd^2)(9ae^2 + cd^2)\text{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{ex+d}\sqrt{1 + \frac{cx^2+a}{ex+d}}}{1155e^4\sqrt{c}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}} \\ & + \frac{8(ae^2 + cd^2)(-15a^2e^4 + 21acd^2e^2 + 4c^2d^4)\text{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{1 + \frac{cx^2+a}{ex+d}}}{1155c^{\frac{3}{2}}e^4\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x+d)^(3/2)*(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(4(4c^3d^6 + 27ac^2d^4e^2 + 234a^2cd^2e^4 - 45a^3e^6)\sqrt{c}e^{\frac{1}{2}}\text{weierstrassPInverse}\left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3 + 9ade^2)e^{(-3)}}{27c}, \dots \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((cex^3 + cdx^2 + aex + ad)\sqrt{cx^2 + a}\sqrt{ex + d}, x\right)$$

22.52 Problem number 664

$$\int \sqrt{d+ex} (a+cx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ex+d)^{\frac{3}{2}}(cx^2+a)^{\frac{3}{2}}}{9e} - \frac{4d(cx^2+a)^{\frac{3}{2}}\sqrt{ex+d}}{21e} \\ & + \frac{4(4d(3ae^2+cd^2)-3e(-7ae^2+cd^2)x)\sqrt{ex+d}\sqrt{cx^2+a}}{315e^3} \\ & + \frac{8(-21a^2e^4+15acd^2e^2+4c^2d^4)\operatorname{EllipticE}\left(\sqrt{\frac{1-\frac{x\sqrt{c}}{\sqrt{-a}}}{2}}\sqrt{2}, \sqrt{\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}}}{315e^4\sqrt{c}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} \\ & + \frac{32d(ae^2+cd^2)(3ae^2+cd^2)\operatorname{EllipticF}\left(\sqrt{\frac{1-\frac{x\sqrt{c}}{\sqrt{-a}}}{2}}\sqrt{2}, \sqrt{\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{e\sqrt{cx^2+a}}{e\sqrt{-a}+d\sqrt{c}}}}{315e^4\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((c*x^2+a)^(3/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(8(2c^2d^5 + 9acd^3e^2 + 39a^2de^4)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe+d)e^{(-1)}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((cx^2+a)^{\frac{3}{2}}\sqrt{ex+d}, x\right)$$

22.53 Problem number 665

$$\int \frac{(a + cx^2)^{3/2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{2(cx^2 + a)^{\frac{3}{2}} \sqrt{ex + d}}{7e} + \frac{4(-3cdex + 5ae^2 + 4cd^2) \sqrt{ex + d} \sqrt{cx^2 + a}}{35e^3}$$

$$+ \frac{32d(2ae^2 + cd^2) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{c} \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}}}{35e^4 \sqrt{cx^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}$$

$$+ \frac{8(ae^2 + cd^2) (5ae^2 + 4cd^2) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{ex}{e\sqrt{-a} + d\sqrt{c}}}}{35e^4 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}$$

command

```
integrate((c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(4(4c^2d^4 + 11acd^2e^2 + 15a^2e^4) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3 + 9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe + d)e^{(-1)} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(cx^2 + a)^{\frac{3}{2}}}{\sqrt{ex + d}}, x \right)$$

22.54 Problem number 666

$$\int \frac{(a + cx^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(cx^2 + a)^{\frac{3}{2}}}{e\sqrt{ex + d}} - \frac{4c(-3ex + 4d)\sqrt{ex + d}\sqrt{cx^2 + a}}{5e^3} \\ & \frac{8(3ae^2 + 4cd^2) \operatorname{EllipticE}\left(\sqrt{\frac{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}{2}}\sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{c}\sqrt{ex + d}\sqrt{1 + \frac{cx^2}{a}}}{5e^4\sqrt{cx^2 + a}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} \\ & + \frac{32d(ae^2 + cd^2) \operatorname{EllipticF}\left(\sqrt{\frac{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}{2}}\sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{c}\sqrt{1 + \frac{cx^2}{a}}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{5e^4\sqrt{ex + d}\sqrt{cx^2 + a}} \end{aligned}$$

command

```
integrate((c*x^2+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(8(2cd^3xe + 2cd^4 + 3adx^3e^3 + 3ad^2e^2)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3 + 9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + a)^{\frac{3}{2}}\sqrt{ex + d}}{e^2x^2 + 2dex + d^2}, x\right)$$

22.55 Problem number 667

$$\int \frac{(a + cx^2)^{3/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(cx^2 + a)^{\frac{3}{2}}}{3e(ex + d)^{\frac{3}{2}}} + \frac{4c(ex + 4d)\sqrt{cx^2 + a}}{3e^3\sqrt{ex + d}} \\ & + \frac{32c^{\frac{3}{2}}d \operatorname{EllipticE}\left(\sqrt{\frac{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}{2}}\sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{ex + d}\sqrt{1 + \frac{cx^2}{a}}}{3e^4\sqrt{cx^2 + a}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} \\ & + \frac{8(ae^2 + 4cd^2) \operatorname{EllipticF}\left(\sqrt{\frac{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}{2}}\sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{c}\sqrt{1 + \frac{cx^2}{a}}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{3e^4\sqrt{ex + d}\sqrt{cx^2 + a}} \end{aligned}$$

command

```
integrate((c*x^2+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(4(8cd^3xe + 4cd^4 + 3ax^2e^4 + 6adx^3e^3 + (4cd^2x^2 + 3ad^2)e^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^2 - 3ae^2)e^{(-2)}}{3c}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + a)^{\frac{3}{2}}\sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

22.56 Problem number 668

$$\int \frac{(a + cx^2)^{3/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(cx^2 + a)^{\frac{3}{2}}}{5e(ex + d)^{\frac{5}{2}}} - \frac{4c(2d(ae^2 + 2cd^2) + e(3ae^2 + 5cd^2)x) \sqrt{cx^2 + a}}{5e^3(ae^2 + cd^2)(ex + d)^{\frac{3}{2}}}$$

$$+ \frac{8c^{\frac{3}{2}}(3ae^2 + 4cd^2) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}}}{5e^4(ae^2 + cd^2) \sqrt{cx^2 + a} \sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}$$

$$+ \frac{32c^{\frac{3}{2}}d \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{5e^4\sqrt{ex + d} \sqrt{cx^2 + a}}$$

command

```
integrate((c*x^2+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(8 (6 c^2 d^5 x e + 2 c^2 d^6 + 3 a c d x^3 e^5 + 9 a c d^2 x^2 e^4 + (2 c^2 d^3 x^3 + 9 a c d^3 x) e^3 + 3 (2 c^2 d^4 x^2 + a c d^4) e^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierst} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + a)^{\frac{3}{2}} \sqrt{ex + d}}{e^4 x^4 + 4 d e^3 x^3 + 6 d^2 e^2 x^2 + 4 d^3 e x + d^4}, x\right)$$

22.57 Problem number 669

$$\int \frac{(a + cx^2)^{3/2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(cx^2 + a)^{\frac{3}{2}}}{7e(ex + d)^{\frac{7}{2}}} - \frac{4c(2d(ae^2 + 2cd^2) + e(5ae^2 + 7cd^2)x) \sqrt{cx^2 + a}}{35e^3(ae^2 + cd^2)(ex + d)^{\frac{5}{2}}} \\ & + \frac{32c^2d(2ae^2 + cd^2) \sqrt{cx^2 + a}}{35e^3(ae^2 + cd^2)^2 \sqrt{ex + d}} \\ & + \frac{32c^{\frac{5}{2}}d(2ae^2 + cd^2) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}}}{35e^4(ae^2 + cd^2)^2 \sqrt{cx^2 + a} \sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} \\ & + \frac{8c^{\frac{3}{2}}(5ae^2 + 4cd^2) \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{35e^4(ae^2 + cd^2) \sqrt{ex + d} \sqrt{cx^2 + a}} \end{aligned}$$

command

```
integrate((c*x^2+a)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(4(16c^3d^7xe + 4c^3d^8 + 15a^2cx^4e^8 + 60a^2cdx^3e^7 + (11ac^2d^2x^4 + 90a^2cd^2x^2)e^6 + 4(11ac^2d^3x^3 + 15a^2cd^3x)e^5 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + a)^{\frac{3}{2}} \sqrt{ex + d}}{e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5}, x\right)$$

22.58 Problem number 670

$$\int \sqrt{d+ex} (a+cx^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ex+d)^{\frac{3}{2}}(cx^2+a)^{\frac{5}{2}}}{13e} + \frac{20(4d(5ae^2+2cd^2)-7e(-11ae^2+cd^2)x)(cx^2+a)^{\frac{3}{2}}\sqrt{ex+d}}{9009e^3} \\ & - \frac{20d(cx^2+a)^{\frac{5}{2}}\sqrt{ex+d}}{143e} \\ & + \frac{8(d(177a^2e^4+113acd^2e^2+32c^2d^4)-3e(-77a^2e^4+27acd^2e^2+8c^2d^4)x)\sqrt{ex+d}\sqrt{cx^2+a}}{9009e^5} \\ & + \frac{16(-231a^3e^6+258a^2cd^2e^4+137ac^2d^4e^2+32c^3d^6)\operatorname{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}}{9009e^6\sqrt{c}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} \\ & + \frac{16d(ae^2+cd^2)(177a^2e^4+113acd^2e^2+32c^2d^4)\operatorname{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}}{9009e^6\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((c*x^2+a)^(5/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(8(32c^3d^7 + 161ac^2d^5e^2 + 354a^2cd^3e^4 + 993a^3de^6) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e}{27c}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((c^2x^4 + 2acx^2 + a^2)\sqrt{cx^2+a}\sqrt{ex+d}, x\right)$$

22.59 Problem number 671

$$\int \frac{(a + cx^2)^{5/2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{20(-7cde x + 9ae^2 + 8cd^2)(cx^2 + a)^{\frac{3}{2}}\sqrt{ex+d}}{693e^3} + \frac{2(cx^2 + a)^{\frac{5}{2}}\sqrt{ex+d}}{11e} \\ & + \frac{8(32c^2d^4 + 69acd^2e^2 + 45a^2e^4 - 24cde(2ae^2 + cd^2)x)\sqrt{ex+d}\sqrt{cx^2+a}}{693e^5} \\ & + \frac{16d(93a^2e^4 + 93acd^2e^2 + 32c^2d^4)\operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{c}\sqrt{ex+d}}{693e^6\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} \\ & + \frac{16(ae^2 + cd^2)(45a^2e^4 + 69acd^2e^2 + 32c^2d^4)\operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{c}\sqrt{ex+d}}{693e^6\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((c*x^2+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(8(32c^3d^6 + 117ac^2d^4e^2 + 156a^2cd^2e^4 + 135a^3e^6)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3 + 9ade^2)e^{(-2)}}{27c}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2acx^2 + a^2)\sqrt{cx^2 + a}}{\sqrt{ex + d}}, x\right)$$

22.60 Problem number 672

$$\int \frac{(a + cx^2)^{5/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(cx^2 + a)^{5/2}}{e\sqrt{ex + d}} - \frac{20c(-7ex + 8d)(cx^2 + a)^{3/2}\sqrt{ex + d}}{63e^3} \\ & - \frac{8c(d(33ae^2 + 32cd^2) - 3e(7ae^2 + 8cd^2)x)\sqrt{ex + d}\sqrt{cx^2 + a}}{63e^5} \\ & - \frac{16(21a^2e^4 + 57acd^2e^2 + 32c^2d^4)\text{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{c}\sqrt{ex + d}}{63e^6\sqrt{cx^2 + a}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} \\ & + \frac{16d(ae^2 + cd^2)(33ae^2 + 32cd^2)\text{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{c}\sqrt{1 + \frac{cx^2}{a}}}{63e^6\sqrt{ex + d}\sqrt{cx^2 + a}} \end{aligned}$$

command

`integrate((c*x^2+a)^(5/2)/(e*x+d)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(8\left(32c^2d^5xe + 32c^2d^6 + 81acd^3xe^3 + 81acd^4e^2 + 57a^2dxe^5 + 57a^2d^2e^4\right)\sqrt{c}e^{\frac{1}{2}}\text{weierstrassPInverse}\left(\frac{4(cd^2-3a)}{3}\right)\right)}{63e^6\sqrt{ex + d}\sqrt{cx^2 + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(c^2x^4 + 2acx^2 + a^2)\sqrt{cx^2 + a}\sqrt{ex + d}}{e^2x^2 + 2dex + d^2}, x\right)$$

22.61 Problem number 673

$$\int \frac{(a + cx^2)^{5/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(cx^2 + a)^{5/2}}{3e(ex + d)^{3/2}} + \frac{20c(ex + 8d)(cx^2 + a)^{3/2}}{21e^3\sqrt{ex + d}} + \frac{8c(-24cdex + 5ae^2 + 32cd^2)\sqrt{ex + d}\sqrt{cx^2 + a}}{21e^5} \\ & + \frac{16c^{3/2}d(29ae^2 + 32cd^2) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{ex + d}\sqrt{1 + \frac{cx^2}{a}}}{21e^6\sqrt{cx^2 + a}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} \\ & + \frac{16(ae^2 + cd^2)(5ae^2 + 32cd^2) \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{c}\sqrt{1 + \frac{cx^2}{a}}}{21e^6\sqrt{ex + d}\sqrt{cx^2 + a}} \end{aligned}$$

command

```
integrate((c*x^2+a)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(8(64c^2d^5xe + 32c^2d^6 + 106acd^3xe^3 + 15a^2x^2e^6 + 30a^2dxe^5 + (53acd^2x^2 + 15a^2d^2)e^4 + (32c^2d^4x^2 + 53acd^4x^2) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2acx^2 + a^2)\sqrt{cx^2 + a}\sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

22.62 Problem number 674

$$\int \frac{(a + cx^2)^{5/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{4c(3ex + 8d)(cx^2 + a)^{\frac{3}{2}}}{15e^3(ex + d)^{\frac{3}{2}}} - \frac{2(cx^2 + a)^{\frac{5}{2}}}{5e(ex + d)^{\frac{5}{2}}} - \frac{8c(8cdex + 9ae^2 + 32cd^2)\sqrt{cx^2 + a}}{15e^5\sqrt{ex + d}}$$

$$16c^{\frac{3}{2}}(9ae^2 + 32cd^2) \text{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}}$$

$$- \frac{15e^6\sqrt{cx^2 + a} \sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{15e^6\sqrt{ex + d} \sqrt{cx^2 + a}} \text{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d)}{e\sqrt{-a} + d\sqrt{c}}}$$

command

```
integrate((c*x^2+a)^(5/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(8(96c^2d^5xe + 32c^2d^6 + 33acd^3x^3e^5 + 99acd^2x^2e^4 + (32c^2d^3x^3 + 99acd^3x)e^3 + 3(32c^2d^4x^2 + 11acd^4)e^2) \sqrt{cx^2 + a} \sqrt{ex + d} \right)}{15e^6\sqrt{ex + d} \sqrt{cx^2 + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(c^2x^4 + 2acx^2 + a^2)\sqrt{cx^2 + a} \sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x \right)$$

22.63 Problem number 675

$$\int \frac{(a + cx^2)^{5/2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4c(2d(ae^2 + 4cd^2) + e(5ae^2 + 11cd^2)x)(cx^2 + a)^{\frac{5}{2}}}{21e^3(ae^2 + cd^2)(ex + d)^{\frac{5}{2}}} - \frac{2(cx^2 + a)^{\frac{5}{2}}}{7e(ex + d)^{\frac{7}{2}}} \\ & + \frac{8c^2(d(29ae^2 + 32cd^2) + e(5ae^2 + 8cd^2)x)\sqrt{cx^2 + a}}{21e^5(ae^2 + cd^2)\sqrt{ex + d}} \\ & + \frac{16c^{\frac{5}{2}}d(29ae^2 + 32cd^2) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{ex + d}\sqrt{1 + \frac{cx^2}{a}}}{21e^6(ae^2 + cd^2)\sqrt{cx^2 + a}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} \\ & + \frac{16c^{\frac{3}{2}}(5ae^2 + 32cd^2) \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{1 + \frac{cx^2}{a}}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{21e^6\sqrt{ex + d}\sqrt{cx^2 + a}} \end{aligned}$$

command

```
integrate((c*x^2+a)^(5/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(8 (128 c^3 d^7 x e + 32 c^3 d^8 + 15 a^2 c x^4 e^8 + 60 a^2 c d x^3 e^7 + (53 a c^2 d^2 x^4 + 90 a^2 c d^2 x^2) e^6 + 4 (53 a c^2 d^3 x^3 + 15 a^2 c d^3 x) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2 x^4 + 2 a c x^2 + a^2) \sqrt{c x^2 + a} \sqrt{e x + d}}{e^5 x^5 + 5 d e^4 x^4 + 10 d^2 e^3 x^3 + 10 d^3 e^2 x^2 + 5 d^4 e x + d^5}, x\right)$$

22.64 Problem number 676

$$\int \frac{(a + cx^2)^{5/2}}{(d + ex)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4c(2d(ae^2 + 4cd^2) + e(7ae^2 + 13cd^2)x)(cx^2 + a)^{\frac{5}{2}}}{63e^3(ae^2 + cd^2)(ex + d)^{\frac{7}{2}}} - \frac{2(cx^2 + a)^{\frac{5}{2}}}{9e(ex + d)^{\frac{9}{2}}} \\ & - \frac{8c^2(d(9a^2e^4 + 49acd^2e^2 + 32c^2d^4) + e(21a^2e^4 + 69acd^2e^2 + 40c^2d^4)x)\sqrt{cx^2 + a}}{63e^5(ae^2 + cd^2)^2(ex + d)^{\frac{3}{2}}} \\ & - \frac{16c^{\frac{5}{2}}(21a^2e^4 + 57acd^2e^2 + 32c^2d^4) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{ex + d} \sqrt{\frac{cx^2 + a}{ex + d}}}{63e^6(ae^2 + cd^2)^2 \sqrt{cx^2 + a} \sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} \\ & + \frac{16c^{\frac{5}{2}}d(33ae^2 + 32cd^2) \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d)}{e\sqrt{-a} + d\sqrt{c}}}}{63e^6(ae^2 + cd^2)\sqrt{ex + d} \sqrt{cx^2 + a}} \end{aligned}$$

command

```
integrate((c*x^2+a)^(5/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(8 (160c^4d^9xe + 32c^4d^{10} + 57a^2c^2dx^5e^9 + 285a^2c^2d^2x^4e^8 + 3(27ac^3d^3x^5 + 190a^2c^2d^3x^3)e^7 + 15(27ac^3d^4x^4 \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2acx^2 + a^2)\sqrt{cx^2 + a}\sqrt{ex + d}}{e^6x^6 + 6de^5x^5 + 15d^2e^4x^4 + 20d^3e^3x^3 + 15d^4e^2x^2 + 6d^5ex + d^6}, x\right)$$

22.65 Problem number 677

$$\int \frac{(d+ex)^{7/2}}{\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{24de(ex+d)^{\frac{3}{2}}\sqrt{cx^2+a}}{35c} + \frac{2e(ex+d)^{\frac{5}{2}}\sqrt{cx^2+a}}{7c} + \frac{2e(-25ae^2+71cd^2)\sqrt{ex+d}\sqrt{cx^2+a}}{105c^2} \\ & + 32d(-13ae^2+11cd^2) \operatorname{EllipticE} \left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a}\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}} \\ & + \frac{105c^{\frac{3}{2}}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{105c^{\frac{3}{2}}\sqrt{cx^2+a}} \\ & + \frac{2(-25ae^2+71cd^2)(ae^2+cd^2) \operatorname{EllipticF} \left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{e}}{105c^{\frac{5}{2}}\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x+d)^(7/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((139c^2d^4 - 554acd^2e^2 + 75a^2e^4)\sqrt{c}e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe+d)e^{(-3)} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{ex+d}}{\sqrt{cx^2+a}}, x \right)$$

22.66 Problem number 678

$$\int \frac{(d+ex)^{5/2}}{\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e(ex+d)^{\frac{3}{2}}\sqrt{cx^2+a}}{5c} + \frac{16de\sqrt{ex+d}\sqrt{cx^2+a}}{15c} \\ & 2(-9ae^2+23cd^2)\operatorname{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}} \\ & - \frac{15c^{\frac{3}{2}}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{15c^{\frac{3}{2}}\sqrt{cx^2+a}} \\ & 16d(ae^2+cd^2)\operatorname{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}} \\ & + \frac{15c^{\frac{3}{2}}\sqrt{ex+d}\sqrt{cx^2+a}}{15c^{\frac{3}{2}}\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

`integrate((e*x+d)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(2(11cd^3-21ade^2)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe+d)e^{(-1)}\right) - 3(23c\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^2x^2+2dex+d^2)\sqrt{ex+d}}{\sqrt{cx^2+a}}, x\right)$$

22.67 Problem number 679

$$\int \frac{(d+ex)^{3/2}}{\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{2e\sqrt{ex+d}\sqrt{cx^2+a}}{3c}$$

$$8d \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{ex+d} \sqrt{1 + \frac{cx^2}{a}}$$

$$+ \frac{3\sqrt{c}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{2(ae^2+cd^2)\operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{3c^{\frac{3}{2}}\sqrt{ex+d}\sqrt{cx^2+a}}$$

command

`integrate((e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(12c^{\frac{3}{2}}de^{\frac{3}{2}}\operatorname{weierstrassZeta} \left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c} \right), \operatorname{weierstrassPInverse} \left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+a}}, x \right)$$

22.68 Problem number 680

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{ex+d} \sqrt{1 + \frac{cx^2}{a}}}{\sqrt{c} \sqrt{cx^2+a} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}$$

command

```
integrate((e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \sqrt{c} d e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3 + 9ade^2)e^{(-3)}}{27c}, \frac{1}{3} (3xe + d)e^{(-1)} \right) - 3 \sqrt{c} e^{\frac{3}{2}} \operatorname{weierstrassZeta} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{ex+d}}{\sqrt{cx^2+a}}, x \right)$$

22.69 Problem number 681

$$\int \frac{1}{\sqrt{d+ex} \sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{\sqrt{c} \sqrt{ex+d} \sqrt{cx^2+a}}$$

command

`integrate(1/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2e^{(-\frac{1}{2})} \text{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe+d)e^{(-1)}\right)}{\sqrt{c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{cex^3+cdx^2+aux+ad}, x\right)$$

22.70 Problem number 682

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{\frac{2e\sqrt{cx^2+a}}{(ae^2+cd^2)\sqrt{ex+d}} + 2 \text{EllipticE}\left(\sqrt{\frac{1-\frac{x\sqrt{c}}{\sqrt{-a}}}{2}} \sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{c}\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}}}{(ae^2+cd^2)\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}$$

command

`integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(dx+e^2)\sqrt{c}e^{\frac{1}{2}} \text{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe+d)e^{(-1)}\right) - 3(xe^2+de)\sqrt{c}\right)}{\sqrt{c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{ce^2x^4+2cdex^3+2adex+ad^2+(cd^2+ae^2)x^2}, x\right)$$

22.71 Problem number 683

$$\int \frac{1}{(d+ex)^{5/2} \sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e\sqrt{cx^2+a}}{3(ae^2+cd^2)(ex+d)^{\frac{3}{2}}} - \frac{8cde\sqrt{cx^2+a}}{3(ae^2+cd^2)^2\sqrt{ex+d}} \\ & 8c^{\frac{3}{2}}d \operatorname{EllipticE} \left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{ex+d} \sqrt{1+\frac{cx^2}{a}} \\ & - \frac{3(ae^2+cd^2)^2\sqrt{cx^2+a} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{3(ae^2+cd^2)^2\sqrt{cx^2+a} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} \\ & 2 \operatorname{EllipticF} \left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{c} \sqrt{1+\frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}} \\ & + \frac{3(ae^2+cd^2)\sqrt{ex+d}\sqrt{cx^2+a}}{3(ae^2+cd^2)\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

`integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((10cd^3xe + 5cd^4 - 3ax^2e^4 - 6adx^3e + (5cd^2x^2 - 3ad^2)e^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3 + 3ade^2)x^2}{3c} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2+a} \sqrt{ex+d}}{ce^3x^5 + 3cde^2x^4 + 3ad^2ex + ad^3 + (3cd^2e + ae^3)x^3 + (cd^3 + 3ade^2)x^2}, x \right)$$

22.72 Problem number 684

$$\int \frac{1}{(d+ex)^{7/2} \sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e\sqrt{cx^2+a}}{5(ae^2+cd^2)(ex+d)^{\frac{5}{2}}} - \frac{16cde\sqrt{cx^2+a}}{15(ae^2+cd^2)^2(ex+d)^{\frac{3}{2}}} - \frac{2ce(-9ae^2+23cd^2)\sqrt{cx^2+a}}{15(ae^2+cd^2)^3\sqrt{ex+d}} \\ & 2c^{\frac{3}{2}}(-9ae^2+23cd^2) \operatorname{EllipticE} \left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a}\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}} \\ & \frac{15(ae^2+cd^2)^3\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{15(ae^2+cd^2)^2\sqrt{ex+d}\sqrt{cx^2+a}} \\ & + \frac{16c^{\frac{3}{2}}d \operatorname{EllipticF} \left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{15(ae^2+cd^2)^2\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

`integrate(1/(e*x+d)^(7/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2(33c^2d^5xe + 11c^2d^6 - 21acdx^3e^5 - 63acd^2x^2e^4 + (11c^2d^3x^3 - 63acd^3x)e^3 + 3(11c^2d^4x^2 - 7acd^4)e^2) \sqrt{c} e \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{ce^4x^6 + 4cde^3x^5 + 4ad^3ex + ad^4 + (6cd^2e^2 + ae^4)x^4 + 4(cd^3e + ade^3)x^3 + (cd^4 + 6ad^2e^2)x^2}, x \right)$$

22.73 Problem number 685

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-cdx+ae)(ex+d)^{\frac{5}{2}}}{ac\sqrt{cx^2+a}} - \frac{de(ex+d)^{\frac{3}{2}}\sqrt{cx^2+a}}{ac} - \frac{e(-5ae^2+3cd^2)\sqrt{ex+d}\sqrt{cx^2+a}}{3ac^2} \\ & \frac{d(-29ae^2+3cd^2)\operatorname{EllipticE}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}}}{3c^{\frac{3}{2}}\sqrt{-a}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} \\ & + \frac{(-5ae^2+3cd^2)(ae^2+cd^2)\operatorname{EllipticF}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(ex+d)}{e\sqrt{-a}+d\sqrt{c}}}}{3c^{\frac{5}{2}}\sqrt{-a}\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x+d)^(7/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left((3c^3d^4x^2 + 3ac^2d^4 - 15(a^2cx^2 + a^3)e^4 + 52(ac^2d^2x^2 + a^2cd^2)e^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{cx^2+a}\sqrt{ex+d}}{c^2x^4 + 2acx^2 + a^2}, x\right)$$

22.74 Problem number 686

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-cdx+ae)(ex+d)^{\frac{3}{2}}}{ac\sqrt{cx^2+a}} - \frac{de\sqrt{ex+d}\sqrt{cx^2+a}}{ac}$$

$$(-3ae^2+cd^2) \operatorname{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}}$$

$$c^{\frac{3}{2}}\sqrt{-a}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}$$

$$d(ae^2+cd^2) \operatorname{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}$$

$$+ \frac{c^{\frac{3}{2}}\sqrt{-a}\sqrt{ex+d}\sqrt{cx^2+a}}$$

command

`integrate((e*x+d)^(5/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\frac{(c^2d^3x^2+acd^3+9(acdx^2+a^2d)e^2)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe+d)\right)}{c^2x^4+2acx^2+a^2}, x\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^2x^2+2dex+d^2)\sqrt{cx^2+a}\sqrt{ex+d}}{c^2x^4+2acx^2+a^2}, x\right)$$

22.75 Problem number 687

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-cdx + ae) \sqrt{ex + d}}{ac \sqrt{cx^2 + a}} + \frac{d \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}}}{\sqrt{-a} \sqrt{c} \sqrt{cx^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} + \frac{(ae^2 + cd^2) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{c^{3/2} \sqrt{-a} \sqrt{ex + d} \sqrt{cx^2 + a}}$$

command

```
integrate((e*x+d)^(3/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left((c^2 d^2 x^2 + acd^2 + 3(acx^2 + a^2)e^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3 + 9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe + d)e^{(-3)} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + a} (ex + d)^{\frac{3}{2}}}{c^2 x^4 + 2acx^2 + a^2}, x \right)$$

22.76 Problem number 688

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{x\sqrt{ex+d}}{a\sqrt{cx^2+a}}}{\text{EllipticE}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}, \sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{ex+d} \sqrt{1+\frac{cx^2}{a}}}$$

$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{\sqrt{-a}\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+a}}$$

$$+ \frac{d \text{EllipticF}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}, \sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{1+\frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{\sqrt{-a}\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+a}}$$

command

```
integrate((e*x+d)^(1/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(3\sqrt{cx^2+a}\sqrt{ex+d}cxe + (cdx^2+ad)\sqrt{c}e^{\frac{1}{2}}\text{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c}, \frac{1}{3}\right)(3xe + \dots)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{c^2x^4+2acx^2+a^2}, x\right)$$

22.77 Problem number 689

$$\int \frac{1}{\sqrt{d+ex} (a+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(cdx+ae)\sqrt{ex+d}}{a(ae^2+cd^2)\sqrt{cx^2+a}} - \frac{d \operatorname{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{c}\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}}}{(ae^2+cd^2)\sqrt{-a}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} + \frac{\operatorname{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{\sqrt{-a}\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+a}}$$

command

`integrate(1/(c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(c^2d^2x^2 + acd^2 + 3(acx^2 + a^2)e^2)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe+d)e^{(-1)}\right)}{c^2ex^5 + c^2dx^4 + 2acex^3 + 2acdx^2 + a^2ex + a^2d}, x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{c^2ex^5 + c^2dx^4 + 2acex^3 + 2acdx^2 + a^2ex + a^2d}, x\right)$$

22.78 Problem number 690

$$\int \frac{1}{(d+ex)^{3/2} (a+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{cdx + ae}{a(ae^2 + cd^2)\sqrt{ex+d}\sqrt{cx^2+a}} + \frac{e(-3ae^2 + cd^2)\sqrt{cx^2+a}}{a(ae^2 + cd^2)^2\sqrt{ex+d}}$$

$$(-3ae^2 + cd^2) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{c} \sqrt{ex+d} \sqrt{1 + \frac{cx^2}{a}}$$

$$\frac{(ae^2 + cd^2)^2 \sqrt{-a} \sqrt{cx^2+a} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{(ae^2 + cd^2)\sqrt{-a}\sqrt{ex+d}\sqrt{cx^2+a}}$$

$$d \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{c} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}$$

$$+$$

command

```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(c^2d^4x^2 + acd^4 + 9(acdx^3 + a^2dx)e^3 + 9(acd^2x^2 + a^2d^2)e^2 + (c^2d^3x^3 + acd^3x)e)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd}{e\sqrt{-a} + d\sqrt{c}}\right)}{(ae^2 + cd^2)\sqrt{-a}\sqrt{ex+d}\sqrt{cx^2+a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{c^2e^2x^6 + 2c^2dex^5 + 4acdex^3 + 2a^2dex + (c^2d^2 + 2ace^2)x^4 + a^2d^2 + (2acd^2 + a^2e^2)x^2}, x\right)$$

22.79 Problem number 691

$$\int \frac{1}{(d+ex)^{5/2} (a+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{cdx + ae}{a(ae^2 + cd^2)(ex + d)^{\frac{3}{2}} \sqrt{cx^2 + a}} \\ & + \frac{e(-5ae^2 + 3cd^2) \sqrt{cx^2 + a}}{3a(ae^2 + cd^2)^2 (ex + d)^{\frac{3}{2}}} + \frac{cde(-29ae^2 + 3cd^2) \sqrt{cx^2 + a}}{3a(ae^2 + cd^2)^3 \sqrt{ex + d}} \\ & + c^{\frac{3}{2}} d(-29ae^2 + 3cd^2) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}} \\ & - \frac{3(ae^2 + cd^2)^3 \sqrt{-a} \sqrt{cx^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{(-5ae^2 + 3cd^2) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{c} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} \\ & + \frac{3(ae^2 + cd^2)^2 \sqrt{-a} \sqrt{ex + d} \sqrt{cx^2 + a}}{3(ae^2 + cd^2)^2 \sqrt{-a} \sqrt{ex + d} \sqrt{cx^2 + a}} \end{aligned}$$

command

```
integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$(3c^3d^6x^2 + 3ac^2d^6 - 15(a^2cx^4 + a^3x^2)e^6 - 30(a^2cdx^3 + a^3dx)e^5 + (52ac^2d^2x^4 + 37a^2cd^2x^2 - 15a^3d^2)e^4 + 104$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + a} \sqrt{ex + d}}{c^2e^3x^7 + 3c^2de^2x^6 + 3a^2d^2ex + (3c^2d^2e + 2ace^3)x^5 + a^2d^3 + (c^2d^3 + 6acde^2)x^4 + (6acd^2e + a^2e^3)x^3 + \dots} \right)$$

22.80 Problem number 692

$$\int \frac{(d+ex)^{9/2}}{(a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-cdx+ae)(ex+d)^{\frac{7}{2}}}{3ac(cx^2+a)^{\frac{3}{2}}} - \frac{(ex+d)^{\frac{3}{2}}(ae(7ae^2+cd^2)-2cd(5ae^2+2cd^2)x)}{6a^2c^2\sqrt{cx^2+a}} \\ & - \frac{2de(3ae^2+cd^2)\sqrt{ex+d}\sqrt{cx^2+a}}{3a^2c^2} \\ & + \frac{(-21a^2e^4+15acd^2e^2+4c^2d^4)\operatorname{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}}}{6(-a)^{\frac{3}{2}}c^{\frac{5}{2}}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} \\ & - \frac{2d(ae^2+cd^2)(3ae^2+cd^2)\operatorname{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{3(-a)^{\frac{3}{2}}c^{\frac{5}{2}}\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x+d)^(9/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(2(2c^4d^5x^4+4ac^3d^5x^2+2a^2c^2d^5+39(a^2c^2dx^4+2a^3cdx^2+a^4d)e^4+9(ac^3d^3x^4+2a^2c^2d^3x^2+a^3cd^3)e^2\right)\sqrt{c}}{\sqrt{cx^2+a}\sqrt{ex+d}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^4x^4+4de^3x^3+6d^2e^2x^2+4d^3ex+d^4)\sqrt{cx^2+a}\sqrt{ex+d}}{c^3x^6+3ac^2x^4+3a^2cx^2+a^3}, x\right)$$

22.81 Problem number 693

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-cdx+ae)(ex+d)^{\frac{5}{2}}}{3ac(cx^2+a)^{\frac{3}{2}}} - \frac{(ae(5ae^2+3cd^2)-2cd(3ae^2+2cd^2)x)\sqrt{ex+d}}{6a^2c^2\sqrt{cx^2+a}} \\ & + \frac{2d(2ae^2+cd^2) \operatorname{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{ex+d} \sqrt{1+\frac{cx^2}{a}}}{3(-a)^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{cx^2+a} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} \\ & + \frac{(ae^2+cd^2)(5ae^2+4cd^2) \operatorname{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{1+\frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{6(-a)^{\frac{3}{2}}c^{\frac{5}{2}}\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x+d)^(7/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left((4c^4d^4x^4 + 8ac^3d^4x^2 + 4a^2c^2d^4 + 15(a^2c^2x^4 + 2a^3cx^2 + a^4)e^4 + 11(ac^3d^2x^4 + 2a^2c^2d^2x^2 + a^3cd^2)e^2) \sqrt{c} e^{\frac{1}{2}} \sqrt{ex+d} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{cx^2+a}\sqrt{ex+d}}{c^3x^6 + 3ac^2x^4 + 3a^2cx^2 + a^3}, x\right)$$

22.82 Problem number 694

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(-cdx+ae)(ex+d)^{\frac{3}{2}}}{3ac(cx^2+a)^{\frac{3}{2}}} - \frac{(ade-(3ae^2+4cd^2)x)\sqrt{ex+d}}{6a^2c\sqrt{cx^2+a}}$$

$$+ \frac{(3ae^2+4cd^2) \operatorname{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{ex+d} \sqrt{1+\frac{cx^2}{a}}}{6(-a)^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{cx^2+a} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}$$

$$+ \frac{2d(ae^2+cd^2) \operatorname{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{1+\frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{3(-a)^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{ex+d}\sqrt{cx^2+a}}$$

command

```
integrate((e*x+d)^(5/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(2(2c^3d^3x^4+4ac^2d^3x^2+2a^2cd^3+3(ac^2dx^4+2a^2cdx^2+a^3d)e^2)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\right)\right)}{3(-a)^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{ex+d}\sqrt{cx^2+a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^2x^2+2dex+d^2)\sqrt{cx^2+a}\sqrt{ex+d}}{c^3x^6+3ac^2x^4+3a^2cx^2+a^3}, x\right)$$

22.83 Problem number 695

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-cdx+ae)\sqrt{ex+d}}{3ac(c x^2+a)^{\frac{3}{2}}} + \frac{(4cdx+ae)\sqrt{ex+d}}{6a^2c\sqrt{cx^2+a}} \\ & + \frac{2d \operatorname{EllipticE}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{ex+d} \sqrt{1+\frac{cx^2}{a}}}{3(-a)^{\frac{3}{2}}\sqrt{c}\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} \\ & + \frac{(ae^2+4cd^2) \operatorname{EllipticF}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right) \sqrt{1+\frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{6(-a)^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{ex+d}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x+d)^(3/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left((4c^3d^2x^4 + 8ac^2d^2x^2 + 4a^2cd^2 + 3(ac^2x^4 + 2a^2cx^2 + a^3)e^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3}{3c}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}(ex+d)^{\frac{3}{2}}}{c^3x^6+3ac^2x^4+3a^2cx^2+a^3}, x\right)$$

22.84 Problem number 696

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{x\sqrt{ex+d}}{3a(cx^2+a)^{\frac{3}{2}}} + \frac{(ade + (3ae^2 + 4cd^2)x)\sqrt{ex+d}}{6a^2(ae^2 + cd^2)\sqrt{cx^2+a}}$$

$$+ \frac{(3ae^2 + 4cd^2) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{ex+d} \sqrt{1 + \frac{cx^2}{a}}}{6(-a)^{\frac{3}{2}}(ae^2 + cd^2)\sqrt{c}\sqrt{cx^2+a} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}$$

$$+ \frac{2d \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{3(-a)^{\frac{3}{2}}\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+a}}$$

command

```
integrate((e*x+d)^(1/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(2c^3d^3x^4 + 4ac^2d^3x^2 + 2a^2cd^3 + 3(ac^2dx^4 + 2a^2cdx^2 + a^3d)e^2)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{c^3x^6 + 3ac^2x^4 + 3a^2cx^2 + a^3}, x\right)$$

22.85 Problem number 697

$$\int \frac{1}{\sqrt{d+ex} (a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(cdx + ae) \sqrt{ex + d}}{3a (ae^2 + cd^2) (cx^2 + a)^{\frac{3}{2}}} + \frac{(ae(5ae^2 + cd^2) + 4cd(2ae^2 + cd^2)x) \sqrt{ex + d}}{6a^2 (ae^2 + cd^2)^2 \sqrt{cx^2 + a}}$$

$$+ \frac{2d(2ae^2 + cd^2) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{c} \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}}}{3(-a)^{\frac{3}{2}} (ae^2 + cd^2)^2 \sqrt{cx^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}$$

$$+ \frac{(5ae^2 + 4cd^2) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{6(-a)^{\frac{3}{2}} (ae^2 + cd^2) \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}$$

command

```
integrate(1/(c*x^2+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(4c^4d^4x^4 + 8ac^3d^4x^2 + 4a^2c^2d^4 + 15(a^2c^2x^4 + 2a^3cx^2 + a^4)e^4 + 11(ac^3d^2x^4 + 2a^2c^2d^2x^2 + a^3cd^2)e^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weier}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + a} \sqrt{ex + d}}{c^3ex^7 + c^3dx^6 + 3ac^2ex^5 + 3ac^2dx^4 + 3a^2cex^3 + 3a^2cdx^2 + a^3ex + a^3d}, x \right)$$

22.86 Problem number 698

$$\int \frac{1}{(d+ex)^{3/2} (a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{cdx + ae}{3a(ae^2 + cd^2)(cx^2 + a)^{\frac{3}{2}} \sqrt{ex+d}} + \frac{-ae(-7ae^2 + cd^2) + 4cd(3ae^2 + cd^2)x}{6a^2(ae^2 + cd^2)^2 \sqrt{ex+d} \sqrt{cx^2+a}} \\ & + \frac{e(-21a^2e^4 + 15acd^2e^2 + 4c^2d^4) \sqrt{cx^2+a}}{6a^2(ae^2 + cd^2)^3 \sqrt{ex+d}} \\ & + \frac{(-21a^2e^4 + 15acd^2e^2 + 4c^2d^4) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{c} \sqrt{ex+d} \sqrt{1 + \frac{cx^2}{a}}}{6(-a)^{\frac{3}{2}} (ae^2 + cd^2)^3 \sqrt{cx^2+a} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} \\ & + \frac{2d(3ae^2 + cd^2) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{c} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{3(-a)^{\frac{3}{2}} (ae^2 + cd^2)^2 \sqrt{ex+d} \sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2c^4d^6x^4 + 4ac^3d^6x^2 + 2a^2c^2d^6 + 39(a^2c^2dx^5 + 2a^3cdx^3 + a^4dx)e^5 + 39(a^2c^2d^2x^4 + 2a^3cd^2x^2 + a^4d^2)e^4 + 9}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2+a} \sqrt{ex+d}}{c^3e^2x^8 + 2c^3dex^7 + 6ac^2dex^5 + 6a^2cdex^3 + (c^3d^2 + 3ac^2e^2)x^6 + 2a^3dex + a^3d^2 + 3(ac^2d^2 + a^2ce^2)x^4 -}$$

22.87 Problem number 1327

$$\int (bd + 2cdx)^{7/2} \sqrt{a + bx + cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-4ac + b^2) d(2cdx + bd)^{\frac{5}{2}} \sqrt{cx^2 + bx + a}}{77c} + \frac{(2cdx + bd)^{\frac{9}{2}} \sqrt{cx^2 + bx + a}}{11cd} \\ & - \frac{10(-4ac + b^2)^2 d^3 \sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{231c} \\ & - \frac{5(-4ac + b^2)^{\frac{13}{4}} d^{\frac{7}{2}} \text{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{231c^2 \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(b^6 - 12ab^4c + 48a^2b^2c^2 - 64a^3c^3)\sqrt{c^2d}d^3\text{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right) - 2(336c^6d^3x^4 + 672b...}{...}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((8c^3d^3x^3 + 12bc^2d^3x^2 + 6b^2cd^3x + b^3d^3)\sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}, x\right)$$

22.88 Problem number 1328

$$\int (bd + 2cdx)^{3/2} \sqrt{a + bx + cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2cdx + bd)^{\frac{5}{2}} \sqrt{cx^2 + bx + a}}{7cd} - \frac{2(-4ac + b^2) d\sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{21c} \\ & - \frac{(-4ac + b^2)^{\frac{9}{4}} d^{\frac{3}{2}} \text{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{21c^2 \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (b^4 - 8 ab^2 c + 16 a^2 c^2) \sqrt{c^2 d} \operatorname{dweierstrassPInverse}\left(\frac{b^2 - 4 ac}{c^2}, 0, \frac{2 cx + b}{2c}\right) - 2 (12 c^4 dx^2 + 12 bc^3 dx + (b^2 c^2 + 8 ac^2))}{42 c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((2 c dx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}, x\right)$$

22.89 Problem number 1329

$$\int \frac{\sqrt{a + bx + cx^2}}{\sqrt{bd + 2cdx}} dx$$

Optimal antiderivative

$$\frac{\sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{3cd} - \frac{(-4ac + b^2)^{\frac{5}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{3c^2 \sqrt{d} \sqrt{cx^2 + bx + a}}$$

command

`integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{2cdx + bd} \sqrt{cx^2 + bx + a} c^2 - \sqrt{2} \sqrt{c^2 d} (b^2 - 4 ac) \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4 ac}{c^2}, 0, \frac{2 cx + b}{2c}\right)}{6 c^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx + a}}{\sqrt{2cdx + bd}}, x\right)$$

22.90 Problem number 1330

$$\int \frac{\sqrt{a + bx + cx^2}}{(bd + 2cdx)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{cx^2 + bx + a}}{3cd(2cdx + bd)^{\frac{3}{2}}} + \frac{(-4ac + b^2)^{\frac{1}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{3c^2 d^{\frac{5}{2}} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2cdx+bd}\sqrt{cx^2+bx+a}c^2 - \sqrt{2}(4c^2x^2+4bcx+b^2)\sqrt{c^2d}\operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right)}{6(4c^5d^3x^2+4bc^4d^3x+b^2c^3d^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx+bd}\sqrt{cx^2+bx+a}}{8c^3d^3x^3+12bc^2d^3x^2+6b^2cd^3x+b^3d^3}, x\right)$$

22.91 Problem number 1331

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{cx^2+bx+a}}{7cd(2cdx+bd)^{7/2}} + \frac{2\sqrt{cx^2+bx+a}}{21c(-4ac+b^2)d^3(2cdx+bd)^{3/2}} \\ & + \frac{\operatorname{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{1/4}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{21c^2(-4ac+b^2)^{3/4}d^{9/2}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(16c^4x^4+32bc^3x^3+24b^2c^2x^2+8b^3cx+b^4)\sqrt{c^2d}\operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right) + 2(8c^4x^2+8bc^3x)}{42(16(b^2c^7-4ac^8)d^5x^4+32(b^3c^6-4abc^7)d^5x^3+24(b^4c^5-4ab^2c^6)d^5x^2+8(b^5c^4-4ab^3c^5))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx+bd}\sqrt{cx^2+bx+a}}{32c^5d^5x^5+80bc^4d^5x^4+80b^2c^3d^5x^3+40b^3c^2d^5x^2+10b^4cd^5x+b^5d^5}, x\right)$$

22.92 Problem number 1332

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{cx^2+bx+a}}{11cd(2cdx+bd)^{\frac{11}{2}}} + \frac{2\sqrt{cx^2+bx+a}}{77c(-4ac+b^2)d^3(2cdx+bd)^{\frac{7}{2}}} \\ & + \frac{10\sqrt{cx^2+bx+a}}{231c(-4ac+b^2)^2d^5(2cdx+bd)^{\frac{3}{2}}} + \frac{5 \operatorname{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{231c^2(-4ac+b^2)^{\frac{7}{4}}d^{\frac{13}{2}}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(64c^6x^6 + 192bc^5x^5 + 240b^2c^4x^4 + 160b^3c^3x^3 + 60b^4c^2x^2 + 12b^5cx + b^6)\sqrt{c^2d} \operatorname{weierstrassPInverse}\left(\frac{b^2-4c^2}{c^2}\right)}{462(64(b^4c^9 - 8ab^2c^{10} + 16a^2c^{11})d^7x^6 + 192(b^5c^8 - 8ab^3c^9 + 16a^2bc^{10})d^7x^5 + 240(b^6c^7 - 8ab^4c^8 + 16a^2b^2c^9)d^7x^4 + \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx+bd}\sqrt{cx^2+bx+a}}{128c^7d^7x^7 + 448bc^6d^7x^6 + 672b^2c^5d^7x^5 + 560b^3c^4d^7x^4 + 280b^4c^3d^7x^3 + 84b^5c^2d^7x^2 + 14b^6cd^7x + b^7d^7}\right)$$

22.93 Problem number 1333

$$\int (bd+2cdx)^{5/2} \sqrt{a+bx+cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-4ac+b^2)d(2cdx+bd)^{\frac{3}{2}}\sqrt{cx^2+bx+a}}{45c} + \frac{(2cdx+bd)^{\frac{7}{2}}\sqrt{cx^2+bx+a}}{9cd} \\ & - \frac{(-4ac+b^2)^{\frac{11}{4}}d^{\frac{5}{2}}\operatorname{EllipticE}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{15c^2\sqrt{cx^2+bx+a}} \\ & + \frac{(-4ac+b^2)^{\frac{11}{4}}d^{\frac{5}{2}}\operatorname{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{15c^2\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{2}(b^4 - 8ab^2c + 16a^2c^2)\sqrt{c^2d}d^2\text{weierstrassZeta}\left(\frac{b^2-4ac}{c^2}, 0, \text{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right)\right) + (40c^4d^2)}{45c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(4c^2d^2x^2 + 4bcd^2x + b^2d^2\right)\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}, x\right)$$

22.94 Problem number 1334

$$\int \sqrt{bd + 2cdx} \sqrt{a + bx + cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{5cd} \\ & - \frac{(-4ac + b^2)^{\frac{7}{4}} \text{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5c^2 \sqrt{cx^2 + bx + a}} \\ & + \frac{(-4ac + b^2)^{\frac{7}{4}} \text{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5c^2 \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}\sqrt{c^2d}(b^2 - 4ac)\text{weierstrassZeta}\left(\frac{b^2-4ac}{c^2}, 0, \text{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right)\right) + (2c^2x + bc)\sqrt{2cdx + bd}}{5c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}, x\right)$$

22.95 Problem number 1335

$$\int \frac{\sqrt{a + bx + cx^2}}{(bd + 2cdx)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{cx^2 + bx + a}}{cd\sqrt{2cdx + bd}} + \frac{(-4ac + b^2)^{\frac{3}{4}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{c^2 d^{\frac{3}{2}} \sqrt{cx^2 + bx + a}} \\ & - \frac{(-4ac + b^2)^{\frac{3}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{c^2 d^{\frac{3}{2}} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{c^2 d} (2cx + b) \operatorname{weierstrassZeta}\left(\frac{b^2 - 4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4ac}{c^2}, 0, \frac{2cx + b}{2c}\right)\right) + \sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{2c^3 d^2 x + bc^2 d^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{4c^2 d^2 x^2 + 4bcd^2 x + b^2 d^2}, x\right)$$

22.96 Problem number 1336

$$\int \frac{\sqrt{a + bx + cx^2}}{(bd + 2cdx)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{cx^2 + bx + a}}{5cd(2cdx + bd)^{\frac{5}{2}}} + \frac{2\sqrt{cx^2 + bx + a}}{5c(-4ac + b^2)d^3\sqrt{2cdx + bd}} \\ & - \frac{\operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5c^2(-4ac + b^2)^{\frac{1}{4}}d^{\frac{7}{2}}\sqrt{cx^2 + bx + a}} \\ & + \frac{\operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5c^2(-4ac + b^2)^{\frac{1}{4}}d^{\frac{7}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (8 c^3 x^3 + 12 b c^2 x^2 + 6 b^2 c x + b^3) \sqrt{c^2 d} \operatorname{weierstrassZeta}\left(\frac{b^2 - 4 a c}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x + b}{2 c}\right)\right) + (8 c^4 d^4 x^4 + 32 b c^3 d^4 x^3 + 24 b^2 c^2 d^4 x^2 + 8 b^3 c d^4 x + b^4 d^4)}{5 (8 (b^2 c^5 - 4 a c^6) d^4 x^3 + 12 (b^3 c^4 - 4 a b c^5) d^4 x^2 + 6 (b^4 c^3 - 4 a b^2 c^4) d^4 x + (b^5 c^2 - 4 a b^3 c^3) d^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{16 c^4 d^4 x^4 + 32 b c^3 d^4 x^3 + 24 b^2 c^2 d^4 x^2 + 8 b^3 c d^4 x + b^4 d^4}, x\right)$$

22.97 Problem number 1337

$$\int (b d + 2 c d x)^{7/2} (a + b x + c x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2 c d x + b d)^{\frac{9}{2}} (c x^2 + b x + a)^{\frac{3}{2}}}{15 c d} + \frac{(-4 a c + b^2)^2 d (2 c d x + b d)^{\frac{5}{2}} \sqrt{c x^2 + b x + a}}{385 c^2} \\ & - \frac{(-4 a c + b^2) (2 c d x + b d)^{\frac{9}{2}} \sqrt{c x^2 + b x + a}}{110 c^2 d} \\ & + \frac{(-4 a c + b^2)^3 d^3 \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{231 c^2} \\ & + \frac{(-4 a c + b^2)^{\frac{17}{4}} d^{\frac{7}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{2 c d x + b d}}{(-4 a c + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c (c x^2 + b x + a)}{-4 a c + b^2}}}{462 c^3 \sqrt{c x^2 + b x + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (b^8 - 16 a b^6 c + 96 a^2 b^4 c^2 - 256 a^3 b^2 c^3 + 256 a^4 c^4) \sqrt{c^2 d} d^3 \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x + b}{2 c}\right) + 2 (2464 c^4 d^4 x^4 + 12288 b c^3 d^4 x^3 + 24640 b^2 c^2 d^4 x^2 + 12288 b^3 c d^4 x + 2464 b^4 d^4)}{5 (8 (b^2 c^5 - 4 a c^6) d^4 x^3 + 12 (b^3 c^4 - 4 a b c^5) d^4 x^2 + 6 (b^4 c^3 - 4 a b^2 c^4) d^4 x + (b^5 c^2 - 4 a b^3 c^3) d^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((8 c^4 d^3 x^5 + 20 b c^3 d^3 x^4 + a b^3 d^3 + 2 (9 b^2 c^2 + 4 a c^3) d^3 x^3 + (7 b^3 c + 12 a b c^2) d^3 x^2 + (b^4 + 6 a b^2 c) d^3 x) \sqrt{2 c d x + b d}, x\right)$$

22.98 Problem number 1338

$$\int (bd + 2cdx)^{3/2} (a + bx + cx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2cdx + bd)^{\frac{5}{2}} (cx^2 + bx + a)^{\frac{3}{2}}}{11cd} - \frac{3(-4ac + b^2) (2cdx + bd)^{\frac{5}{2}} \sqrt{cx^2 + bx + a}}{154c^2d} \\ & + \frac{(-4ac + b^2)^2 d \sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{77c^2} \\ & + \frac{(-4ac + b^2)^{\frac{13}{4}} d^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{154c^3 \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (b^6 - 12ab^4c + 48a^2b^2c^2 - 64a^3c^3) \sqrt{c^2d} \operatorname{dweierstrassPInverse}\left(\frac{b^2 - 4ac}{c^2}, 0, \frac{2cx + b}{2c}\right) + 2(56c^6dx^4 + 112bc^5dx^3 + \dots)}{308c^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((2c^2dx^3 + 3bcdx^2 + abd + (b^2 + 2ac)dx) \sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}, x\right)$$

22.99 Problem number 1339

$$\int \frac{(a + bx + cx^2)^{3/2}}{\sqrt{bd + 2cdx}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(cx^2 + bx + a)^{\frac{3}{2}} \sqrt{2cdx + bd}}{7cd} - \frac{(-4ac + b^2) \sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{14c^2d} \\ & + \frac{(-4ac + b^2)^{\frac{9}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{14c^3 \sqrt{d} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (b^4 - 8 ab^2c + 16 a^2c^2) \sqrt{c^2d} \operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right) + 2 (2c^4x^2 + 2bc^3x - b^2c^2 + 6ac^3) \sqrt{2cdx}}{28c^4d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + bx + a)^{\frac{3}{2}}}{\sqrt{2cdx + bd}}, x\right)$$

22.100 Problem number 1340

$$\int \frac{(a + bx + cx^2)^{3/2}}{(bd + 2cdx)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(cx^2 + bx + a)^{\frac{3}{2}}}{3cd(2cdx + bd)^{\frac{3}{2}}} + \frac{\sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{6c^2d^3} \\ & - \frac{(-4ac + b^2)^{\frac{5}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{6c^3d^{\frac{5}{2}} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (b^4 - 4ab^2c + 4(b^2c^2 - 4ac^3)x^2 + 4(b^3c - 4abc^2)x) \sqrt{c^2d} \operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right) - 2(2c^4x^2 + 2bc^3x - b^2c^2 + 6ac^3) \sqrt{2cdx}}{12(4c^6d^3x^2 + 4bc^5d^3x + b^2c^4d^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx + bd} (cx^2 + bx + a)^{\frac{3}{2}}}{8c^3d^3x^3 + 12bc^2d^3x^2 + 6b^2cd^3x + b^3d^3}, x\right)$$

22.101 Problem number 1341

$$\int \frac{(a + bx + cx^2)^{3/2}}{(bd + 2cdx)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(cx^2 + bx + a)^{\frac{3}{2}}}{7cd(2cdx + bd)^{\frac{7}{2}}} - \frac{\sqrt{cx^2 + bx + a}}{14c^2d^3(2cdx + bd)^{\frac{3}{2}}} \\ & + \frac{(-4ac + b^2)^{\frac{1}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{14c^3d^{\frac{9}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (16c^4x^4 + 32bc^3x^3 + 24b^2c^2x^2 + 8b^3cx + b^4)\sqrt{c^2d} \operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right) - 2(6c^4x^2 + 6bc^3x + b^4)}{28(16c^8d^5x^4 + 32bc^7d^5x^3 + 24b^2c^6d^5x^2 + 8b^3c^5d^5x + b^4c^4d^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx + bd} (cx^2 + bx + a)^{\frac{3}{2}}}{32c^5d^5x^5 + 80bc^4d^5x^4 + 80b^2c^3d^5x^3 + 40b^3c^2d^5x^2 + 10b^4cd^5x + b^5d^5}, x\right)$$

22.102 Problem number 1342

$$\int \frac{(a + bx + cx^2)^{3/2}}{(bd + 2cdx)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(cx^2 + bx + a)^{\frac{3}{2}}}{11cd(2cdx + bd)^{\frac{11}{2}}} - \frac{3\sqrt{cx^2 + bx + a}}{154c^2d^3(2cdx + bd)^{\frac{7}{2}}} + \frac{\sqrt{cx^2 + bx + a}}{77c^2(-4ac + b^2)d^5(2cdx + bd)^{\frac{3}{2}}} \\ & + \frac{\operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{154c^3(-4ac + b^2)^{\frac{3}{4}}d^{\frac{13}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (64 c^6 x^6 + 192 b c^5 x^5 + 240 b^2 c^4 x^4 + 160 b^3 c^3 x^3 + 60 b^4 c^2 x^2 + 12 b^5 c x + b^6) \sqrt{c^2 d} \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4 a c}{c^2}, \dots\right)}{308 (64 (b^2 c^{10} - 4 a c^{11}) d^7 x^6 + 192 (b^3 c^9 - 4 a b c^{10}) d^7 x^5 + 240 (b^4 c^8 - 4 a b^2 c^9) d^7 x^4 + \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2 c d x + b d} (c x^2 + b x + a)^{\frac{3}{2}}}{128 c^7 d^7 x^7 + 448 b c^6 d^7 x^6 + 672 b^2 c^5 d^7 x^5 + 560 b^3 c^4 d^7 x^4 + 280 b^4 c^3 d^7 x^3 + 84 b^5 c^2 d^7 x^2 + 14 b^6 c d^7 x + b^7 d^7}\right)$$

22.103 Problem number 1343

$$\int \frac{(a + b x + c x^2)^{3/2}}{(b d + 2 c d x)^{17/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(c x^2 + b x + a)^{\frac{3}{2}}}{15 c d (2 c d x + b d)^{\frac{15}{2}}} - \frac{\sqrt{c x^2 + b x + a}}{110 c^2 d^3 (2 c d x + b d)^{\frac{11}{2}}} + \frac{\sqrt{c x^2 + b x + a}}{385 c^2 (-4 a c + b^2) d^5 (2 c d x + b d)^{\frac{7}{2}}} \\ & + \frac{\sqrt{c x^2 + b x + a}}{231 c^2 (-4 a c + b^2)^2 d^7 (2 c d x + b d)^{\frac{3}{2}}} + \frac{\operatorname{EllipticF}\left(\frac{\sqrt{2 c d x + b d}}{(-4 a c + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c (c x^2 + b x + a)}{-4 a c + b^2}}}{462 c^3 (-4 a c + b^2)^{\frac{7}{4}} d^{\frac{17}{2}} \sqrt{c x^2 + b x + a}} \end{aligned}$$

command

`integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(17/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (256 c^8 x^8 + 1024 b c^7 x^7 + 1792 b^2 c^6 x^6 + 1792 b^3 c^5 x^5 + 1120 b^4 c^4 x^4 + 448 b^5 c^3 x^3 + 112 b^6 c^2 x^2 + 16 b^7 c x + b^8)}{4620 (256 (b^4 c^{12} - 8 a b^2 c^{13} + 16 a^2 c^{14}) d^9 x^8 + 1024 (b^5 c^{11} - 8 a b^3 c^{12} + 16 a^2 b c^{13}) d^9 x^7 + 1792 (b^6 c^{10} - 8 a b^4 c^{11} + 16 a^2 b^2 c^{12}) d^9 x^6 + \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2 c d x + b d} (c x^2 + b x + a)^{\frac{3}{2}}}{512 c^9 d^9 x^9 + 2304 b c^8 d^9 x^8 + 4608 b^2 c^7 d^9 x^7 + 5376 b^3 c^6 d^9 x^6 + 4032 b^4 c^5 d^9 x^5 + 2016 b^5 c^4 d^9 x^4 + 672 b^6 c^3 d^9 x^3 + \dots}\right)$$

22.104 Problem number 1344

$$\int (bd + 2cdx)^{5/2} (a + bx + cx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2cdx + bd)^{\frac{7}{2}} (cx^2 + bx + a)^{\frac{3}{2}}}{13cd} + \frac{(-4ac + b^2)^2 d(2cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{195c^2} \\ & - \frac{(-4ac + b^2) (2cdx + bd)^{\frac{7}{2}} \sqrt{cx^2 + bx + a}}{78c^2d} \\ & + \frac{(-4ac + b^2)^{\frac{15}{4}} d^{\frac{5}{2}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{130c^3 \sqrt{cx^2 + bx + a}} \\ & - \frac{(-4ac + b^2)^{\frac{15}{4}} d^{\frac{5}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{130c^3 \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{2}(b^6 - 12ab^4c + 48a^2b^2c^2 - 64a^3c^3)\sqrt{c^2d}d^2 \operatorname{weierstrassZeta}\left(\frac{b^2 - 4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4ac}{c^2}, 0, \frac{2cx + a}{2c}\right)\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(4c^3d^2x^4 + 8bc^2d^2x^3 + ab^2d^2 + (5b^2c + 4ac^2)d^2x^2 + (b^3 + 4abc)d^2x\right)\sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}, x\right)$$

22.105 Problem number 1345

$$\int \sqrt{bd + 2cdx} (a + bx + cx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2cdx + bd)^{\frac{3}{2}} (cx^2 + bx + a)^{\frac{3}{2}}}{9cd} - \frac{(-4ac + b^2) (2cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{30c^2d} \\ & + \frac{(-4ac + b^2)^{\frac{11}{4}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{30c^3 \sqrt{cx^2 + bx + a}} \\ & - \frac{(-4ac + b^2)^{\frac{11}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{30c^3 \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{2}(b^4 - 8ab^2c + 16a^2c^2)\sqrt{c^2d} \operatorname{weierstrassZeta}\left(\frac{b^2-4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right)\right) - (20c^4x^3}{90c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{2cdx+bd}(cx^2+bx+a)^{\frac{3}{2}}, x\right)$$

22.106 Problem number 1346

$$\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(cx^2+bx+a)^{\frac{3}{2}}}{cd\sqrt{2cdx+bd}} + \frac{3(2cdx+bd)^{\frac{3}{2}}\sqrt{cx^2+bx+a}}{10c^2d^3} \\ & - \frac{3(-4ac+b^2)^{\frac{7}{4}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{10c^3d^{\frac{3}{2}}\sqrt{cx^2+bx+a}} \\ & + \frac{3(-4ac+b^2)^{\frac{7}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{10c^3d^{\frac{3}{2}}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{2}(b^3 - 4abc + 2(b^2c - 4ac^2)x)\sqrt{c^2d} \operatorname{weierstrassZeta}\left(\frac{b^2-4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right)\right) + (2}{10(2c^4d^2x+bc^3d^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx+bd}(cx^2+bx+a)^{\frac{3}{2}}}{4c^2d^2x^2+4bcd^2x+b^2d^2}, x\right)$$

22.107 Problem number 1347

$$\int \frac{(a + bx + cx^2)^{3/2}}{(bd + 2cdx)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(cx^2 + bx + a)^{\frac{3}{2}}}{5cd(2cdx + bd)^{\frac{5}{2}}} - \frac{3\sqrt{cx^2 + bx + a}}{10c^2d^3\sqrt{2cdx + bd}} \\ & + \frac{3(-4ac + b^2)^{\frac{3}{4}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{10c^3d^{\frac{7}{2}}\sqrt{cx^2 + bx + a}} \\ & - \frac{3(-4ac + b^2)^{\frac{3}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{10c^3d^{\frac{7}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{2}(8c^3x^3 + 12bc^2x^2 + 6b^2cx + b^3)\sqrt{c^2d} \operatorname{weierstrassZeta}\left(\frac{b^2 - 4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4ac}{c^2}, 0, \frac{2cx + b}{2c}\right)\right)}{10(8c^6d^4x^3 + 12bc^5d^4x^2 + 6b^2c^4d^4x + b^3c^3d^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx + bd}(cx^2 + bx + a)^{\frac{3}{2}}}{16c^4d^4x^4 + 32bc^3d^4x^3 + 24b^2c^2d^4x^2 + 8b^3cd^4x + b^4d^4}, x\right)$$

22.108 Problem number 1348

$$\int \frac{(a + bx + cx^2)^{3/2}}{(bd + 2cdx)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(cx^2 + bx + a)^{\frac{3}{2}}}{9cd(2cdx + bd)^{\frac{9}{2}}} - \frac{\sqrt{cx^2 + bx + a}}{30c^2d^3(2cdx + bd)^{\frac{5}{2}}} + \frac{\sqrt{cx^2 + bx + a}}{15c^2(-4ac + b^2)d^5\sqrt{2cdx + bd}} \\ & - \frac{\operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{30c^3(-4ac + b^2)^{\frac{1}{4}}d^{\frac{11}{2}}\sqrt{cx^2 + bx + a}} \\ & + \frac{\operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{30c^3(-4ac + b^2)^{\frac{1}{4}}d^{\frac{11}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{2}(32c^5x^5 + 80bc^4x^4 + 80b^2c^3x^3 + 40b^3c^2x^2 + 10b^4cx + b^5)\sqrt{c^2d}\operatorname{weierstrassZeta}\left(\frac{b^2-4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\right)}{90(32(b^2c^8 - 4ac^9)d^6x^5 + 80(b^3c^7 - 4abc^8)d^6x^4 + 80}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx+bd}(cx^2+bx+a)^{\frac{3}{2}}}{64c^6d^6x^6 + 192bc^5d^6x^5 + 240b^2c^4d^6x^4 + 160b^3c^3d^6x^3 + 60b^4c^2d^6x^2 + 12b^5cd^6x + b^6d^6}, x\right)$$

22.109 Problem number 1349

$$\int (bd + 2cdx)^{7/2} (a + bx + cx^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-4ac + b^2)(2cdx + bd)^{\frac{9}{2}}(cx^2 + bx + a)^{\frac{3}{2}}}{114c^2d} + \frac{(2cdx + bd)^{\frac{9}{2}}(cx^2 + bx + a)^{\frac{5}{2}}}{19cd} \\ & -\frac{(-4ac + b^2)^3 d(2cdx + bd)^{\frac{5}{2}} \sqrt{cx^2 + bx + a}}{2926c^3} \\ & + \frac{(-4ac + b^2)^2 (2cdx + bd)^{\frac{9}{2}} \sqrt{cx^2 + bx + a}}{836c^3d} \\ & -\frac{5(-4ac + b^2)^4 d^3 \sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{8778c^3} \\ & -\frac{5(-4ac + b^2)^{\frac{21}{4}} d^{\frac{7}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{17556c^4 \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(b^{10} - 20ab^8c + 160a^2b^6c^2 - 640a^3b^4c^3 + 1280a^4b^2c^4 - 1024a^5c^5)\sqrt{c^2d}d^3\operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((8c^5d^3x^7 + 28bc^4d^3x^6 + 2(19b^2c^3 + 8ac^4)d^3x^5 + a^2b^3d^3 + 5(5b^3c^2 + 8abc^3)d^3x^4 + 4(2b^4c + 9ab^2c^2 + \right.$$

22.110 Problem number 1350

$$\int (bd + 2cdx)^{3/2} (a + bx + cx^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(-4ac + b^2) (2cdx + bd)^{\frac{5}{2}} (cx^2 + bx + a)^{\frac{3}{2}}}{66c^2d} + \frac{(2cdx + bd)^{\frac{5}{2}} (cx^2 + bx + a)^{\frac{5}{2}}}{15cd} \\ & + \frac{(-4ac + b^2)^2 (2cdx + bd)^{\frac{5}{2}} \sqrt{cx^2 + bx + a}}{308c^3d} \\ & - \frac{(-4ac + b^2)^3 d \sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{462c^3} \\ & - \frac{(-4ac + b^2)^{\frac{17}{4}} d^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{924c^4 \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (b^8 - 16ab^6c + 96a^2b^4c^2 - 256a^3b^2c^3 + 256a^4c^4) \sqrt{c^2d} \operatorname{dweierstrassPInverse}\left(\frac{b^2 - 4ac}{c^2}, 0, \frac{2cx + b}{2c}\right) - 2 (1232$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((2c^3dx^5 + 5bc^2dx^4 + 4(b^2c + ac^2)dx^3 + a^2bd + (b^3 + 6abc)dx^2 + 2(ab^2 + a^2c)dx)\sqrt{2cdx + bd} \sqrt{cx^2 + a}\right)$$

22.111 Problem number 1351

$$\int \frac{(a + bx + cx^2)^{5/2}}{\sqrt{bd + 2cdx}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{5(-4ac + b^2) (cx^2 + bx + a)^{\frac{3}{2}} \sqrt{2cdx + bd}}{154c^2d} + \frac{(cx^2 + bx + a)^{\frac{5}{2}} \sqrt{2cdx + bd}}{11cd} \\ & + \frac{5(-4ac + b^2)^2 \sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{308c^3d} \\ & - \frac{5(-4ac + b^2)^{\frac{13}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{308c^4 \sqrt{d} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(b^6 - 12ab^4c + 48a^2b^2c^2 - 64a^3c^3)\sqrt{c^2d} \operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right) - 2(28c^6x^4 + 56bc^5x^3 + \dots)}{616c^5d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{cx^2 + bx + a}}{\sqrt{2cdx + bd}}, x\right)$$

22.112 Problem number 1352

$$\int \frac{(a + bx + cx^2)^{5/2}}{(bd + 2cdx)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(cx^2 + bx + a)^{5/2}}{3cd(2cdx + bd)^{3/2}} + \frac{5(cx^2 + bx + a)^{3/2}\sqrt{2cdx + bd}}{42c^2d^3} \\ & - \frac{5(-4ac + b^2)\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{84c^3d^3} \\ & + \frac{5(-4ac + b^2)^{9/4}\operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{1/4}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{84c^4d^{5/2}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(b^6 - 8ab^4c + 16a^2b^2c^2 + 4(b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^2 + 4(b^5c - 8ab^3c^2 + 16a^2bc^3)x)\sqrt{c^2d} \operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right) - 2(28c^6x^4 + 56bc^5x^3 + \dots)}{616c^5d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{8c^3d^3x^3 + 12bc^2d^3x^2 + 6b^2cd^3x + b^3d^3}, x\right)$$

22.113 Problem number 1353

$$\int \frac{(a + bx + cx^2)^{5/2}}{(bd + 2cdx)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5(cx^2 + bx + a)^{\frac{3}{2}}}{42c^2d^3(2cdx + bd)^{\frac{3}{2}}} - \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{7cd(2cdx + bd)^{\frac{7}{2}}} + \frac{5\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{84c^3d^5} \\ & - \frac{5(-4ac + b^2)^{\frac{5}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{84c^4d^{\frac{9}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(b^6 - 4ab^4c + 16(b^2c^4 - 4ac^5)x^4 + 32(b^3c^3 - 4abc^4)x^3 + 24(b^4c^2 - 4ab^2c^3)x^2 + 8(b^5c - 4ab^3c^2)x)\sqrt{c^2x^2 + bx + a}}{16}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{32c^5d^5x^5 + 80bc^4d^5x^4 + 80b^2c^3d^5x^3 + 40b^3c^2d^5x^2 + 10b^4cd^5x + b^5d^5}, x\right)$$

22.114 Problem number 1354

$$\int \frac{(a + bx + cx^2)^{5/2}}{(bd + 2cdx)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5(cx^2 + bx + a)^{\frac{3}{2}}}{154c^2d^3(2cdx + bd)^{\frac{7}{2}}} - \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{11cd(2cdx + bd)^{\frac{11}{2}}} - \frac{5\sqrt{cx^2 + bx + a}}{308c^3d^5(2cdx + bd)^{\frac{3}{2}}} \\ & + \frac{5(-4ac + b^2)^{\frac{1}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{308c^4d^{\frac{13}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(64c^6x^6 + 192bc^5x^5 + 240b^2c^4x^4 + 160b^3c^3x^3 + 60b^4c^2x^2 + 12b^5cx + b^6)\sqrt{c^2d}\operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}\right)}{616(64c^{11}d^7x^6 + 192bc^{10}d^7x^5 + \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{128c^7d^7x^7 + 448bc^6d^7x^6 + 672b^2c^5d^7x^5 + 560b^3c^4d^7x^4 + 280b^4c^3d^7x^3 + 84b^5c^2d^7x^2 + 14b^6cd^7x + b^7d^7}\right)$$

22.115 Problem number 1355

$$\int \frac{(a + bx + cx^2)^{5/2}}{(bd + 2cdx)^{17/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(cx^2 + bx + a)^{\frac{3}{2}}}{66c^2d^3(2cdx + bd)^{\frac{11}{2}}} - \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{15cd(2cdx + bd)^{\frac{15}{2}}} - \frac{\sqrt{cx^2 + bx + a}}{308c^3d^5(2cdx + bd)^{\frac{7}{2}}} \\ & + \frac{\sqrt{cx^2 + bx + a}}{462c^3(-4ac + b^2)d^7(2cdx + bd)^{\frac{3}{2}}} + \frac{\operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{924c^4(-4ac + b^2)^{\frac{3}{4}}d^{\frac{17}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(17/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(256c^8x^8 + 1024bc^7x^7 + 1792b^2c^6x^6 + 1792b^3c^5x^5 + 1120b^4c^4x^4 + 448b^5c^3x^3 + 112b^6c^2x^2 + 16b^7cx + b^8)}{9240(256(b^2c^{13} - 4ac^{14})d^9x^8 + 1024(b^3c^{12} - 12ac^{13})d^9x^7 + \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{512c^9d^9x^9 + 2304bc^8d^9x^8 + 4608b^2c^7d^9x^7 + 5376b^3c^6d^9x^6 + 4032b^4c^5d^9x^5 + 2016b^5c^4d^9x^4 + 672b^6c^3d^9x^3 + 192b^7c^2d^9x^2 + 48b^8cd^9x + 48b^9d^9}\right)$$

22.116 Problem number 1356

$$\int \frac{(a + bx + cx^2)^{5/2}}{(bd + 2cdx)^{21/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(cx^2 + bx + a)^{\frac{3}{2}}}{114c^2d^3(2cdx + bd)^{\frac{15}{2}}} - \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{19cd(2cdx + bd)^{\frac{19}{2}}} - \frac{\sqrt{cx^2 + bx + a}}{836c^3d^5(2cdx + bd)^{\frac{11}{2}}} \\ & + \frac{\sqrt{cx^2 + bx + a}}{2926c^3(-4ac + b^2)d^7(2cdx + bd)^{\frac{7}{2}}} + \frac{5\sqrt{cx^2 + bx + a}}{8778c^3(-4ac + b^2)^2d^9(2cdx + bd)^{\frac{3}{2}}} \\ & + \frac{5 \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{17556c^4(-4ac + b^2)^{\frac{7}{4}}d^{\frac{21}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(21/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(1024c^{10}x^{10} + 5120bc^9x^9 + 11520b^2c^8x^8 + 15360b^3c^7x^7 + 13440b^4c^6x^6 + 8064b^5c^5x^5 + 3360b^6c^4x^4 + 960b^7c^3x^3 + 240b^8c^2x^2 + 48b^9cx + 48b^{10})}{35112(1024(b^4c^{15} - 8ab^2c^{16} + 16a^2c^{17})d^{11}x^{10} + 5120(b^5c^{14} - 8ab^3c^{15} + 16a^2c^{16})d^{11}x^9 + 2560(b^6c^{13} - 8ab^4c^{14} + 16a^2c^{15})d^{11}x^8 + 1280(b^7c^{12} - 8ab^5c^{13} + 16a^2c^{14})d^{11}x^7 + 640(b^8c^{11} - 8ab^6c^{12} + 16a^2c^{13})d^{11}x^6 + 320(b^9c^{10} - 8ab^7c^{11} + 16a^2c^{12})d^{11}x^5 + 160(b^{10}c^9 - 8ab^8c^{10} + 16a^2c^{11})d^{11}x^4 + 80(b^{11}c^8 - 8ab^9c^9 + 16a^2c^{10})d^{11}x^3 + 40(b^{12}c^7 - 8ab^{10}c^8 + 16a^2c^9)d^{11}x^2 + 20(b^{13}c^6 - 8ab^{11}c^7 + 16a^2c^8)d^{11}x + 20b^{14}c^5d^{11})}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + 2048c^{11}d^{11}x^{11} + 11264bc^{10}d^{11}x^{10} + 28160b^2c^9d^{11}x^9 + 42240b^3c^8d^{11}x^8 + 42240b^4c^7d^{11}x^7 + 29568b^5c^6d^{11}x^6 + 19968b^6c^5d^{11}x^5 + 11264b^7c^4d^{11}x^4 + 4224b^8c^3d^{11}x^3 + 1024b^9c^2d^{11}x^2 + 128b^{10}cd^{11}x + 128b^{11}d^{11})^{5/2}}{(bd + 2cdx)^{21/2}}\right)$$

22.117 Problem number 1357

$$\int (bd + 2cdx)^{5/2} (a + bx + cx^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{5(-4ac + b^2)(2cdx + bd)^{\frac{7}{2}}(cx^2 + bx + a)^{\frac{3}{2}}}{442c^2d} + \frac{(2cdx + bd)^{\frac{7}{2}}(cx^2 + bx + a)^{\frac{5}{2}}}{17cd} \\
 & -\frac{(-4ac + b^2)^3 d(2cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{1326c^3} \\
 & + \frac{5(-4ac + b^2)^2 (2cdx + bd)^{\frac{7}{2}} \sqrt{cx^2 + bx + a}}{2652c^3d} \\
 & - \frac{(-4ac + b^2)^{\frac{19}{4}} d^{\frac{5}{2}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{884c^4 \sqrt{cx^2 + bx + a}} \\
 & + \frac{(-4ac + b^2)^{\frac{19}{4}} d^{\frac{5}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{884c^4 \sqrt{cx^2 + bx + a}}
 \end{aligned}$$

command

`integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2}(b^8 - 16ab^6c + 96a^2b^4c^2 - 256a^3b^2c^3 + 256a^4c^4)\sqrt{c^2d}d^2\operatorname{weierstrassZeta}\left(\frac{b^2-4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2}{c^2}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((4c^4d^2x^6 + 12bc^3d^2x^5 + (13b^2c^2 + 8ac^3)d^2x^4 + a^2b^2d^2 + 2(3b^3c + 8abc^2)d^2x^3 + (b^4 + 10ab^2c + 4a^2c^2)d^2x^2 + (4ab^3c + 4a^2b^2c^2)d^2x + 4a^3b^2c^2)d^2\right)$$

22.118 Problem number 1358

$$\int \sqrt{bd + 2cdx} (a + bx + cx^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{5(-4ac + b^2)(2cdx + bd)^{\frac{3}{2}}(cx^2 + bx + a)^{\frac{3}{2}}}{234c^2d} + \frac{(2cdx + bd)^{\frac{3}{2}}(cx^2 + bx + a)^{\frac{5}{2}}}{13cd} \\
 & + \frac{(-4ac + b^2)^2 (2cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{156c^3d} \\
 & - \frac{(-4ac + b^2)^{\frac{15}{4}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{156c^4 \sqrt{cx^2 + bx + a}} \\
 & + \frac{(-4ac + b^2)^{\frac{15}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{156c^4 \sqrt{cx^2 + bx + a}}
 \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2} (b^6 - 12ab^4c + 48a^2b^2c^2 - 64a^3c^3) \sqrt{c^2d} \operatorname{weierstrassZeta}\left(\frac{b^2-4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{4c^2d^2x^2 + 4bcd^2x + b^2d^2}, x\right)$$

22.119 Problem number 1359

$$\int \frac{(a + bx + cx^2)^{5/2}}{(bd + 2cdx)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5(2cdx + bd)^{\frac{3}{2}} (cx^2 + bx + a)^{\frac{3}{2}}}{18c^2d^3} - \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{cd\sqrt{2cdx + bd}} \\ & - \frac{(-4ac + b^2)(2cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{12c^3d^3} \\ & + \frac{(-4ac + b^2)^{\frac{11}{4}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{12c^4d^{\frac{3}{2}} \sqrt{cx^2 + bx + a}} \\ & - \frac{(-4ac + b^2)^{\frac{11}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{12c^4d^{\frac{3}{2}} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2} (b^5 - 8ab^3c + 16a^2bc^2 + 2(b^4c - 8ab^2c^2 + 16a^2c^3)x) \sqrt{c^2d} \operatorname{weierstrassZeta}\left(\frac{b^2-4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{4c^2d^2x^2 + 4bcd^2x + b^2d^2}, x\right)$$

22.120 Problem number 1360

$$\int \frac{(a + bx + cx^2)^{5/2}}{(bd + 2cdx)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(cx^2 + bx + a)^{\frac{5}{2}}}{5cd(2cdx + bd)^{\frac{5}{2}}} - \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{2c^2d^3\sqrt{2cdx + bd}} + \frac{3(2cdx + bd)^{\frac{3}{2}}\sqrt{cx^2 + bx + a}}{20c^3d^5} \\ & - \frac{3(-4ac + b^2)^{\frac{7}{4}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{20c^4d^{\frac{7}{2}}\sqrt{cx^2 + bx + a}} \\ & + \frac{3(-4ac + b^2)^{\frac{7}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{20c^4d^{\frac{7}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2}(b^5 - 4ab^3c + 8(b^2c^3 - 4ac^4)x^3 + 12(b^3c^2 - 4abc^3)x^2 + 6(b^4c - 4ab^2c^2)x)\sqrt{c^2d} \operatorname{weierstrassZeta}\left(\frac{b^2 - 4ac}{c^2}, 0\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{16c^4d^4x^4 + 32bc^3d^4x^3 + 24b^2c^2d^4x^2 + 8b^3cd^4x + b^4d^4}, x\right)$$

22.121 Problem number 1361

$$\int \frac{(a + bx + cx^2)^{5/2}}{(bd + 2cdx)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(cx^2 + bx + a)^{\frac{3}{2}}}{18c^2d^3(2cdx + bd)^{\frac{5}{2}}} - \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{9cd(2cdx + bd)^{\frac{9}{2}}} - \frac{\sqrt{cx^2 + bx + a}}{12c^3d^5\sqrt{2cdx + bd}} \\ & + \frac{(-4ac + b^2)^{\frac{3}{4}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{12c^4d^{\frac{11}{2}}\sqrt{cx^2 + bx + a}} \\ & - \frac{(-4ac + b^2)^{\frac{3}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{12c^4d^{\frac{11}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{2}\left(32c^5x^5 + 80bc^4x^4 + 80b^2c^3x^3 + 40b^3c^2x^2 + 10b^4cx + b^5\right)\sqrt{c^2d}\operatorname{weierstrassZeta}\left(\frac{b^2-4ac}{c^2}, 0, \operatorname{weierstrassPI}\right)}{36\left(32c^9d^6x^5 + 80b\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{64c^6d^6x^6 + 192bc^5d^6x^5 + 240b^2c^4d^6x^4 + 160b^3c^3d^6x^3 + 60b^4c^2d^6x^2 + 12b^5cd^6x + b^6d^6}, x\right)$$

22.122 Problem number 1362

$$\int \frac{(a + bx + cx^2)^{5/2}}{(bd + 2cdx)^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5(c^2x^2 + bx + a)^{\frac{3}{2}}}{234c^2d^3(2cdx + bd)^{\frac{9}{2}}} - \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{13cd(2cdx + bd)^{\frac{13}{2}}} - \frac{\sqrt{cx^2 + bx + a}}{156c^3d^5(2cdx + bd)^{\frac{5}{2}}} \\ & + \frac{\sqrt{cx^2 + bx + a}}{78c^3(-4ac + b^2)d^7\sqrt{2cdx + bd}} - \frac{\operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{156c^4(-4ac + b^2)^{\frac{1}{4}}d^{\frac{15}{2}}\sqrt{cx^2 + bx + a}} \\ & + \frac{\operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{156c^4(-4ac + b^2)^{\frac{1}{4}}d^{\frac{15}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(15/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{2}\left(128c^7x^7 + 448bc^6x^6 + 672b^2c^5x^5 + 560b^3c^4x^4 + 280b^4c^3x^3 + 84b^5c^2x^2 + 14b^6cx + b^7\right)\sqrt{c^2d}\operatorname{weierstrassZeta}}{468\left(128(b^2c^{11} - 4ac^{12})\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{256c^8d^8x^8 + 1024bc^7d^8x^7 + 1792b^2c^6d^8x^6 + 1792b^3c^5d^8x^5 + 1120b^4c^4d^8x^4 + 448b^5c^3d^8x^3 + 112b^6c^2d^8x^2 + 112b^7cd^8x + b^8d^8}, x\right)$$

22.123 Problem number 1363

$$\int \frac{(bd + 2cdx)^{7/2}}{\sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{4d(2cdx + bd)^{\frac{5}{2}} \sqrt{cx^2 + bx + a}}{7} + \frac{20(-4ac + b^2) d^3 \sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{21} \\ + \frac{10(-4ac + b^2)^{\frac{9}{4}} d^{\frac{7}{2}} \text{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{21c \sqrt{cx^2 + bx + a}}$$

command

```
integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (b^4 - 8 ab^2c + 16 a^2c^2) \sqrt{c^2d} d^3 \text{weierstrassPInverse}\left(\frac{b^2 - 4ac}{c^2}, 0, \frac{2cx + b}{2c}\right) + 16 (3c^4d^3x^2 + 3bc^3d^3x + (2b^2c^2 - 5c^2d^2)) \sqrt{cx^2 + bx + a}}{21c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(8c^3d^3x^3 + 12bc^2d^3x^2 + 6b^2cd^3x + b^3d^3) \sqrt{2cdx + bd}}{\sqrt{cx^2 + bx + a}}, x\right)$$

22.124 Problem number 1364

$$\int \frac{(bd + 2cdx)^{3/2}}{\sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{4d\sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{3} \\ + \frac{2(-4ac + b^2)^{\frac{5}{4}} d^{\frac{3}{2}} \text{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{3c \sqrt{cx^2 + bx + a}}$$

command

```
integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a} c^2 d + \sqrt{2} \sqrt{c^2 d} (b^2 - 4 a c) d \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x + b}{2 c}\right)}{3 c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(2 c d x + b d)^{\frac{3}{2}}}{\sqrt{c x^2 + b x + a}}, x\right)$$

22.125 Problem number 1365

$$\int \frac{1}{\sqrt{b d + 2 c d x} \sqrt{a + b x + c x^2}} dx$$

Optimal antiderivative

$$\frac{2(-4 a c + b^2)^{\frac{1}{4}} \operatorname{EllipticF}\left(\frac{\sqrt{2 c d x + b d}}{(-4 a c + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(c x^2 + b x + a)}{-4 a c + b^2}}}{c \sqrt{d} \sqrt{c x^2 + b x + a}}$$

command

`integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{c^2 d} \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x + b}{2 c}\right)}{c^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{2 c^2 d x^3 + 3 b c d x^2 + a b d + (b^2 + 2 a c) d x}, x\right)$$

22.126 Problem number 1366

$$\int \frac{1}{(b d + 2 c d x)^{5/2} \sqrt{a + b x + c x^2}} dx$$

Optimal antiderivative

$$\frac{4\sqrt{cx^2+bx+a}}{3(-4ac+b^2)d(2cdx+bd)^{\frac{3}{2}}} + \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{3c(-4ac+b^2)^{\frac{3}{4}}d^{\frac{5}{2}}\sqrt{cx^2+bx+a}}$$

command

`integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4\sqrt{2cdx+bd}\sqrt{cx^2+bx+a}c^2 + \sqrt{2}(4c^2x^2+4bcx+b^2)\sqrt{c^2d} \operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right)}{3(4(b^2c^4-4ac^5)d^3x^2 + 4(b^3c^3-4abc^4)d^3x + (b^4c^2-4ab^2c^3)d^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx+bd}\sqrt{cx^2+bx+a}}{8c^4d^3x^5 + 20bc^3d^3x^4 + ab^3d^3 + 2(9b^2c^2+4ac^3)d^3x^3 + (7b^3c+12abc^2)d^3x^2 + (b^4+6ab^2c)d^3x}, x\right)$$

22.127 Problem number 1367

$$\int \frac{1}{(bd+2cdx)^{9/2}\sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{4\sqrt{cx^2+bx+a}}{7(-4ac+b^2)d(2cdx+bd)^{\frac{7}{2}}} + \frac{20\sqrt{cx^2+bx+a}}{21(-4ac+b^2)^2d^3(2cdx+bd)^{\frac{3}{2}}} + \frac{10 \operatorname{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{21c(-4ac+b^2)^{\frac{7}{4}}d^{\frac{9}{2}}\sqrt{cx^2+bx+a}}$$

command

`integrate(1/(2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(16c^4x^4+32bc^3x^3+24b^2c^2x^2+8b^3cx+b^4)\sqrt{c^2d} \operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right) + 16(5c^4d^5x^7 + 112bc^5d^5x^6 + ab^5d^5 + 32(5b^2c^4+ac^5)d^5x^5 + 40(3b^3c^3+2abc^4)d^5x^4 + 10(5b^4c^2+8ab^2c^3)d^5x^3 + 21(16(b^4c^6-8ab^2c^7+16a^2c^8)d^5x^4 + 32(b^5c^5-8ab^3c^6+16a^2bc^7)d^5x^3 + 24(b^6c^4-8ab^4c^5+16a^2b^2c^6)d^5x^2 + 8(b^7c^3-8ab^5c^4+16a^2b^3c^5)d^5x + 8b^8c^2)}{21(16(b^4c^6-8ab^2c^7+16a^2c^8)d^5x^4 + 32(b^5c^5-8ab^3c^6+16a^2bc^7)d^5x^3 + 24(b^6c^4-8ab^4c^5+16a^2b^2c^6)d^5x^2 + 8(b^7c^3-8ab^5c^4+16a^2b^3c^5)d^5x + 8b^8c^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx+bd}\sqrt{cx^2+bx+a}}{32c^6d^5x^7 + 112bc^5d^5x^6 + ab^5d^5 + 32(5b^2c^4+ac^5)d^5x^5 + 40(3b^3c^3+2abc^4)d^5x^4 + 10(5b^4c^2+8ab^2c^3)d^5x^3 + 21(16(b^4c^6-8ab^2c^7+16a^2c^8)d^5x^4 + 32(b^5c^5-8ab^3c^6+16a^2bc^7)d^5x^3 + 24(b^6c^4-8ab^4c^5+16a^2b^2c^6)d^5x^2 + 8(b^7c^3-8ab^5c^4+16a^2b^3c^5)d^5x + 8b^8c^2}$$

22.128 Problem number 1368

$$\int \frac{(bd + 2cdx)^{9/2}}{\sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{28(-4ac + b^2) d^3 (2cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{45} + \frac{4d(2cdx + bd)^{\frac{7}{2}} \sqrt{cx^2 + bx + a}}{9} \\ & + \frac{14(-4ac + b^2)^{\frac{11}{4}} d^{\frac{9}{2}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{15c \sqrt{cx^2 + bx + a}} \\ & - \frac{14(-4ac + b^2)^{\frac{11}{4}} d^{\frac{9}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{15c \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21 \sqrt{2} (b^4 - 8ab^2c + 16a^2c^2) \sqrt{c^2d} d^4 \operatorname{weierstrassZeta}\left(\frac{b^2 - 4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4ac}{c^2}, 0, \frac{2cx + b}{2c}\right)\right) - 8 \right)}{45c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(16c^4d^4x^4 + 32bc^3d^4x^3 + 24b^2c^2d^4x^2 + 8b^3cd^4x + b^4d^4)\sqrt{2cdx + bd}}{\sqrt{cx^2 + bx + a}}, x\right)$$

22.129 Problem number 1369

$$\int \frac{(bd + 2cdx)^{5/2}}{\sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4d(2cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{5} \\ & + \frac{6(-4ac + b^2)^{\frac{7}{4}} d^{\frac{5}{2}} \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5c \sqrt{cx^2 + bx + a}} \\ & - \frac{6(-4ac + b^2)^{\frac{7}{4}} d^{\frac{5}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5c \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \sqrt{2} \sqrt{c^2 d} (b^2 - 4ac) d^2 \text{weierstrassZeta} \left(\frac{b^2 - 4ac}{c^2}, 0, \text{weierstrassPInverse} \left(\frac{b^2 - 4ac}{c^2}, 0, \frac{2cx+b}{2c} \right) \right) - 2 (2c^2 d^2 x + bcd) \right)}{5c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(4c^2 d^2 x^2 + 4bcd^2 x + b^2 d^2) \sqrt{2cdx + bd}}{\sqrt{cx^2 + bx + a}}, x \right)$$

22.130 Problem number 1370

$$\int \frac{\sqrt{bd + 2cdx}}{\sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(-4ac + b^2)^{\frac{3}{4}} \text{EllipticE} \left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{c \sqrt{cx^2 + bx + a}} - \frac{2(-4ac + b^2)^{\frac{3}{4}} \text{EllipticF} \left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{c \sqrt{cx^2 + bx + a}}$$

command

```
integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{2} \sqrt{c^2 d} \text{weierstrassZeta} \left(\frac{b^2 - 4ac}{c^2}, 0, \text{weierstrassPInverse} \left(\frac{b^2 - 4ac}{c^2}, 0, \frac{2cx+b}{2c} \right) \right)}{c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{2cdx + bd}}{\sqrt{cx^2 + bx + a}}, x \right)$$

22.131 Problem number 1371

$$\int \frac{1}{(bd + 2cdx)^{3/2} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{4\sqrt{cx^2 + bx + a}}{(-4ac + b^2)d\sqrt{2cdx + bd}} - \frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{1/4}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{c(-4ac + b^2)^{1/4}d^{3/2}\sqrt{cx^2 + bx + a}}$$

$$+ \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{1/4}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{c(-4ac + b^2)^{1/4}d^{3/2}\sqrt{cx^2 + bx + a}}$$

command

`integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{2}\sqrt{c^2d}(2cx + b)\operatorname{weierstrassZeta}\left(\frac{b^2 - 4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4ac}{c^2}, 0, \frac{2cx + b}{2c}\right)\right) + 2\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}\right)}{2(b^2c^2 - 4ac^3)d^2x + (b^3c - 4abc^2)d^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{4c^3d^2x^4 + 8bc^2d^2x^3 + ab^2d^2 + (5b^2c + 4ac^2)d^2x^2 + (b^3 + 4abc)d^2x}, x\right)$$

22.132 Problem number 1372

$$\int \frac{1}{(bd + 2cdx)^{7/2} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{4\sqrt{cx^2 + bx + a}}{5(-4ac + b^2)d(2cdx + bd)^{5/2}} + \frac{12\sqrt{cx^2 + bx + a}}{5(-4ac + b^2)^2d^3\sqrt{2cdx + bd}}$$

$$- \frac{6 \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{1/4}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5c(-4ac + b^2)^{5/4}d^{7/2}\sqrt{cx^2 + bx + a}}$$

$$+ \frac{6 \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{1/4}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5c(-4ac + b^2)^{5/4}d^{7/2}\sqrt{cx^2 + bx + a}}$$

command

```
integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \sqrt{2} (8 c^3 x^3 + 12 b c^2 x^2 + 6 b^2 c x + b^3) \sqrt{c^2 d} \operatorname{weierstrassZeta} \left(\frac{b^2 - 4 a c}{c^2}, 0, \operatorname{weierstrassPInverse} \left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x + b}{2 c} \right) \right) \right)}{5 (8 (b^4 c^4 - 8 a b^2 c^5 + 16 a^2 c^6) d^4 x^3 + 12 (b^5 c^3 - 8 a b^3 c^4 + 16 a^2 b c^5) d^4 x^2 + 6 (b^6 c^2 - 8 a b^4 c^3 + 16 a^2 b^2 c^4) d^4 x + 5 (b^7 c - 8 a b^5 c^2 + 16 a^2 b^3 c^3) d^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{16 c^5 d^4 x^6 + 48 b c^4 d^4 x^5 + a b^4 d^4 + 8 (7 b^2 c^3 + 2 a c^4) d^4 x^4 + 32 (b^3 c^2 + a b c^3) d^4 x^3 + 3 (3 b^4 c + 8 a b^2 c^2) d^4 x^2 + 5 (b^5 c - 8 a b^3 c^2 + 16 a^2 b c^3) d^4}, x \right)$$

22.133 Problem number 1373

$$\int \frac{(3 - 2x)^{3/2}}{\sqrt{1 - 3x + x^2}} dx$$

Optimal antiderivative

$$-\frac{2 \cdot 5^{\frac{3}{4}} \operatorname{EllipticF} \left(\frac{\sqrt{3 - 2x}}{5}, \frac{5^{\frac{3}{4}}}{5}, i \right) \sqrt{-x^2 + 3x - 1}}{3 \sqrt{x^2 - 3x + 1}} - \frac{4 \sqrt{3 - 2x} \sqrt{x^2 - 3x + 1}}{3}$$

command

```
integrate((3-2*x)^(3/2)/(x^2-3*x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{4}{3} \sqrt{x^2 - 3x + 1} \sqrt{-2x + 3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(-2x + 3)^{\frac{3}{2}}}{\sqrt{x^2 - 3x + 1}}, x \right)$$

22.134 Problem number 1375

$$\int \frac{1}{(3-2x)^{5/2} \sqrt{1-3x+x^2}} dx$$

Optimal antiderivative

$$-\frac{2 \cdot 5^{3/4} \operatorname{EllipticF}\left(\frac{\sqrt{3-2x}}{5}, i\right) \sqrt{-x^2+3x-1}}{75 \sqrt{x^2-3x+1}} - \frac{4 \sqrt{x^2-3x+1}}{15 (3-2x)^{3/2}}$$

command

`integrate(1/(3-2*x)^(5/2)/(x^2-3*x+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{4 \sqrt{x^2-3x+1} \sqrt{-2x+3}}{15 (4x^2-12x+9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{x^2-3x+1} \sqrt{-2x+3}}{8x^5-60x^4+170x^3-225x^2+135x-27}, x\right)$$

22.135 Problem number 1376

$$\int \frac{(3-2x)^{5/2}}{\sqrt{1-3x+x^2}} dx$$

Optimal antiderivative

$$-\frac{6 \cdot 5^{1/4} \operatorname{EllipticE}\left(\frac{\sqrt{3-2x}}{5}, i\right) \sqrt{-x^2+3x-1}}{\sqrt{x^2-3x+1}} + \frac{6 \cdot 5^{1/4} \operatorname{EllipticF}\left(\frac{\sqrt{3-2x}}{5}, i\right) \sqrt{-x^2+3x-1}}{\sqrt{x^2-3x+1}} - \frac{4(3-2x)^{3/2} \sqrt{x^2-3x+1}}{5}$$

command

`integrate((3-2*x)^(5/2)/(x^2-3*x+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4}{5} \sqrt{x^2-3x+1} (2x-3) \sqrt{-2x+3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(4x^2-12x+9) \sqrt{-2x+3}}{\sqrt{x^2-3x+1}}, x\right)$$

22.136 Problem number 1378

$$\int \frac{1}{(3-2x)^{3/2} \sqrt{1-3x+x^2}} dx$$

Optimal antiderivative

$$\frac{2 \cdot 5^{1/4} \operatorname{EllipticE}\left(\frac{\sqrt{3-2x}}{5} \cdot 5^{3/4}, i\right) \sqrt{-x^2+3x-1}}{5 \sqrt{x^2-3x+1}} - \frac{2 \cdot 5^{1/4} \operatorname{EllipticF}\left(\frac{\sqrt{3-2x}}{5} \cdot 5^{3/4}, i\right) \sqrt{-x^2+3x-1}}{5 \sqrt{x^2-3x+1}} - \frac{4 \sqrt{x^2-3x+1}}{5 \sqrt{3-2x}}$$

command

`integrate(1/(3-2*x)^(3/2)/(x^2-3*x+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \sqrt{x^2-3x+1} \sqrt{-2x+3}}{5(2x-3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^2-3x+1} \sqrt{-2x+3}}{4x^4-24x^3+49x^2-39x+9}, x\right)$$

22.137 Problem number 1379

$$\int \frac{(bd+2cdx)^{11/2}}{(a+bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2d(2cdx+bd)^{9/2}}{\sqrt{cx^2+bx+a}} + \frac{72cd^3(2cdx+bd)^{5/2} \sqrt{cx^2+bx+a}}{7} \\ & + \frac{120c(-4ac+b^2)d^5 \sqrt{2cdx+bd} \sqrt{cx^2+bx+a}}{7} \\ & + \frac{60(-4ac+b^2)^{9/4} d^{11/2} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{1/4} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{7 \sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15 \sqrt{2} \left((b^4 c - 8 a b^2 c^2 + 16 a^2 c^3) d^5 x^2 + (b^5 - 8 a b^3 c + 16 a^2 b c^2) d^5 x + (a b^4 - 8 a^2 b^2 c + 16 a^3 c^2) d^5 \right) \sqrt{c^2 d} \operatorname{weierstrassPInverse} \left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x + b}{2 c} \right) \right) + (8)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(32 c^5 d^5 x^5 + 80 b c^4 d^5 x^4 + 80 b^2 c^3 d^5 x^3 + 40 b^3 c^2 d^5 x^2 + 10 b^4 c d^5 x + b^5 d^5) \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{c^2 x^4 + 2 b c x^3 + 2 a b x + (b^2 + 2 a c) x^2 + a^2}, x \right)$$

22.138 Problem number 1380

$$\int \frac{(b d + 2 c d x)^{7/2}}{(a + b x + c x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 d (2 c d x + b d)^{5/2}}{\sqrt{c x^2 + b x + a}} + \frac{40 c d^3 \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{3} \\ & + \frac{20 (-4 a c + b^2)^{5/4} d^{7/2} \operatorname{EllipticF} \left(\frac{\sqrt{2 c d x + b d}}{(-4 a c + b^2)^{1/4} \sqrt{d}}, i \right) \sqrt{-\frac{c (c x^2 + b x + a)}{-4 a c + b^2}}}{3 \sqrt{c x^2 + b x + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \sqrt{2} \left((b^2 c - 4 a c^2) d^3 x^2 + (b^3 - 4 a b c) d^3 x + (a b^2 - 4 a^2 c) d^3 \right) \sqrt{c^2 d} \operatorname{weierstrassPInverse} \left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x + b}{2 c} \right) \right) + (8)$$

$$3 (c^2 x^2 + b c x + a c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(8 c^3 d^3 x^3 + 12 b c^2 d^3 x^2 + 6 b^2 c d^3 x + b^3 d^3) \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{c^2 x^4 + 2 b c x^3 + 2 a b x + (b^2 + 2 a c) x^2 + a^2}, x \right)$$

22.139 Problem number 1381

$$\int \frac{(bd + 2cdx)^{3/2}}{(a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2d\sqrt{2cdx+bd}}{\sqrt{cx^2+bx+a}} + \frac{4(-4ac+b^2)^{\frac{1}{4}} d^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{\sqrt{cx^2+bx+a}}$$

command

`integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{2cdx+bd}\sqrt{cx^2+bx+a}cd - \sqrt{2}(cdx^2+bdx+ad)\sqrt{c^2d}\operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right)\right)}{c^2x^2+bcx+a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(2cdx+bd)^{\frac{3}{2}}\sqrt{cx^2+bx+a}}{c^2x^4+2bcx^3+2abx+(b^2+2ac)x^2+a^2}, x\right)$$

22.140 Problem number 1382

$$\int \frac{1}{\sqrt{bd+2cdx}(a+bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{2cdx+bd}}{(-4ac+b^2)d\sqrt{cx^2+bx+a}} - \frac{4\operatorname{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{(-4ac+b^2)^{\frac{3}{4}}\sqrt{d}\sqrt{cx^2+bx+a}}$$

command

`integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{2}\sqrt{c^2d}(cx^2+bx+a)\operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right) + \sqrt{2cdx+bd}\sqrt{cx^2+bx+a}c\right)}{(b^2c^2-4ac^3)dx^2+(b^3c-4abc^2)dx+(ab^2c-4a^2c^2)d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx+bd}\sqrt{cx^2+bx+a}}{2c^3dx^5+5bc^2dx^4+4(b^2c+ac^2)dx^3+a^2bd+(b^3+6abc)dx^2+2(ab^2+a^2c)dx}, x\right)$$

22.141 Problem number 1383

$$\int \frac{1}{(bd + 2cdx)^{5/2} (a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2}{(-4ac + b^2) d (2cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}} - \frac{40c \sqrt{cx^2 + bx + a}}{3 (-4ac + b^2)^2 d (2cdx + bd)^{\frac{3}{2}}}$$

$$- \frac{20 \operatorname{EllipticF} \left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{3 (-4ac + b^2)^{\frac{7}{4}} d^{\frac{5}{2}} \sqrt{cx^2 + bx + a}}$$

command

`integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5 \sqrt{2} (4c^3x^4 + 8bc^2x^3 + ab^2 + (5b^2c + 4ac^2)x^2 + (b^3 + 4abc)x) \sqrt{c^2d} \operatorname{weierstrassPInverse} \left(\frac{b^2 - 4ac}{c^2}, 0 \right) \right)}{3 (4(b^4c^4 - 8ab^2c^5 + 16a^2c^6)d^3x^4 + 8(b^5c^3 - 8ab^3c^4 + 16a^2bc^5)d^3x^3 + (5b^6c^2 - 36ab^4c^3 + 48a^2b^2c^4 + 64a^3c^5))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{8c^5d^3x^7 + 28bc^4d^3x^6 + 2(19b^2c^3 + 8ac^4)d^3x^5 + a^2b^3d^3 + 5(5b^3c^2 + 8abc^3)d^3x^4 + 4(2b^4c + 9ab^2c^2 + 2a^2b^2c^3)} \right)$$

22.142 Problem number 1384

$$\int \frac{(bd + 2cdx)^{9/2}}{(a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$- \frac{2d(2cdx + bd)^{\frac{7}{2}}}{\sqrt{cx^2 + bx + a}} + \frac{56c d^3 (2cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{5}$$

$$+ \frac{84(-4ac + b^2)^{\frac{7}{4}} d^{\frac{9}{2}} \operatorname{EllipticE} \left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5 \sqrt{cx^2 + bx + a}}$$

$$- \frac{84(-4ac + b^2)^{\frac{7}{4}} d^{\frac{9}{2}} \operatorname{EllipticF} \left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5 \sqrt{cx^2 + bx + a}}$$

command

```
integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(42 \sqrt{2} \left((b^2 c - 4 a c^2) d^4 x^2 + (b^3 - 4 a b c) d^4 x + (a b^2 - 4 a^2 c) d^4 \right) \sqrt{c^2 d} \operatorname{weierstrassZeta} \left(\frac{b^2 - 4 a c}{c^2}, 0, \operatorname{weierstrassP} \right) \right)$$

5

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(16 c^4 d^4 x^4 + 32 b c^3 d^4 x^3 + 24 b^2 c^2 d^4 x^2 + 8 b^3 c d^4 x + b^4 d^4) \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{c^2 x^4 + 2 b c x^3 + 2 a b x + (b^2 + 2 a c) x^2 + a^2}, x \right)$$

22.143 Problem number 1385

$$\int \frac{(b d + 2 c d x)^{5/2}}{(a + b x + c x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 d(2 c d x + b d)^{\frac{3}{2}}}{\sqrt{c x^2 + b x + a}} + \frac{12(-4 a c + b^2)^{\frac{3}{4}} d^{\frac{5}{2}} \operatorname{EllipticE} \left(\frac{\sqrt{2 c d x + b d}}{(-4 a c + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{-\frac{c(c x^2 + b x + a)}{-4 a c + b^2}}}{\sqrt{c x^2 + b x + a}} \\ & - \frac{12(-4 a c + b^2)^{\frac{3}{4}} d^{\frac{5}{2}} \operatorname{EllipticF} \left(\frac{\sqrt{2 c d x + b d}}{(-4 a c + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{-\frac{c(c x^2 + b x + a)}{-4 a c + b^2}}}{\sqrt{c x^2 + b x + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6 \sqrt{2} \left(c d^2 x^2 + b d^2 x + a d^2 \right) \sqrt{c^2 d} \operatorname{weierstrassZeta} \left(\frac{b^2 - 4 a c}{c^2}, 0, \operatorname{weierstrassPInverse} \left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x + b}{2 c} \right) \right) + (2 c d^2 x^2 + 2 b d^2 x + 2 a d^2) \sqrt{c^2 d} \right) / (c x^2 + b x + a)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(4 c^2 d^2 x^2 + 4 b c d^2 x + b^2 d^2) \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{c^2 x^4 + 2 b c x^3 + 2 a b x + (b^2 + 2 a c) x^2 + a^2}, x \right)$$

22.144 Problem number 1386

$$\int \frac{\sqrt{bd + 2cdx}}{(a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(2cdx + bd)^{\frac{3}{2}}}{(-4ac + b^2) d \sqrt{cx^2 + bx + a}} + \frac{4 \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{cx^2 + bx + a}}$$

$$- \frac{4 \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{cx^2 + bx + a}}$$

command

`integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{2} \sqrt{c^2 d} (cx^2 + bx + a) \operatorname{weierstrassZeta}\left(\frac{b^2 - 4ac}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4ac}{c^2}, 0, \frac{2cx + b}{2c}\right)\right) + \sqrt{2cdx + bd} \right)}{ab^2 - 4a^2c + (b^2c - 4ac^2)x^2 + (b^3 - 4abc)x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x\right)$$

22.145 Problem number 1387

$$\int \frac{1}{(bd + 2cdx)^{3/2} (a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2}{(-4ac + b^2) d \sqrt{2cdx + bd} \sqrt{cx^2 + bx + a}} - \frac{24c \sqrt{cx^2 + bx + a}}{(-4ac + b^2)^2 d \sqrt{2cdx + bd}}$$

$$+ \frac{12 \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{(-4ac + b^2)^{\frac{5}{4}} d^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}$$

$$- \frac{12 \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{(-4ac + b^2)^{\frac{5}{4}} d^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}$$

command

```
integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 \sqrt{2} \left(2 c^2 x^3 + 3 b c x^2 + a b + (b^2 + 2 a c) x \right) \sqrt{c^2 d} \operatorname{weierstrassZeta} \left(\frac{b^2 - 4 a c}{c^2}, 0, \operatorname{weierstrassPInverse} \left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x^2 + b x + a}{d} \right) \right)}{2 (b^4 c^2 - 8 a b^2 c^3 + 16 a^2 c^4) d^2 x^3 + 3 (b^5 c - 8 a b^3 c^2 + 16 a^2 b c^3) d^2 x^2 + (b^6 - 6 a b^4 c + 3 a^2 b^2 c^2) d^2 x + (b^7 c - 8 a b^5 c^2 + 16 a^2 b^3 c^3 - 12 a^3 c^4) d^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{4 c^4 d^2 x^6 + 12 b c^3 d^2 x^5 + (13 b^2 c^2 + 8 a c^3) d^2 x^4 + a^2 b^2 d^2 + 2 (3 b^3 c + 8 a b c^2) d^2 x^3 + (b^4 + 10 a b^2 c + 4 a^2 c^2) d^2 x + (b^5 c - 8 a b^3 c^2 + 16 a^2 b c^3 - 12 a^3 c^4) d^2} \right)$$

22.146 Problem number 1388

$$\int \frac{1}{(b d + 2 c d x)^{7/2} (a + b x + c x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} &-\frac{2}{(-4 a c + b^2) d (2 c d x + b d)^{\frac{5}{2}} \sqrt{c x^2 + b x + a}} - \frac{56 c \sqrt{c x^2 + b x + a}}{5 (-4 a c + b^2)^2 d (2 c d x + b d)^{\frac{5}{2}}} \\ &-\frac{168 c \sqrt{c x^2 + b x + a}}{5 (-4 a c + b^2)^3 d^3 \sqrt{2 c d x + b d}} + \frac{84 \operatorname{EllipticE} \left(\frac{\sqrt{2 c d x + b d}}{(-4 a c + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{-\frac{c (c x^2 + b x + a)}{-4 a c + b^2}}}{5 (-4 a c + b^2)^{\frac{9}{4}} d^{\frac{7}{2}} \sqrt{c x^2 + b x + a}} \\ &-\frac{84 \operatorname{EllipticF} \left(\frac{\sqrt{2 c d x + b d}}{(-4 a c + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{-\frac{c (c x^2 + b x + a)}{-4 a c + b^2}}}{5 (-4 a c + b^2)^{\frac{9}{4}} d^{\frac{7}{2}} \sqrt{c x^2 + b x + a}} \end{aligned}$$

command

```
integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(42 \sqrt{2} \left(8 c^4 x^5 + 20 b c^3 x^4 + a b^3 + 2 (9 b^2 c^2 + 4 a c^3) x^3 + (7 b^3 c + 12 a b c^2) x^2 + (b^4 + 6 a b^2 c) x \right) \sqrt{c^2 d} \operatorname{weierstrassZeta} \left(\frac{b^2 - 4 a c}{c^2}, 0, \operatorname{weierstrassPInverse} \left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x^2 + b x + a}{d} \right) \right)}{5 (8 (b^6 c^4 - 12 a b^4 c^5 + 48 a^2 b^2 c^6 - 64 a^3 c^7) d^4 x^5 + 20 (b^7 c^3 - 12 a b^5 c^4 + 48 a^2 b^3 c^5 - 64 a^3 b c^6) d^4 x^4 + 2 (9 b^8 c^2 - 12 a b^6 c^3 + 48 a^2 b^4 c^4 - 64 a^3 b^2 c^5) d^4 x^3 + (b^9 c - 12 a b^7 c^2 + 48 a^2 b^5 c^3 - 64 a^3 b^3 c^4) d^4 x^2 + (b^{10} - 12 a b^8 c + 48 a^2 b^6 c^2 - 64 a^3 b^4 c^3) d^4 x + (b^{11} c - 12 a b^9 c^2 + 48 a^2 b^7 c^3 - 64 a^3 b^5 c^4) d^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{2 c d x + b d}}{16 c^6 d^4 x^8 + 64 b c^5 d^4 x^7 + 8 (13 b^2 c^4 + 4 a c^5) d^4 x^6 + a^2 b^4 d^4 + 8 (11 b^3 c^3 + 12 a b c^4) d^4 x^5 + (41 b^4 c^2 + 112 a b^2 c^3) d^4 x^4 + (11 b^5 c - 12 a b^3 c^2 + 16 a^2 b c^3) d^4 x^3 + (b^6 - 6 a b^4 c + 3 a^2 b^2 c^2) d^4 x^2 + (b^7 c - 8 a b^5 c^2 + 16 a^2 b^3 c^3 - 12 a^3 c^4) d^4 x + (b^8 c^2 - 12 a b^6 c^3 + 48 a^2 b^4 c^4 - 64 a^3 b^2 c^5) d^4}$$

22.147 Problem number 1389

$$\int \frac{(bd + 2cdx)^{15/2}}{(a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2d(2cdx + bd)^{\frac{13}{2}}}{3(cx^2 + bx + a)^{\frac{3}{2}}} - \frac{52cd^3(2cdx + bd)^{\frac{9}{2}}}{3\sqrt{cx^2 + bx + a}} + \frac{624c^2d^5(2cdx + bd)^{\frac{5}{2}}\sqrt{cx^2 + bx + a}}{7} \\ & + \frac{1040c^2(-4ac + b^2)d^7\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}}{7} \\ & + \frac{520c(-4ac + b^2)^{\frac{9}{4}}d^{\frac{15}{2}}\operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{7\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(390 \sqrt{2} \left((b^4c^2 - 8ab^2c^3 + 16a^2c^4)d^7x^4 + 2(b^5c - 8ab^3c^2 + 16a^2bc^3)d^7x^3 + (b^6 - 6ab^4c + 32a^3c^3)d^7x^2 + 2(a \right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(128c^7d^7x^7 + 448bc^6d^7x^6 + 672b^2c^5d^7x^5 + 560b^3c^4d^7x^4 + 280b^4c^3d^7x^3 + 84b^5c^2d^7x^2 + 14b^6cd^7x + b^7d^7)}{c^3x^6 + 3bc^2x^5 + 3(b^2c + ac^2)x^4 + 3a^2bx + (b^3 + 6abc)x^3 + a^3 + 3(ab^2 + a^2c)}\right)$$

22.148 Problem number 1390

$$\int \frac{(bd + 2cdx)^{11/2}}{(a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2d(2cdx + bd)^{\frac{9}{2}}}{3(cx^2 + bx + a)^{\frac{3}{2}}} - \frac{12cd^3(2cdx + bd)^{\frac{5}{2}}}{\sqrt{cx^2 + bx + a}} + 80c^2d^5\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a} \\ & + \frac{40c(-4ac + b^2)^{\frac{5}{4}}d^{\frac{11}{2}}\operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(30 \sqrt{2} \left((b^2 c^2 - 4 a c^3) d^5 x^4 + 2 (b^3 c - 4 a b c^2) d^5 x^3 + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^5 x^2 + 2 (a b^3 - 4 a^2 b c) d^5 x + (a^2 b^2 - \dots \right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(32 c^5 d^5 x^5 + 80 b c^4 d^5 x^4 + 80 b^2 c^3 d^5 x^3 + 40 b^3 c^2 d^5 x^2 + 10 b^4 c d^5 x + b^5 d^5) \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{c^3 x^6 + 3 b c^2 x^5 + 3 (b^2 c + a c^2) x^4 + 3 a^2 b x + (b^3 + 6 a b c) x^3 + a^3 + 3 (a b^2 + a^2 c) x^2}, x \right)$$

22.149 Problem number 1391

$$\int \frac{(b d + 2 c d x)^{7/2}}{(a + b x + c x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2d(2cdx+bd)^{\frac{5}{2}}}{3(cx^2+bx+a)^{\frac{3}{2}}} - \frac{20cd^3\sqrt{2cdx+bd}}{3\sqrt{cx^2+bx+a}} \\ & + \frac{40c(-4ac+b^2)^{\frac{1}{4}}d^{\frac{7}{2}}\text{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{3\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10 \sqrt{2} \left(c^2 d^3 x^4 + 2 b c d^3 x^3 + 2 a b d^3 x + (b^2 + 2 a c) d^3 x^2 + a^2 d^3 \right) \sqrt{c^2 d} \text{weierstrassPInverse} \left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x + b}{2 c} \right) - \dots \right. \\ \left. \frac{\dots}{3 (c^2 x^4 + 2 b c x^3 + 2 a b x + (b^2 + 2 a c) x^2 + a^2)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(8 c^3 d^3 x^3 + 12 b c^2 d^3 x^2 + 6 b^2 c d^3 x + b^3 d^3) \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{c^3 x^6 + 3 b c^2 x^5 + 3 (b^2 c + a c^2) x^4 + 3 a^2 b x + (b^3 + 6 a b c) x^3 + a^3 + 3 (a b^2 + a^2 c) x^2}, x \right)$$

22.150 Problem number 1392

$$\int \frac{(bd + 2cdx)^{3/2}}{(a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2d\sqrt{2cdx+bd}}{3(cx^2+bx+a)^{\frac{3}{2}}} - \frac{4cd\sqrt{2cdx+bd}}{3(-4ac+b^2)\sqrt{cx^2+bx+a}} \\ & - \frac{8cd^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{3(-4ac+b^2)^{\frac{3}{4}}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{2}\left(c^2dx^4+2bcdx^3+2abdx+(b^2+2ac)dx^2+a^2d\right)\sqrt{c^2d}\operatorname{weierstrassPInverse}\left(\frac{b^2-4ac}{c^2}, 0, \frac{2cx+b}{2c}\right)+(2c^2d\right)}{3\left((b^2c^2-4ac^3)x^4+a^2b^2-4a^3c+2(b^3c-4abc^2)x^3+(b^4-2ab^2c-8a^2c^2)\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(2cdx+bd)^{\frac{3}{2}}\sqrt{cx^2+bx+a}}{c^3x^6+3bc^2x^5+3(b^2c+ac^2)x^4+3a^2bx+(b^3+6abc)x^3+a^3+3(ab^2+a^2c)x^2}, x\right)$$

22.151 Problem number 1393

$$\int \frac{1}{\sqrt{bd + 2cdx} (a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{2cdx+bd}}{3(-4ac+b^2)d(cx^2+bx+a)^{\frac{3}{2}}} + \frac{20c\sqrt{2cdx+bd}}{3(-4ac+b^2)^2d\sqrt{cx^2+bx+a}} \\ & + \frac{40c \operatorname{EllipticF}\left(\frac{\sqrt{2cdx+bd}}{(-4ac+b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{3(-4ac+b^2)^{\frac{7}{4}}\sqrt{d}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10 \sqrt{2} (c^2 x^4 + 2 b c x^3 + 2 a b x + (b^2 + 2 a c) x^2 + a^2) \sqrt{c^2 d} \operatorname{weierstrassPInverse} \left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x + b}{2 c} \right) + (10 c^2 x^2 + 10 c^2 x + 10 c^2) \right)}{3 \left((b^4 c^2 - 8 a b^2 c^3 + 16 a^2 c^4) d x^4 + 2 (b^5 c - 8 a b^3 c^2 + 16 a^2 b c^3) d x^3 + (b^6 - 6 a b^4 c + 32 a^3 c^3) d x^2 + 2 (a b^5 - 8 a^2 b^3 c) d x + (a^6 - 6 a^4 b c + 32 a^3 c^3) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{2 c^4 d x^7 + 7 b c^3 d x^6 + 3 (3 b^2 c^2 + 2 a c^3) d x^5 + 5 (b^3 c + 3 a b c^2) d x^4 + a^3 b d + (b^4 + 12 a b^2 c + 6 a^2 c^2) d x^3 + 3 (2 a b^3 c + a^4) d x^2 + (a^5 - 6 a^3 b c + 32 a^2 c^3) d x + (a^6 - 6 a^4 b c + 32 a^3 c^3)} \right)$$

22.152 Problem number 1394

$$\int \frac{1}{(b d + 2 c d x)^{5/2} (a + b x + c x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2}{3(-4ac + b^2) d (2cdx + bd)^{\frac{3}{2}} (cx^2 + bx + a)^{\frac{3}{2}}} \\ & + \frac{12c}{(-4ac + b^2)^2 d (2cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}} + \frac{80c^2 \sqrt{cx^2 + bx + a}}{(-4ac + b^2)^3 d (2cdx + bd)^{\frac{3}{2}}} \\ & + \frac{40c \operatorname{EllipticF} \left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{(-4ac + b^2)^{\frac{11}{4}} d^{\frac{5}{2}} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(30 \sqrt{2} (4 c^4 x^6 + 12 b c^3 x^5 + (13 b^2 c^2 + 8 a c^3) x^4 + a^2 b^2 + 2 (3 b^3 c^2 + 4 a b c^3) x^3 + (3 b^4 c + 4 a^2 c^2) x^2 + (3 b^5 c + 4 a^3 c^2) x + (3 b^6 c + 4 a^4 c)) \sqrt{c^2 d} \operatorname{weierstrassPInverse} \left(\frac{b^2 - 4 a c}{c^2}, 0, \frac{2 c x + b}{2 c} \right) + (10 c^2 x^2 + 10 c^2 x + 10 c^2) \right)}{3 \left((b^6 c^4 - 12 a b^4 c^5 + 48 a^2 b^2 c^6 - 64 a^3 c^7) d^3 x^6 + 12 (b^7 c^3 - 12 a b^5 c^4 + 48 a^2 b^3 c^5 - 64 a^3 b c^6) d^3 x^5 + (13 b^8 c^2 - 148 a b^6 c^3 + 48 a^2 b^4 c^4 - 64 a^3 c^5) d^3 x^4 + 2 (3 b^9 c - 12 a b^7 c^2 + 48 a^2 b^5 c^3 - 64 a^3 b^3 c^4) d^3 x^3 + (3 b^{10} c^2 - 148 a b^8 c^3 + 48 a^2 b^6 c^4 - 64 a^3 b^4 c^5) d^3 x^2 + 2 (3 b^{11} c^3 - 148 a b^9 c^4 + 48 a^2 b^7 c^5 - 64 a^3 b^5 c^6) d^3 x + (3 b^{12} c^4 - 148 a b^{10} c^5 + 48 a^2 b^8 c^6 - 64 a^3 b^6 c^7) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{2 c d^3 x^9 + 36 b c^5 d^3 x^8 + 6 (11 b^2 c^4 + 4 a c^5) d^3 x^7 + 21 (3 b^3 c^3 + 4 a b c^4) d^3 x^6 + a^3 b^3 d^3 + 3 (11 b^4 c^2 + 38 a b^2 c^3) d^3 x^5 + 2 (3 b^5 c + 4 a^2 c^2) d^3 x^4 + (3 b^6 c + 4 a^3 c^2) d^3 x^3 + (3 b^7 c^2 - 148 a b^5 c^3 + 48 a^2 b^3 c^4 - 64 a^3 b c^5) d^3 x^2 + 2 (3 b^8 c^3 - 148 a b^6 c^4 + 48 a^2 b^4 c^5 - 64 a^3 b^2 c^6) d^3 x + (3 b^9 c^4 - 148 a b^7 c^5 + 48 a^2 b^5 c^6 - 64 a^3 b^3 c^7)} \right)$$

22.153 Problem number 1395

$$\int \frac{(bd + 2cdx)^{13/2}}{(a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2d(2cdx + bd)^{\frac{11}{2}}}{3(cx^2 + bx + a)^{\frac{3}{2}}} - \frac{44cd^3(2cdx + bd)^{\frac{7}{2}}}{3\sqrt{cx^2 + bx + a}} + \frac{1232c^2d^5(2cdx + bd)^{\frac{3}{2}}\sqrt{cx^2 + bx + a}}{15} \\ & + \frac{616c(-4ac + b^2)^{\frac{7}{4}}d^{\frac{13}{2}}\text{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5\sqrt{cx^2 + bx + a}} \\ & - \frac{616c(-4ac + b^2)^{\frac{7}{4}}d^{\frac{13}{2}}\text{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{5\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(924 \sqrt{2} \left((b^2c^3 - 4ac^4)d^6x^4 + 2(b^3c^2 - 4abc^3)d^6x^3 + (b^4c - 2ab^2c^2 - 8a^2c^3)d^6x^2 + 2(ab^3c - 4a^2bc^2)d^6x + \dots \right) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(64c^6d^6x^6 + 192bc^5d^6x^5 + 240b^2c^4d^6x^4 + 160b^3c^3d^6x^3 + 60b^4c^2d^6x^2 + 12b^5cd^6x + b^6d^6)\sqrt{2cdx + bd}}{c^3x^6 + 3bc^2x^5 + 3(b^2c + ac^2)x^4 + 3a^2bx + (b^3 + 6abc)x^3 + a^3 + 3(ab^2 + a^2c)x^2}\right)$$

22.154 Problem number 1396

$$\int \frac{(bd + 2cdx)^{9/2}}{(a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2d(2cdx + bd)^{\frac{7}{2}}}{3(cx^2 + bx + a)^{\frac{3}{2}}} - \frac{28cd^3(2cdx + bd)^{\frac{3}{2}}}{3\sqrt{cx^2 + bx + a}} \\ & + \frac{56c(-4ac + b^2)^{\frac{3}{4}}d^{\frac{9}{2}}\text{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{\sqrt{cx^2 + bx + a}} \\ & - \frac{56c(-4ac + b^2)^{\frac{3}{4}}d^{\frac{9}{2}}\text{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right)\sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(84 \sqrt{2} \left(c^3 d^4 x^4 + 2 b c^2 d^4 x^3 + 2 a b c d^4 x + a^2 c d^4 + (b^2 c + 2 a c^2) d^4 x^2 \right) \sqrt{c^2 d} \operatorname{weierstrassZeta} \left(\frac{b^2 - 4 a c}{c^2}, 0, \operatorname{weierstrassZeta} \right) \right)}{3 (c^2 x^4 + 2 b c x^2 + a^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(16 c^4 d^4 x^4 + 32 b c^3 d^4 x^3 + 24 b^2 c^2 d^4 x^2 + 8 b^3 c d^4 x + b^4 d^4) \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{c^3 x^6 + 3 b c^2 x^5 + 3 (b^2 c + a c^2) x^4 + 3 a^2 b x + (b^3 + 6 a b c) x^3 + a^3 + 3 (a b^2 + a^2 c) x^2}, x \right)$$

22.155 Problem number 1397

$$\int \frac{(b d + 2 c d x)^{5/2}}{(a + b x + c x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 d(2 c d x + b d)^{\frac{3}{2}}}{3 (c x^2 + b x + a)^{\frac{3}{2}}} - \frac{4 c d(2 c d x + b d)^{\frac{3}{2}}}{(-4 a c + b^2) \sqrt{c x^2 + b x + a}} \\ & + \frac{8 c d^{\frac{5}{2}} \operatorname{EllipticE} \left(\frac{\sqrt{2 c d x + b d}}{(-4 a c + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{-\frac{c (c x^2 + b x + a)}{-4 a c + b^2}}}{(-4 a c + b^2)^{\frac{1}{4}} \sqrt{c x^2 + b x + a}} \\ & - \frac{8 c d^{\frac{5}{2}} \operatorname{EllipticF} \left(\frac{\sqrt{2 c d x + b d}}{(-4 a c + b^2)^{\frac{1}{4}} \sqrt{d}}, i \right) \sqrt{-\frac{c (c x^2 + b x + a)}{-4 a c + b^2}}}{(-4 a c + b^2)^{\frac{1}{4}} \sqrt{c x^2 + b x + a}} \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 \sqrt{2} \left(c^3 d^2 x^4 + 2 b c^2 d^2 x^3 + 2 a b c d^2 x + a^2 c d^2 + (b^2 c + 2 a c^2) d^2 x^2 \right) \sqrt{c^2 d} \operatorname{weierstrassZeta} \left(\frac{b^2 - 4 a c}{c^2}, 0, \operatorname{weierstrassZeta} \right) \right)}{3 ((b^2 c^2 - 4 a c^3) x^4 + a^2 b^2 - 4 a^3 c + 2 (b^3 c - a^2 b^2))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(4 c^2 d^2 x^2 + 4 b c d^2 x + b^2 d^2) \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{c^3 x^6 + 3 b c^2 x^5 + 3 (b^2 c + a c^2) x^4 + 3 a^2 b x + (b^3 + 6 a b c) x^3 + a^3 + 3 (a b^2 + a^2 c) x^2}, x \right)$$

22.156 Problem number 1398

$$\int \frac{\sqrt{bd + 2cdx}}{(a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(2cdx + bd)^{\frac{3}{2}}}{3(-4ac + b^2)d(cx^2 + bx + a)^{\frac{3}{2}}} + \frac{4c(2cdx + bd)^{\frac{3}{2}}}{(-4ac + b^2)^2 d \sqrt{cx^2 + bx + a}} \\ & - \frac{8c \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{(-4ac + b^2)^{\frac{5}{4}} \sqrt{cx^2 + bx + a}} \\ & + \frac{8c \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}, i\right) \sqrt{d} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{(-4ac + b^2)^{\frac{5}{4}} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 \sqrt{2} (c^3 x^4 + 2 b c^2 x^3 + 2 a b c x + a^2 c + (b^2 c + 2 a c^2) x^2) \sqrt{c^2 d} \operatorname{weierstrassZeta}\left(\frac{b^2 - 4 a c}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{b^2 - 4 a c}{c^2}, 0, \sqrt{c^2 d}\right)\right) \right)}{3 (a^2 b^4 - 8 a^3 b^2 c + 16 a^4 c^2 + (b^4 c^2 - 8 a b^2 c^3 + 16 a^2 c^4) x^4 + 2 (b^5 c - 8 a b^3 c^2 + 16 a^2 c^3) x^2 + 2 (b^6 c - 8 a b^4 c^2 + 16 a^2 c^4) x + 2 (b^7 c - 8 a b^5 c^2 + 16 a^2 c^5))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{c^3 x^6 + 3 b c^2 x^5 + 3 (b^2 c + a c^2) x^4 + 3 a^2 b x + (b^3 + 6 a b c) x^3 + a^3 + 3 (a b^2 + a^2 c) x^2, x}\right)$$

22.157 Problem number 1399

$$\int \frac{1}{(bd + 2cdx)^{3/2} (a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2}{3(-4ac + b^2)d(cx^2 + bx + a)^{\frac{3}{2}}\sqrt{2cdx + bd}} \\ & + \frac{28c}{3(-4ac + b^2)^2 d\sqrt{2cdx + bd}\sqrt{cx^2 + bx + a}} + \frac{112c^2\sqrt{cx^2 + bx + a}}{(-4ac + b^2)^3 d\sqrt{2cdx + bd}} \\ & - \frac{56c \operatorname{EllipticE}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{(-4ac + b^2)^{\frac{9}{4}} d^{\frac{3}{2}}\sqrt{cx^2 + bx + a}} \\ & + \frac{56c \operatorname{EllipticF}\left(\frac{\sqrt{2cdx + bd}}{(-4ac + b^2)^{\frac{1}{4}}\sqrt{d}}, i\right) \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{(-4ac + b^2)^{\frac{9}{4}} d^{\frac{3}{2}}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(84\sqrt{2}\left(2c^4x^5 + 5bc^3x^4 + a^2bc + 4(b^2c^2 + ac^3)x^3 + (b^3c + 6abc^2)x^2 + 2(ab^2c + a^2c^2)x\right)\sqrt{c^2d}\operatorname{weier}\right)}{3\left(2(b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6)d^2x^5 + 5(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3bc^5)d^2x^4 + 4(b^8c - 11ab^6c\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2cdx + bd}\sqrt{c}}{4c^5d^2x^8 + 16bc^4d^2x^7 + (25b^2c^3 + 12ac^4)d^2x^6 + (19b^3c^2 + 36abc^3)d^2x^5 + a^3b^2d^2 + (7b^4c + 39ab^2c^2 + 1}\right)$$

22.158 Problem number 1400

$$\int \frac{(ce + dex)^{11/2}}{\sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{30e^{\frac{11}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right)}{77d} - \frac{18e^3(dx + ce)^{\frac{5}{2}}\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{77d} \\ & - \frac{2e(dx + ce)^{\frac{9}{2}}\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{11d} - \frac{30e^5\sqrt{dex + ce}\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{77d} \end{aligned}$$

command

```
integrate((d*e*x+c*e)^(11/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((7d^6x^4 + 28cd^5x^3 + 3(14c^2 + 3)d^4x^2 + 2(14c^3 + 9c)d^3x + (7c^4 + 9c^2 + 15)d^2) \sqrt{-d^2x^2 - 2cdx - c^2 + 1} \right)}{77d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{(d^5e^5x^5 + 5cd^4e^5x^4 + 10c^2d^3e^5x^3 + 10c^3d^2e^5x^2 + 5c^4de^5x + c^5e^5) \sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dex + ce}}{d^2x^2 + 2cdx + c^2 - 1} \right)$$

22.159 Problem number 1401

$$\int \frac{(ce + dex)^{7/2}}{\sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$\frac{10e^{\frac{7}{2}} \text{EllipticF} \left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i \right)}{21d} - \frac{2e(dex + ce)^{\frac{5}{2}} \sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{7d} - \frac{10e^3 \sqrt{dex + ce} \sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{21d}$$

command

```
integrate((d*e*x+c*e)^(7/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((3d^4x^2 + 6cd^3x + (3c^2 + 5)d^2) \sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dx + c} e^{\frac{7}{2}} + 5 \sqrt{-d^3e} e^3 \text{weierstrassPInverse} \left(\frac{4}{d^2}, \dots \right) \right)}{21d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{(d^3e^3x^3 + 3cd^2e^3x^2 + 3c^2de^3x + c^3e^3) \sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dex + ce}}{d^2x^2 + 2cdx + c^2 - 1}, x \right)$$

22.160 Problem number 1402

$$\int \frac{(ce + dex)^{3/2}}{\sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$\frac{2e^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right)}{3d} - \frac{2e\sqrt{dex + ce} \sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{3d}$$

command

```
integrate((d*e*x+c*e)^(3/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\left(\sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dx + c} d^2 e^{\frac{3}{2}} + \sqrt{-d^3 e} \operatorname{eweierstrassPInverse}\left(\frac{4}{d^2}, 0, \frac{dx+c}{d}\right)\right)}{3d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-d^2x^2 - 2cdx - c^2 + 1} (dex + ce)^{\frac{3}{2}}}{d^2x^2 + 2cdx + c^2 - 1}, x\right)$$

22.161 Problem number 1403

$$\int \frac{1}{\sqrt{ce + dex} \sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right)}{d\sqrt{e}}$$

command

```
integrate(1/(d*e*x+c*e)^(1/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2\sqrt{-d^3 e} e^{(-1)} \operatorname{weierstrassPInverse}\left(\frac{4}{d^2}, 0, \frac{dx+c}{d}\right)}{d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dex + ce}}{d^3ex^3 + 3cd^2ex^2 + (3c^2 - 1)dex + (c^3 - c)e}, x\right)$$

22.162 Problem number 1404

$$\int \frac{1}{(ce + dex)^{5/2} \sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right)}{3de^{5/2}} - \frac{2\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{3de(dx + ce)^{3/2}}$$

command

```
integrate(1/(d*e*x+c*e)^(5/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dx + c} d^2e^{1/2} + (d^2x^2 + 2cdx + c^2) \sqrt{-d^3e} \operatorname{weierstrassPInverse}\left(\frac{4}{d^2}, 0, \frac{dx+c}{d}\right)\right)e^{-3}}{3(d^5x^2 + 2cd^4x + c^2d^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dex + ce}}{d^5e^3x^5 + 5cd^4e^3x^4 + (10c^2 - 1)d^3e^3x^3 + (10c^3 - 3c)d^2e^3x^2 + (5c^4 - 3c^2)de^3x + (c^5 - c^3)e^3}, x\right)$$

22.163 Problem number 1405

$$\int \frac{1}{(ce + dex)^{9/2} \sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$\frac{10 \operatorname{EllipticF}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right)}{21de^{9/2}} - \frac{2\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{7de(dx + ce)^{7/2}} - \frac{10\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{21de^3(dx + ce)^{3/2}}$$

command

```
integrate(1/(d*e*x+c*e)^(9/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((5d^4x^2 + 10cd^3x + (5c^2 + 3)d^2) \sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dx + c} e^{1/2} + 5(d^4x^4 + 4cd^3x^3 + 6c^2d^2x^2 + 4c^3)\right)}{21(d^7x^4 + 4cd^6x^3 + 6c^2d^5x^2 + 4c^3d^4x + c^4d^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dex + ce}}{d^7e^5x^7 + 7cd^6e^5x^6 + (21c^2 - 1)d^5e^5x^5 + 5(7c^3 - c)d^4e^5x^4 + 5(7c^4 - 2c^2)d^3e^5x^3 + (21c^5 - 10c^3)d^2e^5}, x\right)$$

22.164 Problem number 1406

$$\int \frac{1}{(ce + dex)^{13/2} \sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$\frac{30 \operatorname{EllipticF}\left(\frac{\sqrt{dex+ce}}{\sqrt{e}}, i\right)}{77de^{\frac{13}{2}}} - \frac{2\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{11de(dx+ce)^{\frac{11}{2}}} - \frac{18\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{77de^3(dx+ce)^{\frac{7}{2}}} - \frac{30\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{77de^5(dx+ce)^{\frac{3}{2}}}$$

command

```
integrate(1/(d*e*x+c*e)^(13/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((15d^6x^4 + 60cd^5x^3 + 9(10c^2 + 1)d^4x^2 + 6(10c^3 + 3c)d^3x + (15c^4 + 9c^2 + 7)d^2) \sqrt{-d^2x^2 - 2cdx - c^2 + 1} \right)}{77(d^9x^6 + 6cd^8x^5 + 15c^2d^7x^4 + \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{d^9e^7x^9 + 9cd^8e^7x^8 + (36c^2 - 1)d^7e^7x^7 + 7(12c^3 - c)d^6e^7x^6 + 21(6c^4 - c^2)d^5e^7x^5 + 7(18c^5 - 5c^3)d^4e^7x^4 + \dots}\right)$$

22.165 Problem number 1407

$$\int \frac{(ce + dex)^{9/2}}{\sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$\frac{14e^{\frac{9}{2}} \operatorname{EllipticE}\left(\frac{\sqrt{dex+ce}}{\sqrt{e}}, i\right)}{15d} - \frac{14e^{\frac{9}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{dex+ce}}{\sqrt{e}}, i\right)}{15d} - \frac{14e^3(dx+ce)^{\frac{3}{2}} \sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{45d} - \frac{2e(dx+ce)^{\frac{7}{2}} \sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{9d}$$

command

```
integrate((d*e*x+c*e)^(9/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((5d^4x^3 + 15cd^3x^2 + (15c^2 + 7)d^2x + (5c^3 + 7c)d) \sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dx + c} e^{\frac{9}{2}} - 21 \sqrt{-d^3e} e^4 \operatorname{weierstrassZeta}\left(\frac{4}{d^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{d^2}, dx + c\right)\right) \right)}{45d^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(d^4e^4x^4 + 4cd^3e^4x^3 + 6c^2d^2e^4x^2 + 4c^3de^4x + c^4e^4) \sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dex + ce}}{d^2x^2 + 2cdx + c^2 - 1}, x\right)$$

22.166 Problem number 1408

$$\int \frac{(ce + dex)^{5/2}}{\sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$\frac{6e^{\frac{5}{2}} \operatorname{EllipticE}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right)}{5d} - \frac{6e^{\frac{5}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right)}{5d} - \frac{2e(dex + ce)^{\frac{3}{2}} \sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{5d}$$

command

```
integrate((d*e*x+c*e)^(5/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{-d^2x^2 - 2cdx - c^2 + 1} (d^2x + cd) \sqrt{dx + c} e^{\frac{5}{2}} - 3 \sqrt{-d^3e} e^2 \operatorname{weierstrassZeta}\left(\frac{4}{d^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{d^2}, dx + c\right)\right) \right)}{5d^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(d^2e^2x^2 + 2cde^2x + c^2e^2) \sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dex + ce}}{d^2x^2 + 2cdx + c^2 - 1}, x\right)$$

22.167 Problem number 1409

$$\int \frac{\sqrt{ce + dex}}{\sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right) \sqrt{e}}{d} - \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right) \sqrt{e}}{d}$$

command

`integrate((d*e*x+c*e)^(1/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{-d^3 e} \operatorname{weierstrassZeta}\left(\frac{4}{d^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{d^2}, 0, \frac{dx+c}{d}\right)\right)}{d^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dex + ce}}{d^2x^2 + 2cdx + c^2 - 1}, x\right)$$

22.168 Problem number 1410

$$\int \frac{1}{(ce + dex)^{3/2} \sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{EllipticE}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right)}{de^{3/2}} + \frac{2 \operatorname{EllipticF}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right)}{de^{3/2}} - \frac{2 \sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{de \sqrt{dex + ce}}$$

command

`integrate(1/(d*e*x+c*e)^(3/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dx + c} de^{1/2} + \sqrt{-d^3e} (dx + c) \operatorname{weierstrassZeta}\left(\frac{4}{d^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{d^2}, 0, \frac{dx+c}{d}\right)\right) \right)}{d^3x + cd^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-d^2x^2 - 2cdx - c^2 + 1} \sqrt{dex + ce}}{d^4e^2x^4 + 4cd^3e^2x^3 + (6c^2 - 1)d^2e^2x^2 + 2(2c^3 - c)de^2x + (c^4 - c^2)e^2}, x\right)$$

22.169 Problem number 1411

$$\int \frac{1}{(ce + dex)^{7/2} \sqrt{1 - c^2 - 2cdx - d^2x^2}} dx$$

Optimal antiderivative

$$\frac{6 \operatorname{EllipticE}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right)}{5d e^{7/2}} + \frac{6 \operatorname{EllipticF}\left(\frac{\sqrt{dex + ce}}{\sqrt{e}}, i\right)}{5d e^{7/2}} - \frac{2\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{5de(dex + ce)^{5/2}} - \frac{6\sqrt{-d^2x^2 - 2cdx - c^2 + 1}}{5d e^3 \sqrt{dex + ce}}$$

command

`integrate(1/(d*e*x+c*e)^(7/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((3d^3x^2 + 6cd^2x + (3c^2 + 1)d)\sqrt{-d^2x^2 - 2cdx - c^2 + 1}\sqrt{dx + c}e^{1/2} + 3(d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3)\sqrt{-d^2x^2 - 2cdx - c^2 + 1}\right)}{5(d^5x^3 + 3cd^4x^2 + 3c^2d^3x + c^3d^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-d^2x^2 - 2cdx - c^2 + 1}\sqrt{dex + ce}}{d^6e^4x^6 + 6cd^5e^4x^5 + (15c^2 - 1)d^4e^4x^4 + 4(5c^3 - c)d^3e^4x^3 + 3(5c^4 - 2c^2)d^2e^4x^2 + 2(3c^5 - 2c^3)de^4x + c^6}\right)$$

22.170 Problem number 2294

$$\int \frac{1}{(d + ex)^{5/2} (a + bx + cx^2)} dx$$

Optimal antiderivative

$$\frac{2e}{3(ae^2 - bde + cd^2)(ex + d)^{3/2}} - \frac{2e(-be + 2cd)}{(ae^2 - bde + cd^2)^2 \sqrt{ex + d}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex + d}}{\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}\right) \sqrt{2}\sqrt{c}\left(2c^2d^2 + be^2(b + \sqrt{-4ac + b^2}) - 2ce(bd + ae + d\sqrt{-4ac + b^2})\right)}{(ae^2 - bde + cd^2)^2 \sqrt{-4ac + b^2} \sqrt{2cd - e(b - \sqrt{-4ac + b^2})}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex + d}}{\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \sqrt{2}\sqrt{c}\left(2c^2d^2 + be^2(b - \sqrt{-4ac + b^2}) - 2ce(bd + ae - d\sqrt{-4ac + b^2})\right)}{(ae^2 - bde + cd^2)^2 \sqrt{-4ac + b^2} \sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}$$

command

```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

22.171 Problem number 2299

$$\int \frac{1}{\sqrt{d+ex} (a+bx+cx^2)^2} dx$$

Optimal antiderivative

$$\frac{(bcd - b^2e + 2ace + c(-be + 2cd)x) \sqrt{ex+d}}{(-4ac + b^2)(ae^2 - bde + cd^2)(cx^2 + bx + a)}$$

$$+ \frac{\operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex+d}}{\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}\right) \sqrt{c} \left(8c^2d^2 - be^2(b + \sqrt{-4ac + b^2}) - 2ce(4bd - 6ae - d\sqrt{-4ac + b^2})\right)}{2(-4ac + b^2)^{\frac{3}{2}}(ae^2 - bde + cd^2) \sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}$$

$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex+d}}{\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \sqrt{c} \left(8c^2d^2 - be^2(b - \sqrt{-4ac + b^2}) - 2ce(4bd - 6ae + d\sqrt{-4ac + b^2})\right)}{2(-4ac + b^2)^{\frac{3}{2}}(ae^2 - bde + cd^2) \sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}$$

command

```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

22.172 Problem number 2300

$$\int \frac{1}{(d+ex)^{3/2} (a+bx+cx^2)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{e(2c^2d^2 + 3b^2e^2 - 2ce(5ae + bd))}{(-4ac + b^2)(ae^2 - bde + cd^2)^2 \sqrt{ex + d}} \\ & + \frac{-bcd + b^2e - 2ace - c(-be + 2cd)x}{(-4ac + b^2)(ae^2 - bde + cd^2)(cx^2 + bx + a) \sqrt{ex + d}} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex + d}}{\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}\right) \sqrt{c} \left(8c^3d^3 + 3b^2e^3(b + \sqrt{-4ac + b^2}) - 2c^2de(6bd - 16ae - d\sqrt{-4ac + b^2})\right)}{2(-4ac + b^2)^{\frac{3}{2}}(ae^2 - bde + cd^2)^2 \sqrt{2cd - e(b - \sqrt{-4ac + b^2})}} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex + d}}{\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \sqrt{c} \left(8c^3d^3 + 3b^2e^3(b - \sqrt{-4ac + b^2}) - 2c^2de(6bd - 16ae + d\sqrt{-4ac + b^2})\right)}{2(-4ac + b^2)^{\frac{3}{2}}(ae^2 - bde + cd^2)^2 \sqrt{2cd - e(b + \sqrt{-4ac + b^2})}} \end{aligned}$$

command

```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

22.173 Problem number 2305

$$\int \frac{\sqrt{d+ex}}{(a+bx+cx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(2cx + b) \sqrt{ex + d}}{2(-4ac + b^2)(cx^2 + bx + a)^2} \\
& \frac{(13b^2cde - 4ac^2de - b^3e^2 - 4bc(2ae^2 + 3cd^2) - c(24c^2d^2 + b^2e^2 - 4ce(-5ae + 6bd))x) \sqrt{ex + d}}{4(-4ac + b^2)^2(ae^2 - bde + cd^2)(cx^2 + bx + a)} \\
& \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex+d}}{\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}\right) \sqrt{c}\left(96c^3d^3 + b^2e^3(b + \sqrt{-4ac + b^2}) - 8c^2de(18bd - 13ae - 3d)\right)}{8(-4ac + b^2)^{\frac{5}{2}}(ae^2 - bde + cd^2)\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}} \\
& + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex+d}}{\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \sqrt{c}\left(96c^3d^3 + b^2e^3(b - \sqrt{-4ac + b^2}) - 8c^2de(18bd - 13ae + 3d)\right)}{8(-4ac + b^2)^{\frac{5}{2}}(ae^2 - bde + cd^2)\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}
\end{aligned}$$

command

```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

22.174 Problem number 2440

$$\int (dx)^{5/2} \sqrt{a + bx + cx^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2d(dx)^{\frac{3}{2}}(cx^2+bx+a)^{\frac{3}{2}}}{9c} - \frac{4bd^2(cx^2+bx+a)^{\frac{3}{2}}\sqrt{dx}}{21c^2} \\
& - \frac{4(21a^2c^2-36ab^2c+8b^4)d^3x\sqrt{cx^2+bx+a}}{315c^{\frac{7}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{dx}} \\
& + \frac{2d^2(b(3ac+8b^2)+3c(-7ac+8b^2)x)\sqrt{dx}\sqrt{cx^2+bx+a}}{315c^3} \\
& + \frac{4a^{\frac{1}{4}}(21a^2c^2-36ab^2c+8b^4)d^3\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}}+\frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right),\sqrt{2-\frac{b}{\sqrt{a}\sqrt{c}}}\right)}{315\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{15}{4}}\sqrt{dx}\sqrt{cx^2+bx+a}} \\
& + \frac{a^{\frac{1}{4}}d^3\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}}+\frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right),\sqrt{2-\frac{b}{\sqrt{a}\sqrt{c}}}\right)(\sqrt{a}+x\sqrt{c})}{315\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{15}{4}}\sqrt{dx}\sqrt{cx^2+bx+a}}
\end{aligned}$$

command

```
integrate((d*x)^(5/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left((16b^5-96ab^3c+123a^2bc^2)\sqrt{cd}d^2\operatorname{weierstrassPInverse}\left(\frac{4(b^2-3ac)}{3c^2},-\frac{4(2b^3-9abc)}{27c^3},\frac{3cx+b}{3c}\right)+6(8b^4c-36ab^2c^2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^2+bx+a}\sqrt{dx}d^2x^2,x\right)$$

22.175 Problem number 2441

$$\int (d + ex)^{3/2} \sqrt{a + bx + cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e(cx^2 + bx + a)^{\frac{3}{2}} \sqrt{ex + d}}{7c} \\ & + \frac{2(3c^2d^2 - 4b^2e^2 + ce(-5ae + 9bd) + 12ce(-be + 2cd)x) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}{105c^2e} \\ & (-be + 2cd) (3c^2d^2 + 8b^2e^2 - ce(29ae + 3bd)) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \\ & - \frac{105c^3e^2 \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{105c^3e^2 \sqrt{ex + d} \sqrt{cx^2 + bx + a}} \\ & 4(ae^2 - bde + cd^2) (3c^2d^2 + 2b^2e^2 - ce(5ae + 3bd)) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \\ & + \frac{105c^3e^2 \sqrt{ex + d} \sqrt{cx^2 + bx + a}}{105c^3e^2 \sqrt{ex + d} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((6c^4d^4 - 12bc^3d^3e - (17b^2c^2 - 104ac^3)d^2e^2 + (23b^3c - 104abc^2)de^3 - (8b^4 - 41ab^2c + 30a^2c^2)e^4) \sqrt{c} e^{\frac{1}{2}} \sqrt{ex + d} \sqrt{cx^2 + bx + a} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^2 + bx + a} (ex + d)^{\frac{3}{2}}, x\right)$$

22.176 Problem number 2442

$$\int \sqrt{d+ex} \sqrt{a+bx+cx^2} dx$$

Optimal antiderivative

$$\frac{2(ex+d)^{\frac{3}{2}} \sqrt{cx^2+bx+a}}{5e} - \frac{2(-be+2cd) \sqrt{ex+d} \sqrt{cx^2+bx+a}}{15ce}$$

$$2(c^2d^2 + b^2e^2 - ce(3ae + bd)) \operatorname{EllipticE} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right)$$

$$+ \frac{15c^2e^2 \sqrt{cx^2+bx+a} \sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}}{2(-be+2cd)(ae^2 - bde + cd^2) \operatorname{EllipticF} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right)} + \frac{15c^2e^2 \sqrt{ex+d} \sqrt{cx^2+bx+a}}$$

command

```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^3d^3 - 3bc^2d^2e - 3(b^2c - 6ac^2)de^2 + (2b^3 - 9abc)e^3) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + (b^2 - 3ac)e^2)e^{(-2)}}{3c^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{cx^2+bx+a} \sqrt{ex+d}, x \right)$$

22.177 Problem number 2443

$$\int \frac{\sqrt{a + bx + cx^2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{ex + d} \sqrt{cx^2 + bx + a}}{3e}$$

$$(-be + 2cd) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}$$

$$+ \frac{3ce^2 \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{3ce^2 \sqrt{ex + d} \sqrt{cx^2 + bx + a}} + \frac{4(ae^2 - bde + cd^2) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}}{3ce^2 \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cx^2 + bx + a} \sqrt{ex + d} c^2 e^2 + (2c^2 d^2 - 2bcde - (b^2 - 6ac)e^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(c^2 d^2 - bcde + (b^2 - 3ac)e^2)}{3c^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a}}{\sqrt{ex + d}}, x \right)$$

22.178 Problem number 2444

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{cx^2+bx+a}}{e\sqrt{ex+d}} + \frac{2 \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right) \sqrt{2} \sqrt{-4ac+b^2} \sqrt{ex+d}}{c e^2 \sqrt{ex+d} \sqrt{cx^2+bx+a}} + \frac{2(-be+2cd) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right) \sqrt{2} \sqrt{-4ac+b^2}}{c e^2 \sqrt{ex+d} \sqrt{cx^2+bx+a}}$$

command

```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2cd^2 - bxe^2 + (2cdx - bd)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2 - bcde + (b^2 - 3ac)e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d^3 - 3bc^2d^2e - 3(b^2c - \dots)}{3c^2}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+bx+a} \sqrt{ex+d}}{e^2x^2+2dex+d^2}, x\right)$$

22.179 Problem number 2445

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{cx^2+bx+a}}{3e(ex+d)^{\frac{3}{2}}} + \frac{2(-be+2cd)\sqrt{cx^2+bx+a}}{3e(ae^2-bde+cd^2)\sqrt{ex+d}} \\ & (-be+2cd)\text{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)\sqrt{2}\sqrt{-4ac+b^2} \\ & -\frac{3e^2(ae^2-bde+cd^2)\sqrt{cx^2+bx+a}\sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}}{4\text{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)\sqrt{2}\sqrt{-4ac+b^2}\sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}} \\ & +\frac{3e^2\sqrt{ex+d}\sqrt{cx^2+bx+a}}{3e^2\sqrt{ex+d}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left((2c^2d^4 - (b^2 - 6ac)x^2e^4 - 2(bcdx^2 + (b^2 - 6ac)dx)e^3 + (2c^2d^2x^2 - 4bcd^2x - (b^2 - 6ac)d^2)e^2 + 2(2c^2d^3x - \dots)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{e^3x^3+3de^2x^2+3d^2ex+d^3}, x\right)$$

22.180 Problem number 2446

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{cx^2+bx+a}}{5e(ex+d)^{5/2}} + \frac{2(-be+2cd)\sqrt{cx^2+bx+a}}{15e(ae^2-bde+cd^2)(ex+d)^{3/2}} \\ & + \frac{4(c^2d^2+b^2e^2-ce(3ae+bd))\sqrt{cx^2+bx+a}}{15e(ae^2-bde+cd^2)^2\sqrt{ex+d}} \\ & + \frac{2(c^2d^2+b^2e^2-ce(3ae+bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{15e^2(ae^2-bde+cd^2)^2\sqrt{cx^2+bx+a} \sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}} \\ & + \frac{2(-be+2cd) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right) \sqrt{2}\sqrt{-4ac+b^2}}{15e^2(ae^2-bde+cd^2)\sqrt{ex+d}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^3d^6 + (2b^3 - 9abc)x^3e^6 - 3((b^2c - 6ac^2)dx^3 - (2b^3 - 9abc)dx^2))e^5 - 3(bc^2d^2x^3 + 3(b^2c - 6ac^2)d^2x^2 - \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{e^4x^4+4de^3x^3+6d^2e^2x^2+4d^3ex+d^4}, x\right)$$

22.181 Problem number 2447

$$\int (d + ex)^{3/2} (a + bx + cx^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{2(c^2d^2 - 6b^2e^2 + ce(-3ae + 13bd) + 14ce(-be + 2cd)x)(cx^2 + bx + a)^{\frac{3}{2}}\sqrt{ex + d}}{231c^2e}$$

$$+ \frac{2e(cx^2 + bx + a)^{\frac{5}{2}}\sqrt{ex + d}}{11c}$$

$$+ \frac{2(8c^4d^4 + 8b^4e^4 - c^3d^2e(-42ae + 19bd) - b^2ce^3(21ae + 19bd) + 3c^2e^2(-10a^2e^2 + 17abde + 2b^2d^2) - 3ce(-be + 2cd)x)(cx^2 + bx + a)^{\frac{3}{2}}\sqrt{ex + d}}{1155c^3e^3}$$

$$+ \frac{8(-be + 2cd)(c^2d^2 - 2b^2e^2 - ce(-9ae + bd))(c^2d^2 + b^2e^2 - ce(3ae + bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}\right)}{1155c^4e^4\sqrt{cx^2 + bx + a}\sqrt{\frac{2cd - e}{2}}}$$

$$+ \frac{2(ae^2 - bde + cd^2)(16c^4d^4 - 8b^4e^4 - 4c^3d^2e(-21ae + 8bd) + b^2ce^3(51ae + 13bd) + 3c^2e^2(-20a^2e^2 - 28abde + 16a^2d^2))\sqrt{ex + d}}{1155c^3e^3}$$

command

```
integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^6d^6 - 48bc^5d^5e + 3(11b^2c^4 + 36ac^5)d^4e^2 + 2(7b^3c^3 - 108abc^4)d^3e^3 + 3(11b^4c^2 - 102ab^2c^3 + 312a^2c^4)d^2e^4 + 2(7b^3c^3 - 108abc^4)d^3e^3 + 3(11b^4c^2 - 102ab^2c^3 + 312a^2c^4)d^2e^4 + 2(7b^3c^3 - 108abc^4)d^3e^3 + 3(11b^4c^2 - 102ab^2c^3 + 312a^2c^4)d^2e^4 \right) \sqrt{ex + d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(ce^3x^3 + (cd + be)x^2 + ad + (bd + ae)x\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}, x\right)$$

22.182 Problem number 2448

$$\int \sqrt{d+ex} (a+bx+cx^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{2(ex+d)^{\frac{3}{2}}(cx^2+bx+a)^{\frac{3}{2}}}{9e} - \frac{2(-be+2cd)(cx^2+bx+a)^{\frac{3}{2}}\sqrt{ex+d}}{21ce}$$

$$+ \frac{2(8c^3d^3 - 4b^3e^3 - 3c^2de(-8ae+5bd) + 3bce^2(3ae+bd) - 6ce(c^2d^2 + 2b^2e^2 - ce(7ae+bd))x)\sqrt{ex+d}\sqrt{cx^2+d}}{315c^2e^3}$$

$$- \frac{(16c^4d^4 - 8b^4e^4 - 4c^3d^2e(-15ae+8bd) + b^2ce^3(57ae+7bd) + 3c^2e^2(-28a^2e^2 - 20abde + 3b^2d^2))\text{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}\right)}{315c^3e^4\sqrt{cx^2+bx+a}}$$

$$+ \frac{8(-be+2cd)(ae^2 - bde + cd^2)(2c^2d^2 - b^2e^2 - 2ce(-3ae+bd))\text{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}\right)}{315c^3e^4\sqrt{ex+d}}$$

command

```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^5d^5 - 40bc^4d^4e + 2(11b^2c^3 + 36ac^4)d^3e^2 + (7b^3c^2 - 108abc^3)d^2e^3 + (11b^4c - 102ab^2c^2 + 312a^2c^3)de^4 - \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((cx^2+bx+a)^{\frac{3}{2}}\sqrt{ex+d}, x\right)$$

22.183 Problem number 2449

$$\int \frac{(a + bx + cx^2)^{3/2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(cx^2 + bx + a)^{\frac{3}{2}} \sqrt{ex + d}}{7e} \\ & + \frac{2(8c^2d^2 + b^2e^2 - ce(-10ae + 11bd) - 3ce(-be + 2cd)x) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}{35ce^3} \\ & + \frac{2(-be + 2cd)(4c^2d^2 - b^2e^2 - 4ce(-2ae + bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{35c^2e^4 \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}} \\ & + \frac{2(ae^2 - bde + cd^2)(16c^2d^2 - b^2e^2 - 4ce(-5ae + 4bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{35c^2e^4 \sqrt{ex + d} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^4d^4 - 32bc^3d^3e + (13b^2c^2 + 44ac^3)d^2e^2 + (3b^3c - 44abc^2)de^3 + (2b^4 - 19ab^2c + 60a^2c^2)e^4) \sqrt{c} e^{\frac{1}{2}} \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + bx + a)^{\frac{3}{2}}}{\sqrt{ex + d}}, x\right)$$

22.184 Problem number 2450

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(cx^2 + bx + a)^{\frac{3}{2}}}{e\sqrt{ex + d}} - \frac{2(-6cex - 7be + 8cd) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}{5e^3} \\ & (16c^2d^2 + b^2e^2 - 4ce(-3ae + 4bd)) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \\ & + \frac{5ce^4 \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{16(-be + 2cd)(ae^2 - bde + cd^2) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)} \\ & \frac{2 \left((16c^3d^4 + (b^3 - 12abc)xe^4 + (6(b^2c + 4ac^2)dx + (b^3 - 12abc)d)e^3 - 6(4bc^2d^2x - (b^2c + 4ac^2)d^2)e^2 + 8 \right)}{5ce^4\sqrt{ex + d} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((16c^3d^4 + (b^3 - 12abc)xe^4 + (6(b^2c + 4ac^2)dx + (b^3 - 12abc)d)e^3 - 6(4bc^2d^2x - (b^2c + 4ac^2)d^2)e^2 + 8 \right)}{5ce^4\sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(cx^2 + bx + a)^{\frac{3}{2}} \sqrt{ex + d}}{e^2x^2 + 2dex + d^2}, x \right)$$

22.185 Problem number 2451

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(cx^2 + bx + a)^{\frac{3}{2}}}{3e(ex + d)^{\frac{3}{2}}} + \frac{2(2cex - 3be + 8cd)\sqrt{cx^2 + bx + a}}{3e^3\sqrt{ex + d}} \\ & + \frac{8(-be + 2cd)\operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)\sqrt{2}\sqrt{-4ac + b^2}}{3e^4\sqrt{cx^2 + bx + a}\sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}} \\ & + \frac{2(16c^2d^2 + 3b^2e^2 - 4ce(-ae + 4bd))\operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{3ce^4\sqrt{ex + d}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left((16c^2d^4 + (b^2 + 12ac)x^2e^4 - 2(8bcdx^2 - (b^2 + 12ac)dx)e^3 + (16c^2d^2x^2 - 32bcd^2x + (b^2 + 12ac)d^2)e^2 + 16\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + bx + a)^{\frac{3}{2}}\sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

22.186 Problem number 2452

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(cx^2 + bx + a)^{\frac{3}{2}}}{5e(ex + d)^{\frac{5}{2}}} - \frac{2(8c^2d^3 + abe^3 - cde(-4ae + 7bd) + e(10c^2d^2 + b^2e^2 - 2ce(-3ae + 5bd))x) \sqrt{cx^2 + bx + a}}{5e^3(ae^2 - bde + cd^2)(ex + d)^{\frac{3}{2}}}$$

$$(16c^2d^2 + b^2e^2 - 4ce(-3ae + 4bd)) \text{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)$$

$$+ \frac{5e^4(ae^2 - bde + cd^2) \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{16(-be + 2cd) \text{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}}$$

$$- \frac{5e^4 \sqrt{ex + d} \sqrt{cx^2 + bx + a}}{5e^4 \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^3d^6 + (b^3 - 12abc)x^3e^6 + 3(2(b^2c + 4ac^2)dx^3 + (b^3 - 12abc)dx^2)e^5 - 3(8bc^2d^2x^3 - 6(b^2c + 4ac^2)d^2x^2) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(cx^2 + bx + a)^{\frac{3}{2}} \sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x \right)$$

22.187 Problem number 2453

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(cx^2 + bx + a)^{\frac{3}{2}}}{7e(ex + d)^{\frac{7}{2}}} \\ & - \frac{2(8c^2d^3 - cde(-4ae + 5bd) - be^2(-3ae + 2bd) + e(14c^2d^2 + b^2e^2 - 2ce(-5ae + 7bd))x) \sqrt{cx^2 + bx + a}}{35e^3(ae^2 - bde + cd^2)(ex + d)^{\frac{5}{2}}} \\ & + \frac{4(-be + 2cd)(4c^2d^2 - b^2e^2 - 4ce(-2ae + bd)) \sqrt{cx^2 + bx + a}}{35e^3(ae^2 - bde + cd^2)^2 \sqrt{ex + d}} \\ & - \frac{2(-be + 2cd)(4c^2d^2 - b^2e^2 - 4ce(-2ae + bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{35e^4(ae^2 - bde + cd^2)^2 \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}} \\ & + \frac{2(16c^2d^2 - b^2e^2 - 4ce(-5ae + 4bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{35e^4(ae^2 - bde + cd^2) \sqrt{ex + d} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^2 + bx + a)^{\frac{3}{2}} \sqrt{ex + d}}{e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5}, x\right)$$

22.188 Problem number 2454

$$\int \sqrt{dx} (a + bx + cx^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(dx)^{\frac{3}{2}} (cx^2 + bx + a)^{\frac{5}{2}}}{13d} - \frac{10(3b(-19ac + 6b^2) + 14c(-11ac + 3b^2)x)(cx^2 + bx + a)^{\frac{3}{2}} \sqrt{dx}}{9009c^2} \\ & + \frac{10b(cx^2 + bx + a)^{\frac{5}{2}} \sqrt{dx}}{143c} - \frac{4(-924a^3c^3 + 951a^2b^2c^2 - 268ab^4c + 24b^6) dx \sqrt{cx^2 + bx + a}}{9009c^{\frac{7}{2}} (\sqrt{a} + x\sqrt{c}) \sqrt{dx}} \\ & + \frac{2(b(108a^2c^2 - 151ab^2c + 24b^4) + 3c(308a^2c^2 - 181ab^2c + 24b^4)x) \sqrt{dx} \sqrt{cx^2 + bx + a}}{9009c^3} \\ & + \frac{4a^{\frac{1}{4}}(-924a^3c^3 + 951a^2b^2c^2 - 268ab^4c + 24b^6) d \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)\right)}{9009 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{15}{4}} \sqrt{dx} \sqrt{cx^2 + bx + a}} \\ & + \frac{a^{\frac{1}{4}} d \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \sqrt{2 - \frac{b}{\sqrt{a}\sqrt{c}}}\right) (\sqrt{a} + x\sqrt{c})}{9009 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)} \end{aligned}$$

command

```
integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((48b^7 - 608ab^5c + 2625a^2b^3c^2 - 3972a^3bc^3) \sqrt{cd} \operatorname{weierstrassPInverse}\left(\frac{4(b^2 - 3ac)}{3c^2}, -\frac{4(2b^3 - 9abc)}{27c^3}, \frac{3cx + b}{3c}\right) + 6(24$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2) \sqrt{cx^2 + bx + a} \sqrt{dx}, x\right)$$

22.189 Problem number 2455

$$\int \frac{(a + bx + cx^2)^{5/2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{10(16c^2d^2 + 3b^2e^2 - ce(-18ae + 23bd) - 7ce(-be + 2cd)x)(cx^2 + bx + a)^{\frac{3}{2}}\sqrt{ex + d}}{693c^3e^3}$$

$$+ \frac{2(cx^2 + bx + a)^{\frac{5}{2}}\sqrt{ex + d}}{11e}$$

$$+ \frac{2(128c^4d^4 - 4b^4e^4 - 4c^3d^2e(-69ae + 76bd) - b^2ce^3(-27ae + 7bd) + 3c^2e^2(60a^2e^2 - 124abde + 65b^2d^2) - 12ce(-be + 2cd)(128c^4d^4 + 8b^4e^4 + b^2ce^3(-93ae + 29bd) - 4c^3d^2e(-93ae + 64bd) + 3c^2e^2(124a^2e^2 - 124abde + 33b^2d^2) - 12ce^3(124a^2e^2 - 124abde + 33b^2d^2))}{693c^2e^5}$$

$$(-be + 2cd)(128c^4d^4 + 8b^4e^4 + b^2ce^3(-93ae + 29bd) - 4c^3d^2e(-93ae + 64bd) + 3c^2e^2(124a^2e^2 - 124abde + 33b^2d^2) - 12ce^3(124a^2e^2 - 124abde + 33b^2d^2))$$

$$693c^3e^6\sqrt{cx^2 + a}$$

$$4(ae^2 - bde + cd^2)(128c^4d^4 + 2b^4e^4 - 4c^3d^2e(-69ae + 64bd) + b^2ce^3(-21ae + 5bd) + 3c^2e^2(60a^2e^2 - 92abde + 65b^2d^2) - 12ce^3(124a^2e^2 - 124abde + 33b^2d^2))$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((256c^6d^6 - 768bc^5d^5e + 6(121b^2c^4 + 156ac^5)d^4e^2 - 4(43b^3c^3 + 468abc^4)d^3e^3 - 3(11b^4c^2 - 260ab^2c^3 - 416b^3c^2d + 124a^2b^2c^2 - 124abde + 65b^2d^2))\sqrt{cx^2 + bx + a} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{cx^2 + bx + a}}{\sqrt{ex + d}}, x \right)$$

22.190 Problem number 2456

$$\int \frac{(a + bx + cx^2)^{5/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(cx^2 + bx + a)^{5/2}}{e\sqrt{ex + d}} - \frac{10(-14cex - 15be + 16cd)(cx^2 + bx + a)^{3/2}\sqrt{ex + d}}{63e^3} - \frac{2(128c^3d^3 - b^3e^3 + 3bce^2(-36ae + 37bd) - 12c^2de(-11ae + 20bd) - 3ce(32c^2d^2 + b^2e^2 - 4ce(-7ae + 8bd))x)}{63ce^5}$$

$$2(128c^4d^4 - b^4e^4 - 4c^3d^2e(-57ae + 64bd) - b^2ce^3(-15ae + 7bd) + 3c^2e^2(28a^2e^2 - 76abde + 45b^2d^2)) \text{EllipticE}$$

+

$$\frac{63c^2e^6\sqrt{cx^2 + bx + a}}{2(-be + 2cd)(ae^2 - bde + cd^2)(128c^2d^2 - b^2e^2 - 4ce(-33ae + 32bd)) \text{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, x\right)}{63c^2e^6\sqrt{ex + d}}$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((256c^5d^6 - (2b^5 - 33ab^3c + 228a^2bc^2)xe^6 - ((13b^4c - 258ab^2c^2 - 456a^2c^3)dx + (2b^5 - 33ab^3c + 228a^2bc^2)dx)\right)}{e^2x^2 + 2dex + d^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^2x^2 + 2dex + d^2}, x\right)$$

22.191 Problem number 2457

$$\int \frac{(a + bx + cx^2)^{5/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(cx^2 + bx + a)^{\frac{5}{2}}}{3e(ex + d)^{\frac{3}{2}}} + \frac{10(2cex - 7be + 16cd)(cx^2 + bx + a)^{\frac{3}{2}}}{21e^3\sqrt{ex + d}} \\ & + \frac{2(128c^2d^2 + 51b^2e^2 - 4ce(-5ae + 44bd) - 48ce(-be + 2cd)x)\sqrt{ex + d}\sqrt{cx^2 + bx + a}}{21e^5} \\ & (-be + 2cd)(128c^2d^2 + 3b^2e^2 - 4ce(-29ae + 32bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{cx^2 + bx + a}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \\ & - \frac{21ce^6\sqrt{cx^2 + bx + a}\sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{21ce^6\sqrt{cx^2 + bx + a}} \\ & 4(ae^2 - bde + cd^2)(128c^2d^2 + 27b^2e^2 - 4ce(-5ae + 32bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{cx^2 + bx + a}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \\ & + \frac{21ce^6\sqrt{ex + d}\sqrt{cx^2 + bx + a}}{21ce^6\sqrt{ex + d}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((256c^4d^6 - (3b^4 - 46ab^2c - 120a^2c^2)x^2e^6 - 2((11b^3c + 212abc^2)dx^2 + (3b^4 - 46ab^2c - 120a^2c^2)dx)e^5 + (2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

22.192 Problem number 2458

$$\int \frac{(a + bx + cx^2)^{5/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(6cex - 5be + 16cd)(cx^2 + bx + a)^{\frac{3}{2}}}{15e^3(ex + d)^{\frac{3}{2}}} - \frac{2(cx^2 + bx + a)^{\frac{5}{2}}}{5e(ex + d)^{\frac{5}{2}}}$$

$$- \frac{2(128c^2d^2 + 15b^2e^2 - 4ce(-9ae + 28bd) + 16ce(-be + 2cd)x)\sqrt{cx^2 + bx + a}}{15e^5\sqrt{ex + d}}$$

$$+ \frac{2(128c^2d^2 + 23b^2e^2 - 4ce(-9ae + 32bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{15e^6\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}$$

$$+ \frac{2(-be + 2cd)(128c^2d^2 + 15b^2e^2 - 4ce(-17ae + 32bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{15ce^6\sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((256c^3d^6 + (b^3 - 132abc)x^3e^6 + 3(2(21b^2c + 44ac^2)dx^3 + (b^3 - 132abc)dx^2)e^5 - 3(128bc^2d^2x^3 - 6(21b^2c + 44ac^2)dx^2 + (b^3 - 132abc)dx)e^4 - 3(128bc^2d^2x^3 - 6(21b^2c + 44ac^2)dx^2 + (b^3 - 132abc)dx)e^3 - 3(128bc^2d^2x^3 - 6(21b^2c + 44ac^2)dx^2 + (b^3 - 132abc)dx)e^2 - 3(128bc^2d^2x^3 - 6(21b^2c + 44ac^2)dx^2 + (b^3 - 132abc)dx)e - 3(128bc^2d^2x^3 - 6(21b^2c + 44ac^2)dx^2 + (b^3 - 132abc)dx)\right)}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

22.193 Problem number 2459

$$\int \frac{(a + bx + cx^2)^{5/2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(16c^2d^3 + 3abe^3 - cde(-4ae + 13bd) + e(22c^2d^2 + 3b^2e^2 - 2ce(-5ae + 11bd))x)(cx^2 + bx + a)^{\frac{3}{2}}}{21e^3(ae^2 - bde + cd^2)(ex + d)^{\frac{5}{2}}} \\ & - \frac{2(cx^2 + bx + a)^{\frac{5}{2}}}{7e(ex + d)^{\frac{7}{2}}} \\ & + \frac{2c(128c^2d^3 - 4cde(-29ae + 44bd) + 3b^2e^2(-16ae + 17bd) + e(32c^2d^2 + 3b^2e^2 - 4ce(-5ae + 8bd))x)\sqrt{cx^2 + bx + a}}{21e^5(ae^2 - bde + cd^2)\sqrt{ex + d}} \\ & - \frac{(-be + 2cd)(128c^2d^2 + 3b^2e^2 - 4ce(-29ae + 32bd))\text{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{21e^6(ae^2 - bde + cd^2)\sqrt{cx^2 + bx + a}\sqrt{\frac{c(ea + b + \sqrt{-4ac + b^2})}{2cd - e(b + \sqrt{-4ac + b^2})}}} \\ & + \frac{4(128c^2d^2 + 27b^2e^2 - 4ce(-5ae + 32bd))\text{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{21e^6\sqrt{ex + d}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5}, x\right)$$

22.194 Problem number 2460

$$\int \frac{(a + bx + cx^2)^{5/2}}{(d + ex)^{11/2}} dx$$

Optimal antiderivative

$$\frac{2(16c^2d^3 - be^2(-5ae + 2bd) - cde(-4ae + 11bd) + e(26c^2d^2 + 3b^2e^2 - 2ce(-7ae + 13bd))x)(cx^2 + bx + a)^{\frac{3}{2}}}{63e^3(ae^2 - bde + cd^2)(ex + d)^{\frac{7}{2}}}$$

$$- \frac{2(cx^2 + bx + a)^{\frac{5}{2}}}{9e(ex + d)^{\frac{9}{2}}}$$

$$- \frac{2(128c^4d^5 - 2ab^3e^5 - 4c^3d^3e(-49ae + 60bd) - bce^3(-24a^2e^2 + 9abde + b^2d^2) + 3c^2de^2(12a^2e^2 - 52abde + 37bd^2))}{63e^5(ae^2 - bde + cd^2)}$$

$$2(128c^4d^4 - b^4e^4 - 4c^3d^2e(-57ae + 64bd) - b^2ce^3(-15ae + 7bd) + 3c^2e^2(28a^2e^2 - 76abde + 45b^2d^2)) \text{EllipticE}$$

$$+ \frac{63e^6(ae^2 - bde + cd^2)^2 \sqrt{cx^2 + bx + a}}{63e^6(ae^2 - bde + cd^2) \sqrt{ex + d}}$$

$$- \frac{2(-be + 2cd)(128c^2d^2 - b^2e^2 - 4ce(-33ae + 32bd)) \text{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{cx^2 + bx + a}}{2cd - e(b + 2cx + \sqrt{-4ac + b^2})}}\right)}{63e^6(ae^2 - bde + cd^2) \sqrt{ex + d}}$$

command

```
integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^6x^6 + 6de^5x^5 + 15d^2e^4x^4 + 20d^3e^3x^3 + 15d^4e^2x^2 + 6d^5ex + d^6}, x\right)$$

22.195 Problem number 2461

$$\int \frac{(d+ex)^{7/2}}{\sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{12e(-be+2cd)(ex+d)^{\frac{3}{2}}\sqrt{cx^2+bx+a}}{35c^2} + \frac{2e(ex+d)^{\frac{5}{2}}\sqrt{cx^2+bx+a}}{7c}$$

$$+ \frac{2e(71c^2d^2+24b^2e^2-ce(25ae+71bd))\sqrt{ex+d}\sqrt{cx^2+bx+a}}{105c^3}$$

$$+ \frac{8(-be+2cd)(11c^2d^2+6b^2e^2-ce(13ae+11bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{105c^4\sqrt{cx^2+bx+a}\sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}}$$

$$+ \frac{2(ae^2-bde+cd^2)(71c^2d^2+24b^2e^2-ce(25ae+71bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{2}, \sqrt{\frac{-2cd-e(b+\sqrt{-4ac+b^2})}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{105c^4\sqrt{ex+d}\sqrt{cx^2+bx+a}}$$

command

```
integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((139c^4d^4 - 278bc^3d^3e + (347b^2c^2 - 554ac^3)d^2e^2 - 2(104b^3c - 277abc^2)de^3 + (48b^4 - 176ab^2c + 75a^2c^2)e^4 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}, x\right)$$

22.196 Problem number 2462

$$\int \frac{(d+ex)^{5/2}}{\sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{2e(ex+d)^{\frac{3}{2}} \sqrt{cx^2+bx+a}}{5c} + \frac{8e(-be+2cd) \sqrt{ex+d} \sqrt{cx^2+bx+a}}{15c^2} + \frac{(23c^2d^2 + 8b^2e^2 - ce(9ae + 23bd)) \operatorname{EllipticE} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right)}{15c^3 \sqrt{cx^2+bx+a} \sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}} + \frac{8(-be+2cd) (ae^2 - bde + cd^2) \operatorname{EllipticF} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right)}{15c^3 \sqrt{ex+d} \sqrt{cx^2+bx+a}}$$

command

```
integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((22c^3d^3 - 33bc^2d^2e + 3(9b^2c - 14ac^2)de^2 - (8b^3 - 21abc)e^3) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + (b^2 - 3ac))}{3c^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(e^2x^2 + 2dex + d^2) \sqrt{ex+d}}{\sqrt{cx^2+bx+a}}, x \right)$$

22.197 Problem number 2463

$$\int \frac{(d+ex)^{3/2}}{\sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{2e\sqrt{ex+d}\sqrt{cx^2+bx+a}}{3c}$$

$$+ \frac{2(-be+2cd) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right) \sqrt{2}\sqrt{-4ac+b^2}}{3c^2\sqrt{cx^2+bx+a}\sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}}$$

$$- \frac{2(ae^2-bde+cd^2) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right) \sqrt{2}\sqrt{-4ac+b^2}}{3c^2\sqrt{ex+d}\sqrt{cx^2+bx+a}}$$

command

```
integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(3\sqrt{cx^2+bx+a}\sqrt{xe+d}c^2e^2+(5c^2d^2-5bcde+(2b^2-3ac)e^2)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2-bcde+b^2-3c^2)}{3c^2}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}, x\right)$$

22.198 Problem number 2464

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\text{EllipticE} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-4ac+b^2} \sqrt{ex+d} \sqrt{-\frac{c(cx^2+a)}{2cd-e(b+\sqrt{-4ac+b^2})}}$$

$$c\sqrt{cx^2+bx+a} \sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}$$

command

```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2cd-be)\sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4(c^2d^2-bcde+(b^2-3ac)e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d^3-3bc^2d^2e-3(b^2c-6ac^2)de^2+(2b^3-9abc)e^3)}{27c^3} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}, x \right)$$

22.199 Problem number 2465

$$\int \frac{1}{\sqrt{d+ex} \sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$2 \text{EllipticF} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-4ac+b^2} \sqrt{-\frac{c(cx^2+a)}{-4ac}}$$

$$c\sqrt{ex+d} \sqrt{cx^2+bx+a}$$

command

```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 e^{(-\frac{1}{2})} \text{weierstrassPInverse} \left(\frac{4(c^2 d^2 - bcde + (b^2 - 3ac)e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3 d^3 - 3bc^2 d^2 e - 3(b^2 c - 6ac^2)de^2 + (2b^3 - 9abc)e^3)e^{(-3)}}{27c^3}, \frac{cd + (3ca)}{27c^3} \right) \sqrt{c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx + a} \sqrt{ex + d}}{cex^3 + (cd + be)x^2 + ad + (bd + ae)x}, x \right)$$

22.200 Problem number 2466

$$\int \frac{1}{(d + ex)^{3/2} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2e\sqrt{cx^2 + bx + a}}{(ae^2 - bde + cd^2)\sqrt{ex + d}} + \text{EllipticE} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{-4ac + b^2} + \frac{(ae^2 - bde + cd^2)\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{(ae^2 - bde + cd^2)\sqrt{ex + d}}$$

command

```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2cd^2 - bxe^2 + (2cdx - bd)e) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4(c^2 d^2 - bcde + (b^2 - 3ac)e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3 d^3 - 3bc^2 d^2 e - 3(b^2 c - 6ac^2)de^2 + (2b^3 - 9abc)e^3)e^{(-3)}}{27c^3}, \frac{cd + (3ca)}{27c^3} \right) \sqrt{c} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx + a} \sqrt{ex + d}}{ce^2 x^4 + (2cde + be^2)x^3 + ad^2 + (cd^2 + 2bde + ae^2)x^2 + (bd^2 + 2ade)x}, x \right)$$

22.201 Problem number 2467

$$\int \frac{1}{(d+ex)^{5/2} \sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2e\sqrt{cx^2+bx+a}}{3(ae^2-bde+cd^2)(ex+d)^{\frac{3}{2}}} - \frac{4e(-be+2cd)\sqrt{cx^2+bx+a}}{3(ae^2-bde+cd^2)^2\sqrt{ex+d}} \\ & + 2(-be+2cd) \operatorname{EllipticE} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-4ac+b^2} \\ & + \frac{3(ae^2-bde+cd^2)^2\sqrt{cx^2+bx+a} \sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}}{3(ae^2-bde+cd^2)^2\sqrt{ex+d} \sqrt{cx^2+bx+a}} \\ & - \frac{2 \operatorname{EllipticF} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-4ac+b^2} \sqrt{-\frac{c(cx^2+bx+a)}{2cd-e(b+\sqrt{-4ac+b^2})}}}{3(ae^2-bde+cd^2)\sqrt{ex+d} \sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((5c^2d^4 + (2b^2 - 3ac)x^2e^4 - (5bcdx^2 - 2(2b^2 - 3ac)dx)e^3 + (5c^2d^2x^2 - 10bcd^2x + (2b^2 - 3ac)d^2)e^2 + 5(2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2+bx+a} \sqrt{ex+d}}{ce^3x^5 + (3cde^2 + be^3)x^4 + ad^3 + (3cd^2e + 3bde^2 + ae^3)x^3 + (cd^3 + 3bd^2e + 3ade^2)x^2 + (bd^3 + 3ad^2e)x} \right)$$

22.202 Problem number 2468

$$\int \frac{1}{(d+ex)^{7/2} \sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2e\sqrt{cx^2+bx+a}}{5(ae^2-bde+cd^2)(ex+d)^{5/2}} - \frac{8e(-be+2cd)\sqrt{cx^2+bx+a}}{15(ae^2-bde+cd^2)^2(ex+d)^{3/2}} \\ & - \frac{2e(23c^2d^2+8b^2e^2-ce(9ae+23bd))\sqrt{cx^2+bx+a}}{15(ae^2-bde+cd^2)^3\sqrt{ex+d}} \\ & (23c^2d^2+8b^2e^2-ce(9ae+23bd)) \operatorname{EllipticE} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \\ & + \frac{15(ae^2-bde+cd^2)^3\sqrt{cx^2+bx+a}}{15(ae^2-bde+cd^2)^3\sqrt{ex+d}} \sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}} \\ & 8(-be+2cd) \operatorname{EllipticF} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2}\sqrt{-4ac+b^2} \\ & - \frac{8(-be+2cd) \operatorname{EllipticF} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2}\sqrt{-4ac+b^2}}{15(ae^2-bde+cd^2)^2\sqrt{ex+d}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

`integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((22c^3d^6 - (8b^3 - 21abc)x^3e^6 + 3((9b^2c - 14ac^2)dx^3 - (8b^3 - 21abc)dx^2)e^5 - 3(11bc^2d^2x^3 - 3(9b^2c - 14ac^2)dx^2)e^4 + 3(11bc^2d^2x^3 - 3(9b^2c - 14ac^2)dx^2)e^3 + 3(11bc^2d^2x^3 - 3(9b^2c - 14ac^2)dx^2)e^2 + 3(11bc^2d^2x^3 - 3(9b^2c - 14ac^2)dx^2)e \right) \sqrt{cx^2+bx+a} \sqrt{ex+d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{ce^4x^6 + (4cde^3 + be^4)x^5 + ad^4 + (6cd^2e^2 + 4bde^3 + ae^4)x^4 + 2(2cd^3e + 3bd^2e^2 + 2ade^3)x^3 + (cd^4 + 4ade^3)x^2 + 2cd^3e + 3bd^2e^2 + 2ade^3} \right)$$

22.203 Problem number 2469

$$\int \frac{(d+ex)^{7/2}}{(a+bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ex+d)^{\frac{5}{2}}(bd-2ae+(-be+2cd)x)}{(-4ac+b^2)\sqrt{cx^2+bx+a}} + \frac{2e(-be+2cd)(ex+d)^{\frac{3}{2}}\sqrt{cx^2+bx+a}}{c(-4ac+b^2)} \\ & + \frac{4e(3c^2d^2+2b^2e^2-ce(5ae+3bd))\sqrt{ex+d}\sqrt{cx^2+bx+a}}{3c^2(-4ac+b^2)} \\ & + \frac{(-be+2cd)(3c^2d^2+8b^2e^2-ce(29ae+3bd))\text{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{3c^3\sqrt{-4ac+b^2}\sqrt{cx^2+bx+a}} \\ & + \frac{4(ae^2-bde+cd^2)(3c^2d^2+2b^2e^2-ce(5ae+3bd))\text{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{3c^3\sqrt{-4ac+b^2}\sqrt{ex+d}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

`integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((6c^5d^4x^2 + 6bc^4d^4x + 6ac^4d^4 - (8ab^4 - 41a^2b^2c + 30a^3c^2 + (8b^4c - 41ab^2c^2 + 30a^2c^3)x^2 + (8b^5 - 41ab^3c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x\right)$$

22.204 Problem number 2470

$$\int \frac{(d+ex)^{5/2}}{(a+bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(ex+d)^{\frac{3}{2}}(bd-2ae+(-be+2cd)x)}{(-4ac+b^2)\sqrt{cx^2+bx+a}} + \frac{2e(-be+2cd)\sqrt{ex+d}\sqrt{cx^2+bx+a}}{c(-4ac+b^2)}$$

$$+ \frac{2(c^2d^2+b^2e^2-ce(3ae+bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{c^2\sqrt{-4ac+b^2}\sqrt{cx^2+bx+a}}$$

$$+ \frac{2(-be+2cd)(ae^2-bde+cd^2) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{c^2\sqrt{-4ac+b^2}\sqrt{ex+d}\sqrt{cx^2+bx+a}}$$

command

```
integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((2c^4d^3x^2+2bc^3d^3x+2ac^3d^3+(2ab^3-9a^2bc+(2b^3c-9abc^2)x^2+(2b^4-9ab^2c)x)e^3-3((b^2c^2-6ac^3)\right)}{c^2\sqrt{-4ac+b^2}\sqrt{ex+d}\sqrt{cx^2+bx+a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^2x^2+2dex+d^2)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^2x^4+2bcx^3+2abx+(b^2+2ac)x^2+a^2}, x\right)$$

22.205 Problem number 2471

$$\int \frac{(d+ex)^{3/2}}{(a+bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(bd - 2ae + (-be + 2cd)x) \sqrt{ex+d}}{(-4ac + b^2) \sqrt{cx^2 + bx + a}} + (-be + 2cd) \operatorname{EllipticE} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{ex+d} \sqrt{-c}$$

$$+ \frac{c\sqrt{-4ac + b^2} \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex+d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{c\sqrt{-4ac + b^2} \sqrt{ex+d} \sqrt{cx^2 + bx + a}}$$

$$+ 4(ae^2 - bde + cd^2) \operatorname{EllipticF} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-c}$$

command

`integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((2c^3d^2x^2 + 2bc^2d^2x + 2ac^2d^2 - (ab^2 - 6a^2c + (b^2c - 6ac^2)x^2 + (b^3 - 6abc)x)e^2 - 2(bc^2dx^2 + b^2cdx + abc) \right)}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2, x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a} (ex+d)^{\frac{3}{2}}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x \right)$$

22.206 Problem number 2472

$$\int \frac{\sqrt{d+ex}}{(a+bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(2cx+b)\sqrt{ex+d}}{(-4ac+b^2)\sqrt{cx^2+bx+a}}$$

$$+ 2 \operatorname{EllipticE} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{ex+d} \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}$$

$$+ \frac{\sqrt{-4ac+b^2}\sqrt{cx^2+bx+a}\sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}}{2(-be+2cd) \operatorname{EllipticF} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-\frac{c(cx^2+bx+a)}{-4ac+b^2}}}$$

$$- \frac{c\sqrt{-4ac+b^2}\sqrt{ex+d}\sqrt{cx^2+bx+a}}{2}$$

command

```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((2c^2dx^2 + 2bcdx + 2acd - (bcx^2 + b^2x + ab)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + (b^2 - 3ac)e^2)e^{(-2)}}{3c^2}, -\frac{4(2cd - e(b + \sqrt{-4ac + b^2}))}{3c^2} \right) \right)}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2, x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x \right)$$

22.207 Problem number 2473

$$\int \frac{1}{\sqrt{d+ex} (a+bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(bcd - b^2e + 2ace + c(-be + 2cd)x) \sqrt{ex+d}}{(-4ac + b^2)(ae^2 - bde + cd^2) \sqrt{cx^2 + bx + a}}$$

$$+ \frac{(-be + 2cd) \text{EllipticE} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{ex+d} \sqrt{-\frac{c}{-4ac + b^2}}}{(ae^2 - bde + cd^2) \sqrt{-4ac + b^2} \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex+d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}$$

$$+ \frac{4 \text{EllipticF} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}} \sqrt{-\frac{c}{-4ac + b^2}}}{\sqrt{-4ac + b^2} \sqrt{ex+d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^3d^2x^2 + 2bc^2d^2x + 2ac^2d^2 - (ab^2 - 6a^2c + (b^2c - 6ac^2)x^2 + (b^3 - 6abc)x)e^2 - 2(bc^2dx^2 + b^2cdx + abc) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx + a} \sqrt{ex + d}}{c^2ex^5 + (c^2d + 2bce)x^4 + (2bcd + (b^2 + 2ac)e)x^3 + a^2d + (2abe + (b^2 + 2ac)d)x^2 + (2abd + a^2e)x}, x \right)$$

22.208 Problem number 2474

$$\int \frac{1}{(d+ex)^{3/2} (a+bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(bcd - b^2e + 2ace + c(-be + 2cd)x)}{(-4ac + b^2)(ae^2 - bde + cd^2)\sqrt{ex+d}\sqrt{cx^2+bx+a}} - \frac{4e(c^2d^2 + b^2e^2 - ce(3ae + bd))\sqrt{cx^2+bx+a}}{(-4ac + b^2)(ae^2 - bde + cd^2)^2\sqrt{ex+d}}$$

$$+ \frac{2(c^2d^2 + b^2e^2 - ce(3ae + bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{(ae^2 - bde + cd^2)^2\sqrt{-4ac + b^2}\sqrt{cx^2+bx+a}\sqrt{\frac{c(ex+d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}$$

$$+ \frac{2(-be + 2cd) \operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \sqrt{2}\sqrt{-\frac{c(cx^2 + bx + a)}{-4ac}}}{(ae^2 - bde + cd^2)\sqrt{-4ac + b^2}\sqrt{ex+d}\sqrt{cx^2+bx+a}}$$

command

```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((2c^4d^4x^2 + 2bc^3d^4x + 2ac^3d^4 + ((2b^3c - 9abc^2)x^3 + (2b^4 - 9ab^2c)x^2 + (2ab^3 - 9a^2bc)x)e^4 - (3(b^2c^2 - 6\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^2e^2x^6 + 2(c^2de + bce^2)x^5 + (c^2d^2 + 4bcde + (b^2 + 2ac)e^2)x^4 + a^2d^2 + 2(bcd^2 + abe^2 + (b^2 + 2ac)de)x^3 + \dots}\right)$$

22.209 Problem number 2475

$$\int \frac{1}{(d+ex)^{5/2} (a+bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bcd - b^2e + 2ace + c(-be + 2cd)x)}{(-4ac + b^2)(ae^2 - bde + cd^2)(ex + d)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}} \\ & - \frac{4e(3c^2d^2 + 2b^2e^2 - ce(5ae + 3bd)) \sqrt{cx^2 + bx + a}}{3(-4ac + b^2)(ae^2 - bde + cd^2)^2 (ex + d)^{\frac{3}{2}}} \\ & - \frac{2e(-be + 2cd)(3c^2d^2 + 8b^2e^2 - ce(29ae + 3bd)) \sqrt{cx^2 + bx + a}}{3(-4ac + b^2)(ae^2 - bde + cd^2)^3 \sqrt{ex + d}} \\ & + \frac{(-be + 2cd)(3c^2d^2 + 8b^2e^2 - ce(29ae + 3bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{3(ae^2 - bde + cd^2)^3 \sqrt{-4ac + b^2} \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}} \\ & + \frac{4(3c^2d^2 + 2b^2e^2 - ce(5ae + 3bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{3(ae^2 - bde + cd^2)^2 \sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{c^2e^3x^7 + (3c^2de^2 + 2bce^3)x^6 + (3c^2d^2e + 6bcde^2 + (b^2 + 2ac)e^3)x^5 + a^2d^3 + (c^2d^3 + 6bcd^2e + 2abe^3 + \dots}{\dots}\right)$$

22.210 Problem number 2476

$$\int \frac{(d+ex)^{7/2}}{(a+bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(ex+d)^{\frac{5}{2}}(bd-2ae+(-be+2cd)x)}{3(-4ac+b^2)(cx^2+bx+a)^{\frac{3}{2}}} + \frac{2(8bcd(3ae^2+cd^2)-4ace(5ae^2+3cd^2)-b^2(-ae^3+9cd^2e)+(-be+2cd)(8c^2d^2-b^2e^2-4ce(-3ae+2bd)))}{3c(-4ac+b^2)^2\sqrt{cx^2+bx+a}}$$

$$+ \frac{2(-be+2cd)(4c^2d^2-b^2e^2-4ce(-2ae+bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{3c^2(-4ac+b^2)^{\frac{3}{2}}\sqrt{cx^2+bx+a} \sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}}$$

$$+ \frac{2(ae^2-bde+cd^2)(16c^2d^2-b^2e^2-4ce(-5ae+4bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{3c^2(-4ac+b^2)^{\frac{3}{2}}\sqrt{ex+d}\sqrt{cx^2+bx+a}}$$

command

```
integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^3x^3+3de^2x^2+3d^2ex+d^3)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^3x^6+3bc^2x^5+3(b^2c+ac^2)x^4+3a^2bx+(b^3+6abc)x^3+a^3+3(ab^2+a^2c)x^2}, x\right)$$

22.211 Problem number 2477

$$\int \frac{(d+ex)^{5/2}}{(a+bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(ex+d)^{\frac{3}{2}}(bd-2ae+(-be+2cd)x)}{3(-4ac+b^2)(cx^2+bx+a)^{\frac{3}{2}}}$$

$$\frac{2(7b^2de+4acde-8b(ae^2+cd^2)-(16c^2d^2+b^2e^2-4ce(-3ae+4bd))x)\sqrt{ex+d}}{3(-4ac+b^2)^2\sqrt{cx^2+bx+a}}$$

$$(16c^2d^2+b^2e^2-4ce(-3ae+4bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)$$

$$3c(-4ac+b^2)^{\frac{3}{2}}\sqrt{cx^2+bx+a}\sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}$$

$$16(-be+2cd)(ae^2-bde+cd^2) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)$$

$$+ \frac{3c(-4ac+b^2)^{\frac{3}{2}}\sqrt{ex+d}\sqrt{cx^2+bx+a}}$$

command

```
integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^5d^3x^4 + 32bc^4d^3x^3 + 32abc^3d^3x + 16a^2c^3d^3 + 16(b^2c^3 + 2ac^4)d^3x^2 + (a^2b^3 - 12a^3bc + (b^3c^2 - 12abc^3)x \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^2x^2+2dex+d^2)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^3x^6+3bc^2x^5+3(b^2c+ac^2)x^4+3a^2bx+(b^3+6abc)x^3+a^3+3(ab^2+a^2c)x^2}, x\right)$$

22.212 Problem number 2478

$$\int \frac{(d+ex)^{3/2}}{(a+bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(bd - 2ae + (-be + 2cd)x) \sqrt{ex+d}}{3(-4ac + b^2)(cx^2 + bx + a)^{3/2}} + \frac{2(8bcd - 5b^2e + 4ace + 8c(-be + 2cd)x) \sqrt{ex+d}}{3(-4ac + b^2)^2 \sqrt{cx^2 + bx + a}}$$

$$- \frac{8(-be + 2cd) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{ex+d} \sqrt{-\frac{c(ex+d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{3(-4ac + b^2)^{3/2} \sqrt{cx^2 + bx + a}}$$

$$+ \frac{2(16c^2d^2 + 3b^2e^2 - 4ce(-ae + 4bd)) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{ex+d} \sqrt{cx^2 + bx + a}}{3c(-4ac + b^2)^{3/2} \sqrt{ex+d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^4d^2x^4 + 32bc^3d^2x^3 + 32abc^2d^2x + 16a^2c^2d^2 + 16(b^2c^2 + 2ac^3)d^2x^2 + ((b^2c^2 + 12ac^3)x^4 + a^2b^2 + 12a^3c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a} (ex+d)^{3/2}}{c^3x^6 + 3bc^2x^5 + 3(b^2c + ac^2)x^4 + 3a^2bx + (b^3 + 6abc)x^3 + a^3 + 3(ab^2 + a^2c)x^2, x} \right)$$

22.213 Problem number 2479

$$\int \frac{\sqrt{d+ex}}{(a+bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(2cx+b)\sqrt{ex+d}}{3(-4ac+b^2)(cx^2+bx+a)^{3/2}} - \frac{2(9b^2cde-4ac^2de-b^3e^2-4bc(ae^2+2cd^2)-c(16c^2d^2+b^2e^2-4ce(-3ae+4bd))x)\sqrt{ex+d}}{3(-4ac+b^2)^2(ae^2-bde+cd^2)\sqrt{cx^2+bx+a}}$$

$$(16c^2d^2+b^2e^2-4ce(-3ae+4bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)$$

$$3(-4ac+b^2)^{3/2}(ae^2-bde+cd^2)\sqrt{cx^2+bx+a}\sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}$$

$$16(-be+2cd) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)\sqrt{2}\sqrt{-\frac{c(cx^2+bx+a)}{-4ac}}$$

$$+ \frac{3(-4ac+b^2)^{3/2}\sqrt{ex+d}\sqrt{cx^2+bx+a}}$$

command

```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^5d^3x^4 + 32bc^4d^3x^3 + 32abc^3d^3x + 16a^2c^3d^3 + 16(b^2c^3 + 2ac^4)d^3x^2 + (a^2b^3 - 12a^3bc + (b^3c^2 - 12abc^3)x \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^3x^6+3bc^2x^5+3(b^2c+ac^2)x^4+3a^2bx+(b^3+6abc)x^3+a^3+3(ab^2+a^2c)x^2}, x\right)$$

22.214 Problem number 2480

$$\int \frac{1}{\sqrt{d+ex} (a+bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(bcd - b^2e + 2ace + c(-be + 2cd)x) \sqrt{ex+d}}{3(-4ac + b^2)(ae^2 - bde + cd^2)(cx^2 + bx + a)^{3/2}} - \frac{2(3ace(-be + 2cd)^2 - (2ace - b^2e + bcd)(8c^2d^2 - 2b^2e^2 - 5ce(-2ae + bd)) - 2c(-be + 2cd)(4c^2d^2 - b^2e^2 - 4ce(-2ae + bd))) \sqrt{cx^2 + bx + a}}{3(-4ac + b^2)^2 (ae^2 - bde + cd^2)^2 \sqrt{cx^2 + bx + a}} \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) + \frac{2(16c^2d^2 - b^2e^2 - 4ce(-5ae + 4bd)) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}{3(-4ac + b^2)^{3/2} (ae^2 - bde + cd^2)^2 \sqrt{cx^2 + bx + a} \sqrt{ex+d}}$$

command

```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a} \sqrt{ex+d}}{c^3ex^7 + (c^3d + 3bc^2e)x^6 + 3(bc^2d + (b^2c + ac^2)e)x^5 + (3(b^2c + ac^2)d + (b^3 + 6abc)e)x^4 + a^3d + ((b^3 + 6abc)e)x^3 + (3(b^2c + ac^2)d + (b^3 + 6abc)e)x^2 + (3(bc^2d + (b^2c + ac^2)e)x + a^3d) \sqrt{cx^2 + bx + a}}{c^3ex^7 + (c^3d + 3bc^2e)x^6 + 3(bc^2d + (b^2c + ac^2)e)x^5 + (3(b^2c + ac^2)d + (b^3 + 6abc)e)x^4 + a^3d + ((b^3 + 6abc)e)x^3 + (3(b^2c + ac^2)d + (b^3 + 6abc)e)x^2 + (3(bc^2d + (b^2c + ac^2)e)x + a^3d) \sqrt{cx^2 + bx + a}} \right)$$

22.215 Problem number 2481

$$\int \frac{1}{(d+ex)^{3/2} (a+bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(bcd - b^2e + 2ace + c(-be + 2cd)x)}{3(-4ac + b^2)(ae^2 - bde + cd^2)(cx^2 + bx + a)^{3/2} \sqrt{ex + d}}$$

$$+ \frac{2(5ace(-be + 2cd)^2 - (2ace - b^2e + bcd)(8c^2d^2 - 4b^2e^2 - ce(-14ae + 3bd)) - 4c(-be + 2cd)(2c^2d^2 - b^2e^2 - 2))}{3(-4ac + b^2)^2 (ae^2 - bde + cd^2)^2 \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

$$+ \frac{2e(16c^4d^4 - 8b^4e^4 - 4c^3d^2e(-15ae + 8bd) + b^2ce^3(57ae + 7bd) + 3c^2e^2(-28a^2e^2 - 20abde + 3b^2d^2)) \sqrt{cx^2 + bx + a}}{3(-4ac + b^2)^2 (ae^2 - bde + cd^2)^3 \sqrt{ex + d}}$$

$$+ \frac{(16c^4d^4 - 8b^4e^4 - 4c^3d^2e(-15ae + 8bd) + b^2ce^3(57ae + 7bd) + 3c^2e^2(-28a^2e^2 - 20abde + 3b^2d^2)) \text{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{3(-4ac + b^2)^{3/2} (ae^2 - bde + cd^2)^3 \sqrt{cx^2 + bx + a}}$$

$$+ \frac{8(-be + 2cd)(2c^2d^2 - b^2e^2 - 2ce(-3ae + bd)) \text{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{3(-4ac + b^2)^{3/2} (ae^2 - bde + cd^2)^2 \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{c^3e^2x^8 + (2c^3de + 3bc^2e^2)x^7 + (c^3d^2 + 6bc^2de + 3(b^2c + ac^2)e^2)x^6 + (3bc^2d^2 + 6(b^2c + ac^2)de + (b^3 + 3ac^2d))x^5 + (3c^2d^2e + 6bc^2de + 3(b^2c + ac^2)e^2)x^4 + (3c^2d^2e + 6bc^2de + 3(b^2c + ac^2)e^2)x^3 + (3c^2d^2e + 6bc^2de + 3(b^2c + ac^2)e^2)x^2 + (3c^2d^2e + 6bc^2de + 3(b^2c + ac^2)e^2)x + (3c^2d^2e + 6bc^2de + 3(b^2c + ac^2)e^2)}{(d+ex)^{3/2} (a+bx+cx^2)^{5/2}}\right)$$

23 Test file number 34

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/34_1.2.1.3-d+e_x-^m-f+g_x-a+b_x+c_x^2-^p

23.1 Problem number 433

$$\int (ex)^{7/2}(A+Bx)\sqrt{a+cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{14aB e^2 (ex)^{\frac{3}{2}} (cx^2 + a)^{\frac{3}{2}}}{117c^2} + \frac{2Ae(ex)^{\frac{5}{2}} (cx^2 + a)^{\frac{3}{2}}}{11c} + \frac{2B(ex)^{\frac{7}{2}} (cx^2 + a)^{\frac{3}{2}}}{13c} \\ & -\frac{10aA e^3 (cx^2 + a)^{\frac{3}{2}} \sqrt{ex}}{77c^2} + \frac{28a^3 B e^4 x \sqrt{cx^2 + a}}{195c^{\frac{5}{2}} (\sqrt{a} + x\sqrt{c}) \sqrt{ex}} + \frac{2a^2 e^3 (539Bx + 325A) \sqrt{ex} \sqrt{cx^2 + a}}{15015c^2} \\ & -\frac{28a^{\frac{13}{4}} B e^4 \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{c}) \sqrt{x}}{195 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{11}{4}} \sqrt{ex} \sqrt{cx^2 + a}}}{2a^{\frac{11}{4}} e^4 \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (539B\sqrt{a} + 325A\sqrt{c}) (\sqrt{a} + x\sqrt{c}) \sqrt{x}}{15015 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{11}{4}} \sqrt{ex} \sqrt{cx^2 + a}} \end{aligned}$$

command

`integrate((e*x)^(7/2)*(B*x+A)*(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(1950 Aa^3 \sqrt{c} e^{\frac{7}{2}} \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 3234 Ba^3 \sqrt{c} e^{\frac{7}{2}} \operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right) \right)$$

45

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Be^3x^4 + Ae^3x^3\right)\sqrt{cx^2 + a} \sqrt{ex}, x\right)$$

23.2 Problem number 434

$$\int (ex)^{5/2}(A+Bx)\sqrt{a+cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ae(ex)^{\frac{3}{2}}(cx^2+a)^{\frac{3}{2}}}{9c} + \frac{2B(ex)^{\frac{5}{2}}(cx^2+a)^{\frac{3}{2}}}{11c} - \frac{10aB e^2 (cx^2+a)^{\frac{3}{2}} \sqrt{ex}}{77c^2} \\ & - \frac{4a^2 A e^3 x \sqrt{cx^2+a}}{15c^{\frac{3}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} + \frac{2a e^2 (-77Acx + 25Ba) \sqrt{ex} \sqrt{cx^2+a}}{1155c^2} \\ & + \frac{4a^{\frac{9}{4}} A e^3 \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{c}) \sqrt{x} \sqrt{\frac{a}{c}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{ex} \sqrt{cx^2+a}} \\ & + \frac{2a^{\frac{9}{4}} e^3 \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (25B\sqrt{a} - 77A\sqrt{c}) (\sqrt{a}+x\sqrt{c}) \sqrt{x} \sqrt{\frac{a}{c}}}{1155 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{9}{4}} \sqrt{ex} \sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x)^(5/2)*(B*x+A)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(150 B a^3 \sqrt{c} e^{\frac{5}{2}} \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 462 A a^2 c^{\frac{3}{2}} e^{\frac{5}{2}} \operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}\right)\right) \right)}{3465 c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Be^2x^3 + Ae^2x^2\right)\sqrt{cx^2+a} \sqrt{ex}, x\right)$$

23.3 Problem number 435

$$\int (ex)^{3/2}(A+Bx)\sqrt{a+cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(ex)^{\frac{3}{2}}(cx^2+a)^{\frac{3}{2}}}{9c} + \frac{2Ae(cx^2+a)^{\frac{3}{2}}\sqrt{ex}}{7c} \\ & - \frac{4a^2B e^2 x \sqrt{cx^2+a}}{15c^{\frac{3}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} - \frac{2ae(7Bx+5A)\sqrt{ex}\sqrt{cx^2+a}}{105c} \\ & + \frac{4a^{\frac{9}{4}}B e^2 \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{a+cx^2}{a}}}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}} \\ & - \frac{2a^{\frac{7}{4}}e^2 \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(7B\sqrt{a}+5A\sqrt{c})(\sqrt{a}+x\sqrt{c})\sqrt{ex}}{105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x)^(3/2)*(B*x+A)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(30Aa^2\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c},0,x\right)-42Ba^2\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c},0,\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}\right)\right)\right)}{315c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bex^2+Aex\right)\sqrt{cx^2+a}\sqrt{ex},x\right)$$

23.4 Problem number 436

$$\int \sqrt{ex} (A + Bx) \sqrt{a + cx^2} dx$$

Optimal antiderivative

$$\frac{2B(cx^2 + a)^{\frac{3}{2}} \sqrt{ex}}{7c} + \frac{4aAex \sqrt{cx^2 + a}}{5\sqrt{c} (\sqrt{a} + x\sqrt{c}) \sqrt{ex}} - \frac{2(-21Acx + 5Ba) \sqrt{ex} \sqrt{cx^2 + a}}{105c}$$

$$\frac{4a^{\frac{5}{4}} Ae \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{\frac{cx^2 + a}{a}}}{5 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{3}{4}} \sqrt{ex} \sqrt{cx^2 + a}}$$

$$\frac{2a^{\frac{5}{4}} e \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (5B\sqrt{a} - 21A\sqrt{c}) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{\frac{cx^2 + a}{a}}}{105 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{5}{4}} \sqrt{ex} \sqrt{cx^2 + a}}$$

command

```
integrate((e*x)^(1/2)*(B*x+A)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10 Ba^2 \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 42 Aac^{\frac{3}{2}} e^{\frac{1}{2}} \operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right) \right)}{105 c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^2 + a} (Bx + A) \sqrt{ex}, x\right)$$

23.5 Problem number 437

$$\int \frac{(A + Bx) \sqrt{a + cx^2}}{\sqrt{ex}} dx$$

Optimal antiderivative

$$\frac{4aBx\sqrt{cx^2+a}}{5\sqrt{c}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} + \frac{2(3Bx+5A)\sqrt{ex}\sqrt{cx^2+a}}{15e}$$

$$- \frac{4a^{\frac{5}{4}}B\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{c}{(\sqrt{a}+x\sqrt{c})^2}}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

$$+ \frac{2a^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(3B\sqrt{a}+5A\sqrt{c})(\sqrt{a}+x\sqrt{c})}}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(10Aa\sqrt{c}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 6Ba\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{15c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}(Bx+A)\sqrt{ex}}{ex}, x\right)$$

23.6 Problem number 438

$$\int \frac{(A+Bx)\sqrt{a+cx^2}}{(ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(-Bx + 3A) \sqrt{cx^2 + a}}{3e\sqrt{ex}} + \frac{4Ax\sqrt{c} \sqrt{cx^2 + a}}{e(\sqrt{a} + x\sqrt{c}) \sqrt{ex}}$$

$$\frac{4a^{\frac{1}{4}} A c^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{\frac{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}}}{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) e\sqrt{ex} \sqrt{cx^2 + a}}$$

$$+ \frac{2a^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (B\sqrt{a} + 3A\sqrt{c}) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{\frac{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}}}{3 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{1}{4}} e\sqrt{ex} \sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2Ba\sqrt{c}x\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 6Ac^{\frac{3}{2}}x\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right) + (B\sqrt{a} + 3A\sqrt{c})\sqrt{x}\sqrt{cx^2 + a}}{3cx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a} (Bx + A)\sqrt{ex}}{e^2x^2}, x\right)$$

23.7 Problem number 439

$$\int \frac{(A + Bx)\sqrt{a + cx^2}}{(ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2(3Bx + A) \sqrt{cx^2 + a}}{3e (ex)^{\frac{3}{2}}} + \frac{4Bx\sqrt{c} \sqrt{cx^2 + a}}{e^2 (\sqrt{a} + x\sqrt{c}) \sqrt{ex}} \\
& - \frac{4a^{\frac{1}{4}} B c^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{cx^2 + a}}{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) e^2 \sqrt{ex} \sqrt{cx^2 + a}} \\
& + \frac{2c^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (3B\sqrt{a} + A\sqrt{c}) (\sqrt{a} + x\sqrt{c}) \sqrt{cx^2 + a}}{3 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} e^2 \sqrt{ex} \sqrt{cx^2 + a}}
\end{aligned}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2A\sqrt{c}x^2\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 6B\sqrt{c}x^2\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{3x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a} (Bx + A)\sqrt{ex}}{e^3 x^3}, x\right)$$

23.8 Problem number 440

$$\int \frac{(A + Bx)\sqrt{a + cx^2}}{(ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(5Bx + 3A)\sqrt{cx^2 + a}}{15e(ex)^{\frac{5}{2}}} - \frac{4Ac\sqrt{cx^2 + a}}{5ae^3\sqrt{ex}} + \frac{4Ac^{\frac{3}{2}}x\sqrt{cx^2 + a}}{5ae^3(\sqrt{a} + x\sqrt{c})\sqrt{ex}}$$

$$\frac{4Ac^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\frac{c}{(\sqrt{a} + x\sqrt{c})^2}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}e^3\sqrt{ex}\sqrt{cx^2 + a}}$$

$$+ \frac{2c^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(5B\sqrt{a} + 3A\sqrt{c})(\sqrt{a} + x\sqrt{c})}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}e^3\sqrt{ex}\sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(10Ba\sqrt{c}x^3\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 6Ac^{\frac{3}{2}}x^3\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{15ax^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a}(Bx + A)\sqrt{ex}}{e^4x^4}, x\right)$$

23.9 Problem number 441

$$\int \frac{(A + Bx)\sqrt{a + cx^2}}{(ex)^{9/2}} dx$$

Optimal antiderivative

$$\frac{4Ac\sqrt{cx^2+a}}{21ae^3(ex)^{\frac{3}{2}}} - \frac{2(7Bx+5A)\sqrt{cx^2+a}}{35e(ex)^{\frac{7}{2}}} - \frac{4Bc\sqrt{cx^2+a}}{5ae^4\sqrt{ex}} + \frac{4Bc^{\frac{3}{2}}x\sqrt{cx^2+a}}{5ae^4(\sqrt{a}+x\sqrt{c})\sqrt{ex}}$$

$$- \frac{4Bc^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{c}{a}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}e^4\sqrt{ex}\sqrt{cx^2+a}}$$

$$+ \frac{2c^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(21B\sqrt{a}-5A\sqrt{c})(\sqrt{a}+x\sqrt{c})}{105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{5}{4}}e^4\sqrt{ex}\sqrt{cx^2+a}}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(1/2)/(e*x)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(10Ac^{\frac{3}{2}}x^4\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 42Bc^{\frac{3}{2}}x^4\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{105ax^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}(Bx+A)\sqrt{ex}}{e^5x^5}, x\right)$$

23.10 Problem number 442

$$\int (ex)^{5/2}(A+Bx)(a+cx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ae(ex)^{\frac{3}{2}}(cx^2+a)^{\frac{5}{2}}}{13c} + \frac{2B(ex)^{\frac{5}{2}}(cx^2+a)^{\frac{5}{2}}}{15c} + \frac{2ae^2(-77Acx+13Ba)(cx^2+a)^{\frac{3}{2}}\sqrt{ex}}{3003c^2} \\ & - \frac{2aBe^2(cx^2+a)^{\frac{5}{2}}\sqrt{ex}}{33c^2} - \frac{8a^3Ae^3x\sqrt{cx^2+a}}{65c^{\frac{3}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} + \frac{4a^2e^2(-231Acx+65Ba)\sqrt{ex}\sqrt{cx^2+a}}{15015c^2} \\ & + \frac{8a^{\frac{13}{4}}Ae^3\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{a}{c}}}{65\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}} \\ & + \frac{4a^{\frac{13}{4}}e^3\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(65B\sqrt{a}-231A\sqrt{c})(\sqrt{cx^2+a})}{15015\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{9}{4}}\sqrt{ex}\sqrt{cx^2+a}} \end{aligned}$$

command

`integrate((e*x)^(5/2)*(B*x+A)*(c*x^2+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(260Ba^4\sqrt{c}e^{\frac{5}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 924Aa^3c^{\frac{3}{2}}e^{\frac{5}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}\right)\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bce^2x^5 + Ace^2x^4 + Bae^2x^3 + Aae^2x^2\right)\sqrt{cx^2+a}\sqrt{ex}, x\right)$$

23.11 Problem number 443

$$\int (ex)^{3/2}(A+Bx)(a+cx^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{2B(ex)^{\frac{3}{2}}(cx^2+a)^{\frac{5}{2}}}{13c} - \frac{2ae(77Bx+39A)(cx^2+a)^{\frac{3}{2}}\sqrt{ex}}{3003c} + \frac{2Ae(cx^2+a)^{\frac{5}{2}}\sqrt{ex}}{11c}$$

$$- \frac{8a^3B e^2 x \sqrt{cx^2+a}}{65c^{\frac{3}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} - \frac{4a^2e(77Bx+65A)\sqrt{ex}\sqrt{cx^2+a}}{5005c}$$

$$+ \frac{8a^{\frac{13}{4}}B e^2 \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{cx^2+a}}{65 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

$$- \frac{4a^{\frac{11}{4}}e^2 \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (77B\sqrt{a}+65A\sqrt{c})(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{cx^2+a}}{5005 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

command

```
integrate((e*x)^(3/2)*(B*x+A)*(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(780Aa^3\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c},0,x\right) - 924Ba^3\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c},0,\operatorname{weierstrassPInverse}\left(-\frac{4a}{c},0,x\right)\right)\right)}{15}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bcx^4 + Acex^3 + Baex^2 + Aaex\right)\sqrt{cx^2+a}\sqrt{ex},x\right)$$

23.12 Problem number 444

$$\int \sqrt{ex}(A+Bx)(a+cx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-77Acx + 9Ba)(cx^2 + a)^{\frac{3}{2}}\sqrt{ex}}{693c} + \frac{2B(cx^2 + a)^{\frac{5}{2}}\sqrt{ex}}{11c} \\ & + \frac{8a^2Aex\sqrt{cx^2 + a}}{15\sqrt{c}(\sqrt{a} + x\sqrt{c})\sqrt{ex}} - \frac{4a(-77Acx + 15Ba)\sqrt{ex}\sqrt{cx^2 + a}}{1155c} \\ & - \frac{8a^{\frac{9}{4}}Ae\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\frac{a}{\sqrt{a} + x\sqrt{c}}}}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}} \\ & - \frac{4a^{\frac{9}{4}}e\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(15B\sqrt{a} - 77A\sqrt{c})(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\frac{a}{\sqrt{a} + x\sqrt{c}}}}{1155\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{5}{4}}\sqrt{ex}\sqrt{cx^2 + a}} \end{aligned}$$

command

```
integrate((e*x)^(1/2)*(B*x+A)*(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(180Ba^3\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 924Aa^2c^{\frac{3}{2}}e^{\frac{1}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{3465c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bcx^3 + Acx^2 + Bax + Aa\right)\sqrt{cx^2 + a}\sqrt{ex}, x\right)$$

23.13 Problem number 445

$$\int \frac{(A + Bx)(a + cx^2)^{3/2}}{\sqrt{ex}} dx$$

Optimal antiderivative

$$\frac{2(7Bx + 9A)(cx^2 + a)^{\frac{3}{2}}\sqrt{ex}}{63e} + \frac{8a^2Bx\sqrt{cx^2 + a}}{15\sqrt{c}(\sqrt{a} + x\sqrt{c})\sqrt{ex}} + \frac{4a(7Bx + 15A)\sqrt{ex}\sqrt{cx^2 + a}}{105e}$$

$$8a^{\frac{9}{4}}B\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\frac{c}{(\sqrt{a} + x\sqrt{c})^2}}$$

$$15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}$$

$$4a^{\frac{7}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(7B\sqrt{a} + 15A\sqrt{c})(\sqrt{a} + x\sqrt{c})\sqrt{x}$$

$$+ \frac{105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}}{105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(180Aa^2\sqrt{c}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 84Ba^2\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)$$

315 c

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^3 + Acx^2 + Bax + Aa)\sqrt{cx^2 + a}\sqrt{ex}}{ex}, x\right)$$

23.14 Problem number 446

$$\int \frac{(A + Bx)(a + cx^2)^{3/2}}{(ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(-Bx + 7A)(cx^2 + a)^{\frac{3}{2}}}{7e\sqrt{ex}} + \frac{24aAx\sqrt{c}\sqrt{cx^2 + a}}{5e(\sqrt{a} + x\sqrt{c})\sqrt{ex}} + \frac{4(21Acx + 5Ba)\sqrt{ex}\sqrt{cx^2 + a}}{35e^2} \\
 & -\frac{24a^{\frac{5}{4}}Ac^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\frac{a}{c}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e\sqrt{ex}\sqrt{cx^2 + a}} \\
 & +\frac{4a^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(5B\sqrt{a} + 21A\sqrt{c})(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\frac{a}{c}}}{35\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{1}{4}}e\sqrt{ex}\sqrt{cx^2 + a}}
 \end{aligned}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(3/2)/(e*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(20Ba^2\sqrt{c}x\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 84Aac^{\frac{3}{2}}x\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{35cx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^3 + Acx^2 + Bax + Aa)\sqrt{cx^2 + a}\sqrt{ex}}{e^2x^2}, x\right)$$

23.15 Problem number 447

$$\int \frac{(A + Bx)(a + cx^2)^{3/2}}{(ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(-3Bx + 5A)(cx^2 + a)^{\frac{3}{2}}}{15e(ex)^{\frac{3}{2}}} - \frac{4(-5Acx + 9Ba)\sqrt{cx^2 + a}}{15e^2\sqrt{ex}} + \frac{24aBx\sqrt{c}\sqrt{cx^2 + a}}{5e^2(\sqrt{a} + x\sqrt{c})\sqrt{ex}}$$

$$\frac{24a^{\frac{5}{4}}Bc^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^2\sqrt{ex}\sqrt{cx^2 + a}}$$

$$+ \frac{4a^{\frac{3}{4}}c^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(9B\sqrt{a} + 5A\sqrt{c})(\sqrt{a} + x\sqrt{c})\sqrt{x}}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^2\sqrt{ex}\sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(3/2)/(e*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(20Aa\sqrt{c}x^2\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 36Ba\sqrt{c}x^2\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{15x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^3 + Acx^2 + Bax + Aa)\sqrt{cx^2 + a}\sqrt{ex}}{e^3x^3}, x\right)$$

23.16 Problem number 448

$$\int \frac{(A + Bx)(a + cx^2)^{3/2}}{(ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2(5Bx + 3A)(cx^2 + a)^{\frac{3}{2}}}{15e(ex)^{\frac{5}{2}}} - \frac{4c(-5Bx + 9A)\sqrt{cx^2 + a}}{15e^3\sqrt{ex}} + \frac{24Ac^{\frac{3}{2}}x\sqrt{cx^2 + a}}{5e^3(\sqrt{a} + x\sqrt{c})\sqrt{ex}} \\
& - \frac{24a^{\frac{1}{4}}Ac^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^3\sqrt{ex}\sqrt{cx^2 + a}} \\
& + \frac{4a^{\frac{1}{4}}c^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(5B\sqrt{a} + 9A\sqrt{c})(\sqrt{a} + x\sqrt{c})\sqrt{x}}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^3\sqrt{ex}\sqrt{cx^2 + a}}
\end{aligned}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(3/2)/(e*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(20Ba\sqrt{c}x^3\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 36Ac^{\frac{3}{2}}x^3\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{15x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^3 + Acx^2 + Bax + Aa)\sqrt{cx^2 + a}\sqrt{ex}}{e^4x^4}, x\right)$$

23.17 Problem number 449

$$\int \frac{(A + Bx)(a + cx^2)^{3/2}}{(ex)^{9/2}} dx$$

Optimal antiderivative

$$\frac{2(7Bx + 5A)(cx^2 + a)^{\frac{3}{2}}}{35e(ex)^{\frac{7}{2}}} - \frac{4c(21Bx + 5A)\sqrt{cx^2 + a}}{35e^3(ex)^{\frac{3}{2}}} + \frac{24Bc^{\frac{3}{2}}x\sqrt{cx^2 + a}}{5e^4(\sqrt{a} + x\sqrt{c})\sqrt{ex}}$$

$$\frac{24a^{\frac{1}{4}}Bc^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^4\sqrt{ex}\sqrt{cx^2 + a}}$$

$$+ \frac{4c^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(21B\sqrt{a} + 5A\sqrt{c})(\sqrt{a} + x\sqrt{c})}{35\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{1}{4}}e^4\sqrt{ex}\sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(3/2)/(e*x)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(20Ac^{\frac{3}{2}}x^4\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 84Bc^{\frac{3}{2}}x^4\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{35x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^3 + Acx^2 + Bax + Aa)\sqrt{cx^2 + a}\sqrt{ex}}{e^5x^5}, x\right)$$

23.18 Problem number 450

$$\int (ex)^{3/2}(A + Bx)(a + cx^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{2B(ex)^{\frac{3}{2}}(cx^2+a)^{\frac{7}{2}}}{17c} - \frac{4a^2e(385Bx+221A)(cx^2+a)^{\frac{3}{2}}\sqrt{ex}}{51051c} \\
 & - \frac{2ae(495Bx+221A)(cx^2+a)^{\frac{5}{2}}\sqrt{ex}}{36465c} + \frac{2Ae(cx^2+a)^{\frac{7}{2}}\sqrt{ex}}{15c} \\
 & - \frac{16a^4B e^2 x \sqrt{cx^2+a}}{221c^{\frac{3}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} - \frac{8a^3e(231Bx+221A)\sqrt{ex}\sqrt{cx^2+a}}{51051c} \\
 & + \frac{16a^{\frac{17}{4}}B e^2 \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\dots}}{221 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}} \\
 & - \frac{8a^{\frac{15}{4}}e^2 \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (231B\sqrt{a}+221A\sqrt{c})\sqrt{\dots}}{51051 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}}
 \end{aligned}$$

command

```
integrate((e*x)^(3/2)*(B*x+A)*(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(8840Aa^4\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c},0,x\right) - 9240Ba^4\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c},0,\operatorname{weierstrassPInverse}\right)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bc^2ex^6 + Ac^2ex^5 + 2Bacex^4 + 2Aacex^3 + Ba^2ex^2 + Aa^2ex\right)\sqrt{cx^2+a}\sqrt{ex},x\right)$$

23.19 Problem number 451

$$\int \sqrt{ex}(A+Bx)(a+cx^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(-385Acx + 39Ba)(cx^2 + a)^{\frac{3}{2}}\sqrt{ex}}{9009c} - \frac{2(-165Acx + 13Ba)(cx^2 + a)^{\frac{5}{2}}\sqrt{ex}}{2145c} \\ & + \frac{2B(cx^2 + a)^{\frac{7}{2}}\sqrt{ex}}{15c} + \frac{16a^3Aex\sqrt{cx^2 + a}}{39\sqrt{c}(\sqrt{a} + x\sqrt{c})\sqrt{ex}} - \frac{8a^2(-77Acx + 13Ba)\sqrt{ex}\sqrt{cx^2 + a}}{3003c} \\ & - \frac{16a^{\frac{13}{4}}Ae\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)^2 + 1}}{39\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}} \\ & - \frac{8a^{\frac{13}{4}}e\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(13B\sqrt{a} - 77A\sqrt{c})(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)^2 + 1}}{3003\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{5}{4}}\sqrt{ex}\sqrt{cx^2 + a}} \end{aligned}$$

command

`integrate((e*x)^(1/2)*(B*x+A)*(c*x^2+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(1560Ba^4\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 9240Aa^3c^{\frac{3}{2}}e^{\frac{1}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)\sqrt{cx^2 + a}\sqrt{ex}}{3003c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bc^2x^5 + Ac^2x^4 + 2Bacx^3 + 2Aacx^2 + Ba^2x + Aa^2\right)\sqrt{cx^2 + a}\sqrt{ex}, x\right)$$

23.20 Problem number 452

$$\int \frac{(A + Bx)(a + cx^2)^{5/2}}{\sqrt{ex}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{20a(77Bx + 117A)(cx^2 + a)^{\frac{3}{2}}\sqrt{ex}}{9009e} + \frac{2(11Bx + 13A)(cx^2 + a)^{\frac{5}{2}}\sqrt{ex}}{143e} \\
 & + \frac{16a^3Bx\sqrt{cx^2 + a}}{39\sqrt{c}(\sqrt{a} + x\sqrt{c})\sqrt{ex}} + \frac{8a^2(77Bx + 195A)\sqrt{ex}\sqrt{cx^2 + a}}{3003e} \\
 & - \frac{16a^{\frac{13}{4}}B\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\frac{a}{c}}}}{39\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}} \\
 & + \frac{8a^{\frac{11}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(77B\sqrt{a} + 195A\sqrt{c})(\sqrt{a} + x\sqrt{c})\sqrt{x}}{3003\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}}
 \end{aligned}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4680Aa^3\sqrt{c}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 1848Ba^3\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{9009e}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bc^2x^5 + Ac^2x^4 + 2Bacx^3 + 2Aacx^2 + Ba^2x + Aa^2)\sqrt{cx^2 + a}\sqrt{ex}}{ex}, x\right)$$

23.21 Problem number 453

$$\int \frac{(A + Bx)(a + cx^2)^{5/2}}{(ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(-Bx + 11A)(cx^2 + a)^{\frac{5}{2}}}{11e\sqrt{ex}} + \frac{20(77Acx + 9Ba)(cx^2 + a)^{\frac{3}{2}}\sqrt{ex}}{693e^2} \\
 & + \frac{16a^2Ax\sqrt{c}\sqrt{cx^2 + a}}{3e(\sqrt{a} + x\sqrt{c})\sqrt{ex}} + \frac{8a(77Acx + 15Ba)\sqrt{ex}\sqrt{cx^2 + a}}{231e^2} \\
 & - \frac{16a^{\frac{9}{4}}Ac^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}}{3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e\sqrt{ex}\sqrt{cx^2 + a}} \sqrt{\dots} \\
 & + \frac{8a^{\frac{9}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(15B\sqrt{a} + 77A\sqrt{c})(\sqrt{a} + x\sqrt{c})}{231\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{1}{4}}e\sqrt{ex}\sqrt{cx^2 + a}}
 \end{aligned}$$

command

`integrate((B*x+A)*(c*x^2+a)^(5/2)/(e*x)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(360Ba^3\sqrt{c}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 1848Aa^2c^{\frac{3}{2}}x\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{693cx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bc^2x^5 + Ac^2x^4 + 2Bacx^3 + 2Aacx^2 + Ba^2x + Aa^2)\sqrt{cx^2 + a}\sqrt{ex}}{e^2x^2}, x\right)$$

23.22 Problem number 454

$$\int \frac{(A + Bx)(a + cx^2)^{5/2}}{(ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-Bx + 3A)(cx^2 + a)^{\frac{5}{2}}}{9e(ex)^{\frac{3}{2}}} - \frac{20(-3Acx + 7Ba)(cx^2 + a)^{\frac{3}{2}}}{63e^2\sqrt{ex}} \\ & + \frac{16a^2Bx\sqrt{c}\sqrt{cx^2 + a}}{3e^2(\sqrt{a} + x\sqrt{c})\sqrt{ex}} + \frac{8ac(7Bx + 5A)\sqrt{ex}\sqrt{cx^2 + a}}{21e^3} \\ & - \frac{16a^{\frac{9}{4}}Bc^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}}{3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^2\sqrt{ex}\sqrt{cx^2 + a}} \\ & + \frac{8a^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(7B\sqrt{a} + 5A\sqrt{c})(\sqrt{a} + x\sqrt{c})}{21\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^2\sqrt{ex}\sqrt{cx^2 + a}} \end{aligned}$$

command

`integrate((B*x+A)*(c*x^2+a)^(5/2)/(e*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(120Aa^2\sqrt{c}x^2\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 168Ba^2\sqrt{c}x^2\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{63x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bc^2x^5 + Ac^2x^4 + 2Bacx^3 + 2Aacx^2 + Ba^2x + Aa^2)\sqrt{cx^2 + a}\sqrt{ex}}{e^3x^3}, x\right)$$

23.23 Problem number 455

$$\int \frac{(A + Bx)(a + cx^2)^{5/2}}{(ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{4(-21Acx + 25Ba)(cx^2 + a)^{\frac{3}{2}}}{105e^2(ex)^{\frac{3}{2}}} - \frac{2(-5Bx + 7A)(cx^2 + a)^{\frac{5}{2}}}{35e(ex)^{\frac{5}{2}}} \\
 & - \frac{8ac(-25Bx + 63A)\sqrt{cx^2 + a}}{105e^3\sqrt{ex}} + \frac{48aAc^{\frac{3}{2}}x\sqrt{cx^2 + a}}{5e^3(\sqrt{a} + x\sqrt{c})\sqrt{ex}} \\
 & - \frac{48a^{\frac{5}{4}}Ac^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^3\sqrt{ex}\sqrt{cx^2 + a}} \\
 & + \frac{8a^{\frac{5}{4}}c^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(25B\sqrt{a} + 63A\sqrt{c})(\sqrt{a} + x\sqrt{c})}{105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^3\sqrt{ex}\sqrt{cx^2 + a}}
 \end{aligned}$$

command

`integrate((B*x+A)*(c*x^2+a)^(5/2)/(e*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(200Ba^2\sqrt{c}x^3\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 504Aac^{\frac{3}{2}}x^3\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{105x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bc^2x^5 + Ac^2x^4 + 2Bacx^3 + 2Aacx^2 + Ba^2x + Aa^2)\sqrt{cx^2 + a}\sqrt{ex}}{e^4x^4}, x\right)$$

23.24 Problem number 456

$$\int \frac{(A + Bx)(a + cx^2)^{5/2}}{(ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4(25Acx + 21Ba)(cx^2 + a)^{\frac{3}{2}}}{105e^2(ex)^{\frac{5}{2}}} - \frac{2(-7Bx + 5A)(cx^2 + a)^{\frac{5}{2}}}{35e(ex)^{\frac{7}{2}}} \\ & - \frac{8c(-25Acx + 63Ba)\sqrt{cx^2 + a}}{105e^4\sqrt{ex}} + \frac{48aBc^{\frac{3}{2}}x\sqrt{cx^2 + a}}{5e^4(\sqrt{a} + x\sqrt{c})\sqrt{ex}} \\ & - \frac{48a^{\frac{5}{4}}Bc^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^4\sqrt{ex}\sqrt{cx^2 + a}} \\ & + \frac{8a^{\frac{3}{4}}c^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(63B\sqrt{a} + 25A\sqrt{c})(\sqrt{a} + x\sqrt{c})\sqrt{x}}{105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^4\sqrt{ex}\sqrt{cx^2 + a}} \end{aligned}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(5/2)/(e*x)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(200Aac^{\frac{3}{2}}x^4\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 504Bac^{\frac{3}{2}}x^4\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{105x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bc^2x^5 + Ac^2x^4 + 2Bacx^3 + 2Aacx^2 + Ba^2x + Aa^2)\sqrt{cx^2 + a}\sqrt{ex}}{e^5x^5}, x\right)$$

23.25 Problem number 457

$$\int \frac{(A + Bx)(a + cx^2)^{5/2}}{(ex)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{4c(15Bx+7A)(cx^2+a)^{\frac{3}{2}}}{63e^3(ex)^{\frac{5}{2}}} - \frac{2(9Bx+7A)(cx^2+a)^{\frac{5}{2}}}{63e(ex)^{\frac{9}{2}}} \\
 & -\frac{8c^2(-5Bx+7A)\sqrt{cx^2+a}}{21e^5\sqrt{ex}} + \frac{16Ac^{\frac{5}{2}}x\sqrt{cx^2+a}}{3e^5(\sqrt{a}+x\sqrt{c})\sqrt{ex}} \\
 & -\frac{16a^{\frac{1}{4}}Ac^{\frac{9}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}}{3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^5\sqrt{ex}\sqrt{cx^2+a}} \\
 & +\frac{8a^{\frac{1}{4}}c^{\frac{7}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(5B\sqrt{a}+7A\sqrt{c})(\sqrt{a}+x\sqrt{c})}{21\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)e^5\sqrt{ex}\sqrt{cx^2+a}}
 \end{aligned}$$

command

`integrate((B*x+A)*(c*x^2+a)^(5/2)/(e*x)^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(120Bac^{\frac{3}{2}}x^5\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 168Ac^{\frac{5}{2}}x^5\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{63x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bc^2x^5 + Ac^2x^4 + 2Bacx^3 + 2Aacx^2 + Ba^2x + Aa^2)\sqrt{cx^2+a}\sqrt{ex}}{e^6x^6}, x\right)$$

23.26 Problem number 458

$$\int \frac{(ex)^{7/2}(A+Bx)}{\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{14aB e^2 (ex)^{\frac{3}{2}} \sqrt{cx^2 + a}}{45c^2} + \frac{2Ae(ex)^{\frac{5}{2}} \sqrt{cx^2 + a}}{7c} + \frac{2B(ex)^{\frac{7}{2}} \sqrt{cx^2 + a}}{9c} \\
 & + \frac{14a^2 B e^4 x \sqrt{cx^2 + a}}{15c^{\frac{5}{2}} (\sqrt{a} + x\sqrt{c}) \sqrt{ex}} - \frac{10aA e^3 \sqrt{ex} \sqrt{cx^2 + a}}{21c^2} \\
 & - \frac{14a^{\frac{9}{4}} B e^4 \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{\frac{a}{c}}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{11}{4}} \sqrt{ex} \sqrt{cx^2 + a}} \\
 & + \frac{a^{\frac{7}{4}} e^4 \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (49B\sqrt{a} + 25A\sqrt{c}) (\sqrt{a} + x\sqrt{c}) \sqrt{x}}{105 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{11}{4}} \sqrt{ex} \sqrt{cx^2 + a}}
 \end{aligned}$$

command

```
integrate((e*x)^(7/2)*(B*x+A)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(75 A a^2 \sqrt{c} e^{\frac{7}{2}} \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 147 B a^2 \sqrt{c} e^{\frac{7}{2}} \operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}\right)\right) \right)}{315 c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^3x^4 + Ae^3x^3)\sqrt{ex}}{\sqrt{cx^2 + a}}, x\right)$$

23.27 Problem number 459

$$\int \frac{(ex)^{5/2} (A + Bx)}{\sqrt{a + cx^2}} dx$$

Optimal antiderivative

$$\frac{2Ae(ex)^{\frac{3}{2}}\sqrt{cx^2+a}}{5c} + \frac{2B(ex)^{\frac{5}{2}}\sqrt{cx^2+a}}{7c} - \frac{6aAe^3x\sqrt{cx^2+a}}{5c^{\frac{3}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} - \frac{10aBe^2\sqrt{ex}\sqrt{cx^2+a}}{21c^2}$$

$$+ \frac{6a^{\frac{5}{4}}Ae^3\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{a}{c}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

$$+ \frac{a^{\frac{5}{4}}e^3\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(25B\sqrt{a}-63A\sqrt{c})(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{a}{c}}}{105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{9}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

command

```
integrate((e*x)^(5/2)*(B*x+A)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(25Ba^2\sqrt{c}e^{\frac{5}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 63Aac^{\frac{3}{2}}e^{\frac{5}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{105c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^2x^3 + Ae^2x^2)\sqrt{ex}}{\sqrt{cx^2+a}}, x\right)$$

23.28 Problem number 460

$$\int \frac{(ex)^{3/2}(A+Bx)}{\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{2B(ex)^{\frac{3}{2}} \sqrt{cx^2 + a}}{5c} - \frac{6aB e^2 x \sqrt{cx^2 + a}}{5c^{\frac{3}{2}} (\sqrt{a} + x\sqrt{c}) \sqrt{ex}} + \frac{2Ae\sqrt{ex} \sqrt{cx^2 + a}}{3c}$$

$$+ \frac{6a^{\frac{5}{4}} B e^2 \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{\frac{a}{a+x\sqrt{c}}}}{5 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{ex} \sqrt{cx^2 + a}}$$

$$- \frac{a^{\frac{3}{4}} e^2 \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (9B\sqrt{a} + 5A\sqrt{c}) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{\frac{a}{a+x\sqrt{c}}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{ex} \sqrt{cx^2 + a}}$$

command

```
integrate((e*x)^(3/2)*(B*x+A)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(5Aa\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c},0,x\right) - 9Ba\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c},0,\operatorname{weierstrassPInverse}\left(-\frac{4a}{c},0,x\right)\right)\right)}{15c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bex^2 + Aex)\sqrt{ex}}{\sqrt{cx^2 + a}}, x\right)$$

23.29 Problem number 461

$$\int \frac{\sqrt{ex} (A + Bx)}{\sqrt{a + cx^2}} dx$$

Optimal antiderivative

$$\frac{2Aex\sqrt{cx^2+a}}{\sqrt{c}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} + \frac{2B\sqrt{ex}\sqrt{cx^2+a}}{3c}$$

$$2a^{\frac{1}{4}}Ae\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{cx^2+a}{c}}$$

$$a^{\frac{1}{4}}e\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(B\sqrt{a}-3A\sqrt{c})(\sqrt{a}+x\sqrt{c})\sqrt{x}$$

$$3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2+a}$$

$$3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{5}{4}}\sqrt{ex}\sqrt{cx^2+a}$$

command

```
integrate((e*x)^(1/2)*(B*x+A)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(Ba\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c},0,x\right)+3Ac^{\frac{3}{2}}e^{\frac{1}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c},0,\operatorname{weierstrassPInverse}\left(-\frac{4a}{c},0,x\right)\right)\right)}{3c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bx+A)\sqrt{ex}}{\sqrt{cx^2+a}},x\right)$$

23.30 Problem number 462

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{2Bx\sqrt{cx^2+a}}{\sqrt{c}(\sqrt{a}+x\sqrt{c})\sqrt{ex}}$$

$$- \frac{2a^{\frac{1}{4}}B\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{c}{(\sqrt{a}+x\sqrt{c})^2}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

$$+ \frac{a^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right)\sqrt{x}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

command

`integrate((B*x+A)/(e*x)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(A\sqrt{c}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - B\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)e^{(-\frac{1}{2})}}{c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}(Bx+A)\sqrt{ex}}{cex^3+ae}, x\right)$$

23.31 Problem number 463

$$\int \frac{A+Bx}{(ex)^{3/2}\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{-\frac{2A\sqrt{cx^2+a}}{ae\sqrt{ex}} + \frac{2Ax\sqrt{c}\sqrt{cx^2+a}}{ae(\sqrt{a}+x\sqrt{c})\sqrt{ex}}}{2Ac^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{c}{(\sqrt{a}+x\sqrt{c})^2}}}$$

$$-\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}e\sqrt{ex}\sqrt{cx^2+a}}$$

$$+\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{1}{4}}e\sqrt{ex}\sqrt{cx^2+a}}$$

command

`integrate((B*x+A)/(e*x)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(Ba\sqrt{c}x\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - Ac^{\frac{3}{2}}x\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right) - \sqrt{c}\right)}{acx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}(Bx+A)\sqrt{ex}}{ce^2x^4+ae^2x^2}, x\right)$$

23.32 Problem number 464

$$\int \frac{A+Bx}{(ex)^{5/2}\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{-\frac{2A\sqrt{cx^2+a}}{3ae(ex)^{\frac{3}{2}}} - \frac{2B\sqrt{cx^2+a}}{ae^2\sqrt{ex}} + \frac{2Bx\sqrt{c}\sqrt{cx^2+a}}{ae^2(\sqrt{a}+x\sqrt{c})\sqrt{ex}}}{2Bc^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{a}{(\sqrt{a}+x\sqrt{c})^2}}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}e^2\sqrt{ex}\sqrt{cx^2+a}}$$

$$+ \frac{c^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(3B\sqrt{a}-A\sqrt{c})(\sqrt{a}+x\sqrt{c})}{3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{5}{4}}e^2\sqrt{ex}\sqrt{cx^2+a}}$$

command

```
integrate((B*x+A)/(e*x)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(A\sqrt{c}x^2\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 3B\sqrt{c}x^2\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{3ax^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}(Bx+A)\sqrt{ex}}{ce^3x^5+ae^3x^3}, x\right)$$

23.33 Problem number 465

$$\int \frac{A+Bx}{(ex)^{7/2}\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{-\frac{2A\sqrt{cx^2+a}}{5ae(ex)^{\frac{5}{2}}} - \frac{2B\sqrt{cx^2+a}}{3ae^2(ex)^{\frac{3}{2}}} + \frac{6Ac\sqrt{cx^2+a}}{5a^2e^3\sqrt{ex}} - \frac{6Ac^{\frac{3}{2}}x\sqrt{cx^2+a}}{5a^2e^3(\sqrt{a}+x\sqrt{c})\sqrt{ex}}}{6Ac^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{c}{(\sqrt{a}+x\sqrt{c})^2}}}} + \frac{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}e^3\sqrt{ex}\sqrt{cx^2+a}}{c^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(5B\sqrt{a}+9A\sqrt{c})(\sqrt{a}+x\sqrt{c})\sqrt{cx^2+a}}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}e^3\sqrt{ex}\sqrt{cx^2+a}}$$

command

`integrate((B*x+A)/(e*x)^(7/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(5Ba\sqrt{c}x^3\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 9Ac^{\frac{3}{2}}x^3\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{15a^2x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}(Bx+A)\sqrt{ex}}{ce^4x^6+ae^4x^4}, x\right)$$

23.34 Problem number 466

$$\int \frac{(ex)^{7/2}(A+Bx)}{(a+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{e(ex)^{\frac{5}{2}}(Bx+A)}{c\sqrt{cx^2+a}} + \frac{7Be^2(ex)^{\frac{3}{2}}\sqrt{cx^2+a}}{5c^2} - \frac{21aBe^4x\sqrt{cx^2+a}}{5c^5(\sqrt{a}+x\sqrt{c})\sqrt{ex}} + \frac{5Ae^3\sqrt{ex}\sqrt{cx^2+a}}{3c^2} \\ & + \frac{21a^{\frac{5}{4}}Be^4\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{ex}\sqrt{cx^2+a}} \\ & - \frac{a^{\frac{3}{4}}e^4\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(63B\sqrt{a}+25A\sqrt{c})(\sqrt{a}+x\sqrt{c})\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}}}{30\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{ex}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x)^(7/2)*(B*x+A)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{25(Aacx^2 + Aa^2)\sqrt{c}e^{\frac{7}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 63(Bacx^2 + Ba^2)\sqrt{c}e^{\frac{7}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)}{15(c^4x^2 + ac^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^3x^4 + Ae^3x^3)\sqrt{cx^2+a}\sqrt{ex}}{c^2x^4 + 2acx^2 + a^2}, x\right)$$

23.35 Problem number 467

$$\int \frac{(ex)^{5/2}(A+Bx)}{(a+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{e(ex)^{\frac{3}{2}}(Bx+A)}{c\sqrt{cx^2+a}} + \frac{3Ae^3x\sqrt{cx^2+a}}{c^{\frac{3}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} + \frac{5Be^2\sqrt{ex}\sqrt{cx^2+a}}{3c^2}$$

$$\frac{3a^{\frac{1}{4}}Ae^3\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

$$\frac{a^{\frac{1}{4}}e^3\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(5B\sqrt{a}-9A\sqrt{c})(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}}}{6\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{9}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

command

```
integrate((e*x)^(5/2)*(B*x+A)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(Bacx^2 + Ba^2)\sqrt{c}e^{\frac{5}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 9(Ac^2x^2 + Aac)\sqrt{c}e^{\frac{5}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)}{3(c^4x^2 + ac^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^2x^3 + Ae^2x^2)\sqrt{cx^2+a}\sqrt{ex}}{c^2x^4 + 2acx^2 + a^2}, x\right)$$

23.36 Problem number 468

$$\int \frac{(ex)^{3/2}(A+Bx)}{(a+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{e(Bx+A)\sqrt{ex}}{c\sqrt{cx^2+a}} + \frac{3Be^2x\sqrt{cx^2+a}}{c^{\frac{3}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} \\
& - \frac{3a^{\frac{1}{4}}Be^2\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}} \\
& + \frac{e^2\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(3B\sqrt{a}+A\sqrt{c})(\sqrt{a}+x\sqrt{c})}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{1}{4}}c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}}
\end{aligned}$$

command

```
integrate((e*x)^(3/2)*(B*x+A)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(Acx^2 + Aa)\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 3(Bcx^2 + Ba)\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)}{c^3x^2 + ac^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bex^2 + Aex)\sqrt{cx^2+a}\sqrt{ex}}{c^2x^4 + 2acx^2 + a^2}, x\right)$$

23.37 Problem number 469

$$\int \frac{\sqrt{ex}(A+Bx)}{(a+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{(-Acx + Ba)\sqrt{ex}}{ac\sqrt{cx^2 + a}} - \frac{Aex\sqrt{cx^2 + a}}{a\sqrt{c}(\sqrt{a} + x\sqrt{c})\sqrt{ex}} \\
& + \frac{Ae\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\frac{cx^2}{(\sqrt{a} + x\sqrt{c})^2}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}} \\
& + \frac{e\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(B\sqrt{a} - A\sqrt{c})(\sqrt{a} + x\sqrt{c})}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{5}{4}}\sqrt{ex}\sqrt{cx^2 + a}}
\end{aligned}$$

command

```
integrate((e*x)^(1/2)*(B*x+A)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(Bacx^2 + Ba^2)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + (Ac^2x^2 + Aac)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassP}\left(-\frac{4a}{c}, 0, x\right)\right)}{ac^3x^2 + a^2c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a}(Bx + A)\sqrt{ex}}{c^2x^4 + 2acx^2 + a^2}, x\right)$$

23.38 Problem number 470

$$\int \frac{A + Bx}{\sqrt{ex}(a + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(Bx + A)\sqrt{ex}}{ae\sqrt{cx^2 + a}} - \frac{Bx\sqrt{cx^2 + a}}{a\sqrt{c}(\sqrt{a} + x\sqrt{c})\sqrt{ex}}$$

$$+ \frac{B\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\frac{cx^2 + a}{(\sqrt{a} + x\sqrt{c})^2}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}}$$

$$- \frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(B\sqrt{a} - A\sqrt{c})(\sqrt{a} + x\sqrt{c})}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{5}{4}}c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}}$$

command

`integrate((B*x+A)/(e*x)^(1/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left((Acx^2 + Aa)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + (Bcx^2 + Ba)\sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{ac^2x^2 + a^2c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a}(Bx + A)\sqrt{ex}}{c^2ex^5 + 2acex^3 + a^2ex}, x\right)$$

23.39 Problem number 471

$$\int \frac{A + Bx}{(ex)^{3/2}(a + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{Bx + A}{ae\sqrt{ex} \sqrt{cx^2 + a}} - \frac{3A\sqrt{cx^2 + a}}{a^2e\sqrt{ex}} + \frac{3Ax\sqrt{c} \sqrt{cx^2 + a}}{a^2e(\sqrt{a} + x\sqrt{c}) \sqrt{ex}}$$

$$\frac{3A c^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{\frac{c}{(\sqrt{a} + x\sqrt{c})^2}}}{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{7}{4}} e\sqrt{ex} \sqrt{cx^2 + a}}$$

$$+ \frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (B\sqrt{a} + 3A\sqrt{c}) (\sqrt{a} + x\sqrt{c})}{2 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{7}{4}} c^{\frac{1}{4}} e\sqrt{ex} \sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)/(e*x)^(3/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\left(Bacx^3 + Ba^2x\right)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 3\left(Ac^2x^3 + Aacx\right)\sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{a^2c^2x^3 + a^3cx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a} (Bx + A)\sqrt{ex}}{c^2e^2x^6 + 2ace^2x^4 + a^2e^2x^2}, x\right)$$

23.40 Problem number 472

$$\int \frac{A + Bx}{(ex)^{5/2} (a + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{Bx + A}{ae (ex)^{\frac{3}{2}} \sqrt{cx^2 + a}} - \frac{5A\sqrt{cx^2 + a}}{3a^2e (ex)^{\frac{3}{2}}} - \frac{3B\sqrt{cx^2 + a}}{a^2e^2\sqrt{ex}} + \frac{3Bx\sqrt{c} \sqrt{cx^2 + a}}{a^2e^2(\sqrt{a} + x\sqrt{c})\sqrt{ex}}$$

$$\frac{3Bc^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{\frac{c}{a}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{7}{4}}e^2\sqrt{ex} \sqrt{cx^2 + a}}$$

$$+ \frac{c^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (9B\sqrt{a} - 5A\sqrt{c}) (\sqrt{a} + x\sqrt{c})}{6 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{9}{4}}e^2\sqrt{ex} \sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)/(e*x)^(5/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(5(Acx^4 + Aax^2)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 9(Bcx^4 + Bax^2)\sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)}{3(a^2cx^4 + a^3x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a} (Bx + A)\sqrt{ex}}{c^2e^3x^7 + 2ace^3x^5 + a^2e^3x^3}, x\right)$$

23.41 Problem number 473

$$\int \frac{A + Bx}{(ex)^{7/2} (a + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{Bx + A}{ae (ex)^{\frac{5}{2}} \sqrt{cx^2 + a}} - \frac{7A\sqrt{cx^2 + a}}{5a^2e (ex)^{\frac{5}{2}}} - \frac{5B\sqrt{cx^2 + a}}{3a^2e^2 (ex)^{\frac{3}{2}}}$$

$$+ \frac{21Ac\sqrt{cx^2 + a}}{5a^3e^3\sqrt{ex}} - \frac{21Ac^{\frac{3}{2}}x\sqrt{cx^2 + a}}{5a^3e^3(\sqrt{a} + x\sqrt{c})\sqrt{ex}}$$

$$+ \frac{21Ac^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\sqrt{a} + x\sqrt{c}}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{11}{4}}e^3\sqrt{ex}\sqrt{cx^2 + a}}$$

$$- \frac{c^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(25B\sqrt{a} + 63A\sqrt{c})(\sqrt{a} + x\sqrt{c})}}{30\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{11}{4}}e^3\sqrt{ex}\sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)/(e*x)^(7/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(25(Bacx^5 + Ba^2x^3)\sqrt{c}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 63(Ac^2x^5 + Aacx^3)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)\right)\sqrt{cx^2 + a}}{15(a^3cx^5 + \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a}(Bx + A)\sqrt{ex}}{c^2e^4x^8 + 2ace^4x^6 + a^2e^4x^4}, x\right)$$

23.42 Problem number 474

$$\int \frac{(ex)^{13/2}(A + Bx)}{(a + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{e(ex)^{\frac{11}{2}}(Bx+A)}{3c(cx^2+a)^{\frac{3}{2}}} - \frac{e^3(ex)^{\frac{7}{2}}(13Bx+11A)}{6c^2\sqrt{cx^2+a}} + \frac{77Ae^5(ex)^{\frac{3}{2}}\sqrt{cx^2+a}}{30c^3} \\
& + \frac{39Be^4(ex)^{\frac{5}{2}}\sqrt{cx^2+a}}{14c^3} - \frac{77aAe^7x\sqrt{cx^2+a}}{10c^{\frac{7}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} - \frac{65aBe^6\sqrt{ex}\sqrt{cx^2+a}}{14c^4} \\
& + \frac{77a^{\frac{5}{4}}Ae^7\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{cx^2+a}{a}}}{10\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{15}{4}}\sqrt{ex}\sqrt{cx^2+a}} \\
& + \frac{a^{\frac{5}{4}}e^7\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(325B\sqrt{a}-539A\sqrt{c})(\sqrt{cx^2+a})}{140\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{17}{4}}\sqrt{ex}\sqrt{cx^2+a}}
\end{aligned}$$

command

```
integrate((e*x)^(13/2)*(B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$975(Ba^2c^2x^4 + 2Ba^3cx^2 + Ba^4)\sqrt{c}e^{\frac{13}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 1617(Aac^3x^4 + 2Aa^2c^2x^2 + Aa^3c)\sqrt{c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^6x^7 + Ae^6x^6)\sqrt{cx^2+a}\sqrt{ex}}{c^3x^6 + 3ac^2x^4 + 3a^2cx^2 + a^3}, x\right)$$

23.43 Problem number 475

$$\int \frac{(ex)^{11/2}(A+Bx)}{(a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{e(ex)^{\frac{9}{2}}(Bx+A)}{3c(cx^2+a)^{\frac{3}{2}}} - \frac{e^3(ex)^{\frac{5}{2}}(11Bx+9A)}{6c^2\sqrt{cx^2+a}} + \frac{77Be^4(ex)^{\frac{3}{2}}\sqrt{cx^2+a}}{30c^3} \\
& - \frac{77aBe^6x\sqrt{cx^2+a}}{10c^{\frac{7}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} + \frac{5Ae^5\sqrt{ex}\sqrt{cx^2+a}}{2c^3} \\
& + \frac{77a^{\frac{5}{4}}Be^6\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{1}{c}}}{10\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{15}{4}}\sqrt{ex}\sqrt{cx^2+a}} \\
& + \frac{a^{\frac{3}{4}}e^6\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(77B\sqrt{a}+25A\sqrt{c})(\sqrt{a}}{20\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{15}{4}}\sqrt{ex}\sqrt{cx^2+a}}
\end{aligned}$$

command

```
integrate((e*x)^(11/2)*(B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{75(Aac^2x^4 + 2Aa^2cx^2 + Aa^3)\sqrt{c}e^{\frac{11}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 231(Bac^2x^4 + 2Ba^2cx^2 + Ba^3)\sqrt{c}e^{\frac{11}{2}}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^5x^6 + Ae^5x^5)\sqrt{cx^2+a}\sqrt{ex}}{c^3x^6 + 3ac^2x^4 + 3a^2cx^2 + a^3}, x\right)$$

23.44 Problem number 476

$$\int \frac{(ex)^{9/2}(A+Bx)}{(a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{e(ex)^{\frac{7}{2}}(Bx+A)}{3c(cx^2+a)^{\frac{3}{2}}} - \frac{e^3(ex)^{\frac{3}{2}}(9Bx+7A)}{6c^2\sqrt{cx^2+a}} + \frac{7Ae^5x\sqrt{cx^2+a}}{2c^{\frac{5}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} + \frac{5Be^4\sqrt{ex}\sqrt{cx^2+a}}{2c^3}$$

$$\frac{7a^{\frac{1}{4}}Ae^5\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{1}{c}}}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

$$\frac{a^{\frac{1}{4}}e^5\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(5B\sqrt{a}-7A\sqrt{c})(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{1}{c}}}{4\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{13}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

command

```
integrate((e*x)^(9/2)*(B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15(Bac^2x^4 + 2Ba^2cx^2 + Ba^3)\sqrt{c}e^{\frac{9}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 21(Ac^3x^4 + 2Aac^2x^2 + Aa^2c)\sqrt{c}e^{\frac{9}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)}{6(c^6)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^4x^5 + Ae^4x^4)\sqrt{cx^2+a}\sqrt{ex}}{c^3x^6 + 3ac^2x^4 + 3a^2cx^2 + a^3}, x\right)$$

23.45 Problem number 477

$$\int \frac{(ex)^{7/2}(A+Bx)}{(a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{e(ex)^{\frac{5}{2}}(Bx+A)}{3c(cx^2+a)^{\frac{3}{2}}} - \frac{e^3(7Bx+5A)\sqrt{ex}}{6c^2\sqrt{cx^2+a}} + \frac{7Be^4x\sqrt{cx^2+a}}{2c^{\frac{5}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}}$$

$$\frac{7a^{\frac{1}{4}}Be^4\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{1}{c}}}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

$$+ \frac{e^4\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(21B\sqrt{a}+5A\sqrt{c})(\sqrt{a}+x\sqrt{c})}{12\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{1}{4}}c^{\frac{11}{4}}\sqrt{ex}\sqrt{cx^2+a}}$$

command

```
integrate((e*x)^(7/2)*(B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(Ac^2x^4 + 2Aacx^2 + Aa^2)\sqrt{c}e^{\frac{7}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 21(Bc^2x^4 + 2Bacx^2 + Ba^2)\sqrt{c}e^{\frac{7}{2}}\operatorname{weierstrassP}\left(-\frac{4a}{c}, 0, x\right)}{6(c^5x^4 + 2ac^4x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^3x^4 + Ae^3x^3)\sqrt{cx^2+a}\sqrt{ex}}{c^3x^6 + 3ac^2x^4 + 3a^2cx^2 + a^3}, x\right)$$

23.46 Problem number 478

$$\int \frac{(ex)^{5/2}(A+Bx)}{(a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{e(ex)^{\frac{3}{2}}(Bx+A)}{3c(cx^2+a)^{\frac{3}{2}}} - \frac{e^2(-3Acx+5Ba)\sqrt{ex}}{6ac^2\sqrt{cx^2+a}} - \frac{Ae^3x\sqrt{cx^2+a}}{2ac^{\frac{3}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} \\
& + \frac{Ae^3\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{cx}{(\sqrt{a}+x\sqrt{c})^2}}}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}} \\
& + \frac{e^3\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(5B\sqrt{a}-3A\sqrt{c})(\sqrt{a}+x\sqrt{c})}{12\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{9}{4}}\sqrt{ex}\sqrt{cx^2+a}}
\end{aligned}$$

command

```
integrate((e*x)^(5/2)*(B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(Bac^2x^4 + 2Ba^2cx^2 + Ba^3)\sqrt{c}e^{\frac{5}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 3(Ac^3x^4 + 2Aac^2x^2 + Aa^2c)\sqrt{c}e^{\frac{5}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)}{6(ac^5x^4 + 2a^2c^3x^2 + a^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^2x^3 + Ae^2x^2)\sqrt{cx^2+a}\sqrt{ex}}{c^3x^6 + 3ac^2x^4 + 3a^2cx^2 + a^3}, x\right)$$

23.47 Problem number 479

$$\int \frac{(ex)^{3/2}(A+Bx)}{(a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{e(Bx+A)\sqrt{ex}}{3c(cx^2+a)^{\frac{3}{2}}} + \frac{e(3Bx+A)\sqrt{ex}}{6ac\sqrt{cx^2+a}} - \frac{Be^2x\sqrt{cx^2+a}}{2ac^{\frac{3}{2}}(\sqrt{a}+x\sqrt{c})\sqrt{ex}} \\ & + \frac{Be^2\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x\sqrt{c})\sqrt{x}\sqrt{\frac{cx^2+a}{(\sqrt{a}+x\sqrt{c})^2}}}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}} \\ & - \frac{e^2\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(3B\sqrt{a}-A\sqrt{c})(\sqrt{a}+x\sqrt{c})\sqrt{\frac{cx^2+a}{(\sqrt{a}+x\sqrt{c})^2}}}{12\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{5}{4}}c^{\frac{7}{4}}\sqrt{ex}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x)^(3/2)*(B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(Ac^2x^4 + 2Aacx^2 + Aa^2)\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 3(Bc^2x^4 + 2Bacx^2 + Ba^2)\sqrt{c}e^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(-\frac{4a}{c}, 0, x\right)}{6(ac^4x^4 + 2a^2c^3x^2 + a^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bex^2 + Aex)\sqrt{cx^2+a}\sqrt{ex}}{c^3x^6 + 3ac^2x^4 + 3a^2cx^2 + a^3}, x\right)$$

23.48 Problem number 480

$$\int \frac{\sqrt{ex}(A+Bx)}{(a+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{(-Acx + Ba)\sqrt{ex}}{3ac(c x^2 + a)^{\frac{3}{2}}} + \frac{(3Acx + Ba)\sqrt{ex}}{6a^2c\sqrt{cx^2 + a}} - \frac{Aex\sqrt{cx^2 + a}}{2a^2\sqrt{c}(\sqrt{a} + x\sqrt{c})\sqrt{ex}} \\
 & + \frac{Ae\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\frac{cx^2}{(\sqrt{a} + x\sqrt{c})^2}}}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}} \\
 & + \frac{e\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(B\sqrt{a} - 3A\sqrt{c})(\sqrt{a} + x\sqrt{c})}{12\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}c^{\frac{5}{4}}\sqrt{ex}\sqrt{cx^2 + a}}
 \end{aligned}$$

command

```
integrate((e*x)^(1/2)*(B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(Bac^2x^4 + 2Ba^2cx^2 + Ba^3)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 3(Ac^3x^4 + 2Aac^2x^2 + Aa^2c)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)}{6(a^2c^4x^4 + 2a^3c^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a}(Bx + A)\sqrt{ex}}{c^3x^6 + 3ac^2x^4 + 3a^2cx^2 + a^3}, x\right)$$

23.49 Problem number 481

$$\int \frac{A + Bx}{\sqrt{ex}(a + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(Bx + A)\sqrt{ex}}{3ae(cx^2 + a)^{\frac{3}{2}}} + \frac{(3Bx + 5A)\sqrt{ex}}{6a^2e\sqrt{cx^2 + a}} - \frac{Bx\sqrt{cx^2 + a}}{2a^2\sqrt{c}(\sqrt{a} + x\sqrt{c})\sqrt{ex}}$$

$$+ \frac{B\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\frac{cx^2 + a}{(\sqrt{a} + x\sqrt{c})^2}}}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}}$$

$$- \frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(3B\sqrt{a} - 5A\sqrt{c})(\sqrt{a} + x\sqrt{c})}{12\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{9}{4}}c^{\frac{3}{4}}\sqrt{ex}\sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)/(e*x)^(1/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(5(Ac^2x^4 + 2Aacx^2 + Aa^2)\sqrt{c}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 3(Bc^2x^4 + 2Bacx^2 + Ba^2)\sqrt{c}\operatorname{weierstrassZeta}\left(\frac{x}{\sqrt{c}}, \frac{a}{c}\right)\right)}{6(a^2c^3x^4 + 2a^3c^2x^2 + a^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a}(Bx + A)\sqrt{ex}}{c^3ex^7 + 3ac^2ex^5 + 3a^2cex^3 + a^3ex}, x\right)$$

23.50 Problem number 482

$$\int \frac{A + Bx}{(ex)^{3/2}(a + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{Bx + A}{3ae (cx^2 + a)^{\frac{3}{2}} \sqrt{ex}} + \frac{5Bx + 7A}{6a^2e \sqrt{ex} \sqrt{cx^2 + a}} - \frac{7A\sqrt{cx^2 + a}}{2a^3e \sqrt{ex}} + \frac{7Ax\sqrt{c} \sqrt{cx^2 + a}}{2a^3e (\sqrt{a} + x\sqrt{c}) \sqrt{ex}}$$

$$\frac{7A c^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{\frac{c}{(\sqrt{a} + x\sqrt{c})^2}}}{2 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{11}{4}} e \sqrt{ex} \sqrt{cx^2 + a}}$$

$$+ \frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (5B\sqrt{a} + 21A\sqrt{c}) (\sqrt{a} + x\sqrt{c})}{12 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{11}{4}} c^{\frac{1}{4}} e \sqrt{ex} \sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)/(e*x)^(3/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(5 (Bac^2x^5 + 2Ba^2cx^3 + Ba^3x) \sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 21 (Ac^3x^5 + 2Aac^2x^3 + Aa^2cx) \sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)}{6(a^3c^2e^2x^8 + 3ac^2e^2x^6 + 3a^2ce^2x^4 + a^3e^2x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a} (Bx + A) \sqrt{ex}}{c^3e^2x^8 + 3ac^2e^2x^6 + 3a^2ce^2x^4 + a^3e^2x^2}, x\right)$$

23.51 Problem number 483

$$\int \frac{A + Bx}{(ex)^{5/2} (a + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{Bx + A}{3ae (ex)^{\frac{3}{2}} (cx^2 + a)^{\frac{3}{2}}} + \frac{7Bx + 9A}{6a^2e (ex)^{\frac{3}{2}} \sqrt{cx^2 + a}} \\ & - \frac{5A\sqrt{cx^2 + a}}{2a^3e (ex)^{\frac{3}{2}}} - \frac{7B\sqrt{cx^2 + a}}{2a^3e^2\sqrt{ex}} + \frac{7Bx\sqrt{c}\sqrt{cx^2 + a}}{2a^3e^2(\sqrt{a} + x\sqrt{c})\sqrt{ex}} \\ & - \frac{7Bc^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x\sqrt{c}) \sqrt{x} \sqrt{\frac{c}{\sqrt{a}}}}{2 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{11}{4}} e^2 \sqrt{ex} \sqrt{cx^2 + a}} \\ & + \frac{c^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (7B\sqrt{a} - 5A\sqrt{c}) (\sqrt{a} + x\sqrt{c}) \sqrt{x}}{4 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{13}{4}} e^2 \sqrt{ex} \sqrt{cx^2 + a}} \end{aligned}$$

command

`integrate((B*x+A)/(e*x)^(5/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(15 (Ac^2x^6 + 2Aacx^4 + Aa^2x^2)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) + 21 (Bc^2x^6 + 2Bacx^4 + Ba^2x^2)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a} (Bx + A)\sqrt{ex}}{c^3e^3x^9 + 3ac^2e^3x^7 + 3a^2ce^3x^5 + a^3e^3x^3}, x\right)$$

23.52 Problem number 484

$$\int \frac{A + Bx}{(ex)^{7/2} (a + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{Bx + A}{3ae (ex)^{\frac{5}{2}} (cx^2 + a)^{\frac{3}{2}}} + \frac{9Bx + 11A}{6a^2e (ex)^{\frac{5}{2}} \sqrt{cx^2 + a}} - \frac{77A\sqrt{cx^2 + a}}{30a^3e (ex)^{\frac{5}{2}}} \\ & - \frac{5B\sqrt{cx^2 + a}}{2a^3e^2 (ex)^{\frac{3}{2}}} + \frac{77Ac\sqrt{cx^2 + a}}{10a^4e^3\sqrt{ex}} - \frac{77Ac^{\frac{3}{2}}x\sqrt{cx^2 + a}}{10a^4e^3(\sqrt{a} + x\sqrt{c})\sqrt{ex}} \\ & + \frac{77Ac^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a} + x\sqrt{c})\sqrt{x}\sqrt{\sqrt{a} + x\sqrt{c}}}}{10\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{15}{4}}e^3\sqrt{ex}\sqrt{cx^2 + a}} \\ & - \frac{c^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(25B\sqrt{a} + 77A\sqrt{c})(\sqrt{a} + x\sqrt{c})\sqrt{\sqrt{a} + x\sqrt{c}}}}{20\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{15}{4}}e^3\sqrt{ex}\sqrt{cx^2 + a}} \end{aligned}$$

command

```
integrate((B*x+A)/(e*x)^(7/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(75(Bac^2x^7 + 2Ba^2cx^5 + Ba^3x^3)\sqrt{c}\operatorname{weierstrassPInverse}\left(-\frac{4a}{c}, 0, x\right) - 231(Ac^3x^7 + 2Aac^2x^5 + Aa^2cx^3)\sqrt{c}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + a}(Bx + A)\sqrt{ex}}{c^3e^4x^{10} + 3ac^2e^4x^8 + 3a^2ce^4x^6 + a^3e^4x^4}, x\right)$$

23.53 Problem number 1029

$$\int \sqrt{x}(A + Bx)\sqrt{a + bx + cx^2} dx$$

Optimal antiderivative

$$\frac{2B(cx^2 + bx + a)^{\frac{3}{2}}\sqrt{x}}{7c} - \frac{2(4b^2B - 7Abc + 5aBc + 3c(-7Ac + 4bB)x)\sqrt{x}\sqrt{cx^2 + bx + a}}{105c^2}$$

$$- \frac{2(5abBc - 2(-3ac + b^2)(-7Ac + 4bB))\sqrt{x}\sqrt{cx^2 + bx + a}}{105c^{\frac{5}{2}}(\sqrt{a} + x\sqrt{c})}$$

$$+ \frac{2a^{\frac{1}{4}}(5abBc - 2(-3ac + b^2)(-7Ac + 4bB))\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{\text{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)\right)}{105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^2 + bx + a}}$$

$$- \frac{a^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \sqrt{\frac{2 - \frac{b}{\sqrt{a}\sqrt{c}}}{2}}\right)(\sqrt{a} + x\sqrt{c})}{105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}}$$

command

```
integrate((B*x+A)*x^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$- \frac{2\left((8Bb^4 + 3(10Ba^2 + 21Aab)c^2 - (41Bab^2 + 14Ab^3)c\right)\sqrt{c}\text{weierstrassPInverse}\left(\frac{4(b^2 - 3ac)}{3c^2}, -\frac{4(2b^3 - 9abc)}{27c^3}, \frac{3cx}{3}\right)}{105c^{\frac{11}{4}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{cx^2 + bx + a}(Bx + A)\sqrt{x}, x\right)$$

23.54 Problem number 1030

$$\int \frac{(A + Bx)\sqrt{a + bx + cx^2}}{\sqrt{x}} dx$$

Optimal antiderivative

$$\frac{2(3Bcx + 5Ac + bB) \sqrt{x} \sqrt{cx^2 + bx + a}}{15c} - \frac{2(-5Abc - 6aBc + 2b^2B) \sqrt{x} \sqrt{cx^2 + bx + a}}{15c^{\frac{3}{2}} (\sqrt{a} + x\sqrt{c})}$$

$$+ \frac{2a^{\frac{1}{4}}(-5Abc - 6aBc + 2b^2B) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \sqrt{2 - \frac{\sqrt{a}}{\sqrt{c}}}\right)}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{cx^2 + bx + a}}$$

$$+ \frac{a^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \sqrt{2 - \frac{b}{\sqrt{a}\sqrt{c}}}\right) (\sqrt{a} + x\sqrt{c})}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2Bb^3 + 30Aac^2 - (9Bab + 5Ab^2)c) \sqrt{c} \operatorname{weierstrassPInverse}\left(\frac{4(b^2 - 3ac)}{3c^2}, -\frac{4(2b^3 - 9abc)}{27c^3}, \frac{3cx + b}{3c}\right) + 3(2Bb^2c - \dots) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx + a} (Bx + A)}{\sqrt{x}}, x\right)$$

23.55 Problem number 1031

$$\int \frac{(A + Bx) \sqrt{a + bx + cx^2}}{x^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(-Bx + 3A) \sqrt{cx^2 + bx + a}}{3\sqrt{x}} + \frac{2(6Ac + bB) \sqrt{x} \sqrt{cx^2 + bx + a}}{3\sqrt{c} (\sqrt{a} + x\sqrt{c})} \\
 & 2a^{\frac{1}{4}}(6Ac + bB) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \sqrt{2 - \frac{b}{\sqrt{a}\sqrt{c}}}\right) (\sqrt{c} \\
 & \frac{3 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{3}{4}} \sqrt{cx^2 + bx + a}}{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \sqrt{2 - \frac{b}{\sqrt{a}\sqrt{c}}}\right) (B\sqrt{a} + 3A\sqrt{c}) (\sqrt{c} \\
 & + \frac{3 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} c^{\frac{3}{4}} \sqrt{cx^2 + bx + a}}{
 \end{aligned}$$

command

```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((Bb^2 - 3(2Ba + Ab)c) \sqrt{c} \operatorname{xweierstrassPInverse}\left(\frac{4(b^2 - 3ac)}{3c^2}, -\frac{4(2b^3 - 9abc)}{27c^3}, \frac{3cx + b}{3c}\right) + 3(Bbc + 6Ac^2) \sqrt{c} \operatorname{xweierstrassPInverse}\left(\frac{4(b^2 - 3ac)}{3c^2}, -\frac{4(2b^3 - 9abc)}{27c^3}, \frac{3cx + b}{3c}\right) \right)}{$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx + a} (Bx + A)}{x^{\frac{3}{2}}}, x\right)$$

23.56 Problem number 1032

$$\int \frac{(A + Bx) \sqrt{a + bx + cx^2}}{x^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(aA + (Ab + 3Ba)x)\sqrt{cx^2 + bx + a}}{3ax^{\frac{3}{2}}} + \frac{2(Ab + 6Ba)\sqrt{c}\sqrt{x}\sqrt{cx^2 + bx + a}}{3a(\sqrt{a} + x\sqrt{c})}$$

$$+ \frac{2(Ab + 6Ba)c^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{\text{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \sqrt{2 - \frac{b}{\sqrt{a}\sqrt{c}}}\right)}(\sqrt{a} + x\sqrt{c})$$

$$\frac{3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}\sqrt{cx^2 + bx + a}}{\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \sqrt{2 - \frac{b}{\sqrt{a}\sqrt{c}}}\right)((2Ac + 3bB)\sqrt{a} + x\sqrt{c})$$

$$+ \frac{3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{1}{4}}\sqrt{cx^2 + bx + a}}{3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{1}{4}}\sqrt{cx^2 + bx + a}}$$

command

```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3(6Ba + Ab)c^{\frac{3}{2}}x^2\text{weierstrassZeta}\left(\frac{4(b^2 - 3ac)}{3c^2}, -\frac{4(2b^3 - 9abc)}{27c^3}\right), \text{weierstrassPInverse}\left(\frac{4(b^2 - 3ac)}{3c^2}, -\frac{4(2b^3 - 9abc)}{27c^3}\right), \frac{3cx}{3}\right)}{x^{\frac{5}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}(Bx + A)}{x^{\frac{5}{2}}}, x\right)$$

23.57 Problem number 1033

$$\int \frac{(A + Bx)\sqrt{a + bx + cx^2}}{x^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(3aA + (Ab + 5Ba)x)\sqrt{cx^2 + bx + a}}{15ax^{\frac{5}{2}}} + \frac{2(-6aAc + 2Ab^2 - 5abB)\sqrt{cx^2 + bx + a}}{15a^2\sqrt{x}}$$

$$+ \frac{2(5abB - 2A(-3ac + b^2))\sqrt{c}\sqrt{x}\sqrt{cx^2 + bx + a}}{15a^2(\sqrt{a} + x\sqrt{c})}$$

$$2c^{\frac{1}{4}}(5abB - 2A(-3ac + b^2))\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \sqrt{2 - \frac{\sqrt{a}}{\sqrt{c}}}\right)$$

$$\frac{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}\sqrt{cx^2 + bx + a}}{c^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \sqrt{2 - \frac{b}{\sqrt{a}\sqrt{c}}}\right)(\sqrt{a} + x\sqrt{c})(b)}$$

$$\frac{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}\sqrt{cx^2 + bx + a}}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}\sqrt{cx^2 + bx + a}}$$

command

```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((5Bab^2 - 2Ab^3 - 3(10Ba^2 - 3Aab)c)\sqrt{c}x^3\text{weierstrassPInverse}\left(\frac{4(b^2 - 3ac)}{3c^2}, -\frac{4(2b^3 - 9abc)}{27c^3}, \frac{3cx + b}{3c}\right) + 3(6Aa}\right)}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}\sqrt{cx^2 + bx + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}(Bx + A)}{x^{\frac{7}{2}}}, x\right)$$

23.58 Problem number 1034

$$\int (2 - 5x)x^{7/2}\sqrt{2 + 5x + 3x^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{21620x^{\frac{3}{2}}(3x^2+5x+2)^{\frac{3}{2}}}{34749} + \frac{656x^{\frac{5}{2}}(3x^2+5x+2)^{\frac{3}{2}}}{1287} - \frac{10x^{\frac{7}{2}}(3x^2+5x+2)^{\frac{3}{2}}}{39} \\
& + \frac{157160(3x^2+5x+2)^{\frac{3}{2}}\sqrt{x}}{243243} + \frac{1543648(2+3x)\sqrt{x}}{6567561\sqrt{3x^2+5x+2}} \\
& - \frac{1543648(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{6567561\sqrt{3x^2+5x+2}} \\
& + \frac{349240(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{2189187\sqrt{3x^2+5x+2}} \\
& - \frac{8(397265+502911x)\sqrt{x}\sqrt{3x^2+5x+2}}{2189187}
\end{aligned}$$

command

```
integrate((2-5*x)*x^(7/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
& -\frac{2}{2189187}(841995x^5-270459x^4-185220x^3+167634x^2-162396x+174620)\sqrt{3x^2+5x+2}\sqrt{x} \\
& -\frac{204560}{8444007}\sqrt{3}\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right) \\
& -\frac{1543648}{6567561}\sqrt{3}\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right)\right)
\end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(5x^4-2x^3\right)\sqrt{3x^2+5x+2}\sqrt{x}, x\right)$$

23.59 Problem number 1035

$$\int (2-5x)x^{5/2}\sqrt{2+5x+3x^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{532x^{\frac{3}{2}}(3x^2 + 5x + 2)^{\frac{3}{2}}}{891} - \frac{10x^{\frac{5}{2}}(3x^2 + 5x + 2)^{\frac{3}{2}}}{33} \\
& - \frac{4420(3x^2 + 5x + 2)^{\frac{3}{2}}\sqrt{x}}{6237} - \frac{261784(2 + 3x)\sqrt{x}}{841995\sqrt{3x^2 + 5x + 2}} \\
& + \frac{261784(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{841995\sqrt{3x^2 + 5x + 2}} \\
& - \frac{13016(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{56133\sqrt{3x^2 + 5x + 2}} \\
& + \frac{8(57860 + 74313x)\sqrt{x}\sqrt{3x^2 + 5x + 2}}{280665}
\end{aligned}$$

command

```
integrate((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
& -\frac{2}{280665}(127575x^4 - 38745x^3 - 35550x^2 + 32418x - 32540)\sqrt{3x^2 + 5x + 2}\sqrt{x} \\
& + \frac{3928}{216513}\sqrt{3}\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) \\
& + \frac{261784}{841995}\sqrt{3}\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right)
\end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(5x^3 - 2x^2\right)\sqrt{3x^2 + 5x + 2}\sqrt{x}, x\right)$$

23.60 Problem number 1036

$$\int (2 - 5x)x^{3/2}\sqrt{2 + 5x + 3x^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{10x^{\frac{3}{2}}(3x^2+5x+2)^{\frac{3}{2}}}{27} + \frac{136(3x^2+5x+2)^{\frac{3}{2}}\sqrt{x}}{189} + \frac{2360(2+3x)\sqrt{x}}{5103\sqrt{3x^2+5x+2}} \\
 & - \frac{2360(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{5103\sqrt{3x^2+5x+2}} \\
 & + \frac{668(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{1701\sqrt{3x^2+5x+2}} \\
 & - \frac{4(779+1035x)\sqrt{x}\sqrt{3x^2+5x+2}}{1701}
 \end{aligned}$$

command

`integrate((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
 & -\frac{2}{1701}(945x^3-261x^2-360x+334)\sqrt{3x^2+5x+2}\sqrt{x} \\
 & + \frac{32}{6561}\sqrt{3}\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right) \\
 & - \frac{2360}{5103}\sqrt{3}\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right)\right)
 \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(5x^2-2x\right)\sqrt{3x^2+5x+2}\sqrt{x}, x\right)$$

23.61 Problem number 1037

$$\int (2-5x)\sqrt{x}\sqrt{2+5x+3x^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{10(3x^2+5x+2)^{\frac{3}{2}}\sqrt{x}}{21} - \frac{2476(2+3x)\sqrt{x}}{2835\sqrt{3x^2+5x+2}} \\
 & + \frac{2476(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{2835\sqrt{3x^2+5x+2}} \\
 & - \frac{164(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{189\sqrt{3x^2+5x+2}} \\
 & + \frac{4(430+639x)\sqrt{x}\sqrt{3x^2+5x+2}}{945}
 \end{aligned}$$

command

```
integrate((2-5*x)*x^(1/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{945} (675x^2 - 153x - 410) \sqrt{3x^2 + 5x + 2} \sqrt{x} - \frac{68}{729} \sqrt{3} \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) + \frac{2476}{2835} \sqrt{3} \operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\sqrt{3x^2 + 5x + 2} (5x - 2) \sqrt{x}, x\right)$$

23.62 Problem number 1038

$$\int \frac{(2 - 5x) \sqrt{2 + 5x + 3x^2}}{\sqrt{x}} dx$$

Optimal antiderivative

$$\frac{88(2 + 3x) \sqrt{x}}{27 \sqrt{3x^2 + 5x + 2}} - \frac{88(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1 + x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1 + x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2 + 3x}{1 + x}}}{27 \sqrt{3x^2 + 5x + 2}} + \frac{34(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1 + x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1 + x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2 + 3x}{1 + x}}}{9 \sqrt{3x^2 + 5x + 2}} + \frac{2(1 - 9x) \sqrt{x} \sqrt{3x^2 + 5x + 2}}{9}$$

command

```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{9} \sqrt{3x^2 + 5x + 2} (9x - 1) \sqrt{x} + \frac{172}{243} \sqrt{3} \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - \frac{88}{27} \sqrt{3} \operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} (5x - 2)}{\sqrt{x}}, x\right)$$

23.63 Problem number 1039

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{22(2+3x)\sqrt{x}}{9\sqrt{3x^2+5x+2}} - \frac{22(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{9\sqrt{3x^2+5x+2}} \\ & + \frac{10(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{3\sqrt{3x^2+5x+2}} \\ & - \frac{2(6+5x)\sqrt{3x^2+5x+2}}{3\sqrt{x}} \end{aligned}$$

command

`integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(35\sqrt{3}x\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 99\sqrt{3}x\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right)\right)}{81x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}(5x-2)}{x^{\frac{3}{2}}}, x\right)$$

23.64 Problem number 1040

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{50(2+3x)\sqrt{x}}{3\sqrt{3x^2+5x+2}} + \frac{50(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{3\sqrt{3x^2+5x+2}} \\ & - \frac{21(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \\ & - \frac{4(1-5x)\sqrt{3x^2+5x+2}}{3x^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(64 \sqrt{3} x^2 \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 225 \sqrt{3} x^2 \text{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}\right)\right) \right)}{27 x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2+5x+2}(5x-2)}{x^{\frac{5}{2}}}, x\right)$$

23.65 Problem number 1041

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{139(2+3x)\sqrt{x}}{15\sqrt{3x^2+5x+2}} + \frac{139(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\text{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{15\sqrt{3x^2+5x+2}} \\ & -\frac{11(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\text{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \\ & -\frac{4(3-10x)\sqrt{3x^2+5x+2}}{15x^{\frac{5}{2}}} + \frac{139\sqrt{3x^2+5x+2}}{15\sqrt{x}} \end{aligned}$$

command

```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{295 \sqrt{3} x^3 \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 1251 \sqrt{3} x^3 \text{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}\right)\right)}{135 x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2+5x+2}(5x-2)}{x^{\frac{7}{2}}}, x\right)$$

23.66 Problem number 1042

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{62(2+3x)\sqrt{x}}{21\sqrt{3x^2+5x+2}} + \frac{43(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{14\sqrt{3x^2+5x+2}} \\ & - \frac{62(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{21\sqrt{3x^2+5x+2}} \\ & - \frac{4(1-3x)\sqrt{3x^2+5x+2}}{7x^{\frac{7}{2}}} + \frac{43\sqrt{3x^2+5x+2}}{21x^{\frac{3}{2}}} - \frac{62\sqrt{3x^2+5x+2}}{21\sqrt{x}} \end{aligned}$$

command

`integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{77\sqrt{3}x^4\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 558\sqrt{3}x^4\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right)}{189x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}(5x-2)}{x^{\frac{9}{2}}}, x\right)$$

23.67 Problem number 1043

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1331(2+3x)\sqrt{x}}{630\sqrt{3x^2+5x+2}} + \frac{1331(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{630\sqrt{3x^2+5x+2}} \\ & - \frac{79(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{42\sqrt{3x^2+5x+2}} \\ & - \frac{4(7-20x)\sqrt{3x^2+5x+2}}{63x^{\frac{9}{2}}} + \frac{97\sqrt{3x^2+5x+2}}{105x^{\frac{5}{2}}} \\ & - \frac{79\sqrt{3x^2+5x+2}}{63x^{\frac{3}{2}}} + \frac{1331\sqrt{3x^2+5x+2}}{630\sqrt{x}} \end{aligned}$$

command

`integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{455 \sqrt{3} x^5 \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 11979 \sqrt{3} x^5 \text{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}\right)\right)}{5670 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2+5x+2}(5x-2)}{x^{\frac{11}{2}}}, x\right)$$

23.68 Problem number 1044

$$\int (2-5x)x^{5/2}(2+5x+3x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{136x^{\frac{3}{2}}(3x^2+5x+2)^{\frac{5}{2}}}{351} - \frac{2x^{\frac{5}{2}}(3x^2+5x+2)^{\frac{5}{2}}}{9} + \frac{8(27010+32921x)(3x^2+5x+2)^{\frac{3}{2}}\sqrt{x}}{243243} \\ & - \frac{4660(3x^2+5x+2)^{\frac{5}{2}}\sqrt{x}}{11583} - \frac{497824(2+3x)\sqrt{x}}{32837805\sqrt{3x^2+5x+2}} \\ & + \frac{497824(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\text{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{32837805\sqrt{3x^2+5x+2}} \\ & - \frac{61736(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\text{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{2189187\sqrt{3x^2+5x+2}} \\ & - \frac{8(190465+205407x)\sqrt{x}\sqrt{3x^2+5x+2}}{10945935} \end{aligned}$$

command

`integrate((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{2}{10945935} (10945935 x^6 + 17401230 x^5 + 1199205 x^4 - 5859000 x^3 - 292590 x^2 + 215748 x - 154340) \sqrt{3x^2+5x+2} \\ & - \frac{87632}{8444007} \sqrt{3} \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) \\ & + \frac{497824}{32837805} \sqrt{3} \text{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-((15x^5+19x^4-4x^2)\sqrt{3x^2+5x+2}\sqrt{x}), x\right)$$

23.69 Problem number 1045

$$\int (2 - 5x)x^{3/2}(2 + 5x + 3x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{10x^{\frac{3}{2}}(3x^2 + 5x + 2)^{\frac{5}{2}}}{39} - \frac{4(6959 + 8575x)(3x^2 + 5x + 2)^{\frac{3}{2}}\sqrt{x}}{27027} \\ & + \frac{556(3x^2 + 5x + 2)^{\frac{5}{2}}\sqrt{x}}{1287} + \frac{55112(2 + 3x)\sqrt{x}}{729729\sqrt{3x^2 + 5x + 2}} \\ & - \frac{55112(1 + x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{729729\sqrt{3x^2 + 5x + 2}} \\ & + \frac{25448(1 + x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{243243\sqrt{3x^2 + 5x + 2}} \\ & + \frac{8(6908 + 6381x)\sqrt{x}\sqrt{3x^2 + 5x + 2}}{243243} \end{aligned}$$

command

```
integrate((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{2}{243243}(280665x^5 + 462672x^4 + 40635x^3 - 172818x^2 - 16614x + 12724)\sqrt{3x^2 + 5x + 2}\sqrt{x} \\ & + \frac{26072}{938223}\sqrt{3}\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) \\ & - \frac{55112}{729729}\sqrt{3}\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(- (15x^4 + 19x^3 - 4x)\sqrt{3x^2 + 5x + 2}\sqrt{x}, x\right)$$

23.70 Problem number 1046

$$\int (2 - 5x)\sqrt{x} (2 + 5x + 3x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(65 + 84x)(3x^2 + 5x + 2)^{\frac{3}{2}}\sqrt{x}}{231} - \frac{10(3x^2 + 5x + 2)^{\frac{5}{2}}\sqrt{x}}{33} - \frac{424(2 + 3x)\sqrt{x}}{1155\sqrt{3x^2 + 5x + 2}} \\ & + \frac{424(1 + x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{1155\sqrt{3x^2 + 5x + 2}} \\ & - \frac{36(1 + x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{77\sqrt{3x^2 + 5x + 2}} \\ & - \frac{4(55 + 39x)\sqrt{x}\sqrt{3x^2 + 5x + 2}}{385} \end{aligned}$$

command

```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{2}{1155}(1575x^4 + 2730x^3 + 325x^2 - 1196x - 270)\sqrt{3x^2 + 5x + 2}\sqrt{x} \\ & - \frac{32}{297}\sqrt{3}\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) \\ & + \frac{424}{1155}\sqrt{3}\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(15x^3 + 19x^2 - 4\right)\sqrt{3x^2 + 5x + 2}\sqrt{x}, x\right)$$

23.71 Problem number 1047

$$\int \frac{(2 - 5x)(2 + 5x + 3x^2)^{3/2}}{\sqrt{x}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(1+5x)(3x^2+5x+2)^{\frac{3}{2}}\sqrt{x}}{9} + \frac{860(2+3x)\sqrt{x}}{243\sqrt{3x^2+5x+2}} \\
 & - \frac{860(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{243\sqrt{3x^2+5x+2}} \\
 & + \frac{356(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{81\sqrt{3x^2+5x+2}} \\
 & + \frac{4(82+45x)\sqrt{x}\sqrt{3x^2+5x+2}}{81}
 \end{aligned}$$

command

`integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
 & -\frac{2}{81}(135x^3+252x^2+45x-146)\sqrt{3x^2+5x+2}\sqrt{x} \\
 & + \frac{2108}{2187}\sqrt{3}\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right) \\
 & - \frac{860}{243}\sqrt{3}\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right)\right)
 \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(15x^3+19x^2-4)\sqrt{3x^2+5x+2}}{\sqrt{x}}, x\right)$$

23.72 Problem number 1048

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(14+5x)(3x^2+5x+2)^{\frac{3}{2}}}{7\sqrt{x}} + \frac{5848(2+3x)\sqrt{x}}{315\sqrt{3x^2+5x+2}} \\
 & -\frac{5848(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}},\frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{315\sqrt{3x^2+5x+2}} \\
 & +\frac{482(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}},\frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{21\sqrt{3x^2+5x+2}} \\
 & +\frac{2(1045+531x)\sqrt{x}\sqrt{3x^2+5x+2}}{105}
 \end{aligned}$$

command

`integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(7070\sqrt{3}x\operatorname{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-26316\sqrt{3}x\operatorname{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729}\right),\operatorname{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729}\right)\right)}{2835x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(15x^3+19x^2-4)\sqrt{3x^2+5x+2}}{x^{\frac{3}{2}}},x\right)$$

23.73 Problem number 1049

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(2+3x)(3x^2+5x+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}} - \frac{34(2+3x)\sqrt{x}}{3\sqrt{3x^2+5x+2}} \\
 & +\frac{34(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}},\frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{3\sqrt{3x^2+5x+2}} \\
 & -\frac{14(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}},\frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \\
 & +\frac{2(2-x)\sqrt{3x^2+5x+2}}{\sqrt{x}}
 \end{aligned}$$

command

```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(41 \sqrt{3} x^2 \text{weierstrassPInverse} \left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9} \right) - 153 \sqrt{3} x^2 \text{weierstrassZeta} \left(\frac{28}{27}, \frac{80}{729}, \text{weierstrassPInverse} \left(\frac{28}{27}, \frac{80}{729} \right), x \right) \right)}{27 x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{(15 x^3 + 19 x^2 - 4) \sqrt{3 x^2 + 5 x + 2}}{x^{\frac{5}{2}}}, x \right)$$

23.74 Problem number 1050

$$\int \frac{(2 - 5x) (2 + 5x + 3x^2)^{3/2}}{x^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4(3 - 5x) (3x^2 + 5x + 2)^{\frac{3}{2}}}{15x^{\frac{5}{2}}} - \frac{1418(2 + 3x) \sqrt{x}}{15 \sqrt{3x^2 + 5x + 2}} \\ & + \frac{1418(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \text{EllipticE} \left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2} \right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{15 \sqrt{3x^2 + 5x + 2}} \\ & - \frac{117(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \text{EllipticF} \left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2} \right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2 + 5x + 2}} \\ & + \frac{2(89 - 35x) \sqrt{3x^2 + 5x + 2}}{5\sqrt{x}} \end{aligned}$$

command

```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(1720 \sqrt{3} x^3 \text{weierstrassPInverse} \left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9} \right) - 6381 \sqrt{3} x^3 \text{weierstrassZeta} \left(\frac{28}{27}, \frac{80}{729}, \text{weierstrassPInverse} \left(\frac{28}{27}, \frac{80}{729} \right), x \right) \right)}{135 x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{(15 x^3 + 19 x^2 - 4) \sqrt{3 x^2 + 5 x + 2}}{x^{\frac{7}{2}}}, x \right)$$

23.75 Problem number 1051

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4(1-2x)(3x^2+5x+2)^{\frac{3}{2}}}{7x^{\frac{7}{2}}} - \frac{633(2+3x)\sqrt{x}}{7\sqrt{3x^2+5x+2}} \\ & + \frac{633(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{7\sqrt{3x^2+5x+2}} \\ & - \frac{783(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{7\sqrt{3x^2+5x+2}} \\ & + \frac{3(22+133x)\sqrt{3x^2+5x+2}}{7x^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{511\sqrt{3}x^4\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 1899\sqrt{3}x^4\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}\right)\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}\right)}{21x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(15x^3+19x^2-4)\sqrt{3x^2+5x+2}}{x^{\frac{9}{2}}}, x\right)$$

23.76 Problem number 1052

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{4(7-15x)(3x^2+5x+2)^{\frac{3}{2}}}{63x^{\frac{9}{2}}} - \frac{5438(2+3x)\sqrt{x}}{315\sqrt{3x^2+5x+2}} \\
 & - \frac{899(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{42\sqrt{3x^2+5x+2}} \\
 & + \frac{5438(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{315\sqrt{3x^2+5x+2}} \\
 & + \frac{(1446+4055x)\sqrt{3x^2+5x+2}}{315x^{\frac{5}{2}}} + \frac{5438\sqrt{3x^2+5x+2}}{315\sqrt{x}}
 \end{aligned}$$

command

`integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{13265\sqrt{3}x^5\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 48942\sqrt{3}x^5\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}\right)\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}\right)}{2835x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(15x^3+19x^2-4)\sqrt{3x^2+5x+2}}{x^{\frac{11}{2}}}, x\right)$$

23.77 Problem number 1053

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{4(9-20x)(3x^2+5x+2)^{\frac{3}{2}}}{99x^{\frac{11}{2}}} + \frac{3229(2+3x)\sqrt{x}}{1386\sqrt{3x^2+5x+2}} \\
 & - \frac{3229(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{1386\sqrt{3x^2+5x+2}} \\
 & + \frac{1357(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{462\sqrt{3x^2+5x+2}} \\
 & + \frac{1357\sqrt{3x^2+5x+2}}{693x^{\frac{3}{2}}} + \frac{(634+1367x)\sqrt{3x^2+5x+2}}{231x^{\frac{7}{2}}} - \frac{3229\sqrt{3x^2+5x+2}}{1386\sqrt{x}}
 \end{aligned}$$

command

```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{8281 \sqrt{3} x^6 \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 29061 \sqrt{3} x^6 \text{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}\right)\right)}{12474 x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(15x^3 + 19x^2 - 4)\sqrt{3x^2 + 5x + 2}}{x^{\frac{13}{2}}}, x\right)$$

23.78 Problem number 1054

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4(11-25x)(3x^2+5x+2)^{\frac{3}{2}}}{143x^{\frac{13}{2}}} - \frac{6907(2+3x)\sqrt{x}}{10010\sqrt{3x^2+5x+2}} \\ & + \frac{6907(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\text{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{10010\sqrt{3x^2+5x+2}} \\ & - \frac{3693(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\text{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{4004\sqrt{3x^2+5x+2}} \\ & + \frac{204\sqrt{3x^2+5x+2}}{385x^{\frac{5}{2}}} - \frac{1231\sqrt{3x^2+5x+2}}{2002x^{\frac{3}{2}}} \\ & + \frac{(1834+3445x)\sqrt{3x^2+5x+2}}{1001x^{\frac{9}{2}}} + \frac{6907\sqrt{3x^2+5x+2}}{10010\sqrt{x}} \end{aligned}$$

command

```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{20860 \sqrt{3} x^7 \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 62163 \sqrt{3} x^7 \text{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}\right)\right)}{90090}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(15x^3 + 19x^2 - 4)\sqrt{3x^2 + 5x + 2}}{x^{\frac{15}{2}}}, x\right)$$

23.79 Problem number 1055

$$\int \frac{A + Bx}{\sqrt{ex} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2Bx \sqrt{cx^2 + bx + a}}{\sqrt{c} (\sqrt{a} + x\sqrt{c}) \sqrt{ex}} + \frac{2a^{\frac{1}{4}} B \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{3}{4}} \sqrt{ex} \sqrt{cx^2 + bx + a}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2 - \frac{b}{\sqrt{a} \sqrt{c}}}}{2}\right) (\sqrt{a} + x\sqrt{c})$$

$$+ \frac{a^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right) c^{\frac{3}{4}} \sqrt{ex} \sqrt{cx^2 + bx + a}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2 - \frac{b}{\sqrt{a} \sqrt{c}}}}{2}\right) (\sqrt{a} + x\sqrt{c})$$

command

```
integrate((B*x+A)/(e*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 B c^{\frac{3}{2}} \operatorname{weierstrassZeta}\left(\frac{4(b^2-3ac)}{3c^2}, -\frac{4(2b^3-9abc)}{27c^3}\right), \operatorname{weierstrassPInverse}\left(\frac{4(b^2-3ac)}{3c^2}, -\frac{4(2b^3-9abc)}{27c^3}, \frac{3cx+b}{3c}\right) \right) + (Bb}{3c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx + a} (Bx + A) \sqrt{ex}}{cex^3 + bex^2 + aex}, x\right)$$

23.80 Problem number 1056

$$\int \frac{(2-5x)x^{7/2}}{\sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{68920(2+3x)\sqrt{x}}{15309\sqrt{3x^2+5x+2}} \\ & + \frac{68920(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{15309\sqrt{3x^2+5x+2}} \\ & - \frac{11320(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{5103\sqrt{3x^2+5x+2}} \\ & - \frac{820x^{\frac{3}{2}}\sqrt{3x^2+5x+2}}{567} + \frac{508x^{\frac{5}{2}}\sqrt{3x^2+5x+2}}{567} \\ & - \frac{10x^{\frac{7}{2}}\sqrt{3x^2+5x+2}}{27} + \frac{11320\sqrt{x}\sqrt{3x^2+5x+2}}{5103} \end{aligned}$$

command

`integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{2}{5103}(945x^3 - 2286x^2 + 3690x - 5660)\sqrt{3x^2+5x+2}\sqrt{x} \\ & + \frac{20120}{19683}\sqrt{3}\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) \\ & + \frac{68920}{15309}\sqrt{3}\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x^4 - 2x^3)\sqrt{x}}{\sqrt{3x^2+5x+2}}, x\right)$$

23.81 Problem number 1057

$$\int \frac{(2-5x)x^{5/2}}{\sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{13688(2+3x)\sqrt{x}}{2835\sqrt{3x^2+5x+2}} - \frac{13688(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{2835\sqrt{3x^2+5x+2}} \\ & + \frac{412(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{189\sqrt{3x^2+5x+2}} \\ & + \frac{128x^{\frac{3}{2}}\sqrt{3x^2+5x+2}}{105} - \frac{10x^{\frac{5}{2}}\sqrt{3x^2+5x+2}}{21} - \frac{412\sqrt{x}\sqrt{3x^2+5x+2}}{189} \end{aligned}$$

command

```
integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{2}{945}(225x^2-576x+1030)\sqrt{3x^2+5x+2}\sqrt{x} \\ & -\frac{896}{729}\sqrt{3}\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right) \\ & -\frac{13688}{2835}\sqrt{3}\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right)\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x^3-2x^2)\sqrt{x}}{\sqrt{3x^2+5x+2}}, x\right)$$

23.82 Problem number 1058

$$\int \frac{(2-5x)x^{3/2}}{\sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{412(2+3x)\sqrt{x}}{81\sqrt{3x^2+5x+2}} + \frac{412(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{81\sqrt{3x^2+5x+2}} \\ & - \frac{52(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{27\sqrt{3x^2+5x+2}} \\ & - \frac{2x^{\frac{3}{2}}\sqrt{3x^2+5x+2}}{3} + \frac{52\sqrt{x}\sqrt{3x^2+5x+2}}{27} \end{aligned}$$

command

`integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{27} \sqrt{3x^2 + 5x + 2} (9x - 26)\sqrt{x} + \frac{1124}{729} \sqrt{3} \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) + \frac{412}{81} \sqrt{3} \operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x^2 - 2x)\sqrt{x}}{\sqrt{3x^2 + 5x + 2}}, x\right)$$

23.83 Problem number 1059

$$\int \frac{(2 - 5x)\sqrt{x}}{\sqrt{2 + 5x + 3x^2}} dx$$

Optimal antiderivative

$$\frac{136(2 + 3x)\sqrt{x}}{27\sqrt{3x^2 + 5x + 2}} - \frac{136(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{27\sqrt{3x^2 + 5x + 2}} + \frac{10(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{9\sqrt{3x^2 + 5x + 2}} - \frac{10\sqrt{x} \sqrt{3x^2 + 5x + 2}}{9}$$

command

`integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{500}{243} \sqrt{3} \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - \frac{136}{27} \sqrt{3} \operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right) - \frac{10}{9} \sqrt{3x^2 + 5x + 2} \sqrt{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x - 2)\sqrt{x}}{\sqrt{3x^2 + 5x + 2}}, x\right)$$

23.84 Problem number 1060

$$\int \frac{2-5x}{\sqrt{x}\sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{10(2+3x)\sqrt{x}}{3\sqrt{3x^2+5x+2}} + \frac{10(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{3\sqrt{3x^2+5x+2}} \\ & + \frac{2(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \end{aligned}$$

command

`integrate((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & \frac{86}{27}\sqrt{3}\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) \\ & + \frac{10}{3}\sqrt{3}\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}(5x-2)\sqrt{x}}{3x^3+5x^2+2x}, x\right)$$

23.85 Problem number 1061

$$\int \frac{2-5x}{x^{3/2}\sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(2+3x)\sqrt{x}}{\sqrt{3x^2+5x+2}} - \frac{2(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \\ & - \frac{5(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} - \frac{2\sqrt{3x^2+5x+2}}{\sqrt{x}} \end{aligned}$$

command

```
integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(20 \sqrt{3} \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) + 9 \sqrt{3} \operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right) \right)}{9x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}(5x-2)\sqrt{x}}{3x^4+5x^3+2x^2}, x\right)$$

23.86 Problem number 1062

$$\int \frac{2-5x}{x^{5/2}\sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{25(2+3x)\sqrt{x}}{3\sqrt{3x^2+5x+2}} + \frac{25(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{3\sqrt{3x^2+5x+2}} \\ & -\frac{(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \\ & -\frac{2\sqrt{3x^2+5x+2}}{3x^{\frac{3}{2}}} + \frac{25\sqrt{3x^2+5x+2}}{3\sqrt{x}} \end{aligned}$$

command

```
integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{107 \sqrt{3} x^2 \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) + 225 \sqrt{3} x^2 \operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right)}{27x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}(5x-2)\sqrt{x}}{3x^5+5x^4+2x^3}, x\right)$$

23.87 Problem number 1063

$$\int \frac{2-5x}{x^{7/2}\sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{66(2+3x)\sqrt{x}}{5\sqrt{3x^2+5x+2}} + \frac{9(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{2\sqrt{3x^2+5x+2}} \\ & - \frac{66(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{5\sqrt{3x^2+5x+2}} \\ & - \frac{2\sqrt{3x^2+5x+2}}{5x^{\frac{5}{2}}} + \frac{3\sqrt{3x^2+5x+2}}{x^{\frac{3}{2}}} - \frac{66\sqrt{3x^2+5x+2}}{5\sqrt{x}} \end{aligned}$$

command

`integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{65\sqrt{3}x^3\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) + 198\sqrt{3}x^3\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x\right)\right)}{15x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}(5x-2)\sqrt{x}}{3x^6+5x^5+2x^4}, x\right)$$

23.88 Problem number 1064

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^{\frac{5}{2}}(74+95x)}{3\sqrt{3x^2+5x+2}} - \frac{24(2+3x)\sqrt{x}}{\sqrt{3x^2+5x+2}} \\ & + \frac{24(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \\ & - \frac{20(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \\ & - \frac{64x^{\frac{3}{2}}\sqrt{3x^2+5x+2}}{3} + 20\sqrt{x}\sqrt{3x^2+5x+2} \end{aligned}$$

command

```
integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(36 \sqrt{3} (3x^2 + 5x + 2) \operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right)\right) - (x^3 - 4x^2 - 86x - 60) \sqrt{3x^2 + 5x + 2} \right)}{3(3x^2 + 5x + 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x^4 - 2x^3)\sqrt{3x^2 + 5x + 2}\sqrt{x}}{9x^4 + 30x^3 + 37x^2 + 20x + 4}, x\right)$$

23.89 Problem number 1065

$$\int \frac{(2 - 5x)x^{5/2}}{(2 + 5x + 3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^{\frac{3}{2}}(74 + 95x)}{3\sqrt{3x^2 + 5x + 2}} + \frac{1804(2 + 3x)\sqrt{x}}{81\sqrt{3x^2 + 5x + 2}} \\ & - \frac{1804(1 + x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{81\sqrt{3x^2 + 5x + 2}} \\ & + \frac{580(1 + x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{27\sqrt{3x^2 + 5x + 2}} \\ & - \frac{580\sqrt{x}\sqrt{3x^2 + 5x + 2}}{27} \end{aligned}$$

command

```
integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(710 \sqrt{3} (3x^2 + 5x + 2) \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 8118 \sqrt{3} (3x^2 + 5x + 2) \operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) \right)}{729(3x^2 + 5x + 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x^3 - 2x^2)\sqrt{3x^2 + 5x + 2}\sqrt{x}}{9x^4 + 30x^3 + 37x^2 + 20x + 4}, x\right)$$

23.90 Problem number 1066

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{200(2+3x)\sqrt{x}}{9\sqrt{3x^2+5x+2}} + \frac{2(74+95x)\sqrt{x}}{3\sqrt{3x^2+5x+2}} \\ & + \frac{200(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{9\sqrt{3x^2+5x+2}} \\ & - \frac{74(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{3\sqrt{3x^2+5x+2}} \end{aligned}$$

command

`integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(166\sqrt{3}(3x^2+5x+2)\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right) - 900\sqrt{3}(3x^2+5x+2)\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}\right)\right)}{81(3x^2+5x+2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x^2-2x)\sqrt{3x^2+5x+2}\sqrt{x}}{9x^4+30x^3+37x^2+20x+4}, x\right)$$

23.91 Problem number 1067

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{74(2+3x)\sqrt{x}}{3\sqrt{3x^2+5x+2}} - \frac{2(30+37x)\sqrt{x}}{\sqrt{3x^2+5x+2}} \\ & - \frac{74(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{3\sqrt{3x^2+5x+2}} \\ & + \frac{30(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \end{aligned}$$

command

```
integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(85 \sqrt{3} (3x^2 + 5x + 2) \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 333 \sqrt{3} (3x^2 + 5x + 2) \text{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \text{we}\right) \right)}{27(3x^2 + 5x + 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} (5x - 2)\sqrt{x}}{9x^4 + 30x^3 + 37x^2 + 20x + 4}, x\right)$$

23.92 Problem number 1068

$$\int \frac{2 - 5x}{\sqrt{x} (2 + 5x + 3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{30(2+3x)\sqrt{x}}{\sqrt{3x^2+5x+2}} + \frac{2(38+45x)\sqrt{x}}{\sqrt{3x^2+5x+2}} \\ & + \frac{30(1+x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \text{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \\ & - \frac{37(1+x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \text{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \end{aligned}$$

command

```
integrate((2-5*x)/(3*x^2+5*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 \sqrt{3} (3x^2 + 5x + 2) \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 15 \sqrt{3} (3x^2 + 5x + 2) \text{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}, \text{we}\right) \right)}{3x^2 + 5x + 2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} (5x - 2)\sqrt{x}}{9x^5 + 30x^4 + 37x^3 + 20x^2 + 4x}, x\right)$$

23.93 Problem number 1069

$$\int \frac{2-5x}{x^{3/2}(2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{76+90x}{\sqrt{x}\sqrt{3x^2+5x+2}} + \frac{39(2+3x)\sqrt{x}}{\sqrt{3x^2+5x+2}} \\ & - \frac{39(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \\ & + \frac{45(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} - \frac{39\sqrt{3x^2+5x+2}}{\sqrt{x}} \end{aligned}$$

command

`integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{25\sqrt{3}(3x^3+5x^2+2x)\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right) - 117\sqrt{3}(3x^3+5x^2+2x)\operatorname{weierstrassZeta}\left(\frac{28}{27}, \frac{80}{729}\right)}{3(3x^3+5x^2+2x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}(5x-2)\sqrt{x}}{9x^6+30x^5+37x^4+20x^3+4x^2}, x\right)$$

23.94 Problem number 1070

$$\int \frac{2-5x}{x^{5/2}(2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{76+90x}{x^{\frac{3}{2}}\sqrt{3x^2+5x+2}} - \frac{170(2+3x)\sqrt{x}}{3\sqrt{3x^2+5x+2}} \\ & - \frac{115(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{2\sqrt{3x^2+5x+2}} \\ & + \frac{170(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{3\sqrt{3x^2+5x+2}} \\ & - \frac{115\sqrt{3x^2+5x+2}}{3x^{\frac{3}{2}}} + \frac{170\sqrt{3x^2+5x+2}}{3\sqrt{x}} \end{aligned}$$

command

`integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{185 \sqrt{3} (3x^4 + 5x^3 + 2x^2) \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 1530 \sqrt{3} (3x^4 + 5x^3 + 2x^2) \operatorname{weierstrassZeta}\left(\frac{28}{27}\right)}{27(3x^4 + 5x^3 + 2x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} (5x - 2) \sqrt{x}}{9x^7 + 30x^6 + 37x^5 + 20x^4 + 4x^3}, x\right)$$

23.95 Problem number 1071

$$\int \frac{2 - 5x}{x^{7/2} (2 + 5x + 3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{76 + 90x}{x^{\frac{5}{2}} \sqrt{3x^2 + 5x + 2}} + \frac{2693(2 + 3x) \sqrt{x}}{30 \sqrt{3x^2 + 5x + 2}} \\ & - \frac{2693(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{30 \sqrt{3x^2 + 5x + 2}} \\ & + \frac{157(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{2 \sqrt{3x^2 + 5x + 2}} \\ & - \frac{191 \sqrt{3x^2 + 5x + 2}}{5x^{\frac{5}{2}}} + \frac{157 \sqrt{3x^2 + 5x + 2}}{3x^{\frac{3}{2}}} - \frac{2693 \sqrt{3x^2 + 5x + 2}}{30 \sqrt{x}} \end{aligned}$$

command

`integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{665 \sqrt{3} (3x^5 + 5x^4 + 2x^3) \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 24237 \sqrt{3} (3x^5 + 5x^4 + 2x^3) \operatorname{weierstrassZeta}\left(\frac{28}{27}\right)}{270(3x^5 + 5x^4 + 2x^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} (5x - 2) \sqrt{x}}{9x^8 + 30x^7 + 37x^6 + 20x^5 + 4x^4}, x\right)$$

23.96 Problem number 1072

$$\int \frac{(2-5x)x^{13/2}}{(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^{\frac{11}{2}}(74+95x)}{9(3x^2+5x+2)^{\frac{3}{2}}} - \frac{4x^{\frac{7}{2}}(1484+1685x)}{27\sqrt{3x^2+5x+2}} - \frac{1521056(2+3x)\sqrt{x}}{76545\sqrt{3x^2+5x+2}} \\ & + \frac{1521056(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{76545\sqrt{3x^2+5x+2}} \\ & - \frac{211144(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{5103\sqrt{3x^2+5x+2}} \\ & - \frac{167336x^{\frac{3}{2}}\sqrt{3x^2+5x+2}}{2835} + \frac{45820x^{\frac{5}{2}}\sqrt{3x^2+5x+2}}{567} + \frac{211144\sqrt{x}\sqrt{3x^2+5x+2}}{5103} \end{aligned}$$

command

```
integrate((2-5*x)*x^(13/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(5698840\sqrt{3}\left(9x^4+30x^3+37x^2+20x+4\right)\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right)-6844752\sqrt{3}\left(9x^4+30x^3\right)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x^7-2x^6)\sqrt{3x^2+5x+2}\sqrt{x}}{27x^6+135x^5+279x^4+305x^3+186x^2+60x+8}, x\right)$$

23.97 Problem number 1073

$$\int \frac{(2-5x)x^{11/2}}{(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^{\frac{9}{2}}(74+95x)}{9(3x^2+5x+2)^{\frac{3}{2}}} - \frac{8x^{\frac{5}{2}}(773+905x)}{27\sqrt{3x^2+5x+2}} + \frac{33608(2+3x)\sqrt{x}}{729\sqrt{3x^2+5x+2}} \\ & - \frac{33608(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{729\sqrt{3x^2+5x+2}} \\ & + \frac{16040(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{243\sqrt{3x^2+5x+2}} \\ & + \frac{2348x^{\frac{3}{2}}\sqrt{3x^2+5x+2}}{27} - \frac{16040\sqrt{x}\sqrt{3x^2+5x+2}}{243} \end{aligned}$$

command

`integrate((2-5*x)*x^(11/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(60340 \sqrt{3} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 151236 \sqrt{3} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x^6 - 2x^5)\sqrt{3x^2+5x+2}\sqrt{x}}{27x^6 + 135x^5 + 279x^4 + 305x^3 + 186x^2 + 60x + 8}, x\right)$$

23.98 Problem number 1074

$$\int \frac{(2-5x)x^{9/2}}{(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^{\frac{7}{2}}(74+95x)}{9(3x^2+5x+2)^{\frac{3}{2}}} - \frac{4x^{\frac{3}{2}}(536+645x)}{9\sqrt{3x^2+5x+2}} - \frac{17512(2+3x)\sqrt{x}}{243\sqrt{3x^2+5x+2}} \\ & + \frac{17512(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{243\sqrt{3x^2+5x+2}} \\ & - \frac{7540(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{81\sqrt{3x^2+5x+2}} \\ & + \frac{7540\sqrt{x}\sqrt{3x^2+5x+2}}{81} \end{aligned}$$

command

```
integrate((2-5*x)*x^(9/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(24080 \sqrt{3} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 78804 \sqrt{3} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(5x^5 - 2x^4)\sqrt{3x^2 + 5x + 2}\sqrt{x}}{27x^6 + 135x^5 + 279x^4 + 305x^3 + 186x^2 + 60x + 8}, x\right)$$

23.99 Problem number 1075

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^{5/2}(74+95x)}{9(3x^2+5x+2)^{3/2}} + \frac{8020(2+3x)\sqrt{x}}{81\sqrt{3x^2+5x+2}} - \frac{40(167+206x)\sqrt{x}}{27\sqrt{3x^2+5x+2}} \\ & - \frac{8020(1+x)^{3/2}\sqrt{\frac{1}{1+x}}\text{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{81\sqrt{3x^2+5x+2}} \\ & + \frac{3340(1+x)^{3/2}\sqrt{\frac{1}{1+x}}\text{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{27\sqrt{3x^2+5x+2}} \end{aligned}$$

command

```
integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10010 \sqrt{3} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 36090 \sqrt{3} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \right)$$

729

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(5x^4 - 2x^3)\sqrt{3x^2 + 5x + 2}\sqrt{x}}{27x^6 + 135x^5 + 279x^4 + 305x^3 + 186x^2 + 60x + 8}, x\right)$$

23.100 Problem number 1076

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^{\frac{3}{2}}(74+95x)}{9(3x^2+5x+2)^{\frac{3}{2}}} - \frac{3464(2+3x)\sqrt{x}}{27\sqrt{3x^2+5x+2}} + \frac{4(715+866x)\sqrt{x}}{9\sqrt{3x^2+5x+2}} \\ & + \frac{3464(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{27\sqrt{3x^2+5x+2}} \\ & - \frac{1430(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{9\sqrt{3x^2+5x+2}} \end{aligned}$$

command

`integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4210\sqrt{3}\left(9x^4+30x^3+37x^2+20x+4\right)\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right)-15588\sqrt{3}\left(9x^4+30x^3+37x^2+20x+4\right)\right)}{27\sqrt{3x^2+5x+2}}$$

243

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(5x^3-2x^2)\sqrt{3x^2+5x+2}\sqrt{x}}{27x^6+135x^5+279x^4+305x^3+186x^2+60x+8}, x\right)$$

23.101 Problem number 1077

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(74+95x)\sqrt{x}}{9(3x^2+5x+2)^{\frac{3}{2}}} + \frac{1450(2+3x)\sqrt{x}}{9\sqrt{3x^2+5x+2}} - \frac{2(1831+2175x)\sqrt{x}}{9\sqrt{3x^2+5x+2}} \\ & - \frac{1450(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{9\sqrt{3x^2+5x+2}} \\ & + \frac{598(1+x)^{\frac{3}{2}}\sqrt{\frac{1}{1+x}}\operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right)\sqrt{2}\sqrt{\frac{2+3x}{1+x}}}{3\sqrt{3x^2+5x+2}} \end{aligned}$$

command

```
integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(1757 \sqrt{3} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 6525 \sqrt{3} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \right)}{81 (9x^4 + 30x^3 + 37x^2 + 20x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(5x^2 - 2x)\sqrt{3x^2 + 5x + 2}\sqrt{x}}{27x^6 + 135x^5 + 279x^4 + 305x^3 + 186x^2 + 60x + 8}, x\right)$$

23.102 Problem number 1078

$$\int \frac{(2 - 5x)\sqrt{x}}{(2 + 5x + 3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(30 + 37x)\sqrt{x}}{3(3x^2 + 5x + 2)^{\frac{3}{2}}} - \frac{198(2 + 3x)\sqrt{x}}{\sqrt{3x^2 + 5x + 2}} + \frac{2(250 + 297x)\sqrt{x}}{\sqrt{3x^2 + 5x + 2}} \\ & + \frac{198(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \text{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2 + 5x + 2}} \\ & - \frac{245(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \text{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2 + 5x + 2}} \end{aligned}$$

command

```
integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(80 \sqrt{3} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 297 \sqrt{3} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \right)}{3 (9x^4 + 30x^3 + 37x^2 + 20x + 4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2}(5x - 2)\sqrt{x}}{27x^6 + 135x^5 + 279x^4 + 305x^3 + 186x^2 + 60x + 8}, x\right)$$

23.103 Problem number 1079

$$\int \frac{2-5x}{\sqrt{x}(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(38+45x)\sqrt{x}}{3(3x^2+5x+2)^{\frac{3}{2}}} + \frac{715(2+3x)\sqrt{x}}{3\sqrt{3x^2+5x+2}} - \frac{5(361+429x)\sqrt{x}}{3\sqrt{3x^2+5x+2}} \\ & - \frac{715(1+x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{3\sqrt{3x^2+5x+2}} \\ & + \frac{295(1+x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{\sqrt{3x^2+5x+2}} \end{aligned}$$

command

`integrate((2-5*x)/(3*x^2+5*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1735\sqrt{3}(9x^4+30x^3+37x^2+20x+4)\operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x+\frac{5}{9}\right) - 6435\sqrt{3}(9x^4+30x^3+37x^2+20x+4)}{27(9x^4+30x^3+37x^2+20x+4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}(5x-2)\sqrt{x}}{27x^7+135x^6+279x^5+305x^4+186x^3+60x^2+8x}, x\right)$$

23.104 Problem number 1080

$$\int \frac{2-5x}{x^{3/2}(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\frac{76}{3}+30x}{(3x^2+5x+2)^{\frac{3}{2}}\sqrt{x}} + \frac{-1717-2085x}{3\sqrt{x}\sqrt{3x^2+5x+2}} - \frac{838(2+3x)\sqrt{x}}{3\sqrt{3x^2+5x+2}} \\ & - \frac{695(1+x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{2\sqrt{3x^2+5x+2}} \\ & + \frac{838(1+x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{3\sqrt{3x^2+5x+2}} + \frac{838\sqrt{3x^2+5x+2}}{3\sqrt{x}} \end{aligned}$$

command

`integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2065 \sqrt{3} (9x^5 + 30x^4 + 37x^3 + 20x^2 + 4x) \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 7542 \sqrt{3} (9x^5 + 30x^4 + 37x^3 + 20x^2 + 4x)}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} (5x - 2)\sqrt{x}}{27x^8 + 135x^7 + 279x^6 + 305x^5 + 186x^4 + 60x^3 + 8x^2}, x\right)$$

23.105 Problem number 1081

$$\int \frac{2 - 5x}{x^{5/2} (2 + 5x + 3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\frac{76}{3} + 30x}{x^{\frac{3}{2}} (3x^2 + 5x + 2)^{\frac{3}{2}}} - \frac{3(181 + 225x)}{x^{\frac{3}{2}} \sqrt{3x^2 + 5x + 2}} + \frac{625(2 + 3x)\sqrt{x}}{2\sqrt{3x^2 + 5x + 2}} \\ & - \frac{625(1+x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \text{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{2\sqrt{3x^2 + 5x + 2}} \\ & + \frac{795(1+x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \text{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{2\sqrt{3x^2 + 5x + 2}} \\ & + \frac{265\sqrt{3x^2 + 5x + 2}}{x^{\frac{3}{2}}} - \frac{625\sqrt{3x^2 + 5x + 2}}{2\sqrt{x}} \end{aligned}$$

command

`integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1645 \sqrt{3} (9x^6 + 30x^5 + 37x^4 + 20x^3 + 4x^2) \text{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 5625 \sqrt{3} (9x^6 + 30x^5 + 37x^4 + 20x^3 + 4x^2)}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} (5x - 2)\sqrt{x}}{27x^9 + 135x^8 + 279x^7 + 305x^6 + 186x^5 + 60x^4 + 8x^3}, x\right)$$

23.106 Problem number 1082

$$\int \frac{2 - 5x}{x^{7/2} (2 + 5x + 3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\frac{76}{3} + 30x}{x^{\frac{5}{2}} (3x^2 + 5x + 2)^{\frac{3}{2}}} + \frac{-1541 - 1965x}{3x^{\frac{5}{2}} \sqrt{3x^2 + 5x + 2}} - \frac{9521(2 + 3x) \sqrt{x}}{30 \sqrt{3x^2 + 5x + 2}} \\ & + \frac{9521(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \operatorname{EllipticE}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{30 \sqrt{3x^2 + 5x + 2}} \\ & - \frac{1733(1 + x)^{\frac{3}{2}} \sqrt{\frac{1}{1+x}} \operatorname{EllipticF}\left(\frac{\sqrt{x}}{\sqrt{1+x}}, \frac{i\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{2+3x}{1+x}}}{4 \sqrt{3x^2 + 5x + 2}} \\ & + \frac{1252 \sqrt{3x^2 + 5x + 2}}{5x^{\frac{5}{2}}} - \frac{1733 \sqrt{3x^2 + 5x + 2}}{6x^{\frac{3}{2}}} + \frac{9521 \sqrt{3x^2 + 5x + 2}}{30\sqrt{x}} \end{aligned}$$

command

`integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{30380 \sqrt{3} (9x^7 + 30x^6 + 37x^5 + 20x^4 + 4x^3) \operatorname{weierstrassPInverse}\left(\frac{28}{27}, \frac{80}{729}, x + \frac{5}{9}\right) - 85689 \sqrt{3} (9x^7 + 30x^6 + 30x^5 + 20x^4 + 4x^3)}{270000 x^{\frac{5}{2}} (3x^2 + 5x + 2)^{\frac{3}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} (5x - 2) \sqrt{x}}{27x^{10} + 135x^9 + 279x^8 + 305x^7 + 186x^6 + 60x^5 + 8x^4}, x\right)$$

23.107 Problem number 1255

$$\int (A + Bx) \sqrt{d + ex} \sqrt{bx + cx^2} dx$$

Optimal antiderivative

$$\frac{2B(cx^2 + bx)^{\frac{3}{2}} \sqrt{ex + d}}{7c}$$

$$+ \frac{2(5c(-7Ac + 3bB) de(-be + 2cd) + (7Ace - 4bBe + Bcd) (-2b^2e^2 - 3bcde + 8c^2d^2)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right)}{105c^{\frac{5}{2}}e^3 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}$$

$$+ \frac{2d(-be + cd) (7Ace(-be + 2cd) - B(-4b^2e^2 - bcde + 8c^2d^2)) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}}}{105c^{\frac{5}{2}}e^3 \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

$$+ \frac{2(7Ace(be + cd) - B(4b^2e^2 - 2bcde + 4c^2d^2)) + 3ce(7Ace - 4bBe + Bcd) x \sqrt{ex + d} \sqrt{cx^2 + bx}}{105c^2e^2}$$

command

`integrate((B*x+A)*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((8Bc^4d^4 - (9Bbc^3 + 14Ac^4)d^3e - (4Bb^2c^2 - 21Abc^3)d^2e^2 - 3(3Bb^3c - 7Ab^2c^2)de^3 + 2(4Bb^4 - 7Ab^3c)e^4) \sqrt{ex + d} \sqrt{cx^2 + bx} \right)}{105c^2e^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^2 + bx} (Bx + A)\sqrt{ex + d}, x\right)$$

23.108 Problem number 1256

$$\int \frac{(A + Bx)\sqrt{bx + cx^2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{2(5Ace(-be + 2cd) - B(-2b^2e^2 - 3bcde + 8c^2d^2)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{15c^{\frac{3}{2}}e^3 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}$$

$$+ \frac{2d(-be + cd) (-10Ace + bBe + 8Bcd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{15c^{\frac{3}{2}}e^3 \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

$$+ \frac{2(-3Bcex - 5Ace - bBe + 4Bcd) \sqrt{ex + d} \sqrt{cx^2 + bx}}{15ce^2}$$

command

`integrate((B*x+A)*(c*x^2+b*x)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((8 Bc^3 d^3 - (7 Bbc^2 + 10 Ac^3) d^2 e - 2 (Bb^2 c - 5 Abc^2) de^2 - (2 Bb^3 - 5 Ab^2 c) e^3) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4}{\dots} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx} (Bx + A)}{\sqrt{ex + d}}, x \right)$$

23.109 Problem number 1257

$$\int \frac{(A + Bx) \sqrt{bx + cx^2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(-6Ace - bBe + 8Bcd) \text{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{3e^3 \sqrt{c} \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} + \frac{2(Bd(-5be + 8cd) - 3Ae(-be + 2cd)) \text{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3e^3 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}} + \frac{2(Bex - 3Ae + 4Bd) \sqrt{cx^2 + bx}}{3e^2 \sqrt{ex + d}}$$

command

`integrate((B*x+A)*(c*x^2+b*x)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((8 Bc^2 d^3 - (Bb^2 - 3 Abc) x e^3 - ((5 Bbc + 6 Ac^2) dx + (Bb^2 - 3 Abc) d) e^2 + (8 Bc^2 d^2 x - (5 Bbc + 6 Ac^2) d^2) e \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx} (Bx + A) \sqrt{ex + d}}{e^2 x^2 + 2 dex + d^2}, x \right)$$

23.110 Problem number 1258

$$\int \frac{(A + Bx) \sqrt{bx + cx^2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Bd(-7be + 8cd) - Ae(-be + 2cd)) \operatorname{EllipticE}\left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{3de^3(-be + cd) \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}$$

$$\frac{2(-2Ace - 3bBe + 8Bcd) \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3e^3 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

$$\frac{2(d^2(-Ace - 3bBe + 4Bcd) + e(Bd(-4be + 5cd) - Ae(-be + 2cd))x) \sqrt{cx^2 + bx}}{3de^2(-be + cd)(ex + d)^{\frac{3}{2}}}$$

command

`integrate((B*x+A)*(c*x^2+b*x)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((8Bc^2d^5 + Ab^2x^2e^5 + 2(Ab^2dx + (Bb^2 + Abc)dx^2)e^4 + (Ab^2d^2 - (11Bbc + 2Ac^2)d^2x^2 + 4(Bb^2 + Abc)d^2x \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx} (Bx + A) \sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

23.111 Problem number 1259

$$\int \frac{(A + Bx) \sqrt{bx + cx^2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(2Ae(b^2e^2 - bcde + c^2d^2) + Bd(3b^2e^2 - 13bcde + 8c^2d^2)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}}}{15d^2e^3(-be + cd)^2 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\
+ & \frac{2(Bd(-9be + 8cd) + Ae(-be + 2cd)) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{15de^3(-be + cd) \sqrt{ex + d} \sqrt{cx^2 + bx}} \\
- & \frac{2(d(Bd(-3be + 4cd) + Ae(-2be + cd)) + e(Bd(-6be + 7cd) - Ae(-be + 2cd))x) \sqrt{cx^2 + bx}}{15d^2e^2(-be + cd)(ex + d)^{\frac{5}{2}}} \\
+ & \frac{2(2Ae(b^2e^2 - bcde + c^2d^2) + Bd(3b^2e^2 - 13bcde + 8c^2d^2)) \sqrt{cx^2 + bx}}{15d^2e^2(-be + cd)^2 \sqrt{ex + d}}
\end{aligned}$$

command

```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((8Bc^3d^7 + 2Ab^3x^3e^7 + 3(2Ab^3dx^2 + (Bb^3 - Ab^2c)dx^3)e^6 + (6Ab^3d^2x + (8Bb^2c - 3Abc^2)d^2x^3 + 9(Bb^3 - A$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx} (Bx + A) \sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

23.112 Problem number 1260

$$\int \frac{(A + Bx)(bx + cx^2)^{3/2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{2(-7Bcex - 9Ace - 3bBe + 8Bcd) (cx^2 + bx)^{\frac{3}{2}} \sqrt{ex + d}}{63ce^2}$$

$$+ \frac{2(5bcde(-be + 2cd) (-9Ace - 3bBe + 8Bcd) + (-2b^2e^2 - 3bcde + 8c^2d^2) (9Ace(-be + 2cd) - B(-4b^2e^2 - 7bcde + 8c^2d^2))) \text{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{d+ex}{-b}}\right) + \frac{315c^{\frac{5}{2}}e^5 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}{315c^{\frac{5}{2}}e^5 \sqrt{ex + d} \sqrt{cx^2 + bx}}}{315c^2e^4}$$

$$+ \frac{2d(-be + cd) (9Ace(-b^2e^2 - 16bcde + 16c^2d^2) - B(-4b^3e^3 - 9b^2cde^2 - 120bc^2d^2e + 128c^3d^3)) \text{EllipticF}\left(\frac{\sqrt{c}}{\sqrt{-b}}, \sqrt{\frac{d+ex}{-b}}\right) + \frac{315c^{\frac{5}{2}}e^5 \sqrt{ex + d} \sqrt{cx^2 + bx}}{315c^2e^4}}{315c^2e^4}$$

command

`integrate((B*x+A)*(c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((128 Bc^5d^5 - 8(31 Bbc^4 + 18 Ac^5)d^4e + (95 Bb^2c^3 + 288 Abc^4)d^3e^2 + (20 Bb^3c^2 - 117 Ab^2c^3)d^2e^3 + (7 Bb^4c^2 - 117 Ab^3c^3)d^2e^3 + (7 Bb^4c^2 - 117 Ab^3c^3)d^2e^3 + (7 Bb^4c^2 - 117 Ab^3c^3)d^2e^3 + (7 Bb^4c^2 - 117 Ab^3c^3)d^2e^3 \right)}{315c^2e^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bcx^3 + Abx + (Bb + Ac)x^2) \sqrt{cx^2 + bx}}{\sqrt{ex + d}}, x\right)$$

23.113 Problem number 1261

$$\int \frac{(A + Bx) (bx + cx^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Bex - 7Ae + 8Bd) (cx^2 + bx)^{\frac{3}{2}}}{7e^2 \sqrt{ex + d}}$$

$$+ \frac{2(5bce(-7Ae + 8Bd) (-be + 2cd) - (-14Ace - bBe + 16Bcd) (-2b^2e^2 - 3bcde + 8c^2d^2)) \text{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{d+ex}{-b}}\right) + \frac{35c^{\frac{3}{2}}e^5 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}{35c^{\frac{3}{2}}e^5 \sqrt{ex + d} \sqrt{cx^2 + bx}}}{35c^2e^4}$$

$$+ \frac{2d(-be + cd) (56Ace(-be + 2cd) - B(-b^2e^2 - 72bcde + 128c^2d^2)) \text{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{ex}{d}} + \frac{35c^{\frac{3}{2}}e^5 \sqrt{ex + d} \sqrt{cx^2 + bx}}{35c^2e^4}}{35c^2e^4}$$

$$+ \frac{2(7Ace(-7be + 8cd) - B(b^2e^2 - 60bcde + 64c^2d^2) + 3ce(-14Ace - bBe + 16Bcd) x) \sqrt{ex + d} \sqrt{cx^2 + bx}}{35c^2e^4}$$

command

`integrate((B*x+A)*(c*x^2+b*x)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((128 Bc^4d^5 + (2 Bb^4 - 7 Ab^3c)xe^5 + (2 (5 Bb^3c - 21 Ab^2c^2)dx + (2 Bb^4 - 7 Ab^3c)d)e^4 + ((55 Bb^2c^2 + 168 Abc^3) \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bcx^3 + Abx + (Bb + Ac)x^2) \sqrt{cx^2 + bx} \sqrt{ex + d}}{e^2x^2 + 2dex + d^2}, x \right)$$

23.114 Problem number 1262

$$\int \frac{(A + Bx)(bx + cx^2)^{3/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(3Bex - 5Ae + 8Bd)(cx^2 + bx)^{\frac{3}{2}}}{15e^2(ex + d)^{\frac{3}{2}}}$$

$$\frac{2(40Ace(-be + 2cd) - B(3b^2e^2 - 88bcde + 128c^2d^2)) \text{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{15e^5 \sqrt{c} \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}}$$

$$+ \frac{2(5Ae(3b^2e^2 - 16bcde + 16c^2d^2) - Bd(39b^2e^2 - 152bcde + 128c^2d^2)) \text{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{cx^2 + bx}}{15e^5 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

$$- \frac{2(4Bd(-9be + 16cd) - 5Ae(-3be + 8cd) + e(-10Ace - 3bBe + 16Bcd)x) \sqrt{cx^2 + bx}}{15e^4 \sqrt{ex + d}}$$

command

`integrate((B*x+A)*(c*x^2+b*x)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((128 Bc^3d^5 + (3 Bb^3 - 5 Ab^2c)x^2e^5 + ((23 Bb^2c + 80 Abc^2)dx^2 + 2 (3 Bb^3 - 5 Ab^2c)dx)e^4 - (8 (19 Bbc^2 + 10) \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bcx^3 + Abx + (Bb + Ac)x^2) \sqrt{cx^2 + bx} \sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x \right)$$

23.115 Problem number 1263

$$\int \frac{(A + Bx)(bx + cx^2)^{3/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(d^2(-3Ace - 5bBe + 8Bcd) + e(Bd(-8be + 11cd) - 3Ae(-be + 2cd))x)(cx^2 + bx)^{\frac{3}{2}}}{15de^2(-be + cd)(ex + d)^{\frac{5}{2}}} + \frac{2(3Ae(b^2e^2 - 16bcde + 16c^2d^2) - Bd(43b^2e^2 - 168bcde + 128c^2d^2)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x}}{15de^5(-be + cd) \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} + \frac{2(24Ace(-be + 2cd) - B(15b^2e^2 - 104bcde + 128c^2d^2)) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 - \frac{cx}{b}}}{15e^5 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}} - \frac{2(d(3Ace(-7be + 8cd) - B(15b^2e^2 - 76bcde + 64c^2d^2)) - ce(Bd(-13be + 16cd) - 3Ae(-be + 2cd))x) \sqrt{cx^2 + bx}}{15de^4(-be + cd) \sqrt{ex + d}}$$

command

```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((128 Bc^3d^7 - 3Ab^3x^3e^7 - (9Ab^3dx^2 + 2(Bb^3 + 9Ab^2c)dx^3)e^6 - (9Ab^3d^2x - (103Bb^2c + 72Abc^2)d^2x^3 + 6(L$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^3 + Abx + (Bb + Ac)x^2) \sqrt{cx^2 + bx} \sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

23.116 Problem number 1264

$$\int \frac{(A + Bx)(d + ex)^{5/2}}{\sqrt{bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(7Ace(8b^2e^2 - 23bcde + 23c^2d^2) + B(-48b^3e^3 + 128b^2cde^2 - 103bc^2d^2e + 15c^3d^3)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) + 105c^{\frac{7}{2}}e\sqrt{1 + \frac{ex}{d}}\sqrt{cx^2 + bx} + 2d(-be + cd)(28Ace(-be + 2cd) + B(24b^2e^2 - 43bcde + 15c^2d^2)) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right)\sqrt{-b}\sqrt{x}\sqrt{1 + \frac{ex}{d}}}{105c^3} + \frac{2(7Ace - 6bBe + 5Bcd)(ex + d)^{\frac{3}{2}}\sqrt{cx^2 + bx}}{35c^2} + \frac{2B(ex + d)^{\frac{5}{2}}\sqrt{cx^2 + bx}}{7c} + \frac{2(28Ace(-be + 2cd) + B(24b^2e^2 - 43bcde + 15c^2d^2))\sqrt{ex + d}\sqrt{cx^2 + bx}}{105c^3}$$

command

`integrate((B*x+A)*(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((15Bc^4d^4 + (47Bbc^3 - 154Ac^4)d^3e - (158Bb^2c^2 - 231Abc^3)d^2e^2 + (152Bb^3c - 189Ab^2c^2)de^3 - 8(6Bb^4 - \dots)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^2x^3 + Ad^2 + (2Bde + Ae^2)x^2 + (Bd^2 + 2Ade)x)\sqrt{ex + d}}{\sqrt{cx^2 + bx}}, x\right)$$

23.117 Problem number 1265

$$\int \frac{(A + Bx)(d + ex)^{3/2}}{\sqrt{bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(10Ace(-be + 2cd) + B(8b^2e^2 - 13bcde + 3c^2d^2)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right)\sqrt{-b}\sqrt{x}\sqrt{1 + \frac{cx}{b}}\sqrt{ex + d} + 15c^{\frac{5}{2}}e\sqrt{1 + \frac{ex}{d}}\sqrt{cx^2 + bx} + 2d(-be + cd)(5Ace - 4bBe + 3Bcd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right)\sqrt{-b}\sqrt{x}\sqrt{1 + \frac{cx}{b}}\sqrt{1 + \frac{ex}{d}}}{15c^2} + \frac{2B(ex + d)^{\frac{3}{2}}\sqrt{cx^2 + bx}}{5c} + \frac{2(5Ace - 4bBe + 3Bcd)\sqrt{ex + d}\sqrt{cx^2 + bx}}{15c^2}$$

command

```
integrate((B*x+A)*(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((3 Bc^3 d^3 + (8 Bbc^2 - 25 Ac^3) d^2 e - (17 Bb^2 c - 25 Abc^2) de^2 + 2 (4 Bb^3 - 5 Ab^2 c) e^3) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bex^2 + Ad + (Bd + Ae)x) \sqrt{ex + d}}{\sqrt{cx^2 + bx}}, x \right)$$

23.118 Problem number 1266

$$\int \frac{(A + Bx) \sqrt{d + ex}}{\sqrt{bx + cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3Ace - 2bBe + Bcd) \text{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{3c^{\frac{3}{2}} e \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\ & - \frac{2Bd(-be + cd) \text{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3c^{\frac{3}{2}} e \sqrt{ex + d} \sqrt{cx^2 + bx}} \\ & + \frac{2B \sqrt{ex + d} \sqrt{cx^2 + bx}}{3c} \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cx^2 + bx} \sqrt{xe + d} Bc^2 e^2 - (Bc^2 d^2 + 2 (Bbc - 3 Ac^2) de - (2 Bb^2 - 3 Abc) e^2) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bx + A) \sqrt{ex + d}}{\sqrt{cx^2 + bx}}, x \right)$$

23.119 Problem number 1267

$$\int \frac{A + Bx}{\sqrt{d + ex} \sqrt{bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2B \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{e\sqrt{c} \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} + \frac{2(-Ae + Bd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{e\sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

command

`integrate((B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3Bc^{\frac{3}{2}}e^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(\frac{4(c^2d^2 - bcde + b^2e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d^3 - 3bc^2d^2e - 3b^2cde^2 + 2b^3e^3)e^{(-3)}}{27c^3}\right), \operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2 - bcde + b^2e^2)e^{(-2)}}{3c^2}\right)\right)}{e\sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx} (Bx + A) \sqrt{ex + d}}{cex^3 + bdx + (cd + be)x^2}, x\right)$$

23.120 Problem number 1268

$$\int \frac{A + Bx}{(d + ex)^{3/2} \sqrt{bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(-Ae + Bd) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{de(-be + cd) \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} + \frac{2B \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{e\sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}} + \frac{2(-Ae + Bd) \sqrt{cx^2 + bx}}{d(-be + cd) \sqrt{ex + d}}$$

command

```
integrate((B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((Bcd^3 - Abxe^3 - (Abd + 2(Bb - Ac)dx)e^2 + (Bcd^2x - 2(Bb - Ac)d^2)e) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcd)}{3} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx} (Bx + A) \sqrt{ex + d}}{ce^2x^4 + bd^2x + (2cde + be^2)x^3 + (cd^2 + 2bde)x^2}, x \right)$$

23.121 Problem number 1269

$$\int \frac{A + Bx}{(d + ex)^{5/2} \sqrt{bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(2Ae(-be + 2cd) - Bd(be + cd)) \text{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{3d^2e(-be + cd)^2 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} + \frac{2(-Ae + Bd) \text{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3de(-be + cd) \sqrt{ex + d} \sqrt{cx^2 + bx}} + \frac{2(-Ae + Bd) \sqrt{cx^2 + bx}}{3d(-be + cd)(ex + d)^{\frac{3}{2}}} - \frac{2(2Ae(-be + 2cd) - Bd(be + cd)) \sqrt{cx^2 + bx}}{3d^2(-be + cd)^2 \sqrt{ex + d}}$$

command

```
integrate((B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((Bc^2d^5 + 2Ab^2x^2e^5 + (4Ab^2dx + (Bb^2 - 5Abc)dx^2)e^4 + (2Ab^2d^2 - (4Bbc - 5Ac^2)d^2x^2 + 2(Bb^2 - 5Abc)d^2) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx} (Bx + A) \sqrt{ex + d}}{ce^3x^5 + bd^3x + (3cde^2 + be^3)x^4 + 3(cd^2e + bde^2)x^3 + (cd^3 + 3bd^2e)x^2}, x \right)$$

23.122 Problem number 1270

$$\int \frac{A + Bx}{(d + ex)^{7/2} \sqrt{bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(Bd(-2b^2e^2 + 7bcde + 3c^2d^2) - Ae(8b^2e^2 - 23bcde + 23c^2d^2)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{d}}{15d^3e(-be + cd)^3 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} + \frac{2(4Ae(-be + 2cd) - Bd(be + 3cd)) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{15d^2e(-be + cd)^2 \sqrt{ex + d} \sqrt{cx^2 + bx}} + \frac{2(-Ae + Bd) \sqrt{cx^2 + bx}}{5d(-be + cd)(ex + d)^{\frac{5}{2}}} - \frac{2(4Ae(-be + 2cd) - Bd(be + 3cd)) \sqrt{cx^2 + bx}}{15d^2(-be + cd)^2 (ex + d)^{\frac{3}{2}}} + \frac{2(Bd(-2b^2e^2 + 7bcde + 3c^2d^2) - Ae(8b^2e^2 - 23bcde + 23c^2d^2)) \sqrt{cx^2 + bx}}{15d^3(-be + cd)^3 \sqrt{ex + d}}$$

command

`integrate((B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((3Bc^3d^7 - 8Ab^3x^3e^7 - (24Ab^3dx^2 + (2Bb^3 - 27Ab^2c)dx^3)e^6 - (24Ab^3d^2x - (8Bb^2c - 33Abc^2)d^2x^3 + 3(2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx} (Bx + A) \sqrt{ex + d}}{ce^4x^6 + bd^4x + (4cde^3 + be^4)x^5 + 2(3cd^2e^2 + 2bde^3)x^4 + 2(2cd^3e + 3bd^2e^2)x^3 + (cd^4 + 4bd^3e)x^2}, x\right)$$

23.123 Problem number 1271

$$\int \frac{(A + Bx)(d + ex)^{7/2}}{(bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(ex+d)^{\frac{5}{2}}(Abcd + (2Ac^2d + b^2Be - bc(Ae + Bd))x)}{b^2c\sqrt{cx^2 + bx}} \\
& + \frac{2(30Ac^4d^3 + 48b^4Be^3 - 15bc^3d^2(3Ae + Bd) - 8b^3ce^2(5Ae + 16Bd) + b^2c^2de(95Ae + 103Bd)) \operatorname{EllipticE}\left(\frac{\sqrt{c}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{cx^2 + bx}}{15(-b)^{\frac{3}{2}}c^{\frac{7}{2}}\sqrt{1 + \frac{ex}{d}}\sqrt{cx^2 + bx}} \\
& + \frac{2d(-be + cd)(30Ac^3d^2 - 24b^3Be^2 - 15bc^2d(2Ae + Bd) + b^2ce(20Ae + 43Bd)) \operatorname{EllipticF}\left(\frac{\sqrt{c}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{cx^2 + bx}}{15(-b)^{\frac{3}{2}}c^{\frac{7}{2}}\sqrt{ex+d}\sqrt{cx^2 + bx}} \\
& + \frac{2e(10Ac^2d + 6b^2Be - 5bc(Ae + Bd))(ex+d)^{\frac{3}{2}}\sqrt{cx^2 + bx}}{5b^2c^2} \\
& + \frac{2e(30Ac^3d^2 - 24b^3Be^2 - 15bc^2d(2Ae + Bd) + b^2ce(20Ae + 43Bd))\sqrt{ex+d}\sqrt{cx^2 + bx}}{15b^2c^3}
\end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\left(15(Bbc^5 - 2Ac^6)d^4x^2 + 15(Bb^2c^4 - 2Abc^5)d^4x - 8\left((6Bb^5c - 5Ab^4c^2)x^2 + (6Bb^6 - 5Ab^5c)x\right)e^4 + \left(152Bb^5c - 15Ab^4c^2\right)e^3 + \left(152Bb^6 - 15Ab^5c\right)e^2 + \left(152Bb^7 - 15Ab^6c\right)e + 152Bb^8 - 15Ab^7c\right)\right)}{15b^2c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^3x^4 + Ad^3 + (3Bde^2 + Ae^3)x^3 + 3(Bd^2e + Ade^2)x^2 + (Bd^3 + 3Ad^2e)x)\sqrt{cx^2 + bx}\sqrt{ex+d}}{c^2x^4 + 2bcx^3 + b^2x^2}, x\right)$$

23.124 Problem number 1272

$$\int \frac{(A + Bx)(d + ex)^{5/2}}{(bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{2(ex+d)^{\frac{3}{2}}(Abcd+(2Ac^2d+b^2Be-bc(Ae+Bd))x)}{b^2c\sqrt{cx^2+bx}} \\
 & + \frac{2(6Ac^3d^2-8b^3Be^2-3bc^2d(2Ae+Bd)+b^2ce(6Ae+13Bd))\text{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{ex+d}}{3(-b)^{\frac{3}{2}}c^{\frac{5}{2}}\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}} \\
 & + \frac{2d(-be+cd)(6Ac^2d+4b^2Be-3bc(Ae+Bd))\text{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}}{3(-b)^{\frac{3}{2}}c^{\frac{5}{2}}\sqrt{ex+d}\sqrt{cx^2+bx}} \\
 & + \frac{2e(6Ac^2d+4b^2Be-3bc(Ae+Bd))\sqrt{ex+d}\sqrt{cx^2+bx}}{3b^2c^2}
 \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\left(3(Bbc^4-2Ac^5)d^3x^2+3(Bb^2c^3-2Abc^4)d^3x+2((4Bb^4c-3Ab^3c^2)x^2+(4Bb^5-3Ab^4c)x)e^3-((17Bb^3c^4-12B^2b^2c^3-9Bbc^4+3A^2c^5)d^3x^2+3(2Bb^4c-3Ab^3c^2)d^3x+2(4Bb^5-3Ab^4c)d^3x+3(3Bb^5-3Ab^4c)d^3x+3(3Bb^5-3Ab^4c)d^3x)\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Be^2x^3+Ad^2+(2Bde+ Ae^2)x^2+(Bd^2+2Ade)x)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^2x^4+2bcx^3+b^2x^2},x\right)$$

23.125 Problem number 1273

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{2(Abcd+(2Ac^2d+b^2Be-bc(Ae+Bd))x)\sqrt{ex+d}}{b^2c\sqrt{cx^2+bx}} \\
 & + \frac{2(2Ac^2d+2b^2Be-bc(Ae+Bd))\text{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{ex+d}}{(-b)^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}} \\
 & + \frac{2(-2Ac+bB)d(-be+cd)\text{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}}{(-b)^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{ex+d}\sqrt{cx^2+bx}}
 \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(((Bbc^3 - 2Ac^4)d^2x^2 + (Bb^2c^2 - 2Abc^3)d^2x - ((2Bb^3c - Ab^2c^2)x^2 + (2Bb^4 - Ab^3c)x)e^2 + 2((Bb^2c^2 + Abc^3) \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Bex^2 + Ad + (Bd + Ae)x) \sqrt{cx^2 + bx} \sqrt{ex + d}}{c^2x^4 + 2bcx^3 + b^2x^2}, x \right)$$

23.126 Problem number 1274

$$\int \frac{(A + Bx)\sqrt{d + ex}}{(bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab - (-2Ac + bB)x) \sqrt{ex + d}}{b^2 \sqrt{cx^2 + bx}} \\ & - \frac{2(-2Ac + bB) \text{EllipticE} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{(-b)^{\frac{3}{2}} \sqrt{c} \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\ & + \frac{2(Abe - 2Acd + Bbd) \text{EllipticF} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}} \right) \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{(-b)^{\frac{3}{2}} \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}} \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(((Bbc^2 - 2Ac^3)dx^2 + (Bb^2c - 2Abc^2)dx + ((Bb^2c + Abc^2)x^2 + (Bb^3 + Ab^2c)x)e) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx} (Bx + A) \sqrt{ex + d}}{c^2x^4 + 2bcx^3 + b^2x^2}, x \right)$$

23.127 Problem number 1275

$$\int \frac{A + Bx}{\sqrt{d + ex} (bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab(-be + cd) + c(2Acd - b(Ae + Bd))x) \sqrt{ex + d}}{b^2 d(-be + cd) \sqrt{cx^2 + bx}} \\ & - \frac{2(Abe - 2Acd + Bbd) \operatorname{EllipticE}\left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c} \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex + d}}{(-b)^{\frac{3}{2}} d(-be + cd) \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\ & + \frac{2(-2Ac + bB) \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{(-b)^{\frac{3}{2}} \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + bx}} \end{aligned}$$

command

`integrate((B*x+A)/(c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((Bbc^2 - 2Ac^3)d^2x^2 + (Bb^2c - 2Abc^2)d^2x + (Ab^2cx^2 + Ab^3x)e^2 - 2((Bb^2c - Abc^2)dx^2 + (Bb^3 - Ab^2c)dx) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx} (Bx + A) \sqrt{ex + d}}{c^2ex^5 + b^2dx^2 + (c^2d + 2bce)x^4 + (2bcd + b^2e)x^3}, x\right)$$

23.128 Problem number 1276

$$\int \frac{A + Bx}{(d + ex)^{3/2} (bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(Ab(-be + cd) + c(2Acd - b(Ae + Bd))x)}{b^2d(-be + cd)\sqrt{ex + d}\sqrt{cx^2 + bx}} \\
& + \frac{2(2Ac^2d^2 - b^2e(-2Ae + Bd) - bcd(2Ae + Bd)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x}\sqrt{1 + \frac{cx}{b}}\sqrt{ex + d}}{(-b)^{\frac{3}{2}}d^2(-be + cd)^2\sqrt{1 + \frac{ex}{d}}\sqrt{cx^2 + bx}} \\
& + \frac{2(Abe - 2Acd + Bbd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x}\sqrt{1 + \frac{cx}{b}}\sqrt{1 + \frac{ex}{d}}}{(-b)^{\frac{3}{2}}d(-be + cd)\sqrt{ex + d}\sqrt{cx^2 + bx}} \\
& - \frac{2e(2Ac^2d^2 - b^2e(-2Ae + Bd) - bcd(2Ae + Bd))\sqrt{cx^2 + bx}}{b^2d^2(-be + cd)^2\sqrt{ex + d}}
\end{aligned}$$

command

```
integrate((B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(((Bbc^3 - 2Ac^4)d^4x^2 + (Bb^2c^2 - 2Abc^3)d^4x - 2(Ab^3cx^3 + Ab^4x^2)e^4 - (2Ab^4dx - (Bb^3c + 3Ab^2c^2)dx^3 - (B$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx}(Bx + A)\sqrt{ex + d}}{c^2e^2x^6 + b^2d^2x^2 + 2(c^2de + bce^2)x^5 + (c^2d^2 + 4bcde + b^2e^2)x^4 + 2(bcd^2 + b^2de)x^3}, x\right)$$

23.129 Problem number 1277

$$\int \frac{A + Bx}{(d + ex)^{5/2}(bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab(-be + cd) + c(2Acd - b(Ae + Bd))x)}{b^2d(-be + cd)(ex + d)^{\frac{3}{2}}\sqrt{cx^2 + bx}}$$

$$+ \frac{2(6Ac^3d^3 - b^2cde(-19Ae + 7Bd) + 2b^3e^2(-4Ae + Bd) - 3bc^2d^2(3Ae + Bd)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}}{3(-b)^{\frac{3}{2}}d^3(-be + cd)^3\sqrt{1 + \frac{ex}{d}}\sqrt{cx^2 + bx}}$$

$$- \frac{2(6Ac^2d^2 - b^2e(-4Ae + Bd) - 3bcd(2Ae + Bd)) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x}\sqrt{1 + \frac{cx}{b}}\sqrt{1 + \frac{ex}{d}}}{3(-b)^{\frac{3}{2}}d^2(-be + cd)^2\sqrt{ex + d}\sqrt{cx^2 + bx}}$$

$$- \frac{2e(6Ac^2d^2 - b^2e(-4Ae + Bd) - 3bcd(2Ae + Bd))\sqrt{cx^2 + bx}}{3b^2d^2(-be + cd)^2(ex + d)^{\frac{3}{2}}}$$

$$- \frac{2e(6Ac^3d^3 - b^2cde(-19Ae + 7Bd) + 2b^3e^2(-4Ae + Bd) - 3bc^2d^2(3Ae + Bd))\sqrt{cx^2 + bx}}{3b^2d^3(-be + cd)^3\sqrt{ex + d}}$$

command

```
integrate((B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx}(Bx + A)\sqrt{ex + d}}{c^2e^3x^7 + b^2d^3x^2 + (3c^2de^2 + 2bce^3)x^6 + (3c^2d^2e + 6bcde^2 + b^2e^3)x^5 + (c^2d^3 + 6bcd^2e + 3b^2de^2)x^4 + (2$$

23.130 Problem number 1278

$$\int \frac{(A + Bx)(d + ex)^{7/2}}{(bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(ex+d)^{\frac{5}{2}}(Abcd+(2Ac^2d+b^2Be-bc(Ae+Bd))x)}{3b^2c(cx^2+bx)^{\frac{3}{2}}} + \frac{2(bc d^2(8Ac^2d+b^2Be-bc(9Ae+4Bd))+(16Ac^4d^3-4b^4Be^3+b^3ce^2(Ae+4Bd)-8bc^3d^2(3Ae+Bd)+b^2c^2d^2(8Ac^2d+b^2Be-bc(9Ae+4Bd)))}{3b^4c^2\sqrt{cx^2+bx}} - \frac{2(16Ac^4d^3-8b^4Be^3+b^3ce^2(2Ae+5Bd)-8bc^3d^2(3Ae+Bd)+b^2c^2de(4Ae+5Bd))\text{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{cx+d}{-b}}\right)}{3(-b)^{\frac{7}{2}}c^{\frac{5}{2}}\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}} + \frac{2d(-be+cd)(16Ac^3d^2+4b^3Be^2+b^2ce(-Ae+Bd)-8bc^2d(2Ae+Bd))\text{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}},\sqrt{\frac{be}{cd}}\right)\sqrt{x}\sqrt{1+\frac{ex}{d}}}{3(-b)^{\frac{7}{2}}c^{\frac{5}{2}}\sqrt{ex+d}\sqrt{cx^2+bx}}$$

command

```
integrate((B*x+A)*(e*x+d)^(7/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((8(Bbc^6-2Ac^7)d^4x^4+16(Bb^2c^5-2Abc^6)d^4x^3+8(Bb^3c^4-2Ab^2c^5)d^4x^2+2((4Bb^5c^2-Ab^4c^3)x^4+2\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Be^3x^4+Ad^3+(3Bde^2+ Ae^3)x^3+3(Bd^2e+Ade^2)x^2+(Bd^3+3Ad^2e)x)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3x^6+3bc^2x^5+3b^2cx^4+b^3x^3},x\right)$$

23.131 Problem number 1279

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(ex+d)^{\frac{3}{2}} (Abcd + (2Ac^2d + b^2Be - bc(Ae + Bd)) x)}{3b^2c(cx^2 + bx)^{\frac{3}{2}}} \\
& + \frac{2(bd(8Ac^2d + b^2Be - bc(7Ae + 4Bd)) + (16Ac^3d^2 + 2b^3Be^2 + b^2ce(Ae + 3Bd) - 8bc^2d(2Ae + Bd)) x) \sqrt{ex+d}}{3b^4c\sqrt{cx^2 + bx}} \\
& - \frac{2(16Ac^3d^2 + 2b^3Be^2 + b^2ce(Ae + 3Bd) - 8bc^2d(2Ae + Bd)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{ex+d}}{3(-b)^{\frac{7}{2}} c^{\frac{3}{2}} \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\
& + \frac{2d(-be + cd) (16Ac^2d - b^2Be - 8bc(Ae + Bd)) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3(-b)^{\frac{7}{2}} c^{\frac{3}{2}} \sqrt{ex+d} \sqrt{cx^2 + bx}}
\end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^(5/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((8(Bbc^5 - 2Ac^6)d^3x^4 + 16(Bb^2c^4 - 2Abc^5)d^3x^3 + 8(Bb^3c^3 - 2Ab^2c^4)d^3x^2 - ((2Bb^4c^2 + Ab^3c^3)x^4 + 2(2Bb^4c^2 + Ab^3c^3)x^4 + 2(2Bb^4c^2 + Ab^3c^3)x^4) \right)}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Be^2x^3 + Ad^2 + (2Bde + Ae^2)x^2 + (Bd^2 + 2Ade)x) \sqrt{cx^2 + bx} \sqrt{ex+d}}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}, x\right)$$

23.132 Problem number 1280

$$\int \frac{(A + Bx)(d + ex)^{3/2}}{(bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Abcd + (2Ac^2d + b^2Be - bc(Ae + Bd))x)\sqrt{ex+d}}{3b^2c(cx^2+bx)^{\frac{3}{2}}} \\
+ & \frac{2(b(8Ac^2d + b^2Be - bc(5Ae + 4Bd)) + c(16Ac^2d + b^2Be - 8bc(Ae + Bd))x)\sqrt{ex+d}}{3b^4c\sqrt{cx^2+bx}} \\
& - \frac{2(16Ac^2d + b^2Be - 8bc(Ae + Bd)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{x} \sqrt{1+\frac{cx}{b}} \sqrt{ex+d}}{3(-b)^{\frac{7}{2}} \sqrt{c} \sqrt{1+\frac{ex}{d}} \sqrt{cx^2+bx}} \\
+ & \frac{2(16Ac^2d^2 - 8bcd(2Ae + Bd) + b^2e(3Ae + 5Bd)) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{x} \sqrt{1+\frac{cx}{b}} \sqrt{1+\frac{ex}{d}}}{3(-b)^{\frac{7}{2}} \sqrt{c} \sqrt{ex+d} \sqrt{cx^2+bx}}
\end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((8(Bbc^4 - 2Ac^5)d^2x^4 + 16(Bb^2c^3 - 2Abc^4)d^2x^3 + 8(Bb^3c^2 - 2Ab^2c^3)d^2x^2 - ((Bb^3c^2 + Ab^2c^3)x^4 + 2(Bb^3c^2 + Ab^2c^3)x^3 + (Bb^3c^2 + Ab^2c^3)x^2 + (Bb^3c^2 + Ab^2c^3)x + Bb^3c^2)\right)}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bex^2 + Ad + (Bd + Ae)x)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}, x\right)$$

23.133 Problem number 1281

$$\int \frac{(A + Bx)\sqrt{d + ex}}{(bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - (-2Ac + bB)x)\sqrt{ex+d}}{3b^2(cx^2+bx)^{\frac{3}{2}}}$$

$$\frac{2(b(-be+cd)(Abe-8Acd+4Bbd) - c(16Ac^2d^2 + b^2e(Ae+7Bd) - 8bcd(2Ae+Bd)))x\sqrt{ex+d}}{3b^4d(-be+cd)\sqrt{cx^2+bx}}$$

$$\frac{2(16Ac^2d^2 + b^2e(Ae+7Bd) - 8bcd(2Ae+Bd)) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{ex+d}}{3(-b)^{\frac{7}{2}}d(-be+cd)\sqrt{1+\frac{ex}{d}}\sqrt{cx^2+bx}}$$

$$+ \frac{2(16Ac^2d + 3b^2Be - 8bc(Ae+Bd)) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}}{3(-b)^{\frac{7}{2}}\sqrt{c}\sqrt{ex+d}\sqrt{cx^2+bx}}$$

command

```
integrate((B*x+A)*(e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((8(Bbc^4 - 2Ac^5)d^3x^4 + 16(Bb^2c^3 - 2Abc^4)d^3x^3 + 8(Bb^3c^2 - 2Ab^2c^3)d^3x^2 - (Ab^3c^2x^4 + 2Ab^4cx^3 + Ab^5x^2)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+bx}(Bx+A)\sqrt{ex+d}}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}, x\right)$$

23.134 Problem number 1282

$$\int \frac{A+Bx}{\sqrt{d+ex}(bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Ab(-be + cd) + c(2Acd - b(Ae + Bd))x) \sqrt{ex + d}}{3b^2d(-be + cd)(cx^2 + bx)^{\frac{3}{2}}} \\
+ & \frac{2(b(-be + cd)(8Ac^2d^2 + b^2e(-2Ae + 3Bd)) - bcd(5Ae + 4Bd)) + c(16Ac^3d^3 - b^3e^2(-2Ae + 3Bd)) - 8bc^2d^2(3Ae + Bd)}{3b^4d^2(-be + cd)^2 \sqrt{cx^2 + bx}} \\
& - \frac{2(16Ac^3d^3 - b^3e^2(-2Ae + 3Bd)) - 8bc^2d^2(3Ae + Bd) + b^2cde(4Ae + 13Bd) \operatorname{EllipticE}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}}{3(-b)^{\frac{7}{2}}d^2(-be + cd)^2 \sqrt{1 + \frac{ex}{d}} \sqrt{cx^2 + bx}} \\
+ & \frac{2(16Ac^2d^2 + b^2e(-Ae + 9Bd)) - 8bcd(2Ae + Bd) \operatorname{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right) \sqrt{c}\sqrt{x} \sqrt{1 + \frac{cx}{b}} \sqrt{1 + \frac{ex}{d}}}{3(-b)^{\frac{7}{2}}d(-be + cd) \sqrt{ex + d} \sqrt{cx^2 + bx}}
\end{aligned}$$

command

```
integrate((B*x+A)/(c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((8(Bbc^5 - 2Ac^6)d^4x^4 + 16(Bb^2c^4 - 2Abc^5)d^4x^3 + 8(Bb^3c^3 - 2Ab^2c^4)d^4x^2 - 2(Ab^4c^2x^4 + 2Ab^5cx^3 + Ab^6)) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx} (Bx + A) \sqrt{ex + d}}{c^3ex^7 + b^3dx^3 + (c^3d + 3bc^2e)x^6 + 3(bc^2d + b^2ce)x^5 + (3b^2cd + b^3e)x^4}, x\right)$$

23.135 Problem number 1283

$$\int \frac{A + Bx}{(d + ex)^{3/2} (bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Ab(-be + cd) + c(2Acd - b(Ae + Bd)))x}{3b^2d(-be + cd)(cx^2 + bx)^{\frac{3}{2}}\sqrt{ex + d}} \\
& + \frac{2b(-be + cd)(8Ac^2d^2 + b^2e(-4Ae + 3Bd) - bcd(3Ae + 4Bd))}{3} + \frac{2c(16Ac^3d^3 + 15b^2Bcd^2e - b^3e^2(-4Ae + 3Bd) - 8bc^2d^2(3Ae + Bd))x}{3} \\
& + \frac{b^4d^2(-be + cd)^2\sqrt{ex + d}\sqrt{cx^2 + bx}}{2(16Ac^4d^4 - b^3cde^2(-7Ae + 9Bd) + 2b^4e^3(-4Ae + 3Bd) - 8bc^3d^3(4Ae + Bd) + b^2c^2d^2e(9Ae + 19Bd)) \text{EllipticF}} \\
& - \frac{3(-b)^{\frac{7}{2}}d^3(-be + cd)^3\sqrt{1 + \frac{ex}{d}}\sqrt{cx^2 + bx}}{2(16Ac^3d^3 + 15b^2Bcd^2e - b^3e^2(-4Ae + 3Bd) - 8bc^2d^2(3Ae + Bd)) \text{EllipticF}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}, \sqrt{\frac{be}{cd}}\right)\sqrt{c}\sqrt{x}\sqrt{1 + \frac{ex}{d}}} \\
& + \frac{3(-b)^{\frac{7}{2}}d^2(-be + cd)^2\sqrt{ex + d}\sqrt{cx^2 + bx}}{2e(16Ac^4d^4 - b^3cde^2(-7Ae + 9Bd) - 8bc^3d^3(4Ae + Bd) + b^2c^2d^2e(9Ae + 19Bd) + b^4(-8Ae^4 + 6Bde^3))\sqrt{cx^2 + bx}} \\
& + \frac{2e(16Ac^4d^4 - b^3cde^2(-7Ae + 9Bd) - 8bc^3d^3(4Ae + Bd) + b^2c^2d^2e(9Ae + 19Bd) + b^4(-8Ae^4 + 6Bde^3))\sqrt{cx^2 + bx}}{3b^4d^3(-be + cd)^3\sqrt{ex + d}}
\end{aligned}$$

command

```
integrate((B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx}(Bx + A)\sqrt{ex + d}}{c^3e^2x^8 + b^3d^2x^3 + (2c^3de + 3bc^2e^2)x^7 + (c^3d^2 + 6bc^2de + 3b^2ce^2)x^6 + (3bc^2d^2 + 6b^2cde + b^3e^2)x^5 + (3b^2d^2 + 6bde + b^2e)x^4 + (2bd^2 + 2bde + b^2e)x^3 + (bd^2 + bde + b^2e)x^2 + (bd + be)x + b^2e}\right)$$

23.136 Problem number 1472

$$\int (A + Bx)\sqrt{d + ex}\sqrt{a + cx^2} dx$$

Optimal antiderivative

$$\frac{2B(cx^2 + a)^{\frac{3}{2}} \sqrt{ex + d}}{7c} - \frac{2(4Bcd^2 - 7Acde + 5aBe^2 - 3ce(7Ae + Bd)x) \sqrt{ex + d} \sqrt{cx^2 + a}}{105ce^2} \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{ex + d}$$

$$105e^3 \sqrt{c} \sqrt{cx^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}$$

$$4(ae^2 + cd^2) (-7Acde + 5aBe^2 + 4Bcd^2) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}$$

$$+ \frac{105c^{\frac{3}{2}} e^3 \sqrt{ex + d} \sqrt{cx^2 + a}}{105c^{\frac{3}{2}} e^3 \sqrt{ex + d} \sqrt{cx^2 + a}}$$

command

`integrate((B*x+A)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2(4Bc^2d^4 - 7Ac^2d^3e + 11Bacd^2e^2 - 63Acde^3 + 15Ba^2e^4) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8}{3c} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{cx^2 + a} (Bx + A) \sqrt{ex + d}, x \right)$$

23.137 Problem number 1473

$$\int \frac{(A + Bx) \sqrt{a + cx^2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{2(-3Bex - 5Ae + 4Bd) \sqrt{ex + d} \sqrt{cx^2 + a}}{15e^2} + \frac{4(-5Acde + 3aB e^2 + 4Bc d^2) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}}}{15e^3 \sqrt{c} \sqrt{cx^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} + \frac{4(-5Ae + 4Bd) (ae^2 + cd^2) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{15e^3 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}$$

command

`integrate((B*x+A)*(c*x^2+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 (4 Bcd^3 - 5 Acd^2e + 6 Bade^2 - 15 Aae^3) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4 (cd^2 - 3ae^2) e^{(-2)}}{3c}, -\frac{8 (cd^3 + 9ade^2) e^{(-3)}}{27c}, \frac{1}{3} \right) \right)}{15e^3 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + a} (Bx + A)}{\sqrt{ex + d}}, x \right)$$

23.138 Problem number 1474

$$\int \frac{(A + Bx) \sqrt{a + cx^2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Bex - 3Ae + 4Bd) \sqrt{cx^2 + a}}{3e^2 \sqrt{ex + d}} + \frac{4(-3Ae + 4Bd) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{c} \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}}}{3e^3 \sqrt{cx^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} + \frac{4(-3Acde + aB e^2 + 4Bc d^2) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{e}{e\sqrt{-a} + d\sqrt{c}}}}{3e^3 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2(4Bcd^3 + 3Baxe^3 - 3(Acdx - Bad)e^2 + (4Bcd^2x - 3Acd^2)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, - \right) \right)}{3e^3 \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + a} (Bx + A) \sqrt{ex + d}}{e^2 x^2 + 2dex + d^2}, x \right)$$

23.139 Problem number 1475

$$\int \frac{(A + Bx) \sqrt{a + cx^2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(4Bcd^3 - Acd^2e + 2aBde^2 + aAe^3 + e(-2Acde + 3aBe^2 + 5Bcd^2)x) \sqrt{cx^2 + a}}{3e^2 (ae^2 + cd^2) (ex + d)^{\frac{3}{2}}}$$

$$4(-Acde + 3aBe^2 + 4Bcd^2) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{c} \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}}$$

$$\frac{3e^3 (ae^2 + cd^2) \sqrt{cx^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{3e^3 \sqrt{ex + d} \sqrt{cx^2 + a}}$$

$$4(-Ae + 4Bd) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{c} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}$$

+

command

```
integrate((B*x+A)*(c*x^2+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2(4Bcd^5 - 3Aax^2e^5 + 6(Badx^2 - Aadx)e^4 - (Acd^2x^2 - 12Bad^2x + 3Aad^2)e^3 + 2(2Bcd^3x^2 - Acd^3x + 3Aad^2)) \sqrt{cx^2 + a} \sqrt{ex + d} \right)}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + a} (Bx + A) \sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x \right)$$

23.140 Problem number 1476

$$\int \frac{(A + Bx)(a + cx^2)^{3/2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{2(-7Bex - 9Ae + 8Bd)(cx^2 + a)^{\frac{3}{2}}\sqrt{ex + d}}{63e^2} - \frac{4(32Bcd^3 - 36Ac d^2e + 33aBde^2 - 45aAe^3 - 3e(-9Acde + 7aBe^2 + 8Bcd^2)x)\sqrt{ex + d}\sqrt{cx^2 + a}}{315e^4}$$

$$+ \frac{8(36Acde(2ae^2 + cd^2) - B(21a^2e^4 + 57acd^2e^2 + 32c^2d^4)) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}}}\sqrt{c}\right)}{315e^5\sqrt{c}\sqrt{cx^2 + a}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}$$

$$+ \frac{8(ae^2 + cd^2)(-45aAe^3 - 36Ac d^2e + 33aBde^2 + 32Bcd^3) \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}}}\sqrt{c}\right)}{315e^5\sqrt{c}\sqrt{ex + d}\sqrt{cx^2 + a}}$$

command

`integrate((B*x+A)*(c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4(32Bc^2d^5 - 36Ac^2d^4e + 81Bacd^3e^2 - 99Aacd^2e^3 + 57Ba^2de^4 - 135Aa^2e^5)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4}{c}\right)\right)}{315e^5\sqrt{c}\sqrt{ex + d}\sqrt{cx^2 + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^3 + Acx^2 + Bax + Aa)\sqrt{cx^2 + a}}{\sqrt{ex + d}}, x\right)$$

23.141 Problem number 1477

$$\int \frac{(A + Bx)(a + cx^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Bex - 7Ae + 8Bd)(cx^2 + a)^{\frac{3}{2}}}{7e^2\sqrt{ex + d}} + \frac{4(5aBe^2 + 4cd(-7Ae + 8Bd) - 3ce(-7Ae + 8Bd)x)\sqrt{ex + d}\sqrt{cx^2 + a}}{35e^4}$$

$$+ \frac{8(-21aAe^3 - 28Ac d^2e + 29aBde^2 + 32Bcd^3) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}}{35e^5\sqrt{cx^2 + a} \sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}$$

$$+ \frac{8(ae^2 + cd^2)(-28Acde + 5aBe^2 + 32Bcd^2) \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}}{35e^5\sqrt{c}\sqrt{ex + d}\sqrt{cx^2 + a}}$$

command

`integrate((B*x+A)*(c*x^2+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(4(32Bc^2d^5 + 15Ba^2xe^5 - 3(14Aacdx - 5Ba^2d)e^4 + (53Bacd^2x - 42Aacd^2)e^3 - (28Ac^2d^3x - 53Bacd^3)e^2 \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^3 + Acx^2 + Bax + Aa)\sqrt{cx^2 + a}\sqrt{ex + d}}{e^2x^2 + 2dex + d^2}, x\right)$$

23.142 Problem number 1478

$$\int \frac{(A + Bx)(a + cx^2)^{3/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(3Bex - 5Ae + 8Bd)(cx^2 + a)^{\frac{3}{2}}}{15e^2(ex + d)^{\frac{3}{2}}} - \frac{4(9aBe^2 + 4cd(-5Ae + 8Bd) + ce(-5Ae + 8Bd)x)\sqrt{cx^2 + a}}{15e^4\sqrt{ex + d}}$$

$$- \frac{8(9aBe^2 + 4cd(-5Ae + 8Bd)) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{c}\sqrt{ex + d}\sqrt{cx^2 + a}}{15e^5\sqrt{cx^2 + a}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}$$

$$+ \frac{8(-5aAe^3 - 20Acd^2e + 17aBde^2 + 32Bcd^3) \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{c}\sqrt{ex + d}\sqrt{cx^2 + a}}{15e^5\sqrt{ex + d}\sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4(32Bcd^5 - 15Aax^2e^5 + 3(11Badx^2 - 10Aadx)e^4 - (20Acd^2x^2 - 66Bad^2x + 15Aad^2)e^3 + (32Bcd^3x^2 - \dots)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bcx^3 + Acx^2 + Bax + Aa)\sqrt{cx^2 + a}\sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

23.143 Problem number 1479

$$\int \frac{(A + Bx)(a + cx^2)^{3/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(2B(ad e^2 + 4c d^3) - 3A(-a e^3 + c d^2 e) + e(-6Acde + 5aB e^2 + 11Bc d^2) x) (c x^2 + a)^{\frac{3}{2}}}{15e^2 (a e^2 + c d^2) (ex + d)^{\frac{5}{2}}} + \frac{4c(32Bc d^3 - 12Ac d^2 e + 29aBd e^2 - 9aA e^3 + e(-3Acde + 5aB e^2 + 8Bc d^2) x) \sqrt{c x^2 + a}}{15e^4 (a e^2 + c d^2) \sqrt{ex + d}}$$

$$+ \frac{8c^{\frac{3}{2}}(-9aA e^3 - 12Ac d^2 e + 29aBd e^2 + 32Bc d^3) \text{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}}{15e^5 (a e^2 + c d^2) \sqrt{c x^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}$$

$$+ \frac{8(-12Acde + 5aB e^2 + 32Bc d^2) \text{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{c} \sqrt{1 + \frac{cx^2}{a}}}{15e^5 \sqrt{ex + d} \sqrt{c x^2 + a}}$$

command

```
integrate((B*x+A)*(c*x^2+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(4(32Bc^2d^7 + 15Ba^2x^3e^7 - 9(2Aacd^3 - 5Ba^2dx^2))e^6 + (53Bacd^2x^3 - 54Aacd^2x^2 + 45Ba^2d^2x)e^5 - 3(4A$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Bcx^3 + Acx^2 + Bax + Aa)\sqrt{cx^2 + a}\sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

23.144 Problem number 1480

$$\int \frac{(A + Bx)(d + ex)^{3/2}}{\sqrt{a + cx^2}} dx$$

Optimal antiderivative

$$\frac{2B(ex+d)^{\frac{3}{2}}\sqrt{cx^2+a}}{5c} + \frac{2(5Ae+3Bd)\sqrt{ex+d}\sqrt{cx^2+a}}{15c} \\
+ \frac{2(20Acde-9aBe^2+3Bcd^2)\operatorname{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{ex+d}\sqrt{1+\frac{cx^2}{a}}}{15c^{\frac{3}{2}}e\sqrt{cx^2+a}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}} \\
+ \frac{2(5Ae+3Bd)(ae^2+cd^2)\operatorname{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae+d\sqrt{-a}\sqrt{c}}}\right)\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a}+d\sqrt{c}}}}{15c^{\frac{3}{2}}e\sqrt{ex+d}\sqrt{cx^2+a}}$$

command

```
integrate((B*x+A)*(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left((3Bcd^3-25Acd^2e+27Bade^2+15Aae^3)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd^2-3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3+9ade^2)e^{(-3)}}{27c}, \frac{1}{3}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bex^2+Ad+(Bd+Ae)x)\sqrt{ex+d}}{\sqrt{cx^2+a}}, x\right)$$

23.145 Problem number 1481

$$\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{2B\sqrt{ex+d}\sqrt{cx^2+a}}{3c}$$

$$2(3Ae + Bd) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{ex+d} \sqrt{1 + \frac{cx^2}{a}}$$

$$3e\sqrt{c}\sqrt{cx^2+a} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}$$

$$2B(ae^2 + cd^2) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex+d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}$$

$$+ \frac{3c^{\frac{3}{2}}e\sqrt{ex+d}\sqrt{cx^2+a}}{3c^{\frac{3}{2}}e\sqrt{ex+d}\sqrt{cx^2+a}}$$

command

`integrate((B*x+A)*(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cx^2+a} \sqrt{ex+d} Bce^2 - (Bcd^2 - 6Acde + 3Bae^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3 + 9ae^2)}{27} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Bx + A)\sqrt{ex+d}}{\sqrt{cx^2+a}}, x \right)$$

23.146 Problem number 1482

$$\int \frac{A + Bx}{\sqrt{d + ex} \sqrt{a + cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & 2B \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}} \\
 & \frac{e\sqrt{c} \sqrt{cx^2 + a} \sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{e\sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}} \\
 & + 2(-Ae + Bd) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}} \\
 & + \frac{e\sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}{e\sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}
 \end{aligned}$$

command

```
integrate((B*x+A)/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((Bd - 3Ae)\sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3 + 9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe + d)e^{(-1)} \right) + 3B\sqrt{c} e^{\frac{3}{2}} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + a} (Bx + A)\sqrt{ex + d}}{cex^3 + cdx^2 + aex + ad}, x \right)$$

23.147 Problem number 1483

$$\int \frac{A + Bx}{(d + ex)^{3/2} \sqrt{a + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(-Ae + Bd) \sqrt{cx^2 + a}}{(ae^2 + cd^2) \sqrt{ex + d}}$$

$$+ \frac{2(-Ae + Bd) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{c} \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}}}{e(ae^2 + cd^2) \sqrt{cx^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}$$

$$+ \frac{2B \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{e\sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((Bcd^3 + 3Baxe^3 + (2Ac dx + 3Bad)e^2 + (Bcd^2x + 2Ac d^2)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8}{3c} \right) \right)}{e^2 \sqrt{cx^2 + a} \sqrt{ex + d}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + a} (Bx + A) \sqrt{ex + d}}{ce^2x^4 + 2cde x^3 + 2ade x + ad^2 + (cd^2 + ae^2)x^2}, x \right)$$

23.148 Problem number 1484

$$\int \frac{(A + Bx)(d + ex)^{3/2}}{(a + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(a(Ae + Bd) - (Acd - aBe)x)\sqrt{ex + d}}{ac\sqrt{cx^2 + a}} - \frac{(Acd - 3aBe) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right)\sqrt{ex + d}\sqrt{1 + \frac{cx^2}{a}}}{c^{\frac{3}{2}}\sqrt{-a}\sqrt{cx^2 + a}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} + \frac{A(ae^2 + cd^2) \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a}\sqrt{c}}}\right)\sqrt{1 + \frac{cx^2}{a}}\sqrt{\frac{(ex + d)\sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{c^{\frac{3}{2}}\sqrt{-a}\sqrt{ex + d}\sqrt{cx^2 + a}}$$

command

```
integrate((B*x+A)*(e*x+d)^(3/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left((Ac^2d^2x^2 + Aacd^2 + 3(Aacx^2 + Aa^2)e^2 + 6(Bacdx^2 + Ba^2d)e\right)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -8\right)}{c^{\frac{3}{2}}\sqrt{-a}\sqrt{ex + d}\sqrt{cx^2 + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bex^2 + Ad + (Bd + Ae)x)\sqrt{cx^2 + a}\sqrt{ex + d}}{c^2x^4 + 2acx^2 + a^2}, x\right)$$

23.149 Problem number 1485

$$\int \frac{(A + Bx)\sqrt{d + ex}}{(a + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{(-Acx + Ba) \sqrt{ex + d}}{ac \sqrt{cx^2 + a}} \\
 & A \operatorname{EllipticE} \left(\sqrt{\frac{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}{2}} \sqrt{2}, \sqrt{\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}} \\
 & \frac{\sqrt{-a} \sqrt{c} \sqrt{cx^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{\sqrt{-a} \sqrt{c} \sqrt{cx^2 + a}} \\
 & (Acd + aBe) \operatorname{EllipticF} \left(\sqrt{\frac{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}{2}} \sqrt{2}, \sqrt{\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}} \\
 & + \frac{c^{\frac{3}{2}} \sqrt{-a} \sqrt{ex + d} \sqrt{cx^2 + a}}{c^{\frac{3}{2}} \sqrt{-a} \sqrt{ex + d} \sqrt{cx^2 + a}}
 \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^(1/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left((Ac^2 dx^2 + Aacd + 3(Bacx^2 + Ba^2)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(cd^3 + 9ade^2)e^{(-3)}}{27c}, \frac{1}{3}(3xe + a) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + a} (Bx + A) \sqrt{ex + d}}{c^2 x^4 + 2acx^2 + a^2}, x \right)$$

23.150 Problem number 1486

$$\int \frac{A + Bx}{\sqrt{d + ex} (a + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{(a(-Ae + Bd) - (Acd + aBe)x) \sqrt{ex + d}}{a(ae^2 + cd^2) \sqrt{cx^2 + a}} \\
 & (Acd + aBe) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{ex + d} \sqrt{1 + \frac{cx^2}{a}} \\
 & - \frac{(ae^2 + cd^2) \sqrt{-a} \sqrt{c} \sqrt{cx^2 + a} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}}{A \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ae}{-ae + d\sqrt{-a} \sqrt{c}}} \right) \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(ex + d) \sqrt{c}}{e\sqrt{-a} + d\sqrt{c}}}} \\
 & + \frac{\sqrt{-a} \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}{\sqrt{-a} \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}
 \end{aligned}$$

command

```
integrate((B*x+A)/(c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(Ac^2d^2x^2 + Aacd^2 + 3(Aacx^2 + Aa^2)e^2 - 2(Bacdx^2 + Ba^2d)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(cd^2 - 3ae^2)e^{(-2)}}{3c}, -\frac{8(c}{3} \right)}{\sqrt{-a} \sqrt{c} \sqrt{ex + d} \sqrt{cx^2 + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + a} (Bx + A) \sqrt{ex + d}}{c^2ex^5 + c^2dx^4 + 2acex^3 + 2acdx^2 + a^2ex + a^2d}, x \right)$$

23.151 Problem number 1617

$$\int \frac{b + 2cx}{(d + ex)^{5/2} (a + bx + cx^2)} dx$$

Optimal antiderivative

$$\frac{-\frac{2be}{3} + \frac{4cd}{3}}{(ae^2 - bde + cd^2)(ex + d)^{\frac{3}{2}}} + \frac{4c^2d^2 + 2b^2e^2 - 4ce(ae + bd)}{(ae^2 - bde + cd^2)^2 \sqrt{ex + d}}$$

$$\operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex + d}}{\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}\right) \sqrt{2} \sqrt{c} (b^2e^2(b + \sqrt{-4ac + b^2}) + 2c^2d(4ae + d\sqrt{-4ac + b^2})) -$$

$$(ae^2 - bde + cd^2)^2 \sqrt{-4ac + b^2} \sqrt{2cd - e(b - \sqrt{-4ac + b^2})}$$

$$\operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex + d}}{\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \sqrt{2} \sqrt{c} (b^2e^2(b - \sqrt{-4ac + b^2}) - 2c^2d(-4ae + d\sqrt{-4ac + b^2})) -$$

$$(ae^2 - bde + cd^2)^2 \sqrt{-4ac + b^2} \sqrt{2cd - e(b + \sqrt{-4ac + b^2})}$$

command

`integrate((2*c*x+b)/(e*x+d)^(5/2)/(c*x^2+b*x+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

23.152 Problem number 1622

$$\int \frac{b + 2cx}{(d + ex)^{3/2} (a + bx + cx^2)^2} dx$$

Optimal antiderivative

$$\frac{3e^2(-be + 2cd)}{(ae^2 - bde + cd^2)^2 \sqrt{ex + d}} + \frac{-(-4ac + b^2)(-be + cd) + c(-4ac + b^2)ex}{(-4ac + b^2)(ae^2 - bde + cd^2)(cx^2 + bx + a)\sqrt{ex + d}}$$

$$3e \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex + d}}{\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}\right) \sqrt{c} (2c^2d^2 + be^2(b + \sqrt{-4ac + b^2}) - 2ce(bd + ae + d\sqrt{-4ac + b^2})) -$$

$$2(ae^2 - bde + cd^2)^2 \sqrt{-4ac + b^2} \sqrt{2cd - e(b - \sqrt{-4ac + b^2})}$$

$$3e \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex + d}}{\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \sqrt{c} (2c^2d^2 + be^2(b - \sqrt{-4ac + b^2}) - 2ce(bd + ae - d\sqrt{-4ac + b^2})) -$$

$$2(ae^2 - bde + cd^2)^2 \sqrt{-4ac + b^2} \sqrt{2cd - e(b + \sqrt{-4ac + b^2})}$$

command

```
integrate((2*c*x+b)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

23.153 Problem number 1626

$$\int \frac{(b + 2cx)\sqrt{d + ex}}{(a + bx + cx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{ex+d}}{2(cx^2+bx+a)^2} - \frac{e(bcd - b^2e + 2ace + c(-be + 2cd)x)\sqrt{ex+d}}{4(-4ac+b^2)(ae^2 - bde + cd^2)(cx^2+bx+a)} \\ & + \frac{e \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex+d}}{\sqrt{2cd - e(b - \sqrt{-4ac+b^2})}}\right) \sqrt{c} \left(8c^2d^2 - be^2(b + \sqrt{-4ac+b^2}) - 2ce(4bd - 6ae - d\sqrt{-4ac+b^2})\right)}{8(-4ac+b^2)^{\frac{3}{2}}(ae^2 - bde + cd^2)\sqrt{2cd - e(b - \sqrt{-4ac+b^2})}} \\ & - \frac{e \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex+d}}{\sqrt{2cd - e(b + \sqrt{-4ac+b^2})}}\right) \sqrt{c} \left(8c^2d^2 - be^2(b - \sqrt{-4ac+b^2}) - 2ce(4bd - 6ae + d\sqrt{-4ac+b^2})\right)}{8(-4ac+b^2)^{\frac{3}{2}}(ae^2 - bde + cd^2)\sqrt{2cd - e(b + \sqrt{-4ac+b^2})}} \end{aligned}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

23.154 Problem number 1627

$$\int \frac{b + 2cx}{\sqrt{d + ex} (a + bx + cx^2)^3} dx$$

Optimal antiderivative

$$\frac{((-4ac + b^2)(-be + cd) - c(-4ac + b^2)ex) \sqrt{ex + d}}{2(-4ac + b^2)(ae^2 - bde + cd^2)(cx^2 + bx + a)^2}$$

$$+ \frac{e(5ace(-be + 2cd) + (-3be + cd)(2ace - b^2e + bcd) + c(2c^2d^2 + 3b^2e^2 - 2ce(5ae + bd))x) \sqrt{ex + d}}{4(-4ac + b^2)(ae^2 - bde + cd^2)^2(cx^2 + bx + a)}$$

$$- \frac{e \operatorname{arctanh} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex + d}}{\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}} \right) \sqrt{c} (8c^3d^3 + 3b^2e^3(b + \sqrt{-4ac + b^2}) - 2c^2de(6bd - 16ae - d\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}))}{8(-4ac + b^2)^{\frac{3}{2}}(ae^2 - bde + cd^2)^2 \sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}$$

$$+ \frac{e \operatorname{arctanh} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{ex + d}}{\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{c} (8c^3d^3 + 3b^2e^3(b - \sqrt{-4ac + b^2}) - 2c^2de(6bd - 16ae + d\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}))}{8(-4ac + b^2)^{\frac{3}{2}}(ae^2 - bde + cd^2)^2 \sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}$$

command

```
integrate((2*c*x+b)/(c*x^2+b*x+a)^3/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

23.155 Problem number 1628

$$\int (b + 2cx) \sqrt{d + ex} \sqrt{a + bx + cx^2} dx$$

Optimal antiderivative

$$\frac{4\sqrt{ex+d} (cx^2+bx+a)^{\frac{3}{2}}}{7} - \frac{2(8c^2d^2+b^2e^2-ce(-10ae+11bd)-3ce(-be+2cd)x)\sqrt{ex+d}\sqrt{cx^2+bx+a}}{105ce^2}$$

$$+ \frac{2(-be+2cd)(4c^2d^2-b^2e^2-4ce(-2ae+bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{105c^2e^3\sqrt{cx^2+bx+a}}$$

$$+ \frac{2(ae^2-bde+cd^2)(16c^2d^2-b^2e^2-4ce(-5ae+4bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{105c^2e^3\sqrt{ex+d}\sqrt{cx^2+bx+a}}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((16c^4d^4-32bc^3d^3e+(13b^2c^2+44ac^3)d^2e^2+(3b^3c-44abc^2)de^3+(2b^4-19ab^2c+60a^2c^2)e^4)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrass}\right)}{105c^2e^3\sqrt{ex+d}\sqrt{cx^2+bx+a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^2+bx+a}(2cx+b)\sqrt{ex+d}, x\right)$$

23.156 Problem number 1629

$$\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{\sqrt{d+ex}} dx$$

Optimal antiderivative

$$\frac{2(-6cex - 7be + 8cd) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}{15e^2} + \frac{(16c^2d^2 + b^2e^2 - 4ce(-3ae + 4bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{15ce^3\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}} + \frac{16(-be + 2cd)(ae^2 - bde + cd^2) \operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{15ce^3\sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

`integrate((2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((16c^3d^3 - 24bc^2d^2e + 6(b^2c + 4ac^2)de^2 + (b^3 - 12abc)e^3\right)\sqrt{c}e^{\frac{1}{2}}\operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2 - bcde + (b^2 - 3ac)e^2)}{3c^2}\right)}{15ce^3\sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx + a}(2cx + b)}{\sqrt{ex + d}}, x\right)$$

23.157 Problem number 1630

$$\int \frac{(b + 2cx)\sqrt{a + bx + cx^2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(2cex - 3be + 8cd) \sqrt{cx^2 + bx + a}}{3e^2 \sqrt{ex + d}} + \frac{8(-be + 2cd) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e \sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}}{3e^3 \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}} + \frac{2(16c^2d^2 + 3b^2e^2 - 4ce(-ae + 4bd)) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e \sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}{3ce^3 \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

`integrate((2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16c^2d^3 + (b^2 + 12ac)xe^3 - (16bcdx - (b^2 + 12ac)d)e^2 + 16(c^2d^2x - bcd^2)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(c^2}{e^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a} (2cx + b) \sqrt{ex + d}}{e^2x^2 + 2dex + d^2}, x \right)$$

23.158 Problem number 1631

$$\int \frac{(b + 2cx) \sqrt{a + bx + cx^2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(8c^2d^3 + abe^3 - cde(-4ae + 7bd) + e(10c^2d^2 + b^2e^2 - 2ce(-3ae + 5bd))x) \sqrt{cx^2 + bx + a}}{3e^2(ae^2 - bde + cd^2)(ex + d)^{\frac{3}{2}}}$$

$$+ \frac{(16c^2d^2 + b^2e^2 - 4ce(-3ae + 4bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{3e^3(ae^2 - bde + cd^2) \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}$$

$$+ \frac{16(-be + 2cd) \operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \sqrt{2} \sqrt{-4ac + b^2}}{3e^3 \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

`integrate((2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((16c^3d^5 + (b^3 - 12abc)x^2e^5 + 2(3(b^2c + 4ac^2)dx^2 + (b^3 - 12abc)dx)e^4 - (24bc^2d^2x^2 - 12(b^2c + 4ac^2)d^2x) \right)}{3e^3 \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx + a} (2cx + b) \sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

23.159 Problem number 1632

$$\int \frac{(b + 2cx) \sqrt{a + bx + cx^2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(8c^2d^3 - cde(-4ae + 5bd) - be^2(-3ae + 2bd) + e(14c^2d^2 + b^2e^2 - 2ce(-5ae + 7bd))x) \sqrt{cx^2 + bx + a}}{15e^2(ae^2 - bde + cd^2)(ex + d)^{\frac{5}{2}}} \\
& + \frac{4(-be + 2cd)(4c^2d^2 - b^2e^2 - 4ce(-2ae + bd)) \sqrt{cx^2 + bx + a}}{15e^2(ae^2 - bde + cd^2)^2 \sqrt{ex + d}} \\
& - \frac{2(-be + 2cd)(4c^2d^2 - b^2e^2 - 4ce(-2ae + bd)) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}{15e^3(ae^2 - bde + cd^2)^2 \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}} \\
& + \frac{2(16c^2d^2 - b^2e^2 - 4ce(-5ae + 4bd)) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}{15e^3(ae^2 - bde + cd^2) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}
\end{aligned}$$

command

```
integrate((2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((16c^4d^7 + (2b^4 - 19ab^2c + 60a^2c^2)x^3e^7 + ((3b^3c - 44abc^2)dx^3 + 3(2b^4 - 19ab^2c + 60a^2c^2)dx^2)e^6 + ((13b^2c - 44abc^2)dx^3 + 3(2b^4 - 19ab^2c + 60a^2c^2)dx^2)e^5 + ((13b^2c - 44abc^2)dx^3 + 3(2b^4 - 19ab^2c + 60a^2c^2)dx^2)e^4 + ((13b^2c - 44abc^2)dx^3 + 3(2b^4 - 19ab^2c + 60a^2c^2)dx^2)e^3 + ((13b^2c - 44abc^2)dx^3 + 3(2b^4 - 19ab^2c + 60a^2c^2)dx^2)e^2 + ((13b^2c - 44abc^2)dx^3 + 3(2b^4 - 19ab^2c + 60a^2c^2)dx^2)e \right) \sqrt{cx^2 + bx + a} \sqrt{ex + d}}{15e^3(ae^2 - bde + cd^2) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a} (2cx + b) \sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x \right)$$

23.160 Problem number 1633

$$\int \frac{(b+2cx)(a+bx+cx^2)^{3/2}}{\sqrt{d+ex}} dx$$

Optimal antiderivative

$$\frac{2(-14cex - 15be + 16cd)(cx^2 + bx + a)^{\frac{3}{2}}\sqrt{ex+d}}{63e^2} - \frac{2(128c^3d^3 - b^3e^3 + 3bce^2(-36ae + 37bd) - 12c^2de(-11ae + 20bd) - 3ce(32c^2d^2 + b^2e^2 - 4ce(-7ae + 8bd))x)}{315ce^4}$$

$$2(128c^4d^4 - b^4e^4 - 4c^3d^2e(-57ae + 64bd) - b^2ce^3(-15ae + 7bd) + 3c^2e^2(28a^2e^2 - 76abde + 45b^2d^2)) \text{EllipticE}$$

+

$$\frac{315c^2e^5\sqrt{cx^2+bx+a}}{315c^2e^5\sqrt{ex+d}} + \frac{2(-be + 2cd)(ae^2 - bde + cd^2)(128c^2d^2 - b^2e^2 - 4ce(-33ae + 32bd)) \text{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, x\right)}{315c^2e^5\sqrt{ex+d}}$$

command

```
integrate((2*c*x+b)*(c*x^2+b*x+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((256c^5d^5 - 640bc^4d^4e + 2(239b^2c^3 + 324ac^4)d^3e^2 - (77b^3c^2 + 972abc^3)d^2e^3 - (13b^4c - 258ab^2c^2 - 456a^2\right)}{315c^2e^5\sqrt{ex+d}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(2c^2x^3 + 3bcx^2 + ab + (b^2 + 2ac)x)\sqrt{cx^2 + bx + a}}{\sqrt{ex+d}}, x\right)$$

23.161 Problem number 1634

$$\int \frac{(b+2cx)(a+bx+cx^2)^{3/2}}{(d+ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(2cex - 7be + 16cd)(cx^2 + bx + a)^{\frac{3}{2}}}{7e^2\sqrt{ex+d}} \\ & + \frac{2(128c^2d^2 + 51b^2e^2 - 4ce(-5ae + 44bd) - 48ce(-be + 2cd)x)\sqrt{ex+d}\sqrt{cx^2+bx+a}}{35e^4} \\ & + \frac{(-be + 2cd)(128c^2d^2 + 3b^2e^2 - 4ce(-29ae + 32bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{cx^2+bx+a}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{35ce^5\sqrt{cx^2+bx+a}\sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}} \\ & + \frac{4(ae^2 - bde + cd^2)(128c^2d^2 + 27b^2e^2 - 4ce(-5ae + 32bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{cx^2+bx+a}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{35ce^5\sqrt{ex+d}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((2*c*x+b)*(c*x^2+b*x+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((256c^4d^5 - (3b^4 - 46ab^2c - 120a^2c^2)xe^5 - (2(11b^3c + 212abc^2)dx + (3b^4 - 46ab^2c - 120a^2c^2)d)e^4 + 2((11b^3c + 212abc^2)dx + (3b^4 - 46ab^2c - 120a^2c^2)d)e^3 + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(2c^2x^3 + 3bcx^2 + ab + (b^2 + 2ac)x)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{e^2x^2 + 2dex + d^2}, x\right)$$

23.162 Problem number 1635

$$\int \frac{(b+2cx)(a+bx+cx^2)^{3/2}}{(d+ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(6cex - 5be + 16cd)(cx^2 + bx + a)^{\frac{3}{2}}}{15e^2(ex + d)^{\frac{3}{2}}} - \frac{2(128c^2d^2 + 15b^2e^2 - 4ce(-9ae + 28bd) + 16ce(-be + 2cd)x)\sqrt{cx^2 + bx + a}}{15e^4\sqrt{ex + d}}$$

$$+ \frac{2(128c^2d^2 + 23b^2e^2 - 4ce(-9ae + 32bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{15e^5\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}$$

$$+ \frac{2(-be + 2cd)(128c^2d^2 + 15b^2e^2 - 4ce(-17ae + 32bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{15ce^5\sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((2*c*x+b)*(c*x^2+b*x+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((256c^3d^5 + (b^3 - 132abc)x^2e^5 + 2(3(21b^2c + 44ac^2)dx^2 + (b^3 - 132abc)dx)e^4 - (384bc^2d^2x^2 - 12(21b^2c\right)}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(2c^2x^3 + 3bcx^2 + ab + (b^2 + 2ac)x)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

23.163 Problem number 1636

$$\int \frac{(b+2cx)(a+bx+cx^2)^{3/2}}{(d+ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(16c^2d^3 + 3abe^3 - cde(-4ae + 13bd) + e(22c^2d^2 + 3b^2e^2 - 2ce(-5ae + 11bd))x)(cx^2 + bx + a)^{\frac{3}{2}}}{15e^2(ae^2 - bde + cd^2)(ex + d)^{\frac{5}{2}}} \\ & + \frac{2c(128c^2d^3 - 4cde(-29ae + 44bd) + 3be^2(-16ae + 17bd) + e(32c^2d^2 + 3b^2e^2 - 4ce(-5ae + 8bd))x)\sqrt{cx^2 + bx + a}}{15e^4(ae^2 - bde + cd^2)\sqrt{ex + d}} \\ & + \frac{(-be + 2cd)(128c^2d^2 + 3b^2e^2 - 4ce(-29ae + 32bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}}\right)}{15e^5(ae^2 - bde + cd^2)\sqrt{cx^2 + bx + a}\sqrt{\frac{c(ea+\sqrt{-4ac+b^2})}{2cd-e(b+\sqrt{-4ac+b^2})}}} \\ & + \frac{4(128c^2d^2 + 27b^2e^2 - 4ce(-5ae + 32bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}}\right)}{15e^5\sqrt{ex + d}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*x+b)*(c*x^2+b*x+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((256c^4d^7 - (3b^4 - 46ab^2c - 120a^2c^2)x^3e^7 - (2(11b^3c + 212abc^2)dx^3 + 3(3b^4 - 46ab^2c - 120a^2c^2)dx^2)e^6 + \dots\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(2c^2x^3 + 3bcx^2 + ab + (b^2 + 2ac)x)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

23.164 Problem number 1637

$$\int \frac{(b + 2cx)(d + ex)^{5/2}}{\sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-be + 2cd)(ex + d)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{7c} + \frac{4(ex + d)^{\frac{5}{2}} \sqrt{cx^2 + bx + a}}{7} \\ & + \frac{4(3c^2d^2 + 2b^2e^2 - ce(5ae + 3bd)) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}{21c^2} \\ & + \frac{(-be + 2cd)(3c^2d^2 + 8b^2e^2 - ce(29ae + 3bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{21c^3e\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}} \\ & + \frac{4(ae^2 - bde + cd^2)(3c^2d^2 + 2b^2e^2 - ce(5ae + 3bd)) \operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)}{21c^3e\sqrt{ex + d} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((6c^4d^4 - 12bc^3d^3e - (17b^2c^2 - 104ac^3)d^2e^2 + (23b^3c - 104abc^2)de^3 - (8b^4 - 41ab^2c + 30a^2c^2)e^4) \sqrt{c} e^{\frac{1}{2}} \sqrt{cx^2 + bx + a} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(2ce^2x^3 + bd^2 + (4cde + be^2)x^2 + 2(cd^2 + bde)x)\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}, x\right)$$

23.165 Problem number 1638

$$\int \frac{(b + 2cx)(d + ex)^{3/2}}{\sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{4(ex + d)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{5} + \frac{2(-be + 2cd) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}{5c}$$

$$+ \frac{2(c^2d^2 + b^2e^2 - ce(3ae + bd)) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}{2(c^2d^2 + b^2e^2 - ce(3ae + bd)) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}$$

$$+ \frac{5c^2e\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{5c^2e\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}$$

$$- \frac{2(-be + 2cd) (ae^2 - bde + cd^2) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}{2(-be + 2cd) (ae^2 - bde + cd^2) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}$$

$$- \frac{5c^2e\sqrt{ex + d} \sqrt{cx^2 + bx + a}}{5c^2e\sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

`integrate((2*c*x+b)*(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((2c^3d^3 - 3bc^2d^2e - 3(b^2c - 6ac^2)de^2 + (2b^3 - 9abc)e^3) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + (b^2 - 3ac)e^2)e}{3c^2} \right) \right)}{2 \left((2c^3d^3 - 3bc^2d^2e - 3(b^2c - 6ac^2)de^2 + (2b^3 - 9abc)e^3) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + (b^2 - 3ac)e^2)e}{3c^2} \right) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(2cex^2 + bd + (2cd + be)x) \sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}, x \right)$$

23.166 Problem number 1639

$$\int \frac{(b + 2cx)\sqrt{d + ex}}{\sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{4\sqrt{ex + d} \sqrt{cx^2 + bx + a}}{3} + \frac{(-be + 2cd) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}}{3ce\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}} + \frac{4(ae^2 - bde + cd^2) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}}{3ce\sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6 \sqrt{cx^2 + bx + a} \sqrt{ex + d} c^2 e^2 - (2c^2 d^2 - 2bcde - (b^2 - 6ac)e^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(c^2 d^2 - bcde + (b^2 - 3ac)e^2)}{3c^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(2cx + b)\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}, x \right)$$

23.167 Problem number 1640

$$\int \frac{b + 2cx}{\sqrt{d + ex} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{-\frac{e\sqrt{cx^2 + bx + a}}{2cd - e(b + \sqrt{-4ac + b^2})}}}{2(-be + 2cd) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}} \frac{ce\sqrt{ex + d} \sqrt{cx^2 + bx + a}}{ce\sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

`integrate((2*c*x+b)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2cd - be) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + (b^2 - 3ac)e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d^3 - 3bc^2d^2e - 3(b^2c - 6ac^2)de^2 + (2b^3 - 9abc)e^3)}{27c^3} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a} (2cx + b) \sqrt{ex + d}}{ce x^3 + (cd + be)x^2 + ad + (bd + ae)x}, x \right)$$

23.168 Problem number 1641

$$\int \frac{b + 2cx}{(d + ex)^{3/2} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(-be + 2cd) \sqrt{cx^2 + bx + a}}{(ae^2 - bde + cd^2) \sqrt{ex + d}}$$

$$(-be + 2cd) \operatorname{EllipticE} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e \sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}$$

$$e(ae^2 - bde + cd^2) \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}$$

$$+ \frac{4 \operatorname{EllipticF} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e \sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{-\frac{c(cx^2 - b^2)}{-4ac + b^2}}}{e \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

`integrate((2*c*x+b)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^2d^3 - (b^2 - 6ac)xe^3 - (2bcdx + (b^2 - 6ac)d)e^2 + 2(c^2d^2x - bcd^2)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcd)}{c^2d^2 - bcd} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a} (2cx + b) \sqrt{ex + d}}{ce^2x^4 + (2cde + be^2)x^3 + ad^2 + (cd^2 + 2bde + ae^2)x^2 + (bd^2 + 2ade)x}, x \right)$$

23.169 Problem number 1642

$$\int \frac{b + 2cx}{(d + ex)^{5/2} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(-be + 2cd) \sqrt{cx^2 + bx + a}}{3(ae^2 - bde + cd^2)(ex + d)^{\frac{3}{2}}} + \frac{4(c^2d^2 + b^2e^2 - ce(3ae + bd)) \sqrt{cx^2 + bx + a}}{3(ae^2 - bde + cd^2)^2 \sqrt{ex + d}}$$

$$2(c^2d^2 + b^2e^2 - ce(3ae + bd)) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)$$

$$3e(ae^2 - bde + cd^2)^2 \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}$$

$$2(-be + 2cd) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}$$

$$+ \frac{3e(ae^2 - bde + cd^2) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((2*c*x+b)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^3d^5 + (2b^3 - 9abc)x^2e^5 - (3(b^2c - 6ac^2)dx^2 - 2(2b^3 - 9abc)dx)e^4 - (3bc^2d^2x^2 + 6(b^2c - 6ac^2)d^2x - ($$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a} (2cx + b) \sqrt{ex + d}}{ce^3x^5 + (3cde^2 + be^3)x^4 + ad^3 + (3cd^2e + 3bde^2 + ae^3)x^3 + (cd^3 + 3bd^2e + 3ade^2)x^2 + (bd^3 + 3ad^2e)x + d^4} \right)$$

23.170 Problem number 1643

$$\int \frac{(b+2cx)(d+ex)^{7/2}}{(a+bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(ex+d)^{7/2}}{\sqrt{cx^2+bx+a}} + \frac{14e^2(ex+d)^{3/2}\sqrt{cx^2+bx+a}}{5c} + \frac{56e^2(-be+2cd)\sqrt{ex+d}\sqrt{cx^2+bx+a}}{15c^2} \\ & + \frac{7e(23c^2d^2+8b^2e^2-ce(9ae+23bd)) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{15c^3\sqrt{cx^2+bx+a}\sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}} \\ & + \frac{56e(-be+2cd)(ae^2-bde+cd^2) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)}{15c^3\sqrt{ex+d}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(7(22c^4d^3x^2 + 22bc^3d^3x + 22ac^3d^3 - (8ab^3 - 21a^2bc + (8b^3c - 21abc^2)x^2 + (8b^4 - 21ab^2c)x)e^3 + 3((9b^2c^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(2ce^3x^4 + bd^3 + (6cde^2 + be^3)x^3 + 3(2cd^2e + bde^2)x^2 + (2cd^3 + 3bd^2e)x)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x\right)$$

23.171 Problem number 1644

$$\int \frac{(b+2cx)(d+ex)^{5/2}}{(a+bx+cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(ex+d)^{5/2}}{\sqrt{cx^2+bx+a}} + \frac{10e^2\sqrt{ex+d}\sqrt{cx^2+bx+a}}{3c} \\ & + \frac{10e(-be+2cd) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)\sqrt{2}\sqrt{-4ac+b^2}}{3c^2\sqrt{cx^2+bx+a}\sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}} \\ & + \frac{10e(ae^2-bde+cd^2) \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right)\sqrt{2}\sqrt{-4ac+b^2}}{3c^2\sqrt{ex+d}\sqrt{cx^2+bx+a}} \end{aligned}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5(5c^3d^2x^2 + 5bc^2d^2x + 5ac^2d^2 + (2ab^2 - 3a^2c + (2b^2c - 3ac^2)x^2 + (2b^3 - 3abc)x)e^2 - 5(bc^2dx^2 + b^2cdx + \dots) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(2ce^2x^3 + bd^2 + (4cde + be^2)x^2 + 2(cd^2 + bde)x)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x\right)$$

23.172 Problem number 1645

$$\int \frac{(b + 2cx)(d + ex)^{3/2}}{(a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(ex + d)^{\frac{3}{2}}}{\sqrt{cx^2 + bx + a}} \\ & + \frac{3e \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{ex + d}}{c\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}} \end{aligned}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^2dx^2 + 2bcdx + 2acd - (bcx^2 + b^2x + ab)e) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(\frac{4(c^2d^2 - bcde + (b^2 - 3ac)e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d - b^2e)}{3c^2}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(2cex^2 + bd + (2cd + be)x) \sqrt{cx^2 + bx + a} \sqrt{ex + d}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x\right)$$

23.173 Problem number 1646

$$\int \frac{(b + 2cx)\sqrt{d + ex}}{(a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}} \\ & + \frac{2e \operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{-\frac{c(cx^2 + bx + a)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{c\sqrt{ex + d} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((cx^2 + bx + a) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + (b^2 - 3ac)e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d^3 - 3bc^2d^2e - 3(b^2c - 6ac^2)de^2 + (2b^3 - 9abd))}{27c^3} \right) \right)}{c^2x^2 + bcx + ac}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx + a} (2cx + b) \sqrt{ex + d}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x \right)$$

23.174 Problem number 1647

$$\int \frac{b + 2cx}{\sqrt{d + ex} (a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2((-4ac + b^2)(-be + cd) - c(-4ac + b^2)ex) \sqrt{ex + d}}{(-4ac + b^2)(ae^2 - bde + cd^2) \sqrt{cx^2 + bx + a}}$$

$$e \text{ EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{ex + d}$$

$$\frac{(ae^2 - bde + cd^2) \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{(ae^2 - bde + cd^2) \sqrt{cx^2 + bx + a}}$$

command

```
integrate((2*c*x+b)/(c*x^2+b*x+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((2c^2dx^2 + 2bcdx + 2acd - (bcx^2 + b^2x + ab)e) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4(c^2d^2 - bcde + (b^2 - 3ac)e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3d^3 - 3bc^2d^2e - 3(b^2c - 6ac^2)de^2 + (2b^3 - 9abd))}{27c^3} \right) \right)}{c^2ex^5 + (c^2d + 2bce)x^4 + (2bcd + (b^2 + 2ac)e)x^3 + a^2d + (2abe + (b^2 + 2ac)d)x^2 + (2abd + a^2e)x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx + a} (2cx + b) \sqrt{ex + d}}{c^2ex^5 + (c^2d + 2bce)x^4 + (2bcd + (b^2 + 2ac)e)x^3 + a^2d + (2abe + (b^2 + 2ac)d)x^2 + (2abd + a^2e)x}, x \right)$$

23.175 Problem number 1648

$$\int \frac{b + 2cx}{(d + ex)^{3/2} (a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2((-4ac + b^2)(-be + cd) - c(-4ac + b^2)ex)}{(-4ac + b^2)(ae^2 - bde + cd^2)\sqrt{ex + d}\sqrt{cx^2 + bx + a}} + \frac{4e^2(-be + 2cd)\sqrt{cx^2 + bx + a}}{(ae^2 - bde + cd^2)^2\sqrt{ex + d}} \\ & 2e(-be + 2cd) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \\ & - \frac{(ae^2 - bde + cd^2)^2\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{2e \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{-\frac{c(cx^2 + bx + a)}{4ae^2 - 4bde + 4cd^2}}} \\ & + \frac{2 \left((5c^3d^3x^2 + 5bc^2d^3x + 5ac^2d^3 + ((2b^2c - 3ac^2)x^3 + (2b^3 - 3abc)x^2 + (2ab^2 - 3a^2c)x)e^3 - (5bc^2dx^3 + 3 \right)}{(ae^2 - bde + cd^2)\sqrt{ex + d}\sqrt{cx^2 + bx + a}} \end{aligned}$$

command

`integrate((2*c*x+b)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((5c^3d^3x^2 + 5bc^2d^3x + 5ac^2d^3 + ((2b^2c - 3ac^2)x^3 + (2b^3 - 3abc)x^2 + (2ab^2 - 3a^2c)x)e^3 - (5bc^2dx^3 + 3 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a} (2cx + b) \sqrt{ex + d}}{c^2e^2x^6 + 2(c^2de + bce^2)x^5 + (c^2d^2 + 4bcde + (b^2 + 2ac)e^2)x^4 + a^2d^2 + 2(bcd^2 + abe^2 + (b^2 + 2ac)de)x^3} \right)$$

23.176 Problem number 1649

$$\int \frac{(b+2cx)(d+ex)^{7/2}}{(a+bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(ex+d)^{\frac{7}{2}}}{3(cx^2+bx+a)^{\frac{3}{2}}} - \frac{14e(ex+d)^{\frac{3}{2}}(bd-2ae+(-be+2cd)x)}{3(-4ac+b^2)\sqrt{cx^2+bx+a}} \\ & + \frac{14e^2(-be+2cd)\sqrt{ex+d}\sqrt{cx^2+bx+a}}{3c(-4ac+b^2)} \\ & + 14e(c^2d^2+b^2e^2-ce(3ae+bd)) \operatorname{EllipticE} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \\ & + \frac{14e(-be+2cd)(ae^2-bde+cd^2) \operatorname{EllipticF} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right)}{3c^2\sqrt{-4ac+b^2}\sqrt{cx^2+bx+a}\sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}} \end{aligned}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(7/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(7 \left(2c^5d^3x^4 + 4bc^4d^3x^3 + 4abc^3d^3x + 2a^2c^3d^3 + 2(b^2c^3 + 2ac^4)d^3x^2 + (2a^2b^3 - 9a^3bc + (2b^3c^2 - 9abc^3)x^4 \right) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(2ce^3x^4 + bd^3 + (6cde^2 + be^3)x^3 + 3(2cd^2e + bde^2)x^2 + (2cd^3 + 3bd^2e)x)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^3x^6 + 3bc^2x^5 + 3(b^2c + ac^2)x^4 + 3a^2bx + (b^3 + 6abc)x^3 + a^3 + 3(ab^2 + a^2c)x^2}, x \right)$$

23.177 Problem number 1650

$$\int \frac{(b + 2cx)(d + ex)^{5/2}}{(a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ex + d)^{\frac{5}{2}}}{3(cx^2 + bx + a)^{\frac{3}{2}}} - \frac{10e(bd - 2ae + (-be + 2cd)x) \sqrt{ex + d}}{3(-4ac + b^2) \sqrt{cx^2 + bx + a}} \\ & + 5e(-be + 2cd) \text{EllipticE} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{ex + d} \sqrt{\dots} \\ & + \frac{3c\sqrt{-4ac + b^2} \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{\dots} \\ & + 20e(ae^2 - bde + cd^2) \text{EllipticF} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{\dots} \\ & - \frac{\dots}{3c\sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5 (2c^4d^2x^4 + 4bc^3d^2x^3 + 4abc^2d^2x + 2a^2c^2d^2 + 2(b^2c^2 + 2ac^3)d^2x^2 - ((b^2c^2 - 6ac^3)x^4 + a^2b^2 - 6a^3c + 2) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(2ce^2x^3 + bd^2 + (4cde + be^2)x^2 + 2(cd^2 + bde)x) \sqrt{cx^2 + bx + a} \sqrt{ex + d}}{c^3x^6 + 3bc^2x^5 + 3(b^2c + ac^2)x^4 + 3a^2bx + (b^3 + 6abc)x^3 + a^3 + 3(ab^2 + a^2c)x^2}, x \right)$$

23.178 Problem number 1651

$$\int \frac{(b+2cx)(d+ex)^{3/2}}{(a+bx+cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(ex+d)^{\frac{3}{2}}}{3(cx^2+bx+a)^{\frac{3}{2}}} - \frac{2e(2cx+b)\sqrt{ex+d}}{(-4ac+b^2)\sqrt{cx^2+bx+a}} \\ & + 2e \operatorname{EllipticE} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{ex+d} \sqrt{-\frac{c(cx^2+b)}{-4ac+b^2}} \\ & + \frac{\sqrt{-4ac+b^2} \sqrt{cx^2+bx+a} \sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}}{c\sqrt{-4ac+b^2} \sqrt{ex+d} \sqrt{cx^2+bx+a}} \\ & + 2e(-be+2cd) \operatorname{EllipticF} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-\frac{c(cx^2+b)}{-4ac+b^2}} \end{aligned}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^3dx^4 + 4bc^2dx^3 + 4abcdx + 2a^2cd + 2(b^2c + 2ac^2)dx^2 - (bc^2x^4 + 2b^2cx^3 + 2ab^2x + a^2b + (b^3 + 2abc)x) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(2cex^2 + bd + (2cd + be)x) \sqrt{cx^2 + bx + a} \sqrt{ex + d}}{c^3x^6 + 3bc^2x^5 + 3(b^2c + ac^2)x^4 + 3a^2bx + (b^3 + 6abc)x^3 + a^3 + 3(ab^2 + a^2c)x^2}, x \right)$$

23.179 Problem number 1652

$$\int \frac{(b + 2cx)\sqrt{d + ex}}{(a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{ex+d}}{3(cx^2+bx+a)^{\frac{3}{2}}} - \frac{2e(bcd - b^2e + 2ace + c(-be + 2cd)x)\sqrt{ex+d}}{3(-4ac+b^2)(ae^2 - bde + cd^2)\sqrt{cx^2+bx+a}}$$

$$+ \frac{e(-be + 2cd) \operatorname{EllipticE}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right) \sqrt{2}\sqrt{ex+d} \sqrt{-4ac+b^2}}{3(ae^2 - bde + cd^2)\sqrt{-4ac+b^2}\sqrt{cx^2+bx+a} \sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}}$$

$$+ \frac{4e \operatorname{EllipticF}\left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}}\right) \sqrt{2}\sqrt{\frac{c(cx^2+bx+a)}{-4ac+b^2}}}{3\sqrt{-4ac+b^2}\sqrt{ex+d}\sqrt{cx^2+bx+a}}$$

command

```
integrate((2*c*x+b)*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^4d^2x^4 + 4bc^3d^2x^3 + 4abc^2d^2x + 2a^2c^2d^2 + 2(b^2c^2 + 2ac^3)d^2x^2 - ((b^2c^2 - 6ac^3)x^4 + a^2b^2 - 6a^3c + 2(b^2c^2 + 2ac^3))\sqrt{ex+d}) \sqrt{cx^2+bx+a} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{ex+d}}{c^3x^6 + 3bc^2x^5 + 3(b^2c + ac^2)x^4 + 3a^2bx + (b^3 + 6abc)x^3 + a^3 + 3(ab^2 + a^2c)x^2}, x\right)$$

23.180 Problem number 1653

$$\int \frac{b + 2cx}{\sqrt{d + ex} (a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2((-4ac + b^2)(-be + cd) - c(-4ac + b^2)ex) \sqrt{ex + d}}{3(-4ac + b^2)(ae^2 - bde + cd^2)(cx^2 + bx + a)^{\frac{3}{2}}}$$

$$- \frac{2e(3b^2cde - 8ac^2de - 2b^3e^2 - bc(-7ae^2 + cd^2) - 2c(c^2d^2 + b^2e^2 - ce(3ae + bd))x) \sqrt{ex + d}}{3(-4ac + b^2)(ae^2 - bde + cd^2)^2 \sqrt{cx^2 + bx + a}}$$

$$- 2e(c^2d^2 + b^2e^2 - ce(3ae + bd)) \text{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)$$

$$+ \frac{3(ae^2 - bde + cd^2)^2 \sqrt{-4ac + b^2} \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{3(ae^2 - bde + cd^2)^2 \sqrt{-4ac + b^2} \sqrt{cx^2 + bx + a}}$$

$$+ \frac{2e(-be + 2cd) \text{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{3(ae^2 - bde + cd^2) \sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((2*c*x+b)/(c*x^2+b*x+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx + a} (2cx + b) \sqrt{ex + d}}{c^3ex^7 + (c^3d + 3bc^2e)x^6 + 3(bc^2d + (b^2c + ac^2)e)x^5 + (3(b^2c + ac^2)d + (b^3 + 6abc)e)x^4 + a^3d + ((b^3 + 6abc)e)x^3 + (c^3d + 3bc^2e)x^2 + 3(bc^2d + (b^2c + ac^2)e)x + a^3d} \right)$$

23.181 Problem number 2301

$$\int (1+x)^{3/2} (a+bx) (1-x+x^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{18(55bx^2 + 91ax) \sqrt{1+x} \sqrt{x^2-x+1}}{5005} + \frac{2(11bx^2 + 13ax) (x^3+1) \sqrt{1+x} \sqrt{x^2-x+1}}{143} + \frac{54b\sqrt{1+x} \sqrt{x^2-x+1}}{91(1+x+\sqrt{3})}$$

$$- \frac{273^{\frac{1}{4}} b(1+x)^{\frac{3}{2}} \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{91(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

$$+ \frac{183^{\frac{3}{4}} (1+x)^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) (91a-55b(1-\sqrt{3})) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{5005(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate((1+x)^(3/2)*(b*x+a)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{5005} (385bx^5 + 455ax^4 + 880bx^2 + 1274ax) \sqrt{x^2-x+1} \sqrt{x+1}$$

$$+ \frac{54}{55} a \operatorname{weierstrassPInverse}(0, -4, x)$$

$$- \frac{54}{91} b \operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bx^4 + ax^3 + bx + a) \sqrt{x^2-x+1} \sqrt{x+1}, x\right)$$

23.182 Problem number 2302

$$\int \sqrt{1+x} (a+bx) \sqrt{1-x+x^2} dx$$

Optimal antiderivative

$$\frac{2(5bx^2 + 7ax) \sqrt{1+x} \sqrt{x^2 - x + 1}}{35} + \frac{6b\sqrt{1+x} \sqrt{x^2 - x + 1}}{7(1+x+\sqrt{3})}$$

$$\frac{3 \cdot 3^{\frac{1}{4}} b (1+x)^{\frac{3}{2}} \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{x^2 - x + 1} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}}}{7(x^3 + 1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

$$+ \frac{2 \cdot 3^{\frac{3}{4}} (1+x)^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) (7a - 5b(1-\sqrt{3})) \sqrt{x^2 - x + 1} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}}}{35(x^3 + 1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate((1+x)^(1/2)*(b*x+a)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{35} (5bx^2 + 7ax) \sqrt{x^2 - x + 1} \sqrt{x + 1} + \frac{6}{5} a \operatorname{weierstrassPInverse}(0, -4, x)$$

$$- \frac{6}{7} b \operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bx + a) \sqrt{x^2 - x + 1} \sqrt{x + 1}, x\right)$$

23.183 Problem number 2303

$$\int \frac{a + bx}{\sqrt{1+x} \sqrt{1-x+x^2}} dx$$

Optimal antiderivative

$$\frac{2b(x^3 + 1)}{(1 + x + \sqrt{3}) \sqrt{1+x} \sqrt{x^2 - x + 1}} - \frac{3^{\frac{1}{4}} b \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}}}{\sqrt{x^2 - x + 1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} + \frac{2 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) (a - b(1 - \sqrt{3})) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^2 - x + 1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate((b*x+a)/((1+x)^(1/2)/(x^2-x+1)^(1/2)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 a \operatorname{weierstrassPInverse}(0, -4, x) - 2 b \operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bx + a) \sqrt{x^2 - x + 1} \sqrt{x + 1}}{x^3 + 1}, x\right)$$

23.184 Problem number 2304

$$\int \frac{a + bx}{(1+x)^{3/2} (1-x+x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x(bx+a)}{3\sqrt{1+x}\sqrt{x^2-x+1}} - \frac{2b(x^3+1)}{3(1+x+\sqrt{3})\sqrt{1+x}\sqrt{x^2-x+1}}$$

$$+ \frac{3^{\frac{1}{4}}b \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\sqrt{1+x}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{3\sqrt{x^2-x+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

$$+ \frac{2 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)(a+b-b\sqrt{3})\sqrt{1+x}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{x^2-x+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate((b*x+a)/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((bx^2+ax)\sqrt{x^2-x+1}\sqrt{x+1}+(ax^3+a)\operatorname{weierstrassPInverse}(0,-4,x)+(bx^3+b)\operatorname{weierstrassZeta}(0,-4,x)\right)}{3(x^3+1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bx+a)\sqrt{x^2-x+1}\sqrt{x+1}}{x^6+2x^3+1},x\right)$$

23.185 Problem number 2305

$$\int \frac{a+bx}{(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2x(5bx + 7a)}{27\sqrt{1+x}\sqrt{x^2-x+1}} + \frac{2x(bx+a)}{9(x^3+1)\sqrt{1+x}\sqrt{x^2-x+1}} - \frac{10b(x^3+1)}{27(1+x+\sqrt{3})\sqrt{1+x}\sqrt{x^2-x+1}} + \frac{5 \cdot 3^{\frac{1}{4}} b \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{27\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} + \frac{2 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) (7a+5b(1-\sqrt{3})) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} \cdot 3^{\frac{3}{4}}}{81\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate((b*x+a)/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((5bx^5 + 7ax^4 + 8bx^2 + 10ax) \sqrt{x^2-x+1} \sqrt{x+1} + 7(ax^6 + 2ax^3 + a) \operatorname{weierstrassPInverse}(0, -4, x) + 5(b) \right)}{27(x^6 + 2x^3 + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(bx+a)\sqrt{x^2-x+1}\sqrt{x+1}}{x^9+3x^6+3x^3+1}, x\right)$$

23.186 Problem number 2487

$$\int \frac{(A+Bx)(d+ex)^6}{(a+bx+cx^2)^{7/2}} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((B*x+A)*(e*x+d)^6/(c*x^2+b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

23.187 Problem number 2576

$$\int (5-x)(3+2x)^{5/2} \sqrt{2+5x+3x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{730(3+2x)^{\frac{3}{2}}(3x^2+5x+2)^{\frac{3}{2}}}{891} - \frac{2(3+2x)^{\frac{5}{2}}(3x^2+5x+2)^{\frac{3}{2}}}{33} \\ & + \frac{12130(3x^2+5x+2)^{\frac{3}{2}}\sqrt{3+2x}}{6237} \\ & - \frac{32567 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{48114\sqrt{3x^2+5x+2}} \\ & + \frac{168145 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{336798\sqrt{3x^2+5x+2}} \\ & + \frac{(250447+280359x)\sqrt{3+2x}\sqrt{3x^2+5x+2}}{56133} \end{aligned}$$

command

`integrate((5-x)*(3+2*x)^(5/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{1}{56133} (40824x^4 - 85428x^3 - 878130x^2 - 1465281x - 683491) \sqrt{3x^2+5x+2} \sqrt{2x+3} \\ & - \frac{310447}{6062364} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\ & + \frac{32567}{48114} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(4x^3 - 8x^2 - 51x - 45\right)\sqrt{3x^2+5x+2}\sqrt{2x+3}, x\right)$$

23.188 Problem number 2577

$$\int (5-x)(3+2x)^{3/2} \sqrt{2+5x+3x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(3+2x)^{\frac{3}{2}}(3x^2+5x+2)^{\frac{3}{2}}}{27} + \frac{202(3x^2+5x+2)^{\frac{3}{2}}\sqrt{3+2x}}{189} \\ & - \frac{4729 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2}\sqrt{3}}{7290\sqrt{3x^2+5x+2}} \\ & + \frac{5773 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2}\sqrt{3}}{10206\sqrt{3x^2+5x+2}} \\ & + \frac{(27914+30033x)\sqrt{3+2x}\sqrt{3x^2+5x+2}}{8505} \end{aligned}$$

command

`integrate((5-x)*(3+2*x)^(3/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{1}{8505} (3780x^3 - 15300x^2 - 63513x - 42314) \sqrt{3x^2+5x+2} \sqrt{2x+3} \\ & - \frac{5039}{918540} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\ & + \frac{4729}{7290} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\sqrt{3x^2+5x+2} (2x^2-7x-15) \sqrt{2x+3}, x\right)$$

23.189 Problem number 2578

$$\int (5-x)\sqrt{3+2x} \sqrt{2+5x+3x^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(3x^2 + 5x + 2)^{\frac{3}{2}} \sqrt{3 + 2x}}{21} - \frac{697 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{810 \sqrt{3x^2 + 5x + 2}} \\
 & + \frac{1039 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{1134 \sqrt{3x^2 + 5x + 2}} \\
 & + \frac{(2327 + 2169x) \sqrt{3 + 2x} \sqrt{3x^2 + 5x + 2}}{945}
 \end{aligned}$$

command

`integrate((5-x)*(3+2*x)^(1/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
 & -\frac{1}{945} (270x^2 - 1719x - 2147) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3} \\
 & + \frac{7723}{102060} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\
 & + \frac{697}{810} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)
 \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\sqrt{3x^2 + 5x + 2} \sqrt{2x + 3} (x - 5), x\right)$$

23.190 Problem number 2579

$$\int \frac{(5-x) \sqrt{2+5x+3x^2}}{\sqrt{3+2x}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{761 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{270 \sqrt{3x^2 + 5x + 2}} \\
 & + \frac{191 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{54 \sqrt{3x^2 + 5x + 2}} \\
 & + \frac{(88 - 9x) \sqrt{3 + 2x} \sqrt{3x^2 + 5x + 2}}{45}
 \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(1/2)/(3+2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{45} \sqrt{3x^2 + 5x + 2} (9x - 88) \sqrt{2x + 3} + \frac{2507}{4860} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + \frac{761}{270} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} (x - 5)}{\sqrt{2x + 3}}, x\right)$$

23.191 Problem number 2580

$$\int \frac{(5-x)\sqrt{2+5x+3x^2}}{(3+2x)^{3/2}} dx$$

Optimal antiderivative

$$\frac{121 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{18 \sqrt{3x^2 + 5x + 2}} - \frac{161 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{18 \sqrt{3x^2 + 5x + 2}} - \frac{(21+x) \sqrt{3x^2 + 5x + 2}}{3 \sqrt{3+2x}}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(1/2)/(3+2*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{481 \sqrt{6} (2x + 3) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 2178 \sqrt{6} (2x + 3) \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)}{324 (2x + 3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} \sqrt{2x + 3} (x - 5)}{4x^2 + 12x + 9}, x\right)$$

23.192 Problem number 2581

$$\int \frac{(5-x)\sqrt{2+5x+3x^2}}{(3+2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{67 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{30\sqrt{3x^2+5x+2}} \\ & + \frac{17 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{6\sqrt{3x^2+5x+2}} + \frac{(146+119x)\sqrt{3x^2+5x+2}}{15(3+2x)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((5-x)*(3*x^2+5*x+2)^(1/2)/(3+2*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{229\sqrt{6}(4x^2+12x+9)\operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 1206\sqrt{6}(4x^2+12x+9)\operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)}{540(4x^2+12x+9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}\sqrt{2x+3}(x-5)}{8x^3+36x^2+54x+27}, x\right)$$

23.193 Problem number 2582

$$\int \frac{(5-x)\sqrt{2+5x+3x^2}}{(3+2x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{49 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{250\sqrt{3x^2+5x+2}} \\ & + \frac{9 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{50\sqrt{3x^2+5x+2}} \\ & + \frac{(32+43x)\sqrt{3x^2+5x+2}}{25(3+2x)^{\frac{5}{2}}} + \frac{49\sqrt{3x^2+5x+2}}{125\sqrt{3+2x}} \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(1/2)/(3+2*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{13\sqrt{6}(8x^3 + 36x^2 + 54x + 27)\text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 882\sqrt{6}(8x^3 + 36x^2 + 54x + 27)\text{weiers}}{4500(8x^3 + 36x^2 + 54x + 27)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2}\sqrt{2x + 3}(x - 5)}{16x^4 + 96x^3 + 216x^2 + 216x + 81}, x\right)$$

23.194 Problem number 2583

$$\int \frac{(5-x)\sqrt{2+5x+3x^2}}{(3+2x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{159 \text{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{1250\sqrt{3x^2+5x+2}} \\ & + \frac{183 \text{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{1750\sqrt{3x^2+5x+2}} \\ & + \frac{183\sqrt{3x^2+5x+2}}{875(3+2x)^{\frac{3}{2}}} + \frac{(46+139x)\sqrt{3x^2+5x+2}}{175(3+2x)^{\frac{7}{2}}} + \frac{159\sqrt{3x^2+5x+2}}{625\sqrt{3+2x}} \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(1/2)/(3+2*x)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{223\sqrt{6}(16x^4 + 96x^3 + 216x^2 + 216x + 81)\text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) - 6678\sqrt{6}(16x^4 + 96x^3 + 216x^2 + 216x + 81)}{4500(16x^4 + 96x^3 + 216x^2 + 216x + 81)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2}\sqrt{2x + 3}(x - 5)}{32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243}, x\right)$$

23.195 Problem number 2584

$$\int (5-x)(3+2x)^{5/2} (2+5x+3x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{202(3+2x)^{\frac{3}{2}}(3x^2+5x+2)^{\frac{5}{2}}}{351} - \frac{2(3+2x)^{\frac{5}{2}}(3x^2+5x+2)^{\frac{5}{2}}}{45} \\ & + \frac{(534271+629153x)(3x^2+5x+2)^{\frac{3}{2}}\sqrt{3+2x}}{243243} + \frac{13318(3x^2+5x+2)^{\frac{5}{2}}\sqrt{3+2x}}{11583} \\ & - \frac{207851 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{18764460\sqrt{3x^2+5x+2}} \\ & + \frac{1015187 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{26270244\sqrt{3x^2+5x+2}} \\ & - \frac{(6006884+7817373x)\sqrt{3+2x}\sqrt{3x^2+5x+2}}{21891870} \end{aligned}$$

command

```
integrate((5-x)*(3+2*x)^(5/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{1}{21891870} (35026992x^6 - 4939704x^5 - 749549052x^4 - 2218655502x^3 - 2688028992x^2 - 1484149221x - 307000000) \\ & + \frac{34043759}{2364321960} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\ & + \frac{207851}{18764460} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(12x^5 - 4x^4 - 185x^3 - 406x^2 - 327x - 90\right)\sqrt{3x^2+5x+2}\sqrt{2x+3}, x\right)$$

23.196 Problem number 2585

$$\int (5-x)(3+2x)^{3/2} (2+5x+3x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(3+2x)^{\frac{3}{2}}(3x^2+5x+2)^{\frac{5}{2}}}{39} + \frac{(43822+50771x)(3x^2+5x+2)^{\frac{3}{2}}\sqrt{3+2x}}{27027} \\ & + \frac{886(3x^2+5x+2)^{\frac{5}{2}}\sqrt{3+2x}}{1287} \\ & - \frac{152657 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{2084940\sqrt{3x^2+5x+2}} \\ & + \frac{332459 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{2918916\sqrt{3x^2+5x+2}} \\ & - \frac{(486863+783711x)\sqrt{3+2x}\sqrt{3x^2+5x+2}}{2432430} \end{aligned}$$

command

`integrate((5-x)*(3+2*x)^(3/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{1}{2432430} (2245320x^5 - 4218480x^4 - 43487010x^3 - 77801130x^2 - 53083449x - 12602377)\sqrt{3x^2+5x+2}\sqrt{2} \\ & + \frac{6411863}{262702440}\sqrt{6}\operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\ & + \frac{152657}{2084940}\sqrt{6}\operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(6x^4 - 11x^3 - 76x^2 - 89x - 30\right)\sqrt{3x^2+5x+2}\sqrt{2x+3}, x\right)$$

23.197 Problem number 2586

$$\int (5-x)\sqrt{3+2x} (2+5x+3x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(119+127x)(3x^2+5x+2)^{\frac{3}{2}}\sqrt{3+2x}}{99} - \frac{2(3x^2+5x+2)^{\frac{5}{2}}\sqrt{3+2x}}{33} \\ & - \frac{15283 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{53460\sqrt{3x^2+5x+2}} \\ & + \frac{4153 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{10692\sqrt{3x^2+5x+2}} \\ & - \frac{(1246+3987x)\sqrt{3+2x}\sqrt{3x^2+5x+2}}{8910} \end{aligned}$$

command

`integrate((5-x)*(3*x^2+5*x+2)^(3/2)*(3+2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{1}{8910} (4860x^4 - 18090x^3 - 69300x^2 - 61623x - 18014)\sqrt{3x^2+5x+2}\sqrt{2x+3} \\ & + \frac{64621}{962280}\sqrt{6}\operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\ & + \frac{15283}{53460}\sqrt{6}\operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(3x^3 - 10x^2 - 23x - 10\right)\sqrt{3x^2+5x+2}\sqrt{2x+3}, x\right)$$

23.198 Problem number 2587

$$\int \frac{(5-x)(2+5x+3x^2)^{3/2}}{\sqrt{3+2x}} dx$$

Optimal antiderivative

$$\frac{(52 - 7x)(3x^2 + 5x + 2)^{\frac{3}{2}} \sqrt{3 + 2x}}{63} - \frac{11123 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{4860 \sqrt{3x^2 + 5x + 2}} + \frac{20501 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{6804 \sqrt{3x^2 + 5x + 2}} - \frac{(107 + 12429x) \sqrt{3 + 2x} \sqrt{3x^2 + 5x + 2}}{5670}$$

command

`integrate((5-x)*(3*x^2+5*x+2)^(3/2)/(3+2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{5670} (1890x^3 - 10890x^2 - 9711x - 9253) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3} + \frac{299657}{612360} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + \frac{11123}{4860} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(3x^3 - 10x^2 - 23x - 10) \sqrt{3x^2 + 5x + 2}}{\sqrt{2x + 3}}, x\right)$$

23.199 Problem number 2588

$$\int \frac{(5-x)(2+5x+3x^2)^{3/2}}{(3+2x)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(47+x)(3x^2+5x+2)^{\frac{3}{2}}}{7\sqrt{3+2x}} + \frac{2411 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{180 \sqrt{3x^2 + 5x + 2}} - \frac{4427 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{252 \sqrt{3x^2 + 5x + 2}} - \frac{(136 - 2493x) \sqrt{3 + 2x} \sqrt{3x^2 + 5x + 2}}{210}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(3/2)/(3+2*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{64199 \sqrt{6} (2x + 3) \text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 303786 \sqrt{6} (2x + 3) \text{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \text{weiers}\right)}{22680 (2x + 3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(3x^3 - 10x^2 - 23x - 10) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3}}{4x^2 + 12x + 9}, x\right)$$

23.200 Problem number 2589

$$\int \frac{(5-x)(2+5x+3x^2)^{3/2}}{(3+2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(37+3x)(3x^2+5x+2)^{\frac{3}{2}}}{15(3+2x)^{\frac{3}{2}}} + \frac{289 \text{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{12\sqrt{3x^2+5x+2}} \\ & - \frac{367 \text{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{20\sqrt{3x^2+5x+2}} + \frac{(241+69x)\sqrt{3x^2+5x+2}}{10\sqrt{3+2x}} \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(3/2)/(3+2*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1399 \sqrt{6} (4x^2 + 12x + 9) \text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 6606 \sqrt{6} (4x^2 + 12x + 9) \text{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \text{weiers}\right)}{360 (4x^2 + 12x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(3x^3 - 10x^2 - 23x - 10) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3}}{8x^3 + 36x^2 + 54x + 27}, x\right)$$

23.201 Problem number 2590

$$\int \frac{(5-x)(2+5x+3x^2)^{3/2}}{(3+2x)^{7/2}} dx$$

Optimal antiderivative

$$\frac{(408 + 337x)(3x^2 + 5x + 2)^{\frac{3}{2}}}{75(3 + 2x)^{\frac{5}{2}}} + \frac{2779 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{300\sqrt{3x^2 + 5x + 2}}$$

$$- \frac{243 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{20\sqrt{3x^2 + 5x + 2}} - \frac{(614 + 181x)\sqrt{3x^2 + 5x + 2}}{50\sqrt{3 + 2x}}$$

command

`integrate((5-x)*(3*x^2+5*x+2)^(3/2)/(3+2*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{10573\sqrt{6}(8x^3 + 36x^2 + 54x + 27)\operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 50022\sqrt{6}(8x^3 + 36x^2 + 54x + 27)}{540}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(3x^3 - 10x^2 - 23x - 10)\sqrt{3x^2 + 5x + 2}\sqrt{2x + 3}}{16x^4 + 96x^3 + 216x^2 + 216x + 81}, x\right)$$

23.202 Problem number 2591

$$\int \frac{(5-x)(2+5x+3x^2)^{3/2}}{(3+2x)^{9/2}} dx$$

Optimal antiderivative

$$\frac{(358 + 347x)(3x^2 + 5x + 2)^{\frac{3}{2}}}{175(3 + 2x)^{\frac{7}{2}}} - \frac{721 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{500\sqrt{3x^2 + 5x + 2}}$$

$$+ \frac{1327 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{700\sqrt{3x^2 + 5x + 2}}$$

$$+ \frac{(8561 + 6179x)\sqrt{3x^2 + 5x + 2}}{1750(3 + 2x)^{\frac{3}{2}}}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(3/2)/(3+2*x)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{19339 \sqrt{6} (16x^4 + 96x^3 + 216x^2 + 216x + 81) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 90846 \sqrt{6} (16x^4 + 96x^3)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(3x^3 - 10x^2 - 23x - 10) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3}}{32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243}, x\right)$$

23.203 Problem number 2592

$$\int \frac{(5-x)(2+5x+3x^2)^{3/2}}{(3+2x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(44 + 51x)(3x^2 + 5x + 2)^{\frac{3}{2}}}{45(3 + 2x)^{\frac{9}{2}}} + \frac{23 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{22500 \sqrt{3x^2 + 5x + 2}} \\ & + \frac{7 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{4500 \sqrt{3x^2 + 5x + 2}} \\ & - \frac{(189 + 211x) \sqrt{3x^2 + 5x + 2}}{2250(3 + 2x)^{\frac{5}{2}}} - \frac{23 \sqrt{3x^2 + 5x + 2}}{11250 \sqrt{3 + 2x}} \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(3/2)/(3+2*x)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{499 \sqrt{6} (32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) - 414 \sqrt{6} (32x^5)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(3x^3 - 10x^2 - 23x - 10) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3}}{64x^6 + 576x^5 + 2160x^4 + 4320x^3 + 4860x^2 + 2916x + 729}, x\right)$$

23.204 Problem number 2593

$$\int \frac{(5-x)(2+5x+3x^2)^{3/2}}{(3+2x)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(258 + 367x)(3x^2 + 5x + 2)^{\frac{3}{2}}}{495(3 + 2x)^{\frac{11}{2}}} \\ & - \frac{5861 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{1237500 \sqrt{3x^2 + 5x + 2}} \\ & + \frac{14807 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{1732500 \sqrt{3x^2 + 5x + 2}} \\ & + \frac{14807 \sqrt{3x^2 + 5x + 2}}{866250(3 + 2x)^{\frac{3}{2}}} - \frac{(15647 + 14773x) \sqrt{3x^2 + 5x + 2}}{57750(3 + 2x)^{\frac{7}{2}}} + \frac{5861 \sqrt{3x^2 + 5x + 2}}{618750 \sqrt{3 + 2x}} \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(3/2)/(3+2*x)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{338099 \sqrt{6} (64x^6 + 576x^5 + 2160x^4 + 4320x^3 + 4860x^2 + 2916x + 729) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + \dots}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(3x^3 - 10x^2 - 23x - 10) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3}}{128x^7 + 1344x^6 + 6048x^5 + 15120x^4 + 22680x^3 + 20412x^2 + 10206x + 2187}, x\right)$$

23.205 Problem number 2594

$$\int (5-x)(3+2x)^{5/2} (2+5x+3x^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{430(3+2x)^{\frac{3}{2}}(3x^2+5x+2)^{\frac{7}{2}}}{969} - \frac{2(3+2x)^{\frac{5}{2}}(3x^2+5x+2)^{\frac{7}{2}}}{57} \\
& - \frac{125(64006+79583x)(3x^2+5x+2)^{\frac{3}{2}}\sqrt{3+2x}}{52378326} \\
& + \frac{25(72737+86493x)(3x^2+5x+2)^{\frac{5}{2}}\sqrt{3+2x}}{1247103} + \frac{2350(3x^2+5x+2)^{\frac{7}{2}}\sqrt{3+2x}}{2907} \\
& - \frac{16503475 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{808122744\sqrt{3x^2+5x+2}} \\
& + \frac{142149125 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{5656859208\sqrt{3x^2+5x+2}} \\
& + \frac{25(749099+216603x)\sqrt{3+2x}\sqrt{3x^2+5x+2}}{942809868}
\end{aligned}$$

command

```
integrate((5-x)*(3+2*x)^(5/2)*(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
& -\frac{1}{942809868} (3572753184x^8 + 5989615632x^7 - 68880579768x^6 - 329194523196x^5 - 650694586500x^4 - 69951700000x^3 \\
& + \frac{18691975}{5359129776} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\
& + \frac{16503475}{808122744} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)
\end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(- (36x^7 + 48x^6 - 551x^5 - 2151x^4 - 3381x^3 - 2717x^2 - 1104x - 180) \sqrt{3x^2+5x+2} \sqrt{2x+3}, x\right)$$

23.206 Problem number 2595

$$\int (5-x)(3+2x)^{3/2} (2+5x+3x^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2(3+2x)^{\frac{3}{2}}(3x^2+5x+2)^{\frac{7}{2}}(949997+1332121x)(3x^2+5x+2)^{\frac{3}{2}}\sqrt{3+2x}}{51 \cdot 8270262} \\
& + \frac{(1063774+1253571x)(3x^2+5x+2)^{\frac{5}{2}}\sqrt{3+2x}}{984555} + \frac{1166(3x^2+5x+2)^{\frac{7}{2}}\sqrt{3+2x}}{2295} \\
& - \frac{34355693 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{637991640\sqrt{3x^2+5x+2}} \\
& + \frac{62005241 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{893188296\sqrt{3x^2+5x+2}} \\
& + \frac{(12174838-22593339x)\sqrt{3+2x}\sqrt{3x^2+5x+2}}{744323580}
\end{aligned}$$

command

`integrate((5-x)*(3+2*x)^(3/2)*(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
& -\frac{1}{744323580}(1576214640x^7+35026992x^6-31471976844x^5-98939331792x^4-136604504862x^3-9840176755x^2 \\
& + \frac{866317037}{80386946640}\sqrt{6}\operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\
& + \frac{34355693}{637991640}\sqrt{6}\operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)
\end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(18x^6-3x^5-271x^4-669x^3-687x^2-328x-60\right)\sqrt{3x^2+5x+2}\sqrt{2x+3}, x\right)$$

23.207 Problem number 2596

$$\int (5-x)\sqrt{3+2x}(2+5x+3x^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(15076 + 34643x)(3x^2 + 5x + 2)^{\frac{3}{2}} \sqrt{3 + 2x}}{162162} \\
& + \frac{(15467 + 17193x)(3x^2 + 5x + 2)^{\frac{5}{2}} \sqrt{3 + 2x}}{19305} - \frac{2(3x^2 + 5x + 2)^{\frac{7}{2}} \sqrt{3 + 2x}}{45} \\
& - \frac{2742319 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{12509640 \sqrt{3x^2 + 5x + 2}} \\
& + \frac{5021353 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{17513496 \sqrt{3x^2 + 5x + 2}} \\
& + \frac{(287729 - 2667537x) \sqrt{3 + 2x} \sqrt{3x^2 + 5x + 2}}{14594580}
\end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(5/2)*(3+2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
& - \frac{1}{14594580} (17513496 x^6 - 29413692 x^5 - 314201916 x^4 - 624522906 x^3 - 552292686 x^2 - 231246315 x - 3915707) \\
& + \frac{72391021}{1576214640} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\
& + \frac{2742319}{12509640} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)
\end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(- (9x^5 - 15x^4 - 113x^3 - 165x^2 - 96x - 20) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3}, x\right)$$

23.208 Problem number 2597

$$\int \frac{(5-x)(2+5x+3x^2)^{5/2}}{\sqrt{3+2x}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{5(563 + 4669x)(3x^2 + 5x + 2)^{\frac{3}{2}}\sqrt{3 + 2x}}{18018} + \frac{(224 - 33x)(3x^2 + 5x + 2)^{\frac{5}{2}}\sqrt{3 + 2x}}{429} \\
 & - \frac{651617 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2 - 5x - 2}\sqrt{3}}{277992\sqrt{3x^2 + 5x + 2}} \\
 & + \frac{5983645 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2 - 5x - 2}\sqrt{3}}{1945944\sqrt{3x^2 + 5x + 2}} \\
 & + \frac{(34372 - 676791x)\sqrt{3 + 2x}\sqrt{3x^2 + 5x + 2}}{324324}
 \end{aligned}$$

command

`integrate((5-x)*(3*x^2+5*x+2)^(5/2)/(3+2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
 & -\frac{1}{324324}(224532x^5 - 775656x^4 - 2896614x^3 - 3513708x^2 - 1516527x - 610408)\sqrt{3x^2 + 5x + 2}\sqrt{2x + 3} \\
 & + \frac{17362253}{35026992}\sqrt{6}\operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\
 & + \frac{651617}{277992}\sqrt{6}\operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)
 \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(9x^5 - 15x^4 - 113x^3 - 165x^2 - 96x - 20)\sqrt{3x^2 + 5x + 2}}{\sqrt{2x + 3}}, x\right)$$

23.209 Problem number 2598

$$\int \frac{(5-x)(2+5x+3x^2)^{5/2}}{(3+2x)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{(73+x)(3x^2+5x+2)^{\frac{5}{2}}}{11\sqrt{3+2x}} + \frac{5(218+3031x)(3x^2+5x+2)^{\frac{3}{2}}\sqrt{3+2x}}{1386} \\
 & + \frac{451331 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2 - 5x - 2}\sqrt{3}}{21384\sqrt{3x^2 + 5x + 2}} \\
 & - \frac{4145485 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2 - 5x - 2}\sqrt{3}}{149688\sqrt{3x^2 + 5x + 2}} \\
 & - \frac{(21871 - 471213x)\sqrt{3+2x}\sqrt{3x^2+5x+2}}{24948}
 \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(5/2)/(3+2*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{12034829 \sqrt{6} (2x + 3) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 56867706 \sqrt{6} (2x + 3) \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(9x^5 - 15x^4 - 113x^3 - 165x^2 - 96x - 20)\sqrt{3x^2 + 5x + 2}\sqrt{2x + 3}}{4x^2 + 12x + 9}, x\right)$$

23.210 Problem number 2599

$$\int \frac{(5-x)(2+5x+3x^2)^{5/2}}{(3+2x)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(21+x)(3x^2+5x+2)^{\frac{5}{2}}}{9(3+2x)^{\frac{3}{2}}} + \frac{5(745+121x)(3x^2+5x+2)^{\frac{3}{2}}}{126\sqrt{3+2x}} \\ & - \frac{33335 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{648\sqrt{3x^2+5x+2}} \\ & + \frac{306175 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{4536\sqrt{3x^2+5x+2}} \\ & + \frac{5(326-6957x)\sqrt{3+2x}\sqrt{3x^2+5x+2}}{756} \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(5/2)/(3+2*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{888815 \sqrt{6} (4x^2 + 12x + 9) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 4200210 \sqrt{6} (4x^2 + 12x + 9) \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(9x^5 - 15x^4 - 113x^3 - 165x^2 - 96x - 20)\sqrt{3x^2 + 5x + 2}\sqrt{2x + 3}}{8x^3 + 36x^2 + 54x + 27}, x\right)$$

23.211 Problem number 2600

$$\int \frac{(5-x)(2+5x+3x^2)^{5/2}}{(3+2x)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2291 + 879x)(3x^2 + 5x + 2)^{\frac{3}{2}}}{210(3 + 2x)^{\frac{3}{2}}} - \frac{(53 + 5x)(3x^2 + 5x + 2)^{\frac{5}{2}}}{35(3 + 2x)^{\frac{5}{2}}} \\ & - \frac{12857 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{168 \sqrt{3x^2 + 5x + 2}} \\ & + \frac{2333 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{40 \sqrt{3x^2 + 5x + 2}} \\ & - \frac{(10763 + 3117x) \sqrt{3x^2 + 5x + 2}}{140 \sqrt{3 + 2x}} \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(5/2)/(3+2*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{62207 \sqrt{6} (8x^3 + 36x^2 + 54x + 27) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 293958 \sqrt{6} (8x^3 + 36x^2 + 54x + 27)}{140 \sqrt{3 + 2x}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(9x^5 - 15x^4 - 113x^3 - 165x^2 - 96x - 20) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3}}{16x^4 + 96x^3 + 216x^2 + 216x + 81}, x\right)$$

23.212 Problem number 2601

$$\int \frac{(5-x)(2+5x+3x^2)^{5/2}}{(3+2x)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3354 + 2531x)(3x^2 + 5x + 2)^{\frac{3}{2}}}{210(3 + 2x)^{\frac{5}{2}}} - \frac{(43 + 7x)(3x^2 + 5x + 2)^{\frac{5}{2}}}{35(3 + 2x)^{\frac{7}{2}}} \\ & - \frac{4091 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{120\sqrt{3x^2 + 5x + 2}} \\ & + \frac{2505 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{56\sqrt{3x^2 + 5x + 2}} \\ & + \frac{(6292 + 1823x)\sqrt{3x^2 + 5x + 2}}{140\sqrt{3 + 2x}} \end{aligned}$$

command

`integrate((5-x)*(3*x^2+5*x+2)^(5/2)/(3+2*x)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{109079 \sqrt{6} (16x^4 + 96x^3 + 216x^2 + 216x + 81) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 515466 \sqrt{6} (16x^4 + 96x^3 + 216x^2 + 216x + 81)}{140\sqrt{3 + 2x}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(9x^5 - 15x^4 - 113x^3 - 165x^2 - 96x - 20)\sqrt{3x^2 + 5x + 2}\sqrt{2x + 3}}{32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243}, x\right)$$

23.213 Problem number 2602

$$\int \frac{(5-x)(2+5x+3x^2)^{5/2}}{(3+2x)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(14311 + 10729x)(3x^2 + 5x + 2)^{\frac{3}{2}}}{3150(3 + 2x)^{\frac{5}{2}}} + \frac{(124 + 113x)(3x^2 + 5x + 2)^{\frac{5}{2}}}{63(3 + 2x)^{\frac{9}{2}}} \\ & + \frac{17699 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{1800\sqrt{3x^2 + 5x + 2}} \\ & - \frac{32513 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{2520\sqrt{3x^2 + 5x + 2}} \\ & - \frac{(27213 + 7877x)\sqrt{3x^2 + 5x + 2}}{2100\sqrt{3 + 2x}} \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(5/2)/(3+2*x)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{471941 \sqrt{6} (32 x^5 + 240 x^4 + 720 x^3 + 1080 x^2 + 810 x + 243) \text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 2230074 \sqrt{6} (32 x^5 + 240 x^4 + 720 x^3 + 1080 x^2 + 810 x + 243)}{64 x^6 + 576 x^5 + 2160 x^4 + 4320 x^3 + 4860 x^2 + 2916 x + 729}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(9 x^5 - 15 x^4 - 113 x^3 - 165 x^2 - 96 x - 20) \sqrt{3 x^2 + 5 x + 2} \sqrt{2 x + 3}}{64 x^6 + 576 x^5 + 2160 x^4 + 4320 x^3 + 4860 x^2 + 2916 x + 729}, x\right)$$

23.214 Problem number 2603

$$\int \frac{(5-x)(2+5x+3x^2)^{5/2}}{(3+2x)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(24161 + 18699x)(3x^2 + 5x + 2)^{\frac{3}{2}}}{34650(3 + 2x)^{\frac{7}{2}}} + \frac{(114 + 115x)(3x^2 + 5x + 2)^{\frac{5}{2}}}{99(3 + 2x)^{\frac{11}{2}}} \\ & - \frac{107857 \text{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{99000 \sqrt{3x^2 + 5x + 2}} \\ & + \frac{198109 \text{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{138600 \sqrt{3x^2 + 5x + 2}} \\ & + \frac{(1301762 + 948443x) \sqrt{3x^2 + 5x + 2}}{346500(3 + 2x)^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(5/2)/(3+2*x)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2874913 \sqrt{6} (64 x^6 + 576 x^5 + 2160 x^4 + 4320 x^3 + 4860 x^2 + 2916 x + 729) \text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 2230074 \sqrt{6} (64 x^6 + 576 x^5 + 2160 x^4 + 4320 x^3 + 4860 x^2 + 2916 x + 729)}{128 x^7 + 1344 x^6 + 6048 x^5 + 15120 x^4 + 22680 x^3 + 20412 x^2 + 10206 x + 2187}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(9 x^5 - 15 x^4 - 113 x^3 - 165 x^2 - 96 x - 20) \sqrt{3 x^2 + 5 x + 2} \sqrt{2 x + 3}}{128 x^7 + 1344 x^6 + 6048 x^5 + 15120 x^4 + 22680 x^3 + 20412 x^2 + 10206 x + 2187}, x\right)$$

23.215 Problem number 2604

$$\int \frac{(5-x)(2+5x+3x^2)^{5/2}}{(3+2x)^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(73-33x)(3x^2+5x+2)^{\frac{3}{2}}}{6930(3+2x)^{\frac{9}{2}}} + \frac{(8+9x)(3x^2+5x+2)^{\frac{5}{2}}}{11(3+2x)^{\frac{13}{2}}} \\ & + \frac{5083 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2}\sqrt{3}}{495000\sqrt{3x^2+5x+2}} \\ & - \frac{9421 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2}\sqrt{3}}{693000\sqrt{3x^2+5x+2}} \\ & + \frac{(21492+17833x)\sqrt{3x^2+5x+2}}{346500(3+2x)^{\frac{5}{2}}} - \frac{5083\sqrt{3x^2+5x+2}}{247500\sqrt{3+2x}} \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(5/2)/(3+2*x)^(15/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{139297\sqrt{6}(128x^7+1344x^6+6048x^5+15120x^4+22680x^3+20412x^2+10206x+2187)\operatorname{weierstrassPInverse}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(9x^5-15x^4-113x^3-165x^2-96x-20)\sqrt{3x^2+5x+2}\sqrt{2x+3}}{256x^8+3072x^7+16128x^6+48384x^5+90720x^4+108864x^3+81648x^2+34992x+6561}, x\right)$$

23.216 Problem number 2605

$$\int \frac{(5-x)(2+5x+3x^2)^{5/2}}{(3+2x)^{17/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{(8901 + 8399x)(3x^2 + 5x + 2)^{\frac{3}{2}}}{64350(3 + 2x)^{\frac{11}{2}}} + \frac{(94 + 119x)(3x^2 + 5x + 2)^{\frac{5}{2}}}{195(3 + 2x)^{\frac{15}{2}}} \\
 & - \frac{335723 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{160875000 \sqrt{3x^2 + 5x + 2}} \\
 & + \frac{594851 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{225225000 \sqrt{3x^2 + 5x + 2}} + \frac{594851 \sqrt{3x^2 + 5x + 2}}{112612500(3 + 2x)^{\frac{3}{2}}} \\
 & - \frac{(386846 + 328339x) \sqrt{3x^2 + 5x + 2}}{7507500(3 + 2x)^{\frac{7}{2}}} + \frac{335723 \sqrt{3x^2 + 5x + 2}}{80437500 \sqrt{3 + 2x}}
 \end{aligned}$$

command

```
integrate((5-x)*(3*x^2+5*x+2)^(5/2)/(3+2*x)^(17/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{7967807 \sqrt{6} (256 x^8 + 3072 x^7 + 16128 x^6 + 48384 x^5 + 90720 x^4 + 108864 x^3 + 81648 x^2 + 34992 x + 6561) \operatorname{weiers}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(9x^5 - 15x^4 - 113x^3 - 165x^2 - 96x - 20) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3}}{512x^9 + 6912x^8 + 41472x^7 + 145152x^6 + 326592x^5 + 489888x^4 + 489888x^3 + 314928x^2 + 118098x}\right)$$

23.217 Problem number 2606

$$\int \frac{(5-x)(3+2x)^{5/2}}{\sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{865 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{81 \sqrt{3x^2 + 5x + 2}} \\
 & - \frac{2525 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{567 \sqrt{3x^2 + 5x + 2}} \\
 & + \frac{10(3 + 2x)^{\frac{3}{2}} \sqrt{3x^2 + 5x + 2}}{7} - \frac{2(3 + 2x)^{\frac{5}{2}} \sqrt{3x^2 + 5x + 2}}{21} \\
 & + \frac{1010 \sqrt{3 + 2x} \sqrt{3x^2 + 5x + 2}}{189}
 \end{aligned}$$

command

`integrate((5-x)*(3+2*x)^(5/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{189} (36x^2 - 162x - 829) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3} \\ + \frac{25715}{10206} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\ - \frac{865}{81} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(4x^3 - 8x^2 - 51x - 45)\sqrt{2x + 3}}{\sqrt{3x^2 + 5x + 2}}, x\right)$$

23.218 Problem number 2607

$$\int \frac{(5-x)(3+2x)^{3/2}}{\sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\frac{2743 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{405 \sqrt{3x^2 + 5x + 2}} \\ - \frac{163 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{81 \sqrt{3x^2 + 5x + 2}} \\ - \frac{2(3+2x)^{\frac{3}{2}} \sqrt{3x^2 + 5x + 2}}{15} + \frac{326\sqrt{3+2x} \sqrt{3x^2 + 5x + 2}}{135}$$

command

`integrate((5-x)*(3+2*x)^(3/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{4}{135} \sqrt{3x^2 + 5x + 2} (9x - 68) \sqrt{2x + 3} \\ + \frac{14609}{7290} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) \\ - \frac{2743}{405} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(2x^2 - 7x - 15)\sqrt{2x + 3}}{\sqrt{3x^2 + 5x + 2}}, x\right)$$

23.219 Problem number 2608

$$\int \frac{(5-x)\sqrt{3+2x}}{\sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\frac{101 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{27\sqrt{3x^2+5x+2}} + \frac{5 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{27\sqrt{3x^2+5x+2}} - \frac{2\sqrt{3+2x} \sqrt{3x^2+5x+2}}{9}$$

command

`integrate((5-x)*(3+2*x)^(1/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{853}{486} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) - \frac{101}{27} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right) - \frac{2}{9} \sqrt{3x^2+5x+2} \sqrt{2x+3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{2x+3}(x-5)}{\sqrt{3x^2+5x+2}}, x\right)$$

23.220 Problem number 2609

$$\int \frac{5-x}{\sqrt{3+2x} \sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{3\sqrt{3x^2+5x+2}} + \frac{13 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{3\sqrt{3x^2+5x+2}}$$

command

```
integrate((5-x)/(3+2*x)^(1/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{109}{54} \sqrt{6} \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + \frac{1}{3} \sqrt{6} \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}\sqrt{2x+3}(x-5)}{6x^3+19x^2+19x+6}, x\right)$$

23.221 Problem number 2610

$$\int \frac{5-x}{(3+2x)^{3/2}\sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{3\sqrt{3x^2+5x+2}} + \frac{13\operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{5\sqrt{3x^2+5x+2}} - \frac{26\sqrt{3x^2+5x+2}}{5\sqrt{3+2x}}$$

command

```
integrate((5-x)/(3+2*x)^(3/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{89\sqrt{6}(2x+3)\operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) - 234\sqrt{6}(2x+3)\operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)\right)}{90(2x+3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}\sqrt{2x+3}(x-5)}{12x^4+56x^3+95x^2+69x+18}, x\right)$$

23.222 Problem number 2611

$$\int \frac{5-x}{(3+2x)^{5/2} \sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\frac{193 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{75 \sqrt{3x^2+5x+2}} - \frac{13 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{15 \sqrt{3x^2+5x+2}} - \frac{26 \sqrt{3x^2+5x+2}}{15 (3+2x)^{\frac{3}{2}}} - \frac{386 \sqrt{3x^2+5x+2}}{75 \sqrt{3+2x}}$$

command

`integrate((5-x)/(3+2*x)^(5/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{959 \sqrt{6} (4x^2 + 12x + 9) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) - 3474 \sqrt{6} (4x^2 + 12x + 9) \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}\right)}{1350 (4x^2 + 12x + 9)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2} \sqrt{2x+3} (x-5)}{24x^5 + 148x^4 + 358x^3 + 423x^2 + 243x + 54}, x\right)$$

23.223 Problem number 2612

$$\int \frac{5-x}{(3+2x)^{7/2} \sqrt{2+5x+3x^2}} dx$$

Optimal antiderivative

$$\frac{4501 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{1875 \sqrt{3x^2+5x+2}} - \frac{391 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{375 \sqrt{3x^2+5x+2}} - \frac{26 \sqrt{3x^2+5x+2}}{25 (3+2x)^{\frac{5}{2}}} - \frac{782 \sqrt{3x^2+5x+2}}{375 (3+2x)^{\frac{3}{2}}} - \frac{9002 \sqrt{3x^2+5x+2}}{1875 \sqrt{3+2x}}$$

command

```
integrate((5-x)/(3+2*x)^(7/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{18413 \sqrt{6} (8x^3 + 36x^2 + 54x + 27) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) - 81018 \sqrt{6} (8x^3 + 36x^2 + 54x + 27)}{33750 (8x^3 + 36x^2 + 54x + 27)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} \sqrt{2x + 3} (x - 5)}{48x^6 + 368x^5 + 1160x^4 + 1920x^3 + 1755x^2 + 837x + 162}, x\right)$$

23.224 Problem number 2613

$$\int \frac{(5-x)(3+2x)^{7/2}}{(2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3+2x)^{5/2} (121+139x)}{3\sqrt{3x^2+5x+2}} + \frac{34174 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{405\sqrt{3x^2+5x+2}} \\ & - \frac{6034 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{81\sqrt{3x^2+5x+2}} \\ & + \frac{308(3+2x)^{3/2} \sqrt{3x^2+5x+2}}{5} + \frac{12068\sqrt{3+2x} \sqrt{3x^2+5x+2}}{135} \end{aligned}$$

command

```
integrate((5-x)*(3+2*x)^(7/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{931 \sqrt{6} (3x^2 + 5x + 2) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) - 307566 \sqrt{6} (3x^2 + 5x + 2) \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)}{3645 (3x^2 + 5x + 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(8x^4 - 4x^3 - 126x^2 - 243x - 135) \sqrt{3x^2 + 5x + 2} \sqrt{2x + 3}}{9x^4 + 30x^3 + 37x^2 + 20x + 4}, x\right)$$

23.225 Problem number 2614

$$\int \frac{(5-x)(3+2x)^{5/2}}{(2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(3+2x)^{\frac{3}{2}}(121+139x)}{3\sqrt{3x^2+5x+2}} + \frac{3830 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{81\sqrt{3x^2+5x+2}} \\ & - \frac{4150 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{81\sqrt{3x^2+5x+2}} \\ & + \frac{1660\sqrt{3+2x}\sqrt{3x^2+5x+2}}{27} \end{aligned}$$

command

```
integrate((5-x)*(3+2*x)^(5/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3355\sqrt{6}(3x^2+5x+2)\operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 34470\sqrt{6}(3x^2+5x+2)\operatorname{weierstrassZeta}\left(\frac{19}{27}, 729(3x^2+5x+2)\right)}{729(3x^2+5x+2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(4x^3-8x^2-51x-45)\sqrt{3x^2+5x+2}\sqrt{2x+3}}{9x^4+30x^3+37x^2+20x+4}, x\right)$$

23.226 Problem number 2615

$$\int \frac{(5-x)(3+2x)^{3/2}}{(2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(121+139x)\sqrt{3+2x}}{3\sqrt{3x^2+5x+2}} + \frac{274 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{9\sqrt{3x^2+5x+2}} \\ & - \frac{350 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{9\sqrt{3x^2+5x+2}} \end{aligned}$$

command

```
integrate((5-x)*(3+2*x)^(3/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{479 \sqrt{6} (3x^2 + 5x + 2) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 2466 \sqrt{6} (3x^2 + 5x + 2) \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}\right)}{81 (3x^2 + 5x + 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} (2x^2 - 7x - 15) \sqrt{2x + 3}}{9x^4 + 30x^3 + 37x^2 + 20x + 4}, x\right)$$

23.227 Problem number 2616

$$\int \frac{(5-x)\sqrt{3+2x}}{(2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(29 + 35x) \sqrt{3 + 2x}}{\sqrt{3x^2 + 5x + 2}} + \frac{70 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{3 \sqrt{3x^2 + 5x + 2}} - \frac{94 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{3 \sqrt{3x^2 + 5x + 2}}$$

command

```
integrate((5-x)*(3+2*x)^(1/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{143 \sqrt{6} (3x^2 + 5x + 2) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 630 \sqrt{6} (3x^2 + 5x + 2) \operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}\right)}{27 (3x^2 + 5x + 2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} \sqrt{2x + 3} (x - 5)}{9x^4 + 30x^3 + 37x^2 + 20x + 4}, x\right)$$

23.228 Problem number 2617

$$\int \frac{5-x}{\sqrt{3+2x} (2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6(37+47x)\sqrt{3+2x}}{5\sqrt{3x^2+5x+2}} - \frac{70 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{3\sqrt{3x^2+5x+2}} \\ & + \frac{94 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{5\sqrt{3x^2+5x+2}} \end{aligned}$$

command

`integrate((5-x)/(3*x^2+5*x+2)^(3/2)/(3+2*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{149\sqrt{6}(3x^2+5x+2)\operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 846\sqrt{6}(3x^2+5x+2)\operatorname{weierstrassZeta}\left(\frac{19}{27}, -\frac{28}{729}\right)}{45(3x^2+5x+2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}\sqrt{2x+3}(x-5)}{18x^5+87x^4+164x^3+151x^2+68x+12}, x\right)$$

23.229 Problem number 2618

$$\int \frac{5-x}{(3+2x)^{3/2} (2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6(37+47x)}{5\sqrt{3+2x}\sqrt{3x^2+5x+2}} + \frac{454 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{25\sqrt{3x^2+5x+2}} \\ & - \frac{94 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{5\sqrt{3x^2+5x+2}} - \frac{908\sqrt{3x^2+5x+2}}{25\sqrt{3+2x}} \end{aligned}$$

command

`integrate((5-x)/(3+2*x)^(3/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{299\sqrt{6}(6x^3 + 19x^2 + 19x + 6)\text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 4086\sqrt{6}(6x^3 + 19x^2 + 19x + 6)\text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)}{225(6x^3 + 19x^2 + 19x + 6)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2}\sqrt{2x + 3}(x - 5)}{36x^6 + 228x^5 + 589x^4 + 794x^3 + 589x^2 + 228x + 36}, x\right)$$

23.230 Problem number 2619

$$\int \frac{5 - x}{(3 + 2x)^{5/2}(2 + 5x + 3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6(37 + 47x)}{5(3 + 2x)^{\frac{3}{2}}\sqrt{3x^2 + 5x + 2}} \\ & + \frac{7438 \text{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2 - 5x - 2}\sqrt{3}}{375\sqrt{3x^2 + 5x + 2}} \\ & + \frac{1258 \text{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2 - 5x - 2}\sqrt{3}}{75\sqrt{3x^2 + 5x + 2}} \\ & - \frac{2516\sqrt{3x^2 + 5x + 2}}{75(3 + 2x)^{\frac{3}{2}}} - \frac{14876\sqrt{3x^2 + 5x + 2}}{375\sqrt{3 + 2x}} \end{aligned}$$

command

`integrate((5-x)/(3+2*x)^(5/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1447\sqrt{6}(12x^4 + 56x^3 + 95x^2 + 69x + 18)\text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) - 66942\sqrt{6}(12x^4 + 56x^3 + 95x^2 + 69x + 18)\text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right)}{225(12x^4 + 56x^3 + 95x^2 + 69x + 18)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2}\sqrt{2x + 3}(x - 5)}{72x^7 + 564x^6 + 1862x^5 + 3355x^4 + 3560x^3 + 2223x^2 + 756x + 108}, x\right)$$

23.231 Problem number 2620

$$\int \frac{5-x}{(3+2x)^{7/2} (2+5x+3x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6(37+47x)}{5(3+2x)^{\frac{5}{2}} \sqrt{3x^2+5x+2}} \\ & + \frac{213374 \operatorname{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{9375 \sqrt{3x^2+5x+2}} \\ & - \frac{30734 \operatorname{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2-5x-2} \sqrt{3}}{1875 \sqrt{3x^2+5x+2}} \\ & - \frac{4124 \sqrt{3x^2+5x+2}}{125(3+2x)^{\frac{5}{2}}} - \frac{61468 \sqrt{3x^2+5x+2}}{1875(3+2x)^{\frac{3}{2}}} - \frac{426748 \sqrt{3x^2+5x+2}}{9375 \sqrt{3+2x}} \end{aligned}$$

command

`integrate((5-x)/(3+2*x)^(7/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{161981 \sqrt{6} (24x^5 + 148x^4 + 358x^3 + 423x^2 + 243x + 54) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) - 1920366 \sqrt{6}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2} \sqrt{2x+3} (x-5)}{144x^8 + 1344x^7 + 5416x^6 + 12296x^5 + 17185x^4 + 15126x^3 + 8181x^2 + 2484x + 324}, x\right)$$

23.232 Problem number 2621

$$\int \frac{(5-x)(3+2x)^{9/2}}{(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(3+2x)^{\frac{7}{2}}(121+139x)}{9(3x^2+5x+2)^{\frac{3}{2}}} + \frac{4(3+2x)^{\frac{3}{2}}(2164+2571x)}{9\sqrt{3x^2+5x+2}} \\
 & -\frac{110516 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{243\sqrt{3x^2+5x+2}} \\
 & +\frac{148780 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{243\sqrt{3x^2+5x+2}} \\
 & -\frac{59512\sqrt{3+2x}\sqrt{3x^2+5x+2}}{81}
 \end{aligned}$$

command

```
integrate((5-x)*(3+2*x)^(9/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(113723 \sqrt{6} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 497322 \sqrt{6} (9x^4 + 30x^3 + \dots) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(16x^5 + 16x^4 - 264x^3 - 864x^2 - 999x - 405)\sqrt{3x^2+5x+2}\sqrt{2x+3}}{27x^6 + 135x^5 + 279x^4 + 305x^3 + 186x^2 + 60x + 8}, x\right)$$

23.233 Problem number 2622

$$\int \frac{(5-x)(3+2x)^{7/2}}{(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(3+2x)^{\frac{5}{2}}(121+139x)}{9(3x^2+5x+2)^{\frac{3}{2}}} + \frac{28(1018+1177x)\sqrt{3+2x}}{27\sqrt{3x^2+5x+2}} \\
 & -\frac{31892 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{81\sqrt{3x^2+5x+2}} \\
 & +\frac{41860 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{81\sqrt{3x^2+5x+2}}
 \end{aligned}$$

command

```
integrate((5-x)*(3+2*x)^(7/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(30401 \sqrt{6} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 143514 \sqrt{6} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(8x^4 - 4x^3 - 126x^2 - 243x - 135)\sqrt{3x^2 + 5x + 2}\sqrt{2x + 3}}{27x^6 + 135x^5 + 279x^4 + 305x^3 + 186x^2 + 60x + 8}, x\right)$$

23.234 Problem number 2623

$$\int \frac{(5-x)(3+2x)^{5/2}}{(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(3+2x)^{\frac{3}{2}}(121+139x)}{9(3x^2+5x+2)^{\frac{3}{2}}} + \frac{20(364+431x)\sqrt{3+2x}}{9\sqrt{3x^2+5x+2}} \\ & - \frac{8620 \text{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{27\sqrt{3x^2+5x+2}} \\ & + \frac{11300 \text{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{27\sqrt{3x^2+5x+2}} \end{aligned}$$

command

```
integrate((5-x)*(3+2*x)^(5/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(8185 \sqrt{6} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 38790 \sqrt{6} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(4x^3 - 8x^2 - 51x - 45)\sqrt{3x^2 + 5x + 2}\sqrt{2x + 3}}{27x^6 + 135x^5 + 279x^4 + 305x^3 + 186x^2 + 60x + 8}, x\right)$$

23.235 Problem number 2624

$$\int \frac{(5-x)(3+2x)^{3/2}}{(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(121+139x)\sqrt{3+2x}}{9(3x^2+5x+2)^{\frac{3}{2}}} + \frac{4(1390+1689x)\sqrt{3+2x}}{9\sqrt{3x^2+5x+2}} \\ & - \frac{2252 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{9\sqrt{3x^2+5x+2}} \\ & + \frac{2956 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{9\sqrt{3x^2+5x+2}} \end{aligned}$$

command

```
integrate((5-x)*(3+2*x)^(3/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2147 \sqrt{6} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 10134 \sqrt{6} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}(2x^2-7x-15)\sqrt{2x+3}}{27x^6+135x^5+279x^4+305x^3+186x^2+60x+8}, x\right)$$

23.236 Problem number 2625

$$\int \frac{(5-x)\sqrt{3+2x}}{(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(29+35x)\sqrt{3+2x}}{3(3x^2+5x+2)^{\frac{3}{2}}} + \frac{4(1759+2139x)\sqrt{3+2x}}{15\sqrt{3x^2+5x+2}} \\ & - \frac{2852 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{15\sqrt{3x^2+5x+2}} \\ & + \frac{748 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{3\sqrt{3x^2+5x+2}} \end{aligned}$$

command

```
integrate((5-x)*(3+2*x)^(1/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2711 \sqrt{6} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 12834 \sqrt{6} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} \sqrt{2x + 3} (x - 5)}{27x^6 + 135x^5 + 279x^4 + 305x^3 + 186x^2 + 60x + 8}, x\right)$$

23.237 Problem number 2626

$$\int \frac{5 - x}{\sqrt{3 + 2x} (2 + 5x + 3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(37 + 47x) \sqrt{3 + 2x}}{5(3x^2 + 5x + 2)^{\frac{3}{2}}} + \frac{4(2152 + 2607x) \sqrt{3 + 2x}}{25 \sqrt{3x^2 + 5x + 2}} \\ & - \frac{3476 \text{EllipticE}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{25 \sqrt{3x^2 + 5x + 2}} \\ & + \frac{916 \text{EllipticF}\left(\sqrt{1+x} \sqrt{3}, \frac{i\sqrt{6}}{3}\right) \sqrt{-3x^2 - 5x - 2} \sqrt{3}}{5 \sqrt{3x^2 + 5x + 2}} \end{aligned}$$

command

```
integrate((5-x)/(3*x^2+5*x+2)^(5/2)/(3+2*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3353 \sqrt{6} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 15642 \sqrt{6} (9x^4 + 30x^3 + 37x^2 + 20x + 4) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2 + 5x + 2} \sqrt{2x + 3} (x - 5)}{54x^7 + 351x^6 + 963x^5 + 1447x^4 + 1287x^3 + 678x^2 + 196x + 24}, x\right)$$

23.238 Problem number 2627

$$\int \frac{5-x}{(3+2x)^{3/2} (2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(37+47x)}{5(3x^2+5x+2)^{\frac{3}{2}}\sqrt{3+2x}} + \frac{\frac{8216}{25} + \frac{9636x}{25}}{\sqrt{3+2x}\sqrt{3x^2+5x+2}} \\ & -\frac{11732 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{125\sqrt{3x^2+5x+2}} \\ & + \frac{3212 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{25\sqrt{3x^2+5x+2}} + \frac{23464\sqrt{3x^2+5x+2}}{125\sqrt{3+2x}} \end{aligned}$$

command

`integrate((5-x)/(3+2*x)^(3/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(12671 \sqrt{6} (18x^5 + 87x^4 + 164x^3 + 151x^2 + 68x + 12) \operatorname{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 52794 \sqrt{6} (18x^5 + 87x^4 + 164x^3 + 151x^2 + 68x + 12) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3x^2+5x+2}\sqrt{2x+3}(x-5)}{108x^8+864x^7+2979x^6+5783x^5+6915x^4+5217x^3+2426x^2+636x+72}, x\right)$$

23.239 Problem number 2628

$$\int \frac{5-x}{(3+2x)^{5/2} (2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(37+47x)}{5(3+2x)^{\frac{3}{2}}(3x^2+5x+2)^{\frac{3}{2}}} + \frac{\frac{7824}{25} + \frac{8844x}{25}}{(3+2x)^{\frac{3}{2}}\sqrt{3x^2+5x+2}} \\ & -\frac{95396 \operatorname{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{1875\sqrt{3x^2+5x+2}} \\ & + \frac{30836 \operatorname{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{375\sqrt{3x^2+5x+2}} \\ & + \frac{61672\sqrt{3x^2+5x+2}}{375(3+2x)^{\frac{3}{2}}} + \frac{190792\sqrt{3x^2+5x+2}}{1875\sqrt{3+2x}} \end{aligned}$$

command

```
integrate((5-x)/(3+2*x)^(5/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(156113 \sqrt{6} (36 x^6 + 228 x^5 + 589 x^4 + 794 x^3 + 589 x^2 + 228 x + 36) \text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 42 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2+5x+2}\sqrt{2x+3}(x-5)}{216x^9+2052x^8+8550x^7+20503x^6+31179x^5+31179x^4+20503x^3+8550x^2+2052x+216}, x\right)$$

23.240 Problem number 2629

$$\int \frac{5-x}{(3+2x)^{7/2}(2+5x+3x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(37+47x)}{5(3+2x)^{\frac{5}{2}}(3x^2+5x+2)^{\frac{3}{2}}} + \frac{\frac{7432}{25} + \frac{8052x}{25}}{(3+2x)^{\frac{5}{2}}\sqrt{3x^2+5x+2}} \\ & - \frac{107548 \text{EllipticE}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{15625\sqrt{3x^2+5x+2}} \\ & + \frac{129268 \text{EllipticF}\left(\sqrt{1+x}\sqrt{3}, \frac{i\sqrt{6}}{3}\right)\sqrt{-3x^2-5x-2}\sqrt{3}}{3125\sqrt{3x^2+5x+2}} \\ & + \frac{87144\sqrt{3x^2+5x+2}}{625(3+2x)^{\frac{5}{2}}} + \frac{258536\sqrt{3x^2+5x+2}}{3125(3+2x)^{\frac{3}{2}}} + \frac{215096\sqrt{3x^2+5x+2}}{15625\sqrt{3+2x}} \end{aligned}$$

command

```
integrate((5-x)/(3+2*x)^(7/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(1239169 \sqrt{6} (72 x^7 + 564 x^6 + 1862 x^5 + 3355 x^4 + 3560 x^3 + 2223 x^2 + 756 x + 108) \text{weierstrassPInverse}\left(\frac{19}{27}, -\frac{28}{729}, x + \frac{19}{18}\right) + 42 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3x^2+5x+2}\sqrt{2x+3}(x-5)}{432x^{10}+4752x^9+23256x^8+66656x^7+123867x^6+155895x^5+134543x^4+78609x^3+29754x^2+432x+216}, x\right)$$

23.241 Problem number 2630

$$\int \frac{(A + Bx)(d + ex)^{3/2}}{\sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2B(ex + d)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{5c} + \frac{2(5Ace - 4bBe + 3Bcd) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}{15c^2}$$

$$+ \frac{(10Ace(-be + 2cd) + B(3c^2d^2 + 8b^2e^2 - ce(9ae + 13bd))) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2cd}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}{15c^3e \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}$$

$$+ \frac{2(5Ace - 4bBe + 3Bcd) (ae^2 - bde + cd^2) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e \sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}{15c^3e \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((B*x+A)*(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((3Bc^3d^3 + (8Bbc^2 - 25Ac^3)d^2e - (17Bb^2c - (27Ba + 25Ab)c^2)de^2 + (8Bb^3 + 15Aac^2 - (21Bab + 10Ab^2)c^2)e^3 \right)}{15c^3e \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Bex^2 + Ad + (Bd + Ae)x) \sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}, x \right)$$

23.242 Problem number 2631

$$\int \frac{(A + Bx)\sqrt{d + ex}}{\sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2B\sqrt{ex + d} \sqrt{cx^2 + bx + a}}{3c}$$

$$\begin{aligned}
 & (3Ace - 2bBe + Bcd) \operatorname{EllipticE} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \\
 + & \frac{3c^2e\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{3c^2e\sqrt{ex + d} \sqrt{cx^2 + bx + a}} \\
 & 2B(ae^2 - bde + cd^2) \operatorname{EllipticF} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}
 \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cx^2 + bx + a} \sqrt{ex + d} Bc^2e^2 - (Bc^2d^2 + 2(Bbc - 3Ac^2)de - (2Bb^2 - 3(Ba + Ab)c)e^2) \sqrt{c} e^{\frac{1}{2}} \operatorname{weierstrass} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Bx + A)\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}, x \right)$$

23.243 Problem number 2632

$$\int \frac{A + Bx}{\sqrt{d + ex} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$B \text{EllipticE} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{\frac{ce\sqrt{cx^2 + bx + a}}{2cd - e(b + \sqrt{-4ac + b^2})} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}$$

$$2(-Ae + Bd) \text{EllipticF} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{cx^2 + bx + a}$$

command

`integrate((B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 B c^{\frac{3}{2}} e^{\frac{3}{2}} \text{weierstrassZeta} \left(\frac{4(c^2 d^2 - bcde + (b^2 - 3ac)e^2)e^{(-2)}}{3c^2}, -\frac{4(2c^3 d^3 - 3bc^2 d^2 e - 3(b^2 c - 6ac^2)de^2 + (2b^3 - 9abc)e^3)e^{(-3)}}{27c^3} \right), \text{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx + a} (Bx + A) \sqrt{ex + d}}{cex^3 + (cd + be)x^2 + ad + (bd + ae)x}, x \right)$$

23.244 Problem number 2633

$$\int \frac{A + Bx}{(d + ex)^{3/2} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(-Ae + Bd) \sqrt{cx^2 + bx + a}}{(ae^2 - bde + cd^2) \sqrt{ex + d}}$$

$$(-Ae + Bd) \text{EllipticE} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e \sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}$$

$$e(ae^2 - bde + cd^2) \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}$$

$$+ 2B \text{EllipticF} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e \sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{-\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}$$

$$+ \frac{ce \sqrt{ex + d} \sqrt{cx^2 + bx + a}}{ce \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((Bcd^3 + (3Ba - Ab)xe^3 - (2(Bb - Ac)dx - (3Ba - Ab)d)e^2 + (Bcd^2x - 2(Bb - Ac)d^2)e) \sqrt{c} e^{\frac{1}{2}} \text{weierstrassF} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx + a} (Bx + A) \sqrt{ex + d}}{ce^2x^4 + (2cde + be^2)x^3 + ad^2 + (cd^2 + 2bde + ae^2)x^2 + (bd^2 + 2ade)x}, x \right)$$

23.245 Problem number 2634

$$\int \frac{A + Bx}{(d + ex)^{5/2} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(-Ae + Bd) \sqrt{cx^2 + bx + a}}{3(ae^2 - bde + cd^2)(ex + d)^{\frac{3}{2}}} - \frac{2(2Ae(-be + 2cd) - B(cd^2 + e(-3ae + bd))) \sqrt{cx^2 + bx + a}}{3(ae^2 - bde + cd^2)^2 \sqrt{ex + d}}$$

$$+ \frac{(2Ae(-be + 2cd) - B(cd^2 + e(-3ae + bd))) \text{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}{3e(ae^2 - bde + cd^2)^2 \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}$$

$$+ \frac{2(-Ae + Bd) \text{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}}{3e(ae^2 - bde + cd^2) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((Bc^2d^5 - (3Bab - 2Ab^2 + 3Aac)x^2e^5 + ((Bb^2 + (9Ba - 5Ab)c)dx^2 - 2(3Bab - 2Ab^2 + 3Aac)dx)e^4 - ((4$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx + a} (Bx + A) \sqrt{ex + d}}{ce^3x^5 + (3cde^2 + be^3)x^4 + ad^3 + (3cd^2e + 3bde^2 + ae^3)x^3 + (cd^3 + 3bd^2e + 3ade^2)x^2 + (bd^3 + 3ad^2e)x + d^4} \right)$$

23.246 Problem number 2635

$$\int \frac{(A + Bx)(d + ex)^{5/2}}{(a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(ex + d)^{\frac{3}{2}} (2ac(Ae + Bd) - b(Acd + aBe) - (b^2Be - bc(Ae + Bd) + 2c(Acd - aBe)) x)}{c(-4ac + b^2) \sqrt{cx^2 + bx + a}}$$

$$+ \frac{2e(4b^2Be - 3bc(Ae + Bd) + 2c(3Acd - 5aBe)) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}{3c^2(-4ac + b^2)}$$

$$(8b^3B e^2 - b^2ce(6Ae + 13Bd) - 2c^2(-9Aa e^2 + 3Ac d^2 - 20aBde) + bc(6Acde - 29aB e^2 + 3Bc d^2)) \text{EllipticE}$$

$$2(a e^2 - bde + c d^2) (4b^2Be - 3bc(Ae + Bd) + 2c(3Acd - 5aBe)) \text{EllipticF} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \frac{3c^3 \sqrt{-4ac + b^2} \sqrt{cx^2 + bx + a}}{3c^3 \sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{c}}$$

command

```
integrate((B*x+A)*(e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((3 (Bbc^4 - 2Ac^5)d^3x^2 + 3 (Bb^2c^3 - 2Abc^4)d^3x + 3 (Babc^3 - 2Aac^4)d^3 + (8Bab^4 + 3(10Ba^3 + 9Aa^2b)c^2 + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Be^2x^3 + Ad^2 + (2Bde + Ae^2)x^2 + (Bd^2 + 2Ade)x) \sqrt{cx^2 + bx + a} \sqrt{ex + d}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x \right)$$

23.247 Problem number 2636

$$\int \frac{(A + Bx)(d + ex)^{3/2}}{(a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(2ac(Ae + Bd) - b(ACd + aBe) - (b^2Be - bc(Ae + Bd) + 2c(ACd - aBe))x) \sqrt{ex + d}}{c(-4ac + b^2) \sqrt{cx^2 + bx + a}}$$

$$+ \frac{(2Ac^2d + 2b^2Be - c(Abe + 6aBe + Bbd)) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}{c^2 \sqrt{-4ac + b^2} \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}$$

$$+ \frac{2(-2Ac + bB)(ae^2 - bde + cd^2) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)}{c^2 \sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((B*x+A)*(e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\frac{((Bbc^3 - 2Ac^4)d^2x^2 + (Bb^2c^2 - 2Abc^3)d^2x + (Babc^2 - 2Aac^3)d^2 - (2Bab^3 + 6Aa^2c^2 + (2Bb^3c + 6Aac^3 -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Bex^2 + Ad + (Bd + Ae)x) \sqrt{cx^2 + bx + a} \sqrt{ex + d}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x \right)$$

23.248 Problem number 2637

$$\int \frac{(A + Bx)\sqrt{d + ex}}{(a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - 2Ba - (-2Ac + bB)x)\sqrt{ex + d}}{(-4ac + b^2)\sqrt{cx^2 + bx + a}} + \frac{(-2Ac + bB)\text{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)\sqrt{2}\sqrt{ex + d}\sqrt{cx^2 + bx + a}}{c\sqrt{-4ac + b^2}\sqrt{cx^2 + bx + a}\sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}} + \frac{2(Abe - 2Acd - 2aBe + Bbd)\text{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}\sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}}\right)\sqrt{2}\sqrt{ex + d}\sqrt{cx^2 + bx + a}}{c\sqrt{-4ac + b^2}\sqrt{ex + d}\sqrt{cx^2 + bx + a}}$$

command

```
integrate((B*x+A)*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\frac{((Bbc^2 - 2Ac^3)dx^2 + (Bb^2c - 2Abc^2)dx + (Babc - 2Aac^2)d + (Bab^2 + (Bb^2c - (6Ba - Ab)c^2)x^2 - (6Ba^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}(Bx + A)\sqrt{ex + d}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x\right)$$

23.249 Problem number 2638

$$\int \frac{A + Bx}{\sqrt{d + ex} (a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(aB(-be + 2cd) - A(2ace - b^2e + bcd) + c(Abe - 2Acd - 2aBe + Bbd)x) \sqrt{ex + d}}{(-4ac + b^2)(ae^2 - bde + cd^2) \sqrt{cx^2 + bx + a}} \\ (Abe - 2Acd - 2aBe + Bbd) \text{EllipticE} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}} \\ - \frac{2(-2Ac + bB) \text{EllipticF} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-\frac{c(cx^2 + bx + a)}{-4ac + b^2}}}{c\sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{cx^2 + bx + a}} \\ +$$

command

```
integrate((B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\frac{((Bbc^2 - 2Ac^3)d^2x^2 + (Bb^2c - 2Abc^2)d^2x + (Babc - 2Aac^2)d^2 + (Ba^2b + Aab^2 - 6Aa^2c - (6Aac^2 - (Bab$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx + a} (Bx + A) \sqrt{ex + d}}{c^2ex^5 + (c^2d + 2bce)x^4 + (2bcd + (b^2 + 2ac)e)x^3 + a^2d + (2abe + (b^2 + 2ac)d)x^2 + (2abd + a^2e)x}, x \right)$$

23.250 Problem number 2639

$$\int \frac{A + Bx}{(d + ex)^{3/2} (a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2aB(-be + 2cd) - 2A(2ace - b^2e + bcd) + 2c(Abe - 2Acd - 2aBe + Bbd) x}{(-4ac + b^2) (a e^2 - bde + c d^2) \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

$$+ \frac{2e(b^2e(-2Ae + Bd) - 2c(-3Aae^2 + Acd^2 + 4aBde) + b(2Acde + aBe^2 + Bcd^2)) \sqrt{cx^2 + bx + a}}{(-4ac + b^2) (a e^2 - bde + c d^2)^2 \sqrt{ex + d}}$$

$$(b^2e(-2Ae + Bd) - 2c(-3Aae^2 + Acd^2 + 4aBde) + b(2Acde + aBe^2 + Bcd^2)) \text{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \right)$$

$$\frac{(a e^2 - bde + c d^2)^2 \sqrt{-4ac + b^2} \sqrt{cx^2 + bx + a}}{(a e^2 - bde + c d^2) \sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

$$+ \frac{2(Abe - 2Acd - 2aBe + Bbd) \text{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2e \sqrt{-4ac + b^2}}{2cd - e (b + \sqrt{-4ac + b^2})}} \right)}{(a e^2 - bde + c d^2) \sqrt{-4ac + b^2} \sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{cx^2 + bx + a} (Bx + A) \sqrt{ex + d}}{c^2 e^2 x^6 + 2(c^2 de + bce^2) x^5 + (c^2 d^2 + 4bcde + (b^2 + 2ac)e^2) x^4 + a^2 d^2 + 2(bcd^2 + abe^2 + (b^2 + 2ac)de) x^3 + \dots} \right)$$

24 Test file number 35

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/35_1.2.1.4-d+e_x-^m-f+g_x-^n-a+b_x+c_x^2-^p

24.1 Problem number 489

$$\int x^3 \sqrt{1+x} \sqrt{1-x+x^2} dx$$

Optimal antiderivative

$$\frac{6x\sqrt{1+x} \sqrt{x^2-x+1}}{55} + \frac{2x^4\sqrt{1+x} \sqrt{x^2-x+1}}{11} + 43^{\frac{3}{4}}(1+x)^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}$$

$$55(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}$$

command

```
integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{55} (5x^4 + 3x) \sqrt{x^2-x+1} \sqrt{x+1} - \frac{12}{55} \operatorname{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{x^2-x+1} \sqrt{x+1} x^3, x\right)$$

24.2 Problem number 491

$$\int x \sqrt{1+x} \sqrt{1-x+x^2} dx$$

Optimal antiderivative

$$\frac{2x^2\sqrt{1+x}\sqrt{x^2-x+1}}{7} + \frac{6\sqrt{1+x}\sqrt{x^2-x+1}}{7(1+x+\sqrt{3})}$$

$$+ \frac{23^{\frac{3}{4}}(1+x)^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{2}\sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{7(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

$$- \frac{33^{\frac{1}{4}}(1+x)^{\frac{3}{2}} \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{7(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{7} \sqrt{x^2-x+1} \sqrt{x+1} x^2 - \frac{6}{7} \operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{x^2-x+1} \sqrt{x+1} x, x\right)$$

24.3 Problem number 492

$$\int \sqrt{1+x} \sqrt{1-x+x^2} dx$$

Optimal antiderivative

$$\frac{2x\sqrt{1+x}\sqrt{x^2-x+1}}{5}$$

$$+ \frac{23^{\frac{3}{4}}(1+x)^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{5(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{5} \sqrt{x^2 - x + 1} \sqrt{x + 1} x + \frac{6}{5} \text{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{x^2 - x + 1} \sqrt{x + 1}, x\right)$$

24.4 Problem number 494

$$\int \frac{\sqrt{1+x} \sqrt{1-x+x^2}}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{1+x} \sqrt{x^2-x+1}}{x} + \frac{3\sqrt{1+x} \sqrt{x^2-x+1}}{1+x+\sqrt{3}} \\ & + \frac{3^{\frac{3}{4}}(1+x)^{\frac{3}{2}} \text{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \\ & - \frac{3^{\frac{3}{4}}(1+x)^{\frac{3}{2}} \text{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{2(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3x \text{weierstrassZeta}(0, -4, \text{weierstrassPInverse}(0, -4, x)) + \sqrt{x^2 - x + 1} \sqrt{x + 1}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^2 - x + 1} \sqrt{x + 1}}{x^2}, x\right)$$

24.5 Problem number 495

$$\int \frac{\sqrt{1+x} \sqrt{1-x+x^2}}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{1+x} \sqrt{x^2-x+1}}{2x^2} \\ & + \frac{3^{\frac{3}{4}}(1+x)^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{2(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

`integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3x^2 \operatorname{weierstrassPInverse}(0, -4, x) - \sqrt{x^2-x+1} \sqrt{x+1}}{2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^2-x+1} \sqrt{x+1}}{x^3}, x\right)$$

24.6 Problem number 496

$$\int x^3(1+x)^{3/2} (1-x+x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{54x\sqrt{1+x} \sqrt{x^2-x+1}}{935} + \frac{18x^4\sqrt{1+x} \sqrt{x^2-x+1}}{187} + \frac{2x^4(x^3+1) \sqrt{1+x} \sqrt{x^2-x+1}}{17} \\ & + \frac{36 \cdot 3^{\frac{3}{4}}(1+x)^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{935(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{935} (55x^7 + 100x^4 + 27x) \sqrt{x^2 - x + 1} \sqrt{x + 1} - \frac{108}{935} \text{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((x^6 + x^3) \sqrt{x^2 - x + 1} \sqrt{x + 1}, x\right)$$

24.7 Problem number 498

$$\int x(1+x)^{3/2} (1-x+x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{18x^2\sqrt{1+x}\sqrt{x^2-x+1}}{91} + \frac{2x^2(x^3+1)\sqrt{1+x}\sqrt{x^2-x+1}}{13} + \frac{54\sqrt{1+x}\sqrt{x^2-x+1}}{91(1+x+\sqrt{3})} \\ & + \frac{183^{\frac{3}{4}}(1+x)^{\frac{3}{2}} \text{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{2}\sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{91(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \\ & - \frac{273^{\frac{1}{4}}(1+x)^{\frac{3}{2}} \text{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{91(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{91} (7x^5 + 16x^2) \sqrt{x^2 - x + 1} \sqrt{x + 1} - \frac{54}{91} \text{weierstrassZeta}(0, -4, \text{weierstrassPInverse}(0, -4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((x^4 + x) \sqrt{x^2 - x + 1} \sqrt{x + 1}, x\right)$$

24.8 Problem number 499

$$\int (1+x)^{3/2} (1-x+x^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{18x\sqrt{1+x}\sqrt{x^2-x+1}}{55} + \frac{2x(x^3+1)\sqrt{1+x}\sqrt{x^2-x+1}}{11}$$

$$+ \frac{18 \cdot 3^{\frac{3}{4}} (1+x)^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{55(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{55} (5x^4 + 14x) \sqrt{x^2 - x + 1} \sqrt{x + 1} + \frac{54}{55} \operatorname{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((x^3 + 1) \sqrt{x^2 - x + 1} \sqrt{x + 1}, x\right)$$

24.9 Problem number 501

$$\int \frac{(1+x)^{3/2} (1-x+x^2)^{3/2}}{x^2} dx$$

Optimal antiderivative

$$\frac{9x^2\sqrt{1+x}\sqrt{x^2-x+1}}{7} - \frac{(x^3+1)\sqrt{1+x}\sqrt{x^2-x+1}}{x} + \frac{27\sqrt{1+x}\sqrt{x^2-x+1}}{7(1+x+\sqrt{3})}$$

$$+ \frac{9 \cdot 3^{\frac{3}{4}}(1+x)^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{2}\sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{7(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

$$- \frac{27 \cdot 3^{\frac{1}{4}}(1+x)^{\frac{3}{2}} \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{14(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2x^3 - 7)\sqrt{x^2 - x + 1}\sqrt{x + 1} - 27x \operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x))}{7x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(x^3 + 1)\sqrt{x^2 - x + 1}\sqrt{x + 1}}{x^2}, x\right)$$

24.10 Problem number 502

$$\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^3} dx$$

Optimal antiderivative

$$\frac{9x\sqrt{1+x}\sqrt{x^2-x+1}}{10} - \frac{(x^3+1)\sqrt{1+x}\sqrt{x^2-x+1}}{2x^2}$$

$$+ \frac{9 \cdot 3^{\frac{3}{4}}(1+x)^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{x^2-x+1} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{10(x^3+1) \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{27 x^2 \text{weierstrassPInverse}(0, -4, x) + (4 x^3 - 5) \sqrt{x^2 - x + 1} \sqrt{x + 1}}{10 x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(x^3 + 1) \sqrt{x^2 - x + 1} \sqrt{x + 1}}{x^3}, x\right)$$

24.11 Problem number 503

$$\int \frac{x^3}{\sqrt{1+x} \sqrt{1-x+x^2}} dx$$

Optimal antiderivative

$$\frac{\frac{2x(x^3 + 1)}{5\sqrt{1+x} \sqrt{x^2 - x + 1}}{4 \text{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{15\sqrt{x^2 - x + 1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{5} \sqrt{x^2 - x + 1} \sqrt{x + 1} x - \frac{4}{5} \text{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^2 - x + 1} \sqrt{x + 1} x^3}{x^3 + 1}, x\right)$$

24.12 Problem number 505

$$\int \frac{x}{\sqrt{1+x} \sqrt{1-x+x^2}} dx$$

Optimal antiderivative

$$\frac{\frac{2x^3 + 2}{(1+x+\sqrt{3}) \sqrt{1+x} \sqrt{x^2-x+1}} + \frac{2 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{1+x} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} - \frac{3^{\frac{1}{4}} \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2 \operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^2-x+1} \sqrt{x+1} x}{x^3+1}, x\right)$$

24.13 Problem number 506

$$\int \frac{1}{\sqrt{1+x} \sqrt{1-x+x^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

2 weierstrassPInverse(0, -4, x)

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3+1}, x\right)$$

24.14 Problem number 508

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{-x^3-1}{x\sqrt{1+x}\sqrt{x^2-x+1}} + \frac{x^3+1}{(1+x+\sqrt{3})\sqrt{1+x}\sqrt{x^2-x+1}} \\ & \frac{\text{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{1+x}\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^2-x+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \\ & - \frac{3^{\frac{1}{4}}\text{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\sqrt{1+x}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{2\sqrt{x^2-x+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{x\text{weierstrassZeta}(0, -4, \text{weierstrassPInverse}(0, -4, x)) + \sqrt{x^2-x+1}\sqrt{x+1}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^5+x^2}, x\right)$$

24.15 Problem number 509

$$\int \frac{1}{x^3 \sqrt{1+x} \sqrt{1-x+x^2}} dx$$

Optimal antiderivative

$$\frac{-x^3 - 1}{2x^2 \sqrt{1+x} \sqrt{x^2 - x + 1}} \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}} - \frac{6\sqrt{x^2 - x + 1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}{\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{x^2 \operatorname{weierstrassPInverse}(0, -4, x) + \sqrt{x^2 - x + 1} \sqrt{x + 1}}{2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^2 - x + 1} \sqrt{x + 1}}{x^6 + x^3}, x\right)$$

24.16 Problem number 510

$$\int \frac{x^3}{(1+x)^{3/2} (1-x+x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2x}{3\sqrt{1+x} \sqrt{x^2 - x + 1}} 4 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}} + \frac{9\sqrt{x^2 - x + 1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}{\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{x^2 - x + 1} \sqrt{x + 1} x - 2 (x^3 + 1) \text{weierstrassPInverse}(0, -4, x) \right)}{3 (x^3 + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{x^2 - x + 1} \sqrt{x + 1} x^3}{x^6 + 2x^3 + 1}, x \right)$$

24.17 Problem number 512

$$\int \frac{x}{(1+x)^{3/2} (1-x+x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^2}{3\sqrt{1+x} \sqrt{x^2-x+1}} - \frac{2(x^3+1)}{3(1+x+\sqrt{3}) \sqrt{1+x} \sqrt{x^2-x+1}} \\ & - \frac{2 \text{EllipticF} \left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{1+x} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \\ & + \frac{3^{\frac{1}{4}} \text{EllipticE} \left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i \right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{3\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

`integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{x^2 - x + 1} \sqrt{x + 1} x^2 + (x^3 + 1) \text{weierstrassZeta}(0, -4, \text{weierstrassPInverse}(0, -4, x)) \right)}{3 (x^3 + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{x^2 - x + 1} \sqrt{x + 1} x}{x^6 + 2x^3 + 1}, x \right)$$

24.18 Problem number 513

$$\int \frac{1}{(1+x)^{3/2} (1-x+x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x}{3\sqrt{1+x} \sqrt{x^2-x+1}} + \frac{2 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{x^2-x+1} \sqrt{x+1} x + (x^3+1) \operatorname{weierstrassPInverse}(0, -4, x) \right)}{3(x^3+1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^2-x+1} \sqrt{x+1}}{x^6+2x^3+1}, x\right)$$

24.19 Problem number 515

$$\int \frac{1}{x^2(1+x)^{3/2} (1-x+x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2}{3x\sqrt{1+x}\sqrt{x^2-x+1}} - \frac{5(x^3+1)}{3x\sqrt{1+x}\sqrt{x^2-x+1}}$$

$$+ \frac{\frac{5x^3}{3} + \frac{5}{3}}{(1+x+\sqrt{3})\sqrt{1+x}\sqrt{x^2-x+1}}$$

$$+ \frac{5 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{2}\sqrt{1+x} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

$$- \frac{5 \cdot 3^{\frac{1}{4}} \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{6\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(5x^3+3)\sqrt{x^2-x+1}\sqrt{x+1} + 5(x^4+x)\operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x))}{3(x^4+x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^8+2x^5+x^2}, x\right)$$

24.20 Problem number 516

$$\int \frac{1}{x^3(1+x)^{3/2}(1-x+x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2}{3x^2\sqrt{1+x}\sqrt{x^2-x+1}} - \frac{7(x^3+1)}{6x^2\sqrt{1+x}\sqrt{x^2-x+1}}$$

$$+ \frac{7 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{18\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(7x^3 + 3)\sqrt{x^2 - x + 1}\sqrt{x + 1} + 7(x^5 + x^2)\text{weierstrassPInverse}(0, -4, x)}{6(x^5 + x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^2 - x + 1}\sqrt{x + 1}}{x^9 + 2x^6 + x^3}, x\right)$$

24.21 Problem number 517

$$\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4x}{27\sqrt{1+x}\sqrt{x^2-x+1}} - \frac{2x}{9(x^3+1)\sqrt{1+x}\sqrt{x^2-x+1}} \\ & 4 \text{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}} \\ & + \frac{81\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}{81\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

`integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((2x^4 - x)\sqrt{x^2 - x + 1}\sqrt{x + 1} + 2(x^6 + 2x^3 + 1)\text{weierstrassPInverse}(0, -4, x)\right)}{27(x^6 + 2x^3 + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^2 - x + 1}\sqrt{x + 1}x^3}{x^9 + 3x^6 + 3x^3 + 1}, x\right)$$

24.22 Problem number 519

$$\int \frac{x}{(1+x)^{5/2} (1-x+x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10x^2}{27\sqrt{1+x} \sqrt{x^2-x+1}} + \frac{2x^2}{9(x^3+1)\sqrt{1+x} \sqrt{x^2-x+1}} \\ & - \frac{10(x^3+1)}{27(1+x+\sqrt{3})\sqrt{1+x} \sqrt{x^2-x+1}} \\ & - \frac{10 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{1+x} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{81\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \\ & + \frac{5 \cdot 3^{3/4} \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{27\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((5x^5 + 8x^2) \sqrt{x^2-x+1} \sqrt{x+1} + 5(x^6 + 2x^3 + 1) \operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x)) \right)}{27(x^6 + 2x^3 + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^2-x+1} \sqrt{x+1} x}{x^9 + 3x^6 + 3x^3 + 1}, x\right)$$

24.23 Problem number 520

$$\int \frac{1}{(1+x)^{5/2} (1-x+x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{14x}{27\sqrt{1+x}\sqrt{x^2-x+1}} + \frac{2x}{9(x^3+1)\sqrt{1+x}\sqrt{x^2-x+1}}$$

$$+ \frac{14 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{81\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((7x^4 + 10x) \sqrt{x^2 - x + 1} \sqrt{x + 1} + 7(x^6 + 2x^3 + 1) \operatorname{weierstrassPInverse}(0, -4, x) \right)}{27(x^6 + 2x^3 + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^9+3x^6+3x^3+1}, x\right)$$

24.24 Problem number 522

$$\int \frac{1}{x^2(1+x)^{5/2} (1-x+x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{22}{27x\sqrt{1+x}\sqrt{x^2-x+1}} + \frac{2}{9x(x^3+1)\sqrt{1+x}\sqrt{x^2-x+1}} \\
 & - \frac{55(x^3+1)}{27x\sqrt{1+x}\sqrt{x^2-x+1}} + \frac{\frac{55x^3}{27} + \frac{55}{27}}{(1+x+\sqrt{3})\sqrt{1+x}\sqrt{x^2-x+1}} \\
 & + \frac{55 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{2}\sqrt{1+x} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{81\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} 3^{\frac{3}{4}} \\
 & - \frac{55 \cdot 3^{\frac{1}{4}} \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{1+x} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{54\sqrt{x^2-x+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}
 \end{aligned}$$

command

```
integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(55x^6 + 88x^3 + 27)\sqrt{x^2-x+1}\sqrt{x+1} + 55(x^7 + 2x^4 + x)\operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4))}{27(x^7 + 2x^4 + x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^{11} + 3x^8 + 3x^5 + x^2}, x\right)$$

24.25 Problem number 523

$$\int \frac{1}{x^3(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{\frac{26}{27x^2\sqrt{1+x}\sqrt{x^2-x+1}} + \frac{2}{9x^2(x^3+1)\sqrt{1+x}\sqrt{x^2-x+1}}}{\frac{91(x^3+1)}{54x^2\sqrt{1+x}\sqrt{x^2-x+1}} - \frac{91 \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\sqrt{1+x}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{162\sqrt{x^2-x+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}} 3^{\frac{3}{4}}$$

command

```
integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(91x^6 + 130x^3 + 27)\sqrt{x^2-x+1}\sqrt{x+1} + 91(x^8 + 2x^5 + x^2)\operatorname{weierstrassPInverse}(0, -4, x)}{54(x^8 + 2x^5 + x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^{12} + 3x^9 + 3x^6 + x^3}, x\right)$$

24.26 Problem number 622

$$\int (d + ex)^3 \sqrt{f + gx} \sqrt{a + cx^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(2a e^2 g^2 (-231dg + 74ef) - c(-567d^3 g^3 + 1107d^2 ef g^2 - 843d e^2 f^2 g + 233e^3 f^3)) (gx + f)^{\frac{3}{2}} \sqrt{cx^2 + a}}{3465c g^4} \\
& + \frac{2e(18a e^2 g^2 - c(81d^2 g^2 - 96defg + 29e^2 f^2)) (gx + f)^{\frac{5}{2}} \sqrt{cx^2 + a}}{693c g^4} \\
& + \frac{2e^2(-3dg + ef) (gx + f)^{\frac{7}{2}} \sqrt{cx^2 + a}}{99g^4} \\
& - \frac{2(150a^2 e^4 g^4 - 6ac e^2 g^2 (165d^2 g^2 - 33defg + 2e^2 f^2) + c^2(315d^4 g^4 - 798d^3 ef g^3 + 1098d^2 e^2 f^2 g^2 - 732d e^3 f^3 g + 111e^4 f^4)) \sqrt{cx^2 + a}}{3465c^2 e g^4} \\
& + \frac{2(ex + d)^4 \sqrt{gx + f} \sqrt{cx^2 + a}}{11e}
\end{aligned}$$

$$\begin{aligned}
& 4(3a^2 e^2 g^4 (231dg + 26ef) - c^2 f^2 (-231d^3 g^3 + 396d^2 ef g^2 - 264d e^2 f^2 g + 64e^3 f^3) - 9ac g^2 (77d^3 g^3 + 88d^2 ef g^2 - 64d e^2 f^2 g + 11e^3 f^3)) \sqrt{cx^2 + a} \\
& + \frac{3465c^{\frac{3}{2}} g^5 \sqrt{cx^2 + a}}{3465c^{\frac{3}{2}} g^5 \sqrt{cx^2 + a}}
\end{aligned}$$

$$\begin{aligned}
& 4(a g^2 + c f^2) (75a^2 e^3 g^4 - 3ace g^2 (165d^2 g^2 - 33defg + 2e^2 f^2) - c^2 f (-231d^3 g^3 + 396d^2 ef g^2 - 264d e^2 f^2 g + 64e^3 f^3)) \sqrt{cx^2 + a} \\
& - \frac{3465c^{\frac{5}{2}} g^5 \sqrt{gx + f}}{3465c^{\frac{5}{2}} g^5 \sqrt{gx + f}}
\end{aligned}$$

command

```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2(231 c^3 d^3 f^3 g^3 + 2079 ac^2 d^3 f g^5 - (64 c^3 f^6 + 102 ac^2 f^4 g^2 - 51 a^2 c f^2 g^4 - 225 a^3 g^6)) e^3 + 33 (8 c^3 d f^5 g + 15 ac^2 d f^4 g^2 + 11 a^2 c f^3 g^3 + 3 a^3 f^4 g^4) \right) \sqrt{cx^2 + a} \sqrt{gx + f}}{3465c^2 e g^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3\right) \sqrt{cx^2 + a} \sqrt{gx + f}, x\right)$$

24.27 Problem number 623

$$\int (d + ex)^2 \sqrt{f + gx} \sqrt{a + cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(7ae^2g^2 - c(21d^2g^2 - 24defg + 8e^2f^2))(gx + f)^{\frac{3}{2}} \sqrt{cx^2 + a}}{315cg^3} \\ & + \frac{2e(-3dg + ef)(gx + f)^{\frac{5}{2}} \sqrt{cx^2 + a}}{63g^3} \\ & - \frac{2(6ae^2g^2(-10dg + ef) - c(-35d^3g^3 + 63d^2efg^2 - 57de^2f^2g + 19e^3f^3)) \sqrt{gx + f} \sqrt{cx^2 + a}}{315ceg^3} \\ & + \frac{2(ex + d)^3 \sqrt{gx + f} \sqrt{cx^2 + a}}{9e} \\ & + \frac{4(21a^2e^2g^4 + 3acg^2(-21d^2g^2 - 16defg + 3e^2f^2) + c^2f^2(21d^2g^2 - 24defg + 8e^2f^2)) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2}\right)}{315c^{\frac{3}{2}}g^4 \sqrt{cx^2 + a} \sqrt{\frac{(gx + f)\sqrt{c}}{g\sqrt{-a} + f\sqrt{c}}}} \\ & + \frac{4(ag^2 + cf^2)(3aeg^2(-10dg + ef) + cf(21d^2g^2 - 24defg + 8e^2f^2)) \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2}, \sqrt{-\frac{2}{-ag + \dots}}\right)}{315c^{\frac{3}{2}}g^4 \sqrt{gx + f} \sqrt{cx^2 + a}} \end{aligned}$$

command

```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2(21c^2d^2f^3g^2 + 189acd^2fg^4 + (8c^2f^5 + 15acf^3g^2 - 33a^2fg^4)e^2 - 6(4c^2df^4g + 11acdf^2g^3 + 15a^2dg^5)e \right) \sqrt{cx^2 + a} \sqrt{gx + f}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((e^2x^2 + 2dex + d^2)\sqrt{cx^2 + a} \sqrt{gx + f}, x\right)$$

24.28 Problem number 624

$$\int (d + ex) \sqrt{f + gx} \sqrt{a + cx^2} dx$$

Optimal antiderivative

$$\frac{2e(cx^2 + a)^{\frac{3}{2}} \sqrt{gx + f}}{7c} - \frac{2(5ae g^2 + cf(-7dg + 4ef) - 3cg(7dg + ef)x) \sqrt{gx + f} \sqrt{cx^2 + a}}{105c g^2}$$

$$4(cf^2(-7dg + 4ef) + ag^2(21dg + 8ef)) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag + f\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{gx + f}$$

$$105g^3 \sqrt{c} \sqrt{cx^2 + a} \sqrt{\frac{(gx + f)\sqrt{c}}{g\sqrt{-a} + f\sqrt{c}}}$$

$$4(ag^2 + cf^2)(5ae g^2 + cf(-7dg + 4ef)) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag + f\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2 + a}{gx + f}}$$

$$+ \frac{105c^{\frac{3}{2}} g^3 \sqrt{gx + f} \sqrt{cx^2 + a}}{105c^{\frac{3}{2}} g^3 \sqrt{gx + f} \sqrt{cx^2 + a}}$$

command

```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2(7c^2df^3g + 63acdfg^3 - (4c^2f^4 + 11acf^2g^2 + 15a^2g^4)e) \sqrt{cg} \operatorname{weierstrassPInverse} \left(\frac{4(cf^2 - 3ag^2)}{3cg^2}, -\frac{8(cf^3 + 9afg^2)}{27cg^3} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{cx^2 + a} (ex + d) \sqrt{gx + f}, x \right)$$

24.29 Problem number 625

$$\int \sqrt{f+gx} \sqrt{a+cx^2} dx$$

Optimal antiderivative

$$\frac{2(gx+f)^{\frac{3}{2}} \sqrt{cx^2+a}}{5g} - \frac{4f \sqrt{gx+f} \sqrt{cx^2+a}}{15g}$$

$$+ \frac{4(-3ag^2+cf^2) \operatorname{EllipticE}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}, \sqrt{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{gx+f} \sqrt{1+\frac{cx^2}{a}}}{15g^2\sqrt{c} \sqrt{cx^2+a} \sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a}+f\sqrt{c}}}}$$

$$+ \frac{4f(a g^2+cf^2) \operatorname{EllipticF}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}, \sqrt{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{1+\frac{cx^2}{a}} \sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a}+f\sqrt{c}}}}{15g^2\sqrt{c} \sqrt{gx+f} \sqrt{cx^2+a}}$$

command

```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((cf^3 + 9afg^2) \sqrt{cg} \operatorname{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3}, \frac{3gx+f}{3g}\right) + 6(cf^2g - 3ag^3) \sqrt{cg} \operatorname{weierstrassP}\left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3}, \frac{3gx+f}{3g}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^2+a} \sqrt{gx+f}, x\right)$$

24.30 Problem number 629

$$\int \frac{(d+ex)^3 \sqrt{a+cx^2}}{\sqrt{f+gx}} dx$$

Optimal antiderivative

$$\frac{4e(7ae^2g^2 + c(42d^2g^2 - 111defg + 64e^2f^2))(gx+f)^{\frac{3}{2}}\sqrt{cx^2+a}}{315cg^4}$$

$$- \frac{4e^2(-3dg + 4ef)(gx+f)^{\frac{5}{2}}\sqrt{cx^2+a}}{63g^4}$$

$$- \frac{4(9ae^2g^2(-5dg + 2ef) + c(-35d^3g^3 + 168d^2efg^2 - 204de^2f^2g + 76e^3f^3))\sqrt{gx+f}\sqrt{cx^2+a}}{315cg^4}$$

$$+ \frac{2(ex+d)^3\sqrt{gx+f}\sqrt{cx^2+a}}{9g}$$

$$4(21a^2e^3g^4 - 3aceg^2(63d^2g^2 - 39defg + 10e^2f^2) - c^2f(-105d^3g^3 + 252d^2efg^2 - 216de^2f^2g + 64e^3f^3)) \text{EllipticF}$$

$$+ \frac{315c^{\frac{3}{2}}g^5\sqrt{cx^2+a}\sqrt{\frac{(gx+f)\sqrt{cx^2+a}}{g\sqrt{-a}+f}}}{315c^{\frac{3}{2}}g^5\sqrt{gx+f}\sqrt{cx^2+a}} \left(\frac{\sqrt{1-\frac{x\sqrt{cx^2+a}}{\sqrt{-a}}}}{2} \right)$$

$$4(ag^2 + cf^2)(9ae^2g^2(-5dg + 2ef) - c(-105d^3g^3 + 252d^2efg^2 - 216de^2f^2g + 64e^3f^3)) \text{EllipticF}$$

command

`integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2(105c^2d^3f^2g^3 + 315acd^3g^5 - 2(32c^2f^5 + 39acf^3g^2 - 6a^2fg^4)e^3 + 9(24c^2df^4g + 31acdf^2g^3 - 15a^2dg^5)e^2 - \dots\right)}{315c^{\frac{3}{2}}g^5\sqrt{gx+f}\sqrt{cx^2+a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{cx^2+a}}{\sqrt{gx+f}}, x\right)$$

24.31 Problem number 630

$$\int \frac{(d+ex)^2 \sqrt{a+cx^2}}{\sqrt{f+gx}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4e(-2dg+3ef)(gx+f)^{\frac{3}{2}}\sqrt{cx^2+a}}{35g^3} \\ & + \frac{4(5ae^2g^2+c(10d^2g^2-34defg+21e^2f^2))\sqrt{gx+f}\sqrt{cx^2+a}}{105cg^3} \\ & + \frac{2(ex+d)^2\sqrt{gx+f}\sqrt{cx^2+a}}{7g} \\ & + \frac{4(aeg^2(-42dg+13ef)+cf(35d^2g^2-56defg+24e^2f^2))\operatorname{EllipticE}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}, \sqrt{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}}\right)}{105g^4\sqrt{c}\sqrt{cx^2+a}\sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a}+f\sqrt{c}}}} \\ & + \frac{4(ag^2+cf^2)(5ae^2g^2-c(35d^2g^2-56defg+24e^2f^2))\operatorname{EllipticF}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}, \sqrt{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}}\right)}{105c^{\frac{3}{2}}g^4\sqrt{gx+f}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2(35c^2d^2f^2g^2 + 105acd^2g^4 + (24c^2f^4 + 31acf^2g^2 - 15a^2g^4)e^2 - 28(2c^2df^3g + 3acdfg^3)e) \sqrt{cg} \operatorname{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^2x^2+2dex+d^2)\sqrt{cx^2+a}}{\sqrt{gx+f}}, x\right)$$

24.32 Problem number 631

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}} dx$$

Optimal antiderivative

$$\frac{2(-3egx - 5dg + 4ef)\sqrt{gx+f}\sqrt{cx^2+a}}{15g^2} + \frac{4(3aeg^2 + cf(-5dg + 4ef)) \operatorname{EllipticE}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag + f\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{gx+f}\sqrt{1 + \frac{cx^2}{a}}}{15g^3\sqrt{c}\sqrt{cx^2+a}\sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a} + f\sqrt{c}}}} + \frac{4(-5dg + 4ef)(ag^2 + cf^2) \operatorname{EllipticF}\left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag + f\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{1 + \frac{cx^2}{a}}\sqrt{\frac{gx+f}{g\sqrt{-a}}}}{15g^3\sqrt{c}\sqrt{gx+f}\sqrt{cx^2+a}}$$

command

```
integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(2(5cdf^2g + 15adg^3 - 2(2cf^3 + 3afg^2)e)\sqrt{cg} \operatorname{weierstrassPInverse}\left(\frac{4(cf^2 - 3ag^2)}{3cg^2}, -\frac{8(cf^3 + 9afg^2)}{27cg^3}, \frac{3gx+f}{3g}\right) + 6\left(\frac{2d\sqrt{cx^2+a}}{g\sqrt{f+gx}} + \frac{2e}{g}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}(ex+d)}{\sqrt{gx+f}}, x\right)$$

24.33 Problem number 632

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{gx+f}\sqrt{cx^2+a}}{3g} + \frac{4f \operatorname{EllipticE}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{c}\sqrt{gx+f}\sqrt{1+\frac{cx^2}{a}}}{3g^2\sqrt{cx^2+a}\sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a}+f\sqrt{c}}}} + \frac{4(a g^2 + c f^2) \operatorname{EllipticF}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a}+f\sqrt{c}}}}{3g^2\sqrt{c}\sqrt{gx+f}\sqrt{cx^2+a}}$$

command

```
integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6\sqrt{cg}c f g \operatorname{weierstrassZeta}\left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3}\right), \operatorname{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3}, \frac{3gx+f}{3g}\right)\right)}{9cg}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}, x\right)$$

24.34 Problem number 636

$$\int \frac{(d+ex)^3 \sqrt{f+gx}}{\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e^2(11dg+ef)(gx+f)^{\frac{3}{2}}\sqrt{cx^2+a}}{35c^2g^2} \\ & - \frac{2e(25ae^2g^2+c(-90d^2g^2+12defg+7e^2f^2))\sqrt{gx+f}\sqrt{cx^2+a}}{105c^2g^2} \\ & + \frac{2e(ex+d)^2\sqrt{gx+f}\sqrt{cx^2+a}}{7c} \\ & + \frac{2(ae^2g^2(189dg+19ef)-c(105d^3g^3+105d^2efg^2-42de^2f^2g+8e^3f^3))\operatorname{EllipticE}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2},\sqrt{-\frac{cx^2+a}{-a}}\right)}{105c^{\frac{3}{2}}g^3\sqrt{cx^2+a}\sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a}+f\sqrt{c}}}} \\ & + \frac{2e(ag^2+cf^2)(25ae^2g^2-c(105d^2g^2-42defg+8e^2f^2))\operatorname{EllipticF}\left(\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2},\sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}}\right)}{105c^{\frac{5}{2}}g^3\sqrt{gx+f}\sqrt{cx^2+a}} \end{aligned}$$

command

```
integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((210c^2d^3fg^3 - (8c^2f^4 - 13acf^2g^2 - 75a^2g^4)e^3 + 42(c^2df^3g - 6acdfg^3)e^2 - 105(c^2d^2f^2g^2 + 3acd^2g^4)e)\sqrt{cg} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^3x^3+3de^2x^2+3d^2ex+d^3)\sqrt{gx+f}}{\sqrt{cx^2+a}},x\right)$$

24.35 Problem number 637

$$\int \frac{(d+ex)^2 \sqrt{f+gx}}{\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{2e(7dg+ef)\sqrt{gx+f}\sqrt{cx^2+a}}{15cg} + \frac{2e(ex+d)\sqrt{gx+f}\sqrt{cx^2+a}}{5c}$$

$$+ \frac{2(9ae^2g^2 + c(-15d^2g^2 - 10defg + 2e^2f^2)) \operatorname{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{g}}{15c^{\frac{3}{2}}g^2\sqrt{cx^2+a}}$$

$$+ \frac{4e(-5dg+ef)(ag^2+cf^2) \operatorname{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{gx+f}{g\sqrt{-a}}}}{15c^{\frac{3}{2}}g^2\sqrt{gx+f}\sqrt{cx^2+a}}$$

command

```
integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2(15cd^2fg^2 + (cf^3 - 6afg^2)e^2 - 5(cdf^2g + 3adg^3)e) \sqrt{cg} \operatorname{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3}, 3\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^2x^2 + 2dex + d^2)\sqrt{gx+f}}{\sqrt{cx^2+a}}, x\right)$$

24.36 Problem number 638

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{2e\sqrt{gx+f}\sqrt{cx^2+a}}{3c}$$

$$2(3dg+ef) \operatorname{EllipticE} \left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a}\sqrt{gx+f}\sqrt{1+\frac{cx^2}{a}}$$

$$\frac{3g\sqrt{c}\sqrt{cx^2+a}\sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a}+f\sqrt{c}}}}{2e(ag^2+cf^2) \operatorname{EllipticF} \left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a}+f\sqrt{c}}}}$$

$$+ \frac{3c^{\frac{3}{2}}g\sqrt{gx+f}\sqrt{cx^2+a}}$$

command

```
integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3\sqrt{cx^2+a}\sqrt{gx+f}cg^2e + (6cdfg - (cf^2 + 3ag^2)e)\sqrt{cg} \operatorname{weierstrassPInverse} \left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3}, 3 \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}, x \right)$$

24.37 Problem number 639

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ag}{-ag + f\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{gx+f} \sqrt{1 + \frac{cx^2}{a}}}{\sqrt{c} \sqrt{cx^2+a} \sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a} + f\sqrt{c}}}}$$

command

```
integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{cg} f \operatorname{weierstrassPInverse} \left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3}, \frac{3gx+f}{3g} \right) - 3 \sqrt{cg} g \operatorname{weierstrassZeta} \left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3} \right) \right)}{3cg}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}, x \right)$$

24.38 Problem number 645

$$\int \frac{(d+ex)^3}{\sqrt{f+gx} \sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{8e^2(-3dg + ef) \sqrt{gx + f} \sqrt{cx^2 + a}}{15cg^2} + \frac{2e^2(ex + d) \sqrt{gx + f} \sqrt{cx^2 + a}}{5cg}$$

$$+ \frac{2e(9ae^2g^2 - c(45d^2g^2 - 30defg + 8e^2f^2)) \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag + f\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{gx + f}}{15c^{\frac{3}{2}}g^3 \sqrt{cx^2 + a} \sqrt{\frac{(gx + f)\sqrt{c}}{g\sqrt{-a} + f\sqrt{c}}}}$$

$$+ \frac{2(ae^2g^2(-15dg + 7ef) - c(-15d^3g^3 + 45d^2efg^2 - 30de^2f^2g + 8e^3f^3)) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}} \sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag + f\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{gx + f}}{15c^{\frac{3}{2}}g^3 \sqrt{gx + f} \sqrt{cx^2 + a}}$$

command

`integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((45cd^3g^3 - 45cd^2fg^2e - (8cf^3 - 3afg^2)e^3 + 15(2cdf^2g - 3adg^3)e^2) \sqrt{cg} \operatorname{weierstrassPInverse} \left(\frac{4(cf^2 - 3ag^2)}{3cg^2}, - \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3) \sqrt{cx^2 + a} \sqrt{gx + f}}{cgx^3 + cfx^2 + agx + af}, x \right)$$

24.39 Problem number 646

$$\int \frac{(d + ex)^2}{\sqrt{f + gx} \sqrt{a + cx^2}} dx$$

Optimal antiderivative

$$\frac{2e^2 \sqrt{gx+f} \sqrt{cx^2+a}}{3cg}$$

$$+ \frac{4e(-3dg+ef) \operatorname{EllipticE} \left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{gx+f} \sqrt{1+\frac{cx^2}{a}}}{3g^2 \sqrt{c} \sqrt{cx^2+a} \sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a}+f\sqrt{c}}}} + \frac{2((-ae^2+3cd^2)g^2+2cef(-3dg+ef)) \operatorname{EllipticF} \left(\frac{\sqrt{1-\frac{x\sqrt{c}}{\sqrt{-a}}}\sqrt{2}}{2}, \sqrt{-\frac{2ag}{-ag+f\sqrt{-a}\sqrt{c}}}\right) \sqrt{-a} \sqrt{1+\frac{cx^2}{a}}}{3c^{\frac{3}{2}}g^2 \sqrt{gx+f} \sqrt{cx^2+a}}$$

command

`integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cx^2+a} \sqrt{gx+f} cg^2 e^2 + (9cd^2g^2 - 6cdfge + (2cf^2 - 3ag^2)e^2) \sqrt{cg} \operatorname{weierstrassPInverse} \left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8}{3cg^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(e^2x^2 + 2dex + d^2) \sqrt{cx^2+a} \sqrt{gx+f}}{cgx^3 + cfx^2 + agx + af}, x \right)$$

24.40 Problem number 647

$$\int \frac{d+ex}{\sqrt{f+gx} \sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & 2e \operatorname{EllipticE} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ag}{-ag + f\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{gx+f} \sqrt{1 + \frac{cx^2}{a}} \\
 & \frac{g\sqrt{c} \sqrt{cx^2+a} \sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a} + f\sqrt{c}}}}{\sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a} + f\sqrt{c}}}} \\
 & 2(-dg + ef) \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ag}{-ag + f\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a} + f\sqrt{c}}} \\
 & + \frac{g\sqrt{c} \sqrt{gx+f} \sqrt{cx^2+a}}{\sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a} + f\sqrt{c}}}}
 \end{aligned}$$

command

`integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \sqrt{cg} \operatorname{geweierstrassZeta} \left(\frac{4(cf^2 - 3ag^2)}{3cg^2}, -\frac{8(cf^3 + 9afg^2)}{27cg^3} \right), \operatorname{weierstrassPInverse} \left(\frac{4(cf^2 - 3ag^2)}{3cg^2}, -\frac{8(cf^3 + 9afg^2)}{27cg^3}, \frac{3gx+f}{3g} \right) \right)}{3cg^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2+a} (ex+d) \sqrt{gx+f}}{cgx^3 + cfx^2 + agx + af}, x \right)$$

24.41 Problem number 648

$$\int \frac{1}{\sqrt{f+gx} \sqrt{a+cx^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF} \left(\frac{\sqrt{1 - \frac{x\sqrt{c}}{\sqrt{-a}}}}{2} \sqrt{2}, \sqrt{-\frac{2ag}{-ag + f\sqrt{-a}\sqrt{c}}} \right) \sqrt{-a} \sqrt{1 + \frac{cx^2}{a}} \sqrt{\frac{(gx+f)\sqrt{c}}{g\sqrt{-a} + f\sqrt{c}}}}{\sqrt{c} \sqrt{gx+f} \sqrt{cx^2+a}}$$

command

```
integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{cg} \operatorname{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3}, \frac{3gx+f}{3g}\right)}{cg}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{gx+f}}{cgx^3+cfx^2+agx+af}, x\right)$$

24.42 Problem number 656

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticF}\left(x, \sqrt{2}\right)\sqrt{-2x^2+1}\sqrt{-x^2+1}}{\sqrt{-1+x}\sqrt{1+x}\sqrt{2x^2-1}}$$

command

```
integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\operatorname{ellipticF}(x, 2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2x^2-1}\sqrt{x+1}\sqrt{x-1}}{2x^4-3x^2+1}, x\right)$$

24.43 Problem number 796

$$\int \frac{1}{\sqrt{1-dx} \sqrt{1+dx} (a+bx+cx^2)^2} dx$$

Optimal antiderivative

$$\frac{(b(b^2d^2 - c(3ad^2 + c)) - c(2acd^2 - b^2d^2 + 2c^2)x) \sqrt{-d^2x^2 + 1}}{(-4ac + b^2) (b^2d^2 - (ad^2 + c)^2) (cx^2 + bx + a)}$$

$$- \frac{c \operatorname{arctanh} \left(\frac{(2c+d^2x(b - \sqrt{-4ac + b^2})) \sqrt{2}}{2\sqrt{-d^2x^2 + 1} \sqrt{2c^2 + 2acd^2 - b^2d^2(b - \sqrt{-4ac + b^2})}} \right) (4c^3 + 12ac^2d^2 - ab^2d^4(b + \sqrt{-4ac + b^2}))}{2(-4ac + b^2)^{\frac{3}{2}} (b^2d^2 - (ad^2 + c)^2) \sqrt{2c^2 + 2acd^2 - b^2d^2(b - \sqrt{-4ac + b^2})}}$$

$$+ \frac{c \operatorname{arctanh} \left(\frac{(2c+d^2x(b + \sqrt{-4ac + b^2})) \sqrt{2}}{2\sqrt{-d^2x^2 + 1} \sqrt{2c^2 + 2acd^2 - b^2d^2(b + \sqrt{-4ac + b^2})}} \right) (4c^3 + 12ac^2d^2 - 2ab^2d^4 - 4cd^2(-2a^2d^2 + b^2))}{2(-4ac + b^2)^{\frac{3}{2}} (b^2d^2 - (ad^2 + c)^2) \sqrt{2c^2 + 2acd^2 - b^2d^2(b + \sqrt{-4ac + b^2})}}$$

command

```
integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

24.44 Problem number 800

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)} dx$$

Optimal antiderivative

$$\frac{d^2(b - (ad^2 + c)x)}{(b^2d^2 - (ad^2 + c)^2)\sqrt{-d^2x^2 + 1}}$$

$$+ \frac{c \operatorname{arctanh}\left(\frac{(2c+d^2x(b-\sqrt{-4ac+b^2}))\sqrt{2}}{2\sqrt{-d^2x^2+1}\sqrt{2c^2+2acd^2-bd^2(b-\sqrt{-4ac+b^2})}}\right)(2c^2+2acd^2-bd^2(b+\sqrt{-4ac+b^2}))}{2(b^2d^2-(ad^2+c)^2)\sqrt{-4ac+b^2}\sqrt{2c^2+2acd^2-bd^2(b-\sqrt{-4ac+b^2})}}$$

$$- \frac{c \operatorname{arctanh}\left(\frac{(2c+d^2x(b+\sqrt{-4ac+b^2}))\sqrt{2}}{2\sqrt{-d^2x^2+1}\sqrt{2c^2+2acd^2-bd^2(b+\sqrt{-4ac+b^2})}}\right)(2c^2+2acd^2-bd^2(b-\sqrt{-4ac+b^2}))}{2(b^2d^2-(ad^2+c)^2)\sqrt{-4ac+b^2}\sqrt{2c^2+2acd^2-bd^2(b+\sqrt{-4ac+b^2})}}$$

command

```
integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

24.45 Problem number 886

$$\int (d + ex)^3 \sqrt{f + gx} \sqrt{a + bx + cx^2} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{cx^2 + bx + a}\sqrt{gx + f}, x\right)$$

24.46 Problem number 887

$$\int (d + ex)^2 \sqrt{f + gx} \sqrt{a + bx + cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(3b^2e^2g^2 + ceg(-7aeg - 9bdg + 4bef) + c^2(21d^2g^2 - 24defg + 8e^2f^2))(gx + f)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}}{315c^2g^3} \\ & + \frac{2e(beg - 3cdg + cef)(gx + f)^{\frac{5}{2}} \sqrt{cx^2 + bx + a}}{63cg^3} \\ & + \frac{2(8b^3e^3g^3 + 3bce^2g^2(-9aeg - 8bdg + bef) + c^3(-35d^3g^3 + 63d^2efg^2 - 57de^2f^2g + 19e^3f^3) - 3c^2e^2g^2(2ae(-10d^2g^2 - 24defg + 8e^2f^2) - 2aefg^2))}{315c^3eg^3} \\ & + \frac{2(ex + d)^3 \sqrt{gx + f} \sqrt{cx^2 + bx + a}}{9e} \end{aligned}$$

$$2(8b^4e^2g^4 - 4b^2ce^3g^3(9aeg + 6bdg + bef) + c^4f^2(21d^2g^2 - 24defg + 8e^2f^2) + 3c^2g^2(7a^2e^2g^2 + abeg(29dg + 5efg^2) - 2aefg^2))$$

$$2(a^2g^2 - bfg + cf^2)(8b^3e^2g^3 + 3bce^2g^2(-9aeg - 8bdg + bef) - 2c^3f(21d^2g^2 - 24defg + 8e^2f^2) - 3c^2g^2(2ae(-10d^2g^2 - 24defg + 8e^2f^2) - 2aefg^2))$$

command

```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((42c^5d^2f^3g^2 - 63bc^4d^2f^2g^3 - 63(b^2c^3 - 6ac^4)d^2fg^4 + 21(2b^3c^2 - 9abc^3)d^2g^5 + (16c^5f^5 - 16bc^4f^4g - 5(b^2c^3 - 6ac^4)df^3g^2) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(e^2x^2 + 2dex + d^2\right)\sqrt{cx^2 + bx + a} \sqrt{gx + f}, x\right)$$

24.47 Problem number 888

$$\int (d + ex) \sqrt{f + gx} \sqrt{a + bx + cx^2} dx$$

Optimal antiderivative

$$\frac{2e(cx^2 + bx + a)^{\frac{3}{2}} \sqrt{gx + f}}{7c} - \frac{2(4b^2eg^2 + c^2f(-7dg + 4ef) - cg(-5aeg + 7bdg + 2bef) - 3cg(-4beg + 7cdg + cef)x) \sqrt{gx + f} \sqrt{cx^2 + bx + a}}{105c^2g^2}$$

$$+ \frac{((-4beg + 7cdg + cef)(8c^2f^2 - 2b^2g^2 - 3cg(-2ag + bf)) - 5cg(-bg + 2cf)(7cdf - e(ag + 3bf))) \text{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac}}{\sqrt{-4ac + b^2}}} \right)}{105c^3g^3 \sqrt{cx^2 + bx + a}}$$

$$+ \frac{2(a g^2 - bfg + c f^2) (4b^2e g^2 - 2c^2 f(-7dg + 4ef) + cg(-10aeg - 7bdg + bef)) \text{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac}}{\sqrt{-4ac + b^2}}} \right)}{105c^3g^3 \sqrt{gx}}$$

command

```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((14c^4df^3g - 21bc^3df^2g^2 - 21(b^2c^2 - 6ac^3)dfg^3 + 7(2b^3c - 9abc^2)dg^4 - (8c^4f^4 - 9bc^3f^3g - 2(2b^2c^2 - 11ac^3)df^2g^2) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{cx^2 + bx + a} (ex + d) \sqrt{gx + f}, x \right)$$

24.48 Problem number 889

$$\int \sqrt{f+gx} \sqrt{a+bx+cx^2} dx$$

Optimal antiderivative

$$\frac{2(gx+f)^{\frac{3}{2}} \sqrt{cx^2+bx+a}}{5g} - \frac{2(-bg+2cf) \sqrt{gx+f} \sqrt{cx^2+bx+a}}{15cg}$$

$$2(c^2f^2 + b^2g^2 - cg(3ag + bf)) \operatorname{EllipticE} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}, \sqrt{-\frac{2g\sqrt{-4ac+b^2}}{2cf-g(b+\sqrt{-4ac+b^2})}} \right)$$

$$- \frac{15c^2g^2 \sqrt{cx^2+bx+a} \sqrt{\frac{c(gx+f)}{2cf-g(b+\sqrt{-4ac+b^2})}}}{2(-bg+2cf)(ag^2 - bfg + cf^2) \operatorname{EllipticF} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}, \sqrt{-\frac{2g\sqrt{-4ac+b^2}}{2cf-g(b+\sqrt{-4ac+b^2})}} \right)}$$

$$+ \frac{15c^2g^2 \sqrt{gx+f} \sqrt{cx^2+bx+a}}{15c^2g^2 \sqrt{gx+f} \sqrt{cx^2+bx+a}}$$

command

```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2c^3f^3 - 3bc^2f^2g - 3(b^2c - 6ac^2)f^2 + (2b^3 - 9abc)g^3) \sqrt{cg} \operatorname{weierstrassPInverse} \left(\frac{4(c^2f^2 - bcfg + (b^2 - 3ac)g^2)}{3c^2g^2}, - \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{cx^2+bx+a} \sqrt{gx+f}, x \right)$$

24.49 Problem number 893

$$\int \frac{(d+ex)^3 \sqrt{a+bx+cx^2}}{\sqrt{f+gx}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e(6b^2e^2g^2 + ceg(-14aeg - 27bdg + 17bef) - 2c^2(42d^2g^2 - 111defg + 64e^2f^2))(gx+f)^{\frac{3}{2}}\sqrt{cx^2+bx+a}}{315c^2g^4} \\ & - \frac{2e^2(-beg - 6cdg + 8cef)(gx+f)^{\frac{5}{2}}\sqrt{cx^2+bx+a}}{63cg^4} \\ & + \frac{2(8b^3e^3g^3 + 3bce^2g^2(-9aeg - 12bdg + 5bef) - c^3(-70d^3g^3 + 336d^2efg^2 - 408de^2f^2g + 152e^3f^3) - 3c^2eg(6aeg - 2c^2d^2))}{315c^3g^4} \\ & + \frac{2(ex+d)^3\sqrt{gx+f}\sqrt{cx^2+bx+a}}{9g} \end{aligned}$$

$$(16b^4e^3g^4 + 8b^2ce^2g^3(-9aeg - 9bdg + 2bef) - 2c^4f(-105d^3g^3 + 252d^2efg^2 - 216de^2f^2g + 64e^3f^3) + 3c^2eg^2($$

$$2(a^2g^2 - bfg + cf^2)(8b^3e^3g^3 + 3bce^2g^2(-9aeg - 12bdg + 5bef) + 2c^3(-105d^3g^3 + 252d^2efg^2 - 216de^2f^2g +$$

command

```
integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((210c^5d^3f^2g^3 - 210bc^4d^3fg^4 - 105(b^2c^3 - 6ac^4)d^3g^5 - (128c^5f^5 - 104bc^4f^4g - (25b^2c^3 - 156ac^4)f^3g^2 - 5$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{cx^2+bx+a}}{\sqrt{gx+f}}, x\right)$$

24.50 Problem number 894

$$\int \frac{(d+ex)^2 \sqrt{a+bx+cx^2}}{\sqrt{f+gx}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e(-beg - 4cdg + 6cef)(gx+f)^{\frac{3}{2}} \sqrt{cx^2+bx+a}}{35c^3g^3} \\ & - \frac{4(2b^2e^2g^2 + ceg(-5aeg - 7bdg + 4bef) - c^2(10d^2g^2 - 34defg + 21e^2f^2)) \sqrt{gx+f} \sqrt{cx^2+bx+a}}{105c^2g^3} \\ & + \frac{2(ex+d)^2 \sqrt{gx+f} \sqrt{cx^2+bx+a}}{7g} \\ & (8b^3e^2g^3 + bce g^2(-29aeg - 28bdg + 9bef) - 2c^3f(35d^2g^2 - 56defg + 24e^2f^2) - c^2g(2aeg(-42dg + 13ef) - b(3 \\ & + \frac{4(a g^2 - bfg + c f^2) (2b^2e^2g^2 + ceg(-5aeg - 7bdg + 4bef) + c^2(35d^2g^2 - 56defg + 24e^2f^2)) \text{EllipticF} \left(\frac{\sqrt{\frac{b+2}{\dots}}}{\dots}}{105c^3g} \right) \end{aligned}$$

command

```
integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((70c^4d^2f^2g^2 - 70bc^3d^2fg^3 - 35(b^2c^2 - 6ac^3)d^2g^4 + (48c^4f^4 - 40bc^3f^3g - 2(5b^2c^2 - 31ac^3)f^2g^2 - (5b^3c - \dots) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(e^2x^2 + 2dex + d^2) \sqrt{cx^2 + bx + a}}{\sqrt{gx + f}}, x \right)$$

24.51 Problem number 895

$$\int \frac{(d + ex) \sqrt{a + bx + cx^2}}{\sqrt{f + gx}} dx$$

Optimal antiderivative

$$\frac{2(-3cegx - beg - 5cdg + 4cef) \sqrt{gx + f} \sqrt{cx^2 + bx + a}}{15cg^2}$$

$$(2b^2eg^2 - 2c^2f(-5dg + 4ef) + cg(-6aeg - 5bdg + 3bef)) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{c(gx + f)}{2cf - g(b + \sqrt{-4ac + b^2})}} \right)$$

$$15c^2g^3 \sqrt{cx^2 + bx + a} \sqrt{\frac{c(gx + f)}{2cf - g(b + \sqrt{-4ac + b^2})}}$$

$$2(beg - 10cdg + 8cef) (ag^2 - bfg + cf^2) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2g\sqrt{-4ac + b^2}}{2cf - g(b + \sqrt{-4ac + b^2})}} \right)$$

$$15c^2g^3 \sqrt{gx + f} \sqrt{cx^2 + bx + a}$$

command

```
integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((10c^3df^2g - 10bc^2dfg^2 - 5(b^2c - 6ac^2)dg^3 - (8c^3f^3 - 7bc^2f^2g - 2(b^2c - 6ac^2)fg^2 - (2b^3 - 9abc)g^3)e \right) \sqrt{cx^2 + bx + a} \sqrt{gx + f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a} (ex + d)}{\sqrt{gx + f}}, x \right)$$

24.52 Problem number 896

$$\int \frac{\sqrt{a + bx + cx^2}}{\sqrt{f + gx}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{gx + f} \sqrt{cx^2 + bx + a}}{3g}$$

$$(-bg + 2cf) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2g\sqrt{-4ac + b^2}}{2cf - g(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}$$

$$+ \frac{3cg^2 \sqrt{cx^2 + bx + a} \sqrt{\frac{c(gx + f)}{2cf - g(b + \sqrt{-4ac + b^2})}}{4(a g^2 - bfg + c f^2) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2g\sqrt{-4ac + b^2}}{2cf - g(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}}$$

$$+ \frac{3cg^2 \sqrt{gx + f} \sqrt{cx^2 + bx + a}}{3cg^2 \sqrt{gx + f} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cx^2 + bx + a} \sqrt{gx + f} c^2 g^2 + (2c^2 f^2 - 2bcfg - (b^2 - 6ac)g^2) \sqrt{cg} \operatorname{weierstrassPInverse} \left(\frac{4(c^2 f^2 - bcfg + (b^2 - 3ac)g^2)}{3c^2 g^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2 + bx + a}}{\sqrt{gx + f}}, x \right)$$

24.53 Problem number 900

$$\int \frac{(d+ex)^3 \sqrt{f+gx}}{\sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e^2(-6beg + 11cdg + cef)(gx+f)^{\frac{3}{2}} \sqrt{cx^2+bx+a}}{35c^2g^2} \\ & + \frac{2e(24b^2e^2g^2 + ceg(-25aeg - 84bdg + 13bef) - c^2(-90d^2g^2 + 12defg + 7e^2f^2)) \sqrt{gx+f} \sqrt{cx^2+bx+a}}{105c^3g^2} \\ & + \frac{2e(ex+d)^2 \sqrt{gx+f} \sqrt{cx^2+bx+a}}{7c} \\ & (48b^3e^3g^3 - 8bc^2e^2g^2(13aeg + 21bdg + 2bef) - c^3(105d^3g^3 + 105d^2efg^2 - 42de^2f^2g + 8e^3f^3) + c^2eg(aeg(189dg \end{aligned}$$

$$2e(ag^2 - bfg + cf^2)(24b^2e^2g^2 + ceg(-25aeg - 84bdg + 13bef) + c^2(105d^2g^2 - 42defg + 8e^2f^2)) \text{EllipticF} \left(\begin{array}{l} \sqrt{cx^2+bx+a} \\ \sqrt{gx+f} \end{array}, x \right)$$

command

```
integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((210c^4d^3fg^3 - 105bc^3d^3g^4 - (8c^4f^4 + 5bc^3f^3g + (10b^2c^2 - 13ac^3)f^2g^2 + (40b^3c - 113abc^2)fg^3 - (48b^4 - 105b^2c^2)g^4) \sqrt{cx^2+bx+a} \sqrt{gx+f} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3) \sqrt{gx+f}}{\sqrt{cx^2+bx+a}}, x \right)$$

24.54 Problem number 901

$$\int \frac{(d+ex)^2 \sqrt{f+gx}}{\sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{2e(-4beg + 7cdg + cef) \sqrt{gx+f} \sqrt{cx^2+bx+a}}{15c^2g} + \frac{2e(ex+d) \sqrt{gx+f} \sqrt{cx^2+bx+a}}{5c}$$

$$+ \frac{(8b^2e^2g^2 - ceg(9aeg + 20bdg + 3bef) - c^2(-15d^2g^2 - 10defg + 2e^2f^2)) \text{EllipticE} \left(\sqrt{\frac{b+2cx + \sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \frac{b+2cx + \sqrt{-4ac+b^2}}{2} \right)}{15c^3g^2 \sqrt{cx^2+bx+a} \sqrt{\frac{c}{2cf-g(b+\sqrt{-4ac+b^2})}}}$$

$$+ \frac{4e(2beg - 5cdg + cef) (ag^2 - bfg + cf^2) \text{EllipticF} \left(\sqrt{\frac{b+2cx + \sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}, \sqrt{2}, \sqrt{-\frac{2g\sqrt{-4ac+b^2}}{2cf-g(b+\sqrt{-4ac+b^2})}} \right)}{15c^3g^2 \sqrt{gx+f} \sqrt{cx^2+bx+a}}$$

command

`integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((30c^3d^2fg^2 - 15bc^2d^2g^3 + (2c^3f^3 + 2bc^2f^2g + (7b^2c - 12ac^2)fg^2 - (8b^3 - 21abc)g^3)e^2 - 10(c^3df^2g + 2bc^2d^2fg) \right) \sqrt{gx+f} \sqrt{cx^2+bx+a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(e^2x^2 + 2dex + d^2) \sqrt{gx+f}}{\sqrt{cx^2+bx+a}}, x \right)$$

24.55 Problem number 902

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{2e\sqrt{gx+f}\sqrt{cx^2+bx+a}}{3c}$$

$$\begin{aligned}
 & (-2beg + 3cdg + cef) \operatorname{EllipticE} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}}{2}, \sqrt{\frac{2g\sqrt{-4ac+b^2}}{2cf-g(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-} \\
 + & \frac{3c^2g\sqrt{cx^2+bx+a}\sqrt{\frac{c(gx+f)}{2cf-g(b+\sqrt{-4ac+b^2})}}}{\sqrt{2}\sqrt{-}} \\
 & 2e(ag^2 - bfg + cf^2) \operatorname{EllipticF} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2}}{2}, \sqrt{\frac{2g\sqrt{-4ac+b^2}}{2cf-g(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-} \\
 - & \frac{3c^2g\sqrt{gx+f}\sqrt{cx^2+bx+a}}{\sqrt{2}\sqrt{-}}
 \end{aligned}$$

command

```
integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cx^2+bx+a} \sqrt{gx+f} c^2 g^2 e + (6c^2 dfg - 3bcdg^2 - (c^2 f^2 + 2bcfg - (2b^2 - 3ac)g^2) e) \sqrt{cg} \operatorname{weierstrassPI} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}, x \right)$$

24.56 Problem number 903

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\text{EllipticE} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2} \sqrt{2}, \sqrt{-\frac{2g\sqrt{-4ac+b^2}}{2cf-g(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-4ac+b^2} \sqrt{gx+f} \sqrt{-}$$

$$c\sqrt{cx^2+bx+a} \sqrt{\frac{c(gx+f)}{2cf-g(b+\sqrt{-4ac+b^2})}}$$

command

```
integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cg} \text{cgweierstrassZeta} \left(\frac{4(c^2 f^2 - bcfg + (b^2 - 3ac)g^2)}{3c^2 g^2}, -\frac{4(2c^3 f^3 - 3bc^2 f^2 g - 3(b^2 c - 6ac^2)fg^2 + (2b^3 - 9abc)g^3)}{27c^3 g^3} \right), \text{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}, x \right)$$

24.57 Problem number 909

$$\int \frac{(d+ex)^3}{\sqrt{f+gx} \sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{8e^2(beg - 3cdg + cef) \sqrt{gx + f} \sqrt{cx^2 + bx + a}}{15c^2g^2} + \frac{2e^2(ex + d) \sqrt{gx + f} \sqrt{cx^2 + bx + a}}{5cg}$$

$$+ \frac{e(8b^2e^2g^2 + ceg(-9aeg - 30bdg + 7bef) + c^2(45d^2g^2 - 30defg + 8e^2f^2)) \operatorname{EllipticE}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \frac{c}{2}\right)}{15c^3g^3 \sqrt{cx^2 + bx + a} \sqrt{\frac{c}{2cf - g(b + 2cx + \sqrt{-4ac + b^2})}}}$$

$$+ \frac{2(4be^3g^2(-ag + bf) + c^2(-15d^3g^3 + 45d^2efg^2 - 30de^2f^2g + 8e^3f^3) - ce^2g(ag(-15dg + 7ef) - 3bf(-5dg + e^2f)))}{15c^3g^3 \sqrt{cx^2 + bx + a} \sqrt{\frac{c}{2cf - g(b + 2cx + \sqrt{-4ac + b^2})}}}$$

command

`integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((45c^3d^3g^3 - (8c^3f^3 + 3bc^2f^2g + 3(b^2c - ac^2)fg^2 + (8b^3 - 21abc)g^3)e^3 + 15(2c^3df^2g + bc^2dfg^2 + (2b^2c - 3bc^2)fg^2) \right) \sqrt{cx^2 + bx + a} \sqrt{gx + f} / (cgx^3 + (cf + bg)x^2 + af + (bf + ag)x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3) \sqrt{cx^2 + bx + a} \sqrt{gx + f}}{cgx^3 + (cf + bg)x^2 + af + (bf + ag)x}, x\right)$$

24.58 Problem number 910

$$\int \frac{(d + ex)^2}{\sqrt{f + gx} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2e^2 \sqrt{gx+f} \sqrt{cx^2+bx+a}}{3cg}$$

$$2e(beg - 3cdg + cef) \text{EllipticE} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2g\sqrt{-4ac+b^2}}{2cf-g(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-4ac+b^2}$$

$$\frac{3c^2g^2 \sqrt{cx^2+bx+a} \sqrt{\frac{c(gx+f)}{2cf-g(b+\sqrt{-4ac+b^2})}}}{3c^2g^2 \sqrt{cx^2+bx+a}}$$

$$2(e^2g(-ag+bf) + c(3d^2g^2 - 6defg + 2e^2f^2)) \text{EllipticF} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2g\sqrt{-4ac+b^2}}{2cf-g(b+\sqrt{-4ac+b^2})}} \right)$$

$$+ \frac{3c^2g^2 \sqrt{gx+f} \sqrt{cx^2+bx+a}}{3c^2g^2 \sqrt{gx+f} \sqrt{cx^2+bx+a}}$$

command

`integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cx^2+bx+a} \sqrt{gx+f} c^2g^2e^2 + (9c^2d^2g^2 + (2c^2f^2 + bcfg + (2b^2 - 3ac)g^2)e^2 - 6(c^2dfg + bcdg^2)e) \sqrt{cg} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(e^2x^2 + 2dex + d^2) \sqrt{cx^2+bx+a} \sqrt{gx+f}}{cgx^3 + (cf+bg)x^2 + af + (bf+ag)x}, x \right)$$

24.59 Problem number 911

$$\int \frac{d+ex}{\sqrt{f+gx} \sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{e \operatorname{EllipticE} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2} \sqrt{2}, \sqrt{-\frac{2g\sqrt{-4ac+b^2}}{2cf-g(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-4ac+b^2} \sqrt{gx+f} \sqrt{cx^2+bx+a}}{2(-dg+ef) \operatorname{EllipticF} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2} \sqrt{2}, \sqrt{-\frac{2g\sqrt{-4ac+b^2}}{2cf-g(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-4ac+b^2} \sqrt{cx^2+bx+a}}$$

command

`integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cg} \operatorname{cgewierstrassZeta} \left(\frac{4(c^2f^2-bcfg+(b^2-3ac)g^2)}{3c^2g^2}, -\frac{4(2c^3f^3-3bc^2f^2g-3(b^2c-6ac^2)fg^2+(2b^3-9abc)g^3)}{27c^3g^3} \right), \operatorname{weierstrass} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{cx^2+bx+a} (ex+d) \sqrt{gx+f}}{cgx^3+(cf+bg)x^2+af+(bf+ag)x}, x \right)$$

24.60 Problem number 912

$$\int \frac{1}{\sqrt{f+gx} \sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF} \left(\frac{\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2} \sqrt{2}, \sqrt{-\frac{2g\sqrt{-4ac+b^2}}{2cf-g(b+\sqrt{-4ac+b^2})}} \right) \sqrt{2} \sqrt{-4ac+b^2} \sqrt{-\frac{c(cx^2+bx+a)}{-4ac}}}{c \sqrt{gx+f} \sqrt{cx^2+bx+a}}$$

command

```
integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\sqrt{cg} \operatorname{weierstrassPInverse}\left(\frac{4(c^2f^2-bcfg+(b^2-3ac)g^2)}{3c^2g^2}, -\frac{4(2c^3f^3-3bc^2f^2g-3(b^2c-6ac^2)fg^2+(2b^3-9abc)g^3)}{27c^3g^3}, \frac{3cgx+cf+bg}{3cg}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{cgx^3+(cf+bg)x^2+af+(bf+ag)x}, x\right)$$

25 Test file number 37

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/37_1.2.1.6-g+h_x-^m-a+b_x+c_x^2-^p-d+e_x+f_x^2-^q

25.1 Problem number 21

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\left(4cd-e\left(b+\sqrt{-4ac+b^2}\right)+2x\left(ce-f\left(b+\sqrt{-4ac+b^2}\right)\right)\right)\sqrt{2}}{4\sqrt{fx^2+ex+d}\sqrt{2c^2d-bce+b^2f-2acf-(-bf+ce)\sqrt{-4ac+b^2}}}\right)\left(2Ac-B\left(b+\sqrt{-4ac+b^2}\right)\right)}{2\sqrt{-4ac+b^2}\sqrt{2c^2d-bce+b^2f-2acf-(-bf+ce)\sqrt{-4ac+b^2}}}$$

$$+\frac{\operatorname{arctanh}\left(\frac{\left(4cd+2x\left(ce-f\left(b-\sqrt{-4ac+b^2}\right)\right)\right)-e\left(b-\sqrt{-4ac+b^2}\right)\right)\sqrt{2}}{4\sqrt{fx^2+ex+d}\sqrt{2c^2d-bce+b^2f-2acf+(-bf+ce)\sqrt{-4ac+b^2}}}\right)\left(bB-2Ac-B\sqrt{-4ac+b^2}\right)}{2\sqrt{-4ac+b^2}\sqrt{2c^2d-bce+b^2f-2acf+(-bf+ce)\sqrt{-4ac+b^2}}}$$

command

```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.2 Problem number 54

$$\int \frac{\sqrt{a + cx^2}}{d + ex + fx^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{c}}{\sqrt{cx^2 + a}}\right)\sqrt{c}}{f} - \frac{\operatorname{arctanh}\left(\frac{(2af - cx(e - \sqrt{-4df + e^2}))\sqrt{2}}{2\sqrt{cx^2 + a}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{-4df + e^2})}}\right)\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{-4df + e^2})}\sqrt{2}}{2f\sqrt{-4df + e^2}} + \frac{\operatorname{arctanh}\left(\frac{(2af - cx(e + \sqrt{-4df + e^2}))\sqrt{2}}{2\sqrt{cx^2 + a}\sqrt{2af^2 + c(e^2 - 2df + e\sqrt{-4df + e^2})}}\right)\sqrt{2af^2 + c(e^2 - 2df + e\sqrt{-4df + e^2})}\sqrt{2}}{2f\sqrt{-4df + e^2}}$$

command

```
integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.3 Problem number 56

$$\int \frac{\sqrt{a + cx^2}}{x^2(d + ex + fx^2)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{e \operatorname{arctanh}\left(\frac{\sqrt{cx^2+a}}{\sqrt{a}}\right) \sqrt{a}}{d^2} - \frac{\sqrt{cx^2+a}}{dx} \\
& - \frac{f \operatorname{arctanh}\left(\frac{(2af-cx(e-\sqrt{-4df+e^2}))\sqrt{2}}{2\sqrt{cx^2+a}\sqrt{2af^2+c(e^2-2df-e\sqrt{-4df+e^2})}}\right) (2cd^2+a(e^2-2df+e\sqrt{-4df+e^2}))\sqrt{2}}{2d^2\sqrt{-4df+e^2}\sqrt{2af^2+c(e^2-2df-e\sqrt{-4df+e^2})}} \\
& + \frac{f \operatorname{arctanh}\left(\frac{(2af-cx(e+\sqrt{-4df+e^2}))\sqrt{2}}{2\sqrt{cx^2+a}\sqrt{2af^2+c(e^2-2df+e\sqrt{-4df+e^2})}}\right) (2cd^2+a(e^2-2df-e\sqrt{-4df+e^2}))\sqrt{2}}{2d^2\sqrt{-4df+e^2}\sqrt{2af^2+c(e^2-2df+e\sqrt{-4df+e^2})}}
\end{aligned}$$

command

```
integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.4 Problem number 68

$$\int \frac{1}{x\sqrt{a+cx^2}(d+ex+fx^2)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{\operatorname{arctanh}\left(\frac{\sqrt{cx^2+a}}{\sqrt{a}}\right)}{d\sqrt{a}} \\
& + \frac{f \operatorname{arctanh}\left(\frac{(2af-cx(e-\sqrt{-4df+e^2}))\sqrt{2}}{2\sqrt{cx^2+a}\sqrt{2af^2+c(e^2-2df-e\sqrt{-4df+e^2})}}\right)(e+\sqrt{-4df+e^2})\sqrt{2}}{2d\sqrt{-4df+e^2}\sqrt{2af^2+c(e^2-2df-e\sqrt{-4df+e^2})}} \\
& - \frac{f \operatorname{arctanh}\left(\frac{(2af-cx(e+\sqrt{-4df+e^2}))\sqrt{2}}{2\sqrt{cx^2+a}\sqrt{2af^2+c(e^2-2df+e\sqrt{-4df+e^2})}}\right)(e-\sqrt{-4df+e^2})\sqrt{2}}{2d\sqrt{-4df+e^2}\sqrt{2af^2+c(e^2-2df+e\sqrt{-4df+e^2})}}
\end{aligned}$$

command

```
integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.5 Problem number 71

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{1}{cf\sqrt{cx^2+a}} - \frac{ex}{af^2\sqrt{cx^2+a}} + \frac{af(cd^2+a(-df+e^2))+ce(cd^2+a(-2df+e^2))x}{af^2(ac e^2+(-af+cd)^2)\sqrt{cx^2+a}} \\
 & -\frac{\operatorname{arctanh}\left(\frac{(2af-cx(e-\sqrt{-4df+e^2}))\sqrt{2}}{2\sqrt{cx^2+a}\sqrt{2af^2+c(e^2-2df-e\sqrt{-4df+e^2})}}\right)(2adef-(cd^2+a(-df+e^2))(e-\sqrt{-4df+e^2}))}{2(ac e^2+(-af+cd)^2)\sqrt{-4df+e^2}\sqrt{2af^2+c(e^2-2df-e\sqrt{-4df+e^2})}} \\
 & +\frac{\operatorname{arctanh}\left(\frac{(2af-cx(e+\sqrt{-4df+e^2}))\sqrt{2}}{2\sqrt{cx^2+a}\sqrt{2af^2+c(e^2-2df+e\sqrt{-4df+e^2})}}\right)(2adef-(cd^2+a(-df+e^2))(e+\sqrt{-4df+e^2}))}{2(ac e^2+(-af+cd)^2)\sqrt{-4df+e^2}\sqrt{2af^2+c(e^2-2df+e\sqrt{-4df+e^2})}}
 \end{aligned}$$

command

```
integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.6 Problem number 72

$$\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal antiderivative

$$\frac{-ae - (-af + cd)x}{(ace^2 + (-af + cd)^2) \sqrt{cx^2 + a}}$$

$$f \operatorname{arctanh} \left(\frac{(2af - cx(e - \sqrt{-4df + e^2})) \sqrt{2}}{2\sqrt{cx^2 + a} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{-4df + e^2})}} \right) (2d(-af + cd) + ae(e - \sqrt{-4df + e^2})) \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{-4df + e^2})}$$

$$f \operatorname{arctanh} \left(\frac{(2af - cx(e + \sqrt{-4df + e^2})) \sqrt{2}}{2\sqrt{cx^2 + a} \sqrt{2af^2 + c(e^2 - 2df + e\sqrt{-4df + e^2})}} \right) (2d(-af + cd) + ae(e + \sqrt{-4df + e^2})) \sqrt{2af^2 + c(e^2 - 2df + e\sqrt{-4df + e^2})}$$

$$+ \frac{2(ace^2 + (-af + cd)^2) \sqrt{-4df + e^2} \sqrt{2af^2 + c(e^2 - 2df + e\sqrt{-4df + e^2})}}{2(ace^2 + (-af + cd)^2) \sqrt{-4df + e^2} \sqrt{2af^2 + c(e^2 - 2df + e\sqrt{-4df + e^2})}}$$

command

```
integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.7 Problem number 73

$$\int \frac{x}{(a + cx^2)^{3/2} (d + ex + fx^2)} dx$$

Optimal antiderivative

$$\frac{cex + af - cd}{(ace^2 + (-af + cd)^2) \sqrt{cx^2 + a}}$$

$$+ \frac{f \operatorname{arctanh} \left(\frac{(2af - cx(e - \sqrt{-4df + e^2})) \sqrt{2}}{2\sqrt{cx^2 + a} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{-4df + e^2})}} \right) (2cde - (-af + cd)(e - \sqrt{-4df + e^2}))}{2(ace^2 + (-af + cd)^2) \sqrt{-4df + e^2} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{-4df + e^2})}}$$

$$- \frac{f \operatorname{arctanh} \left(\frac{(2af - cx(e + \sqrt{-4df + e^2})) \sqrt{2}}{2\sqrt{cx^2 + a} \sqrt{2af^2 + c(e^2 - 2df + e\sqrt{-4df + e^2})}} \right) (2cde - (-af + cd)(e + \sqrt{-4df + e^2}))}{2(ace^2 + (-af + cd)^2) \sqrt{-4df + e^2} \sqrt{2af^2 + c(e^2 - 2df + e\sqrt{-4df + e^2})}}$$

command

```
integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.8 Problem number 74

$$\int \frac{1}{(a + cx^2)^{3/2} (d + ex + fx^2)} dx$$

Optimal antiderivative

$$\frac{c(ae + (-af + cd)x)}{a \left(ace^2 + (-af + cd)^2 \right) \sqrt{cx^2 + a}}$$

$$- \frac{f \operatorname{arctanh} \left(\frac{(2af - cx(e - \sqrt{-4df + e^2}))\sqrt{2}}{2\sqrt{cx^2 + a} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{-4df + e^2})}} \right) (2af^2 + c(e^2 - 2df + e\sqrt{-4df + e^2}))\sqrt{2}}{2 \left(ace^2 + (-af + cd)^2 \right) \sqrt{-4df + e^2} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{-4df + e^2})}}$$

$$+ \frac{f \operatorname{arctanh} \left(\frac{(2af - cx(e + \sqrt{-4df + e^2}))\sqrt{2}}{2\sqrt{cx^2 + a} \sqrt{2af^2 + c(e^2 - 2df + e\sqrt{-4df + e^2})}} \right) (2af^2 + c(e^2 - 2df - e\sqrt{-4df + e^2}))\sqrt{2}}{2 \left(ace^2 + (-af + cd)^2 \right) \sqrt{-4df + e^2} \sqrt{2af^2 + c(e^2 - 2df + e\sqrt{-4df + e^2})}}$$

command

`integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.9 Problem number 79

$$\int \frac{x \sqrt{a + bx + cx^2}}{d - fx^2} dx$$

Optimal antiderivative

$$- \frac{b \operatorname{arctanh} \left(\frac{2cx+b}{2\sqrt{c} \sqrt{cx^2 + bx + a}} \right) - \frac{\sqrt{cx^2 + bx + a}}{f}}{2f\sqrt{c}}$$

$$- \frac{\operatorname{arctanh} \left(\frac{b\sqrt{d} - 2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f})}{2\sqrt{cx^2 + bx + a} \sqrt{cd + af - b\sqrt{d}\sqrt{f}}} \right) \sqrt{cd + af - b\sqrt{d}\sqrt{f}}}{2f^{\frac{3}{2}}}$$

$$+ \frac{\operatorname{arctanh} \left(\frac{b\sqrt{d} + 2a\sqrt{f} + x(2c\sqrt{d} + b\sqrt{f})}{2\sqrt{cx^2 + bx + a} \sqrt{cd + af + b\sqrt{d}\sqrt{f}}} \right) \sqrt{cd + af + b\sqrt{d}\sqrt{f}}}{2f^{\frac{3}{2}}}$$

command

```
integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$cf \sqrt{\frac{f^3 \sqrt{\frac{b^2 d}{f^5}} + cd + af}{f^3}} \log \left(\frac{2 \sqrt{cx^2 + bx + a} f^4 \sqrt{\frac{b^2 d}{f^5}} \sqrt{\frac{f^3 \sqrt{\frac{b^2 d}{f^5}} + cd + af}{f^3}} + 2bcdx + b^2 d + (bf^3 x + 2af^3) \sqrt{\frac{b^2 d}{f^5}}}{x} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.10 Problem number 100

$$\int \frac{1}{x^2 \sqrt{a + bx + cx^2} (d - fx^2)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b \operatorname{arctanh} \left(\frac{bx+2a}{2\sqrt{a} \sqrt{cx^2 + bx + a}} \right)}{2a^{\frac{3}{2}}d} - \frac{\sqrt{cx^2 + bx + a}}{adx} \\ & + \frac{f \operatorname{arctanh} \left(\frac{b\sqrt{d} - 2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f})}{2\sqrt{cx^2 + bx + a} \sqrt{cd + af - b\sqrt{d} \sqrt{f}}} \right)}{2d^{\frac{3}{2}} \sqrt{cd + af - b\sqrt{d} \sqrt{f}}} \\ & + \frac{f \operatorname{arctanh} \left(\frac{b\sqrt{d} + 2a\sqrt{f} + x(2c\sqrt{d} + b\sqrt{f})}{2\sqrt{cx^2 + bx + a} \sqrt{cd + af + b\sqrt{d} \sqrt{f}}} \right)}{2d^{\frac{3}{2}} \sqrt{cd + af + b\sqrt{d} \sqrt{f}}} \end{aligned}$$

command

```
integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.11 Problem number 103

$$\int \frac{x^3}{(a + bx + cx^2)^{3/2} (d - fx^2)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{d \operatorname{arctanh} \left(\frac{b\sqrt{d} - 2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f})}{2\sqrt{cx^2 + bx + a} \sqrt{cd + af - b\sqrt{d}\sqrt{f}}} \right)}{2\sqrt{f} (cd + af - b\sqrt{d}\sqrt{f})^{\frac{3}{2}}} \\ & + \frac{d \operatorname{arctanh} \left(\frac{b\sqrt{d} + 2a\sqrt{f} + x(2c\sqrt{d} + b\sqrt{f})}{2\sqrt{cx^2 + bx + a} \sqrt{cd + af + b\sqrt{d}\sqrt{f}}} \right)}{2\sqrt{f} (cd + af + b\sqrt{d}\sqrt{f})^{\frac{3}{2}}} \\ & - \frac{2(bx + 2a)}{(-4ac + b^2) f \sqrt{cx^2 + bx + a}} - \frac{2d(a(2acf - b^2f + 2c^2d) + bc(-af + cd)x)}{(-4ac + b^2) f (b^2df - (af + cd)^2) \sqrt{cx^2 + bx + a}} \end{aligned}$$

command

```
integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.12 Problem number 104

$$\int \frac{x^2}{(a + bx + cx^2)^{3/2} (d - fx^2)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{b\sqrt{d} - 2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f})}{2\sqrt{cx^2 + bx + a} \sqrt{cd + af - b\sqrt{d}\sqrt{f}}}\right) \sqrt{d}}{2(cd + af - b\sqrt{d}\sqrt{f})^{\frac{3}{2}}} + \frac{\operatorname{arctanh}\left(\frac{b\sqrt{d} + 2a\sqrt{f} + x(2c\sqrt{d} + b\sqrt{f})}{2\sqrt{cx^2 + bx + a} \sqrt{cd + af + b\sqrt{d}\sqrt{f}}}\right) \sqrt{d}}{2(cd + af + b\sqrt{d}\sqrt{f})^{\frac{3}{2}}} + \frac{2ab(-af + cd) + 2c(b^2d - 2a(af + cd))x}{(-4ac + b^2)(b^2df - (af + cd)^2) \sqrt{cx^2 + bx + a}}$$

command

```
integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.13 Problem number 105

$$\int \frac{x}{(a + bx + cx^2)^{3/2} (d - fx^2)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})}{2\sqrt{cx^2+bx+a}\sqrt{cd+af-b\sqrt{d}\sqrt{f}}}\right)\sqrt{f}}{2\left(cd+af-b\sqrt{d}\sqrt{f}\right)^{\frac{3}{2}}} \\
& + \frac{\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+x(2c\sqrt{d}+b\sqrt{f})}{2\sqrt{cx^2+bx+a}\sqrt{cd+af+b\sqrt{d}\sqrt{f}}}\right)\sqrt{f}}{2\left(cd+af+b\sqrt{d}\sqrt{f}\right)^{\frac{3}{2}}} \\
& - \frac{2\left(a(2acf-b^2f+2c^2d)+bc(-af+cd)x\right)}{(-4ac+b^2)\left(b^2df-(af+cd)^2\right)\sqrt{cx^2+bx+a}}
\end{aligned}$$

command

```
integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

25.14 Problem number 106

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal antiderivative

$$\begin{aligned}
& f \operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})}{2\sqrt{cx^2+bx+a}\sqrt{cd+af-b\sqrt{d}\sqrt{f}}}\right) \\
& \frac{2\sqrt{d}\left(cd+af-b\sqrt{d}\sqrt{f}\right)^{\frac{3}{2}}}{2\sqrt{d}\left(cd+af-b\sqrt{d}\sqrt{f}\right)^{\frac{3}{2}}} \\
& + \frac{f \operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+x(2c\sqrt{d}+b\sqrt{f})}{2\sqrt{cx^2+bx+a}\sqrt{cd+af+b\sqrt{d}\sqrt{f}}}\right)}{2\sqrt{d}\left(cd+af+b\sqrt{d}\sqrt{f}\right)^{\frac{3}{2}}} \\
& - \frac{2\left(b(b^2f-c(3af+cd))-c(2acf-b^2f+2c^2d)x\right)}{(-4ac+b^2)\left(b^2df-(af+cd)^2\right)\sqrt{cx^2+bx+a}}
\end{aligned}$$

command

```
integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

26 Test file number 38

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/38_1.2.1.9_P-x-d+e_x-^m-a+b_x+c_x^2-^p

26.1 Problem number 49

$$\int \frac{A + Bx + Cx^2}{(d + ex)^3 (a + cx^2)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{-Ae^2 + Bde - Cd^2}{2e(ae^2 + cd^2)(ex + d)^2} + \frac{-2Acde - aBe^2 + Bcd^2 + 2aCde}{(ae^2 + cd^2)^2(ex + d)} \\ & - \frac{(Bcd(-3ae^2 + cd^2) - (Ac - aC)e(-ae^2 + 3cd^2)) \ln(ex + d)}{(ae^2 + cd^2)^3} \\ & + \frac{(Bcd(-3ae^2 + cd^2) - (Ac - aC)e(-ae^2 + 3cd^2)) \ln(cx^2 + a)}{2(ae^2 + cd^2)^3} \\ & + \frac{(Acd(-3ae^2 + cd^2) - a(cd^2(-3Be + Cd) - ae^2(-Be + 3Cd))) \arctan\left(\frac{x\sqrt{c}}{\sqrt{a}}\right) \sqrt{c}}{(ae^2 + cd^2)^3 \sqrt{a}} \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$Cc^2d^6 - 3Bc^2d^5e - 3(Ca^2 - 2Aac)d^2e^4 - ((Cac - Ac^2)d^5e + Ba^2x^2e^6 + (2Ba^2dx - 3(Ca^2 - Aac)dx^2)e^5 -$$

$$Cc^2d^6 - 3Bc^2d^5e - 3(Ca^2 - 2Aac)d^2e^4 + 2((Cac - Ac^2)d^5e + Ba^2x^2e^6 + (2Ba^2dx - 3(Ca^2 - Aac)dx^2)e^5 -$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

26.2 Problem number 55

$$\int \frac{A + Bx + Cx^2}{(d + ex)^2 (a + cx^2)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{e(Ae^2 - Bde + Cd^2)}{(ae^2 + cd^2)^2 (ex + d)} \\ & + \frac{-a(-2Acde - aBe^2 + Bcd^2 + 2aCde) + (Ac(-ae^2 + cd^2) + a(aCe^2 - cd(-2Be + Cd)))x}{2a(ae^2 + cd^2)^2 (cx^2 + a)} \\ & - \frac{e(ae^2(-Be + 2Cd) - cd(2Cd^2 - e(-4Ae + 3Bd))) \ln(ex + d)}{(ae^2 + cd^2)^3} \\ & + \frac{e(ae^2(-Be + 2Cd) - cd(2Cd^2 - e(-4Ae + 3Bd))) \ln(cx^2 + a)}{2(ae^2 + cd^2)^3} \\ & + \frac{(Ac(-3a^2e^4 + 6acd^2e^2 + c^2d^4) + a(a^2Ce^4 + c^2d^3(-2Be + Cd) - 6acd e^2(-Be + Cd))) \arctan\left(\frac{x\sqrt{c}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}(ae^2 + cd^2)^3 \sqrt{c}} \end{aligned}$$

command

`integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

26.3 Problem number 61

$$\int \frac{A + Bx + Cx^2}{(d + ex)(a + cx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{-a(-Ace + Bcd + aCe) + c(Acd + aBe - aCd)x}{4ac(ae^2 + cd^2)(cx^2 + a)^2} \\ & + \frac{4a^2e(Ae^2 - Bde + Cd^2) + (a(-Be + Cd)(-3ae^2 + cd^2) + Acd(7ae^2 + 3cd^2))x}{8a^2(ae^2 + cd^2)^2(cx^2 + a)} \\ & + \frac{e^3(Ae^2 - Bde + Cd^2)\ln(ex + d)}{(ae^2 + cd^2)^3} - \frac{e^3(Ae^2 - Bde + Cd^2)\ln(cx^2 + a)}{2(ae^2 + cd^2)^3} \\ & + \frac{(a(-Be + Cd)(-3a^2e^4 + 6acd^2e^2 + c^2d^4) + Acd(15a^2e^4 + 10acd^2e^2 + 3c^2d^4))\arctan\left(\frac{x\sqrt{c}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}(ae^2 + cd^2)^3\sqrt{c}} \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

26.4 Problem number 86

$$\int \frac{\sqrt{a + cx^2} (d + ex + fx^2)}{(g + hx)^5} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(dh^2 - egh + fg^2)(cx^2 + a)^{\frac{3}{2}}}{4h(a h^2 + cg^2)(hx + g)^4} \\
& + \frac{(4ah^2(-eh + 2fg) + cg(3fg^2 + h(-5dh + eg)))(cx^2 + a)^{\frac{3}{2}}}{12h(a h^2 + cg^2)^2(hx + g)^3} \\
& - \frac{ac(4c^2dg^2 + 4a^2fh^2 - ac(fg^2 - h(-dh + 5eg))) \operatorname{arctanh}\left(\frac{-cgx + ah}{\sqrt{ah^2 + cg^2}\sqrt{cx^2 + a}}\right)}{8(a h^2 + cg^2)^{\frac{7}{2}}} \\
& - \frac{(4c^2dg^2 + 4a^2fh^2 - ac(fg^2 - h(-dh + 5eg)))(-cgx + ah)\sqrt{cx^2 + a}}{8(a h^2 + cg^2)^3(hx + g)^2}
\end{aligned}$$

command

```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

26.5 Problem number 87

$$\int \frac{\sqrt{a + cx^2} (d + ex + fx^2)}{(g + hx)^6} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(dh^2 - egh + fg^2)(cx^2 + a)^{\frac{3}{2}}}{5h(a h^2 + cg^2)(hx + g)^5} + \frac{(5ah^2(-eh + 2fg) + cg(3fg^2 + h(-7dh + 2eg)))(cx^2 + a)^{\frac{3}{2}}}{20h(a h^2 + cg^2)^2(hx + g)^4} \\
& - \frac{(20a^2fh^4 - c^2g^2(3fg^2 + h(-27dh + 2eg)) - ach^2(18fg^2 - h(-8dh + 33eg)))(cx^2 + a)^{\frac{3}{2}}}{60h(a h^2 + cg^2)^3(hx + g)^3} \\
& - \frac{ac^2(4c^2dg^3 + a^2h^2(-eh + 6fg) - acg(fg^2 - 3h(-dh + 2eg))) \operatorname{arctanh}\left(\frac{-cgx + ah}{\sqrt{ah^2 + cg^2}\sqrt{cx^2 + a}}\right)}{8(a h^2 + cg^2)^{\frac{9}{2}}} \\
& - \frac{c(4c^2dg^3 + a^2h^2(-eh + 6fg) - acg(fg^2 - 3h(-dh + 2eg)))(-cgx + ah)\sqrt{cx^2 + a}}{8(a h^2 + cg^2)^4(hx + g)^2}
\end{aligned}$$

command

`integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

26.6 Problem number 105

$$\int \frac{d + ex + fx^2}{(g + hx)\sqrt{a + cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(-eh + fg) \operatorname{arctanh}\left(\frac{x\sqrt{c}}{\sqrt{cx^2 + a}}\right)}{h^2\sqrt{c}} \\ & - \frac{(dh^2 - egh + fg^2) \operatorname{arctanh}\left(\frac{-cgx+ah}{\sqrt{ah^2 + cg^2}\sqrt{cx^2 + a}}\right)}{h^2\sqrt{ah^2 + cg^2}} + \frac{f\sqrt{cx^2 + a}}{ch} \end{aligned}$$

command

`integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & \left[\frac{(cfg^3 + afg^2 - (cg^2h + ah^3)e)\sqrt{c} \log\left(-2cx^2 - 2\sqrt{cx^2 + a}\sqrt{c}x - a\right) + (cfg^2 + cdh^2 - cghe)\sqrt{cg^2 + ah^2}}{2(c^2g^2h^2 + ach^4)} \right. \\ & - \frac{2(cfg^2 + cdh^2 - cghe)\sqrt{-cg^2 - ah^2} \arctan\left(\frac{\sqrt{-cg^2 - ah^2}(cgx-ah)\sqrt{cx^2 + a}}{acg^2 + a^2h^2 + (c^2g^2 + ach^2)x^2}\right) + (cfg^3 + afg^2 - (cg^2h + ah^3)e)\sqrt{cx^2 + a}}{2(c^2g^2h^2 + ach^4)} \\ & \left. - \frac{(cfg^2 + cdh^2 - cghe)\sqrt{-cg^2 - ah^2} \arctan\left(\frac{\sqrt{-cg^2 - ah^2}(cgx-ah)\sqrt{cx^2 + a}}{acg^2 + a^2h^2 + (c^2g^2 + ach^2)x^2}\right) - (cfg^3 + afg^2 - (cg^2h + ah^3)e)\sqrt{cx^2 + a}}{c^2g^2h^2 + ach^4} \right] \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

26.7 Problem number 232

$$\int \frac{d + ex + fx^2}{(g + hx)^3 \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{(8c^2dg^2 + 8a^2fh^2 - 4abh(eh + 2fg) + b^2(3dh^2 + egh + 3fg^2) - 4c(bg(2dh + eg) + a(dh^2 - 3egh + fg^2))) \arctan\left(\frac{\sqrt{cx^2 + bx + a}}{hx + g}\right) + \frac{(fg^2 - h(-dh + eg)) \sqrt{cx^2 + bx + a}}{2h(a h^2 - bgh + cg^2)(hx + g)^2} + \frac{(2cg(fg^2 + h(-3dh + eg)) + h(4ah(-eh + 2fg) - b(-3dh^2 - egh + 5fg^2))) \sqrt{cx^2 + bx + a}}{4h(a h^2 - bgh + cg^2)^2(hx + g)}}{8(a h^2 - bgh + cg^2)^{\frac{5}{2}}}$$

command

```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

26.8 Problem number 238

$$\int \frac{d + ex + fx^2}{(g + hx)^2 (a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(2cg(fg^2 - h(-3dh + 2eg)) - h(2ah(-eh + 2fg) - b(-3dh^2 + egh + fg^2))) \operatorname{arctanh}\left(\frac{bg - 2ah + (-bh + 2cg)\sqrt{cx^2 + bx + a}}{2\sqrt{a h^2 - bgh + cg^2}}\right) + \frac{2(b^3dh^2 - b^2h(aeh + 2cdg) - 2ac(cg(-2dh + eg) + ah(-eh + 2fg)) + b(c^2dg^2 + a^2fh^2 + ac(-3dh^2 + 2egh + fg^2))) \sqrt{cx^2 + bx + a}}{2(a h^2 - bgh + cg^2)^{\frac{5}{2}}}}{(-4ac + b^2)(a h^2 - bgh)} - \frac{h(fg^2 - h(-dh + eg)) \sqrt{cx^2 + bx + a}}{(a h^2 - bgh + cg^2)^2(hx + g)}$$

command

```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

26.9 Problem number 239

$$\int \frac{d + ex + fx^2}{(g + hx)^3 (a + bx + cx^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(8c^2g^2(6dh^2 - 3egh + fg^2) + h^2(8a^2fh^2 + 4abh(-3eh + 2fg) - b^2(fg^2 + 3h(-5dh + eg))) - 4ch(ah(3dh^2 - 9gh + fg^2) + h^2(-dh + eg)))}{8(a h^2 - bgh + cg^2)} - \frac{2b^4dh^3 - 2b^3h^2(aeh + 3cdg) + 2b^2h(3c^2dg^2 + a^2fh^2 + ach(-4dh + 3eg)) - 2bc(c^2dg^3 + 3a^2h^2(-eh + fg) + acd^2)}{8(a h^2 - bgh + cg^2)} - \frac{h(fg^2 - h(-dh + eg)) \sqrt{cx^2 + bx + a}}{2(a h^2 - bgh + cg^2)^2 (hx + g)^2} - \frac{h(2cg(3fg^2 - h(-7dh + 5eg)) - h(4ah(-eh + 2fg) - b(-7dh^2 + 3egh + fg^2))) \sqrt{cx^2 + bx + a}}{4(a h^2 - bgh + cg^2)^3 (hx + g)}$$

command

```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

26.10 Problem number 259

$$\int \sqrt{d+ex} \sqrt{a+bx+cx^2} (A+Bx+Cx^2) dx$$

Optimal antiderivative

$$\frac{2C(ex+d)^{\frac{3}{2}}(cx^2+bx+a)^{\frac{3}{2}}}{9ce} - \frac{2(-3Bce+2Cbe+2Ccd)(cx^2+bx+a)^{\frac{3}{2}}\sqrt{ex+d}}{21c^2e}$$

$$+ \frac{2(8b^3C e^3 - 3bc e^2(4bBe - aCe + bCd) + c^3d(8C d^2 - 3e(-7Ae + 4Bd)) + 3c^2e(ae(-5Be + Cd) - b(-7Ae^2 - 3$$

$$\left(2\left(4c^2d^2 - b^2e^2 - \frac{3ce(-2ae+bd)}{2}\right)(8b^2C e^2 - ce(12bBe + 7aCe + bCd) - c^2(2C d^2 - 3e(7Ae + Bd))) - 5ce(-be +$$

+

$$2(ae^2 - bde + cd^2)(8b^3C e^3 - 3c^2e^2(-7Abe - 10aBe + Bbd + 2aCd) + 3bc e^2(-4bBe - 9aCe + bCd) - 2c^3d(8$$

command

```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16 C^5 d^5 - 8 (2 C b c^4 + 3 B c^5) d^4 e - (5 C b^2 c^3 - 42 A c^5 - 3 (10 C a + 9 B b) c^4) d^3 e^2 - (5 C b^3 c^2 + 3 (22 B a + 21 A b) c^3) d^2 e^3 - (5 C b^4 c + 3 (11 B a^2 + 10 A b a) c^2) d e^4 - 3 C b^5 e^5 \right) \sqrt{ex+d} \sqrt{cx^2+bx+a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((Cx^2 + Bx + A)\sqrt{cx^2 + bx + a}\sqrt{ex + d}, x\right)$$

26.11 Problem number 260

$$\int \frac{\sqrt{a + bx + cx^2} (A + Bx + Cx^2)}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{2C(cx^2 + bx + a)^{\frac{3}{2}} \sqrt{ex + d}}{7ce} - \frac{2(5ce(-7Ace + aCe + 3bCd) - (-be + 4cd)(-7Bce + 4Cbe + 6Ccd) + 3ce(-7Bce + 4Cbe + 6Ccd)x) \sqrt{ex + d}}{105c^2e^3}$$

$$(5ce(-be + 2cd)(-7Ace + aCe + 3bCd) - (-7Bce + 4Cbe + 6Ccd)(8c^2d^2 - 2b^2e^2 - 3ce(-2ae + bd))) \text{EllipticF}$$

+

$$\frac{105c^3e^4 \sqrt{cx^2 + bx + a}}{105c}$$

$$2(ae^2 - bde + cd^2)(4b^2Ce^2 + ce(-7bBe - 10aCe + 8bCd) + c^2(48Cd^2 - 14e(-5Ae + 4Bd))) \text{EllipticF} \left(\frac{\sqrt{\frac{b \cdot \dots}{\dots}}}{\dots} \right)$$

+

$$\frac{105c}{105c}$$

command

```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((48C^4d^4 - 8(5Cbc^3 + 7Bc^4)d^3e - (10Cb^2c^2 - 70Ac^4 - (62Ca + 49Bb)c^3)d^2e^2 - (5Cb^3c + 14(6Ba + 5A) \dots) \right)}{105c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cx^2 + Bx + A) \sqrt{cx^2 + bx + a}}{\sqrt{ex + d}}, x \right)$$

26.12 Problem number 261

$$\int \frac{\sqrt{a + bx + cx^2} (A + Bx + Cx^2)}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Cd^2 - e(-Ae + Bd)) (cx^2 + bx + a)^{\frac{3}{2}}}{e(ae^2 - bde + cd^2) \sqrt{ex + d}}$$

$$\frac{2(bCe^2(-ae + bd) + c^2d(24Cd^2 - 5e(-3Ae + 4Bd)) + ce(ae(-5Be + 9Cd) - 5b(3Ae^2 - 4Bde + 5Cd^2)) + 3e^2d^2)}{15ce^3(ae^2 - bde + cd^2)}$$

$$(2b^2Ce^2 + ce(-5bBe - 6aCe + 8bCd) - c^2(48Cd^2 - 10e(-3Ae + 4Bd))) \text{EllipticE} \left(\frac{\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}}{2} \right)$$

$$15c^2e^4 \sqrt{cx^2 + bx + a} \sqrt{\frac{c}{2cd - e(b + 2cx + a)}}$$

$$2(bCe^2(-ae + bd) - 2c^2d(24Cd^2 - 5e(-3Ae + 4Bd)) - ce(2ae(-5Be + 9Cd) - b(32Cd^2 - 5e(-3Ae + 5Bd))))$$

+

command

```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((48Cc^3d^4 - (2Cb^3 + 15(2Ba + Ab)c^2 - (9Cab + 5Bb^2)c)xe^4 - ((7Cb^2c - 30Ac^3 - (42Ca + 25Bb)c^2)dx \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cx^2 + Bx + A) \sqrt{cx^2 + bx + a} \sqrt{ex + d}}{e^2x^2 + 2dex + d^2}, x \right)$$

26.13 Problem number 262

$$\int \frac{\sqrt{a+bx+cx^2} (A+Bx+Cx^2)}{(d+ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Cd^2 - e(-Ae + Bd)) (cx^2 + bx + a)^{\frac{3}{2}}}{3e(ae^2 - bde + cd^2)(ex + d)^{\frac{3}{2}}}$$

$$- \frac{2\left(e(-ae + bd)(-3Be + 7Cd) - cd(8Cd^2 - e(-Ae + 4Bd)) + e^2\left(Bcd + bCd - \frac{2cCd^2}{e} - Ace - aCe\right)x\right) \sqrt{cx^2}}{3e^3(ae^2 - bde + cd^2)\sqrt{ex + d}}$$

$$+ \frac{\left(2\left(4cd - \frac{be}{2}\right)\left(Bcd + bCd - \frac{2cCd^2}{e} - Ace - aCe\right) + 6c(bd(-Be + Cd) + e(Acd + aBe - aCd))\right) \text{EllipticE}\left(\sqrt{\frac{b}{\dots}}\right)}{3ce^3(ae^2 - bde + cd^2)\sqrt{cx^2 + bx}}$$

$$- \frac{2\left(e(-3bBe - 2aCe + 8bCd) - 2c(8Cd^2 - e(-Ae + 4Bd))\right) \text{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}}, \sqrt{\frac{\dots}{2c}}\right)}{3ce^4\sqrt{ex + d}\sqrt{cx^2}}$$

command

```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\left(16C^3d^6 - (Cab^2 - 6Aac^2 - (6Ca^2 + 3Bab - Ab^2)c\right)x^2e^6 + \left((Cb^3 - 2(6Ba + Ab)c^2 - 2(7Cab + Bb^2)c\right)dx\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

26.14 Problem number 263

$$\int \frac{\sqrt{a+bx+cx^2} (A+Bx+Cx^2)}{(d+ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(Cd^2 - e(-Ae + Bd)) (cx^2 + bx + a)^{\frac{3}{2}}}{5e(ae^2 - bde + cd^2)(ex + d)^{\frac{5}{2}}}$$

$$- \frac{2(c^2d^3(24Cd^2 - e(Ae + 4Bd)) + e^2(15b^2Cd^3 + 5a^2e^2(Be + Cd) - abe(2Ae^2 + 3Bde + 22Cd^2)) - cde(bd(Ae^2 - bde + cd^2) + ce(10ae(-Be + 5Cd) - b(-Ae^2 - 9Bde + 64Cd^2)))}{15ce^4}$$

$$(2c^2d^2(24Cd^2 - e(Ae + 4Bd)) + e^2(30a^2Ce^2 - 5abe(-Be + 14Cd) + b^2(-2Ae^2 - 3Bde + 38Cd^2)) - ce(bd(-Ae^2 - bde + cd^2) + ce(10ae(-Be + 5Cd) - b(-Ae^2 - 9Bde + 64Cd^2)))) EL$$

+

$$2(15bCe^2(-ae + bd) + 2c^2d(24Cd^2 - e(Ae + 4Bd)) + ce(10ae(-Be + 5Cd) - b(-Ae^2 - 9Bde + 64Cd^2))) EL$$

15ce⁴

command

```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

26.15 Problem number 264

$$\int \frac{\sqrt{a + bx + cx^2} (A + Bx + Cx^2)}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5}, x\right)$$

26.16 Problem number 265

$$\int \frac{\sqrt{a + bx + cx^2} (A + Bx + Cx^2)}{(d + ex)^{11/2}} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^6x^6 + 6de^5x^5 + 15d^2e^4x^4 + 20d^3e^3x^3 + 15d^4e^2x^2 + 6d^5ex + d^6}, x\right)$$

26.17 Problem number 266

$$\int \frac{(d+ex)^{3/2} (A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-7Bce+6Cbe+2Ccd)(ex+d)^{\frac{3}{2}}\sqrt{cx^2+bx+a}}{35c^2e} + \frac{2C(ex+d)^{\frac{5}{2}}\sqrt{cx^2+bx+a}}{7ce} \\ & + \frac{2(24b^2C e^2 - ce(28bBe + 25aCe + 15bCd) - c^2(6C d^2 - 7e(5Ae + 3Bd)))\sqrt{ex+d}\sqrt{cx^2+bx+a}}{105c^3e} \end{aligned}$$

$$(48b^3C e^3 - 8bc e^2(7bBe + 13aCe + 9bCd) + c^3d(6C d^2 - 7e(20Ae + 3Bd)) + c^2e(ae(63Be + 82Cd) + b(70A e^2$$

105c^4e^2

$$2(a e^2 - bde + c d^2) (24b^2C e^2 - ce(28bBe + 25aCe + 15bCd) - c^2(6C d^2 - 7e(5Ae + 3Bd))) \text{EllipticF} \left(\frac{\sqrt{\frac{b+}{\dots}}}{\dots} \right)$$

105c^4

command

```
integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((6C c^4 d^4 + 3(3C b c^3 - 7B c^4) d^3 e + (39C b^2 c^2 + 175A c^4 - (71C a + 56B b) c^3) d^2 e^2 - (96C b^3 c + 7(27B a + 25$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cex^3 + (Cd + Be)x^2 + Ad + (Bd + Ae)x)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}, x \right)$$

26.18 Problem number 267

$$\int \frac{\sqrt{d+ex} (A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{2C(ex+d)^{\frac{3}{2}} \sqrt{cx^2+bx+a}}{5ce} - \frac{2(-5Bce+4Cbe+2Ccd) \sqrt{ex+d} \sqrt{cx^2+bx+a}}{15c^2e}$$

$$+ \frac{(8b^2C e^2 - ce(10bBe + 9aCe + 3bCd) - c^2(2C d^2 - 5e(3Ae + Bd))) \text{EllipticE} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2} \right)}{15c^3e^2 \sqrt{cx^2+bx+a} \sqrt{\frac{c(ex+d)}{2cd-e(b+\sqrt{-4ac+b^2})}}}$$

$$+ \frac{2(-5Bce+4Cbe+2Ccd) (ae^2 - bde + cd^2) \text{EllipticF} \left(\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2cd-e(b+\sqrt{-4ac+b^2})}} \right)}{15c^3e^2 \sqrt{ex+d} \sqrt{cx^2+bx+a}}$$

command

```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2Cc^3d^3 + (2Cbc^2 - 5Bc^3)d^2e + (7Cb^2c + 30Ac^3 - 2(6Ca + 5Bb)c^2)de^2 - (8Cb^3 + 15(Ba + Ab)c^2 - (21C^2d^2 + 10Cbd + 5B^2c^2)e^2) \sqrt{ex+d} \sqrt{cx^2+bx+a} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cx^2 + Bx + A) \sqrt{ex+d}}{\sqrt{cx^2+bx+a}}, x \right)$$

26.19 Problem number 268

$$\int \frac{A + Bx + Cx^2}{\sqrt{d + ex} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2C\sqrt{ex + d} \sqrt{cx^2 + bx + a}}{3ce}$$

$$(-3Bce + 2Cbe + 2Ccd) \text{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \sqrt{2} \right) \\ - \frac{3c^2e^2\sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}}{2(Ce(-ae + bd) + c(2Cd^2 - 3e(-Ae + Bd))) \text{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)} \\ + \frac{3c^2e^2\sqrt{ex + d} \sqrt{cx^2 + bx + a}}{3c^2e^2\sqrt{ex + d} \sqrt{cx^2 + bx + a}}$$

command

```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{cx^2 + bx + a} \sqrt{ex + d} Cc^2e^2 + (2Cc^2d^2 + (Cbc - 3Bc^2)de + (2Cb^2 + 9Ac^2 - 3(Ca + Bb)c)e^2) \sqrt{c} e^{\frac{1}{2}} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Cx^2 + Bx + A) \sqrt{cx^2 + bx + a} \sqrt{ex + d}}{cex^3 + (cd + be)x^2 + ad + (bd + ae)x}, x \right)$$

26.20 Problem number 269

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{3/2} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(Cd^2 - e(-Ae + Bd)) \sqrt{cx^2 + bx + a}}{e(ae^2 - bde + cd^2) \sqrt{ex + d}}$$

$$(Ce(-ae + bd) - c(2Cd^2 - e(-Ae + Bd))) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)$$

$$2(-Be + 2Cd) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2}$$

$$ce^2(ae^2 - bde + cd^2) \sqrt{cx^2 + bx + a} \sqrt{\frac{c(ex + d)}{2cd - e(b + \sqrt{-4ac + b^2})}}$$

$$ce^2 \sqrt{ex + d} \sqrt{cx^2 + bx + a}$$

command

```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2Cc^2d^4 + (Cab - (3Ba - Ab)c)xe^4 - ((Cb^2 + 2Ac^2 - 2(2Ca + Bb)c)dx - (Cab - (3Ba - Ab)c)d)e^3 - (($$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Cx^2 + Bx + A) \sqrt{cx^2 + bx + a} \sqrt{ex + d}}{ce^2x^4 + (2cde + be^2)x^3 + ad^2 + (cd^2 + 2bde + ae^2)x^2 + (bd^2 + 2ade)x}, x \right)$$

26.21 Problem number 270

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{5/2} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(Cd^2 - e(-Ae + Bd)) \sqrt{cx^2 + bx + a}}{3e(ae^2 - bde + cd^2)(ex + d)^{3/2}} + \frac{2(cd(2Cd^2 + e(-4Ae + Bd)) + e(3ae(-Be + 2Cd) - b(-2Ae^2 - Bde + 4Cd^2))) \sqrt{cx^2 + bx + a}}{3e(ae^2 - bde + cd^2)^2 \sqrt{ex + d}}$$

$$(cd(2Cd^2 + e(-4Ae + Bd)) + e(3ae(-Be + 2Cd) - b(-2Ae^2 - Bde + 4Cd^2))) \operatorname{EllipticE} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \right)$$

$$2(3Ce(-ae + bd) - c(2Cd^2 + e(-Ae + Bd))) \operatorname{EllipticF} \left(\sqrt{\frac{b + 2cx + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2cd - e(b + \sqrt{-4ac + b^2})}} \right)$$

$$3e^2(ae^2 - bde + cd^2)^2 \sqrt{cx^2 + bx + a} \sqrt{ex + d}$$

command

```
integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2Cc^2d^6 + (9Ca^2 - 3Bab + 2Ab^2 - 3Aac)x^2e^6 - ((12Cab - Bb^2 - (9Ba - 5Ab)c)dx^2 - 2(9Ca^2 - 3Bab + \dots) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(Cx^2 + Bx + A) \sqrt{cx^2 + bx + a} \sqrt{ex + d}}{ce^3x^5 + (3cde^2 + be^3)x^4 + ad^3 + (3cd^2e + 3bde^2 + ae^3)x^3 + (cd^3 + 3bd^2e + 3ade^2)x^2 + (bd^3 + 3ad^2e)x + d^4} \right)$$

26.22 Problem number 271

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{7/2} \sqrt{a + bx + cx^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Cd^2 - e(-Ae + Bd)) \sqrt{cx^2 + bx + a}}{5e(ae^2 - bde + cd^2)(ex + d)^{\frac{5}{2}}} \\ & + \frac{2(cd(2Cd^2 + e(-8Ae + 3Bd)) + e(5ae(-Be + 2Cd) - b(-4Ae^2 - Bde + 6Cd^2))) \sqrt{cx^2 + bx + a}}{15e(ae^2 - bde + cd^2)^2(ex + d)^{\frac{3}{2}}} \\ & + \frac{2(c^2d^2(2Cd^2 + e(-23Ae + 3Bd)) - e^2(15a^2Ce^2 - 10abe(Be + Cd) + b^2(8Ae^2 + 2Bde + 3Cd^2)) - ce(bd(-23Ae + 3Bd) + e^2d^2)) \sqrt{ex + d}}{15e(ae^2 - bde + cd^2)^3 \sqrt{ex + d}} \end{aligned}$$

$$(c^2d^2(2Cd^2 + e(-23Ae + 3Bd)) - e^2(15a^2Ce^2 - 10abe(Be + Cd) + b^2(8Ae^2 + 2Bde + 3Cd^2)) - ce(bd(-23Ae + 3Bd) + e^2d^2)) \sqrt{ex + d}$$

$$+ \frac{2(cd(2Cd^2 + e(-8Ae + 3Bd)) + e(5ae(-Be + 2Cd) - b(-4Ae^2 - Bde + 6Cd^2))) \operatorname{EllipticF}\left(\sqrt{\frac{b + 2cx + \sqrt{-4acx - 4a^2}}{\sqrt{-4acx - 4a^2}}}\right)}{15e^2(ae^2 - bde + cd^2)}$$

command

```
integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Cx^2 + Bx + A) \sqrt{cx^2 + bx + a} \sqrt{ex + d}}{ce^4x^6 + (4cde^3 + be^4)x^5 + ad^4 + (6cd^2e^2 + 4bde^3 + ae^4)x^4 + 2(2cd^3e + 3bd^2e^2 + 2ade^3)x^3 + (cd^4 + 4ace^3)x^2 + (bd^3 + 3ade^2)x + ad^2}, x\right)$$

27 Test file number 39

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/39_1.2.2.2-d_x-
 $\hat{m}-a+b_x^2+c_x^4-\hat{p}$

27.1 Problem number 16

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}} dx$$

Optimal antiderivative

$$\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{6}\right)$$

command

`integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\text{ellipticF}\left(\frac{1}{2}\sqrt{2}x, -6\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-3x^4+5x^2+2}}{3x^4-5x^2-2}, x\right)$$

27.2 Problem number 17

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{-4+2\sqrt{10}}}{2}, \frac{i\sqrt{6}}{3} + \frac{i\sqrt{15}}{3}\right) \sqrt{12+6\sqrt{10}}}{6}$$

command

`integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \left(\sqrt{10} + 2 \right) \sqrt{\sqrt{10} - 2} \operatorname{ellipticF} \left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{10} - 2}, -\frac{2}{3} \sqrt{10} - \frac{7}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-3x^4 + 4x^2 + 2}}{3x^4 - 4x^2 - 2}, x \right)$$

27.3 Problem number 18

$$\int \frac{1}{\sqrt{2 + 3x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticF} \left(\frac{x\sqrt{6}}{\sqrt{3 + \sqrt{33}}}, \frac{i\sqrt{6}}{4} + \frac{i\sqrt{22}}{4} \right) \sqrt{2}}{\sqrt{-3 + \sqrt{33}}}$$

command

```
integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{24} \left(\sqrt{33} \sqrt{2} + 3 \sqrt{2} \right) \sqrt{\sqrt{33} - 3} \operatorname{ellipticF} \left(\frac{1}{2} x \sqrt{\sqrt{33} - 3}, -\frac{1}{4} \sqrt{33} - \frac{7}{4} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-3x^4 + 3x^2 + 2}}{3x^4 - 3x^2 - 2}, x \right)$$

27.4 Problem number 19

$$\int \frac{1}{\sqrt{2 + 2x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{3}}{\sqrt{1+\sqrt{7}}}, \frac{i\sqrt{6}}{6} + \frac{i\sqrt{42}}{6}\right)}{\sqrt{-1+\sqrt{7}}}$$

command

```
integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} (\sqrt{7} + 1) \sqrt{\sqrt{7} - 1} \text{ellipticF}\left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{7} - 1}, -\frac{1}{3} \sqrt{7} - \frac{4}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-3x^4 + 2x^2 + 2}}{3x^4 - 2x^2 - 2}, x\right)$$

27.5 Problem number 20

$$\int \frac{1}{\sqrt{2 + x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x, \frac{i\sqrt{6}}{2}\right) \sqrt{2}}{2}$$

command

```
integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}\left(x, -\frac{3}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-3x^4 + x^2 + 2}}{3x^4 - x^2 - 2}, x\right)$$

27.6 Problem number 21

$$\int \frac{1}{\sqrt{2-3x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}, i\right) 6^{\frac{3}{4}}}{6}$$

command

`integrate(1/(-3*x^4+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \cdot 6^{\frac{3}{4}} \text{ellipticF}\left(\frac{1}{2} \cdot 6^{\frac{1}{4}} \sqrt{2} x, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-3x^4+2}}{3x^4-2}, x\right)$$

27.7 Problem number 22

$$\int \frac{1}{\sqrt{2-x^2-3x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{6}}{2}, \frac{i\sqrt{6}}{3}\right) \sqrt{3}}{3}$$

command

`integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{3} \text{ellipticF}\left(\frac{1}{2} \sqrt{3} \sqrt{2} x, -\frac{2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-3x^4-x^2+2}}{3x^4+x^2-2}, x\right)$$

27.8 Problem number 23

$$\int \frac{1}{\sqrt{2 - 2x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{3}}{\sqrt{-1 + \sqrt{7}}}, \frac{i\sqrt{42}}{6} - \frac{i\sqrt{6}}{6}\right)}{\sqrt{1 + \sqrt{7}}}$$

command

`integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{\sqrt{7} + 1} (\sqrt{7} - 1) \text{ellipticF}\left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{7} + 1}, \frac{1}{3} \sqrt{7} - \frac{4}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-3x^4 - 2x^2 + 2}}{3x^4 + 2x^2 - 2}, x\right)$$

27.9 Problem number 24

$$\int \frac{1}{\sqrt{2 - 3x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{6}}{\sqrt{-3 + \sqrt{33}}}, \frac{i\sqrt{22}}{4} - \frac{i\sqrt{6}}{4}\right) \sqrt{2}}{\sqrt{3 + \sqrt{33}}}$$

command

`integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{24} (\sqrt{33} \sqrt{2} - 3 \sqrt{2}) \sqrt{\sqrt{33} + 3} \text{ellipticF}\left(\frac{1}{2} x \sqrt{\sqrt{33} + 3}, \frac{1}{4} \sqrt{33} - \frac{7}{4}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-3x^4 - 3x^2 + 2}}{3x^4 + 3x^2 - 2}, x\right)$$

27.10 Problem number 25

$$\int \frac{1}{\sqrt{2 - 4x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{4+2\sqrt{10}}}{2}, \frac{i\sqrt{15}}{3} - \frac{i\sqrt{6}}{3}\right) \sqrt{-12+6\sqrt{10}}}{6}$$

command

```
integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{\sqrt{10} + 2} (\sqrt{10} - 2) \text{ellipticF}\left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{10} + 2}, \frac{2}{3} \sqrt{10} - \frac{7}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-3x^4 - 4x^2 + 2}}{3x^4 + 4x^2 - 2}, x\right)$$

27.11 Problem number 26

$$\int \frac{1}{\sqrt{2 - 5x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x\sqrt{3}, \frac{i\sqrt{6}}{6}\right) \sqrt{6}}{6}$$

command

```
integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{3} \sqrt{2} \text{ellipticF}\left(\sqrt{3} x, -\frac{1}{6}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-3x^4 - 5x^2 + 2}}{3x^4 + 5x^2 - 2}, x\right)$$

27.12 Problem number 27

$$\int \frac{1}{\sqrt{3 + 7x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{2x}{\sqrt{7 + \sqrt{73}}}, \frac{7i\sqrt{6}}{12} + \frac{i\sqrt{438}}{12}\right) \sqrt{2}}{\sqrt{-7 + \sqrt{73}}}$$

command

`integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{72} \left(\sqrt{73} \sqrt{6} \sqrt{3} + 7 \sqrt{6} \sqrt{3} \right) \sqrt{\sqrt{73} - 7} \text{ellipticF}\left(\frac{1}{6} \sqrt{6} x \sqrt{\sqrt{73} - 7}, -\frac{7}{12} \sqrt{73} - \frac{61}{12}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 + 7x^2 + 3}}{2x^4 - 7x^2 - 3}, x\right)$$

27.13 Problem number 28

$$\int \frac{1}{\sqrt{3 + 6x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{-9 + 3\sqrt{15}}}{3}, \frac{i\sqrt{6}}{2} + \frac{i\sqrt{10}}{2}\right) \sqrt{18 + 6\sqrt{15}}}{6}$$

command

`integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \left(\sqrt{5} \sqrt{3} + 3 \right) \sqrt{\sqrt{5} \sqrt{3} - 3} \text{ellipticF}\left(\frac{1}{3} \sqrt{3} \sqrt{\sqrt{5} \sqrt{3} - 3} x, -\sqrt{5} \sqrt{3} - 4\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 + 6x^2 + 3}}{2x^4 - 6x^2 - 3}, x\right)$$

27.14 Problem number 29

$$\int \frac{1}{\sqrt{3 + 5x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\text{EllipticF}\left(\frac{x\sqrt{3}}{3}, i\sqrt{6}\right)$$

command

`integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\text{ellipticF}\left(\frac{1}{3}\sqrt{3}x, -6\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 + 5x^2 + 3}}{2x^4 - 5x^2 - 3}, x\right)$$

27.15 Problem number 30

$$\int \frac{1}{\sqrt{3 + 4x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{2 + \sqrt{10}}}, \frac{i\sqrt{6}}{3} + \frac{i\sqrt{15}}{3}\right)}{\sqrt{-2 + \sqrt{10}}}$$

command

`integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6}(\sqrt{10} + 2)\sqrt{\sqrt{10} - 2} \text{ellipticF}\left(\frac{1}{3}\sqrt{3}x\sqrt{\sqrt{10} - 2}, -\frac{2}{3}\sqrt{10} - \frac{7}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 + 4x^2 + 3}}{2x^4 - 4x^2 - 3}, x\right)$$

27.16 Problem number 31

$$\int \frac{1}{\sqrt{3 + 3x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{2x}{\sqrt{3 + \sqrt{33}}}, \frac{i\sqrt{6}}{4} + \frac{i\sqrt{22}}{4}\right) \sqrt{2}}{\sqrt{-3 + \sqrt{33}}}$$

command

`integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{24} \left(\sqrt{11} \sqrt{6} + \sqrt{6} \sqrt{3} \right) \sqrt{\sqrt{11} \sqrt{3} - 3} \text{ellipticF}\left(\frac{1}{6} \sqrt{6} \sqrt{\sqrt{11} \sqrt{3} - 3} x, -\frac{1}{4} \sqrt{11} \sqrt{3} - \frac{7}{4}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 + 3x^2 + 3}}{2x^4 - 3x^2 - 3}, x\right)$$

27.17 Problem number 32

$$\int \frac{1}{\sqrt{3 + 2x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{1 + \sqrt{7}}}, \frac{i\sqrt{6}}{6} + \frac{i\sqrt{42}}{6}\right)}{\sqrt{-1 + \sqrt{7}}}$$

command

`integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \left(\sqrt{7} + 1 \right) \sqrt{\sqrt{7} - 1} \text{ellipticF}\left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{7} - 1}, -\frac{1}{3} \sqrt{7} - \frac{4}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 + 2x^2 + 3}}{2x^4 - 2x^2 - 3}, x\right)$$

27.18 Problem number 33

$$\int \frac{1}{\sqrt{3+x^2-2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{6}}{3}, \frac{i\sqrt{6}}{2}\right) \sqrt{2}}{2}$$

command

```
integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}\left(\frac{1}{3} \sqrt{3} \sqrt{2} x, -\frac{3}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4+x^2+3}}{2x^4-x^2-3}, x\right)$$

27.19 Problem number 34

$$\int \frac{1}{\sqrt{3-2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} x}{3}, i\right) 6^{\frac{3}{4}}}{6}$$

command

```
integrate(1/(-2*x^4+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{3} \sqrt{2} \sqrt{\sqrt{3} \sqrt{2}} \text{ellipticF}\left(\frac{1}{3} \sqrt{3} \sqrt{\sqrt{3} \sqrt{2}} x, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4+3}}{2x^4-3}, x\right)$$

27.20 Problem number 35

$$\int \frac{1}{\sqrt{3 - x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x, \frac{i\sqrt{6}}{3}\right) \sqrt{3}}{3}$$

command

```
integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{3} \text{ellipticF}\left(x, -\frac{2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 - x^2 + 3}}{2x^4 + x^2 - 3}, x\right)$$

27.21 Problem number 36

$$\int \frac{1}{\sqrt{3 - 2x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{-1 + \sqrt{7}}}, \frac{i\sqrt{42}}{6} - \frac{i\sqrt{6}}{6}\right)}{\sqrt{1 + \sqrt{7}}}$$

command

```
integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{\sqrt{7} + 1} (\sqrt{7} - 1) \text{ellipticF}\left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{7} + 1}, \frac{1}{3} \sqrt{7} - \frac{4}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 - 2x^2 + 3}}{2x^4 + 2x^2 - 3}, x\right)$$

27.22 Problem number 37

$$\int \frac{1}{\sqrt{3 - 3x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{2x}{\sqrt{-3 + \sqrt{33}}}, \frac{i\sqrt{22}}{4} - \frac{i\sqrt{6}}{4}\right) \sqrt{2}}{\sqrt{3 + \sqrt{33}}}$$

command

```
integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{24} \left(\sqrt{11} \sqrt{6} - \sqrt{6} \sqrt{3} \right) \sqrt{\sqrt{11} \sqrt{3} + 3} \text{ellipticF}\left(\frac{1}{6} \sqrt{6} \sqrt{\sqrt{11} \sqrt{3} + 3} x, \frac{1}{4} \sqrt{11} \sqrt{3} - \frac{7}{4}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 - 3x^2 + 3}}{2x^4 + 3x^2 - 3}, x\right)$$

27.23 Problem number 38

$$\int \frac{1}{\sqrt{3 - 4x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{-2 + \sqrt{10}}}, \frac{i\sqrt{15}}{3} - \frac{i\sqrt{6}}{3}\right)}{\sqrt{2 + \sqrt{10}}}$$

command

```
integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{\sqrt{10} + 2} \left(\sqrt{10} - 2 \right) \text{ellipticF}\left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{10} + 2}, \frac{2}{3} \sqrt{10} - \frac{7}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 - 4x^2 + 3}}{2x^4 + 4x^2 - 3}, x\right)$$

27.24 Problem number 39

$$\int \frac{1}{\sqrt{3 - 5x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x\sqrt{2}, \frac{i\sqrt{6}}{6}\right) \sqrt{6}}{6}$$

command

`integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{3} \sqrt{2} \text{ellipticF}\left(\sqrt{2} x, -\frac{1}{6}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 - 5x^2 + 3}}{2x^4 + 5x^2 - 3}, x\right)$$

27.25 Problem number 40

$$\int \frac{1}{\sqrt{3 - 6x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{9+3\sqrt{15}}}{3}, \frac{i\sqrt{10}}{2} - \frac{i\sqrt{6}}{2}\right) \sqrt{-18+6\sqrt{15}}}{6}$$

command

`integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{\sqrt{5} \sqrt{3} + 3} (\sqrt{5} \sqrt{3} - 3) \text{ellipticF}\left(\frac{1}{3} \sqrt{3} \sqrt{\sqrt{5} \sqrt{3} + 3} x, \sqrt{5} \sqrt{3} - 4\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4 - 6x^2 + 3}}{2x^4 + 6x^2 - 3}, x\right)$$

27.26 Problem number 41

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}, \frac{i\sqrt{438}}{12} - \frac{7i\sqrt{6}}{12}\right) \sqrt{2}}{\sqrt{7+\sqrt{73}}}$$

command

`integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{72} \left(\sqrt{73} \sqrt{6} \sqrt{3} - 7 \sqrt{6} \sqrt{3} \right) \sqrt{\sqrt{73} + 7} \text{ellipticF}\left(\frac{1}{6} \sqrt{6} x \sqrt{\sqrt{73} + 7}, \frac{7}{12} \sqrt{73} - \frac{61}{12}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2x^4-7x^2+3}}{2x^4+7x^2-3}, x\right)$$

27.27 Problem number 66

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}} dx$$

Optimal antiderivative

$$\frac{(x^2+1)^{\frac{3}{2}} \sqrt{\frac{1}{x^2+1}} \text{EllipticF}\left(\frac{x}{\sqrt{x^2+1}}, \frac{i\sqrt{2}}{2}\right) \sqrt{\frac{3x^2+2}{x^2+1}} \sqrt{2}}{2\sqrt{3x^4+5x^2+2}}$$

command

`integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2}i\sqrt{2} \text{ellipticF}\left(ix, \frac{3}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{1}{\sqrt{3x^4+5x^2+2}}, x\right)$$

27.28 Problem number 67

$$\int \frac{1}{\sqrt{2 + 4x^2 + 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x2^{4\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{18 - 6\sqrt{6}}}{6}\right) (2 + x^2\sqrt{6}) \sqrt{\frac{3x^4 + 4x^2}{(2 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{3x^4 + 4x^2 + 2}}$$

command

```
integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} (\sqrt{-2} + 2) \sqrt{\sqrt{-2} - 2} \operatorname{ellipticF}\left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{-2} - 2}, \frac{2}{3} \sqrt{-2} + \frac{1}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{3x^4 + 4x^2 + 2}}, x\right)$$

27.29 Problem number 68

$$\int \frac{1}{\sqrt{2 + 3x^2 + 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x2^{4\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{8 - 2\sqrt{6}}}{4}\right) (2 + x^2\sqrt{6}) \sqrt{\frac{3x^4 + 3x^2}{(2 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{3x^4 + 3x^2 + 2}}$$

command

```
integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{24} \sqrt{2} (\sqrt{-15} + 3) \sqrt{\sqrt{-15} - 3} \operatorname{ellipticF} \left(\frac{1}{2} x \sqrt{\sqrt{-15} - 3}, \frac{1}{4} \sqrt{-15} - \frac{1}{4} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{3x^4 + 3x^2 + 2}}, x \right)$$

27.30 Problem number 69

$$\int \frac{1}{\sqrt{2 + 2x^2 + 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos \left(4 \arctan \left(\frac{x 2^{\frac{1}{4}}}{2} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{3^{\frac{1}{4}} 2^{\frac{3}{4}} x}{2} \right) \right), \frac{\sqrt{18 - 3\sqrt{6}}}{6} \right) (2 + x^2 \sqrt{6}) \sqrt{\frac{3x^4 + 2x^2}{(2 + x^2 \sqrt{6})}}}{12 \cos \left(2 \arctan \left(\frac{3^{\frac{1}{4}} 2^{\frac{3}{4}} x}{2} \right) \right) \sqrt{3x^4 + 2x^2 + 2}}$$

command

```
integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} (\sqrt{-5} + 1) \sqrt{\sqrt{-5} - 1} \operatorname{ellipticF} \left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{-5} - 1}, \frac{1}{3} \sqrt{-5} - \frac{2}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{3x^4 + 2x^2 + 2}}, x \right)$$

27.31 Problem number 70

$$\int \frac{1}{\sqrt{2+x^2+3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{x24^{\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{72-6\sqrt{6}}}{12}\right) (2+x^2\sqrt{6}) \sqrt{\frac{3x^4+x^2}{(2+x^2\sqrt{6})^2}}}{12 \cos\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{3x^4+x^2+2}}$$

command

```
integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{24} \sqrt{2} (\sqrt{-23} + 1) \sqrt{\sqrt{-23} - 1} \operatorname{ellipticF}\left(\frac{1}{2} x \sqrt{\sqrt{-23} - 1}, \frac{1}{12} \sqrt{-23} - \frac{11}{12}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{3x^4+x^2+2}}, x\right)$$

27.32 Problem number 71

$$\int \frac{1}{\sqrt{2+3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{x24^{\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{2}}{2}\right) (2+x^2\sqrt{6}) \sqrt{\frac{3x^4+2}{(2+x^2\sqrt{6})^2}} 6^{\frac{3}{4}}}{12 \cos\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{3x^4+2}}$$

command

```
integrate(1/(3*x^4+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6}(-6)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{1}{2}\sqrt{2}(-6)^{\frac{1}{4}}x, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{3x^4+2}}, x\right)$$

27.33 Problem number 72

$$\int \frac{1}{\sqrt{2-x^2+3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{x24^{\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{72+6\sqrt{6}}}{12}\right) (2+x^2\sqrt{6}) \sqrt{\frac{3x^4-x^2}{(2+x^2\sqrt{6})}}}{12 \cos\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{3x^4-x^2+2}}$$

command

```
integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{24}\sqrt{2}\sqrt{\sqrt{-23}+1}(\sqrt{-23}-1)\operatorname{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{-23}+1}, -\frac{1}{12}\sqrt{-23}-\frac{11}{12}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{3x^4-x^2+2}}, x\right)$$

27.34 Problem number 73

$$\int \frac{1}{\sqrt{2 - 2x^2 + 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x2^{4\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{18 + 3\sqrt{6}}}{6}\right) (2 + x^2\sqrt{6}) \sqrt{\frac{3x^4 - 2x^2}{(2 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{3x^4 - 2x^2 + 2}}$$

command

```
integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{\sqrt{-5} + 1} (\sqrt{-5} - 1) \operatorname{ellipticF}\left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{-5} + 1}, -\frac{1}{3} \sqrt{-5} - \frac{2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{3x^4 - 2x^2 + 2}}, x\right)$$

27.35 Problem number 74

$$\int \frac{1}{\sqrt{2 - 3x^2 + 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x2^{4\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{8 + 2\sqrt{6}}}{4}\right) (2 + x^2\sqrt{6}) \sqrt{\frac{3x^4 - 3x^2}{(2 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{3x^4 - 3x^2 + 2}}$$

command

```
integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{24} \sqrt{2} \sqrt{\sqrt{-15} + 3} (\sqrt{-15} - 3) \text{ellipticF} \left(\frac{1}{2} x \sqrt{\sqrt{-15} + 3}, -\frac{1}{4} \sqrt{-15} - \frac{1}{4} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\sqrt{3x^4 - 3x^2 + 2}}, x \right)$$

27.36 Problem number 75

$$\int \frac{1}{\sqrt{2 - 4x^2 + 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos \left(4 \arctan \left(\frac{x^{2\frac{1}{4}}}{2} \right) \right)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(2 \arctan \left(\frac{3^{\frac{1}{4}} 2^{\frac{3}{4}} x}{2} \right) \right), \frac{\sqrt{18 + 6\sqrt{6}}}{6} \right) (2 + x^2 \sqrt{6}) \sqrt{\frac{3x^4 - 4x^2}{(2 + x^2 \sqrt{6})}}}{12 \cos \left(2 \arctan \left(\frac{3^{\frac{1}{4}} 2^{\frac{3}{4}} x}{2} \right) \right) \sqrt{3x^4 - 4x^2 + 2}}$$

command

```
integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{\sqrt{-2} + 2} (\sqrt{-2} - 2) \text{ellipticF} \left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{-2} + 2}, -\frac{2}{3} \sqrt{-2} + \frac{1}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\sqrt{3x^4 - 4x^2 + 2}}, x \right)$$

27.37 Problem number 76

$$\int \frac{1}{\sqrt{2 - 5x^2 + 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x24^{\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{72 + 30\sqrt{6}}}{12}\right) (2 + x^2\sqrt{6}) \sqrt{\frac{3x^4 - 5x^2 + 2}{(2 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{3x^4 - 5x^2 + 2}}$$

command

```
integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \operatorname{ellipticF}\left(x, \frac{3}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{3x^4 - 5x^2 + 2}}, x\right)$$

27.38 Problem number 77

$$\int \frac{1}{\sqrt{2 - 6x^2 + 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x24^{\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{2 + \sqrt{6}}}{2}\right) (2 + x^2\sqrt{6}) \sqrt{\frac{3x^4 - 6x^2 + 2}{(2 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{3x^4 - 6x^2 + 2}}$$

command

```
integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{\sqrt{3} + 3} (\sqrt{3} - 3) \operatorname{ellipticF}\left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{3} + 3}, -\sqrt{3} + 2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{3x^4 - 6x^2 + 2}}, x\right)$$

27.39 Problem number 78

$$\int \frac{1}{\sqrt{3 + 9x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{36 + x^2(54 + 6\sqrt{57})}} \sqrt{36 + x^2(54 + 6\sqrt{57})} \operatorname{EllipticF}\left(\frac{x\sqrt{54 + 6\sqrt{57}}}{\sqrt{36 + x^2(54 + 6\sqrt{57})}}, \frac{\sqrt{-19 + 3\sqrt{57}}}{2}\right)}{\sqrt{2x^4 + 9x^2 + 3} \sqrt{54 + 6\sqrt{57}}} \quad (6)$$

command

```
integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{24} (\sqrt{19} \sqrt{6} + 3 \sqrt{6} \sqrt{3}) \sqrt{\sqrt{19} \sqrt{3} - 9} \operatorname{ellipticF}\left(\frac{1}{6} \sqrt{6} \sqrt{\sqrt{19} \sqrt{3} - 9} x, \frac{3}{4} \sqrt{19} \sqrt{3} + \frac{23}{4}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4 + 9x^2 + 3}}, x\right)$$

27.40 Problem number 79

$$\int \frac{1}{\sqrt{3 + 8x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{9 + x^2(12 + 3\sqrt{10})}} \sqrt{9 + x^2(12 + 3\sqrt{10})} \operatorname{EllipticF}\left(\frac{x\sqrt{12 + 3\sqrt{10}}}{\sqrt{9 + x^2(12 + 3\sqrt{10})}}, \frac{\sqrt{-30 + 12\sqrt{10}}}{3}\right) (3 + \sqrt{2x^4 + 8x^2 + 3}) \sqrt{12 + 3\sqrt{10}}}{\sqrt{2x^4 + 8x^2 + 3} \sqrt{12 + 3\sqrt{10}}}$$

command

```
integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} (\sqrt{10} + 4) \sqrt{\sqrt{10} - 4} \operatorname{ellipticF}\left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{10} - 4}, \frac{4}{3} \sqrt{10} + \frac{13}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4 + 8x^2 + 3}}, x\right)$$

27.41 Problem number 80

$$\int \frac{1}{\sqrt{3 + 7x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{(2x^2 + 1)^{\frac{3}{2}} \sqrt{\frac{1}{2x^2 + 1}} \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{2x^2 + 1}}, \frac{\sqrt{30}}{6}\right) \sqrt{\frac{x^2 + 3}{2x^2 + 1}} \sqrt{6}}{6 \sqrt{2x^4 + 7x^2 + 3}}$$

command

```
integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \operatorname{ellipticF}\left(\frac{1}{3} i \sqrt{3} x, 6\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4 + 7x^2 + 3}}, x\right)$$

27.42 Problem number 81

$$\int \frac{1}{\sqrt{3 + 6x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{9 + x^2(9 + 3\sqrt{3})}} \sqrt{9 + x^2(9 + 3\sqrt{3})} \operatorname{EllipticF}\left(\frac{x\sqrt{9 + 3\sqrt{3}}}{\sqrt{9 + x^2(9 + 3\sqrt{3})}}, \sqrt{\sqrt{3} - 1}\right) (3 + x^2(3 + \sqrt{3}))}{\sqrt{2x^4 + 6x^2 + 3} \sqrt{9 + 3\sqrt{3}}}$$

command

```
integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} (\sqrt{3} + 3) \sqrt{\sqrt{3} - 3} \operatorname{ellipticF}\left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{3} - 3}, \sqrt{3} + 2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4 + 6x^2 + 3}}, x\right)$$

27.43 Problem number 82

$$\int \frac{1}{\sqrt{3 + 5x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{(x^2 + 1)^{\frac{3}{2}} \sqrt{\frac{1}{x^2 + 1}} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2 + 1}}, \frac{\sqrt{3}}{3}\right) \sqrt{\frac{2x^2 + 3}{x^2 + 1}} \sqrt{3}}{3\sqrt{2x^4 + 5x^2 + 3}}$$

command

```
integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2} \sqrt{-2} \operatorname{ellipticF}\left(\frac{1}{3} \sqrt{3} \sqrt{-2} x, \frac{3}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4 + 5x^2 + 3}}, x\right)$$

27.44 Problem number 83

$$\int \frac{1}{\sqrt{3 + 4x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{18 - 6\sqrt{6}}}{6}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 + 4x^2}{(3 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{2x^4 + 4x^2 + 3}}$$

command

```
integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} (\sqrt{-2} + 2) \sqrt{\sqrt{-2} - 2} \operatorname{ellipticF}\left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{-2} - 2}, \frac{2}{3} \sqrt{-2} + \frac{1}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4 + 4x^2 + 3}}, x\right)$$

27.45 Problem number 84

$$\int \frac{1}{\sqrt{3 + 3x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{8 - 2\sqrt{6}}}{4}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 + 3x^2}{(3 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{2x^4 + 3x^2 + 3}}$$

command

```
integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{24} \sqrt{6} \sqrt{\sqrt{3} \sqrt{-5} - 3} (\sqrt{3} + \sqrt{-5}) \operatorname{ellipticF} \left(\frac{1}{6} \sqrt{6} \sqrt{\sqrt{3} \sqrt{-5} - 3} x, \frac{1}{4} \sqrt{3} \sqrt{-5} - \frac{1}{4} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{2x^4 + 3x^2 + 3}}, x \right)$$

27.46 Problem number 85

$$\int \frac{1}{\sqrt{3 + 2x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos \left(4 \arctan \left(\frac{x^{5/4}}{3} \right) \right)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{2^{1/4} 3^{3/4} x}{3} \right) \right), \frac{\sqrt{18 - 3\sqrt{6}}}{6} \right) (3 + x^2 \sqrt{6}) \sqrt{\frac{2x^4 + 2x^2}{(3 + x^2 \sqrt{6})}}}{12 \cos \left(2 \arctan \left(\frac{2^{1/4} 3^{3/4} x}{3} \right) \right) \sqrt{2x^4 + 2x^2 + 3}}$$

command

```
integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} (\sqrt{-5} + 1) \sqrt{\sqrt{-5} - 1} \operatorname{ellipticF} \left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{-5} - 1}, \frac{1}{3} \sqrt{-5} - \frac{2}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{2x^4 + 2x^2 + 3}}, x \right)$$

27.47 Problem number 86

$$\int \frac{1}{\sqrt{3+x^2+2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{x5^{1/4}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{2^{1/4}3^{3/4}x}{3}\right)\right), \frac{\sqrt{72-6\sqrt{6}}}{12}\right) (3+x^2\sqrt{6}) \sqrt{\frac{2x^4+x^2}{(3+x^2\sqrt{6})^2}}}{12 \cos\left(2\arctan\left(\frac{2^{1/4}3^{3/4}x}{3}\right)\right) \sqrt{2x^4+x^2+3}}$$

command

```
integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{72} \sqrt{6} \sqrt{3} (\sqrt{-23} + 1) \sqrt{\sqrt{-23} - 1} \operatorname{ellipticF}\left(\frac{1}{6} \sqrt{6} x \sqrt{\sqrt{-23} - 1}, \frac{1}{12} \sqrt{-23} - \frac{11}{12}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4+x^2+3}}, x\right)$$

27.48 Problem number 87

$$\int \frac{1}{\sqrt{3+2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{x5^{1/4}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{2^{1/4}3^{3/4}x}{3}\right)\right), \frac{\sqrt{2}}{2}\right) (3+x^2\sqrt{6}) \sqrt{\frac{2x^4+3}{(3+x^2\sqrt{6})^2}} 6^{3/4}}{12 \cos\left(2\arctan\left(\frac{2^{1/4}3^{3/4}x}{3}\right)\right) \sqrt{2x^4+3}}$$

command

```
integrate(1/(2*x^4+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{3} \sqrt{-2} \sqrt{\sqrt{3} \sqrt{-2}} \operatorname{ellipticF}\left(\frac{1}{3} \sqrt{3} \sqrt{\sqrt{3} \sqrt{-2}} x, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4+3}}, x\right)$$

27.49 Problem number 88

$$\int \frac{1}{\sqrt{3-x^2+2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x^{54\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} x}{3}\right)\right), \frac{\sqrt{72+6\sqrt{6}}}{12}\right) (3+x^2\sqrt{6}) \sqrt{\frac{2x^4-x^2}{(3+x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} x}{3}\right)\right) \sqrt{2x^4-x^2+3}}$$

command

```
integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{72} \sqrt{6} \sqrt{3} \sqrt{\sqrt{-23} + 1} (\sqrt{-23} - 1) \operatorname{ellipticF}\left(\frac{1}{6} \sqrt{6} x \sqrt{\sqrt{-23} + 1}, -\frac{1}{12} \sqrt{-23} - \frac{11}{12}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4-x^2+3}}, x\right)$$

27.50 Problem number 89

$$\int \frac{1}{\sqrt{3 - 2x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{18 + 3\sqrt{6}}}{6}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 - 2x^2}{(3 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{2x^4 - 2x^2 + 3}}$$

command

```
integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{\sqrt{-5} + 1} (\sqrt{-5} - 1) \operatorname{ellipticF}\left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{-5} + 1}, -\frac{1}{3} \sqrt{-5} - \frac{2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4 - 2x^2 + 3}}, x\right)$$

27.51 Problem number 90

$$\int \frac{1}{\sqrt{3 - 3x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{8 + 2\sqrt{6}}}{4}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 - 3x^2}{(3 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{2x^4 - 3x^2 + 3}}$$

command

```
integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{24} \sqrt{6} \sqrt{\sqrt{3} \sqrt{-5} + 3} (\sqrt{3} - \sqrt{-5}) \operatorname{ellipticF} \left(\frac{1}{6} \sqrt{6} \sqrt{\sqrt{3} \sqrt{-5} + 3} x, -\frac{1}{4} \sqrt{3} \sqrt{-5} - \frac{1}{4} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{2x^4 - 3x^2 + 3}}, x \right)$$

27.52 Problem number 91

$$\int \frac{1}{\sqrt{3 - 4x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos \left(4 \arctan \left(\frac{x^{5/4}}{3} \right) \right)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{2^{1/4} 3^{3/4} x}{3} \right) \right), \frac{\sqrt{18 + 6\sqrt{6}}}{6} \right) (3 + x^2 \sqrt{6}) \sqrt{\frac{2x^4 - 4x^2}{(3 + x^2 \sqrt{6})}}}{12 \cos \left(2 \arctan \left(\frac{2^{1/4} 3^{3/4} x}{3} \right) \right) \sqrt{2x^4 - 4x^2 + 3}}$$

command

```
integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{\sqrt{-2} + 2} (\sqrt{-2} - 2) \operatorname{ellipticF} \left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{-2} + 2}, -\frac{2}{3} \sqrt{-2} + \frac{1}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{2x^4 - 4x^2 + 3}}, x \right)$$

27.53 Problem number 92

$$\int \frac{1}{\sqrt{3 - 5x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{72 + 30\sqrt{6}}}{12}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 - 5x^2 + 3}{(3 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{2x^4 - 5x^2 + 3}}$$

command

```
integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{3} \operatorname{ellipticF}\left(x, \frac{2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4 - 5x^2 + 3}}, x\right)$$

27.54 Problem number 93

$$\int \frac{1}{\sqrt{3 - 6x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{2 + \sqrt{6}}}{2}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 - 6x^2 + 3}{(3 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{2x^4 - 6x^2 + 3}}$$

command

```
integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{\sqrt{3} + 3} (\sqrt{3} - 3) \operatorname{ellipticF}\left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{3} + 3}, -\sqrt{3} + 2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4 - 6x^2 + 3}}, x\right)$$

27.55 Problem number 94

$$\int \frac{1}{\sqrt{3 - 7x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x^{5/4}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{1/4} 3^{3/4} x}{3}\right)\right), \frac{\sqrt{72 + 42\sqrt{6}}}{12}\right) (3 + x^2 \sqrt{6}) \sqrt{\frac{2x^4 - 7x^2 + 3}{(3 + x^2 \sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{1/4} 3^{3/4} x}{3}\right)\right) \sqrt{2x^4 - 7x^2 + 3}}$$

command

```
integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{3} \sqrt{2} \operatorname{ellipticF}\left(\sqrt{2} x, \frac{1}{6}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4 - 7x^2 + 3}}, x\right)$$

27.56 Problem number 98

$$\int \frac{1}{\sqrt{-3 + 4x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{18 + 6\sqrt{6}}}{6}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 - 4x^2}{(3 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{-2x^4 + 4x^2 - 3}}$$

command

```
integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{18} \sqrt{3} (\sqrt{-2} \sqrt{-3} - 2 \sqrt{-3}) \sqrt{\sqrt{-2} + 2} \operatorname{ellipticF}\left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{-2} + 2}, -\frac{2}{3} \sqrt{-2} + \frac{1}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2x^4 + 4x^2 - 3}}{2x^4 - 4x^2 + 3}, x\right)$$

27.57 Problem number 99

$$\int \frac{1}{\sqrt{-3 + 3x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{8 + 2\sqrt{6}}}{4}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 - 3x^2}{(3 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{-2x^4 + 3x^2 - 3}}$$

command

```
integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{72} \sqrt{6} \left(\sqrt{3} \sqrt{-3} \sqrt{-5} - 3 \sqrt{-3} \right) \sqrt{\sqrt{3} \sqrt{-5} + 3} \operatorname{ellipticF} \left(\frac{1}{6} \sqrt{6} \sqrt{\sqrt{3} \sqrt{-5} + 3} x, \right. \\ \left. -\frac{1}{4} \sqrt{3} \sqrt{-5} - \frac{1}{4} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-2x^4 + 3x^2 - 3}}{2x^4 - 3x^2 + 3}, x \right)$$

27.58 Problem number 100

$$\int \frac{1}{\sqrt{-3 + 2x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos \left(4 \arctan \left(\frac{x^{54\frac{1}{4}}}{3} \right) \right)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} x}{3} \right) \right), \frac{\sqrt{18 + 3\sqrt{6}}}{6} \right) (3 + x^2 \sqrt{6}) \sqrt{\frac{2x^4 - 2x^2}{(3 + x^2 \sqrt{6})}}}{12 \cos \left(2 \arctan \left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} x}{3} \right) \right) \sqrt{-2x^4 + 2x^2 - 3}}$$

command

```
integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{18} \sqrt{3} \sqrt{-3} \sqrt{\sqrt{-5} + 1} (\sqrt{-5} - 1) \operatorname{ellipticF} \left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{-5} + 1}, -\frac{1}{3} \sqrt{-5} - \frac{2}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-2x^4 + 2x^2 - 3}}{2x^4 - 2x^2 + 3}, x \right)$$

27.59 Problem number 101

$$\int \frac{1}{\sqrt{-3 + x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{72 + 6\sqrt{6}}}{12}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 - x^2}{(3 + x^2\sqrt{6})^2}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{-2x^4 + x^2 - 3}}$$

command

```
integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{72} \sqrt{6} \sqrt{-3} \sqrt{\sqrt{-23} + 1} (\sqrt{-23} - 1) \operatorname{ellipticF}\left(\frac{1}{6} \sqrt{6} x \sqrt{\sqrt{-23} + 1}, -\frac{1}{12} \sqrt{-23} - \frac{11}{12}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2x^4 + x^2 - 3}}{2x^4 - x^2 + 3}, x\right)$$

27.60 Problem number 102

$$\int \frac{1}{\sqrt{-3 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{2}}{2}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 + 3}{(3 + x^2\sqrt{6})^2}} 6^{\frac{3}{4}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{-2x^4 - 3}}$$

command

```
integrate(1/(-2*x^4-3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{-2} \sqrt{-3} \sqrt{\sqrt{3} \sqrt{-2}} \operatorname{ellipticF}\left(\frac{1}{3} \sqrt{3} \sqrt{\sqrt{3} \sqrt{-2}} x, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2x^4-3}}{2x^4+3}, x\right)$$

27.61 Problem number 103

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x^{5/4}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{1/4} 3^{3/4} x}{3}\right)\right), \frac{\sqrt{72-6\sqrt{6}}}{12}\right) (3+x^2\sqrt{6}) \sqrt{\frac{2x^4+x^2}{(3+x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{1/4} 3^{3/4} x}{3}\right)\right) \sqrt{-2x^4-x^2-3}}$$

command

```
integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{72} \sqrt{6} \sqrt{-3} (\sqrt{-23} + 1) \sqrt{\sqrt{-23} - 1} \operatorname{ellipticF}\left(\frac{1}{6} \sqrt{6} x \sqrt{\sqrt{-23} - 1}, \frac{1}{12} \sqrt{-23} - \frac{11}{12}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2x^4-x^2-3}}{2x^4+x^2+3}, x\right)$$

27.62 Problem number 104

$$\int \frac{1}{\sqrt{-3 - 2x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{18 - 3\sqrt{6}}}{6}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 + 2x^2}{(3 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{-2x^4 - 2x^2 - 3}}$$

command

```
integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{18} \sqrt{3} \sqrt{-3} (\sqrt{-5} + 1) \sqrt{\sqrt{-5} - 1} \operatorname{ellipticF}\left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{-5} - 1}, \frac{1}{3} \sqrt{-5} - \frac{2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2x^4 - 2x^2 - 3}}{2x^4 + 2x^2 + 3}, x\right)$$

27.63 Problem number 105

$$\int \frac{1}{\sqrt{-3 - 3x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x5^{4\frac{1}{4}}}{3}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right), \frac{\sqrt{8 - 2\sqrt{6}}}{4}\right) (3 + x^2\sqrt{6}) \sqrt{\frac{2x^4 + 3x^2}{(3 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{2^{\frac{1}{4}}3^{\frac{3}{4}}x}{3}\right)\right) \sqrt{-2x^4 - 3x^2 - 3}}$$

command

```
integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{72} \sqrt{6} \left(\sqrt{3} \sqrt{-3} \sqrt{-5} + 3 \sqrt{-3} \right) \sqrt{\sqrt{3} \sqrt{-5} - 3} \operatorname{ellipticF} \left(\frac{1}{6} \sqrt{6} \sqrt{\sqrt{3} \sqrt{-5} - 3} x, \frac{1}{4} \sqrt{3} \sqrt{-5} - \frac{1}{4} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-2x^4 - 3x^2 - 3}}{2x^4 + 3x^2 + 3}, x \right)$$

27.64 Problem number 106

$$\int \frac{1}{\sqrt{-3 - 4x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos \left(4 \arctan \left(\frac{x^{5/4}}{3} \right) \right)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{2^{1/4} 3^{3/4} x}{3} \right) \right), \frac{\sqrt{18 - 6\sqrt{6}}}{6} \right) (3 + x^2 \sqrt{6}) \sqrt{\frac{2x^4 + 4x^2}{(3 + x^2 \sqrt{6})}}}{12 \cos \left(2 \arctan \left(\frac{2^{1/4} 3^{3/4} x}{3} \right) \right) \sqrt{-2x^4 - 4x^2 - 3}}$$

command

```
integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{18} \sqrt{3} \left(\sqrt{-2} \sqrt{-3} + 2 \sqrt{-3} \right) \sqrt{\sqrt{-2} - 2} \operatorname{ellipticF} \left(\frac{1}{3} \sqrt{3} x \sqrt{\sqrt{-2} - 2}, \frac{2}{3} \sqrt{-2} + \frac{1}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-2x^4 - 4x^2 - 3}}{2x^4 + 4x^2 + 3}, x \right)$$

27.65 Problem number 107

$$\int \frac{1}{\sqrt{-3 - 5x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{x^2+1}} \sqrt{x^2+1} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2+1}}, \frac{\sqrt{3}}{3}\right) \sqrt{2x^2+3} \sqrt{3}}{3\sqrt{-x^2-1} \sqrt{\frac{2x^2+3}{x^2+1}}}$$

command

```
integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{3} \sqrt{-2} \sqrt{-3} \operatorname{ellipticF}\left(\frac{1}{3} \sqrt{3} \sqrt{-2} x, \frac{3}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2x^4 - 5x^2 - 3}}{2x^4 + 5x^2 + 3}, x\right)$$

27.66 Problem number 110

$$\int \frac{1}{\sqrt{-2 + 4x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x^2 2^{\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{3^{\frac{1}{4}} 2^{\frac{3}{4}} x}{2}\right)\right), \frac{\sqrt{18 + 6\sqrt{6}}}{6}\right) (2 + x^2 \sqrt{6}) \sqrt{\frac{3x^4 - 4x^2}{(2 + x^2 \sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{3^{\frac{1}{4}} 2^{\frac{3}{4}} x}{2}\right)\right) \sqrt{-3x^4 + 4x^2 - 2}}$$

command

```
integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{2} \sqrt{\sqrt{-2} + 2} (\sqrt{-2} + 1) \operatorname{ellipticF} \left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{-2} + 2}, -\frac{2}{3} \sqrt{-2} + \frac{1}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-3x^4 + 4x^2 - 2}}{3x^4 - 4x^2 + 2}, x \right)$$

27.67 Problem number 111

$$\int \frac{1}{\sqrt{-2 + 3x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos \left(4 \arctan \left(\frac{x 2^{3/4}}{2} \right) \right)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{3^{1/4} 2^{3/4} x}{2} \right) \right), \frac{\sqrt{8 + 2\sqrt{6}}}{4} \right) (2 + x^2 \sqrt{6}) \sqrt{\frac{3x^4 - 3x^2 - 2}{(2 + x^2 \sqrt{6})}}}{12 \cos \left(2 \arctan \left(\frac{3^{1/4} 2^{3/4} x}{2} \right) \right) \sqrt{-3x^4 + 3x^2 - 2}}$$

command

```
integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{24} \sqrt{-2} \sqrt{\sqrt{-15} + 3} (\sqrt{-15} - 3) \operatorname{ellipticF} \left(\frac{1}{2} x \sqrt{\sqrt{-15} + 3}, -\frac{1}{4} \sqrt{-15} - \frac{1}{4} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-3x^4 + 3x^2 - 2}}{3x^4 - 3x^2 + 2}, x \right)$$

27.68 Problem number 112

$$\int \frac{1}{\sqrt{-2 + 2x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{x^{24\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{18+3\sqrt{6}}}{6}\right) (2+x^2\sqrt{6}) \sqrt{\frac{3x^4-2x^2}{(2+x^2\sqrt{6})}}}{12\cos\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{-3x^4+2x^2-2}}$$

command

```
integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{12} \sqrt{2} \sqrt{-2} \sqrt{\sqrt{-5} + 1} (\sqrt{-5} - 1) \operatorname{ellipticF}\left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{-5} + 1}, -\frac{1}{3} \sqrt{-5} - \frac{2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-3x^4+2x^2-2}}{3x^4-2x^2+2}, x\right)$$

27.69 Problem number 113

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{x^{24\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{72+6\sqrt{6}}}{12}\right) (2+x^2\sqrt{6}) \sqrt{\frac{3x^4-x^2}{(2+x^2\sqrt{6})}}}{12\cos\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{-3x^4+x^2-2}}$$

command

```
integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{24} \sqrt{-2} \sqrt{\sqrt{-23} + 1} (\sqrt{-23} - 1) \operatorname{ellipticF}\left(\frac{1}{2} x \sqrt{\sqrt{-23} + 1}, -\frac{1}{12} \sqrt{-23} - \frac{11}{12}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-3x^4+x^2-2}}{3x^4-x^2+2}, x\right)$$

27.70 Problem number 114

$$\int \frac{1}{\sqrt{-2-3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{x2^{4\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{2}}{2}\right) (2+x^2\sqrt{6}) \sqrt{\frac{3x^4+2}{(2+x^2\sqrt{6})^2}} 6^{\frac{3}{4}}}{12 \cos\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{-3x^4-2}}$$

command

```
integrate(1/(-3*x^4-2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{12} \sqrt{2} \sqrt{-2} (-6)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{1}{2} \sqrt{2} (-6)^{\frac{1}{4}} x, -1\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-3x^4-2}}{3x^4+2}, x\right)$$

27.71 Problem number 115

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{x2^{4\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{72-6\sqrt{6}}}{12}\right) (2+x^2\sqrt{6}) \sqrt{\frac{3x^4+x^2}{(2+x^2\sqrt{6})^2}}}{12 \cos\left(2\arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{-3x^4-x^2-2}}$$

command

```
integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{24} \sqrt{-2} (\sqrt{-23} + 1) \sqrt{\sqrt{-23} - 1} \operatorname{ellipticF} \left(\frac{1}{2} x \sqrt{\sqrt{-23} - 1}, \frac{1}{12} \sqrt{-23} - \frac{11}{12} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-3x^4 - x^2 - 2}}{3x^4 + x^2 + 2}, x \right)$$

27.72 Problem number 116

$$\int \frac{1}{\sqrt{-2 - 2x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos \left(4 \arctan \left(\frac{x 2^{3/4}}{2} \right) \right)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{3^{1/4} 2^{3/4} x}{2} \right) \right), \frac{\sqrt{18 - 3\sqrt{6}}}{6} \right) (2 + x^2 \sqrt{6}) \sqrt{\frac{3x^4 + 2x^2}{(2 + x^2 \sqrt{6})}}}{12 \cos \left(2 \arctan \left(\frac{3^{1/4} 2^{3/4} x}{2} \right) \right) \sqrt{-3x^4 - 2x^2 - 2}}$$

command

```
integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{12} \sqrt{2} \sqrt{-2} (\sqrt{-5} + 1) \sqrt{\sqrt{-5} - 1} \operatorname{ellipticF} \left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{-5} - 1}, \frac{1}{3} \sqrt{-5} - \frac{2}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-3x^4 - 2x^2 - 2}}{3x^4 + 2x^2 + 2}, x \right)$$

27.73 Problem number 117

$$\int \frac{1}{\sqrt{-2 - 3x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x2^{4\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{8 - 2\sqrt{6}}}{4}\right) (2 + x^2\sqrt{6}) \sqrt{\frac{3x^4 + 3x^2 - 2}{(2 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{-3x^4 - 3x^2 - 2}}$$

command

```
integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{24} \sqrt{-2} (\sqrt{-15} + 3) \sqrt{\sqrt{-15} - 3} \operatorname{ellipticF}\left(\frac{1}{2} x \sqrt{\sqrt{-15} - 3}, \frac{1}{4} \sqrt{-15} - \frac{1}{4}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-3x^4 - 3x^2 - 2}}{3x^4 + 3x^2 + 2}, x\right)$$

27.74 Problem number 118

$$\int \frac{1}{\sqrt{-2 - 4x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{x2^{4\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{18 - 6\sqrt{6}}}{6}\right) (2 + x^2\sqrt{6}) \sqrt{\frac{3x^4 + 4x^2 - 2}{(2 + x^2\sqrt{6})}}}{12 \cos\left(2 \arctan\left(\frac{3^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{-3x^4 - 4x^2 - 2}}$$

command

```
integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{2} (\sqrt{-2} - 1) \sqrt{\sqrt{-2} - 2} \operatorname{ellipticF} \left(\frac{1}{2} \sqrt{2} x \sqrt{\sqrt{-2} - 2}, \frac{2}{3} \sqrt{-2} + \frac{1}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-3x^4 - 4x^2 - 2}}{3x^4 + 4x^2 + 2}, x \right)$$

27.75 Problem number 119

$$\int \frac{1}{\sqrt{-2 - 5x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{x^2 + 1}} \operatorname{EllipticF} \left(\frac{x}{\sqrt{x^2 + 1}}, \frac{i\sqrt{2}}{2} \right) \sqrt{-3x^2 - 2} \sqrt{2}}{2 \sqrt{\frac{3x^2 + 2}{x^2 + 1}}}$$

command

```
integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} i \sqrt{-2} \operatorname{ellipticF} \left(i x, \frac{3}{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-3x^4 - 5x^2 - 2}}{3x^4 + 5x^2 + 2}, x \right)$$

27.76 Problem number 120

$$\int \frac{1}{\sqrt{2 + 5x^2 + 5x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{x40^{\frac{1}{4}}}{2}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{5^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right), \frac{\sqrt{8-2\sqrt{10}}}{4}\right) (2+x^2\sqrt{10}) \sqrt{\frac{5x^4+5x^2+2}{(2+x^2\sqrt{10})^2}}}{20\cos\left(2\arctan\left(\frac{5^{\frac{1}{4}}2^{\frac{3}{4}}x}{2}\right)\right) \sqrt{5x^4+5x^2+2}}$$

command

`integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{40}\sqrt{2}(\sqrt{-15}+5)\sqrt{\sqrt{-15}-5}\operatorname{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{-15}-5}, \frac{1}{4}\sqrt{-15}+\frac{1}{4}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{5x^4+5x^2+2}}, x\right)$$

27.77 Problem number 121

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4\arctan\left(2^{\frac{1}{4}}x\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(2^{\frac{1}{4}}x\right)\right), \frac{\sqrt{8-5\sqrt{2}}}{4}\right) (1+x^2\sqrt{2}) \sqrt{\frac{4x^4+5x^2+2}{(1+x^2\sqrt{2})^2}}}{4\cos\left(2\arctan\left(2^{\frac{1}{4}}x\right)\right) \sqrt{4x^4+5x^2+2}}$$

command

`integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{32}\sqrt{2}(\sqrt{-7}+5)\sqrt{\sqrt{-7}-5}\operatorname{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{-7}-5}, \frac{5}{16}\sqrt{-7}+\frac{9}{16}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{4x^4+5x^2+2}}, x\right)$$

27.78 Problem number 122

$$\int \frac{1}{\sqrt{2 + 5x^2 + 3x^4}} dx$$

Optimal antiderivative

$$\frac{(x^2 + 1)^{\frac{3}{2}} \sqrt{\frac{1}{x^2 + 1}} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2 + 1}}, \frac{i\sqrt{2}}{2}\right) \sqrt{\frac{3x^2 + 2}{x^2 + 1}} \sqrt{2}}{2\sqrt{3x^4 + 5x^2 + 2}}$$

command

```
integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2}i\sqrt{2} \operatorname{ellipticF}\left(ix, \frac{3}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{3x^4 + 5x^2 + 2}}, x\right)$$

27.79 Problem number 123

$$\int \frac{1}{\sqrt{2 + 5x^2 + 2x^4}} dx$$

Optimal antiderivative

$$\frac{(2x^2 + 1)^{\frac{3}{2}} \sqrt{\frac{1}{2x^2 + 1}} \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{2x^2 + 1}}, \frac{\sqrt{3}}{2}\right) \sqrt{\frac{x^2 + 2}{2x^2 + 1}}}{2\sqrt{2x^4 + 5x^2 + 2}}$$

command

```
integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \operatorname{ellipticF}\left(\frac{1}{2}i\sqrt{2}x, 4\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2x^4 + 5x^2 + 2}}, x\right)$$

27.80 Problem number 124

$$\int \frac{1}{\sqrt{2+5x^2+x^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{4+x^2(5+\sqrt{17})}} (4+x^2(5+\sqrt{17}))^{\frac{3}{2}} \text{EllipticF}\left(\frac{x\sqrt{5+\sqrt{17}}}{\sqrt{4+x^2(5+\sqrt{17})}}, \frac{\sqrt{-17+5\sqrt{17}}}{2}\right) \sqrt{\frac{4+x^2(5+\sqrt{17})}{4+x^2(5+\sqrt{17})}}}{2\sqrt{x^4+5x^2+2} \sqrt{5+\sqrt{17}}}$$

command

```
integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{8} (\sqrt{17} \sqrt{2} + 5 \sqrt{2}) \sqrt{\sqrt{17} - 5} \text{ellipticF}\left(\frac{1}{2} x \sqrt{\sqrt{17} - 5}, \frac{5}{4} \sqrt{17} + \frac{21}{4}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{1}{\sqrt{x^4+5x^2+2}}, x\right)$$

27.81 Problem number 125

$$\int \frac{1}{\sqrt{2+5x^2-x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{5+\sqrt{33}}}, \frac{5i\sqrt{2}}{4} + \frac{i\sqrt{66}}{4}\right) \sqrt{2}}{\sqrt{-5+\sqrt{33}}}$$

command

```
integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{8} \left(\sqrt{33} \sqrt{2} + 5 \sqrt{2} \right) \sqrt{\sqrt{33} - 5} \operatorname{ellipticF} \left(\frac{1}{2} x \sqrt{\sqrt{33} - 5}, -\frac{5}{4} \sqrt{33} - \frac{29}{4} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-x^4 + 5x^2 + 2}}{x^4 - 5x^2 - 2}, x \right)$$

27.82 Problem number 126

$$\int \frac{1}{\sqrt{2 + 5x^2 - 2x^4}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticF} \left(\frac{2x}{\sqrt{5 + \sqrt{41}}}, \frac{5i}{4} + \frac{i\sqrt{41}}{4} \right) \sqrt{2}}{\sqrt{-5 + \sqrt{41}}}$$

command

```
integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{16} \left(\sqrt{41} \sqrt{2} + 5 \sqrt{2} \right) \sqrt{\sqrt{41} - 5} \operatorname{ellipticF} \left(\frac{1}{2} x \sqrt{\sqrt{41} - 5}, -\frac{5}{8} \sqrt{41} - \frac{33}{8} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{-2x^4 + 5x^2 + 2}}{2x^4 - 5x^2 - 2}, x \right)$$

27.83 Problem number 127

$$\int \frac{1}{\sqrt{2 + 5x^2 - 3x^4}} dx$$

Optimal antiderivative

$$\operatorname{EllipticF} \left(\frac{x\sqrt{2}}{2}, i\sqrt{6} \right)$$

command

`integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\text{ellipticF}\left(\frac{1}{2}\sqrt{2}x, -6\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-3x^4+5x^2+2}}{3x^4-5x^2-2}, x\right)$$

27.84 Problem number 128

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{2x\sqrt{2}}{\sqrt{5+\sqrt{57}}}, \frac{5i\sqrt{2}}{8} + \frac{i\sqrt{114}}{8}\right)\sqrt{2}}{\sqrt{-5+\sqrt{57}}}$$

command

`integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{32} \left(\sqrt{57} \sqrt{2} + 5 \sqrt{2} \right) \sqrt{\sqrt{57} - 5} \text{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{57} - 5}, -\frac{5}{16}\sqrt{57} - \frac{41}{16}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-4x^4+5x^2+2}}{4x^4-5x^2-2}, x\right)$$

27.85 Problem number 129

$$\int \frac{1}{\sqrt{2 + 5x^2 - 5x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{10}}{\sqrt{5 + \sqrt{65}}}, \frac{i\sqrt{10}}{4} + \frac{i\sqrt{26}}{4}\right) \sqrt{2}}{\sqrt{-5 + \sqrt{65}}}$$

command

`integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{40} \left(\sqrt{65} \sqrt{2} + 5 \sqrt{2} \right) \sqrt{\sqrt{65} - 5} \text{ellipticF}\left(\frac{1}{2} x \sqrt{\sqrt{65} - 5}, -\frac{1}{4} \sqrt{65} - \frac{9}{4}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-5x^4 + 5x^2 + 2}}{5x^4 - 5x^2 - 2}, x\right)$$

27.86 Problem number 130

$$\int \frac{1}{\sqrt{2 + 5x^2 - 6x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{2x\sqrt{3}}{\sqrt{5 + \sqrt{73}}}, \frac{5i\sqrt{3}}{12} + \frac{i\sqrt{219}}{12}\right) \sqrt{2}}{\sqrt{-5 + \sqrt{73}}}$$

command

`integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{48} \left(\sqrt{73} \sqrt{2} + 5 \sqrt{2} \right) \sqrt{\sqrt{73} - 5} \text{ellipticF}\left(\frac{1}{2} x \sqrt{\sqrt{73} - 5}, -\frac{5}{24} \sqrt{73} - \frac{49}{24}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-6x^4 + 5x^2 + 2}}{6x^4 - 5x^2 - 2}, x\right)$$

27.87 Problem number 131

$$\int \frac{1}{\sqrt{2 + 5x^2 - 7x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(x, \frac{i\sqrt{14}}{2}\right) \sqrt{2}}{2}$$

command

`integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{2} \text{ellipticF}\left(x, -\frac{7}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-7x^4 + 5x^2 + 2}}{7x^4 - 5x^2 - 2}, x\right)$$

27.88 Problem number 132

$$\int \frac{1}{\sqrt{2 + 5x^2 - 8x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{4x}{\sqrt{5 + \sqrt{89}}}, \frac{5i}{8} + \frac{i\sqrt{89}}{8}\right) \sqrt{2}}{\sqrt{-5 + \sqrt{89}}}$$

command

`integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{64} \left(\sqrt{89} \sqrt{2} + 5 \sqrt{2} \right) \sqrt{\sqrt{89} - 5} \text{ellipticF}\left(\frac{1}{2} x \sqrt{\sqrt{89} - 5}, -\frac{5}{32} \sqrt{89} - \frac{57}{32}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-8x^4 + 5x^2 + 2}}{8x^4 - 5x^2 - 2}, x\right)$$

27.89 Problem number 133

$$\int \frac{1}{\sqrt{2 + 5x^2 - 9x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{3x\sqrt{2}}{\sqrt{5 + \sqrt{97}}}, \frac{5i\sqrt{2}}{12} + \frac{i\sqrt{194}}{12}\right) \sqrt{2}}{\sqrt{-5 + \sqrt{97}}}$$

command

```
integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{72} \left(\sqrt{97} \sqrt{2} + 5 \sqrt{2} \right) \sqrt{\sqrt{97} - 5} \text{ellipticF}\left(\frac{1}{2} x \sqrt{\sqrt{97} - 5}, -\frac{5}{36} \sqrt{97} - \frac{61}{36}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-9x^4 + 5x^2 + 2}}{9x^4 - 5x^2 - 2}, x\right)$$

27.90 Problem number 352

$$\int x^{7/2} \sqrt{bx^2 + cx^4} dx$$

Optimal antiderivative

$$\frac{28b^3x^{\frac{3}{2}}(cx^2+b)}{195c^{\frac{5}{2}}(\sqrt{b}+x\sqrt{c})\sqrt{cx^4+bx^2}} + \frac{4bx^{\frac{5}{2}}\sqrt{cx^4+bx^2}}{117c}$$

$$+ \frac{2x^{\frac{9}{2}}\sqrt{cx^4+bx^2}}{13} - \frac{28b^2\sqrt{x}\sqrt{cx^4+bx^2}}{585c^2}$$

$$28b^{\frac{13}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b}+x\sqrt{c})}}$$

$$195\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4+bx^2}$$

$$14b^{\frac{13}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b}+x\sqrt{c})}}$$

$$195\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4+bx^2}$$

command

`integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(42b^3\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c},0,\operatorname{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)-(45c^3x^4+10bc^2x^2-14b^2c)\sqrt{cx^4+bx^2}\right)}{585c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^4+bx^2}x^{\frac{7}{2}},x\right)$$

27.91 Problem number 353

$$\int x^{5/2}\sqrt{bx^2+cx^4}dx$$

Optimal antiderivative

$$\frac{4bx^{\frac{3}{2}}\sqrt{cx^4+bx^2}}{77c} + \frac{2x^{\frac{7}{2}}\sqrt{cx^4+bx^2}}{11} - \frac{20b^2\sqrt{cx^4+bx^2}}{231c^2\sqrt{x}}$$

$$10b^{\frac{11}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b}+x\sqrt{c})}}$$

$$231\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{9}{4}}\sqrt{cx^4+bx^2}$$

command

```
integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10 b^3 \sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (21 c^3 x^4 + 6 b c^2 x^2 - 10 b^2 c) \sqrt{c x^4 + b x^2} \sqrt{x} \right)}{231 c^3 x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{c x^4 + b x^2} x^{\frac{5}{2}}, x\right)$$

27.92 Problem number 354

$$\int x^{3/2} \sqrt{b x^2 + c x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4 b^2 x^{\frac{3}{2}} (c x^2 + b)}{15 c^{\frac{3}{2}} (\sqrt{b} + x \sqrt{c}) \sqrt{c x^4 + b x^2}} + \frac{2 x^{\frac{5}{2}} \sqrt{c x^4 + b x^2}}{9} + \frac{4 b \sqrt{x} \sqrt{c x^4 + b x^2}}{45 c} \\ & + \frac{4 b^{\frac{9}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2 + b}{(\sqrt{b} + x \sqrt{c})}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{c x^4 + b x^2}} \\ & + \frac{2 b^{\frac{9}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2 - b}{(\sqrt{b} + x \sqrt{c})}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{c x^4 + b x^2}} \end{aligned}$$

command

```
integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 b^2 \sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{c x^4 + b x^2} (5 c^2 x^2 + 2 b c) \sqrt{x} \right)}{45 c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{c x^4 + b x^2} x^{\frac{3}{2}}, x\right)$$

27.93 Problem number 355

$$\int \sqrt{x} \sqrt{bx^2 + cx^4} dx$$

Optimal antiderivative

$$\frac{2x^{\frac{3}{2}} \sqrt{cx^4 + bx^2}}{7} + \frac{4b \sqrt{cx^4 + bx^2}}{21c\sqrt{x}}$$

$$\frac{2b^{\frac{7}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 - (\sqrt{b} + x\sqrt{c})}{(\sqrt{b} + x\sqrt{c})}}}{21 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{5}{4}} \sqrt{cx^4 + bx^2}}$$

command

```
integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2b^2 \sqrt{c} \operatorname{xweierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - \sqrt{cx^4 + bx^2} (3c^2x^2 + 2bc) \sqrt{x} \right)}{21c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^4 + bx^2} \sqrt{x}, x\right)$$

27.94 Problem number 356

$$\int \frac{\sqrt{bx^2 + cx^4}}{\sqrt{x}} dx$$

Optimal antiderivative

$$\frac{4bx^{\frac{3}{2}}(cx^2+b)}{5\sqrt{c}(\sqrt{b}+x\sqrt{c})\sqrt{cx^4+bx^2}} + \frac{2\sqrt{x}\sqrt{cx^4+bx^2}}{5}$$

$$4b^{\frac{5}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b}+x\sqrt{c})}}$$

$$5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{cx^4+bx^2}$$

$$2b^{\frac{5}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b}+x\sqrt{c})}}$$

$$5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{cx^4+bx^2}$$

command

```
integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2b\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c},0,\operatorname{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}c\sqrt{x}\right)}{5c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}}{\sqrt{x}},x\right)$$

27.95 Problem number 357

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{cx^4+bx^2}}{3\sqrt{x}}$$

$$2b^{\frac{3}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b}+x\sqrt{c})}}$$

$$3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{1}{4}}\sqrt{cx^4+bx^2}$$

command

```
integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2b\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + \sqrt{cx^4 + bx^2} c\sqrt{x} \right)}{3cx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{3}{2}}}, x\right)$$

27.96 Problem number 358

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^{5/2}} dx$$

Optimal antiderivative

$$\frac{4x^{\frac{3}{2}}(cx^2 + b)\sqrt{c}}{(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{2\sqrt{cx^4 + bx^2}}{x^{\frac{3}{2}}}$$

$$\frac{4b^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)\sqrt{cx^4 + bx^2}}$$

$$+ \frac{2b^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)\sqrt{cx^4 + bx^2}}$$

command

```
integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2\sqrt{c} x^2 \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{cx^4 + bx^2} \sqrt{x} \right)}{x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{5}{2}}}, x\right)$$

27.97 Problem number 359

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^{7/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{cx^4 + bx^2}}{3x^{\frac{5}{2}}} + \frac{2c^{\frac{3}{4}}x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})^2}}}{3 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{1}{4}} \sqrt{cx^4 + bx^2}}$$

command

```
integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{c} x^3 \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - \sqrt{cx^4 + bx^2} \sqrt{x} \right)}{3x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{7}{2}}}, x\right)$$

27.98 Problem number 360

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^{9/2}} dx$$

Optimal antiderivative

$$\frac{4c^{\frac{3}{2}}x^{\frac{3}{2}}(cx^2+b)}{5b(\sqrt{b}+x\sqrt{c})\sqrt{cx^4+bx^2}} - \frac{2\sqrt{cx^4+bx^2}}{5x^{\frac{7}{2}}} - \frac{4c\sqrt{cx^4+bx^2}}{5bx^{\frac{3}{2}}}$$

$$\frac{4c^{\frac{5}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2-b}{(\sqrt{b}+x\sqrt{c})^2}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{3}{4}}\sqrt{cx^4+bx^2}}$$

$$+ \frac{2c^{\frac{5}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2+b}{(\sqrt{b}+x\sqrt{c})^2}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{3}{4}}\sqrt{cx^4+bx^2}}$$

command

```
integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2c^{\frac{3}{2}}x^4\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{cx^4+bx^2}(2cx^2+b)\sqrt{x}\right)}{5bx^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}}{x^{\frac{9}{2}}}, x\right)$$

27.99 Problem number 361

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{cx^4+bx^2}}{7x^{\frac{9}{2}}} - \frac{4c\sqrt{cx^4+bx^2}}{21bx^{\frac{5}{2}}}$$

$$\frac{2c^{\frac{7}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2-b}{(\sqrt{b}+x\sqrt{c})^2}}}{21\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{5}{4}}\sqrt{cx^4+bx^2}}$$

command

`integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 c^{\frac{3}{2}} x^5 \text{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + \sqrt{cx^4 + bx^2} (2cx^2 + 3b) \sqrt{x} \right)}{21 bx^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{11}{2}}}, x\right)$$

27.100 Problem number 362

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4c^{\frac{5}{2}} x^{\frac{3}{2}} (cx^2 + b)}{15b^2 (\sqrt{b} + x\sqrt{c}) \sqrt{cx^4 + bx^2}} - \frac{2\sqrt{cx^4 + bx^2}}{9x^{\frac{11}{2}}} - \frac{4c\sqrt{cx^4 + bx^2}}{45bx^{\frac{7}{2}}} + \frac{4c^2\sqrt{cx^4 + bx^2}}{15b^2x^{\frac{3}{2}}} \\ & + \frac{4c^{\frac{9}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{7}{4}} \sqrt{cx^4 + bx^2}} \\ & - \frac{2c^{\frac{9}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 - b}{(\sqrt{b} + x\sqrt{c})}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{7}{4}} \sqrt{cx^4 + bx^2}} \end{aligned}$$

command

`integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 c^{\frac{5}{2}} x^6 \text{weierstrassZeta}\left(-\frac{4b}{c}, 0, \text{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (6c^2x^4 - 2bcx^2 - 5b^2) \sqrt{cx^4 + bx^2} \sqrt{x} \right)}{45 b^2 x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{13}{2}}}, x\right)$$

27.101 Problem number 363

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2\sqrt{cx^4 + bx^2}}{11x^{\frac{13}{2}}} - \frac{4c\sqrt{cx^4 + bx^2}}{77bx^{\frac{9}{2}}} + \frac{20c^2\sqrt{cx^4 + bx^2}}{231b^2x^{\frac{5}{2}}} \\ & + \frac{10c^{\frac{11}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}{231 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{9}{4}} \sqrt{cx^4 + bx^2}} \end{aligned}$$

command

`integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10 c^{\frac{5}{2}} x^7 \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (10 c^2 x^4 - 6 bcx^2 - 21 b^2) \sqrt{cx^4 + bx^2} \sqrt{x} \right)}{231 b^2 x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{15}{2}}}, x\right)$$

27.102 Problem number 364

$$\int x^{3/2} (bx^2 + cx^4)^{3/2} dx$$

Optimal antiderivative

$$\frac{2x^{\frac{5}{2}}(cx^4 + bx^2)^{\frac{3}{2}}}{17} + \frac{56b^4x^{\frac{3}{2}}(cx^2 + b)}{1105c^{\frac{5}{2}}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}}$$

$$+ \frac{8b^2x^{\frac{5}{2}}\sqrt{cx^4 + bx^2}}{663c} + \frac{12bx^{\frac{9}{2}}\sqrt{cx^4 + bx^2}}{221} - \frac{56b^3\sqrt{x}\sqrt{cx^4 + bx^2}}{3315c^2}$$

$$56b^{\frac{17}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})}}$$

$$1105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4 + bx^2}$$

$$28b^{\frac{17}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})}}$$

$$1105\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4 + bx^2}$$

command

`integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(84b^4\sqrt{c}\text{weierstrassZeta}\left(-\frac{4b}{c}, 0, \text{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) - (195c^4x^6 + 285bc^3x^4 + 20b^2c^2x^2 - 28b^3c)\right)}{3315c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((cx^5 + bx^3)\sqrt{cx^4 + bx^2}\sqrt{x}, x\right)$$

27.103 Problem number 365

$$\int \sqrt{x}(bx^2 + cx^4)^{3/2} dx$$

Optimal antiderivative

$$\frac{2x^{\frac{3}{2}}(cx^4 + bx^2)^{\frac{3}{2}}}{15} + \frac{8b^2x^{\frac{3}{2}}\sqrt{cx^4 + bx^2}}{385c} + \frac{4bx^{\frac{7}{2}}\sqrt{cx^4 + bx^2}}{55} - \frac{8b^3\sqrt{cx^4 + bx^2}}{231c^2\sqrt{x}}$$

$$4b^{\frac{15}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})}}$$

$$231\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{9}{4}}\sqrt{cx^4 + bx^2}$$

command

```
integrate((c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(20 b^4 \sqrt{c} \operatorname{xweierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (77 c^4 x^6 + 119 b c^3 x^4 + 12 b^2 c^2 x^2 - 20 b^3 c) \sqrt{c x^4 + b x^2} \sqrt{x} \right)}{1155 c^3 x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((c x^4 + b x^2)^{\frac{3}{2}} \sqrt{x}, x\right)$$

27.104 Problem number 366

$$\int \frac{(b x^2 + c x^4)^{3/2}}{\sqrt{x}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(c x^4 + b x^2)^{\frac{3}{2}} \sqrt{x}}{13} - \frac{8 b^3 x^{\frac{3}{2}} (c x^2 + b)}{65 c^{\frac{3}{2}} (\sqrt{b} + x \sqrt{c}) \sqrt{c x^4 + b x^2}} \\ & + \frac{4 b x^{\frac{5}{2}} \sqrt{c x^4 + b x^2}}{39} + \frac{8 b^2 \sqrt{x} \sqrt{c x^4 + b x^2}}{195 c} \\ & + \frac{8 b^{\frac{13}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2 - b}{(\sqrt{b} + x \sqrt{c})}}}{65 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{c x^4 + b x^2}} \\ & - \frac{4 b^{\frac{13}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2 + b}{(\sqrt{b} + x \sqrt{c})}}}{65 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{c x^4 + b x^2}} \end{aligned}$$

command

```
integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 b^3 \sqrt{c} \operatorname{weierstrassZeta} \left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse} \left(-\frac{4b}{c}, 0, x \right) \right) + (15 c^3 x^4 + 25 b c^2 x^2 + 4 b^2 c) \sqrt{c x^4 + b x^2} \sqrt{x} \right)}{195 c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{c x^4 + b x^2} (c x^3 + b x) \sqrt{x}, x \right)$$

27.105 Problem number 367

$$\int \frac{(b x^2 + c x^4)^{3/2}}{x^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(c x^4 + b x^2)^{\frac{3}{2}}}{11 \sqrt{x}} + \frac{12 b x^{\frac{3}{2}} \sqrt{c x^4 + b x^2}}{77} + \frac{8 b^2 \sqrt{c x^4 + b x^2}}{77 c \sqrt{x}} + \frac{4 b^{\frac{11}{4}} x \sqrt{\frac{\cos \left(4 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2}{(\sqrt{b} + x \sqrt{c})^2}}}}{77 \cos \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right) c^{\frac{5}{4}} \sqrt{c x^4 + b x^2}}$$

command

```
integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 b^3 \sqrt{c} x \operatorname{weierstrassPInverse} \left(-\frac{4b}{c}, 0, x \right) - (7 c^3 x^4 + 13 b c^2 x^2 + 4 b^2 c) \sqrt{c x^4 + b x^2} \sqrt{x} \right)}{77 c^2 x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{c x^4 + b x^2} (c x^2 + b) \sqrt{x}, x \right)$$

27.106 Problem number 368

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(cx^4 + bx^2)^{\frac{3}{2}}}{9x^{\frac{3}{2}}} + \frac{8b^2x^{\frac{3}{2}}(cx^2 + b)}{15\sqrt{c}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} + \frac{4b\sqrt{x}\sqrt{cx^4 + bx^2}}{15} \\ & - \frac{8b^{\frac{9}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 - (\sqrt{b} + x\sqrt{c})}{(\sqrt{b} + x\sqrt{c})}}}{15 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{3}{4}} \sqrt{cx^4 + bx^2}} \\ & + \frac{4b^{\frac{9}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 + (\sqrt{b} + x\sqrt{c})}{(\sqrt{b} + x\sqrt{c})}}}{15 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{3}{4}} \sqrt{cx^4 + bx^2}} \end{aligned}$$

command

```
integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 b^2 \sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) - \sqrt{cx^4 + bx^2} (5c^2x^2 + 11bc)\sqrt{x} \right)}{45c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}(cx^2 + b)}{\sqrt{x}}, x\right)$$

27.107 Problem number 369

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(cx^4 + bx^2)^{\frac{3}{2}}}{7x^{\frac{5}{2}}} + \frac{4b\sqrt{cx^4 + bx^2}}{7\sqrt{x}}$$

$$+ \frac{4b^{\frac{7}{4}}x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})^2}}}{7 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{1}{4}} \sqrt{cx^4 + bx^2}}$$

command

```
integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4b^2\sqrt{c} \operatorname{xweierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + \sqrt{cx^4 + bx^2} (c^2x^2 + 3bc)\sqrt{x}\right)}{7cx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} (cx^2 + b)}{x^{\frac{3}{2}}}, x\right)$$

27.108 Problem number 370

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{7}{2}}} + \frac{24bx^{\frac{3}{2}}(cx^2 + b)\sqrt{c}}{5(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} + \frac{12c\sqrt{x}\sqrt{cx^4 + bx^2}}{5} \\
 & - \frac{24b^{\frac{5}{4}}c^{\frac{1}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{c}{(\sqrt{b} + x\sqrt{c})^2}}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)\sqrt{cx^4 + bx^2}} \\
 & + \frac{12b^{\frac{5}{4}}c^{\frac{1}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)\sqrt{cx^4 + bx^2}}
 \end{aligned}$$

command

```
integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12b\sqrt{c}x^2\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) - \sqrt{cx^4 + bx^2}(cx^2 - 5b)\sqrt{x}\right)}{5x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}(cx^2 + b)}{x^{\frac{5}{2}}}, x\right)$$

27.109 Problem number 371

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2(cx^4 + bx^2)^{\frac{3}{2}}}{3x^{\frac{9}{2}}} + \frac{4c\sqrt{cx^4 + bx^2}}{3\sqrt{x}} \\
 & - \frac{4b^{\frac{3}{4}}c^{\frac{3}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}}{3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)\sqrt{cx^4 + bx^2}}
 \end{aligned}$$

command

`integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4b\sqrt{c} x^3 \text{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + \sqrt{cx^4 + bx^2} (cx^2 - b) \sqrt{x} \right)}{3x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2} (cx^2 + b)}{x^{\frac{7}{2}}}, x\right)$$

27.110 Problem number 372

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(cx^4 + bx^2)^{\frac{3}{2}}}{5x^{\frac{11}{2}}} + \frac{24c^{\frac{3}{2}}x^{\frac{3}{2}}(cx^2 + b)}{5(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{12c\sqrt{cx^4 + bx^2}}{5x^{\frac{3}{2}}} \\ & \frac{24b^{\frac{1}{4}}c^{\frac{5}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{c}{(\sqrt{b} + x\sqrt{c})^2}}}{5 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) \sqrt{cx^4 + bx^2}} \\ & + \frac{12b^{\frac{1}{4}}c^{\frac{5}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{c}{(\sqrt{b} + x\sqrt{c})^2}}}{5 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) \sqrt{cx^4 + bx^2}} \end{aligned}$$

command

`integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12c^{\frac{3}{2}}x^4 \text{weierstrassZeta}\left(-\frac{4b}{c}, 0, \text{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{cx^4 + bx^2} (7cx^2 + b) \sqrt{x} \right)}{5x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2} (cx^2 + b)}{x^{\frac{9}{2}}}, x\right)$$

27.111 Problem number 373

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{15/2}} dx$$

Optimal antiderivative

$$\frac{2(cx^4 + bx^2)^{\frac{3}{2}}}{7x^{\frac{13}{2}}} - \frac{4c\sqrt{cx^4 + bx^2}}{7x^{\frac{5}{2}}} + \frac{4c^{\frac{7}{4}}x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})}}}{7 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{1}{4}} \sqrt{cx^4 + bx^2}}$$

command

`integrate((c*x^4+b*x^2)^(3/2)/x^(15/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 c^{\frac{3}{2}} x^5 \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - \sqrt{cx^4 + bx^2} (3cx^2 + b) \sqrt{x} \right)}{7 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} (cx^2 + b)}{x^{\frac{11}{2}}}, x\right)$$

27.112 Problem number 374

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{17/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(c x^4 + b x^2)^{\frac{3}{2}}}{9 x^{\frac{15}{2}}} + \frac{8 c^{\frac{5}{2}} x^{\frac{3}{2}} (c x^2 + b)}{15 b (\sqrt{b} + x \sqrt{c}) \sqrt{c x^4 + b x^2}} - \frac{4 c \sqrt{c x^4 + b x^2}}{15 x^{\frac{7}{2}}} - \frac{8 c^2 \sqrt{c x^4 + b x^2}}{15 b x^{\frac{3}{2}}} \\ & - \frac{8 c^{\frac{9}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2 + b}{(\sqrt{b} + x \sqrt{c})^2}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{c x^4 + b x^2}} \\ & + \frac{4 c^{\frac{9}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2 + b}{(\sqrt{b} + x \sqrt{c})^2}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{c x^4 + b x^2}} \end{aligned}$$

command

`integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 c^{\frac{5}{2}} x^6 \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (12 c^2 x^4 + 11 b c x^2 + 5 b^2) \sqrt{c x^4 + b x^2} \sqrt{x} \right)}{45 b x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c x^4 + b x^2} (c x^2 + b)}{x^{\frac{13}{2}}}, x\right)$$

27.113 Problem number 375

$$\int \frac{(b x^2 + c x^4)^{3/2}}{x^{19/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(c x^4 + b x^2)^{\frac{3}{2}}}{11 x^{\frac{17}{2}}} - \frac{12 c \sqrt{c x^4 + b x^2}}{77 x^{\frac{9}{2}}} - \frac{8 c^2 \sqrt{c x^4 + b x^2}}{77 b x^{\frac{5}{2}}} \\ & - \frac{4 c^{\frac{11}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2 + b}{(\sqrt{b} + x \sqrt{c})^2}}}{77 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{5}{4}} \sqrt{c x^4 + b x^2}} \end{aligned}$$

command

```
integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 c^{\frac{5}{2}} x^7 \text{weierstrassPInverse} \left(-\frac{4b}{c}, 0, x \right) + (4 c^2 x^4 + 13 b c x^2 + 7 b^2) \sqrt{c x^4 + b x^2} \sqrt{x} \right)}{77 b x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{c x^4 + b x^2} (c x^2 + b)}{x^{\frac{15}{2}}}, x \right)$$

27.114 Problem number 376

$$\int \frac{(b x^2 + c x^4)^{3/2}}{x^{21/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(c x^4 + b x^2)^{\frac{3}{2}}}{13 x^{\frac{19}{2}}} - \frac{8 c^{\frac{7}{2}} x^{\frac{3}{2}} (c x^2 + b)}{65 b^2 (\sqrt{b} + x \sqrt{c}) \sqrt{c x^4 + b x^2}} \\ & - \frac{4 c \sqrt{c x^4 + b x^2}}{39 x^{\frac{11}{2}}} - \frac{8 c^2 \sqrt{c x^4 + b x^2}}{195 b x^{\frac{7}{2}}} + \frac{8 c^3 \sqrt{c x^4 + b x^2}}{65 b^2 x^{\frac{3}{2}}} \\ & + \frac{8 c^{\frac{13}{4}} x \sqrt{\frac{\cos \left(4 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2 -}{(\sqrt{b} + x \sqrt{c})}}}{65 \cos \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right) b^{\frac{7}{4}} \sqrt{c x^4 + b x^2}} \\ & - \frac{4 c^{\frac{13}{4}} x \sqrt{\frac{\cos \left(4 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2}{(\sqrt{b} + x \sqrt{c})}}}{65 \cos \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right) b^{\frac{7}{4}} \sqrt{c x^4 + b x^2}} \end{aligned}$$

command

```
integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 c^{\frac{7}{2}} x^8 \text{weierstrassZeta} \left(-\frac{4b}{c}, 0, \text{weierstrassPInverse} \left(-\frac{4b}{c}, 0, x \right) \right) + (12 c^3 x^6 - 4 b c^2 x^4 - 25 b^2 c x^2 - 15 b^3) \sqrt{c x^4 + b x^2} \right)}{195 b^2 x^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{c x^4 + b x^2} (c x^2 + b)}{x^{\frac{17}{2}}}, x \right)$$

27.115 Problem number 377

$$\int \frac{(b x^2 + c x^4)^{3/2}}{x^{23/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(c x^4 + b x^2)^{\frac{3}{2}}}{15 x^{\frac{21}{2}}} - \frac{4c \sqrt{c x^4 + b x^2}}{55 x^{\frac{13}{2}}} - \frac{8c^2 \sqrt{c x^4 + b x^2}}{385 b x^{\frac{9}{2}}} + \frac{8c^3 \sqrt{c x^4 + b x^2}}{231 b^2 x^{\frac{5}{2}}} \\ & + \frac{4c^{\frac{15}{4}} x \sqrt{\frac{\cos \left(4 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2}{(\sqrt{b} + x \sqrt{c})}}}{231 \cos \left(2 \arctan \left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}} \right) \right) b^{\frac{9}{4}} \sqrt{c x^4 + b x^2}} \end{aligned}$$

command

`integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(20 c^{\frac{7}{2}} x^9 \text{weierstrassPInverse} \left(-\frac{4b}{c}, 0, x \right) + (20 c^3 x^6 - 12 b c^2 x^4 - 119 b^2 c x^2 - 77 b^3) \sqrt{c x^4 + b x^2} \sqrt{x} \right)}{1155 b^2 x^9}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{c x^4 + b x^2} (c x^2 + b)}{x^{\frac{19}{2}}}, x \right)$$

27.116 Problem number 378

$$\int \frac{x^{13/2}}{\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{-\frac{18bx^{\frac{3}{2}}\sqrt{cx^4 + bx^2}}{77c^2} + \frac{2x^{\frac{7}{2}}\sqrt{cx^4 + bx^2}}{11c} + \frac{30b^2\sqrt{cx^4 + bx^2}}{77c^3\sqrt{x}}}{15b^{\frac{11}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}}{77\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{13}{4}}\sqrt{cx^4 + bx^2}}$$

command

`integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(15b^3\sqrt{c}x\operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - (7c^3x^4 - 9bc^2x^2 + 15b^2c)\sqrt{cx^4 + bx^2}\sqrt{x}\right)}{77c^4x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}x^{\frac{9}{2}}}{cx^2 + b}, x\right)$$

27.117 Problem number 379

$$\int \frac{x^{11/2}}{\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{14b^2x^{\frac{3}{2}}(cx^2+b)}{15c^{\frac{5}{2}}(\sqrt{b}+x\sqrt{c})\sqrt{cx^4+bx^2}} + \frac{2x^{\frac{5}{2}}\sqrt{cx^4+bx^2}}{9c} - \frac{14b\sqrt{x}\sqrt{cx^4+bx^2}}{45c^2}$$

$$+ \frac{14b^{\frac{9}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b}+x\sqrt{c})}}}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4+bx^2}}$$

$$+ \frac{7b^{\frac{9}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b}+x\sqrt{c})}}}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4+bx^2}}$$

command

`integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(21b^2\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) - \sqrt{cx^4+bx^2}(5c^2x^2-7bc)\sqrt{x}\right)}{45c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}x^{\frac{7}{2}}}{cx^2+b}, x\right)$$

27.118 Problem number 380

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}} dx$$

Optimal antiderivative

$$\frac{2x^{\frac{3}{2}}\sqrt{cx^4+bx^2}}{7c} - \frac{10b\sqrt{cx^4+bx^2}}{21c^2\sqrt{x}}$$

$$+ \frac{5b^{\frac{7}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b}+x\sqrt{c})}}}{21\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{9}{4}}\sqrt{cx^4+bx^2}}$$

command

`integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5 b^2 \sqrt{c} \operatorname{xweierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + \sqrt{c x^4 + b x^2} (3 c^2 x^2 - 5 b c) \sqrt{x} \right)}{21 c^3 x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c x^4 + b x^2} x^{\frac{5}{2}}}{c x^2 + b}, x\right)$$

27.119 Problem number 381

$$\int \frac{x^{7/2}}{\sqrt{b x^2 + c x^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6 b x^{\frac{3}{2}} (c x^2 + b)}{5 c^{\frac{3}{2}} (\sqrt{b} + x \sqrt{c}) \sqrt{c x^4 + b x^2}} + \frac{2 \sqrt{x} \sqrt{c x^4 + b x^2}}{5 c} \\ & + \frac{6 b^{\frac{5}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2 + b}{(\sqrt{b} + x \sqrt{c})}}}{5 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{c x^4 + b x^2}} \\ & + \frac{3 b^{\frac{5}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2 - b}{(\sqrt{b} + x \sqrt{c})}}}{5 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{7}{4}} \sqrt{c x^4 + b x^2}} \end{aligned}$$

command

`integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 b \sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{c x^4 + b x^2} c \sqrt{x} \right)}{5 c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c x^4 + b x^2} x^{\frac{3}{2}}}{c x^2 + b}, x\right)$$

27.120 Problem number 382

$$\int \frac{x^{5/2}}{\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{cx^4 + bx^2}}{3c\sqrt{x}} \left(b^{\frac{3}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})^2}} \right) - \frac{3 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{5}{4}} \sqrt{cx^4 + bx^2}}{3c^2x}$$

command

```
integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(b\sqrt{c} x \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - \sqrt{cx^4 + bx^2} c\sqrt{x} \right)}{3c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} \sqrt{x}}{cx^2 + b}, x\right)$$

27.121 Problem number 383

$$\int \frac{x^{3/2}}{\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2x^{\frac{3}{2}}(cx^2 + b)}{\sqrt{c}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}}$$

$$\frac{2b^{\frac{1}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2 - b}{(\sqrt{b} + x\sqrt{c})^2}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{cx^4 + bx^2}}$$

$$+ \frac{b^{\frac{1}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})^2}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{3}{4}}\sqrt{cx^4 + bx^2}}$$

command

`integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right)}{\sqrt{c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{x}}{cx^3 + bx}, x\right)$$

27.122 Problem number 384

$$\int \frac{\sqrt{x}}{\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})^2}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{cx^4 + bx^2}}$$

command

`integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)}{\sqrt{c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^4 + bx^2}}, x\right)$$

27.123 Problem number 385

$$\int \frac{1}{\sqrt{x} \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2x^{\frac{3}{2}}(cx^2 + b)\sqrt{c}}{b(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{2\sqrt{cx^4 + bx^2}}{bx^{\frac{3}{2}}}$$

$$- \frac{2c^{\frac{1}{4}}x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 - (\sqrt{b} + x\sqrt{c})}{(\sqrt{b} + x\sqrt{c})}}}{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{cx^4 + bx^2}}$$

$$+ \frac{c^{\frac{1}{4}}x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 + (\sqrt{b} + x\sqrt{c})}{(\sqrt{b} + x\sqrt{c})}}}{\cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}} \sqrt{cx^4 + bx^2}}$$

command

`integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{c}x^2 \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{cx^4 + bx^2} \sqrt{x}\right)}{bx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} \sqrt{x}}{cx^5 + bx^3}, x\right)$$

27.124 Problem number 386

$$\int \frac{1}{x^{3/2} \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{cx^4 + bx^2}}{3bx^{\frac{5}{2}}} + \frac{c^{\frac{3}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})^2}}}{3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{5}{4}} \sqrt{cx^4 + bx^2}}$$

command

```
integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{c}x^3\operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + \sqrt{cx^4 + bx^2}\sqrt{x}\right)}{3bx^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{x}}{cx^6 + bx^4}, x\right)$$

27.125 Problem number 387

$$\int \frac{1}{x^{5/2} \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{6c^{\frac{3}{2}}x^{\frac{3}{2}}(cx^2+b)}{5b^2(\sqrt{b}+x\sqrt{c})\sqrt{cx^4+bx^2}} - \frac{2\sqrt{cx^4+bx^2}}{5bx^{\frac{7}{2}}} + \frac{6c\sqrt{cx^4+bx^2}}{5b^2x^{\frac{3}{2}}}$$

$$+ \frac{6c^{\frac{5}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b}+x\sqrt{c}) \sqrt{\frac{cx^2+b}{(\sqrt{b}+x\sqrt{c})}}}{5 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{7}{4}} \sqrt{cx^4+bx^2}}$$

$$+ \frac{3c^{\frac{5}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b}+x\sqrt{c}) \sqrt{\frac{cx^2-b}{(\sqrt{b}+x\sqrt{c})}}}{5 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{7}{4}} \sqrt{cx^4+bx^2}}$$

command

`integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3c^{\frac{3}{2}}x^4 \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{cx^4+bx^2}(3cx^2-b)\sqrt{x}\right)}{5b^2x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{cx^7+bx^5}, x\right)$$

27.126 Problem number 388

$$\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{cx^4+bx^2}}{7bx^{\frac{9}{2}}} + \frac{10c\sqrt{cx^4+bx^2}}{21b^2x^{\frac{5}{2}}}$$

$$+ \frac{5c^{\frac{7}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b}+x\sqrt{c}) \sqrt{\frac{cx^2+b}{(\sqrt{b}+x\sqrt{c})}}}{21 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{9}{4}} \sqrt{cx^4+bx^2}}$$

command

`integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5 c^{\frac{3}{2}} x^5 \text{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + \sqrt{cx^4 + bx^2} (5cx^2 - 3b) \sqrt{x} \right)}{21 b^2 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2} \sqrt{x}}{cx^8 + bx^6}, x\right)$$

27.127 Problem number 389

$$\int \frac{1}{x^{9/2} \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{14c^{\frac{5}{2}} x^{\frac{3}{2}} (cx^2 + b)}{15b^3 (\sqrt{b} + x\sqrt{c}) \sqrt{cx^4 + bx^2}} - \frac{2\sqrt{cx^4 + bx^2}}{9bx^{\frac{11}{2}}} + \frac{14c\sqrt{cx^4 + bx^2}}{45b^2 x^{\frac{7}{2}}} - \frac{14c^2 \sqrt{cx^4 + bx^2}}{15b^3 x^{\frac{3}{2}}}$$

$$\frac{14c^{\frac{9}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{11}{4}} \sqrt{cx^4 + bx^2}}$$

$$+ \frac{7c^{\frac{9}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{11}{4}} \sqrt{cx^4 + bx^2}}$$

command

`integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21 c^{\frac{5}{2}} x^6 \text{weierstrassZeta}\left(-\frac{4b}{c}, 0, \text{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (21 c^2 x^4 - 7bcx^2 + 5b^2) \sqrt{cx^4 + bx^2} \sqrt{x} \right)}{45 b^3 x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2} \sqrt{x}}{cx^9 + bx^7}, x\right)$$

27.128 Problem number 390

$$\int \frac{1}{x^{11/2} \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{cx^4 + bx^2}}{11bx^{\frac{13}{2}}} + \frac{18c\sqrt{cx^4 + bx^2}}{77b^2x^{\frac{9}{2}}} - \frac{30c^2\sqrt{cx^4 + bx^2}}{77b^3x^{\frac{5}{2}}} - \frac{15c^{\frac{11}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}{77 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{13}{4}} \sqrt{cx^4 + bx^2}}$$

command

`integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(15c^{\frac{5}{2}}x^7 \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (15c^2x^4 - 9bcx^2 + 7b^2)\sqrt{cx^4 + bx^2}\sqrt{x}\right)}{77b^3x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{x}}{cx^{10} + bx^8}, x\right)$$

27.129 Problem number 391

$$\int \frac{x^{17/2}}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x^{\frac{11}{2}}}{c\sqrt{cx^4 + bx^2}} + \frac{9x^{\frac{3}{2}}\sqrt{cx^4 + bx^2}}{7c^2} - \frac{15b\sqrt{cx^4 + bx^2}}{7c^3\sqrt{x}} + \frac{15b^{\frac{7}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}{14 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{13}{4}} \sqrt{cx^4 + bx^2}}$$

command

`integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15(b^2cx^3 + b^3x)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (2c^3x^4 - 6bc^2x^2 - 15b^2c)\sqrt{cx^4 + bx^2}\sqrt{x}}{7(c^5x^3 + bc^4x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} x^{\frac{9}{2}}}{c^2x^4 + 2bcx^2 + b^2}, x\right)$$

27.130 Problem number 392

$$\int \frac{x^{15/2}}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x^{\frac{9}{2}}}{c\sqrt{cx^4 + bx^2}} - \frac{21bx^{\frac{3}{2}}(cx^2 + b)}{5c^{\frac{5}{2}}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} + \frac{7\sqrt{x}\sqrt{cx^4 + bx^2}}{5c^2} \\ & + \frac{21b^{\frac{5}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}{5\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4 + bx^2}} \\ & + \frac{21b^{\frac{5}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}{10\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4 + bx^2}} \end{aligned}$$

command

`integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21(bc^2x^2 + b^2)\sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{cx^4 + bx^2}(2c^2x^2 + 7bc)\sqrt{x}}{5(c^4x^2 + bc^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} x^{\frac{7}{2}}}{c^2x^4 + 2bcx^2 + b^2}, x\right)$$

27.131 Problem number 393

$$\int \frac{x^{13/2}}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x^7}{c\sqrt{cx^4 + bx^2}} + \frac{5\sqrt{cx^4 + bx^2}}{3c^2\sqrt{x}}$$

$$5b^{\frac{3}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 - (\sqrt{b} + x\sqrt{c})^2}{(\sqrt{b} + x\sqrt{c})^2}}$$

$$6\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) c^{\frac{9}{4}}\sqrt{cx^4 + bx^2}$$

command

```
integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(bc^3 + b^2x)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - \sqrt{cx^4 + bx^2}(2c^2x^2 + 5bc)\sqrt{x}}{3(c^4x^3 + bc^3x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} x^{\frac{5}{2}}}{c^2x^4 + 2bcx^2 + b^2}, x\right)$$

27.132 Problem number 394

$$\int \frac{x^{11/2}}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{x^{\frac{5}{2}}}{c\sqrt{cx^4+bx^2}} + \frac{3x^{\frac{3}{2}}(cx^2+b)}{c^{\frac{3}{2}}(\sqrt{b}+x\sqrt{c})\sqrt{cx^4+bx^2}} \\
 & 3b^{\frac{1}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2-b}{(\sqrt{b}+x\sqrt{c})}} \\
 & \frac{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{cx^4+bx^2}}{3b^{\frac{1}{4}}x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2+b}{(\sqrt{b}+x\sqrt{c})}} \\
 & + \frac{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{cx^4+bx^2}}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{cx^4+bx^2}}
 \end{aligned}$$

command

`integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(cx^2+b)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c},0,\operatorname{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}c\sqrt{x}}{c^3x^2+bc^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}x^{\frac{3}{2}}}{c^2x^4+2bcx^2+b^2},x\right)$$

27.133 Problem number 395

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{x^{\frac{3}{2}}}{c\sqrt{cx^4+bx^2}} \\
 & x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2}}{2}\right)(\sqrt{b}+x\sqrt{c})\sqrt{\frac{cx^2+b}{(\sqrt{b}+x\sqrt{c})}} \\
 & + \frac{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{1}{4}}c^{\frac{5}{4}}\sqrt{cx^4+bx^2}}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{1}{4}}c^{\frac{5}{4}}\sqrt{cx^4+bx^2}}
 \end{aligned}$$

command

`integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(cx^3 + bx)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) - \sqrt{cx^4 + bx^2} c\sqrt{x}}{c^3x^3 + bc^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} \sqrt{x}}{c^2x^4 + 2bcx^2 + b^2}, x\right)$$

27.134 Problem number 396

$$\int \frac{x^{7/2}}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x^{\frac{5}{2}}}{b\sqrt{cx^4 + bx^2}} - \frac{x^{\frac{3}{2}}(cx^2 + b)}{b\sqrt{c}(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}}$$

$$+ \frac{x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{3}{4}}c^{\frac{3}{4}}\sqrt{cx^4 + bx^2}}$$

$$- \frac{x\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{b} + x\sqrt{c})\sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})}}}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)b^{\frac{3}{4}}c^{\frac{3}{4}}\sqrt{cx^4 + bx^2}}$$

command

`integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(cx^2 + b)\sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{cx^4 + bx^2} c\sqrt{x}}{bc^2x^2 + b^2c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} \sqrt{x}}{c^2x^5 + 2bcx^3 + b^2x}, x\right)$$

27.135 Problem number 397

$$\int \frac{x^{5/2}}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x^{\frac{3}{2}}}{b\sqrt{cx^4 + bx^2}} + \frac{x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2 + b}{(\sqrt{b} + x\sqrt{c})}}}{2 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{5}{4}} c^{\frac{1}{4}} \sqrt{cx^4 + bx^2}}$$

command

```
integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(cx^3 + bx)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + \sqrt{cx^4 + bx^2} c\sqrt{x}}{bc^2x^3 + b^2cx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} \sqrt{x}}{c^2x^6 + 2bcx^4 + b^2x^2}, x\right)$$

27.136 Problem number 398

$$\int \frac{x^{3/2}}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{3x^{\frac{3}{2}}(cx^2+b)\sqrt{c}}{b^2(\sqrt{b}+x\sqrt{c})\sqrt{cx^4+bx^2}} + \frac{\sqrt{x}}{b\sqrt{cx^4+bx^2}} - \frac{3\sqrt{cx^4+bx^2}}{b^2x^{\frac{3}{2}}}$$

$$+ \frac{3c^{\frac{1}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b}+x\sqrt{c}) \sqrt{\frac{cx^2-}{(\sqrt{b}+x\sqrt{c})}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{7}{4}}\sqrt{cx^4+bx^2}}$$

$$+ \frac{3c^{\frac{1}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b}+x\sqrt{c}) \sqrt{\frac{cx^2+}{(\sqrt{b}+x\sqrt{c})}}}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{7}{4}}\sqrt{cx^4+bx^2}}$$

command

`integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(cx^4+bx^2)\sqrt{c}\operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + \sqrt{cx^4+bx^2}(3cx^2+2b)\sqrt{x}}{b^2cx^4+b^3x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{c^2x^7+2bcx^5+b^2x^3}, x\right)$$

27.137 Problem number 399

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{b\sqrt{x}\sqrt{cx^4+bx^2}} - \frac{5\sqrt{cx^4+bx^2}}{3b^2x^{\frac{5}{2}}}$$

$$+ \frac{5c^{\frac{3}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b}+x\sqrt{c}) \sqrt{\frac{cx^2-}{(\sqrt{b}+x\sqrt{c})}}}{6\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{9}{4}}\sqrt{cx^4+bx^2}}$$

command

`integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 (cx^5 + bx^3) \sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + \sqrt{cx^4 + bx^2} (5cx^2 + 2b) \sqrt{x}}{3 (b^2cx^5 + b^3x^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} \sqrt{x}}{c^2x^8 + 2bcx^6 + b^2x^4}, x\right)$$

27.138 Problem number 400

$$\int \frac{1}{\sqrt{x} (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1}{bx^{\frac{3}{2}} \sqrt{cx^4 + bx^2}} - \frac{21c^{\frac{3}{2}}x^{\frac{3}{2}}(cx^2 + b)}{5b^3(\sqrt{b} + x\sqrt{c})\sqrt{cx^4 + bx^2}} - \frac{7\sqrt{cx^4 + bx^2}}{5b^2x^{\frac{7}{2}}} + \frac{21c\sqrt{cx^4 + bx^2}}{5b^3x^{\frac{3}{2}}} \\ & + \frac{21c^{\frac{5}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}{5 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{11}{4}} \sqrt{cx^4 + bx^2}} \\ & - \frac{21c^{\frac{5}{4}}x \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}{10 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{11}{4}} \sqrt{cx^4 + bx^2}} \end{aligned}$$

command

`integrate(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21 (c^2x^6 + bcx^4) \sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (21c^2x^4 + 14bcx^2 - 2b^2) \sqrt{cx^4 + bx^2}}{5 (b^3cx^6 + b^4x^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2} \sqrt{x}}{c^2x^9 + 2bcx^7 + b^2x^5}, x\right)$$

27.139 Problem number 401

$$\int \frac{1}{x^{3/2} (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{bx^{\frac{5}{2}} \sqrt{cx^4 + bx^2}} - \frac{9\sqrt{cx^4 + bx^2}}{7b^2x^{\frac{9}{2}}} + \frac{15c\sqrt{cx^4 + bx^2}}{7b^3x^{\frac{5}{2}}}$$

$$+ \frac{15c^{\frac{7}{4}}x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x\sqrt{c}) \sqrt{\frac{cx^2}{(\sqrt{b} + x\sqrt{c})^2}}}{14 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}\sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{13}{4}} \sqrt{cx^4 + bx^2}}$$

command

`integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15(c^2x^7 + bcx^5)\sqrt{c} \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (15c^2x^4 + 6bcx^2 - 2b^2)\sqrt{cx^4 + bx^2}\sqrt{x}}{7(b^3cx^7 + b^4x^5)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{x}}{c^2x^{10} + 2bcx^8 + b^2x^6}, x\right)$$

27.140 Problem number 402

$$\int \frac{1}{x^{5/2} (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1}{b x^{\frac{7}{2}} \sqrt{c x^4 + b x^2}} + \frac{77 c^{\frac{5}{2}} x^{\frac{3}{2}} (c x^2 + b)}{15 b^4 (\sqrt{b} + x \sqrt{c}) \sqrt{c x^4 + b x^2}} \\ & - \frac{11 \sqrt{c x^4 + b x^2}}{9 b^2 x^{\frac{11}{2}}} + \frac{77 c \sqrt{c x^4 + b x^2}}{45 b^3 x^{\frac{7}{2}}} - \frac{77 c^2 \sqrt{c x^4 + b x^2}}{15 b^4 x^{\frac{3}{2}}} \\ & - \frac{77 c^{\frac{9}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2}{(\sqrt{b} + x \sqrt{c})^2}}}{15 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{15}{4}} \sqrt{c x^4 + b x^2}} \\ & + \frac{77 c^{\frac{9}{4}} x \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{b} + x \sqrt{c}) \sqrt{\frac{c x^2}{(\sqrt{b} + x \sqrt{c})^2}}}{30 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} \sqrt{x}}{b^{\frac{1}{4}}}\right)\right) b^{\frac{15}{4}} \sqrt{c x^4 + b x^2}} \end{aligned}$$

command

`integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{231 (c^3 x^8 + b c^2 x^6) \sqrt{c} \operatorname{weierstrassZeta}\left(-\frac{4b}{c}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right)\right) + (231 c^3 x^6 + 154 b c^2 x^4 - 22 b^2 c x^2)}{45 (b^4 c x^8 + b^5 x^6)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c x^4 + b x^2} \sqrt{x}}{c^2 x^{11} + 2 b c x^9 + b^2 x^7}, x\right)$$

27.141 Problem number 954

$$\int \sqrt{3 - 2x^2 - x^4} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{EllipticE}\left(x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3} + \frac{4 \operatorname{EllipticF}\left(x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3} + \frac{x \sqrt{-x^4 - 2x^2 + 3}}{3}$$

command

```
integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^4 - 2x^2 + 3} (x^2 + 2)}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{-x^4 - 2x^2 + 3}, x\right)$$

27.142 Problem number 965

$$\int \frac{1}{\sqrt{a + bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2 - \frac{b}{\sqrt{a}\sqrt{c}}}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4 + b}{(\sqrt{a} + x^2\sqrt{c})}}}{2 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{cx^4 + bx^2 + a}}$$

command

```
integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{\frac{1}{2}} \left(a\sqrt{\frac{b^2 - 4ac}{a^2}} + b\right) \sqrt{\frac{a\sqrt{\frac{b^2 - 4ac}{a^2}} - b}{a}} \text{ellipticF}\left(\sqrt{\frac{1}{2}} x \sqrt{\frac{a\sqrt{\frac{b^2 - 4ac}{a^2}} - b}{a}}, \frac{ab\sqrt{\frac{b^2 - 4ac}{a^2}} + b^2 - 2ac}{2ac}\right)}{2\sqrt{a}c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{1}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

27.143 Problem number 978

$$\int \frac{1}{\sqrt{a + bx^2 - cx^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{4ac+b^2}}}, \sqrt{\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}}\right) \sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}} \sqrt{b+\sqrt{4ac+b^2}} \sqrt{1-\frac{2cx^2}{b+\sqrt{4ac+b^2}}}}{2\sqrt{c}\sqrt{-cx^4+bx^2+a}}$$

command

`integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{\frac{1}{2}} \left(a^{\frac{3}{2}} \sqrt{\frac{b^2+4ac}{a^2}} + \sqrt{a} b \right) \sqrt{\frac{a \sqrt{\frac{b^2+4ac}{a^2}} - b}{a}} \text{ellipticF} \left(\sqrt{\frac{1}{2}} x \sqrt{\frac{a \sqrt{\frac{b^2+4ac}{a^2}} - b}{a}}, -\frac{ab \sqrt{\frac{b^2+4ac}{a^2}} + b^2}{2ac} \right)}{2ac}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-cx^4+bx^2+a}}{cx^4-bx^2-a}, x\right)$$

27.144 Problem number 1003

$$\int \frac{x^4}{\sqrt{a + (2 + 2b - 2(1 + b))x^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{\frac{x\sqrt{cx^4+a}}{3c} + \frac{1}{2} \sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4+a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{6 \cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) c^{\frac{5}{4}} \sqrt{cx^4+a}}$$

command

```
integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{c} \left(-\frac{a}{c}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}, -1\right) - \sqrt{cx^4 + a} x}{3c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^4}{\sqrt{cx^4 + a}}, x\right)$$

27.145 Problem number 1007

$$\int \frac{1}{\sqrt{a + (2 + 2b - 2(1 + b))x^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4 + a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{2 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{cx^4 + a}}$$

command

```
integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{a} \left(-\frac{c}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}, -1\right)}{c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{cx^4 + a}}, x\right)$$

27.146 Problem number 1011

$$\int \frac{1}{x^4 \sqrt{a + (2 + 2b - 2(1 + b))x^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{\frac{\sqrt{cx^4 + a}}{3ax^3} + c^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4 + a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{6 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{5}{4}} \sqrt{cx^4 + a}}$$

command

```
integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{a} x^3 \left(-\frac{c}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x \left(-\frac{c}{a}\right)^{\frac{1}{4}}, -1\right) - \sqrt{cx^4 + a}}{3ax^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + a}}{cx^8 + ax^4}, x\right)$$

27.147 Problem number 1039

$$\int \frac{1}{\sqrt{3 - 2x^2 - x^4}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticF}\left(x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3}$$

command

```
integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{3} \operatorname{ellipticF}\left(x, -\frac{1}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^4 - 2x^2 + 3}}{x^4 + 2x^2 - 3}, x\right)$$

27.148 Problem number 1071

$$\int \frac{x^{13/2}}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{bx^{\frac{3}{2}}}{2c(-4ac + b^2)} + \frac{x^{\frac{7}{2}}(bx^2 + 2a)}{2(-4ac + b^2)(cx^4 + bx^2 + a)} \\ & \frac{\operatorname{arctan}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(3b^3 - 20abc - (-14ac + 3b^2)\sqrt{-4ac + b^2}\right) 2^{\frac{1}{4}}}{8c^{\frac{7}{4}}(-4ac + b^2)^{\frac{3}{2}}(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \\ & + \frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(3b^3 - 20abc - (-14ac + 3b^2)\sqrt{-4ac + b^2}\right) 2^{\frac{1}{4}}}{8c^{\frac{7}{4}}(-4ac + b^2)^{\frac{3}{2}}(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \\ & + \frac{\operatorname{arctan}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(3b^3 - 20abc + (-14ac + 3b^2)\sqrt{-4ac + b^2}\right) 2^{\frac{1}{4}}}{8c^{\frac{7}{4}}(-4ac + b^2)^{\frac{3}{2}}(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \\ & - \frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(3b^3 - 20abc + (-14ac + 3b^2)\sqrt{-4ac + b^2}\right) 2^{\frac{1}{4}}}{8c^{\frac{7}{4}}(-4ac + b^2)^{\frac{3}{2}}(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \end{aligned}$$

command

`integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

27.149 Problem number 1078

$$\int \frac{1}{\sqrt{x} (a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c^{\frac{3}{4}} \arctan \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (3b^2 - 28ac - 3b\sqrt{-4ac + b^2}) 2^{\frac{3}{4}}}{8a(-4ac + b^2)^{\frac{3}{2}} (-b - \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & + \frac{c^{\frac{3}{4}} \operatorname{arctanh} \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (3b^2 - 28ac - 3b\sqrt{-4ac + b^2}) 2^{\frac{3}{4}}}{8a(-4ac + b^2)^{\frac{3}{2}} (-b - \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & - \frac{c^{\frac{3}{4}} \arctan \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (3b^2 - 28ac + 3b\sqrt{-4ac + b^2}) 2^{\frac{3}{4}}}{8a(-4ac + b^2)^{\frac{3}{2}} (-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & - \frac{c^{\frac{3}{4}} \operatorname{arctanh} \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (3b^2 - 28ac + 3b\sqrt{-4ac + b^2}) 2^{\frac{3}{4}}}{8a(-4ac + b^2)^{\frac{3}{2}} (-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & + \frac{(bcx^2 - 2ac + b^2) \sqrt{x}}{2a(-4ac + b^2)(cx^4 + bx^2 + a)} \end{aligned}$$

command

```
integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

27.150 Problem number 1079

$$\int \frac{1}{x^{3/2} (a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c^{\frac{1}{4}} \arctan \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) \left(5b^3 - 28abc - (-18ac + 5b^2) \sqrt{-4ac + b^2} \right) 2^{\frac{1}{4}}}{8a^2 (-4ac + b^2)^{\frac{3}{2}} (-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \\ & - \frac{c^{\frac{1}{4}} \operatorname{arctanh} \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) \left(5b^3 - 28abc - (-18ac + 5b^2) \sqrt{-4ac + b^2} \right) 2^{\frac{1}{4}}}{8a^2 (-4ac + b^2)^{\frac{3}{2}} (-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \\ & - \frac{c^{\frac{1}{4}} \arctan \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) \left(5b^3 - 28abc + (-18ac + 5b^2) \sqrt{-4ac + b^2} \right) 2^{\frac{1}{4}}}{8a^2 (-4ac + b^2)^{\frac{3}{2}} (-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \\ & + \frac{c^{\frac{1}{4}} \operatorname{arctanh} \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) \left(5b^3 - 28abc + (-18ac + 5b^2) \sqrt{-4ac + b^2} \right) 2^{\frac{1}{4}}}{8a^2 (-4ac + b^2)^{\frac{3}{2}} (-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \\ & + \frac{18ac - 5b^2}{2a^2 (-4ac + b^2) \sqrt{x}} + \frac{bcx^2 - 2ac + b^2}{2a (-4ac + b^2) (cx^4 + bx^2 + a) \sqrt{x}} \end{aligned}$$

command

```
integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

27.151 Problem number 1080

$$\int \frac{x^{15/2}}{(a + bx^2 + cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^{\frac{9}{2}}(bx^2 + 2a)}{4(-4ac + b^2)(cx^4 + bx^2 + a)^2} + \frac{3x^{\frac{5}{2}}(8ab + (12ac + b^2)x^2)}{16(-4ac + b^2)^2(cx^4 + bx^2 + a)} \\ & - \frac{3 \arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(b^3 - 28abc + \frac{-24a^2c^2 - 30ab^2c + b^4}{\sqrt{-4ac + b^2}}\right) 2^{\frac{3}{4}}}{64c^{\frac{5}{4}}(-4ac + b^2)^2(-b - \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & - \frac{3 \operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(b^3 - 28abc + \frac{-24a^2c^2 - 30ab^2c + b^4}{\sqrt{-4ac + b^2}}\right) 2^{\frac{3}{4}}}{64c^{\frac{5}{4}}(-4ac + b^2)^2(-b - \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & - \frac{3 \arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(b^3 - 28abc + \frac{24a^2c^2 + 30ab^2c - b^4}{\sqrt{-4ac + b^2}}\right) 2^{\frac{3}{4}}}{64c^{\frac{5}{4}}(-4ac + b^2)^2(-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & - \frac{3 \operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(b^3 - 28abc + \frac{24a^2c^2 + 30ab^2c - b^4}{\sqrt{-4ac + b^2}}\right) 2^{\frac{3}{4}}}{64c^{\frac{5}{4}}(-4ac + b^2)^2(-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & - \frac{3(12ac + b^2)\sqrt{x}}{16c(-4ac + b^2)^2} \end{aligned}$$

command

```
integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

27.152 Problem number 1082

$$\int \frac{x^{11/2}}{(a + bx^2 + cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^{\frac{5}{2}}(bx^2 + 2a)}{4(-4ac + b^2)(cx^4 + bx^2 + a)^2} \\ & - \frac{3 \arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(7b^2 + 20ac + \frac{-36abc - 7b^3}{\sqrt{-4ac + b^2}}\right) 2^{\frac{3}{4}}}{64c^{\frac{1}{4}}(-4ac + b^2)^2(-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & - \frac{3 \operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(7b^2 + 20ac + \frac{-36abc - 7b^3}{\sqrt{-4ac + b^2}}\right) 2^{\frac{3}{4}}}{64c^{\frac{1}{4}}(-4ac + b^2)^2(-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & - \frac{3 \arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(7b^3 + 36abc + (20ac + 7b^2)\sqrt{-4ac + b^2}\right) 2^{\frac{3}{4}}}{64c^{\frac{1}{4}}(-4ac + b^2)^{\frac{5}{2}}(-b - \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & - \frac{3 \operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(7b^3 + 36abc + (20ac + 7b^2)\sqrt{-4ac + b^2}\right) 2^{\frac{3}{4}}}{64c^{\frac{1}{4}}(-4ac + b^2)^{\frac{5}{2}}(-b - \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & + \frac{(24ab + (20ac + 7b^2)x^2)\sqrt{x}}{16(-4ac + b^2)^2(cx^4 + bx^2 + a)} \end{aligned}$$

command

`integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

27.153 Problem number 1083

$$\int \frac{x^{9/2}}{(a + bx^2 + cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^{\frac{3}{2}}(bx^2 + 2a)}{4(-4ac + b^2)(cx^4 + bx^2 + a)^2} - \frac{3x^{\frac{3}{2}}(8bcx^2 - 4ac + 5b^2)}{16(-4ac + b^2)^2(cx^4 + bx^2 + a)} \\ & + \frac{3c^{\frac{1}{4}} \arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(11b^2 + 20ac - 4b\sqrt{-4ac + b^2}\right) 2^{\frac{1}{4}}}{32(-4ac + b^2)^{\frac{5}{2}} \left(-b + \sqrt{-4ac + b^2}\right)^{\frac{1}{4}}} \\ & - \frac{3c^{\frac{1}{4}} \operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(11b^2 + 20ac - 4b\sqrt{-4ac + b^2}\right) 2^{\frac{1}{4}}}{32(-4ac + b^2)^{\frac{5}{2}} \left(-b + \sqrt{-4ac + b^2}\right)^{\frac{1}{4}}} \\ & - \frac{3c^{\frac{1}{4}} \arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(11b^2 + 20ac + 4b\sqrt{-4ac + b^2}\right) 2^{\frac{1}{4}}}{32(-4ac + b^2)^{\frac{5}{2}} \left(-b - \sqrt{-4ac + b^2}\right)^{\frac{1}{4}}} \\ & + \frac{3c^{\frac{1}{4}} \operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) \left(11b^2 + 20ac + 4b\sqrt{-4ac + b^2}\right) 2^{\frac{1}{4}}}{32(-4ac + b^2)^{\frac{5}{2}} \left(-b - \sqrt{-4ac + b^2}\right)^{\frac{1}{4}}} \end{aligned}$$

command

```
integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

27.154 Problem number 1084

$$\int \frac{x^{7/2}}{(a + bx^2 + cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c^{\frac{3}{4}} \arctan \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (41b^2 + 28ac - 36b\sqrt{-4ac + b^2}) 2^{\frac{3}{4}}}{32(-4ac + b^2)^{\frac{5}{2}} (-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & - \frac{c^{\frac{3}{4}} \operatorname{arctanh} \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (41b^2 + 28ac - 36b\sqrt{-4ac + b^2}) 2^{\frac{3}{4}}}{32(-4ac + b^2)^{\frac{5}{2}} (-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & + \frac{c^{\frac{3}{4}} \arctan \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (41b^2 + 28ac + 36b\sqrt{-4ac + b^2}) 2^{\frac{3}{4}}}{32(-4ac + b^2)^{\frac{5}{2}} (-b - \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & + \frac{c^{\frac{3}{4}} \operatorname{arctanh} \left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (41b^2 + 28ac + 36b\sqrt{-4ac + b^2}) 2^{\frac{3}{4}}}{32(-4ac + b^2)^{\frac{5}{2}} (-b - \sqrt{-4ac + b^2})^{\frac{3}{4}}} \\ & + \frac{(bx^2 + 2a)\sqrt{x}}{4(-4ac + b^2)(cx^4 + bx^2 + a)^2} - \frac{(24bcx^2 - 4ac + 13b^2)\sqrt{x}}{16(-4ac + b^2)^2(cx^4 + bx^2 + a)} \end{aligned}$$

command

```
integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

27.155 Problem number 1086

$$\int \frac{x^{3/2}}{(a + bx^2 + cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3c^{\frac{3}{4}} \arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b-\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right) \left(b^2 + 44ac - \frac{b^3}{\sqrt{-4ac+b^2}} + \frac{68abc}{\sqrt{-4ac+b^2}}\right) 2^{\frac{3}{4}}}{64a(-4ac+b^2)^2(-b-\sqrt{-4ac+b^2})^{\frac{3}{4}}} \\ & - \frac{3c^{\frac{3}{4}} \operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b-\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right) \left(b^2 + 44ac - \frac{b^3}{\sqrt{-4ac+b^2}} + \frac{68abc}{\sqrt{-4ac+b^2}}\right) 2^{\frac{3}{4}}}{64a(-4ac+b^2)^2(-b-\sqrt{-4ac+b^2})^{\frac{3}{4}}} \\ & - \frac{3c^{\frac{3}{4}} \arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b+\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right) \left(b^3 - 68abc + (44ac+b^2)\sqrt{-4ac+b^2}\right) 2^{\frac{3}{4}}}{64a(-4ac+b^2)^{\frac{5}{2}}(-b+\sqrt{-4ac+b^2})^{\frac{3}{4}}} \\ & - \frac{3c^{\frac{3}{4}} \operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}}{(-b+\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right) \left(b^3 - 68abc + (44ac+b^2)\sqrt{-4ac+b^2}\right) 2^{\frac{3}{4}}}{64a(-4ac+b^2)^{\frac{5}{2}}(-b+\sqrt{-4ac+b^2})^{\frac{3}{4}}} \\ & - \frac{(2cx^2+b)\sqrt{x}}{4(-4ac+b^2)(cx^4+bx^2+a)^2} + \frac{(b(20ac+b^2)+c(44ac+b^2)x^2)\sqrt{x}}{16a(-4ac+b^2)^2(cx^4+bx^2+a)} \end{aligned}$$

command

```
integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

28 Test file number 40

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/40_1.2.2.3-d+e_x^2-
^m-a+b_x^2+c_x^4-^p

28.1 Problem number 14

$$\int \frac{1 + bx^2}{\sqrt{1 - b^2x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(x\sqrt{b}, i\right)}{\sqrt{b}}$$

command

```
integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{-b^2x^4 + 1}}{bx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-b^2x^4 + 1}}{bx^2 - 1}, x\right)$$

28.2 Problem number 15

$$\int \frac{1 - bx^2}{\sqrt{1 - b^2x^4}} dx$$

Optimal antiderivative

$$-\frac{\text{EllipticE}\left(x\sqrt{b}, i\right)}{\sqrt{b}} + \frac{2 \text{EllipticF}\left(x\sqrt{b}, i\right)}{\sqrt{b}}$$

command

```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-b^2x^4 + 1}}{bx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-b^2x^4 + 1}}{bx^2 + 1}, x\right)$$

28.3 Problem number 22

$$\int \frac{\sqrt{1 + c^2x^2}}{\sqrt{1 - c^2x^2}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}(cx, i)}{c}$$

command

```
integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{c^2x^2 + 1} \sqrt{-c^2x^2 + 1}}{c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{c^2x^2 + 1} \sqrt{-c^2x^2 + 1}}{c^2x^2 - 1}, x\right)$$

28.4 Problem number 23

$$\int \frac{1 + c^2x^2}{\sqrt{1 - c^4x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}(cx, i)}{c}$$

command

```
integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\sqrt{-c^4x^4+1}}{c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-c^4x^4+1}}{c^2x^2-1}, x\right)$$

28.5 Problem number 24

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}} dx$$

Optimal antiderivative

$$-\frac{\text{EllipticE}(cx, i)}{c} + \frac{2 \text{EllipticF}(cx, i)}{c}$$

command

`integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{c^2x^2+1} \sqrt{-c^2x^2+1}}{c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}, x\right)$$

28.6 Problem number 25

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}} dx$$

Optimal antiderivative

$$-\frac{\text{EllipticE}(cx, i)}{c} + \frac{2 \text{EllipticF}(cx, i)}{c}$$

command

```
integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-c^4x^4 + 1}}{c^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-c^4x^4 + 1}}{c^2x^2 + 1}, x\right)$$

28.7 Problem number 113

$$\int \frac{3 - x^2}{\sqrt{3 + x^2 - x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(\frac{x\sqrt{2}}{\sqrt{1 + \sqrt{13}}}, \frac{i\sqrt{3}}{6} + \frac{i\sqrt{39}}{6}\right) \sqrt{-2 + 2\sqrt{13}}}{2} + \text{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{1 + \sqrt{13}}}, \frac{i\sqrt{3}}{6} + \frac{i\sqrt{39}}{6}\right) \sqrt{7 + 2\sqrt{13}}$$

command

```
integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^4 + x^2 + 3}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^4 + x^2 + 3} (x^2 - 3)}{x^4 - x^2 - 3}, x\right)$$

28.8 Problem number 114

$$\int \frac{3 - x^2}{\sqrt{3 + 2x^2 - x^4}} dx$$

Optimal antiderivative

$$- \text{EllipticE} \left(\frac{x\sqrt{3}}{3}, i\sqrt{3} \right) + 4 \text{EllipticF} \left(\frac{x\sqrt{3}}{3}, i\sqrt{3} \right)$$

command

`integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^4 + 2x^2 + 3}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{-x^4 + 2x^2 + 3}}{x^2 + 1}, x \right)$$

28.9 Problem number 115

$$\int \frac{3 - x^2}{\sqrt{3 + 3x^2 - x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE} \left(\frac{x\sqrt{2}}{\sqrt{3 + \sqrt{21}}}, \frac{i\sqrt{3}}{2} + \frac{i\sqrt{7}}{2} \right) \sqrt{-6 + 2\sqrt{21}}}{2} + \text{EllipticF} \left(\frac{x\sqrt{2}}{\sqrt{3 + \sqrt{21}}}, \frac{i\sqrt{3}}{2} + \frac{i\sqrt{7}}{2} \right) \sqrt{9 + 2\sqrt{21}}$$

command

`integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^4 + 3x^2 + 3}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{-x^4 + 3x^2 + 3} (x^2 - 3)}{x^4 - 3x^2 - 3}, x \right)$$

28.10 Problem number 116

$$\int \frac{3 - x^2}{\sqrt{3 - x^2 - x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(\frac{x\sqrt{2}}{\sqrt{-1 + \sqrt{13}}}, \frac{i\sqrt{39}}{6} - \frac{i\sqrt{3}}{6}\right) \sqrt{2 + 2\sqrt{13}}}{2} + \text{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{-1 + \sqrt{13}}}, \frac{i\sqrt{39}}{6} - \frac{i\sqrt{3}}{6}\right) \sqrt{5 + 2\sqrt{13}}$$

command

```
integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^4 - x^2 + 3}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^4 - x^2 + 3} (x^2 - 3)}{x^4 + x^2 - 3}, x\right)$$

28.11 Problem number 117

$$\int \frac{3 - x^2}{\sqrt{3 - 2x^2 - x^4}} dx$$

Optimal antiderivative

$$-\text{EllipticE}\left(x, \frac{i\sqrt{3}}{3}\right) \sqrt{3} + 2 \text{EllipticF}\left(x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}$$

command

```
integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^4 - 2x^2 + 3}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^4 - 2x^2 + 3} (x^2 - 3)}{x^4 + 2x^2 - 3}, x\right)$$

28.12 Problem number 118

$$\int \frac{3 - x^2}{\sqrt{3 - 3x^2 - x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(\frac{x\sqrt{2}}{\sqrt{-3 + \sqrt{21}}}, \frac{i\sqrt{7}}{2} - \frac{i\sqrt{3}}{2}\right) \sqrt{6 + 2\sqrt{21}}}{2} + \text{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{-3 + \sqrt{21}}}, \frac{i\sqrt{7}}{2} - \frac{i\sqrt{3}}{2}\right) \sqrt{3 + 2\sqrt{21}}$$

command

```
integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^4 - 3x^2 + 3}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^4 - 3x^2 + 3} (x^2 - 3)}{x^4 + 3x^2 - 3}, x\right)$$

28.13 Problem number 149

$$\int \frac{1}{(d + ex^2)^2 (a + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{e^4 x}{2d(ae^2 + cd^2)^2(e^2 x^2 + d)} + \frac{cx(-2cde x^2 - ae^2 + cd^2)}{4a(ae^2 + cd^2)^2(cx^4 + a)} + \frac{e^{\frac{7}{2}} \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}(ae^2 + cd^2)^2} \\
& + \frac{c^{\frac{3}{4}} e^2 \arctan\left(-1 + \frac{c^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (3cd^2 - ae^2 - 4de\sqrt{a}\sqrt{c}) \sqrt{2}}{4a^{\frac{3}{4}}(ae^2 + cd^2)^3} \\
& + \frac{c^{\frac{3}{4}} e^2 \arctan\left(1 + \frac{c^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (3cd^2 - ae^2 - 4de\sqrt{a}\sqrt{c}) \sqrt{2}}{4a^{\frac{3}{4}}(ae^2 + cd^2)^3} \\
& + \frac{c^{\frac{3}{4}} \arctan\left(-1 + \frac{c^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (3cd^2 - 3ae^2 - 2de\sqrt{a}\sqrt{c}) \sqrt{2}}{16a^{\frac{7}{4}}(ae^2 + cd^2)^2} \\
& + \frac{c^{\frac{3}{4}} \arctan\left(1 + \frac{c^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (3cd^2 - 3ae^2 - 2de\sqrt{a}\sqrt{c}) \sqrt{2}}{16a^{\frac{7}{4}}(ae^2 + cd^2)^2} \\
& - \frac{c^{\frac{3}{4}} \ln\left(-a^{\frac{1}{4}} c^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{c}\right) (3cd^2 - 3ae^2 + 2de\sqrt{a}\sqrt{c}) \sqrt{2}}{32a^{\frac{7}{4}}(ae^2 + cd^2)^2} \\
& + \frac{c^{\frac{3}{4}} \ln\left(a^{\frac{1}{4}} c^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{c}\right) (3cd^2 - 3ae^2 + 2de\sqrt{a}\sqrt{c}) \sqrt{2}}{32a^{\frac{7}{4}}(ae^2 + cd^2)^2} \\
& - \frac{c^{\frac{3}{4}} e^2 \ln\left(-a^{\frac{1}{4}} c^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{c}\right) (3cd^2 - ae^2 + 4de\sqrt{a}\sqrt{c}) \sqrt{2}}{8a^{\frac{3}{4}}(ae^2 + cd^2)^3} \\
& + \frac{c^{\frac{3}{4}} e^2 \ln\left(a^{\frac{1}{4}} c^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{c}\right) (3cd^2 - ae^2 + 4de\sqrt{a}\sqrt{c}) \sqrt{2}}{8a^{\frac{3}{4}}(ae^2 + cd^2)^3} \\
& + \frac{4ce^{\frac{7}{2}} \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) \sqrt{d}}{(ae^2 + cd^2)^3}
\end{aligned}$$

command

```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

28.14 Problem number 235

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)}{2} + \frac{(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}}{4\cos(2\arctan(x))\sqrt{x^4+x^2+1}}$$

command

`integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{8}\sqrt{2}(\sqrt{-3}+1)\sqrt{\sqrt{-3}-1}\operatorname{ellipticF}\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}, \frac{1}{2}\sqrt{-3}-\frac{1}{2}\right) + \frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^6+2x^4+2x^2+1}, x\right)$$

28.15 Problem number 263

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4} dx$$

Optimal antiderivative

$$\frac{e^2(6c^2d^2+b^2e^2-ce(ae+4bd))x}{c^3} + \frac{e^3(-be+4cd)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(e(-be+2cd)(2c^2d^2+b^2e^2-2ce(ae+bd)) + \frac{2c^4d^4+b^4e^4-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)}{\sqrt{-4ac+b^2}}\right)}{2c^{\frac{7}{2}}\sqrt{b-\sqrt{-4ac+b^2}}} + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(e(-be+2cd)(2c^2d^2+b^2e^2-2ce(ae+bd)) + \frac{-2c^4d^4-b^4e^4+4b^2ce^3(ae+bd)+4c^3d^2e(3ae+bd)}{\sqrt{-4ac+b^2}}\right)}{2c^{\frac{7}{2}}\sqrt{b+\sqrt{-4ac+b^2}}}$$

command

```
integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

28.16 Problem number 268

$$\int \frac{1}{(d + ex^2)(a + bx^2 + cx^4)} dx$$

Optimal antiderivative

$$\frac{e^{\frac{3}{2}} \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{(ae^2 - bde + cd^2)\sqrt{d}} - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2(ae^2 - bde + cd^2)\sqrt{b - \sqrt{-4ac + b^2}}}$$

$$- \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2(ae^2 - bde + cd^2)\sqrt{b + \sqrt{-4ac + b^2}}}$$

command

```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

28.17 Problem number 303

$$\int \frac{1}{\sqrt{2+3x^2+x^4}} dx$$

Optimal antiderivative

$$\frac{(x^2+1)^{\frac{3}{2}} \sqrt{\frac{1}{x^2+1}} \operatorname{EllipticF}\left(\frac{x}{\sqrt{x^2+1}}, \frac{\sqrt{2}}{2}\right) \sqrt{2} \sqrt{\frac{x^2+2}{x^2+1}}}{2\sqrt{x^4+3x^2+2}}$$

command

```
integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \operatorname{ellipticF}\left(\frac{1}{2}i \sqrt{2} x, 2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{x^4+3x^2+2}}, x\right)$$

28.18 Problem number 316

$$\int (7+5x^2)^4 \sqrt{2+x^2-x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{116100x(-x^4+x^2+2)^{\frac{3}{2}}}{77} - \frac{14500x^3(-x^4+x^2+2)^{\frac{3}{2}}}{33} \\ & - \frac{625x^5(-x^4+x^2+2)^{\frac{3}{2}}}{11} + \frac{3764813 \operatorname{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{231} \\ & - \frac{539419 \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{77} + \frac{x(717372x^2+177953) \sqrt{-x^4+x^2+2}}{231} \end{aligned}$$

command

```
integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(13125x^{10} + 88375x^8 + 220550x^6 + 166072x^4 - 518647x^2 - 3764813)\sqrt{-x^4 + x^2 + 2}}{231x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((625x^8 + 3500x^6 + 7350x^4 + 6860x^2 + 2401)\sqrt{-x^4 + x^2 + 2}, x\right)$$

28.19 Problem number 317

$$\int (7 + 5x^2)^3 \sqrt{2 + x^2 - x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1825x(-x^4 + x^2 + 2)^{\frac{3}{2}}}{21} - \frac{125x^3(-x^4 + x^2 + 2)^{\frac{3}{2}}}{9} + \frac{79411 \text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{63} \\ & - \frac{8735 \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{21} + \frac{x(14691x^2 + 5956)\sqrt{-x^4 + x^2 + 2}}{63} \end{aligned}$$

command

`integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(875x^8 + 4600x^6 + 7466x^4 - 4994x^2 - 79411)\sqrt{-x^4 + x^2 + 2}}{63x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((125x^6 + 525x^4 + 735x^2 + 343)\sqrt{-x^4 + x^2 + 2}, x\right)$$

28.20 Problem number 318

$$\int (7 + 5x^2)^2 \sqrt{2 + x^2 - x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{25x(-x^4 + x^2 + 2)^{\frac{3}{2}}}{7} + \frac{2045 \text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{21} \\ & - \frac{79 \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{7} + \frac{x(354x^2 + 275)\sqrt{-x^4 + x^2 + 2}}{21} \end{aligned}$$

command

```
integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(75x^6 + 279x^4 + 125x^2 - 2045)\sqrt{-x^4 + x^2 + 2}}{21x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((25x^4 + 70x^2 + 49)\sqrt{-x^4 + x^2 + 2}, x\right)$$

28.21 Problem number 319

$$\int (7 + 5x^2) \sqrt{2 + x^2 - x^4} dx$$

Optimal antiderivative

$$7 \text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) + 3 \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) + x(x^2 + 2) \sqrt{-x^4 + x^2 + 2}$$

command

```
integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(x^4 + 2x^2 - 7)\sqrt{-x^4 + x^2 + 2}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{-x^4 + x^2 + 2} (5x^2 + 7), x\right)$$

28.22 Problem number 320

$$\int \sqrt{2 + x^2 - x^4} dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{3} + \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) + \frac{x\sqrt{-x^4 + x^2 + 2}}{3}$$

command

`integrate((-x^4+x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^4 + x^2 + 2} (x^2 - 1)}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{-x^4 + x^2 + 2}, x\right)$$

28.23 Problem number 324

$$\int (7 + 5x^2)^4 (2 + x^2 - x^4)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x(-1581440x^2 + 69817)(-x^4 + x^2 + 2)^{\frac{3}{2}}}{1001} - \frac{132300x(-x^4 + x^2 + 2)^{\frac{5}{2}}}{143} \\ & - \frac{11750x^3(-x^4 + x^2 + 2)^{\frac{5}{2}}}{39} - \frac{125x^5(-x^4 + x^2 + 2)^{\frac{5}{2}}}{3} + \frac{124141422 \text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{5005} \\ & - \frac{50794416 \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{5005} + \frac{3x(7837383x^2 + 2193559)\sqrt{-x^4 + x^2 + 2}}{5005} \end{aligned}$$

command

`integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(625625x^{14} + 3272500x^{12} + 2967125x^{10} - 15130150x^8 - 45845855x^6 - 43271392x^4 + 37918479x^2 + 3724242)}{15015x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(- (625x^{12} + 2875x^{10} + 2600x^8 - 7490x^6 - 19159x^4 - 16121x^2 - 4802)\sqrt{-x^4 + x^2 + 2}, x\right)$$

28.24 Problem number 325

$$\int (7 + 5x^2)^3 (2 + x^2 - x^4)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(374045x^2 + 33792)(-x^4 + x^2 + 2)^{\frac{3}{2}}}{3003} - \frac{7825x(-x^4 + x^2 + 2)^{\frac{5}{2}}}{143} \\ & - \frac{125x^3(-x^4 + x^2 + 2)^{\frac{5}{2}}}{13} + \frac{31072528 \operatorname{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{15015} \\ & - \frac{3199778 \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{5005} + \frac{x(5712051x^2 + 2512273)\sqrt{-x^4 + x^2 + 2}}{15015} \end{aligned}$$

command

`integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(144375x^{12} + 532875x^{10} - 206150x^8 - 3588640x^6 - 5757461x^4 + 436307x^2 + 31072528)\sqrt{-x^4 + x^2 + 2}}{15015x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(125x^{10} + 400x^8 - 40x^6 - 1442x^4 - 1813x^2 - 686\right)\sqrt{-x^4 + x^2 + 2}, x\right)$$

28.25 Problem number 326

$$\int (7 + 5x^2)^2 (2 + x^2 - x^4)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(920x^2 + 363)(-x^4 + x^2 + 2)^{\frac{3}{2}}}{99} - \frac{25x(-x^4 + x^2 + 2)^{\frac{5}{2}}}{11} + \frac{85942 \operatorname{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{495} \\ & - \frac{3392 \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{165} + \frac{x(14889x^2 + 11497)\sqrt{-x^4 + x^2 + 2}}{495} \end{aligned}$$

command

`integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(1125x^{10} + 2350x^8 - 6160x^6 - 21404x^4 - 10627x^2 + 85942)\sqrt{-x^4 + x^2 + 2}}{495x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(25x^8 + 45x^6 - 71x^4 - 189x^2 - 98\right)\sqrt{-x^4 + x^2 + 2}, x\right)$$

28.26 Problem number 327

$$\int (7 + 5x^2)(2 + x^2 - x^4)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(35x^2 + 48)(-x^4 + x^2 + 2)^{\frac{3}{2}}}{63} + \frac{4432 \text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{315} \\ & + \frac{418 \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{105} + \frac{x(669x^2 + 1087)\sqrt{-x^4 + x^2 + 2}}{315} \end{aligned}$$

command

`integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(175x^8 + 65x^6 - 1259x^4 - 1567x^2 + 4432)\sqrt{-x^4 + x^2 + 2}}{315x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(5x^6 + 2x^4 - 17x^2 - 14\right)\sqrt{-x^4 + x^2 + 2}, x\right)$$

28.27 Problem number 328

$$\int (2 + x^2 - x^4)^{3/2} dx$$

Optimal antiderivative

$$\frac{x(-x^4 + x^2 + 2)^{\frac{3}{2}}}{7} + \frac{34 \operatorname{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{35} \\ + \frac{48 \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{35} + \frac{x(3x^2 + 19)\sqrt{-x^4 + x^2 + 2}}{35}$$

command

`integrate((-x^4+x^2+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(5x^6 - 8x^4 - 29x^2 + 34)\sqrt{-x^4 + x^2 + 2}}{35x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((-x^4 + x^2 + 2)^{\frac{3}{2}}, x\right)$$

28.28 Problem number 332

$$\int \frac{(7 + 5x^2)^3}{\sqrt{2 + x^2 - x^4}} dx$$

Optimal antiderivative

$$\frac{3905 \operatorname{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{3} - 542 \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) \\ - \frac{625x\sqrt{-x^4 + x^2 + 2}}{3} - 25x^3\sqrt{-x^4 + x^2 + 2}$$

command

`integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5(15x^4 + 125x^2 + 781)\sqrt{-x^4 + x^2 + 2}}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(125x^6 + 525x^4 + 735x^2 + 343)\sqrt{-x^4 + x^2 + 2}}{x^4 - x^2 - 2}, x\right)$$

28.29 Problem number 333

$$\int \frac{(7 + 5x^2)^2}{\sqrt{2 + x^2 - x^4}} dx$$

Optimal antiderivative

$$\frac{260 \operatorname{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{3} - 21 \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) - \frac{25x\sqrt{-x^4 + x^2 + 2}}{3}$$

command

`integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{5\sqrt{-x^4+x^2+2}(5x^2+52)}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(25x^4+70x^2+49)\sqrt{-x^4+x^2+2}}{x^4-x^2-2}, x\right)$$

28.30 Problem number 334

$$\int \frac{7 + 5x^2}{\sqrt{2 + x^2 - x^4}} dx$$

Optimal antiderivative

$$5 \operatorname{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right) + 2 \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)$$

command

`integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{5\sqrt{-x^4+x^2+2}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^4+x^2+2}(5x^2+7)}{x^4-x^2-2}, x\right)$$

28.31 Problem number 335

$$\int \frac{1}{\sqrt{2+x^2-x^4}} dx$$

Optimal antiderivative

$$\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)$$

command

`integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\text{ellipticF}\left(\frac{1}{2}\sqrt{2}x, -2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4+x^2+2}}{x^4-x^2-2}, x\right)$$

28.32 Problem number 339

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3482293 \text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{18} + \frac{627857 \text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{6} \\ & + \frac{x(1419793x^2 + 1419985)}{18\sqrt{-x^4+x^2+2}} + \frac{27500x\sqrt{-x^4+x^2+2}}{3} + 625x^3\sqrt{-x^4+x^2+2} \end{aligned}$$

command

`integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(5625x^8 + 76875x^6 + 937500x^4 - 2616139x^2 - 3482293)\sqrt{-x^4+x^2+2}}{9(x^5-x^3-2x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(3125x^{10} + 21875x^8 + 61250x^6 + 85750x^4 + 60025x^2 + 16807)\sqrt{-x^4+x^2+2}}{x^8-2x^6-3x^4+4x^2+4}, x\right)$$

28.33 Problem number 340

$$\int \frac{(7 + 5x^2)^4}{(2 + x^2 - x^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{165239 \operatorname{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{18} + \frac{31921 \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{6} + \frac{x(83489x^2 + 83585)}{18\sqrt{-x^4 + x^2 + 2}} + \frac{625x\sqrt{-x^4 + x^2 + 2}}{3}$$

command

`integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(1875x^6 + 39000x^4 - 128162x^2 - 165239)\sqrt{-x^4 + x^2 + 2}}{9(x^5 - x^3 - 2x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(625x^8 + 3500x^6 + 7350x^4 + 6860x^2 + 2401)\sqrt{-x^4 + x^2 + 2}}{x^8 - 2x^6 - 3x^4 + 4x^2 + 4}, x\right)$$

28.34 Problem number 341

$$\int \frac{(7 + 5x^2)^3}{(2 + x^2 - x^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{7147 \operatorname{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{18} + \frac{1763 \operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{6} + \frac{x(4897x^2 + 4945)}{18\sqrt{-x^4 + x^2 + 2}}$$

command

`integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(1125x^4 - 6046x^2 - 7147)\sqrt{-x^4 + x^2 + 2}}{9(x^5 - x^3 - 2x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(125x^6 + 525x^4 + 735x^2 + 343)\sqrt{-x^4 + x^2 + 2}}{x^8 - 2x^6 - 3x^4 + 4x^2 + 4}, x\right)$$

28.35 Problem number 367

$$\int \frac{1}{\sqrt{4 + 3x^2 + x^4}} dx$$

Optimal antiderivative

$$\frac{(x^2 + 2) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{x\sqrt{2}}{2}\right)\right), \frac{\sqrt{2}}{4}\right) \sqrt{\frac{x^4 + 3x^2 + 4}{(x^2 + 2)^2}} \sqrt{2}}{4 \cos\left(2 \arctan\left(\frac{x\sqrt{2}}{2}\right)\right) \sqrt{x^4 + 3x^2 + 4}}$$

command

```
integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{16} \sqrt{2} (\sqrt{-7} + 3) \sqrt{\sqrt{-7} - 3} \operatorname{ellipticF}\left(\frac{1}{4} \sqrt{2} x \sqrt{\sqrt{-7} - 3}, \frac{3}{8} \sqrt{-7} + \frac{1}{8}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{x^4 + 3x^2 + 4}}, x\right)$$

29 Test file number 41

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/41_1.2.2.4-f_x-
~m-d+e_x^2-~q-a+b_x^2+c_x^4-~p

29.1 Problem number 236

$$\int \frac{1}{x^5 (d + ex^2) (a + cx^4)} dx$$

Optimal antiderivative

$$-\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} + \frac{c^{\frac{3}{2}} e \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}} (ae^2 + cd^2)} - \frac{(-ae^2 + cd^2) \ln(x)}{a^2d^3} - \frac{e^4 \ln(ex^2 + d)}{2d^3 (ae^2 + cd^2)} + \frac{c^2d \ln(cx^4 + a)}{4a^2 (ae^2 + cd^2)}$$

command

```
integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{acd^3x^4\sqrt{-\frac{c}{a}}e\log\left(\frac{cx^4+2ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+c^2d^4x^4\log(cx^4+a)+2acd^3x^2e-2a^2x^4e^4\log(x^2e+d)-acd^4+2a^2d^2x^2e^3}{4(a^2cd^5x^4+a^3d^3x^4e^2)}$$

$$\frac{2acd^3x^4\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)e-c^2d^4x^4\log(cx^4+a)-2acd^3x^2e+2a^2x^4e^4\log(x^2e+d)+acd^4-2a^2dx^2e^3}{4(a^2cd^5x^4+a^3d^3x^4e^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.2 Problem number 249

$$\int \frac{1}{x(d+ex^2)(a+cx^4)^2} dx$$

Optimal antiderivative

$$\frac{c(-ex^2+d)}{4a(ae^2+cd^2)(cx^4+a)} + \frac{\ln(x)}{a^2d} - \frac{e^4\ln(ex^2+d)}{2d(ae^2+cd^2)^2} - \frac{cd(2ae^2+cd^2)\ln(cx^4+a)}{4a^2(ae^2+cd^2)^2}$$

$$- \frac{e\arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)\sqrt{c}}{4a^{\frac{3}{2}}(ae^2+cd^2)} - \frac{e^3\arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)\sqrt{c}}{2(ae^2+cd^2)^2\sqrt{a}}$$

command

```
integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2ac^2d^3x^2e - 2ac^2d^4 + 2a^2cdx^2e^3 - 2a^2cd^2e^2 + 4(a^2cx^4 + a^3)e^4 \log(x^2e + d) - (3(a^2cdx^4 + a^3d)e^3 + (ac^2d^3x^4 + a^2d^4)e^2 + (ac^2d^3x^4 + a^2d^4)e)$$

$$ac^2d^3x^2e - ac^2d^4 + a^2cdx^2e^3 - a^2cd^2e^2 + 2(a^2cx^4 + a^3)e^4 \log(x^2e + d) - (3(a^2cdx^4 + a^3d)e^3 + (ac^2d^3x^4 + a^2d^4)e^2 + (ac^2d^3x^4 + a^2d^4)e)$$

$$4(a^2c^3d^5x^5 + a^3cd^4x^4 + a^4cd^3x^3 + a^5cd^2x^2 + a^6cdx + a^7c)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.3 Problem number 250

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1}{2a^2d^2x^2} - \frac{c(cd^2x^2 + ae)}{4a^2(ae^2 + cd^2)(cx^4 + a)} - \frac{c^{\frac{3}{2}}d \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}(ae^2 + cd^2)} \\ & - \frac{c^{\frac{3}{2}}d(2ae^2 + cd^2) \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}(ae^2 + cd^2)^2} - \frac{e \ln(x)}{a^2d^2} \\ & + \frac{e^5 \ln(ex^2 + d)}{2d^2(ae^2 + cd^2)^2} + \frac{ce(2ae^2 + cd^2) \ln(cx^4 + a)}{4a^2(ae^2 + cd^2)^2} \end{aligned}$$

command

```
integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6c^3d^5x^4 + 2ac^2d^4x^2e + 4ac^2d^5 + 2a^2cd^2x^2e^3 - 4(a^2cx^6 + a^3x^2)e^5 \log(x^2e + d) - (3c^3d^5x^6 + 3ac^2d^5x^2 + 5(ac^2d$$

$$3c^3d^5x^4 + ac^2d^4x^2e + 2ac^2d^5 + a^2cd^2x^2e^3 - 2(a^2cx^6 + a^3x^2)e^5 \log(x^2e + d) - (3c^3d^5x^6 + 3ac^2d^5x^2 + 5(ac^2d$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.4 Problem number 258

$$\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{1}{3a^2 d x^3} + \frac{e}{a^2 d^2 x} - \frac{c^2 x(-e x^2 + d)}{4a^2 (a e^2 + c d^2)(c x^4 + a)} + \frac{e^{\frac{11}{2}} \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{d^{\frac{5}{2}} (a e^2 + c d^2)^2} \\
& - \frac{c^{\frac{5}{4}} (2a e^2 + c d^2) \arctan\left(-1 + \frac{c^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (-e\sqrt{a} + d\sqrt{c}) \sqrt{2}}{4a^{\frac{11}{4}} (a e^2 + c d^2)^2} \\
& - \frac{c^{\frac{5}{4}} (2a e^2 + c d^2) \arctan\left(1 + \frac{c^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (-e\sqrt{a} + d\sqrt{c}) \sqrt{2}}{4a^{\frac{11}{4}} (a e^2 + c d^2)^2} \\
& + \frac{c^{\frac{5}{4}} (2a e^2 + c d^2) \ln\left(-a^{\frac{1}{4}} c^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{c}\right) (e\sqrt{a} + d\sqrt{c}) \sqrt{2}}{8a^{\frac{11}{4}} (a e^2 + c d^2)^2} \\
& - \frac{c^{\frac{5}{4}} (2a e^2 + c d^2) \ln\left(a^{\frac{1}{4}} c^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{c}\right) (e\sqrt{a} + d\sqrt{c}) \sqrt{2}}{8a^{\frac{11}{4}} (a e^2 + c d^2)^2} \\
& - \frac{c^{\frac{5}{4}} \arctan\left(-1 + \frac{c^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (-e\sqrt{a} + 3d\sqrt{c}) \sqrt{2}}{16a^{\frac{11}{4}} (a e^2 + c d^2)} \\
& - \frac{c^{\frac{5}{4}} \arctan\left(1 + \frac{c^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (-e\sqrt{a} + 3d\sqrt{c}) \sqrt{2}}{16a^{\frac{11}{4}} (a e^2 + c d^2)} \\
& + \frac{c^{\frac{5}{4}} \ln\left(-a^{\frac{1}{4}} c^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{c}\right) (e\sqrt{a} + 3d\sqrt{c}) \sqrt{2}}{32a^{\frac{11}{4}} (a e^2 + c d^2)} \\
& - \frac{c^{\frac{5}{4}} \ln\left(a^{\frac{1}{4}} c^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{c}\right) (e\sqrt{a} + 3d\sqrt{c}) \sqrt{2}}{32a^{\frac{11}{4}} (a e^2 + c d^2)}
\end{aligned}$$

command

```
integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.5 Problem number 259

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)\sqrt{2}}{4} + \frac{(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^4+1}{(x^2+1)^2}}}{4\cos(2\arctan(x))\sqrt{x^4+1}}$$

command

```
integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right) - \frac{1}{2}i\sqrt{i}\operatorname{ellipticF}(\sqrt{i}x, -1)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^4+1}x^2}{x^6+x^4+x^2+1}, x\right)$$

29.6 Problem number 260

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)\sqrt{2}}{4} - \frac{(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^4+1}{(x^2+1)^2}}}{4\cos(2\arctan(x))\sqrt{x^4+1}}$$

command

```
integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2}i\sqrt{i}\operatorname{ellipticF}\left(\sqrt{i}x,-1\right)+\frac{1}{8}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{x^4+1}x^2}{x^6-x^4+x^2-1},x\right)$$

29.7 Problem number 263

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(-x^2+1)}{2\sqrt{x^4-1}} - \frac{\operatorname{EllipticE}(x,i)\sqrt{-x^2+1}\sqrt{x^2+1}}{2\sqrt{x^4-1}} \\ & + \frac{\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}},\frac{\sqrt{2}}{2}\right)\sqrt{x^2-1}\sqrt{x^2+1}\sqrt{2}}{2\sqrt{x^4-1}} \end{aligned}$$

command

```
integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{x^4-1}x}{2(x^2+1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^4-1}x^2}{x^6+x^4-x^2-1},x\right)$$

29.8 Problem number 264

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}} dx$$

Optimal antiderivative

$$\frac{x(x^2+1)}{2\sqrt{x^4-1}} - \frac{\text{EllipticE}(x, i) \sqrt{-x^2+1} \sqrt{x^2+1}}{2\sqrt{x^4-1}}$$

command

```
integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{x^4-1} x}{2(x^2-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{x^4-1} x^2}{x^6-x^4-x^2+1}, x\right)$$

29.9 Problem number 265

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}} dx$$

Optimal antiderivative

$$\frac{\text{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{-x^4-1}}\right) \sqrt{2}}{4} + \frac{(x^2+1) \sqrt{\frac{\cos(4 \arctan(x))}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin(2 \arctan(x)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^4+1}{(x^2+1)^2}}}{4 \cos(2 \arctan(x)) \sqrt{-x^4-1}}$$

command

```
integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2} \sqrt{i} \operatorname{ellipticF}(\sqrt{i} x, -1) - \frac{1}{8} \sqrt{2} \log\left(\frac{\sqrt{2} x + \sqrt{-x^4 - 1}}{x^2 + 1}\right) \\ + \frac{1}{8} \sqrt{2} \log\left(-\frac{\sqrt{2} x - \sqrt{-x^4 - 1}}{x^2 + 1}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$-\frac{1}{8} \sqrt{2} \log\left(\frac{\sqrt{2} x + \sqrt{-x^4 - 1}}{x^2 + 1}\right) \\ + \frac{1}{8} \sqrt{2} \log\left(-\frac{\sqrt{2} x - \sqrt{-x^4 - 1}}{x^2 + 1}\right) + \operatorname{integral}\left(-\frac{\sqrt{-x^4 - 1}}{2(x^4 + 1)}, x\right)$$

29.10 Problem number 266

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{-x^4-1}}\right)\sqrt{2}}{4} \\ \frac{(x^2+1)\sqrt{\frac{\cos(4\arctan(x))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2\arctan(x)), \frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^4+1}{(x^2+1)^2}}}{4\cos(2\arctan(x))\sqrt{-x^4-1}}$$

command

`integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{i} \operatorname{ellipticF}(\sqrt{i} x, -1) - \frac{1}{8} i \sqrt{2} \log\left(\frac{i \sqrt{2} x + \sqrt{-x^4 - 1}}{x^2 - 1}\right) \\ + \frac{1}{8} i \sqrt{2} \log\left(\frac{-i \sqrt{2} x + \sqrt{-x^4 - 1}}{x^2 - 1}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$-\frac{1}{8} i \sqrt{2} \log\left(\frac{i \sqrt{2} x + \sqrt{-x^4 - 1}}{x^2 - 1}\right) \\ + \frac{1}{8} i \sqrt{2} \log\left(\frac{-i \sqrt{2} x + \sqrt{-x^4 - 1}}{x^2 - 1}\right) + \operatorname{integral}\left(\frac{\sqrt{-x^4 - 1}}{2(x^4 + 1)}, x\right)$$

29.11 Problem number 296

$$\int \frac{x^7}{(d + ex^2)(a + bx^2 + cx^4)} dx$$

Optimal antiderivative

$$\frac{x^2}{2ce} - \frac{d^3 \ln(ex^2 + d)}{2e^2(ae^2 - bde + cd^2)} + \frac{(-abe - acd + b^2d) \ln(cx^4 + bx^2 + a)}{4c^2(ae^2 - bde + cd^2)}$$

$$+ \frac{(2a^2ce - ab^2e - 3abcd + b^3d) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{2c^2(ae^2 - bde + cd^2)\sqrt{-4ac + b^2}}$$

command

`integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{2(b^2c^2 - 4ac^3)d^2x^2e - 2(b^3c - 4abc^2)dx^2e^2 - 2(b^2c^2 - 4ac^3)d^3 \log(x^2e + d) + 2(ab^2c - 4a^2c^2)x^2e^3 + ((b^3 - 4ac^2)d^2 - 2abcd + b^3d)e^4}{4((b^2c^3 - 4ac^4)d^2 - 2abcd + b^3d)e^5} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.12 Problem number 303

$$\int \frac{x^8}{(d + ex^2)(a + bx^2 + cx^4)} dx$$

Optimal antiderivative

$$-\frac{(be + cd)x}{c^2e^2} + \frac{x^3}{3ce} + \frac{d^{\frac{7}{2}} \operatorname{arctan}\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{e^{\frac{5}{2}}(ae^2 - bde + cd^2)}$$

$$-\frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(b^3d - 2abcd - ab^2e + a^2ce + \frac{-3a^2bce - 2a^2c^2d + ab^3e + 4ab^2cd - b^4d}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2c^{\frac{5}{2}}(ae^2 - bde + cd^2)\sqrt{b - \sqrt{-4ac + b^2}}}$$

$$-\frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(b^3d - 2abcd - ab^2e + a^2ce + \frac{3a^2bce + 2a^2c^2d - ab^3e - 4ab^2cd + b^4d}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2c^{\frac{5}{2}}(ae^2 - bde + cd^2)\sqrt{b + \sqrt{-4ac + b^2}}}$$

command

```
integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.13 Problem number 304

$$\int \frac{x^6}{(d + ex^2)(a + bx^2 + cx^4)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{ce} - \frac{d^{\frac{5}{2}} \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{e^{\frac{3}{2}}(ae^2 - bde + cd^2)} \\ & + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(b^2d - acd - abe + \frac{-2a^2ce + ab^2e + 3abcd - b^3d}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2c^{\frac{3}{2}}(ae^2 - bde + cd^2) \sqrt{b - \sqrt{-4ac + b^2}}} \\ & + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(b^2d - acd - abe + \frac{2a^2ce - ab^2e - 3abcd + b^3d}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2c^{\frac{3}{2}}(ae^2 - bde + cd^2) \sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

command

```
integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.14 Problem number 307

$$\int \frac{1}{(d + ex^2)(a + bx^2 + cx^4)} dx$$

Optimal antiderivative

$$\frac{e^{\frac{3}{2}} \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{(ae^2 - bde + cd^2)\sqrt{d}} - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2(ae^2 - bde + cd^2)\sqrt{b - \sqrt{-4ac + b^2}}}$$

$$- \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2(ae^2 - bde + cd^2)\sqrt{b + \sqrt{-4ac + b^2}}}$$

command

`integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.15 Problem number 308

$$\int \frac{1}{x^2(d + ex^2)(a + bx^2 + cx^4)} dx$$

Optimal antiderivative

$$-\frac{1}{adx} - \frac{e^{\frac{5}{2}} \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}(ae^2 - bde + cd^2)}$$

$$- \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(cd - be + \frac{2ace - b^2e + bcd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2a(ae^2 - bde + cd^2)\sqrt{b - \sqrt{-4ac + b^2}}}$$

$$- \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(cd - be + \frac{-2ace + b^2e - bcd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2a(ae^2 - bde + cd^2)\sqrt{b + \sqrt{-4ac + b^2}}}$$

command

```
integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.16 Problem number 311

$$\int \frac{x^5 \sqrt{a + bx^2 + cx^4}}{d + ex^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(cx^4 + bx^2 + a)^{\frac{3}{2}}}{6ce} \\ & - \frac{(16c^3d^3 - b^3e^3 - 2bce^2(-2ae + bd) - 8c^2de(-ae + bd)) \operatorname{arctanh}\left(\frac{2cx^2 + b}{2\sqrt{c}\sqrt{cx^4 + bx^2 + a}}\right)}{32c^{\frac{5}{2}}e^4} \\ & + \frac{d^2 \operatorname{arctanh}\left(\frac{bd - 2ae + (-be + 2cd)x^2}{2\sqrt{ae^2 - bde + cd^2}\sqrt{cx^4 + bx^2 + a}}\right) \sqrt{ae^2 - bde + cd^2}}{2e^4} \\ & + \frac{((-be + 2cd)(be + 4cd) - 2ce(be + 2cd)x^2) \sqrt{cx^4 + bx^2 + a}}{16c^2e^3} \end{aligned}$$

command

```
integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\left(48 \sqrt{cd^2 - bde + ae^2} c^3 d^2 \log \left(-\frac{8c^2d^2x^4 + 8bcd^2x^2 + (b^2 + 4ac)d^2 + 4\sqrt{cx^4 + bx^2 + a} (2cdx^2 + bd - (bx^2 + 2a)e) \sqrt{cd^2 - bde}}{x^4e^2 + 2dx^2e + d^2} \right) \right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.17 Problem number 314

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x(d + ex^2)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{bx^2+2a}{2\sqrt{a}\sqrt{cx^4+bx^2+a}}\right)\sqrt{a}}{2d} + \frac{\operatorname{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\sqrt{cx^4+bx^2+a}}\right)\sqrt{c}}{2e} - \frac{\operatorname{arctanh}\left(\frac{bd-2ae+(-be+2cd)x^2}{2\sqrt{ae^2-bde+cd^2}\sqrt{cx^4+bx^2+a}}\right)\sqrt{ae^2-bde+cd^2}}{2de}$$

command

`integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.18 Problem number 324

$$\int \frac{x(a + bx^2 + cx^4)^{3/2}}{d + ex^2} dx$$

Optimal antiderivative

$$\frac{(cx^4 + bx^2 + a)^{\frac{3}{2}}}{6e} - \frac{(-be + 2cd)(8c^2d^2 - b^2e^2 - 4ce(-3ae + 2bd)) \operatorname{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\sqrt{cx^4+bx^2+a}}\right)}{32c^{\frac{3}{2}}e^4} + \frac{(ae^2 - bde + cd^2)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{bd-2ae+(-be+2cd)x^2}{2\sqrt{ae^2-bde+cd^2}\sqrt{cx^4+bx^2+a}}\right)}{2e^4} + \frac{(8c^2d^2 + b^2e^2 - 2ce(-4ae + 5bd) - 2ce(-be + 2cd)x^2)\sqrt{cx^4 + bx^2 + a}}{16ce^3}$$

command

`integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{\left(3(16c^3d^3 - 24bc^2d^2e + 6(b^2c + 4ac^2)de^2 + (b^3 - 12abc)e^3) \sqrt{c} \log\left(-8c^2x^4 - 8bcx^2 - b^2 - 4\sqrt{cx^4 + bx^2}\right) \right)}{\dots} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.19 Problem number 355

$$\int \frac{x^5 \sqrt{d + ex^2}}{a + bx^2 + cx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(ex^2 + d)^{\frac{3}{2}}}{3ce} - \frac{b\sqrt{ex^2 + d}}{c^2} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex^2 + d}}{\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}\right) \left(bcd - b^2e + ace + \frac{-3abce + 2ac^2d + b^3e - b^2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2c^{\frac{5}{2}}\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex^2 + d}}{\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}\right) \left(bcd - b^2e + ace + \frac{3abce - 2ac^2d - b^3e + b^2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2c^{\frac{5}{2}}\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}} \end{aligned}$$

command

`integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.20 Problem number 358

$$\int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{ex^2+d}}{\sqrt{d}}\right)\sqrt{d}}{a} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex^2+d}}{\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}\right)\sqrt{c}\left(bd-2ae+d\sqrt{-4ac+b^2}\right)\sqrt{2}}{2a\sqrt{-4ac+b^2}\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex^2+d}}{\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}\right)\sqrt{c}\left(bd-2ae-d\sqrt{-4ac+b^2}\right)\sqrt{2}}{2a\sqrt{-4ac+b^2}\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}$$

command

```
integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.21 Problem number 368

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$$

Optimal antiderivative

$$\frac{e\sqrt{ex^2+d}}{c} \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex^2+d}}{\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}\right) \frac{(2c^2d^2+be^2(b-\sqrt{-4ac+b^2})-2ce(bd+ae-d\sqrt{-4ac+b^2}))}{2c^{\frac{3}{2}}\sqrt{-4ac+b^2}\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}$$

$$+ \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex^2+d}}{\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}\right) (2c^2d^2+be^2(b+\sqrt{-4ac+b^2})-2ce(bd+ae+d\sqrt{-4ac+b^2}))}{2c^{\frac{3}{2}}\sqrt{-4ac+b^2}\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}$$

command

```
integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.22 Problem number 372

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right) \sqrt{e}}{2c} + \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right) \left(cd - be + \frac{-2ace+b^2e-bcd}{\sqrt{-4ac+b^2}}\right) \sqrt{e}}{2c^2} \\ & + \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right) \left(cd - be + \frac{2ace-b^2e+bcd}{\sqrt{-4ac+b^2}}\right) \sqrt{e}}{2c^2} + \frac{ex\sqrt{ex^2+d}}{2c} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}{\sqrt{ex^2+d}\sqrt{b-\sqrt{-4ac+b^2}}}\right) \left(cd - be + \frac{-2ace+b^2e-bcd}{\sqrt{-4ac+b^2}}\right) \sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}{2c^2\sqrt{b-\sqrt{-4ac+b^2}}} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}{\sqrt{ex^2+d}\sqrt{b+\sqrt{-4ac+b^2}}}\right) \left(cd - be + \frac{2ace-b^2e+bcd}{\sqrt{-4ac+b^2}}\right) \sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}{2c^2\sqrt{b+\sqrt{-4ac+b^2}}} \end{aligned}$$

command

```
integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.23 Problem number 386

$$\int \frac{x^8}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{3d^2 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{8ce^{\frac{5}{2}}} + \frac{bd \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{2c^2e^{\frac{3}{2}}} \\
 & + \frac{(-ac+b^2) \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{c^3\sqrt{e}} - \frac{3dx\sqrt{ex^2+d}}{8ce^2} - \frac{bx\sqrt{ex^2+d}}{2c^2e} + \frac{x^3\sqrt{ex^2+d}}{4ce} \\
 & - \frac{\operatorname{arctan}\left(\frac{x\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}{\sqrt{ex^2+d}\sqrt{b-\sqrt{-4ac+b^2}}}\right) \left(b^3-2abc+\frac{-2a^2c^2+4ab^2c-b^4}{\sqrt{-4ac+b^2}}\right)}{c^3\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}\sqrt{b-\sqrt{-4ac+b^2}}} \\
 & - \frac{\operatorname{arctan}\left(\frac{x\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}{\sqrt{ex^2+d}\sqrt{b+\sqrt{-4ac+b^2}}}\right) \left(b^3-2abc+\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{-4ac+b^2}}\right)}{c^3\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}
 \end{aligned}$$

command

```
integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.24 Problem number 387

$$\int \frac{x^6}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{d \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{2ce^{\frac{3}{2}}} - \frac{b \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{c^2\sqrt{e}} + \frac{x\sqrt{ex^2+d}}{2ce} \\
 & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}{\sqrt{ex^2+d}\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(b^2-ac-\frac{b(-3ac+b^2)}{\sqrt{-4ac+b^2}}\right)}{c^2\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}\sqrt{b-\sqrt{-4ac+b^2}}} \\
 & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}{\sqrt{ex^2+d}\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(b^2-ac+\frac{b(-3ac+b^2)}{\sqrt{-4ac+b^2}}\right)}{c^2\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}
 \end{aligned}$$

command

```
integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.25 Problem number 392

$$\int \frac{1}{x^4\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{\sqrt{ex^2+d}}{3adx^3} + \frac{b\sqrt{ex^2+d}}{a^2dx} + \frac{2e\sqrt{ex^2+d}}{3ad^2x} \\
& + \frac{c \arctan\left(\frac{x\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}{\sqrt{ex^2+d}\sqrt{b-\sqrt{-4ac+b^2}}}\right) \left(b + \frac{-2ac+b^2}{\sqrt{-4ac+b^2}}\right)}{a^2\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}\sqrt{b-\sqrt{-4ac+b^2}}} \\
& + \frac{c \arctan\left(\frac{x\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}{\sqrt{ex^2+d}\sqrt{b+\sqrt{-4ac+b^2}}}\right) \left(b + \frac{2ac-b^2}{\sqrt{-4ac+b^2}}\right)}{a^2\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}
\end{aligned}$$

command

```
integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.26 Problem number 393

$$\int \frac{1}{x^6 \sqrt{d+ex^2} (a+bx^2+cx^4)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sqrt{ex^2+d}}{5adx^5} + \frac{b\sqrt{ex^2+d}}{3a^2dx^3} + \frac{4e\sqrt{ex^2+d}}{15ad^2x^3} \\ & -\frac{(-ac+b^2)\sqrt{ex^2+d}}{a^3dx} - \frac{2be\sqrt{ex^2+d}}{3a^2d^2x} - \frac{8e^2\sqrt{ex^2+d}}{15ad^3x} \\ & - \frac{\operatorname{c arctan}\left(\frac{x\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}{\sqrt{ex^2+d}\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(b^2-ac+\frac{b(-3ac+b^2)}{\sqrt{-4ac+b^2}}\right)}{a^3\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}\sqrt{b-\sqrt{-4ac+b^2}}} \\ & - \frac{\operatorname{c arctan}\left(\frac{x\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}{\sqrt{ex^2+d}\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(b^2-ac-\frac{b(-3ac+b^2)}{\sqrt{-4ac+b^2}}\right)}{a^3\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}} \end{aligned}$$

command

```
integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.27 Problem number 395

$$\int \frac{x^4}{(d+ex^2)^{3/2} (a+bx^2+cx^4)} dx$$

Optimal antiderivative

$$\frac{dx}{(ae^2 - bde + cd^2) \sqrt{ex^2 + d}}$$

$$- \frac{\arctan\left(\frac{x\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}{\sqrt{ex^2 + d} \sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(bd - ae + \frac{abe + 2acd - b^2d}{\sqrt{-4ac + b^2}}\right)}{(ae^2 - bde + cd^2) \sqrt{2cd - e(b - \sqrt{-4ac + b^2})} \sqrt{b - \sqrt{-4ac + b^2}}}$$

$$- \frac{\arctan\left(\frac{x\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}{\sqrt{ex^2 + d} \sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(bd - ae + \frac{-abe - 2acd + b^2d}{\sqrt{-4ac + b^2}}\right)}{(ae^2 - bde + cd^2) \sqrt{b + \sqrt{-4ac + b^2}} \sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}$$

command

`integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.28 Problem number 396

$$\int \frac{x^2}{(d + ex^2)^{3/2} (a + bx^2 + cx^4)} dx$$

Optimal antiderivative

$$- \frac{ex}{(ae^2 - bde + cd^2) \sqrt{ex^2 + d}}$$

$$+ \frac{c \arctan\left(\frac{x\sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}{\sqrt{ex^2 + d} \sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(d + \frac{2ae - bd}{\sqrt{-4ac + b^2}}\right)}{(ae^2 - bde + cd^2) \sqrt{2cd - e(b - \sqrt{-4ac + b^2})} \sqrt{b - \sqrt{-4ac + b^2}}}$$

$$+ \frac{c \arctan\left(\frac{x\sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}{\sqrt{ex^2 + d} \sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(d + \frac{-2ae + bd}{\sqrt{-4ac + b^2}}\right)}{(ae^2 - bde + cd^2) \sqrt{b + \sqrt{-4ac + b^2}} \sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}$$

command

```
integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

29.29 Problem number 397

$$\int \frac{1}{(d + ex^2)^{3/2} (a + bx^2 + cx^4)} dx$$

Optimal antiderivative

$$\frac{\frac{e^2 x}{d(ae^2 - bde + cd^2) \sqrt{ex^2 + d}}{\operatorname{arctan}\left(\frac{x \sqrt{2cd - e(b - \sqrt{-4ac + b^2})}}{\sqrt{ex^2 + d} \sqrt{b - \sqrt{-4ac + b^2}}}\right)} \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right)}{(ae^2 - bde + cd^2) \sqrt{2cd - e(b - \sqrt{-4ac + b^2})} \sqrt{b - \sqrt{-4ac + b^2}}} - \frac{\operatorname{arctan}\left(\frac{x \sqrt{2cd - e(b + \sqrt{-4ac + b^2})}}{\sqrt{ex^2 + d} \sqrt{b + \sqrt{-4ac + b^2}}}\right)}{(ae^2 - bde + cd^2) \sqrt{b + \sqrt{-4ac + b^2}} \sqrt{2cd - e(b + \sqrt{-4ac + b^2})}} \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right)$$

command

```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

30 Test file number 42

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/42_1.2.2.5_P-x-a+b_x^2+c_x^4-^p

30.1 Problem number 20

$$\int \frac{d + ex}{a + bx^2 + cx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{e \operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}} + \frac{d \operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right) \sqrt{2}\sqrt{c}}{\sqrt{-4ac+b^2} \sqrt{b-\sqrt{-4ac+b^2}}} \\ & -\frac{d \operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right) \sqrt{2}\sqrt{c}}{\sqrt{-4ac+b^2} \sqrt{b+\sqrt{-4ac+b^2}}} \end{aligned}$$

command

```
integrate((e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

30.2 Problem number 21

$$\int \frac{d + ex + fx^2}{a + bx^2 + cx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{e \operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}} + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right) \left(f + \frac{-bf+2cd}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{2\sqrt{c} \sqrt{b-\sqrt{-4ac+b^2}}} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right) \left(f + \frac{bf-2cd}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{2\sqrt{c} \sqrt{b+\sqrt{-4ac+b^2}}} \end{aligned}$$

command

```
integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

30.3 Problem number 64

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{e \operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}} + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right) \left(f + \frac{-bf+2cd}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right) \left(f + \frac{bf-2cd}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}} \end{aligned}$$

command

```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+b*x^2+a)^2,x
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

31 Test file number 43

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/43_1.2.2.6_P-x-d_x^-m-a+b_x^2+c_x^4-^p

31.1 Problem number 22

$$\int \frac{x^3(A + Bx + Cx^2)}{a + bx^2 + cx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{Bx}{c} + \frac{Cx^2}{2c} + \frac{(Ac - bC) \ln(cx^4 + bx^2 + a)}{4c^2} \\ & + \frac{(Abc + 2acC - b^2C) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{2c^2 \sqrt{-4ac + b^2}} \\ & - \frac{B \operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(b + \frac{2ac - b^2}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2c^{\frac{3}{2}} \sqrt{b - \sqrt{-4ac + b^2}}} \\ & - \frac{B \operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(b + \frac{-2ac + b^2}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2c^{\frac{3}{2}} \sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

command

```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

31.2 Problem number 23

$$\int \frac{x^2(A + Bx + Cx^2)}{a + bx^2 + cx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{Cx}{c} + \frac{B \ln(cx^4 + bx^2 + a)}{4c} + \frac{bB \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{2c\sqrt{-4ac + b^2}} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(Ac - bC + \frac{-Abc + (-2ac + b^2)C}{\sqrt{-4ac + b^2}} \right) \sqrt{2}}{2c^{\frac{3}{2}} \sqrt{b - \sqrt{-4ac + b^2}}} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(Ac - bC + \frac{Abc + 2acC - b^2C}{\sqrt{-4ac + b^2}} \right) \sqrt{2}}{2c^{\frac{3}{2}} \sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

command

`integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

31.3 Problem number 24

$$\int \frac{x(A + Bx + Cx^2)}{a + bx^2 + cx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{C \ln(cx^4 + bx^2 + a)}{4c} - \frac{(2Ac - bC) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{2c\sqrt{-4ac + b^2}} \\ & - \frac{B \operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{b - \sqrt{-4ac + b^2}} \sqrt{2}}{2\sqrt{c} \sqrt{-4ac + b^2}} \\ & + \frac{B \operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{b + \sqrt{-4ac + b^2}} \sqrt{2}}{2\sqrt{c} \sqrt{-4ac + b^2}} \end{aligned}$$

command

```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

31.4 Problem number 25

$$\int \frac{A + Bx + Cx^2}{a + bx^2 + cx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B \operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}} + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right) \left(C + \frac{2Ac-bC}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right) \left(C + \frac{-2Ac+bC}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}} \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

31.5 Problem number 68

$$\int \frac{x^6 (d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-2bf + ce)x}{c^3} + \frac{fx^3}{3c^2} \\ & + \frac{x(a(b^2ce - 2ac^2e - b^3f - bc(-3af + cd)) + (b^3ce - 3abc^2e - b^4f - b^2c(-4af + cd) + 2ac^2(-af + cd))x^2)}{2c^3(-4ac + b^2)(cx^4 + bx^2 + a)} \\ & - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(3b^3ce - 13abc^2e - 5b^4f - b^2c(-24af + cd) + 2ac^2(-7af + 3cd) + \frac{-3b^4ce + 19ab^2c^2}{4c^{\frac{7}{2}}(-4ac + b^2)}\sqrt{b - \sqrt{-4ac + b^2}}\right)}{4c^{\frac{7}{2}}(-4ac + b^2)\sqrt{b - \sqrt{-4ac + b^2}}} \\ & - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(3b^3ce - 13abc^2e - 5b^4f - b^2c(-24af + cd) + 2ac^2(-7af + 3cd) + \frac{3b^4ce - 19ab^2c^2}{4c^{\frac{7}{2}}(-4ac + b^2)}\sqrt{b + \sqrt{-4ac + b^2}}\right)}{4c^{\frac{7}{2}}(-4ac + b^2)\sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

command

```
integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

31.6 Problem number 73

$$\int \frac{d + ex^2 + fx^4}{x^4 (a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{d}{3a^2x^3} + \frac{-ae + 2bd}{a^3x} \\
 & + \frac{x\left(a^2\left(\frac{b^4d}{a^2} + 2c^2d + 3bce - \frac{b^2(be+4cd)}{a} + b^2f - 2acf\right) + c(b^3d - ab^2e + 2a^2ce - ab(-af + 3cd))x^2\right)}{2a^3(-4ac + b^2)(cx^4 + bx^2 + a)} \\
 & + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c}\left(5b^4d + b^3(-3ae + 5d\sqrt{-4ac + b^2}) - ab^2(29cd - af + 3e\sqrt{-4ac + b^2})\right)}{4a^3(-4ac + b^2)^{\frac{3}{2}}\sqrt{b - \sqrt{-4ac + b^2}}} \\
 & - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c}\left(5b^4d - b^3(3ae + 5d\sqrt{-4ac + b^2}) + 2a^2c(14cd - 6af - 5e\sqrt{-4ac + b^2})\right)}{4a^3(-4ac + b^2)^{\frac{3}{2}}\sqrt{b + \sqrt{-4ac + b^2}}}
 \end{aligned}$$

command

`integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

31.7 Problem number 126

$$\int \frac{x^4(d + ex^2 + fx^4 + gx^6)}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{(-2bg + cf)x}{c^3} + \frac{gx^3}{3c^2} \\
 & + \frac{x\left(a(2c^3d - c^2(2af + be) - b^3g + bc(3ag + bf)) + (b^3cf + bc^2(-3af + cd) - b^4g - b^2c(-4ag + ce) + 2ac^2(-ag - 3b^2d))\right)}{2c^3(-4ac + b^2)(cx^4 + bx^2 + a)} \\
 & - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(3b^3cf - bc^2(13af + cd) - 5b^4g - b^2c(-24ag + ce) + 2ac^2(-7ag + 3ce) + \frac{-3b^4d}{c}\right)}{4c^{\frac{7}{2}}(-4ac + b^2)\sqrt{b - \sqrt{-4ac + b^2}}} \\
 & - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(3b^3cf - bc^2(13af + cd) - 5b^4g - b^2c(-24ag + ce) + 2ac^2(-7ag + 3ce) + \frac{3b^4d}{c}\right)}{4c^{\frac{7}{2}}(-4ac + b^2)\sqrt{b + \sqrt{-4ac + b^2}}}
 \end{aligned}$$

command

```
integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

31.8 Problem number 127

$$\int \frac{x^2(d + ex^2 + fx^4 + gx^6)}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\frac{gx}{c^2} - \frac{x(bc(af + cd) - ab^2g - 2ac(-ag + ce) + (2c^3d - c^2(2af + be) - b^3g + bc(3ag + bf))x^2)}{2c^2(-4ac + b^2)(cx^4 + bx^2 + a)}$$

$$\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(2c^3d - c^2(-6af + be) + 3b^3g - bc(13ag + bf) + \frac{b^3cf - 4bc^2(2af + cd) - 3b^4g + 4ac^2(-5}{\sqrt{-4ac + b^2}}\right)}{4c^{\frac{5}{2}}(-4ac + b^2)\sqrt{b - \sqrt{-4ac + b^2}}}$$

$$\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(2c^3d - c^2(-6af + be) + 3b^3g - bc(13ag + bf) + \frac{-b^3cf + 4bc^2(2af + cd) + 3b^4g - 4ac^2(-}{\sqrt{-4ac + b^2}}\right)}{4c^{\frac{5}{2}}(-4ac + b^2)\sqrt{b + \sqrt{-4ac + b^2}}}$$

command

```
integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

31.9 Problem number 128

$$\int \frac{d + ex^2 + fx^4 + gx^6}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\frac{x \left(c \left(b^2 d - 2a(-af + cd) - \frac{ab(ag+ce)}{c} \right) + (bc(af + cd) - ab^2g - 2ac(-ag + ce)) x^2 \right)}{2ac(-4ac + b^2)(cx^4 + bx^2 + a)}$$

$$+ \frac{\arctan \left(\frac{x\sqrt{2} \sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}} \right) \left(b(af + cd) + \frac{ab^2g}{c} - 2a(3ag + ce) + \frac{b^2c(-af+cd) - 4ac^2(af+3cd) - ab^3g + 4abc(2ag+ce)}{c\sqrt{-4ac + b^2}} \right)}{4a(-4ac + b^2) \sqrt{c} \sqrt{b - \sqrt{-4ac + b^2}}}$$

$$+ \frac{\arctan \left(\frac{x\sqrt{2} \sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}} \right) \left(b(af + cd) + \frac{ab^2g}{c} - 2a(3ag + ce) + \frac{-b^2c(-af+cd) + 4ac^2(af+3cd) + ab^3g - 4abc(2ag+ce)}{c\sqrt{-4ac + b^2}} \right)}{4a(-4ac + b^2) \sqrt{c} \sqrt{b + \sqrt{-4ac + b^2}}}$$

command

```
integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

Fracas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fracas 1.3.7 via sagemath 9.3 output

Timed out

31.10 Problem number 129

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^2(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\frac{d}{a^2x} \frac{x \left(a \left(\frac{b^3d}{a} - b(be + 3cd) + a(bf + 2ce) - 2a^2g \right) + (b^2cd - 2ac(-af + cd) - ab(ag + ce)) x^2 \right)}{2a^2(-4ac + b^2)(cx^4 + bx^2 + a)}$$

$$- \frac{\arctan \left(\frac{x\sqrt{2} \sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}} \right) \left(3b^2cd - 2ac(-af + 5cd) - ab(ag + ce) + \frac{3b^3cd - 4abc(af+4cd) - ab^2(-ag+ce) + 4a^2c(ag+ce)}{\sqrt{-4ac + b^2}} \right)}{4a^2(-4ac + b^2) \sqrt{c} \sqrt{b - \sqrt{-4ac + b^2}}}$$

$$- \frac{\arctan \left(\frac{x\sqrt{2} \sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}} \right) \left(3b^2cd - 2ac(-af + 5cd) - ab(ag + ce) + \frac{-3b^3cd + 4abc(af+4cd) + ab^2(-ag+ce) - 4a^2c(ag+ce)}{\sqrt{-4ac + b^2}} \right)}{4a^2(-4ac + b^2) \sqrt{c} \sqrt{b + \sqrt{-4ac + b^2}}}$$

command

```
integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

31.11 Problem number 130

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^4 (a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{d}{3a^2x^3} + \frac{-ae + 2bd}{a^3x} \\ & + \frac{x \left(a^2 \left(\frac{b^4d}{a^2} + 2c^2d + 3bce - \frac{b^2(be+4cd)}{a} + b^2f - a(bg + 2cf) \right) + c(b^3d - ab^2e - ab(-af + 3cd) + 2a^2(-ag + ce)) \right) x^2}{2a^3(-4ac + b^2)(cx^4 + bx^2 + a)} \\ & + \frac{\arctan \left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}} \right) \sqrt{c} \left(5b^3d - 3ab^2e - ab(-af + 19cd) + 2a^2(-ag + 5ce) + \frac{5b^4d - 3ab^3e + 4a^2c(-3af + \dots)}{\dots} \right)}{4a^3(-4ac + b^2)\sqrt{b - \sqrt{-4ac + b^2}}} \\ & + \frac{\arctan \left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}} \right) \sqrt{c} \left(5b^3d - 3ab^2e - ab(-af + 19cd) + 2a^2(-ag + 5ce) + \frac{-5b^4d + 3ab^3e - 4a^2c(-3af + \dots)}{\dots} \right)}{4a^3(-4ac + b^2)\sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

command

```
integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

32 Test file number 47

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.3_General/47_1.2.3.3-d+e_x^n-
^q-a+b_x^n+c_x^-2_n-^p

32.1 Problem number 41

$$\int \frac{d + \frac{e}{x^4}}{c + \frac{a}{x^8} + \frac{b}{x^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{dx}{c} + \frac{\arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}x}{(-b-\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right)\left(bd-ce+\frac{-2acd+b^2d-bce}{\sqrt{-4ac+b^2}}\right)2^{\frac{3}{4}}}{4c^{\frac{5}{4}}(-b-\sqrt{-4ac+b^2})^{\frac{3}{4}}} \\ & + \frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}x}{(-b-\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right)\left(bd-ce+\frac{-2acd+b^2d-bce}{\sqrt{-4ac+b^2}}\right)2^{\frac{3}{4}}}{4c^{\frac{5}{4}}(-b-\sqrt{-4ac+b^2})^{\frac{3}{4}}} \\ & + \frac{\arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}x}{(-b+\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right)\left(bd-ce+\frac{2acd-b^2d+bce}{\sqrt{-4ac+b^2}}\right)2^{\frac{3}{4}}}{4c^{\frac{5}{4}}(-b+\sqrt{-4ac+b^2})^{\frac{3}{4}}} \\ & + \frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}x}{(-b+\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right)\left(bd-ce+\frac{2acd-b^2d+bce}{\sqrt{-4ac+b^2}}\right)2^{\frac{3}{4}}}{4c^{\frac{5}{4}}(-b+\sqrt{-4ac+b^2})^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

33 Test file number 48

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.3_General/48_1.2.3.4-f_x-
 $\hat{m}-d+e_x\hat{n}-\hat{q}-a+b_x\hat{n}+c_x\hat{-2}_n-\hat{p}$

33.1 Problem number 35

$$\int (d + ex^3)^{5/2} (a + bx^3 + cx^6) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{30d(667ae^2 - 58bde + 16cd^2)x(e x^3 + d)^{\frac{3}{2}}}{124729e^2} + \frac{2(667ae^2 - 58bde + 16cd^2)x(e x^3 + d)^{\frac{5}{2}}}{11339e^2} \\ & - \frac{2(-29be + 8cd)x(e x^3 + d)^{\frac{7}{2}}}{667e^2} + \frac{2cx^4(e x^3 + d)^{\frac{7}{2}}}{29e} + \frac{54d^2(667ae^2 - 58bde + 16cd^2)x\sqrt{ex^3 + d}}{124729e^2} \\ & + \frac{54 \cdot 3^{\frac{3}{4}} d^3 (667ae^2 - 58bde + 16cd^2) \left(d^{\frac{1}{3}} + e^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 - \sqrt{3})}{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{d^{\frac{2}{3}}}{\left(e^{\frac{1}{3}}x\right)}}}{124729e^{\frac{7}{3}}\sqrt{ex^3 + d} \sqrt{\frac{d^{\frac{1}{3}}(d^{\frac{1}{3}} + e^{\frac{1}{3}}x)}{\left(e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{124729} \left(81 (16cd^5 - 58bd^4e + 667ad^3e^2) e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(0, -4de^{(-1)}, x\right) - (648cd^4xe - 11(391cx^{13} + 4\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left((ce^2x^{12} + (2cde + be^2)x^9 + (cd^2 + 2bde + ae^2)x^6 + (bd^2 + 2ade)x^3 + ad^2\right)\sqrt{ex^3 + d}, x\right)$$

33.2 Problem number 36

$$\int (d + ex^3)^{3/2} (a + bx^3 + cx^6) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(391ae^2 - 46bde + 16cd^2)x(e^3x^3 + d)^{\frac{3}{2}}}{4301e^2} - \frac{2(-23be + 8cd)x(e^3x^3 + d)^{\frac{5}{2}}}{391e^2} \\ & + \frac{2cx^4(e^3x^3 + d)^{\frac{5}{2}}}{23e} + \frac{18d(391ae^2 - 46bde + 16cd^2)x\sqrt{e^3x^3 + d}}{21505e^2} \\ & + \frac{18 \cdot 3^{\frac{3}{4}} d^2 (391ae^2 - 46bde + 16cd^2) \left(d^{\frac{1}{3}} + e^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 - \sqrt{3})}{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{d^{\frac{2}{3}}}{\left(e^{\frac{1}{3}}x + d^{\frac{1}{3}}\right)^2}}}{21505e^{\frac{7}{3}}\sqrt{e^3x^3 + d} \sqrt{\frac{d^{\frac{1}{3}}\left(d^{\frac{1}{3}} + e^{\frac{1}{3}}x\right)}{\left(e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

`integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{21505} \left(27(16cd^4 - 46bd^3e + 391ad^2e^2)e^{\frac{1}{2}} \operatorname{weierstrassPInverse}\left(0, -4de^{(-1)}, x\right) - (216cd^3xe - 5(187cx^{10} + 253$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((cex^9 + (cd + be)x^6 + (bd + ae)x^3 + ad)\sqrt{ex^3 + d}, x\right)$$

33.3 Problem number 37

$$\int \sqrt{d + ex^3} (a + bx^3 + cx^6) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-17be + 8cd)x(e^3x^3 + d)^{\frac{3}{2}}}{187e^2} + \frac{2cx^4(e^3x^3 + d)^{\frac{3}{2}}}{17e} + \frac{2(187ae^2 - 34bde + 16cd^2)x\sqrt{e^3x^3 + d}}{935e^2} \\ & + \frac{23 \cdot 3^{\frac{3}{4}} d(187ae^2 - 34bde + 16cd^2) \left(d^{\frac{1}{3}} + e^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 - \sqrt{3})}{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{d^{\frac{2}{3}} -}{\left(e^{\frac{1}{3}}x + d^{\frac{1}{3}}\right)^2}}}{935e^{\frac{7}{3}}\sqrt{e^3x^3 + d} \sqrt{\frac{d^{\frac{1}{3}}\left(d^{\frac{1}{3}} + e^{\frac{1}{3}}x\right)}{\left(e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}} \end{aligned}$$

command

```
integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{935} \left(3 (16 cd^3 - 34 bd^2 e + 187 ade^2) e^{\frac{1}{2}} \text{weierstrassPInverse} \left(0, -4 de^{(-1)}, x \right) - (24 cd^2 x e - (55 cx^7 + 85 bx^4 + 187 \dots) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((cx^6 + bx^3 + a) \sqrt{ex^3 + d}, x \right)$$

33.4 Problem number 38

$$\int \frac{a + bx^3 + cx^6}{\sqrt{d + ex^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-11be + 8cd)x\sqrt{ex^3 + d}}{55e^2} + \frac{2cx^4\sqrt{ex^3 + d}}{11e} \\ & + \frac{2(55ae^2 - 22bde + 16cd^2)(d^{\frac{1}{3}} + e^{\frac{1}{3}}x) \text{EllipticF} \left(\frac{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 - \sqrt{3})}{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{d^{\frac{2}{3}} - d^{\frac{1}{3}}e^{\frac{1}{3}}}{(e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3}))^2}}}{165e^{\frac{7}{3}}\sqrt{ex^3 + d} \sqrt{\frac{d^{\frac{1}{3}}(d^{\frac{1}{3}} + e^{\frac{1}{3}}x)}{(e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{55} \left((16 cd^2 - 22 bde + 55 ae^2) e^{\frac{1}{2}} \text{weierstrassPInverse} \left(0, -4 de^{(-1)}, x \right) - (8 cdxe - (5 cx^4 + 11 bx) e^2) \sqrt{x^3 e + d} \right) e$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}}, x \right)$$

33.5 Problem number 39

$$\int \frac{a + bx^3 + cx^6}{(d + ex^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(ae^2 - bde + cd^2)x}{3de^2\sqrt{ex^3 + d}} + \frac{2cx\sqrt{ex^3 + d}}{5e^2}$$

$$2(16cd^2 - 5e(ae + 2bd)) \left(d^{\frac{1}{3}} + e^{\frac{1}{3}}x\right) \text{EllipticF} \left(\frac{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 - \sqrt{3})}{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{d^{\frac{2}{3}} - d^{\frac{1}{3}}e^{\frac{1}{3}}x}{\left(e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

$$+ \frac{45de^{\frac{7}{3}}\sqrt{ex^3 + d}}{\sqrt{\frac{d^{\frac{1}{3}}(d^{\frac{1}{3}} + e^{\frac{1}{3}}x)}{\left(e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

`integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((5ax^3e^3 - 16cd^3 + 5(2bdx^3 + ad)e^2 - 2(8cd^2x^3 - 5bd^2)e\right)e^{\frac{1}{2}}\text{weierstrassPInverse}(0, -4de^{(-1)}, x) + (8cd^2xe^{\frac{1}{2}})}{15(dx^3e^4 + d^2e^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(cx^6 + bx^3 + a)\sqrt{ex^3 + d}}{e^2x^6 + 2dex^3 + d^2}, x\right)$$

33.6 Problem number 40

$$\int \frac{a + bx^3 + cx^6}{(d + ex^3)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(ae^2 - bde + cd^2)x}{9de^2(e^{\frac{1}{3}}x + d)^{\frac{3}{2}}} - \frac{2(-7ae^2 - 2bde + 11cd^2)x}{27d^2e^2\sqrt{ex^3 + d}}$$

$$2(16cd^2 + e(7ae + 2bd)) \left(d^{\frac{1}{3}} + e^{\frac{1}{3}}x\right) \text{EllipticF} \left(\frac{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 - \sqrt{3})}{e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{d^{\frac{2}{3}} - d^{\frac{1}{3}}e^{\frac{1}{3}}x}{\left(e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}$$

$$+ \frac{81d^2e^{\frac{7}{3}}\sqrt{ex^3 + d}}{\sqrt{\frac{d^{\frac{1}{3}}(d^{\frac{1}{3}} + e^{\frac{1}{3}}x)}{\left(e^{\frac{1}{3}}x + d^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((7 a x^6 e^4 + 16 c d^4 + 2 (b d x^6 + 7 a d x^3) e^3 + (16 c d^2 x^6 + 4 b d^2 x^3 + 7 a d^2) e^2 + 2 (16 c d^3 x^3 + b d^3) e \right) e^{\frac{1}{2}} \text{weierstrassP}}{27 (d^2 x^6 e^5 + 2 d^3 x^3 e^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(c x^6 + b x^3 + a) \sqrt{e x^3 + d}}{e^3 x^9 + 3 d e^2 x^6 + 3 d^2 e x^3 + d^3}, x \right)$$

33.7 Problem number 41

$$\int \frac{a + b x^3 + c x^6}{(d + e x^3)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(a e^2 - b d e + c d^2) x}{15 d e^2 (e x^3 + d)^{\frac{5}{2}}} - \frac{2(-13 a e^2 - 2 b d e + 17 c d^2) x}{135 d^2 e^2 (e x^3 + d)^{\frac{3}{2}}} + \frac{2(91 a e^2 + 14 b d e + 16 c d^2) x}{405 d^3 e^2 \sqrt{e x^3 + d}} \\ & + \frac{2(91 a e^2 + 14 b d e + 16 c d^2) \left(d^{\frac{1}{3}} + e^{\frac{1}{3}} x \right) \text{EllipticF} \left(\frac{e^{\frac{1}{3}} x + d^{\frac{1}{3}} (1 - \sqrt{3})}{e^{\frac{1}{3}} x + d^{\frac{1}{3}} (1 + \sqrt{3})}, i \sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{d^{\frac{2}{3}} - d^{\frac{1}{3}} e^{\frac{1}{3}}}{\left(e^{\frac{1}{3}} x + d^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{1215 d^3 e^{\frac{7}{3}} \sqrt{e x^3 + d} \sqrt{\frac{d^{\frac{1}{3}} (d^{\frac{1}{3}} + e^{\frac{1}{3}} x)}{\left(e^{\frac{1}{3}} x + d^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}} \end{aligned}$$

command

```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((91 a x^9 e^5 + 16 c d^5 + 7 (2 b d x^9 + 39 a d x^6) e^4 + (16 c d^2 x^9 + 42 b d^2 x^6 + 273 a d^2 x^3) e^3 + (48 c d^3 x^6 + 42 b d^3 x^3 + 91 a d^3) e^2 + (16 c d^4 x^3 + 42 b d^4 x) e + 7 a d^4 \right) e^{\frac{5}{2}}}{27 (d^2 x^6 e^5 + 2 d^3 x^3 e^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(c x^6 + b x^3 + a) \sqrt{e x^3 + d}}{e^4 x^{12} + 4 d e^3 x^9 + 6 d^2 e^2 x^6 + 4 d^3 e x^3 + d^4}, x \right)$$

33.8 Problem number 42

$$\int \frac{a + bx^3 + cx^6}{(d + ex^3)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ae^2 - bde + cd^2)x}{21de^2(e^3x + d)^{7/2}} - \frac{2(-19ae^2 - 2bde + 23cd^2)x}{315d^2e^2(e^3x + d)^{5/2}} \\ & + \frac{2(247ae^2 + 26bde + 16cd^2)x}{2835d^3e^2(e^3x + d)^{3/2}} + \frac{2(247ae^2 + 26bde + 16cd^2)x}{1215d^4e^2\sqrt{ex^3 + d}} \\ & + \frac{2(247ae^2 + 26bde + 16cd^2)(d^{1/3} + e^{1/3}x) \operatorname{EllipticF}\left(\frac{e^{1/3}x + d^{1/3}(1 - \sqrt{3})}{e^{1/3}x + d^{1/3}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{d^{2/3} - d^{1/3}e}{e^{1/3}x + d^{1/3}}}}{3645d^4e^{7/3}\sqrt{ex^3 + d} \sqrt{\frac{d^{1/3}(d^{1/3} + e^{1/3}x)}{(e^{1/3}x + d^{1/3}(1 + \sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(7(247ax^{12}e^6 + 16cd^6 + 26(bdx^{12} + 38adx^9))e^5 + 2(8cd^2x^{12} + 52bd^2x^9 + 741ad^2x^6)e^4 + 4(16cd^3x^9 + 39bd^3) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(cx^6 + bx^3 + a)\sqrt{ex^3 + d}}{e^5x^{15} + 5de^4x^{12} + 10d^2e^3x^9 + 10d^3e^2x^6 + 5d^4ex^3 + d^5}, x\right)$$

33.9 Problem number 43

$$\int \frac{x^4(d + ex^4)}{a + bx^4 + cx^8} dx$$

Optimal antiderivative

$$\frac{ex}{c} - \frac{\arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}x}{(-b-\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right)\left(cd-be+\frac{2ace-b^2e+bcd}{\sqrt{-4ac+b^2}}\right)2^{\frac{3}{4}}}{4c^{\frac{5}{4}}(-b-\sqrt{-4ac+b^2})^{\frac{3}{4}}}$$

$$- \frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}x}{(-b-\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right)\left(cd-be+\frac{2ace-b^2e+bcd}{\sqrt{-4ac+b^2}}\right)2^{\frac{3}{4}}}{4c^{\frac{5}{4}}(-b-\sqrt{-4ac+b^2})^{\frac{3}{4}}}$$

$$- \frac{\arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}x}{(-b+\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right)\left(cd-be+\frac{-2ace+b^2e-bcd}{\sqrt{-4ac+b^2}}\right)2^{\frac{3}{4}}}{4c^{\frac{5}{4}}(-b+\sqrt{-4ac+b^2})^{\frac{3}{4}}}$$

$$- \frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}x}{(-b+\sqrt{-4ac+b^2})^{\frac{1}{4}}}\right)\left(cd-be+\frac{-2ace+b^2e-bcd}{\sqrt{-4ac+b^2}}\right)2^{\frac{3}{4}}}{4c^{\frac{5}{4}}(-b+\sqrt{-4ac+b^2})^{\frac{3}{4}}}$$

command

```
integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

33.10 Problem number 67

$$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right) x^3(d+ex)} dx$$

Optimal antiderivative

$$\frac{\ln(x)}{cd} - \frac{e^2 \ln(ex+d)}{d(ad^2 - bde + ce^2)} - \frac{(ad-be)\ln(ax^2+bx+c)}{2c(ad^2 - e(bd-ce))}$$

$$+ \frac{(abd + 2ace - b^2e) \operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{-4ac+b^2}}\right)}{c(ad^2 - e(bd-ce))\sqrt{-4ac+b^2}}$$

command

```
integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(b^2c - 4ac^2)e^2 \log(xe + d) - (abd^2 - (b^2 - 2ac)de) \sqrt{b^2 - 4ac} \log\left(\frac{2a^2x^2 + 2abx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2ax + b)}{ax^2 + bx + c}\right)}{2((ab^2c - 4a^2c^2)d^3 - (b^3c - 4a^2c^2)d^2)} + \frac{2(b^2c - 4ac^2)e^2 \log(xe + d) - 2(abd^2 - (b^2 - 2ac)de) \sqrt{-b^2 + 4ac} \arctan\left(-\frac{\sqrt{-b^2 + 4ac}(2ax + b)}{b^2 - 4ac}\right) + ((ab^2c - 4a^2c^2)d^3 - (b^3c - 4a^2c^2)d^2)}{2((ab^2c - 4a^2c^2)d^3 - (b^3c - 4a^2c^2)d^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

33.11 Problem number 70

$$\int \frac{x^3}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(2ad + be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4(a d^2 - e(bd - ce))(ex + d)} + \frac{d^4(3a d^2 - e(4bd - 5ce)) \ln(ex + d)}{e^4(a d^2 - e(bd - ce))^2} \\ & + \frac{(b^4d^2 - 2b^3cde + 4abc^2de + ac^2(ad^2 - ce^2) - b^2c(3ad^2 - ce^2)) \ln(ax^2 + bx + c)}{2a^3(ad^2 - e(bd - ce))^2} \\ & + \frac{(b^5d^2 - 2b^4cde + 8ab^2c^2de - 4a^2c^3de + abc^2(5ad^2 - 3ce^2) - b^3c(5ad^2 - ce^2)) \operatorname{arctanh}\left(\frac{2ax + b}{\sqrt{-4ac + b^2}}\right)}{a^3(ad^2 - e(bd - ce))^2 \sqrt{-4ac + b^2}} \end{aligned}$$

command

```
integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

33.12 Problem number 79

$$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^4 \sqrt{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(233a^3d^3 + 48b^3e^3 + abe^2(67bd - 157ce) + 4a^2de(18bd - 37ce)) x(ex + d)^{\frac{3}{2}} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{3465a^3e^4} \\ & - \frac{2(29a^2d^2 + 8b^2e^2 + ae(19bd - 18ce)) x(ex + d)^{\frac{5}{2}} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{693a^2e^4} \\ & + \frac{2(ad + be) x(ex + d)^{\frac{7}{2}} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{99ae^4} \\ & - \frac{2(187a^4d^4 + 64b^4e^4 + 4ab^2e^3(7bd - 69ce) - 4a^3d^2e(2bd + 3ce) + 3a^2e^2(3b^2d^2 - 29bcde + 50c^2e^2)) x \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{3465a^4e^4} \\ & + \frac{2x^5 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex + d}}{11} \end{aligned}$$

$$(128a^5d^5 + 128b^5e^5 - 4a^4d^3e(14bd - 27ce) - 8ab^3e^4(7bd + 87ce) - a^2b^3e^3(37b^2d^2 - 258bcde - 771c^2e^2) - a^3d^2e^2)$$

+

$$2(a^2d^2 - e(bd - ce)) (128a^4d^4 - 64b^4e^4 - 4ab^2e^3(7bd - 69ce) + 4a^3d^2e(2bd + 3ce) - 3a^2e^2(3b^2d^2 - 29bcde + 50c^2e^2) - a^3d^2e^2)$$

command

`integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((128a^6d^6 - 120a^5bd^5e - 3(11a^4b^2 - 68a^5c)d^4e^2 - (20a^3b^3 - 87a^4bc)d^3e^3 - 3(11a^2b^4 - 53a^3b^2c + 34a^4c^2) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{ex+d} x^4 \sqrt{\frac{ax^2+bx+c}{x^2}}, x\right)$$

33.13 Problem number 80

$$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^3 \sqrt{d+ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(8a^2d^2 + 3b^2e^2 + ae(4bd - 7ce)) x(ex+d)^{\frac{3}{2}} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{315a^2e^3} \\ & + \frac{2(ad+be) x(ex+d)^{\frac{5}{2}} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{63ae^3} \\ & + \frac{2(19a^3d^3 - 6a^2cde^2 + 8b^3e^3 + 3abe^2(bd - 9ce)) x \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex+d}}{315a^3e^3} \\ & + \frac{2x^4 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex+d}}{9} \\ & + \frac{2(8a^4d^4 + 8b^4e^4 - a^3d^2e(4bd - 9ce) - 4ab^2e^3(bd + 9ce) - 3a^2e^2(b^2d^2 - 5bcde - 7c^2e^2)) x \text{EllipticE}\left(\sqrt{\frac{b+2ax}{\sqrt{-4ac+b^2}}}\right)}{315a^4e^4(a^2x^2 + bx + c)} \\ & + \frac{2(16a^3d^3 + 6a^2cde^2 - 8b^3e^3 - 3abe^2(bd - 9ce)) (ad^2 - e(bd - ce)) x \text{EllipticF}\left(\sqrt{\frac{b+2ax + \sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\right) \sqrt{2}}{315a^4e^4(a^2x^2 + bx + c)} \end{aligned}$$

command

```
integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((16 a^5 d^5 - 16 a^4 b d^4 e - 5 (a^3 b^2 - 6 a^4 c) d^3 e^2 - (5 a^2 b^3 - 21 a^3 b c) d^2 e^3 - 2 (8 a b^4 - 42 a^2 b^2 c + 33 a^3 c^2) d e^4 + (16 a^4 d^4 - 16 a^3 b d^3 e - 5 (a^2 b^2 - 6 a^3 c) d^2 e^2 - (5 a^2 b^3 - 21 a^3 b c) d e^3 - 2 (8 a b^4 - 42 a^2 b^2 c + 33 a^3 c^2) d e^4 + (16 a^4 d^4 - 16 a^3 b d^3 e - 5 (a^2 b^2 - 6 a^3 c) d^2 e^2 - (5 a^2 b^3 - 21 a^3 b c) d e^3 - 2 (8 a b^4 - 42 a^2 b^2 c + 33 a^3 c^2) d e^4 + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{e x + d} x^3 \sqrt{\frac{a x^2 + b x + c}{x^2}}, x \right)$$

33.14 Problem number 81

$$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^2 \sqrt{d + e x} dx$$

Optimal antiderivative

$$\frac{2x(4a^2d^2 + 4b^2e^2 - ae(2bd - 5ce) - 3ae(ad - 4be)x) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex + d}}{105a^2e^2} + \frac{2x(ax^2 + bx + c) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex + d}}{7a}$$

$$+ (8a^3d^3 + 8b^3e^3 - a^2de(5bd - 16ce) - abe^2(5bd + 29ce)) x \text{EllipticE} \left(\sqrt{\frac{b + 2ax + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2ad - e(b - 2ax)}{2ad - e(b - 2ax)}} \right)$$

$$+ \frac{105a^3e^3(ax^2 + bx + c) \sqrt{\frac{a}{2ad - e(b - 2ax)}}}{105a^3e^3(ax^2 + bx + c) \sqrt{\frac{a}{2ad - e(b - 2ax)}}}$$

$$- \frac{2(8a^2d^2 - 4b^2e^2 - ae(bd - 10ce)) (ad^2 - e(bd - ce)) x \text{EllipticF} \left(\sqrt{\frac{b + 2ax + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}, \sqrt{\frac{2ad - e(b - 2ax)}{2ad - e(b - 2ax)}} \right)}{105a^3e^3(ax^2 + bx + c) \sqrt{\frac{a}{2ad - e(b - 2ax)}}}$$

command

```
integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((8a^4d^4 - 9a^3bd^3e - 2(2a^2b^2 - 11a^3c)d^2e^2 - (9ab^3 - 41a^2bc)de^3 + (8b^4 - 41ab^2c + 30a^2c^2)e^4) \sqrt{a} e^{\frac{1}{2}} \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{ex+d} x^2 \sqrt{\frac{ax^2+bx+c}{x^2}}, x \right)$$

33.15 Problem number 82

$$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} dx$$

Optimal antiderivative

$$\frac{2x(ex+d)^{\frac{3}{2}} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{5e} - \frac{2(2ad-be)x \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex+d}}{15ae}$$

$$2(a^2d^2 + b^2e^2 - ae(bd + 3ce)) x \text{EllipticE} \left(\frac{\sqrt{\frac{b+2ax + \sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2ad-e(b+\sqrt{-4ac+b^2})}} \right)$$

$$15a^2e^2(a^2x^2 + b^2x + c) \sqrt{\frac{a(ex+d)}{2ad-e(b+\sqrt{-4ac+b^2})}}$$

$$2(2ad-be)(ad^2 - e(bd - ce)) x \text{EllipticF} \left(\frac{\sqrt{\frac{b+2ax + \sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2ad-e(b+\sqrt{-4ac+b^2})}} \right)$$

$$+ \frac{15a^2e^2(a^2x^2 + b^2x + c) \sqrt{ex+d}}{15a^2e^2(a^2x^2 + b^2x + c) \sqrt{ex+d}}$$

command

`integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((2a^3d^3 - 3a^2bd^2e - 3(ab^2 - 6a^2c)de^2 + (2b^3 - 9abc)e^3) \sqrt{a} e^{\frac{1}{2}} \text{weierstrassPInverse} \left(\frac{4(a^2d^2 - abde + (b^2 - 3ac)e^2)e^{\frac{1}{2}}}{3a^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{ex+d} x \sqrt{\frac{ax^2+bx+c}{x^2}}, x\right)$$

34 Test file number 50

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.4_Improper/50_1.2.4.2-d_x~m-a_x^q+b_x^n+c_x^-2_n-q~p

34.1 Problem number 114

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2 - \frac{b}{\sqrt{a}\sqrt{c}}}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{x} \sqrt{\frac{cx^4}{(\sqrt{a}})}}{2 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{cx^5+bx^3+ax}}$$

command

`integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{\frac{1}{2}} \left(c \sqrt{\frac{b^2-4ac}{c^2}} + b\right) \sqrt{\frac{c \sqrt{\frac{b^2-4ac}{c^2}} - b}{c}} \text{ellipticF}\left(\frac{\sqrt{\frac{1}{2}} \sqrt{\frac{c \sqrt{\frac{b^2-4ac}{c^2}} - b}{c}}}{x}, \frac{bc \sqrt{\frac{b^2-4ac}{c^2}} + b^2 - 2ac}{2ac}\right)}{2a\sqrt{c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}}, x\right)$$

35 Test file number 51

Test folder name:

test_cases/1_Algebraic_functions/1.3_Miscellaneous/51_1.3.1_Rational_functions

35.1 Problem number 61

$$\int \frac{1}{8 + 24x + 8x^2 - 15x^3 + 8x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\arctan\left(\frac{(-5x^2+12x+8)\sqrt{39}}{39x^2}\right)\sqrt{39}}{52} \\ & - \frac{\ln\left(\left(3+\frac{4}{x}\right)^2 + \sqrt{517} - \left(3+\frac{4}{x}\right)\sqrt{38+2\sqrt{517}}\right)\sqrt{-208364442+9476610\sqrt{517}}}{322608} \\ & + \frac{\ln\left(\left(3+\frac{4}{x}\right)^2 + \sqrt{517} + \left(3+\frac{4}{x}\right)\sqrt{38+2\sqrt{517}}\right)\sqrt{-208364442+9476610\sqrt{517}}}{322608} \\ & - \frac{\arctan\left(\frac{6+\frac{8}{x}-\sqrt{38+2\sqrt{517}}}{\sqrt{-38+2\sqrt{517}}}\right)\sqrt{208364442+9476610\sqrt{517}}}{161304} \\ & - \frac{\arctan\left(\frac{6+\frac{8}{x}+\sqrt{38+2\sqrt{517}}}{\sqrt{-38+2\sqrt{517}}}\right)\sqrt{208364442+9476610\sqrt{517}}}{161304} \end{aligned}$$

command

```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.2 Problem number 62

$$\int \frac{1}{(8 + 24x + 8x^2 - 15x^3 + 8x^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3\left(3359 - 107\left(3 + \frac{4}{x}\right)^2\right)}{208\left(517 - 38\left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4\right)} + \frac{\left(3327931 - 129631\left(3 + \frac{4}{x}\right)^2\right)\left(3 + \frac{4}{x}\right)}{166788336 - 12259104\left(3 + \frac{4}{x}\right)^2 + 322608\left(3 + \frac{4}{x}\right)^4} \\ & + \frac{73 \arctan\left(\frac{(-5x^2+12x+8)\sqrt{39}}{39x^2}\right) \sqrt{39}}{2704} \\ & - \frac{\arctan\left(\frac{6+\frac{8}{x}-\sqrt{38+2\sqrt{517}}}{\sqrt{-38+2\sqrt{517}}}\right) \left(1678181 + 74897\sqrt{517}\right) \sqrt{766194 + 40326\sqrt{517}}}{26018980416} \\ & - \frac{\arctan\left(\frac{6+\frac{8}{x}+\sqrt{38+2\sqrt{517}}}{\sqrt{-38+2\sqrt{517}}}\right) \left(1678181 + 74897\sqrt{517}\right) \sqrt{766194 + 40326\sqrt{517}}}{26018980416} \\ & - \frac{\ln\left(\left(3 + \frac{4}{x}\right)^2 + \sqrt{517} - \left(3 + \frac{4}{x}\right) \sqrt{38 + 2\sqrt{517}}\right) \sqrt{-2405208568240933026 + 105781971094684170\sqrt{517}}}{26018980416} \\ & + \frac{\ln\left(\left(3 + \frac{4}{x}\right)^2 + \sqrt{517} + \left(3 + \frac{4}{x}\right) \sqrt{38 + 2\sqrt{517}}\right) \sqrt{-2405208568240933026 + 105781971094684170\sqrt{517}}}{26018980416} \end{aligned}$$

command

```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.3 Problem number 111

$$\int \frac{x^2}{a + b(c + dx)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c \arctan\left(\frac{(dx+c)^2 \sqrt{b}}{\sqrt{a}}\right)}{d^3 \sqrt{a} \sqrt{b}} \\ & + \frac{\ln\left(-a^{\frac{1}{4}} b^{\frac{1}{4}} (dx+c) \sqrt{2} + \sqrt{a} + (dx+c)^2 \sqrt{b}\right) (\sqrt{a} - \sqrt{b} c^2) \sqrt{2}}{8a^{\frac{3}{4}} b^{\frac{3}{4}} d^3} \\ & - \frac{\ln\left(a^{\frac{1}{4}} b^{\frac{1}{4}} (dx+c) \sqrt{2} + \sqrt{a} + (dx+c)^2 \sqrt{b}\right) (\sqrt{a} - \sqrt{b} c^2) \sqrt{2}}{8a^{\frac{3}{4}} b^{\frac{3}{4}} d^3} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}(dx+c)\sqrt{2}}{a^{\frac{1}{4}}}\right) (\sqrt{a} + \sqrt{b} c^2) \sqrt{2}}{4a^{\frac{3}{4}} b^{\frac{3}{4}} d^3} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}(dx+c)\sqrt{2}}{a^{\frac{1}{4}}}\right) (\sqrt{a} + \sqrt{b} c^2) \sqrt{2}}{4a^{\frac{3}{4}} b^{\frac{3}{4}} d^3} \end{aligned}$$

command

```
integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.4 Problem number 112

$$\int \frac{x}{a + b(c + dx)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c \arctan\left(-1 + \frac{b^{\frac{1}{4}}(dx+c)\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{4a^{\frac{3}{4}} b^{\frac{1}{4}} d^2} - \frac{c \arctan\left(1 + \frac{b^{\frac{1}{4}}(dx+c)\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{4a^{\frac{3}{4}} b^{\frac{1}{4}} d^2} \\ & + \frac{c \ln\left(-a^{\frac{1}{4}} b^{\frac{1}{4}} (dx+c) \sqrt{2} + \sqrt{a} + (dx+c)^2 \sqrt{b}\right) \sqrt{2}}{8a^{\frac{3}{4}} b^{\frac{1}{4}} d^2} \\ & - \frac{c \ln\left(a^{\frac{1}{4}} b^{\frac{1}{4}} (dx+c) \sqrt{2} + \sqrt{a} + (dx+c)^2 \sqrt{b}\right) \sqrt{2}}{8a^{\frac{3}{4}} b^{\frac{1}{4}} d^2} + \frac{\arctan\left(\frac{(dx+c)^2 \sqrt{b}}{\sqrt{a}}\right)}{2d^2 \sqrt{a} \sqrt{b}} \end{aligned}$$

command

```
integrate(x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.5 Problem number 114

$$\int \frac{1}{x(a+b(c+dx)^4)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\ln(x)}{bc^4+a} - \frac{\ln(a+b(dx+c)^4)}{4(bc^4+a)} - \frac{c^2 \arctan\left(\frac{(dx+c)^2\sqrt{b}}{\sqrt{a}}\right)\sqrt{b}}{2(bc^4+a)\sqrt{a}} \\ & - \frac{b^{\frac{1}{4}}c \ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}(dx+c)\sqrt{2} + \sqrt{a} + (dx+c)^2\sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}c^2\right)\sqrt{2}}{8a^{\frac{3}{4}}(bc^4+a)} \\ & + \frac{b^{\frac{1}{4}}c \ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}(dx+c)\sqrt{2} + \sqrt{a} + (dx+c)^2\sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}c^2\right)\sqrt{2}}{8a^{\frac{3}{4}}(bc^4+a)} \\ & - \frac{b^{\frac{1}{4}}c \arctan\left(-1 + \frac{b^{\frac{1}{4}}(dx+c)\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(\sqrt{a} + \sqrt{b}c^2\right)\sqrt{2}}{4a^{\frac{3}{4}}(bc^4+a)} \\ & - \frac{b^{\frac{1}{4}}c \arctan\left(1 + \frac{b^{\frac{1}{4}}(dx+c)\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(\sqrt{a} + \sqrt{b}c^2\right)\sqrt{2}}{4a^{\frac{3}{4}}(bc^4+a)} \end{aligned}$$

command

```
integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.6 Problem number 115

$$\int \frac{1}{x^2 (a + b(c + dx)^4)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1}{(bc^4 + a)x} - \frac{4bc^3 d \ln(x)}{(bc^4 + a)^2} + \frac{bc^3 d \ln(a + b(dx + c)^4)}{(bc^4 + a)^2} - \frac{c(-bc^4 + a) d \arctan\left(\frac{(dx+c)^2 \sqrt{b}}{\sqrt{a}}\right) \sqrt{b}}{(bc^4 + a)^2 \sqrt{a}} \\ & - \frac{b^{\frac{1}{4}} d \ln\left(-a^{\frac{1}{4}} b^{\frac{1}{4}} (dx + c) \sqrt{2} + \sqrt{a} + (dx + c)^2 \sqrt{b}\right) \left((-3bc^4 + a) \sqrt{a} - c^2(-bc^4 + 3a) \sqrt{b}\right) \sqrt{2}}{8a^{\frac{3}{4}} (bc^4 + a)^2} \\ & + \frac{b^{\frac{1}{4}} d \ln\left(a^{\frac{1}{4}} b^{\frac{1}{4}} (dx + c) \sqrt{2} + \sqrt{a} + (dx + c)^2 \sqrt{b}\right) \left((-3bc^4 + a) \sqrt{a} - c^2(-bc^4 + 3a) \sqrt{b}\right) \sqrt{2}}{8a^{\frac{3}{4}} (bc^4 + a)^2} \\ & - \frac{b^{\frac{1}{4}} d \arctan\left(-1 + \frac{b^{\frac{1}{4}}(dx+c)\sqrt{2}}{a^{\frac{1}{4}}}\right) \left((-3bc^4 + a) \sqrt{a} + c^2(-bc^4 + 3a) \sqrt{b}\right) \sqrt{2}}{4a^{\frac{3}{4}} (bc^4 + a)^2} \\ & - \frac{b^{\frac{1}{4}} d \arctan\left(1 + \frac{b^{\frac{1}{4}}(dx+c)\sqrt{2}}{a^{\frac{1}{4}}}\right) \left((-3bc^4 + a) \sqrt{a} + c^2(-bc^4 + 3a) \sqrt{b}\right) \sqrt{2}}{4a^{\frac{3}{4}} (bc^4 + a)^2} \end{aligned}$$

command

```
integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.7 Problem number 127

$$\int \frac{x}{a + 8x - 8x^2 + 4x^3 - x^4} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2\sqrt{4+a}} - \frac{\operatorname{arctan}\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a} \sqrt{1-\sqrt{4+a}}} + \frac{\operatorname{arctan}\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a} \sqrt{1+\sqrt{4+a}}}$$

command

```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.8 Problem number 134

$$\int \frac{x^2}{a + 8x - 8x^2 + 4x^3 - x^4} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{\sqrt{4+a}} - \frac{\operatorname{arctan}\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}} - \frac{\operatorname{arctan}\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}}$$

command

```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.9 Problem number 138

$$\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left(\frac{(3a^{\frac{2}{3}}c^{\frac{1}{3}}+2bx)\sqrt{3}}{3\sqrt{a}\sqrt{4b-3a^{\frac{1}{3}}c^{\frac{2}{3}}}} \right) \sqrt{3}}{81a^{\frac{11}{6}}c^{\frac{2}{3}}\sqrt{4b-3a^{\frac{1}{3}}c^{\frac{2}{3}}}} + \frac{2(-1)^{\frac{2}{3}} \arctan \left(\frac{(3(-1)^{\frac{2}{3}}a^{\frac{2}{3}}c^{\frac{1}{3}}+2bx)\sqrt{3}}{3\sqrt{a}\sqrt{4b+3(-1)^{\frac{1}{3}}a^{\frac{1}{3}}c^{\frac{2}{3}}}} \right) \sqrt{3}}{27 \left(1 - (-1)^{\frac{1}{3}}\right) \left(1 + (-1)^{\frac{1}{3}}\right)^2 a^{\frac{11}{6}}c^{\frac{2}{3}}\sqrt{4b+3(-1)^{\frac{1}{3}}a^{\frac{1}{3}}c^{\frac{2}{3}}}} + \frac{2(-1)^{\frac{2}{3}} \arctan \left(\frac{(3(-1)^{\frac{1}{3}}a^{\frac{2}{3}}c^{\frac{1}{3}}-2bx)\sqrt{3}}{3\sqrt{a}\sqrt{4b-3(-1)^{\frac{2}{3}}a^{\frac{1}{3}}c^{\frac{2}{3}}}} \right) \sqrt{3}}{27 \left(1 + (-1)^{\frac{1}{3}}\right)^2 a^{\frac{11}{6}}c^{\frac{2}{3}}\sqrt{4b-3(-1)^{\frac{2}{3}}a^{\frac{1}{3}}c^{\frac{2}{3}}}}$$

command

```
integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.10 Problem number 337

$$\int \frac{a + bx + cx^2}{d + ex^2 + fx^4} dx$$

Optimal antiderivative

$$-\frac{b \operatorname{arctanh} \left(\frac{2fx^2+e}{\sqrt{-4df+e^2}} \right)}{\sqrt{-4df+e^2}} + \frac{\operatorname{arctan} \left(\frac{x\sqrt{2}\sqrt{f}}{\sqrt{e-\sqrt{-4df+e^2}}} \right) \left(c + \frac{2af-ce}{\sqrt{-4df+e^2}} \right) \sqrt{2}}{2\sqrt{f}\sqrt{e-\sqrt{-4df+e^2}}} + \frac{\operatorname{arctan} \left(\frac{x\sqrt{2}\sqrt{f}}{\sqrt{e+\sqrt{-4df+e^2}}} \right) \left(c + \frac{-2af+ce}{\sqrt{-4df+e^2}} \right) \sqrt{2}}{2\sqrt{f}\sqrt{e+\sqrt{-4df+e^2}}}$$

command

`integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.11 Problem number 338

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2de \operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right) + \operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right) \left(e^2 + \frac{-be^2+2cd^2}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{\sqrt{-4ac+b^2}} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right) \left(e^2 + \frac{be^2-2cd^2}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}} \end{aligned}$$

command

`integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.12 Problem number 342

$$\int \frac{x^2}{(c+dx)(a+bx^4)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{c^2 d \ln(dx+c)}{a d^4 + b c^4} - \frac{c^2 d \ln(bx^4+a)}{4(a d^4 + b c^4)} + \frac{d^3 \arctan\left(\frac{x^2 \sqrt{b}}{\sqrt{a}}\right) \sqrt{a}}{2(a d^4 + b c^4) \sqrt{b}} \\
& + \frac{c \arctan\left(-1 + \frac{b^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (-d^2 \sqrt{a} + \sqrt{b} c^2) \sqrt{2}}{4a^{\frac{1}{4}} b^{\frac{1}{4}} (a d^4 + b c^4)} \\
& + \frac{c \arctan\left(1 + \frac{b^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (-d^2 \sqrt{a} + \sqrt{b} c^2) \sqrt{2}}{4a^{\frac{1}{4}} b^{\frac{1}{4}} (a d^4 + b c^4)} \\
& + \frac{c \ln\left(-a^{\frac{1}{4}} b^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{b}\right) (d^2 \sqrt{a} + \sqrt{b} c^2) \sqrt{2}}{8a^{\frac{1}{4}} b^{\frac{1}{4}} (a d^4 + b c^4)} \\
& - \frac{c \ln\left(a^{\frac{1}{4}} b^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{b}\right) (d^2 \sqrt{a} + \sqrt{b} c^2) \sqrt{2}}{8a^{\frac{1}{4}} b^{\frac{1}{4}} (a d^4 + b c^4)}
\end{aligned}$$

command

```
integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.13 Problem number 391

$$\int \frac{1-x^2}{a+b(1-x^2)^4} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{\arctan\left(\frac{b^{\frac{1}{8}}x}{\sqrt{(-a)^{\frac{1}{4}} - b^{\frac{1}{4}}}}\right)}{4b^{\frac{3}{8}}\sqrt{-a}\sqrt{(-a)^{\frac{1}{4}} - b^{\frac{1}{4}}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{8}}x}{\sqrt{(-a)^{\frac{1}{4}} + b^{\frac{1}{4}}}}\right)}{4b^{\frac{3}{8}}\sqrt{-a}\sqrt{(-a)^{\frac{1}{4}} + b^{\frac{1}{4}}}} \\
& - \frac{\arctan\left(\frac{-b^{\frac{1}{8}}x\sqrt{2} + \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}}{\sqrt{-b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}}\right) \sqrt{-b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}} \sqrt{2}}{8b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt{-a} + \sqrt{b}}} \\
& + \frac{\arctan\left(\frac{b^{\frac{1}{8}}x\sqrt{2} + \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}}{\sqrt{-b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}}\right) \sqrt{-b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}} \sqrt{2}}{8b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt{-a} + \sqrt{b}}} \\
& + \frac{\ln\left(b^{\frac{1}{4}}x^2 + \sqrt{\sqrt{-a} + \sqrt{b}} - b^{\frac{1}{8}}x\sqrt{2} \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}\right) \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}} \sqrt{2}}{16b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt{-a} + \sqrt{b}}} \\
& + \frac{\ln\left(b^{\frac{1}{4}}x^2 + \sqrt{\sqrt{-a} + \sqrt{b}} + b^{\frac{1}{8}}x\sqrt{2} \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}\right) \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}} \sqrt{2}}{16b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt{-a} + \sqrt{b}}}
\end{aligned}$$

command

```
integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.14 Problem number 392

$$\int \frac{1-x^2}{a+b(-1+x^2)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\arctan\left(\frac{b^{\frac{1}{8}}x}{\sqrt{(-a)^{\frac{1}{4}}-b^{\frac{1}{4}}}}\right)}{4b^{\frac{3}{8}}\sqrt{-a}\sqrt{(-a)^{\frac{1}{4}}-b^{\frac{1}{4}}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{8}}x}{\sqrt{(-a)^{\frac{1}{4}}+b^{\frac{1}{4}}}}\right)}{4b^{\frac{3}{8}}\sqrt{-a}\sqrt{(-a)^{\frac{1}{4}}+b^{\frac{1}{4}}}} \\ & - \frac{\arctan\left(\frac{-b^{\frac{1}{8}}x\sqrt{2} + \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}}{\sqrt{-b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}}\right) \sqrt{-b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}} \sqrt{2}}{8b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt{-a} + \sqrt{b}}} \\ & + \frac{\arctan\left(\frac{b^{\frac{1}{8}}x\sqrt{2} + \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}}{\sqrt{-b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}}\right) \sqrt{-b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}} \sqrt{2}}{8b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt{-a} + \sqrt{b}}} \\ & + \frac{\ln\left(b^{\frac{1}{4}}x^2 + \sqrt{\sqrt{-a} + \sqrt{b}} - b^{\frac{1}{8}}x\sqrt{2} \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}\right) \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}} \sqrt{2}}{16b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt{-a} + \sqrt{b}}} \\ & - \frac{\ln\left(b^{\frac{1}{4}}x^2 + \sqrt{\sqrt{-a} + \sqrt{b}} + b^{\frac{1}{8}}x\sqrt{2} \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}}\right) \sqrt{b^{\frac{1}{4}} + \sqrt{\sqrt{-a} + \sqrt{b}}} \sqrt{2}}{16b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt{-a} + \sqrt{b}}} \end{aligned}$$

command

```
integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.15 Problem number 394

$$\int \frac{(d+ex)^3}{a+cx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{e^3 \ln(cx^4+a)}{4c} + \frac{3d^2 e \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} \\ & - \frac{d \ln\left(-a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right) (-3e^2\sqrt{a} + d^2\sqrt{c}) \sqrt{2}}{8a^{\frac{3}{4}}c^{\frac{3}{4}}} \\ & + \frac{d \ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right) (-3e^2\sqrt{a} + d^2\sqrt{c}) \sqrt{2}}{8a^{\frac{3}{4}}c^{\frac{3}{4}}} \\ & + \frac{d \arctan\left(-1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (3e^2\sqrt{a} + d^2\sqrt{c}) \sqrt{2}}{4a^{\frac{3}{4}}c^{\frac{3}{4}}} \\ & + \frac{d \arctan\left(1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (3e^2\sqrt{a} + d^2\sqrt{c}) \sqrt{2}}{4a^{\frac{3}{4}}c^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.16 Problem number 395

$$\int \frac{(d+ex)^2}{a+cx^4} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{de \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - \frac{\ln\left(-a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right)(-e^2\sqrt{a} + d^2\sqrt{c})\sqrt{2}}{8a^{\frac{3}{4}}c^{\frac{3}{4}}} \\
& + \frac{\ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right)(-e^2\sqrt{a} + d^2\sqrt{c})\sqrt{2}}{8a^{\frac{3}{4}}c^{\frac{3}{4}}} \\
& + \frac{\arctan\left(-1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)(e^2\sqrt{a} + d^2\sqrt{c})\sqrt{2}}{4a^{\frac{3}{4}}c^{\frac{3}{4}}} \\
& + \frac{\arctan\left(1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)(e^2\sqrt{a} + d^2\sqrt{c})\sqrt{2}}{4a^{\frac{3}{4}}c^{\frac{3}{4}}}
\end{aligned}$$

command

```
integrate((e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.17 Problem number 396

$$\int \frac{d + ex}{a + cx^4} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{d \arctan\left(-1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\sqrt{2}}{4a^{\frac{3}{4}}c^{\frac{1}{4}}} + \frac{d \arctan\left(1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\sqrt{2}}{4a^{\frac{3}{4}}c^{\frac{1}{4}}} \\
& - \frac{d \ln\left(-a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right)\sqrt{2}}{8a^{\frac{3}{4}}c^{\frac{1}{4}}} \\
& + \frac{d \ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right)\sqrt{2}}{8a^{\frac{3}{4}}c^{\frac{1}{4}}} + \frac{e \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}
\end{aligned}$$

command

```
integrate((e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.18 Problem number 398

$$\int \frac{1}{(d+ex)(a+cx^4)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{e^3 \ln(ex+d)}{ae^4+cd^4} - \frac{e^3 \ln(cx^4+a)}{4(ae^4+cd^4)} - \frac{d^2 e \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right) \sqrt{c}}{2(ae^4+cd^4)\sqrt{a}} \\ & - \frac{c^{\frac{1}{4}} d \ln\left(-a^{\frac{1}{4}} c^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{c}\right) (-e^2 \sqrt{a} + d^2 \sqrt{c}) \sqrt{2}}{8a^{\frac{3}{4}}(ae^4+cd^4)} \\ & + \frac{c^{\frac{1}{4}} d \ln\left(a^{\frac{1}{4}} c^{\frac{1}{4}} x \sqrt{2} + \sqrt{a} + x^2 \sqrt{c}\right) (-e^2 \sqrt{a} + d^2 \sqrt{c}) \sqrt{2}}{8a^{\frac{3}{4}}(ae^4+cd^4)} \\ & + \frac{c^{\frac{1}{4}} d \arctan\left(-1 + \frac{c^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (e^2 \sqrt{a} + d^2 \sqrt{c}) \sqrt{2}}{4a^{\frac{3}{4}}(ae^4+cd^4)} \\ & + \frac{c^{\frac{1}{4}} d \arctan\left(1 + \frac{c^{\frac{1}{4}} x \sqrt{2}}{a^{\frac{1}{4}}}\right) (e^2 \sqrt{a} + d^2 \sqrt{c}) \sqrt{2}}{4a^{\frac{3}{4}}(ae^4+cd^4)} \end{aligned}$$

command

```
integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.19 Problem number 401

$$\int \frac{(d+ex)^3}{(a+cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{-ae^3 + cx(3de^2x^2 + 3d^2ex + d^3)}{4ac(cx^4 + a)} + \frac{3d^2e \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{c}} \\ & - \frac{3d \ln\left(-a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right) (-e^2\sqrt{a} + d^2\sqrt{c}) \sqrt{2}}{32a^{\frac{7}{4}}c^{\frac{3}{4}}} \\ & + \frac{3d \ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right) (-e^2\sqrt{a} + d^2\sqrt{c}) \sqrt{2}}{32a^{\frac{7}{4}}c^{\frac{3}{4}}} \\ & + \frac{3d \arctan\left(-1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (e^2\sqrt{a} + d^2\sqrt{c}) \sqrt{2}}{16a^{\frac{7}{4}}c^{\frac{3}{4}}} \\ & + \frac{3d \arctan\left(1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (e^2\sqrt{a} + d^2\sqrt{c}) \sqrt{2}}{16a^{\frac{7}{4}}c^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.20 Problem number 402

$$\int \frac{(d+ex)^2}{(a+cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex+d)^2}{4a(cx^4+a)} + \frac{de \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\sqrt{c}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right)(-e^2\sqrt{a} + 3d^2\sqrt{c})\sqrt{2}}{32a^{\frac{7}{4}}c^{\frac{3}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right)(-e^2\sqrt{a} + 3d^2\sqrt{c})\sqrt{2}}{32a^{\frac{7}{4}}c^{\frac{3}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)(e^2\sqrt{a} + 3d^2\sqrt{c})\sqrt{2}}{16a^{\frac{7}{4}}c^{\frac{3}{4}}} \\ & + \frac{\arctan\left(1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)(e^2\sqrt{a} + 3d^2\sqrt{c})\sqrt{2}}{16a^{\frac{7}{4}}c^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.21 Problem number 403

$$\int \frac{d+ex}{(a+cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex+d)}{4a(cx^4+a)} + \frac{3d \arctan\left(-1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\sqrt{2}}{16a^{\frac{7}{4}}c^{\frac{1}{4}}} \\ & + \frac{3d \arctan\left(1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\sqrt{2}}{16a^{\frac{7}{4}}c^{\frac{1}{4}}} - \frac{3d \ln\left(-a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right)\sqrt{2}}{32a^{\frac{7}{4}}c^{\frac{1}{4}}} \\ & + \frac{3d \ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right)\sqrt{2}}{32a^{\frac{7}{4}}c^{\frac{1}{4}}} + \frac{e \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{c}} \end{aligned}$$

command

```
integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.22 Problem number 408

$$\int \frac{(d+ex)^3}{(a+cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(15de^2x^2 + 18d^2ex + 7d^3)}{32a^2(cx^4 + a)} + \frac{-ae^3 + cx(3de^2x^2 + 3d^2ex + d^3)}{8ac(cx^4 + a)^2} \\ & + \frac{9d^2e \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{c}} - \frac{3d \ln\left(-a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right) (-5e^2\sqrt{a} + 7d^2\sqrt{c}) \sqrt{2}}{256a^{\frac{11}{4}}c^{\frac{3}{4}}} \\ & + \frac{3d \ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right) (-5e^2\sqrt{a} + 7d^2\sqrt{c}) \sqrt{2}}{256a^{\frac{11}{4}}c^{\frac{3}{4}}} \\ & + \frac{3d \arctan\left(-1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (5e^2\sqrt{a} + 7d^2\sqrt{c}) \sqrt{2}}{128a^{\frac{11}{4}}c^{\frac{3}{4}}} \\ & + \frac{3d \arctan\left(1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (5e^2\sqrt{a} + 7d^2\sqrt{c}) \sqrt{2}}{128a^{\frac{11}{4}}c^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.23 Problem number 409

$$\int \frac{(d+ex)^2}{(a+cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex+d)^2}{8a(cx^4+a)^2} + \frac{x(5e^2x^2+12dex+7d^2)}{32a^2(cx^4+a)} + \frac{3de \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}\sqrt{c}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right) (-5e^2\sqrt{a} + 21d^2\sqrt{c}) \sqrt{2}}{256a^{\frac{11}{4}}c^{\frac{3}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right) (-5e^2\sqrt{a} + 21d^2\sqrt{c}) \sqrt{2}}{256a^{\frac{11}{4}}c^{\frac{3}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (5e^2\sqrt{a} + 21d^2\sqrt{c}) \sqrt{2}}{128a^{\frac{11}{4}}c^{\frac{3}{4}}} \\ & + \frac{\arctan\left(1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (5e^2\sqrt{a} + 21d^2\sqrt{c}) \sqrt{2}}{128a^{\frac{11}{4}}c^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.24 Problem number 410

$$\int \frac{d+ex}{(a+cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex+d)}{8a(cx^4+a)^2} + \frac{x(6ex+7d)}{32a^2(cx^4+a)} + \frac{21d \arctan\left(-1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{128a^{\frac{11}{4}}c^{\frac{1}{4}}} \\ & + \frac{21d \arctan\left(1 + \frac{c^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{128a^{\frac{11}{4}}c^{\frac{1}{4}}} - \frac{21d \ln\left(-a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right) \sqrt{2}}{256a^{\frac{11}{4}}c^{\frac{1}{4}}} \\ & + \frac{21d \ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{c}\right) \sqrt{2}}{256a^{\frac{11}{4}}c^{\frac{1}{4}}} + \frac{3e \arctan\left(\frac{x^2\sqrt{c}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{c}} \end{aligned}$$

command

```
integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

35.25 Problem number 483

$$\int \left(\frac{1}{2}(3 - \sqrt{37}) + x \right) \left(\frac{1}{2}(3 + \sqrt{37}) + x \right) dx$$

Optimal antiderivative

$$-7x + \frac{3}{2}x^2 + \frac{1}{3}x^3$$

command

```
integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: SyntaxError

36 Test file number 52

Test folder name:

test_cases/1_Algebraic_functions/1.3_Miscellaneous/52_1.3.2_Algebraic_functions

36.1 Problem number 1

$$\int \frac{1}{(2^{2/3} + x) \sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{(1+2^{1/3}x)\sqrt{3}}{\sqrt{x^3+1}}\right) \sqrt{3}}{9} + \frac{2 \cdot 2^{1/3} (1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} \cdot 3^{3/4}}{9 \sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{9} \sqrt{3} \arctan\left(-\frac{\sqrt{3} \left(5x^3 - 2^{2/3}(x^5 + x^2) + 2^{1/3}(7x^4 + 4x) + 2\right) \sqrt{x^3+1}}{6(2x^6 + 3x^3 + 1)}\right) + \frac{2}{3} \cdot 2^{1/3} \operatorname{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^3+1} \left(x^2 - 2^{2/3}x + 2 \cdot 2^{1/3}\right)}{x^6 + 5x^3 + 4}, x\right)$$

36.2 Problem number 2

$$\int \frac{1}{(2^{2/3} - x) \sqrt{1 - x^3}} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left(\frac{(1-2^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{-x^3+1}} \right) \sqrt{3}}{9} - \frac{2 \cdot 2^{\frac{1}{3}}(1-x) \operatorname{EllipticF} \left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}} \cdot 3^{\frac{3}{4}}}{9 \sqrt{-x^3+1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}$$

command

`integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{9} \sqrt{3} \arctan \left(-\frac{\sqrt{3} \left(5x^3 - 2^{\frac{2}{3}}(x^5 - x^2) - 2^{\frac{1}{3}}(7x^4 - 4x) - 2 \right) \sqrt{-x^3+1}}{6(2x^6 - 3x^3 + 1)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{-x^3+1} \left(x^2 + 2^{\frac{2}{3}}x + 2 \cdot 2^{\frac{1}{3}} \right)}{x^6 - 5x^3 + 4}, x \right)$$

36.3 Problem number 9

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left(\frac{(2dx+c)\sqrt{3}\sqrt{c}}{\sqrt{4d^3x^3+c^3}} \right) \sqrt{3}}{9c^{\frac{3}{2}}d} + \frac{2 \cdot 2^{\frac{1}{3}}(c+2^{\frac{2}{3}}dx) \operatorname{EllipticF} \left(\frac{2^{\frac{2}{3}}dx+c(1-\sqrt{3})}{2^{\frac{2}{3}}dx+c(1+\sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{c^2 - 2^{\frac{2}{3}}cdx + 2 \cdot 2^{\frac{1}{3}}d^2x^2}{(2^{\frac{2}{3}}dx+c(1+\sqrt{3}))^2}} \cdot 3^{\frac{3}{4}}}{9cd\sqrt{4d^3x^3+c^3} \sqrt{\frac{c(c+2^{\frac{2}{3}}dx)}{(2^{\frac{2}{3}}dx+c(1+\sqrt{3}))^2}}}$$

command

```
integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{\sqrt{3} \sqrt{-c} d^2 \log \left(\frac{2d^6x^6 - 36cd^5x^5 - 18c^2d^4x^4 + 28c^3d^3x^3 + 18c^4d^2x^2 - c^6 + \sqrt{3} (4d^4x^4 - 10cd^3x^3 - 18c^2d^2x^2 - 8c^3dx - c^4) \sqrt{4d^3x^3 + c^3}}{d^6x^6 + 6cd^5x^5 + 15c^2d^4x^4 + 20c^3d^3x^3 + 15c^4d^2x^2 + 6c^5dx + c^6} \right)}{18c^2d^3} \right. \\ \left. - \frac{\sqrt{3} \sqrt{c} d^2 \arctan \left(\frac{\sqrt{3} \sqrt{4d^3x^3 + c^3} (2d^3x^3 - 6cd^2x^2 - 6c^2dx - c^3) \sqrt{c}}{3(8cd^4x^4 + 4c^2d^3x^3 + 2c^4dx + c^5)} \right) - 6c\sqrt{d^3} \text{weierstrassPInverse} \left(0, -\frac{c^3}{d^3}, x \right)}{9c^2d^3} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{4d^3x^3 + c^3}}{4d^4x^4 + 4cd^3x^3 + c^3dx + c^4}, x \right)$$

36.4 Problem number 10

$$\int \frac{1}{(1 + \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\frac{3^{\frac{1}{4}}(1+x) \text{EllipticF} \left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}}}{3\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \\ + \frac{\arctan \left(\frac{(1+x)\sqrt{3+2\sqrt{3}}}{\sqrt{x^3+1}} \right)}{\sqrt{9+6\sqrt{3}}}$$

command

```
integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{2\sqrt{3}-3} \arctan\left(\frac{(\sqrt{3}(x^2-4x-2)-6x-6)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right) + \frac{1}{3} \sqrt{3} \operatorname{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^3+1}(x-\sqrt{3}+1)}{x^5+2x^4-2x^3+x^2+2x-2}, x\right)$$

36.5 Problem number 11

$$\int \frac{1}{(1+\sqrt{3}-x)\sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\frac{3^{\frac{1}{4}}(1-x) \operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}}}{3\sqrt{-x^3+1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}} - \frac{\arctan\left(\frac{(1-x)\sqrt{3+2\sqrt{3}}}{\sqrt{-x^3+1}}\right)}{\sqrt{9+6\sqrt{3}}}$$

command

`integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{2\sqrt{3}-3} \arctan\left(\frac{\sqrt{-x^3+1}(\sqrt{3}(x^2+4x-2)+6x-6)\sqrt{2\sqrt{3}-3}}{6(x^3-1)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^3+1}(x+\sqrt{3}-1)}{x^5-2x^4-2x^3-x^2+2x+2}, x\right)$$

36.6 Problem number 12

$$\int \frac{1}{(1 + \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{3^{\frac{1}{4}}(1-x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}}}{3\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}} - \frac{\operatorname{arctanh}\left(\frac{(1-x)\sqrt{3+2\sqrt{3}}}{\sqrt{x^3-1}}\right)}{\sqrt{9+6\sqrt{3}}}$$

command

```
integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{12} \sqrt{2\sqrt{3}-3} \log\left(\frac{x^8 + 16x^7 + 112x^6 + 16x^5 + 112x^4 - 224x^3 + 64x^2 + 4(2x^6 + 18x^5 + 42x^4 + 8x^3 + \sqrt{3}x^2 + 2x + 2)}{x^8}\right) + \frac{1}{3} \sqrt{3} \operatorname{weierstrassPInverse}(0, 4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{x^3-1}(x+\sqrt{3}-1)}{x^5-2x^4-2x^3-x^2+2x+2}, x\right)$$

36.7 Problem number 13

$$\int \frac{1}{(1 + \sqrt{3} + x) \sqrt{-1 - x^3}} dx$$

Optimal antiderivative

$$\frac{3^{\frac{1}{4}}(1+x) \operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}}}{3\sqrt{-x^3-1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}} + \frac{\operatorname{arctanh}\left(\frac{(1+x)\sqrt{3+2\sqrt{3}}}{\sqrt{-x^3-1}}\right)}{\sqrt{9+6\sqrt{3}}}$$

command

```
integrate(1/(1+x*3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{12} \sqrt{2\sqrt{3}-3} \log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4(2x^6-18x^5+42x^4-8x^3+\sqrt{3}x^2-x+1)}{x}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3-1}(x-\sqrt{3}+1)}{x^5+2x^4-2x^3+x^2+2x-2}, x\right)$$

36.8 Problem number 52

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\frac{2\left(2-3\cdot 2^{\frac{2}{3}}\right) \operatorname{arctan}\left(\frac{(1+2^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{x^3+1}}\right) \sqrt{3}}{9} + \frac{2\left(3+2\cdot 2^{\frac{1}{3}}\right) (1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} \cdot 3^{\frac{3}{4}}}{9\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{9} \sqrt{3} \sqrt{-12 \cdot 2^{\frac{2}{3}} + 18 \cdot 2^{\frac{1}{3}} + 4} \arctan \left(\frac{\sqrt{3} (18x^5 - 42x^4 - 10x^3 + 18x^2 + 2^{\frac{2}{3}}(2x^5 - 63x^4 - 15x^3 + 2x^2 - 36))}{x^6 + 5x^3 + 4} \right) + \frac{2}{3} (2 \cdot 2^{\frac{1}{3}} + 3) \text{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(3x^3 + 2x^2 - 2^{\frac{2}{3}}(3x^2 + 2x) + 2 \cdot 2^{\frac{1}{3}}(3x + 2)) \sqrt{x^3 + 1}}{x^6 + 5x^3 + 4}, x \right)$$

36.9 Problem number 53

$$\int \frac{2 + 3x}{(2^{2/3} - x) \sqrt{1 - x^3}} dx$$

Optimal antiderivative

$$\frac{2(2 + 3 \cdot 2^{\frac{2}{3}}) \arctan \left(\frac{(1 - 2^{\frac{1}{3}}x) \sqrt{3}}{\sqrt{-x^3 + 1}} \right) \sqrt{3}}{9} + \frac{2(3 - 2 \cdot 2^{\frac{1}{3}}) (1 - x) \text{EllipticF} \left(\frac{1 - x - \sqrt{3}}{1 - x + \sqrt{3}}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2 + x + 1}{(1 - x + \sqrt{3})^2}} \cdot 3^{\frac{3}{4}}}{9 \sqrt{-x^3 + 1} \sqrt{\frac{1 - x}{(1 - x + \sqrt{3})^2}}}$$

command

`integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{9} \sqrt{3} \sqrt{12 \cdot 2^{\frac{2}{3}} + 18 \cdot 2^{\frac{1}{3}} + 4} \arctan \left(\frac{\sqrt{3} (18x^5 - 42x^4 - 10x^3 - 18x^2 + 2^{\frac{2}{3}}(2x^5 + 63x^4 + 15x^3 - 2x^2 - 36))}{x^6 - 5x^3 + 4} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(3x^3 + 2x^2 + 2^{\frac{2}{3}}(3x^2 + 2x) + 2 \cdot 2^{\frac{1}{3}}(3x + 2)) \sqrt{-x^3 + 1}}{x^6 - 5x^3 + 4}, x \right)$$

36.10 Problem number 55

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}} dx$$

Optimal antiderivative

$$\frac{2\left(2-3\cdot 2^{\frac{2}{3}}\right) \operatorname{arctanh}\left(\frac{\left(1+2^{\frac{1}{3}}x\right)\sqrt{3}}{\sqrt{-x^3-1}}\right) \sqrt{3}}{9} + \frac{2\left(3+2\cdot 2^{\frac{1}{3}}\right)(1+x) \operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{-x^3-1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

command

`integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{18} \sqrt{3} \sqrt{-12 \cdot 2^{\frac{2}{3}} + 18 \cdot 2^{\frac{1}{3}} + 4} \log \left(\frac{25x^{18} - 36000x^{15} + 435000x^{12} + 526400x^9 - 259200x^6 - 384000x^3 + 2\sqrt{3}}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(3x^3+2x^2-2^{\frac{2}{3}}(3x^2+2x)+2\cdot 2^{\frac{1}{3}}(3x+2)\right)\sqrt{-x^3-1}}{x^6+5x^3+4}, x\right)$$

36.11 Problem number 56

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\frac{2\left(e-2^{\frac{2}{3}}f\right) \operatorname{arctan}\left(\frac{\left(1+2^{\frac{1}{3}}x\right)\sqrt{3}}{\sqrt{x^3+1}}\right) \sqrt{3}}{9} + \frac{2\left(2^{\frac{1}{3}}e+f\right)(1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1}{18} \sqrt{3} \sqrt{2 \cdot 2^{\frac{2}{3}} f e - 2 \cdot 2^{\frac{1}{3}} f^2 - e^2} \log \left(-\frac{4 f^3 x^{18} - 5760 f^3 x^{15} + 69600 f^3 x^{12} + 84224 f^3 x^9 - 41472 f^3 x^6 - 6144 f^3 x^3 + 216 e^2}{(2^{\frac{2}{3}} + x)^3 (x^3 + 1)^3} \right) \right. \\ \left. + \frac{2}{3} \left(2^{\frac{1}{3}} e + f \right) \text{weierstrassPInverse}(0, -4, x), \right. \\ \left. - \frac{1}{9} \sqrt{3} \sqrt{-2 \cdot 2^{\frac{2}{3}} f e + 2 \cdot 2^{\frac{1}{3}} f^2 + e^2} \arctan \left(-\frac{\sqrt{3} (4 f^2 x^5 + 4 f^2 x^2 - (5 x^3 + 2) e^2 - 2 (7 f x^4 + 4 f x) e - 2^{\frac{2}{3}} (14 f^2 x^3 + 7 f^2 x - 2 e^2))}{(2^{\frac{2}{3}} + x)^3 (x^3 + 1)^3} \right) \right. \\ \left. + \frac{2}{3} \left(2^{\frac{1}{3}} e + f \right) \text{weierstrassPInverse}(0, -4, x) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

36.12 Problem number 57

$$\int \frac{e + f x}{(2^{2/3} - x) \sqrt{1 - x^3}} dx$$

Optimal antiderivative

$$\frac{2 \left(e + 2^{\frac{2}{3}} f \right) \arctan \left(\frac{(1 - 2^{\frac{1}{3}} x) \sqrt{3}}{\sqrt{-x^3 + 1}} \right) \sqrt{3}}{9} \\ - \frac{2 \left(2^{\frac{1}{3}} e - f \right) (1 - x) \text{EllipticF} \left(\frac{1 - x - \sqrt{3}}{1 - x + \sqrt{3}}, i \sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2 + x + 1}{(1 - x + \sqrt{3})^2}} 3^{\frac{3}{4}}}{9 \sqrt{-x^3 + 1} \sqrt{\frac{1 - x}{(1 - x + \sqrt{3})^2}}}$$

command

```
integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1}{18} \sqrt{3} \sqrt{-2 \cdot 2^{\frac{2}{3}} f e - 2 \cdot 2^{\frac{1}{3}} f^2 - e^2} \log \left(\frac{4 f^3 x^{18} + 5760 f^3 x^{15} + 69600 f^3 x^{12} - 84224 f^3 x^9 - 41472 f^3 x^6 + 61440 f^3 x^3 - 41472 f^3 x^0}{\sqrt{3} (4 f^2 x^5 - 4 f^2 x^2 - (5 x^3 - 2) e^2 - 2 (7 f x^4 - 4 f x) e + 2^{\frac{2}{3}} (14 f^2 x^4 - 14 f^2 x^1 - 2 e^2))} \right) \right. \\ \left. - \frac{1}{9} \sqrt{3} \sqrt{2 \cdot 2^{\frac{2}{3}} f e + 2 \cdot 2^{\frac{1}{3}} f^2 + e^2} \arctan \left(\frac{\sqrt{3} (4 f^2 x^5 - 4 f^2 x^2 - (5 x^3 - 2) e^2 - 2 (7 f x^4 - 4 f x) e + 2^{\frac{2}{3}} (14 f^2 x^4 - 14 f^2 x^1 - 2 e^2))}{\sqrt{3} (4 f^2 x^5 - 4 f^2 x^2 - (5 x^3 - 2) e^2 - 2 (7 f x^4 - 4 f x) e + 2^{\frac{2}{3}} (14 f^2 x^4 - 14 f^2 x^1 - 2 e^2))} \right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(f x^3 + e x^2 + 2^{\frac{2}{3}} (f x^2 + e x) + 2 \cdot 2^{\frac{1}{3}} (f x + e) \right) \sqrt{-x^3 + 1}}{x^6 - 5 x^3 + 4}, x \right)$$

36.13 Problem number 58

$$\int \frac{e + f x}{(2^{2/3} - x) \sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2(e + 2^{\frac{2}{3}} f) \operatorname{arctanh} \left(\frac{\left(1 - 2^{\frac{1}{3}} x\right) \sqrt{3}}{\sqrt{x^3 - 1}} \right) \sqrt{3}}{9} \\ - \frac{2 \left(2^{\frac{1}{3}} e - f\right) (1 - x) \operatorname{EllipticF} \left(\frac{1 - x + \sqrt{3}}{1 - x - \sqrt{3}}, 2i - i\sqrt{3} \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2 + x + 1}{(1 - x - \sqrt{3})^2}} 3^{\frac{3}{4}}}{9 \sqrt{x^3 - 1} \sqrt{\frac{-1 + x}{(1 - x - \sqrt{3})^2}}}$$

command

`integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1}{18} \sqrt{3} \sqrt{2 \cdot 2^{\frac{2}{3}} f e + 2 \cdot 2^{\frac{1}{3}} f^2 + e^2} \log \left(\frac{4 f^3 x^{18} + 5760 f^3 x^{15} + 69600 f^3 x^{12} - 84224 f^3 x^9 - 41472 f^3 x^6 + 61440 f^3 x^3 - 41472 f^3 x^0}{\sqrt{3} (4 f^2 x^5 - 4 f^2 x^2 - (5 x^3 - 2) e^2 - 2 (7 f x^4 - 4 f x) e + 2^{\frac{2}{3}} (14 f^2 x^4 - 14 f^2 x^1 - 2 e^2))} \right) \right. \\ \left. + \frac{2}{3} \left(2^{\frac{1}{3}} e - f\right) \operatorname{weierstrassPInverse}(0, 4, x), \right. \\ \left. - \frac{1}{9} \sqrt{3} \sqrt{-2 \cdot 2^{\frac{2}{3}} f e - 2 \cdot 2^{\frac{1}{3}} f^2 - e^2} \arctan \left(\frac{\sqrt{3} (4 f^2 x^5 - 4 f^2 x^2 - (5 x^3 - 2) e^2 - 2 (7 f x^4 - 4 f x) e + 2^{\frac{2}{3}} (14 f^2 x^4 - 14 f^2 x^1 - 2 e^2))}{\sqrt{3} (4 f^2 x^5 - 4 f^2 x^2 - (5 x^3 - 2) e^2 - 2 (7 f x^4 - 4 f x) e + 2^{\frac{2}{3}} (14 f^2 x^4 - 14 f^2 x^1 - 2 e^2))} \right) \right. \\ \left. + \frac{2}{3} \left(2^{\frac{1}{3}} e - f\right) \operatorname{weierstrassPInverse}(0, 4, x) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{(fx^3 + ex^2 + 2^{\frac{2}{3}}(fx^2 + ex) + 2 \cdot 2^{\frac{1}{3}}(fx + e))\sqrt{x^3 - 1}}{x^6 - 5x^3 + 4}, x \right)$$

36.14 Problem number 64

$$\int \frac{e + fx}{(c + dx)\sqrt{c^3 + 4d^3x^3}} dx$$

Optimal antiderivative

$$\frac{2(-cf + de) \arctan\left(\frac{(2dx+c)\sqrt{3}\sqrt{c}}{\sqrt{4d^3x^3+c^3}}\right)\sqrt{3}}{9c^{\frac{3}{2}}d^2} + \frac{2^{\frac{1}{3}}(cf + 2de)\left(c + 2^{\frac{2}{3}}dx\right) \text{EllipticF}\left(\frac{2^{\frac{2}{3}}dx+c(1-\sqrt{3})}{2^{\frac{2}{3}}dx+c(1+\sqrt{3})}, i\sqrt{3} + 2i\right)\left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)\sqrt{\frac{c^2 - 2^{\frac{2}{3}}cdx + 2 \cdot 2^{\frac{1}{3}}d^2x^2}{\left(2^{\frac{2}{3}}dx + c(1 + \sqrt{3})\right)^2}}}{9cd^2\sqrt{4d^3x^3+c^3}\sqrt{\frac{c(c+2^{\frac{2}{3}}dx)}{\left(2^{\frac{2}{3}}dx+c(1+\sqrt{3})\right)^2}}}$$

command

`integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{\sqrt{3}(cd^2f - d^3e)\sqrt{-c} \log\left(\frac{2d^6x^6 - 36cd^5x^5 - 18c^2d^4x^4 + 28c^3d^3x^3 + 18c^4d^2x^2 - c^6 + \sqrt{3}(4d^4x^4 - 10cd^3x^3 - 18c^2d^2x^2 - 8c^3dx - c^4)\sqrt{c}}{d^6x^6 + 6cd^5x^5 + 15c^2d^4x^4 + 20c^3d^3x^3 + 15c^4d^2x^2 + 6c^5dx + c^6}\right)}{18c^2d^4} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{4d^3x^3 + c^3}(fx + e)}{4d^4x^4 + 4cd^3x^3 + c^3dx + c^4}, x \right)$$

36.15 Problem number 65

$$\int \frac{x}{(2^{2/3} + x) \sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2 \cdot 2^{2/3} \arctan\left(\frac{(1+2^{1/3}x)\sqrt{3}}{\sqrt{x^3+1}}\right) \sqrt{3}}{9} + \frac{2(1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} \cdot 3^{3/4}}{9\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{9} \sqrt{3} 2^{2/3} \arctan\left(-\frac{\sqrt{3} 2^{2/3} (2x^5 + 2x^2 - 2^{2/3}(7x^4 + 4x) - 2^{1/3}(5x^3 + 2)) \sqrt{x^3 + 1}}{12(2x^6 + 3x^3 + 1)}\right) + \frac{2}{3} \operatorname{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(x^3 - 2^{2/3}x^2 + 2 \cdot 2^{1/3}x) \sqrt{x^3 + 1}}{x^6 + 5x^3 + 4}, x\right)$$

36.16 Problem number 66

$$\int \frac{x}{(2^{2/3} - x) \sqrt{1 - x^3}} dx$$

Optimal antiderivative

$$\frac{2 \cdot 2^{\frac{2}{3}} \arctan\left(\frac{(1-2^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{-x^3+1}}\right) \sqrt{3}}{9} + \frac{2(1-x) \operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}} \cdot 3^{\frac{3}{4}}}{9\sqrt{-x^3+1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}$$

command

`integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{9} \sqrt{3} \cdot 2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3} \cdot 2^{\frac{2}{3}} (2x^5 - 2x^2 + 2^{\frac{2}{3}}(7x^4 - 4x) - 2^{\frac{1}{3}}(5x^3 - 2)) \sqrt{-x^3+1}}{12(2x^6 - 3x^3 + 1)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(x^3 + 2^{\frac{2}{3}}x^2 + 2 \cdot 2^{\frac{1}{3}}x) \sqrt{-x^3+1}}{x^6 - 5x^3 + 4}, x\right)$$

36.17 Problem number 67

$$\int \frac{x}{(2^{2/3} - x) \sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2 \cdot 2^{\frac{2}{3}} \operatorname{arctanh}\left(\frac{(1-2^{\frac{1}{3}}x)\sqrt{3}}{\sqrt{x^3-1}}\right) \sqrt{3}}{9} + \frac{2(1-x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}} \cdot 3^{\frac{3}{4}}}{9\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

command

```
integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{18} \sqrt{3} 2^{\frac{2}{3}} \log \left(\frac{x^{18} + 1440 x^{15} + 17400 x^{12} - 21056 x^9 - 10368 x^6 + 15360 x^3 + 2 \sqrt{3} 2^{\frac{2}{3}} (126 x^{14} + 2664 x^{11} - 460}{- \frac{2}{3} \text{weierstrassPInverse}(0, 4, x)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{(x^3 + 2^{\frac{2}{3}} x^2 + 2 \cdot 2^{\frac{1}{3}} x) \sqrt{x^3 - 1}}{x^6 - 5 x^3 + 4}, x \right)$$

36.18 Problem number 68

$$\int \frac{x}{(2^{2/3} + x) \sqrt{-1 - x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \cdot 2^{\frac{2}{3}} \operatorname{arctanh} \left(\frac{(1 + 2^{\frac{1}{3}} x) \sqrt{3}}{\sqrt{-x^3 - 1}} \right) \sqrt{3}}{9} \\ & + \frac{2(1+x) \operatorname{EllipticF} \left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i - i\sqrt{3} \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2 - x + 1}{(1+x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{9 \sqrt{-x^3 - 1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{18} \sqrt{3} 2^{\frac{2}{3}} \log \left(\frac{x^{18} - 1440 x^{15} + 17400 x^{12} + 21056 x^9 - 10368 x^6 - 15360 x^3 + 2 \sqrt{3} 2^{\frac{2}{3}} (126 x^{14} - 2664 x^{11} + 460}{- \frac{2}{3} \text{weierstrassPInverse}(0, 4, x)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{(x^3 - 2^{\frac{2}{3}} x^2 + 2 \cdot 2^{\frac{1}{3}} x) \sqrt{-x^3 - 1}}{x^6 + 5 x^3 + 4}, x \right)$$

36.19 Problem number 73

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{(2dx+c)\sqrt{3}\sqrt{c}}{\sqrt{4d^3x^3+c^3}}\right)\sqrt{3}}{9d^2\sqrt{c}} + \frac{2^{\frac{1}{3}}\left(c+2^{\frac{2}{3}}dx\right) \operatorname{EllipticF}\left(\frac{2^{\frac{2}{3}}dx+c(1-\sqrt{3})}{2^{\frac{2}{3}}dx+c(1+\sqrt{3})}, i\sqrt{3}+2i\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{c^2-2^{\frac{2}{3}}cdx+2\cdot 2^{\frac{1}{3}}d^2x^2}{\left(2^{\frac{2}{3}}dx+c(1+\sqrt{3})\right)^2}}}{9d^2\sqrt{4d^3x^3+c^3}\sqrt{\frac{c(c+2^{\frac{2}{3}}dx)}{\left(2^{\frac{2}{3}}dx+c(1+\sqrt{3})\right)^2}}} 3^{\frac{3}{4}}$$

command

`integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{\sqrt{3}\sqrt{-c}d^2 \log\left(\frac{2d^6x^6-36cd^5x^5-18c^2d^4x^4+28c^3d^3x^3+18c^4d^2x^2-c^6-\sqrt{3}(4d^4x^4-10cd^3x^3-18c^2d^2x^2-8c^3dx-c^4)\sqrt{4d^3x^3+c^3}}{d^6x^6+6cd^5x^5+15c^2d^4x^4+20c^3d^3x^3+15c^4d^2x^2+6c^5dx+c^6}\right)}{18cd^4} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{4d^3x^3+c^3}x}{4d^4x^4+4cd^3x^3+c^3dx+c^4}, x\right)$$

36.20 Problem number 78

$$\int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{\left(2\sqrt[3]{a} - \sqrt[3]{b}x\right)\sqrt{a+bx^3}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}\left(\frac{\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)^2}{3a^{\frac{1}{6}}\sqrt{bx^3+a}}\right)}{3a^{\frac{1}{6}}b^{\frac{1}{3}}}$$

command

```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1}{6} a^{\frac{1}{3}} \sqrt{\frac{1}{ab^{\frac{2}{3}}}} \log \left(\frac{b^6 x^{18} + 7800 ab^5 x^{15} + 535272 a^2 b^4 x^{12} + 5147264 a^3 b^3 x^9 + 10516992 a^4 b^2 x^6 + 5922816 a^5 b x^3 + \dots}{\dots} \right) \right. \\ \left. - \frac{1}{3} a^{\frac{1}{3}} \sqrt{-\frac{1}{ab^{\frac{2}{3}}}} \arctan \left(\frac{\sqrt{bx^3 + a} \left((11bx^4 - 16ax) a^{\frac{2}{3}} b^{\frac{2}{3}} + (b^2 x^5 + 28 abx^2) a^{\frac{1}{3}} - (17 abx^3 - 10 a^2) b^{\frac{1}{3}} \right) \sqrt{-\frac{1}{ab^{\frac{2}{3}}}}}{6(b^2 x^6 + 2 abx^3 + a^2)} \right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

36.21 Problem number 79

$$\int \frac{\sqrt[3]{a} - \sqrt[3]{b} x}{(2\sqrt[3]{a} + \sqrt[3]{b} x) \sqrt{a - bx^3}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh} \left(\frac{(a^{\frac{1}{3}} - b^{\frac{1}{3}} x)^2}{3a^{\frac{1}{6}} \sqrt{-bx^3 + a}} \right)}{3a^{\frac{1}{6}} b^{\frac{1}{3}}}$$

command

```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1}{6} a^{\frac{1}{3}} \sqrt{\frac{1}{ab^{\frac{2}{3}}}} \log \left(\frac{b^6 x^{18} - 7800 ab^5 x^{15} + 535272 a^2 b^4 x^{12} - 5147264 a^3 b^3 x^9 + 10516992 a^4 b^2 x^6 - 5922816 a^5 b x^3 + \dots}{\dots} \right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

36.22 Problem number 80

$$\int \frac{\sqrt[3]{a} - \sqrt[3]{b} x}{(2\sqrt[3]{a} + \sqrt[3]{b} x) \sqrt{-a + bx^3}} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left(\frac{(a^{\frac{1}{3}} - b^{\frac{1}{3}} x)^2}{3a^{\frac{1}{6}} \sqrt{bx^3 - a}} \right)}{3a^{\frac{1}{6}} b^{\frac{1}{3}}}$$

command

```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1}{6} a^{\frac{1}{3}} \sqrt{-\frac{1}{ab^{\frac{2}{3}}}} \log \left(\frac{b^6 x^{18} - 7800 ab^5 x^{15} + 535272 a^2 b^4 x^{12} - 5147264 a^3 b^3 x^9 + 10516992 a^4 b^2 x^6 - 5922816 a^5 b x^3}{\dots} \right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

36.23 Problem number 81

$$\int \frac{\sqrt[3]{a} + \sqrt[3]{b} x}{(2\sqrt[3]{a} - \sqrt[3]{b} x) \sqrt{-a - bx^3}} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left(\frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}} x)^2}{3a^{\frac{1}{6}} \sqrt{-bx^3 - a}} \right)}{3a^{\frac{1}{6}} b^{\frac{1}{3}}}$$

command

```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1}{6} a^{\frac{1}{3}} \sqrt{-\frac{1}{ab^{\frac{2}{3}}}} \log \left(\frac{b^6 x^{18} + 7800 ab^5 x^{15} + 535272 a^2 b^4 x^{12} + 5147264 a^3 b^3 x^9 + 10516992 a^4 b^2 x^6 + 5922816 a^5 b x^3 + 10516992 a^6}{6 (b^2 x^6 + 2 abx^3 + a^2)} \right) \right. \\ \left. - \frac{1}{3} a^{\frac{1}{3}} \sqrt{\frac{1}{ab^{\frac{2}{3}}}} \arctan \left(\frac{\left((11 bx^4 - 16 ax) \sqrt{-bx^3 - a} a^{\frac{2}{3}} b^{\frac{2}{3}} + (b^2 x^5 + 28 abx^2) \sqrt{-bx^3 - a} a^{\frac{1}{3}} - (17 abx^3 - 10 a^2) \right)}{6 (b^2 x^6 + 2 abx^3 + a^2)} \right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

36.24 Problem number 83

$$\int \frac{e + fx}{(2-x)\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\frac{2(e+2f) \operatorname{arctanh} \left(\frac{(1+x)^2}{3\sqrt{x^3+1}} \right)}{9} \\ + \frac{2(e-f)(1+x) \operatorname{EllipticF} \left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{9} (2f + e) \log \left(\frac{x^3 + 12x^2 + 6\sqrt{x^3+1}(x+1) - 6x + 10}{x^3 - 6x^2 + 12x - 8} \right) \\ - \frac{2}{3} (f - e) \operatorname{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{x^3+1}(fx+e)}{x^4-2x^3+x-2}, x \right)$$

36.25 Problem number 84

$$\int \frac{e + fx}{(2+x)\sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\frac{2(e-2f) \operatorname{arctanh}\left(\frac{(1-x)^2}{3\sqrt{-x^3+1}}\right)}{9} - \frac{2(e+f)(1-x) \operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{-x^3+1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}$$

command

`integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{9}(2f-e) \log\left(-\frac{x^3-12x^2-6\sqrt{-x^3+1}(x-1)-6x-10}{x^3+6x^2+12x+8}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3+1}(fx+e)}{x^4+2x^3-x-2}, x\right)$$

36.26 Problem number 85

$$\int \frac{e + fx}{(2+x)\sqrt{-1+x^3}} dx$$

Optimal antiderivative

$$\frac{2(e-2f) \operatorname{arctan}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)}{9} - \frac{2(e+f)(1-x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

command

```
integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{9}(2f - e) \arctan\left(\frac{(x^3 - 12x^2 - 6x - 10)\sqrt{x^3 - 1}}{6(x^4 - x^3 - x + 1)}\right) + \frac{2}{3}(f + e)\text{weierstrassPInverse}(0, 4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3 - 1}(fx + e)}{x^4 + 2x^3 - x - 2}, x\right)$$

36.27 Problem number 86

$$\int \frac{e + fx}{(2 - x)\sqrt{-1 - x^3}} dx$$

Optimal antiderivative

$$\frac{2(e + 2f) \arctan\left(\frac{(1+x)^2}{3\sqrt{-x^3 - 1}}\right)}{9} + \frac{2(e - f)(1 + x) \text{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i - i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{-x^3 - 1} \sqrt{\frac{-1 - x}{(1+x-\sqrt{3})^2}}}$$

command

```
integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{9}(2f + e) \arctan\left(\frac{(x^3 + 12x^2 - 6x + 10)\sqrt{-x^3 - 1}}{6(x^4 + x^3 + x + 1)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^3 - 1}(fx + e)}{x^4 - 2x^3 + x - 2}, x\right)$$

36.28 Problem number 91

$$\int \frac{e + fx}{(c + dx)\sqrt{c^3 - 8d^3x^3}} dx$$

Optimal antiderivative

$$\frac{2(-cf + de) \operatorname{arctanh}\left(\frac{(-2dx+c)^2}{3\sqrt{c}\sqrt{-8d^3x^3+c^3}}\right) + (cf + 2de)(-2dx + c) \operatorname{EllipticF}\left(\frac{-2dx+c(1-\sqrt{3})}{-2dx+c(1+\sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{4d^2x^2 + 2cdx + c^2}{(-2dx + c(1 + \sqrt{3}))^2}}}{9c^{\frac{3}{2}}d^2} \cdot \frac{3^{\frac{3}{4}}}{9c d^2 \sqrt{-8d^3x^3 + c^3} \sqrt{\frac{c(-2dx + c)}{(-2dx + c(1 + \sqrt{3}))^2}}}$$

command

```
integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{3\sqrt{2}\sqrt{-d^3}(c^2f + 2cde)\operatorname{weierstrassPInverse}\left(0, \frac{c^3}{2d^3}, x\right) + (cd^2f - d^3e)\sqrt{c}\log\left(\frac{8d^6x^6 - 240cd^5x^5 + 408c^2d^4x^4 + 88c^3d^3x^3 - 24c^2d^2x^2 - 6c^2dx - 5c^2}{3(16cd^4x^4 - 8c^2d^3x^3 + c^2d^2x^2 - c^2dx - c^2)}\right)}{18c^2d^4}, \right.$$

$$\left. \frac{3\sqrt{2}\sqrt{-d^3}(c^2f + 2cde)\operatorname{weierstrassPInverse}\left(0, \frac{c^3}{2d^3}, x\right) - 2(cd^2f - d^3e)\sqrt{-c}\arctan\left(\frac{(4d^3x^3 - 24cd^2x^2 - 6c^2dx - 5c^2)}{3(16cd^4x^4 - 8c^2d^3x^3 + c^2d^2x^2 - c^2dx - c^2)}\right)}{18c^2d^4} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-8d^3x^3 + c^3}(fx + e)}{8d^4x^4 + 8cd^3x^3 - c^3dx - c^4}, x\right)$$

36.29 Problem number 92

$$\int \frac{x}{(2-x)\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\frac{4 \operatorname{arctanh}\left(\frac{(1+x)^2}{3\sqrt{x^3+1}}\right)}{9} + \frac{2(1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{9} \log\left(\frac{x^3 + 12x^2 + 6\sqrt{x^3+1}(x+1) - 6x + 10}{x^3 - 6x^2 + 12x - 8}\right) - \frac{2}{3} \operatorname{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{x^3+1}x}{x^4-2x^3+x-2}, x\right)$$

36.30 Problem number 93

$$\int \frac{x}{(2+x)\sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\frac{4 \operatorname{arctanh}\left(\frac{(1-x)^2}{3\sqrt{-x^3+1}}\right)}{9} + \frac{2(1-x) \operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{9\sqrt{-x^3+1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}$$

command

```
integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{9} \log \left(-\frac{x^3 - 12x^2 + 6\sqrt{-x^3 + 1}(x - 1) - 6x - 10}{x^3 + 6x^2 + 12x + 8} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{-x^3 + 1} x}{x^4 + 2x^3 - x - 2}, x \right)$$

36.31 Problem number 94

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}} dx$$

Optimal antiderivative

$$\frac{4 \arctan \left(\frac{(1-x)^2}{3\sqrt{x^3-1}} \right)}{9} - \frac{2(1-x) \operatorname{EllipticF} \left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3} \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}}}{9\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}} 3^{\frac{3}{4}}$$

command

```
integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{9} \arctan \left(\frac{(x^3 - 12x^2 - 6x - 10)\sqrt{x^3 - 1}}{6(x^4 - x^3 - x + 1)} \right) + \frac{2}{3} \operatorname{weierstrassPInverse}(0, 4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{x^3 - 1} x}{x^4 + 2x^3 - x - 2}, x \right)$$

36.32 Problem number 95

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}} dx$$

Optimal antiderivative

$$\frac{4 \arctan\left(\frac{(1+x)^2}{3\sqrt{-x^3-1}}\right)}{9} - \frac{2(1+x) \operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}}}{9\sqrt{-x^3-1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}} 3^{\frac{3}{4}}$$

command

`integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{9} \arctan\left(\frac{(x^3+12x^2-6x+10)\sqrt{-x^3-1}}{6(x^4+x^3+x+1)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^3-1}x}{x^4-2x^3+x-2}, x\right)$$

36.33 Problem number 100

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}\left(\frac{(-2dx+c)^2}{3\sqrt{c}\sqrt{-8d^3x^3+c^3}}\right)}{9d^2\sqrt{c}} - \frac{(-2dx+c) \operatorname{EllipticF}\left(\frac{-2dx+c(1-\sqrt{3})}{-2dx+c(1+\sqrt{3})}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right) \sqrt{\frac{4d^2x^2+2cdx+c^2}{(-2dx+c(1+\sqrt{3}))^2}}}{9d^2\sqrt{-8d^3x^3+c^3} \sqrt{\frac{c(-2dx+c)}{(-2dx+c(1+\sqrt{3}))^2}}} 3^{\frac{3}{4}}$$

command

```
integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{\sqrt{c} d^2 \log \left(\frac{8 d^6 x^6 - 240 c d^5 x^5 + 408 c^2 d^4 x^4 + 88 c^3 d^3 x^3 + 156 c^4 d^2 x^2 + 12 c^5 d x + 17 c^6 + 3 (8 d^4 x^4 - 52 c d^3 x^3 + 12 c^2 d^2 x^2 - 4 c^3 d x + 5 c^4) \sqrt{-8 d^3 x^3 + c^3}}{d^6 x^6 + 6 c d^5 x^5 + 15 c^2 d^4 x^4 + 20 c^3 d^3 x^3 + 15 c^4 d^2 x^2 + 6 c^5 d x + c^6} \right)}{18 c d^4} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{-8 d^3 x^3 + c^3} x}{8 d^4 x^4 + 8 c d^3 x^3 - c^3 d x - c^4}, x \right)$$

36.34 Problem number 125

$$\int \frac{1+x}{(1+\sqrt{3}+x)\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\frac{(1+x) \text{EllipticF} \left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} - \frac{\arctan \left(\frac{(1+x)\sqrt{3+2\sqrt{3}}}{\sqrt{x^3+1}} \right)}{\sqrt{3+2\sqrt{3}}}$$

command

```
integrate((1+x)/((1+x+3^(1/2)))/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \sqrt{3} \sqrt{2\sqrt{3}-3} \arctan \left(\frac{(\sqrt{3}(x^2-4x-2)-6x-6)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}} \right) + \text{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3+1}(x-\sqrt{3}+1)}{x^4+x^3-3x^2+4x-2}, x\right)$$

36.35 Problem number 126

$$\int \frac{1+x}{(1-\sqrt{3}+x)\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\frac{(1+x) \text{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}} - \frac{\text{arctanh}\left(\frac{(1+x)\sqrt{-3+2\sqrt{3}}}{\sqrt{x^3+1}}\right)}{\sqrt{-3+2\sqrt{3}}}}$$

command

```
integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{12} \sqrt{3} \sqrt{2\sqrt{3}+3} \log\left(\frac{x^8 - 16x^7 + 112x^6 - 16x^5 + 112x^4 + 224x^3 + 64x^2 - 4(2x^6 - 18x^5 + 42x^4 - 8x^3 - \dots)}{\dots}\right) + \text{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3+1}(x+\sqrt{3}+1)}{x^4+x^3-3x^2+4x-2}, x\right)$$

36.36 Problem number 127

$$\int \frac{e + fx}{(1 + \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\frac{(1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) (e - f(1 - \sqrt{3})) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}} 3^{\frac{1}{4}}}{3\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} + \frac{\arctan\left(\frac{(1+x)\sqrt{3+2\sqrt{3}}}{\sqrt{x^3+1}}\right) (e - f - f\sqrt{3})}{\sqrt{9+6\sqrt{3}}}$$

command

`integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[-\frac{1}{3} (\sqrt{3}(f-e) - 3f) \operatorname{weierstrassPInverse}(0, -4, x) \right. \\ \left. + \frac{1}{12} \sqrt{6fe - 2\sqrt{3}(f^2 + fe + e^2) + 3e^2} \log\left(-\frac{2f^2x^8 - 32f^2x^7 + 224f^2x^6 - 32f^2x^5 + 224f^2x^4 + 448f^2x^3 + \dots}{\dots}\right) \right. \\ \left. - \frac{1}{3} (\sqrt{3}(f-e) - 3f) \operatorname{weierstrassPInverse}(0, -4, x) \right. \\ \left. - \frac{1}{6} \sqrt{-6fe + 2\sqrt{3}(f^2 + fe + e^2) - 3e^2} \arctan\left(-\frac{(3fx^2 - 6fx - 6(x+1)e - \sqrt{3}(fx^2 + 2fx - (x^2 - 4x - \dots)))}{6(2f^2x^3 + 2f^2 - (x^3 - \dots))}\right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(fx^2 + (e+f)x - \sqrt{3}(fx+e) + e)\sqrt{x^3+1}}{x^5 + 2x^4 - 2x^3 + x^2 + 2x - 2}, x\right)$$

36.37 Problem number 128

$$\int \frac{e + fx}{(1 + \sqrt{3} - x) \sqrt{1 - x^3}} dx$$

Optimal antiderivative

$$\frac{(1-x) \operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3} + 2i\right) (e + f(1 - \sqrt{3})) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 + x + 1}{(1-x+\sqrt{3})^2}} 3^{\frac{1}{4}}}{3\sqrt{-x^3+1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}$$

$$\frac{\arctan\left(\frac{(1-x)\sqrt{3+2\sqrt{3}}}{\sqrt{-x^3+1}}\right) (e + f + f\sqrt{3})}{\sqrt{9+6\sqrt{3}}}$$

command

```
integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1}{12} \sqrt{-6fe - 2\sqrt{3}(f^2 - fe + e^2) + 3e^2} \log\left(-\frac{2f^2x^8 + 32f^2x^7 + 224f^2x^6 + 32f^2x^5 + 224f^2x^4 - 448f^2x^3 - \dots}{\dots} \right) \right.$$

$$\left. - \frac{1}{6} \sqrt{6fe + 2\sqrt{3}(f^2 - fe + e^2) - 3e^2} \arctan\left(\frac{(3fx^2 + 6fx - 6(x-1)e - \sqrt{3}(fx^2 - 2fx + (x^2 + 4x - 2)e))}{6(2f^2x^3 - 2f^2 - (x^3 - 1))} \right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(fx^2 + (e - f)x + \sqrt{3}(fx + e) - e)\sqrt{-x^3 + 1}}{x^5 - 2x^4 - 2x^3 - x^2 + 2x + 2}, x\right)$$

36.38 Problem number 129

$$\int \frac{e + fx}{(1 + \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{(1-x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) (e+f(1-\sqrt{3})) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}} 3^{\frac{1}{4}}}{3\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

$$\frac{\operatorname{arctanh}\left(\frac{(1-x)\sqrt{3+2\sqrt{3}}}{\sqrt{x^3-1}}\right) (e+f+f\sqrt{3})}{\sqrt{9+6\sqrt{3}}}$$

command

`integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1}{3} (\sqrt{3}(f+e) - 3f) \operatorname{weierstrassPInverse}(0, 4, x) \right.$$

$$+ \frac{1}{12} \sqrt{6fe + 2\sqrt{3}(f^2 - fe + e^2) - 3e^2} \log \left(-\frac{2f^2x^8 + 32f^2x^7 + 224f^2x^6 + 32f^2x^5 + 224f^2x^4 - 448f^2x^3 + \dots}{\dots} \right.$$

$$\left. - \frac{1}{6} \sqrt{-6fe - 2\sqrt{3}(f^2 - fe + e^2) + 3e^2} \operatorname{arctan} \left(\frac{(3fx^2 + 6fx - 6(x-1)e - \sqrt{3}(fx^2 - 2fx + (x^2 + 4x - 2)))}{6(2f^2x^3 - 2f^2 - (x^3 - \dots))} \right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{(fx^2 + (e-f)x + \sqrt{3}(fx+e) - e) \sqrt{x^3-1}}{x^5 - 2x^4 - 2x^3 - x^2 + 2x + 2}, x \right)$$

36.39 Problem number 130

$$\int \frac{e + fx}{(1 + \sqrt{3} + x) \sqrt{-1 - x^3}} dx$$

Optimal antiderivative

$$\frac{(1+x) \operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right) (e-f(1-\sqrt{3})) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}} 3^{\frac{1}{4}}}{3\sqrt{-x^3-1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}} + \frac{\operatorname{arctanh}\left(\frac{(1+x)\sqrt{3+2\sqrt{3}}}{\sqrt{-x^3-1}}\right) (e-f(1+\sqrt{3}))}{\sqrt{9+6\sqrt{3}}}$$

command

```
integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{1}{12} \sqrt{-6fe + 2\sqrt{3}(f^2 + fe + e^2) - 3e^2} \log\left(-\frac{2f^2x^8 - 32f^2x^7 + 224f^2x^6 - 32f^2x^5 + 224f^2x^4 + 448f^2x^3 - \dots}{\dots}\right) - \frac{1}{6} \sqrt{6fe - 2\sqrt{3}(f^2 + fe + e^2) + 3e^2} \operatorname{arctan}\left(-\frac{(3fx^2 - 6fx - 6(x+1)e - \sqrt{3}(fx^2 + 2fx - (x^2 - 4x - 2)))}{6(2f^2x^3 + 2f^2 - (x^3 + \dots))}\right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(fx^2 + (e+f)x - \sqrt{3}(fx+e) + e)\sqrt{-x^3-1}}{x^5 + 2x^4 - 2x^3 + x^2 + 2x - 2}, x\right)$$

36.40 Problem number 131

$$\int \frac{e + fx}{\left((1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{b} x \right) \sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh} \left(\frac{a^{\frac{1}{6}} (a^{\frac{1}{3}} + b^{\frac{1}{3}} x) \sqrt{-3 + 2\sqrt{3}}}{\sqrt{bx^3 + a}} \right) \left(b^{\frac{1}{3}} e - a^{\frac{1}{3}} f (1 - \sqrt{3}) \right)}{b^{\frac{2}{3}} \sqrt{a} \sqrt{-9 + 6\sqrt{3}}} \\ + \frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(b^{\frac{1}{3}} e - a^{\frac{1}{3}} f (1 + \sqrt{3}) \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}{3a^{\frac{1}{3}} b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} (a^{\frac{1}{3}} + b^{\frac{1}{3}} x)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}) \right)^2}}}$$

command

```
integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

36.41 Problem number 133

$$\int \frac{e + fx}{\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{b} x \right) \sqrt{-a + bx^3}} dx$$

Optimal antiderivative

$$\frac{\left(a^{\frac{1}{3}} - b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}, 2i - i\sqrt{3}\right) \left(b^{\frac{1}{3}}e + a^{\frac{1}{3}}f(1 + \sqrt{3})\right) \sqrt{\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)^2}} \left(\frac{\sqrt{6}}{2}\right) + \frac{3a^{\frac{1}{3}}b^{\frac{2}{3}}\sqrt{bx^3 - a} \sqrt{-\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} - b^{\frac{1}{3}}x)}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)^2}}}{\arctan\left(\frac{a^{\frac{1}{6}}(a^{\frac{1}{3}} - b^{\frac{1}{3}}x)\sqrt{-3 + 2\sqrt{3}}}{\sqrt{bx^3 - a}}\right) \left(b^{\frac{1}{3}}e + a^{\frac{1}{3}}f(1 - \sqrt{3})\right)}{b^{\frac{2}{3}}\sqrt{a} \sqrt{-9 + 6\sqrt{3}}}}$$

command

```
integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{bx^3 - a} \left(2 \left(2bfx^4 + 2bex^3 - 2afx - 2ae - \sqrt{3}(bfx^4 + bex^3 + 2afx + 2ae)\right)a^{\frac{2}{3}} + (bfx^5 + bex^4)\right)}{b^3x^9}\right)$$

36.42 Problem number 134

$$\int \frac{e + fx}{\left(\left(1 - \sqrt{3}\right) \sqrt[3]{a} + \sqrt[3]{b}x\right) \sqrt{-a - bx^3}} dx$$

Optimal antiderivative

$$\frac{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}, 2i - i\sqrt{3}\right) \left(b^{\frac{1}{3}}e - a^{\frac{1}{3}}f(1 + \sqrt{3})\right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)^2}} \left(\frac{\sqrt{6}}{2}\right) - \frac{3a^{\frac{1}{3}}b^{\frac{2}{3}}\sqrt{-bx^3 - a} \sqrt{-\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})\right)^2}}}{\arctan\left(\frac{a^{\frac{1}{6}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)\sqrt{-3 + 2\sqrt{3}}}{\sqrt{-bx^3 - a}}\right) \left(b^{\frac{1}{3}}e - a^{\frac{1}{3}}f(1 - \sqrt{3})\right)}{b^{\frac{2}{3}}\sqrt{a} \sqrt{-9 + 6\sqrt{3}}}}$$

command

```
integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{2 \left(2 b f x^4 + 2 b e x^3 + 2 a f x + 2 a e - \sqrt{3} (b f x^4 + b e x^3 - 2 a f x - 2 a e) \right) \sqrt{-b x^3 - a} a^{\frac{2}{3}} - (b f x^5 + b e x^4}{\dots} \right)$$

36.43 Problem number 135

$$\int \frac{x}{(1 + \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\frac{\arctan \left(\frac{(1+x)\sqrt{3+2\sqrt{3}}}{\sqrt{x^3+1}} \right) \sqrt{2} 3^{\frac{1}{4}}}{3} + \frac{(1+x) \text{EllipticF} \left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i \right) \sqrt{2} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{1}{4}}}{3\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{3} (\sqrt{3} - 3) \text{weierstrassPInverse}(0, -4, x) - \frac{1}{6} \cdot 3^{\frac{1}{4}} \sqrt{2} \arctan \left(-\frac{3^{\frac{1}{4}} \sqrt{2} (3x^2 - \sqrt{3}(x^2 + 2x + 4) - 6x)}{12\sqrt{x^3+1}} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{x^3+1} (x^2 - \sqrt{3}x + x)}{x^5 + 2x^4 - 2x^3 + x^2 + 2x - 2}, x \right)$$

36.44 Problem number 136

$$\int \frac{x}{(1 + \sqrt{3} - x) \sqrt{1 - x^3}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{(1-x)\sqrt{3+2\sqrt{3}}}{\sqrt{-x^3+1}}\right) \sqrt{2} 3^{\frac{1}{4}}}{3} + \frac{(1-x) \operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3} + 2i\right) \sqrt{2} \sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}} 3^{\frac{1}{4}}}{3\sqrt{-x^3+1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}$$

command

`integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{6} \cdot 3^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{3^{\frac{1}{4}} \sqrt{2} \sqrt{-x^3+1} (3x^2 - \sqrt{3}(x^2 - 2x + 4) + 6x)}{12(x^3 - 1)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{-x^3+1} (x^2 + \sqrt{3}x - x)}{x^5 - 2x^4 - 2x^3 - x^2 + 2x + 2}, x\right)$$

36.45 Problem number 137

$$\int \frac{x}{(1 + \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{(1-x)\sqrt{3+2\sqrt{3}}}{\sqrt{x^3-1}}\right)\sqrt{2}3^{\frac{1}{4}}}{3} + \frac{2(1-x)\operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right)\left(\frac{\sqrt{6}}{3}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}}3^{\frac{3}{4}}}{3\sqrt{x^3-1}\sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

command

`integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3}(\sqrt{3}-3)\operatorname{weierstrassPInverse}(0,4,x)+\frac{1}{12}\cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}(x^6+18x^5+12x^4+40x^3-36x^2+\sqrt{3})}{x^8-8}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{x^3-1}(x^2+\sqrt{3}x-x)}{x^5-2x^4-2x^3-x^2+2x+2},x\right)$$

36.46 Problem number 138

$$\int \frac{x}{(1+\sqrt{3}+x)\sqrt{-1-x^3}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{(1+x)\sqrt{3+2\sqrt{3}}}{\sqrt{-x^3-1}}\right)\sqrt{2}3^{\frac{1}{4}}}{3} + \frac{2(1+x)\operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i-i\sqrt{3}\right)\left(\frac{\sqrt{6}}{3}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x-\sqrt{3})^2}}3^{\frac{3}{4}}}{3\sqrt{-x^3-1}\sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

command

```
integrate(x/(1+x*3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{12}$$

$$\cdot 3^{\frac{1}{4}} \sqrt{2} \log \left(\frac{x^8 - 16x^7 + 112x^6 - 16x^5 + 112x^4 + 224x^3 + 2 \cdot 3^{\frac{1}{4}} \sqrt{2} (x^6 - 18x^5 + 12x^4 - 40x^3 - 36x^2 + \sqrt{3} x^8 + \dots}{x^8 + \dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{-x^3-1} (x^2 - \sqrt{3}x + x)}{x^5 + 2x^4 - 2x^3 + x^2 + 2x - 2}, x \right)$$

36.47 Problem number 139

$$\int \frac{x}{(1 - \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh} \left(\frac{(1+x)\sqrt{-3+2\sqrt{3}}}{\sqrt{x^3+1}} \right) \sqrt{2} 3^{\frac{1}{4}}}{3} \\ & + \frac{2(1+x) \operatorname{EllipticF} \left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{3} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate(x/(1+x*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} (\sqrt{3} + 3) \operatorname{weierstrassPInverse}(0, -4, x) + \frac{1}{12}$$

$$\cdot 3^{\frac{1}{4}} \sqrt{2} \log \left(\frac{x^8 - 16x^7 + 112x^6 - 16x^5 + 112x^4 + 224x^3 + 2 \cdot 3^{\frac{1}{4}} \sqrt{2} (x^6 - 18x^5 + 12x^4 - 40x^3 - 36x^2 - \sqrt{3} x^8 + 8x^8 + \dots)}{x^8 + 8x^8 + \dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{x^3+1} (x^2 + \sqrt{3} x + x)}{x^5 + 2x^4 - 2x^3 + x^2 + 2x - 2}, x \right)$$

36.48 Problem number 140

$$\int \frac{x}{\left((1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{b} x \right) \sqrt{a + bx^3}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh} \left(\frac{a^{\frac{1}{6}} (a^{\frac{1}{3}} + b^{\frac{1}{3}} x) \sqrt{-3 + 2\sqrt{3}}}{\sqrt{bx^3 + a}} \right) \sqrt{2} 3^{\frac{1}{4}}}{3a^{\frac{1}{6}} b^{\frac{2}{3}}} + \frac{2(a^{\frac{1}{3}} + b^{\frac{1}{3}} x) \operatorname{EllipticF} \left(\frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}, i\sqrt{3} + 2i \right) \left(\frac{\sqrt{6}}{3} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}))^2}}}{3b^{\frac{2}{3}} \sqrt{bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}} (a^{\frac{1}{3}} + b^{\frac{1}{3}} x)}{(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}))^2}}} 3^{\frac{3}{4}}$$

command

```
integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\sqrt{2} a^{\frac{1}{3}} b^{\frac{4}{3}} \sqrt{\frac{\sqrt{3}}{a}} \log \left(\frac{b^8 x^{24} - 1840 ab^7 x^{21} + 67264 a^2 b^6 x^{18} - 58624 a^3 b^5 x^{15} + 504064 a^4 b^4 x^{12} + 2140160 a^5 b^3 x^9 + 3100672 a^6 b^2 x^6 + 1089536 a^7 b x^3 + 1089536 a^8}{(b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3}))^2} \right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{bx^3 + a} \left(2 \left(2bx^4 + 2ax - \sqrt{3} (bx^4 - 2ax) \right) a^{\frac{2}{3}} - \left(bx^5 - 8ax^2 - \sqrt{3} (bx^5 + 4ax^2) \right) a^{\frac{1}{3}} b^{\frac{1}{3}} + (bx^6 + 6ax^3 - 6a^2) a^{\frac{2}{3}} \right)}{b^3 x^9 + 21 ab^2 x^6 + 12 a^2 b x^3 - 8 a^3}, x \right)$$

36.49 Problem number 141

$$\int \frac{x}{\left(\left(1 - \sqrt{3}\right) \sqrt[3]{a} - \sqrt[3]{b} x\right) \sqrt{a - bx^3}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{6}}(a^{\frac{1}{3}} - b^{\frac{1}{3}}x)\sqrt{-3 + 2\sqrt{3}}}{\sqrt{-bx^3 + a}}\right) \sqrt{2} 3^{\frac{1}{4}}}{3a^{\frac{1}{6}}b^{\frac{2}{3}}} + \frac{2\left(a^{\frac{1}{3}} - b^{\frac{1}{3}}x\right) \operatorname{EllipticF}\left(\frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{3} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}} 3^{\frac{3}{4}}}{3b^{\frac{2}{3}}\sqrt{-bx^3 + a} \sqrt{\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} - b^{\frac{1}{3}}x)}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})\right)^2}}}$$

command

`integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\sqrt{2} a^{\frac{1}{3}} b^{\frac{4}{3}} \sqrt{\frac{\sqrt{3}}{a}} \log\left(\frac{b^8 x^{24} + 1840 ab^7 x^{21} + 67264 a^2 b^6 x^{18} + 58624 a^3 b^5 x^{15} + 504064 a^4 b^4 x^{12} - 2140160 a^5 b^3 x^9 + 3100672 a^6 b^2 x^6 - 1089536 a^7 b x^3 + 1089536 a^8}{12(bx^3 - a)^2}\right) \right]$$

$$\sqrt{2} a^{\frac{1}{3}} b^{\frac{4}{3}} \sqrt{-\frac{\sqrt{3}}{a}} \operatorname{arctan}\left(\frac{2\sqrt{2}\sqrt{-bx^3 + a}(\sqrt{3}x - 3x)a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{-\frac{\sqrt{3}}{a}} - \sqrt{2}\sqrt{-bx^3 + a}(\sqrt{3}x^2 + 3x^2)a^{\frac{1}{3}}b^{\frac{2}{3}}\sqrt{-\frac{\sqrt{3}}{a}}}{12(bx^3 - a)}\right)$$

$6b^2$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{2 \left(2bx^4 - 2ax - \sqrt{3} (bx^4 + 2ax) \right) \sqrt{-bx^3 + a} a^{\frac{2}{3}} + \left(bx^5 + 8ax^2 - \sqrt{3} (bx^5 - 4ax^2) \right) \sqrt{-bx^3 + a} a^{\frac{1}{3}}}{b^3x^9 - 21ab^2x^6 + 12a^2bx^3 + 8a^3} \right)$$

36.50 Problem number 142

$$\int \frac{x}{\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{b} x \right) \sqrt{-a + bx^3}} dx$$

Optimal antiderivative

$$\frac{\arctan \left(\frac{a^{\frac{1}{6}} (a^{\frac{1}{3}} - b^{\frac{1}{3}} x) \sqrt{-3 + 2\sqrt{3}}}{\sqrt{bx^3 - a}} \right) \sqrt{2} 3^{\frac{1}{4}}}{3a^{\frac{1}{6}} b^{\frac{2}{3}}} + \frac{\left(a^{\frac{1}{3}} - b^{\frac{1}{3}} x \right) \text{EllipticF} \left(\frac{-b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 + \sqrt{3})}{-b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3})}, 2i - i\sqrt{3} \right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2}{\left(-b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3}) \right)^2}} 3^{\frac{1}{4}}}{3b^{\frac{2}{3}} \sqrt{bx^3 - a} \sqrt{-\frac{a^{\frac{1}{3}} (a^{\frac{1}{3}} - b^{\frac{1}{3}} x)}{\left(-b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1 - \sqrt{3}) \right)^2}}}$$

command

```
integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\sqrt{2} a^{\frac{1}{3}} b^{\frac{4}{3}} \sqrt{-\frac{\sqrt{3}}{a}} \log \left(\frac{b^8 x^{24} + 1840 ab^7 x^{21} + 67264 a^2 b^6 x^{18} + 58624 a^3 b^5 x^{15} + 504064 a^4 b^4 x^{12} - 2140160 a^5 b^3 x^9 + 3100672 a^6 b^2 x^6 - 1089536 a^7 b x^3 + 1024 a^8}{6 b^2} \right) \right.$$

$$\left. \sqrt{2} a^{\frac{1}{3}} b^{\frac{4}{3}} \sqrt{\frac{\sqrt{3}}{a}} \arctan \left(-\frac{\sqrt{2} \left(2 \left(\sqrt{3} x - 3x \right) a^{\frac{2}{3}} b^{\frac{1}{3}} - \left(\sqrt{3} x^2 + 3x^2 \right) a^{\frac{1}{3}} b^{\frac{2}{3}} - 4 \sqrt{3} a \right) \sqrt{\frac{\sqrt{3}}{a}}}{12 \sqrt{bx^3 - a}} \right) + 2 b^{\frac{7}{6}} \left(\sqrt{3} + 3 \right) \operatorname{weier} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{bx^3 - a} \left(2 \left(2bx^4 - 2ax - \sqrt{3} (bx^4 + 2ax) \right) a^{\frac{2}{3}} + \left(bx^5 + 8ax^2 - \sqrt{3} (bx^5 - 4ax^2) \right) a^{\frac{1}{3}} b^{\frac{1}{3}} + (bx^6 - \dots) \right)}{b^3 x^9 - 21 ab^2 x^6 + 12 a^2 b x^3 + 8 a^3} \right)$$

36.51 Problem number 143

$$\int \frac{x}{\left((1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{b} x \right) \sqrt{-a - bx^3}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{a^{\frac{1}{6}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)\sqrt{-3 + 2\sqrt{3}}}{\sqrt{-bx^3 - a}}\right) \sqrt{2} 3^{\frac{1}{4}}}{3a^{\frac{1}{6}}b^{\frac{2}{3}}} + \frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}}x) \operatorname{EllipticF}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 + \sqrt{3})}{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3})}, 2i - i\sqrt{3}\right) \sqrt{2} \sqrt{\frac{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))^2}} 3^{\frac{1}{4}}}{3b^{\frac{2}{3}}\sqrt{-bx^3 - a} \sqrt{-\frac{a^{\frac{1}{3}}(a^{\frac{1}{3}} + b^{\frac{1}{3}}x)}{(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1 - \sqrt{3}))^2}}}$$

command

```
integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\sqrt{2} a^{\frac{1}{3}} b^{\frac{4}{3}} \sqrt{-\frac{\sqrt{3}}{a}} \log\left(\frac{b^8 x^{24} - 1840 ab^7 x^{21} + 67264 a^2 b^6 x^{18} - 58624 a^3 b^5 x^{15} + 504064 a^4 b^4 x^{12} + 2140160 a^5 b^3 x^9 + 3100672 a^6 b^2 x^6 + 1089536 a^7 b x^3 + 1089536 a^8}{\dots} \right) \right]$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{2(2bx^4 + 2ax - \sqrt{3}(bx^4 - 2ax))\sqrt{-bx^3 - a} a^{\frac{2}{3}} - (bx^5 - 8ax^2 - \sqrt{3}(bx^5 + 4ax^2))\sqrt{-bx^3 - a} a}{b^3 x^9 + 21 ab^2 x^6 + 12 a^2 b x^3 - 8 a^3} \right)$$

36.52 Problem number 152

$$\int \frac{1 + \sqrt{3} + x}{x \sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}(\sqrt{x^3 + 1})(1 + \sqrt{3})}{3} + \frac{2(1 + x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3 + 1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} (\sqrt{3} + 1) \log\left(\frac{x^3 - 2\sqrt{x^3 + 1} + 2}{x^3}\right) + 2 \operatorname{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{x^3 + 1}(x + \sqrt{3} + 1)}{x^4 + x}, x\right)$$

36.53 Problem number 153

$$\int \frac{1 + \sqrt{3} - x}{x \sqrt{1 - x^3}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}(\sqrt{-x^3 + 1})(1 + \sqrt{3})}{3} + \frac{2(1 - x) \operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 + x + 1}{(1-x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{-x^3 + 1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}}$$

command

`integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \left(\sqrt{3} + 1 \right) \log \left(-\frac{x^3 + 2\sqrt{-x^3 + 1} - 2}{x^3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{-x^3 + 1} (x - \sqrt{3} - 1)}{x^4 - x}, x \right)$$

36.54 Problem number 154

$$\int \frac{1 + \sqrt{3} - x}{x \sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left(\sqrt{x^3 - 1} \right) (1 + \sqrt{3})}{3} + \frac{2(1-x) \text{EllipticF} \left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i - i\sqrt{3} \right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \sqrt{\frac{x^2 + x + 1}{(1-x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}}$$

command

`integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{2\sqrt{3} + 4} \arctan \left(-\frac{(x^3 - \sqrt{3}(x^3 - 2) - 2)\sqrt{2\sqrt{3} + 4}}{4\sqrt{x^3 - 1}} \right) - 2 \text{weierstrassPInverse}(0, 4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{x^3 - 1} (x - \sqrt{3} - 1)}{x^4 - x}, x \right)$$

36.55 Problem number 155

$$\int \frac{1 + \sqrt{3} + x}{x \sqrt{-1 - x^3}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\sqrt{-x^3 - 1}\right) (1 + \sqrt{3})}{3} + \frac{2(1+x) \operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i - i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{-x^3-1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

command

`integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{2\sqrt{3} + 4} \arctan\left(-\frac{(x^3 - \sqrt{3}(x^3 + 2) + 2)\sqrt{-x^3 - 1} \sqrt{2\sqrt{3} + 4}}{4(x^3 + 1)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3 - 1} (x + \sqrt{3} + 1)}{x^4 + x}, x\right)$$

36.56 Problem number 156

$$\int \frac{1 - \sqrt{3} + x}{x \sqrt{1 + x^3}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh}\left(\sqrt{x^3 + 1}\right) (1 - \sqrt{3})}{3} + \frac{2(1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

```
integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} (\sqrt{3} - 1) \log \left(\frac{x^3 + 2 \sqrt{x^3 + 1} + 2}{x^3} \right) + 2 \text{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{x^3 + 1} (x - \sqrt{3} + 1)}{x^4 + x}, x \right)$$

36.57 Problem number 157

$$\int \frac{1 - \sqrt{3} - x}{x \sqrt{1 - x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2 \operatorname{arctanh}(\sqrt{-x^3 + 1}) (1 - \sqrt{3})}{3} \\ & + \frac{2(1 - x) \operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 + x + 1}{(1-x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{-x^3 + 1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} (\sqrt{3} - 1) \log \left(-\frac{x^3 - 2 \sqrt{-x^3 + 1} - 2}{x^3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{-x^3 + 1} (x + \sqrt{3} - 1)}{x^4 - x}, x \right)$$

36.58 Problem number 158

$$\int \frac{1 - \sqrt{3} - x}{x \sqrt{-1 + x^3}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\sqrt{x^3 - 1}\right) (1 - \sqrt{3})}{3} + \frac{2(1 - x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i - i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 + x + 1}{(1 - x - \sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3 - 1} \sqrt{\frac{-1 + x}{(1 - x - \sqrt{3})^2}}}$$

command

`integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{3} \sqrt{-2\sqrt{3} + 4} \arctan\left(\frac{(x^3 + \sqrt{3}(x^3 - 2) - 2)\sqrt{-2\sqrt{3} + 4}}{4\sqrt{x^3 - 1}}\right) - 2 \operatorname{weierstrassPInverse}(0, 4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{x^3 - 1}(x + \sqrt{3} - 1)}{x^4 - x}, x\right)$$

36.59 Problem number 159

$$\int \frac{1 - \sqrt{3} + x}{x \sqrt{-1 - x^3}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\sqrt{-x^3 - 1}\right) (1 - \sqrt{3})}{3} + \frac{2(1 + x) \operatorname{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i - i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1 + x - \sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{-x^3 - 1} \sqrt{\frac{-1 - x}{(1 + x - \sqrt{3})^2}}}$$

command

```
integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{3} \sqrt{-2\sqrt{3}+4} \arctan\left(\frac{(x^3 + \sqrt{3}(x^3 + 2) + 2) \sqrt{-x^3 - 1} \sqrt{-2\sqrt{3} + 4}}{4(x^3 + 1)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^3 - 1} (x - \sqrt{3} + 1)}{x^4 + x}, x\right)$$

36.60 Problem number 168

$$\int \frac{e + fx}{x\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2e \operatorname{arctanh}\left(\sqrt{x^3 + 1}\right)}{3} \\ & + \frac{2f(1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3} + 2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{x^3 + 1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} e \log\left(\frac{x^3 - 2\sqrt{x^3 + 1} + 2}{x^3}\right) + 2f \operatorname{weierstrassPInverse}(0, -4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3 + 1} (fx + e)}{x^4 + x}, x\right)$$

36.61 Problem number 169

$$\int \frac{e + fx}{x\sqrt{1-x^3}} dx$$

Optimal antiderivative

$$\frac{2e \operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{3} - \frac{2f(1-x) \operatorname{EllipticF}\left(\frac{1-x-\sqrt{3}}{1-x+\sqrt{3}}, i\sqrt{3}+2i\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x+\sqrt{3})^2}}}{3\sqrt{-x^3+1} \sqrt{\frac{1-x}{(1-x+\sqrt{3})^2}}} 3^{\frac{3}{4}}$$

command

`integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} e \log\left(-\frac{x^3 + 2\sqrt{-x^3+1} - 2}{x^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3+1}(fx+e)}{x^4-x}, x\right)$$

36.62 Problem number 170

$$\int \frac{e + fx}{x\sqrt{-1+x^3}} dx$$

Optimal antiderivative

$$\frac{2e \operatorname{arctan}\left(\sqrt{x^3-1}\right)}{3} - \frac{2f(1-x) \operatorname{EllipticF}\left(\frac{1-x+\sqrt{3}}{1-x-\sqrt{3}}, 2i-i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2+x+1}{(1-x-\sqrt{3})^2}}}{3\sqrt{x^3-1} \sqrt{\frac{-1+x}{(1-x-\sqrt{3})^2}}} 3^{\frac{3}{4}}$$

command

```
integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \arctan\left(\frac{x^3 - 2}{2\sqrt{x^3 - 1}}\right) e + 2 f \text{weierstrassPInverse}(0, 4, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{x^3 - 1} (fx + e)}{x^4 - x}, x\right)$$

36.63 Problem number 171

$$\int \frac{e + fx}{x \sqrt{-1 - x^3}} dx$$

Optimal antiderivative

$$\frac{2e \arctan\left(\sqrt{-x^3 - 1}\right)}{3} + \frac{2f(1+x) \text{EllipticF}\left(\frac{1+x+\sqrt{3}}{1+x-\sqrt{3}}, 2i - i\sqrt{3}\right) \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{(1+x-\sqrt{3})^2}} 3^{\frac{3}{4}}}{3\sqrt{-x^3 - 1} \sqrt{\frac{-1-x}{(1+x-\sqrt{3})^2}}}$$

command

```
integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \arctan\left(\frac{(x^3 + 2)\sqrt{-x^3 - 1}}{2(x^3 + 1)}\right) e$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^3 - 1} (fx + e)}{x^4 + x}, x\right)$$

36.64 Problem number 208

$$\int (d + ex) \sqrt{a + cx^4} dx$$

Optimal antiderivative

$$\frac{ae \operatorname{arctanh}\left(\frac{x^2\sqrt{c}}{\sqrt{cx^4+a}}\right)}{4\sqrt{c}} + \frac{dx\sqrt{cx^4+a}}{3} + \frac{ex^2\sqrt{cx^4+a}}{4}$$

$$+ \frac{a^{\frac{3}{4}}d\sqrt{\frac{\cos\left(4\operatorname{arctan}\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\operatorname{arctan}\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4+a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{3\cos\left(2\operatorname{arctan}\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) c^{\frac{1}{4}}\sqrt{cx^4+a}}$$

command

`integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{16c^{\frac{3}{2}}d\left(-\frac{a}{c}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}, -1\right) + 3a\sqrt{c}e \log\left(-2cx^4 - 2\sqrt{cx^4+a}\sqrt{c}x^2 - a\right) + 2\sqrt{cx^4+a}(3cx^2e + 4c)}{24c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{cx^4+a}(ex+d), x\right)$$

36.65 Problem number 209

$$\int \sqrt{a + cx^4} dx$$

Optimal antiderivative

$$\frac{x\sqrt{cx^4+a}}{3}$$

$$+ \frac{a^{\frac{3}{4}}\sqrt{\frac{\cos\left(4\operatorname{arctan}\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\operatorname{arctan}\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4+a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{3\cos\left(2\operatorname{arctan}\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) c^{\frac{1}{4}}\sqrt{cx^4+a}}$$

command

```
integrate((c*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2}{3} \sqrt{c} \left(-\frac{a}{c}\right)^{\frac{3}{4}} \text{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}, -1\right) + \frac{1}{3} \sqrt{cx^4 + a} x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{cx^4 + a}, x\right)$$

36.66 Problem number 214

$$\int \frac{d + ex}{\sqrt{a + cx^4}} dx$$

Optimal antiderivative

$$\frac{e \operatorname{arctanh}\left(\frac{x^2 \sqrt{c}}{\sqrt{cx^4 + a}}\right)}{2\sqrt{c}} + \frac{d \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}} x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}} x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2 \sqrt{c}) \sqrt{\frac{cx^4 + a}{(\sqrt{a} + x^2 \sqrt{c})^2}}}{2 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}} x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{cx^4 + a}}$$

command

```
integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 c^{\frac{3}{2}} d \left(-\frac{a}{c}\right)^{\frac{3}{4}} \text{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}, -1\right) + a \sqrt{c} e \log\left(-2 cx^4 - 2 \sqrt{cx^4 + a} \sqrt{c} x^2 - a\right)}{4 ac}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{ex + d}{\sqrt{cx^4 + a}}, x\right)$$

36.67 Problem number 215

$$\int \frac{1}{\sqrt{a + cx^4}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4 + a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{2 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{cx^4 + a}}$$

command

`integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{a} \left(-\frac{c}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x \left(-\frac{c}{a}\right)^{\frac{1}{4}}, -1\right)}{c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{cx^4 + a}}, x\right)$$

36.68 Problem number 221

$$\int \frac{d + ex}{(a + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x(ex + d)}{2a\sqrt{cx^4 + a}} + \frac{d \sqrt{\frac{\cos\left(4 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4 + a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{4 \cos\left(2 \arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{5}{4}} c^{\frac{1}{4}} \sqrt{cx^4 + a}}$$

command

```
integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(cdx^4 + ad)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}, -1\right) - \sqrt{cx^4 + a}(cx^2e + cdx)}{2(ac^2x^4 + a^2c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + a}(ex + d)}{c^2x^8 + 2acx^4 + a^2}, x\right)$$

36.69 Problem number 222

$$\int \frac{1}{(a + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x}{2a\sqrt{cx^4 + a}} + \frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{a} + x^2\sqrt{c}) \sqrt{\frac{cx^4 + a}{(\sqrt{a} + x^2\sqrt{c})^2}}}{4\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right) a^{\frac{5}{4}} c^{\frac{1}{4}} \sqrt{cx^4 + a}}$$

command

```
integrate(1/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(cx^4 + a)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}, -1\right) - \sqrt{cx^4 + a}cx}{2(ac^2x^4 + a^2c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + a}}{c^2x^8 + 2acx^4 + a^2}, x\right)$$

36.70 Problem number 260

$$\int \sqrt{(1-x^2)(3+x^2)} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{EllipticE}\left(x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3} + \frac{4 \operatorname{EllipticF}\left(x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3} + \frac{x \sqrt{-x^4 - 2x^2 + 3}}{3}$$

command

```
integrate((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^4 - 2x^2 + 3} (x^2 + 2)}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{-x^4 - 2x^2 + 3}, x\right)$$

36.71 Problem number 262

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{EllipticF}\left(x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3}$$

command

```
integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} \sqrt{3} \operatorname{ellipticF}\left(x, -\frac{1}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-x^4 - 2x^2 + 3}}{x^4 + 2x^2 - 3}, x\right)$$

36.72 Problem number 378

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{ax^3} \sqrt{x^2+1}}{3x} - \frac{(1+x) \sqrt{\frac{\cos(4 \arctan(\sqrt{x}))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \arctan(\sqrt{x})), \frac{\sqrt{2}}{2}\right) \sqrt{ax^3} \sqrt{\frac{x^2+1}{(1+x)^2}}}{3 \cos(2 \arctan(\sqrt{x})) x^{\frac{3}{2}} \sqrt{x^2+1}}$$

command

`integrate((a*x^3)^(1/2)/(x^2+1)^(1/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{a} \operatorname{xweierstrassPInverse}(-4, 0, x) - \sqrt{ax^3} \sqrt{x^2+1} \right)}{3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{ax^3}}{\sqrt{x^2+1}}, x\right)$$

36.73 Problem number 380

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{ax} \sqrt{x^2+1}}{1+x} - \frac{2(1+x) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{a} \sqrt{\frac{x^2+1}{(1+x)^2}}}{\cos\left(2 \arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right) \sqrt{x^2+1}} + \frac{(1+x) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{a} \sqrt{\frac{x^2+1}{(1+x)^2}}}{\cos\left(2 \arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right) \sqrt{x^2+1}}$$

command

```
integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2\sqrt{a} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{ax}}{\sqrt{x^2+1}}, x\right)$$

36.74 Problem number 381

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} dx$$

Optimal antiderivative

$$\frac{(1+x) \sqrt{\frac{\cos(4 \arctan(\sqrt{x}))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \arctan(\sqrt{x})), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a}{x}} \sqrt{x} \sqrt{\frac{x^2+1}{(1+x)^2}}}{\cos(2 \arctan(\sqrt{x})) \sqrt{x^2+1}}$$

command

```
integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\sqrt{a} \operatorname{weierstrassPInverse}(-4, 0, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}, x\right)$$

36.75 Problem number 383

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -2x \sqrt{\frac{a}{x^3}} \sqrt{x^2+1} + \frac{2x^2 \sqrt{\frac{a}{x^3}} \sqrt{x^2+1}}{1+x} \\
 & - \frac{2x^{\frac{3}{2}}(1+x) \sqrt{\frac{\cos(4 \arctan(\sqrt{x}))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin(2 \arctan(\sqrt{x})), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a}{x^3}} \sqrt{\frac{x^2+1}{(1+x)^2}}}{\cos(2 \arctan(\sqrt{x})) \sqrt{x^2+1}} \\
 & + \frac{x^{\frac{3}{2}}(1+x) \sqrt{\frac{\cos(4 \arctan(\sqrt{x}))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \arctan(\sqrt{x})), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a}{x^3}} \sqrt{\frac{x^2+1}{(1+x)^2}}}{\cos(2 \arctan(\sqrt{x})) \sqrt{x^2+1}}
 \end{aligned}$$

command

```
integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2 \sqrt{x^2+1} x \sqrt{\frac{a}{x^3}} - 2 \sqrt{a} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, x))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}, x\right)$$

36.76 Problem number 387

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{ax^2} \sqrt{x^3+1}}{x(1+x+\sqrt{3})} + \frac{2(1+x) \operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{2} \sqrt{ax^2} \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}} 3^{\frac{3}{4}}}{3x\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} - \frac{3^{\frac{1}{4}}(1+x) \operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right) \sqrt{ax^2} \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{x\sqrt{x^3+1} \sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

command

`integrate((a*x^2)^(1/2)/(x^3+1)^(1/2), x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{ax^2} \operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x))}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{ax^2}}{\sqrt{x^3+1}}, x\right)$$

36.77 Problem number 389

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\frac{x(1+x) \sqrt{\frac{(1+x(1-\sqrt{3}))^2}{(1+x(1+\sqrt{3}))^2}} (1+x(1+\sqrt{3})) \operatorname{EllipticF}\left(\sqrt{1-\frac{(1+x(1-\sqrt{3}))^2}{(1+x(1+\sqrt{3}))^2}}, \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right) \sqrt{\frac{a}{x}}}{3(1+x(1-\sqrt{3})) \sqrt{x^3+1} \sqrt{\frac{x(1+x)}{(1+x(1+\sqrt{3}))^2}}}$$

command

```
integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2\sqrt{a} \operatorname{weierstrassPInverse}\left(0, -4, \frac{1}{x}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}, x\right)$$

36.78 Problem number 391

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -2x\sqrt{\frac{a}{x^3}}\sqrt{x^3+1} + \frac{2x^2(1+\sqrt{3})\sqrt{\frac{a}{x^3}}\sqrt{x^3+1}}{1+x(1+\sqrt{3})} \\ & \frac{2\sqrt[4]{3}x^2(1+x)\sqrt{\frac{(1+x(1-\sqrt{3}))^2}{(1+x(1+\sqrt{3}))^2}}(1+x(1+\sqrt{3}))\operatorname{EllipticE}\left(\sqrt{1-\frac{(1+x(1-\sqrt{3}))^2}{(1+x(1+\sqrt{3}))^2}}, \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right)}{(1+x(1-\sqrt{3}))\sqrt{x^3+1}\sqrt{\frac{x(1+x)}{(1+x(1+\sqrt{3}))^2}}} \\ & \frac{x^2(1+x)\sqrt{\frac{(1+x(1-\sqrt{3}))^2}{(1+x(1+\sqrt{3}))^2}}(1+x(1+\sqrt{3}))\operatorname{EllipticF}\left(\sqrt{1-\frac{(1+x(1-\sqrt{3}))^2}{(1+x(1+\sqrt{3}))^2}}, \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right)}{3(1+x(1-\sqrt{3}))\sqrt{x^3+1}\sqrt{\frac{x(1+x)}{(1+x(1+\sqrt{3}))^2}}} \end{aligned}$$

command

```
integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\sqrt{a} \operatorname{weierstrassZeta}\left(0, -4, \operatorname{weierstrassPInverse}\left(0, -4, \frac{1}{x}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}, x\right)$$

36.79 Problem number 392

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & -x\sqrt{\frac{a}{x^4}}\sqrt{x^3+1} + \frac{x^2\sqrt{\frac{a}{x^4}}\sqrt{x^3+1}}{1+x+\sqrt{3}} \\ & \frac{x^2(1+x)\operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\sqrt{2}\sqrt{\frac{a}{x^4}}\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}3^{\frac{3}{4}}}{3\sqrt{x^3+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \\ & - \frac{3^{\frac{1}{4}}x^2(1+x)\operatorname{EllipticE}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, i\sqrt{3}+2i\right)\sqrt{\frac{a}{x^4}}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}}{2\sqrt{x^3+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}} \end{aligned}$$

command

```
integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-x^2\sqrt{\frac{a}{x^4}}\operatorname{weierstrassZeta}(0, -4, \operatorname{weierstrassPInverse}(0, -4, x)) - \sqrt{x^3+1}x\sqrt{\frac{a}{x^4}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}, x\right)$$

36.80 Problem number 397

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex} \sqrt{e+fx}} dx$$

Optimal antiderivative

$$\frac{2 \text{EllipticE}\left(\frac{\sqrt{f} \sqrt{ex+d}}{\sqrt{df-e^2}}, \sqrt{1-\frac{e^2}{df}}\right) \sqrt{df-e^2} \sqrt{ax} \sqrt{\frac{e(fx+e)}{-df+e^2}}}{e \sqrt{f} \sqrt{-\frac{ex}{d}} \sqrt{fx+e}}$$

command

`integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{af} (df + e^2) e^{\frac{1}{2}} \text{weierstrassPInverse}\left(\frac{4(d^2 f^2 - df e^2 + e^4) e^{(-2)}}{3 f^2}, -\frac{4(2 d^3 f^3 - 3 d^2 f^2 e^2 - 3 df e^4 + 2 e^6) e^{(-3)}}{27 f^3}, \frac{(3 f x e + df + e^2) e^{(-1)}}{3 f}\right) \right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{ax} \sqrt{ex+d} \sqrt{fx+e}}{efx^2+de+(e^2+df)x}, x\right)$$

36.81 Problem number 601

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(dx+c)^{\frac{3}{2}}(ax^2+b)}{5ax\sqrt{a+\frac{b}{x^2}}} + \frac{2c(ax^2+b)\sqrt{dx+c}}{5ax\sqrt{a+\frac{b}{x^2}}} \\
& + \frac{2(ac^2-3bd^2)\text{EllipticE}\left(\frac{\sqrt{1-\frac{x\sqrt{-a}}{\sqrt{b}}}\sqrt{2}}{2}, \sqrt{-\frac{2d\sqrt{-a}\sqrt{b}}{ac-d\sqrt{-a}\sqrt{b}}}\right)\sqrt{b}\sqrt{dx+c}\sqrt{1+\frac{ax^2}{b}}}{5(-a)^{\frac{3}{2}}dx\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{a(dx+c)}{ac-d\sqrt{-a}\sqrt{b}}}} \\
& + \frac{2c(ac^2+bd^2)\text{EllipticF}\left(\frac{\sqrt{1-\frac{x\sqrt{-a}}{\sqrt{b}}}\sqrt{2}}{2}, \sqrt{-\frac{2d\sqrt{-a}\sqrt{b}}{ac-d\sqrt{-a}\sqrt{b}}}\right)\sqrt{b}\sqrt{1+\frac{ax^2}{b}}\sqrt{\frac{a(dx+c)}{ac-d\sqrt{-a}\sqrt{b}}}}{5(-a)^{\frac{3}{2}}dx\sqrt{a+\frac{b}{x^2}}\sqrt{dx+c}}
\end{aligned}$$

command

```
integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((ac^3 + 9bcd^2)\sqrt{ad} \text{weierstrassPInverse}\left(\frac{4(ac^2-3bd^2)}{3ad^2}, -\frac{8(ac^3+9bcd^2)}{27ad^3}, \frac{3dx+c}{3d}\right) + 3(ac^2d - 3bd^3)\sqrt{ad} \text{weierstrassP}\left(\frac{4(ac^2-3bd^2)}{3ad^2}, -\frac{8(ac^3+9bcd^2)}{27ad^3}, \frac{3dx+c}{3d}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(dx^3+cx^2)\sqrt{dx+c}\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}, x\right)$$

36.82 Problem number 764

$$\int (8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3 - 2(-1 + x)^2 - (-1 + x)^4)^{\frac{3}{2}} (-1 + x)}{7} \\ & - \frac{16 \operatorname{EllipticE}\left(-1 + x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{5} + \frac{176 \operatorname{EllipticF}\left(-1 + x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{35} \\ & + \frac{2(13 - 3(-1 + x)^2)(-1 + x) \sqrt{3 - 2(-1 + x)^2 - (-1 + x)^4}}{35} \end{aligned}$$

command

```
integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(5x^6 - 30x^5 + 91x^4 - 164x^3 + 130x^2 - 12x - 132) \sqrt{-x^4 + 4x^3 - 8x^2 + 8x}}{35(x - 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((-x^4 + 4x^3 - 8x^2 + 8x)^{\frac{3}{2}}, x\right)$$

36.83 Problem number 765

$$\int \sqrt{8x - 8x^2 + 4x^3 - x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \operatorname{EllipticE}\left(-1 + x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3} + \frac{4 \operatorname{EllipticF}\left(-1 + x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3} \\ & + \frac{(-1 + x) \sqrt{3 - 2(-1 + x)^2 - (-1 + x)^4}}{3} \end{aligned}$$

command

```
integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^4 + 4x^3 - 8x^2 + 8x} (x^2 - 2x + 3)}{3(x - 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{-x^4 + 4x^3 - 8x^2 + 8x}, x\right)$$

36.84 Problem number 766

$$\int \frac{1}{\sqrt{8x - 8x^2 + 4x^3 - x^4}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(-1 + x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3}$$

command

`integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2} \sqrt{2} \text{weierstrassPInverse}\left(-\frac{2}{3}, \frac{7}{54}, -\frac{x-3}{3x}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4 + 4x^3 - 8x^2 + 8x}}{x^4 - 4x^3 + 8x^2 - 8x}, x\right)$$

36.85 Problem number 767

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\text{EllipticE}\left(-1 + x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{24} + \frac{\text{EllipticF}\left(-1 + x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{12} + \frac{\left(5 + (-1 + x)^2\right) (-1 + x)}{24\sqrt{3 - 2(-1 + x)^2 - (-1 + x)^4}}$$

command

```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(x^4 - 4x^3 + 8x^2 - 8x)\text{weierstrassPInverse}\left(-\frac{2}{3}, \frac{7}{54}, -\frac{x-3}{3x}\right) - 6\sqrt{2}(x^4 - 4x^3 + 8x^2 - 8x)\text{weierstrassZeta}}{72(x^4 - 4x^3 + 8x^2 - 8x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^4 + 4x^3 - 8x^2 + 8x}}{x^8 - 8x^7 + 32x^6 - 80x^5 + 128x^4 - 128x^3 + 64x^2}, x\right)$$

36.86 Problem number 768

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5 + (-1 + x)^2)(-1 + x)}{72(3 - 2(-1 + x)^2 - (-1 + x)^4)^{\frac{3}{2}}} - \frac{7\text{EllipticE}\left(-1 + x, \frac{i\sqrt{3}}{3}\right)\sqrt{3}}{432} \\ & + \frac{11\text{EllipticF}\left(-1 + x, \frac{i\sqrt{3}}{3}\right)\sqrt{3}}{432} + \frac{(26 + 7(-1 + x)^2)(-1 + x)}{432\sqrt{3 - 2(-1 + x)^2 - (-1 + x)^4}} \end{aligned}$$

command

```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{43\sqrt{2}(x^8 - 8x^7 + 32x^6 - 80x^5 + 128x^4 - 128x^3 + 64x^2)\text{weierstrassPInverse}\left(-\frac{2}{3}, \frac{7}{54}, -\frac{x-3}{3x}\right) - 84\sqrt{2}(x^8 - 8x^7 + 32x^6 - 80x^5 + 128x^4 - 128x^3 + 64x^2)\text{weierstrassZeta}}{72(x^8 - 8x^7 + 32x^6 - 80x^5 + 128x^4 - 128x^3 + 64x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4 + 4x^3 - 8x^2 + 8x}}{x^{12} - 12x^{11} + 72x^{10} - 280x^9 + 768x^8 - 1536x^7 + 2240x^6 - 2304x^5 + 1536x^4 - 512x^3}, x\right)$$

36.87 Problem number 769

$$\int ((2-x)x(4-2x+x^2))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3-2(-1+x)^2-(-1+x)^4)^{\frac{3}{2}}(-1+x)}{7} \\ & - \frac{16 \operatorname{EllipticE}\left(-1+x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{5} + \frac{176 \operatorname{EllipticF}\left(-1+x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{35} \\ & + \frac{2(13-3(-1+x)^2)(-1+x) \sqrt{3-2(-1+x)^2-(-1+x)^4}}{35} \end{aligned}$$

command

```
integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{(5x^6 - 30x^5 + 91x^4 - 164x^3 + 130x^2 - 12x - 132) \sqrt{-x^4 + 4x^3 - 8x^2 + 8x}}{35(x-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((-x^4 + 4x^3 - 8x^2 + 8x)^{\frac{3}{2}}, x\right)$$

36.88 Problem number 770

$$\int \sqrt{(2-x)x(4-2x+x^2)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \operatorname{EllipticE}\left(-1+x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3} + \frac{4 \operatorname{EllipticF}\left(-1+x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3} \\ & + \frac{(-1+x) \sqrt{3-2(-1+x)^2-(-1+x)^4}}{3} \end{aligned}$$

command

```
integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-x^4 + 4x^3 - 8x^2 + 8x} (x^2 - 2x + 3)}{3(x - 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{-x^4 + 4x^3 - 8x^2 + 8x}, x\right)$$

36.89 Problem number 771

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}} dx$$

Optimal antiderivative

$$\frac{\text{EllipticF}\left(-1 + x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{3}$$

command

```
integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2} \sqrt{2} \text{weierstrassPInverse}\left(-\frac{2}{3}, \frac{7}{54}, -\frac{x-3}{3x}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4 + 4x^3 - 8x^2 + 8x}}{x^4 - 4x^3 + 8x^2 - 8x}, x\right)$$

36.90 Problem number 772

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\text{EllipticE}\left(-1 + x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{24} + \frac{\text{EllipticF}\left(-1 + x, \frac{i\sqrt{3}}{3}\right) \sqrt{3}}{12} \\ & + \frac{\left(5 + (-1 + x)^2\right) (-1 + x)}{24\sqrt{3 - 2(-1 + x)^2 - (-1 + x)^4}} \end{aligned}$$

command

```
integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(x^4 - 4x^3 + 8x^2 - 8x)\text{weierstrassPInverse}\left(-\frac{2}{3}, \frac{7}{54}, -\frac{x-3}{3x}\right) - 6\sqrt{2}(x^4 - 4x^3 + 8x^2 - 8x)\text{weierstrassZeta}}{72(x^4 - 4x^3 + 8x^2 - 8x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{-x^4 + 4x^3 - 8x^2 + 8x}}{x^8 - 8x^7 + 32x^6 - 80x^5 + 128x^4 - 128x^3 + 64x^2}, x\right)$$

36.91 Problem number 773

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5 + (-1 + x)^2)(-1 + x)}{72(3 - 2(-1 + x)^2 - (-1 + x)^4)^{\frac{3}{2}}} - \frac{7\text{EllipticE}\left(-1 + x, \frac{i\sqrt{3}}{3}\right)\sqrt{3}}{432} \\ & + \frac{11\text{EllipticF}\left(-1 + x, \frac{i\sqrt{3}}{3}\right)\sqrt{3}}{432} + \frac{(26 + 7(-1 + x)^2)(-1 + x)}{432\sqrt{3 - 2(-1 + x)^2 - (-1 + x)^4}} \end{aligned}$$

command

```
integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{43\sqrt{2}(x^8 - 8x^7 + 32x^6 - 80x^5 + 128x^4 - 128x^3 + 64x^2)\text{weierstrassPInverse}\left(-\frac{2}{3}, \frac{7}{54}, -\frac{x-3}{3x}\right) - 84\sqrt{2}(x^8 - 8x^7 + 32x^6 - 80x^5 + 128x^4 - 128x^3 + 64x^2)\text{weierstrassZeta}}{72(x^8 - 8x^7 + 32x^6 - 80x^5 + 128x^4 - 128x^3 + 64x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-x^4 + 4x^3 - 8x^2 + 8x}}{x^{12} - 12x^{11} + 72x^{10} - 280x^9 + 768x^8 - 1536x^7 + 2240x^6 - 2304x^5 + 1536x^4 - 512x^3}, x\right)$$

36.92 Problem number 996

$$\int \frac{\sqrt{-x + \sqrt{x} \sqrt{1+x}}}{\sqrt{1+x}} dx$$

Optimal antiderivative

$$-\frac{3 \arcsin(\sqrt{x} - \sqrt{1+x}) \sqrt{2}}{4} + \frac{(\sqrt{x} + 3\sqrt{1+x}) \sqrt{-x + \sqrt{x} \sqrt{1+x}}}{2}$$

command

```
integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \sqrt{\sqrt{x+1} \sqrt{x} - x} (3 \sqrt{x+1} + \sqrt{x}) + \frac{3}{8} \sqrt{2} \arctan \left(\frac{2 \left(13230146471646497941753920 \sqrt{2} (4x+1) \sqrt{x+1} \sqrt{x} + 472818412040 \sqrt{2} (1119258145177 \dots) \right)}{13921 \dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

37 Test file number 61

Test folder name:

test_cases/3_Logarithms/61_3.2.3_u_log-e-f-a+b_x-^p-c+d_x-^q-r-^s

37.1 Problem number 31

$$\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^2} dx$$

Optimal antiderivative

$$\frac{bpr \ln(bx+a)}{h(-ah+bg)} + \frac{dqr \ln(dx+c)}{h(-ch+dg)} - \frac{\ln(e(f(bx+a)^p(dx+c)^q)^r)}{h(hx+g)} - \frac{bpr \ln(hx+g)}{h(-ah+bg)} - \frac{dqr \ln(hx+g)}{h(-ch+dg)}$$

command

`integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bdg^2 + ach^2 - (bc + ad)gh + (bdg^2 + ach^2 - (bc + ad)gh)r \log(f) - ((bdgh - bch^2)prx + (adgh - ach^2)pr) \log}{bdg^3h + a}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

37.2 Problem number 106

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\ln\left(\frac{(-af+be)(dx+c)}{(-cf+de)(bx+a)}\right)^2 \ln\left(1 - \frac{(-af+be)(dx+c)}{(-cf+de)(bx+a)}\right)}{-af+be} \\ & -\frac{2 \ln\left(\frac{(-af+be)(dx+c)}{(-cf+de)(bx+a)}\right) \operatorname{polylog}\left(2, \frac{(-af+be)(dx+c)}{(-cf+de)(bx+a)}\right)}{-af+be} + \frac{2 \operatorname{polylog}\left(3, \frac{(-af+be)(dx+c)}{(-cf+de)(bx+a)}\right)}{-af+be} \end{aligned}$$

command

`integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\log\left(\frac{adf+acf-(bdx+bc)e}{bcfx+acf-(bdx+ad)e}\right)^2 \log\left(\frac{(bc-ad)fx+(bc-ad)e}{bcfx+acf-(bdx+ad)e}\right) + 2 \operatorname{Li}_2\left(-\frac{(bc-ad)fx+(bc-ad)e}{bcfx+acf-(bdx+ad)e} + 1\right) \log\left(\frac{adf+acf-(bdx+bc)e}{bcfx+acf-(bdx+ad)e}\right) - 2 \operatorname{Li}_2\left(\frac{(bc-ad)fx+(bc-ad)e}{bcfx+acf-(bdx+ad)e}\right)}{af-be}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\log\left(\frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x}\right)^2}{bfx^2 + ae + (be+af)x}, x\right)$$

38 Test file number 65

Test folder name:

test_cases/4_Trig_functions/4.1_Sine/65_4.1.0-a_sin-^m-b_trg-^n

38.1 Problem number 9

$$\int \sin^{\frac{7}{2}}(bx) dx$$

Optimal antiderivative

$$\frac{10 \sqrt{\frac{1}{2} + \frac{\sin(bx)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{21 \sin\left(\frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx) \left(\sin^{\frac{5}{2}}(bx)\right)}{7b} - \frac{10 \cos(bx) \left(\sqrt{\sin(bx)}\right)}{21b}$$

command

`integrate(sin(b*x)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} \sqrt{-i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx) + i \sin(bx)) + 5 \sqrt{2} \sqrt{i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx) - i \sin(bx))}{21b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(\cos(bx)^2 - 1\right) \sin(bx)^{\frac{3}{2}}, x\right)$$

38.2 Problem number 10

$$\int \sin^{\frac{5}{2}}(bx) dx$$

Optimal antiderivative

$$\frac{6 \sqrt{\frac{1}{2} + \frac{\sin(bx)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx) \left(\sin^{\frac{3}{2}}(bx)\right)}{5b}$$

command

`integrate(sin(b*x)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \cos(bx) \sin(bx)^{\frac{3}{2}} - 3i \sqrt{2} \sqrt{-i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx) + i \sin(bx))) + 3i \sqrt{2} \sqrt{i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx) - i \sin(bx)))}{5b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(\cos(bx)^2 - 1\right) \sqrt{\sin(bx)}, x\right)$$

38.3 Problem number 11

$$\int \sin^{\frac{3}{2}}(bx) dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx) \left(\sqrt{\sin(bx)}\right)}{3b}$$

command

```
integrate(sin(b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx) + i \sin(bx)) + \sqrt{2} \sqrt{i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx) - i \sin(bx))}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sin(bx)^{\frac{3}{2}}, x\right)$$

38.4 Problem number 12

$$\int \sqrt{\sin(bx)} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(sin(b*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \sqrt{-i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx) + i \sin(bx))) - i \sqrt{2} \sqrt{i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx) - i \sin(bx)))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{\sin(bx)}, x\right)$$

38.5 Problem number 13

$$\int \frac{1}{\sqrt{\sin(bx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/sin(b*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx) + i \sin(bx)) + \sqrt{2} \sqrt{i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx) - i \sin(bx))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{\sin(bx)}}, x\right)$$

38.6 Problem number 14

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx)}{b \sqrt{\sin(bx)}}$$

command

```
integrate(1/sin(b*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{-i} \sin(bx) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx) + i \sin(bx))) + i \sqrt{2} \sqrt{i} \sin(bx) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx) - i \sin(bx)))}{b \sin(bx)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{\sin(bx)}}{\cos(bx)^2 - 1}, x\right)$$

38.7 Problem number 15

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx)}{3b \sin(bx)^{\frac{3}{2}}}$$

command

```
integrate(1/sin(b*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-i} \left(\sqrt{2} \cos(bx)^2 - \sqrt{2} \right) \operatorname{weierstrassPInverse}(4, 0, \cos(bx) + i \sin(bx)) + \sqrt{i} \left(\sqrt{2} \cos(bx)^2 - \sqrt{2} \right) \operatorname{weierstrassPInverse}(4, 0, \cos(bx) - i \sin(bx))}{3 \left(b \cos(bx)^2 - b \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{1}{\left(\cos(bx)^2 - 1\right) \sqrt{\sin(bx)}}, x\right)$$

38.8 Problem number 16

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)} dx$$

Optimal antiderivative

$$\frac{6\sqrt{\frac{1}{2} + \frac{\sin(bx)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx)}{5b \sin(bx)^{\frac{5}{2}}} - \frac{6 \cos(bx)}{5b \sqrt{\sin(bx)}}$$

command

```
integrate(1/sin(b*x)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \sqrt{-i} \left(i \sqrt{2} \cos(bx)^2 - i \sqrt{2} \right) \sin(bx) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx) + i \sin(bx))) + 3 \sqrt{i} \left(i \sqrt{2} \cos(bx)^2 - i \sqrt{2} \right) \sin(bx) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx) - i \sin(bx)))}{3 \left(b \cos(bx)^2 - b \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sin(bx)}}{\cos(bx)^4 - 2 \cos(bx)^2 + 1}, x\right)$$

38.9 Problem number 17

$$\int \sin^{\frac{7}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{10 \sqrt{\frac{1}{2} + \frac{\sin(bx+a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{21 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx+a) \left(\sin^{\frac{5}{2}}(bx+a)\right)}{7b} - \frac{10 \cos(bx+a) \left(\sqrt{\sin(bx+a)}\right)}{21b}$$

command

```
integrate(sin(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} \sqrt{-i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx+a) + i \sin(bx+a)) + 5 \sqrt{2} \sqrt{i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx+a) - i \sin(bx+a))}{21b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(\cos(bx+a)^2 - 1\right) \sin(bx+a)^{\frac{3}{2}}, x\right)$$

38.10 Problem number 18

$$\int \sin^{\frac{5}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{6 \sqrt{\frac{1}{2} + \frac{\sin(bx+a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx+a) \left(\sin^{\frac{3}{2}}(bx+a)\right)}{5b}$$

command

```
integrate(sin(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \cos(bx+a) \sin(bx+a)^{\frac{3}{2}} - 3i \sqrt{2} \sqrt{-i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx+a) + i \sin(bx+a))) + 3i \sqrt{2} \sqrt{i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx+a) - i \sin(bx+a)))}{5b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(\cos(bx+a)^2 - 1\right) \sqrt{\sin(bx+a)}, x\right)$$

38.11 Problem number 19

$$\int \sin^{\frac{3}{2}}(a + bx) dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx+a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx+a) \left(\sqrt{\sin(bx+a)}\right)}{3b}$$

command

`integrate(sin(b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx+a) + i \sin(bx+a)) + \sqrt{2} \sqrt{i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx+a) - i \sin(bx+a))}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sin(bx+a)^{\frac{3}{2}}, x\right)$$

38.12 Problem number 20

$$\int \sqrt{\sin(a + bx)} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx+a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

`integrate(sin(b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \sqrt{-i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx+a) + i \sin(bx+a))) - i \sqrt{2} \sqrt{i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx+a) - i \sin(bx+a)))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{\sin(bx+a)}, x\right)$$

38.13 Problem number 21

$$\int \frac{1}{\sqrt{\sin(ax + bx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a)) + \sqrt{2} \sqrt{i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) - i \sin(bx + a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{\sin(bx + a)}}, x\right)$$

38.14 Problem number 22

$$\int \frac{1}{\sin^{\frac{3}{2}}(ax + bx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx + a)}{b \sqrt{\sin(bx + a)}}$$

command

```
integrate(1/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{-i} \sin(bx + a) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a))) + i \sqrt{2} \sqrt{i} \sin(bx + a) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) - i \sin(bx + a)))}{b \sin(bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{\sin(bx + a)}}{\cos(bx + a)^2 - 1}, x\right)$$

38.15 Problem number 23

$$\int \frac{1}{\sin^{\frac{5}{2}}(a + bx)} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx + a)}{3b \sin(bx + a)^{\frac{3}{2}}}$$

command

```
integrate(1/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-i} \left(\sqrt{2} \cos(bx + a)^2 - \sqrt{2} \right) \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a)) + \sqrt{i} \left(\sqrt{2} \cos(bx + a)^2 - \sqrt{2} \right)}{3 \left(b \cos(bx + a)^2 - b \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{1}{\left(\cos(bx + a)^2 - 1\right) \sqrt{\sin(bx + a)}}, x\right)$$

38.16 Problem number 24

$$\int \frac{1}{\sin^{\frac{7}{2}}(a + bx)} dx$$

Optimal antiderivative

$$\frac{6\sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b} - \frac{2 \cos(bx + a)}{5b \sin(bx + a)^{\frac{5}{2}}} - \frac{6 \cos(bx + a)}{5b \sqrt{\sin(bx + a)}}$$

command

```
integrate(1/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \sqrt{-i} \left(i \sqrt{2} \cos(bx + a)^2 - i \sqrt{2} \right) \sin(bx + a) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a)))}{3 \left(b \cos(bx + a)^2 - b \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sin(bx + a)}}{\cos(bx + a)^4 - 2 \cos(bx + a)^2 + 1}, x\right)$$

38.17 Problem number 25

$$\int (c \sin(a + bx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2c \cos(bx + a) (c \sin(bx + a))^{\frac{5}{2}}}{7b} \\ & - \frac{10c^4 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(bx + a)}\right)}{21 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b \sqrt{c \sin(bx + a)}} \\ & - \frac{10c^3 \cos(bx + a) \sqrt{c \sin(bx + a)}}{21b} \end{aligned}$$

command

```
integrate((c*sin(b*x+a))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} \sqrt{-i c} c^3 \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a)) + 5 \sqrt{2} \sqrt{i c} c^3 \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) - i \sin(bx + a))}{21 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(c^3 \cos(bx + a)^2 - c^3\right) \sqrt{c \sin(bx + a)} \sin(bx + a), x\right)$$

38.18 Problem number 26

$$\int (c \sin(a + bx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2c \cos(bx + a) (c \sin(bx + a))^{\frac{3}{2}}}{5b} \\ & - \frac{6c^2 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \sin(bx + a)}}{5 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b \sqrt{\sin(bx + a)}} \end{aligned}$$

command

```
integrate((c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{c \sin (bx+a)} c^2 \cos (bx+a) \sin (bx+a) - 3i \sqrt{2} \sqrt{-i c} c^2 \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(c^2 \cos (bx+a)^2 - c^2\right) \sqrt{c \sin (bx+a)}, x\right)$$

38.19 Problem number 27

$$\int (c \sin (a+bx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2c^2 \sqrt{\frac{1}{2} + \frac{\sin (bx+a)}{2}} \text{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin (bx+a)}\right)}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b \sqrt{c \sin (bx+a)}} - \frac{2c \cos (bx+a) \sqrt{c \sin (bx+a)}}{3b}$$

command

`integrate((c*sin(b*x+a))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-i c} c \text{weierstrassPInverse}(4, 0, \cos (bx+a) + i \sin (bx+a)) + \sqrt{2} \sqrt{i c} c \text{weierstrassPInverse}(4, 0, \cos (bx+a) - i \sin (bx+a))}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{c \sin (bx+a)} c \sin (bx+a), x\right)$$

38.20 Problem number 28

$$\int \sqrt{c \sin (a+bx)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{1}{2} + \frac{\sin (bx+a)}{2}} \text{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \sin (bx+a)}}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b \sqrt{\sin (bx+a)}}$$

command

```
integrate((c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \sqrt{-ic} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + i \sin (bx + a))) - i \sqrt{2} \sqrt{ic} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + i \sin (bx + a)))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{c \sin (bx + a)}, x\right)$$

38.21 Problem number 29

$$\int \frac{1}{\sqrt{c \sin (a + bx)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{1}{2} + \frac{\sin (bx + a)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin (bx + a)}\right)}{\sin \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b \sqrt{c \sin (bx + a)}}$$

command

```
integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-ic} \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + i \sin (bx + a)) + \sqrt{2} \sqrt{ic} \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + i \sin (bx + a))}{bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \sin (bx + a)}}{c \sin (bx + a)}, x\right)$$

38.22 Problem number 30

$$\int \frac{1}{(c \sin(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2 \cos(bx + a)}{bc \sqrt{c \sin(bx + a)}} + \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \sin(bx + a)}}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b c^2 \sqrt{\sin(bx + a)}}$$

command

`integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{-ic} \sin(bx + a) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a))) + i \sqrt{2} \sqrt{ic}}{bc^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{c \sin(bx + a)}}{c^2 \cos(bx + a)^2 - c^2}, x\right)$$

38.23 Problem number 31

$$\int \frac{1}{(c \sin(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \cos(bx + a)}{3bc (c \sin(bx + a))^{3/2}} - \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(bx + a)}\right)}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b c^2 \sqrt{c \sin(bx + a)}}$$

command

`integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{2} \cos(bx + a)^2 - \sqrt{2}\right) \sqrt{-ic} \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a)) + \left(\sqrt{2} \cos(bx + a)^2 - \sqrt{2}\right) \sqrt{ic}}{3 \left(bc^3 \cos(bx + a)^2 - bc^3\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{c \sin(bx + a)}}{\left(c^3 \cos(bx + a)^2 - c^3\right) \sin(bx + a)}, x\right)$$

38.24 Problem number 32

$$\int \frac{1}{(c \sin(a + bx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cos(bx + a)}{5bc (c \sin(bx + a))^{5/2}} - \frac{6 \cos(bx + a)}{5b c^3 \sqrt{c \sin(bx + a)}} \\ & + \frac{6 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \sin(bx + a)}}{5 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b c^4 \sqrt{\sin(bx + a)}} \end{aligned}$$

command

```
integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(i \sqrt{2} \cos(bx + a)^2 - i \sqrt{2} \right) \sqrt{-ic} \sin(bx + a) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \sin(bx + a)}}{c^4 \cos(bx + a)^4 - 2c^4 \cos(bx + a)^2 + c^4}, x\right)$$

38.25 Problem number 194

$$\int (d \cos(a + bx))^{9/2} \sin^2(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{28d^3 (d \cos(bx + a))^{3/2} \sin(bx + a)}{585b} + \frac{4d (d \cos(bx + a))^{7/2} \sin(bx + a)}{117b} \\ & - \frac{2(d \cos(bx + a))^{11/2} \sin(bx + a)}{13bd} \\ & + \frac{28d^4 \sqrt{\frac{\cos(bx + a)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos(bx + a)}}{195 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos(bx + a)}} \end{aligned}$$

command

`integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(-21i \sqrt{2} d^{\frac{9}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) + 21i \sqrt{2} d^{\frac{9}{2}} \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(d^4 \cos(bx + a)^6 - d^4 \cos(bx + a)^4\right) \sqrt{d \cos(bx + a)}, x\right)$$

38.26 Problem number 195

$$\int (d \cos(a + bx))^{7/2} \sin^2(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4d(d \cos(bx + a))^{\frac{5}{2}} \sin(bx + a)}{77b} - \frac{2(d \cos(bx + a))^{\frac{9}{2}} \sin(bx + a)}{11bd} \\ & + \frac{20d^4 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{231 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{d \cos(bx + a)}} \\ & + \frac{20d^3 \sin(bx + a) \sqrt{d \cos(bx + a)}}{231b} \end{aligned}$$

command

`integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} d^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) - 5i \sqrt{2} d^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a)) \right)$$

231 b

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(d^3 \cos(bx + a)^5 - d^3 \cos(bx + a)^3\right) \sqrt{d \cos(bx + a)}, x\right)$$

38.27 Problem number 196

$$\int (d \cos(a + bx))^{5/2} \sin^2(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4d(d \cos(bx + a))^{\frac{3}{2}} \sin(bx + a)}{45b} - \frac{2(d \cos(bx + a))^{\frac{7}{2}} \sin(bx + a)}{9bd} \\ & + \frac{4d^2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos(bx + a)}}{15 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos(bx + a)}} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-3i \sqrt{2} d^{\frac{5}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) + 3i \sqrt{2} d^{\frac{5}{2}} \operatorname{weierst} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(d^2 \cos(bx + a)^4 - d^2 \cos(bx + a)^2\right) \sqrt{d \cos(bx + a)}, x\right)$$

38.28 Problem number 197

$$\int (d \cos(a + bx))^{3/2} \sin^2(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(d \cos(bx + a))^{\frac{5}{2}} \sin(bx + a)}{7bd} \\ & + \frac{4d^2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{21 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{d \cos(bx + a)}} \\ & + \frac{4d \sin(bx + a) \sqrt{d \cos(bx + a)}}{21b} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} d^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos (bx + a) + i \sin (bx + a)) - i \sqrt{2} d^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos (bx - a) + i \sin (bx - a)) \right)}{21 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(d \cos (bx + a)^3 - d \cos (bx + a)\right) \sqrt{d \cos (bx + a)}, x\right)$$

38.29 Problem number 198

$$\int \sqrt{d \cos (a + bx)} \sin ^2(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(d \cos (bx + a))^{\frac{3}{2}} \sin (bx + a)}{5bd} \\ & + \frac{4 \sqrt{\frac{\cos (bx + a)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin \left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos (bx + a)}}{5 \cos \left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos (bx + a)}} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{d \cos (bx + a)} \cos (bx + a) \sin (bx + a) - i \sqrt{2} \sqrt{d} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos (bx + a) + i \sin (bx + a))) \right)}{21 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\sqrt{d \cos (bx + a)} \left(\cos (bx + a)^2 - 1\right), x\right)$$

38.30 Problem number 199

$$\int \frac{\sin ^2(a + bx)}{\sqrt{d \cos (a + bx)}} dx$$

Optimal antiderivative

$$\frac{4\sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx+a)})}{3 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{d \cos(bx+a)}} - \frac{2 \sin(bx+a) \sqrt{d \cos(bx+a)}}{3bd}$$

command

`integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} \sqrt{d} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) + i \sin(bx+a)) - i \sqrt{2} \sqrt{d} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) - i \sin(bx+a)) \right)}{3bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{d \cos(bx+a)} (\cos(bx+a)^2 - 1)}{d \cos(bx+a)}, x\right)$$

38.31 Problem number 200

$$\int \frac{\sin^2(a+bx)}{(d \cos(a+bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(bx+a)}{bd \sqrt{d \cos(bx+a)}} - \frac{4\sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos(bx+a)}}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b d^2 \sqrt{\cos(bx+a)}}$$

command

`integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} \sqrt{d} \cos(bx+a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) + i \sin(bx+a))) - i \sqrt{2} \sqrt{d} \cos(bx+a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) - i \sin(bx+a))) \right)}{3bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{d \cos(bx+a)} (\cos(bx+a)^2 - 1)}{d^2 \cos(bx+a)^2}, x\right)$$

38.32 Problem number 201

$$\int \frac{\sin^2(a + bx)}{(d \cos(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(bx + a)}{3bd(d \cos(bx + a))^{\frac{3}{2}}} - \frac{4 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{3 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b d^2 \sqrt{d \cos(bx + a)}}$$

command

```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-i \sqrt{2} \sqrt{d} \cos(bx + a)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) + i \sqrt{2} \sqrt{d} \cos(bx + a)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a)) \right)}{3bd^3 \cos(bx + a)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{d \cos(bx + a)} (\cos(bx + a)^2 - 1)}{d^3 \cos(bx + a)^3}, x\right)$$

38.33 Problem number 202

$$\int \frac{\sin^2(a + bx)}{(d \cos(a + bx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(bx + a)}{5bd(d \cos(bx + a))^{\frac{5}{2}}} - \frac{4 \sin(bx + a)}{5bd^3 \sqrt{d \cos(bx + a)}} + \frac{4 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos(bx + a)}}{5 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b d^4 \sqrt{\cos(bx + a)}}$$

command

```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-i \sqrt{2} \sqrt{d} \cos (bx+a)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a))) + i \sqrt{2} \sqrt{d} \cos (bx+a)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a)) \right)}{21 b d^5 \cos (bx+a)^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{d \cos (bx+a)} \left(\cos (bx+a)^2 - 1 \right)}{d^4 \cos (bx+a)^4}, x \right)$$

38.34 Problem number 203

$$\int \frac{\sin^2(a+bx)}{(d \cos(a+bx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin (bx+a)}{7 b d (d \cos (bx+a))^{\frac{7}{2}}} - \frac{4 \sin (bx+a)}{21 b d^3 (d \cos (bx+a))^{\frac{3}{2}}} - \frac{4 \sqrt{\frac{\cos (bx+a)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(\frac{a}{2} + \frac{bx}{2} \right), \sqrt{2} \right) (\sqrt{\cos (bx+a)})}{21 \cos \left(\frac{a}{2} + \frac{bx}{2} \right) b d^4 \sqrt{d \cos (bx+a)}}$$

command

`integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-i \sqrt{2} \sqrt{d} \cos (bx+a)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a)) + i \sqrt{2} \sqrt{d} \cos (bx+a)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a)) \right)}{21 b d^5 \cos (bx+a)^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{d \cos (bx+a)} \left(\cos (bx+a)^2 - 1 \right)}{d^5 \cos (bx+a)^5}, x \right)$$

38.35 Problem number 211

$$\int (d \cos(a + bx))^{9/2} \sin^4(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{56d^3(d \cos(bx + a))^{\frac{3}{2}} \sin(bx + a)}{3315b} + \frac{8d(d \cos(bx + a))^{\frac{7}{2}} \sin(bx + a)}{663b} \\ & - \frac{12(d \cos(bx + a))^{\frac{11}{2}} \sin(bx + a)}{221bd} - \frac{2(d \cos(bx + a))^{\frac{11}{2}} (\sin^3(bx + a))}{17bd} \\ & + \frac{56d^4 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos(bx + a)}}{1105 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos(bx + a)}} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(-42i \sqrt{2} d^{\frac{9}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) + 42i \sqrt{2} d^{\frac{9}{2}} \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(d^4 \cos(bx + a)^8 - 2d^4 \cos(bx + a)^6 + d^4 \cos(bx + a)^4\right) \sqrt{d \cos(bx + a)}, x\right)$$

38.36 Problem number 212

$$\int (d \cos(a + bx))^{7/2} \sin^4(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8d(d \cos(bx + a))^{\frac{5}{2}} \sin(bx + a)}{385b} - \frac{4(d \cos(bx + a))^{\frac{9}{2}} \sin(bx + a)}{55bd} \\ & - \frac{2(d \cos(bx + a))^{\frac{9}{2}} (\sin^3(bx + a))}{15bd} \\ & + \frac{8d^4 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{231 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{d \cos(bx + a)}} \\ & + \frac{8d^3 \sin(bx + a) \sqrt{d \cos(bx + a)}}{231b} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} d^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) - 10i \sqrt{2} d^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a)) \right) \sqrt{d \cos(bx + a)}, x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(d^3 \cos(bx + a)^7 - 2d^3 \cos(bx + a)^5 + d^3 \cos(bx + a)^3\right) \sqrt{d \cos(bx + a)}, x\right)$$

38.37 Problem number 213

$$\int (d \cos(a + bx))^{5/2} \sin^4(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8d(d \cos(bx + a))^{\frac{3}{2}} \sin(bx + a)}{195b} - \frac{4(d \cos(bx + a))^{\frac{7}{2}} \sin(bx + a)}{39bd} \\ & - \frac{2(d \cos(bx + a))^{\frac{7}{2}} (\sin^3(bx + a))}{13bd} \\ & + \frac{8d^2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos(bx + a)}}{65 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos(bx + a)}} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(-6i \sqrt{2} d^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) + 6i \sqrt{2} d^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a))) \right) \sqrt{d \cos(bx + a)}, x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(d^2 \cos(bx + a)^6 - 2d^2 \cos(bx + a)^4 + d^2 \cos(bx + a)^2\right) \sqrt{d \cos(bx + a)}, x\right)$$

38.38 Problem number 214

$$\int (d \cos(a + bx))^{3/2} \sin^4(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{12(d \cos(bx + a))^{\frac{5}{2}} \sin(bx + a)}{77bd} - \frac{2(d \cos(bx + a))^{\frac{5}{2}} (\sin^3(bx + a))}{11bd} \\ & + \frac{8d^2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{77 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{d \cos(bx + a)}} \\ & + \frac{8d \sin(bx + a) \sqrt{d \cos(bx + a)}}{77b} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2i \sqrt{2} d^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) - 2i \sqrt{2} d^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a)) \right)}{77b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(d \cos(bx + a)^5 - 2d \cos(bx + a)^3 + d \cos(bx + a)\right) \sqrt{d \cos(bx + a)}, x\right)$$

38.39 Problem number 215

$$\int \sqrt{d \cos(a + bx)} \sin^4(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4(d \cos(bx + a))^{\frac{3}{2}} \sin(bx + a)}{15bd} - \frac{2(d \cos(bx + a))^{\frac{3}{2}} (\sin^3(bx + a))}{9bd} \\ & + \frac{8 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos(bx + a)}}{15 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos(bx + a)}} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(5 \cos (bx + a)^3 - 11 \cos (bx + a) \right) \sqrt{d \cos (bx + a)} \sin (bx + a) + 6i \sqrt{2} \sqrt{d} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrass} \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(\cos (bx + a)^4 - 2 \cos (bx + a)^2 + 1 \right) \sqrt{d \cos (bx + a)}, x \right)$$

38.40 Problem number 216

$$\int \frac{\sin^4(a + bx)}{\sqrt{d \cos(a + bx)}} dx$$

Optimal antiderivative

$$\frac{8 \sqrt{\frac{\cos (bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{a}{2} + \frac{bx}{2} \right), \sqrt{2} \right) (\sqrt{\cos (bx + a)})}{7 \cos \left(\frac{a}{2} + \frac{bx}{2} \right) b \sqrt{d \cos (bx + a)}} - \frac{4 \sin (bx + a) \sqrt{d \cos (bx + a)}}{7bd} - \frac{2(\sin^3 (bx + a)) \sqrt{d \cos (bx + a)}}{7bd}$$

command

`integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{d \cos (bx + a)} \left(\cos (bx + a)^2 - 3 \right) \sin (bx + a) - 2i \sqrt{2} \sqrt{d} \operatorname{weierstrassPInverse}(-4, 0, \cos (bx + a) + i \sin (bx + a)) \right)$$

$7bd$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(\cos (bx + a)^4 - 2 \cos (bx + a)^2 + 1 \right) \sqrt{d \cos (bx + a)}}{d \cos (bx + a)}, x \right)$$

38.41 Problem number 217

$$\int \frac{\sin^4(a + bx)}{(d \cos(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{12(d \cos(bx + a))^{\frac{3}{2}} \sin(bx + a)}{5bd^3} + \frac{2(\sin^3(bx + a))}{bd\sqrt{d \cos(bx + a)}} - \frac{24\sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos(bx + a)}}{5 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b d^2 \sqrt{\cos(bx + a)}}$$

command

```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6i\sqrt{2}\sqrt{d}\cos(bx+a)\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))) - 6i\sqrt{2}\sqrt{d}\cos(bx+a)\right)}{5bd^2\sqrt{\cos(bx+a)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\sqrt{d \cos(bx+a)}}{d^2 \cos(bx+a)^2}, x\right)$$

38.42 Problem number 218

$$\int \frac{\sin^4(a + bx)}{(d \cos(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(\sin^3(bx + a))}{3bd(d \cos(bx + a))^{\frac{3}{2}}} - \frac{8\sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{3 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b d^2 \sqrt{d \cos(bx + a)}} + \frac{4 \sin(bx + a) \sqrt{d \cos(bx + a)}}{3bd^3}$$

command

```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-2i \sqrt{2} \sqrt{d} \cos (bx + a)^2 \text{weierstrassPInverse}(-4, 0, \cos (bx + a) + i \sin (bx + a)) + 2i \sqrt{2} \sqrt{d} \cos (bx + a)^2 \right)}{3 b d^3 \cos (bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(\cos (bx + a)^4 - 2 \cos (bx + a)^2 + 1 \right) \sqrt{d \cos (bx + a)}}{d^3 \cos (bx + a)^3}, x \right)$$

38.43 Problem number 219

$$\int \frac{\sin^4(a + bx)}{(d \cos(a + bx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(\sin^3(bx + a))}{5bd(d \cos(bx + a))^{\frac{5}{2}}} - \frac{12 \sin(bx + a)}{5bd^3 \sqrt{d \cos(bx + a)}} + \frac{24 \sqrt{\frac{\cos(bx + a)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos(bx + a)}}{5 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b d^4 \sqrt{\cos(bx + a)}}$$

command

```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-6i \sqrt{2} \sqrt{d} \cos (bx + a)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos (bx + a) + i \sin (bx + a))) + 6 \right)}{3 b d^3 \cos (bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(\cos (bx + a)^4 - 2 \cos (bx + a)^2 + 1 \right) \sqrt{d \cos (bx + a)}}{d^4 \cos (bx + a)^4}, x \right)$$

38.44 Problem number 220

$$\int \frac{\sin^4(a + bx)}{(d \cos(a + bx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4 \sin(bx + a)}{7b d^3 (d \cos(bx + a))^{\frac{3}{2}}} + \frac{2(\sin^3(bx + a))}{7bd (d \cos(bx + a))^{\frac{7}{2}}} \\ & + \frac{8 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{7 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b d^4 \sqrt{d \cos(bx + a)}} \end{aligned}$$

command

`integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2i \sqrt{2} \sqrt{d} \cos(bx + a)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) - 2i \sqrt{2} \sqrt{d} \cos(bx + a)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a)) \right)}{7bd^5 \cos(bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(\cos(bx + a)^4 - 2 \cos(bx + a)^2 + 1\right) \sqrt{d \cos(bx + a)}}{d^5 \cos(bx + a)^5}, x\right)$$

38.45 Problem number 232

$$\int (d \cos(a + bx))^{11/2} \csc^2(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{d(d \cos(bx + a))^{\frac{9}{2}} \csc(bx + a)}{b} - \frac{9d^3(d \cos(bx + a))^{\frac{5}{2}} \sin(bx + a)}{7b} \\ & - \frac{15d^6 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{7 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{d \cos(bx + a)}} \\ & - \frac{15d^5 \sin(bx + a) \sqrt{d \cos(bx + a)}}{7b} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$15i \sqrt{2} d^{\frac{11}{2}} \sin (bx + a)$ weierstrassPInverse(-4, 0, cos (bx + a) + i sin (bx + a)) - $15i \sqrt{2} d^{\frac{11}{2}} \sin (bx + a)$ weierstra
14 b sin

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{d \cos (bx + a)} d^5 \cos (bx + a)^5 \csc (bx + a)^2, x\right)$$

38.46 Problem number 233

$$\int (d \cos (a + bx))^{9/2} \csc ^2(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{d(d \cos (bx + a))^{\frac{7}{2}} \csc (bx + a)}{b} - \frac{7d^3(d \cos (bx + a))^{\frac{3}{2}} \sin (bx + a)}{5b} \\ & - \frac{21d^4 \sqrt{\frac{\cos (bx + a)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin \left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos (bx + a)}}{5 \cos \left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos (bx + a)}} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-21i \sqrt{2} d^{\frac{9}{2}} \sin (bx + a)$ weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos (bx + a) + i sin (bx + a))) + $21i \sqrt{2}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{d \cos (bx + a)} d^4 \cos (bx + a)^4 \csc (bx + a)^2, x\right)$$

38.47 Problem number 234

$$\int (d \cos(a + bx))^{7/2} \csc^2(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d(d \cos(bx + a))^{\frac{5}{2}} \csc(bx + a)}{b} \\ & - \frac{5d^4 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{3 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{d \cos(bx + a)}} \\ & - \frac{5d^3 \sin(bx + a) \sqrt{d \cos(bx + a)}}{3b} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5i \sqrt{2} d^{\frac{7}{2}} \sin(bx + a) \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) - 5i \sqrt{2} d^{\frac{7}{2}} \sin(bx + a) \operatorname{weierstrassP}}{6 b \sin(bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \cos(bx + a)} d^3 \cos(bx + a)^3 \csc(bx + a)^2, x\right)$$

38.48 Problem number 235

$$\int (d \cos(a + bx))^{5/2} \csc^2(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d(d \cos(bx + a))^{\frac{3}{2}} \csc(bx + a)}{b} \\ & - \frac{3d^2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos(bx + a)}}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos(bx + a)}} \end{aligned}$$

command

```
integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-3i \sqrt{2} d^{\frac{5}{2}} \sin (bx+a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a))) + 3i \sqrt{2} d^{\frac{5}{2}} \sin (bx+a) \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a))}{2 b \sin (bx+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \cos (bx+a)} d^2 \cos (bx+a)^2 \csc (bx+a)^2, x\right)$$

38.49 Problem number 236

$$\int (d \cos (a+bx))^{3/2} \csc ^2(a+bx) dx$$

Optimal antiderivative

$$\frac{d^2 \sqrt{\frac{\cos (bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin \left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos (bx+a)})}{\cos \left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{d \cos (bx+a)}} - \frac{d \csc (bx+a) \sqrt{d \cos (bx+a)}}{b}$$

command

```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} d^{\frac{3}{2}} \sin (bx+a) \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a)) - i \sqrt{2} d^{\frac{3}{2}} \sin (bx+a) \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a))}{2 b \sin (bx+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \cos (bx+a)} d \cos (bx+a) \csc (bx+a)^2, x\right)$$

38.50 Problem number 237

$$\int \sqrt{d \cos (a+bx)} \csc ^2(a+bx) dx$$

Optimal antiderivative

$$\frac{(d \cos (bx+a))^{\frac{3}{2}} \csc (bx+a)}{bd} - \frac{\sqrt{\frac{\cos (bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin \left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos (bx+a)}}{\cos \left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos (bx+a)}}$$

command

```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{d} \sin (bx+a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a))) + i \sqrt{2} \sqrt{d} \sin (bx+a) \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \cos (bx+a)} \operatorname{csc}(bx+a)^2, x\right)$$

38.51 Problem number 238

$$\int \frac{\operatorname{csc}^2(a+bx)}{\sqrt{d \cos(a+bx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos (bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin \left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos (bx+a)})}{\cos \left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{d \cos (bx+a)}} - \frac{\operatorname{csc}(bx+a) \sqrt{d \cos (bx+a)}}{bd}$$

command

```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{d} \sin (bx+a) \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a)) + i \sqrt{2} \sqrt{d} \sin (bx+a) \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a))}{2bd \sin (bx+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \cos (bx+a)} \operatorname{csc}(bx+a)^2}{d \cos (bx+a)}, x\right)$$

38.52 Problem number 239

$$\int \frac{\csc^2(a + bx)}{(d \cos(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\csc(bx + a)}{bd\sqrt{d \cos(bx + a)}} + \frac{3 \sin(bx + a)}{bd\sqrt{d \cos(bx + a)}} \\ & - \frac{3\sqrt{\frac{\cos(bx + a)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos(bx + a)}}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b d^2 \sqrt{\cos(bx + a)}} \end{aligned}$$

command

```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} \sqrt{d} \cos(bx + a) \sin(bx + a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \cos(bx + a)} \csc(bx + a)^2}{d^2 \cos(bx + a)^2}, x\right)$$

38.53 Problem number 240

$$\int \frac{\csc^2(a + bx)}{(d \cos(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\csc(bx + a)}{bd(d \cos(bx + a))^{3/2}} + \frac{5 \sin(bx + a)}{3bd(d \cos(bx + a))^{3/2}} \\ & + \frac{5\sqrt{\frac{\cos(bx + a)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{3 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b d^2 \sqrt{d \cos(bx + a)}} \end{aligned}$$

command

```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} \sqrt{d} \cos (bx+a)^2 \sin (bx+a) \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a)) + 5i \sqrt{2} \sqrt{d} \cos (bx+a)}{6 b d^3 \cos (bx+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \cos (bx+a)} \csc (bx+a)^2}{d^3 \cos (bx+a)^3}, x\right)$$

38.54 Problem number 241

$$\int \frac{\csc^2(a+bx)}{(d \cos(a+bx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\csc (bx+a)}{bd(d \cos (bx+a))^{\frac{5}{2}}} + \frac{7 \sin (bx+a)}{5bd(d \cos (bx+a))^{\frac{5}{2}}} + \frac{21 \sin (bx+a)}{5b d^3 \sqrt{d \cos (bx+a)}} \\ & - \frac{21 \sqrt{\frac{\cos (bx+a)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{d \cos (bx+a)}}{5 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b d^4 \sqrt{\cos (bx+a)}} \end{aligned}$$

command

```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-21i \sqrt{2} \sqrt{d} \cos (bx+a)^3 \sin (bx+a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a) + i \sin (bx+a)))}{6 b d^3 \cos (bx+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \cos (bx+a)} \csc (bx+a)^2}{d^4 \cos (bx+a)^4}, x\right)$$

38.55 Problem number 269

$$\int \frac{(c \sin(a + bx))^{3/2}}{(d \cos(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2c\sqrt{c \sin(bx + a)}}{3bd(d \cos(bx + a))^{3/2}} + \frac{c^2 \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin(2bx + 2a)}\right)}{3 \sin\left(a + \frac{\pi}{4} + bx\right) b d^2 \sqrt{d \cos(bx + a)} \sqrt{c \sin(bx + a)}}$$

command

`integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{icd} c \cos(bx + a)^2 \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + \sqrt{-icd} c \cos(bx + a)^2 \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1)}{3 bd^3 \cos(bx + a)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \cos(bx + a)} \sqrt{c \sin(bx + a)} c \sin(bx + a)}{d^3 \cos(bx + a)^3}, x\right)$$

38.56 Problem number 270

$$\int \frac{(c \sin(a + bx))^{3/2}}{(d \cos(a + bx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2c\sqrt{c \sin(bx + a)}}{7bd(d \cos(bx + a))^{7/2}} - \frac{2c\sqrt{c \sin(bx + a)}}{21bd^3(d \cos(bx + a))^{3/2}} + \frac{2c^2 \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin(2bx + 2a)}\right)}{21 \sin\left(a + \frac{\pi}{4} + bx\right) b d^4 \sqrt{d \cos(bx + a)} \sqrt{c \sin(bx + a)}}$$

command

`integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{icd} c \cos(bx + a)^4 \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + \sqrt{-icd} c \cos(bx + a)^4 \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1) \right)}{21 bd^5 \cos(bx + a)^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \cos(bx + a)} \sqrt{c \sin(bx + a)} c \sin(bx + a)}{d^5 \cos(bx + a)^5}, x\right)$$

38.57 Problem number 293

$$\int \frac{1}{\sqrt{d \cos(a + bx)} \sqrt{c \sin(a + bx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin(2bx + 2a)}\right)}{\sin\left(a + \frac{\pi}{4} + bx\right) b \sqrt{d \cos(bx + a)} \sqrt{c \sin(bx + a)}}$$

command

`integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{icd} \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + \sqrt{-icd} \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1)}{bcd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \cos(bx + a)} \sqrt{c \sin(bx + a)}}{cd \cos(bx + a) \sin(bx + a)}, x\right)$$

38.58 Problem number 294

$$\int \frac{1}{(d \cos(a + bx))^{5/2} \sqrt{c \sin(a + bx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{c \sin(bx + a)}}{3bcd (d \cos(bx + a))^{\frac{3}{2}}} \frac{2\sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin(2bx + 2a)}\right)}{3 \sin\left(a + \frac{\pi}{4} + bx\right) b d^2 \sqrt{d \cos(bx + a)} \sqrt{c \sin(bx + a)}}$$

command

`integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{icd} \cos(bx + a)^2 \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + \sqrt{-icd} \cos(bx + a)^2 \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1) \right)}{3bcd^3 \cos(bx + a)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \cos(bx + a)} \sqrt{c \sin(bx + a)}}{cd^3 \cos(bx + a)^3 \sin(bx + a)}, x\right)$$

38.59 Problem number 295

$$\int \frac{1}{(d \cos(a + bx))^{9/2} \sqrt{c \sin(a + bx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{c \sin(bx + a)}}{7bcd (d \cos(bx + a))^{7/2}} + \frac{4\sqrt{c \sin(bx + a)}}{7bc d^3 (d \cos(bx + a))^{3/2}} - \frac{4\sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin(2bx + 2a)}\right)}{7 \sin\left(a + \frac{\pi}{4} + bx\right) b d^4 \sqrt{d \cos(bx + a)} \sqrt{c \sin(bx + a)}}$$

command

```
integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{i cd} \cos(bx + a)^4 \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + 2 \sqrt{-i cd} \cos(bx + a)^4 \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1) \right)}{7 bcd^5 \cos(bx + a)^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \cos(bx + a)} \sqrt{c \sin(bx + a)}}{cd^5 \cos(bx + a)^5 \sin(bx + a)}, x\right)$$

38.60 Problem number 378

$$\int \sqrt{b \sec(e + fx)} \sin^6(e + fx) dx$$

Optimal antiderivative

$$-\frac{40b \sin(fx + e)}{77f \sqrt{b \sec(fx + e)}} - \frac{20b(\sin^3(fx + e))}{77f \sqrt{b \sec(fx + e)}} - \frac{2b(\sin^5(fx + e))}{11f \sqrt{b \sec(fx + e)}} + \frac{80\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) \left(\sqrt{\cos(fx + e)}\right) \sqrt{b \sec(fx + e)}}{77 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f}$$

command

```
integrate(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(7 \cos (fx + e)^5 - 24 \cos (fx + e)^3 + 37 \cos (fx + e) \right) \sqrt{\frac{b}{\cos (fx + e)}} \sin (fx + e) + 20i \sqrt{2} \sqrt{b} \text{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \left(\cos (fx + e)^6 - 3 \cos (fx + e)^4 + 3 \cos (fx + e)^2 - 1 \right) \sqrt{b \sec (fx + e)}, x \right)$$

38.61 Problem number 379

$$\int \sqrt{b \sec (e + fx)} \sin^4 (e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4b \sin (fx + e)}{7f \sqrt{b \sec (fx + e)}} - \frac{2b (\sin^3 (fx + e))}{7f \sqrt{b \sec (fx + e)}} \\ & + \frac{8 \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right) (\sqrt{\cos (fx + e)}) \sqrt{b \sec (fx + e)}}{7 \cos \left(\frac{fx}{2} + \frac{e}{2} \right) f} \end{aligned}$$

command

```
integrate(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\cos (fx + e)^3 - 3 \cos (fx + e) \right) \sqrt{\frac{b}{\cos (fx + e)}} \sin (fx + e) - 2i \sqrt{2} \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos (fx + e)) \right)$$

7f

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(\cos (fx + e)^4 - 2 \cos (fx + e)^2 + 1 \right) \sqrt{b \sec (fx + e)}, x \right)$$

38.62 Problem number 380

$$\int \sqrt{b \sec(e + fx)} \sin^2(e + fx) dx$$

Optimal antiderivative

$$\frac{2b \sin(fx + e)}{3f \sqrt{b \sec(fx + e)}} + \frac{4 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f}$$

command

```
integrate(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{\frac{b}{\cos(fx + e)}} \cos(fx + e) \sin(fx + e) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) \right)}{3f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(\cos(fx + e)^2 - 1\right) \sqrt{b \sec(fx + e)}, x\right)$$

38.63 Problem number 381

$$\int \sqrt{b \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f}$$

command

```
integrate((b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e))}{f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(fx + e)}, x\right)$$

38.64 Problem number 382

$$\int \csc^2(e + fx) \sqrt{b \sec(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b \csc(fx + e)}{f \sqrt{b \sec(fx + e)}} \\ & + \frac{\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \sin(fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + i \sqrt{2} \sqrt{b} \sin(fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e))}{2 f \sin(fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(fx + e)} \csc(fx + e)^2, x\right)$$

38.65 Problem number 383

$$\int \csc^4(e + fx) \sqrt{b \sec(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5b \csc(fx + e)}{6f \sqrt{b \sec(fx + e)}} - \frac{b(\csc^3(fx + e))}{3f \sqrt{b \sec(fx + e)}} \\ & + \frac{5 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{6 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} \left(i \cos (fx + e)^2 - i \right) \sqrt{b} \sin (fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e)) + 5\sqrt{2} \left(-i \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec (fx + e)} \csc (fx + e)^4, x\right)$$

38.66 Problem number 384

$$\int \csc^6(e + fx) \sqrt{b \sec(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3b \csc (fx + e)}{4f \sqrt{b \sec (fx + e)}} - \frac{3b(\csc^3 (fx + e))}{10f \sqrt{b \sec (fx + e)}} - \frac{b(\csc^5 (fx + e))}{5f \sqrt{b \sec (fx + e)}} \\ & + \frac{3\sqrt{\frac{\cos (fx + e)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos (fx + e)}) \sqrt{b \sec (fx + e)}}{4 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15\sqrt{2} \left(i \cos (fx + e)^4 - 2i \cos (fx + e)^2 + i \right) \sqrt{b} \sin (fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec (fx + e)} \csc (fx + e)^6, x\right)$$

38.67 Problem number 391

$$\int (b \sec(e + fx))^{3/2} \sin^6(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8b^3 \sin(fx + e)}{3f (b \sec(fx + e))^{3/2}} + \frac{20b^3 (\sin^3(fx + e))}{9f (b \sec(fx + e))^{3/2}} \\ & - \frac{16b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}} \\ & + \frac{2b (\sin^5(fx + e)) \sqrt{b \sec(fx + e)}}{f} \end{aligned}$$

command

```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(12i \sqrt{2} b^{3/2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) - 12i \sqrt{2} b^{3/2} \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(b \cos(fx + e)^6 - 3b \cos(fx + e)^4 + 3b \cos(fx + e)^2 - b\right) \sqrt{b \sec(fx + e)} \sec(fx + e), x\right)$$

38.68 Problem number 392

$$\int (b \sec(e + fx))^{3/2} \sin^4(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{12b^3 \sin(fx + e)}{5f (b \sec(fx + e))^{3/2}} - \frac{24b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}} \\ & + \frac{2b (\sin^3(fx + e)) \sqrt{b \sec(fx + e)}}{f} \end{aligned}$$

command

```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6i \sqrt{2} b^{\frac{3}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) - 6i \sqrt{2} b^{\frac{3}{2}} \text{weierstrass} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(b \cos(fx + e)^4 - 2b \cos(fx + e)^2 + b \right) \sqrt{b \sec(fx + e)} \sec(fx + e), x \right)$$

38.69 Problem number 393

$$\int (b \sec(e + fx))^{3/2} \sin^2(e + fx) dx$$

Optimal antiderivative

$$-\frac{4b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{fx}{2} + \frac{e}{2} \right) f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}} + \frac{2b \sin(fx + e) \sqrt{b \sec(fx + e)}}{f}$$

command

```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} b^{\frac{3}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) - i \sqrt{2} b^{\frac{3}{2}} \text{weierstrass} \right)$$

f

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \left(b \cos(fx + e)^2 - b \right) \sqrt{b \sec(fx + e)} \sec(fx + e), x \right)$$

38.70 Problem number 394

$$\int (b \sec(e + fx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{2b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}} + \frac{2b \sin(fx + e) \sqrt{b \sec(fx + e)}}{f}$$

command

```
integrate((b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}$$

f

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(fx + e)} b \sec(fx + e), x\right)$$

38.71 Problem number 395

$$\int \csc^2(e + fx)(b \sec(e + fx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{3b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}} - \frac{b \csc(fx + e) \sqrt{b \sec(fx + e)}}{f} + \frac{3b \sin(fx + e) \sqrt{b \sec(fx + e)}}{f}$$

command

```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} b^{\frac{3}{2}} \sin (fx + e) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e))) + 3i \sqrt{2} b^{\frac{3}{2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \sec (fx + e)} b \csc (fx + e)^2 \sec (fx + e), x\right)$$

38.72 Problem number 396

$$\int \csc^4(e + fx)(b \sec(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{7b^2 \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos (fx + e)} \sqrt{b \sec (fx + e)}} - \frac{7b \csc (fx + e) \sqrt{b \sec (fx + e)}}{6f} \\ & - \frac{b(\csc^3 (fx + e)) \sqrt{b \sec (fx + e)}}{3f} + \frac{7b \sin (fx + e) \sqrt{b \sec (fx + e)}}{2f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21 \sqrt{2} \left(i b \cos (fx + e)^2 - i b \right) \sqrt{b} \sin (fx + e) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \sec (fx + e)} b \csc (fx + e)^4 \sec (fx + e), x\right)$$

38.73 Problem number 404

$$\int (b \sec(e + fx))^{5/2} \sin^6(e + fx) dx$$

Optimal antiderivative

$$\frac{2b(b \sec(fx + e))^{\frac{3}{2}} (\sin^5(fx + e))}{3f} + \frac{40b^3 \sin(fx + e)}{21f \sqrt{b \sec(fx + e)}} + \frac{20b^3 (\sin^3(fx + e))}{21f \sqrt{b \sec(fx + e)}} - \frac{80b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{21 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f}$$

command

```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(-20i \sqrt{2} b^{\frac{5}{2}} \cos(fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + 20i \sqrt{2} b^{\frac{5}{2}} \cos(fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(b^2 \cos(fx + e)^6 - 3b^2 \cos(fx + e)^4 + 3b^2 \cos(fx + e)^2 - b^2\right) \sqrt{b \sec(fx + e)} \sec(fx + e)^2, x\right)$$

38.74 Problem number 405

$$\int (b \sec(e + fx))^{5/2} \sin^4(e + fx) dx$$

Optimal antiderivative

$$\frac{2b(b \sec(fx + e))^{\frac{3}{2}} (\sin^3(fx + e))}{3f} + \frac{4b^3 \sin(fx + e)}{3f \sqrt{b \sec(fx + e)}} - \frac{8b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f}$$

command

```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(-2i \sqrt{2} b^{\frac{5}{2}} \cos (fx + e) \text{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e)) + 2i \sqrt{2} b^{\frac{5}{2}} \cos (fx + e) \text{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e)) \right) / (3 f \cos (fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(b^2 \cos (fx + e)^4 - 2 b^2 \cos (fx + e)^2 + b^2 \right) \sqrt{b \sec (fx + e)} \sec (fx + e)^2, x \right)$$

38.75 Problem number 406

$$\int (b \sec (e + fx))^{5/2} \sin ^2(e + fx) dx$$

Optimal antiderivative

$$\frac{2b(b \sec (fx + e))^{\frac{3}{2}} \sin (fx + e)}{3f} - \frac{4b^2 \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right) (\sqrt{\cos (fx + e)}) \sqrt{b \sec (fx + e)}}{3 \cos \left(\frac{fx}{2} + \frac{e}{2} \right) f}$$

command

```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(-i \sqrt{2} b^{\frac{5}{2}} \cos (fx + e) \text{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e)) + i \sqrt{2} b^{\frac{5}{2}} \cos (fx + e) \text{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e)) \right) / (3 f \cos (fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \left(b^2 \cos (fx + e)^2 - b^2 \right) \sqrt{b \sec (fx + e)} \sec (fx + e)^2, x \right)$$

38.76 Problem number 407

$$\int (b \sec(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b(b \sec(fx + e))^{\frac{3}{2}} \sin(fx + e)}{3f} + \frac{2b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f}$$

command

```
integrate((b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b^{\frac{5}{2}} \cos(fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + i \sqrt{2} b^{\frac{5}{2}} \cos(fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e))}{3 f \cos(fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(fx + e)} b^2 \sec(fx + e)^2, x\right)$$

38.77 Problem number 408

$$\int \csc^2(e + fx)(b \sec(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b \csc(fx + e) (b \sec(fx + e))^{\frac{3}{2}}}{3f} - \frac{5b^3 \csc(fx + e)}{3f \sqrt{b \sec(fx + e)}} + \frac{5b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f}$$

command

```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} b^{\frac{5}{2}} \cos (fx + e) \sin (fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e)) + 5i \sqrt{2} b^{\frac{5}{2}} \cos (fx + e) \sin (fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e)) + 6 f \cos (fx + e) \sin (fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec (fx + e)} b^2 \csc (fx + e)^2 \sec (fx + e)^2, x\right)$$

38.78 Problem number 409

$$\int \csc^4(e + fx)(b \sec(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{b \csc (fx + e) (b \sec (fx + e))^{\frac{3}{2}}}{f} - \frac{b (\csc^3 (fx + e)) (b \sec (fx + e))^{\frac{3}{2}}}{3f} - \frac{5b^3 \csc (fx + e)}{2f \sqrt{b \sec (fx + e)}} + \frac{5b^2 \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos (fx + e)}) \sqrt{b \sec (fx + e)}}{2 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f}$$

command

`integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} \left(i b^2 \cos (fx + e)^3 - i b^2 \cos (fx + e) \right) \sqrt{b} \sin (fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec (fx + e)} b^2 \csc (fx + e)^4 \sec (fx + e)^2, x\right)$$

38.79 Problem number 417

$$\int \frac{\sin^6(e + fx)}{\sqrt{b \sec(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{8b \sin(fx + e)}{39f (b \sec(fx + e))^{\frac{3}{2}}} - \frac{20b(\sin^3(fx + e))}{117f (b \sec(fx + e))^{\frac{3}{2}}} - \frac{2b(\sin^5(fx + e))}{13f (b \sec(fx + e))^{\frac{3}{2}}} \\ & + \frac{16\sqrt{\frac{\cos(fx + e)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{39 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}} \end{aligned}$$

command

```
integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(9 \cos^6(fx + e) - 28 \cos^4(fx + e) + 31 \cos^2(fx + e) \right) \sqrt{\frac{b}{\cos(fx + e)}} \sin(fx + e) - 12i \sqrt{2} \sqrt{b} \operatorname{weierstrass} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(\cos^6(fx + e) - 3 \cos^4(fx + e) + 3 \cos^2(fx + e) - 1\right) \sqrt{b \sec(fx + e)}}{b \sec(fx + e)}, x\right)$$

38.80 Problem number 418

$$\int \frac{\sin^4(e + fx)}{\sqrt{b \sec(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4b \sin(fx + e)}{15f (b \sec(fx + e))^{\frac{3}{2}}} - \frac{2b(\sin^3(fx + e))}{9f (b \sec(fx + e))^{\frac{3}{2}}} + \frac{8\sqrt{\frac{\cos(fx + e)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}} \end{aligned}$$

command

```
integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(5 \cos (fx + e)^4 - 11 \cos (fx + e)^2 \right) \sqrt{\frac{b}{\cos (fx + e)}} \sin (fx + e) + 6i \sqrt{2} \sqrt{b} \text{weierstrassZeta}(-4, 0, \text{weierstras}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(\cos (fx + e)^4 - 2 \cos (fx + e)^2 + 1 \right) \sqrt{b \sec (fx + e)}}{b \sec (fx + e)}, x \right)$$

38.81 Problem number 419

$$\int \frac{\sin^2(e + fx)}{\sqrt{b \sec(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{2b \sin (fx + e)}{5f (b \sec (fx + e))^{\frac{3}{2}}} + \frac{4 \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{fx}{2} + \frac{e}{2} \right) f \sqrt{\cos (fx + e)} \sqrt{b \sec (fx + e)}}$$

command

`integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{\frac{b}{\cos (fx + e)}} \cos (fx + e)^2 \sin (fx + e) - i \sqrt{2} \sqrt{b} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos (f$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\left(\cos (fx + e)^2 - 1 \right) \sqrt{b \sec (fx + e)}}{b \sec (fx + e)}, x \right)$$

38.82 Problem number 420

$$\int \frac{1}{\sqrt{b \sec(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}}$$

command

`integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) - i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \cos(fx + e) - i \sin(fx + e))}{bf}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(fx + e)}}{b \sec(fx + e)}, x\right)$$

38.83 Problem number 421

$$\int \frac{\csc^2(e + fx)}{\sqrt{b \sec(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{b \csc(fx + e)}{f (b \sec(fx + e))^{\frac{3}{2}}} - \frac{\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}}$$

command

`integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{b} \sin(fx + e) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + i \sqrt{2} \sqrt{b} \sin(fx + e) \operatorname{weierstrassZeta}(-4, 0, \cos(fx + e) - i \sin(fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(fx + e)} \csc(fx + e)^2}{b \sec(fx + e)}, x\right)$$

38.84 Problem number 422

$$\int \frac{\csc^4(e + fx)}{\sqrt{b \sec(e + fx)}} dx$$

Optimal antiderivative

$$\frac{b \csc(fx + e)}{2f (b \sec(fx + e))^{\frac{3}{2}}} - \frac{b(\csc^3(fx + e))}{3f (b \sec(fx + e))^{\frac{3}{2}}} - \frac{\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}}$$

command

```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2} \left(i \cos(fx + e)^2 - i \right) \sqrt{b} \sin(fx + e) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(fx + e)} \csc(fx + e)^4}{b \sec(fx + e)}, x\right)$$

38.85 Problem number 423

$$\int \frac{\csc^6(e + fx)}{\sqrt{b \sec(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{7b \csc(fx + e)}{20f (b \sec(fx + e))^{\frac{3}{2}}} - \frac{7b(\csc^3(fx + e))}{30f (b \sec(fx + e))^{\frac{3}{2}}} - \frac{b(\csc^5(fx + e))}{5f (b \sec(fx + e))^{\frac{3}{2}}} \\ & - \frac{7\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{20 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}} \end{aligned}$$

command

```
integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21 \sqrt{2} \left(i \cos (fx + e)^4 - 2i \cos (fx + e)^2 + i \right) \sqrt{b} \sin (fx + e) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \sec (fx + e)} \csc (fx + e)^6}{b \sec (fx + e)}, x \right)$$

38.86 Problem number 431

$$\int \frac{\sin^4(e + fx)}{(b \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{12b \sin (fx + e)}{77f (b \sec (fx + e))^{\frac{5}{2}}} - \frac{2b(\sin^3 (fx + e))}{11f (b \sec (fx + e))^{\frac{5}{2}}} + \frac{8 \sin (fx + e)}{77bf \sqrt{b \sec (fx + e)}} \\ & + \frac{8 \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right) (\sqrt{\cos (fx + e)}) \sqrt{b \sec (fx + e)}}{77 \cos \left(\frac{fx}{2} + \frac{e}{2} \right) b^2 f} \end{aligned}$$

command

```
integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(7 \cos (fx + e)^5 - 13 \cos (fx + e)^3 + 4 \cos (fx + e) \right) \sqrt{\frac{b}{\cos (fx + e)}} \sin (fx + e) - 2i \sqrt{2} \sqrt{b} \text{weierstrassPInv}$$

77 b^2

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(\cos (fx + e)^4 - 2 \cos (fx + e)^2 + 1 \right) \sqrt{b \sec (fx + e)}}{b^2 \sec (fx + e)^2}, x \right)$$

38.87 Problem number 432

$$\int \frac{\sin^2(e + fx)}{(b \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b \sin(fx + e)}{7f (b \sec(fx + e))^{\frac{5}{2}}} + \frac{4 \sin(fx + e)}{21bf \sqrt{b \sec(fx + e)}} \\ & + \frac{4 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{21 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f} \end{aligned}$$

command

`integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((3 \cos(fx + e)^3 - 2 \cos(fx + e)) \sqrt{\frac{b}{\cos(fx + e)}} \sin(fx + e) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e)) \right)}{21 b^2 f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(\cos(fx + e)^2 - 1) \sqrt{b \sec(fx + e)}}{b^2 \sec(fx + e)^2}, x\right)$$

38.88 Problem number 433

$$\int \frac{1}{(b \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \sin(fx + e)}{3bf \sqrt{b \sec(fx + e)}} \\ & + \frac{2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f} \end{aligned}$$

command

`integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{\frac{b}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) - i \sqrt{2} \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(fx+e) + i \sin(fx+e)) + i$$

$$3b^2f$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \sec(fx+e)}}{b^2 \sec(fx+e)^2}, x\right)$$

38.89 Problem number 434

$$\int \frac{\csc^2(e+fx)}{(b \sec(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\csc(fx+e)}{bf \sqrt{b \sec(fx+e)}} - \frac{\sqrt{\frac{\cos(fx+e)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx+e)}) \sqrt{b \sec(fx+e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f}$$

command

`integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$i \sqrt{2} \sqrt{b} \sin(fx+e) \text{weierstrassPInverse}(-4, 0, \cos(fx+e) + i \sin(fx+e)) - i \sqrt{2} \sqrt{b} \sin(fx+e) \text{weierstrassPInverse}(-4, 0, \cos(fx+e) + i \sin(fx+e))$$

$$2b^2f \sin(fx+e)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \sec(fx+e)} \csc(fx+e)^2}{b^2 \sec(fx+e)^2}, x\right)$$

38.90 Problem number 435

$$\int \frac{\csc^4(e + fx)}{(b \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\csc(fx + e)}{6bf \sqrt{b \sec(fx + e)}} - \frac{\csc^3(fx + e)}{3bf \sqrt{b \sec(fx + e)}} - \frac{\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{6 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f}$$

command

```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(i \cos(fx + e)^2 - i \right) \sqrt{b} \sin(fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + \sqrt{2} \left(-i \cos(fx + e) \right) \sqrt{b^2 f \cos(fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(fx + e)} \csc(fx + e)^4}{b^2 \sec(fx + e)^2}, x\right)$$

38.91 Problem number 436

$$\int \frac{\csc^6(e + fx)}{(b \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\csc(fx + e)}{12bf \sqrt{b \sec(fx + e)}} + \frac{\csc^3(fx + e)}{30bf \sqrt{b \sec(fx + e)}} - \frac{\csc^5(fx + e)}{5bf \sqrt{b \sec(fx + e)}} - \frac{\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{b \sec(fx + e)}}{12 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f}$$

command

`integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}\left(-i\cos(fx+e)^4+2i\cos(fx+e)^2-i\right)\sqrt{b}\sin(fx+e)\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b\sec(fx+e)}\csc(fx+e)^6}{b^2\sec(fx+e)^2},x\right)$$

38.92 Problem number 444

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} &-\frac{4b\sin(fx+e)}{39f(b\sec(fx+e))^{\frac{7}{2}}} + \frac{8\sin(fx+e)}{195bf(b\sec(fx+e))^{\frac{3}{2}}} - \frac{2b(\sin^3(fx+e))}{13f(b\sec(fx+e))^{\frac{7}{2}}} \\ &+ \frac{8\sqrt{\frac{\cos(fx+e)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{65\cos\left(\frac{fx}{2} + \frac{e}{2}\right)b^2f\sqrt{\cos(fx+e)}\sqrt{b\sec(fx+e)}} \end{aligned}$$

command

`integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\left(15\cos(fx+e)^6-25\cos(fx+e)^4+4\cos(fx+e)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin(fx+e)+6i\sqrt{2}\sqrt{b}\text{weierstrassZeta}\left(\frac{fx+e}{2},\sqrt{\frac{b}{\cos(fx+e)}}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(\cos(fx+e)^4-2\cos(fx+e)^2+1\right)\sqrt{b\sec(fx+e)}}{b^3\sec(fx+e)^3},x\right)$$

38.93 Problem number 445

$$\int \frac{\sin^2(e + fx)}{(b \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2b \sin(fx + e)}{9f (b \sec(fx + e))^{7/2}} + \frac{4 \sin(fx + e)}{45bf (b \sec(fx + e))^{3/2}} + \frac{4 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}}$$

command

```
integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((5 \cos(fx + e)^4 - 2 \cos(fx + e)^2) \sqrt{\frac{b}{\cos(fx + e)}} \sin(fx + e) - 3i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstra} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(\cos(fx + e)^2 - 1) \sqrt{b \sec(fx + e)}}{b^3 \sec(fx + e)^3}, x\right)$$

38.94 Problem number 446

$$\int \frac{1}{(b \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(fx + e)}{5bf (b \sec(fx + e))^{3/2}} + \frac{6 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}}$$

command

```
integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{\frac{b}{\cos(fx+e)}} \cos(fx+e)^2 \sin(fx+e) + 3i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx+e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(fx+e)}}{b^3 \sec(fx+e)^3}, x\right)$$

38.95 Problem number 447

$$\int \frac{\csc^2(e+fx)}{(b \sec(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{\csc(fx+e)}{bf(b \sec(fx+e))^{3/2}} - \frac{3 \sqrt{\frac{\cos(fx+e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f \sqrt{\cos(fx+e)} \sqrt{b \sec(fx+e)}}$$

command

`integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} \sqrt{b} \sin(fx+e) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx+e) + i \sin(fx+e))) + 3i \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(fx+e)} \csc(fx+e)^2}{b^3 \sec(fx+e)^3}, x\right)$$

38.96 Problem number 448

$$\int \frac{\csc^4(e+fx)}{(b \sec(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\csc(fx + e)}{2bf(b \sec(fx + e))^{\frac{3}{2}}} - \frac{\csc^3(fx + e)}{3bf(b \sec(fx + e))^{\frac{3}{2}}} + \frac{\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}}$$

command

```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2} \left(-i \cos(fx + e)^2 + i \right) \sqrt{b} \sin(fx + e) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e)) + i s$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(fx + e)} \csc(fx + e)^4}{b^3 \sec(fx + e)^3}, x\right)$$

38.97 Problem number 449

$$\int \frac{\csc^6(e + fx)}{(b \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3 \csc(fx + e)}{20bf(b \sec(fx + e))^{\frac{3}{2}}} + \frac{\csc^3(fx + e)}{10bf(b \sec(fx + e))^{\frac{3}{2}}} - \frac{\csc^5(fx + e)}{5bf(b \sec(fx + e))^{\frac{3}{2}}} \\ & + \frac{3\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{20 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f \sqrt{\cos(fx + e)} \sqrt{b \sec(fx + e)}} \end{aligned}$$

command

```
integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2} \left(-i \cos(fx + e)^4 + 2i \cos(fx + e)^2 - i \right) \sqrt{b} \sin(fx + e) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e)) + i s$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(fx + e)} \csc(fx + e)^6}{b^3 \sec(fx + e)^3}, x\right)$$

38.98 Problem number 458

$$\int \frac{\sqrt{b \sec(e + fx)}}{\sqrt{a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{2} + \frac{\sin(2fx + 2e)}{2}} \operatorname{EllipticF}\left(\cos\left(e + \frac{\pi}{4} + fx\right), \sqrt{2}\right) \sqrt{b \sec(fx + e)} \left(\sqrt{\sin(2fx + 2e)}\right)}{\sin\left(e + \frac{\pi}{4} + fx\right) f \sqrt{a \sin(fx + e)}}$$

command

```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{iab} \operatorname{ellipticF}(\cos(fx + e) + i \sin(fx + e), -1) + \sqrt{-iab} \operatorname{ellipticF}(\cos(fx + e) - i \sin(fx + e), -1)}{af}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(fx + e)} \sqrt{a \sin(fx + e)}}{a \sin(fx + e)}, x\right)$$

38.99 Problem number 459

$$\int \frac{\sqrt{b \sec(e + fx)}}{(a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2b}{3af (a \sin(fx + e))^{\frac{3}{2}} \sqrt{b \sec(fx + e)}}}{2\sqrt{\frac{1}{2} + \frac{\sin(2fx + 2e)}{2}} \operatorname{EllipticF}\left(\cos\left(e + \frac{\pi}{4} + fx\right), \sqrt{2}\right) \sqrt{b \sec(fx + e)} \left(\sqrt{\sin(2fx + 2e)}\right)}{3 \sin\left(e + \frac{\pi}{4} + fx\right) a^2 f \sqrt{a \sin(fx + e)}}$$

command

```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{i ab} \left(\cos (fx + e)^2 - 1 \right) \text{ellipticF}(\cos (fx + e) + i \sin (fx + e), -1) + \sqrt{-i ab} \left(\cos (fx + e)^2 - 1 \right) \text{ellipticF}(\cos (fx + e) - i \sin (fx + e), -1) \right)}{3 \left(a^3 f \cos (fx + e)^2 - a^3 f \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{b \sec (fx + e)} \sqrt{a \sin (fx + e)}}{\left(a^3 \cos (fx + e)^2 - a^3 \right) \sin (fx + e)}, x \right)$$

38.100 Problem number 460

$$\int \frac{\sqrt{b \sec (e + fx)}}{(a \sin (e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2b}{7af (a \sin (fx + e))^{\frac{7}{2}} \sqrt{b \sec (fx + e)}} - \frac{4b}{7a^3 f (a \sin (fx + e))^{\frac{3}{2}} \sqrt{b \sec (fx + e)}}}{4 \sqrt{\frac{1}{2} + \frac{\sin (2fx + 2e)}{2}} \text{EllipticF} \left(\cos \left(e + \frac{\pi}{4} + fx \right), \sqrt{2} \right) \sqrt{b \sec (fx + e)} \left(\sqrt{\sin (2fx + 2e)} \right)}{7 \sin \left(e + \frac{\pi}{4} + fx \right) a^4 f \sqrt{a \sin (fx + e)}}$$

command

```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \left(\cos (fx + e)^4 - 2 \cos (fx + e)^2 + 1 \right) \sqrt{i ab} \text{ellipticF}(\cos (fx + e) + i \sin (fx + e), -1) + 2 \left(\cos (fx + e)^4 - 2 \cos (fx + e)^2 + 1 \right) \sqrt{-i ab} \text{ellipticF}(\cos (fx + e) - i \sin (fx + e), -1) \right)}{7 \left(a^5 f \cos (fx + e)^2 - a^5 f \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \sec (fx + e)} \sqrt{a \sin (fx + e)}}{\left(a^5 \cos (fx + e)^4 - 2 a^5 \cos (fx + e)^2 + a^5 \right) \sin (fx + e)}, x \right)$$

38.101 Problem number 480

$$\int \frac{1}{(b \sec(e + fx))^{3/2} (a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2}{3abf (a \sin(fx + e))^{\frac{3}{2}} \sqrt{b \sec(fx + e)}} \\ & + \frac{\sqrt{\frac{1}{2} + \frac{\sin(2fx + 2e)}{2}} \operatorname{EllipticF}\left(\cos\left(e + \frac{\pi}{4} + fx\right), \sqrt{2}\right) \sqrt{b \sec(fx + e)} \left(\sqrt{\sin(2fx + 2e)}\right)}{3 \sin\left(e + \frac{\pi}{4} + fx\right) a^2 b^2 f \sqrt{a \sin(fx + e)}} \end{aligned}$$

command

`integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{iab} \left(\cos(fx + e)^2 - 1\right) \operatorname{ellipticF}(\cos(fx + e) + i \sin(fx + e), -1) + \sqrt{-iab} \left(\cos(fx + e)^2 - 1\right) \operatorname{ellipticF}(\cos(fx + e) - i \sin(fx + e), -1)}{3 \left(a^3 b^2 f \cos(fx + e)^2 - a^3 b^2 f\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{b \sec(fx + e)} \sqrt{a \sin(fx + e)}}{\left(a^3 b^2 \cos(fx + e)^2 - a^3 b^2\right) \sec(fx + e)^2 \sin(fx + e)}, x\right)$$

38.102 Problem number 481

$$\int \frac{1}{(b \sec(e + fx))^{3/2} (a \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2}{7abf (a \sin(fx + e))^{\frac{7}{2}} \sqrt{b \sec(fx + e)}} + \frac{2}{21a^3bf (a \sin(fx + e))^{\frac{3}{2}} \sqrt{b \sec(fx + e)}} \\ & + \frac{2\sqrt{\frac{1}{2} + \frac{\sin(2fx + 2e)}{2}} \operatorname{EllipticF}\left(\cos\left(e + \frac{\pi}{4} + fx\right), \sqrt{2}\right) \sqrt{b \sec(fx + e)} \left(\sqrt{\sin(2fx + 2e)}\right)}{21 \sin\left(e + \frac{\pi}{4} + fx\right) a^4 b^2 f \sqrt{a \sin(fx + e)}} \end{aligned}$$

command

`integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((\cos(fx + e)^4 - 2 \cos(fx + e)^2 + 1) \sqrt{iab} \operatorname{ellipticF}(\cos(fx + e) + i \sin(fx + e), -1) + (\cos(fx + e)^4 - 2 \cos(fx + e)^2 + 1) \sqrt{iab} \operatorname{ellipticF}(\cos(fx + e) - i \sin(fx + e), -1) \right)$$

$$21 \left(a^5 b^2 f \cos(fx + e) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \sec(fx + e)} \sqrt{a \sin(fx + e)}}{\left(a^5 b^2 \cos(fx + e)^4 - 2 a^5 b^2 \cos(fx + e)^2 + a^5 b^2 \right) \sec(fx + e)^2 \sin(fx + e)}, x \right)$$

38.103 Problem number 482

$$\int \frac{1}{(b \sec(e + fx))^{3/2} (a \sin(e + fx))^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2}{11abf (a \sin(fx + e))^{\frac{11}{2}} \sqrt{b \sec(fx + e)}} + \frac{2}{77a^3bf (a \sin(fx + e))^{\frac{7}{2}} \sqrt{b \sec(fx + e)}} \\ & + \frac{4}{77a^5bf (a \sin(fx + e))^{\frac{3}{2}} \sqrt{b \sec(fx + e)}} \\ & + \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(2fx + 2e)}{2}} \operatorname{EllipticF} \left(\cos \left(e + \frac{\pi}{4} + fx \right), \sqrt{2} \right) \sqrt{b \sec(fx + e)} \left(\sqrt{\sin(2fx + 2e)} \right)}{77 \sin \left(e + \frac{\pi}{4} + fx \right) a^6 b^2 f \sqrt{a \sin(fx + e)}} \end{aligned}$$

command

`integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \left(\cos(fx + e)^6 - 3 \cos(fx + e)^4 + 3 \cos(fx + e)^2 - 1 \right) \sqrt{iab} \operatorname{ellipticF}(\cos(fx + e) + i \sin(fx + e), -1) + 2 \left(\cos(fx + e)^6 - 3 \cos(fx + e)^4 + 3 \cos(fx + e)^2 - 1 \right) \sqrt{iab} \operatorname{ellipticF}(\cos(fx + e) - i \sin(fx + e), -1) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{b \sec(fx + e)} \sqrt{a \sin(fx + e)}}{\left(a^7 b^2 \cos(fx + e)^6 - 3 a^7 b^2 \cos(fx + e)^4 + 3 a^7 b^2 \cos(fx + e)^2 - a^7 b^2 \right) \sec(fx + e)^2 \sin(fx + e)}, x \right)$$

38.104 Problem number 507

$$\int \sqrt{d \csc(e + fx)} \sin^4(e + fx) dx$$

Optimal antiderivative

$$\frac{-\frac{2d^3 \cos(fx + e)}{7f (d \csc(fx + e))^{\frac{5}{2}}} - \frac{10d \cos(fx + e)}{21f \sqrt{d \csc(fx + e)}}}{10 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{21 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f}$$

command

```
integrate(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \cos(fx + e)^3 - 8 \cos(fx + e) \right) \sqrt{\frac{d}{\sin(fx + e)}} \sin(fx + e) - 5i \sqrt{2id} \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e))}{21 f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(\cos(fx + e)^4 - 2 \cos(fx + e)^2 + 1\right) \sqrt{d \csc(fx + e)}, x\right)$$

38.105 Problem number 508

$$\int \sqrt{d \csc(e + fx)} \sin^3(e + fx) dx$$

Optimal antiderivative

$$\frac{-\frac{2d^2 \cos(fx + e)}{5f (d \csc(fx + e))^{\frac{3}{2}}} - \frac{6d \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \csc(fx + e)} \sqrt{\sin(fx + e)}}}{}$$

command

```
integrate(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\cos (fx + e)^3 - \cos (fx + e) \right) \sqrt{\frac{d}{\sin (fx + e)}} + 3 \sqrt{2i d} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e)))}{5 f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(\cos (fx + e)^2 - 1\right) \sqrt{d \operatorname{csc}(fx + e)} \sin (fx + e), x\right)$$

38.106 Problem number 509

$$\int \sqrt{d \operatorname{csc}(e + fx)} \sin^2(e + fx) dx$$

Optimal antiderivative

$$\frac{-\frac{2d \cos (fx + e)}{3f \sqrt{d \operatorname{csc}(fx + e)}} - 2 \sqrt{\frac{1}{2} + \frac{\sin (fx + e)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \operatorname{csc}(fx + e)} \left(\sqrt{\sin (fx + e)}\right)}{3 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f}$$

command

```
integrate(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{\frac{d}{\sin (fx + e)}} \cos (fx + e) \sin (fx + e) + i \sqrt{2i d} \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e)) - i}{3 f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(\cos (fx + e)^2 - 1\right) \sqrt{d \operatorname{csc}(fx + e)}, x\right)$$

38.107 Problem number 510

$$\int \sqrt{d \operatorname{csc}(e + fx)} \sin(e + fx) dx$$

Optimal antiderivative

$$\frac{2d\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \csc(fx + e)} \sqrt{\sin(fx + e)}}$$

command

```
integrate(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2id} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e))) + \sqrt{-2id} \operatorname{weierstrassZeta}(4, 0, \cos(fx + e) + i \sin(fx + e))}{f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc(fx + e)} \sin(fx + e), x\right)$$

38.108 Problem number 511

$$\int \sqrt{d \csc(e + fx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f}$$

command

```
integrate((d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2id} \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)) + i\sqrt{-2id} \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e))}{f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc(fx + e)}, x\right)$$

38.109 Problem number 512

$$\int \csc(e + fx) \sqrt{d \csc(e + fx)} dx$$

Optimal antiderivative

$$-\frac{2 \cos(fx + e) \sqrt{d \csc(fx + e)}}{f} + \frac{2d \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \csc(fx + e)} \sqrt{\sin(fx + e)}}$$

command

```
integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2 \sqrt{\frac{d}{\sin(fx + e)}} \cos(fx + e) + \sqrt{2id} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)))}{f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc(fx + e)} \csc(fx + e), x\right)$$

38.110 Problem number 513

$$\int \csc^2(e + fx) \sqrt{d \csc(e + fx)} dx$$

Optimal antiderivative

$$-\frac{2 \cos(fx + e) (d \csc(fx + e))^{\frac{3}{2}}}{3df} - \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f}$$

command

```
integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2i d} \sin (fx + e) \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e)) + i \sqrt{-2i d} \sin (fx + e) \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e))}{3 f \sin (fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc (fx + e)} \csc (fx + e)^2, x\right)$$

38.111 Problem number 514

$$\int \csc^3(e + fx) \sqrt{d \csc(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cos (fx + e) (d \csc (fx + e))^{\frac{5}{2}}}{5 d^2 f} - \frac{6 \cos (fx + e) \sqrt{d \csc (fx + e)}}{5 f} \\ & + \frac{6 d \sqrt{\frac{1}{2} + \frac{\sin (fx + e)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{5 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \csc (fx + e)} \sqrt{\sin (fx + e)}} \end{aligned}$$

command

`integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(\cos (fx + e)^2 - 1 \right) \sqrt{2i d} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e))) + 3 \left(\cos (fx + e)^2 - 1 \right) \sqrt{-2i d} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc (fx + e)} \csc (fx + e)^3, x\right)$$

38.112 Problem number 515

$$\int (d \csc(e + fx))^{3/2} \sin^5(e + fx) dx$$

Optimal antiderivative

$$\frac{2d^4 \cos(fx + e)}{7f (d \csc(fx + e))^{\frac{5}{2}}} - \frac{10d^2 \cos(fx + e)}{21f \sqrt{d \csc(fx + e)}} - \frac{10d \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{21 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f}$$

command

```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3d \cos(fx + e)^3 - 8d \cos(fx + e) \right) \sqrt{\frac{d}{\sin(fx + e)}} \sin(fx + e) - 5i \sqrt{2id} \operatorname{dweierstrassPInverse}(4, 0, \cos(fx + e))}{21f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(d \cos(fx + e)^4 - 2d \cos(fx + e)^2 + d\right) \sqrt{d \csc(fx + e)} \csc(fx + e) \sin(fx + e), x\right)$$

38.113 Problem number 516

$$\int (d \csc(e + fx))^{3/2} \sin^4(e + fx) dx$$

Optimal antiderivative

$$\frac{2d^3 \cos(fx + e)}{5f (d \csc(fx + e))^{\frac{3}{2}}} - \frac{6d^2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \csc(fx + e)} \sqrt{\sin(fx + e)}}$$

command

```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \sqrt{2i d} \operatorname{dweierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e))) + 3 \sqrt{-2i d} \operatorname{dweierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(d \cos (fx + e)^4 - 2 d \cos (fx + e)^2 + d\right) \sqrt{d \operatorname{csc}(fx + e)} \operatorname{csc}(fx + e), x\right)$$

38.114 Problem number 517

$$\int (d \operatorname{csc}(e + fx))^{3/2} \sin^3(e + fx) dx$$

Optimal antiderivative

$$\frac{\frac{2d^2 \cos (fx + e)}{3f \sqrt{d \operatorname{csc}(fx + e)}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \operatorname{csc}(fx + e)} \left(\sqrt{\sin}(fx + e)\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f}$$

command

```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2d \sqrt{\frac{d}{\sin (fx + e)}} \cos (fx + e) \sin (fx + e) + i \sqrt{2i d} \operatorname{dweierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e))}{3f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(d \cos (fx + e)^2 - d\right) \sqrt{d \operatorname{csc}(fx + e)} \operatorname{csc}(fx + e) \sin (fx + e), x\right)$$

38.115 Problem number 518

$$\int (d \operatorname{csc}(e + fx))^{3/2} \sin^2(e + fx) dx$$

Optimal antiderivative

$$\frac{2d^2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \csc(fx + e)} \sqrt{\sin(fx + e)}}$$

command

```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2id} \operatorname{dweierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e))) + \sqrt{-2id} \operatorname{dweierstrassZeta}(4, 0, \cos(fx + e) + i \sin(fx + e))}{f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(d \cos(fx + e)^2 - d\right) \sqrt{d \csc(fx + e)} \csc(fx + e), x\right)$$

38.116 Problem number 519

$$\int (d \csc(e + fx))^{3/2} \sin(e + fx) dx$$

Optimal antiderivative

$$\frac{2d \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f}$$

command

```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2id} \operatorname{dweierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)) + i \sqrt{-2id} \operatorname{dweierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e))}{f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc(fx + e)} d \csc(fx + e) \sin(fx + e), x\right)$$

38.117 Problem number 520

$$\int (d \csc(e + fx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{2d \cos(fx + e) \sqrt{d \csc(fx + e)}}{f} + \frac{2d^2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \csc(fx + e)} \sqrt{\sin(fx + e)}}$$

command

```
integrate((d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2d \sqrt{\frac{d}{\sin(fx + e)}} \cos(fx + e) + \sqrt{2id} d \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)))}{f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc(fx + e)} d \csc(fx + e), x\right)$$

38.118 Problem number 521

$$\int \csc(e + fx)(d \csc(e + fx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{2 \cos(fx + e) (d \csc(fx + e))^{\frac{3}{2}}}{3f} - \frac{2d \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f}$$

command

```
integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2i d} d \sin (fx + e) \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e)) + i \sqrt{-2i d} d \sin (fx + e) \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e))}{3 f \sin (fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc (fx + e)} d \csc (fx + e)^2, x\right)$$

38.119 Problem number 522

$$\int \csc^2(e + fx)(d \csc(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2 \cos (fx + e) (d \csc (fx + e))^{\frac{5}{2}}}{5df} - \frac{6d \cos (fx + e) \sqrt{d \csc (fx + e)}}{5f} + \frac{6d^2 \sqrt{\frac{1}{2} + \frac{\sin (fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \csc (fx + e)} \sqrt{\sin (fx + e)}}$$

command

```
integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(d \cos (fx + e)^2 - d \right) \sqrt{2i d} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e))) + 3 \left(d \cos (fx + e)^2 - d \right) \sqrt{-2i d} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc (fx + e)} d \csc (fx + e)^3, x\right)$$

38.120 Problem number 523

$$\int \frac{\sin^3(e + fx)}{\sqrt{d \csc(e + fx)}} dx$$

Optimal antiderivative

$$\frac{-\frac{2d^2 \cos(fx + e)}{7f (d \csc(fx + e))^{\frac{5}{2}}} - \frac{10 \cos(fx + e)}{21f \sqrt{d \csc(fx + e)}}}{10 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{21 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df}$$

command

```
integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \cos(fx + e)^3 - 8 \cos(fx + e) \right) \sqrt{\frac{d}{\sin(fx + e)}} \sin(fx + e) - 5i \sqrt{2id} \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e))}{21 df}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(\cos(fx + e)^2 - 1\right) \sqrt{d \csc(fx + e)} \sin(fx + e)}{d \csc(fx + e)}, x\right)$$

38.121 Problem number 524

$$\int \frac{\sin^2(e + fx)}{\sqrt{d \csc(e + fx)}} dx$$

Optimal antiderivative

$$\frac{-\frac{2d \cos(fx + e)}{5f (d \csc(fx + e))^{\frac{3}{2}}} - \frac{6 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \csc(fx + e)} \sqrt{\sin(fx + e)}}}{21 df}$$

command

```
integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\cos(fx + e)^3 - \cos(fx + e) \right) \sqrt{\frac{d}{\sin(fx + e)}} + 3 \sqrt{2id} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e))) - i \sqrt{d} \text{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{(\cos(fx + e)^2 - 1) \sqrt{d \csc(fx + e)}}{d \csc(fx + e)}, x \right)$$

38.122 Problem number 525

$$\int \frac{\sin(e + fx)}{\sqrt{d \csc(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2 \cos(fx + e)}{3f \sqrt{d \csc(fx + e)}} - \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticF} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right), \sqrt{2} \right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)} \right)}{3 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) df}$$

command

```
integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{\frac{d}{\sin(fx + e)}} \cos(fx + e) \sin(fx + e) + i \sqrt{2id} \text{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)) - i \sqrt{d} \text{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{d \csc(fx + e)} \sin(fx + e)}{d \csc(fx + e)}, x \right)$$

38.123 Problem number 526

$$\int \frac{1}{\sqrt{d \csc(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \csc(fx + e)} \sqrt{\sin(fx + e)}}$$

command

```
integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2id} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e))) + \sqrt{-2id} \operatorname{weierstrassZeta}(4, 0, \cos(fx + e) + i \sin(fx + e))}{df}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \csc(fx + e)}}{d \csc(fx + e)}, x\right)$$

38.124 Problem number 527

$$\int \frac{\csc(e + fx)}{\sqrt{d \csc(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df}$$

command

```
integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2id} \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)) + i \sqrt{-2id} \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e))}{df}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \csc(fx + e)}}{d}, x\right)$$

38.125 Problem number 528

$$\int \frac{\csc^2(e + fx)}{\sqrt{d \csc(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{2 \cos(fx + e) \sqrt{d \csc(fx + e)}}{df} + \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \csc(fx + e)} \sqrt{\sin(fx + e)}}$$

command

```
integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{\frac{d}{\sin(fx + e)}} \cos(fx + e) + \sqrt{2id} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)))$$

df

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \csc(fx + e)} \csc(fx + e)}{d}, x\right)$$

38.126 Problem number 529

$$\int \frac{\csc^3(e + fx)}{\sqrt{d \csc(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{2 \cos(fx + e) (d \csc(fx + e))^{\frac{3}{2}}}{3d^2 f} - \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df}$$

command

```
integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2i d} \sin (fx + e) \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e)) + i \sqrt{-2i d} \sin (fx + e) \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) - i \sin (fx + e))}{3 df \sin (fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \csc (fx + e)} \csc (fx + e)^2}{d}, x\right)$$

38.127 Problem number 530

$$\int \frac{\sin^2(e + fx)}{(d \csc(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2d \cos (fx + e)}{7f (d \csc (fx + e))^{\frac{5}{2}}} - \frac{10 \cos (fx + e)}{21df \sqrt{d \csc (fx + e)}}}{10 \sqrt{\frac{1}{2} + \frac{\sin (fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc (fx + e)} \left(\sqrt{\sin (fx + e)}\right)}{21 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f}$$

command

`integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \cos (fx + e)^3 - 8 \cos (fx + e)\right) \sqrt{\frac{d}{\sin (fx + e)}} \sin (fx + e) - 5i \sqrt{2i d} \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e)) + 5i \sqrt{-2i d} \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) - i \sin (fx + e))}{21 d^2 f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(\cos (fx + e)^2 - 1\right) \sqrt{d \csc (fx + e)}}{d^2 \csc (fx + e)^2}, x\right)$$

38.128 Problem number 531

$$\int \frac{\sin(e + fx)}{(d \csc(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2 \cos(fx + e)}{5f (d \csc(fx + e))^{3/2}} - \frac{6 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{d \csc(fx + e)} \sqrt{\sin(fx + e)}}$$

command

```
integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\cos(fx + e)^3 - \cos(fx + e) \right) \sqrt{\frac{d}{\sin(fx + e)}} + 3 \sqrt{2id} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e)))$$

$$5 d^2 f$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \csc(fx + e)} \sin(fx + e)}{d^2 \csc(fx + e)^2}, x\right)$$

38.129 Problem number 532

$$\int \frac{1}{(d \csc(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2 \cos(fx + e)}{3df \sqrt{d \csc(fx + e)}} - \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f}$$

command

```
integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{\frac{d}{\sin(fx+e)}} \cos(fx+e) \sin(fx+e) + i \sqrt{2id} \operatorname{weierstrassPInverse}(4, 0, \cos(fx+e) + i \sin(fx+e)) - i \sqrt{2id} \operatorname{weierstrassPInverse}(4, 0, \cos(fx+e) - i \sin(fx+e))}{3d^2f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \csc(fx+e)}}{d^2 \csc(fx+e)^2}, x\right)$$

38.130 Problem number 533

$$\int \frac{\csc(e+fx)}{(d \csc(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{d \csc(fx+e)} \sqrt{\sin(fx+e)}}$$

command

`integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2id} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx+e) + i \sin(fx+e))) + \sqrt{-2id} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx+e) - i \sin(fx+e)))}{d^2f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \csc(fx+e)}}{d^2 \csc(fx+e)}, x\right)$$

38.131 Problem number 534

$$\int \frac{\csc^2(e+fx)}{(d \csc(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx+e)} \left(\sqrt{\sin(fx+e)}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2f}$$

command

```
integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2id}\operatorname{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{-2id}\operatorname{weierstrassPInverse}(4,0,\cos(fx+e))}{d^2f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d\csc(fx+e)}}{d^2},x\right)$$

38.132 Problem number 535

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2\cos(fx+e)\sqrt{d\csc(fx+e)}}{d^2f} + \frac{2\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{d\csc(fx+e)} \sqrt{\sin(fx+e)}}$$

command

```
integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)+\sqrt{2id}\operatorname{weierstrassZeta}(4,0,\operatorname{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e)))}{d^2f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d\csc(fx+e)}\csc(fx+e)}{d^2},x\right)$$

38.133 Problem number 536

$$\int \frac{\csc^4(e + fx)}{(d \csc(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2 \cos(fx + e) (d \csc(fx + e))^{\frac{3}{2}}}{3d^3 f} \\ & - \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \csc(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f} \end{aligned}$$

command

`integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -i \sqrt{2id} \sin(fx + e) \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)) + i \sqrt{-2id} \sin(fx + e) \operatorname{weierstrassP} \\ & \hline & 3 d^2 f \sin(fx + e) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \csc(fx + e)} \csc(fx + e)^2}{d^2}, x\right)$$

38.134 Problem number 537

$$\int \frac{\csc^5(e + fx)}{(d \csc(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2 \cos(fx + e) (d \csc(fx + e))^{\frac{5}{2}}}{5d^4 f} - \frac{6 \cos(fx + e) \sqrt{d \csc(fx + e)}}{5d^2 f} \\ & + \frac{6 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{d \csc(fx + e)} \sqrt{\sin(fx + e)}} \end{aligned}$$

command

`integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(\cos (fx + e)^2 - 1 \right) \sqrt{2id} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e))) + 3 \left(\cos (fx + e) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{d \csc (fx + e)} \csc (fx + e)^3}{d^2}, x \right)$$

39 Test file number 67

Test folder name:

test_cases/4_Trig_functions/4.1_Sine/67_4.1.1.1-a+b_sin-^n

39.1 Problem number 50

$$\int (a + b \sin (c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{24ab \cos (dx + c) (a + b \sin (dx + c))^{3/2}}{35d} - \frac{2b \cos (dx + c) (a + b \sin (dx + c))^{5/2}}{7d} \\ & - \frac{2b(71a^2 + 25b^2) \cos (dx + c) \sqrt{a + b \sin (dx + c)}}{105d} \\ & - \frac{32a(11a^2 + 13b^2) \sqrt{\frac{1}{2} + \frac{\sin (dx + c)}{2}} \text{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{a + b \sin (dx + c)}}{105 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) d \sqrt{\frac{a + b \sin (dx + c)}{a + b}}} \\ & + \frac{2(71a^4 - 46a^2b^2 - 25b^4) \sqrt{\frac{1}{2} + \frac{\sin (dx + c)}{2}} \text{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{\frac{a + b \sin (dx + c)}{a + b}}}{105 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) d \sqrt{a + b \sin (dx + c)}} \end{aligned}$$

command

`integrate((a+b*sin(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (37a^4 - 346a^2b^2 - 75b^4) \sqrt{ib} \operatorname{weierstrassPInverse} \left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(dx+c) - 3ib \sin(dx+c) - 2ia}{3b} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\left(3ab^2 \cos(dx+c)^2 - a^3 - 3ab^2 + \left(b^3 \cos(dx+c)^2 - 3a^2b - b^3 \right) \sin(dx+c) \right) \sqrt{b \sin(dx+c) + a}, x \right)$$

39.2 Problem number 51

$$\int (a + b \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b \cos(dx+c) (a + b \sin(dx+c))^{3/2}}{5d} - \frac{16ab \cos(dx+c) \sqrt{a + b \sin(dx+c)}}{15d} \\ + \frac{2(23a^2 + 9b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a + b \sin(dx+c)}}{15 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) d \sqrt{\frac{a + b \sin(dx+c)}{a+b}}} \\ + \frac{16a(a^2 - b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{\frac{a + b \sin(dx+c)}{a+b}}}{15 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) d \sqrt{a + b \sin(dx+c)}}$$

command

`integrate((a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (a^3 - 33ab^2) \sqrt{ib} \operatorname{weierstrassPInverse} \left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(dx+c) - 3ib \sin(dx+c) - 2ia}{3b} \right) + \sqrt{2} (a^3$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\left(b^2 \cos(dx+c)^2 - 2ab \sin(dx+c) - a^2 - b^2 \right) \sqrt{b \sin(dx+c) + a}, x \right)$$

39.3 Problem number 52

$$\int (a + b \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b \cos(dx + c) \sqrt{a + b \sin(dx + c)}}{3d} - \frac{8a \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} + \frac{2(a^2 - b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{a + b \sin(dx + c)}}$$

command

`integrate((a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-12i \sqrt{2} a \sqrt{ib} b \operatorname{weierstrassZeta}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8ia^3 - 9iab^2)}{27b^3}\right), \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8ia^3 - 9iab^2)}{27b^3}\right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((b \sin(dx + c) + a)^{\frac{3}{2}}, x\right)$$

39.4 Problem number 53

$$\int \sqrt{a + b \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}$$

command

`integrate((a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} a \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(dx+c)-3ib \sin(dx+c)-2ia}{3b}\right) + \sqrt{2} a \sqrt{-i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib \sin(dx+c)-2ia}{3b}\right)}{bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sin(dx+c)+a}, x\right)$$

39.5 Problem number 54

$$\int \frac{1}{\sqrt{a+b \sin(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \sin(dx+c)}{a+b}}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{a+b \sin(dx+c)}}$$

command

`integrate(1/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(dx+c)-3ib \sin(dx+c)-2ia}{3b}\right) + \sqrt{2} \sqrt{-i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib \sin(dx+c)-2ia}{3b}\right)}{bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{b \sin(dx+c)+a}}, x\right)$$

39.6 Problem number 55

$$\int \frac{1}{(a + b \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2b \cos(dx + c)}{(a^2 - b^2) d \sqrt{a + b \sin(dx + c)}}{2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) (a^2 - b^2) d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}$$

command

`integrate(1/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \sin(dx + c) + a} b^2 \cos(dx + c) + \left(\sqrt{2} ab \sin(dx + c) + \sqrt{2} a^2\right) \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{b \sin(dx + c) + a}}{b^2 \cos^2(dx + c) - 2ab \sin(dx + c) - a^2 - b^2}, x\right)$$

39.7 Problem number 56

$$\int \frac{1}{(a + b \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2b \cos(dx + c)}{3(a^2 - b^2) d (a + b \sin(dx + c))^{3/2}} + \frac{8ab \cos(dx + c)}{3(a^2 - b^2)^2 d \sqrt{a + b \sin(dx + c)}}{8a \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) (a^2 - b^2)^2 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}$$

$$+ \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) (a^2 - b^2) d \sqrt{a + b \sin(dx + c)}}$$

command

`integrate(1/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (a^2 b^2 + 3 b^4) \cos(dx + c)^2 - 2 \sqrt{2} (a^3 b + 3 a b^3) \sin(dx + c) - \sqrt{2} (a^4 + 4 a^2 b^2 + 3 b^4) \right) \sqrt{i b} \text{weierstrassPInv}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\sqrt{b \sin(dx + c) + a}}{3 a b^2 \cos(dx + c)^2 - a^3 - 3 a b^2 + (b^3 \cos(dx + c)^2 - 3 a^2 b - b^3) \sin(dx + c)}, x \right)$$

39.8 Problem number 57

$$\int \frac{1}{(a + b \sin(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b \cos(dx + c)}{5(a^2 - b^2) d(a + b \sin(dx + c))^{5/2}} + \frac{16ab \cos(dx + c)}{15(a^2 - b^2)^2 d(a + b \sin(dx + c))^{3/2}} \\ & + \frac{2b(23a^2 + 9b^2) \cos(dx + c)}{15(a^2 - b^2)^3 d \sqrt{a + b \sin(dx + c)}} \\ & \frac{2(23a^2 + 9b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{a + b \sin(dx + c)}}{15 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) (a^2 - b^2)^3 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\ & + \frac{16a \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{15 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) (a^2 - b^2)^2 d \sqrt{a + b \sin(dx + c)}} \end{aligned}$$

command

`integrate(1/(a+b*sin(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(3 \sqrt{2} (a^4 b^2 - 33 a^2 b^4) \cos(dx + c)^2 + \left(\sqrt{2} (a^3 b^3 - 33 a b^5) \cos(dx + c)^2 - \sqrt{2} (3 a^5 b - 98 a^3 b^3 - 33 a b^5) \right) \sin(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \sin(dx + c) + a}}{b^4 \cos(dx + c)^4 + a^4 + 6 a^2 b^2 + b^4 - 2(3 a^2 b^2 + b^4) \cos(dx + c)^2 - 4(ab^3 \cos(dx + c)^2 - a^3 b - ab^3) \sin(dx + c)}, x \right)$$

40 Test file number 70

Test folder name:

test_cases/4_Trig_functions/4.1_Sine/70_4.1.1.2-g_cos-^p-a+b_sin-^m

40.1 Problem number 196

$$\int (e \cos(c + dx))^{7/2} (a + a \sin(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(e \cos(dx + c))^{\frac{9}{2}}}{9de} + \frac{2ae(e \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7d} \\ & + \frac{10a e^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & + \frac{10a e^3 \sin(dx + c) \sqrt{e \cos(dx + c)}}{21d} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i \sqrt{2} a e^{\frac{7}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} a e^{\frac{7}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

63

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(ae^3 \cos(dx + c)^3 \sin(dx + c) + ae^3 \cos(dx + c)^3\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.2 Problem number 197

$$\int (e \cos(c + dx))^{5/2} (a + a \sin(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(e \cos(dx + c))^{\frac{7}{2}}}{7de} + \frac{2ae(e \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5d} \\ & + \frac{6a e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i \sqrt{2} a e^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2} a e^{\frac{5}{2}} \text{weierstra}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(ae^2 \cos(dx + c)^2 \sin(dx + c) + ae^2 \cos(dx + c)^2\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.3 Problem number 198

$$\int (e \cos(c + dx))^{3/2} (a + a \sin(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(e \cos(dx + c))^{\frac{5}{2}}}{5de} \\ & + \frac{2a e^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & + \frac{2ae \sin(dx + c) \sqrt{e \cos(dx + c)}}{3d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} a e^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} a e^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

15 d

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(ae \cos(dx + c) \sin(dx + c) + ae \cos(dx + c)\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.4 Problem number 199

$$\int \sqrt{e \cos(c + dx)} (a + a \sin(c + dx)) dx$$

Optimal antiderivative

$$-\frac{2a(e \cos(dx + c))^{\frac{3}{2}}}{3de} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2a \cos(dx + c)^{\frac{3}{2}} e^{\frac{1}{2}} - 3i \sqrt{2} a e^{\frac{1}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{e \cos(dx + c)} (a \sin(dx + c) + a), x\right)$$

40.5 Problem number 200

$$\int \frac{a + a \sin(c + dx)}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} - \frac{2a \sqrt{e \cos(dx + c)}}{de}$$

command

```
integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(-i \sqrt{2} a \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)} (a \sin(dx + c) + a)}{e \cos(dx + c)}, x\right)$$

40.6 Problem number 201

$$\int \frac{a + a \sin(c + dx)}{(e \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2a}{de \sqrt{e \cos(dx + c)}} + \frac{2a \sin(dx + c)}{de \sqrt{e \cos(dx + c)}} - \frac{2a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^2 \sqrt{\cos(dx + c)}}$$

command

```
integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(i \sqrt{2} a \cos(dx + c) - i \sqrt{2} a \sin(dx + c) + i \sqrt{2} a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)))}{e^2 \sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)} (a \sin(dx + c) + a)}{e^2 \cos(dx + c)^2}, x\right)$$

40.7 Problem number 202

$$\int \frac{a + a \sin(c + dx)}{(e \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a}{3de (e \cos(dx + c))^{3/2}} + \frac{2a \sin(dx + c)}{3de (e \cos(dx + c))^{3/2}} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^2 \sqrt{e \cos(dx + c)}}$$

command

```
integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i\sqrt{2}a\sin(dx+c)+i\sqrt{2}a\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}a\sin(dx+c)-i\sqrt{2}a\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{3\left(de^{\frac{5}{2}}\sin(dx+c)-de^{\frac{5}{2}}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)}{e^3\cos(dx+c)^3},x\right)$$

40.8 Problem number 203

$$\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2a}{5de(e\cos(dx+c))^{\frac{5}{2}}} + \frac{2a\sin(dx+c)}{5de(e\cos(dx+c))^{\frac{5}{2}}} + \frac{6a\sin(dx+c)}{5de^3\sqrt{e\cos(dx+c)}} - \frac{6a\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\sqrt{e\cos(dx+c)}}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)de^4\sqrt{\cos(dx+c)}}$$

command

```
integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\left(i\sqrt{2}a\cos(dx+c)\sin(dx+c)-i\sqrt{2}a\cos(dx+c)\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(-i\sqrt{2}a\cos(dx+c)\sin(dx+c)+i\sqrt{2}a\cos(dx+c)\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{3\left(de^{\frac{5}{2}}\sin(dx+c)-de^{\frac{5}{2}}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)}{e^4\cos(dx+c)^4},x\right)$$

40.9 Problem number 204

$$\int (e \cos(c + dx))^{7/2} (a + a \sin(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{26a^2(e \cos(dx + c))^{\frac{9}{2}}}{99de} + \frac{26a^2e(e \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{77d} \\ & - \frac{2(e \cos(dx + c))^{\frac{9}{2}} (a^2 + a^2 \sin(dx + c))}{11de} \\ & + \frac{130a^2e^4 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & + \frac{130a^2e^3 \sin(dx + c) \sqrt{e \cos(dx + c)}}{231d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-195i \sqrt{2} a^2 e^{\frac{7}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 195i \sqrt{2} a^2 e^{\frac{7}{2}} \operatorname{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(a^2 e^3 \cos(dx + c)^5 - 2a^2 e^3 \cos(dx + c)^3 \sin(dx + c) - 2a^2 e^3 \cos(dx + c)^3\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.10 Problem number 205

$$\int (e \cos(c + dx))^{5/2} (a + a \sin(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{22a^2(e \cos(dx + c))^{\frac{7}{2}}}{63de} + \frac{22a^2e(e \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45d} \\ & - \frac{2(e \cos(dx + c))^{\frac{7}{2}} (a^2 + a^2 \sin(dx + c))}{9de} \\ & + \frac{22a^2e^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231i \sqrt{2} a^2 e^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 231i \sqrt{2} a^2 e^{\frac{5}{2}} \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(a^2 e^2 \cos(dx + c)^4 - 2 a^2 e^2 \cos(dx + c)^2 \sin(dx + c) - 2 a^2 e^2 \cos(dx + c)^2\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.11 Problem number 206

$$\int (e \cos(c + dx))^{3/2} (a + a \sin(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{18a^2(e \cos(dx + c))^{\frac{5}{2}}}{35de} - \frac{2(e \cos(dx + c))^{\frac{5}{2}}(a^2 + a^2 \sin(dx + c))}{7de} \\ & + \frac{6a^2 e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & + \frac{6a^2 e \sin(dx + c) \sqrt{e \cos(dx + c)}}{7d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i \sqrt{2} a^2 e^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} a^2 e^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(a^2 e \cos(dx + c)^3 - 2 a^2 e \cos(dx + c) \sin(dx + c) - 2 a^2 e \cos(dx + c)\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.12 Problem number 207

$$\int \sqrt{e \cos(c + dx)} (a + a \sin(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{14a^2(e \cos(dx + c))^{\frac{3}{2}}}{15de} - \frac{2(e \cos(dx + c))^{\frac{3}{2}}(a^2 + a^2 \sin(dx + c))}{5de} \\ & + \frac{14a^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i \sqrt{2} a^2 e^{\frac{1}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2} a^2 e^{\frac{1}{2}} \operatorname{weierst}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(a^2 \cos(dx + c)^2 - 2a^2 \sin(dx + c) - 2a^2\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.13 Problem number 208

$$\int \frac{(a + a \sin(c + dx))^2}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10a^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & - \frac{10a^2 \sqrt{e \cos(dx + c)}}{3de} - \frac{2(a^2 + a^2 \sin(dx + c)) \sqrt{e \cos(dx + c)}}{3de} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-5i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c))+i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c))\right)}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\left(a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2\right)\sqrt{e\cos(dx+c)}}{e\cos(dx+c)},x\right)$$

40.14 Problem number 209

$$\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{4a^4(e\cos(dx+c))^{\frac{3}{2}}}{de^3(a^2-a^2\sin(dx+c))} - \frac{6a^2\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\sqrt{e\cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)de^2\sqrt{\cos(dx+c)}}$$

command

`integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\left(-i\sqrt{2}a^2\cos(dx+c)+i\sqrt{2}a^2\sin(dx+c)-i\sqrt{2}a^2\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\left(a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2\right)\sqrt{e\cos(dx+c)}}{e^2\cos(dx+c)^2},x\right)$$

40.15 Problem number 210

$$\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2a^2\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)de^2\sqrt{e\cos(dx+c)}} + \frac{4a^4\sqrt{e\cos(dx+c)}}{3de^3(a^2-a^2\sin(dx+c))}$$

command

```
integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 a^2 \sqrt{\cos(dx+c)} - \left(i \sqrt{2} a^2 \sin(dx+c) - i \sqrt{2} a^2\right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{3 \left(d e^{\frac{5}{2}} \sin(dx+c) - d e^{\frac{5}{2}}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\left(a^2 \cos(dx+c)^2 - 2 a^2 \sin(dx+c) - 2 a^2\right) \sqrt{e \cos(dx+c)}}{e^3 \cos(dx+c)^3}, x\right)$$

40.16 Problem number 211

$$\int \frac{(a + a \sin(c + dx))^2}{(e \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2 a^4 (e \cos(dx+c))^{\frac{3}{2}}}{5 d e^5 (a - a \sin(dx+c))^2} + \frac{2 a^4 (e \cos(dx+c))^{\frac{3}{2}}}{5 d e^5 (a^2 - a^2 \sin(dx+c))} - \frac{2 a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4 \sqrt{\cos(dx+c)}}$$

command

```
integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i \sqrt{2} a^2 \cos(dx+c)^2 + i \sqrt{2} a^2 \cos(dx+c) + 2 i \sqrt{2} a^2 + \left(-i \sqrt{2} a^2 \cos(dx+c) - 2 i \sqrt{2} a^2\right) \sin(dx+c)\right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{3 \left(d e^{\frac{7}{2}} \sin(dx+c) - d e^{\frac{7}{2}}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\left(a^2 \cos(dx+c)^2 - 2 a^2 \sin(dx+c) - 2 a^2\right) \sqrt{e \cos(dx+c)}}{e^4 \cos(dx+c)^4}, x\right)$$

40.17 Problem number 212

$$\int \frac{(a + a \sin(c + dx))^2}{(e \cos(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sin(dx + c)}{7d e^3 (e \cos(dx + c))^{\frac{3}{2}}} + \frac{\frac{4a^2}{7} + \frac{4a^2 \sin(dx+c)}{7}}{de (e \cos(dx + c))^{\frac{7}{2}}} + \frac{2a^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4 \sqrt{e \cos(dx + c)}}$$

command

`integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i \sqrt{2} a^2 \cos(dx + c)^2 - 2i \sqrt{2} a^2 \sin(dx + c) + 2i \sqrt{2} a^2\right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(a^2 \cos(dx + c)^2 - 2a^2 \sin(dx + c) - 2a^2\right) \sqrt{e \cos(dx + c)}}{e^5 \cos(dx + c)^5}, x\right)$$

40.18 Problem number 213

$$\int \frac{(a + a \sin(c + dx))^2}{(e \cos(c + dx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sin(dx + c)}{9d e^3 (e \cos(dx + c))^{\frac{5}{2}}} + \frac{\frac{4a^2}{9} + \frac{4a^2 \sin(dx+c)}{9}}{de (e \cos(dx + c))^{\frac{9}{2}}} + \frac{2a^2 \sin(dx + c)}{3d e^5 \sqrt{e \cos(dx + c)}} - \frac{2a^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^6 \sqrt{\cos(dx + c)}}$$

command

`integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(i \sqrt{2} a^2 \cos(dx+c)^3 + 2i \sqrt{2} a^2 \cos(dx+c) \sin(dx+c) - 2i \sqrt{2} a^2 \cos(dx+c) \right) \text{weierstrassZeta}(-4, 0, \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\left(a^2 \cos(dx+c)^2 - 2 a^2 \sin(dx+c) - 2 a^2 \right) \sqrt{e \cos(dx+c)}}{e^6 \cos(dx+c)^6}, x \right)$$

40.19 Problem number 214

$$\int (e \cos(c+dx))^{7/2} (a+a \sin(c+dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{34a^3 (e \cos(dx+c))^{\frac{9}{2}}}{99de} + \frac{34a^3 e (e \cos(dx+c))^{\frac{5}{2}} \sin(dx+c)}{77d} \\ & - \frac{2a (e \cos(dx+c))^{\frac{9}{2}} (a+a \sin(dx+c))^2}{13de} - \frac{34 (e \cos(dx+c))^{\frac{9}{2}} (a^3+a^3 \sin(dx+c))}{143de} \\ & + \frac{170a^3 e^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)})}{231 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d \sqrt{e \cos(dx+c)}} \\ & + \frac{170a^3 e^3 \sin(dx+c) \sqrt{e \cos(dx+c)}}{231d} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3315i \sqrt{2} a^3 e^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 3315i \sqrt{2} a^3 e^{\frac{7}{2}} \text{weierstrassPInverse}(-4,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \left(3 a^3 e^3 \cos(dx+c)^5 - 4 a^3 e^3 \cos(dx+c)^3 + \left(a^3 e^3 \cos(dx+c)^5 - 4 a^3 e^3 \cos(dx+c)^3 \right) \sin(dx+c) \right) \sqrt{\right.$$

40.20 Problem number 215

$$\int (e \cos(c + dx))^{5/2} (a + a \sin(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{10a^3(e \cos(dx + c))^{\frac{7}{2}}}{21de} + \frac{2a^3e(e \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3d} \\ & -\frac{2a(e \cos(dx + c))^{\frac{7}{2}} (a + a \sin(dx + c))^2}{11de} - \frac{10(e \cos(dx + c))^{\frac{7}{2}} (a^3 + a^3 \sin(dx + c))}{33de} \\ & + \frac{2a^3e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231i \sqrt{2} a^3 e^{\frac{5}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 231i \sqrt{2} a^3 e^{\frac{5}{2}} \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(3a^3e^2 \cos(dx + c)^4 - 4a^3e^2 \cos(dx + c)^2 + \left(a^3e^2 \cos(dx + c)^4 - 4a^3e^2 \cos(dx + c)^2\right) \sin(dx + c)\right) \sqrt{\cos(dx + c)}\right)$$

40.21 Problem number 216

$$\int (e \cos(c + dx))^{3/2} (a + a \sin(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{26a^3(e \cos(dx + c))^{\frac{5}{2}}}{35de} - \frac{2a(e \cos(dx + c))^{\frac{5}{2}} (a + a \sin(dx + c))^2}{9de} \\ & -\frac{26(e \cos(dx + c))^{\frac{5}{2}} (a^3 + a^3 \sin(dx + c))}{63de} \\ & + \frac{26a^3e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & + \frac{26a^3e \sin(dx + c) \sqrt{e \cos(dx + c)}}{21d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-195i \sqrt{2} a^3 e^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 195i \sqrt{2} a^3 e^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(3 a^3 e \cos(dx + c)^3 - 4 a^3 e \cos(dx + c) + \left(a^3 e \cos(dx + c)^3 - 4 a^3 e \cos(dx + c)\right) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.22 Problem number 217

$$\int \sqrt{e \cos(c + dx)} (a + a \sin(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{22a^3(e \cos(dx + c))^{\frac{3}{2}}}{15de} - \frac{2a(e \cos(dx + c))^{\frac{3}{2}}(a + a \sin(dx + c))^2}{7de} \\ & - \frac{22(e \cos(dx + c))^{\frac{3}{2}}(a^3 + a^3 \sin(dx + c))}{35de} \\ & + \frac{22a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231i \sqrt{2} a^3 e^{\frac{1}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 231i \sqrt{2} a^3 e^{\frac{1}{2}} \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(3 a^3 \cos(dx + c)^2 - 4 a^3 + \left(a^3 \cos(dx + c)^2 - 4 a^3\right) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.23 Problem number 218

$$\int \frac{(a + a \sin(c + dx))^3}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{6a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx+c)}} - \frac{6a^3 \sqrt{e \cos(dx+c)}}{de} - \frac{2a(a + a \sin(dx+c))^2 \sqrt{e \cos(dx+c)}}{5de} - \frac{6(a^3 + a^3 \sin(dx+c)) \sqrt{e \cos(dx+c)}}{5de}$$

command

`integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(-15i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 15i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))) \sqrt{e \cos(dx+c)}}{5d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(3a^3 \cos(dx+c)^2 - 4a^3 + (a^3 \cos(dx+c)^2 - 4a^3) \sin(dx+c)) \sqrt{e \cos(dx+c)}}{e \cos(dx+c)}, x\right)$$

40.24 Problem number 219

$$\int \frac{(a + a \sin(c + dx))^3}{(e \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{14a^3 (e \cos(dx+c))^{3/2}}{3de^3} + \frac{4a^5 (e \cos(dx+c))^{7/2}}{de^5 (a - a \sin(dx+c))^2} - \frac{14a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^2 \sqrt{\cos(dx+c)}}$$

command

```
integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21 \left(-i \sqrt{2} a^3 \cos(dx + c) + i \sqrt{2} a^3 \sin(dx + c) - i \sqrt{2} a^3 \right) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\left(3 a^3 \cos(dx + c)^2 - 4 a^3 + \left(a^3 \cos(dx + c)^2 - 4 a^3 \right) \sin(dx + c) \right) \sqrt{e \cos(dx + c)}}{e^2 \cos(dx + c)^2}, x \right)$$

40.25 Problem number 220

$$\int \frac{(a + a \sin(c + dx))^3}{(e \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^5 (e \cos(dx + c))^{\frac{5}{2}}}{3d e^5 (a - a \sin(dx + c))^2} \\ & - \frac{10a^3 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d e^2 \sqrt{e \cos(dx + c)}} \\ & + \frac{10a^3 \sqrt{e \cos(dx + c)}}{3d e^3} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \left(-i \sqrt{2} a^3 \sin(dx + c) + i \sqrt{2} a^3 \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \left(i \sqrt{2} a^3 \sin(dx + c) - i \sqrt{2} a^3 \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{3 \left(d e^{\frac{5}{2}} \sin(dx + c) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\left(3 a^3 \cos(dx + c)^2 - 4 a^3 + \left(a^3 \cos(dx + c)^2 - 4 a^3 \right) \sin(dx + c) \right) \sqrt{e \cos(dx + c)}}{e^3 \cos(dx + c)^3}, x \right)$$

40.26 Problem number 221

$$\int \frac{(a + a \sin(c + dx))^3}{(e \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{4a^5(e \cos(dx + c))^{\frac{3}{2}}}{5d e^5 (a - a \sin(dx + c))^2} - \frac{6a^6(e \cos(dx + c))^{\frac{3}{2}}}{5d e^5 (a^3 - a^3 \sin(dx + c))} + \frac{6a^3 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4 \sqrt{\cos(dx + c)}}$$

command

`integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \left(-i \sqrt{2} a^3 \cos(dx + c)^2 + i \sqrt{2} a^3 \cos(dx + c) + 2i \sqrt{2} a^3 + \left(-i \sqrt{2} a^3 \cos(dx + c) - 2i \sqrt{2} a^3 \right) \sin(dx + c) \right)}{e^4 \cos(dx + c)^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(3a^3 \cos(dx + c)^2 - 4a^3 + \left(a^3 \cos(dx + c)^2 - 4a^3\right) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}}{e^4 \cos(dx + c)^4}, x\right)$$

40.27 Problem number 222

$$\int \frac{(a + a \sin(c + dx))^3}{(e \cos(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2a^3 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4 \sqrt{e \cos(dx + c)}} + \frac{4a^5 \sqrt{e \cos(dx + c)}}{7d e^5 (a - a \sin(dx + c))^2} - \frac{2a^6 \sqrt{e \cos(dx + c)}}{21d e^5 (a^3 - a^3 \sin(dx + c))}$$

command

`integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(i\sqrt{2}a^3\cos(dx+c)^2 + 2i\sqrt{2}a^3\sin(dx+c) - 2i\sqrt{2}a^3)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{e^5\cos(dx+c)^5}$$

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Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(3a^3\cos(dx+c)^2 - 4a^3 + (a^3\cos(dx+c)^2 - 4a^3)\sin(dx+c))\sqrt{e\cos(dx+c)}}{e^5\cos(dx+c)^5},x\right)$$

40.28 Problem number 223

$$\int \frac{(a + a\sin(c + dx))^3}{(e\cos(c + dx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{2a^6(e\cos(dx+c))^{\frac{3}{2}}}{9de^7(a-a\sin(dx+c))^3} + \frac{2a^5(e\cos(dx+c))^{\frac{3}{2}}}{15de^7(a-a\sin(dx+c))^2} + \frac{2a^6(e\cos(dx+c))^{\frac{3}{2}}}{15de^7(a^3-a^3\sin(dx+c))} - \frac{2a^3\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right),\sqrt{2}\right)\sqrt{e\cos(dx+c)}}{15\cos\left(\frac{dx}{2} + \frac{c}{2}\right)de^6\sqrt{\cos(dx+c)}}$$

command

`integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\left(-i\sqrt{2}a^3\cos(dx+c)^3 - 3i\sqrt{2}a^3\cos(dx+c)^2 + 2i\sqrt{2}a^3\cos(dx+c) + 4i\sqrt{2}a^3 + (i\sqrt{2}a^3\cos(dx+c))^2 - 2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(3a^3\cos(dx+c)^2 - 4a^3 + (a^3\cos(dx+c)^2 - 4a^3)\sin(dx+c))\sqrt{e\cos(dx+c)}}{e^6\cos(dx+c)^6},x\right)$$

40.29 Problem number 224

$$\int (e \cos(c + dx))^{3/2} (a + a \sin(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{442a^4(e \cos(dx + c))^{5/2}}{385de} - \frac{2a(e \cos(dx + c))^{5/2} (a + a \sin(dx + c))^3}{11de} \\ & - \frac{34(e \cos(dx + c))^{5/2} (a^2 + a^2 \sin(dx + c))^2}{99de} - \frac{442(e \cos(dx + c))^{5/2} (a^4 + a^4 \sin(dx + c))}{693de} \\ & + \frac{442a^4 e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & + \frac{442a^4 e \sin(dx + c) \sqrt{e \cos(dx + c)}}{231d} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3315i \sqrt{2} a^4 e^{3/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 3315i \sqrt{2} a^4 e^{3/2} \operatorname{weierstrassPInverse}(-4,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^4 e \cos(dx + c)^5 - 8a^4 e \cos(dx + c)^3 + 8a^4 e \cos(dx + c) - 4\left(a^4 e \cos(dx + c)^3 - 2a^4 e \cos(dx + c)\right) \sin\right) \sin\right)$$

40.30 Problem number 225

$$\int \sqrt{e \cos(c + dx)} (a + a \sin(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{22a^4(e \cos(dx + c))^{3/2}}{9de} - \frac{2a(e \cos(dx + c))^{3/2} (a + a \sin(dx + c))^3}{9de} \\ & - \frac{10(e \cos(dx + c))^{3/2} (a^2 + a^2 \sin(dx + c))^2}{21de} - \frac{22(e \cos(dx + c))^{3/2} (a^4 + a^4 \sin(dx + c))}{21de} \\ & + \frac{22a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231i \sqrt{2} a^4 e^{\frac{1}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 231i \sqrt{2} a^4 e^{\frac{1}{2}} \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(a^4 \cos(dx + c)^4 - 8a^4 \cos(dx + c)^2 + 8a^4 - 4\left(a^4 \cos(dx + c)^2 - 2a^4\right) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}}{e \cos(dx + c)}, x\right)$$

40.31 Problem number 226

$$\int \frac{(a + a \sin(c + dx))^4}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{78a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} - \frac{78a^4 \sqrt{e \cos(dx + c)}}{7de} - \frac{2a(a + a \sin(dx + c))^3 \sqrt{e \cos(dx + c)}}{7de} - \frac{26(a^2 + a^2 \sin(dx + c))^2 \sqrt{e \cos(dx + c)}}{35de} - \frac{78(a^4 + a^4 \sin(dx + c)) \sqrt{e \cos(dx + c)}}{35de}$$

command

```
integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-195i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 195i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))\right) \sqrt{e \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(a^4 \cos(dx + c)^4 - 8a^4 \cos(dx + c)^2 + 8a^4 - 4\left(a^4 \cos(dx + c)^2 - 2a^4\right) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}}{e \cos(dx + c)}, x\right)$$

40.32 Problem number 227

$$\int \frac{(a + a \sin(c + dx))^4}{(e \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{154a^4(e \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{15d e^3} + \frac{4a^7(e \cos(dx + c))^{\frac{11}{2}}}{d e^7 (a - a \sin(dx + c))^3} + \frac{44a^8(e \cos(dx + c))^{\frac{7}{2}}}{3d e^5 (a^4 - a^4 \sin(dx + c))} \\ & - \frac{154a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^2 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231 \left(-i \sqrt{2} a^4 \cos(dx + c) + i \sqrt{2} a^4 \sin(dx + c) - i \sqrt{2} a^4 \right) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(a^4 \cos(dx + c)^4 - 8a^4 \cos(dx + c)^2 + 8a^4 - 4\left(a^4 \cos(dx + c)^2 - 2a^4\right) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}}{e^2 \cos(dx + c)^2}, dx\right)$$

40.33 Problem number 228

$$\int \frac{(a + a \sin(c + dx))^4}{(e \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^7(e \cos(dx + c))^{\frac{9}{2}}}{3d e^7 (a - a \sin(dx + c))^3} + \frac{12a^8(e \cos(dx + c))^{\frac{5}{2}}}{d e^5 (a^4 - a^4 \sin(dx + c))} \\ & - \frac{10a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^2 \sqrt{e \cos(dx + c)}} \\ & - \frac{10a^4 \sin(dx + c) \sqrt{e \cos(dx + c)}}{d e^3} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \left(-i \sqrt{2} a^4 \sin(dx + c) + i \sqrt{2} a^4 \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \left(i \sqrt{2} a^4 \sin(dx + c) - i \sqrt{2} a^4 \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{e^3 \cos(dx + c)^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(a^4 \cos(dx + c)^4 - 8 a^4 \cos(dx + c)^2 + 8 a^4 - 4 \left(a^4 \cos(dx + c)^2 - 2 a^4 \right) \sin(dx + c) \right) \sqrt{e \cos(dx + c)}}{e^3 \cos(dx + c)^3}, dx \right)$$

40.34 Problem number 229

$$\int \frac{(a + a \sin(c + dx))^4}{(e \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{4a^7 (e \cos(dx + c))^{\frac{7}{2}}}{5de^7 (a - a \sin(dx + c))^3} - \frac{28a^8 (e \cos(dx + c))^{\frac{3}{2}}}{5de^5 (a^4 - a^4 \sin(dx + c))} + \frac{42a^4 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) \sqrt{e \cos(dx + c)}}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) de^4 \sqrt{\cos(dx + c)}}$$

command

```
integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21 \left(-i \sqrt{2} a^4 \cos(dx + c)^2 + i \sqrt{2} a^4 \cos(dx + c) + 2i \sqrt{2} a^4 + \left(-i \sqrt{2} a^4 \cos(dx + c) - 2i \sqrt{2} a^4 \right) \sin(dx + c) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 21 \left(i \sqrt{2} a^4 \cos(dx + c)^2 - i \sqrt{2} a^4 \cos(dx + c) - 2i \sqrt{2} a^4 + \left(i \sqrt{2} a^4 \cos(dx + c) + 2i \sqrt{2} a^4 \right) \sin(dx + c) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{e^4 \cos(dx + c)^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(a^4 \cos(dx + c)^4 - 8 a^4 \cos(dx + c)^2 + 8 a^4 - 4 \left(a^4 \cos(dx + c)^2 - 2 a^4 \right) \sin(dx + c) \right) \sqrt{e \cos(dx + c)}}{e^4 \cos(dx + c)^4}, dx \right)$$

40.35 Problem number 230

$$\int \frac{(a + a \sin(c + dx))^4}{(e \cos(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^7 (e \cos(dx + c))^{\frac{5}{2}}}{7d e^7 (a - a \sin(dx + c))^3} \\ & + \frac{10a^4 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4 \sqrt{e \cos(dx + c)}} \\ & - \frac{20a^8 \sqrt{e \cos(dx + c)}}{21d e^5 (a^4 - a^4 \sin(dx + c))} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \left(i \sqrt{2} a^4 \cos(dx + c)^2 + 2i \sqrt{2} a^4 \sin(dx + c) - 2i \sqrt{2} a^4 \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(a^4 \cos(dx + c)^4 - 8a^4 \cos(dx + c)^2 + 8a^4 - 4\left(a^4 \cos(dx + c)^2 - 2a^4\right) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}}{e^5 \cos(dx + c)^5}, x\right)$$

40.36 Problem number 231

$$\int \frac{(a + a \sin(c + dx))^4}{(e \cos(c + dx))^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^7 (e \cos(dx + c))^{\frac{3}{2}}}{9d e^7 (a - a \sin(dx + c))^3} - \frac{2a^8 (e \cos(dx + c))^{\frac{3}{2}}}{15d e^7 (a^2 - a^2 \sin(dx + c))^2} - \frac{2a^8 (e \cos(dx + c))^{\frac{3}{2}}}{15d e^7 (a^4 - a^4 \sin(dx + c))} \\ & + \frac{2a^4 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^6 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(i \sqrt{2} a^4 \cos(dx+c)^3 + 3i \sqrt{2} a^4 \cos(dx+c)^2 - 2i \sqrt{2} a^4 \cos(dx+c) - 4i \sqrt{2} a^4 + \left(-i \sqrt{2} a^4 \cos(dx+c)^2 + 2 \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(a^4 \cos(dx+c)^4 - 8 a^4 \cos(dx+c)^2 + 8 a^4 - 4 \left(a^4 \cos(dx+c)^2 - 2 a^4 \right) \sin(dx+c) \right) \sqrt{e \cos(dx+c)}}{e^6 \cos(dx+c)^6}, x \right)$$

40.37 Problem number 232

$$\int \frac{(a + a \sin(c + dx))^4}{(e \cos(c + dx))^{13/2}} dx$$

Optimal antiderivative

$$\frac{2a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^6 \sqrt{e \cos(dx+c)}} + \frac{4a^7 \sqrt{e \cos(dx+c)}}{11 d e^7 (a - a \sin(dx+c))^3} - \frac{2a^8 \sqrt{e \cos(dx+c)}}{77 d e^7 (a^2 - a^2 \sin(dx+c))^2} - \frac{2a^8 \sqrt{e \cos(dx+c)}}{77 d e^7 (a^4 - a^4 \sin(dx+c))}$$

command

```
integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-3i \sqrt{2} a^4 \cos(dx+c)^2 + 4i \sqrt{2} a^4 + \left(i \sqrt{2} a^4 \cos(dx+c)^2 - 4i \sqrt{2} a^4 \right) \sin(dx+c) \right) \text{weierstrassPInverse}(-4, \dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(a^4 \cos(dx+c)^4 - 8 a^4 \cos(dx+c)^2 + 8 a^4 - 4 \left(a^4 \cos(dx+c)^2 - 2 a^4 \right) \sin(dx+c) \right) \sqrt{e \cos(dx+c)}}{e^7 \cos(dx+c)^7}, x \right)$$

40.38 Problem number 233

$$\int \frac{(e \cos(c + dx))^{11/2}}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e(e \cos(dx + c))^{\frac{9}{2}}}{9ad} + \frac{2e^3(e \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7ad} \\ & + \frac{10e^6 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{e \cos(dx + c)}} \\ & + \frac{10e^5 \sin(dx + c) \sqrt{e \cos(dx + c)}}{21ad} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i \sqrt{2} e^{\frac{11}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} e^{\frac{11}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

63 ad

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)} e^5 \cos(dx + c)^5}{a \sin(dx + c) + a}, x\right)$$

40.39 Problem number 234

$$\int \frac{(e \cos(c + dx))^{9/2}}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e(e \cos(dx + c))^{\frac{7}{2}}}{7ad} + \frac{2e^3(e \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5ad} \\ & + \frac{6e^4 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$21i \sqrt{2} e^{\frac{9}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2} e^{\frac{9}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{e \cos(dx + c)} e^4 \cos(dx + c)^4}{a \sin(dx + c) + a}, x\right)$$

40.40 Problem number 235

$$\int \frac{(e \cos(c + dx))^{7/2}}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{2e(e \cos(dx + c))^{\frac{5}{2}}}{5ad} + \frac{2e^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{e \cos(dx + c)}} + \frac{2e^3 \sin(dx + c) \sqrt{e \cos(dx + c)}}{3ad}$$

command

`integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} e^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} e^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

15 ad

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{e \cos(dx + c)} e^3 \cos(dx + c)^3}{a \sin(dx + c) + a}, x\right)$$

40.41 Problem number 236

$$\int \frac{(e \cos(c + dx))^{5/2}}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{2e(e \cos(dx + c))^{3/2}}{3ad} + \frac{2e^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx + c)}}$$

command

```
integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \cos(dx + c)^{\frac{3}{2}} e^{\frac{5}{2}} + 3i \sqrt{2} e^{\frac{5}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 3i}{3ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)} e^2 \cos(dx + c)^2}{a \sin(dx + c) + a}, x\right)$$

40.42 Problem number 237

$$\int \frac{(e \cos(c + dx))^{3/2}}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{2e^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{e \cos(dx + c)}} + \frac{2e \sqrt{e \cos(dx + c)}}{ad}$$

command

```
integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} e^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} e^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)} e \cos(dx + c)}{a \sin(dx + c) + a}, x\right)$$

40.43 Problem number 238

$$\int \frac{\sqrt{e \cos(c + dx)}}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(e \cos(dx + c))^{\frac{3}{2}}}{de(a + a \sin(dx + c))} - \frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx + c)}}$$

command

`integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-i \sqrt{2} \cos(dx + c) e^{\frac{1}{2}} - i \sqrt{2} e^{\frac{1}{2}} \sin(dx + c) - i \sqrt{2} e^{\frac{1}{2}}\right) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)}}{a \sin(dx + c) + a}, x\right)$$

40.44 Problem number 239

$$\int \frac{1}{\sqrt{e \cos(c + dx)} (a + a \sin(c + dx))} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{e \cos(dx + c)}} - \frac{2\sqrt{e \cos(dx + c)}}{3de(a + a \sin(dx + c))}$$

command

`integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i \sqrt{2} \sin(dx + c) - i \sqrt{2}\right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \left(i \sqrt{2} \sin(dx + c) + i \sqrt{2}\right)}{3 \left(ade^{\frac{1}{2}} \sin(dx + c) + ade^{\frac{1}{2}}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)}}{ae \cos(dx + c) \sin(dx + c) + ae \cos(dx + c)}, x\right)$$

40.45 Problem number 240

$$\int \frac{1}{(e \cos(c + dx))^{3/2}(a + a \sin(c + dx))} dx$$

Optimal antiderivative

$$\frac{6 \sin(dx + c)}{5ade \sqrt{e \cos(dx + c)}} - \frac{2}{5de (a + a \sin(dx + c)) \sqrt{e \cos(dx + c)}} \\ - \frac{6 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad e^2 \sqrt{\cos(dx + c)}}$$

command

```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \left(i \sqrt{2} \cos(dx + c) \sin(dx + c) + i \sqrt{2} \cos(dx + c) \right) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)}}{ae^2 \cos(dx + c)^2 \sin(dx + c) + ae^2 \cos(dx + c)^2}, x\right)$$

40.46 Problem number 241

$$\int \frac{1}{(e \cos(c + dx))^{5/2}(a + a \sin(c + dx))} dx$$

Optimal antiderivative

$$\frac{10 \sin(dx + c)}{21ade (e \cos(dx + c))^{\frac{3}{2}}} - \frac{2}{7de (e \cos(dx + c))^{\frac{3}{2}} (a + a \sin(dx + c))} \\ + \frac{10 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad e^2 \sqrt{e \cos(dx + c)}}$$

command

```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx+c)^2 \sin(dx+c) + i \sqrt{2} \cos(dx+c)^2 \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \cos(dx+c)}}{ae^3 \cos(dx+c)^3 \sin(dx+c) + ae^3 \cos(dx+c)^3}, x \right)$$

40.47 Problem number 242

$$\int \frac{1}{(e \cos(c+dx))^{7/2} (a + a \sin(c+dx))} dx$$

Optimal antiderivative

$$\frac{14 \sin(dx+c)}{45ade (e \cos(dx+c))^{\frac{5}{2}}} - \frac{2}{9de (e \cos(dx+c))^{\frac{5}{2}} (a + a \sin(dx+c))} + \frac{14 \sin(dx+c)}{15ad e^3 \sqrt{e \cos(dx+c)}} \\ - \frac{14 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) \sqrt{e \cos(dx+c)}}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad e^4 \sqrt{\cos(dx+c)}}$$

command

`integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21 \left(i \sqrt{2} \cos(dx+c)^3 \sin(dx+c) + i \sqrt{2} \cos(dx+c)^3 \right) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \cos(dx+c)}}{ae^4 \cos(dx+c)^4 \sin(dx+c) + ae^4 \cos(dx+c)^4}, x \right)$$

40.48 Problem number 243

$$\int \frac{(e \cos(c + dx))^{11/2}}{(a + a \sin(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{18e^3(e \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{35a^2d} + \frac{4e(e \cos(dx + c))^{\frac{9}{2}}}{5d(a^2 + a^2 \sin(dx + c))} \\ & + \frac{6e^6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{e \cos(dx + c)}} \\ & + \frac{6e^5 \sin(dx + c) \sqrt{e \cos(dx + c)}}{7a^2d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-15i \sqrt{2} e^{\frac{11}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} e^{\frac{11}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{35 a^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{e \cos(dx + c)} e^5 \cos(dx + c)^5}{a^2 \cos(dx + c)^2 - 2 a^2 \sin(dx + c) - 2 a^2}, x\right)$$

40.49 Problem number 244

$$\int \frac{(e \cos(c + dx))^{9/2}}{(a + a \sin(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14e^3(e \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{15a^2d} + \frac{4e(e \cos(dx + c))^{\frac{7}{2}}}{3d(a^2 + a^2 \sin(dx + c))} \\ & + \frac{14e^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$21i\sqrt{2}e^{\frac{9}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 21i\sqrt{2}e^{\frac{9}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}e^4\cos(dx+c)^4}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2},x\right)$$

40.50 Problem number 245

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4e(e\cos(dx+c))^{\frac{5}{2}}}{d(a^2+a^2\sin(dx+c))} \\ & + \frac{10e^4\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^2d\sqrt{e\cos(dx+c)}} \\ & + \frac{10e^3\sin(dx+c)\sqrt{e\cos(dx+c)}}{3a^2d} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i\sqrt{2}e^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}e^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$
 $3a^2d$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}e^3\cos(dx+c)^3}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2},x\right)$$

40.51 Problem number 246

$$\int \frac{(e \cos(c + dx))^{5/2}}{(a + a \sin(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{4e(e \cos(dx + c))^{\frac{3}{2}}}{d(a^2 + a^2 \sin(dx + c))} - \frac{6e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx + c)}}$$

command

```
integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\left(i\sqrt{2}\cos(dx+c)e^{\frac{5}{2}} + i\sqrt{2}e^{\frac{5}{2}}\sin(dx+c) + i\sqrt{2}e^{\frac{5}{2}}\right)\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)))}{3(a^2d\sin(dx+c) + a^2d)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{e \cos(dx + c)} e^2 \cos(dx + c)^2}{a^2 \cos(dx + c)^2 - 2a^2 \sin(dx + c) - 2a^2}, x\right)$$

40.52 Problem number 247

$$\int \frac{(e \cos(c + dx))^{3/2}}{(a + a \sin(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{2e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{e \cos(dx + c)}} - \frac{4e \sqrt{e \cos(dx + c)}}{3d(a^2 + a^2 \sin(dx + c))}$$

command

```
integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(i\sqrt{2}e^{\frac{3}{2}}\sin(dx+c) + i\sqrt{2}e^{\frac{3}{2}}\right)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c) + i\sin(dx+c)) + \left(-i\sqrt{2}e^{\frac{3}{2}}\sin(dx+c) + i\sqrt{2}e^{\frac{3}{2}}\right)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c) - i\sin(dx+c))}{3(a^2d\sin(dx+c) + a^2d)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{e \cos(dx + c)} e \cos(dx + c)}{a^2 \cos(dx + c)^2 - 2a^2 \sin(dx + c) - 2a^2}, x\right)$$

40.53 Problem number 248

$$\int \frac{\sqrt{e \cos(c + dx)}}{(a + a \sin(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(e \cos(dx + c))^{\frac{3}{2}}}{5de(a + a \sin(dx + c))^2} - \frac{2(e \cos(dx + c))^{\frac{3}{2}}}{5de(a^2 + a^2 \sin(dx + c))} \\ & - \frac{2\sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(i \sqrt{2} \cos(dx + c)^2 e^{\frac{1}{2}} - i \sqrt{2} \cos(dx + c) e^{\frac{1}{2}} + (-i \sqrt{2} \cos(dx + c) e^{\frac{1}{2}} - 2i \sqrt{2} e^{\frac{1}{2}}) \sin(dx + c) - 2i \sqrt{2} e^{\frac{1}{2}})w}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{e \cos(dx + c)}}{a^2 \cos(dx + c)^2 - 2a^2 \sin(dx + c) - 2a^2}, x\right)$$

40.54 Problem number 249

$$\int \frac{1}{\sqrt{e \cos(c + dx)} (a + a \sin(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2\sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{e \cos(dx + c)}} \\ & - \frac{2\sqrt{e \cos(dx + c)}}{7de(a + a \sin(dx + c))^2} - \frac{2\sqrt{e \cos(dx + c)}}{7de(a^2 + a^2 \sin(dx + c))} \end{aligned}$$

command

```
integrate(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(i\sqrt{2}\cos(dx+c)^2 - 2i\sqrt{2}\sin(dx+c) - 2i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + (-7(a^2d\cos(dx+c) - 2a^2d\sin(dx+c) - 2a^2d)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)))}{7(a^2d\cos(dx+c) - 2a^2d\sin(dx+c) - 2a^2d)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}}{a^2e\cos(dx+c)^3 - 2a^2e\cos(dx+c)\sin(dx+c) - 2a^2e\cos(dx+c)}, x\right)$$

40.55 Problem number 250

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2\sin(dx+c)}{3a^2de\sqrt{e\cos(dx+c)}} - \frac{2}{9de(a+a\sin(dx+c))^2\sqrt{e\cos(dx+c)}} \\ & - \frac{9de(a^2+a^2\sin(dx+c))\sqrt{e\cos(dx+c)}}{2} \\ & - \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\sqrt{e\cos(dx+c)}}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^2de^2\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3(-i\sqrt{2}\cos(dx+c)^3 + 2i\sqrt{2}\cos(dx+c)\sin(dx+c) + 2i\sqrt{2}\cos(dx+c))\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))) + (-7(a^2d\cos(dx+c) - 2a^2d\sin(dx+c) - 2a^2d)\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))))}{7(a^2d\cos(dx+c) - 2a^2d\sin(dx+c) - 2a^2d)\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}}{a^2e^2\cos(dx+c)^4 - 2a^2e^2\cos(dx+c)^2\sin(dx+c) - 2a^2e^2\cos(dx+c)^2}, x\right)$$

40.56 Problem number 251

$$\int \frac{1}{(e \cos(c + dx))^{5/2} (a + a \sin(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10 \sin(dx + c)}{33a^2 de (e \cos(dx + c))^{3/2}} - \frac{2}{11de (e \cos(dx + c))^{3/2} (a + a \sin(dx + c))^2} \\ & - \frac{2}{11de (e \cos(dx + c))^{3/2} (a^2 + a^2 \sin(dx + c))} \\ & + \frac{10 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{33 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d e^2 \sqrt{e \cos(dx + c)}} \end{aligned}$$

command

`integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(-i \sqrt{2} \cos(dx + c)^4 + 2i \sqrt{2} \cos(dx + c)^2 \sin(dx + c) + 2i \sqrt{2} \cos(dx + c)^2 \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{e \cos(dx + c)}}{a^2 e^3 \cos(dx + c)^5 - 2 a^2 e^3 \cos(dx + c)^3 \sin(dx + c) - 2 a^2 e^3 \cos(dx + c)^3}, x\right)$$

40.57 Problem number 252

$$\int \frac{1}{(e \cos(c + dx))^{7/2} (a + a \sin(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14 \sin(dx + c)}{65a^2 de (e \cos(dx + c))^{5/2}} - \frac{2}{13de (e \cos(dx + c))^{5/2} (a + a \sin(dx + c))^2} \\ & - \frac{2}{13de (e \cos(dx + c))^{5/2} (a^2 + a^2 \sin(dx + c))} + \frac{42 \sin(dx + c)}{65a^2 d e^3 \sqrt{e \cos(dx + c)}} \\ & - \frac{42 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{65 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d e^4 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21 \left(-i \sqrt{2} \cos(dx + c)^5 + 2i \sqrt{2} \cos(dx + c)^3 \sin(dx + c) + 2i \sqrt{2} \cos(dx + c)^3 \right) \text{weierstrassZeta}(-4, 0, \text{weierstrassZeta}(-4, 0, \text{weierstrassZeta}(-4, 0, \text{weierstrassZeta}(-4, 0, \dots)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\sqrt{e \cos(dx + c)}}{a^2 e^4 \cos(dx + c)^6 - 2 a^2 e^4 \cos(dx + c)^4 \sin(dx + c) - 2 a^2 e^4 \cos(dx + c)^4}, x \right)$$

40.58 Problem number 253

$$\int \frac{(e \cos(c + dx))^{15/2}}{(a + a \sin(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{26e^3(e \cos(dx + c))^{\frac{9}{2}}}{45a^3d} + \frac{26e^5(e \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{35a^3d} + \frac{4e(e \cos(dx + c))^{\frac{13}{2}}}{5ad(a + a \sin(dx + c))^2}$$

$$+ \frac{26e^8 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)})}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d \sqrt{e \cos(dx + c)}}$$

$$+ \frac{26e^7 \sin(dx + c) \sqrt{e \cos(dx + c)}}{21a^3d}$$

command

`integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-195i \sqrt{2} e^{\frac{15}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 195i \sqrt{2} e^{\frac{15}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\sqrt{e \cos(dx + c)} e^7 \cos(dx + c)^7}{3 a^3 \cos(dx + c)^2 - 4 a^3 + (a^3 \cos(dx + c)^2 - 4 a^3) \sin(dx + c)}, x \right)$$

40.59 Problem number 254

$$\int \frac{(e \cos(c + dx))^{13/2}}{(a + a \sin(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{22e^3(e \cos(dx + c))^{7/2}}{21a^3d} + \frac{22e^5(e \cos(dx + c))^{3/2} \sin(dx + c)}{15a^3d} + \frac{4e(e \cos(dx + c))^{11/2}}{3ad(a + a \sin(dx + c))^2} \\ & + \frac{22e^6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$231i \sqrt{2} e^{\frac{13}{2}}$ weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))) - $231i \sqrt{2} e^{\frac{13}{2}}$ weierstr

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{e \cos(dx + c)} e^6 \cos(dx + c)^6}{3a^3 \cos(dx + c)^2 - 4a^3 + (a^3 \cos(dx + c)^2 - 4a^3) \sin(dx + c)}, x\right)$$

40.60 Problem number 255

$$\int \frac{(e \cos(c + dx))^{11/2}}{(a + a \sin(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{18e^3(e \cos(dx + c))^{5/2}}{5a^3d} + \frac{4e(e \cos(dx + c))^{9/2}}{ad(a + a \sin(dx + c))^2} \\ & + \frac{6e^6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{e \cos(dx + c)}} \\ & + \frac{6e^5 \sin(dx + c) \sqrt{e \cos(dx + c)}}{a^3 d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-15i\sqrt{2}e^{\frac{11}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}e^{\frac{11}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{5a^3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}e^5\cos(dx+c)^5}{3a^3\cos(dx+c)^2-4a^3+(a^3\cos(dx+c)^2-4a^3)\sin(dx+c)},x\right)$$

40.61 Problem number 256

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} &-\frac{14e^3(e\cos(dx+c))^{\frac{3}{2}}}{3a^3d}-\frac{4e(e\cos(dx+c))^{\frac{7}{2}}}{ad(a+a\sin(dx+c))^2} \\ &-\frac{14e^4\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{e\cos(dx+c)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21\left(i\sqrt{2}\cos(dx+c)e^{\frac{9}{2}}+i\sqrt{2}e^{\frac{9}{2}}\sin(dx+c)+i\sqrt{2}e^{\frac{9}{2}}\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))}{5a^3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}e^4\cos(dx+c)^4}{3a^3\cos(dx+c)^2-4a^3+(a^3\cos(dx+c)^2-4a^3)\sin(dx+c)},x\right)$$

40.62 Problem number 257

$$\int \frac{(e \cos(c + dx))^{7/2}}{(a + a \sin(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4e(e \cos(dx + c))^{\frac{5}{2}}}{3ad(a + a \sin(dx + c))^2} \\ & -\frac{10e^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{e \cos(dx + c)}} \\ & -\frac{10e^3 \sqrt{e \cos(dx + c)}}{3a^3 d} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \left(-i \sqrt{2} e^{\frac{7}{2}} \sin(dx + c) - i \sqrt{2} e^{\frac{7}{2}} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \left(i \sqrt{2} e^{\frac{7}{2}} \sin(dx + c) + i \sqrt{2} e^{\frac{7}{2}} \right)}{3 (a^3 d \sin(dx + c))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{e \cos(dx + c)} e^3 \cos(dx + c)^3}{3 a^3 \cos(dx + c)^2 - 4 a^3 + (a^3 \cos(dx + c)^2 - 4 a^3) \sin(dx + c)}, x\right)$$

40.63 Problem number 258

$$\int \frac{(e \cos(c + dx))^{5/2}}{(a + a \sin(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4e(e \cos(dx + c))^{\frac{3}{2}}}{5ad(a + a \sin(dx + c))^2} + \frac{6e(e \cos(dx + c))^{\frac{3}{2}}}{5d(a^3 + a^3 \sin(dx + c))} \\ & + \frac{6e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(i \sqrt{2} \cos(dx+c)^2 e^{\frac{5}{2}} - i \sqrt{2} \cos(dx+c) e^{\frac{5}{2}} + \left(-i \sqrt{2} \cos(dx+c) e^{\frac{5}{2}} - 2i \sqrt{2} e^{\frac{5}{2}} \right) \sin(dx+c) - 2i \sqrt{2} e^{\frac{5}{2}} \right) w$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\sqrt{e \cos(dx+c)} e^2 \cos(dx+c)^2}{3 a^3 \cos(dx+c)^2 - 4 a^3 + \left(a^3 \cos(dx+c)^2 - 4 a^3 \right) \sin(dx+c)}, x \right)$$

40.64 Problem number 259

$$\int \frac{(e \cos(c+dx))^{3/2}}{(a+a \sin(c+dx))^3} dx$$

Optimal antiderivative

$$\frac{2e^2 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)})}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d \sqrt{e \cos(dx+c)}} - \frac{4e \sqrt{e \cos(dx+c)}}{7ad (a+a \sin(dx+c))^2} + \frac{2e \sqrt{e \cos(dx+c)}}{21d (a^3 + a^3 \sin(dx+c))}$$

command

```
integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-i \sqrt{2} \cos(dx+c)^2 e^{\frac{3}{2}} + 2i \sqrt{2} e^{\frac{3}{2}} \sin(dx+c) + 2i \sqrt{2} e^{\frac{3}{2}} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\sqrt{e \cos(dx+c)} e \cos(dx+c)}{3 a^3 \cos(dx+c)^2 - 4 a^3 + \left(a^3 \cos(dx+c)^2 - 4 a^3 \right) \sin(dx+c)}, x \right)$$

40.65 Problem number 260

$$\int \frac{\sqrt{e \cos(c + dx)}}{(a + a \sin(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(e \cos(dx + c))^{\frac{3}{2}}}{9de(a + a \sin(dx + c))^3} - \frac{2(e \cos(dx + c))^{\frac{3}{2}}}{15ade(a + a \sin(dx + c))^2} - \frac{2(e \cos(dx + c))^{\frac{3}{2}}}{15de(a^3 + a^3 \sin(dx + c))} \\ & - \frac{2\sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(i \sqrt{2} \cos(dx + c)^3 e^{\frac{1}{2}} + 3i \sqrt{2} \cos(dx + c)^2 e^{\frac{1}{2}} - 2i \sqrt{2} \cos(dx + c) e^{\frac{1}{2}} + \left(i \sqrt{2} \cos(dx + c)^2 e^{\frac{1}{2}} - 2i \sqrt{2} \cos(dx + c) e^{\frac{1}{2}} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{e \cos(dx + c)}}{3a^3 \cos(dx + c)^2 - 4a^3 + (a^3 \cos(dx + c)^2 - 4a^3) \sin(dx + c)}, x\right)$$

40.66 Problem number 261

$$\int \frac{1}{\sqrt{e \cos(c + dx)} (a + a \sin(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10\sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{e \cos(dx + c)}} \\ & - \frac{2\sqrt{e \cos(dx + c)}}{11de(a + a \sin(dx + c))^3} - \frac{10\sqrt{e \cos(dx + c)}}{77ade(a + a \sin(dx + c))^2} - \frac{10\sqrt{e \cos(dx + c)}}{77de(a^3 + a^3 \sin(dx + c))} \end{aligned}$$

command

`integrate(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(3i \sqrt{2} \cos(dx + c)^2 + \left(i \sqrt{2} \cos(dx + c)^2 - 4i \sqrt{2} \right) \sin(dx + c) - 4i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{e \cos(dx + c)}}{3 a^3 e \cos(dx + c)^3 - 4 a^3 e \cos(dx + c) + \left(a^3 e \cos(dx + c)^3 - 4 a^3 e \cos(dx + c) \right) \sin(dx + c)}, x \right)$$

40.67 Problem number 262

$$\int \frac{1}{(e \cos(c + dx))^{3/2} (a + a \sin(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14 \sin(dx + c)}{39 a^3 d e \sqrt{e \cos(dx + c)}} - \frac{2}{13 d e (a + a \sin(dx + c))^3 \sqrt{e \cos(dx + c)}} \\ & - \frac{117 a d e (a + a \sin(dx + c))^2 \sqrt{e \cos(dx + c)}}{14} \\ & - \frac{117 d e (a^3 + a^3 \sin(dx + c)) \sqrt{e \cos(dx + c)}}{14} \\ & - \frac{14 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) \sqrt{e \cos(dx + c)}}{39 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d e^2 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21 \left(3i \sqrt{2} \cos(dx + c)^3 + \left(i \sqrt{2} \cos(dx + c)^3 - 4i \sqrt{2} \cos(dx + c) \right) \sin(dx + c) - 4i \sqrt{2} \cos(dx + c) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{e \cos(dx + c)}}{3 a^3 e^2 \cos(dx + c)^4 - 4 a^3 e^2 \cos(dx + c)^2 + \left(a^3 e^2 \cos(dx + c)^4 - 4 a^3 e^2 \cos(dx + c)^2 \right) \sin(dx + c)}, x \right)$$

40.68 Problem number 263

$$\int \frac{(e \cos(c + dx))^{15/2}}{(a + a \sin(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{234e^5(e \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{35a^4d} + \frac{4e(e \cos(dx + c))^{\frac{13}{2}}}{ad(a + a \sin(dx + c))^3} + \frac{52e^3(e \cos(dx + c))^{\frac{9}{2}}}{5d(a^4 + a^4 \sin(dx + c))} \\ & + \frac{78e^8 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{e \cos(dx + c)}} \\ & + \frac{78e^7 \sin(dx + c) \sqrt{e \cos(dx + c)}}{7a^4d} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-195i \sqrt{2} e^{\frac{15}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 195i \sqrt{2} e^{\frac{15}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)} e^7 \cos(dx + c)^7}{a^4 \cos(dx + c)^4 - 8a^4 \cos(dx + c)^2 + 8a^4 - 4(a^4 \cos(dx + c)^2 - 2a^4) \sin(dx + c)}, x\right)$$

40.69 Problem number 264

$$\int \frac{(e \cos(c + dx))^{13/2}}{(a + a \sin(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{154e^5(e \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{15a^4d} - \frac{4e(e \cos(dx + c))^{\frac{11}{2}}}{ad(a + a \sin(dx + c))^3} - \frac{44e^3(e \cos(dx + c))^{\frac{7}{2}}}{3d(a^4 + a^4 \sin(dx + c))} \\ & - \frac{154e^6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{231 \left(i \sqrt{2} \cos(dx+c) e^{\frac{13}{2}} + i \sqrt{2} e^{\frac{13}{2}} \sin(dx+c) + i \sqrt{2} e^{\frac{13}{2}} \right) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \cos(dx+c)} e^6 \cos(dx+c)^6}{a^4 \cos(dx+c)^4 - 8a^4 \cos(dx+c)^2 + 8a^4 - 4(a^4 \cos(dx+c)^2 - 2a^4) \sin(dx+c)}, x \right)$$

40.70 Problem number 265

$$\int \frac{(e \cos(c+dx))^{11/2}}{(a+a \sin(c+dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4e(e \cos(dx+c))^{\frac{9}{2}}}{3ad(a+a \sin(dx+c))^3} - \frac{12e^3(e \cos(dx+c))^{\frac{5}{2}}}{d(a^4+a^4 \sin(dx+c))} \\ & - \frac{10e^6 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^4 d \sqrt{e \cos(dx+c)}} \\ & - \frac{10e^5 \sin(dx+c) \sqrt{e \cos(dx+c)}}{a^4 d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \left(-i \sqrt{2} e^{\frac{11}{2}} \sin(dx+c) - i \sqrt{2} e^{\frac{11}{2}} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 15 \left(i \sqrt{2} e^{\frac{11}{2}} \sin(dx+c) + i \sqrt{2} e^{\frac{11}{2}} \right) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \cos(dx+c)} e^5 \cos(dx+c)^5}{a^4 \cos(dx+c)^4 - 8a^4 \cos(dx+c)^2 + 8a^4 - 4(a^4 \cos(dx+c)^2 - 2a^4) \sin(dx+c)}, x \right)$$

40.71 Problem number 266

$$\int \frac{(e \cos(c + dx))^{9/2}}{(a + a \sin(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4e(e \cos(dx + c))^{\frac{7}{2}}}{5ad(a + a \sin(dx + c))^3} + \frac{28e^3(e \cos(dx + c))^{\frac{3}{2}}}{5d(a^4 + a^4 \sin(dx + c))} \\ & + \frac{42e^4 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21 \left(i \sqrt{2} \cos(dx + c)^2 e^{\frac{9}{2}} - i \sqrt{2} \cos(dx + c) e^{\frac{9}{2}} + \left(-i \sqrt{2} \cos(dx + c) e^{\frac{9}{2}} - 2i \sqrt{2} e^{\frac{9}{2}} \right) \sin(dx + c) - 2i \sqrt{2} e^{\frac{9}{2}} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)} e^4 \cos(dx + c)^4}{a^4 \cos(dx + c)^4 - 8 a^4 \cos(dx + c)^2 + 8 a^4 - 4 \left(a^4 \cos(dx + c)^2 - 2 a^4 \right) \sin(dx + c)}, x\right)$$

40.72 Problem number 267

$$\int \frac{(e \cos(c + dx))^{7/2}}{(a + a \sin(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4e(e \cos(dx + c))^{\frac{5}{2}}}{7ad(a + a \sin(dx + c))^3} \\ & + \frac{10e^4 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{e \cos(dx + c)}} \\ & + \frac{20e^3 \sqrt{e \cos(dx + c)}}{21d(a^4 + a^4 \sin(dx + c))} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(-i \sqrt{2} \cos(dx+c)^2 e^{\frac{7}{2}} + 2i \sqrt{2} e^{\frac{7}{2}} \sin(dx+c) + 2i \sqrt{2} e^{\frac{7}{2}} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \cos(dx+c)} e^3 \cos(dx+c)^3}{a^4 \cos(dx+c)^4 - 8 a^4 \cos(dx+c)^2 + 8 a^4 - 4 \left(a^4 \cos(dx+c)^2 - 2 a^4 \right) \sin(dx+c)}, x \right)$$

40.73 Problem number 268

$$\int \frac{(e \cos(c+dx))^{5/2}}{(a+a \sin(c+dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4e(e \cos(dx+c))^{\frac{3}{2}}}{9ad(a+a \sin(dx+c))^3} + \frac{2e(e \cos(dx+c))^{\frac{3}{2}}}{15d(a^2+a^2 \sin(dx+c))^2} + \frac{2e(e \cos(dx+c))^{\frac{3}{2}}}{15d(a^4+a^4 \sin(dx+c))} \\ & + \frac{2e^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(-i \sqrt{2} \cos(dx+c)^3 e^{\frac{5}{2}} - 3i \sqrt{2} \cos(dx+c)^2 e^{\frac{5}{2}} + 2i \sqrt{2} \cos(dx+c) e^{\frac{5}{2}} + \left(-i \sqrt{2} \cos(dx+c)^2 e^{\frac{5}{2}} + 2i \sqrt{2} \cos(dx+c) e^{\frac{5}{2}} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \cos(dx+c)} e^2 \cos(dx+c)^2}{a^4 \cos(dx+c)^4 - 8 a^4 \cos(dx+c)^2 + 8 a^4 - 4 \left(a^4 \cos(dx+c)^2 - 2 a^4 \right) \sin(dx+c)}, x \right)$$

40.74 Problem number 269

$$\int \frac{(e \cos(c + dx))^{3/2}}{(a + a \sin(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2e^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{e \cos(dx+c)}} \\ & - \frac{4e \sqrt{e \cos(dx+c)}}{11ad (a + a \sin(dx+c))^3} + \frac{2e \sqrt{e \cos(dx+c)}}{77d (a^2 + a^2 \sin(dx+c))^2} + \frac{2e \sqrt{e \cos(dx+c)}}{77d (a^4 + a^4 \sin(dx+c))} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(3i \sqrt{2} \cos(dx+c)^2 e^{\frac{3}{2}} + \left(i \sqrt{2} \cos(dx+c)^2 e^{\frac{3}{2}} - 4i \sqrt{2} e^{\frac{3}{2}}\right) \sin(dx+c) - 4i \sqrt{2} e^{\frac{3}{2}}\right) \operatorname{weierstrassPInverse}(-4, 0)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx+c)} e \cos(dx+c)}{a^4 \cos(dx+c)^4 - 8a^4 \cos(dx+c)^2 + 8a^4 - 4(a^4 \cos(dx+c)^2 - 2a^4) \sin(dx+c)}, x\right)$$

40.75 Problem number 270

$$\int \frac{\sqrt{e \cos(c + dx)}}{(a + a \sin(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(e \cos(dx+c))^{\frac{3}{2}}}{13de (a + a \sin(dx+c))^4} - \frac{10(e \cos(dx+c))^{\frac{3}{2}}}{117ade (a + a \sin(dx+c))^3} \\ & - \frac{2(e \cos(dx+c))^{\frac{3}{2}}}{39de (a^2 + a^2 \sin(dx+c))^2} - \frac{2(e \cos(dx+c))^{\frac{3}{2}}}{39de (a^4 + a^4 \sin(dx+c))} \\ & - \frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx+c)}}{39 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(-i \sqrt{2} \cos(dx+c)^4 e^{\frac{1}{2}} + 3i \sqrt{2} \cos(dx+c)^3 e^{\frac{1}{2}} + 8i \sqrt{2} \cos(dx+c)^2 e^{\frac{1}{2}} - 4i \sqrt{2} \cos(dx+c) e^{\frac{1}{2}} + \left(i \sqrt{2} \cos \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \cos(dx+c)}}{a^4 \cos(dx+c)^4 - 8a^4 \cos(dx+c)^2 + 8a^4 - 4 \left(a^4 \cos(dx+c)^2 - 2a^4 \right) \sin(dx+c)}, x \right)$$

40.76 Problem number 271

$$\int \frac{1}{\sqrt{e \cos(c+dx)} (a + a \sin(c+dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)})}{33 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^4 d \sqrt{e \cos(dx+c)}} \\ & - \frac{2 \sqrt{e \cos(dx+c)}}{15de (a + a \sin(dx+c))^4} - \frac{14 \sqrt{e \cos(dx+c)}}{165ade (a + a \sin(dx+c))^3} \\ & - \frac{2 \sqrt{e \cos(dx+c)}}{33de (a^2 + a^2 \sin(dx+c))^2} - \frac{2 \sqrt{e \cos(dx+c)}}{33de (a^4 + a^4 \sin(dx+c))} \end{aligned}$$

command

```
integrate(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(-i \sqrt{2} \cos(dx+c)^4 + 8i \sqrt{2} \cos(dx+c)^2 + 4 \left(i \sqrt{2} \cos(dx+c)^2 - 2i \sqrt{2} \right) \sin(dx+c) - 8i \sqrt{2} \right) \text{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \cos(dx+c)}}{a^4 e \cos(dx+c)^5 - 8a^4 e \cos(dx+c)^3 + 8a^4 e \cos(dx+c) - 4 \left(a^4 e \cos(dx+c)^3 - 2a^4 e \cos(dx+c) \right) \sin} \right)$$

40.77 Problem number 272

$$\int \frac{1}{(e \cos(c + dx))^{3/2} (a + a \sin(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{42 \sin(dx + c)}{221 a^4 d e \sqrt{e \cos(dx + c)}} - \frac{2}{17 d e (a + a \sin(dx + c))^4 \sqrt{e \cos(dx + c)}} \\ & - \frac{221 a d e (a + a \sin(dx + c))^3 \sqrt{e \cos(dx + c)}}{14} \\ & - \frac{221 d e (a^2 + a^2 \sin(dx + c))^2 \sqrt{e \cos(dx + c)}}{14} \\ & - \frac{221 d e (a^4 + a^4 \sin(dx + c)) \sqrt{e \cos(dx + c)}}{14} \\ & - \frac{42 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{221 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d e^2 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21 \left(-i \sqrt{2} \cos(dx + c)^5 + 8i \sqrt{2} \cos(dx + c)^3 + 4 \left(i \sqrt{2} \cos(dx + c)^3 - 2i \sqrt{2} \cos(dx + c) \right) \sin(dx + c) - 8i \sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)}}{a^4 e^2 \cos(dx + c)^6 - 8 a^4 e^2 \cos(dx + c)^4 + 8 a^4 e^2 \cos(dx + c)^2 - 4 \left(a^4 e^2 \cos(dx + c)^4 - 2 a^4 e^2 \cos(dx + c) \right)}\right)$$

40.78 Problem number 273

$$\int (e \cos(c + dx))^{3/2} \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{a(e \cos(dx+c))^{\frac{5}{2}}}{2de \sqrt{a+a \sin(dx+c)}} + \frac{3e \sqrt{e \cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{4d} \\
 & - \frac{3e^{\frac{3}{2}} \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx+c)}}{\sqrt{e}}\right) \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{4d(1+\cos(dx+c)+\sin(dx+c))} \\
 & + \frac{3e^{\frac{3}{2}} \operatorname{arctan}\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx+c)} \sqrt{1+\cos(dx+c)}}\right) \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{4d(1+\cos(dx+c)+\sin(dx+c))}
 \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.79 Problem number 274

$$\int \sqrt{e \cos(c+dx)} \sqrt{a+a \sin(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{a(e \cos(dx+c))^{\frac{3}{2}}}{de \sqrt{a+a \sin(dx+c)}} \\
 & + \frac{\operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx+c)}}{\sqrt{e}}\right) \sqrt{e} \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{d(1+\cos(dx+c)+\sin(dx+c))} \\
 & + \frac{\operatorname{arctan}\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx+c)} \sqrt{1+\cos(dx+c)}}\right) \sqrt{e} \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{d(1+\cos(dx+c)+\sin(dx+c))}
 \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.80 Problem number 275

$$\int \frac{\sqrt{a + a \sin(c + dx)}}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2 \operatorname{arcsinh} \left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}} \right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{d(1 + \cos(dx + c) + \sin(dx + c)) \sqrt{e}} \\ & + \frac{2 \arctan \left(\frac{\sin(dx + c) \sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}} \right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{d(1 + \cos(dx + c) + \sin(dx + c)) \sqrt{e}} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.81 Problem number 280

$$\int (e \cos(c + dx))^{5/2} (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{15a^3 (e \cos(dx + c))^{7/2}}{32de (a + a \sin(dx + c))^{3/2}} + \frac{15a^2 e (e \cos(dx + c))^{3/2}}{64d \sqrt{a + a \sin(dx + c)}} \\ & - \frac{3a^2 (e \cos(dx + c))^{7/2}}{8de \sqrt{a + a \sin(dx + c)}} - \frac{a (e \cos(dx + c))^{7/2} \sqrt{a + a \sin(dx + c)}}{4de} \\ & + \frac{45a e^{5/2} \operatorname{arcsinh} \left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}} \right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{64d(1 + \cos(dx + c) + \sin(dx + c))} \\ & + \frac{45a e^{5/2} \arctan \left(\frac{\sin(dx + c) \sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}} \right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{64d(1 + \cos(dx + c) + \sin(dx + c))} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.82 Problem number 281

$$\int (e \cos(c + dx))^{3/2} (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{7a^2(e \cos(dx+c))^{\frac{5}{2}}}{12de \sqrt{a+a \sin(dx+c)}} - \frac{a(e \cos(dx+c))^{\frac{5}{2}} \sqrt{a+a \sin(dx+c)}}{3de} \\ & + \frac{7ae \sqrt{e \cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{8d} \\ & - \frac{7ae^{\frac{3}{2}} \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx+c)}}{\sqrt{e}}\right) \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{8d(1+\cos(dx+c)+\sin(dx+c))} \\ & + \frac{7ae^{\frac{3}{2}} \operatorname{arctan}\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx+c)} \sqrt{1+\cos(dx+c)}}\right) \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{8d(1+\cos(dx+c)+\sin(dx+c))} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.83 Problem number 282

$$\int \sqrt{e \cos(c + dx)} (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{5a^2 (e \cos(dx + c))^{\frac{3}{2}}}{4de \sqrt{a + a \sin(dx + c)}} - \frac{a (e \cos(dx + c))^{\frac{3}{2}} \sqrt{a + a \sin(dx + c)}}{2de} \\ & + \frac{5a \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{e} \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{4d(1 + \cos(dx + c) + \sin(dx + c))} \\ & + \frac{5a \arctan\left(\frac{\sin(dx + c) \sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{e} \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{4d(1 + \cos(dx + c) + \sin(dx + c))} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.84 Problem number 283

$$\int \frac{(a + a \sin(c + dx))^{3/2}}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a \sqrt{e \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{de} \\ & - \frac{3a \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{d(1 + \cos(dx + c) + \sin(dx + c)) \sqrt{e}} \\ & + \frac{3a \arctan\left(\frac{\sin(dx + c) \sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{d(1 + \cos(dx + c) + \sin(dx + c)) \sqrt{e}} \end{aligned}$$

command

`integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.85 Problem number 284

$$\int \frac{(a + a \sin(c + dx))^{3/2}}{(e \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{4a \sqrt{a + a \sin(dx + c)}}{de \sqrt{e \cos(dx + c)}} - \frac{2a^2 \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{de^{\frac{3}{2}} (a + a \cos(dx + c) + a \sin(dx + c))} - \frac{2a^2 \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{de^{\frac{3}{2}} (a + a \cos(dx + c) + a \sin(dx + c))}$$

command

`integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.86 Problem number 289

$$\int (e \cos(c + dx))^{3/2} (a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{a(e \cos(dx+c))^{\frac{5}{2}}(a+a \sin(dx+c))^{\frac{3}{2}}}{4de} - \frac{77a^3(e \cos(dx+c))^{\frac{5}{2}}}{96de \sqrt{a+a \sin(dx+c)}} \\
 & - \frac{11a^2(e \cos(dx+c))^{\frac{5}{2}} \sqrt{a+a \sin(dx+c)}}{24de} + \frac{77a^2e \sqrt{e \cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{64d} \\
 & - \frac{77a^2e^{\frac{3}{2}} \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx+c)}}{\sqrt{e}}\right) \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{64d(1+\cos(dx+c)+\sin(dx+c))} \\
 & + \frac{77a^2e^{\frac{3}{2}} \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx+c)} \sqrt{1+\cos(dx+c)}}\right) \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{64d(1+\cos(dx+c)+\sin(dx+c))}
 \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.87 Problem number 290

$$\int \sqrt{e \cos(c+dx)} (a+a \sin(c+dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{a(e \cos(dx+c))^{\frac{3}{2}}(a+a \sin(dx+c))^{\frac{3}{2}}}{3de} - \frac{15a^3(e \cos(dx+c))^{\frac{3}{2}}}{8de \sqrt{a+a \sin(dx+c)}} \\
 & - \frac{3a^2(e \cos(dx+c))^{\frac{3}{2}} \sqrt{a+a \sin(dx+c)}}{4de} \\
 & + \frac{15a^2 \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx+c)}}{\sqrt{e}}\right) \sqrt{e} \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{8d(1+\cos(dx+c)+\sin(dx+c))} \\
 & + \frac{15a^2 \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx+c)} \sqrt{1+\cos(dx+c)}}\right) \sqrt{e} \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{8d(1+\cos(dx+c)+\sin(dx+c))}
 \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.88 Problem number 291

$$\int \frac{(a + a \sin(c + dx))^{5/2}}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a(a + a \sin(dx + c))^{\frac{3}{2}} \sqrt{e \cos(dx + c)}}{2de} - \frac{7a^2 \sqrt{e \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{4de} \\ & - \frac{21a^2 \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{4d(1 + \cos(dx + c) + \sin(dx + c)) \sqrt{e}} \\ & + \frac{21a^2 \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{4d(1 + \cos(dx + c) + \sin(dx + c)) \sqrt{e}} \end{aligned}$$

command

`integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.89 Problem number 292

$$\int \frac{(a + a \sin(c + dx))^{5/2}}{(e \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(a + a \sin(dx + c))^{\frac{3}{2}}}{de \sqrt{e \cos(dx + c)}} + \frac{5a^3(e \cos(dx + c))^{\frac{3}{2}}}{de^3 \sqrt{a + a \sin(dx + c)}} \\ & - \frac{5a^2 \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{de^{\frac{3}{2}}(1 + \cos(dx + c) + \sin(dx + c))} \\ & - \frac{5a^2 \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{de^{\frac{3}{2}}(1 + \cos(dx + c) + \sin(dx + c))} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.90 Problem number 293

$$\int \frac{(a + a \sin(c + dx))^{5/2}}{(e \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{4a(a + a \sin(dx + c))^{\frac{3}{2}}}{3de(e \cos(dx + c))^{\frac{3}{2}}} + \frac{2a^2 \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{de^{\frac{5}{2}}(1 + \cos(dx + c) + \sin(dx + c))}$$

$$- \frac{2a^2 \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{de^{\frac{5}{2}}(1 + \cos(dx + c) + \sin(dx + c))}$$

command

```
integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.91 Problem number 298

$$\int \frac{(e \cos(c + dx))^{5/2}}{\sqrt{a + a \sin(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{a(e \cos(dx+c))^{\frac{7}{2}}}{2de(a+a \sin(dx+c))^{\frac{3}{2}}} + \frac{e(e \cos(dx+c))^{\frac{3}{2}}}{4d\sqrt{a+a \sin(dx+c)}} \\
 & + \frac{3e^{\frac{5}{2}} \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx+c)}}{\sqrt{e}}\right) \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{4d(a+a \cos(dx+c)+a \sin(dx+c))} \\
 & + \frac{3e^{\frac{5}{2}} \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx+c)} \sqrt{1+\cos(dx+c)}}\right) \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{4d(a+a \cos(dx+c)+a \sin(dx+c))}
 \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.92 Problem number 299

$$\int \frac{(e \cos(c+dx))^{3/2}}{\sqrt{a+a \sin(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{e \sqrt{e \cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{ad} \\
 & - \frac{e^{\frac{3}{2}} \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx+c)}}{\sqrt{e}}\right) \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{ad(1+\cos(dx+c)+\sin(dx+c))} \\
 & + \frac{e^{\frac{3}{2}} \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx+c)} \sqrt{1+\cos(dx+c)}}\right) \sqrt{1+\cos(dx+c)} \sqrt{a+a \sin(dx+c)}}{ad(1+\cos(dx+c)+\sin(dx+c))}
 \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.93 Problem number 300

$$\int \frac{\sqrt{e \cos(c + dx)}}{\sqrt{a + a \sin(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{e} \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{d(a + a \cos(dx + c) + a \sin(dx + c))} + \frac{2 \arctan\left(\frac{\sin(dx + c) \sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{e} \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{d(a + a \cos(dx + c) + a \sin(dx + c))}$$

command

```
integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.94 Problem number 305

$$\int \frac{(e \cos(c + dx))^{7/2}}{(a + a \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{e(e \cos(dx + c))^{5/2}}{2ad \sqrt{a + a \sin(dx + c)}} + \frac{5e^3 \sqrt{e \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{4a^2 d} - \frac{5e^{7/2} \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{4a^2 d (1 + \cos(dx + c) + \sin(dx + c))} + \frac{5e^{7/2} \arctan\left(\frac{\sin(dx + c) \sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{4a^2 d (1 + \cos(dx + c) + \sin(dx + c))}$$

command

```
integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.95 Problem number 306

$$\int \frac{(e \cos(c + dx))^{5/2}}{(a + a \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{e(e \cos(dx + c))^{\frac{3}{2}}}{ad\sqrt{a + a \sin(dx + c)}} + \frac{3e^{\frac{5}{2}} \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{d(a^2 + a^2 \cos(dx + c) + a^2 \sin(dx + c))}$$

$$+ \frac{3e^{\frac{5}{2}} \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{d(a^2 + a^2 \cos(dx + c) + a^2 \sin(dx + c))}$$

command

```
integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.96 Problem number 307

$$\int \frac{(e \cos(c + dx))^{3/2}}{(a + a \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2(e \cos(dx + c))^{\frac{5}{2}}}{de(a + a \sin(dx + c))^{\frac{3}{2}}} - \frac{2e\sqrt{e \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{a^2d}$$

$$+ \frac{2e^{\frac{3}{2}} \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{a^2d(1 + \cos(dx + c) + \sin(dx + c))}$$

$$- \frac{2e^{\frac{3}{2}} \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{a^2d(1 + \cos(dx + c) + \sin(dx + c))}$$

command

```
integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.97 Problem number 313

$$\int \frac{(e \cos(c + dx))^{9/2}}{(a + a \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{e(e \cos(dx + c))^{7/2}}{2ad(a + a \sin(dx + c))^{3/2}} + \frac{7e^3(e \cos(dx + c))^{3/2}}{4a^2d\sqrt{a + a \sin(dx + c)}} \\ & + \frac{21e^{9/2} \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{4d(a^3 + a^3 \cos(dx + c) + a^3 \sin(dx + c))} \\ & + \frac{21e^{9/2} \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{4d(a^3 + a^3 \cos(dx + c) + a^3 \sin(dx + c))} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.98 Problem number 314

$$\int \frac{(e \cos(c + dx))^{7/2}}{(a + a \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4e(e \cos(dx + c))^{5/2}}{ad(a + a \sin(dx + c))^{3/2}} - \frac{5e^3 \sqrt{e \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{a^3d} \\ & + \frac{5e^{7/2} \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{a^3d(1 + \cos(dx + c) + \sin(dx + c))} \\ & - \frac{5e^{7/2} \arctan\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{a^3d(1 + \cos(dx + c) + \sin(dx + c))} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.99 Problem number 315

$$\int \frac{(e \cos(c + dx))^{5/2}}{(a + a \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{4e(e \cos(dx + c))^{\frac{3}{2}}}{3ad(a + a \sin(dx + c))^{\frac{3}{2}}} - \frac{2e^{\frac{5}{2}} \operatorname{arcsinh}\left(\frac{\sqrt{e \cos(dx + c)}}{\sqrt{e}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{d(a^3 + a^3 \cos(dx + c) + a^3 \sin(dx + c))} - \frac{2e^{\frac{5}{2}} \operatorname{arctan}\left(\frac{\sin(dx+c)\sqrt{e}}{\sqrt{e \cos(dx + c)} \sqrt{1 + \cos(dx + c)}}\right) \sqrt{1 + \cos(dx + c)} \sqrt{a + a \sin(dx + c)}}{d(a^3 + a^3 \cos(dx + c) + a^3 \sin(dx + c))}$$

command

`integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

40.100 Problem number 479

$$\int \cos^4(c + dx) \sqrt{a + b \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(\cos^3(dx+c))(a+b\sin(dx+c))^{\frac{3}{2}}}{9bd} - \frac{4a(\cos^3(dx+c))\sqrt{a+b\sin(dx+c)}}{21bd}$$

$$- \frac{4\cos(dx+c)(4a(a^2-3b^2)-3b(a^2+7b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{315b^3d}$$

$$+ \frac{8(4a^4-15a^2b^2-21b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{315\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}$$

$$- \frac{32a(a^4-4a^2b^2+3b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{315\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{a+b\sin(dx+c)}}$$

command

`integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{2}\left(8a^5-33a^3b^2+57ab^4\right)\sqrt{ib}\operatorname{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8ia^3-9iab^2\right)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2}{3b}\right)\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b\sin(dx+c)+a}\cos(dx+c)^4,x\right)$$

40.101 Problem number 480

$$\int \cos^2(c+dx)\sqrt{a+b\sin(c+dx)}dx$$

Optimal antiderivative

$$\frac{2\cos(dx+c)(a+b\sin(dx+c))^{\frac{3}{2}}}{5bd} - \frac{4a\cos(dx+c)\sqrt{a+b\sin(dx+c)}}{15bd}$$

$$- \frac{4(a^2+3b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{15\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^2d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}$$

$$+ \frac{4a(a^2-b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{15\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^2d\sqrt{a+b\sin(dx+c)}}$$

command

```
integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{2} (a^3 - 3ab^2) \sqrt{ib} \operatorname{weierstrassPInverse} \left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8ia^3 - 9iab^2)}{27b^3}, \frac{3b \cos(dx+c) - 3ib \sin(dx+c) - 2ia}{3b} \right) + 2 \sqrt{2} a \right)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{b \sin(dx+c) + a} \cos(dx+c)^2, x \right)$$

40.102 Problem number 481

$$\int \sec^2(c+dx) \sqrt{a+b \sin(c+dx)} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a+b \sin(dx+c)}}{\sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) d \sqrt{\frac{a+b \sin(dx+c)}{a+b}}} - \frac{a \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{\frac{a+b \sin(dx+c)}{a+b}}}{\sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) d \sqrt{a+b \sin(dx+c)}} + \frac{\sqrt{a+b \sin(dx+c)} \tan(dx+c)}{d}$$

command

```
integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{2} a \sqrt{ib} \cos(dx+c) \operatorname{weierstrassPInverse} \left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8ia^3 - 9iab^2)}{27b^3}, \frac{3b \cos(dx+c) - 3ib \sin(dx+c) - 2ia}{3b} \right) + 2 \sqrt{2} a}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{b \sin(dx+c) + a} \sec(dx+c)^2, x \right)$$

40.103 Problem number 482

$$\int \sec^4(c + dx) \sqrt{a + b \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sec(dx + c) (ab - (4a^2 - 3b^2) \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{6(a^2 - b^2)d} \\ & + \frac{(4a^2 - 3b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{6 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) (a^2 - b^2) d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\ & - \frac{2a \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{a + b \sin(dx + c)}} \\ & + \frac{(\sec^2(dx + c)) \sqrt{a + b \sin(dx + c)} \tan(dx + c)}{3d} \end{aligned}$$

command

`integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (8a^3 - 9ab^2) \sqrt{ib} \cos(dx + c)^3 \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8ia^3 - 9iab^2)}{27b^3}, \frac{3b \cos(dx + c) - 3ib \sin(dx + c) - 2ia}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sin(dx + c) + a} \sec(dx + c)^4, x\right)$$

40.104 Problem number 489

$$\int \cos^4(c + dx) (a + b \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2b(\cos^5(dx+c))\sqrt{a+b\sin(dx+c)}}{11d} \\
& + \frac{2(\cos^3(dx+c))(a^2+3b^2+28ab\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{231bd} \\
& - \frac{4\cos(dx+c)(4a^4-21a^2b^2-15b^4-3ab(a^2+31b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{1155b^3d} \\
& + \frac{32a(a^4-6a^2b^2-27b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{1155\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}} \\
& - \frac{8(4a^6-25a^4b^2+6a^2b^4+15b^6)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{1155\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{a+b\sin(dx+c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(2\sqrt{2}\left(8a^6-51a^4b^2+126a^2b^4+45b^6\right)\sqrt{ib}\operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib^2}{3b}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b\cos(dx+c)^4\sin(dx+c)+a\cos(dx+c)^4\right)\sqrt{b\sin(dx+c)+a},x\right)$$

40.105 Problem number 490

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b(\cos^3(dx+c))\sqrt{a+b\sin(dx+c)}}{7d} + \frac{2\cos(dx+c)(3a^2+5b^2+24ab\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{105bd}$$

$$\frac{4a(3a^2+29b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{105\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^2d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}$$

$$+ \frac{4(3a^4+2a^2b^2-5b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{105\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^2d\sqrt{a+b\sin(dx+c)}}$$

command

`integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{2}\left(6a^4-23a^2b^2-15b^4\right)\sqrt{ib}\operatorname{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8ia^3-9iab^2\right)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2i}{3b}\right)\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b\cos(dx+c)^2\sin(dx+c)+a\cos(dx+c)^2\right)\sqrt{b\sin(dx+c)+a},x\right)$$

40.106 Problem number 491

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{\sec(dx+c)(b+a\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{d}$$

$$+ \frac{a\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}$$

$$+ \frac{(a^2-b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)d\sqrt{a+b\sin(dx+c)}}$$

command

```
integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i \sqrt{2} a \sqrt{i b} b \cos(dx + c) \operatorname{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}\right), \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}\right), x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b \sec(dx + c)^2 \sin(dx + c) + a \sec(dx + c)^2\right) \sqrt{b \sin(dx + c) + a}, x\right)$$

40.107 Problem number 492

$$\int \sec^4(c + dx)(a + b \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sec(dx + c)(b - 4a \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{6d} \\ + & \frac{(\sec^3(dx + c)(b + a \sin(dx + c)) \sqrt{a + b \sin(dx + c)})}{3d} \\ + & \frac{2a \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \sin(dx + c)}}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\frac{a + b \sin(dx + c)}{a+b}}} \\ + & \frac{(4a^2 - b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a+b}}}{6 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{a + b \sin(dx + c)}} \end{aligned}$$

command

```
integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12i \sqrt{2} a \sqrt{i b} b \cos(dx + c)^3 \operatorname{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}\right), \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}\right), x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b \sec(dx + c)^4 \sin(dx + c) + a \sec(dx + c)^4\right) \sqrt{b \sin(dx + c) + a}, x\right)$$

40.108 Problem number 493

$$\int \sec^6(c + dx)(a + b \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(\sec^3(dx + c)(b - 8a \sin(dx + c)) \sqrt{a + b \sin(dx + c)})}{30d} \\ + & \frac{(\sec^5(dx + c)(b + a \sin(dx + c)) \sqrt{a + b \sin(dx + c)})}{5d} \\ & \frac{\sec(dx + c)(b(8a^4 - 13a^2b^2 + 5b^4) - a(32a^4 - 61a^2b^2 + 29b^4) \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{60(a^2 - b^2)^2 d} \\ + & \frac{a(32a^2 - 29b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{60 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) (a^2 - b^2) d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\ & \frac{(32a^2 - 5b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{60 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{a + b \sin(dx + c)}} \end{aligned}$$

command

```
integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (64a^4 - 82a^2b^2 + 15b^4) \sqrt{ib} \cos(dx + c)^5 \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8ia^3 - 9iab^2)}{27b^3}, \frac{3b \cos(dx + c) - 3ib \sin(dx + c)}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b \sec(dx + c)^6 \sin(dx + c) + a \sec(dx + c)^6\right) \sqrt{b \sin(dx + c) + a}, x\right)$$

40.109 Problem number 500

$$\int \cos^4(c + dx)(a + b \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2b(\cos^5(dx+c))(a+b\sin(dx+c))^{\frac{3}{2}}}{13d} - \frac{32ab(\cos^5(dx+c))\sqrt{a+b\sin(dx+c)}}{143d} \\
& + \frac{2(\cos^3(dx+c))(a(5a^2+59b^2)+7b(53a^2+11b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{3003bd} \\
& - \frac{4\cos(dx+c)(4a(5a^4-40a^2b^2-93b^4)-3b(5a^4+430a^2b^2+77b^4)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{15015b^3d} \\
& + \frac{8(20a^6-175a^4b^2-1662a^2b^4-231b^6)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{15015b^3d} \\
& + \frac{15015\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{15015\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{a+b\sin(dx+c)}} \\
& - \frac{32a(5a^6-45a^4b^2-53a^2b^4+93b^6)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a}}}{15015\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{a+b\sin(dx+c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(2\sqrt{2}(40a^7-365a^5b^2+1026a^3b^4+1347ab^6)\sqrt{ib}\operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)}{a}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(b^2\cos(dx+c)^6-2ab\cos(dx+c)^4\sin(dx+c)-(a^2+b^2)\cos(dx+c)^4\right)\sqrt{b\sin(dx+c)+a},x\right)$$

40.110 Problem number 501

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b(\cos^3(dx+c))(a+b\sin(dx+c))^{\frac{3}{2}}}{9d} - \frac{8ab(\cos^3(dx+c))\sqrt{a+b\sin(dx+c)}}{21d}$$

$$+ \frac{2\cos(dx+c)(a(5a^2+27b^2)+3b(25a^2+7b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{315bd}$$

$$- \frac{4(5a^4+102a^2b^2+21b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{315\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^2d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}$$

$$+ \frac{4a(5a^4+22a^2b^2-27b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{315\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^2d\sqrt{a+b\sin(dx+c)}}$$

command

`integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{2}\left(5a^5-18a^3b^2-51ab^4\right)\sqrt{ib}\operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b\cos(dx+c)-3ib\sin(dx+c)}{3b}\right)\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(b^2\cos(dx+c)^4-2ab\cos(dx+c)^2\sin(dx+c)-(a^2+b^2)\cos(dx+c)^2\right)\sqrt{b\sin(dx+c)+a}, x\right)$$

40.111 Problem number 502

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{\sec(dx+c)(b+a\sin(dx+c))(a+b\sin(dx+c))^{\frac{3}{2}}}{d} + \frac{ab\cos(dx+c)\sqrt{a+b\sin(dx+c)}}{d}$$

$$+ \frac{(a^2+3b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}$$

$$- \frac{a(a^2-b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)d\sqrt{a+b\sin(dx+c)}}$$

command

```
integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\sqrt{2}(a^3 - 3ab^2)\sqrt{ib}\cos(dx+c)\text{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(2ab\sec(dx+c)^2\sin(dx+c) - \left(b^2\cos(dx+c)^2 - a^2 - b^2\right)\sec(dx+c)^2\right)\sqrt{b\sin(dx+c)+a}, x\right)$$

40.112 Problem number 503

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(\sec^3(dx+c))(b+a\sin(dx+c))(a+b\sin(dx+c))^{3/2}}{3d} \\ & + \frac{\sec(dx+c)(ab+(4a^2-3b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{6d} \\ & + \frac{(4a^2-3b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\text{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{6\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}} \\ & + \frac{2a(a^2-b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\text{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{3\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)d\sqrt{a+b\sin(dx+c)}} \end{aligned}$$

command

```
integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(8a^3 - 9ab^2)\sqrt{ib}\cos(dx+c)^3\text{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(2ab\sec(dx+c)^4\sin(dx+c) - \left(b^2\cos(dx+c)^2 - a^2 - b^2\right)\sec(dx+c)^4\right)\sqrt{b\sin(dx+c)+a}, x\right)$$

40.113 Problem number 504

$$\int \sec^6(c + dx)(a + b \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(\sec^5(dx + c))(b + a \sin(dx + c))(a + b \sin(dx + c))^{3/2}}{5d} \\ & + \frac{(\sec^3(dx + c))(5ab + (8a^2 - 3b^2) \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{30d} \\ & - \frac{\sec(dx + c)(8ab(a^2 - b^2) - (32a^4 - 41a^2b^2 + 9b^4) \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{60(a^2 - b^2)d} \\ & + \frac{(32a^2 - 9b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{60 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\ & - \frac{a(32a^2 - 17b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{60 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{a + b \sin(dx + c)}} \end{aligned}$$

command

```
integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{2} (32 a^3 - 21 a b^2) \sqrt{i b} \cos(dx + c)^5 \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8ia^3 - 9iab^2)}{27b^3}, \frac{3b \cos(dx + c) - 3ib \sin(dx + c)}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(2ab \sec(dx + c)^6 \sin(dx + c) - (b^2 \cos(dx + c)^2 - a^2 - b^2) \sec(dx + c)^6\right) \sqrt{b \sin(dx + c) + a}, x\right)$$

40.114 Problem number 505

$$\int \sec^8(c + dx)(a + b \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(\sec^7(dx+c))(b+a\sin(dx+c))(a+b\sin(dx+c))^{\frac{3}{2}}}{7d} \\
+ & \frac{3(\sec^5(dx+c))(3ab+(4a^2-b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{70d} \\
- & \frac{(\sec^3(dx+c))(4ab(a^2-b^2)-(32a^4-39a^2b^2+7b^4)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{140(a^2-b^2)d} \\
- & \frac{\sec(dx+c)(ab(32a^4-59a^2b^2+27b^4)-(128a^6-272a^4b^2+165a^2b^4-21b^6)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{280(a^2-b^2)^2d} \\
+ & \frac{(128a^4-144a^2b^2+21b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{280\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)(a^2-b^2)d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}} \\
- & \frac{2a(8a^2-3b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{35\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)d\sqrt{a+b\sin(dx+c)}}
\end{aligned}$$

command

```
integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(256a^5-384a^3b^2+123ab^4)\sqrt{ib}\cos(dx+c)^7\operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)}{27b^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(2ab\sec(dx+c)^8\sin(dx+c)-\left(b^2\cos(dx+c)^2-a^2-b^2\right)\sec(dx+c)^8\right)\sqrt{b\sin(dx+c)+a},x\right)$$

40.115 Problem number 512

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2(\cos^3(dx+c))\sqrt{a+b\sin(dx+c)}}{7bd} - \frac{4\cos(dx+c)(4a^2-5b^2-3ab\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{35b^3d}$$

$$+ \frac{32a(a^2-2b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{35\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}$$

$$- \frac{8(4a^4-9a^2b^2+5b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{35\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{a+b\sin(dx+c)}}$$

command

`integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(2\sqrt{2}\left(8a^4-19a^2b^2+15b^4\right)\sqrt{ib}\operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2i}{3b}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx+c)^4}{\sqrt{b\sin(dx+c)+a}},x\right)$$

40.116 Problem number 513

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sin(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\cos(dx+c)\sqrt{a+b\sin(dx+c)}}{3bd} - \frac{4a\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{3\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^2d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}$$

$$+ \frac{4(a^2-b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{3\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^2d\sqrt{a+b\sin(dx+c)}}$$

command

```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3i \sqrt{2} a \sqrt{i b} b \text{weierstrassZeta} \left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3} \right), \text{weierstrassPInverse} \left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx+c)^2}{\sqrt{b \sin(dx+c) + a}}, x \right)$$

40.117 Problem number 514

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b \sin(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sec(dx+c)(b-a \sin(dx+c)) \sqrt{a+b \sin(dx+c)}}{(a^2-b^2)d} \\ & + \frac{a \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \text{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a+b \sin(dx+c)}}{\sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) (a^2-b^2) d \sqrt{\frac{a+b \sin(dx+c)}{a+b}}} \\ & - \frac{\sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \text{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{\frac{a+b \sin(dx+c)}{a+b}}}{\sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) d \sqrt{a+b \sin(dx+c)}} \end{aligned}$$

command

```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} a \sqrt{i b} b \cos(dx+c) \text{weierstrassZeta} \left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3} \right), \text{weierstrassPInverse} \left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sec(dx+c)^2}{\sqrt{b \sin(dx+c) + a}}, x \right)$$

40.118 Problem number 515

$$\int \frac{\sec^4(c + dx)}{\sqrt{a + b \sin(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(\sec^3(dx + c))(b - a \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{3(a^2 - b^2)d} \\ & - \frac{\sec(dx + c)(b(a^2 - 5b^2) - 4a(a^2 - 2b^2) \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{6(a^2 - b^2)^2 d} \\ & + \frac{2a(a^2 - 2b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) (a^2 - b^2)^2 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\ & - \frac{(4a^2 - 5b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{6 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) (a^2 - b^2) d \sqrt{a + b \sin(dx + c)}} \end{aligned}$$

command

`integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (8a^4 - 19a^2b^2 + 15b^4) \sqrt{ib} \cos(dx + c)^3 \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8ia^3 - 9iab^2)}{27b^3}, \frac{3b \cos(dx + c) - 3ib \sin(dx + c)}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(dx + c)^4}{\sqrt{b \sin(dx + c) + a}}, x\right)$$

40.119 Problem number 522

$$\int \frac{\cos^6(c + dx)}{(a + b \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(\cos^5(dx+c))}{bd\sqrt{a+b\sin(dx+c)}} + \frac{20(\cos^3(dx+c))(8a-7b\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{63b^3d}$$

$$\frac{8\cos(dx+c)(a(32a^2-33b^2)-3b(8a^2-7b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{63b^5d}$$

$$+ \frac{16(32a^4-57a^2b^2+21b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{63\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^6d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}$$

$$- \frac{16a(32a^4-65a^2b^2+33b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{63\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^6d\sqrt{a+b\sin(dx+c)}}$$

command

`integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(8\left(\sqrt{2}(32a^5b-69a^3b^3+39ab^5)\sin(dx+c)+\sqrt{2}(32a^6-69a^4b^2+39a^2b^4)\right)\sqrt{ib}\operatorname{weierstrassPInverse}\left(-\frac{4}{\sqrt{2}(32a^5b-69a^3b^3+39ab^5)\sin(dx+c)+\sqrt{2}(32a^6-69a^4b^2+39a^2b^4)}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\cos(dx+c)^6}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2},x\right)$$

40.120 Problem number 523

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(\cos^3(dx+c))}{bd\sqrt{a+b\sin(dx+c)}} + \frac{4\cos(dx+c)(4a-3b\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{5b^3d}$$

$$\frac{8(4a^2-3b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{5\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}$$

$$+ \frac{32a(a^2-b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{5\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{a+b\sin(dx+c)}}$$

command

```
integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \left(\sqrt{2} (8a^3b - 9ab^3) \sin(dx + c) + \sqrt{2} (8a^4 - 9a^2b^2) \right) \sqrt{ib} \operatorname{weierstrassPInverse} \left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8ia^3 - 9iab)}{27b^3} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{b \sin(dx + c) + a} \cos(dx + c)^4}{b^2 \cos(dx + c)^2 - 2ab \sin(dx + c) - a^2 - b^2}, x \right)$$

40.121 Problem number 524

$$\int \frac{\cos^2(c + dx)}{(a + b \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2 \cos(dx + c)}{bd \sqrt{a + b \sin(dx + c)}} + 4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{a + b \sin(dx + c)}}{\sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) b^2 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} + \frac{4a \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{\sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) b^2 d \sqrt{a + b \sin(dx + c)}}$$

command

```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{b \sin(dx + c) + a} b^2 \cos(dx + c) - 2 \left(\sqrt{2} ab \sin(dx + c) + \sqrt{2} a^2 \right) \sqrt{ib} \operatorname{weierstrassPInverse} \left(-\frac{4(4a^2 - 3b^2)}{3b^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{\sqrt{b \sin(dx + c) + a} \cos(dx + c)^2}{b^2 \cos(dx + c)^2 - 2ab \sin(dx + c) - a^2 - b^2}, x \right)$$

40.122 Problem number 525

$$\int \frac{\sec^2(c + dx)}{(a + b \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b \sec(dx + c)}{(a^2 - b^2) d \sqrt{a + b \sin(dx + c)}} - \frac{\sec(dx + c) (4ab - (a^2 + 3b^2) \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{(a^2 - b^2)^2 d}$$

$$+ \frac{(a^2 + 3b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) (a^2 - b^2)^2 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}$$

$$- \frac{a \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) (a^2 - b^2) d \sqrt{a + b \sin(dx + c)}}$$

command

`integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (a^3 b - 3 a b^3) \cos(dx + c) \sin(dx + c) + \sqrt{2} (a^4 - 3 a^2 b^2) \cos(dx + c) \right) \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{b \sin(dx + c) + a} \sec(dx + c)^2}{b^2 \cos(dx + c)^2 - 2ab \sin(dx + c) - a^2 - b^2}, x\right)$$

40.123 Problem number 526

$$\int \frac{\sec^4(c + dx)}{(a + b \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b(\sec^3(dx+c))}{(a^2-b^2)d\sqrt{a+b\sin(dx+c)}} \\
& - \frac{(\sec^3(dx+c))(8ab-(a^2+7b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{3(a^2-b^2)^2d} \\
& - \frac{\sec(dx+c)(ab(a^2-33b^2)-(4a^4-15a^2b^2-21b^4)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{6(a^2-b^2)^3d} \\
& + \frac{(4a^4-15a^2b^2-21b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{6\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)(a^2-b^2)^3d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}} \\
& - \frac{2a(a^2-3b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{3\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)(a^2-b^2)^2d\sqrt{a+b\sin(dx+c)}}
\end{aligned}$$

command

```
integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(8a^5b-33a^3b^3+57ab^5)\cos(dx+c)^3\sin(dx+c)+\sqrt{2}(8a^6-33a^4b^2+57a^2b^4)\cos(dx+c)^3\right)\sqrt{ib}\operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\sec(dx+c)^4}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2},x\right)$$

40.124 Problem number 533

$$\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(\cos^7(dx+c))}{3bd(a+b\sin(dx+c))^{\frac{3}{2}}} - \frac{28(\cos^5(dx+c))(12a+b\sin(dx+c))}{33b^3d\sqrt{a+b\sin(dx+c)}} \\
& + \frac{40(\cos^3(dx+c))(32a^2-3b^2-28ab\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{99b^5d} \\
& - \frac{16\cos(dx+c)(128a^4-144a^2b^2+15b^4-3ab(32a^2-31b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{99b^7d} \\
& + \frac{128a(8a^2-9b^2)(4a^2-3b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{99\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^8d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}} \\
& - \frac{32(128a^6-272a^4b^2+159a^2b^4-15b^6)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{99\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^8d\sqrt{a+b\sin(dx+c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(8\left(\sqrt{2}\left(256a^6b^2-576a^4b^4+369a^2b^6-45b^8\right)\cos(dx+c)^2-2\sqrt{2}\left(256a^7b-576a^5b^3+369a^3b^5-45ab^7\right)\sin(dx+c)\right)\sqrt{a+b\sin(dx+c)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\cos(dx+c)^8}{3ab^2\cos(dx+c)^2-a^3-3ab^2+(b^3\cos(dx+c)^2-3a^2b-b^3)\sin(dx+c)},x\right)$$

40.125 Problem number 534

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(\cos^5(dx+c))}{3bd(a+b\sin(dx+c))^{\frac{3}{2}}} - \frac{20(\cos^3(dx+c))(8a+b\sin(dx+c))}{21b^3d\sqrt{a+b\sin(dx+c)}} + \frac{8\cos(dx+c)(32a^2-5b^2-24ab\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{21b^5d} - \frac{16a(32a^2-29b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{21\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^6d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}} + \frac{16(32a^4-37a^2b^2+5b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{21\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^6d\sqrt{a+b\sin(dx+c)}}$$

command

`integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4\left(\sqrt{2}(64a^4b^2-82a^2b^4+15b^6)\cos(dx+c)^2-2\sqrt{2}(64a^5b-82a^3b^3+15ab^5)\sin(dx+c)-\sqrt{2}(64a^6-\right.\right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sin(dx+c)+a}\cos(dx+c)^6}{3ab^2\cos(dx+c)^2-a^3-3ab^2+(b^3\cos(dx+c)^2-3a^2b-b^3)\sin(dx+c)},x\right)$$

40.126 Problem number 535

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(\cos^3(dx+c))}{3bd(a+b\sin(dx+c))^{\frac{3}{2}}} - \frac{4\cos(dx+c)(4a+b\sin(dx+c))}{3b^3d\sqrt{a+b\sin(dx+c)}} + \frac{32a\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{3\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}} + \frac{8(4a^2-b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{3\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^4d\sqrt{a+b\sin(dx+c)}}$$

command

```
integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (8a^2b^2 - 3b^4) \cos(dx + c)^2 - 2\sqrt{2} (8a^3b - 3ab^3) \sin(dx + c) - \sqrt{2} (8a^4 + 5a^2b^2 - 3b^4) \right) \sqrt{ib} \text{ weierstr}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{b \sin(dx + c) + a} \cos(dx + c)^4}{3ab^2 \cos(dx + c)^2 - a^3 - 3ab^2 + (b^3 \cos(dx + c)^2 - 3a^2b - b^3) \sin(dx + c)}, x \right)$$

40.127 Problem number 536

$$\int \frac{\cos^2(c + dx)}{(a + b \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cos(dx + c)}{3bd(a + b \sin(dx + c))^{3/2}} + \frac{4a \cos(dx + c)}{3b(a^2 - b^2) d \sqrt{a + b \sin(dx + c)}} \\ & - \frac{4a \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{a + b \sin(dx + c)}}{3 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) b^2 (a^2 - b^2) d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\ & + \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{3 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) b^2 d \sqrt{a + b \sin(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} (2a^2b^2 - 3b^4) \cos(dx + c)^2 - 2\sqrt{2} (2a^3b - 3ab^3) \sin(dx + c) - \sqrt{2} (2a^4 - a^2b^2 - 3b^4) \right) \sqrt{ib} \text{ weierstr}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{b \sin(dx + c) + a} \cos(dx + c)^2}{3ab^2 \cos(dx + c)^2 - a^3 - 3ab^2 + (b^3 \cos(dx + c)^2 - 3a^2b - b^3) \sin(dx + c)}, x \right)$$

40.128 Problem number 537

$$\int \frac{\sec^2(c + dx)}{(a + b \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2b \sec(dx + c)}{3(a^2 - b^2)d(a + b \sin(dx + c))^{3/2}} + \frac{16ab \sec(dx + c)}{3(a^2 - b^2)^2 d \sqrt{a + b \sin(dx + c)}} - \frac{\sec(dx + c)(b(27a^2 + 5b^2) - a(3a^2 + 29b^2) \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{3(a^2 - b^2)^3 d} + \frac{a(3a^2 + 29b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) (a^2 - b^2)^3 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} + \frac{(3a^2 + 5b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) (a^2 - b^2)^2 d \sqrt{a + b \sin(dx + c)}}$$

command

`integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(6a^4b^2 - 23a^2b^4 - 15b^6) \cos(dx + c)^3 - 2\sqrt{2}(6a^5b - 23a^3b^3 - 15ab^5) \cos(dx + c) \sin(dx + c) - \sqrt{2}(6a^6 - 23a^4b^2 + 15b^6) \sin(dx + c)^3\right) \sqrt{a + b \sin(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{b \sin(dx + c) + a} \sec(dx + c)^2}{3ab^2 \cos(dx + c)^2 - a^3 - 3ab^2 + (b^3 \cos(dx + c)^2 - 3a^2b - b^3) \sin(dx + c)}, x\right)$$

40.129 Problem number 538

$$\int \frac{\sec^4(c + dx)}{(a + b \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2b(\sec^3(dx+c))}{3(a^2-b^2)d(a+b\sin(dx+c))^{\frac{3}{2}}} + \frac{8ab(\sec^3(dx+c))}{(a^2-b^2)^2 d \sqrt{a+b\sin(dx+c)}}$$

$$\frac{(\sec^3(dx+c))(b(29a^2+3b^2)-a(a^2+31b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{3(a^2-b^2)^3 d}$$

$$\frac{\sec(dx+c)(b(a^4-114a^2b^2-15b^4)-4a(a^4-6a^2b^2-27b^4)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{6(a^2-b^2)^4 d}$$

$$+ \frac{2a(a^4-6a^2b^2-27b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{3\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)(a^2-b^2)^4 d \sqrt{\frac{a+b\sin(dx+c)}{a+b}}}$$

$$\frac{(4a^4-21a^2b^2-15b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{6\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)(a^2-b^2)^3 d \sqrt{a+b\sin(dx+c)}}$$

command

`integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(8a^6b^2-51a^4b^4+126a^2b^6+45b^8)\cos(dx+c)^5-2\sqrt{2}(8a^7b-51a^5b^3+126a^3b^5+45ab^7)\cos(dx+c)\right)^3$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\sec(dx+c)^4}{3ab^2\cos(dx+c)^2-a^3-3ab^2+(b^3\cos(dx+c)^2-3a^2b-b^3)\sin(dx+c)}, x\right)$$

40.130 Problem number 539

$$\int (e \cos(c+dx))^{7/2} (a+b\sin(c+dx)) dx$$

Optimal antiderivative

$$-\frac{2b(e\cos(dx+c))^{\frac{9}{2}}}{9de} + \frac{2ae(e\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{7d}$$

$$+ \frac{10ae^4\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{e\cos(dx+c)}}$$

$$+ \frac{10ae^3\sin(dx+c)\sqrt{e\cos(dx+c)}}{21d}$$

command

```
integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i \sqrt{2} a e^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} a e^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

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Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b e^3 \cos(dx + c)^3 \sin(dx + c) + a e^3 \cos(dx + c)^3\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.131 Problem number 540

$$\int (e \cos(c + dx))^{5/2} (a + b \sin(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(e \cos(dx + c))^{\frac{7}{2}}}{7de} + \frac{2ae(e \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5d} \\ & + \frac{6a e^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i \sqrt{2} a e^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2} a e^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b e^2 \cos(dx + c)^2 \sin(dx + c) + a e^2 \cos(dx + c)^2\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.132 Problem number 541

$$\int (e \cos(c + dx))^{3/2} (a + b \sin(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(e \cos(dx + c))^{\frac{5}{2}}}{5de} \\ & + \frac{2ae^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & + \frac{2ae \sin(dx + c) \sqrt{e \cos(dx + c)}}{3d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} a e^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} a e^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{15d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((be \cos(dx + c) \sin(dx + c) + ae \cos(dx + c)) \sqrt{e \cos(dx + c)}, x\right)$$

40.133 Problem number 542

$$\int \sqrt{e \cos(c + dx)} (a + b \sin(c + dx)) dx$$

Optimal antiderivative

$$-\frac{2b(e \cos(dx + c))^{\frac{3}{2}}}{3de} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2b \cos(dx + c)^{\frac{3}{2}} e^{\frac{1}{2}} - 3i \sqrt{2} a e^{\frac{1}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{e \cos(dx + c)} (b \sin(dx + c) + a), x\right)$$

40.134 Problem number 543

$$\int \frac{a + b \sin(c + dx)}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} - \frac{2b \sqrt{e \cos(dx + c)}}{de}$$

command

`integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c))\right)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)} (b \sin(dx + c) + a)}{e \cos(dx + c)}, x\right)$$

40.135 Problem number 544

$$\int \frac{a + b \sin(c + dx)}{(e \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b}{de \sqrt{e \cos(dx + c)}} + \frac{2a \sin(dx + c)}{de \sqrt{e \cos(dx + c)}} - \frac{2a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^2 \sqrt{\cos(dx + c)}}$$

command

`integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i \sqrt{2} a \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} a \cos(dx + c)\right)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)} (b \sin(dx + c) + a)}{e^2 \cos(dx + c)^2}, x\right)$$

40.136 Problem number 545

$$\int \frac{a + b \sin(c + dx)}{(e \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2b}{3de (e \cos(dx + c))^{\frac{3}{2}}} + \frac{2a \sin(dx + c)}{3de (e \cos(dx + c))^{\frac{3}{2}}} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^2 \sqrt{e \cos(dx + c)}}$$

command

```
integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i \sqrt{2} a \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \cos(dx + c)^2 \operatorname{weierstrassP}\right)}{3 d \cos(dx + c)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \cos(dx + c)} (b \sin(dx + c) + a)}{e^3 \cos(dx + c)^3}, x\right)$$

40.137 Problem number 546

$$\int \frac{a + b \sin(c + dx)}{(e \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2b}{5de (e \cos(dx + c))^{\frac{5}{2}}} + \frac{2a \sin(dx + c)}{5de (e \cos(dx + c))^{\frac{5}{2}}} + \frac{6a \sin(dx + c)}{5d e^3 \sqrt{e \cos(dx + c)}} - \frac{6a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4 \sqrt{\cos(dx + c)}}$$

command

```
integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-3i \sqrt{2} a \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2} a\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{e \cos(dx + c)} (b \sin(dx + c) + a)}{e^4 \cos(dx + c)^4}, x\right)$$

40.138 Problem number 547

$$\int (e \cos(c + dx))^{7/2} (a + b \sin(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{26ab(e \cos(dx + c))^{\frac{9}{2}}}{99de} + \frac{2(11a^2 + 2b^2) e(e \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{77d} \\ & - \frac{2b(e \cos(dx + c))^{\frac{9}{2}} (a + b \sin(dx + c))}{11de} \\ & + \frac{10(11a^2 + 2b^2) e^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & + \frac{10(11a^2 + 2b^2) e^3 \sin(dx + c) \sqrt{e \cos(dx + c)}}{231d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i \sqrt{2} (11a^2 + 2b^2) e^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} (11a^2 + 2b^2) e^{\frac{7}{2}} \text{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(b^2 e^3 \cos(dx + c)^5 - 2abe^3 \cos(dx + c)^3 \sin(dx + c) - (a^2 + b^2) e^3 \cos(dx + c)^3\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.139 Problem number 548

$$\int (e \cos(c + dx))^{5/2} (a + b \sin(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{22ab(e \cos(dx + c))^{7/2}}{63de} + \frac{2(9a^2 + 2b^2) e(e \cos(dx + c))^{3/2} \sin(dx + c)}{45d} \\ & - \frac{2b(e \cos(dx + c))^{7/2} (a + b \sin(dx + c))}{9de} \\ & + \frac{2(9a^2 + 2b^2) e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i \sqrt{2} (9a^2 + 2b^2) e^{5/2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2} (9$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(b^2 e^2 \cos(dx + c)^4 - 2abe^2 \cos(dx + c)^2 \sin(dx + c) - (a^2 + b^2) e^2 \cos(dx + c)^2\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.140 Problem number 549

$$\int (e \cos(c + dx))^{3/2} (a + b \sin(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{18ab(e \cos(dx + c))^{5/2}}{35de} - \frac{2b(e \cos(dx + c))^{5/2} (a + b \sin(dx + c))}{7de} \\ & + \frac{2(7a^2 + 2b^2) e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & + \frac{2(7a^2 + 2b^2) e \sin(dx + c) \sqrt{e \cos(dx + c)}}{21d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(7a^2+2b^2)e^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(7a^2+2b^2)e^{\frac{3}{2}}\text{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(b^2e\cos(dx+c)^3-2abe\cos(dx+c)\sin(dx+c)-(a^2+b^2)e\cos(dx+c)\right)\sqrt{e\cos(dx+c)},x\right)$$

40.141 Problem number 550

$$\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} &-\frac{14ab(e\cos(dx+c))^{\frac{3}{2}}}{15de}-\frac{2b(e\cos(dx+c))^{\frac{3}{2}}(a+b\sin(dx+c))}{5de} \\ &+\frac{2(5a^2+2b^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{e\cos(dx+c)}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i\sqrt{2}(5a^2+2b^2)e^{\frac{1}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}(5a$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2\right)\sqrt{e\cos(dx+c)},x\right)$$

40.142 Problem number 551

$$\int \frac{(a + b \sin(c + dx))^2}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(3a^2 + 2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} - \frac{10ab \sqrt{e \cos(dx + c)}}{3de} - \frac{2b(a + b \sin(dx + c)) \sqrt{e \cos(dx + c)}}{3de}$$

command

```
integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\sqrt{2}(-3ia^2 - 2ib^2)\operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(3ia^2 + 2ib^2)\operatorname{weierstrassPInverse}(\dots))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(b^2 \cos(dx + c)^2 - 2ab \sin(dx + c) - a^2 - b^2) \sqrt{e \cos(dx + c)}}{e \cos(dx + c)}, x\right)$$

40.143 Problem number 552

$$\int \frac{(a + b \sin(c + dx))^2}{(e \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2ab(e \cos(dx + c))^{\frac{3}{2}}}{de^3} + \frac{2(b + a \sin(dx + c))(a + b \sin(dx + c))}{de \sqrt{e \cos(dx + c)}} - \frac{2(a^2 + 2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^2 \sqrt{\cos(dx + c)}}$$

command

```
integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-i a^2 - 2i b^2) \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{(b^2 \cos(dx + c)^2 - 2ab \sin(dx + c) - a^2 - b^2) \sqrt{e \cos(dx + c)}}{e^2 \cos(dx + c)^2}, x \right)$$

40.144 Problem number 553

$$\int \frac{(a + b \sin(c + dx))^2}{(e \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b + a \sin(dx + c))(a + b \sin(dx + c))}{3de (e \cos(dx + c))^{\frac{3}{2}}} \\ & + \frac{2(a^2 - 2b^2) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^2 \sqrt{e \cos(dx + c)}} \\ & + \frac{2ab \sqrt{e \cos(dx + c)}}{3d e^3} \end{aligned}$$

command

```
integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-i a^2 + 2i b^2) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i a^2 - 2i b^2) \cos(dx + c) \right.$$

$$3 d \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{(b^2 \cos(dx + c)^2 - 2ab \sin(dx + c) - a^2 - b^2) \sqrt{e \cos(dx + c)}}{e^3 \cos(dx + c)^3}, x \right)$$

40.145 Problem number 554

$$\int \frac{(a + b \sin(c + dx))^2}{(e \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(b + a \sin(dx + c))(a + b \sin(dx + c))}{5de(e \cos(dx + c))^{\frac{5}{2}}} + \frac{2ab}{5de^3 \sqrt{e \cos(dx + c)}} + \frac{2(3a^2 - 2b^2) \sin(dx + c)}{5de^3 \sqrt{e \cos(dx + c)}} - \frac{2(3a^2 - 2b^2) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4 \sqrt{\cos(dx + c)}}$$

command

`integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-3ia^2 + 2ib^2) \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(b^2 \cos(dx + c)^2 - 2ab \sin(dx + c) - a^2 - b^2) \sqrt{e \cos(dx + c)}}{e^4 \cos(dx + c)^4}, x\right)$$

40.146 Problem number 555

$$\int (e \cos(c + dx))^{7/2} (a + b \sin(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(177a^2 + 44b^2)(e \cos(dx + c))^{\frac{9}{2}}}{1287de} + \frac{2a(11a^2 + 6b^2)e(e \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{77d} \\ & -\frac{34ab(e \cos(dx + c))^{\frac{9}{2}}(a + b \sin(dx + c))}{143de} - \frac{2b(e \cos(dx + c))^{\frac{9}{2}}(a + b \sin(dx + c))^2}{13de} \\ & + \frac{10a(11a^2 + 6b^2)e^4 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})^2}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & + \frac{10a(11a^2 + 6b^2)e^3 \sin(dx + c) \sqrt{e \cos(dx + c)}}{231d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-195i \sqrt{2} (11 a^3 + 6 a b^2) e^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 195i \sqrt{2} (11 a^3 + 6 a b^2) e^{\frac{7}{2}} w$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(3 a b^2 e^3 \cos(dx + c)^5 - (a^3 + 3 a b^2) e^3 \cos(dx + c)^3 + (b^3 e^3 \cos(dx + c)^5 - (3 a^2 b + b^3) e^3 \cos(dx + c)^3\right.\right.$$

40.147 Problem number 556

$$\int (e \cos(c + dx))^{5/2} (a + b \sin(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(43a^2 + 12b^2) (e \cos(dx + c))^{\frac{7}{2}}}{231de} + \frac{2a(3a^2 + 2b^2) e (e \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{15d} \\ & - \frac{10ab(e \cos(dx + c))^{\frac{7}{2}} (a + b \sin(dx + c))}{33de} - \frac{2b(e \cos(dx + c))^{\frac{7}{2}} (a + b \sin(dx + c))^2}{11de} \\ & + \frac{2a(3a^2 + 2b^2) e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231i \sqrt{2} (3 a^3 + 2 a b^2) e^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 231i \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(3 a b^2 e^2 \cos(dx + c)^4 - (a^3 + 3 a b^2) e^2 \cos(dx + c)^2 + (b^3 e^2 \cos(dx + c)^4 - (3 a^2 b + b^3) e^2 \cos(dx + c)^2\right.\right.$$

40.148 Problem number 557

$$\int (e \cos(c + dx))^{3/2} (a + b \sin(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(89a^2 + 28b^2) (e \cos(dx + c))^{5/2}}{315de} - \frac{26ab(e \cos(dx + c))^{5/2} (a + b \sin(dx + c))}{63de} \\ & - \frac{2b(e \cos(dx + c))^{5/2} (a + b \sin(dx + c))^2}{9de} \\ & + \frac{2a(7a^2 + 6b^2) e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & + \frac{2a(7a^2 + 6b^2) e \sin(dx + c) \sqrt{e \cos(dx + c)}}{21d} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i \sqrt{2} (7a^3 + 6ab^2) e^{3/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} (7a^3 + 6ab^2) e^{3/2} \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(3ab^2 e \cos(dx + c)\right)^3 - (a^3 + 3ab^2) e \cos(dx + c) + \left(b^3 e \cos(dx + c)\right)^3 - (3a^2b + b^3) e \cos(dx + c)\right) \sin(dx + c)$$

40.149 Problem number 558

$$\int \sqrt{e \cos(c + dx)} (a + b \sin(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(57a^2 + 20b^2) (e \cos(dx + c))^{3/2}}{105de} - \frac{22ab(e \cos(dx + c))^{3/2} (a + b \sin(dx + c))}{35de} \\ & - \frac{2b(e \cos(dx + c))^{3/2} (a + b \sin(dx + c))^2}{7de} \\ & + \frac{2a(5a^2 + 6b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i\sqrt{2}(5a^3 + 6ab^2)e^{\frac{1}{2}}\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i\sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(3ab^2 \cos(dx + c)^2 - a^3 - 3ab^2 + \left(b^3 \cos(dx + c)^2 - 3a^2b - b^3\right) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}, x\right)$$

40.150 Problem number 559

$$\int \frac{(a + b \sin(c + dx))^3}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2a(a^2 + 2b^2) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} - \frac{2b(11a^2 + 4b^2) \sqrt{e \cos(dx + c)}}{5de} - \frac{6ab(a + b \sin(dx + c)) \sqrt{e \cos(dx + c)}}{5de} - \frac{2b(a + b \sin(dx + c))^2 \sqrt{e \cos(dx + c)}}{5de}$$

command

```
integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$(5\sqrt{2}(ia^3 + 2iab^2)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-ia^3 - 2iab^2)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 5\sqrt{2}(-ia^3 - 2iab^2)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\left(3ab^2 \cos(dx + c)^2 - a^3 - 3ab^2 + \left(b^3 \cos(dx + c)^2 - 3a^2b - b^3\right) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}}{e \cos(dx + c)}, x\right)$$

40.151 Problem number 560

$$\int \frac{(a + b \sin(c + dx))^3}{(e \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(3a^2 + 4b^2)(e \cos(dx + c))^{\frac{3}{2}}}{3de^3} + \frac{2ab(e \cos(dx + c))^{\frac{3}{2}}(a + b \sin(dx + c))}{de^3} \\ & + \frac{2(b + a \sin(dx + c))(a + b \sin(dx + c))^2}{de \sqrt{e \cos(dx + c)}} \\ & - \frac{2a(a^2 + 6b^2) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^2 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(3\sqrt{2}(ia^3 + 6iab^2) \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(3ab^2 \cos(dx + c)^2 - a^3 - 3ab^2 + \left(b^3 \cos(dx + c)^2 - 3a^2b - b^3\right) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}}{e^2 \cos(dx + c)^2}, x\right)$$

40.152 Problem number 561

$$\int \frac{(a + b \sin(c + dx))^3}{(e \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b + a \sin(dx + c))(a + b \sin(dx + c))^2}{3de(e \cos(dx + c))^{\frac{3}{2}}} \\ & + \frac{2a(a^2 - 6b^2) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^2 \sqrt{e \cos(dx + c)}} \\ & + \frac{2b(a^2 + 4b^2) \sqrt{e \cos(dx + c)}}{3de^3} + \frac{2ab(a + b \sin(dx + c)) \sqrt{e \cos(dx + c)}}{3de^3} \end{aligned}$$

command

```
integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-i a^3 + 6i ab^2) \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i a^3 - 6i ab^2) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\left(3 ab^2 \cos(dx + c)^2 - a^3 - 3 ab^2 + \left(b^3 \cos(dx + c)^2 - 3 a^2 b - b^3 \right) \sin(dx + c) \right) \sqrt{e \cos(dx + c)}}{e^3 \cos(dx + c)^3}, x \right)$$

40.153 Problem number 562

$$\int \frac{(a + b \sin(c + dx))^3}{(e \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(3a^2 - 4b^2) (e \cos(dx + c))^{\frac{3}{2}}}{5d e^5} + \frac{2(b + a \sin(dx + c)) (a + b \sin(dx + c))^2}{5de (e \cos(dx + c))^{\frac{5}{2}}} \\ & - \frac{2(a + b \sin(dx + c)) (ab - (3a^2 - 4b^2) \sin(dx + c))}{5d e^3 \sqrt{e \cos(dx + c)}} \\ & - \frac{6a(a^2 - 2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(3 \sqrt{2} (i a^3 - 2i ab^2) \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\left(3 ab^2 \cos(dx + c)^2 - a^3 - 3 ab^2 + \left(b^3 \cos(dx + c)^2 - 3 a^2 b - b^3 \right) \sin(dx + c) \right) \sqrt{e \cos(dx + c)}}{e^4 \cos(dx + c)^4}, x \right)$$

40.154 Problem number 563

$$\int \frac{(a + b \sin(c + dx))^3}{(e \cos(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b + a \sin(dx + c))(a + b \sin(dx + c))^2}{7de (e \cos(dx + c))^{\frac{7}{2}}} \\ & + \frac{2(a + b \sin(dx + c))(ab + (5a^2 - 4b^2) \sin(dx + c))}{21de^3 (e \cos(dx + c))^{\frac{3}{2}}} \\ & + \frac{2a(5a^2 - 6b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^4 \sqrt{e \cos(dx + c)}} \\ & + \frac{2b(5a^2 - 4b^2) \sqrt{e \cos(dx + c)}}{21de^5} \end{aligned}$$

command

```
integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{2}(-5ia^3 + 6iab^2) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(5ia^3 - 6iab^2)\right)}{e^5 \cos(dx + c)^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(3ab^2 \cos(dx + c)^2 - a^3 - 3ab^2 + \left(b^3 \cos(dx + c)^2 - 3a^2b - b^3\right) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}}{e^5 \cos(dx + c)^5}, x\right)$$

40.155 Problem number 564

$$\int (e \cos(c + dx))^{7/2} (a + b \sin(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{34ab(53a^2 + 38b^2) (e \cos(dx + c))^{\frac{9}{2}}}{6435de} + \frac{2(55a^4 + 60a^2b^2 + 4b^4) e(e \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{385d} \\
 & - \frac{2b(93a^2 + 26b^2) (e \cos(dx + c))^{\frac{9}{2}} (a + b \sin(dx + c))}{715de} \\
 & - \frac{14ab(e \cos(dx + c))^{\frac{9}{2}} (a + b \sin(dx + c))^2}{65de} - \frac{2b(e \cos(dx + c))^{\frac{9}{2}} (a + b \sin(dx + c))^3}{15de} \\
 & + \frac{2(55a^4 + 60a^2b^2 + 4b^4) e^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\
 & + \frac{2(55a^4 + 60a^2b^2 + 4b^4) e^3 \sin(dx + c) \sqrt{e \cos(dx + c)}}{231d}
 \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-195i \sqrt{2} (55a^4 + 60a^2b^2 + 4b^4) e^{\frac{7}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 195i \sqrt{2} (55a^4 + 60a^2b^2 + 4b^4) e^{\frac{7}{2}} \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^4 e^3 \cos(dx + c)^7 - 2(3a^2b^2 + b^4) e^3 \cos(dx + c)^5 + (a^4 + 6a^2b^2 + b^4) e^3 \cos(dx + c)^3 - 4(ab^3 e^3 \cos(dx + c) + a^3 b e^3 \cos(dx + c))\right) dx\right)$$

40.156 Problem number 565

$$\int (e \cos(c + dx))^{5/2} (a + b \sin(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{10ab(115a^2 + 94b^2) (e \cos(dx + c))^{\frac{7}{2}}}{3003de} + \frac{2(39a^4 + 52a^2b^2 + 4b^4) e(e \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{195d} \\
 & - \frac{2b(73a^2 + 22b^2) (e \cos(dx + c))^{\frac{7}{2}} (a + b \sin(dx + c))}{429de} \\
 & - \frac{38ab(e \cos(dx + c))^{\frac{7}{2}} (a + b \sin(dx + c))^2}{143de} - \frac{2b(e \cos(dx + c))^{\frac{7}{2}} (a + b \sin(dx + c))^3}{13de} \\
 & + \frac{2(39a^4 + 52a^2b^2 + 4b^4) e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{65 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}
 \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231i\sqrt{2}(39a^4 + 52a^2b^2 + 4b^4)e^{\frac{5}{2}}\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^4e^2\cos(dx + c)^6 - 2(3a^2b^2 + b^4)e^2\cos(dx + c)^4 + (a^4 + 6a^2b^2 + b^4)e^2\cos(dx + c)^2 - 4(ab^3e^2\cos(dx + c)^2 - 2ab^2e^2\cos(dx + c) + a^2e^2)\right)\right)$$

40.157 Problem number 566

$$\int (e \cos(c + dx))^{3/2} (a + b \sin(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{26ab(79a^2 + 74b^2)(e \cos(dx + c))^{\frac{5}{2}}}{3465de} - \frac{2b(167a^2 + 54b^2)(e \cos(dx + c))^{\frac{5}{2}}(a + b \sin(dx + c))}{693de} \\ & - \frac{34ab(e \cos(dx + c))^{\frac{5}{2}}(a + b \sin(dx + c))^2}{99de} - \frac{2b(e \cos(dx + c))^{\frac{5}{2}}(a + b \sin(dx + c))^3}{11de} \\ & + \frac{2(77a^4 + 132a^2b^2 + 12b^4)e^2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})}{231\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d\sqrt{e\cos(dx + c)}} \\ & + \frac{2(77a^4 + 132a^2b^2 + 12b^4)e\sin(dx + c)\sqrt{e\cos(dx + c)}}{231d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i\sqrt{2}(77a^4 + 132a^2b^2 + 12b^4)e^{\frac{3}{2}}\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + 15i\sqrt{2}(77a^4 + 132a^2b^2 + 12b^4)e^{\frac{3}{2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^4e\cos(dx + c)^5 - 2(3a^2b^2 + b^4)e\cos(dx + c)^3 + (a^4 + 6a^2b^2 + b^4)e\cos(dx + c) - 4(ab^3e\cos(dx + c)^2 - 2ab^2e\cos(dx + c) + a^2e)\right)\right)$$

40.158 Problem number 567

$$\int \sqrt{e \cos(c + dx)} (a + b \sin(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{22ab(17a^2 + 18b^2) (e \cos(dx + c))^{\frac{3}{2}}}{315de} - \frac{2b(41a^2 + 14b^2) (e \cos(dx + c))^{\frac{3}{2}} (a + b \sin(dx + c))}{105de} \\ & - \frac{10ab(e \cos(dx + c))^{\frac{3}{2}} (a + b \sin(dx + c))^2}{21de} - \frac{2b(e \cos(dx + c))^{\frac{3}{2}} (a + b \sin(dx + c))^3}{9de} \\ & + \frac{2(15a^4 + 36a^2b^2 + 4b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i \sqrt{2} (15a^4 + 36a^2b^2 + 4b^4) e^{\frac{1}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^4 \cos(dx + c)^4 + a^4 + 6a^2b^2 + b^4 - 2(3a^2b^2 + b^4) \cos(dx + c)^2 - 4(ab^3 \cos(dx + c)^2 - a^3b - ab^3) \sin(dx + c)\right) \sqrt{e \cos(dx + c)}\right)$$

40.159 Problem number 568

$$\int \frac{(a + b \sin(c + dx))^4}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7a^4 + 28a^2b^2 + 4b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}} \\ & - \frac{6ab(31a^2 + 34b^2) \sqrt{e \cos(dx + c)}}{35de} - \frac{2b(29a^2 + 10b^2) (a + b \sin(dx + c)) \sqrt{e \cos(dx + c)}}{35de} \\ & - \frac{26ab(a + b \sin(dx + c))^2 \sqrt{e \cos(dx + c)}}{35de} - \frac{2b(a + b \sin(dx + c))^3 \sqrt{e \cos(dx + c)}}{7de} \end{aligned}$$

command

```
integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(5\sqrt{2}(7ia^4 + 28ia^2b^2 + 4ib^4)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-7ia^4 - 28ia^2b^2 - 4ib^4)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))}{e \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b^4 \cos(dx + c)^4 + a^4 + 6a^2b^2 + b^4 - 2(3a^2b^2 + b^4) \cos(dx + c)^2 - 4(ab^3 \cos(dx + c)^2 - a^3b - ab^3) \sin(dx + c))}{e \cos(dx + c)} \right)$$

40.160 Problem number 569

$$\int \frac{(a + b \sin(c + dx))^4}{(e \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ab(15a^2 + 62b^2)(e \cos(dx + c))^{\frac{3}{2}}}{15de^3} + \frac{2b(5a^2 + 6b^2)(e \cos(dx + c))^{\frac{3}{2}}(a + b \sin(dx + c))}{5de^3} \\ & + \frac{2ab(e \cos(dx + c))^{\frac{3}{2}}(a + b \sin(dx + c))^2}{de^3} + \frac{2(b + a \sin(dx + c))(a + b \sin(dx + c))^3}{de \sqrt{e \cos(dx + c)}} \\ & - \frac{2(5a^4 + 60a^2b^2 + 12b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^2 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(3\sqrt{2}(5ia^4 + 60ia^2b^2 + 12ib^4) \cos(dx + c) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3\sqrt{2}(-5ia^4 - 60ia^2b^2 - 12ib^4) \cos(dx + c) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))))}{e^2 \cos(dx + c)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b^4 \cos(dx + c)^4 + a^4 + 6a^2b^2 + b^4 - 2(3a^2b^2 + b^4) \cos(dx + c)^2 - 4(ab^3 \cos(dx + c)^2 - a^3b - ab^3) \sin(dx + c))}{e^2 \cos(dx + c)^2} \right)$$

40.161 Problem number 570

$$\int \frac{(a + b \sin(c + dx))^4}{(e \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b + a \sin(dx + c))(a + b \sin(dx + c))^3}{3de(e \cos(dx + c))^{\frac{3}{2}}} \\ & + \frac{2(a^4 - 12a^2b^2 - 4b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^2 \sqrt{e \cos(dx + c)}} \\ & + \frac{2ab(a^2 + 14b^2) \sqrt{e \cos(dx + c)}}{3de^3} + \frac{2b(a^2 + 2b^2)(a + b \sin(dx + c)) \sqrt{e \cos(dx + c)}}{3de^3} \\ & + \frac{2ab(a + b \sin(dx + c))^2 \sqrt{e \cos(dx + c)}}{3de^3} \end{aligned}$$

command

`integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-i a^4 + 12i a^2 b^2 + 4i b^4) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(i a^4 -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^4 \cos(dx + c)^4 + a^4 + 6a^2b^2 + b^4 - 2(3a^2b^2 + b^4) \cos(dx + c)^2 - 4(ab^3 \cos(dx + c)^2 - a^3b - ab^3) \sin(dx + c))}{e^3 \cos(dx + c)^3}\right)$$

40.162 Problem number 571

$$\int \frac{(a + b \sin(c + dx))^4}{(e \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ab(3a^2 - 10b^2)(e \cos(dx + c))^{\frac{3}{2}}}{5de^5} + \frac{6b(a^2 - 2b^2)(e \cos(dx + c))^{\frac{3}{2}}(a + b \sin(dx + c))}{5de^5} \\ & + \frac{2(b + a \sin(dx + c))(a + b \sin(dx + c))^3}{5de(e \cos(dx + c))^{\frac{5}{2}}} \\ & - \frac{6(a + b \sin(dx + c))^2(ab - (a^2 - 2b^2) \sin(dx + c))}{5de^3 \sqrt{e \cos(dx + c)}} \\ & - \frac{6(a^4 - 4a^2b^2 - 4b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^4 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(3\sqrt{2}(ia^4 - 4ia^2b^2 - 4ib^4)\cos(dx+c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)))}{e^4 \cos(dx+c)^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^4 \cos(dx+c)^4 + a^4 + 6a^2b^2 + b^4 - 2(3a^2b^2 + b^4)\cos(dx+c)^2 - 4(ab^3 \cos(dx+c)^2 - a^3b - ab^3)\sin(dx+c))}{e^4 \cos(dx+c)^4}\right)$$

40.163 Problem number 572

$$\int \frac{(a + b \sin(c + dx))^4}{(e \cos(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b + a \sin(dx+c))(a + b \sin(dx+c))^3}{7de(e \cos(dx+c))^{\frac{7}{2}}} - \frac{2(a + b \sin(dx+c))^2(ab - (5a^2 - 6b^2)\sin(dx+c))}{21de^3(e \cos(dx+c))^{\frac{3}{2}}} \\ & + \frac{2(5a^4 - 12a^2b^2 + 12b^4)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^4 \sqrt{e \cos(dx+c)}} \\ & + \frac{10ab(a^2 - 2b^2)\sqrt{e \cos(dx+c)}}{21de^5} + \frac{2b(5a^2 - 6b^2)(a + b \sin(dx+c))\sqrt{e \cos(dx+c)}}{21de^5} \end{aligned}$$

command

```
integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\sqrt{2}(-5ia^4 + 12ia^2b^2 - 12ib^4)\cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + \sqrt{2}(5ia^4 - 12ia^2b^2 + 12ib^4)\sin(dx+c))}{e^5 \cos(dx+c)^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^4 \cos(dx+c)^4 + a^4 + 6a^2b^2 + b^4 - 2(3a^2b^2 + b^4)\cos(dx+c)^2 - 4(ab^3 \cos(dx+c)^2 - a^3b - ab^3)\sin(dx+c))}{e^5 \cos(dx+c)^5}\right)$$

40.164 Problem number 573

$$\int \frac{(a + b \sin(c + dx))^4}{(e \cos(c + dx))^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ab(21a^2 - 22b^2)(e \cos(dx + c))^{\frac{3}{2}}}{45de^7} + \frac{2(b + a \sin(dx + c))(a + b \sin(dx + c))^3}{9de(e \cos(dx + c))^{\frac{9}{2}}} \\ & + \frac{2(a + b \sin(dx + c))^2(ab + (7a^2 - 6b^2)\sin(dx + c))}{45de^3(e \cos(dx + c))^{\frac{5}{2}}} \\ & - \frac{2(a + b \sin(dx + c))(b(7a^2 - 6b^2) - a(21a^2 - 22b^2)\sin(dx + c))}{45de^5\sqrt{e \cos(dx + c)}} \\ & - \frac{2(7a^4 - 12a^2b^2 + 4b^4)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\sqrt{e \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^6\sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(3\sqrt{2}(7ia^4 - 12ia^2b^2 + 4ib^4)\cos(dx + c)^5 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sqrt{2})) + \dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^4 \cos(dx + c)^4 + a^4 + 6a^2b^2 + b^4 - 2(3a^2b^2 + b^4)\cos(dx + c)^2 - 4(ab^3 \cos(dx + c)^2 - a^3b - ab^3)\sin(dx + c))}{e^6 \cos(dx + c)^6}\right)$$

41 Test file number 73

Test folder name:

test_cases/4_Trig_functions/4.1_Sine/73_4.1.2.1-a+b_sin-^m-c+d_sin-^n

41.1 Problem number 203

$$\int \sin(e + fx) \sqrt{a + b \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2 \cos(fx + e) \sqrt{a + b \sin(fx + e)}}{3f} - \frac{2a \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \sin(fx + e)}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) b f \sqrt{\frac{a + b \sin(fx + e)}{a+b}}} + \frac{2(a^2 - b^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \sin(fx + e)}{a+b}}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) b f \sqrt{a + b \sin(fx + e)}}$$

command

`integrate(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} a \sqrt{ib} b \operatorname{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}\right), \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, 3\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sin(fx + e) + a} \sin(fx + e), x\right)$$

41.2 Problem number 204

$$\int \sqrt{a + b \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \sin(fx + e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{\frac{a + b \sin(fx + e)}{a+b}}}$$

command

```
integrate((a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} a \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(fx+e)-3ib \sin(fx+e)-2ia}{3b}\right) + \sqrt{2} a \sqrt{-i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(fx+e)+3ib \sin(fx+e)-2ia}{3b}\right)}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sin(fx+e)+a}, x\right)$$

41.3 Problem number 207

$$\int \frac{\sin(e+fx)}{\sqrt{a+b \sin(e+fx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \sin(fx+e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) b f \sqrt{\frac{a+b \sin(fx+e)}{a+b}}} + \frac{2a \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \sin(fx+e)}{a+b}}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) b f \sqrt{a+b \sin(fx+e)}}$$

command

```
integrate(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2} a \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(fx+e)-3ib \sin(fx+e)-2ia}{3b}\right) + 2\sqrt{2} a \sqrt{-i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(fx+e)+3ib \sin(fx+e)-2ia}{3b}\right)}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sin(fx+e)}{\sqrt{b \sin(fx+e)+a}}, x\right)$$

41.4 Problem number 208

$$\int \frac{1}{\sqrt{a + b \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \sin(fx + e)}{a+b}}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{a + b \sin(fx + e)}}$$

command

`integrate(1/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{ib} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(fx+e)-3ib \sin(fx+e)-2ia}{3b}\right) + \sqrt{2} \sqrt{-ib} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b \cos(fx+e)-3ib \sin(fx+e)-2ia}{3b}\right)}{bf}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{b \sin(fx + e) + a}}, x\right)$$

41.5 Problem number 482

$$\int (a + a \sin(e + fx))(c + d \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2a(5c + 7d) \cos(fx + e) (c + d \sin(fx + e))^{\frac{3}{2}}}{35f} - \frac{2a \cos(fx + e) (c + d \sin(fx + e))^{\frac{5}{2}}}{7f} - \frac{2a(15c^2 + 56cd + 25d^2) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{105f} - \frac{2a(15c^3 + 161c^2d + 145cd^2 + 63d^3) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d \sin(fx + e)}}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{\frac{c + d \sin(fx + e)}{c+d}}} + \frac{2a(c^2 - d^2) (15c^2 + 56cd + 25d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{c + d \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (30 ac^4 + 7 ac^3 d - 115 ac^2 d^2 - 231 acd^3 - 75 ad^4) \sqrt{i d} \operatorname{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2}, -\frac{8(8ic^3-9icd^2)}{27d^3}, \frac{3d}{27d^3}\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(ac^2 + 2acd + ad^2 - (2acd + ad^2) \cos(fx + e)^2 - (ad^2 \cos(fx + e)^2 - ac^2 - 2acd - ad^2) \sin(fx + e)\right)\right)$$

41.6 Problem number 483

$$\int (a + a \sin(e + fx))(c + d \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \cos(fx + e) (c + d \sin(fx + e))^{\frac{3}{2}}}{5f} - \frac{2a(3c + 5d) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{15f} \\ & \frac{2a(3c^2 + 20cd + 9d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d \sin(fx + e)}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{\frac{c + d \sin(fx + e)}{c+d}}} \\ & + \frac{2a(3c + 5d) (c^2 - d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (6 ac^3 - 5 ac^2 d - 18 acd^2 - 15 ad^3) \sqrt{i d} \operatorname{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2}, -\frac{8(8ic^3-9icd^2)}{27d^3}, \frac{3d \cos(fx+e) - 3i d \sin(fx+e)}{3d}\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(ad \cos(fx + e)^2 - ac - ad - (ac + ad) \sin(fx + e)\right) \sqrt{d \sin(fx + e) + c}, x\right)$$

41.7 Problem number 484

$$\int (a + a \sin(e + fx)) \sqrt{c + d \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2a \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{3f} - \frac{2a(c + 3d) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{c + d \sin(fx + e)}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} + \frac{2a(c^2 - d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{c + d \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{d \sin(fx + e) + c} ad^2 \cos(fx + e) + \sqrt{2} (2ac^2 - 3acd - 3ad^2) \sqrt{id} \operatorname{weierstrassPInverse}\left(-\frac{4(4c^2 - 3d^2)}{3d^2}, -\frac{8}{3d}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((a \sin(fx + e) + a) \sqrt{d \sin(fx + e) + c}, x\right)$$

41.8 Problem number 485

$$\int \frac{a + a \sin(e + fx)}{\sqrt{c + d \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2a \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{c + d \sin(fx + e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} + \frac{2a(c - d) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{c + d \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-3i\sqrt{2}a\sqrt{id}d\text{weierstrassZeta}\left(-\frac{4(4c^2-3d^2)}{3d^2}, -\frac{8(8ic^3-9icd^2)}{27d^3}, \text{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2}, -\frac{8(8ic^3-9icd^2)}{27d^3}, 3\right)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{a\sin(fx+e)+a}{\sqrt{d\sin(fx+e)+c}}, x\right)$$

41.9 Problem number 486

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a\cos(fx+e)}{(c+d)f\sqrt{c+d\sin(fx+e)}} \\ & + \frac{2a\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}} \text{EllipticE}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right) \sqrt{c+d\sin(fx+e)}}{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) d(c+d)f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}} \\ & - \frac{2a\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}} \text{EllipticF}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) df\sqrt{c+d\sin(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6\sqrt{d\sin(fx+e)+c}ad^2\cos(fx+e) - \left(\sqrt{2}(2acd+3ad^2)\sin(fx+e) + \sqrt{2}(2ac^2+3acd)\right)\sqrt{id}\text{weierstrass}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(a\sin(fx+e)+a)\sqrt{d\sin(fx+e)+c}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}, x\right)$$

41.10 Problem number 487

$$\int \frac{a + a \sin(e + fx)}{(c + d \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a \cos(fx + e)}{3(c + d) f (c + d \sin(fx + e))^{3/2}} - \frac{2a(c - 3d) \cos(fx + e)}{3(c - d)(c + d)^2 f \sqrt{c + d \sin(fx + e)}} \\ & + \frac{2a(c - 3d) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{c + d \sin(fx + e)}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) (c - d) d (c + d)^2 f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & + \frac{2a \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d (c + d) f \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (2ac^2d^2 + 3acd^3 - 3ad^4) \cos(fx + e)^2 - 2\sqrt{2} (2ac^3d + 3ac^2d^2 - 3acd^3) \sin(fx + e) - \sqrt{2} (2ac^4 + 3ac^3d) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(a \sin(fx + e) + a) \sqrt{d \sin(fx + e) + c}}{3cd^2 \cos(fx + e)^2 - c^3 - 3cd^2 + (d^3 \cos(fx + e)^2 - 3c^2d - d^3) \sin(fx + e)}, x\right)$$

41.11 Problem number 488

$$\int \frac{a + a \sin(e + fx)}{(c + d \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2a \cos(fx + e)}{5(c+d)f(c+d \sin(fx+e))^{\frac{5}{2}}} - \frac{2a(3c-5d)\cos(fx+e)}{15(c-d)(c+d)^2 f(c+d \sin(fx+e))^{\frac{3}{2}}} \\
& - \frac{2a(3c^2-20cd+9d^2)\cos(fx+e)}{15(c-d)^2(c+d)^3 f \sqrt{c+d \sin(fx+e)}} \\
& + \frac{2a(3c^2-20cd+9d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c+d \sin(fx+e)}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) (c-d)^2 d(c+d)^3 f \sqrt{\frac{c+d \sin(fx+e)}{c+d}}} \\
& + \frac{2a(3c-5d) \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin(fx+e)}{c+d}}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) (c-d) d(c+d)^2 f \sqrt{c+d \sin(fx+e)}}
\end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(3\sqrt{2}(6ac^4d^2+5ac^3d^3-18ac^2d^4+15acd^5)\cos(fx+e)^2 + \left(\sqrt{2}(6ac^3d^3+5ac^2d^4-18acd^5+15ad^6)\cos(fx+e)\right)^2\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(a \sin(fx + e) + a) \sqrt{d \sin(fx + e) + c}}{d^4 \cos(fx + e)^4 + c^4 + 6c^2d^2 + d^4 - 2(3c^2d^2 + d^4) \cos(fx + e)^2 - 4(cd^3 \cos(fx + e)^2 - c^3d - cd^3) \sin(fx + e)}\right)$$

41.12 Problem number 489

$$\int (a + a \sin(e + fx))^2 (c + d \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{4a^2(5c(c-9d) - 56d^2) \cos(fx+e) (c+d \sin(fx+e))^{\frac{3}{2}}}{315df} \\
 & + \frac{4a^2(c-9d) \cos(fx+e) (c+d \sin(fx+e))^{\frac{5}{2}}}{63df} - \frac{2a^2 \cos(fx+e) (c+d \sin(fx+e))^{\frac{7}{2}}}{9df} \\
 & + \frac{4a^2(5c^3 - 45c^2d - 141cd^2 - 75d^3) \cos(fx+e) \sqrt{c+d \sin(fx+e)}}{315df} \\
 & + \frac{4a^2(5c^4 - 45c^3d - 381c^2d^2 - 435cd^3 - 168d^4) \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right)}{315 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{\frac{c+d \sin(fx+e)}{c+d}}} \\
 & - \frac{4a^2(c^2 - d^2) (5c^3 - 45c^2d - 141cd^2 - 75d^3) \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right)}{315 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{c+d \sin(fx+e)}}
 \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} (10 a^2 c^5 - 90 a^2 c^4 d - 57 a^2 c^3 d^2 + 345 a^2 c^2 d^3 + 591 a^2 c d^4 + 225 a^2 d^5) \sqrt{i d} \operatorname{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2}\right) \right)}{315}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^2 d^2 \cos(fx+e)^4 + 2 a^2 c^2 + 4 a^2 c d + 2 a^2 d^2 - (a^2 c^2 + 4 a^2 c d + 3 a^2 d^2) \cos(fx+e)^2 + 2 (a^2 c^2 + 2 a^2 c d\right.\right.$$

41.13 Problem number 490

$$\int (a + a \sin(e + fx))^2 (c + d \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{4a^2(c-7d)\cos(fx+e)(c+d\sin(fx+e))^{\frac{3}{2}}}{35df} - \frac{2a^2\cos(fx+e)(c+d\sin(fx+e))^{\frac{5}{2}}}{7df}$$

$$+ \frac{4a^2(c^2-7cd-10d^2)\cos(fx+e)\sqrt{c+d\sin(fx+e)}}{35df}$$

$$+ \frac{4a^2(c+3d)(c^2-10cd-7d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right),\sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{35\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^2f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}$$

$$+ \frac{4a^2(c^2-7cd-10d^2)(c^2-d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right),\sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{35\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^2f\sqrt{c+d\sin(fx+e)}}$$

command

`integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(2\sqrt{2}(a^2c^4-7a^2c^3d+2a^2c^2d^2+21a^2cd^3+15a^2d^4)\sqrt{id}\operatorname{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3}{d}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(2a^2c+2a^2d-(a^2c+2a^2d)\cos(fx+e)^2-(a^2d\cos(fx+e)^2-2a^2c-2a^2d)\sin(fx+e)\right)\sqrt{d\sin(fx+e)}\right)$$

41.14 Problem number 491

$$\int (a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}dx$$

Optimal antiderivative

$$\frac{2a^2\cos(fx+e)(c+d\sin(fx+e))^{\frac{3}{2}}}{5df} + \frac{4a^2(c-5d)\cos(fx+e)\sqrt{c+d\sin(fx+e)}}{15df}$$

$$+ \frac{4a^2(c^2-5cd-12d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right),\sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{15\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^2f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}$$

$$+ \frac{4a^2(c-5d)(c^2-d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right),\sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{15\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^2f\sqrt{c+d\sin(fx+e)}}$$

command

```
integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (2 a^2 c^3 - 10 a^2 c^2 d + 9 a^2 c d^2 + 15 a^2 d^3) \sqrt{i d} \operatorname{weierstrassPInverse} \left(-\frac{4(4 c^2 - 3 d^2)}{3 d^2}, -\frac{8(8 i c^3 - 9 i c d^2)}{27 d^3}, \frac{3 d \cos(f x + e)}{c + d} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\left(a^2 \cos(f x + e)^2 - 2 a^2 \sin(f x + e) - 2 a^2 \right) \sqrt{d \sin(f x + e) + c}, x \right)$$

41.15 Problem number 492

$$\int \frac{(a + a \sin(e + f x))^2}{\sqrt{c + d \sin(e + f x)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 a^2 \cos(f x + e) \sqrt{c + d \sin(f x + e)}}{3 d f} \\ & + \frac{4 a^2 (c - 3 d) \sqrt{\frac{1}{2} + \frac{\sin(f x + e)}{2}} \operatorname{EllipticE} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right), \sqrt{2} \sqrt{\frac{d}{c + d}} \right) \sqrt{c + d \sin(f x + e)}}{3 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right) d^2 f \sqrt{\frac{c + d \sin(f x + e)}{c + d}}} \\ & - \frac{4 a^2 (c - 2 d) (c - d) \sqrt{\frac{1}{2} + \frac{\sin(f x + e)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right), \sqrt{2} \sqrt{\frac{d}{c + d}} \right) \sqrt{\frac{c + d \sin(f x + e)}{c + d}}}{3 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right) d^2 f \sqrt{c + d \sin(f x + e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{d \sin(f x + e) + c} a^2 d^2 \cos(f x + e) - 2 \sqrt{2} (a^2 c^2 - 3 a^2 c d + 3 a^2 d^2) \sqrt{i d} \operatorname{weierstrassPInverse} \left(-\frac{4(4 c^2 - 3 d^2)}{3 d^2}, -\frac{8(8 i c^3 - 9 i c d^2)}{27 d^3}, \frac{3 d \cos(f x + e)}{c + d} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{a^2 \cos(f x + e)^2 - 2 a^2 \sin(f x + e) - 2 a^2}{\sqrt{d \sin(f x + e) + c}}, x \right)$$

41.16 Problem number 493

$$\int \frac{(a + a \sin(e + fx))^2}{(c + d \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2a^2(c-d)\cos(fx+e)}{d(c+d)f\sqrt{c+d\sin(fx+e)}} + \frac{4a^2c\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)d^2(c+d)f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}$$

$$+ \frac{4a^2(c-d)\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)d^2f\sqrt{c+d\sin(fx+e)}}$$

command

```
integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(a^2cd^2 - a^2d^3)\sqrt{d\sin(fx+e)+c}\cos(fx+e) - \left(\sqrt{2}(2a^2c^2d - 3a^2d^3)\sin(fx+e) + \sqrt{2}(2a^2c^3 - 3a^2cd)\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(a^2\cos(fx+e)^2 - 2a^2\sin(fx+e) - 2a^2\right)\sqrt{d\sin(fx+e)+c}}{d^2\cos(fx+e)^2 - 2cd\sin(fx+e) - c^2 - d^2}, x\right)$$

41.17 Problem number 494

$$\int \frac{(a + a \sin(e + fx))^2}{(c + d \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a^2(c-d)\cos(fx+e)}{3d(c+d)f(c+d\sin(fx+e))^{\frac{3}{2}}} - \frac{4a^2(c+3d)\cos(fx+e)}{3d(c+d)^2f\sqrt{c+d\sin(fx+e)}}$$

$$+ \frac{4a^2(c+3d)\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{3\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)d^2(c+d)^2f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}$$

$$- \frac{4a^2(c+2d)\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{3\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)d^2(c+d)f\sqrt{c+d\sin(fx+e)}}$$

command

`integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(2\left(\sqrt{2}\left(a^2c^2d^2+3a^2cd^3+3a^2d^4\right)\cos(fx+e)^2-2\sqrt{2}\left(a^2c^3d+3a^2c^2d^2+3a^2cd^3\right)\sin(fx+e)-\sqrt{2}\left(a^2c^4+\right.\right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(a^2\cos(fx+e)^2-2a^2\sin(fx+e)-2a^2\right)\sqrt{d\sin(fx+e)+c}}{3cd^2\cos(fx+e)^2-c^3-3cd^2+\left(d^3\cos(fx+e)^2-3c^2d-d^3\right)\sin(fx+e)},x\right)$$

41.18 Problem number 495

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2a^2(c-d)\cos(fx+e)}{5d(c+d)f(c+d\sin(fx+e))^{\frac{5}{2}}} - \frac{4a^2(c+5d)\cos(fx+e)}{15d(c+d)^2f(c+d\sin(fx+e))^{\frac{3}{2}}}$$

$$- \frac{4a^2(c^2+5cd-12d^2)\cos(fx+e)}{15(c-d)d(c+d)^3f\sqrt{c+d\sin(fx+e)}}$$

$$+ \frac{4a^2(c^2+5cd-12d^2)\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{15\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)(c-d)d^2(c+d)^3f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}$$

$$- \frac{4a^2(c+5d)\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{15\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)d^2(c+d)^2f\sqrt{c+d\sin(fx+e)}}$$

command

```
integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\left(a^2 \cos (f x+e)^2-2 a^2 \sin (f x+e)-2 a^2\right) \sqrt{d \sin (f x+e)+c}}{d^4 \cos (f x+e)^4+c^4+6 c^2 d^2+d^4-2\left(3 c^2 d^2+d^4\right) \cos (f x+e)^2-4\left(c d^3 \cos (f x+e)^2-c^3 d-c d^3\right) \sin (f x+e)} dx \right)$$

41.19 Problem number 496

$$\int (a+a \sin (e+f x))^3(c+d \sin (e+f x))^{5 / 2} d x$$

Optimal antiderivative

$$\begin{aligned} & \frac{4 a^3\left(4 c^3-33 c^2 d+182 c d^2+231 d^3\right) \cos (f x+e)(c+d \sin (f x+e))^{\frac{3}{2}}}{693 d^2 f} \\ & - \frac{4 a^3\left(4 c^2-33 c d+189 d^2\right) \cos (f x+e)(c+d \sin (f x+e))^{\frac{5}{2}}}{693 d^2 f} \\ & + \frac{8 a^3(c-6 d) \cos (f x+e)(c+d \sin (f x+e))^{\frac{7}{2}}}{99 d^2 f} \\ & - \frac{2 \cos (f x+e)\left(a^3+a^3 \sin (f x+e)\right)(c+d \sin (f x+e))^{\frac{7}{2}}}{11 d f} \\ & - \frac{4 a^3\left(4 c^4-33 c^3 d+177 c^2 d^2+561 c d^3+315 d^4\right) \cos (f x+e) \sqrt{c+d \sin (f x+e)}}{693 d^2 f} \\ & - \frac{4 a^3(c+3 d)\left(4 c^4-45 c^3 d+309 c^2 d^2+525 c d^3+231 d^4\right) \sqrt{\frac{1}{2}+\frac{\sin (f x+e)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right), \sqrt{2}\right)}{693 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right) d^3 f \sqrt{\frac{c+d \sin (f x+e)}{c+d}}} \\ & + \frac{4 a^3\left(c^2-d^2\right)\left(4 c^4-33 c^3 d+177 c^2 d^2+561 c d^3+315 d^4\right) \sqrt{\frac{1}{2}+\frac{\sin (f x+e)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right), \sqrt{2}\right)}{693 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right) d^3 f \sqrt{c+d \sin (f x+e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (8 a^3 c^6 - 66 a^3 c^5 d + 345 a^3 c^4 d^2 + 330 a^3 c^3 d^3 - 1392 a^3 c^2 d^4 - 2376 a^3 c d^5 - 945 a^3 d^6) \sqrt{i d} \text{weierstrassPInverse} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(4 a^3 c^2 + 8 a^3 c d + 4 a^3 d^2 + (2 a^3 c d + 3 a^3 d^2) \cos (f x + e) \right)^4 - (3 a^3 c^2 + 10 a^3 c d + 7 a^3 d^2) \cos (f x + e) \right)^2 + \dots$$

41.20 Problem number 497

$$\int (a + a \sin(e + f x))^3 (c + d \sin(e + f x))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4 a^3 (4 c^2 - 27 c d + 119 d^2) \cos (f x + e) (c + d \sin (f x + e))^{3/2}}{315 d^2 f} \\ & + \frac{8 a^3 (c - 5 d) \cos (f x + e) (c + d \sin (f x + e))^{5/2}}{63 d^2 f} \\ & - \frac{2 \cos (f x + e) (a^3 + a^3 \sin (f x + e)) (c + d \sin (f x + e))^{5/2}}{9 d f} \\ & - \frac{4 a^3 (4 c^3 - 27 c^2 d + 114 c d^2 + 165 d^3) \cos (f x + e) \sqrt{c + d \sin (f x + e)}}{315 d^2 f} \\ & - \frac{4 a^3 (4 c^4 - 27 c^3 d + 111 c^2 d^2 + 579 c d^3 + 357 d^4) \sqrt{\frac{1}{2} + \frac{\sin (f x + e)}{2}} \text{EllipticE} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right), \sqrt{2} \sqrt{\frac{d}{c + d}} \right)}{315 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right) d^3 f \sqrt{\frac{c + d \sin (f x + e)}{c + d}}} \\ & + \frac{4 a^3 (c^2 - d^2) (4 c^3 - 27 c^2 d + 114 c d^2 + 165 d^3) \sqrt{\frac{1}{2} + \frac{\sin (f x + e)}{2}} \text{EllipticF} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right), \sqrt{2} \sqrt{\frac{d}{c + d}} \right)}{315 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right) d^3 f \sqrt{c + d \sin (f x + e)}} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (8 a^3 c^5 - 54 a^3 c^4 d + 219 a^3 c^3 d^2 - 3 a^3 c^2 d^3 - 699 a^3 c d^4 - 495 a^3 d^5) \sqrt{i d} \text{weierstrassPInverse} \left(-\frac{4(4 c^2 - 3 d^2)}{3 d^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(a^3 d \cos (f x + e) \right)^4 + 4 a^3 c + 4 a^3 d - (3 a^3 c + 5 a^3 d) \cos (f x + e) \right)^2 + \left(4 a^3 c + 4 a^3 d - (a^3 c + 3 a^3 d) \cos (f x + e) \right)^2 + \dots$$

41.21 Problem number 498

$$\int (a + a \sin(e + fx))^3 \sqrt{c + d \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^3(c-4d)\cos(fx+e)(c+d\sin(fx+e))^{\frac{3}{2}}}{35d^2f} \\ & - \frac{2\cos(fx+e)(a^3+a^3\sin(fx+e))(c+d\sin(fx+e))^{\frac{3}{2}}}{7df} \\ & - \frac{4a^3(4c^2-21cd+65d^2)\cos(fx+e)\sqrt{c+d\sin(fx+e)}}{105d^2f} \\ & - \frac{4a^3(4c^3-21c^2d+62cd^2+147d^3)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right),\sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{105\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^3f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}} \\ & + \frac{4a^3(c^2-d^2)(4c^2-21cd+65d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right),\sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{105\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^3f\sqrt{c+d\sin(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\sqrt{2}\left(8a^3c^4-42a^3c^3d+121a^3c^2d^2-84a^3cd^3-195a^3d^4\right)\sqrt{id}\operatorname{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9d^3)}{27d^3}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(3a^3\cos(fx+e)^2-4a^3+\left(a^3\cos(fx+e)^2-4a^3\right)\sin(fx+e)\right)\sqrt{d\sin(fx+e)+c},x\right)$$

41.22 Problem number 499

$$\int \frac{(a + a \sin(e + fx))^3}{\sqrt{c + d \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^3(c-3d)\cos(fx+e)\sqrt{c+d\sin(fx+e)}}{15d^2f} \\ & - \frac{2\cos(fx+e)(a^3+a^3\sin(fx+e))\sqrt{c+d\sin(fx+e)}}{5df} \\ & - \frac{4a^3(4c^2-15cd+27d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right),\sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{15\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^3f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}} \\ & + \frac{4a^3(c-d)(4c^2-11cd+15d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right),\sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{15\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^3f\sqrt{c+d\sin(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{2}\left(8a^3c^3-30a^3c^2d+51a^3cd^2-45a^3d^3\right)\sqrt{id}\operatorname{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2},-\frac{8(8ic^3-9icd^2)}{27d^3},\frac{3d\cos(fx+e)}{c+d}\right)\right)}{\sqrt{d\sin(fx+e)+c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{3a^3\cos(fx+e)^2-4a^3+(a^3\cos(fx+e)^2-4a^3)\sin(fx+e)}{\sqrt{d\sin(fx+e)+c}},x\right)$$

41.23 Problem number 500

$$\int \frac{(a + a \sin(e + fx))^3}{(c + d \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(c-d)\cos(fx+e)(a^3+a^3\sin(fx+e))}{d(c+d)f\sqrt{c+d\sin(fx+e)}} - \frac{4a^3(2c-d)\cos(fx+e)\sqrt{c+d\sin(fx+e)}}{3d^2(c+d)f}$$

$$+ \frac{4a^3(4c^2-5cd-3d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{3\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^3(c+d)f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}$$

$$- \frac{4a^3(4c-5d)(c-d)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{3\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^3f\sqrt{c+d\sin(fx+e)}}$$

command

```
integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\left(\sqrt{2}(8a^3c^3d-10a^3c^2d^2-9a^3cd^3+15a^3d^4)\sin(fx+e)+\sqrt{2}(8a^3c^4-10a^3c^3d-9a^3c^2d^2+15a^3cd^3)\right)\sqrt{\dots}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(3a^3\cos(fx+e)^2-4a^3+(a^3\cos(fx+e)^2-4a^3)\sin(fx+e)\right)\sqrt{d\sin(fx+e)+c}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}, x\right)$$

41.24 Problem number 501

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(c-d)\cos(fx+e)(a^3+a^3\sin(fx+e))}{3d(c+d)f(c+d\sin(fx+e))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(fx+e)}{3d^2(c+d)^2f\sqrt{c+d\sin(fx+e)}}$$

$$- \frac{4a^3(4c^2+5cd-3d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{3\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^3(c+d)^2f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}$$

$$+ \frac{4a^3(c-d)(4c+5d)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{3\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)d^3(c+d)f\sqrt{c+d\sin(fx+e)}}$$

command

`integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} (8 a^3 c^3 d^2 + 10 a^3 c^2 d^3 - 9 a^3 c d^4 - 15 a^3 d^5) \cos (f x + e)^2 - 2 \sqrt{2} (8 a^3 c^4 d + 10 a^3 c^3 d^2 - 9 a^3 c^2 d^3 - 15 a^3 c d^4 - 15 a^3 d^5) \sin (f x + e) \right) \sqrt{d \sin (f x + e) + c} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(3 a^3 \cos (f x + e)^2 - 4 a^3 + \left(a^3 \cos (f x + e)^2 - 4 a^3 \right) \sin (f x + e) \right) \sqrt{d \sin (f x + e) + c}}{3 c d^2 \cos (f x + e)^2 - c^3 - 3 c d^2 + \left(d^3 \cos (f x + e)^2 - 3 c^2 d - d^3 \right) \sin (f x + e)}, x \right)$$

41.25 Problem number 502

$$\int \frac{(a + a \sin(e + fx))^3}{(c + d \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(c-d) \cos (f x + e) \left(a^3 + a^3 \sin (f x + e) \right)}{5 d (c+d) f (c+d \sin (f x + e))^{5/2}} \\ & + \frac{8 a^3 (c-d) (c+3 d) \cos (f x + e)}{15 d^2 (c+d)^2 f (c+d \sin (f x + e))^{3/2}} - \frac{4 a^3 (4 c^2 + 15 c d + 27 d^2) \cos (f x + e)}{15 d^2 (c+d)^3 f \sqrt{c+d \sin (f x + e)}} \\ & + \frac{4 a^3 (4 c^2 + 15 c d + 27 d^2) \sqrt{\frac{1}{2} + \frac{\sin (f x + e)}{2}} \operatorname{EllipticE} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right), \sqrt{2} \sqrt{\frac{d}{c+d}} \right) \sqrt{c+d \sin (f x + e)}}{15 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right) d^3 (c+d)^3 f \sqrt{\frac{c+d \sin (f x + e)}{c+d}}} \\ & - \frac{4 a^3 (4 c^2 + 11 c d + 15 d^2) \sqrt{\frac{1}{2} + \frac{\sin (f x + e)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right), \sqrt{2} \sqrt{\frac{d}{c+d}} \right) \sqrt{\frac{c+d \sin (f x + e)}{c+d}}}{15 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right) d^3 (c+d)^2 f \sqrt{c+d \sin (f x + e)}} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\left(3 a^3 \cos (f x + e)^2 - 4 a^3 + \left(a^3 \cos (f x + e)^2 - 4 a^3 \right) \sin (f x + e) \right) \sqrt{d \sin (f x + e) + c}}{d^4 \cos (f x + e)^4 + c^4 + 6 c^2 d^2 + d^4 - 2 (3 c^2 d^2 + d^4) \cos (f x + e)^2 - 4 (c d^3 \cos (f x + e)^2 - c^3 d - c d^3) \sin (f x + e)}, x \right)$$

41.26 Problem number 503

$$\int \frac{(a + a \sin(e + fx))^3}{(c + d \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(c-d) \cos(fx+e) (a^3 + a^3 \sin(fx+e))}{7d(c+d) f (c+d \sin(fx+e))^{7/2}} + \frac{8a^3(c-d)(c+4d) \cos(fx+e)}{35d^2(c+d)^2 f (c+d \sin(fx+e))^{5/2}} \\ & - \frac{4a^3(4c^2 + 21cd + 65d^2) \cos(fx+e)}{105d^2(c+d)^3 f (c+d \sin(fx+e))^{3/2}} - \frac{4a^3(4c^3 + 21c^2d + 62cd^2 - 147d^3) \cos(fx+e)}{105(c-d)d^2(c+d)^4 f \sqrt{c+d \sin(fx+e)}} \\ & + \frac{4a^3(4c^3 + 21c^2d + 62cd^2 - 147d^3) \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c+d \sin(fx+e)}}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) (c-d) d^3 (c+d)^4 f \sqrt{\frac{c+d \sin(fx+e)}{c+d}}} \\ & - \frac{4a^3(4c^2 + 21cd + 65d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin(fx+e)}{c+d}}}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 (c+d)^3 f \sqrt{c+d \sin(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(3a^3 \cos(fx+e)^2 - 4a^3 + (a^3 \cos(fx+e)^2 - 4a^3) \sin(fx+e))}{5cd^4 \cos(fx+e)^4 + c^5 + 10c^3d^2 + 5cd^4 - 10(c^3d^2 + cd^4) \cos(fx+e)^2 + (d^5 \cos(fx+e)^4 + 5c^4d + \dots)}\right)$$

41.27 Problem number 504

$$\int \frac{(c + d \sin(e + fx))^{5/2}}{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{(c-d)\cos(fx+e)(c+d\sin(fx+e))^{\frac{3}{2}}}{f(a+a\sin(fx+e))} + \frac{(3c-5d)d\cos(fx+e)\sqrt{c+d\sin(fx+e)}}{3af}$$

$$+ \frac{(3c^2-20cd+9d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{3\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)af\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}$$

$$- \frac{(3c-5d)(c^2-d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{3\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)af\sqrt{c+d\sin(fx+e)}}$$

command

```
integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\sqrt{2}(6c^3+5c^2d-18cd^2+15d^3)\cos(fx+e)+\sqrt{2}(6c^3+5c^2d-18cd^2+15d^3)\sin(fx+e)+\sqrt{2}(6c^3+5c^2d-18cd^2+15d^3))}{3\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)af\sqrt{c+d\sin(fx+e)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2)\sqrt{d\sin(fx+e)+c}}{a\sin(fx+e)+a}, x\right)$$

41.28 Problem number 505

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{a+a\sin(e+fx)} dx$$

Optimal antiderivative

$$\frac{(c-d)\cos(fx+e)\sqrt{c+d\sin(fx+e)}}{f(a+a\sin(fx+e))}$$

$$+ \frac{(c-3d)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)af\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}$$

$$- \frac{(c^2-d^2)\sqrt{\frac{1}{2}+\frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{\sin\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)af\sqrt{c+d\sin(fx+e)}}$$

command

```
integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(2c^2 + 3cd - 3d^2)\cos(fx + e) + \sqrt{2}(2c^2 + 3cd - 3d^2)\sin(fx + e) + \sqrt{2}(2c^2 + 3cd - 3d^2)\right)\sqrt{id} \text{ weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(d \sin(fx + e) + c)^{\frac{3}{2}}}{a \sin(fx + e) + a}, x\right)$$

41.29 Problem number 506

$$\int \frac{\sqrt{c + d \sin(e + fx)}}{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e) \sqrt{c + d \sin(fx + e)}}{f(a + a \sin(fx + e))} \\ & + \frac{\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d \sin(fx + e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) af \sqrt{\frac{c + d \sin(fx + e)}{c+d}}} \\ & - \frac{(c+d) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) af \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(2c + 3d)\cos(fx + e) + \sqrt{2}(2c + 3d)\sin(fx + e) + \sqrt{2}(2c + 3d)\right)\sqrt{id} \text{ weierstrassPInverse}\left(-\frac{4(4c^2 - 3d^2)}{3d^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{d \sin(fx + e) + c}}{a \sin(fx + e) + a}, x\right)$$

41.30 Problem number 507

$$\int \frac{1}{(a + a \sin(e + fx)) \sqrt{c + d \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e) \sqrt{c + d \sin(fx + e)}}{(c - d) f (a + a \sin(fx + e))} \\ & + \frac{\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{c + d \sin(fx + e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) a (c - d) f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & - \frac{\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) a f \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(2c - 3d) \cos(fx + e) + \sqrt{2}(2c - 3d) \sin(fx + e) + \sqrt{2}(2c - 3d)\right) \sqrt{id} \operatorname{weierstrassPInverse}\left(-\frac{4(4c^2 - 3d^2)}{3d^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{d \sin(fx + e) + c}}{ad \cos(fx + e)^2 - ac - ad - (ac + ad) \sin(fx + e)}, x\right)$$

41.31 Problem number 508

$$\int \frac{1}{(a + a \sin(e + fx))(c + d \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{d(c+3d)\cos(fx+e)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(fx+e)}} - \frac{\cos(fx+e)}{(c-d)f(a+a\sin(fx+e))\sqrt{c+d\sin(fx+e)}}$$

$$+ \frac{(c+3d)\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)a(c-d)^2(c+d)f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}$$

$$+ \frac{\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)a(c-d)f\sqrt{c+d\sin(fx+e)}}$$

command

`integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(2c^2d - 3cd^2 - 3d^3)\cos(fx+e)^2 - \sqrt{2}(2c^3 - 3c^2d - 3cd^2)\cos(fx+e) - \left(\sqrt{2}(2c^2d - 3cd^2 - 3d^3)\cos\right.\right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d\sin(fx+e)+c}}{ac^2+2acd+ad^2-(2acd+ad^2)\cos(fx+e)^2-(ad^2\cos(fx+e)^2-ac^2-2acd-ad^2)\sin(fx+e)}, x\right)$$

41.32 Problem number 509

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{d(3c+5d)\cos(fx+e)}{3a(c-d)^2(c+d)f(c+d\sin(fx+e))^{3/2}} - \frac{\cos(fx+e)}{(c-d)f(a+a\sin(fx+e))(c+d\sin(fx+e))^{3/2}}$$

$$+ \frac{d(3c^2+20cd+9d^2)\cos(fx+e)}{3a(c-d)^3(c+d)^2f\sqrt{c+d\sin(fx+e)}}$$

$$+ \frac{(3c^2+20cd+9d^2)\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{c+d\sin(fx+e)}}{3\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)a(c-d)^3(c+d)^2f\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}$$

$$+ \frac{(3c+5d)\sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\sqrt{\frac{d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c+d}}}{3\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)a(c-d)^2(c+d)f\sqrt{c+d\sin(fx+e)}}$$

command

```
integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{d \sin(fx + e) + c}}{ad^3 \cos(fx + e)^4 + ac^3 + 3ac^2d + 3acd^2 + ad^3 - (3ac^2d + 3acd^2 + 2ad^3) \cos(fx + e)^2 + (ac^3 + 3ac^2d} \right)$$

41.33 Problem number 510

$$\int \frac{(c + d \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(c - d) \cos(fx + e) (c + d \sin(fx + e))^{3/2}}{3f(a + a \sin(fx + e))^2} - \frac{(c - d)(c + 5d) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{3a^2 f(1 + \sin(fx + e))} \\ & + \frac{(c^2 + 5cd - 12d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticE} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right), \sqrt{2} \sqrt{\frac{d}{c + d}} \right) \sqrt{c + d \sin(fx + e)}}{3 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) a^2 f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & - \frac{(c + 5d)(c^2 - d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticF} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right), \sqrt{2} \sqrt{\frac{d}{c + d}} \right) \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}{3 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) a^2 f \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\sqrt{2}(2c^3 + 10c^2d + 9cd^2 - 15d^3) \cos(fx + e)^2 - \sqrt{2}(2c^3 + 10c^2d + 9cd^2 - 15d^3) \cos(fx + e) - (\sqrt{2}(2c^3 +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(d^2 \cos(fx + e)^2 - 2cd \sin(fx + e) - c^2 - d^2 \right) \sqrt{d \sin(fx + e) + c}}{a^2 \cos(fx + e)^2 - 2a^2 \sin(fx + e) - 2a^2}, x \right)$$

41.34 Problem number 511

$$\int \frac{(c + d \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(c + 3d) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{3a^2 f (1 + \sin(fx + e))} - \frac{(c - d) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{3f (a + a \sin(fx + e))^2} \\ & + \frac{(c + 3d) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{c + d \sin(fx + e)}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) a^2 f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & - \frac{(c + d)(c + 2d) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) a^2 f \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (c^2 + 3cd + 3d^2) \cos(fx + e)^2 - \sqrt{2} (c^2 + 3cd + 3d^2) \cos(fx + e) - \left(\sqrt{2} (c^2 + 3cd + 3d^2) \cos(fx + e) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(d \sin(fx + e) + c)^{\frac{3}{2}}}{a^2 \cos(fx + e)^2 - 2a^2 \sin(fx + e) - 2a^2}, x\right)$$

41.35 Problem number 512

$$\int \frac{\sqrt{c + d \sin(e + fx)}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{-\frac{c \cos (fx+e) \sqrt{c+d \sin (fx+e)}}{3 a^2(c-d) f(1+\sin (fx+e))}-\frac{\cos (fx+e) \sqrt{c+d \sin (fx+e)}}{3 f(a+a \sin (fx+e))^2}}{c \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c+d \sin (fx+e)}} + \frac{3 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^2(c-d) f \sqrt{\frac{c+d \sin (fx+e)}{c+d}}}{(c+d) \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin (fx+e)}{c+d}}}$$

$$3 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^2 f \sqrt{c+d \sin (fx+e)}$$

command

```
integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{2}\left(2 c^2-3 d^2\right) \cos (f x+e)^2-\sqrt{2}\left(2 c^2-3 d^2\right) \cos (f x+e)-\left(\sqrt{2}\left(2 c^2-3 d^2\right) \cos (f x+e)+2 \sqrt{2}\left(2 c^2-3 d^2\right)\right)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{d \sin (f x+e)+c}}{a^2 \cos (f x+e)^2-2 a^2 \sin (f x+e)-2 a^2}, x\right)$$

41.36 Problem number 513

$$\int \frac{1}{(a+a \sin (e+f x))^2 \sqrt{c+d \sin (e+f x)}} dx$$

Optimal antiderivative

$$\frac{-\frac{(c-3 d) \cos (fx+e) \sqrt{c+d \sin (fx+e)}}{3 a^2(c-d)^2 f(1+\sin (fx+e))}-\frac{\cos (fx+e) \sqrt{c+d \sin (fx+e)}}{3(c-d) f(a+a \sin (fx+e))^2}}{(c-3 d) \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c+d \sin (fx+e)}} + \frac{3 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^2(c-d)^2 f \sqrt{\frac{c+d \sin (fx+e)}{c+d}}}{(c-2 d) \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin (fx+e)}{c+d}}}$$

$$3 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^2(c-d) f \sqrt{c+d \sin (fx+e)}$$

command

`integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (c^2 - 3cd + 3d^2) \cos(fx + e)^2 - \sqrt{2} (c^2 - 3cd + 3d^2) \cos(fx + e) - \left(\sqrt{2} (c^2 - 3cd + 3d^2) \cos(fx + e) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{d \sin(fx + e) + c}}{2a^2c + 2a^2d - (a^2c + 2a^2d) \cos(fx + e)^2 - (a^2d \cos(fx + e))^2 - 2a^2c - 2a^2d} \sin(fx + e), x \right)$$

41.37 Problem number 514

$$\int \frac{1}{(a + a \sin(e + fx))^2 (c + d \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d(c^2 - 5cd - 12d^2) \cos(fx + e)}{3a^2(c - d)^3(c + d) f \sqrt{c + d \sin(fx + e)}} \\ & - \frac{(c - 5d) \cos(fx + e)}{3a^2(c - d)^2 f (1 + \sin(fx + e)) \sqrt{c + d \sin(fx + e)}} \\ & - \frac{\cos(fx + e)}{3(c - d) f (a + a \sin(fx + e))^2 \sqrt{c + d \sin(fx + e)}} \\ & + \frac{(c^2 - 5cd - 12d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticE} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right), \sqrt{2} \sqrt{\frac{d}{c + d}} \right) \sqrt{c + d \sin(fx + e)}}{3 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) a^2 (c - d)^3 (c + d) f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & + \frac{(c - 5d) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticF} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right), \sqrt{2} \sqrt{\frac{d}{c + d}} \right) \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}{3 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) a^2 (c - d)^2 f \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

`integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{d \sin(fx + e) + c}}{a^2d^2 \cos(fx + e)^4 + 2a^2c^2 + 4a^2cd + 2a^2d^2 - (a^2c^2 + 4a^2cd + 3a^2d^2) \cos(fx + e)^2 + 2(a^2c^2 + 2a^2cd}$$

41.38 Problem number 515

$$\int \frac{1}{(a + a \sin(e + fx))^2 (c + d \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d(c^2 - 7cd - 10d^2) \cos(fx + e)}{3a^2 (c - d)^3 (c + d) f (c + d \sin(fx + e))^{\frac{3}{2}}} - \frac{(c - 7d) \cos(fx + e)}{3a^2 (c - d)^2 f (1 + \sin(fx + e)) (c + d \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{\cos(fx + e)}{3(c - d) f (a + a \sin(fx + e))^2 (c + d \sin(fx + e))^{\frac{3}{2}}} - \frac{d(c + 3d) (c^2 - 10cd - 7d^2) \cos(fx + e)}{3a^2 (c - d)^4 (c + d)^2 f \sqrt{c + d \sin(fx + e)}} \\ & + \frac{(c + 3d) (c^2 - 10cd - 7d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{c + d \sin(fx + e)}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) a^2 (c - d)^4 (c + d)^2 f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & - \frac{(c^2 - 7cd - 10d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) a^2 (c - d)^3 (c + d) f \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{2a^2c^3 + 6a^2c^2d + 6a^2cd^2 + 2a^2d^3 + (3a^2cd^2 + 2a^2d^3) \cos(fx + e)^4 - (a^2c^3 + 6a^2c^2d + 9a^2cd^2 + 4a^2d^3) \cos(fx + e)^5}{(a + a \sin(e + fx))^2 (c + d \sin(e + fx))^{5/2}} dx\right)$$

41.39 Problem number 516

$$\int \frac{(c + d \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(c-d) \cos (fx+e) (c+d \sin (fx+e))^{\frac{3}{2}}}{5 f (a+a \sin (fx+e))^3} - \frac{2(c-d) (c+3 d) \cos (fx+e) \sqrt{c+d \sin (fx+e)}}{15 a f (a+a \sin (fx+e))^2} \\
& - \frac{(4 c^2+15 c d+27 d^2) \cos (fx+e) \sqrt{c+d \sin (fx+e)}}{30 f (a^3+a^3 \sin (fx+e))} \\
& + \frac{(4 c^2+15 c d+27 d^2) \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c+d \sin (fx+e)}}{30 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^3 f \sqrt{\frac{c+d \sin (fx+e)}{c+d}}} \\
& - \frac{(c+d) (4 c^2+11 c d+15 d^2) \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin (fx+e)}{c+d}}}{30 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^3 f \sqrt{c+d \sin (fx+e)}}
\end{aligned}$$

command

```
integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(d^2 \cos (fx+e)^2-2 c d \sin (fx+e)-c^2-d^2\right) \sqrt{d \sin (fx+e)+c}}{3 a^3 \cos (fx+e)^2-4 a^3+\left(a^3 \cos (fx+e)^2-4 a^3\right) \sin (fx+e)}, x\right)$$

41.40 Problem number 517

$$\int \frac{(c+d \sin (e+fx))^{3/2}}{(a+a \sin (e+fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(c-d) \cos (fx+e) \sqrt{c+d \sin (fx+e)}}{5 f(a+a \sin (fx+e))^3} - \frac{2(c+2d) \cos (fx+e) \sqrt{c+d \sin (fx+e)}}{15 a f(a+a \sin (fx+e))^2} \\
& - \frac{(4 c^2+5 c d-3 d^2) \cos (fx+e) \sqrt{c+d \sin (fx+e)}}{30(c-d) f\left(a^3+a^3 \sin (fx+e)\right)} \\
& + \frac{(4 c^2+5 c d-3 d^2) \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c+d \sin (fx+e)}}{30 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^3(c-d) f \sqrt{\frac{c+d \sin (fx+e)}{c+d}}} \\
& - \frac{(c+d)(4 c+5 d) \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin (fx+e)}{c+d}}}{30 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^3 f \sqrt{c+d \sin (fx+e)}}
\end{aligned}$$

command

```
integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(d \sin (fx+e)+c)^{\frac{3}{2}}}{3 a^3 \cos (fx+e)^2-4 a^3+\left(a^3 \cos (fx+e)^2-4 a^3\right) \sin (fx+e)}, x\right)$$

41.41 Problem number 518

$$\int \frac{\sqrt{c+d \sin (e+fx)}}{(a+a \sin (e+fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{\cos (fx+e) \sqrt{c+d \sin (fx+e)}}{5 f\left(a+a \sin (fx+e)\right)^3}-\frac{(2 c-d) \cos (fx+e) \sqrt{c+d \sin (fx+e)}}{15 a(c-d) f\left(a+a \sin (fx+e)\right)^2} \\
& -\frac{\left(4 c^2-5 c d-3 d^2\right) \cos (fx+e) \sqrt{c+d \sin (fx+e)}}{30(c-d)^2 f\left(a^3+a^3 \sin (fx+e)\right)} \\
& +\frac{\left(4 c^2-5 c d-3 d^2\right) \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c+d \sin (fx+e)}}{30 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^3(c-d)^2 f \sqrt{\frac{c+d \sin (fx+e)}{c+d}}} \\
& -\frac{\left(4 c-5 d\right)(c+d) \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin (fx+e)}{c+d}}}{30 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^3(c-d) f \sqrt{c+d \sin (fx+e)}}
\end{aligned}$$

command

```
integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{d \sin (fx+e)+c}}{3 a^3 \cos (fx+e)^2-4 a^3+\left(a^3 \cos (fx+e)^2-4 a^3\right) \sin (fx+e)}, x\right)$$

41.42 Problem number 519

$$\int \frac{1}{\left(a+a \sin (e+fx)\right)^3 \sqrt{c+d \sin (e+fx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{\cos (fx+e) \sqrt{c+d \sin (fx+e)}}{5(c-d) f(a+a \sin (fx+e))^3} - \frac{2(c-3d) \cos (fx+e) \sqrt{c+d \sin (fx+e)}}{15 a(c-d)^2 f(a+a \sin (fx+e))^2} \\
& - \frac{(4 c^2-15 c d+27 d^2) \cos (fx+e) \sqrt{c+d \sin (fx+e)}}{30(c-d)^3 f\left(a^3+a^3 \sin (fx+e)\right)} \\
& + \frac{(4 c^2-15 c d+27 d^2) \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c+d \sin (fx+e)}}{30 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^3(c-d)^3 f \sqrt{\frac{c+d \sin (fx+e)}{c+d}}} \\
& - \frac{(4 c^2-11 c d+15 d^2) \sqrt{\frac{1}{2}+\frac{\sin (fx+e)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin (fx+e)}{c+d}}}{30 \sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right) a^3(c-d)^2 f \sqrt{c+d \sin (fx+e)}}
\end{aligned}$$

command

```
integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sin (fx+e)+c}}{a^3 d \cos (fx+e)^4+4 a^3 c+4 a^3 d-(3 a^3 c+5 a^3 d) \cos (fx+e)^2+\left(4 a^3 c+4 a^3 d-\left(a^3 c+3 a^3 d\right) \cos (fx+e)\right) \sqrt{d \sin (fx+e)+c}}\right)$$

41.43 Problem number 520

$$\int \frac{1}{(a+a \sin (e+fx))^3(c+d \sin (e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{d(4c^3 - 21c^2d + 62cd^2 + 147d^3) \cos(fx + e)}{30a^3 (c - d)^4 (c + d) f \sqrt{c + d \sin(fx + e)}} \\
& - \frac{\cos(fx + e)}{5(c - d) f (a + a \sin(fx + e))^3 \sqrt{c + d \sin(fx + e)}} \\
& - \frac{2(c - 4d) \cos(fx + e)}{15a(c - d)^2 f (a + a \sin(fx + e))^2 \sqrt{c + d \sin(fx + e)}} \\
& - \frac{(4c^2 - 21cd + 65d^2) \cos(fx + e)}{30(c - d)^3 f (a^3 + a^3 \sin(fx + e)) \sqrt{c + d \sin(fx + e)}} \\
& + \frac{(4c^3 - 21c^2d + 62cd^2 + 147d^3) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{c + d \sin(fx + e)}}{30 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) a^3 (c - d)^4 (c + d) f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\
& + \frac{(4c^2 - 21cd + 65d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}{30 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) a^3 (c - d)^3 f \sqrt{c + d \sin(fx + e)}}
\end{aligned}$$

command

```
integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sin(fx + e)}}{4a^3c^2 + 8a^3cd + 4a^3d^2 + (2a^3cd + 3a^3d^2) \cos(fx + e)^4 - (3a^3c^2 + 10a^3cd + 7a^3d^2) \cos(fx + e)^2 + (c + d \sin(fx + e))^5}\right) dx$$

41.44 Problem number 521

$$\int \frac{1}{(a + a \sin(e + fx))^3 (c + d \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{d(4c^3 - 27c^2d + 114cd^2 + 165d^3) \cos(fx + e)}{30a^3(c-d)^4(c+d)f(c+d \sin(fx+e))^{\frac{3}{2}} \cos(fx+e)} \\
& - \frac{5(c-d)f(a+a \sin(fx+e))^3(c+d \sin(fx+e))^{\frac{3}{2}}}{2(c-5d) \cos(fx+e)} \\
& - \frac{15a(c-d)^2 f(a+a \sin(fx+e))^2(c+d \sin(fx+e))^{\frac{3}{2}}}{(4c^2 - 27cd + 119d^2) \cos(fx+e)} \\
& - \frac{30(c-d)^3 f(a^3 + a^3 \sin(fx+e))(c+d \sin(fx+e))^{\frac{3}{2}}}{d(4c^4 - 27c^3d + 111c^2d^2 + 579cd^3 + 357d^4) \cos(fx+e)} \\
& - \frac{30a^3(c-d)^5(c+d)^2 f \sqrt{c+d \sin(fx+e)}}{(4c^4 - 27c^3d + 111c^2d^2 + 579cd^3 + 357d^4) \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin(fx+e)}{c+d}} \\
& + \frac{30 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) a^3(c-d)^5(c+d)^2 f \sqrt{\frac{c+d \sin(fx+e)}{c+d}}}{(4c^3 - 27c^2d + 114cd^2 + 165d^3) \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin(fx+e)}{c+d}} \\
& - \frac{30 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) a^3(c-d)^4(c+d) f \sqrt{c+d \sin(fx+e)}}{30 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) a^3(c-d)^4(c+d) f \sqrt{c+d \sin(fx+e)}}
\end{aligned}$$

command

```
integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{a^3d^3 \cos(fx+e)^6 - 4a^3c^3 - 12a^3c^2d - 12a^3cd^2 - 4a^3d^3 - 3(a^3c^2d + 3a^3cd^2 + 2a^3d^3) \cos(fx+e)^4}{a^3d^3 \cos(fx+e)^6 - 4a^3c^3 - 12a^3c^2d - 12a^3cd^2 - 4a^3d^3 - 3(a^3c^2d + 3a^3cd^2 + 2a^3d^3) \cos(fx+e)^4}\right)$$

41.45 Problem number 704

$$\int \frac{1}{(a+b \sin(e+fx))(c+d \sin(e+fx))^2} dx$$

Optimal antiderivative

$$\frac{2d(acd - b(2c^2 - d^2)) \arctan\left(\frac{d+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{c^2-d^2}}\right)}{(-ad+bc)^2(c^2-d^2)^{\frac{3}{2}}f} - \frac{d^2 \cos(fx+e)}{(-ad+bc)(c^2-d^2)f(c+d\sin(fx+e))} + \frac{2b^2 \arctan\left(\frac{b+a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(-ad+bc)^2 f \sqrt{a^2-b^2}}$$

command

```
integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

41.46 Problem number 711

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))} dx$$

Optimal antiderivative

$$\frac{2b(-2a^2d+abc+b^2d) \arctan\left(\frac{b+a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}(-ad+bc)^2f} + \frac{b^2 \cos(fx+e)}{(a^2-b^2)(-ad+bc)f(a+b\sin(fx+e))} + \frac{2d^2 \arctan\left(\frac{d+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{c^2-d^2}}\right)}{(-ad+bc)^2 f \sqrt{c^2-d^2}}$$

command

```
integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

41.47 Problem number 723

$$\int (a + b \sin(e + fx))(c + d \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7ad + 5bc) \cos(fx + e) (c + d \sin(fx + e))^{\frac{3}{2}}}{35f} - \frac{2b \cos(fx + e) (c + d \sin(fx + e))^{\frac{5}{2}}}{7f} \\ & - \frac{2(56acd + 15b^2c^2 + 25bd^2) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{105f} \\ & - \frac{2(161ac^2d + 63ad^3 + 15b^2c^3 + 145bcd^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d \sin(fx + e)}}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & + \frac{2(c^2 - d^2) (56acd + 15b^2c^2 + 25bd^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d \sin(fx + e)}}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (30bc^4 + 7ac^3d - 115bc^2d^2 - 231acd^3 - 75bd^4) \sqrt{id} \operatorname{weierstrassPInverse}\left(-\frac{4(4c^2-3d^2)}{3d^2}, -\frac{8(8ic^3-9icd^2)}{27d^3}, \frac{3dc}{27d^3}\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(ac^2 + 2bcd + ad^2 - (2bcd + ad^2) \cos(fx + e)^2 - (bd^2 \cos(fx + e)^2 - bc^2 - 2acd - bd^2) \sin(fx + e)\right) \sqrt{c + d \sin(fx + e)}\right)$$

41.48 Problem number 724

$$\int (a + b \sin(e + fx))(c + d \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b \cos(fx + e) (c + d \sin(fx + e))^{\frac{3}{2}}}{5f} - \frac{2(5ad + 3bc) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{15f}$$

$$- \frac{2(20acd + 3b(c^2 + 3d^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d \sin(fx + e)}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}$$

$$+ \frac{2(5ad + 3bc) (c^2 - d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{c + d \sin(fx + e)}}$$

command

`integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (6bc^3 - 5ac^2d - 18bcd^2 - 15ad^3) \sqrt{id} \operatorname{weierstrassPInverse}\left(-\frac{4(4c^2 - 3d^2)}{3d^2}, -\frac{8(8ic^3 - 9icd^2)}{27d^3}, \frac{3d \cos(fx+e) - 3id \sin(fx+e)}{3d}\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(bd \cos(fx + e)^2 - ac - bd - (bc + ad) \sin(fx + e)\right) \sqrt{d \sin(fx + e) + c}, x\right)$$

41.49 Problem number 725

$$\int (a + b \sin(e + fx)) \sqrt{c + d \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2b \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{3f}$$

$$- \frac{2(3ad + bc) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d \sin(fx + e)}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}$$

$$+ \frac{2b(c^2 - d^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{c + d \sin(fx + e)}}$$

command

```
integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \sqrt{d \sin (f x+e)+c} b d^2 \cos (f x+e)+\sqrt{2}\left(2 b c^2-3 a c d-3 b d^2\right) \sqrt{i d} \operatorname{weierstrassPInverse}\left(-\frac{4\left(4 c^2-3 d^2\right)}{3 d^2},-\frac{8\left(8 i c^3-9 i c d^2\right)}{27 d^3}\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b \sin (f x+e)+a\right) \sqrt{d \sin (f x+e)+c}, x\right)$$

41.50 Problem number 726

$$\int \frac{a+b \sin (e+f x)}{\sqrt{c+d \sin (e+f x)}} d x$$

Optimal antiderivative

$$\frac{2 b \sqrt{\frac{1}{2}+\frac{\sin (f x+e)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c+d \sin (f x+e)}}{\sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right) d f \sqrt{\frac{c+d \sin (f x+e)}{c+d}}}$$

$$+\frac{2(-a d+b c) \sqrt{\frac{1}{2}+\frac{\sin (f x+e)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin (f x+e)}{c+d}}}{\sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right) d f \sqrt{c+d \sin (f x+e)}}$$

command

```
integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-3 i \sqrt{2} b \sqrt{i d} d \operatorname{weierstrassZeta}\left(-\frac{4\left(4 c^2-3 d^2\right)}{3 d^2},-\frac{8\left(8 i c^3-9 i c d^2\right)}{27 d^3}\right), \operatorname{weierstrassPInverse}\left(-\frac{4\left(4 c^2-3 d^2\right)}{3 d^2},-\frac{8\left(8 i c^3-9 i c d^2\right)}{27 d^3}\right), 3}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b \sin (f x+e)+a}{\sqrt{d \sin (f x+e)+c}}, x\right)$$

41.51 Problem number 727

$$\int \frac{a + b \sin(e + fx)}{(c + d \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(-ad + bc) \cos(fx + e)}{(c^2 - d^2) f \sqrt{c + d \sin(fx + e)}} + \frac{2(-ad + bc) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d \sin(fx + e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d(c^2 - d^2) f \sqrt{\frac{c + d \sin(fx + e)}{c+d}}} + \frac{2b \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) df \sqrt{c + d \sin(fx + e)}}$$

command

```
integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(bcd^2 - ad^3) \sqrt{d \sin(fx + e) + c} \cos(fx + e) - \left(\sqrt{2} (2bc^2d + acd^2 - 3bd^3) \sin(fx + e) + \sqrt{2} (2bc^3 + ac^2d - \dots)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(b \sin(fx + e) + a) \sqrt{d \sin(fx + e) + c}}{d^2 \cos(fx + e)^2 - 2cd \sin(fx + e) - c^2 - d^2}, x\right)$$

41.52 Problem number 728

$$\int \frac{a + b \sin(e + fx)}{(c + d \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(-ad + bc) \cos(fx + e)}{3(c^2 - d^2) f (c + d \sin(fx + e))^{\frac{3}{2}}} + \frac{2(4acd - b(c^2 + 3d^2)) \cos(fx + e)}{3(c^2 - d^2)^2 f \sqrt{c + d \sin(fx + e)}}$$

$$\frac{2(4acd - b(c^2 + 3d^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d \sin(fx + e)}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d (c^2 - d^2)^2 f \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}$$

$$\frac{2(-ad + bc) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d (c^2 - d^2) f \sqrt{c + d \sin(fx + e)}}$$

command

`integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (2bc^3d^2 + ac^2d^3 - 6bcd^4 + 3ad^5) \cos(fx + e)^2 - 2\sqrt{2} (2bc^4d + ac^3d^2 - 6bc^2d^3 + 3acd^4) \sin(fx + e) - \sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(b \sin(fx + e) + a) \sqrt{d \sin(fx + e) + c}}{3cd^2 \cos(fx + e)^2 - c^3 - 3cd^2 + (d^3 \cos(fx + e)^2 - 3c^2d - d^3) \sin(fx + e)}, x\right)$$

41.53 Problem number 729

$$\int \frac{a + b \sin(e + fx)}{(c + d \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(-ad + bc) \cos(fx + e)}{5(c^2 - d^2) f (c + d \sin(fx + e))^{\frac{5}{2}}} - \frac{2(-8acd + 3bc^2 + 5bd^2) \cos(fx + e)}{15(c^2 - d^2)^2 f (c + d \sin(fx + e))^{\frac{3}{2}}}$$

$$\frac{2(-23ac^2d - 9ad^3 + 3bc^3 + 29bcd^2) \cos(fx + e)}{15(c^2 - d^2)^3 f \sqrt{c + d \sin(fx + e)}}$$

$$2(-23ac^2d - 9ad^3 + 3bc^3 + 29bcd^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d \sin(fx + e)}$$

$$+ \frac{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d (c^2 - d^2)^3 f \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}{2(-8acd + 3bc^2 + 5bd^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c+d}}}$$

$$\frac{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d (c^2 - d^2)^2 f \sqrt{c + d \sin(fx + e)}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d (c^2 - d^2)^2 f \sqrt{c + d \sin(fx + e)}}$$

command

```
integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b \sin(fx + e) + a) \sqrt{d \sin(fx + e) + c}}{d^4 \cos(fx + e)^4 + c^4 + 6c^2d^2 + d^4 - 2(3c^2d^2 + d^4) \cos(fx + e)^2 - 4(cd^3 \cos(fx + e)^2 - c^3d - cd^3) \sin(fx + e)} \right)$$

41.54 Problem number 730

$$\int (a + b \sin(e + fx))^2 (c + d \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7(9a^2 + 7b^2)d^2 - 10bc(-9ad + bc)) \cos(fx + e) (c + d \sin(fx + e))^{3/2}}{315df} \\ & + \frac{4b(-9ad + bc) \cos(fx + e) (c + d \sin(fx + e))^{5/2}}{63df} - \frac{2b^2 \cos(fx + e) (c + d \sin(fx + e))^{7/2}}{9df} \\ & - \frac{4(84a^2cd^2 + 15abd(3c^2 + 5d^2) - b^2(5c^3 - 57cd^2)) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{315df} \\ & - \frac{2(21a^2d^2(23c^2 + 9d^2) + 30abd(3c^3 + 29cd^2) - b^2(10c^4 - 279c^2d^2 - 147d^4)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticE} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) \right)}{315 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) d^2 f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & - \frac{4(c^2 - d^2) (-84a^2cd^2 - 45abc^2d - 75abd^3 + 5b^2c^3 - 57b^2cd^2) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticF} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) \right)}{315 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) d^2 f \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (20b^2c^5 - 180abc^4d + 690abc^2d^3 + 450abd^5 - 3(7a^2 + 31b^2)c^3d^2 + 3(231a^2 + 163b^2)cd^4) \sqrt{id} \text{weierstrassF}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(b^2d^2 \cos(fx + e)^4 + 4abcd + (a^2 + b^2)c^2 + (a^2 + b^2)d^2 - (b^2c^2 + 4abcd + (a^2 + 2b^2)d^2) \cos(fx + e)^2 - \right) \right)$$

41.55 Problem number 731

$$\int (a + b \sin(e + fx))^2 (c + d \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4b(-7ad + bc) \cos(fx + e) (c + d \sin(fx + e))^{\frac{3}{2}}}{35df} - \frac{2b^2 \cos(fx + e) (c + d \sin(fx + e))^{\frac{5}{2}}}{7df} \\ & - \frac{2(5(7a^2 + 5b^2)d^2 - 6bc(-7ad + bc)) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{105df} \\ & - \frac{4(70a^2cd^2 + 21abd(c^2 + 3d^2) - b^2(3c^3 - 41cd^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right)}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & + \frac{2(c^2 - d^2)(42abcd + 35a^2d^2 - b^2(6c^2 - 25d^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right)}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (12b^2c^4 - 84abc^3d + 252abcd^3 + (35a^2 - 11b^2)c^2d^2 + 15(7a^2 + 5b^2)d^4) \sqrt{id} \operatorname{weierstrassPInverse}\left(-\frac{4(4c^2 - 3d^2)}{3d^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(2abd - (b^2c + 2abd) \cos(fx + e)^2 + (a^2 + b^2)c - (b^2d \cos(fx + e)^2 - 2abc - (a^2 + b^2)d) \sin(fx + e)\right)\right)$$

41.56 Problem number 732

$$\int (a + b \sin(e + fx))^2 \sqrt{c + d \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2b^2 \cos (fx + e) (c + d \sin (fx + e))^{\frac{3}{2}}}{5df} + \frac{4b(-5ad + bc) \cos (fx + e) \sqrt{c + d \sin (fx + e)}}{15df}$$

$$\frac{2(3(5a^2 + 3b^2) d^2 - 2bc(-5ad + bc)) \sqrt{\frac{1}{2} + \frac{\sin (fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{\frac{c + d \sin (fx + e)}{c + d}}}$$

$$\frac{4b(-5ad + bc) (c^2 - d^2) \sqrt{\frac{1}{2} + \frac{\sin (fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c + d \sin (fx + e)}{c + d}}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{c + d \sin (fx + e)}}$$

command

```
integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (4b^2c^3 - 20abc^2d + 30abd^3 + 3(5a^2 + b^2)cd^2) \sqrt{id} \operatorname{weierstrassPInverse}\left(-\frac{4(4c^2 - 3d^2)}{3d^2}, -\frac{8(8ic^3 - 9icd^2)}{27d^3}, \frac{3d \cos(fx + e)}{c + d}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(b^2 \cos (fx + e)^2 - 2ab \sin (fx + e) - a^2 - b^2\right) \sqrt{d \sin (fx + e) + c}, x\right)$$

41.57 Problem number 733

$$\int \frac{(a + b \sin(e + fx))^2}{\sqrt{c + d \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2b^2 \cos (fx + e) \sqrt{c + d \sin (fx + e)}}{3df}$$

$$4b(-3ad + bc) \sqrt{\frac{1}{2} + \frac{\sin (fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c + d \sin (fx + e)}$$

$$3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{\frac{c + d \sin (fx + e)}{c + d}}$$

$$2((3a^2 + b^2) d^2 + 2bc(-3ad + bc)) \sqrt{\frac{1}{2} + \frac{\sin (fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c + d \sin (fx + e)}{c}}$$

$$3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{c + d \sin (fx + e)}$$

command

```
integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{d \sin (f x+e)+c} b^2 d^2 \cos (f x+e)-\sqrt{2}\left(4 b^2 c^2-12 a b c d+3\left(3 a^2+b^2\right) d^2\right) \sqrt{i d} \text {weierstrassPInverse}\left(-\frac{4}{3}\left(\frac{a+b \sin (f x+e)}{c+d \sin (f x+e)}\right)^2\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text {integral}\left(-\frac{b^2 \cos (f x+e)^2-2 a b \sin (f x+e)-a^2-b^2}{\sqrt{d \sin (f x+e)+c}}, x\right)$$

41.58 Problem number 734

$$\int \frac{(a+b \sin (e+f x))^2}{(c+d \sin (e+f x))^{3 / 2}} d x$$

Optimal antiderivative

$$\frac{2(-a d+b c)^2 \cos (f x+e)}{d\left(c^2-d^2\right) f \sqrt{c+d \sin (f x+e)}} \\ + \frac{2\left(2 b^2 c^2-2 a b c d+\left(a^2-b^2\right) d^2\right) \sqrt{\frac{1}{2}+\frac{\sin (f x+e)}{2}} \text {EllipticE}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c+d \sin (f x+e)}}{\sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right) d^2\left(c^2-d^2\right) f \sqrt{\frac{c+d \sin (f x+e)}{c+d}}} \\ + \frac{4 b(-a d+b c) \sqrt{\frac{1}{2}+\frac{\sin (f x+e)}{2}} \text {EllipticF}\left(\cos \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{\frac{c+d \sin (f x+e)}{c+d}}}{\sin \left(\frac{e}{2}+\frac{\pi}{4}+\frac{f x}{2}\right) d^2 f \sqrt{c+d \sin (f x+e)}}$$

command

```
integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6\left(b^2 c^2 d^2-2 a b c d^3+a^2 d^4\right) \sqrt{d \sin (f x+e)+c} \cos (f x+e)-\left(\sqrt{2}\left(4 b^2 c^3 d-4 a b c^2 d^2+6 a b d^4-\left(a^2+5 b^2\right) c d^3\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text {integral}\left(\frac{\left(b^2 \cos (f x+e)^2-2 a b \sin (f x+e)-a^2-b^2\right) \sqrt{d \sin (f x+e)+c}}{d^2 \cos (f x+e)^2-2 c d \sin (f x+e)-c^2-d^2}, x\right)$$

41.59 Problem number 735

$$\int \frac{(a + b \sin(e + fx))^2}{(c + d \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(-ad + bc)^2 \cos(fx + e)}{3d(c^2 - d^2) f (c + d \sin(fx + e))^{\frac{3}{2}}} - \frac{4(-ad + bc)(2acd + b(c^2 - 3d^2)) \cos(fx + e)}{3d(c^2 - d^2)^2 f \sqrt{c + d \sin(fx + e)}}$$

$$+ \frac{4(-ad + bc)(2acd + b(c^2 - 3d^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{c + d \sin(fx + e)}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 (c^2 - d^2)^2 f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}$$

$$- \frac{2(2abcd - a^2 d^2 + b^2(2c^2 - 3d^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 (c^2 - d^2) f \sqrt{c + d \sin(fx + e)}}$$

command

```
integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{2} (4b^2 c^4 d^2 + 4abc^3 d^3 - 12abcd^5 + (a^2 - 9b^2)c^2 d^4 + 3(a^2 + 3b^2)d^6) \cos(fx + e)^2 - 2\sqrt{2} (4b^2 c^5 d + 4abc^4 d^2)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(b^2 \cos(fx + e)^2 - 2ab \sin(fx + e) - a^2 - b^2\right) \sqrt{d \sin(fx + e) + c}}{3cd^2 \cos(fx + e)^2 - c^3 - 3cd^2 + \left(d^3 \cos(fx + e)^2 - 3c^2 d - d^3\right) \sin(fx + e)}, x\right)$$

41.60 Problem number 736

$$\int \frac{(a + b \sin(e + fx))^2}{(c + d \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(-ad + bc)^2 \cos(fx + e)}{5d(c^2 - d^2) f(c + d \sin(fx + e))^{\frac{5}{2}}} - \frac{4(-ad + bc)(4acd + b(c^2 - 5d^2)) \cos(fx + e)}{15d(c^2 - d^2)^2 f(c + d \sin(fx + e))^{\frac{3}{2}}} \\
& + \frac{2(a^2 d^2(23c^2 + 9d^2) - ab(6c^3 d + 58c d^3) - b^2(2c^4 - 19c^2 d^2 - 15d^4)) \cos(fx + e)}{15d(c^2 - d^2)^3 f \sqrt{c + d \sin(fx + e)}} \\
& - \frac{2(a^2 d^2(23c^2 + 9d^2) - ab(6c^3 d + 58c d^3) - b^2(2c^4 - 19c^2 d^2 - 15d^4)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4}\right)\right)}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 (c^2 - d^2)^3 f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\
& - \frac{4(-ad + bc)(4acd + b(c^2 - 5d^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c - d}}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 (c^2 - d^2)^2 f \sqrt{c + d \sin(fx + e)}}
\end{aligned}$$

command

```
integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(b^2 \cos(fx + e)^2 - 2ab \sin(fx + e) - a^2 - b^2) \sqrt{d \sin(fx + e) + c}}{d^4 \cos(fx + e)^4 + c^4 + 6c^2 d^2 + d^4 - 2(3c^2 d^2 + d^4) \cos(fx + e)^2 - 4(cd^3 \cos(fx + e)^2 - c^3 d - cd^3) \sin(fx + e)}\right)$$

41.61 Problem number 737

$$\int (a + b \sin(e + fx))^3 (c + d \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(1485a^2bc d^2 + 693a^3d^3 - 33ab^2d(10c^2 - 49d^2) + 5b^3(8c^3 + 67cd^2)) \cos(fx + e)(c + d \sin(fx + e))^{\frac{3}{2}}}{3465d^2f} \\
& + \frac{2b(66abcd - 297a^2d^2 - b^2(8c^2 + 81d^2)) \cos(fx + e)(c + d \sin(fx + e))^{\frac{5}{2}}}{693d^2f} \\
& + \frac{8b^2(-6ad + bc) \cos(fx + e)(c + d \sin(fx + e))^{\frac{7}{2}}}{99d^2f} \\
& - \frac{2b^2 \cos(fx + e)(a + b \sin(fx + e))(c + d \sin(fx + e))^{\frac{7}{2}}}{11df} \\
& - \frac{2(1848a^3cd^3 + 495a^2bd^2(3c^2 + 5d^2) - 66ab^2d(5c^3 - 57cd^2) + 5b^3(8c^4 + 57c^2d^2 + 135d^4)) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{3465d^2f} \\
& - \frac{2(231a^3d^3(23c^2 + 9d^2) + 495a^2bcd^2(3c^2 + 29d^2) - 33ab^2d(10c^4 - 279c^2d^2 - 147d^4) + 5b^3(8c^5 + 51c^3d^2 + 741cd^3)) \sqrt{c + d \sin(fx + e)}}{3465 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 f \sqrt{c + d \sin(fx + e)}} \\
& + \frac{2(c^2 - d^2)(1848a^3cd^3 + 495a^2bd^2(3c^2 + 5d^2) - 66ab^2d(5c^3 - 57cd^2) + 5b^3(8c^4 + 57c^2d^2 + 135d^4)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}}}{3465 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 f \sqrt{c + d \sin(fx + e)}}
\end{aligned}$$

command

```
integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (80b^3c^6 - 660ab^2c^5d + 30(99a^2b + 16b^3)c^4d^2 + 33(7a^3 + 93ab^2)c^3d^3 - 15(759a^2b + 169b^3)c^2d^4 - 99(77a^3 + 169ab^2)cd^5 + 33(11a^4 + 169ab^3)d^6)}{3465 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 f \sqrt{c + d \sin(fx + e)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left((2b^3cd + 3ab^2d^2) \cos(fx + e)^4 + (a^3 + 3ab^2)c^2 + 2(3a^2b + b^3)cd + (a^3 + 3ab^2)d^2 - (3ab^2c^2 + 2(3a^2b + b^3)d^2)\right) \sqrt{c + d \sin(fx + e)}\right)$$

41.62 Problem number 738

$$\int (a + b \sin(e + fx))^3 (c + d \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b(54abcd - 189a^2d^2 - b^2(8c^2 + 49d^2)) \cos(fx + e) (c + d \sin(fx + e))^{\frac{3}{2}}}{315d^2f} \\
& + \frac{8b^2(-5ad + bc) \cos(fx + e) (c + d \sin(fx + e))^{\frac{5}{2}}}{63d^2f} \\
& - \frac{2b^2 \cos(fx + e) (a + b \sin(fx + e)) (c + d \sin(fx + e))^{\frac{5}{2}}}{9df} \\
& - \frac{2(189a^2bcd^2 + 105a^3d^3 - 9ab^2d(6c^2 - 25d^2) + b^3(8c^3 + 39cd^2)) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{315d^2f} \\
& - \frac{2(420a^3cd^3 + 189a^2bd^2(c^2 + 3d^2) - ab^2(54c^3d - 738cd^3) + b^3(8c^4 + 33c^2d^2 + 147d^4)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \frac{c + d \sin(fx + e)}{c + d}\right)}{315 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 f \sqrt{c + d \sin(fx + e)}} \\
& + \frac{2(c^2 - d^2) (189a^2bcd^2 + 105a^3d^3 - 9ab^2d(6c^2 - 25d^2) + b^3(8c^3 + 39cd^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \frac{c + d \sin(fx + e)}{c + d}\right)}{315 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 f \sqrt{c + d \sin(fx + e)}}
\end{aligned}$$

command

```
integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (16b^3c^5 - 108ab^2c^4d + 6(63a^2b + 10b^3)c^3d^2 - 3(35a^3 - 33ab^2)c^2d^3 - 6(189a^2b + 44b^3)cd^4 - 45(7a^3 + 3ab^2)d^5)}{315d^2f \sqrt{c + d \sin(fx + e)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^3d \cos(fx + e)^4 - (3ab^2c + (3a^2b + 2b^3)d) \cos(fx + e)^2 + (a^3 + 3ab^2)c + (3a^2b + b^3)d - ((b^3c + 3ab^2)d^2)\right) \sqrt{c + d \sin(fx + e)}\right) dx$$

41.63 Problem number 739

$$\int (a + b \sin(e + fx))^3 \sqrt{c + d \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{8b^2(-4ad + bc) \cos(fx + e) (c + d \sin(fx + e))^{\frac{3}{2}}}{35d^2f}$$

$$- \frac{2b^2 \cos(fx + e) (a + b \sin(fx + e)) (c + d \sin(fx + e))^{\frac{3}{2}}}{7df}$$

$$+ \frac{2b(42abcd - 105a^2d^2 - b^2(8c^2 + 25d^2)) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{105d^2f}$$

$$- \frac{2(105a^2bcd^2 + 105a^3d^3 - 21ab^2d(2c^2 - 9d^2) + b^3(8c^3 + 19cd^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\right)}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}}$$

$$- \frac{2b(c^2 - d^2) (42abcd - 105a^2d^2 - b^2(8c^2 + 25d^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right)}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 f \sqrt{c + d \sin(fx + e)}}$$

command

```
integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16b^3c^4 - 84ab^2c^3d + 2(105a^2b + 16b^3)c^2d^2 - 21(5a^3 + 3ab^2)cd^3 - 15(21a^2b + 5b^3)d^4) \sqrt{id} \operatorname{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(3ab^2 \cos(fx + e)^2 - a^3 - 3ab^2 + \left(b^3 \cos(fx + e)^2 - 3a^2b - b^3\right) \sin(fx + e)\right) \sqrt{d \sin(fx + e) + c}, x\right)$$

41.64 Problem number 740

$$\int \frac{(a + b \sin(e + fx))^3}{\sqrt{c + d \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{8b^2(-3ad + bc) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{15d^2 f} \\
& - \frac{2b^2 \cos(fx + e) (a + b \sin(fx + e)) \sqrt{c + d \sin(fx + e)}}{5df} \\
& + \frac{2b(30abcd - 45a^2d^2 - b^2(8c^2 + 9d^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c+d}}\right) \sqrt{c+d}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 f \sqrt{\frac{c + d \sin(fx + e)}{c+d}}} \\
& + \frac{2(45a^2bc d^2 - 15a^3d^3 - 15ab^2d(2c^2 + d^2) + b^3(8c^3 + 7cd^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \right)}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 f \sqrt{c + d \sin(fx + e)}}
\end{aligned}$$

command

```
integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (16b^3c^3 - 60ab^2c^2d + 6(15a^2b + 2b^3)cd^2 - 45(a^3 + ab^2)d^3) \sqrt{i d} \operatorname{weierstrassPInverse}\left(-\frac{4(4c^2 - 3d^2)}{3d^2}, -\frac{8(8i)}{3d^2}\right)}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 f \sqrt{c + d \sin(fx + e)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{3ab^2 \cos(fx + e)^2 - a^3 - 3ab^2 + (b^3 \cos(fx + e)^2 - 3a^2b - b^3) \sin(fx + e)}{\sqrt{d \sin(fx + e) + c}}, x\right)$$

41.65 Problem number 741

$$\int \frac{(a + b \sin(e + fx))^3}{(c + d \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-ad + bc)^2 \cos(fx + e) (a + b \sin(fx + e))}{d(c^2 - d^2) f \sqrt{c + d \sin(fx + e)}} \\ & + \frac{2b(6abcd - 3a^2d^2 - b^2(4c^2 - d^2)) \cos(fx + e) \sqrt{c + d \sin(fx + e)}}{3d^2(c^2 - d^2) f} \\ & + \frac{2(9a^2bcd^2 - 3a^3d^3 - 9ab^2d(2c^2 - d^2) + b^3(8c^3 - 5cd^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 (c^2 - d^2) f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & + \frac{2b(18abcd - 9a^2d^2 - b^2(8c^2 + d^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c}}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 f \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

`integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (16b^3c^4d - 36ab^2c^3d^2 + 2(9a^2b - 8b^3)c^2d^3 + 3(a^3 + 15ab^2)cd^4 - 3(9a^2b + b^3)d^5) \sin(fx + e) + \sqrt{2} (16b^3c^4d - 36ab^2c^3d^2 + 2(9a^2b - 8b^3)c^2d^3 + 3(a^3 + 15ab^2)cd^4 - 3(9a^2b + b^3)d^5) \sin(fx + e) + \sqrt{2} (16b^3c^4d - 36ab^2c^3d^2 + 2(9a^2b - 8b^3)c^2d^3 + 3(a^3 + 15ab^2)cd^4 - 3(9a^2b + b^3)d^5) \sin(fx + e) + \dots\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(3ab^2 \cos(fx + e)^2 - a^3 - 3ab^2 + \left(b^3 \cos(fx + e)^2 - 3a^2b - b^3\right) \sin(fx + e)\right) \sqrt{d \sin(fx + e) + c}}{d^2 \cos(fx + e)^2 - 2cd \sin(fx + e) - c^2 - d^2}, x\right)$$

41.66 Problem number 742

$$\int \frac{(a + b \sin(e + fx))^3}{(c + d \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-ad + bc)^2 \cos(fx + e) (a + b \sin(fx + e))}{3d(c^2 - d^2) f (c + d \sin(fx + e))^{3/2}} + \frac{8(-ad + bc)^2 (acd + b(c^2 - 2d^2)) \cos(fx + e)}{3d^2(c^2 - d^2)^2 f \sqrt{c + d \sin(fx + e)}} \\ & - \frac{2(4a^3cd^3 - 6ab^2cd(c^2 - 3d^2) - 3a^2bd^2(c^2 + 3d^2) + b^3(8c^4 - 15c^2d^2 + 3d^4)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 (c^2 - d^2)^2 f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & + \frac{2(-ad + bc) (2abcd - a^2d^2 + b^2(8c^2 - 9d^2)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2} \sqrt{\frac{d}{c + d}}\right) \sqrt{\frac{c + d \sin(fx + e)}{c}}}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 (c^2 - d^2) f \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(3ab^2 \cos(fx + e)^2 - a^3 - 3ab^2 + \left(b^3 \cos(fx + e)^2 - 3a^2b - b^3 \right) \sin(fx + e) \right) \sqrt{d \sin(fx + e) + c}}{3cd^2 \cos(fx + e)^2 - c^3 - 3cd^2 + \left(d^3 \cos(fx + e)^2 - 3c^2d - d^3 \right) \sin(fx + e)}, x \right)$$

41.67 Problem number 743

$$\int \frac{(a + b \sin(e + fx))^3}{(c + d \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-ad + bc)^2 \cos(fx + e) (a + b \sin(fx + e))}{5d(c^2 - d^2) f (c + d \sin(fx + e))^{5/2}} + \frac{8(-ad + bc)^2 (2acd + b(c^2 - 3d^2)) \cos(fx + e)}{15d^2 (c^2 - d^2)^2 f (c + d \sin(fx + e))^{3/2}} \\ & - \frac{2(-ad + bc) (a^2 d^2 (23c^2 + 9d^2) + 2abd(7c^3 - 39cd^2) + b^2(8c^4 - 21c^2 d^2 + 45d^4)) \cos(fx + e)}{15d^2 (c^2 - d^2)^3 f \sqrt{c + d \sin(fx + e)}} \\ & + \frac{2(-ad + bc) (a^2 d^2 (23c^2 + 9d^2) + 2abd(7c^3 - 39cd^2) + b^2(8c^4 - 21c^2 d^2 + 45d^4)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticE} \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right)}{15 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) d^3 (c^2 - d^2)^3 f \sqrt{\frac{c + d \sin(fx + e)}{c + d}}} \\ & + \frac{2(8a^3 c d^3 - 6a b^2 c d (c^2 - 5d^2) - 3a^2 b d^2 (3c^2 + 5d^2) - b^3 (8c^4 - 15c^2 d^2 + 15d^4)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticF} \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right)}{15 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) d^3 (c^2 - d^2)^2 f \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\left(3ab^2 \cos(fx + e)^2 - a^3 - 3ab^2 + \left(b^3 \cos(fx + e)^2 - 3a^2b - b^3 \right) \sin(fx + e) \right) \sqrt{d \sin(fx + e)}}{d^4 \cos(fx + e)^4 + c^4 + 6c^2 d^2 + d^4 - 2(3c^2 d^2 + d^4) \cos(fx + e)^2 - 4(cd^3 \cos(fx + e)^2 - c^3 d - cd^3) \sin(fx + e)}, x \right)$$

41.68 Problem number 744

$$\int \frac{(a + b \sin(e + fx))^3}{(c + d \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2(-ad + bc)^2 \cos(fx + e) (a + b \sin(fx + e))}{7d(c^2 - d^2) f (c + d \sin(fx + e))^{\frac{7}{2}}} + \frac{8(-ad + bc)^2 (3acd + b(c^2 - 4d^2)) \cos(fx + e)}{35d^2 (c^2 - d^2)^2 f (c + d \sin(fx + e))^{\frac{5}{2}}}$$

$$\frac{2(-ad + bc) (a^2 d^2 (71c^2 + 25d^2) + ab(26c^3 d - 218c d^3) + b^2(8c^4 - 17c^2 d^2 + 105d^4)) \cos(fx + e)}{105d^2 (c^2 - d^2)^3 f (c + d \sin(fx + e))^{\frac{3}{2}}}$$

$$+ \frac{2(16a^3 c d^3 (11c^2 + 13d^2) - 6a b^2 c d (3c^4 - 62c^2 d^2 - 133d^4) - 9a^2 b d^2 (5c^4 + 102c^2 d^2 + 21d^4) - b^3 (8c^6 - 23c^4 d^2 + 105d^2 c^2)) \sqrt{c + d \sin(fx + e)}}{105d^2 (c^2 - d^2)^4 f \sqrt{c + d \sin(fx + e)}}$$

$$\frac{2(16a^3 c d^3 (11c^2 + 13d^2) - 6a b^2 c d (3c^4 - 62c^2 d^2 - 133d^4) - 9a^2 b d^2 (5c^4 + 102c^2 d^2 + 21d^4) - b^3 (8c^6 - 23c^4 d^2 + 105d^2 c^2)) \sqrt{c + d \sin(fx + e)}}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 (c^2 - d^2)^3 f \sqrt{c + d \sin(fx + e)}}$$

$$\frac{2(-ad + bc) (a^2 d^2 (71c^2 + 25d^2) + ab(26c^3 d - 218c d^3) + b^2(8c^4 - 17c^2 d^2 + 105d^4)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)}{105 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^3 (c^2 - d^2)^3 f \sqrt{c + d \sin(fx + e)}}$$

command

```
integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(3ab^2 \cos(fx + e)^2 - a^3 - 3ab^2 + \left(b^3 \cos(fx + e)^2 - 3a^2b - b^3\right) \sin(fx + e)\right)}{5cd^4 \cos(fx + e)^4 + c^5 + 10c^3d^2 + 5cd^4 - 10(c^3d^2 + cd^4) \cos(fx + e)^2 + \left(d^5 \cos(fx + e)^4 + 5c^4d + \dots\right)}\right)$$

42 Test file number 74

Test folder name:

test_cases/4_Trig_functions/4.1_Sine/74_4.1.2.2-g_cos-^p-a+b_sin-^m-c+d_sin-^n

42.1 Problem number 88

$$\int (g \cos(e + fx))^{3/2} \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10a c^2 (g \cos(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{3}{2}}}{77fg \sqrt{a + a \sin(fx + e)}} + \frac{2ac (g \cos(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{5}{2}}}{33fg \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a (g \cos(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{7}{2}}}{11fg \sqrt{a + a \sin(fx + e)}} + \frac{2a c^4 (g \cos(fx + e))^{\frac{5}{2}}}{3fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{2a c^4 g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)} \sqrt{g \cos(fx + e)})}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{2a c^3 (g \cos(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}}{7fg \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-231i \sqrt{2} \sqrt{acg} c^3 g \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 231i \sqrt{2} \sqrt{g \cos(fx + e)}}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(3c^3g \cos(fx + e)^3 - 4c^3g \cos(fx + e) - \left(c^3g \cos(fx + e)^3 - 4c^3g \cos(fx + e)\right) \sin(fx + e)\right) \sqrt{g \cos(fx + e)}\right)$$

42.2 Problem number 89

$$\int (g \cos(e + fx))^{3/2} \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ac(g \cos(fx + e))^{5/2} (c - c \sin(fx + e))^{3/2}}{21fg \sqrt{a + a \sin(fx + e)}} - \frac{2a(g \cos(fx + e))^{5/2} (c - c \sin(fx + e))^{5/2}}{9fg \sqrt{a + a \sin(fx + e)}} \\ & + \frac{22ac^3(g \cos(fx + e))^{5/2}}{45fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{22ac^3g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{22ac^2(g \cos(fx + e))^{5/2} \sqrt{c - c \sin(fx + e)}}{105fg \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-231i \sqrt{2} \sqrt{acg} c^2 g \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 231i \sqrt{2} \sqrt{acg} c^2 g \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(c^2g \cos(fx + e)^3 + 2c^2g \cos(fx + e) \sin(fx + e) - 2c^2g \cos(fx + e)\right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}\right)$$

42.3 Problem number 90

$$\int (g \cos(e + fx))^{3/2} \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(g \cos(fx + e))^{5/2} (c - c \sin(fx + e))^{3/2}}{7fg \sqrt{a + a \sin(fx + e)}} + \frac{2ac^2(g \cos(fx + e))^{5/2}}{5fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{6ac^2g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{6ac(g \cos(fx + e))^{5/2} \sqrt{c - c \sin(fx + e)}}{35fg \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-21i\sqrt{2}\sqrt{acg}cg\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))) + 21i\sqrt{2}\sqrt{acg}}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-cg\cos(fx+e)\sin(fx+e) - cg\cos(fx+e)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)} + \dots\right)$$

42.4 Problem number 91

$$\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ac(g\cos(fx+e))^{\frac{5}{2}}}{5fg\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} \\ & + \frac{6acg\sqrt{\frac{\cos(fx+e)}{2} + \frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)(\sqrt{\cos(fx+e)}\sqrt{g\cos(fx+e)})}{5\cos\left(\frac{fx}{2} + \frac{e}{2}\right)f\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} \\ & - \frac{2a(g\cos(fx+e))^{\frac{5}{2}}\sqrt{c-c\sin(fx+e)}}{5fg\sqrt{a+a\sin(fx+e)}} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}g\sin(fx+e) - 3i\sqrt{2}\sqrt{acg}g\text{weierstrassZeta}(-4,0, \dots)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}g\cos(fx+e),x\right)$$

42.5 Problem number 92

$$\int \frac{(g \cos(e + fx))^{3/2} \sqrt{a + a \sin(e + fx)}}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(g \cos(fx + e))^{\frac{5}{2}}}{3fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{2ag \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} \sqrt{acg} g \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 3i \sqrt{2} \sqrt{acg} g w$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} g \cos(fx + e)}{c \sin(fx + e) - c}, x\right)$$

42.6 Problem number 93

$$\int \frac{(g \cos(e + fx))^{3/2} \sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos(fx + e))^{\frac{5}{2}}}{fg (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{6ag \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fr`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \sqrt{g \cos (f x+e)} \sqrt{a \sin (f x+e)+a} \sqrt{-c \sin (f x+e)+c} g+3 \sqrt{a c g}\left(-i \sqrt{2} g \sin (f x+e)+i \sqrt{2} g\right)}{\text{weiers}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{g \cos (f x+e)} \sqrt{a \sin (f x+e)+a} \sqrt{-c \sin (f x+e)+c} g \cos (f x+e)}{c^2 \cos (f x+e)^2+2 c^2 \sin (f x+e)-2 c^2}, x\right)$$

42.7 Problem number 94

$$\int \frac{(g \cos (e+f x))^{3 / 2} \sqrt{a+a \sin (e+f x)}}{(c-c \sin (e+f x))^{5 / 2}} d x$$

Optimal antiderivative

$$\begin{aligned} & \frac{4 a(g \cos (f x+e))^{\frac{5}{2}}}{5 f g(c-c \sin (f x+e))^{\frac{5}{2}} \sqrt{a+a \sin (f x+e)}} \\ & - \frac{6 a(g \cos (f x+e))^{\frac{5}{2}}}{5 c f g(c-c \sin (f x+e))^{\frac{3}{2}} \sqrt{a+a \sin (f x+e)}} \\ & + \frac{6 a g \sqrt{\frac{\cos (f x+e)}{2}+\frac{1}{2}} \operatorname{EllipticE}\left(\sin \left(\frac{f x}{2}+\frac{e}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos (f x+e)}\right) \sqrt{g \cos (f x+e)}}{5 \cos \left(\frac{f x}{2}+\frac{e}{2}\right) c^2 f \sqrt{a+a \sin (f x+e)} \sqrt{c-c \sin (f x+e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fr`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{g \cos (f x+e)} \sqrt{a \sin (f x+e)+a} \sqrt{-c \sin (f x+e)+c}\left(3 g \sin (f x+e)-g\right)+3\left(i \sqrt{2} g \cos (f x+e)^2+2\right)}{\text{weiers}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{g \cos (f x+e)} \sqrt{a \sin (f x+e)+a} \sqrt{-c \sin (f x+e)+c} g \cos (f x+e)}{3 c^3 \cos (f x+e)^2-4 c^3-\left(c^3 \cos (f x+e)^2-4 c^3\right) \sin (f x+e)}, x\right)$$

42.8 Problem number 95

$$\int \frac{(g \cos(e + fx))^{3/2} \sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos(fx + e))^{\frac{5}{2}}}{9fg(c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a(g \cos(fx + e))^{\frac{5}{2}}}{15c^2fg(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a(g \cos(fx + e))^{\frac{5}{2}}}{15c^2fg(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{2ag \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^3 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3g \cos(fx + e)^2 + 9g \sin(fx + e) + g \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} - 3 \left(-3i \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} g \cos(fx + e)}{c^4 \cos(fx + e)^4 - 8c^4 \cos(fx + e)^2 + 8c^4 + 4 \left(c^4 \cos(fx + e)^2 - 2c^4 \right) \sin(fx + e)}, x\right)$$

42.9 Problem number 96

$$\int \frac{(g \cos(e + fx))^{3/2} \sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{4a(g \cos (fx + e))^{\frac{5}{2}}}{13fg(c - c \sin (fx + e))^{\frac{9}{2}} \sqrt{a + a \sin (fx + e)}} - \frac{2a(g \cos (fx + e))^{\frac{5}{2}}}{39c^3fg(c - c \sin (fx + e))^{\frac{7}{2}} \sqrt{a + a \sin (fx + e)}} \\ - \frac{2a(g \cos (fx + e))^{\frac{5}{2}}}{65c^2fg(c - c \sin (fx + e))^{\frac{5}{2}} \sqrt{a + a \sin (fx + e)}} \\ - \frac{2a(g \cos (fx + e))^{\frac{5}{2}}}{65c^3fg(c - c \sin (fx + e))^{\frac{3}{2}} \sqrt{a + a \sin (fx + e)}} \\ + \frac{2ag \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos (fx + e)}) \sqrt{g \cos (fx + e)}}{65 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^4 f \sqrt{a + a \sin (fx + e)} \sqrt{c - c \sin (fx + e)}}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(12 g \cos (fx + e)^2 - \left(3 g \cos (fx + e)^2 - 23 g \right) \sin (fx + e) + 7 g \right) \sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a} \sqrt{-c \sin (fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a} \sqrt{-c \sin (fx + e) + c} g \cos (fx + e)}{5 c^5 \cos (fx + e)^4 - 20 c^5 \cos (fx + e)^2 + 16 c^5 - \left(c^5 \cos (fx + e)^4 - 12 c^5 \cos (fx + e)^2 + 16 c^5\right) \sin (fx + e)}\right)$$

42.10 Problem number 97

$$\int (g \cos (e + fx))^{3/2} (a + a \sin (e + fx))^{3/2} (c - c \sin (e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2a^2c(g \cos (fx + e))^{\frac{5}{2}} (c - c \sin (fx + e))^{\frac{3}{2}}}{33fg \sqrt{a + a \sin (fx + e)}} - \frac{14a^2(g \cos (fx + e))^{\frac{5}{2}} (c - c \sin (fx + e))^{\frac{5}{2}}}{99fg \sqrt{a + a \sin (fx + e)}} \\ - \frac{2a(g \cos (fx + e))^{\frac{5}{2}} (c - c \sin (fx + e))^{\frac{5}{2}} \sqrt{a + a \sin (fx + e)}}{11fg} \\ + \frac{14a^2c^3(g \cos (fx + e))^{\frac{5}{2}}}{45fg \sqrt{a + a \sin (fx + e)} \sqrt{c - c \sin (fx + e)}} \\ + \frac{14a^2c^3g \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos (fx + e)}) \sqrt{g \cos (fx + e)}}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin (fx + e)} \sqrt{c - c \sin (fx + e)}} \\ + \frac{2a^2c^2(g \cos (fx + e))^{\frac{5}{2}} \sqrt{c - c \sin (fx + e)}}{15fg \sqrt{a + a \sin (fx + e)}}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-231i\sqrt{2}\sqrt{acg}ac^2g\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))) + 231i\sqrt{2}\sqrt{acg}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(ac^2g\cos(fx+e)^3\sin(fx+e) - ac^2g\cos(fx+e)^3\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)}\right)$$

42.11 Problem number 98

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^2(g\cos(fx+e))^{\frac{5}{2}}(c-c\sin(fx+e))^{\frac{3}{2}}}{9fg\sqrt{a+a\sin(fx+e)}} \\ & -\frac{2a(g\cos(fx+e))^{\frac{5}{2}}(c-c\sin(fx+e))^{\frac{3}{2}}\sqrt{a+a\sin(fx+e)}}{9fg} \\ & +\frac{14a^2c^2(g\cos(fx+e))^{\frac{5}{2}}}{45fg\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} \\ & +\frac{14a^2c^2g\sqrt{\frac{\cos(fx+e)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{fx}{2}+\frac{e}{2}\right),\sqrt{2}\right)(\sqrt{\cos(fx+e)}\sqrt{g\cos(fx+e)})}{15\cos\left(\frac{fx}{2}+\frac{e}{2}\right)f\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} \\ & +\frac{2a^2c(g\cos(fx+e))^{\frac{5}{2}}\sqrt{c-c\sin(fx+e)}}{15fg\sqrt{a+a\sin(fx+e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-21i\sqrt{2}\sqrt{acg}acg\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))) + 21i\sqrt{2}\sqrt{acg}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}acg\cos(fx+e)^3,x\right)$$

42.12 Problem number 99

$$\int (g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2c(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}}}{7fg \sqrt{c - c \sin(fx + e)}} - \frac{2a^2c(g \cos(fx + e))^{\frac{5}{2}}}{5fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{6a^2cg \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{6ac(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{35fg \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-21i \sqrt{2} \sqrt{acg} agweierstrassZeta(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 21i \sqrt{2} \sqrt{acg}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((ag \cos(fx + e) \sin(fx + e) + ag \cos(fx + e)) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}\right)$$

42.13 Problem number 100

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{3/2}}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{14a^2(g \cos(fx + e))^{\frac{5}{2}}}{15fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{14a^2g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{2a(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{5fg \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-21i\sqrt{2}\sqrt{acg}ag\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))) + 21i\sqrt{2}\sqrt{acg}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(ag\cos(fx+e)\sin(fx+e)+ag\cos(fx+e))\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)}}{c\sin(fx+e)-c}\right)$$

42.14 Problem number 101

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{4a(g\cos(fx+e))^{\frac{5}{2}}\sqrt{a+a\sin(fx+e)}}{fg(c-c\sin(fx+e))^{\frac{3}{2}}} + \frac{14a^2(g\cos(fx+e))^{\frac{5}{2}}}{3cfg\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} - \frac{14a^2g\sqrt{\frac{\cos(fx+e)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{fx}{2}+\frac{e}{2}\right),\sqrt{2}\right)(\sqrt{\cos(fx+e)}\sqrt{g\cos(fx+e)})}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)cf\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(ag\sin(fx+e)-13ag)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}-21(-i\sqrt{2}ag\sin(fx+e))}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{(ag\cos(fx+e)\sin(fx+e)+ag\cos(fx+e))\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)}}{c^2\cos(fx+e)^2+2c^2\sin(fx+e)-2c^2}\right)$$

42.15 Problem number 102

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{28a^2(g \cos(fx + e))^{\frac{5}{2}}}{5c f g (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} + \frac{4a(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{5 f g (c - c \sin(fx + e))^{\frac{5}{2}}} \\ & + \frac{42a^2 g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{8(4ag \sin(fx + e) - 3ag) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} + 21(i \sqrt{2} ag \cos(fx + e) \dots}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(ag \cos(fx + e) \sin(fx + e) + ag \cos(fx + e)) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e)}}{3c^3 \cos(fx + e)^2 - 4c^3 - (c^3 \cos(fx + e)^2 - 4c^3) \sin(fx + e)}\right)$$

42.16 Problem number 103

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{28a^2(g \cos(fx + e))^{\frac{5}{2}}}{45c f g (c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{14a^2(g \cos(fx + e))^{\frac{5}{2}}}{15c^2 f g (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} + \frac{4a(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{9 f g (c - c \sin(fx + e))^{\frac{7}{2}}} \\ & - \frac{14a^2 g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^3 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21 ag \cos (fx + e)^2 + 18 ag \sin (fx + e) - 38 ag \right) \sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a} \sqrt{-c \sin (fx + e) + c} +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(ag \cos (fx + e) \sin (fx + e) + ag \cos (fx + e)) \sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a} \sqrt{-c \sin (fx + e) + c}}{c^4 \cos (fx + e)^4 - 8 c^4 \cos (fx + e)^2 + 8 c^4 + 4 \left(c^4 \cos (fx + e)^2 - 2 c^4 \right) \sin (fx + e)} \right)$$

42.17 Problem number 104

$$\int \frac{(g \cos (e + fx))^{3/2} (a + a \sin (e + fx))^{3/2}}{(c - c \sin (e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{28 a^2 (g \cos (fx + e))^{\frac{5}{2}}}{117 c f g (c - c \sin (fx + e))^{\frac{7}{2}} \sqrt{a + a \sin (fx + e)}} \\ & + \frac{14 a^2 (g \cos (fx + e))^{\frac{5}{2}}}{195 c^2 f g (c - c \sin (fx + e))^{\frac{5}{2}} \sqrt{a + a \sin (fx + e)}} \\ & + \frac{14 a^2 (g \cos (fx + e))^{\frac{5}{2}}}{195 c^3 f g (c - c \sin (fx + e))^{\frac{3}{2}} \sqrt{a + a \sin (fx + e)}} + \frac{4 a (g \cos (fx + e))^{\frac{5}{2}} \sqrt{a + a \sin (fx + e)}}{13 f g (c - c \sin (fx + e))^{\frac{9}{2}}} \\ & - \frac{14 a^2 g \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right) (\sqrt{\cos (fx + e)}) \sqrt{g \cos (fx + e)}}{195 \cos \left(\frac{fx}{2} + \frac{e}{2} \right) c^4 f \sqrt{a + a \sin (fx + e)} \sqrt{c - c \sin (fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(84 ag \cos (fx + e)^2 - 146 ag - \left(21 ag \cos (fx + e)^2 + 34 ag \right) \sin (fx + e) \right) \sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a} +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(ag \cos (fx + e) \sin (fx + e) + ag \cos (fx + e)) \sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a} \sqrt{-c \sin (fx + e) + c}}{5 c^5 \cos (fx + e)^4 - 20 c^5 \cos (fx + e)^2 + 16 c^5 - \left(c^5 \cos (fx + e)^4 - 12 c^5 \cos (fx + e)^2 + 16 c^5 \right) \sin (fx + e)} \right)$$

42.18 Problem number 105

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{28a^2(g \cos(fx + e))^{\frac{5}{2}}}{221c^2fg(c - c \sin(fx + e))^{\frac{9}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{14a^2(g \cos(fx + e))^{\frac{5}{2}}}{663c^2fg(c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{14a^2(g \cos(fx + e))^{\frac{5}{2}}}{1105c^3fg(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{14a^2(g \cos(fx + e))^{\frac{5}{2}}}{1105c^4fg(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} + \frac{4a(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{17fg(c - c \sin(fx + e))^{\frac{11}{2}}} \\ & - \frac{14a^2g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{1105 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^5 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(21 ag \cos(fx + e)^4 - 266 ag \cos(fx + e)^2 + 502 ag + (105 ag \cos(fx + e)^2 + 278 ag) \sin(fx + e) \right) \sqrt{g \cos(fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(ag \cos(fx + e) \sin(fx + e) + ag \cos(fx + e)) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e)}}{c^6 \cos(fx + e)^6 - 18 c^6 \cos(fx + e)^4 + 48 c^6 \cos(fx + e)^2 - 32 c^6 + 2 (3 c^6 \cos(fx + e)^4 - 16 c^6 \cos(fx + e)^2)}\right)$$

42.19 Problem number 106

$$\int (g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{5}{2}}}{13fg} \\ & + \frac{2a^3c(g \cos(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{3}{2}}}{39fg \sqrt{a + a \sin(fx + e)}} - \frac{14a^3(g \cos(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{5}{2}}}{117fg \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a^2(g \cos(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{13fg} \\ & + \frac{154a^3c^3(g \cos(fx + e))^{\frac{5}{2}}}{585fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{154a^3c^3g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{195 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{22a^3c^2(g \cos(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}}{195fg \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-231i \sqrt{2} \sqrt{acg} a^2 c^2 g \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 231i \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} a^2 c^2 g \cos(fx + e)^5, x\right)$$

42.20 Problem number 107

$$\int (g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a^2 c^2 (g \cos(fx + e))^{5/2} (a + a \sin(fx + e))^{3/2}}{33fg \sqrt{c - c \sin(fx + e)}} + \frac{14c^2 (g \cos(fx + e))^{5/2} (a + a \sin(fx + e))^{5/2}}{99fg \sqrt{c - c \sin(fx + e)}} \\ & - \frac{14a^3 c^2 (g \cos(fx + e))^{5/2}}{45fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{14a^3 c^2 g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{2a^2 c^2 (g \cos(fx + e))^{5/2} \sqrt{a + a \sin(fx + e)}}{15fg \sqrt{c - c \sin(fx + e)}} \\ & + \frac{2c (g \cos(fx + e))^{5/2} (a + a \sin(fx + e))^{5/2} \sqrt{c - c \sin(fx + e)}}{11fg} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-231i \sqrt{2} \sqrt{acg} a^2 cg \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 231i \sqrt{2} \sqrt{acg} a^2 cg \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e)))}{11fg}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^2cg \cos(fx + e)^3 \sin(fx + e) + a^2cg \cos(fx + e)^3\right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e)}\right)$$

42.21 Problem number 108

$$\int (g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ac(g \cos (fx+e))^{\frac{5}{2}}(a+a \sin (fx+e))^{\frac{3}{2}}}{21fg \sqrt{c-c \sin (fx+e)}}+\frac{2c(g \cos (fx+e))^{\frac{5}{2}}(a+a \sin (fx+e))^{\frac{5}{2}}}{9fg \sqrt{c-c \sin (fx+e)}} \\ & -\frac{22a^3c(g \cos (fx+e))^{\frac{5}{2}}}{45fg \sqrt{a+a \sin (fx+e)} \sqrt{c-c \sin (fx+e)}} \\ & +\frac{22a^3cg \sqrt{\frac{\cos (fx+e)}{2}+\frac{1}{2}} \operatorname{EllipticE}\left(\sin \left(\frac{fx}{2}+\frac{e}{2}\right), \sqrt{2}\right)(\sqrt{\cos (fx+e)}) \sqrt{g \cos (fx+e)}}{15 \cos \left(\frac{fx}{2}+\frac{e}{2}\right) f \sqrt{a+a \sin (fx+e)} \sqrt{c-c \sin (fx+e)}} \\ & -\frac{22a^2c(g \cos (fx+e))^{\frac{5}{2}} \sqrt{a+a \sin (fx+e)}}{105fg \sqrt{c-c \sin (fx+e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="fr`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$\frac{-231i \sqrt{2} \sqrt{acg} a^2 g \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos (fx+e)+i \sin (fx+e))) + 231i \sqrt{2} \sqrt{a} \sqrt{g \cos (fx+e)}}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(a^2 g \cos (fx+e)^3-2 a^2 g \cos (fx+e) \sin (fx+e)-2 a^2 g \cos (fx+e)\right) \sqrt{g \cos (fx+e)} \sqrt{a \sin (fx+e)}\right)$$

42.22 Problem number 109

$$\int \frac{(g \cos (e+fx))^{3/2}(a+a \sin (e+fx))^{5/2}}{\sqrt{c-c \sin (e+fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(g \cos (fx+e))^{\frac{5}{2}}(a+a \sin (fx+e))^{\frac{3}{2}}}{7fg \sqrt{c-c \sin (fx+e)}}-\frac{22a^3(g \cos (fx+e))^{\frac{5}{2}}}{15fg \sqrt{a+a \sin (fx+e)} \sqrt{c-c \sin (fx+e)}} \\ & +\frac{22a^3g \sqrt{\frac{\cos (fx+e)}{2}+\frac{1}{2}} \operatorname{EllipticE}\left(\sin \left(\frac{fx}{2}+\frac{e}{2}\right), \sqrt{2}\right)(\sqrt{\cos (fx+e)}) \sqrt{g \cos (fx+e)}}{5 \cos \left(\frac{fx}{2}+\frac{e}{2}\right) f \sqrt{a+a \sin (fx+e)} \sqrt{c-c \sin (fx+e)}} \\ & -\frac{22a^2(g \cos (fx+e))^{\frac{5}{2}} \sqrt{a+a \sin (fx+e)}}{35fg \sqrt{c-c \sin (fx+e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fr`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-231i \sqrt{2} \sqrt{acg} a^2 g \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 231i \sqrt{2} \sqrt{acg} a^2 g \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e)))}{c \sin(fx + e) - c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(a^2 g \cos(fx + e)^3 - 2 a^2 g \cos(fx + e) \sin(fx + e) - 2 a^2 g \cos(fx + e) \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}}{c \sin(fx + e) - c} \right)$$

42.23 Problem number 110

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}}}{fg(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{154a^3(g \cos(fx + e))^{\frac{5}{2}}}{15c f g \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{154a^3 g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{22a^2(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{5c f g \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fr`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 a^2 g \cos(fx + e)^2 - 17 a^2 g \sin(fx + e) + 137 a^2 g \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{c^2 \cos(fx + e)^2 + 2 c^2 \sin(fx + e) - 2 c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(a^2 g \cos(fx + e)^3 - 2 a^2 g \cos(fx + e) \sin(fx + e) - 2 a^2 g \cos(fx + e) \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}}{c^2 \cos(fx + e)^2 + 2 c^2 \sin(fx + e) - 2 c^2} \right)$$

42.24 Problem number 111

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}}}{5fg(c - c \sin(fx + e))^{\frac{5}{2}}} - \frac{44a^2(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{5c^2fg(c - c \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{154a^3(g \cos(fx + e))^{\frac{5}{2}}}{15c^2fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{154a^3g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^2f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5a^2g \cos(fx + e)^2 + 166a^2g \sin(fx + e) - 142a^2g \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(a^2g \cos(fx + e)^3 - 2a^2g \cos(fx + e) \sin(fx + e) - 2a^2g \cos(fx + e)\right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}}{3c^3 \cos(fx + e)^2 - 4c^3 - \left(c^3 \cos(fx + e)^2 - 4c^3\right) \sin(fx + e)}\right)$$

42.25 Problem number 112

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}}}{9fg(c - c \sin(fx + e))^{\frac{7}{2}}} + \frac{308a^3(g \cos(fx + e))^{\frac{5}{2}}}{45c^2fg(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{44a^2(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{45c^2fg(c - c \sin(fx + e))^{\frac{5}{2}}} \\ & - \frac{154a^3g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^3f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \frac{\left(93 a^2 g \cos (f x+e)^2+144 a^2 g \sin (f x+e)-164 a^2 g\right) \sqrt{g \cos (f x+e)} \sqrt{a \sin (f x+e)+a} \sqrt{-c \sin (f x+e)+c}}{c^4 \cos (f x+e)^4-8 c^4 \cos (f x+e)^2+8 c^4+4\left(c^4 \cos (f x+e)^2-2 c^4\right) \sin (f x+e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\left(a^2 g \cos (f x+e)^3-2 a^2 g \cos (f x+e) \sin (f x+e)-2 a^2 g \cos (f x+e)\right) \sqrt{g \cos (f x+e)} \sqrt{a \sin (f x+e)+a}}{c^4 \cos (f x+e)^4-8 c^4 \cos (f x+e)^2+8 c^4+4\left(c^4 \cos (f x+e)^2-2 c^4\right) \sin (f x+e)}, x \right)$$

42.26 Problem number 113

$$\int \frac{(g \cos (e+f x))^{3 / 2}(a+a \sin (e+f x))^{5 / 2}}{(c-c \sin (e+f x))^{9 / 2}} d x$$

Optimal antiderivative

$$\begin{aligned} & \frac{4 a(g \cos (f x+e))^{\frac{5}{2}}(a+a \sin (f x+e))^{\frac{3}{2}}}{13 f g(c-c \sin (f x+e))^{\frac{9}{2}}} + \frac{308 a^3(g \cos (f x+e))^{\frac{5}{2}}}{585 c^2 f g(c-c \sin (f x+e))^{\frac{5}{2}} \sqrt{a+a \sin (f x+e)}} \\ & - \frac{154 a^3(g \cos (f x+e))^{\frac{5}{2}}}{195 c^3 f g(c-c \sin (f x+e))^{\frac{3}{2}} \sqrt{a+a \sin (f x+e)}} - \frac{44 a^2(g \cos (f x+e))^{\frac{5}{2}} \sqrt{a+a \sin (f x+e)}}{117 c f g(c-c \sin (f x+e))^{\frac{7}{2}}} \\ & + \frac{154 a^3 g \sqrt{\frac{\cos (f x+e)}{2}+\frac{1}{2}} \operatorname{EllipticE}\left(\sin \left(\frac{f x}{2}+\frac{e}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos (f x+e)}\right) \sqrt{g \cos (f x+e)}}{195 \cos \left(\frac{f x}{2}+\frac{e}{2}\right) c^4 f \sqrt{a+a \sin (f x+e)} \sqrt{c-c \sin (f x+e)}} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \frac{\left(339 a^2 g \cos (f x+e)^2-436 a^2 g-\left(231 a^2 g \cos (f x+e)^2-796 a^2 g\right) \sin (f x+e)\right) \sqrt{g \cos (f x+e)} \sqrt{a \sin (f x+e)+a}}{5 c^5 \cos (f x+e)^4-20 c^5 \cos (f x+e)^2+16 c^5-\left(c^5 \cos (f x+e)^4-12 c^5 \cos (f x+e)^2+16 c^5\right) \sin (f x+e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\left(a^2 g \cos (f x+e)^3-2 a^2 g \cos (f x+e) \sin (f x+e)-2 a^2 g \cos (f x+e)\right) \sqrt{g \cos (f x+e)} \sqrt{a \sin (f x+e)+a}}{5 c^5 \cos (f x+e)^4-20 c^5 \cos (f x+e)^2+16 c^5-\left(c^5 \cos (f x+e)^4-12 c^5 \cos (f x+e)^2+16 c^5\right) \sin (f x+e)}, x \right)$$

42.27 Problem number 114

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}}}{17fg(c - c \sin(fx + e))^{\frac{11}{2}}} + \frac{308a^3(g \cos(fx + e))^{\frac{5}{2}}}{1989c^2 fg(c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{154a^3(g \cos(fx + e))^{\frac{5}{2}}}{3315c^3 fg(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{154a^3(g \cos(fx + e))^{\frac{5}{2}}}{3315c^4 fg(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{44a^2(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{221c fg(c - c \sin(fx + e))^{\frac{9}{2}}} \\ & + \frac{154a^3 g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{3315 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^5 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(231 a^2 g \cos(fx + e)^4 + 389 a^2 g \cos(fx + e)^2 - 1108 a^2 g + \left(1155 a^2 g \cos(fx + e)^2 - 3572 a^2 g \right) \sin(fx + e) \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(a^2 g \cos(fx + e)^3 - 2 a^2 g \cos(fx + e) \sin(fx + e) - 2 a^2 g \cos(fx + e)\right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}}{c^6 \cos(fx + e)^6 - 18 c^6 \cos(fx + e)^4 + 48 c^6 \cos(fx + e)^2 - 32 c^6 + 2 \left(3 c^6 \cos(fx + e)^4 - 16 c^6 \cos(fx + e)^2\right)}\right)$$

42.28 Problem number 115

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}}}{21fg(c - c \sin(fx + e))^{\frac{13}{2}}} + \frac{44a^3(g \cos(fx + e))^{\frac{5}{2}}}{663c^2fg(c - c \sin(fx + e))^{\frac{9}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{22a^3(g \cos(fx + e))^{\frac{5}{2}}}{1989c^3fg(c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{22a^3(g \cos(fx + e))^{\frac{5}{2}}}{3315c^4fg(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{22a^3(g \cos(fx + e))^{\frac{5}{2}}}{3315c^5fg(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{44a^2(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{357c^6fg(c - c \sin(fx + e))^{\frac{11}{2}}} \\ & + \frac{22a^3g \sqrt{\frac{\cos(fx + e)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{3315 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^6 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(1386 a^2 g \cos(fx + e)^4 + 5607 a^2 g \cos(fx + e)^2 - 10796 a^2 g - \left(231 a^2 g \cos(fx + e)^4 - 4081 a^2 g \cos(fx + e)^2 + \dots \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(a^2 g \cos(fx + e)^3 - 2 a^2 g \cos(fx + e) \sin(fx + e) - 2 a^2 g \cos(fx + e)\right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}}{7 c^7 \cos(fx + e)^6 - 56 c^7 \cos(fx + e)^4 + 112 c^7 \cos(fx + e)^2 - 64 c^7 - \left(c^7 \cos(fx + e)^6 - 24 c^7 \cos(fx + e)^4 + \dots\right)}\right)$$

42.29 Problem number 116

$$\int (g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{7/2} (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2c(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{7}{2}} (c - c \sin(fx + e))^{\frac{3}{2}}}{15fg} \\ & - \frac{2a^2c^3(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}}}{39fg\sqrt{c - c \sin(fx + e)}} - \frac{14ac^3(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{5}{2}}}{585fg\sqrt{c - c \sin(fx + e)}} \\ & + \frac{14c^3(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{7}{2}}}{195fg\sqrt{c - c \sin(fx + e)}} - \frac{154a^4c^3(g \cos(fx + e))^{\frac{5}{2}}}{585fg\sqrt{a + a \sin(fx + e)}\sqrt{c - c \sin(fx + e)}} \\ & + \frac{154a^4c^3g\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{195 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{22a^3c^3(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{195fg\sqrt{c - c \sin(fx + e)}} \\ & + \frac{22c^2(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{7}{2}} \sqrt{c - c \sin(fx + e)}}{195fg} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-231i \sqrt{2} \sqrt{acg} a^3 c^2 g \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 231i \sqrt{2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^3 c^2 g \cos(fx + e)^5 \sin(fx + e) + a^3 c^2 g \cos(fx + e)^5\right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e)}\right)$$

42.30 Problem number 117

$$\int (g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{7/2} (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{10a^2c^2(g \cos(fx + e))^{\frac{5}{2}}(a + a \sin(fx + e))^{\frac{3}{2}}}{143fg\sqrt{c - c \sin(fx + e)}} - \frac{14ac^2(g \cos(fx + e))^{\frac{5}{2}}(a + a \sin(fx + e))^{\frac{5}{2}}}{429fg\sqrt{c - c \sin(fx + e)}} \\ & + \frac{14c^2(g \cos(fx + e))^{\frac{5}{2}}(a + a \sin(fx + e))^{\frac{7}{2}}}{143fg\sqrt{c - c \sin(fx + e)}} - \frac{14a^4c^2(g \cos(fx + e))^{\frac{5}{2}}}{39fg\sqrt{a + a \sin(fx + e)}\sqrt{c - c \sin(fx + e)}} \\ & + \frac{14a^4c^2g\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{13 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{2a^3c^2(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{13fg\sqrt{c - c \sin(fx + e)}} \\ & + \frac{2c(g \cos(fx + e))^{\frac{5}{2}}(a + a \sin(fx + e))^{\frac{7}{2}} \sqrt{c - c \sin(fx + e)}}{13fg} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-231i \sqrt{2} \sqrt{acg} a^3 cg \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 231i \sqrt{2} \sqrt{acg} a^3 cg \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e)))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(a^3cg \cos(fx + e)^5 - 2a^3cg \cos(fx + e)^3 \sin(fx + e) - 2a^3cg \cos(fx + e)^3\right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}\right)$$

42.31 Problem number 118

$$\int (g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{7/2} \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{10a^2c(g \cos (fx + e))^{\frac{5}{2}}(a + a \sin (fx + e))^{\frac{3}{2}}}{77fg\sqrt{c - c \sin (fx + e)}} - \frac{2ac(g \cos (fx + e))^{\frac{5}{2}}(a + a \sin (fx + e))^{\frac{5}{2}}}{33fg\sqrt{c - c \sin (fx + e)}} \\
& + \frac{2c(g \cos (fx + e))^{\frac{5}{2}}(a + a \sin (fx + e))^{\frac{7}{2}}}{11fg\sqrt{c - c \sin (fx + e)}} - \frac{2a^4c(g \cos (fx + e))^{\frac{5}{2}}}{3fg\sqrt{a + a \sin (fx + e)}\sqrt{c - c \sin (fx + e)}} \\
& + \frac{2a^4cg\sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos (fx + e)}) \sqrt{g \cos (fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin (fx + e)} \sqrt{c - c \sin (fx + e)}} \\
& - \frac{2a^3c(g \cos (fx + e))^{\frac{5}{2}}\sqrt{a + a \sin (fx + e)}}{7fg\sqrt{c - c \sin (fx + e)}}
\end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-231i \sqrt{2} \sqrt{acg} a^3 g \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e))) + 231i \sqrt{2} \sqrt{g \cos (fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(3a^3g \cos (fx + e)^3 - 4a^3g \cos (fx + e) + \left(a^3g \cos (fx + e)^3 - 4a^3g \cos (fx + e)\right) \sin (fx + e)\right) \sqrt{g \cos (fx + e)}\right)$$

42.32 Problem number 119

$$\int \frac{(g \cos (e + fx))^{3/2}(a + a \sin (e + fx))^{7/2}}{\sqrt{c - c \sin (e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{10a^2(g \cos (fx + e))^{\frac{5}{2}}(a + a \sin (fx + e))^{\frac{3}{2}}}{21fg\sqrt{c - c \sin (fx + e)}} - \frac{2a(g \cos (fx + e))^{\frac{5}{2}}(a + a \sin (fx + e))^{\frac{5}{2}}}{9fg\sqrt{c - c \sin (fx + e)}} \\
& - \frac{22a^4(g \cos (fx + e))^{\frac{5}{2}}}{9fg\sqrt{a + a \sin (fx + e)}\sqrt{c - c \sin (fx + e)}} \\
& + \frac{22a^4g\sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos (fx + e)}) \sqrt{g \cos (fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin (fx + e)} \sqrt{c - c \sin (fx + e)}} \\
& - \frac{22a^3(g \cos (fx + e))^{\frac{5}{2}}\sqrt{a + a \sin (fx + e)}}{21fg\sqrt{c - c \sin (fx + e)}}
\end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-231i \sqrt{2} \sqrt{acg} a^3 g \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 231i \sqrt{2} \sqrt{g \cos(fx + e)}}{c \sin(fx + e) - c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(3a^3g \cos(fx + e)^3 - 4a^3g \cos(fx + e) + \left(a^3g \cos(fx + e)^3 - 4a^3g \cos(fx + e) \right) \sin(fx + e) \right) \sqrt{g \cos(fx + e)}}{c \sin(fx + e) - c} \right)$$

42.33 Problem number 120

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{5}{2}}}{fg(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{30a^2(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}}}{7cfg \sqrt{c - c \sin(fx + e)}} \\ & + \frac{22a^4(g \cos(fx + e))^{\frac{5}{2}}}{cfg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{66a^4g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{\cos \left(\frac{fx}{2} + \frac{e}{2} \right) cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{66a^3(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{7cfg \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6a^3g \cos(fx + e)^2 + 133a^3g + \left(a^3g \cos(fx + e)^2 - 21a^3g \right) \sin(fx + e) \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}}{c^2 \cos(fx + e)^2 + 2c^2 \sin(fx + e) - 2c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(3a^3g \cos(fx + e)^3 - 4a^3g \cos(fx + e) + \left(a^3g \cos(fx + e)^3 - 4a^3g \cos(fx + e) \right) \sin(fx + e) \right) \sqrt{g \cos(fx + e)}}{c^2 \cos(fx + e)^2 + 2c^2 \sin(fx + e) - 2c^2} \right)$$

42.34 Problem number 121

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{5}{2}}}{5fg(c - c \sin(fx + e))^{\frac{5}{2}}} - \frac{12a^2(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}}}{c^2fg(c - c \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{154a^4(g \cos(fx + e))^{\frac{5}{2}}}{5c^2fg\sqrt{a + a \sin(fx + e)}\sqrt{c - c \sin(fx + e)}} \\ & + \frac{462a^4g\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^2f\sqrt{a + a \sin(fx + e)}\sqrt{c - c \sin(fx + e)}} \\ & - \frac{66a^3(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{5c^2fg\sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(8a^3g \cos(fx + e)^2 - 146a^3g + \left(a^3g \cos(fx + e)^2 + 162a^3g \right) \sin(fx + e) \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(3a^3g \cos(fx + e)^3 - 4a^3g \cos(fx + e) + \left(a^3g \cos(fx + e)^3 - 4a^3g \cos(fx + e)\right) \sin(fx + e)\right) \sqrt{g \cos(fx + e)}}{3c^3 \cos(fx + e)^2 - 4c^3 - \left(c^3 \cos(fx + e)^2 - 4c^3\right) \sin(fx + e)}\right)$$

42.35 Problem number 122

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{4a(g \cos (fx+e))^{\frac{5}{2}}(a+a \sin (fx+e))^{\frac{5}{2}}}{9fg(c-c \sin (fx+e))^{\frac{7}{2}}}-\frac{4a^2(g \cos (fx+e))^{\frac{5}{2}}(a+a \sin (fx+e))^{\frac{3}{2}}}{3cfg(c-c \sin (fx+e))^{\frac{5}{2}}} \\ +\frac{44a^3(g \cos (fx+e))^{\frac{5}{2}}\sqrt{a+a \sin (fx+e)}}{3c^2fg(c-c \sin (fx+e))^{\frac{3}{2}}}+\frac{154a^4(g \cos (fx+e))^{\frac{5}{2}}}{9c^3fg\sqrt{a+a \sin (fx+e)}\sqrt{c-c \sin (fx+e)}} \\ -\frac{154a^4g\sqrt{\frac{\cos (fx+e)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin \left(\frac{fx}{2}+\frac{e}{2}\right),\sqrt{2}\right)(\sqrt{\cos (fx+e)})\sqrt{g \cos (fx+e)}}{3 \cos \left(\frac{fx}{2}+\frac{e}{2}\right)c^3f\sqrt{a+a \sin (fx+e)}\sqrt{c-c \sin (fx+e)}}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(177a^3g \cos (fx+e)^2-316a^3g-3\left(a^3g \cos (fx+e)^2-100a^3g\right) \sin (fx+e)\right) \sqrt{g \cos (fx+e)} \sqrt{a \sin (fx+e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(3a^3g \cos (fx+e)^3-4a^3g \cos (fx+e)+\left(a^3g \cos (fx+e)^3-4a^3g \cos (fx+e)\right) \sin (fx+e)\right) \sqrt{g \cos (fx+e)}}{c^4 \cos (fx+e)^4-8c^4 \cos (fx+e)^2+8c^4+4\left(c^4 \cos (fx+e)^2-\right.}\right.$$

42.36 Problem number 123

$$\int \frac{(g \cos (e+fx))^{3/2}(a+a \sin (e+fx))^{7/2}}{(c-c \sin (e+fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{4a(g \cos (fx+e))^{\frac{5}{2}}(a+a \sin (fx+e))^{\frac{5}{2}}}{13fg(c-c \sin (fx+e))^{\frac{9}{2}}}-\frac{20a^2(g \cos (fx+e))^{\frac{5}{2}}(a+a \sin (fx+e))^{\frac{3}{2}}}{39cfg(c-c \sin (fx+e))^{\frac{7}{2}}} \\ -\frac{308a^4(g \cos (fx+e))^{\frac{5}{2}}}{39c^3fg(c-c \sin (fx+e))^{\frac{3}{2}}\sqrt{a+a \sin (fx+e)}}+\frac{44a^3(g \cos (fx+e))^{\frac{5}{2}}\sqrt{a+a \sin (fx+e)}}{39c^2fg(c-c \sin (fx+e))^{\frac{5}{2}}} \\ +\frac{154a^4g\sqrt{\frac{\cos (fx+e)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin \left(\frac{fx}{2}+\frac{e}{2}\right),\sqrt{2}\right)(\sqrt{\cos (fx+e)})\sqrt{g \cos (fx+e)}}{13 \cos \left(\frac{fx}{2}+\frac{e}{2}\right)c^4f\sqrt{a+a \sin (fx+e)}\sqrt{c-c \sin (fx+e)}}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$16 \left(57 a^3 g \cos (fx + e)^2 - 74 a^3 g - 8 \left(3 a^3 g \cos (fx + e)^2 - 10 a^3 g \right) \sin (fx + e) \right) \sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\left(3 a^3 g \cos (fx + e)^3 - 4 a^3 g \cos (fx + e) + \left(a^3 g \cos (fx + e)^3 - 4 a^3 g \cos (fx + e) \right) \sin (fx + e) \right) \sqrt{g \cos (fx + e)}}{5 c^5 \cos (fx + e)^4 - 20 c^5 \cos (fx + e)^2 + 16 c^5 - \left(c^5 \cos (fx + e)^4 - 12 c^5 \cos (fx + e)^2 \right) \sin (fx + e)} dx \right)$$

42.37 Problem number 124

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos (fx + e))^{\frac{5}{2}} (a + a \sin (fx + e))^{\frac{5}{2}}}{17fg(c - c \sin (fx + e))^{\frac{11}{2}}} - \frac{60a^2(g \cos (fx + e))^{\frac{5}{2}} (a + a \sin (fx + e))^{\frac{3}{2}}}{221c^2fg(c - c \sin (fx + e))^{\frac{9}{2}}} \\ & - \frac{308a^4(g \cos (fx + e))^{\frac{5}{2}}}{663c^3fg(c - c \sin (fx + e))^{\frac{5}{2}} \sqrt{a + a \sin (fx + e)}} \\ & + \frac{154a^4(g \cos (fx + e))^{\frac{5}{2}}}{221c^4fg(c - c \sin (fx + e))^{\frac{3}{2}} \sqrt{a + a \sin (fx + e)}} + \frac{220a^3(g \cos (fx + e))^{\frac{5}{2}} \sqrt{a + a \sin (fx + e)}}{663c^2fg(c - c \sin (fx + e))^{\frac{7}{2}}} \\ & - \frac{154a^4g \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right) (\sqrt{\cos (fx + e)}) \sqrt{g \cos (fx + e)}}{221 \cos \left(\frac{fx}{2} + \frac{e}{2} \right) c^5 f \sqrt{a + a \sin (fx + e)} \sqrt{c - c \sin (fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(231 a^3 g \cos (fx + e)^4 - 1600 a^3 g \cos (fx + e)^2 + 1544 a^3 g + 4 \left(123 a^3 g \cos (fx + e)^2 - 230 a^3 g \right) \sin (fx + e) \right) \sqrt{g \cos (fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(3 a^3 g \cos (fx + e)^3 - 4 a^3 g \cos (fx + e) + \left(a^3 g \cos (fx + e)^3 - 4 a^3 g \cos (fx + e) \right) \sin (fx + e) \right) \sqrt{g \cos (fx + e)}}{c^6 \cos (fx + e)^6 - 18 c^6 \cos (fx + e)^4 + 48 c^6 \cos (fx + e)^2 - 32 c^6 + 2 \left(3 c^6 \cos (fx + e)^4 - 12 c^6 \cos (fx + e)^2 \right) \sin (fx + e)} dx \right)$$

42.38 Problem number 125

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{5}{2}}}{21fg(c - c \sin(fx + e))^{\frac{13}{2}}} - \frac{20a^2(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}}}{119c^2fg(c - c \sin(fx + e))^{\frac{11}{2}}} \\ & - \frac{220a^4(g \cos(fx + e))^{\frac{5}{2}}}{1989c^3fg(c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{22a^4(g \cos(fx + e))^{\frac{5}{2}}}{663c^4fg(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{22a^4(g \cos(fx + e))^{\frac{5}{2}}}{663c^5fg(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} + \frac{220a^3(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{1547c^2fg(c - c \sin(fx + e))^{\frac{9}{2}}} \\ & - \frac{22a^4g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{663 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^6 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(1386 a^3 g \cos(fx + e)^4 - 8316 a^3 g \cos(fx + e)^2 + 7768 a^3 g - \left(231 a^3 g \cos(fx + e)^4 + 560 a^3 g \cos(fx + e)^2 - \right. \right.}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(3 a^3 g \cos(fx + e)^3 - 4 a^3 g \cos(fx + e) + \left(a^3 g \cos(fx + e)^3 - 4 a^3 g \cos(fx + e)\right) \sin(fx + e)\right) \sqrt{g \cos}}{7 c^7 \cos(fx + e)^6 - 56 c^7 \cos(fx + e)^4 + 112 c^7 \cos(fx + e)^2 - 64 c^7 - \left(c^7 \cos(fx + e)^6 - 24 c^7 \cos(fx + e)^2\right)}\right)$$

42.39 Problem number 126

$$\int \frac{(g \cos(e + fx))^{3/2} (a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{5}{2}}}{25fg(c - c \sin(fx + e))^{\frac{15}{2}}} - \frac{4a^2(g \cos(fx + e))^{\frac{5}{2}} (a + a \sin(fx + e))^{\frac{3}{2}}}{35c^2fg(c - c \sin(fx + e))^{\frac{13}{2}}} \\ & - \frac{44a^4(g \cos(fx + e))^{\frac{5}{2}}}{1105c^3fg(c - c \sin(fx + e))^{\frac{9}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{22a^4(g \cos(fx + e))^{\frac{5}{2}}}{3315c^4fg(c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{22a^4(g \cos(fx + e))^{\frac{5}{2}}}{5525c^5fg(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{22a^4(g \cos(fx + e))^{\frac{5}{2}}}{5525c^6fg(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} + \frac{44a^3(g \cos(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{595c^2fg(c - c \sin(fx + e))^{\frac{11}{2}}} \\ & - \frac{22a^4g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5525 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^7 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(231 a^3 g \cos(fx + e)^6 - 5698 a^3 g \cos(fx + e)^4 + 42044 a^3 g \cos(fx + e)^2 - 42056 a^3 g + 7 \left(231 a^3 g \cos(fx + e)^4 \right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(3 a^3 g \cos(fx + e)^3 - 4 a^3 g \cos(fx + e) + \left(a^3 g \cos(fx + e)^3 - 4 a^3 g \cos(fx + e)\right) \sin(fx + e)\right)}{c^8 \cos(fx + e)^8 - 32 c^8 \cos(fx + e)^6 + 160 c^8 \cos(fx + e)^4 - 256 c^8 \cos(fx + e)^2 + 128 c^8 + 8 \left(c^8 \cos(fx + e)\right)}\right)$$

42.40 Problem number 127

$$\int \frac{(g \cos(e + fx))^{3/2} (c - c \sin(e + fx))^{5/2}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2c(g \cos(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{3}{2}}}{7fg \sqrt{a + a \sin(fx + e)}} + \frac{22c^3(g \cos(fx + e))^{\frac{5}{2}}}{15fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{22c^3g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{22c^2(g \cos(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}}{35fg \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-231i \sqrt{2} \sqrt{acg} c^2 g \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 231i \sqrt{2} \sqrt{c - c \sin(fx + e)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(c^2g \cos(fx + e)^3 + 2c^2g \cos(fx + e) \sin(fx + e) - 2c^2g \cos(fx + e)\right) \sqrt{g \cos(fx + e)} \sqrt{-c \sin(fx + e)}}{\sqrt{a \sin(fx + e) + a}}\right)$$

42.41 Problem number 128

$$\int \frac{(g \cos(e + fx))^{3/2} (c - c \sin(e + fx))^{3/2}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14c^2(g \cos(fx + e))^{\frac{5}{2}}}{15fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{14c^2g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{2c(g \cos(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}}{5fg \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-21i \sqrt{2} \sqrt{acg} \operatorname{cgweierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 21i \sqrt{2} \sqrt{acg}}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(cg \cos(fx + e) \sin(fx + e) - cg \cos(fx + e)) \sqrt{g \cos(fx + e)} \sqrt{-c \sin(fx + e) + c}}{\sqrt{a \sin(fx + e) + a}}, x\right)$$

42.42 Problem number 129

$$\int \frac{(g \cos(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2c(g \cos(fx + e))^{\frac{5}{2}}}{3fg \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{2cg \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-3i \sqrt{2} \sqrt{acg} \operatorname{gweierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 3i \sqrt{2} \sqrt{acg} \operatorname{gw}}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{g \cos(fx + e)} \sqrt{-c \sin(fx + e) + c} g \cos(fx + e)}{\sqrt{a \sin(fx + e) + a}}, x\right)$$

42.43 Problem number 130

$$\int \frac{(g \cos(e + fx))^{3/2}}{\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{acg} g \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + i \sqrt{2} \sqrt{acg} g \operatorname{weierstrassZeta}(-4, 0, \cos(fx + e) - i \sin(fx + e))}{acf}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} g}{ac \cos(fx + e)}, x\right)$$

42.44 Problem number 131

$$\int \frac{(g \cos(e + fx))^{3/2}}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(g \cos(fx + e))^{\frac{5}{2}}}{fg(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} - \frac{2g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{g \cos (fx+e)} \sqrt{a \sin (fx+e)+a} \sqrt{-c \sin (fx+e)+c} g - \sqrt{acg} \left(i \sqrt{2} g \sin (fx+e) - i \sqrt{2} g \right) \text{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{g \cos (fx+e)} \sqrt{a \sin (fx+e)+a} \sqrt{-c \sin (fx+e)+c} g}{ac^2 \cos (fx+e) \sin (fx+e) - ac^2 \cos (fx+e)}, x \right)$$

42.45 Problem number 132

$$\int \frac{(g \cos (e+fx))^{3/2}}{\sqrt{a+a \sin (e+fx)} (c-c \sin (e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(g \cos (fx+e))^{\frac{5}{2}}}{5fg(c-c \sin (fx+e))^{\frac{5}{2}} \sqrt{a+a \sin (fx+e)}} \\ & + \frac{2(g \cos (fx+e))^{\frac{5}{2}}}{5cfg(c-c \sin (fx+e))^{\frac{3}{2}} \sqrt{a+a \sin (fx+e)}} \\ & - \frac{2g \sqrt{\frac{\cos (fx+e)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right) (\sqrt{\cos (fx+e)}) \sqrt{g \cos (fx+e)}}{5 \cos \left(\frac{fx}{2} + \frac{e}{2} \right) c^2 f \sqrt{a+a \sin (fx+e)} \sqrt{c-c \sin (fx+e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{g \cos (fx+e)} \sqrt{a \sin (fx+e)+a} \sqrt{-c \sin (fx+e)+c} (g \sin (fx+e) - 2g) + \left(i \sqrt{2} g \cos (fx+e) \right)^2 + 2i \sqrt{2} g \cos (fx+e)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{g \cos (fx+e)} \sqrt{a \sin (fx+e)+a} \sqrt{-c \sin (fx+e)+c} g}{ac^3 \cos (fx+e)^3 + 2ac^3 \cos (fx+e) \sin (fx+e) - 2ac^3 \cos (fx+e)}, x \right)$$

42.46 Problem number 133

$$\int \frac{(g \cos(e + fx))^{3/2}}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(g \cos(fx + e))^{\frac{5}{2}}}{9fg(c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{2(g \cos(fx + e))^{\frac{5}{2}}}{15c^2fg(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{2(g \cos(fx + e))^{\frac{5}{2}}}{15c^2fg(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) c^3 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3g \cos(fx + e)^2 + 9g \sin(fx + e) - 14g \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} + 3 \left(3i \sqrt{3} \operatorname{arctan}\left(\frac{\sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a}}{\sqrt{-c \sin(fx + e) + c}}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} g}{3ac^4 \cos(fx + e)^3 - 4ac^4 \cos(fx + e) - (ac^4 \cos(fx + e)^3 - 4ac^4 \cos(fx + e)) \sin(fx + e)}, x\right)$$

42.47 Problem number 134

$$\int \frac{(g \cos(e + fx))^{3/2} (c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4c(g \cos (fx + e))^{\frac{5}{2}}(c - c \sin (fx + e))^{\frac{5}{2}}}{fg(a + a \sin (fx + e))^{\frac{3}{2}}} - \frac{30c^2(g \cos (fx + e))^{\frac{5}{2}}(c - c \sin (fx + e))^{\frac{3}{2}}}{7afg\sqrt{a + a \sin (fx + e)}} \\ & - \frac{22c^4(g \cos (fx + e))^{\frac{5}{2}}}{afg\sqrt{a + a \sin (fx + e)}\sqrt{c - c \sin (fx + e)}} \\ & - \frac{66c^4g\sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)(\sqrt{\cos (fx + e)})\sqrt{g \cos (fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right)af\sqrt{a + a \sin (fx + e)}\sqrt{c - c \sin (fx + e)}} \\ & - \frac{66c^3(g \cos (fx + e))^{\frac{5}{2}}\sqrt{c - c \sin (fx + e)}}{7afg\sqrt{a + a \sin (fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(6c^3g \cos (fx + e)^2 + 133c^3g - \left(c^3g \cos (fx + e)^2 - 21c^3g\right) \sin (fx + e)\right) \sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(3c^3g \cos (fx + e)^3 - 4c^3g \cos (fx + e) - \left(c^3g \cos (fx + e)^3 - 4c^3g \cos (fx + e)\right) \sin (fx + e)\right) \sqrt{g \cos (fx + e)}}{a^2 \cos (fx + e)^2 - 2a^2 \sin (fx + e) - 2a^2}\right)$$

42.48 Problem number 135

$$\int \frac{(g \cos (e + fx))^{3/2}(c - c \sin (e + fx))^{5/2}}{(a + a \sin (e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4c(g \cos (fx + e))^{\frac{5}{2}}(c - c \sin (fx + e))^{\frac{3}{2}}}{fg(a + a \sin (fx + e))^{\frac{3}{2}}} - \frac{154c^3(g \cos (fx + e))^{\frac{5}{2}}}{15afg\sqrt{a + a \sin (fx + e)}\sqrt{c - c \sin (fx + e)}} \\ & - \frac{154c^3g\sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)(\sqrt{\cos (fx + e)})\sqrt{g \cos (fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right)af\sqrt{a + a \sin (fx + e)}\sqrt{c - c \sin (fx + e)}} \\ & - \frac{22c^2(g \cos (fx + e))^{\frac{5}{2}}\sqrt{c - c \sin (fx + e)}}{5afg\sqrt{a + a \sin (fx + e)}} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 c^2 g \cos (f x + e)^2 + 17 c^2 g \sin (f x + e) + 137 c^2 g \right) \sqrt{g \cos (f x + e)} \sqrt{a \sin (f x + e) + a} \sqrt{-c \sin (f x + e) + c}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(c^2 g \cos (f x + e)^3 + 2 c^2 g \cos (f x + e) \sin (f x + e) - 2 c^2 g \cos (f x + e) \right) \sqrt{g \cos (f x + e)} \sqrt{a \sin (f x + e)}}{a^2 \cos (f x + e)^2 - 2 a^2 \sin (f x + e) - 2 a^2} \right)$$

42.49 Problem number 136

$$\int \frac{(g \cos (e + f x))^{3/2} (c - c \sin (e + f x))^{3/2}}{(a + a \sin (e + f x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{14 c^2 (g \cos (f x + e))^{\frac{5}{2}}}{3 a f g \sqrt{a + a \sin (f x + e)} \sqrt{c - c \sin (f x + e)}} + \frac{14 c^2 g \sqrt{\frac{\cos (f x + e)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{f x}{2} + \frac{e}{2} \right), \sqrt{2} \right) (\sqrt{\cos (f x + e)}) \sqrt{g \cos (f x + e)}}{\cos \left(\frac{f x}{2} + \frac{e}{2} \right) a f \sqrt{a + a \sin (f x + e)} \sqrt{c - c \sin (f x + e)}} - \frac{4 c (g \cos (f x + e))^{\frac{5}{2}} \sqrt{c - c \sin (f x + e)}}{f g (a + a \sin (f x + e))^{\frac{3}{2}}}}$$

command

```
integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 (c g \sin (f x + e) + 13 c g) \sqrt{g \cos (f x + e)} \sqrt{a \sin (f x + e) + a} \sqrt{-c \sin (f x + e) + c} + 21 \left(-i \sqrt{2} c g \sin (f x + e) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(c g \cos (f x + e) \sin (f x + e) - c g \cos (f x + e)) \sqrt{g \cos (f x + e)} \sqrt{a \sin (f x + e) + a} \sqrt{-c \sin (f x + e) + c}}{a^2 \cos (f x + e)^2 - 2 a^2 \sin (f x + e) - 2 a^2} \right)$$

42.50 Problem number 137

$$\int \frac{(g \cos(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{4c(g \cos(fx + e))^{\frac{5}{2}}}{fg(a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}} - \frac{6cg \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) af \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} g + 3 \sqrt{acg} \left(-i \sqrt{2} g \sin(fx + e) - i \sqrt{2} g\right) \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} g \cos(fx + e)}{a^2 \cos(fx + e)^2 - 2a^2 \sin(fx + e) - 2a^2}, x\right)$$

42.51 Problem number 138

$$\int \frac{(g \cos(e + fx))^{3/2}}{(a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2(g \cos(fx + e))^{\frac{5}{2}}}{fg(a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}} - \frac{2g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) af \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fr`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{g \cos (f x+e)} \sqrt{a \sin (f x+e)+a} \sqrt{-c \sin (f x+e)+c} g-\sqrt{a c g}\left(i \sqrt{2} g \sin (f x+e)+i \sqrt{2} g\right) \text{weierstrassZeta}(-4,0, \text{weierstrassPInverse}(-4,0, \cos (f x+e)+i \sin (f x+e)))}{-}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{g \cos (f x+e)} \sqrt{a \sin (f x+e)+a} \sqrt{-c \sin (f x+e)+c} g}{a^2 c \cos (f x+e) \sin (f x+e)+a^2 c \cos (f x+e)}, x\right)$$

42.52 Problem number 139

$$\int \frac{(g \cos (e+f x))^{3/2}}{(a+a \sin (e+f x))^{3/2}(c-c \sin (e+f x))^{3/2}} d x$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(g \cos (f x+e))^{\frac{5}{2}}}{f g(a+a \sin (f x+e))^{\frac{3}{2}}(c-c \sin (f x+e))^{\frac{3}{2}}} \\ & +\frac{2(g \cos (f x+e))^{\frac{5}{2}}}{a f g(c-c \sin (f x+e))^{\frac{3}{2}} \sqrt{a+a \sin (f x+e)}} \\ & -\frac{2 g \sqrt{\frac{\cos (f x+e)}{2}+\frac{1}{2}} \text{EllipticE}\left(\sin \left(\frac{f x}{2}+\frac{e}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos (f x+e)}\right) \sqrt{g \cos (f x+e)}}{\cos \left(\frac{f x}{2}+\frac{e}{2}\right) a c f \sqrt{a+a \sin (f x+e)} \sqrt{c-c \sin (f x+e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fr`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \sqrt{a c g} g \cos (f x+e)^2 \text{weierstrassZeta}(-4,0, \text{weierstrassPInverse}(-4,0, \cos (f x+e)+i \sin (f x+e)))}{-}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{g \cos (f x+e)} \sqrt{a \sin (f x+e)+a} \sqrt{-c \sin (f x+e)+c} g}{a^2 c^2 \cos (f x+e)^3}, x\right)$$

42.53 Problem number 140

$$\int \frac{(g \cos(e + fx))^{3/2}}{(a + a \sin(e + fx))^{3/2} (c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(g \cos(fx + e))^{\frac{5}{2}}}{fg(a + a \sin(fx + e))^{\frac{3}{2}}(c - c \sin(fx + e))^{\frac{5}{2}}} \\ & + \frac{6(g \cos(fx + e))^{\frac{5}{2}}}{5afg(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{6(g \cos(fx + e))^{\frac{5}{2}}}{5acfg(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{6g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^2 c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3g \cos(fx + e)^2 + 3g \sin(fx + e) - 2g \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} + 3 \left(-i \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} g}{a^2 c^3 \cos(fx + e)^3 \sin(fx + e) - a^2 c^3 \cos(fx + e)^3}, x\right)$$

42.54 Problem number 141

$$\int \frac{(g \cos(e + fx))^{3/2}}{(a + a \sin(e + fx))^{3/2} (c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(g \cos (fx + e))^{\frac{5}{2}}}{fg(a + a \sin (fx + e))^{\frac{3}{2}}(c - c \sin (fx + e))^{\frac{7}{2}}} \\ & + \frac{10(g \cos (fx + e))^{\frac{5}{2}}}{9afg(c - c \sin (fx + e))^{\frac{7}{2}}\sqrt{a + a \sin (fx + e)}} \\ & + \frac{2(g \cos (fx + e))^{\frac{5}{2}}}{3acfg(c - c \sin (fx + e))^{\frac{5}{2}}\sqrt{a + a \sin (fx + e)}} \\ & + \frac{2(g \cos (fx + e))^{\frac{5}{2}}}{3ac^2fg(c - c \sin (fx + e))^{\frac{3}{2}}\sqrt{a + a \sin (fx + e)}} \\ & + \frac{2g\sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos (fx + e)}) \sqrt{g \cos (fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) ac^3f\sqrt{a + a \sin (fx + e)} \sqrt{c - c \sin (fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="fr`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$\frac{2\left(6g \cos (fx + e)^2 - \left(3g \cos (fx + e)^2 - 5g\right) \sin (fx + e) - 4g\right) \sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a} \sqrt{-c \sin (fx + e)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a} \sqrt{-c \sin (fx + e) + c} g}{a^2c^4 \cos (fx + e)^5 + 2a^2c^4 \cos (fx + e)^3 \sin (fx + e) - 2a^2c^4 \cos (fx + e)^3}, x\right)$$

42.55 Problem number 142

$$\int \frac{(g \cos (e + fx))^{\frac{3}{2}}(c - c \sin (e + fx))^{\frac{9}{2}}}{(a + a \sin (e + fx))^{\frac{5}{2}}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{76c^2(g \cos (fx + e))^{\frac{5}{2}}(c - c \sin (fx + e))^{\frac{5}{2}}}{5afg(a + a \sin (fx + e))^{\frac{3}{2}}} - \frac{4c(g \cos (fx + e))^{\frac{5}{2}}(c - c \sin (fx + e))^{\frac{7}{2}}}{5fg(a + a \sin (fx + e))^{\frac{5}{2}}} \\ & + \frac{114c^3(g \cos (fx + e))^{\frac{5}{2}}(c - c \sin (fx + e))^{\frac{3}{2}}}{7a^2fg\sqrt{a + a \sin (fx + e)}} + \frac{418c^5(g \cos (fx + e))^{\frac{5}{2}}}{5a^2fg\sqrt{a + a \sin (fx + e)} \sqrt{c - c \sin (fx + e)}} \\ & + \frac{1254c^5g\sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos (fx + e)}) \sqrt{g \cos (fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^2f\sqrt{a + a \sin (fx + e)} \sqrt{c - c \sin (fx + e)}} \\ & + \frac{1254c^4(g \cos (fx + e))^{\frac{5}{2}} \sqrt{c - c \sin (fx + e)}}{35a^2fg\sqrt{a + a \sin (fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="fr`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5 c^4 g \cos (f x + e)^4 - 192 c^4 g \cos (f x + e)^2 + 2814 c^4 g + \left(39 c^4 g \cos (f x + e)^2 + 3038 c^4 g \right) \sin (f x + e) \right) \sqrt{g \cos (f x + e)}}{3 a^3 \cos (f x + e)^2 - 4 a^3 + \left(a^3 \cos (f x + e)^2 - 4 a^3 \right) \sin (f x + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\left(c^4 g \cos (f x + e)^5 - 8 c^4 g \cos (f x + e)^3 + 8 c^4 g \cos (f x + e) + 4 \left(c^4 g \cos (f x + e)^3 - 2 c^4 g \cos (f x + e) \right) \sin (f x + e) \right) \sqrt{g \cos (f x + e)}}{3 a^3 \cos (f x + e)^2 - 4 a^3 + \left(a^3 \cos (f x + e)^2 - 4 a^3 \right) \sin (f x + e)} \right)$$

42.56 Problem number 143

$$\int \frac{(g \cos (e + f x))^{3/2} (c - c \sin (e + f x))^{7/2}}{(a + a \sin (e + f x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{12 c^2 (g \cos (f x + e))^{\frac{5}{2}} (c - c \sin (f x + e))^{\frac{3}{2}}}{a f g (a + a \sin (f x + e))^{\frac{3}{2}}} - \frac{4 c (g \cos (f x + e))^{\frac{5}{2}} (c - c \sin (f x + e))^{\frac{5}{2}}}{5 f g (a + a \sin (f x + e))^{\frac{5}{2}}} \\ & + \frac{154 c^4 (g \cos (f x + e))^{\frac{5}{2}}}{5 a^2 f g \sqrt{a + a \sin (f x + e)} \sqrt{c - c \sin (f x + e)}} \\ & + \frac{462 c^4 g \sqrt{\frac{\cos (f x + e)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{f x}{2} + \frac{e}{2} \right), \sqrt{2} \right) (\sqrt{\cos (f x + e)}) \sqrt{g \cos (f x + e)}}{5 \cos \left(\frac{f x}{2} + \frac{e}{2} \right) a^2 f \sqrt{a + a \sin (f x + e)} \sqrt{c - c \sin (f x + e)}} \\ & + \frac{66 c^3 (g \cos (f x + e))^{\frac{5}{2}} \sqrt{c - c \sin (f x + e)}}{5 a^2 f g \sqrt{a + a \sin (f x + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="fr`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(8 c^3 g \cos (f x + e)^2 - 146 c^3 g - \left(c^3 g \cos (f x + e)^2 + 162 c^3 g \right) \sin (f x + e) \right) \sqrt{g \cos (f x + e)} \sqrt{a \sin (f x + e) + a}}{3 a^3 \cos (f x + e)^2 - 4 a^3 + \left(a^3 \cos (f x + e)^2 - 4 a^3 \right) \sin (f x + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(3 c^3 g \cos (f x + e)^3 - 4 c^3 g \cos (f x + e) - \left(c^3 g \cos (f x + e)^3 - 4 c^3 g \cos (f x + e) \right) \sin (f x + e) \right) \sqrt{g \cos (f x + e)}}{3 a^3 \cos (f x + e)^2 - 4 a^3 + \left(a^3 \cos (f x + e)^2 - 4 a^3 \right) \sin (f x + e)} \right)$$

42.57 Problem number 144

$$\int \frac{(g \cos(e + fx))^{3/2} (c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4c(g \cos(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{3}{2}}}{5fg(a + a \sin(fx + e))^{\frac{5}{2}}} + \frac{154c^3(g \cos(fx + e))^{\frac{5}{2}}}{15a^2fg\sqrt{a + a \sin(fx + e)}\sqrt{c - c \sin(fx + e)}} \\ & + \frac{154c^3g\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^2f\sqrt{a + a \sin(fx + e)}\sqrt{c - c \sin(fx + e)}} \\ & + \frac{44c^2(g \cos(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}}{5a^2fg(a + a \sin(fx + e))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5c^2g \cos(fx + e)^2 - 166c^2g \sin(fx + e) - 142c^2g \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(c^2g \cos(fx + e)^3 + 2c^2g \cos(fx + e) \sin(fx + e) - 2c^2g \cos(fx + e)\right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}}{3a^3 \cos(fx + e)^2 - 4a^3 + \left(a^3 \cos(fx + e)^2 - 4a^3\right) \sin(fx + e)}\right)$$

42.58 Problem number 145

$$\int \frac{(g \cos(e + fx))^{3/2} (c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{28c^2(g \cos(fx + e))^{\frac{5}{2}}}{5a^2fg(a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{42c^2g\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^2f\sqrt{a + a \sin(fx + e)}\sqrt{c - c \sin(fx + e)}} \\ & - \frac{4c(g \cos(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}}{5a^2fg(a + a \sin(fx + e))^{\frac{5}{2}}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{8(4cg \sin(fx + e) + 3cg) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} - 21 \left(-i \sqrt{2} cg \cos(fx + e) + \right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(cg \cos(fx + e) \sin(fx + e) - cg \cos(fx + e)) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{3a^3 \cos(fx + e)^2 - 4a^3 + (a^3 \cos(fx + e)^2 - 4a^3) \sin(fx + e)} \right)$$

42.59 Problem number 146

$$\int \frac{(g \cos(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4c(g \cos(fx + e))^{\frac{5}{2}}}{5fg(a + a \sin(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{6c(g \cos(fx + e))^{\frac{5}{2}}}{5afg(a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{6cg \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos \left(\frac{fx}{2} + \frac{e}{2} \right) a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} (3g \sin(fx + e) + g) - 3 \left(-i \sqrt{2} g \cos(fx + e)^2 + \right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} g \cos(fx + e)}{3a^3 \cos(fx + e)^2 - 4a^3 + (a^3 \cos(fx + e)^2 - 4a^3) \sin(fx + e)}, x \right)$$

42.60 Problem number 147

$$\int \frac{(g \cos(e + fx))^{3/2}}{(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(g \cos(fx + e))^{\frac{5}{2}}}{5fg(a + a \sin(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{2(g \cos(fx + e))^{\frac{5}{2}}}{5afg(a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{2g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} (g \sin(fx + e) + 2g) - \left(-i \sqrt{2} g \cos(fx + e)\right)^2 + 2i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} g}{a^3 c \cos(fx + e)^3 - 2 a^3 c \cos(fx + e) \sin(fx + e) - 2 a^3 c \cos(fx + e)}, x\right)$$

42.61 Problem number 148

$$\int \frac{(g \cos(e + fx))^{3/2}}{(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(g \cos(fx + e))^{\frac{5}{2}}}{fg(a + a \sin(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{6(g \cos(fx + e))^{\frac{5}{2}}}{5cfg(a + a \sin(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{6(g \cos(fx + e))^{\frac{5}{2}}}{5acfg(a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{6g \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^2 c f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3g \cos (fx + e)^2 - 3g \sin (fx + e) - 2g \right) \sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a} \sqrt{-c \sin (fx + e) + c} + 3 \left(-i \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a} \sqrt{-c \sin (fx + e) + c} g}{a^3 c^2 \cos (fx + e)^3 \sin (fx + e) + a^3 c^2 \cos (fx + e)^3}, x \right)$$

42.62 Problem number 149

$$\int \frac{(g \cos (e + fx))^{3/2}}{(a + a \sin (e + fx))^{5/2} (c - c \sin (e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(g \cos (fx + e))^{\frac{5}{2}}}{5fg(a + a \sin (fx + e))^{\frac{5}{2}}(c - c \sin (fx + e))^{\frac{5}{2}}} \\ & - \frac{2(g \cos (fx + e))^{\frac{5}{2}}}{afg(a + a \sin (fx + e))^{\frac{3}{2}}(c - c \sin (fx + e))^{\frac{5}{2}}} \\ & + \frac{6(g \cos (fx + e))^{\frac{5}{2}}}{5a^2fg(c - c \sin (fx + e))^{\frac{5}{2}}\sqrt{a + a \sin (fx + e)}} \\ & + \frac{6(g \cos (fx + e))^{\frac{5}{2}}}{5a^2cfg(c - c \sin (fx + e))^{\frac{3}{2}}\sqrt{a + a \sin (fx + e)}} \\ & - \frac{6g \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right) (\sqrt{\cos (fx + e)}) \sqrt{g \cos (fx + e)}}{5 \cos \left(\frac{fx}{2} + \frac{e}{2} \right) a^2 c^2 f \sqrt{a + a \sin (fx + e)} \sqrt{c - c \sin (fx + e)}} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fr`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i \sqrt{2} \sqrt{acg} g \cos (fx + e)^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e))) - 3i \sqrt{2} \sqrt{acg} g \cos (fx + e)^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos (fx + e) - i \sin (fx + e))) + 3i \sqrt{2} \sqrt{acg} g \cos (fx + e)^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e))) - 3i \sqrt{2} \sqrt{acg} g \cos (fx + e)^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos (fx + e) - i \sin (fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{g \cos (fx + e)} \sqrt{a \sin (fx + e) + a} \sqrt{-c \sin (fx + e) + c} g}{a^3 c^3 \cos (fx + e)^5}, x \right)$$

42.63 Problem number 150

$$\int \frac{(g \cos(e + fx))^{3/2}}{(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(g \cos(fx + e))^{\frac{5}{2}}}{5fg(a + a \sin(fx + e))^{\frac{5}{2}}(c - c \sin(fx + e))^{\frac{7}{2}}} - \frac{14(g \cos(fx + e))^{\frac{5}{2}}}{5afg(a + a \sin(fx + e))^{\frac{3}{2}}(c - c \sin(fx + e))^{\frac{7}{2}}} \\ & + \frac{14(g \cos(fx + e))^{\frac{5}{2}}}{9a^2fg(c - c \sin(fx + e))^{\frac{7}{2}}\sqrt{a + a \sin(fx + e)}} \\ & + \frac{14(g \cos(fx + e))^{\frac{5}{2}}}{15a^2c^2fg(c - c \sin(fx + e))^{\frac{5}{2}}\sqrt{a + a \sin(fx + e)}} \\ & + \frac{14(g \cos(fx + e))^{\frac{5}{2}}}{15a^2c^2fg(c - c \sin(fx + e))^{\frac{3}{2}}\sqrt{a + a \sin(fx + e)}} \\ & - \frac{14g\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{g \cos(fx + e)}}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^2 c^3 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

```
integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21 g \cos(fx + e)^4 - 14 g \cos(fx + e)^2 + 7 \left(3 g \cos(fx + e)^2 + g \right) \sin(fx + e) - 2 g \right) \sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{g \cos(fx + e)} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} g}{a^3 c^4 \cos(fx + e)^5 \sin(fx + e) - a^3 c^4 \cos(fx + e)^5}, x\right)$$

42.64 Problem number 1143

$$\int \cos^4(c + dx) \sin^2(c + dx) \sqrt{a + b \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{8(480a^4 - 937a^2b^2 + 231b^4) \cos(dx + c) (a + b \sin(dx + c))^{\frac{3}{2}}}{45045b^5d} \\ & + \frac{8a(40a^2 - 81b^2) \cos(dx + c) \sin(dx + c) (a + b \sin(dx + c))^{\frac{3}{2}}}{3003b^4d} \\ & - \frac{10(16a^2 - 33b^2) \cos(dx + c) (\sin^2(dx + c)) (a + b \sin(dx + c))^{\frac{3}{2}}}{1287b^3d} \\ & + \frac{20a \cos(dx + c) (\sin^3(dx + c)) (a + b \sin(dx + c))^{\frac{3}{2}}}{143b^2d} \\ & - \frac{2 \cos(dx + c) (\sin^4(dx + c)) (a + b \sin(dx + c))^{\frac{3}{2}}}{13bd} \\ & + \frac{16a(160a^4 - 279a^2b^2 + 27b^4) \cos(dx + c) \sqrt{a + b \sin(dx + c)}}{45045b^5d} \\ & + \frac{8(320a^6 - 798a^4b^2 + 435a^2b^4 - 693b^6) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b}}{45045 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^6 d \sqrt{\frac{a+b \sin(dx+c)}{a+b}}} \\ & - \frac{16a(160a^6 - 439a^4b^2 + 306a^2b^4 - 27b^6) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b}}{45045 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^6 d \sqrt{a+b \sin(dx+c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \sqrt{2} (640 a^7 - 1836 a^5 b^2 + 1401 a^3 b^4 + 531 a b^6) \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8i a^3 - 9i a b^2)}{27b^3}, \frac{3b \cos(dx+c)}{27b^3}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(\cos(dx + c)^6 - \cos(dx + c)^4\right) \sqrt{b \sin(dx + c) + a}, x\right)$$

42.65 Problem number 1144

$$\int \cos^4(c + dx) \sin(c + dx) \sqrt{a + b \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(\cos^5(dx + c)) \sqrt{a + b \sin(dx + c)}}{11d} \\ & - \frac{2(\cos^3(dx + c)) (8a^2 - 9b^2 - 7ab \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{693b^2d} \\ & + \frac{4 \cos(dx + c) (32a^4 - 69a^2b^2 + 45b^4 - 24ab(a^2 - 2b^2) \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{3465b^4d} \\ & - \frac{8a(32a^4 - 93a^2b^2 + 93b^4) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \sin(dx + c)}}{3465 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^5 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\ & + \frac{8(32a^6 - 101a^4b^2 + 114a^2b^4 - 45b^6) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a}}}{3465 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^5 d \sqrt{a + b \sin(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{2} (64 a^6 - 210 a^4 b^2 + 249 a^2 b^4 - 135 b^6) \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8ia^3 - 9iab^2)}{27b^3}, \frac{3b \cos(dx + c)}{a}\right) \right)}{3465 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^5 d \sqrt{a + b \sin(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sin(dx + c) + a} \cos(dx + c)^4 \sin(dx + c), x\right)$$

42.66 Problem number 1151

$$\int \cos^4(c + dx) \sin^2(c + dx) (a + b \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{16a(32a^4 - 47a^2b^2 - 27b^4) \cos(dx + c) (a + b \sin(dx + c))^{\frac{3}{2}}}{45045b^5d} \\
& - \frac{8(160a^4 - 375a^2b^2 + 117b^4) \cos(dx + c) (a + b \sin(dx + c))^{\frac{5}{2}}}{45045b^5d} \\
& + \frac{8a(8a^2 - 21b^2) \cos(dx + c) \sin(dx + c) (a + b \sin(dx + c))^{\frac{5}{2}}}{1287b^4d} \\
& - \frac{2(80a^2 - 221b^2) \cos(dx + c) (\sin^2(dx + c)) (a + b \sin(dx + c))^{\frac{5}{2}}}{2145b^3d} \\
& + \frac{4a \cos(dx + c) (\sin^3(dx + c)) (a + b \sin(dx + c))^{\frac{5}{2}}}{39b^2d} \\
& - \frac{2 \cos(dx + c) (\sin^4(dx + c)) (a + b \sin(dx + c))^{\frac{5}{2}}}{15bd} \\
& + \frac{8(64a^6 - 174a^4b^2 + 81a^2b^4 - 195b^6) \cos(dx + c) \sqrt{a + b \sin(dx + c)}}{45045b^5d} \\
& + \frac{16a(32a^6 - 111a^4b^2 + 102a^2b^4 - 471b^6) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \sin(dx + c)}}{45045 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^6 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\
& - \frac{8(64a^8 - 238a^6b^2 + 255a^4b^4 - 276a^2b^6 + 195b^8) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \sin(dx + c)}}{45045 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^6 d \sqrt{a + b \sin(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{2} (128 a^8 - 492 a^6 b^2 + 561 a^4 b^4 + 114 a^2 b^6 + 585 b^8) \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8ia^3 - 9iab^2)}{27b^3}\right) \right)}{45045 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^6 d \sqrt{a + b \sin(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(a \cos(dx + c)^6 - a \cos(dx + c)^4 + \left(b \cos(dx + c)^6 - b \cos(dx + c)^4\right) \sin(dx + c)\right) \sqrt{b \sin(dx + c) + a}\right)$$

42.67 Problem number 1152

$$\int \cos^4(c + dx) \sin(c + dx) (a + b \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(\cos^5(dx + c))(a + b \sin(dx + c))^{3/2}}{13d} - \frac{6a(\cos^5(dx + c))\sqrt{a + b \sin(dx + c)}}{143d} \\ & - \frac{2(\cos^3(dx + c))(4a(2a^2 - 5b^2) - 7b(a^2 + 11b^2)\sin(dx + c))\sqrt{a + b \sin(dx + c)}}{3003b^2d} \\ & + \frac{4\cos(dx + c)(a(32a^4 - 113a^2b^2 + 177b^4) - 3b(8a^4 - 27a^2b^2 - 77b^4)\sin(dx + c))\sqrt{a + b \sin(dx + c)}}{15015b^4d} \\ & + \frac{8(32a^6 - 137a^4b^2 + 258a^2b^4 + 231b^6)\sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b}}{15015\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right)b^5d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}} \\ & + \frac{8a(32a^6 - 145a^4b^2 + 290a^2b^4 - 177b^6)\sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b}{a+b\sin(dx+c)}}}{15015\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right)b^5d\sqrt{a+b\sin(dx+c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(4\sqrt{2}\left(32a^7 - 149a^5b^2 + 306a^3b^4 - 381ab^6\right)\sqrt{ib}\operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b\cos(dx+c)}{a+b}\right)\right)}{15015\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right)b^5d\sqrt{a+b\sin(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(b\cos(dx+c)^6 - a\cos(dx+c)^4\sin(dx+c) - b\cos(dx+c)^4\right)\sqrt{b\sin(dx+c)+a}, x\right)$$

42.68 Problem number 1160

$$\int \cos^4(c + dx) \sin(c + dx) (a + b \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2a(\cos^5(dx+c))(a+b\sin(dx+c))^{\frac{3}{2}}}{39d} - \frac{2(\cos^5(dx+c))(a+b\sin(dx+c))^{\frac{5}{2}}}{15d} \\
& - \frac{2(3a^2+13b^2)(\cos^5(dx+c))\sqrt{a+b\sin(dx+c)}}{429d} \\
& - \frac{2(\cos^3(dx+c))(8a^4-33a^2b^2-39b^4-7ab(a^2+63b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{9009b^2d} \\
& + \frac{4\cos(dx+c)(32a^6-165a^4b^2+450a^2b^4+195b^6-24ab(a^4-5a^2b^2-60b^4)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{45045b^4d} \\
& + \frac{8a(32a^6-189a^4b^2+570a^2b^4+1635b^6)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\sin(dx+c)}}{45045b^4d} \\
& + \frac{45045\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^5d\sqrt{\frac{a+b\sin(dx+c)}{a+b}}}{45045\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^5d\sqrt{a+b\sin(dx+c)}} \\
& + \frac{8(32a^8-197a^6b^2+615a^4b^4-255a^2b^6-195b^8)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)}{45045\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^5d\sqrt{a+b\sin(dx+c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{2}(64a^8-402a^6b^2+1275a^4b^4-2400a^2b^6-585b^8)\sqrt{ib}\operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3}\right)\right)}{45045\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^5d\sqrt{a+b\sin(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(2ab\cos(dx+c)^6-2ab\cos(dx+c)^4+\left(b^2\cos(dx+c)^6-(a^2+b^2)\cos(dx+c)^4\right)\sin(dx+c)\right)\sqrt{b\sin(dx+c)}\right)$$

42.69 Problem number 1168

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{\sqrt{a+b\sin(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{64a(80a^4 - 118a^2b^2 + 17b^4) \cos(dx + c) \sqrt{a + b \sin(dx + c)}}{15015b^6d} \\
& - \frac{8(480a^4 - 683a^2b^2 + 77b^4) \cos(dx + c) \sin(dx + c) \sqrt{a + b \sin(dx + c)}}{15015b^5d} \\
& + \frac{4a(160a^2 - 223b^2) \cos(dx + c) (\sin^2(dx + c)) \sqrt{a + b \sin(dx + c)}}{3003b^4d} \\
& - \frac{10(8a^2 - 11b^2) \cos(dx + c) (\sin^3(dx + c)) \sqrt{a + b \sin(dx + c)}}{429b^3d} \\
& + \frac{24a \cos(dx + c) (\sin^4(dx + c)) \sqrt{a + b \sin(dx + c)}}{143b^2d} \\
& - \frac{2 \cos(dx + c) (\sin^5(dx + c)) \sqrt{a + b \sin(dx + c)}}{13bd} \\
& - \frac{8(1280a^6 - 2048a^4b^2 + 453a^2b^4 + 231b^6) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a}}{15015 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^7 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\
& + \frac{8a(1280a^6 - 2368a^4b^2 + 875a^2b^4 + 213b^6) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a}}{15015 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^7 d \sqrt{a + b \sin(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(8 \sqrt{2} (640 a^7 - 1264 a^5 b^2 + 543 a^3 b^4 + 102 a b^6) \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8i a^3 - 9i a b^2)}{27b^3}, \frac{3b \cos(c)}{27b^3}\right) \right)}{15015 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^7 d \sqrt{a + b \sin(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(\cos(dx + c))^6 - \cos(dx + c)^4}{\sqrt{b \sin(dx + c) + a}} \sin(dx + c), x\right)$$

42.70 Problem number 1169

$$\int \frac{\cos^4(c + dx) \sin^2(c + dx)}{\sqrt{a + b \sin(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{8(160a^4 - 247a^2b^2 + 45b^4) \cos(dx + c) \sqrt{a + b \sin(dx + c)}}{3465b^5d} \\ & + \frac{8a(120a^2 - 179b^2) \cos(dx + c) \sin(dx + c) \sqrt{a + b \sin(dx + c)}}{3465b^4d} \\ & - \frac{2(80a^2 - 117b^2) \cos(dx + c) (\sin^2(dx + c)) \sqrt{a + b \sin(dx + c)}}{693b^3d} \\ & + \frac{20a \cos(dx + c) (\sin^3(dx + c)) \sqrt{a + b \sin(dx + c)}}{99b^2d} \\ & - \frac{2 \cos(dx + c) (\sin^4(dx + c)) \sqrt{a + b \sin(dx + c)}}{11bd} \\ & + \frac{16a(160a^4 - 267a^2b^2 + 69b^4) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \sin(dx + c)}}{3465 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^6 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\ & - \frac{8(320a^6 - 614a^4b^2 + 249a^2b^4 + 45b^6) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \sin(dx + c)}}{3465 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^6 d \sqrt{a + b \sin(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \sqrt{2} (640 a^6 - 1308 a^4 b^2 + 609 a^2 b^4 + 135 b^6) \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4 a^2 - 3 b^2)}{3 b^2}, -\frac{8(8 i a^3 - 9 i a b^2)}{27 b^3}, \frac{3 b \cos(dx + c)}{27 b^3}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\cos(dx + c)^6 - \cos(dx + c)^4}{\sqrt{b \sin(dx + c) + a}}, x\right)$$

42.71 Problem number 1170

$$\int \frac{\cos^4(c + dx) \sin(c + dx)}{\sqrt{a + b \sin(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(\cos^3(dx + c))(8a - 7b \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{63b^2d} \\ + & \frac{4 \cos(dx + c) (a(32a^2 - 33b^2) - 3b(8a^2 - 7b^2) \sin(dx + c)) \sqrt{a + b \sin(dx + c)}}{315b^4d} \\ & \frac{8(32a^4 - 57a^2b^2 + 21b^4) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{315 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^5 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\ + & \frac{8a(32a^4 - 65a^2b^2 + 33b^4) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{315 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^5 d \sqrt{a + b \sin(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 \sqrt{2} (32 a^5 - 69 a^3 b^2 + 39 a b^4) \sqrt{i b} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8i a^3 - 9i a b^2)}{27b^3}, \frac{3b \cos(dx+c) - 3i b \sin(dx+c)}{3b}\right) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx + c)^4 \sin(dx + c)}{\sqrt{b \sin(dx + c) + a}}, x\right)$$

42.72 Problem number 1176

$$\int \frac{\cos^4(c + dx) \sin^3(c + dx)}{(a + b \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2) \cos(dx + c) (\sin^4(dx + c))}{ab^2d\sqrt{a + b \sin(dx + c)}} \\
& - \frac{8(640a^4 - 592a^2b^2 + 15b^4) \cos(dx + c) \sqrt{a + b \sin(dx + c)}}{1155b^6d} \\
& + \frac{8a(480a^2 - 419b^2) \cos(dx + c) \sin(dx + c) \sqrt{a + b \sin(dx + c)}}{1155b^5d} \\
& - \frac{20(32a^2 - 27b^2) \cos(dx + c) (\sin^2(dx + c)) \sqrt{a + b \sin(dx + c)}}{231b^4d} \\
& + \frac{2(40a^2 - 33b^2) \cos(dx + c) (\sin^3(dx + c)) \sqrt{a + b \sin(dx + c)}}{33ab^3d} \\
& - \frac{2 \cos(dx + c) (\sin^4(dx + c)) \sqrt{a + b \sin(dx + c)}}{11b^2d} \\
& + \frac{8a(1280a^4 - 1344a^2b^2 + 123b^4) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \sin(dx + c)}}{1155 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^7 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\
& - \frac{8(1280a^6 - 1664a^4b^2 + 369a^2b^4 + 15b^6) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \sin(dx + c)}}{1155 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^7 d \sqrt{a + b \sin(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \left(\sqrt{2} (2560 a^6 b - 3648 a^4 b^3 + 984 a^2 b^5 + 45 b^7) \sin(dx + c) + \sqrt{2} (2560 a^7 - 3648 a^5 b^2 + 984 a^3 b^4 + 45 a b^6) \right) \sqrt{b \sin(dx + c) + a} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(\cos(dx + c)^6 - \cos(dx + c)^4\right) \sqrt{b \sin(dx + c) + a} \sin(dx + c)}{b^2 \cos(dx + c)^2 - 2ab \sin(dx + c) - a^2 - b^2}, x\right)$$

42.73 Problem number 1177

$$\int \frac{\cos^4(c + dx) \sin^2(c + dx)}{(a + b \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(a^2 - b^2) \cos(dx + c) (\sin^3(dx + c))}{a b^2 d \sqrt{a + b \sin(dx + c)}} + \frac{8a(160a^2 - 139b^2) \cos(dx + c) \sqrt{a + b \sin(dx + c)}}{315b^5 d} \\ & - \frac{16(60a^2 - 49b^2) \cos(dx + c) \sin(dx + c) \sqrt{a + b \sin(dx + c)}}{315b^4 d} \\ & + \frac{2(80a^2 - 63b^2) \cos(dx + c) (\sin^2(dx + c)) \sqrt{a + b \sin(dx + c)}}{63a b^3 d} \\ & - \frac{2 \cos(dx + c) (\sin^3(dx + c)) \sqrt{a + b \sin(dx + c)}}{9b^2 d} \\ & - \frac{8(320a^4 - 318a^2b^2 + 21b^4) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \sin(dx + c)}}{315 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^6 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\ & + \frac{16a(160a^4 - 199a^2b^2 + 39b^4) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{315 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^6 d \sqrt{a + b \sin(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \left(\sqrt{2} (640 a^5 b - 876 a^3 b^3 + 213 a b^5) \sin(dx + c) + \sqrt{2} (640 a^6 - 876 a^4 b^2 + 213 a^2 b^4) \right) \sqrt{i b} \operatorname{weierstrassP} \operatorname{Invc} \right)}{b^2 \cos^2(dx + c) - 2 a b \sin(dx + c) - a^2 - b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(\cos(dx + c)^6 - \cos(dx + c)^4\right) \sqrt{b \sin(dx + c) + a}}{b^2 \cos^2(dx + c) - 2 a b \sin(dx + c) - a^2 - b^2}, x\right)$$

42.74 Problem number 1178

$$\int \frac{\cos^4(c+dx) \sin(c+dx)}{(a+b \sin(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(\cos^3(dx+c))(8a+b \sin(dx+c))}{7b^2d\sqrt{a+b \sin(dx+c)}} - \frac{4 \cos(dx+c)(32a^2-5b^2-24ab \sin(dx+c))\sqrt{a+b \sin(dx+c)}}{35b^4d} + \frac{8a(32a^2-29b^2)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b \sin(dx+c)}}{35 \sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^5d\sqrt{\frac{a+b \sin(dx+c)}{a+b}}} + \frac{8(32a^4-37a^2b^2+5b^4)\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b \sin(dx+c)}{a+b}}}{35 \sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)b^5d\sqrt{a+b \sin(dx+c)}}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \left(\sqrt{2} (64a^4b - 82a^2b^3 + 15b^5) \sin(dx+c) + \sqrt{2} (64a^5 - 82a^3b^2 + 15ab^4) \right) \sqrt{ib} \operatorname{weierstrassPInverse}\left(-\frac{4(4a^4b - 82a^2b^3 + 15b^5)}{b^5}, \sqrt{ib}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{b \sin(dx+c)+a} \cos(dx+c)^4 \sin(dx+c)}{b^2 \cos(dx+c)^2 - 2ab \sin(dx+c) - a^2 - b^2}, x\right)$$

42.75 Problem number 1183

$$\int \frac{\cos^4(c+dx) \sin^3(c+dx)}{(a+b \sin(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(a^2 - b^2) \cos(dx + c) (\sin^4(dx + c))}{3a b^2 d (a + b \sin(dx + c))^{\frac{3}{2}}} + \frac{2(13a^2 - 5b^2) \cos(dx + c) (\sin^4(dx + c))}{3a^2 b^2 d \sqrt{a + b \sin(dx + c)}} \\
& + \frac{128a(40a^2 - 19b^2) \cos(dx + c) \sqrt{a + b \sin(dx + c)}}{315b^6 d} \\
& - \frac{8(480a^2 - 203b^2) \cos(dx + c) \sin(dx + c) \sqrt{a + b \sin(dx + c)}}{315b^5 d} \\
& + \frac{4(160a^2 - 63b^2) \cos(dx + c) (\sin^2(dx + c)) \sqrt{a + b \sin(dx + c)}}{63a b^4 d} \\
& - \frac{10(8a^2 - 3b^2) \cos(dx + c) (\sin^3(dx + c)) \sqrt{a + b \sin(dx + c)}}{9a^2 b^3 d} \\
& - \frac{8(1280a^4 - 768a^2 b^2 + 21b^4) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \sin(dx + c)}}{315 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^7 d \sqrt{\frac{a + b \sin(dx + c)}{a + b}}} \\
& + \frac{8a(1280a^4 - 1088a^2 b^2 + 123b^4) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \sin(dx + c)}{a + b}}}{315 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^7 d \sqrt{a + b \sin(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(8 \left(\sqrt{2} (640 a^5 b^2 - 624 a^3 b^4 + 87 a b^6) \cos(dx + c)^2 - 2 \sqrt{2} (640 a^6 b - 624 a^4 b^3 + 87 a^2 b^5) \sin(dx + c) - \sqrt{2} (\right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(\cos(dx + c)^6 - \cos(dx + c)^4\right) \sqrt{b \sin(dx + c) + a} \sin(dx + c)}{3 a b^2 \cos(dx + c)^2 - a^3 - 3 a b^2 + \left(b^3 \cos(dx + c)^2 - 3 a^2 b - b^3\right) \sin(dx + c)}, x\right)$$

42.76 Problem number 1184

$$\int \frac{\cos^4(c+dx) \sin^2(c+dx)}{(a+b \sin(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(a^2 - b^2) \cos(dx+c) (\sin^3(dx+c))}{3ab^2d(a+b \sin(dx+c))^{\frac{3}{2}}} + \frac{2(11a^2 - 3b^2) \cos(dx+c) (\sin^3(dx+c))}{3a^2b^2d\sqrt{a+b \sin(dx+c)}} \\ & - \frac{8(32a^2 - 11b^2) \cos(dx+c) \sqrt{a+b \sin(dx+c)}}{21b^5d} \\ & + \frac{8(24a^2 - 7b^2) \cos(dx+c) \sin(dx+c) \sqrt{a+b \sin(dx+c)}}{21ab^4d} \\ & - \frac{2(80a^2 - 21b^2) \cos(dx+c) (\sin^2(dx+c)) \sqrt{a+b \sin(dx+c)}}{21a^2b^3d} \\ & + \frac{16a(32a^2 - 15b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \sin(dx+c)}}{21 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^6 d \sqrt{\frac{a+b \sin(dx+c)}{a+b}}} \\ & - \frac{8(64a^4 - 46a^2b^2 + 3b^4) \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \sin(dx+c)}{a+b}}}{21 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^6 d \sqrt{a+b \sin(dx+c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \left(\sqrt{2} (128 a^4 b^2 - 108 a^2 b^4 + 9 b^6) \cos(dx+c)^2 - 2 \sqrt{2} (128 a^5 b - 108 a^3 b^3 + 9 a b^5) \sin(dx+c) - \sqrt{2} (128 a^6 \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(\cos(dx+c)^6 - \cos(dx+c)^4\right) \sqrt{b \sin(dx+c) + a}}{3ab^2 \cos(dx+c)^2 - a^3 - 3ab^2 + \left(b^3 \cos(dx+c)^2 - 3a^2b - b^3\right) \sin(dx+c)}, x\right)$$

42.77 Problem number 1185

$$\int \frac{\cos^4(c+dx) \sin(c+dx)}{(a+b \sin(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(\cos^3(dx+c))(8a+3b \sin(dx+c))}{15b^2d(a+b \sin(dx+c))^{\frac{3}{2}}} + \frac{4 \cos(dx+c)(32a^2-9b^2+8ab \sin(dx+c))}{15b^4d \sqrt{a+b \sin(dx+c)}}$$

$$\frac{8(32a^2-9b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \sin(dx+c)}}{15 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^5 d \sqrt{\frac{a+b \sin(dx+c)}{a+b}}}$$

$$+ \frac{8a(32a^2-17b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \sin(dx+c)}{a+b}}}{15 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^5 d \sqrt{a+b \sin(dx+c)}}$$

command

`integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 \left(\sqrt{2} (32 a^3 b^2 - 21 a b^4) \cos(dx+c)^2 - 2 \sqrt{2} (32 a^4 b - 21 a^2 b^3) \sin(dx+c) - \sqrt{2} (32 a^5 + 11 a^3 b^2 - 21 a b^4) \right) \right)}{15 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) b^5 d \sqrt{a+b \sin(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{b \sin(dx+c)+a} \cos(dx+c)^4 \sin(dx+c)}{3 a b^2 \cos(dx+c)^2 - a^3 - 3 a b^2 + (b^3 \cos(dx+c)^2 - 3 a^2 b - b^3) \sin(dx+c)}, x\right)$$

42.78 Problem number 1277

$$\int \frac{(a+b \sin(e+fx))^2}{(g \cos(e+fx))^{5/2} \sqrt{d \sin(e+fx)}} dx$$

Optimal antiderivative

$$\frac{4ab(d \sin(fx+e))^{\frac{3}{2}}}{3d^2fg(g \cos(fx+e))^{\frac{3}{2}}} + \frac{2(a^2+b^2) \sqrt{d \sin(fx+e)}}{3dfg(g \cos(fx+e))^{\frac{3}{2}}}$$

$$- \frac{(2a^2-b^2) \sqrt{\frac{1}{2} + \frac{\sin(2fx+2e)}{2}} \operatorname{EllipticF}\left(\cos\left(e + \frac{\pi}{4} + fx\right), \sqrt{2}\right) \left(\sqrt{\sin(2fx+2e)}\right)}{3 \sin\left(e + \frac{\pi}{4} + fx\right) f g^2 \sqrt{g \cos(fx+e)} \sqrt{d \sin(fx+e)}}$$

command

```
integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2a^2 - b^2)\sqrt{idg} \cos(fx + e)^2 \operatorname{ellipticF}(\cos(fx + e) + i \sin(fx + e), -1) + (2a^2 - b^2)\sqrt{-idg} \cos(fx + e)^2 \operatorname{ellipticF}(\cos(fx + e) - i \sin(fx + e), -1)}{3dfg^3 \cos(fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(b^2 \cos(fx + e)^2 - 2ab \sin(fx + e) - a^2 - b^2)\sqrt{g \cos(fx + e)}\sqrt{d \sin(fx + e)}}{dg^3 \cos(fx + e)^3 \sin(fx + e)}, x\right)$$

43 Test file number 75

Test folder name:

test_cases/4_Trig_functions/4.1_Sine/75_4.1.2.3-g_sin-^p-a+b_sin-^m-c+d_sin-^n

43.1 Problem number 39

$$\int \frac{\sin^2(e + fx)}{(a + b \sin(e + fx))^2(c + d \sin(e + fx))} dx$$

Optimal antiderivative

$$\frac{2a(a^2c + abd - 2b^2c) \arctan\left(\frac{b+a \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{\frac{3}{2}} (-ad + bc)^2 f} + \frac{a^2 \cos(fx + e)}{(a^2 - b^2) (-ad + bc) f (a + b \sin(fx + e))} + \frac{2c^2 \arctan\left(\frac{d+c \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{c^2 - d^2}}\right)}{(-ad + bc)^2 f \sqrt{c^2 - d^2}}$$

command

```
integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

44 Test file number 78

Test folder name:

test_cases/4_Trig_functions/4.1_Sine/78_4.1.4.2-a+b_sin-^m-c+d_sin-^n-A+B_sin+C_sin^2-

44.1 Problem number 33

$$\int \frac{(a + b \sin(e + fx))(A + B \sin(e + fx) + C \sin^2(e + fx))}{\sin^{\frac{3}{2}}(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bB - a(A - C)) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f} \\ & - \frac{2(3Ab + 3Ba + bC) \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f} \\ & - \frac{2aA \cos(fx + e)}{f \sqrt{\sin(fx + e)}} - \frac{2bC \cos(fx + e) \left(\sqrt{\sin(fx + e)}\right)}{3f} \end{aligned}$$

command

```
integrate((a+b*sin(f*x+e))*(A+B*sin(f*x+e)+C*sin(f*x+e)^2)/sin(f*x+e)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-i} (3Ba + (3A + C)b) \sin(fx + e) \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)) + \sqrt{2} \sqrt{i} (3Ba + (3A + C)b) \sin(fx + e)}{\sin^{\frac{3}{2}}(fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left((Ca + Bb) \cos(fx + e)^2 - (A + C)a - Bb + (Cb \cos(fx + e)^2 - Ba - (A + C)b) \sin(fx + e)\right) \sqrt{\sin(fx + e)}}{\cos(fx + e)^2 - 1}\right)$$

45 Test file number 79

Test folder name:

test_cases/4_Trig_functions/4.1_Sine/79_4.1.7-d_trig-^m-a+b-c_sin-ⁿ-^p

45.1 Problem number 7

$$\int (a \sin^3(x))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{26a^2 \cot(x) \sqrt{a(\sin^3(x))}}{77} \\ & - \frac{26a^2 \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right) \sqrt{a(\sin^3(x))}}{77 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \sin(x)^{\frac{3}{2}}} \\ & - \frac{78a^2 \cos(x) \sin(x) \sqrt{a(\sin^3(x))}}{385} - \frac{26a^2 \cos(x) (\sin^3(x)) \sqrt{a(\sin^3(x))}}{165} \\ & - \frac{2a^2 \cos(x) (\sin^5(x)) \sqrt{a(\sin^3(x))}}{15} \end{aligned}$$

command

```
integrate((a*sin(x)^3)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$195 \sqrt{2} \sqrt{-i a} a^2 \sin(x) \operatorname{weierstrassPInverse}(4, 0, \cos(x) + i \sin(x)) + 195 \sqrt{2} \sqrt{i a} a^2 \sin(x) \operatorname{weierstrassPInverse}$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(a^2 \cos(x)^6 - 3a^2 \cos(x)^4 + 3a^2 \cos(x)^2 - a^2\right) \sqrt{-\left(a \cos(x)^2 - a\right) \sin(x)}, x\right)$$

45.2 Problem number 8

$$\int (a \sin^3(x))^{3/2} dx$$

Optimal antiderivative

$$\frac{14a \cos(x) \sqrt{a (\sin^3(x))}}{45} - \frac{14a \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right) \sqrt{a (\sin^3(x))}}{15 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \sin(x)^{\frac{3}{2}}} - \frac{2a \cos(x) (\sin^2(x)) \sqrt{a (\sin^3(x))}}{9}$$

command

`integrate((a*sin(x)^3)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{7}{15}i \sqrt{2} \sqrt{-ia} \operatorname{aweierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(x) + i \sin(x))) - \frac{7}{15}i \sqrt{2} \sqrt{ia} \operatorname{aweierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(x) - i \sin(x))) + \frac{2}{45} (5a \cos(x)^3 - 12a \cos(x)) \sqrt{-(a \cos(x)^2 - a) \sin(x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(a \cos(x)^2 - a\right) \sqrt{-(a \cos(x)^2 - a) \sin(x)} \sin(x), x\right)$$

45.3 Problem number 9

$$\int \sqrt{a \sin^3(x)} dx$$

Optimal antiderivative

$$\frac{2 \cot(x) \sqrt{a (\sin^3(x))}}{3} - \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right) \sqrt{a (\sin^3(x))}}{3 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \sin(x)^{\frac{3}{2}}}$$

command

```
integrate((a*sin(x)^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-ia} \sin(x) \operatorname{weierstrassPInverse}(4, 0, \cos(x) + i \sin(x)) + \sqrt{2} \sqrt{ia} \sin(x) \operatorname{weierstrassPInverse}(4, 0, \cos(x))}{3 \sin(x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{-(a \cos(x)^2 - a) \sin(x)}, x\right)$$

45.4 Problem number 10

$$\int \frac{1}{\sqrt{a \sin^3(x)}} dx$$

Optimal antiderivative

$$-\frac{2 \cos(x) \sin(x)}{\sqrt{a (\sin^3(x))}} + \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right) \left(\sin^{\frac{3}{2}}(x)\right)}{\sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \sqrt{a (\sin^3(x))}}$$

command

```
integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i \sqrt{2} \cos(x)^2 + i \sqrt{2}\right) \sqrt{-ia} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(x) + i \sin(x))) + \left(i \sqrt{2} \cos(x)\right)}{a \cos(x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-(a \cos(x)^2 - a) \sin(x)}}{(a \cos(x)^2 - a) \sin(x)}, x\right)$$

45.5 Problem number 11

$$\int \frac{1}{(a \sin^3(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{10 \cos(x)}{21a \sqrt{a (\sin^3(x))}} - \frac{2 \cot(x) \csc(x)}{7a \sqrt{a (\sin^3(x))}} - \frac{10 \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right) \left(\sin^{\frac{3}{2}}(x)\right)}{21 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) a \sqrt{a (\sin^3(x))}}$$

command

```
integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \left(\sqrt{2} \cos(x)^4 - 2 \sqrt{2} \cos(x)^2 + \sqrt{2} \right) \sqrt{-ia} \sin(x) \operatorname{weierstrassPInverse}(4, 0, \cos(x) + i \sin(x)) + 5 \left(\sqrt{2} \cos(x) \right)}{21 \left(a^2 \cos(x)^6 - 3 a^2 \cos(x)^4 + 3 a^2 \cos(x)^2 - a^2 \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-(a \cos(x)^2 - a) \sin(x)}}{a^2 \cos(x)^6 - 3 a^2 \cos(x)^4 + 3 a^2 \cos(x)^2 - a^2}, x\right)$$

45.6 Problem number 12

$$\int \frac{1}{(a \sin^3(x))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{154 \cot(x)}{585 a^2 \sqrt{a (\sin^3(x))}} - \frac{22 \cot(x) (\csc^2(x))}{117 a^2 \sqrt{a (\sin^3(x))}} - \frac{2 \cot(x) (\csc^4(x))}{13 a^2 \sqrt{a (\sin^3(x))}} - \frac{154 \cos(x) \sin(x)}{195 a^2 \sqrt{a (\sin^3(x))}} + \frac{154 \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right) \left(\sin^{\frac{3}{2}}(x)\right)}{195 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) a^2 \sqrt{a (\sin^3(x))}}$$

command

`integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231 \left(i \sqrt{2} \cos(x)^8 - 4i \sqrt{2} \cos(x)^6 + 6i \sqrt{2} \cos(x)^4 - 4i \sqrt{2} \cos(x)^2 + i \sqrt{2} \right) \sqrt{-i a} \text{weierstrassZeta}(4, 0, \text{wei})$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{-(a \cos(x)^2 - a) \sin(x)}}{(a^3 \cos(x)^8 - 4 a^3 \cos(x)^6 + 6 a^3 \cos(x)^4 - 4 a^3 \cos(x)^2 + a^3) \sin(x)}, x \right)$$

45.7 Problem number 150

$$\int \frac{1}{\sqrt{a + b \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos(2fx + 2e)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin(fx + e), \sqrt{-\frac{b}{a}} \right) \sqrt{1 + \frac{b(\sin^2(fx + e))}{a}}}{\cos(fx + e) f \sqrt{a + b(\sin^2(fx + e))}}$$

command

`integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(2i \sqrt{-b} b \sqrt{\frac{a^2 + ab}{b^2}} + (-2ia - ib) \sqrt{-b} \right) \sqrt{\frac{2b \sqrt{\frac{a^2 + ab}{b^2}} + 2a + b}{b}} \text{ellipticF} \left(\sqrt{\frac{2b \sqrt{\frac{a^2 + ab}{b^2}} + 2a + b}{b}} \right) \text{ (c$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{-b \cos(fx + e)^2 + a + b}}{b \cos(fx + e)^2 - a - b}, x \right)$$

45.8 Problem number 182

$$\int \frac{\sin^7(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3x}{8b} + \frac{a \cos(dx + c)}{b^2 d} - \frac{3 \cos(dx + c) \sin(dx + c)}{8bd} - \frac{\cos(dx + c) (\sin^3(dx + c))}{4bd} \\ & - \frac{2a^{\frac{5}{3}} \arctan\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3b^{\frac{7}{3}} d \sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} + \frac{2(-1)^{\frac{1}{3}} a^{\frac{5}{3}} \arctan\left(\frac{(-1)^{\frac{2}{3}} b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}}\right)}{3b^{\frac{7}{3}} d \sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} \\ & + \frac{2(-1)^{\frac{2}{3}} a^{\frac{5}{3}} \arctan\left(\frac{(-1)^{\frac{1}{3}} b^{\frac{1}{3}} - a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3b^{\frac{7}{3}} d \sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.9 Problem number 183

$$\int \frac{\sin^5(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{2b} - \frac{\cos(dx + c) \sin(dx + c)}{2bd} - \frac{2a \arctan\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3b^{\frac{5}{3}} d \sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \\ & + \frac{2a \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} + (-1)^{\frac{2}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3b^{\frac{5}{3}} d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}} + \frac{2a \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - (-1)^{\frac{1}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3b^{\frac{5}{3}} d \sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.10 Problem number 184

$$\int \frac{\sin^3(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} \frac{x}{b} & - \frac{2a^{\frac{1}{3}} \arctan\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3bd\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} - \frac{2a^{\frac{1}{3}} \arctan\left(\frac{(-1)^{\frac{2}{3}} b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}}\right)}{3bd\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} \\ & + \frac{2a^{\frac{1}{3}} \arctan\left(\frac{(-1)^{\frac{1}{3}} (b^{\frac{1}{3}} + (-1)^{\frac{2}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right))}{\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3bd\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.11 Problem number 185

$$\int \frac{\sin(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \arctan\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3a^{\frac{1}{3}}b^{\frac{1}{3}}d\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} + \frac{2(-1)^{\frac{1}{3}} \arctan\left(\frac{(-1)^{\frac{2}{3}}b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}}b^{\frac{2}{3}}}}\right)}{3a^{\frac{1}{3}}b^{\frac{1}{3}}d\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}}b^{\frac{2}{3}}}} \\ & + \frac{2(-1)^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{1}{3}}b^{\frac{1}{3}} - a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}}b^{\frac{2}{3}}}}\right)}{3a^{\frac{1}{3}}b^{\frac{1}{3}}d\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}}b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.12 Problem number 186

$$\int \frac{\csc(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}(\cos(dx + c))}{ad} - \frac{2b^{\frac{1}{3}} \arctan\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3ad\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \\ & + \frac{2b^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} + (-1)^{\frac{2}{3}}a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3ad\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}} + b^{\frac{2}{3}}}} + \frac{2b^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - (-1)^{\frac{1}{3}}a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{-(-1)^{\frac{2}{3}}a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3ad\sqrt{-(-1)^{\frac{2}{3}}a^{\frac{2}{3}} + b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.13 Problem number 187

$$\int \frac{\csc^3(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}(\cos(dx+c))}{2ad} - \frac{\cot(dx+c)\csc(dx+c)}{2ad} - \frac{2b \operatorname{arctan}\left(\frac{b^{\frac{1}{3}}+a^{\frac{1}{3}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}}-b^{\frac{2}{3}}}}\right)}{3a^{\frac{5}{3}}d\sqrt{a^{\frac{2}{3}}-b^{\frac{2}{3}}}} \\ & - \frac{2b \operatorname{arctan}\left(\frac{(-1)^{\frac{2}{3}}b^{\frac{1}{3}}+a^{\frac{1}{3}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}}+(-1)^{\frac{1}{3}}b^{\frac{2}{3}}}}\right)}{3a^{\frac{5}{3}}d\sqrt{a^{\frac{2}{3}}+(-1)^{\frac{1}{3}}b^{\frac{2}{3}}}} + \frac{2b \operatorname{arctan}\left(\frac{(-1)^{\frac{1}{3}}\left(b^{\frac{1}{3}}+(-1)^{\frac{2}{3}}a^{\frac{1}{3}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sqrt{a^{\frac{2}{3}}-(-1)^{\frac{2}{3}}b^{\frac{2}{3}}}}\right)}{3a^{\frac{5}{3}}d\sqrt{a^{\frac{2}{3}}-(-1)^{\frac{2}{3}}b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.14 Problem number 188

$$\int \frac{\csc^5(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3 \operatorname{arctanh}(\cos(dx + c))}{8ad} + \frac{b \cot(dx + c)}{a^2 d} - \frac{3 \cot(dx + c) \csc(dx + c)}{8ad} \\ & - \frac{\cot(dx + c) (\csc^3(dx + c))}{4ad} - \frac{2b^{\frac{5}{3}} \operatorname{arctan}\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3a^{\frac{7}{3}} d \sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \\ & + \frac{2(-1)^{\frac{1}{3}} b^{\frac{5}{3}} \operatorname{arctan}\left(\frac{(-1)^{\frac{2}{3}} b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}}\right)}{3a^{\frac{7}{3}} d \sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} + \frac{2(-1)^{\frac{2}{3}} b^{\frac{5}{3}} \operatorname{arctan}\left(\frac{(-1)^{\frac{1}{3}} b^{\frac{1}{3}} - a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3a^{\frac{7}{3}} d \sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.15 Problem number 189

$$\int \frac{\sin^6(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{ax}{b^2} - \frac{\cos(dx + c)}{bd} + \frac{\cos^3(dx + c)}{3bd} + \frac{2a^{\frac{4}{3}} \operatorname{arctan}\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3b^2 d \sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \\ & + \frac{2a^{\frac{4}{3}} \operatorname{arctan}\left(\frac{(-1)^{\frac{2}{3}} b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}}\right)}{3b^2 d \sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} - \frac{2a^{\frac{4}{3}} \operatorname{arctan}\left(\frac{(-1)^{\frac{1}{3}} (b^{\frac{1}{3}} + (-1)^{\frac{2}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right))}{\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3b^2 d \sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.16 Problem number 190

$$\int \frac{\sin^4(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\cos(dx+c)}{bd} + \frac{2a^{\frac{2}{3}} \arctan\left(\frac{b^{\frac{1}{3}}+a^{\frac{1}{3}} \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}}-b^{\frac{2}{3}}}}\right)}{3b^{\frac{4}{3}}d\sqrt{a^{\frac{2}{3}}-b^{\frac{2}{3}}}} \\ & - \frac{2(-1)^{\frac{1}{3}} a^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{2}{3}} b^{\frac{1}{3}}+a^{\frac{1}{3}} \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}}+(-1)^{\frac{1}{3}} b^{\frac{2}{3}}}}\right)}{3b^{\frac{4}{3}}d\sqrt{a^{\frac{2}{3}}+(-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} - \frac{2(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{1}{3}} b^{\frac{1}{3}}-a^{\frac{1}{3}} \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}}-(-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3b^{\frac{4}{3}}d\sqrt{a^{\frac{2}{3}}-(-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.17 Problem number 191

$$\int \frac{\sin^2(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3b^{\frac{2}{3}}d\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} - \frac{2 \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} + (-1)^{\frac{2}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3b^{\frac{2}{3}}d\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}} - \frac{2 \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - (-1)^{\frac{1}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3b^{\frac{2}{3}}d\sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}}$$

command

```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.18 Problem number 192

$$\int \frac{1}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3a^{\frac{2}{3}}d\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} + \frac{2 \arctan\left(\frac{(-1)^{\frac{2}{3}} b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}}\right)}{3a^{\frac{2}{3}}d\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} - \frac{2 \arctan\left(\frac{(-1)^{\frac{1}{3}} (b^{\frac{1}{3}} + (-1)^{\frac{2}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right))}{\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3a^{\frac{2}{3}}d\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}$$

command

```
integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.19 Problem number 193

$$\int \frac{\csc^2(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\cot(dx + c)}{ad} + \frac{2b^{\frac{2}{3}} \arctan\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3a^{\frac{4}{3}}d\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \\ & - \frac{2(-1)^{\frac{1}{3}} b^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{2}{3}} b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}}\right)}{3a^{\frac{4}{3}}d\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} - \frac{2(-1)^{\frac{2}{3}} b^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{1}{3}} b^{\frac{1}{3}} - a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3a^{\frac{4}{3}}d\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.20 Problem number 194

$$\int \frac{\csc^4(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\frac{b \operatorname{arctanh}(\cos(dx + c))}{a^2 d} - \frac{\cot(dx + c)}{ad} - \frac{\cot^3(dx + c)}{3ad} + \frac{2b^{\frac{4}{3}} \operatorname{arctan}\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3a^2 d \sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}$$

$$- \frac{2b^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} + (-1)^{\frac{2}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3a^2 d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}}$$

$$- \frac{2b^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - (-1)^{\frac{1}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3a^2 d \sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}}$$

command

```
integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.21 Problem number 250

$$\int \frac{1}{a + b \sin^6(x)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctan}\left(\frac{\sqrt{a^{\frac{1}{3}} + b^{\frac{1}{3}} \tan(x)}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} + b^{\frac{1}{3}}}} + \frac{\operatorname{arctan}\left(\frac{\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{1}{3}} b^{\frac{1}{3}} \tan(x)}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} - (-1)^{\frac{1}{3}} b^{\frac{1}{3}}}}$$

$$+ \frac{\operatorname{arctan}\left(\frac{\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{2}{3}} b^{\frac{1}{3}} \tan(x)}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} + (-1)^{\frac{2}{3}} b^{\frac{1}{3}}}}$$

command

```
integrate(1/(a+b*sin(x)^6),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.22 Problem number 251

$$\int \frac{1}{a + b \sin^8(x)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{(-a)^{\frac{1}{4}} - b^{\frac{1}{4}}}\tan(x)}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} - b^{\frac{1}{4}}}} - \frac{\arctan\left(\frac{\sqrt{(-a)^{\frac{1}{4}} - ib^{\frac{1}{4}}}\tan(x)}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} - ib^{\frac{1}{4}}}}$$

$$\frac{\arctan\left(\frac{\sqrt{(-a)^{\frac{1}{4}} + ib^{\frac{1}{4}}}\tan(x)}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} + ib^{\frac{1}{4}}}} - \frac{\arctan\left(\frac{\sqrt{(-a)^{\frac{1}{4}} + b^{\frac{1}{4}}}\tan(x)}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} + b^{\frac{1}{4}}}}$$

command

```
integrate(1/(a+b*sin(x)^8),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.23 Problem number 253

$$\int \frac{1}{a - b \sin^6(x)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{a^{\frac{1}{3}} - b^{\frac{1}{3}} \tan(x)}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} - b^{\frac{1}{3}}}} + \frac{\arctan\left(\frac{\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{1}{3}} b^{\frac{1}{3}} \tan(x)}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} + (-1)^{\frac{1}{3}} b^{\frac{1}{3}}}}$$

$$+ \frac{\arctan\left(\frac{\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{2}{3}} b^{\frac{1}{3}} \tan(x)}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} - (-1)^{\frac{2}{3}} b^{\frac{1}{3}}}}$$

command

```
integrate(1/(a-b*sin(x)^6),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.24 Problem number 254

$$\int \frac{1}{a - b \sin^8(x)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{a^{\frac{1}{4}} - b^{\frac{1}{4}} \tan(x)}}{a^{\frac{1}{8}}}\right)}{4a^{\frac{7}{8}} \sqrt{a^{\frac{1}{4}} - b^{\frac{1}{4}}}} + \frac{\arctan\left(\frac{\sqrt{a^{\frac{1}{4}} - ib^{\frac{1}{4}} \tan(x)}}{a^{\frac{1}{8}}}\right)}{4a^{\frac{7}{8}} \sqrt{a^{\frac{1}{4}} - ib^{\frac{1}{4}}}}$$

$$+ \frac{\arctan\left(\frac{\sqrt{a^{\frac{1}{4}} + ib^{\frac{1}{4}} \tan(x)}}{a^{\frac{1}{8}}}\right)}{4a^{\frac{7}{8}} \sqrt{a^{\frac{1}{4}} + ib^{\frac{1}{4}}}} + \frac{\arctan\left(\frac{\sqrt{a^{\frac{1}{4}} + b^{\frac{1}{4}} \tan(x)}}{a^{\frac{1}{8}}}\right)}{4a^{\frac{7}{8}} \sqrt{a^{\frac{1}{4}} + b^{\frac{1}{4}}}}$$

command

```
integrate(1/(a-b*sin(x)^8),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.25 Problem number 351

$$\int \frac{1}{\sqrt{a + b \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos(2fx + 2e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(fx + e), \sqrt{-\frac{b}{a}}\right) \sqrt{1 + \frac{b(\sin^2(fx + e))}{a}}}{\cos(fx + e) f \sqrt{a + b(\sin^2(fx + e))}}$$

command

```
integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(2i \sqrt{-b} b \sqrt{\frac{a^2 + ab}{b^2}} + (-2ia - ib) \sqrt{-b}\right) \sqrt{\frac{2b \sqrt{\frac{a^2 + ab}{b^2}} + 2a + b}{b}} \operatorname{ellipticF}\left(\sqrt{\frac{2b \sqrt{\frac{a^2 + ab}{b^2}} + 2a + b}{b}}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-b \cos^2(fx + e) + a + b}}{b \cos^2(fx + e) - a - b}, x\right)$$

45.26 Problem number 388

$$\int \frac{\cos^4(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\cos(dx + c)}{bd} + \frac{2 \arctan\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3a^{\frac{2}{3}}d\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \\ & + \frac{2a^{\frac{2}{3}} \arctan\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3b^{\frac{4}{3}}d\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} - \frac{4 \arctan\left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3b^{\frac{2}{3}}d\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \\ & + \frac{4 \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} + (-1)^{\frac{2}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3b^{\frac{2}{3}}d\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}} + \frac{4 \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - (-1)^{\frac{1}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3b^{\frac{2}{3}}d\sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} + b^{\frac{2}{3}}}} \\ & + \frac{2 \arctan\left(\frac{(-1)^{\frac{2}{3}} b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}}\right)}{3a^{\frac{2}{3}}d\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} - \frac{2(-1)^{\frac{1}{3}} a^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{2}{3}} b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}}\right)}{3b^{\frac{4}{3}}d\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} \\ & - \frac{2(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{1}{3}} b^{\frac{1}{3}} - a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3b^{\frac{4}{3}}d\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} - \frac{2 \arctan\left(\frac{(-1)^{\frac{1}{3}} (b^{\frac{1}{3}} + (-1)^{\frac{2}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right))}{\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3a^{\frac{2}{3}}d\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.27 Problem number 390

$$\int \frac{1}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \right)}{3a^{\frac{2}{3}} d \sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} + \frac{2 \arctan \left(\frac{(-1)^{\frac{2}{3}} b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} \right)}{3a^{\frac{2}{3}} d \sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} - \frac{2 \arctan \left(\frac{(-1)^{\frac{1}{3}} (b^{\frac{1}{3}} + (-1)^{\frac{2}{3}} a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right))}{\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} \right)}{3a^{\frac{2}{3}} d \sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}$$

command

```
integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.28 Problem number 391

$$\int \frac{\sec^2(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(-1)^{\frac{2}{3}} b^{\frac{2}{3}} \arctan \left(\frac{(-1)^{\frac{1}{3}} b^{\frac{1}{3}} - a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} \right)}{3a^{\frac{2}{3}} \left(a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}} \right)^{\frac{3}{2}} d} - \frac{2b^{\frac{2}{3}} \arctan \left(\frac{b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \right)}{3a^{\frac{2}{3}} \left(a^{\frac{2}{3}} - b^{\frac{2}{3}} \right)^{\frac{3}{2}} d} + \frac{2(-1)^{\frac{1}{3}} b^{\frac{2}{3}} \arctan \left(\frac{(-1)^{\frac{2}{3}} b^{\frac{1}{3}} + a^{\frac{1}{3}} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}}}} \right)}{3a^{\frac{2}{3}} \left(a^{\frac{2}{3}} + (-1)^{\frac{1}{3}} b^{\frac{2}{3}} \right)^{\frac{3}{2}} d} + \frac{\sec(dx + c) (b - a \sin(dx + c))}{(-a^2 + b^2) d}$$

command

```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.29 Problem number 392

$$\int \frac{\sec^4(c + dx)}{a + b \sin^3(c + dx)} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.30 Problem number 393

$$\int \frac{\cos^7(c + dx)}{(a + b \sin^3(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{2\left(2a^2 - 3a^{\frac{4}{3}}b^{\frac{2}{3}} + b^2\right) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} \sin(dx + c)\right)}{9a^{\frac{5}{3}}b^{\frac{7}{3}}d} - \frac{\left(2a^2 - 3a^{\frac{4}{3}}b^{\frac{2}{3}} + b^2\right) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} \sin(dx + c) + b^{\frac{2}{3}}(\sin^2(dx + c))\right)}{9a^{\frac{5}{3}}b^{\frac{7}{3}}d} - \frac{\sin(dx + c)}{b^2d} - \frac{\sin(dx + c)(a^2 - b^2 + 3ab \sin(dx + c) + 3b^2(\sin^2(dx + c)))}{3ab^2d(a + b(\sin^3(dx + c)))} - \frac{2\left(2a^2 + 3a^{\frac{4}{3}}b^{\frac{2}{3}} + b^2\right) \arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}} \sin(dx + c))\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{9a^{\frac{5}{3}}b^{\frac{7}{3}}d}$$

command

```
integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.31 Problem number 394

$$\int \frac{\cos^5(c + dx)}{(a + b \sin^3(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2 \left(a^{\frac{4}{3}} - b^{\frac{4}{3}} \right) \ln \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} \sin(dx + c) \right)}{9 a^{\frac{5}{3}} b^{\frac{5}{3}} d} \\ & + \frac{\left(a^{\frac{4}{3}} - b^{\frac{4}{3}} \right) \ln \left(a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} \sin(dx + c) + b^{\frac{2}{3}} (\sin^2(dx + c)) \right)}{9 a^{\frac{5}{3}} b^{\frac{5}{3}} d} \\ & + \frac{\sin(dx + c) (b - a \sin(dx + c) - 2b (\sin^2(dx + c)))}{3abd (a + b (\sin^3(dx + c)))} \\ & - \frac{2 \left(a^{\frac{4}{3}} + b^{\frac{4}{3}} \right) \arctan \left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}} \sin(dx + c) \right) \sqrt{3}}{3a^{\frac{1}{3}}} \right) \sqrt{3}}{9 a^{\frac{5}{3}} b^{\frac{5}{3}} d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.32 Problem number 398

$$\int \frac{\sec^3(c + dx)}{(a + b \sin^3(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(a + 7b) \ln(1 - \sin(dx + c))}{4(a + b)^3 d} + \frac{(a - 7b) \ln(1 + \sin(dx + c))}{4(a - b)^3 d} \\ & + \frac{b^{\frac{5}{3}} \left(4a^2 + 3a^{\frac{4}{3}} b^{\frac{2}{3}} + 2b^2\right) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} \sin(dx + c)\right)}{9a^{\frac{5}{3}} (a^2 - b^2)^2 d} \\ & + \frac{b^{\frac{5}{3}} \left(3b^{\frac{2}{3}} (3a^2 + b^2) + 4a^{\frac{2}{3}} (a^2 + 2b^2)\right) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} \sin(dx + c)\right)}{3a^{\frac{1}{3}} (a^2 - b^2)^3 d} \\ & - \frac{b^{\frac{5}{3}} \left(4a^2 + 3a^{\frac{4}{3}} b^{\frac{2}{3}} + 2b^2\right) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} \sin(dx + c) + b^{\frac{2}{3}} (\sin^2(dx + c))\right)}{18a^{\frac{5}{3}} (a^2 - b^2)^2 d} \\ & - \frac{b^{\frac{5}{3}} \left(3b^{\frac{2}{3}} (3a^2 + b^2) + 4a^{\frac{2}{3}} (a^2 + 2b^2)\right) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} \sin(dx + c) + b^{\frac{2}{3}} (\sin^2(dx + c))\right)}{6a^{\frac{1}{3}} (a^2 - b^2)^3 d} \\ & + \frac{2ab(a^2 + 5b^2) \ln(a + b(\sin^3(dx + c)))}{3(a^2 - b^2)^3 d} + \frac{1}{4(a + b)^2 d (1 - \sin(dx + c))} \\ & - \frac{1}{4(a - b)^2 d (1 + \sin(dx + c))} - \frac{b(a^2 + 2b^2) - b \sin(dx + c) (2a^2 + b^2 - 3ab \sin(dx + c))}{3a(a^2 - b^2)^2 d (a + b(\sin^3(dx + c)))} \\ & - \frac{b^{\frac{5}{3}} \left(4a^2 - 3a^{\frac{4}{3}} b^{\frac{2}{3}} + 2b^2\right) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}} \sin(dx + c)\right) \sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{9a^{\frac{5}{3}} (a^2 - b^2)^2 d} \\ & - \frac{b^{\frac{5}{3}} \left(4a^{\frac{8}{3}} - 9a^2 b^{\frac{2}{3}} + 8a^{\frac{2}{3}} b^2 - 3b^{\frac{8}{3}}\right) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}} \sin(dx + c)\right) \sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{3a^{\frac{1}{3}} (a^2 - b^2)^3 d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.33 Problem number 400

$$\int \frac{\cos^2(c + dx)}{(a + b \sin^3(c + dx))^2} dx$$

Optimal antiderivative

$$\text{Unintegrable}\left(\frac{\cos^2(dx + c)}{(a + b(\sin^3(dx + c)))^2}, x\right)$$

command

```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.34 Problem number 401

$$\int \frac{1}{(a + b \sin^3(c + dx))^2} dx$$

Optimal antiderivative

$$\text{Unintegrable}\left(\frac{1}{(a + b(\sin^3(dx + c)))^2}, x\right)$$

command

```
integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.35 Problem number 402

$$\int \frac{\sec^2(c + dx)}{(a + b \sin^3(c + dx))^2} dx$$

Optimal antiderivative

$$\text{Unintegrable}\left(\frac{\sec^2(dx + c)}{(a + b(\sin^3(dx + c)))^2}, x\right)$$

command

```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.36 Problem number 403

$$\int \frac{\sec^4(c + dx)}{(a + b \sin^3(c + dx))^2} dx$$

Optimal antiderivative

$$\text{Unintegrable}\left(\frac{\sec^4(dx + c)}{(a + b(\sin^3(dx + c)))^2}, x\right)$$

command

```
integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.37 Problem number 407

$$\int \frac{\cos(c + dx)}{a - b \sin^4(c + dx)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{b^{\frac{1}{4}} \sin(dx+c)}{a^{\frac{1}{4}}}\right)}{2a^{\frac{3}{4}}b^{\frac{1}{4}}d} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}} \sin(dx+c)}{a^{\frac{1}{4}}}\right)}{2a^{\frac{3}{4}}b^{\frac{1}{4}}d}$$

command

```
integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{2} \left(\frac{1}{a^3 b d^4} \right)^{\frac{1}{4}} \arctan \left(a^2 b d^3 \left(\frac{1}{a^3 b d^4} \right)^{\frac{3}{4}} \sin(dx + c) \right. \\ & + \left. \sqrt{a^2 d^2 \sqrt{\frac{1}{a^3 b d^4}} - \cos(dx + c)^2 + 1} a^2 b d^3 \left(\frac{1}{a^3 b d^4} \right)^{\frac{3}{4}} \right) \\ & - \frac{1}{2} \left(\frac{1}{a^3 b d^4} \right)^{\frac{1}{4}} \arctan \left(-a^2 b d^3 \left(\frac{1}{a^3 b d^4} \right)^{\frac{3}{4}} \sin(dx + c) \right. \\ & + \left. \sqrt{a^2 d^2 \sqrt{\frac{1}{a^3 b d^4}} - \cos(dx + c)^2 + 1} a^2 b d^3 \left(\frac{1}{a^3 b d^4} \right)^{\frac{3}{4}} \right) \\ & + \frac{1}{8} \left(\frac{1}{a^3 b d^4} \right)^{\frac{1}{4}} \log \left(\frac{1}{4} a^2 d^2 \sqrt{\frac{1}{a^3 b d^4}} + \frac{1}{2} a d \left(\frac{1}{a^3 b d^4} \right)^{\frac{1}{4}} \sin(dx + c) - \frac{1}{4} \cos(dx + c)^2 + \frac{1}{4} \right) \\ & - \frac{1}{8} \left(\frac{1}{a^3 b d^4} \right)^{\frac{1}{4}} \log \left(\frac{1}{4} a^2 d^2 \sqrt{\frac{1}{a^3 b d^4}} - \frac{1}{2} a d \left(\frac{1}{a^3 b d^4} \right)^{\frac{1}{4}} \sin(dx + c) - \frac{1}{4} \cos(dx + c)^2 + \frac{1}{4} \right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

45.38 Problem number 518

$$\int \frac{1}{\sqrt{a + b \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos(2fx + 2e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(fx + e), \sqrt{-\frac{b}{a}}\right) \sqrt{1 + \frac{b(\sin^2(fx + e))}{a}}}{\cos(fx + e) f \sqrt{a + b(\sin^2(fx + e))}}$$

command

`integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(2i \sqrt{-b} b \sqrt{\frac{a^2 + ab}{b^2}} + (-2ia - ib) \sqrt{-b}\right) \sqrt{\frac{2b \sqrt{\frac{a^2 + ab}{b^2}} + 2a + b}{b}} \operatorname{ellipticF}\left(\sqrt{\frac{2b \sqrt{\frac{a^2 + ab}{b^2}} + 2a + b}{b}}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-b \cos^2(fx + e) + a + b}}{b \cos^2(fx + e) - a - b}, x\right)$$

45.39 Problem number 591

$$\int \frac{a + b \sin^2(e + fx)}{(g \cos(e + fx))^{5/2} \sqrt{d \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2(a + b) \sqrt{d \sin(fx + e)}}{3dfg (g \cos(fx + e))^{\frac{3}{2}}} - \frac{(2a - b) \sqrt{\frac{1}{2} + \frac{\sin(2fx + 2e)}{2}} \operatorname{EllipticF}\left(\cos\left(e + \frac{\pi}{4} + fx\right), \sqrt{2}\right) \left(\sqrt{\sin(2fx + 2e)}\right)}{3 \sin\left(e + \frac{\pi}{4} + fx\right) f g^2 \sqrt{g \cos(fx + e)} \sqrt{d \sin(fx + e)}}$$

command

```
integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{d g} (2 a - b) \cos (f x + e)^2 \operatorname{ellipticF}(\cos (f x + e) + i \sin (f x + e), -1) + \sqrt{-i d g} (2 a - b) \cos (f x + e)^2 \operatorname{ellipticE}(\cos (f x + e) + i \sin (f x + e), -1)}{3 d f g^3 \cos (f x + e)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(b \cos (f x + e)^2 - a - b\right) \sqrt{g \cos (f x + e)} \sqrt{d \sin (f x + e)}}{d g^3 \cos (f x + e)^3 \sin (f x + e)}, x\right)$$

46 Test file number 81

Test folder name:

test_cases/4_Trig_functions/4.1_Sine/81_4.1.9_trig^m-a+b_sin^n+c_sin^-2_n-^p

46.1 Problem number 6

$$\int \frac{\csc(x)}{a + b \sin(x) + c \sin^2(x)} dx$$

Optimal antiderivative

$$\frac{\frac{\operatorname{arctanh}(\cos(x))}{a} + c \operatorname{arctan}\left(\frac{\left(2c + \left(b - \sqrt{-4ac + b^2}\right) \tan\left(\frac{x}{2}\right)\right) \sqrt{2}}{2\sqrt{b^2 - 2c(a+c) - b\sqrt{-4ac + b^2}}}\right) \sqrt{2} \left(1 + \frac{b}{\sqrt{-4ac + b^2}}\right)}{a\sqrt{b^2 - 2c(a+c) - b\sqrt{-4ac + b^2}}} + \frac{c \operatorname{arctan}\left(\frac{\left(2c + \left(b + \sqrt{-4ac + b^2}\right) \tan\left(\frac{x}{2}\right)\right) \sqrt{2}}{2\sqrt{b^2 - 2c(a+c) + b\sqrt{-4ac + b^2}}}\right) \sqrt{2} \left(1 - \frac{b}{\sqrt{-4ac + b^2}}\right)}{a\sqrt{b^2 - 2c(a+c) + b\sqrt{-4ac + b^2}}}$$

command

```
integrate(csc(x)/(a+b*sin(x)+c*sin(x)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

46.2 Problem number 13

$$\int \frac{\sec^2(x)}{a + b \sin(x) + c \sin^2(x)} dx$$

Optimal antiderivative

$$\frac{\frac{\cos(x)}{2(a+b+c)(1-\sin(x))} - \frac{\cos(x)}{2(a-b+c)(1+\sin(x))} + bc \arctan\left(\frac{(2c+(b-\sqrt{-4ac+b^2})\tan(\frac{x}{2}))\sqrt{2}}{2\sqrt{b^2-2c(a+c)}-b\sqrt{-4ac+b^2}}\right)\sqrt{2}\left(1+\frac{b^2-2c(a+c)}{b\sqrt{-4ac+b^2}}\right)}{(a-b+c)(a+b+c)\sqrt{b^2-2c(a+c)}-b\sqrt{-4ac+b^2}} - \frac{bc \arctan\left(\frac{(2c+(b+\sqrt{-4ac+b^2})\tan(\frac{x}{2}))\sqrt{2}}{2\sqrt{b^2-2c(a+c)}+b\sqrt{-4ac+b^2}}\right)\sqrt{2}\left(1+\frac{-b^2+2c(a+c)}{b\sqrt{-4ac+b^2}}\right)}{(a-b+c)(a+b+c)\sqrt{b^2-2c(a+c)}+b\sqrt{-4ac+b^2}}$$

command

```
integrate(sec(x)^2/(a+b*sin(x)+c*sin(x)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

47 Test file number 82

Test folder name:

test_cases/4_Trig_functions/4.2_Cosine/82_4.2.0-a_cos-^m-b_trg-^n

47.1 Problem number 9

$$\int \cos^{\frac{7}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{10\sqrt{\frac{\cos(bx+a)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b} + \frac{2\left(\cos^{\frac{5}{2}}(bx+a)\right) \sin(bx+a)}{7b} + \frac{10 \sin(bx+a) (\sqrt{\cos(bx+a)})}{21b}$$

command

```
integrate(cos(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \cos (bx + a)^2 + 5 \right) \sqrt{\cos (bx + a)} \sin (bx + a) - 5i \sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos (bx + a) + i \sin (bx + a))}{21 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\cos (bx + a)^{\frac{7}{2}}, x\right)$$

47.2 Problem number 10

$$\int \cos^{\frac{5}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{6 \sqrt{\frac{\cos (bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin \left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{5 \cos \left(\frac{a}{2} + \frac{bx}{2}\right) b} + \frac{2 \left(\cos^{\frac{3}{2}}(bx + a)\right) \sin (bx + a)}{5 b}$$

command

```
integrate(cos(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \cos (bx + a)^{\frac{3}{2}} \sin (bx + a) + 3i \sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos (bx + a) + i \sin (bx + a)))}{5 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\cos (bx + a)^{\frac{5}{2}}, x\right)$$

47.3 Problem number 11

$$\int \cos^{\frac{3}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{a}{2} + \frac{bx}{2}\right)b} + \frac{2\sin(bx+a)(\sqrt{\cos(bx+a)})}{3b}$$

command

```
integrate(cos(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{\cos(bx+a)}\sin(bx+a) - i\sqrt{2}\operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) + i\sin(bx+a)) + i\sqrt{2}\operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) - i\sin(bx+a))}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\cos(bx+a)^{\frac{3}{2}}, x\right)$$

47.4 Problem number 12

$$\int \sqrt{\cos(a+bx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right)b}$$

command

```
integrate(cos(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i\sqrt{2}\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) + i\sin(bx+a))) - i\sqrt{2}\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) - i\sin(bx+a)))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{\cos(bx+a)}, x\right)$$

47.5 Problem number 13

$$\int \frac{1}{\sqrt{\cos(ax + bx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(bx + a)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) + i\sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{\cos(bx + a)}}, x\right)$$

47.6 Problem number 14

$$\int \frac{1}{\cos^{\frac{3}{2}}(ax + bx)} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{\cos(bx + a)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b} + \frac{2 \sin(bx + a)}{b\sqrt{\cos(bx + a)}}$$

command

```
integrate(1/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2} \cos(bx + a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) + i\sqrt{2} \cos(bx + a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a)))}{b \cos(bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\cos^{\frac{3}{2}}(bx + a)}, x\right)$$

47.7 Problem number 15

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(bx+a)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b} + \frac{2 \sin(bx+a)}{3b \cos(bx+a)^{\frac{3}{2}}}$$

command

```
integrate(1/cos(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2} \cos(bx+a)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) + i \sin(bx+a)) + i\sqrt{2} \cos(bx+a)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) - i \sin(bx+a))}{3b \cos(bx+a)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\cos(bx+a)^{\frac{5}{2}}}, x\right)$$

47.8 Problem number 16

$$\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)} dx$$

Optimal antiderivative

$$-\frac{6\sqrt{\frac{\cos(bx+a)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b} + \frac{2 \sin(bx+a)}{5b \cos(bx+a)^{\frac{5}{2}}} + \frac{6 \sin(bx+a)}{5b \sqrt{\cos(bx+a)}}$$

command

```
integrate(1/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-3i\sqrt{2} \cos(bx+a)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) + i \sin(bx+a))) + 3i\sqrt{2} \cos(bx+a)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) - i \sin(bx+a)))}{3b \cos(bx+a)^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\cos(bx+a)^{\frac{7}{2}}}, x\right)$$

47.9 Problem number 17

$$\int (c \cos(a + bx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2c(c \cos(bx + a))^{\frac{5}{2}} \sin(bx + a)}{7b} \\ & + \frac{10c^4 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{21 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{c \cos(bx + a)}} \\ & + \frac{10c^3 \sin(bx + a) \sqrt{c \cos(bx + a)}}{21b} \end{aligned}$$

command

```
integrate((c*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} c^{\frac{7}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) + 5i \sqrt{2} c^{\frac{7}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a))}{21b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{c \cos(bx + a)} c^3 \cos(bx + a)^3, x\right)$$

47.10 Problem number 18

$$\int (c \cos(a + bx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2c(c \cos(bx + a))^{\frac{3}{2}} \sin(bx + a)}{5b} \\ & + \frac{6c^2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \cos(bx + a)}}{5 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos(bx + a)}} \end{aligned}$$

command

```
integrate((c*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{c \cos (bx+a)} c^2 \cos (bx+a) \sin (bx+a)+3 i \sqrt{2} c^{\frac{5}{2}} \text{weierstrassZeta}(-4,0, \text{weierstrassPInverse}(-4,0, \cos (bx+a)))}{5 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{c \cos (bx+a)} c^2 \cos (bx+a)^2, x\right)$$

47.11 Problem number 19

$$\int (c \cos (a+bx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2 c^2 \sqrt{\frac{\cos (bx+a)}{2}+\frac{1}{2}} \text{EllipticF}\left(\sin \left(\frac{a}{2}+\frac{bx}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos (bx+a)}\right)}{3 \cos \left(\frac{a}{2}+\frac{bx}{2}\right) b \sqrt{c \cos (bx+a)}}+\frac{2 c \sin (bx+a) \sqrt{c \cos (bx+a)}}{3 b}$$

command

`integrate((c*cos(b*x+a))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} c^{\frac{3}{2}} \text{weierstrassPInverse}(-4,0, \cos (bx+a))+i \sin (bx+a)+i \sqrt{2} c^{\frac{3}{2}} \text{weierstrassPInverse}(-4,0, \cos (bx+a))}{3 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{c \cos (bx+a)} c \cos (bx+a), x\right)$$

47.12 Problem number 20

$$\int \sqrt{c \cos (a+bx)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos (bx+a)}{2}+\frac{1}{2}} \text{EllipticE}\left(\sin \left(\frac{a}{2}+\frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \cos (bx+a)}}{\cos \left(\frac{a}{2}+\frac{bx}{2}\right) b \sqrt{\cos (bx+a)}}$$

command

```
integrate((c*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \sqrt{c} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) - i \sqrt{2} \sqrt{c} \operatorname{weierstrassZeta}(-4, 0, \cos(bx + a) - i \sin(bx + a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{c \cos(bx + a)}, x\right)$$

47.13 Problem number 21

$$\int \frac{1}{\sqrt{c \cos(a + bx)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{c \cos(bx + a)}}$$

command

```
integrate(1/(c*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{c} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) + i \sqrt{2} \sqrt{c} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a))}{bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \cos(bx + a)}}{c \cos(bx + a)}, x\right)$$

47.14 Problem number 22

$$\int \frac{1}{(c \cos(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(bx + a)}{bc \sqrt{c \cos(bx + a)}} - \frac{2 \sqrt{\frac{\cos(bx + a)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \cos(bx + a)}}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b c^2 \sqrt{\cos(bx + a)}}$$

command

`integrate(1/(c*cos(b*x+a))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{c} \cos(bx + a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) + i \sqrt{2} \sqrt{c} \cos(bx + a)}{bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \cos(bx + a)}}{c^2 \cos(bx + a)^2}, x\right)$$

47.15 Problem number 23

$$\int \frac{1}{(c \cos(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(bx + a)}{3bc (c \cos(bx + a))^{\frac{3}{2}}} + \frac{2 \sqrt{\frac{\cos(bx + a)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)})}{3 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b c^2 \sqrt{c \cos(bx + a)}}$$

command

`integrate(1/(c*cos(b*x+a))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{c} \cos(bx + a)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) + i \sqrt{2} \sqrt{c} \cos(bx + a)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))}{3 bc^3 \cos(bx + a)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \cos(bx + a)}}{c^3 \cos(bx + a)^3}, x\right)$$

47.16 Problem number 24

$$\int \frac{1}{(c \cos(a + bx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(bx + a)}{5bc(c \cos(bx + a))^{\frac{5}{2}}} + \frac{6 \sin(bx + a)}{5b c^3 \sqrt{c \cos(bx + a)}} - \frac{6 \sqrt{\frac{\cos(bx + a)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \cos(bx + a)}}{5 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b c^4 \sqrt{\cos(bx + a)}}$$

command

`integrate(1/(c*cos(b*x+a))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} \sqrt{c} \cos(bx + a)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) + 3i \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \cos(bx + a)}}{c^4 \cos(bx + a)^4}, x\right)$$

47.17 Problem number 45

$$\int (a \cos^3(x))^{5/2} dx$$

Optimal antiderivative

$$\frac{26a^2 \sqrt{\frac{\cos(x)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a(\cos^3(x))}}{77 \cos\left(\frac{x}{2}\right) \cos(x)^{\frac{3}{2}}} + \frac{78a^2 \cos(x) \sin(x) \sqrt{a(\cos^3(x))}}{385} + \frac{26a^2 (\cos^3(x)) \sin(x) \sqrt{a(\cos^3(x))}}{165} + \frac{2a^2 (\cos^5(x)) \sin(x) \sqrt{a(\cos^3(x))}}{15} + \frac{26a^2 \sqrt{a(\cos^3(x))} \tan(x)}{77}$$

command

`integrate((a*cos(x)^3)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{195i \sqrt{2} a^{\frac{5}{2}} \cos(x) \operatorname{weierstrassPInverse}(-4, 0, \cos(x) + i \sin(x)) - 195i \sqrt{2} a^{\frac{5}{2}} \cos(x) \operatorname{weierstrassPInverse}(-4, 0, \cos(x) - i \sin(x))}{1155 \cos(x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \cos(x)^3} a^2 \cos(x)^6, x\right)$$

47.18 Problem number 46

$$\int (a \cos^3(x))^{3/2} dx$$

Optimal antiderivative

$$\frac{14a \sqrt{\frac{\cos(x)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a(\cos^3(x))}}{15 \cos\left(\frac{x}{2}\right) \cos(x)^{\frac{3}{2}}} + \frac{14a \sin(x) \sqrt{a(\cos^3(x))}}{45} + \frac{2a(\cos^2(x)) \sin(x) \sqrt{a(\cos^3(x))}}{9}$$

command

`integrate((a*cos(x)^3)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{7}{15}i \sqrt{2} a^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(x) + i \sin(x))) \\ & + \frac{7}{15}i \sqrt{2} a^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(x) - i \sin(x))) \\ & + \frac{2}{45} \sqrt{a \cos(x)^3} (5a \cos(x)^2 + 7a) \sin(x) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \cos(x)^3} a \cos(x)^3, x\right)$$

47.19 Problem number 47

$$\int \sqrt{a \cos^3(x)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a(\cos^3(x))}}{3 \cos\left(\frac{x}{2}\right) \cos(x)^{\frac{3}{2}}} + \frac{2\sqrt{a(\cos^3(x))} \tan(x)}{3}$$

command

```
integrate((a*cos(x)^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i\sqrt{2}\sqrt{a}\cos(x)\operatorname{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x)) - i\sqrt{2}\sqrt{a}\cos(x)\operatorname{weierstrassPInverse}(-4,0,\cos(x)-i\sin(x))}{3\cos(x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a\cos(x)^3}, x\right)$$

47.20 Problem number 48

$$\int \frac{1}{\sqrt{a \cos^3(x)}} dx$$

Optimal antiderivative

$$-\frac{2\left(\cos^{\frac{3}{2}}(x)\right) \sqrt{\frac{\cos(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{x}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{x}{2}\right) \sqrt{a(\cos^3(x))}} + \frac{2\cos(x)\sin(x)}{\sqrt{a(\cos^3(x))}}$$

command

```
integrate(1/(a*cos(x)^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i\sqrt{2}\sqrt{a}\cos(x)^2\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x))) - i\sqrt{2}\sqrt{a}\cos(x)^2\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(x)-i\sin(x)))}{a\cos(x)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a\cos(x)^3}}{a\cos(x)^3}, x\right)$$

47.21 Problem number 49

$$\int \frac{1}{(a \cos^3(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{10 \left(\cos^{\frac{3}{2}}(x) \right) \sqrt{\frac{\cos(x)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{x}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{x}{2} \right) a \sqrt{a (\cos^3(x))}} + \frac{10 \sin(x)}{21a \sqrt{a (\cos^3(x))}} + \frac{2 \sec(x) \tan(x)}{7a \sqrt{a (\cos^3(x))}}$$

command

```
integrate(1/(a*cos(x)^3)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5i \sqrt{2} \sqrt{a} \cos(x)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(x) + i \sin(x)) - 5i \sqrt{2} \sqrt{a} \cos(x)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(x) - i \sin(x))}{21 a^2 \cos(x)^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{a \cos(x)^3}}{a^2 \cos(x)^6}, x \right)$$

47.22 Problem number 50

$$\int \frac{1}{(a \cos^3(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{154 \left(\cos^{\frac{3}{2}}(x) \right) \sqrt{\frac{\cos(x)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{x}{2} \right), \sqrt{2} \right)}{195 \cos \left(\frac{x}{2} \right) a^2 \sqrt{a (\cos^3(x))}} + \frac{154 \cos(x) \sin(x)}{195 a^2 \sqrt{a (\cos^3(x))}} \\ & + \frac{154 \tan(x)}{585 a^2 \sqrt{a (\cos^3(x))}} + \frac{22 (\sec^2(x)) \tan(x)}{117 a^2 \sqrt{a (\cos^3(x))}} + \frac{2 (\sec^4(x)) \tan(x)}{13 a^2 \sqrt{a (\cos^3(x))}} \end{aligned}$$

command

```
integrate(1/(a*cos(x)^3)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231i \sqrt{2} \sqrt{a} \cos(x)^8 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(x) + i \sin(x))) - 231i \sqrt{2} \sqrt{a} \cos(x)^8$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \cos(x)^3}}{a^3 \cos(x)^9}, x\right)$$

47.23 Problem number 66

$$\int \cos^5(c + dx) \sqrt{b \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{18(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{77b^2d} + \frac{2(b \cos(dx + c))^{\frac{9}{2}} \sin(dx + c)}{11b^4d} \\ & + \frac{30b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{30 \sin(dx + c) \sqrt{b \cos(dx + c)}}{77d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(7 \cos(dx + c)^4 + 9 \cos(dx + c)^2 + 15 \right) \sqrt{b \cos(dx + c)} \sin(dx + c) - 15i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

77 d

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} \cos(dx + c)^5, x\right)$$

47.24 Problem number 67

$$\int \cos^4(c + dx) \sqrt{b \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45bd} + \frac{2(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^3d} \\ & + \frac{14 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \cos(dx + c)^3 + 7 \cos(dx + c) \right) \sqrt{b \cos(dx + c)} \sin(dx + c) + 21i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassP}(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} \cos(dx + c)^4, x\right)$$

47.25 Problem number 68

$$\int \cos^3(c + dx) \sqrt{b \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^2d} \\ & + \frac{10b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{10 \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{b \cos(dx+c)} \left(3 \cos(dx+c)^2 + 5 \right) \sin(dx+c) - 5i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{21 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx+c)} \cos(dx+c)^3, x\right)$$

47.26 Problem number 69

$$\int \cos^2(c+dx) \sqrt{b \cos(c+dx)} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{5bd} + \frac{6 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}}$$

command

`integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{b \cos(dx+c)} \cos(dx+c) \sin(dx+c) + 3i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))}{5 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx+c)} \cos(dx+c)^2, x\right)$$

47.27 Problem number 70

$$\int \cos(c+dx) \sqrt{b \cos(c+dx)} dx$$

Optimal antiderivative

$$\frac{2b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}} + \frac{2 \sin(dx+c) \sqrt{b \cos(dx+c)}}{3d}$$

command

```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx-c)+i\sin(dx-c))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b\cos(dx+c)}\cos(dx+c),x\right)$$

47.28 Problem number 71

$$\int \sqrt{b\cos(c+dx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{b\cos(dx+c)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{\cos(dx+c)}}$$

command

```
integrate((b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i\sqrt{2}\sqrt{b}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\sqrt{b}\operatorname{weierstrassZeta}(-4,0,\cos(dx-c)+i\sin(dx-c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b\cos(dx+c)},x\right)$$

47.29 Problem number 72

$$\int \sqrt{b\cos(c+dx)}\sec(c+dx) dx$$

Optimal antiderivative

$$\frac{2b\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{b\cos(dx+c)}}$$

command

```
integrate(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx-c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b\cos(dx+c)}\sec(dx+c),x\right)$$

47.30 Problem number 73

$$\int \sqrt{b\cos(c+dx)}\sec^2(c+dx)dx$$

Optimal antiderivative

$$\frac{2b\sin(dx+c)}{d\sqrt{b\cos(dx+c)}} - \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right),\sqrt{2}\right)\sqrt{b\cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d\sqrt{\cos(dx+c)}}$$

command

```
integrate(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx-c)+i\sin(dx-c)))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b\cos(dx+c)}\sec(dx+c)^2,x\right)$$

47.31 Problem number 74

$$\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx+c)}{3d(b \cos(dx+c))^{\frac{3}{2}}} + \frac{2b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}}$$

command

```
integrate(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} \sqrt{b} \cos(dx+c)^2 \operatorname{weierstrassPInverse}(4, 0, \cos(dx+c) - i \sin(dx+c))}{3 d \cos(dx+c)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx+c)} \sec(dx+c)^3, x\right)$$

47.32 Problem number 75

$$\int \sqrt{b \cos(c+dx)} \sec^4(c+dx) dx$$

Optimal antiderivative

$$\frac{2b^3 \sin(dx+c)}{5d(b \cos(dx+c))^{\frac{5}{2}}} + \frac{6b \sin(dx+c)}{5d \sqrt{b \cos(dx+c)}} - \frac{6 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}}$$

command

```
integrate(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-3i \sqrt{2} \sqrt{b} \cos(dx+c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 3i \sqrt{2} \sqrt{b} \cos(dx+c)^3 \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx+c) - i \sin(dx+c)))}{3 d \cos(dx+c)^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx+c)} \sec(dx+c)^4, x\right)$$

47.33 Problem number 76

$$\int \sqrt{b \cos(c + dx)} \sec^5(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^4 \sin(dx + c)}{7d(b \cos(dx + c))^{\frac{7}{2}}} + \frac{10b^2 \sin(dx + c)}{21d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{10b \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

command

```
integrate(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} \sqrt{b} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} \sqrt{b} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))}{21 d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} \sec(dx + c)^5, x\right)$$

47.34 Problem number 77

$$\int \sqrt{b \cos(c + dx)} \sec^6(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^5 \sin(dx + c)}{9d(b \cos(dx + c))^{\frac{9}{2}}} + \frac{14b^3 \sin(dx + c)}{45d(b \cos(dx + c))^{\frac{5}{2}}} + \frac{14b \sin(dx + c)}{15d \sqrt{b \cos(dx + c)}} - \frac{14 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-21i\sqrt{2}\sqrt{b}\cos(dx+c)^5 \operatorname{weierstrassZeta}(-4,0, \operatorname{weierstrassPInverse}(-4,0, \cos(dx+c) + i\sin(dx+c))) + 21i\sqrt{2}\sqrt{b}\cos(dx+c)^5 \operatorname{weierstrassPInverse}(-4,0, \cos(dx+c) + i\sin(dx+c))}{77d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b\cos(dx+c)} \sec(dx+c)^6, x\right)$$

47.35 Problem number 78

$$\int \cos^4(c+dx)(b\cos(c+dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{18(b\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{77bd} + \frac{2(b\cos(dx+c))^{\frac{9}{2}}\sin(dx+c)}{11b^3d} \\ & + \frac{30b^2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{77\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b\cos(dx+c)}} \\ & + \frac{30b\sin(dx+c)\sqrt{b\cos(dx+c)}}{77d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-15i\sqrt{2}b^{\frac{3}{2}}\operatorname{weierstrassPInverse}(-4,0, \cos(dx+c) + i\sin(dx+c)) + 15i\sqrt{2}b^{\frac{3}{2}}\operatorname{weierstrassPInverse}(-4,0, \cos(dx+c) + i\sin(dx+c))}{77d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b\cos(dx+c)} b\cos(dx+c)^5, x\right)$$

47.36 Problem number 79

$$\int \cos^3(c+dx)(b\cos(c+dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{14(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45d} + \frac{2(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^2d}$$

$$+ \frac{14b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} b \cos(dx + c)^4, x\right)$$

47.37 Problem number 80

$$\int \cos^2(c + dx)(b \cos(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7bd}$$

$$+ \frac{10b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

$$+ \frac{10b \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d}$$

command

```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

21 d

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} b \cos(dx + c)^3, x\right)$$

47.38 Problem number 81

$$\int \cos(c + dx)(b \cos(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5d} + \frac{6b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{b \cos(dx + c)} b \cos(dx + c) \sin(dx + c) + 3i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)))}{5d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} b \cos(dx + c)^2, x\right)$$

47.39 Problem number 82

$$\int (b \cos(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} + \frac{2b \sin(dx + c) \sqrt{b \cos(dx + c)}}{3d}$$

command

```
integrate((b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)) + i \sin(dx + c) + i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} b \cos(dx + c), x\right)$$

47.40 Problem number 83

$$\int (b \cos(c + dx))^{3/2} \sec(c + dx) dx$$

Optimal antiderivative

$$\frac{2b \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) - i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}(0, -4, 0, \cos(dx+c) + i \sin(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx+c)} b \cos(dx+c) \sec(dx+c), x\right)$$

47.41 Problem number 84

$$\int (b \cos(c + dx))^{3/2} \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^2 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx+c)} b \cos(dx+c) \sec(dx+c)^2, x\right)$$

47.42 Problem number 85

$$\int (b \cos(c + dx))^{3/2} \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx + c)}{d\sqrt{b \cos(dx + c)}} - \frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b^{\frac{3}{2}} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} b^{\frac{3}{2}} \cos(dx + c)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} b \cos(dx + c) \sec(dx + c)^3, x\right)$$

47.43 Problem number 86

$$\int (b \cos(c + dx))^{3/2} \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^3 \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b^{\frac{3}{2}} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} b^{\frac{3}{2}} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3 d \cos(dx + c)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} b \cos(dx + c) \sec(dx + c)^4, x\right)$$

47.44 Problem number 87

$$\int (b \cos(c + dx))^{3/2} \sec^5(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^4 \sin(dx + c)}{5d (b \cos(dx + c))^{5/2}} + \frac{6b^2 \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} - \frac{6b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} b^{3/2} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2} b$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} b \cos(dx + c) \sec(dx + c)^5, x\right)$$

47.45 Problem number 88

$$\int (b \cos(c + dx))^{3/2} \sec^6(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^5 \sin(dx + c)}{7d (b \cos(dx + c))^{7/2}} + \frac{10b^3 \sin(dx + c)}{21d (b \cos(dx + c))^{3/2}} + \frac{10b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} b^{\frac{3}{2}} \cos(dx+c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} b^{\frac{3}{2}} \cos(dx+c)^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))}{21 d \cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \cos(dx+c)} b \cos(dx+c) \sec(dx+c)^6, x\right)$$

47.46 Problem number 89

$$\int (b \cos(c+dx))^{3/2} \sec^7(c+dx) dx$$

Optimal antiderivative

$$\frac{2b^6 \sin(dx+c)}{9d (b \cos(dx+c))^{\frac{9}{2}}} + \frac{14b^4 \sin(dx+c)}{45d (b \cos(dx+c))^{\frac{5}{2}}} + \frac{14b^2 \sin(dx+c)}{15d \sqrt{b \cos(dx+c)}} - \frac{14b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}}$$

command

`integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-21i \sqrt{2} b^{\frac{3}{2}} \cos(dx+c)^5 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 21i \sqrt{2} b^{\frac{3}{2}} \cos(dx+c)^5 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{21 d \cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \cos(dx+c)} b \cos(dx+c) \sec(dx+c)^7, x\right)$$

47.47 Problem number 90

$$\int \cos^3(c+dx)(b \cos(c+dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{18(b \cos(dx+c))^{\frac{5}{2}} \sin(dx+c)}{77d} + \frac{2(b \cos(dx+c))^{\frac{9}{2}} \sin(dx+c)}{11b^2d} + \frac{30b^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}} + \frac{30b^2 \sin(dx+c) \sqrt{b \cos(dx+c)}}{77d}$$

command

```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i \sqrt{2} b^{\frac{5}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} b^{\frac{5}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

77 d

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \cos(dx + c)} b^2 \cos(dx + c)^5, x\right)$$

47.48 Problem number 91

$$\int \cos^2(c + dx)(b \cos(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14b(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45d} + \frac{2(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9bd} \\ & + \frac{14b^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i \sqrt{2} b^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2} b^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \cos(dx + c)} b^2 \cos(dx + c)^4, x\right)$$

47.49 Problem number 92

$$\int \cos(c + dx)(b \cos(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7d} \\ & + \frac{10b^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{10b^2 \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} b^{\frac{5}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} b^{\frac{5}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{21d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} b^2 \cos(dx + c)^3, x\right)$$

47.50 Problem number 93

$$\int (b \cos(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5d} \\ & + \frac{6b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{b \cos(dx+c)} b^2 \cos(dx+c) \sin(dx+c) + 3i \sqrt{2} b^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c)))}{5d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \cos(dx+c)} b^2 \cos(dx+c)^2, x\right)$$

47.51 Problem number 94

$$\int (b \cos(c+dx))^{\frac{5}{2}} \sec(c+dx) dx$$

Optimal antiderivative

$$\frac{2b^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}} + \frac{2b^2 \sin(dx+c) \sqrt{b \cos(dx+c)}}{3d}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b^{\frac{5}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} b^{\frac{5}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx+c))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \cos(dx+c)} b^2 \cos(dx+c)^2 \sec(dx+c), x\right)$$

47.52 Problem number 95

$$\int (b \cos(c+dx))^{\frac{5}{2}} \sec^2(c+dx) dx$$

Optimal antiderivative

$$\frac{2b^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} b^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - i \sqrt{2} b^{\frac{5}{2}} \text{weierstrassZeta}(0, -4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \cos(dx + c)} b^2 \cos(dx + c)^2 \sec(dx + c)^2, x\right)$$

47.53 Problem number 96

$$\int (b \cos(c + dx))^{5/2} \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b^{\frac{5}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} b^{\frac{5}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \cos(dx + c)} b^2 \cos(dx + c)^2 \sec(dx + c)^3, x\right)$$

47.54 Problem number 97

$$\int (b \cos(c + dx))^{5/2} \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^3 \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} - \frac{2b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b^{\frac{5}{2}} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} b^{\frac{5}{2}} \cos(dx + c)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} b^2 \cos(dx + c)^2 \sec(dx + c)^4, x\right)$$

47.55 Problem number 98

$$\int (b \cos(c + dx))^{5/2} \sec^5(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^4 \sin(dx + c)}{3d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{2b^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b^{\frac{5}{2}} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} b^{\frac{5}{2}} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{3 d \cos(dx + c)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} b^2 \cos(dx + c)^2 \sec(dx + c)^5, x\right)$$

47.56 Problem number 99

$$\int (b \cos(c + dx))^{5/2} \sec^6(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^5 \sin(dx + c)}{5d (b \cos(dx + c))^{5/2}} + \frac{6b^3 \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} - \frac{6b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} b^{5/2} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2} b$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c)} b^2 \cos(dx + c)^2 \sec(dx + c)^6, x\right)$$

47.57 Problem number 100

$$\int (b \cos(c + dx))^{5/2} \sec^7(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^6 \sin(dx + c)}{7d (b \cos(dx + c))^{7/2}} + \frac{10b^4 \sin(dx + c)}{21d (b \cos(dx + c))^{3/2}} + \frac{10b^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} b^{\frac{5}{2}} \cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} b^{\frac{5}{2}} \cos(dx+c)^4 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))}{21 d \cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx+c)} b^2 \cos(dx+c)^2 \sec(dx+c)^7, x\right)$$

47.58 Problem number 101

$$\int (b \cos(c+dx))^{5/2} \sec^8(c+dx) dx$$

Optimal antiderivative

$$\frac{2b^7 \sin(dx+c)}{9d (b \cos(dx+c))^{\frac{9}{2}}} + \frac{14b^5 \sin(dx+c)}{45d (b \cos(dx+c))^{\frac{5}{2}}} + \frac{14b^3 \sin(dx+c)}{15d \sqrt{b \cos(dx+c)}} - \frac{14b^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}}$$

command

`integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-21i \sqrt{2} b^{\frac{5}{2}} \cos(dx+c)^5 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 21i \sqrt{2} b^{\frac{5}{2}} \cos(dx+c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{21 d \cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx+c)} b^2 \cos(dx+c)^2 \sec(dx+c)^8, x\right)$$

47.59 Problem number 102

$$\int (b \cos(c+dx))^{7/2} dx$$

Optimal antiderivative

$$\frac{2b(b \cos(dx+c))^{\frac{5}{2}} \sin(dx+c)}{7d} + \frac{10b^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}} + \frac{10b^3 \sin(dx+c) \sqrt{b \cos(dx+c)}}{21d}$$

command

```
integrate((b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} b^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} b^{\frac{7}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{21 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \cos(dx + c)} b^3 \cos(dx + c)^3, x\right)$$

47.60 Problem number 103

$$\int \frac{\cos^6(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{18(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{77b^3d} + \frac{2(b \cos(dx + c))^{\frac{9}{2}} \sin(dx + c)}{11b^5d} \\ & + \frac{30 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{30 \sin(dx + c) \sqrt{b \cos(dx + c)}}{77bd} \end{aligned}$$

command

```
integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(7 \cos(dx + c)^4 + 9 \cos(dx + c)^2 + 15\right) \sqrt{b \cos(dx + c)} \sin(dx + c) - 15i \sqrt{2} \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{77bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \cos(dx + c)} \cos(dx + c)^5}{b}, x\right)$$

47.61 Problem number 104

$$\int \frac{\cos^5(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45b^2d} + \frac{2(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^4d} \\ & + \frac{14 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \cos(dx + c)^3 + 7 \cos(dx + c) \right) \sqrt{b \cos(dx + c)} \sin(dx + c) + 21i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassP}(dx + c, b))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \cos(dx + c)^4}{b}, x\right)$$

47.62 Problem number 105

$$\int \frac{\cos^4(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^3d} \\ & + \frac{10 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{10 \sin(dx + c) \sqrt{b \cos(dx + c)}}{21bd} \end{aligned}$$

command

`integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{b \cos(dx+c)} \left(3 \cos(dx+c)^2 + 5 \right) \sin(dx+c) - 5i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{21bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \cos(dx+c)} \cos(dx+c)^3}{b}, x \right)$$

47.63 Problem number 106

$$\int \frac{\cos^3(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{5b^2d} + \frac{6 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) \sqrt{b \cos(dx+c)}}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) bd \sqrt{\cos(dx+c)}}$$

command

`integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{b \cos(dx+c)} \cos(dx+c) \sin(dx+c) + 3i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))}{5b^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \cos(dx+c)} \cos(dx+c)^2}{b}, x \right)$$

47.64 Problem number 107

$$\int \frac{\cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}} + \frac{2 \sin(dx+c) \sqrt{b \cos(dx+c)}}{3bd}$$

command

`integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx-c) + i \sin(dx-c))}{3bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c)} \cos(dx+c)}{b}, x\right)$$

47.65 Problem number 108

$$\int \frac{\cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx+c)}}$$

command

`integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) - i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx-c) + i \sin(dx-c)))}{bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c)}}{b}, x\right)$$

47.66 Problem number 109

$$\int \frac{1}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx+c)}}$$

command

```
integrate(1/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx-c)+i\sin(dx-c))}{bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c)}}{b \cos(dx+c)}, x\right)$$

47.67 Problem number 110

$$\int \frac{\sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx+c)}{d\sqrt{b \cos(dx+c)}} - \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd\sqrt{\cos(dx+c)}}$$

command

```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\cos(dx-c)\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx-c)+i\sin(dx-c)))}{bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c)} \sec(dx+c)}{b \cos(dx+c)}, x\right)$$

47.68 Problem number 111

$$\int \frac{\sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2b \sin(dx + c)}{3d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

command

```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{3bd \cos(dx + c)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b \cos(dx + c)}, x\right)$$

47.69 Problem number 112

$$\int \frac{\sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx + c)}{5d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{6 \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} - \frac{6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}}$$

command

```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-3i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3bd \cos(dx + c)^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \sec(dx + c)^3}{b \cos(dx + c)}, x\right)$$

47.70 Problem number 113

$$\int \frac{\sec^4(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2b^3 \sin(dx + c)}{7d(b \cos(dx + c))^{\frac{7}{2}}} + \frac{10b \sin(dx + c)}{21d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{10\sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx + c)}}$$

command

```
integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} \sqrt{b} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} \sqrt{b} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))}{21bd \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \sec(dx + c)^4}{b \cos(dx + c)}, x\right)$$

47.71 Problem number 114

$$\int \frac{\sec^5(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2b^4 \sin(dx + c)}{9d(b \cos(dx + c))^{\frac{9}{2}}} + \frac{14b^2 \sin(dx + c)}{45d(b \cos(dx + c))^{\frac{5}{2}}} + \frac{14 \sin(dx + c)}{15d\sqrt{b \cos(dx + c)}} - \frac{14\sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd\sqrt{\cos(dx + c)}}$$

command

```
integrate(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-21i \sqrt{2} \sqrt{b} \cos(dx + c)^5 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 21i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \sec(dx + c)^5}{b \cos(dx + c)}, x\right)$$

47.72 Problem number 115

$$\int \frac{\cos^7(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{18(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{77b^4d} + \frac{2(b \cos(dx + c))^{\frac{9}{2}} \sin(dx + c)}{11b^6d} \\ & + \frac{30 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}} \\ & + \frac{30 \sin(dx + c) \sqrt{b \cos(dx + c)}}{77b^2d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(7 \cos(dx + c)^4 + 9 \cos(dx + c)^2 + 15 \right) \sqrt{b \cos(dx + c)} \sin(dx + c) - 15i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \cos(dx + c)^5}{b^2}, x\right)$$

47.73 Problem number 116

$$\int \frac{\cos^6(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45b^3d} + \frac{2(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^5d} \\ & + \frac{14 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \cos(dx + c)^3 + 7 \cos(dx + c) \right) \sqrt{b \cos(dx + c)} \sin(dx + c) + 21i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassP}(dx + c, b))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \cos(dx + c)^4}{b^2}, x\right)$$

47.74 Problem number 117

$$\int \frac{\cos^5(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^4d} \\ & + \frac{10 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b d \sqrt{b \cos(dx + c)}} \\ & + \frac{10 \sin(dx + c) \sqrt{b \cos(dx + c)}}{21b^2d} \end{aligned}$$

command

`integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{b \cos(dx + c)} \left(3 \cos(dx + c)^2 + 5 \right) \sin(dx + c) - 5i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{21 b^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \cos(dx + c)^3}{b^2}, x\right)$$

47.75 Problem number 118

$$\int \frac{\cos^4(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5b^3 d} + \frac{6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

`integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{b \cos(dx + c)} \cos(dx + c) \sin(dx + c) + 3i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{5 b^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \cos(dx + c)^2}{b^2}, x\right)$$

47.76 Problem number 119

$$\int \frac{\cos^3(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx+c)}} + \frac{2 \sin(dx+c) \sqrt{b \cos(dx+c)}}{3b^2d}$$

command

`integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx-c) + i \sin(dx-c))}{3b^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c)} \cos(dx+c)}{b^2}, x\right)$$

47.77 Problem number 120

$$\int \frac{\cos^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{\cos(dx+c)}}$$

command

`integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) - i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx-c) + i \sin(dx-c)))}{b^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c)}}{b^2}, x\right)$$

47.78 Problem number 121

$$\int \frac{\cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd\sqrt{b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4, 0, \cos(dx - b^2d))}{b^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)}, x\right)$$

47.79 Problem number 122

$$\int \frac{1}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c)}{bd\sqrt{b \cos(dx + c)}} - \frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d\sqrt{\cos(dx + c)}}$$

command

`integrate(1/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx + c)\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c))) + i\sqrt{2}\sqrt{b}}{b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)^2}, x\right)$$

47.80 Problem number 123

$$\int \frac{\sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}}$$

command

`integrate(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{3 b^2 d \cos(dx + c)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \sec(dx + c)}{b^2 \cos(dx + c)^2}, x\right)$$

47.81 Problem number 124

$$\int \frac{\sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b \sin(dx + c)}{5d(b \cos(dx + c))^{\frac{5}{2}}} + \frac{6 \sin(dx + c)}{5bd \sqrt{b \cos(dx + c)}} - \frac{6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

`integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-3i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3 b^2 d \cos(dx + c)^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b^2 \cos(dx + c)^2}, x\right)$$

47.82 Problem number 125

$$\int \frac{\sec^3(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx + c)}{7d (b \cos(dx + c))^{\frac{7}{2}}} + \frac{10 \sin(dx + c)}{21d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{10 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}}$$

command

```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} \sqrt{b} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} \sqrt{b} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{21 b^2 d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \sec(dx + c)^3}{b^2 \cos(dx + c)^2}, x\right)$$

47.83 Problem number 126

$$\int \frac{\sec^4(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b^3 \sin(dx + c)}{9d (b \cos(dx + c))^{\frac{9}{2}}} + \frac{14b \sin(dx + c)}{45d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{14 \sin(dx + c)}{15bd \sqrt{b \cos(dx + c)}} - \frac{14 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

```
integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-21i \sqrt{2} \sqrt{b} \cos(dx + c)^5 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 21i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \sec(dx + c)^4}{b^2 \cos(dx + c)^2}, x\right)$$

47.84 Problem number 127

$$\int \frac{\cos^8(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{18(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{77b^5d} + \frac{2(b \cos(dx + c))^{\frac{9}{2}} \sin(dx + c)}{11b^7d} \\ & + \frac{30 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx + c)}} \\ & + \frac{30 \sin(dx + c) \sqrt{b \cos(dx + c)}}{77b^3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(7 \cos(dx + c)^4 + 9 \cos(dx + c)^2 + 15 \right) \sqrt{b \cos(dx + c)} \sin(dx + c) - 15i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \cos(dx + c)^5}{b^3}, x\right)$$

47.85 Problem number 128

$$\int \frac{\cos^7(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45b^4d} + \frac{2(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^6d} \\ & + \frac{14 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \cos(dx + c)^3 + 7 \cos(dx + c) \right) \sqrt{b \cos(dx + c)} \sin(dx + c) + 21i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassP}(dx + c, b))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \cos(dx + c)^4}{b^3}, x\right)$$

47.86 Problem number 129

$$\int \frac{\cos^6(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^5d} \\ & + \frac{10 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx + c)}} \\ & + \frac{10 \sin(dx + c) \sqrt{b \cos(dx + c)}}{21b^3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{b \cos(dx+c)} \left(3 \cos(dx+c)^2 + 5 \right) \sin(dx+c) - 5i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{21 b^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \cos(dx+c)} \cos(dx+c)^3}{b^3}, x \right)$$

47.87 Problem number 130

$$\int \frac{\cos^5(c+dx)}{(b \cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{5b^4 d} + \frac{6 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) \sqrt{b \cos(dx+c)}}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) b^3 d \sqrt{\cos(dx+c)}}$$

command

`integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{b \cos(dx+c)} \cos(dx+c) \sin(dx+c) + 3i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))}{5 b^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \cos(dx+c)} \cos(dx+c)^2}{b^3}, x \right)$$

47.88 Problem number 131

$$\int \frac{\cos^4(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx+c)}} + \frac{2 \sin(dx+c) \sqrt{b \cos(dx+c)}}{3b^3 d}$$

command

`integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx-c) + i \sin(dx-c))}{3b^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c)} \cos(dx+c)}{b^3}, x\right)$$

47.89 Problem number 132

$$\int \frac{\cos^3(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx+c)}}$$

command

`integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) - i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx-c) + i \sin(dx-c)))}{b^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c)}}{b^3}, x\right)$$

47.90 Problem number 133

$$\int \frac{\cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx+c)}}$$

command

```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx-c)+i\sin(dx-c))}{b^3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c)}}{b^3 \cos(dx+c)}, x\right)$$

47.91 Problem number 134

$$\int \frac{\cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx+c)}{b^2 d \sqrt{b \cos(dx+c)}} - \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx+c)}}$$

command

```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\cos(dx-c)\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx-c)+i\sin(dx-c)))}{b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c)}}{b^3 \cos(dx+c)^2}, x\right)$$

47.92 Problem number 135

$$\int \frac{1}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c)}{3bd (b \cos(dx + c))^{3/2}} + \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx + c)}}$$

command

`integrate(1/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3 b^3 d \cos(dx + c)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)^3}, x\right)$$

47.93 Problem number 136

$$\int \frac{\sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c)}{5d (b \cos(dx + c))^{5/2}} + \frac{6 \sin(dx + c)}{5b^2 d \sqrt{b \cos(dx + c)}} - \frac{6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx + c)}}$$

command

`integrate(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-3i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3 b^3 d \cos(dx + c)^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \sec(dx + c)}{b^3 \cos(dx + c)^3}, x\right)$$

47.94 Problem number 137

$$\int \frac{\sec^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2b \sin(dx + c)}{7d (b \cos(dx + c))^{\frac{7}{2}}} + \frac{10 \sin(dx + c)}{21bd (b \cos(dx + c))^{\frac{3}{2}}} + \frac{10 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx + c)}}$$

command

```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} \sqrt{b} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} \sqrt{b} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{21 b^3 d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b^3 \cos(dx + c)^3}, x\right)$$

47.95 Problem number 138

$$\int \frac{\sec^3(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx + c)}{9d (b \cos(dx + c))^{\frac{9}{2}}} + \frac{14 \sin(dx + c)}{45d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{14 \sin(dx + c)}{15b^2 d \sqrt{b \cos(dx + c)}} - \frac{14 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx + c)}}$$

command

```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-21i \sqrt{2} \sqrt{b} \cos(dx + c)^5 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 21i \sqrt{2} \sqrt{b} \cos(dx + c)^5 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 21i \sqrt{2} \sqrt{b} \cos(dx + c)^5 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 21i \sqrt{2} \sqrt{b} \cos(dx + c)^5 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \cos(dx + c)} \sec(dx + c)^3}{b^3 \cos(dx + c)^3}, x\right)$$

47.96 Problem number 139

$$\int \frac{1}{(b \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c)}{5bd (b \cos(dx + c))^{5/2}} + \frac{6 \sin(dx + c)}{5b^3 d \sqrt{b \cos(dx + c)}} - \frac{6 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{\cos(dx + c)}}$$

command

`integrate(1/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 3i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \cos(dx + c)}}{b^4 \cos(dx + c)^4}, x\right)$$

47.97 Problem number 263

$$\int \cos^2(a + bx) \sqrt{\csc(a + bx)} dx$$

Optimal antiderivative

$$\frac{2 \cos(bx + a)}{3b \sqrt{\csc(bx + a)}} - \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\csc(bx + a)}) (\sqrt{\sin(bx + a)})}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(cos(b*x+a)^2*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\cos(bx + a) \sqrt{\sin(bx + a)} - i \sqrt{2i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a)) + i \sqrt{-2i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) - i \sin(bx + a)) \right)}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\cos(bx + a)^2 \sqrt{\csc(bx + a)}, x\right)$$

47.98 Problem number 264

$$\int \frac{\cos^2(a + bx)}{\sqrt{\csc(a + bx)}} dx$$

Optimal antiderivative

$$\frac{2 \cos(bx + a)}{5b \csc(bx + a)^{\frac{3}{2}}} - \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\csc(bx + a)}) (\sqrt{\sin(bx + a)})}{5 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(cos(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + i \sin (bx + a))) + \sqrt{-2i} \operatorname{weierstrassZeta}(4, 0, \dots) \right)}{5b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\cos (bx + a)^2}{\sqrt{\csc (bx + a)}}, x \right)$$

47.99 Problem number 268

$$\int \cos^4(a + bx) \sqrt{\csc(a + bx)} dx$$

Optimal antiderivative

$$\frac{\frac{4 \cos (bx + a)}{7b \sqrt{\csc (bx + a)}} + \frac{2(\cos^3 (bx + a))}{7b \sqrt{\csc (bx + a)}}}{8 \sqrt{\frac{1}{2} + \frac{\sin (bx + a)}{2}}} \operatorname{EllipticF} \left(\cos \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2} \right), \sqrt{2} \right) (\sqrt{\csc (bx + a)}) (\sqrt{\sin (bx + a)})}{7 \sin \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2} \right) b}$$

command

```
integrate(cos(b*x+a)^4*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\left(\cos (bx + a)^3 + 2 \cos (bx + a) \right) \sqrt{\sin (bx + a)} - 2i \sqrt{2i} \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + i \sin (bx + a)) \right)}{7b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\cos (bx + a)^4 \sqrt{\csc (bx + a)}, x \right)$$

47.100 Problem number 269

$$\int \frac{\cos^4(a + bx)}{\sqrt{\csc(a + bx)}} dx$$

Optimal antiderivative

$$\frac{\frac{4 \cos (bx+a)}{15 b \csc (bx+a)^{\frac{3}{2}}} + \frac{2\left(\cos ^3 (bx+a)\right)}{9 b \csc (bx+a)^{\frac{3}{2}}}}{8 \sqrt{\frac{1}{2} + \frac{\sin (bx+a)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \left(\sqrt{\csc (bx+a)}\right) \left(\sqrt{\sin (bx+a)}\right)} - \frac{15 \sin \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(cos(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6 \sqrt{2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (bx+a) + i \sin (bx+a))) + 6 \sqrt{-2i} \operatorname{weierstrassZeta}(4, 0, \cos (bx+a) - i \sin (bx+a)) \right) / 45 b$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos (bx+a)^4}{\sqrt{\csc (bx+a)}}, x\right)$$

47.101 Problem number 273

$$\int \sqrt{\csc (a+bx)} \sec ^2(a+bx) dx$$

Optimal antiderivative

$$\frac{\frac{\sec (bx+a)}{b \sqrt{\csc (bx+a)}}}{\sqrt{\frac{1}{2} + \frac{\sin (bx+a)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \left(\sqrt{\csc (bx+a)}\right) \left(\sqrt{\sin (bx+a)}\right)} - \frac{\sin \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2i} \cos (bx+a) \operatorname{weierstrassPInverse}(4, 0, \cos (bx+a) + i \sin (bx+a)) + i \sqrt{-2i} \cos (bx+a) \operatorname{weierstrassPInverse}(4, 0, \cos (bx+a) - i \sin (bx+a))}{2 b \cos (bx+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{\csc (bx+a)} \sec (bx+a)^2, x\right)$$

47.102 Problem number 274

$$\int \frac{\sec^2(a + bx)}{\sqrt{\csc(a + bx)}} dx$$

Optimal antiderivative

$$\frac{\sec(bx + a)}{b \csc(bx + a)^{\frac{3}{2}}} + \frac{\sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\csc(bx + a)}) (\sqrt{\sin(bx + a)})}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(sec(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2i} \cos(bx + a) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a))) + \sqrt{-2i} \cos(bx + a)}{2b \cos(bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(bx + a)^2}{\sqrt{\csc(bx + a)}}, x\right)$$

47.103 Problem number 277

$$\int \sqrt{\csc(a + bx)} \sec^4(a + bx) dx$$

Optimal antiderivative

$$\frac{5 \sec(bx + a)}{6b \sqrt{\csc(bx + a)}} + \frac{\sec^3(bx + a)}{3b \sqrt{\csc(bx + a)}} - \frac{5 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\csc(bx + a)}) (\sqrt{\sin(bx + a)})}{6 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2i} \cos (bx+a)^3 \text{weierstrassPInverse}(4,0, \cos (bx+a)+i \sin (bx+a))+5i \sqrt{-2i} \cos (bx+a)^3 \text{weierstrassPInverse}(4,0, \cos (bx+a)-i \sin (bx+a))}{12 b \cos (bx+a)^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{\csc (bx+a)} \sec (bx+a)^4, x\right)$$

47.104 Problem number 278

$$\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}} dx$$

Optimal antiderivative

$$\frac{\sec (bx+a)}{2 b \csc (bx+a)^{\frac{3}{2}}}+\frac{\sec ^3(bx+a)}{3 b \csc (bx+a)^{\frac{3}{2}}}+\frac{\sqrt{\frac{1}{2}+\frac{\sin (bx+a)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{a}{2}+\frac{\pi}{4}+\frac{bx}{2}\right), \sqrt{2}\right)\left(\sqrt{\csc (bx+a)}\right)\left(\sqrt{\sin (bx+a)}\right)}{2 \sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{bx}{2}\right) b}$$

command

```
integrate(sec(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \sqrt{2i} \cos (bx+a)^3 \text{weierstrassZeta}(4,0, \text{weierstrassPInverse}(4,0, \cos (bx+a)+i \sin (bx+a)))+3 \sqrt{-2i} \cos (bx+a)^3 \text{weierstrassZeta}(4,0, \text{weierstrassPInverse}(4,0, \cos (bx+a)-i \sin (bx+a)))}{12 b \cos (bx+a)^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sec (bx+a)^4}{\sqrt{\csc (bx+a)}}, x\right)$$

48 Test file number 83

Test folder name:

test_cases/4_Trig_functions/4.2_Cosine/83_4.2.10-c+d_x-^m-a+b_cos-^n

48.1 Problem number 76

$$\int \sqrt{\cos(a + bx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(cos(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i\sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) - i\sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \cos(bx + a) + i \sin(bx + a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{\cos(bx + a)}, x\right)$$

48.2 Problem number 79

$$\int \cos^{\frac{3}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b} + \frac{2 \sin(bx + a) (\sqrt{\cos(bx + a)})}{3b}$$

command

```
integrate(cos(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{\cos(bx + a)} \sin(bx + a) - i\sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) + i\sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\cos(bx + a)^{\frac{3}{2}}, x\right)$$

48.3 Problem number 84

$$\int \frac{1}{\sqrt{\cos(ax + bx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(bx + a)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) + i\sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{\cos(bx + a)}}, x\right)$$

48.4 Problem number 87

$$\int \frac{1}{\cos^{\frac{3}{2}}(ax + bx)} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{\cos(bx + a)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b} + \frac{2 \sin(bx + a)}{b\sqrt{\cos(bx + a)}}$$

command

```
integrate(1/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2} \cos(bx + a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) + i\sqrt{2} \cos(bx + a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a)))}{b \cos(bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\cos^{\frac{3}{2}}(bx + a)}, x\right)$$

49 Test file number 84

Test folder name:

test_cases/4_Trig_functions/4.2_Cosine/84_4.2.1.1-a+b_cos-^n

49.1 Problem number 50

$$\int (a + b \cos(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5d} + \frac{16ab \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15d} \\ & + \frac{2(23a^2 + 9b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{16a(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (i a^3 - 33i ab^2) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3ib \sin(dx+c) + 2a}{3b}\right) + \sqrt{2} (-i a^3 +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

49.2 Problem number 51

$$\int (a + b \cos(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3d} + \frac{8a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} - \frac{2(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12i \sqrt{2} ab^{\frac{3}{2}} \operatorname{weierstrassZeta}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}\right), \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx + c)}{a + b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((b \cos(dx + c) + a)^{\frac{3}{2}}, x\right)$$

49.3 Problem number 52

$$\int \sqrt{a + b \cos(c + dx)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

command

`integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}a\sqrt{b}\operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right) + i\sqrt{2}a\sqrt{b}\operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right)}{bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b\cos(dx+c)+a}, x\right)$$

49.4 Problem number 53

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{a+b\cos(dx+c)}}$$

command

`integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right) + i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right)}{bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

49.5 Problem number 54

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b \sin(dx + c)}{(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \end{aligned}$$

command

`integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \cos(dx + c) + a} b^2 \sin(dx + c) + \left(i \sqrt{2} ab \cos(dx + c) + i \sqrt{2} a^2\right) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8}{3b^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c) + a}}{b^2 \cos^2(dx + c) + 2ab \cos(dx + c) + a^2}, x\right)$$

49.6 Problem number 55

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b \sin(dx + c)}{3(a^2 - b^2) d (a + b \cos(dx + c))^{3/2}} - \frac{8ab \sin(dx + c)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{8a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(4ab^3 \cos(dx+c) + 5a^2b^2 - b^4) \sqrt{b \cos(dx+c) + a} \sin(dx+c) - \left(\sqrt{2}(-ia^2b^2 - 3ib^4) \cos(dx+c)^2 - 2\sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \cos(dx+c) + a}}{b^3 \cos(dx+c)^3 + 3ab^2 \cos(dx+c)^2 + 3a^2b \cos(dx+c) + a^3}, x\right)$$

50 Test file number 86

Test folder name:

test_cases/4_Trig_functions/4.2_Cosine/86_4.2.1.2-g_sin~p-a+b_cos~m

50.1 Problem number 33

$$\int (a + b \cos(c + dx))(e \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ae \cos(dx+c)(e \sin(dx+c))^{5/2}}{7d} + \frac{2b(e \sin(dx+c))^{9/2}}{9de} \\ & - \frac{10ae^4 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \text{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx+c)}\right)}{21 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{e \sin(dx+c)}} \\ & - \frac{10ae^3 \cos(dx+c) \sqrt{e \sin(dx+c)}}{21d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15\sqrt{2}\sqrt{-i}ae^{7/2}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+15\sqrt{2}\sqrt{i}ae^{7/2}\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(be^3 \cos(dx+c)^3 + ae^3 \cos(dx+c)^2 - be^3 \cos(dx+c) - ae^3\right) \sqrt{e \sin(dx+c)} \sin(dx+c), x\right)$$

50.2 Problem number 34

$$\int (a + b \cos(c + dx))(e \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ae \cos(dx + c) (e \sin(dx + c))^{\frac{3}{2}}}{5d} + \frac{2b(e \sin(dx + c))^{\frac{7}{2}}}{7de} \\ & - \frac{6ae^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\sin(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21i \sqrt{2} \sqrt{-i} a e^{\frac{5}{2}} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2} \sqrt{i} a e^{\frac{5}{2}} \operatorname{weierstrassZeta}(4, 0, \cos(dx + c) - i \sin(dx + c))}{d \sqrt{\sin(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(b e^2 \cos(dx + c)^3 + a e^2 \cos(dx + c)^2 - b e^2 \cos(dx + c) - a e^2\right) \sqrt{e \sin(dx + c)}, x\right)$$

50.3 Problem number 35

$$\int (a + b \cos(c + dx))(e \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(e \sin(dx + c))^{\frac{5}{2}}}{5de} \\ & - \frac{2ae^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{e \sin(dx + c)}} \\ & - \frac{2ae \cos(dx + c) \sqrt{e \sin(dx + c)}}{3d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}\sqrt{-i}ae^{\frac{3}{2}}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}\sqrt{i}ae^{\frac{3}{2}}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))}{15d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left((be\cos(dx+c)+ae)\sqrt{e\sin(dx+c)}\sin(dx+c),x\right)\right)$$

50.4 Problem number 36

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}dx$$

Optimal antiderivative

$$\frac{2b(e\sin(dx+c))^{\frac{3}{2}}}{3de} - \frac{2a\sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \text{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e\sin(dx+c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d\sqrt{\sin(dx+c)}}$$

command

`integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3i\sqrt{2}\sqrt{-i}ae^{\frac{1}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{i}ae^{\frac{1}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left((b\cos(dx+c)+a)\sqrt{e\sin(dx+c)},x\right)\right)$$

50.5 Problem number 37

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}dx$$

Optimal antiderivative

$$-\frac{2a\sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \text{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx+c)}\right)}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d\sqrt{e\sin(dx+c)}} + \frac{2b\sqrt{e\sin(dx+c)}}{de}$$

command

```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{2} \sqrt{-i} \operatorname{aweierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} \sqrt{i} \operatorname{aweierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))\right)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b \cos(dx + c) + a) \sqrt{e \sin(dx + c)}}{e \sin(dx + c)}, x\right)$$

50.6 Problem number 38

$$\int \frac{a + b \cos(c + dx)}{(e \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b}{de \sqrt{e \sin(dx + c)}} - \frac{2a \cos(dx + c)}{de \sqrt{e \sin(dx + c)}} \\ & + \frac{2a \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d e^2 \sqrt{\sin(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i \sqrt{2} \sqrt{-i} a \sin(dx + c) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} \sqrt{i} a \sin(dx + c) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)))\right)}{d e^2 \sqrt{\sin(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(b \cos(dx + c) + a) \sqrt{e \sin(dx + c)}}{e^2 \cos(dx + c)^2 - e^2}, x\right)$$

50.7 Problem number 39

$$\int \frac{a + b \cos(c + dx)}{(e \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b}{3de (e \sin(dx + c))^{\frac{3}{2}}} - \frac{2a \cos(dx + c)}{3de (e \sin(dx + c))^{\frac{3}{2}}} \\ & - \frac{2a \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d e^2 \sqrt{e \sin(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-i} \left(\sqrt{2} a \cos(dx + c)^2 - \sqrt{2} a \right) \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{i} \left(\sqrt{2} a \cos(dx + c) \right)}{3 \left(d \cos(dx + c) \right)^2 e^{\frac{5}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(b \cos(dx + c) + a) \sqrt{e \sin(dx + c)}}{(e^3 \cos(dx + c)^2 - e^3) \sin(dx + c)}, x\right)$$

50.8 Problem number 40

$$\int \frac{a + b \cos(c + dx)}{(e \sin(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b}{5de (e \sin(dx + c))^{\frac{5}{2}}} - \frac{2a \cos(dx + c)}{5de (e \sin(dx + c))^{\frac{5}{2}}} - \frac{6a \cos(dx + c)}{5de^3 \sqrt{e \sin(dx + c)}} \\ & + \frac{6a \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d e^4 \sqrt{\sin(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \sqrt{-i} \left(i \sqrt{2} a \cos(dx + c)^2 - i \sqrt{2} a \right) \sin(dx + c) \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx + c) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b \cos(dx + c) + a) \sqrt{e \sin(dx + c)}}{e^4 \cos(dx + c)^4 - 2e^4 \cos(dx + c)^2 + e^4}, x \right)$$

50.9 Problem number 41

$$\int (a + b \cos(c + dx))^2 (e \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(11a^2 + 2b^2) e \cos(dx + c) (e \sin(dx + c))^{5/2}}{77d} \\ & + \frac{26ab(e \sin(dx + c))^{9/2}}{99de} + \frac{2b(a + b \cos(dx + c)) (e \sin(dx + c))^{9/2}}{11de} \\ & - \frac{10(11a^2 + 2b^2) e^4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right) \left(\sqrt{\sin(dx + c)} \right)}{231 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) d \sqrt{e \sin(dx + c)}} \\ & - \frac{10(11a^2 + 2b^2) e^3 \cos(dx + c) \sqrt{e \sin(dx + c)}}{231d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} \sqrt{-i} (11 a^2 + 2 b^2) e^{7/2} \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \sqrt{2} \sqrt{i} (11 a^2 + 2 b^2) e^{7/2} \text{we}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \left(b^2 e^3 \cos(dx + c)^4 + 2 a b e^3 \cos(dx + c)^3 - 2 a b e^3 \cos(dx + c) + (a^2 - b^2) e^3 \cos(dx + c)^2 - a^2 e^3 \right) \sqrt{e \sin(dx + c)}, x \right)$$

50.10 Problem number 42

$$\int (a + b \cos(c + dx))^2 (e \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(9a^2 + 2b^2) e \cos(dx + c) (e \sin(dx + c))^{3/2}}{45d} \\ & + \frac{22ab(e \sin(dx + c))^{7/2}}{63de} + \frac{2b(a + b \cos(dx + c)) (e \sin(dx + c))^{7/2}}{9de} \\ & - \frac{2(9a^2 + 2b^2) e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{15 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\sin(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i \sqrt{2} \sqrt{-i} (9a^2 + 2b^2) e^{5/2} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(b^2 e^2 \cos(dx + c)^4 + 2abe^2 \cos(dx + c)^3 - 2abe^2 \cos(dx + c) + (a^2 - b^2)e^2 \cos(dx + c)^2 - a^2 e^2\right) \sqrt{e \sin(dx + c)}\right)$$

50.11 Problem number 43

$$\int (a + b \cos(c + dx))^2 (e \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{18ab(e \sin(dx + c))^{5/2}}{35de} + \frac{2b(a + b \cos(dx + c)) (e \sin(dx + c))^{5/2}}{7de} \\ & - \frac{2(7a^2 + 2b^2) e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{21 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{e \sin(dx + c)}} \\ & - \frac{2(7a^2 + 2b^2) e \cos(dx + c) \sqrt{e \sin(dx + c)}}{21d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} \sqrt{-i} (7a^2 + 2b^2) e^{\frac{3}{2}} \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} \sqrt{i} (7a^2 + 2b^2) e^{\frac{3}{2}} \text{weierst}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 e \cos(dx + c)^2 + 2abe \cos(dx + c) + a^2 e\right) \sqrt{e \sin(dx + c)} \sin(dx + c), x\right)$$

50.12 Problem number 44

$$\int (a + b \cos(c + dx))^2 \sqrt{e \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{14ab(e \sin(dx + c))^{\frac{3}{2}}}{15de} + \frac{2b(a + b \cos(dx + c))(e \sin(dx + c))^{\frac{3}{2}}}{5de} - \frac{2(5a^2 + 2b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\sin(dx + c)}}$$

command

```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i \sqrt{2} \sqrt{-i} (5a^2 + 2b^2) e^{\frac{1}{2}} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) - 3i \sqrt{2} \sqrt{i} (5a^2 + 2b^2) e^{\frac{1}{2}} \text{weierst}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2\right) \sqrt{e \sin(dx + c)}, x\right)$$

50.13 Problem number 45

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{e \sin(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(3a^2 + 2b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{e \sin(dx + c)}} + \frac{10ab \sqrt{e \sin(dx + c)}}{3de} + \frac{2b(a + b \cos(dx + c)) \sqrt{e \sin(dx + c)}}{3de}$$

command

```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{2} \sqrt{-i} (3a^2 + 2b^2) \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} \sqrt{i} (3a^2 + 2b^2) \operatorname{weierstrassP}\right)}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2\right) \sqrt{e \sin(dx + c)}}{e \sin(dx + c)}, x\right)$$

50.14 Problem number 46

$$\int \frac{(a + b \cos(c + dx))^2}{(e \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2ab(e \sin(dx + c))^{3/2}}{de^3} - \frac{2(b + a \cos(dx + c))(a + b \cos(dx + c))}{de \sqrt{e \sin(dx + c)}} + \frac{2(a^2 + 2b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) de^2 \sqrt{\sin(dx + c)}}$$

command

```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} \sqrt{-i} (-i a^2 - 2i b^2) \sin(dx + c) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)))\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2\right) \sqrt{e \sin(dx + c)}}{e^2 \cos(dx + c)^2 - e^2}, x\right)$$

50.15 Problem number 47

$$\int \frac{(a + b \cos(c + dx))^2}{(e \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(b + a \cos(dx + c))(a + b \cos(dx + c))}{3de(e \sin(dx + c))^{\frac{3}{2}}} \\ & -\frac{2(a^2 - 2b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d e^2 \sqrt{e \sin(dx + c)}} \\ & -\frac{2ab \sqrt{e \sin(dx + c)}}{3d e^3} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{-i} \left(\sqrt{2} (a^2 - 2b^2) \cos(dx + c)^2 - \sqrt{2} (a^2 - 2b^2)\right) \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{i}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2\right) \sqrt{e \sin(dx + c)}}{\left(e^3 \cos(dx + c)^2 - e^3\right) \sin(dx + c)}, x\right)$$

50.16 Problem number 48

$$\int \frac{(a + b \cos(c + dx))^2}{(e \sin(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b + a \cos(dx + c))(a + b \cos(dx + c))}{5de(e \sin(dx + c))^{\frac{5}{2}}} - \frac{2ab}{5de^3 \sqrt{e \sin(dx + c)}} - \frac{2(3a^2 - 2b^2) \cos(dx + c)}{5de^3 \sqrt{e \sin(dx + c)}} \\ & + \frac{2(3a^2 - 2b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d e^4 \sqrt{\sin(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-i} \left(\sqrt{2} (-3i a^2 + 2i b^2) \cos(dx + c)^2 + \sqrt{2} (3i a^2 - 2i b^2) \right) \sin(dx + c) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(\dots))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2) \sqrt{e \sin(dx + c)}}{e^4 \cos(dx + c)^4 - 2e^4 \cos(dx + c)^2 + e^4}, x\right)$$

50.17 Problem number 49

$$\int (a + b \cos(c + dx))^3 (e \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(11a^2 + 6b^2) e \cos(dx + c) (e \sin(dx + c))^{\frac{5}{2}}}{77d} + \frac{2b(177a^2 + 44b^2) (e \sin(dx + c))^{\frac{9}{2}}}{1287de} \\ & + \frac{34ab(a + b \cos(dx + c)) (e \sin(dx + c))^{\frac{9}{2}}}{143de} + \frac{2b(a + b \cos(dx + c))^2 (e \sin(dx + c))^{\frac{9}{2}}}{13de} \\ & - \frac{10a(11a^2 + 6b^2) e^4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{231 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{e \sin(dx + c)}} \\ & - \frac{10a(11a^2 + 6b^2) e^3 \cos(dx + c) \sqrt{e \sin(dx + c)}}{231d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$195 \sqrt{2} \sqrt{-i} (11 a^3 + 6 a b^2) e^{\frac{7}{2}} \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + 195 \sqrt{2} \sqrt{i} (11 a^3 + 6 a b^2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(b^3 e^3 \cos(dx + c)^5 + 3 a b^2 e^3 \cos(dx + c)^4 - 3 a^2 b e^3 \cos(dx + c) + (3 a^2 b - b^3) e^3 \cos(dx + c)^3 - a^3 e^3 + c\right), x\right)$$

50.18 Problem number 50

$$\int (a + b \cos(c + dx))^3 (e \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(3a^2 + 2b^2) e \cos(dx + c) (e \sin(dx + c))^{\frac{3}{2}}}{15d} + \frac{2b(43a^2 + 12b^2) (e \sin(dx + c))^{\frac{7}{2}}}{231de} \\ & + \frac{10ab(a + b \cos(dx + c)) (e \sin(dx + c))^{\frac{7}{2}}}{33de} + \frac{2b(a + b \cos(dx + c))^2 (e \sin(dx + c))^{\frac{7}{2}}}{11de} \\ & - \frac{2a(3a^2 + 2b^2) e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\sin(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231i \sqrt{2} \sqrt{-i} (3 a^3 + 2 a b^2) e^{\frac{5}{2}} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) - 231i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(b^3 e^2 \cos(dx + c)^5 + 3 a b^2 e^2 \cos(dx + c)^4 - 3 a^2 b e^2 \cos(dx + c) + (3 a^2 b - b^3) e^2 \cos(dx + c)^3 - a^3 e^2 + c\right), x\right)$$

50.19 Problem number 51

$$\int (a + b \cos(c + dx))^3 (e \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(89a^2 + 28b^2) (e \sin(dx + c))^{\frac{5}{2}}}{315de} + \frac{26ab(a + b \cos(dx + c)) (e \sin(dx + c))^{\frac{5}{2}}}{63de} \\ & + \frac{2b(a + b \cos(dx + c))^2 (e \sin(dx + c))^{\frac{5}{2}}}{9de} \\ & - \frac{2a(7a^2 + 6b^2) e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{21 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{e \sin(dx + c)}} \\ & - \frac{2a(7a^2 + 6b^2) e \cos(dx + c) \sqrt{e \sin(dx + c)}}{21d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} \sqrt{-i} (7a^3 + 6ab^2) e^{\frac{3}{2}} \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \sqrt{2} \sqrt{i} (7a^3 + 6ab^2) e^{\frac{3}{2}} \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^3 e \cos(dx + c)^3 + 3ab^2 e \cos(dx + c)^2 + 3a^2 b e \cos(dx + c) + a^3 e\right) \sqrt{e \sin(dx + c)} \sin(dx + c), x\right)$$

50.20 Problem number 52

$$\int (a + b \cos(c + dx))^3 \sqrt{e \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(57a^2 + 20b^2) (e \sin(dx + c))^{\frac{3}{2}}}{105de} + \frac{22ab(a + b \cos(dx + c)) (e \sin(dx + c))^{\frac{3}{2}}}{35de} \\ & + \frac{2b(a + b \cos(dx + c))^2 (e \sin(dx + c))^{\frac{3}{2}}}{7de} \\ & - \frac{2a(5a^2 + 6b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\sin(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i \sqrt{2} \sqrt{-i} (5a^3 + 6ab^2) e^{\frac{1}{2}} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3\right) \sqrt{e \sin(dx + c)}, x\right)$$

50.21 Problem number 53

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{e \sin(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(a^2 + 2b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{e \sin(dx + c)}} \\ & + \frac{2b(11a^2 + 4b^2) \sqrt{e \sin(dx + c)}}{5de} + \frac{6ab(a + b \cos(dx + c)) \sqrt{e \sin(dx + c)}}{5de} \\ & + \frac{2b(a + b \cos(dx + c))^2 \sqrt{e \sin(dx + c)}}{5de} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(5 \sqrt{2} \sqrt{-i} (a^3 + 2ab^2) \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} \sqrt{i} (a^3 + 2ab^2) \text{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3\right) \sqrt{e \sin(dx + c)}}{e \sin(dx + c)}, x\right)$$

50.22 Problem number 54

$$\int \frac{(a + b \cos(c + dx))^3}{(e \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(3a^2 + 4b^2)(e \sin(dx + c))^{\frac{3}{2}}}{3de^3} - \frac{2ab(a + b \cos(dx + c))(e \sin(dx + c))^{\frac{3}{2}}}{de^3} \\ & - \frac{2(b + a \cos(dx + c))(a + b \cos(dx + c))^2}{de \sqrt{e \sin(dx + c)}} \\ & + \frac{2a(a^2 + 6b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) de^2 \sqrt{\sin(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(3\sqrt{2}\sqrt{-i}(ia^3 + 6iab^2)\sin(dx + c)\operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i\sin(dx + c)))}{\dots}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3) \sqrt{e \sin(dx + c)}}{e^2 \cos(dx + c)^2 - e^2}, x\right)$$

50.23 Problem number 55

$$\int \frac{(a + b \cos(c + dx))^3}{(e \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(b + a \cos(dx + c))(a + b \cos(dx + c))^2}{3de(e \sin(dx + c))^{\frac{3}{2}}} \\ & - \frac{2a(a^2 - 6b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) (\sqrt{\sin(dx + c)})}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) de^2 \sqrt{e \sin(dx + c)}} \\ & - \frac{2b(a^2 + 4b^2) \sqrt{e \sin(dx + c)}}{3de^3} - \frac{2ab(a + b \cos(dx + c)) \sqrt{e \sin(dx + c)}}{3de^3} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-i} \left(\sqrt{2} (a^3 - 6ab^2) \cos(dx + c)^2 - \sqrt{2} (a^3 - 6ab^2) \right) \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\left(b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3 \right) \sqrt{e \sin(dx + c)}}{\left(e^3 \cos(dx + c)^2 - e^3 \right) \sin(dx + c)}, x \right)$$

50.24 Problem number 56

$$\int \frac{(a + b \cos(c + dx))^3}{(e \sin(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(b + a \cos(dx + c)) (a + b \cos(dx + c))^2}{5de (e \sin(dx + c))^{5/2}} - \frac{2b(3a^2 - 4b^2) (e \sin(dx + c))^{3/2}}{5de^5} \\ & + \frac{2(a + b \cos(dx + c)) (ab - (3a^2 - 4b^2) \cos(dx + c))}{5de^3 \sqrt{e \sin(dx + c)}} \\ & + \frac{6a(a^2 - 2b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right) \sqrt{e \sin(dx + c)}}{5 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) de^4 \sqrt{\sin(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \sqrt{-i} \left(\sqrt{2} (i a^3 - 2i ab^2) \cos(dx + c)^2 + \sqrt{2} (-i a^3 + 2i ab^2) \right) \sin(dx + c) \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(\dots))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3 \right) \sqrt{e \sin(dx + c)}}{e^4 \cos(dx + c)^4 - 2e^4 \cos(dx + c)^2 + e^4}, x \right)$$

50.25 Problem number 57

$$\int \frac{(a + b \cos(c + dx))^3}{(e \sin(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(b + a \cos(dx + c))(a + b \cos(dx + c))^2}{7de (e \sin(dx + c))^{\frac{7}{2}}} \\ & - \frac{2(a + b \cos(dx + c))(ab + (5a^2 - 4b^2) \cos(dx + c))}{21de^3 (e \sin(dx + c))^{\frac{3}{2}}} \\ & - \frac{2a(5a^2 - 6b^2) \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{21 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) de^4 \sqrt{e \sin(dx + c)}} \\ & - \frac{2b(5a^2 - 4b^2) \sqrt{e \sin(dx + c)}}{21de^5} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-i} \left(\sqrt{2} (5a^3 - 6ab^2) \cos(dx + c)^4 - 2\sqrt{2} (5a^3 - 6ab^2) \cos(dx + c)^2 + \sqrt{2} (5a^3 - 6ab^2) \right) \operatorname{weierstrassPInvers}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3\right) \sqrt{e \sin(dx + c)}}{\left(e^5 \cos(dx + c)^4 - 2e^5 \cos(dx + c)^2 + e^5\right) \sin(dx + c)}, x\right)$$

51 Test file number 89

Test folder name:

test_cases/4_Trig_functions/4.2_Cosine/89_4.2.2.1-a+b_cos-^m-c+d_cos-^n

51.1 Problem number 146

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} \\ & + \frac{2a \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} + \frac{10a \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15 a \cos(dx+c)^2 + 21 a \cos(dx+c) + 25 a \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 25i \sqrt{2} a \operatorname{weierstrassPInverse}(-4, 0, c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a \cos(dx+c)^3 + a \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}, x\right)$$

51.2 Problem number 147

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2a \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3a \cos(dx+c) + 5a) \sqrt{\cos(dx+c)} \sin(dx+c) - 5i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a \cos(dx+c)^2 + a \cos(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

51.3 Problem number 148

$$\int \sqrt{\cos(c+dx)} (a + a \cos(c+dx)) dx$$

Optimal antiderivative

$$\frac{2a \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2a \sqrt{\cos(dx+c)} \sin(dx+c) - i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a \cos(dx+c) + a\right) \sqrt{\cos(dx+c)}, x\right)$$

51.4 Problem number 149

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c))}{d \sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a \cos(dx + c) + a}{\sqrt{\cos(dx + c)}}, x\right)$$

51.5 Problem number 150

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$-\frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sin(dx + c)}{d \sqrt{\cos(dx + c)}}$$

command

`integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} a \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a \cos(dx + c) + a}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

51.6 Problem number 151

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} a \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a \cos(dx + c) + a}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

51.7 Problem number 152

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6a \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2a \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{6a \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} a \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} a \cos(dx+c)^3 \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a \cos(dx+c) + a}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

51.8 Problem number 153

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32a^2 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{20a^2 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{32a^2 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} + \frac{4a^2 \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} \\ & + \frac{2a^2 \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{9d} + \frac{20a^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(75i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 75i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(a^2 \cos(dx + c)^4 + 2a^2 \cos(dx + c)^3 + a^2 \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}, x\right)$$

51.9 Problem number 154

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{12a^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{4a^2 \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2a^2 \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{8a^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(a^2 \cos(dx + c)^3 + 2a^2 \cos(dx + c)^2 + a^2 \cos(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

51.10 Problem number 155

$$\int \sqrt{\cos(c+dx)} (a+a\cos(c+dx))^2 dx$$

Optimal antiderivative

$$\frac{16a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a^2 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{4a^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d}$$

command

`integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2\right) \sqrt{\cos(dx+c)}, x\right)$$

51.11 Problem number 156

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{8a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d}$$

command

`integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(a^2 \sqrt{\cos(dx+c)} \sin(dx+c) - 2i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 2i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2}{\sqrt{\cos(dx+c)}}, x \right)$$

51.12 Problem number 157

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} + \frac{2a^2 \sin(dx+c)}{d \sqrt{\cos(dx+c)}}$$

command

`integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} a^2 \cos(dx+c) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - i \sqrt{2} a^2 \cos(dx+c) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) \right)}{d \cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2}{\cos(dx+c)^{\frac{3}{2}}}, x \right)$$

51.13 Problem number 158

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a^2 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{4a^2 \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2i \sqrt{2} a^2 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 2i \sqrt{2} a^2 \cos(dx+c)^2 \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

51.14 Problem number 159

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{16a^2 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{4a^2 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{16a^2 \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^2 \cos(dx+c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} a^2 \cos(dx+c)^3 \text{weiers} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^2 \cos(dx+c)^2 + 2 a^2 \cos(dx+c) + a^2}{\cos(dx+c)^{\frac{7}{2}}}, x \right)$$

51.15 Problem number 160

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{68a^3 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{44a^3 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{68a^3 \left(\cos^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{45d} + \frac{6a^3 \left(\cos^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{7d} \\ & + \frac{2a^3 \left(\cos^{\frac{7}{2}}(dx+c) \right) \sin(dx+c)}{9d} + \frac{44a^3 \sin(dx+c) \left(\sqrt{\cos(dx+c)} \right)}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(165i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 165i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(a^3 \cos(dx+c)^4 + 3 a^3 \cos(dx+c)^3 + 3 a^3 \cos(dx+c)^2 + a^3 \cos(dx+c) \right) \sqrt{\cos(dx+c)}, x \right)$$

51.16 Problem number 161

$$\int \sqrt{\cos(c+dx)} (a+a\cos(c+dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{28a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{52a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{6a^3 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} \\ & + \frac{2a^3 \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} + \frac{52a^3 \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(65i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 65i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3\right) \sqrt{\cos(dx+c)}, x\right)$$

51.17 Problem number 162

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{36a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^3 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2a^3 \sin(dx+c) (\sqrt{\cos(dx+c)})}{d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3}{\sqrt{\cos(dx + c)}}, x \right)$$

51.18 Problem number 163

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{20a^3 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a^3 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} + \frac{2a^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3}{\cos(dx + c)^{\frac{3}{2}}}, x \right)$$

51.19 Problem number 164

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{20a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^3 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{6a^3 \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} a^3 \cos(dx+c)^2 \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

51.20 Problem number 165

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{36a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^3 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2a^3 \sin(dx+c)}{d \cos(dx+c)^{\frac{3}{2}}} + \frac{36a^3 \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \cos(dx + c)^3 \text{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}{\cos(dx + c)^{\frac{7}{2}}}, x \right)$$

51.21 Problem number 166

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{28a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{52a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a^3 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{6a^3 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{52a^3 \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} + \frac{28a^3 \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(65i \sqrt{2} a^3 \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 65i \sqrt{2} a^3 \cos(dx + c)^4 \text{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}{\cos(dx + c)^{\frac{9}{2}}}, x \right)$$

51.22 Problem number 167

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{128a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{904a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{128a^4 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} + \frac{150a^4 \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{77d} \\ & + \frac{8a^4 \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{9d} + \frac{2a^4 \left(\cos^{\frac{9}{2}}(dx+c)\right) \sin(dx+c)}{11d} \\ & + \frac{904a^4 \sin(dx+c) (\sqrt{\cos(dx+c)})}{231d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3390i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 3390i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)}{231d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^4 \cos(dx+c)^5 + 4a^4 \cos(dx+c)^4 + 6a^4 \cos(dx+c)^3 + 4a^4 \cos(dx+c)^2 + a^4 \cos(dx+c)\right) \sqrt{\cos(dx+c)}\right)$$

51.23 Problem number 168

$$\int \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{152a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{32a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{122a^4 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} + \frac{8a^4 \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} \\ & + \frac{2a^4 \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{9d} + \frac{32a^4 \sin(dx+c) (\sqrt{\cos(dx+c)})}{7d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(360i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 360i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^4 \cos(dx+c)^4 + 4a^4 \cos(dx+c)^3 + 6a^4 \cos(dx+c)^2 + 4a^4 \cos(dx+c) + a^4\right) \sqrt{\cos(dx+c)}, x\right)$$

51.24 Problem number 169

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{64a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{136a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^4 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2a^4 \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} \\ & + \frac{94a^4 \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(170i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 170i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^4 \cos(dx + c)^4 + 4 a^4 \cos(dx + c)^3 + 6 a^4 \cos(dx + c)^2 + 4 a^4 \cos(dx + c) + a^4}{\sqrt{\cos(dx + c)}}, x \right)$$

51.25 Problem number 170

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{56a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{32a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a^4 \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5d} + \frac{2a^4 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} + \frac{8a^4 \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(40i \sqrt{2} a^4 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 40i \sqrt{2} a^4 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^4 \cos(dx + c)^4 + 4 a^4 \cos(dx + c)^3 + 6 a^4 \cos(dx + c)^2 + 4 a^4 \cos(dx + c) + a^4}{\cos(dx + c)^{\frac{3}{2}}}, x \right)$$

51.26 Problem number 171

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{40a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a^4 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} \\ + \frac{8a^4 \sin(dx+c)}{d \sqrt{\cos(dx+c)}} + \frac{2a^4 \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d}$$

command

`integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} a^4 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 10i \sqrt{2} a^4 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^4 \cos(dx+c)^4 + 4a^4 \cos(dx+c)^3 + 6a^4 \cos(dx+c)^2 + 4a^4 \cos(dx+c) + a^4}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

51.27 Problem number 172

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{56a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ + \frac{32a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ + \frac{2a^4 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{8a^4 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{66a^4 \sin(dx+c)}{5d \sqrt{\cos(dx+c)}}$$

command

```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(40i \sqrt{2} a^4 \cos(dx+c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 40i \sqrt{2} a^4 \cos(dx+c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^4 \cos(dx+c)^4 + 4a^4 \cos(dx+c)^3 + 6a^4 \cos(dx+c)^2 + 4a^4 \cos(dx+c) + a^4}{\cos(dx+c)^{\frac{7}{2}}}, x \right)$$

51.28 Problem number 173

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{64a^4 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{136a^4 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a^4 \sin(dx+c)}{7d \cos(dx+c)^{\frac{7}{2}}} + \frac{8a^4 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{94a^4 \sin(dx+c)}{21d \cos(dx+c)^{\frac{3}{2}}} + \frac{64a^4 \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(170i \sqrt{2} a^4 \cos(dx+c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 170i \sqrt{2} a^4 \cos(dx+c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^4 \cos(dx+c)^4 + 4a^4 \cos(dx+c)^3 + 6a^4 \cos(dx+c)^2 + 4a^4 \cos(dx+c) + a^4}{\cos(dx+c)^{\frac{9}{2}}}, x \right)$$

51.29 Problem number 174

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)} dx$$

Optimal antiderivative

$$\frac{21\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} - \frac{5\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{7\left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5ad} - \frac{\left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{d(a+a\cos(dx+c))} - \frac{5\sin(dx+c)(\sqrt{\cos(dx+c)})}{3ad}$$

command

`integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(6\cos(dx+c)^2 - 4\cos(dx+c) - 25\right)\sqrt{\cos(dx+c)}\sin(dx+c) - 25\left(-i\sqrt{2}\cos(dx+c) - i\sqrt{2}\right)\operatorname{weierstrass}}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx+c)^{\frac{7}{2}}}{a\cos(dx+c)+a}, x\right)$$

51.30 Problem number 175

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)} dx$$

Optimal antiderivative

$$\frac{3\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{5\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} - \frac{\left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{d(a+a\cos(dx+c))} + \frac{5\sin(dx+c)(\sqrt{\cos(dx+c)})}{3ad}$$

command

```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(2 \cos(dx + c) + 5) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx + c)^{\frac{5}{2}}}{a \cos(dx + c) + a}, x \right)$$

51.31 Problem number 176

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\frac{3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} - \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} - \frac{\sin(dx + c) (\sqrt{\cos(dx + c)})}{d(a + a \cos(dx + c))}$$

command

```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \left(-i \sqrt{2} \cos(dx + c) - i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx + c)^{\frac{3}{2}}}{a \cos(dx + c) + a}, x \right)$$

51.32 Problem number 177

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{\sin(dx+c) (\sqrt{\cos(dx+c)})}{d(a+a\cos(dx+c))} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-i\sqrt{2}\cos(dx+c) - i\sqrt{2}\right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + \left(i\sqrt{2}\cos(dx+c) + i\sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}, x\right)$$

51.33 Problem number 178

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} - \frac{\sin(dx+c) (\sqrt{\cos(dx+c)})}{d(a+a\cos(dx+c))} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)^2+a\cos(dx+c)},x\right)$$

51.34 Problem number 179

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \\ & -\frac{\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \\ & +\frac{3\sin(dx+c)}{ad\sqrt{\cos(dx+c)}}-\frac{\sin(dx+c)}{d(a+a\cos(dx+c))\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c)+\left(i\sqrt{2}\cos(dx+c)^2+i\sqrt{2}\cos(dx+c)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)^3+a\cos(dx+c)^2},x\right)$$

51.35 Problem number 180

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{5\sin(dx+c)}{3ad\cos(dx+c)^{\frac{3}{2}}} \\ & - \frac{\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}(a+a\cos(dx+c))} - \frac{3\sin(dx+c)}{ad\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(9\cos(dx+c)^2 + 4\cos(dx+c) - 2\right)\sqrt{\cos(dx+c)}\sin(dx+c) + 5\left(i\sqrt{2}\cos(dx+c)^3 + i\sqrt{2}\cos(dx+c)\right)^2}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)^4 + a\cos(dx+c)^3}, x\right)$$

51.36 Problem number 181

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{56 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{56 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15 a^2 d} - \frac{3 \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{a^2 d (1 + \cos(dx+c))} \\ & - \frac{\left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{3d (a + a \cos(dx+c))^2} - \frac{5 \sin(dx+c) (\sqrt{\cos(dx+c)})}{a^2 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 \cos(dx+c)^3 - 8 \cos(dx+c)^2 - 94 \cos(dx+c) - 75\right) \sqrt{\cos(dx+c)} \sin(dx+c) - 75 \left(-i \sqrt{2} \cos(dx+c)\right)^2}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx+c)^{\frac{9}{2}}}{a^2 \cos(dx+c)^2 + 2 a^2 \cos(dx+c) + a^2}, x\right)$$

51.37 Problem number 182

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{7 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{10 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} - \frac{7 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3 a^2 d (1 + \cos(dx+c))} \\ & - \frac{\left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{3d (a + a \cos(dx+c))^2} + \frac{10 \sin(dx+c) (\sqrt{\cos(dx+c)})}{3 a^2 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \cos(dx + c)^2 + 13 \cos(dx + c) + 10 \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 10 \left(i \sqrt{2} \cos(dx + c)^2 + 2i \sqrt{2} \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx + c)^{\frac{7}{2}}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x \right)$$

51.38 Problem number 183

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & - \frac{5 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & - \frac{\left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d(a + a \cos(dx + c))^2} - \frac{5 \sin(dx + c) (\sqrt{\cos(dx + c)})}{3a^2 d (1 + \cos(dx + c))} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(6 \cos(dx + c) + 5) \sqrt{\cos(dx + c)} \sin(dx + c) + 5 \left(-i \sqrt{2} \cos(dx + c)^2 - 2i \sqrt{2} \cos(dx + c) - i \sqrt{2} \right) \text{weierst}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx + c)^{\frac{5}{2}}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x \right)$$

51.39 Problem number 184

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{\sin(dx+c) (\sqrt{\cos(dx+c)})}{a^2 d (1 + \cos(dx+c))} - \frac{\sin(dx+c) (\sqrt{\cos(dx+c)})}{3d (a + a \cos(dx+c))^2} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3 \cos(dx+c) + 2) \sqrt{\cos(dx+c)} \sin(dx+c) - 2(i \sqrt{2} \cos(dx+c)^2 + 2i \sqrt{2} \cos(dx+c) + i \sqrt{2}) \operatorname{weierstrass}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx+c)^{\frac{3}{2}}}{a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2}, x\right)$$

51.40 Problem number 185

$$\int \frac{\sqrt{\cos(c + dx)}}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} + \frac{\sin(dx+c) (\sqrt{\cos(dx+c)})}{3d (a + a \cos(dx+c))^2}$$

command

`integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i\sqrt{2}\cos(dx+c)^2 - 2i\sqrt{2}\cos(dx+c) - i\sqrt{2}\right)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + (i}{6\left(a^2d\cos(dx+c)\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^2 + 2a^2\cos(dx+c) + a^2}, x\right)$$

51.41 Problem number 186

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{\sin(dx+c)(\sqrt{\cos(dx+c)})}{a^2 d(1+\cos(dx+c))} - \frac{\sin(dx+c)(\sqrt{\cos(dx+c)})}{3d(a+a\cos(dx+c))^2} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3\cos(dx+c) + 4)\sqrt{\cos(dx+c)}\sin(dx+c) + 2\left(i\sqrt{2}\cos(dx+c)^2 + 2i\sqrt{2}\cos(dx+c) + i\sqrt{2}\right)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))}{6\left(a^2d\cos(dx+c)\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^3 + 2a^2\cos(dx+c)^2 + a^2\cos(dx+c)}, x\right)$$

51.42 Problem number 187

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} + \frac{4\sin(dx+c)}{a^2 d \sqrt{\cos(dx+c)}} \\ & - \frac{5\sin(dx+c)}{3a^2 d (1+\cos(dx+c)) \sqrt{\cos(dx+c)}} - \frac{\sin(dx+c)}{3d (a+a\cos(dx+c))^2 \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(12 \cos(dx+c)^2 + 19 \cos(dx+c) + 6 \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 5 \left(-i \sqrt{2} \cos(dx+c)^3 - 2i \sqrt{2} \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^4 + 2a^2 \cos(dx+c)^3 + a^2 \cos(dx+c)^2}, x\right)$$

51.43 Problem number 188

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{10\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{10\sin(dx+c)}{3a^2 d \cos(dx+c)^{\frac{3}{2}}} - \frac{7\sin(dx+c)}{3a^2 d \cos(dx+c)^{\frac{3}{2}} (1+\cos(dx+c))} \\ & - \frac{\sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}} (a+a\cos(dx+c))^2} - \frac{7\sin(dx+c)}{a^2 d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21 \cos(dx+c)^3 + 32 \cos(dx+c)^2 + 8 \cos(dx+c) - 2 \right) \sqrt{\cos(dx+c)} \sin(dx+c) + 10 \left(i \sqrt{2} \cos(dx+c) \right)^4}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^5 + 2a^2 \cos(dx+c)^4 + a^2 \cos(dx+c)^3}, x \right)$$

51.44 Problem number 189

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{231 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & - \frac{21 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} + \frac{77 \left(\cos^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{10a^3 d} \\ & - \frac{\left(\cos^{\frac{9}{2}}(dx+c) \right) \sin(dx+c)}{5d(a+a\cos(dx+c))^3} - \frac{4 \left(\cos^{\frac{7}{2}}(dx+c) \right) \sin(dx+c)}{5ad(a+a\cos(dx+c))^2} \\ & - \frac{63 \left(\cos^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{10d(a^3+a^3\cos(dx+c))} - \frac{21 \sin(dx+c) \left(\sqrt{\cos(dx+c)} \right)}{2a^3 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(4 \cos(dx+c)^4 - 8 \cos(dx+c)^3 - 147 \cos(dx+c)^2 - 238 \cos(dx+c) - 105 \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 10}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx+c)^{\frac{11}{2}}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x \right)$$

51.45 Problem number 190

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{119\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{11\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{\left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{5d(a+a\cos(dx+c))^3} - \frac{2\left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{3ad(a+a\cos(dx+c))^2} \\ & - \frac{119\left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{30d(a^3+a^3\cos(dx+c))} + \frac{11\sin(dx+c)(\sqrt{\cos(dx+c)})}{2a^3 d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(20\cos(dx+c)^3 + 237\cos(dx+c)^2 + 376\cos(dx+c) + 165\right)\sqrt{\cos(dx+c)}\sin(dx+c) - 165\left(i\sqrt{2}\cos(dx+c)\right)}{a^3\cos(dx+c)^3 + 3a^3\cos(dx+c)^2 + 3a^3\cos(dx+c) + a^3}, x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx+c)^{\frac{9}{2}}}{a^3\cos(dx+c)^3 + 3a^3\cos(dx+c)^2 + 3a^3\cos(dx+c) + a^3}, x\right)$$

51.46 Problem number 191

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\frac{49 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} - \frac{13 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} - \frac{\left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{5d(a+a\cos(dx+c))^3} - \frac{8\left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15ad(a+a\cos(dx+c))^2} - \frac{13 \sin(dx+c) (\sqrt{\cos(dx+c)})}{6d(a^3+a^3\cos(dx+c))}$$

command

```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(87 \cos(dx+c)^2 + 146 \cos(dx+c) + 65\right) \sqrt{\cos(dx+c)} \sin(dx+c) + 65\left(-i\sqrt{2} \cos(dx+c)^3 - 3i\sqrt{2} \cos(dx+c)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx+c)^{\frac{7}{2}}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x\right)$$

51.47 Problem number 192

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\frac{9 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} - \frac{\left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d(a+a\cos(dx+c))^3} - \frac{2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{5ad(a+a\cos(dx+c))^2} + \frac{9 \sin(dx+c) (\sqrt{\cos(dx+c)})}{10d(a^3+a^3\cos(dx+c))}$$

command

`integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(9 \cos(dx+c)^2 + 12 \cos(dx+c) + 5 \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 5 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx+c)^{\frac{5}{2}}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x \right)$$

51.48 Problem number 193

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} - \frac{\sin(dx+c) (\sqrt{\cos(dx+c)})}{5d (a+a\cos(dx+c))^3} \\ & + \frac{4 \sin(dx+c) (\sqrt{\cos(dx+c)})}{15ad (a+a\cos(dx+c))^2} + \frac{\sin(dx+c) (\sqrt{\cos(dx+c)})}{10d (a^3 + a^3 \cos(dx+c))} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \cos(dx+c)^2 + 14 \cos(dx+c) + 5 \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 5 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx+c)^{\frac{3}{2}}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x \right)$$

51.49 Problem number 194

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} + \frac{\sin(dx+c) (\sqrt{\cos(dx+c)})}{5d (a+a\cos(dx+c))^3} + \frac{\sin(dx+c) (\sqrt{\cos(dx+c)})}{15ad (a+a\cos(dx+c))^2} - \frac{\sin(dx+c) (\sqrt{\cos(dx+c)})}{10d (a^3 + a^3 \cos(dx+c))}$$

command

`integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \cos(dx+c)^2 + 4 \cos(dx+c) - 5 \right) \sqrt{\cos(dx+c)} \sin(dx+c) + 5 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x\right)$$

51.50 Problem number 195

$$\int \frac{1}{\sqrt{\cos(c+dx)} (a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\frac{9 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} - \frac{\sin(dx+c) (\sqrt{\cos(dx+c)})}{5d (a+a\cos(dx+c))^3} - \frac{2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{5ad (a+a\cos(dx+c))^2} - \frac{9 \sin(dx+c) (\sqrt{\cos(dx+c)})}{10d (a^3 + a^3 \cos(dx+c))}$$

command

```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(9 \cos(dx+c)^2 + 22 \cos(dx+c) + 15 \right) \sqrt{\cos(dx+c)} \sin(dx+c) + 5 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^4 + 3a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + a^3 \cos(dx+c)}, x \right)$$

51.51 Problem number 196

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{49 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & - \frac{13 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{49 \sin(dx+c)}{10a^3 d \sqrt{\cos(dx+c)}} - \frac{\sin(dx+c)}{5d(a+a\cos(dx+c))^3 \sqrt{\cos(dx+c)}} \\ & - \frac{8 \sin(dx+c)}{15ad(a+a\cos(dx+c))^2 \sqrt{\cos(dx+c)}} - \frac{13 \sin(dx+c)}{6d(a^3+a^3\cos(dx+c)) \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(147 \cos(dx+c)^3 + 376 \cos(dx+c)^2 + 295 \cos(dx+c) + 60 \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 65 \left(-i \sqrt{2} \cos(dx+c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^5 + 3a^3 \cos(dx+c)^4 + 3a^3 \cos(dx+c)^3 + a^3 \cos(dx+c)^2}, x \right)$$

51.52 Problem number 197

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{119 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{11 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} + \frac{11 \sin(dx+c)}{2a^3 d \cos(dx+c)^{\frac{3}{2}}} \\ & - \frac{\sin(dx+c)}{5d \cos(dx+c)^{\frac{3}{2}} (a+a\cos(dx+c))^3} - \frac{2 \sin(dx+c)}{3ad \cos(dx+c)^{\frac{3}{2}} (a+a\cos(dx+c))^2} \\ & - \frac{119 \sin(dx+c)}{30d \cos(dx+c)^{\frac{3}{2}} (a^3+a^3\cos(dx+c))} - \frac{119 \sin(dx+c)}{10a^3 d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(357 \cos(dx+c)^4 + 906 \cos(dx+c)^3 + 695 \cos(dx+c)^2 + 120 \cos(dx+c) - 20 \right) \sqrt{\cos(dx+c)} \sin(dx+c)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^6 + 3a^3 \cos(dx+c)^5 + 3a^3 \cos(dx+c)^4 + a^3 \cos(dx+c)^3}, x\right)$$

51.53 Problem number 291

$$\int (a+a\cos(c+dx)) \sec^{\frac{7}{2}}(c+dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2a \left(\sec^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{5d} \\ & + \frac{6a \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d} \\ & - \frac{6a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} a \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} a \cos(dx+c)^2 \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left((a \cos(dx+c) + a) \sec(dx+c)^{\frac{7}{2}}, x \right)$$

51.54 Problem number 292

$$\int (a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2a \sin(dx+c) (\sqrt{\sec(dx+c)})}{d} \\ & - \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} a \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((a \cos(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}, x\right)$$

51.55 Problem number 293

$$\int (a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} a \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((a \cos(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

51.56 Problem number 294

$$\int (a + a \cos(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c)) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((a \cos(dx + c) + a) \sqrt{\sec(dx + c)}, x\right)$$

51.57 Problem number 295

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2a \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$2a\sqrt{\cos(dx+c)}\sin(dx+c) - i\sqrt{2}\operatorname{aweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}\operatorname{aweierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a\cos(dx+c)+a}{\sqrt{\sec(dx+c)}},x\right)$$

51.58 Problem number 296

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a\sin(dx+c)}{5d\sec(dx+c)^{\frac{3}{2}}} + \frac{2a\sin(dx+c)}{3d\sqrt{\sec(dx+c)}} \\ & + \frac{6a\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2a\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i\sqrt{2}\operatorname{aweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}\operatorname{aweierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

51.59 Problem number 297

$$\int \frac{a + a \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{10a \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{6a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-25i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a \cos(dx + c) + a}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

51.60 Problem number 298

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} + \frac{2a^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{16a^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{16a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^2 \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^2 \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2 \right) \sec(dx + c)^{\frac{7}{2}}, x \right)$$

51.61 Problem number 299

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{4a^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{4a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{8a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2i \sqrt{2} a^2 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 2i \sqrt{2} a^2 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2 \right) \sec(dx + c)^{\frac{5}{2}}, x \right)$$

51.62 Problem number 300

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2a^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} + \frac{4a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

51.63 Problem number 301

$$\int (a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2a^2 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{4a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{8a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(a^2 \sqrt{\cos(dx+c)} \sin(dx+c) - 2i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 2i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2 \right) \sqrt{\sec(dx+c)}, x \right)$$

51.64 Problem number 302

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \sin(dx+c)}{5d \sec(dx+c)^{\frac{3}{2}}} + \frac{4a^2 \sin(dx+c)}{3d \sqrt{\sec(dx+c)}} \\ & + \frac{16a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2}{\sqrt{\sec(dx+c)}}, x \right)$$

51.65 Problem number 303

$$\int \frac{(a + a \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{8a^2 \sin(dx + c)}{7d \sqrt{\sec(dx + c)}} \\ & + \frac{12a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

51.66 Problem number 304

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{52a^3 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} + \frac{6a^3 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2a^3 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{28a^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{28a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{52a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(65i \sqrt{2} a^3 \cos(dx+c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 65i \sqrt{2} a^3 \cos(dx+c)^3 \text{wei} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3\right) \sec(dx+c)^{\frac{9}{2}}, x\right)$$

51.67 Problem number 305

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{d} + \frac{2a^3 \left(\sec^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{5d} \\ & + \frac{36a^3 \sin(dx+c) (\sqrt{\sec}(dx+c))}{5d} \\ & - \frac{36a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \cos(dx+c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} a^3 \cos(dx+c)^2 \text{weiers} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3\right) \sec(dx+c)^{\frac{7}{2}}, x\right)$$

51.68 Problem number 306

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{6a^3 \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & - \frac{4a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{20a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \cos(dx + c) \operatorname{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3 \right) \sec(dx + c)^{\frac{5}{2}}, x \right)$$

51.69 Problem number 307

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2a^3 \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & + \frac{4a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{20a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

51.70 Problem number 308

$$\int (a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a^3 \sin(dx + c)}{d \sqrt{\sec(dx + c)}} \\ & + \frac{36a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3\right) \sqrt{\sec(dx + c)}, x\right)$$

51.71 Problem number 309

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{6a^3 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{52a^3 \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{28a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{52a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(65i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 65i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}{\sqrt{\sec(dx + c)}}, x\right)$$

51.72 Problem number 310

$$\int \frac{(a + a \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{6a^3 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{68a^3 \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{44a^3 \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{68a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{44a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(165i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 165i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}{\sec(dx + c)^{\frac{3}{2}}}, x \right)$$

51.73 Problem number 311

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{94a^4 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} + \frac{8a^4 \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2a^4 \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{64a^4 \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{64a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{136a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(170i \sqrt{2} a^4 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 170i \sqrt{2} a^4 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(a^4 \cos(dx + c)^4 + 4a^4 \cos(dx + c)^3 + 6a^4 \cos(dx + c)^2 + 4a^4 \cos(dx + c) + a^4 \right) \sec(dx + c)^{\frac{9}{2}}, x \right)$$

51.74 Problem number 312

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^4 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2a^4 \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{66a^4 \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{56a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{32a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(40i \sqrt{2} a^4 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 40i \sqrt{2} a^4 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(a^4 \cos(dx + c)^4 + 4a^4 \cos(dx + c)^3 + 6a^4 \cos(dx + c)^2 + 4a^4 \cos(dx + c) + a^4 \right) \sec(dx + c)^{\frac{7}{2}}, x \right)$$

51.75 Problem number 313

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^4 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2a^4 \sin(dx + c)}{3d \sqrt{\sec}(dx + c)} + \frac{8a^4 \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & + \frac{40a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} a^4 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} a^4 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right) \frac{1}{3d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(a^4 \cos(dx + c)^4 + 4a^4 \cos(dx + c)^3 + 6a^4 \cos(dx + c)^2 + 4a^4 \cos(dx + c) + a^4 \right) \sec(dx + c)^{\frac{5}{2}}, x \right)$$

51.76 Problem number 314

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^4 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{8a^4 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2a^4 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & + \frac{56a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{32a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(40i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 40i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right) \frac{1}{3d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(a^4 \cos(dx + c)^4 + 4a^4 \cos(dx + c)^3 + 6a^4 \cos(dx + c)^2 + 4a^4 \cos(dx + c) + a^4 \right) \sec(dx + c)^{\frac{3}{2}}, x \right)$$

51.77 Problem number 315

$$\int (a + a \cos(c + dx))^4 \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^4 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{8a^4 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{94a^4 \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{64a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{136a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(170i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 170i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^4 \cos(dx + c)^4 + 4a^4 \cos(dx + c)^3 + 6a^4 \cos(dx + c)^2 + 4a^4 \cos(dx + c) + a^4\right) \sqrt{\sec(dx + c)}, x\right)$$

51.78 Problem number 316

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^4 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{8a^4 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{122a^4 \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{32a^4 \sin(dx + c)}{7d \sqrt{\sec(dx + c)}} \\ & + \frac{152a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{32a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(360i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 360i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^4 \cos(dx + c)^4 + 4a^4 \cos(dx + c)^3 + 6a^4 \cos(dx + c)^2 + 4a^4 \cos(dx + c) + a^4}{\sqrt{\sec(dx + c)}}, x \right)$$

51.79 Problem number 317

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3ad} - \frac{\left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \sec(dx + c))} - \frac{3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{ad} \\ & + \frac{3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{5 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx + c)^2 + i \sqrt{2} \cos(dx + c) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \left(-i \sqrt{2} \cos(dx + c)^2 - i \sqrt{2} \cos(dx + c) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sec(dx + c)^{\frac{5}{2}}}{a \cos(dx + c) + a}, x \right)$$

51.80 Problem number 318

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{d(a+a\sec(dx+c))} + \frac{3\sin(dx+c)(\sqrt{\sec}(dx+c))}{ad} \\ & - \frac{3\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(i\sqrt{2}\cos(dx+c) + i\sqrt{2}\right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + \left(-i\sqrt{2}\cos(dx+c) - i\sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}, x\right)$$

51.81 Problem number 319

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sin(dx+c)(\sqrt{\sec}(dx+c))}{d(a+a\sec(dx+c))} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a},x\right)$$

51.82 Problem number 320

$$\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\sin(dx+c)(\sqrt{\sec(dx+c)})}{d(a+a\sec(dx+c))} - \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad}$$

command

```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{1}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}},x\right)$$

51.83 Problem number 321

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\sin(dx + c) (\sqrt{\sec(dx + c)})}{d(a + a \sec(dx + c))} \\ & + \frac{3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(i \sqrt{2} \cos(dx + c) + i \sqrt{2}\right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \left(-i \sqrt{2} \cos(dx + c) - i \sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}}, x\right)$$

51.84 Problem number 322

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5 \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}} - \frac{\sin(dx + c)}{d(a + a \sec(dx + c)) \sqrt{\sec(dx + c)}} \\ & - \frac{3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \left(-i \sqrt{2} \cos(dx + c) - \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}}, x \right)$$

51.85 Problem number 323

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7 \sin(dx + c)}{5ad \sec(dx + c)^{\frac{3}{2}}} - \frac{\sin(dx + c)}{d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))} - \frac{5 \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}} \\ & + \frac{21 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{5 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \end{aligned}$$

command

```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25 \left(-i \sqrt{2} \cos(dx + c) - i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25 \left(i \sqrt{2} \cos(dx + c) - \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{7}{2}}}, x \right)$$

51.86 Problem number 324

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d} - \frac{7 \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d (1 + \sec(dx + c))} \\ & - \frac{\left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{3d (a + a \sec(dx + c))^2} - \frac{7 \sin(dx + c) (\sqrt{\sec}(dx + c))}{a^2d} \\ & + \frac{7 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & + \frac{10 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$10 \left(i \sqrt{2} \cos(dx + c)^3 + 2i \sqrt{2} \cos(dx + c)^2 + i \sqrt{2} \cos(dx + c) \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(dx + c)^{\frac{5}{2}}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x\right)$$

51.87 Problem number 325

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3a^2d(1+\sec(dx+c))} - \frac{\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{3d(a+a\sec(dx+c))^2} \\ & + \frac{4\sin(dx+c)(\sqrt{\sec(dx+c)})}{a^2d} \\ & - \frac{4\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & - \frac{5\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\left(-i\sqrt{2}\cos(dx+c)^2 - 2i\sqrt{2}\cos(dx+c) - i\sqrt{2}\right)\operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(dx+c)^{\frac{3}{2}}}{a^2\cos(dx+c)^2 + 2a^2\cos(dx+c) + a^2}, x\right)$$

51.88 Problem number 326

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3d(a+a\sec(dx+c))^2} - \frac{\sin(dx+c)(\sqrt{\sec(dx+c)})}{a^2d(1+\sec(dx+c))} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & + \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} \cos(dx+c)^2 + 2i \sqrt{2} \cos(dx+c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\sec(dx+c)}}{a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2}, x \right)$$

51.89 Problem number 327

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{\left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d(a+a \sec(dx+c))^2} + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d}$$

command

```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i \sqrt{2} \cos(dx+c)^2 - 2i \sqrt{2} \cos(dx+c) - i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \left(i \sqrt{2} \cos(dx+c)^2 + 2i \sqrt{2} \cos(dx+c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{6 \left(a^2 d \cos(dx+c) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\left(a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2 \right) \sqrt{\sec(dx+c)}}, x \right)$$

51.90 Problem number 328

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(\sec^{\frac{3}{2}}(dx + c)) \sin(dx + c)}{3d(a + a \sec(dx + c))^2} + \frac{\sin(dx + c) (\sqrt{\sec}(dx + c))}{a^2 d (1 + \sec(dx + c))} \\ & - \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

`integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} \cos(dx + c)^2 + 2i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\left(a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2\right) \sec(dx + c)^{\frac{3}{2}}}, x\right)$$

51.91 Problem number 329

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{5 \sin(dx + c) (\sqrt{\sec}(dx + c))}{3a^2 d (1 + \sec(dx + c))} - \frac{\sin(dx + c) (\sqrt{\sec}(dx + c))}{3d(a + a \sec(dx + c))^2} \\ & + \frac{4\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

`integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(-i \sqrt{2} \cos(dx + c)^2 - 2i \sqrt{2} \cos(dx + c) - i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\left(a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2 \right) \sec(dx + c)^{\frac{5}{2}}, x \right)$$

51.92 Problem number 330

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10 \sin(dx + c)}{3a^2 d \sqrt{\sec(dx + c)}} - \frac{7 \sin(dx + c)}{3a^2 d (1 + \sec(dx + c)) \sqrt{\sec(dx + c)}} \\ & - \frac{\sin(dx + c)}{3d (a + a \sec(dx + c))^2 \sqrt{\sec(dx + c)}} \\ & - \frac{7 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & + \frac{10 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \end{aligned}$$

command

`integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$10 \left(i \sqrt{2} \cos(dx + c)^2 + 2i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\left(a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2 \right) \sec(dx + c)^{\frac{7}{2}}, x \right)$$

51.93 Problem number 331

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{56 \sin(dx + c)}{15a^2 d \sec(dx + c)^{\frac{3}{2}}} - \frac{3 \sin(dx + c)}{a^2 d \sec(dx + c)^{\frac{3}{2}} (1 + \sec(dx + c))} \\ & - \frac{\sin(dx + c)}{3d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))^2} - \frac{5 \sin(dx + c)}{a^2 d \sqrt{\sec(dx + c)}} \\ & + \frac{56 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$75 \left(-i \sqrt{2} \cos(dx + c)^2 - 2i \sqrt{2} \cos(dx + c) - i \sqrt{2} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\left(a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2\right) \sec(dx + c)^{\frac{9}{2}}}, x\right)$$

51.94 Problem number 332

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\sec^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{5d(a+a\sec(dx+c))^3} - \frac{8\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{15ad(a+a\sec(dx+c))^2} \\ & - \frac{13\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{6d(a^3+a^3\sec(dx+c))} + \frac{49\sin(dx+c)(\sqrt{\sec(dx+c)})}{10a^3d} \\ & - \frac{49\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{10\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^3d} \\ & - \frac{13\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{6\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^3d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$65\left(-i\sqrt{2}\cos(dx+c)^3 - 3i\sqrt{2}\cos(dx+c)^2 - 3i\sqrt{2}\cos(dx+c) - i\sqrt{2}\right)\operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(dx+c)^{\frac{3}{2}}}{a^3\cos(dx+c)^3 + 3a^3\cos(dx+c)^2 + 3a^3\cos(dx+c) + a^3}, x\right)$$

51.95 Problem number 333

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{5d(a+a\sec(dx+c))^3} - \frac{2\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{5ad(a+a\sec(dx+c))^2} \\ & - \frac{9\sin(dx+c)(\sqrt{\sec(dx+c)})}{10d(a^3+a^3\sec(dx+c))} \\ & + \frac{9\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{10\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^3d} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{2\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^3d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx + c)^3 + 3i \sqrt{2} \cos(dx + c)^2 + 3i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\sec(dx + c)}}{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}, x \right)$$

51.96 Problem number 334

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(\sec^{\frac{3}{2}}(dx + c)) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} - \frac{4 \sin(dx + c) (\sqrt{\sec(dx + c)})}{15ad(a + a \sec(dx + c))^2} \\ & + \frac{\sin(dx + c) (\sqrt{\sec(dx + c)})}{6d(a^3 + a^3 \sec(dx + c))} \\ & + \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx + c)^3 + 3i \sqrt{2} \cos(dx + c)^2 + 3i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{(a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3) \sqrt{\sec(dx + c)}}, x \right)$$

51.97 Problem number 335

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(\sec^{\frac{3}{2}}(dx + c)) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} - \frac{\sin(dx + c) (\sqrt{\sec}(dx + c))}{15ad(a + a \sec(dx + c))^2} + \frac{\sin(dx + c) (\sqrt{\sec}(dx + c))}{6d(a^3 + a^3 \sec(dx + c))} \\ & - \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx + c)^3 + 3i \sqrt{2} \cos(dx + c)^2 + 3i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{(a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3) \sec(dx + c)^{\frac{3}{2}}}, x\right)$$

51.98 Problem number 336

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{(\sec^{\frac{3}{2}}(dx+c)) \sin(dx+c)}{5d(a+a \sec(dx+c))^3} + \frac{2 \sin(dx+c) (\sqrt{\sec(dx+c)})}{5ad(a+a \sec(dx+c))^2} \\
 & + \frac{\sin(dx+c) (\sqrt{\sec(dx+c)})}{2d(a^3+a^3 \sec(dx+c))} \\
 & - \frac{9 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
 & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d}
 \end{aligned}$$

command

```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c)^2 + 3i \sqrt{2} \cos(dx+c) + i \sqrt{2} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\left(a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3\right) \sec(dx+c)^{\frac{5}{2}}, x}\right)$$

51.99 Problem number 337

$$\int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{7}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{\sin(dx+c) (\sqrt{\sec(dx+c)})}{5d(a+a \sec(dx+c))^3} - \frac{8 \sin(dx+c) (\sqrt{\sec(dx+c)})}{15ad(a+a \sec(dx+c))^2} \\
 & - \frac{13 \sin(dx+c) (\sqrt{\sec(dx+c)})}{6d(a^3+a^3 \sec(dx+c))} \\
 & + \frac{49 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
 & - \frac{13 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d}
 \end{aligned}$$

command

`integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$65 \left(-i \sqrt{2} \cos(dx+c)^3 - 3i \sqrt{2} \cos(dx+c)^2 - 3i \sqrt{2} \cos(dx+c) - i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\left(a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3 \right) \sec(dx+c)^{\frac{7}{2}}}, x \right)$$

51.100 Problem number 338

$$\int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{9}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{11 \sin(dx+c)}{2a^3 d \sqrt{\sec(dx+c)}} - \frac{\sin(dx+c)}{5d(a+a \sec(dx+c))^3 \sqrt{\sec(dx+c)}} \\ & - \frac{2 \sin(dx+c)}{3ad(a+a \sec(dx+c))^2 \sqrt{\sec(dx+c)}} - \frac{119 \sin(dx+c)}{30d(a^3+a^3 \sec(dx+c)) \sqrt{\sec(dx+c)}} \\ & \frac{119 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{11 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \end{aligned}$$

command

`integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$165 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c)^2 + 3i \sqrt{2} \cos(dx+c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\left(a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3 \right) \sec(dx+c)^{\frac{9}{2}}}, x \right)$$

51.101 Problem number 486

$$\int \cos^3(c + dx) \sqrt{a + b \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{8a(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{35b^2d} + \frac{2 \cos(dx + c) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{7bd} \\ & + \frac{2(8a^2 + 25b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105b^2d} \\ & + \frac{2a(8a^2 + 19b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2(8a^4 + 17a^2b^2 - 25b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16i a^4 + 32i a^2 b^2 - 75i b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3ib \sin(dx+c) + 2a}{3b}\right) + \dots$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c) + a} \cos(dx + c)^3, x\right)$$

51.102 Problem number 487

$$\int \cos^2(c + dx) \sqrt{a + b \cos(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5bd} - \frac{4a \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15bd}$$

$$- \frac{2(2a^2 - 9b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{4a(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-4i a^3 - 3i ab^2) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx + c) + 3i b \sin(dx + c) + 2a}{3b}\right) + \sqrt{2}(4i a^3}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c) + a} \cos(dx + c)^2, x\right)$$

51.103 Problem number 488

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3d}$$

$$+ \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i\sqrt{2}ab^{\frac{3}{2}}\text{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3}{3}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b\cos(dx+c)+a}\cos(dx+c), x\right)$$

51.104 Problem number 489

$$\int \sqrt{a+b\cos(c+dx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right) \sqrt{a+b\cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\frac{a+b\cos(dx+c)}{a+b}}}$$

command

```
integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i\sqrt{2}a\sqrt{b}\text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right) + i\sqrt{2}a\sqrt{b}\text{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b\cos(dx+c)+a}, x\right)$$

51.105 Problem number 493

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(8a^2 + 49b^2)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{315b^2d} - \frac{8a(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{63b^2d} \\ & + \frac{2 \cos(dx + c)(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{9bd} + \frac{2a(8a^2 + 39b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315b^2d} \\ & + \frac{2(8a^4 + 33a^2b^2 + 147b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{2a(8a^4 + 31a^2b^2 - 39b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \sqrt{2} (-4i a^5 - 15i a^3 b^2 + 66i a b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i b \sin(dx+c) + 2a}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b \cos(dx + c)^4 + a \cos(dx + c)^3\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

51.106 Problem number 494

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{4a(a+b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{35bd} + \frac{2(a+b\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{7bd}$$

$$- \frac{2(6a^2-25b^2)\sin(dx+c)\sqrt{a+b\cos(dx+c)}}{105bd}$$

$$- \frac{4a(3a^2-41b^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{105\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^2d\sqrt{\frac{a+b\cos(dx+c)}{a+b}}}$$

$$+ \frac{2(6a^4-31a^2b^2+25b^4)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{105\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^2d\sqrt{a+b\cos(dx+c)}}$$

command

`integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-12ia^4+11ia^2b^2-75ib^4)\sqrt{b}\operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b\cos(dx+c)^3+a\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)+a},x\right)$$

51.107 Problem number 495

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2(a+b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{5d} + \frac{2a\sin(dx+c)\sqrt{a+b\cos(dx+c)}}{5d}$$

$$+ \frac{2(a^2+3b^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)bd\sqrt{\frac{a+b\cos(dx+c)}{a+b}}}$$

$$- \frac{2a(a^2-b^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)bd\sqrt{a+b\cos(dx+c)}}$$

command

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\sqrt{2}(-ia^3 + 3iab^2)\sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right) + 2\sqrt{2}(i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b\cos(dx+c)^2 + a\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}, x\right)$$

51.108 Problem number 496

$$\int (a + b\cos(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b\sin(dx+c)\sqrt{a+b\cos(dx+c)}}{3d} + \frac{8a\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d\sqrt{\frac{a+b\cos(dx+c)}{a+b}}} + \frac{2(a^2-b^2)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d\sqrt{a+b\cos(dx+c)}}$$

command

```
integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12i\sqrt{2}ab^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b\cos(dx+c)+a\right)^{\frac{3}{2}}, x\right)$$

51.109 Problem number 500

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(8a^2 + 67b^2)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{693b^2d} + \frac{2(8a^2 + 81b^2)(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{693b^2d} \\ & - \frac{8a(a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{99b^2d} + \frac{2 \cos(dx + c)(a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{11bd} \\ & + \frac{2(8a^4 + 57a^2b^2 + 135b^4) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{693b^2d} \\ & + \frac{2a(8a^4 + 51a^2b^2 + 741b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{693 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{2(8a^6 + 49a^4b^2 + 78a^2b^4 - 135b^6) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{693 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (16i a^6 + 96i a^4 b^2 - 507i a^2 b^4 - 405i b^6) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i b \sin(dx+c)}{3b}\right)}{693 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx + c)^5 + 2ab \cos(dx + c)^4 + a^2 \cos(dx + c)^3\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

51.110 Problem number 501

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2(10a^2 - 49b^2)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{315bd} - \frac{4a(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{63bd}$$

$$+ \frac{2(a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9bd} - \frac{4a(5a^2 - 57b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315bd}$$

$$- \frac{2(10a^4 - 279a^2b^2 - 147b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{4a(5a^4 - 62a^2b^2 + 57b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-20i a^5 + 93i a^3 b^2 - 489i a b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx + c) + 3i b \sin(dx + c) + 2a}{3b}\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx + c)^4 + 2ab \cos(dx + c)^3 + a^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

51.111 Problem number 502

$$\int \cos(c + dx)(a + b \cos(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2a(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{7d} + \frac{2(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7d}$$

$$+ \frac{2(3a^2 + 5b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{21d}$$

$$+ \frac{2a(3a^2 + 29b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(3a^4 + 2a^2b^2 - 5b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (6i a^4 - 23i a^2 b^2 - 15i b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i b \sin(dx+c) + 2a}{3b}\right) + \sqrt{2} (-i a^3 + 3i a b^2 - 3i b^3)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx+c)^3 + 2ab \cos(dx+c)^2 + a^2 \cos(dx+c)\right) \sqrt{b \cos(dx+c) + a}, x\right)$$

51.112 Problem number 503

$$\int (a + b \cos(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(a + b \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{5d} + \frac{16ab \sin(dx+c) \sqrt{a + b \cos(dx+c)}}{15d} \\ & + \frac{2(23a^2 + 9b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx+c)}{a+b}}} \\ & - \frac{16a(a^2 - b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (i a^3 - 33i a b^2) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i b \sin(dx+c) + 2a}{3b}\right) + \sqrt{2} (-i a^3 + 3i a b^2 - 3i b^3)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2\right) \sqrt{b \cos(dx+c) + a}, x\right)$$

51.113 Problem number 508

$$\int (a + b \cos(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{24ab(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{35d} + \frac{2b(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7d} \\ & + \frac{2b(71a^2 + 25b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105d} \\ & + \frac{32a(11a^2 + 13b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx + c)}{a+b}}} \\ & + \frac{2(71a^4 - 46a^2b^2 - 25b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (37i a^4 - 346i a^2 b^2 - 75i b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i b \sin(dx+c) + 2a}{3b}\right) + \dots}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

51.114 Problem number 509

$$\int \cos^3(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{3(3+4\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{70d} + \frac{\cos(dx+c)(3+4\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{14d} \\
 & + \frac{47\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right)\sqrt{7}}{140\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\
 & + \frac{59\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right)\sqrt{7}}{420\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\
 & + \frac{59\sin(dx+c)\sqrt{3+4\cos(dx+c)}}{105d}
 \end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4\left(60\cos(dx+c)^2+9\cos(dx+c)+91\right)\sqrt{4\cos(dx+c)+3}\sin(dx+c)-277i\sqrt{2}\operatorname{weierstrassPInverse}(-1,1,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{4\cos(dx+c)+3}\cos(dx+c)^3,x\right)$$

51.115 Problem number 510

$$\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{(3+4\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{10d} \\
 & + \frac{21\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right)\sqrt{7}}{20\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\
 & - \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right)\sqrt{7}}{20\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\
 & - \frac{\sin(dx+c)\sqrt{3+4\cos(dx+c)}}{5d}
 \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \sqrt{4 \cos(dx + c) + 3} (4 \cos(dx + c) + 1) \sin(dx + c) - 7i \sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{4 \cos(dx + c) + 3} \cos(dx + c)^2, x\right)$$

51.116 Problem number 511

$$\int \cos(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2 \sin(dx + c) \sqrt{3 + 4 \cos(dx + c)}}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$8 \sqrt{4 \cos(dx + c) + 3} \sin(dx + c) - 5i \sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c) + \frac{1}{2}) + 5i \sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2})$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{4 \cos(dx + c) + 3} \cos(dx + c), x\right)$$

51.117 Problem number 512

$$\int \sqrt{3 + 4 \cos(c + dx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx+c) + i \sin(dx+c) + \frac{1}{2}) + i\sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{4 \cos(dx+c) + 3}, x\right)$$

51.118 Problem number 516

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^3(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3(3-4\cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{70d} - \frac{(3-4\cos(dx+c))^{\frac{3}{2}} \cos(dx+c) \sin(dx+c)}{14d} \\ & + \frac{47\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{140 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{59\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{420 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{59 \sin(dx+c) \sqrt{3-4\cos(dx+c)}}{105d} \end{aligned}$$

command

`integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$4 \left(60 \cos(dx + c)^2 - 9 \cos(dx + c) + 91 \right) \sqrt{-4 \cos(dx + c) + 3} \sin(dx + c) + 277 \sqrt{2} \operatorname{weierstrassPInverse}(-1, -$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{-4 \cos(dx + c) + 3} \cos(dx + c)^3, x\right)$$

51.119 Problem number 517

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^2(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3 - 4 \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{10d} \\ & - \frac{21 \sqrt{\frac{1}{2} - \frac{\cos(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{20 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{\sqrt{\frac{1}{2} - \frac{\cos(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{20 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{\sin(dx + c) \sqrt{3 - 4 \cos(dx + c)}}{5d} \end{aligned}$$

command

`integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$4(4 \cos(dx + c) - 1) \sqrt{-4 \cos(dx + c) + 3} \sin(dx + c) - 7 \sqrt{2} \operatorname{weierstrassPInverse}(-1, -1, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{-4 \cos(dx + c) + 3} \cos(dx + c)^2, x\right)$$

51.120 Problem number 518

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{\frac{1}{2} - \frac{\cos(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{2 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{\sqrt{\frac{1}{2} - \frac{\cos(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{6 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2 \sin(dx + c) \sqrt{3 - 4 \cos(dx + c)}}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$8 \sqrt{-4 \cos(dx + c) + 3} \sin(dx + c) + 5 \sqrt{2} \operatorname{weierstrassPInverse}\left(-1, -1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2}\right) + 5 \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{-4 \cos(dx + c) + 3} \cos(dx + c), x\right)$$

51.121 Problem number 519

$$\int \sqrt{3 - 4 \cos(c + dx)} dx$$

Optimal antiderivative

$$-\frac{2 \sqrt{\frac{1}{2} - \frac{\cos(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{\sin\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \operatorname{weierstrassPInverse}\left(-1, -1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2}\right) + \sqrt{2} \operatorname{weierstrassPInverse}\left(-1, -1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{-4 \cos(dx + c) + 3}, x\right)$$

51.122 Problem number 523

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{8a \sin(dx+c) \sqrt{a+b\cos(dx+c)}}{15b^2d} + \frac{2 \cos(dx+c) \sin(dx+c) \sqrt{a+b\cos(dx+c)}}{5bd} \\ & + \frac{2(8a^2+9b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b\cos(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{\frac{a+b\cos(dx+c)}{a+b}}} \\ & - \frac{2a(8a^2+7b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{a+b\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4\sqrt{2}(-4ia^3 - 3iab^2)\sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right) + 4\sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

51.123 Problem number 524

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \sin(dx+c) \sqrt{a+b\cos(dx+c)}}{3bd} \\ & - \frac{4a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b\cos(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{\frac{a+b\cos(dx+c)}{a+b}}} \\ & + \frac{2(2a^2+b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{a+b\cos(dx+c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-6i \sqrt{2} ab^{\frac{3}{2}} \text{weierstrassZeta} \left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \text{weierstrassPInverse} \left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)}{3b} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx+c)^2}{\sqrt{b \cos(dx+c) + a}}, x \right)$$

51.124 Problem number 525

$$\int \frac{\cos(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a+b \cos(dx+c)}}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) bd \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}$$

$$- \frac{2a\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) bd \sqrt{a+b \cos(dx+c)}}$$

command

```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2i \sqrt{2} a \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib \sin(dx+c)+2a}{3b} \right) - 2i \sqrt{2} a \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib \sin(dx+c)+2a}{3b} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx+c)}{\sqrt{b \cos(dx+c) + a}}, x \right)$$

51.125 Problem number 526

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a+b \cos(dx+c)}}$$

command

`integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib \sin(dx+c)+2a}{3b}\right) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib \sin(dx+c)+2a}{3b}\right)}{bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{b \cos(dx+c) + a}}, x\right)$$

51.126 Problem number 530

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(\cos^2(dx+c)) \sin(dx+c)}{b(a^2-b^2)d\sqrt{a+b \cos(dx+c)}} - \frac{2a(8a^2-3b^2) \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{5b^3(a^2-b^2)d} \\ & + \frac{2(6a^2-b^2) \cos(dx+c) \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{5b^2(a^2-b^2)d} \\ & + \frac{2(16a^4-8a^2b^2-3b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \cos(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2-b^2) d \sqrt{\frac{a+b \cos(dx+c)}{a+b}}} \\ & - \frac{8a(4a^2+b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a+b \cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(8 a^4 b^2 - 3 a^2 b^4 - (a^2 b^4 - b^6) \cos(dx + c)^2 + 2 (a^3 b^3 - ab^5) \cos(dx + c) \right) \sqrt{b \cos(dx + c) + a} \sin(dx + c) + \left(\dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \cos(dx + c) + a} \cos(dx + c)^4}{b^2 \cos(dx + c)^2 + 2 ab \cos(dx + c) + a^2}, x \right)$$

51.127 Problem number 531

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2a^2 \cos(dx + c) \sin(dx + c)}{b(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} + \frac{2(4a^2 - b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3b^2(a^2 - b^2) d}}{\dots} + \frac{2a(8a^2 - 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{a + b \cos(dx + c)}}{\dots} + \frac{2(8a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) b^3 d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(4 a^3 b^2 - ab^4 + (a^2 b^3 - b^5) \cos(dx + c) \right) \sqrt{b \cos(dx + c) + a} \sin(dx + c) - \left(\sqrt{2} (16i a^4 b - 16i a^2 b^3 - 3i b^5) \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \cos(dx + c) + a} \cos(dx + c)^3}{b^2 \cos(dx + c)^2 + 2 ab \cos(dx + c) + a^2}, x \right)$$

51.128 Problem number 532

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a^2 \sin(dx + c)}{b(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{2(2a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{4a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \cos(dx + c) + a} a^2 b^2 \sin(dx + c) + \left(\sqrt{2} (-4i a^3 b + 5i a b^3) \cos(dx + c) + \sqrt{2} (-4i a^4 + 5i a^2 b^2)\right) \sqrt{b} \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c) + a} \cos(dx + c)^2}{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}, x\right)$$

51.129 Problem number 533

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \sin(dx + c)}{(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \\ & - \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \cos(dx+c) + a} ab^2 \sin(dx+c) - \left(\sqrt{2} (2i a^2 b - 3i b^3) \cos(dx+c) + \sqrt{2} (2i a^3 - 3i ab^2) \right) \sqrt{b} \text{weierstrassPI}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \cos(dx+c) + a} \cos(dx+c)}{b^2 \cos^2(dx+c) + 2ab \cos(dx+c) + a^2}, x \right)$$

51.130 Problem number 534

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b \sin(dx+c)}{(a^2 - b^2) d \sqrt{a + b \cos(dx+c)}} \\ & + \frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a + b \cos(dx+c)}}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx+c)}{a+b}}} \end{aligned}$$

command

```
integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \cos(dx+c) + a} b^2 \sin(dx+c) + \left(i \sqrt{2} ab \cos(dx+c) + i \sqrt{2} a^2 \right) \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8}{3b^2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \cos(dx+c) + a}}{b^2 \cos^2(dx+c) + 2ab \cos(dx+c) + a^2}, x \right)$$

51.131 Problem number 538

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(\cos^3(dx + c)) \sin(dx + c)}{3b(a^2 - b^2)d(a + b \cos(dx + c))^{\frac{3}{2}}} - \frac{8a^2(2a^2 - 3b^2)(\cos^2(dx + c)) \sin(dx + c)}{3b^2(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\ & - \frac{4a(32a^4 - 49a^2b^2 + 7b^4) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^4(a^2 - b^2)^2 d} \\ & + \frac{2(48a^4 - 71a^2b^2 + 3b^4) \cos(dx + c) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^3(a^2 - b^2)^2 d} \\ & + \frac{2(128a^6 - 212a^4b^2 + 55a^2b^4 + 9b^6) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^5 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{2a(128a^4 - 116a^2b^2 - 17b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^5 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(64 a^7 b^2 - 98 a^5 b^4 + 14 a^3 b^6 - 3 (a^4 b^5 - 2 a^2 b^7 + b^9) \cos(dx + c)^3 + 8 (a^5 b^4 - 2 a^3 b^6 + a b^8) \cos(dx + c)^2 + 5 (1 \right)}{15 b^5 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c) + a} \cos(dx + c)^5}{b^3 \cos(dx + c)^3 + 3 a b^2 \cos(dx + c)^2 + 3 a^2 b \cos(dx + c) + a^3}, x\right)$$

51.132 Problem number 539

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a^2(\cos^2(dx + c)) \sin(dx + c)}{3b(a^2 - b^2)d(a + b \cos(dx + c))^{3/2}} + \frac{4a^3(3a^2 - 5b^2) \sin(dx + c)}{3b^3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{2(2a^2 - b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3b^3(a^2 - b^2)d} \\ & - \frac{8a(4a^4 - 7a^2b^2 + 2b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2(16a^4 - 16a^2b^2 - b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(8a^6b^2 - 13a^4b^4 + a^2b^6 + (a^4b^4 - 2a^2b^6 + b^8) \cos(dx + c)^2 + 2(5a^5b^3 - 8a^3b^5 + ab^7) \cos(dx + c) \right) \sqrt{b \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c) + a} \cos(dx + c)^4}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

51.133 Problem number 540

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \cos(dx+c) \sin(dx+c)}{3b(a^2-b^2)d(a+b \cos(dx+c))^{\frac{3}{2}}} - \frac{8a^2(a^2-2b^2) \sin(dx+c)}{3b^2(a^2-b^2)^2 d \sqrt{a+b \cos(dx+c)}}$$

$$+ \frac{2(8a^4-15a^2b^2+3b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \cos(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2-b^2)^2 d \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}$$

$$- \frac{2a(8a^2-9b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2-b^2) d \sqrt{a+b \cos(dx+c)}}$$

command

`integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(4a^5b^2 - 8a^3b^4 + (5a^4b^3 - 9a^2b^5) \cos(dx+c)) \sqrt{b \cos(dx+c) + a} \sin(dx+c) + 4(\sqrt{2}(-4ia^5b^2 + 9ia^3b^4))}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c) + a} \cos(dx+c)^3}{b^3 \cos(dx+c)^3 + 3ab^2 \cos(dx+c)^2 + 3a^2b \cos(dx+c) + a^3}, x\right)$$

51.134 Problem number 541

$$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sin(dx+c)}{3b(a^2-b^2)d(a+b \cos(dx+c))^{\frac{3}{2}}} + \frac{4a(a^2-3b^2) \sin(dx+c)}{3b(a^2-b^2)^2 d \sqrt{a+b \cos(dx+c)}}$$

$$- \frac{4a(a^2-3b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \cos(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2-b^2)^2 d \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}$$

$$+ \frac{2(2a^2-3b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2-b^2) d \sqrt{a+b \cos(dx+c)}}$$

command

```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (a^4 b^2 - 5 a^2 b^4 + 2 (a^3 b^3 - 3 a b^5) \cos(dx + c)) \sqrt{b \cos(dx + c) + a} \sin(dx + c) + \left(\sqrt{2} (-4i a^4 b^2 + 9i a^2 b^4 - 9i b^6)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \cos(dx + c) + a} \cos(dx + c)^2}{b^3 \cos(dx + c)^3 + 3 a b^2 \cos(dx + c)^2 + 3 a^2 b \cos(dx + c) + a^3}, x\right)$$

51.135 Problem number 542

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a \sin(dx + c)}{3(a^2 - b^2) d(a + b \cos(dx + c))^{3/2}} + \frac{2(a^2 + 3b^2) \sin(dx + c)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

$$\frac{2(a^2 + 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (2 a^3 b^2 + 2 a b^4 + (a^2 b^3 + 3 b^5) \cos(dx + c)) \sqrt{b \cos(dx + c) + a} \sin(dx + c) - 2 \left(\sqrt{2} (i a^3 b^2 - 3i a b^4) \cos(dx + c) + \dots\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \cos(dx + c) + a} \cos(dx + c)}{b^3 \cos(dx + c)^3 + 3 a b^2 \cos(dx + c)^2 + 3 a^2 b \cos(dx + c) + a^3}, x\right)$$

51.136 Problem number 543

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b \sin(dx + c)}{3(a^2 - b^2) d (a + b \cos(dx + c))^{3/2}} - \frac{8abs \sin(dx + c)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{8a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(4ab^3 \cos(dx + c) + 5a^2b^2 - b^4) \sqrt{b \cos(dx + c) + a} \sin(dx + c) - (\sqrt{2}(-ia^2b^2 - 3ib^4) \cos(dx + c)^2 - 2\sqrt{2}ab^3 \cos(dx + c) + 5a^2b^2 - b^4) \sqrt{b \cos(dx + c) + a}}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

51.137 Problem number 546

$$\int \frac{1}{(a + b \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b \sin(dx+c)}{5(a^2-b^2)d(a+b \cos(dx+c))^{\frac{5}{2}}} - \frac{16ab \sin(dx+c)}{15(a^2-b^2)^2 d(a+b \cos(dx+c))^{\frac{3}{2}}} \\ & - \frac{2b(23a^2+9b^2) \sin(dx+c)}{15(a^2-b^2)^3 d \sqrt{a+b \cos(dx+c)}} \\ & + \frac{2(23a^2+9b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \cos(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2-b^2)^3 d \sqrt{\frac{a+b \cos(dx+c)}{a+b}}} \\ & - \frac{16a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2-b^2)^2 d \sqrt{a+b \cos(dx+c)}} \end{aligned}$$

command

`integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(34 a^4 b^2 - 5 a^2 b^4 + 3 b^6 + (23 a^2 b^4 + 9 b^6) \cos(dx+c)^2 + 2 (27 a^3 b^3 + 5 a b^5) \cos(dx+c) \right) \sqrt{b \cos(dx+c) + a}}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cos(dx+c) + a}}{b^4 \cos(dx+c)^4 + 4 a b^3 \cos(dx+c)^3 + 6 a^2 b^2 \cos(dx+c)^2 + 4 a^3 b \cos(dx+c) + a^4}, x\right)$$

51.138 Problem number 547

$$\int \frac{\cos^3(c+dx)}{\sqrt{3+4 \cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{23 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{140 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{9 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{20 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{\sin(dx+c) \sqrt{3+4 \cos(dx+c)}}{10d} + \frac{\cos(dx+c) \sin(dx+c) \sqrt{3+4 \cos(dx+c)}}{10d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \sqrt{4 \cos(dx + c) + 3} (\cos(dx + c) - 1) \sin(dx + c) + 7i \sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c) + \frac{1}{2}) + 7i \sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2})$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx + c)^3}{\sqrt{4 \cos(dx + c) + 3}}, x\right)$$

51.139 Problem number 548

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{17 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{84 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{4 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{\sin(dx + c) \sqrt{3 + 4 \cos(dx + c)}}{6d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \sqrt{4 \cos(dx + c) + 3} \sin(dx + c) - 7i \sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c) + \frac{1}{2}) + 7i \sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2})$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx + c)^2}{\sqrt{4 \cos(dx + c) + 3}}, x\right)$$

51.140 Problem number 549

$$\int \frac{\cos(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{14 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c) + \frac{1}{2}) - i \sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx + c)}{\sqrt{4 \cos(dx + c) + 3}}, x\right)$$

51.141 Problem number 550

$$\int \frac{1}{\sqrt{3 + 4 \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx+c) + i \sin(dx+c) + \frac{1}{2}) + i\sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx+c))}{2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{4 \cos(dx+c) + 3}}, x\right)$$

51.142 Problem number 554

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{23 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{140 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{9 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{20 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{\sin(dx+c) \sqrt{3-4\cos(dx+c)}}{10d} - \frac{\cos(dx+c) \sin(dx+c) \sqrt{3-4\cos(dx+c)}}{10d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4(\cos(dx+c)+1)\sqrt{-4\cos(dx+c)+3}\sin(dx+c)+7\sqrt{2}\operatorname{weierstrassPInverse}(-1,-1,\cos(dx+c)+i\sin(dx+c))}{2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-4\cos(dx+c)+3}\cos(dx+c)^3}{4\cos(dx+c)-3}, x\right)$$

51.143 Problem number 555

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{17 \sqrt{\frac{1}{2} - \frac{\cos(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{84 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{\sqrt{\frac{1}{2} - \frac{\cos(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{4 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{\sin(dx + c) \sqrt{3 - 4 \cos(dx + c)}}{6d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \sqrt{-4 \cos(dx + c) + 3} \sin(dx + c) + 7 \sqrt{2} \operatorname{weierstrassPInverse}(-1, -1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2}) + 7 \sqrt{2}}{6d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-4 \cos(dx + c) + 3} \cos(dx + c)^2}{4 \cos(dx + c) - 3}, x\right)$$

51.144 Problem number 556

$$\int \frac{\cos(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3 \sqrt{\frac{1}{2} - \frac{\cos(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{14 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{\sqrt{\frac{1}{2} - \frac{\cos(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{2 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$\frac{\sqrt{2} \operatorname{weierstrassPInverse}(-1, -1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2}) + \sqrt{2} \operatorname{weierstrassPInverse}(-1, -1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2})}{2d}$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-4 \cos(dx + c) + 3} \cos(dx + c)}{4 \cos(dx + c) - 3}, x\right)$$

51.145 Problem number 557

$$\int \frac{1}{\sqrt{3 - 4 \cos(c + dx)}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{1}{2} - \frac{\cos(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right) \sqrt{7}}{7 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$\frac{\sqrt{2} \operatorname{weierstrassPInverse}(-1, -1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2}) + \sqrt{2} \operatorname{weierstrassPInverse}(-1, -1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2})}{2d}$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-4 \cos(dx + c) + 3}}{4 \cos(dx + c) - 3}, x\right)$$

51.146 Problem number 561

$$\int \cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2A \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} \\ & + \frac{2B \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} + \frac{10B \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15 B \cos(dx+c)^2 + 21 A \cos(dx+c) + 25 B \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 25i \sqrt{2} B \operatorname{weierstrassPInverse}(-4, C)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(B \cos(dx+c)^3 + A \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}, x\right)$$

51.147 Problem number 562

$$\int \cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2A \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3B \cos(dx+c) + 5A) \sqrt{\cos(dx+c)} \sin(dx+c) - 5i \sqrt{2} \operatorname{AweierstrassPInverse}(-4, 0, c) + i \sin(dx+c)}{dx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(B \cos(dx+c)^2 + A \cos(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

51.148 Problem number 563

$$\int \sqrt{\cos(c+dx)} (A + B \cos(c+dx)) dx$$

Optimal antiderivative

$$\frac{2A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d}$$

command

```
integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2B \sqrt{\cos(dx+c)} \sin(dx+c) - i \sqrt{2} \operatorname{BweierstrassPInverse}(-4, 0, c) + i \sqrt{2} \operatorname{BweierstrassPInverse}(-4, 0, c) + i \sin(dx+c)}{dx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(B \cos(dx+c) + A\right) \sqrt{\cos(dx+c)}, x\right)$$

51.149 Problem number 564

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \operatorname{AweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \operatorname{AweierstrassPInverse}(-4, 0, \cos(dx + c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B \cos(dx + c) + A}{\sqrt{\cos(dx + c)}}, x\right)$$

51.150 Problem number 565

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2A \sin(dx + c)}{d \sqrt{\cos(dx + c)}}$$

command

`integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B \cos(dx + c) + A}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

51.151 Problem number 566

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2A \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2B \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} A \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B \cos(dx + c) + A}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

51.152 Problem number 567

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2B \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{6A \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} B \cos(dx+c)^3 \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B \cos(dx+c) + A}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

51.153 Problem number 568

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9a^2 + 7b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{20ab \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(9a^2 + 7b^2) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} + \frac{4ab \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} \\ & + \frac{2b^2 \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{9d} + \frac{20ab \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-150i \sqrt{2} \text{ abweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 150i \sqrt{2} \text{ abweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

integral($(b^2 \cos(dx + c)^4 + 2ab \cos(dx + c)^3 + a^2 \cos(dx + c)^2) \sqrt{\cos(dx + c)}$, x)

51.154 Problem number 569

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{12ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2b^2 \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2(7a^2 + 5b^2) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$126i \sqrt{2} \text{ abweierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 126i \sqrt{2} \text{ abweierstrassZeta}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

integral($(b^2 \cos(dx + c)^3 + 2ab \cos(dx + c)^2 + a^2 \cos(dx + c)) \sqrt{\cos(dx + c)}$, x)

51.155 Problem number 570

$$\int \sqrt{\cos(c+dx)} (a+b\cos(c+dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5a^2+3b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b^2 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{4ab \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-10i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 10i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos^2(dx+c) + 2ab \cos(dx+c) + a^2\right) \sqrt{\cos(dx+c)}, x\right)$$

51.156 Problem number 571

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4ab \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3a^2+b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 b^2 \sqrt{\cos(dx+c)} \sin(dx+c) + 6i \sqrt{2} ab \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2}{\sqrt{\cos(dx+c)}}, x\right)$$

51.157 Problem number 572

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(a^2 - b^2) \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a^2 \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2i \sqrt{2} ab \cos(dx+c) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 2i \sqrt{2} ab \cos(dx+c) \text{weierstrassZeta}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

51.158 Problem number 573

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4ab \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^2 + 3b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{4ab \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-6i \sqrt{2} ab \cos(dx+c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 6i \sqrt{2} a$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

51.159 Problem number 574

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3a^2 + 5b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{4ab \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2(3a^2 + 5b^2) \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-10i \sqrt{2} ab \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} ab \cos(dx + c)^3 \text{weier}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

51.160 Problem number 575

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(27a^2 + 7b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7a^2 + 15b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(27a^2 + 7b^2) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} + \frac{40ab^2 \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\ & + \frac{2b^2 \left(\cos^{\frac{5}{2}}(dx + c)\right) (a + b \cos(dx + c)) \sin(dx + c)}{9d} \\ & + \frac{2a(7a^2 + 15b^2) \sin(dx + c) \left(\sqrt{\cos(dx + c)}\right)}{21d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$2 \left(35b^3 \cos(dx + c)^3 + 135ab^2 \cos(dx + c)^2 + 105a^3 + 225ab^2 + 7(27a^2b + 7b^3) \cos(dx + c) \right) \sqrt{\cos(dx + c)} \sin$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^3 \cos(dx + c)^4 + 3ab^2 \cos(dx + c)^3 + 3a^2b \cos(dx + c)^2 + a^3 \cos(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

51.161 Problem number 576

$$\int \sqrt{\cos(c+dx)} (a+b\cos(c+dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5a^2+9b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(21a^2+5b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{32ab^2 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{35d} + \frac{2b^2 \left(\cos^{\frac{3}{2}}(dx+c)\right) (a+b\cos(dx+c)) \sin(dx+c)}{7d} \\ & + \frac{2b(21a^2+5b^2) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15b^3 \cos(dx+c)^2 + 63ab^2 \cos(dx+c) + 105a^2b + 25b^3 \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 5\sqrt{2} (21ia^2b + 5ib^3)w$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^3 \cos(dx+c)^3 + 3ab^2 \cos(dx+c)^2 + 3a^2b \cos(dx+c) + a^3\right) \sqrt{\cos(dx+c)}, x\right)$$

51.162 Problem number 577

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6b(5a^2+b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(a^2+b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{5d} + \frac{2b^2(a+b\cos(dx+c)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{5d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 (b^3 \cos(dx + c) + 5 ab^2) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \sqrt{2} (i a^3 + i ab^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c)) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{b^3 \cos(dx + c)^3 + 3 ab^2 \cos(dx + c)^2 + 3 a^2 b \cos(dx + c) + a^3}{\sqrt{\cos(dx + c)}}, x \right)$$

51.163 Problem number 578

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(a^2 - 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2b(9a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a^2(a + b \cos(dx + c)) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} - \frac{2b(3a^2 - b^2) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-9i a^2 b - i b^3) \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (9i a^2 b + i b^3) \cos(dx + c) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{b^3 \cos(dx + c)^3 + 3 ab^2 \cos(dx + c)^2 + 3 a^2 b \cos(dx + c) + a^3}{\cos(dx + c)^{\frac{3}{2}}}, x \right)$$

51.164 Problem number 579

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(3a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(a^2 + 9b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(a + b \cos(dx + c)) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{16a^2b \sin(dx + c)}{3d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-i a^3 - 9i ab^2) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(i a^3 + 9i ab^2) \cos(dx + c)}{\cos(dx + c)^{\frac{5}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

51.165 Problem number 580

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{6a(a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2b \sin(dx + c)}{5d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a^2(a + b \cos(dx + c)) \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{6a(a^2 + 5b^2) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (i a^2 b + i b^3) \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2} (-i a^2 b - i b^3) \cos(dx + c)^3$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

51.166 Problem number 581

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(9a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5a^2 + 21b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{32a^2b \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{2a(5a^2 + 21b^2) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2a^2(a + b \cos(dx + c)) \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{2b(9a^2 + 5b^2) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (5i a^3 + 21i ab^2) \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2} (-5i a^3 - 21i ab^2) \cos(dx + c)^4$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}{\cos(dx + c)^{\frac{9}{2}}}, x\right)$$

51.167 Problem number 690

$$\int (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2A \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{6A \sin(dx + c) (\sqrt{\sec}(dx + c)})}{5d} \\ & - \frac{6A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \cos(dx + c)^2 \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left((B \cos(dx + c) + A) \sec(dx + c)^{\frac{7}{2}}, x \right)$$

51.168 Problem number 691

$$\int (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2B \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & - \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} A \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((B \cos(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}, x\right)$$

51.169 Problem number 692

$$\int (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((B \cos(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

51.170 Problem number 693

$$\int (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \operatorname{AweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \operatorname{AweierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(B \cos(dx + c) + A\right) \sqrt{\sec(dx + c)}, x\right)$$

51.171 Problem number 694

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2B \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 B \sqrt{\cos(dx+c)} \sin(dx+c) - i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{B \cos(dx+c) + A}{\sqrt{\sec(dx+c)}}, x\right)$$

51.172 Problem number 695

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \sin(dx+c)}{5d \sec(dx+c)^{\frac{3}{2}}} + \frac{2A \sin(dx+c)}{3d \sqrt{\sec(dx+c)}} \\ & + \frac{6B \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} \text{AweierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} \text{AweierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{B \cos(dx+c) + A}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

51.173 Problem number 696

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2A \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{10B \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{6A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-25i \sqrt{2} B \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} B \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B \cos(dx + c) + A}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

51.174 Problem number 697

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5a^2 + 7b^2) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} + \frac{4ab \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2a^2 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{12ab \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{12ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5a^2 + 7b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-126i \sqrt{2} ab \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 126i \sqrt{2} ab \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2\right) \sec(dx + c)^{\frac{9}{2}}, x\right)$$

51.175 Problem number 698

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4ab \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} + \frac{2a^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2(3a^2 + 5b^2) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2(3a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-10i \sqrt{2} ab \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} ab \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2\right) \sec(dx + c)^{\frac{7}{2}}, x\right)$$

51.176 Problem number 699

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{4ab \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & - \frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^2 + 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-6i \sqrt{2} ab \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 6i \sqrt{2} ab$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2\right) \sec(dx + c)^{\frac{5}{2}}, x\right)$$

51.177 Problem number 700

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & - \frac{2(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2i \sqrt{2} \operatorname{abweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2i \sqrt{2} \operatorname{abweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

51.178 Problem number 701

$$\int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2b^2 \sqrt{\cos(dx + c)} \sin(dx + c) + 6i \sqrt{2} \operatorname{abweierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2\right) \sqrt{\sec(dx + c)}, x\right)$$

51.179 Problem number 702

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{4ab \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2(5a^2 + 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-10i \sqrt{2} \operatorname{abweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} \operatorname{abweierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}{\sqrt{\sec(dx + c)}}, x\right)$$

51.180 Problem number 703

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{4ab \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(7a^2 + 5b^2) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{12ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$126i \sqrt{2} ab \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 126i \sqrt{2} ab \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

51.181 Problem number 704

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{4ab \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(9a^2 + 7b^2) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{20ab \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(9a^2 + 7b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{20ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-150i \sqrt{2} ab \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 150i \sqrt{2} ab \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

51.182 Problem number 705

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5a^2 + 21b^2) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} + \frac{32a^2b \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{35d} \\ & + \frac{2a^2 \left(\sec^{\frac{5}{2}}(dx + c) \right) (b + a \sec(dx + c)) \sin(dx + c)}{7d} + \frac{2b(9a^2 + 5b^2) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{2b(9a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5a^2 + 21b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (5i a^3 + 21i ab^2) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-5i a^3 - 21i ab^2) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3\right) \sec(dx + c)^{\frac{9}{2}}, x\right)$$

51.183 Problem number 706

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^2b \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2a^2 \left(\sec^{\frac{3}{2}}(dx+c)\right) (b+a \sec(dx+c)) \sin(dx+c)}{5d} \\ & + \frac{6a(a^2+5b^2) \sin(dx+c) (\sqrt{\sec}(dx+c))}{5d} \\ & - \frac{6a(a^2+5b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(a^2+b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (i a^2 b + i b^3) \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5\sqrt{2} (-i a^2 b - i b^3) \cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^3 \cos(dx+c)^3 + 3ab^2 \cos(dx+c)^2 + 3a^2b \cos(dx+c) + a^3\right) \sec(dx+c)^{\frac{7}{2}}, x\right)$$

51.184 Problem number 707

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16a^2b \sin(dx+c) (\sqrt{\sec}(dx+c))}{3d} + \frac{2a^2(b+a \sec(dx+c)) \sin(dx+c) (\sqrt{\sec}(dx+c))}{3d} \\ & - \frac{2b(3a^2-b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(a^2+9b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i a^3 - 9i ab^2) \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i a^3 + 9i ab^2) \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3\right) \sec(dx + c)^{\frac{5}{2}}, x\right)$$

51.185 Problem number 708

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(b + a \sec(dx + c)) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2a(3a^2 - b^2) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & - \frac{2a(a^2 - 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(9a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-9i a^2b - i b^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (9i a^2b + i b^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

51.186 Problem number 709

$$\int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(b + a \sec(dx + c)) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{8a b^2 \sin(dx + c)}{5d \sqrt{\sec(dx + c)}} \\ & + \frac{6b(5a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i a^3 + i a b^2) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-i a^3 - i a b^2) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^3 \cos(dx + c)^3 + 3 a b^2 \cos(dx + c)^2 + 3 a^2 b \cos(dx + c) + a^3\right) \sqrt{\sec(dx + c)}, x\right)$$

51.187 Problem number 710

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32a b^2 \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{2b^2(b + a \sec(dx + c)) \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2b(21a^2 + 5b^2) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(5a^2 + 9b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(21a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (21i a^2 b + 5i b^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-21i a^2 b - 5i b^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^3 \cos(dx + c)^3 + 3 ab^2 \cos(dx + c)^2 + 3 a^2 b \cos(dx + c) + a^3}{\sqrt{\sec(dx + c)}}, x\right)$$

51.188 Problem number 711

$$\int \frac{(a + b \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{40a b^2 \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{2b(27a^2 + 7b^2) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2b^2(b + a \sec(dx + c)) \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2a(7a^2 + 15b^2) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2b(27a^2 + 7b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7a^2 + 15b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (7i a^3 + 15i ab^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \sqrt{2} (-7i a^3 - 15i ab^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^3 \cos(dx + c)^3 + 3 ab^2 \cos(dx + c)^2 + 3 a^2 b \cos(dx + c) + a^3}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

51.189 Problem number 794

$$\int \frac{aB + bB \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}$$

command

`integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} Ba \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i b \sin(dx+c) + 2a}{3b}\right) + i \sqrt{2} Ba \sqrt{b} \operatorname{weierstra}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(dx + c) + a} B, x\right)$$

51.190 Problem number 799

$$\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5bd} + \frac{2B(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^2d} \\ & + \frac{10bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{10B \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \\ & + \frac{6A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-25i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(B \cos(dx + c)^3 + A \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}, x\right)$$

51.191 Problem number 800

$$\int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5bd} \\ & + \frac{2Ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2A \sin(dx + c) \sqrt{b \cos(dx + c)}}{3d} \\ & + \frac{6B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} A \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} A \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(B \cos(dx + c)^2 + A \cos(dx + c)\right) \sqrt{b \cos(dx + c)}, x\right)$$

51.192 Problem number 801

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bB \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}} \\ & + \frac{2B \sin(dx+c) \sqrt{b \cos(dx+c)}}{3d} \\ & + \frac{2A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((B \cos(dx+c) + A) \sqrt{b \cos(dx+c)}, x\right)$$

51.193 Problem number 802

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}A\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i\sqrt{2}A\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c)} \sec(dx + c), x\right)$$

51.194 Problem number 803

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{2Ab \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} + \frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} - \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

`integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}B\sqrt{b}$ cos(dx + c) weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i\sqrt{2}B\sqrt{b}$ cos(dx + c) weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^2, x\right)$$

51.195 Problem number 804

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab^2 \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2bB \sin(dx + c)}{d\sqrt{b \cos(dx + c)}} \\ & + \frac{2Ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx + c)}} \\ & - \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \operatorname{weier}$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^3, x\right)$$

51.196 Problem number 805

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab^3 \sin(dx + c)}{5d(b \cos(dx + c))^{\frac{5}{2}}} + \frac{2b^2B \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{6Ab \sin(dx + c)}{5d\sqrt{b \cos(dx + c)}} \\ & + \frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx + c)}} \\ & - \frac{6A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(B\cos(dx+c)+A\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^4,x\right)$$

51.197 Problem number 806

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{5d} + \frac{2B(b\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{7bd} \\ & + \frac{10b^2B\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})}{21\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d\sqrt{b\cos(dx+c)}} \\ & + \frac{10bB\sin(dx+c)\sqrt{b\cos(dx+c)}}{21d} \\ & + \frac{6Ab\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\sqrt{b\cos(dx+c)}}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-25i\sqrt{2}Bb^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+25i\sqrt{2}Bb^{\frac{3}{2}}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb\cos(dx+c)^3+Ab\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)},x\right)$$

51.198 Problem number 807

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(b \cos(dx + c))^{3/2} \sin(dx + c)}{5d} \\ & + \frac{2A b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2Ab \sin(dx + c) \sqrt{b \cos(dx + c)}}{3d} \\ & + \frac{6bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} Ab^{3/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} Ab^{3/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)}, x\right)$$

51.199 Problem number 808

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2bB \sin(dx + c) \sqrt{b \cos(dx + c)}}{3d} \\ & + \frac{2Ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} B b^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B b^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c), x\right)$$

51.200 Problem number 809

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{2Ab^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} + \frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} A b^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A b^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^2, x\right)$$

51.201 Problem number 810

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{2Ab^2 \sin(dx + c)}{d\sqrt{b \cos(dx + c)}} + \frac{2b^2 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx + c)}} - \frac{2Ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\cos(dx + c)}}$$

command

`integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} B b^{\frac{3}{2}} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B b^{\frac{3}{2}} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^3, x\right)$$

51.202 Problem number 811

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{2Ab^3 \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2b^2 B \sin(dx + c)}{d\sqrt{b \cos(dx + c)}} + \frac{2Ab^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx + c)}} - \frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} A b^{\frac{3}{2}} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A b^{\frac{3}{2}} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^4, x\right)$$

51.203 Problem number 812

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A b^4 \sin(dx + c)}{5d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{2b^3 B \sin(dx + c)}{3d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{6A b^2 \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} \\ & + \frac{2b^2 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{6Ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B b^{\frac{3}{2}} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B b^{\frac{3}{2}} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^5, x\right)$$

51.204 Problem number 813

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab(b \cos(dx + c))^{3/2} \sin(dx + c)}{5d} + \frac{2B(b \cos(dx + c))^{5/2} \sin(dx + c)}{7d} \\ & + \frac{10b^3 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{10b^2 B \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \\ & + \frac{6A b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-25i \sqrt{2} B b^{5/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} B b^{5/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}, x\right)$$

51.205 Problem number 814

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bB(b \cos(dx + c))^{3/2} \sin(dx + c)}{5d} \\ & + \frac{2A b^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2A b^2 \sin(dx + c) \sqrt{b \cos(dx + c)}}{3d} \\ & + \frac{6b^2 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}Ab^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}Ab^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c),x\right)$$

51.206 Problem number 815

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^3B\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{b\cos(dx+c)}} \\ & + \frac{2b^2B\sin(dx+c)\sqrt{b\cos(dx+c)}}{3d} \\ & + \frac{2Ab^2\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{b\cos(dx+c)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i\sqrt{2}Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^2,x\right)$$

51.207 Problem number 816

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{2Ab^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx+c)}} + \frac{2b^2 B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\cos(dx+c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} Ab^{\frac{5}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} Ab^{\frac{5}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx+c)^3 + Ab^2 \cos(dx+c)^2\right) \sqrt{b \cos(dx+c)} \sec(dx+c)^3, x\right)$$

51.208 Problem number 817

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{2Ab^3 \sin(dx+c)}{d\sqrt{b \cos(dx+c)}} + \frac{2b^3 B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx+c)}} - \frac{2Ab^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\cos(dx+c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} B b^{\frac{5}{2}} \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B b^{\frac{5}{2}} \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^4, x\right)$$

51.209 Problem number 818

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab^4 \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2b^3 B \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} \\ & + \frac{2Ab^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{2b^2 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} A b^{\frac{5}{2}} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A b^{\frac{5}{2}} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^5, x\right)$$

51.210 Problem number 819

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^6(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab^5 \sin(dx + c)}{5d(b \cos(dx + c))^{5/2}} + \frac{2b^4 B \sin(dx + c)}{3d(b \cos(dx + c))^{3/2}} + \frac{6Ab^3 \sin(dx + c)}{5d\sqrt{b \cos(dx + c)}} \\ & + \frac{2b^3 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{6Ab^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B b^{5/2} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B b^{5/2} \cos(dx + c)^3 \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^6, x\right)$$

51.211 Problem number 820

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(b \cos(dx + c))^{3/2} \sin(dx + c)}{5b^2 d} + \frac{2B(b \cos(dx + c))^{5/2} \sin(dx + c)}{7b^3 d} \\ & + \frac{10B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{10B \sin(dx + c) \sqrt{b \cos(dx + c)}}{21bd} \\ & + \frac{6A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-25i\sqrt{2}B\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $25i\sqrt{2}B\sqrt{b}$ weierstrassPInverse(-4, 0,

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^3 + A\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}}{b}, x\right)$$

51.212 Problem number 821

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{5b^2d} \\ & + \frac{2A\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d\sqrt{b\cos(dx+c)}} \\ & + \frac{2A\sin(dx+c)\sqrt{b\cos(dx+c)}}{3bd} \\ & + \frac{6B\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\sqrt{b\cos(dx+c)}}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)bd\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i\sqrt{2}A\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $5i\sqrt{2}A\sqrt{b}$ weierstrassPInverse(-4, 0, co

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^2 + A\cos(dx+c)\right)\sqrt{b\cos(dx+c)}}{b}, x\right)$$

51.213 Problem number 822

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sin(dx + c) \sqrt{b \cos(dx + c)}}{3bd} \\ & + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{3bd \sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b}, x\right)$$

51.214 Problem number 823

$$\int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} A \sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i \sqrt{2} A \sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b \cos(dx + c)}, x\right)$$

51.215 Problem number 824

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} - \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b d \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \sqrt{b} \cos(dx + c)$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i \sqrt{2} B \sqrt{b} \cos(dx + c)$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)}{b \cos(dx + c)}, x\right)$$

51.216 Problem number 825

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2B \sin(dx + c)}{d\sqrt{b \cos(dx + c)}} \\ & + \frac{2A \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx + c)}} \\ & - \frac{2B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd\sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \operatorname{weier}$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b \cos(dx + c)}, x\right)$$

51.217 Problem number 826

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A b^2 \sin(dx + c)}{5d(b \cos(dx + c))^{\frac{5}{2}}} + \frac{2bB \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{6A \sin(dx + c)}{5d\sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx + c)}} \\ & - \frac{6A \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd\sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3}{b\cos(dx+c)},x\right)$$

51.218 Problem number 827

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{5b^3d} + \frac{2B(b\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{7b^4d} \\ & + \frac{10B\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})}{21\cos\left(\frac{dx}{2} + \frac{c}{2}\right)bd\sqrt{b\cos(dx+c)}} \\ & + \frac{10B\sin(dx+c)\sqrt{b\cos(dx+c)}}{21b^2d} \\ & + \frac{6A\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\sqrt{b\cos(dx+c)}}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)b^2d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-25i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+25i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B\cos(dx+c)^3+A\cos(dx+c)^2)\sqrt{b\cos(dx+c)}}{b^2},x\right)$$

51.219 Problem number 828

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5b^3d} \\ & + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}} \\ & + \frac{2A \sin(dx + c) \sqrt{b \cos(dx + c)}}{3b^2d} \\ & + \frac{6B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} A \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} A \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B \cos(dx + c)^2 + A \cos(dx + c)\right) \sqrt{b \cos(dx + c)}}{b^2}, x\right)$$

51.220 Problem number 829

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sin(dx + c) \sqrt{b \cos(dx + c)}}{3b^2d} \\ & + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i \sqrt{2} B \sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^2}, x\right)$$

51.221 Problem number 830

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b d \sqrt{b \cos(dx + c)}} + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} A \sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i \sqrt{2} A \sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)}, x\right)$$

51.222 Problem number 831

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{bd \sqrt{b \cos(dx + c)}} + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}} - \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

`integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \sqrt{b} \cos(dx + c) \operatorname{weiers}$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)^2}, x\right)$$

51.223 Problem number 832

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{3d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{2B \sin(dx + c)}{bd \sqrt{b \cos(dx + c)}} + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}} - \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \text{weier}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)}{b^2 \cos(dx + c)^2}, x\right)$$

51.224 Problem number 833

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab \sin(dx + c)}{5d (b \cos(dx + c))^{5/2}} + \frac{2B \sin(dx + c)}{3d (b \cos(dx + c))^{3/2}} + \frac{6A \sin(dx + c)}{5bd \sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}} \\ & - \frac{6A \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{we}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b^2 \cos(dx + c)^2}, x\right)$$

51.225 Problem number 834

$$\int \frac{\cos^5(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5b^4d} + \frac{2B(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^5d} \\ & + \frac{10B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{b \cos(dx + c)}} \\ & + \frac{10B \sin(dx + c) \sqrt{b \cos(dx + c)}}{21b^3d} \\ & + \frac{6A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-25i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B \cos(dx + c)^3 + A \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}}{b^3}, x\right)$$

51.226 Problem number 835

$$\int \frac{\cos^4(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2B(b \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{5b^4d} \\
& + \frac{2A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{b \cos(dx+c)}} \\
& + \frac{2A \sin(dx+c) \sqrt{b \cos(dx+c)}}{3b^3d} \\
& + \frac{6B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{\cos(dx+c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} A \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} A \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))}{10}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B \cos(dx+c)^2 + A \cos(dx+c)\right) \sqrt{b \cos(dx+c)}}{b^3}, x\right)$$

51.227 Problem number 836

$$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{b \cos(dx+c)}} \\
& + \frac{2B \sin(dx+c) \sqrt{b \cos(dx+c)}}{3b^3d} \\
& + \frac{2A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{\cos(dx+c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}B\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i\sqrt{2}B\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx + c) + A)\sqrt{b \cos(dx + c)}}{b^3}, x\right)$$

51.228 Problem number 837

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2A\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx + c)}} + \frac{2B\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}A\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i\sqrt{2}A\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx + c) + A)\sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)}, x\right)$$

51.229 Problem number 838

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx+c)}{b^2 d \sqrt{b \cos(dx+c)}} + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx+c)}} - \frac{2A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx+c)}}$$

command

```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} B \sqrt{b} \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} B \sqrt{b} \cos(dx+c) \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx+c) + A) \sqrt{b \cos(dx+c)}}{b^3 \cos(dx+c)^2}, x\right)$$

51.230 Problem number 839

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx+c)}{3bd (b \cos(dx+c))^{\frac{3}{2}}} + \frac{2B \sin(dx+c)}{b^2 d \sqrt{b \cos(dx+c)}} + \frac{2A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx+c)}} - \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx+c)}}$$

command

```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \text{weier}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)^3}, x\right)$$

51.231 Problem number 840

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \sin(dx + c)}{5d (b \cos(dx + c))^{5/2}} + \frac{2B \sin(dx + c)}{3bd (b \cos(dx + c))^{3/2}} + \frac{6A \sin(dx + c)}{5b^2 d \sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx + c)}} \\ & - \frac{6A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{we}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)}{b^3 \cos(dx + c)^3}, x\right)$$

51.232 Problem number 841

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \sin(dx + c)}{5bd (b \cos(dx + c))^{\frac{5}{2}}} + \frac{2B \sin(dx + c)}{3b^2 d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{6A \sin(dx + c)}{5b^3 d \sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{b \cos(dx + c)}} \\ & - \frac{6A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^4 \cos(dx + c)^4}, x\right)$$

52 Test file number 92

Test folder name:

test_cases/4_Trig_functions/4.2_Cosine/92_4.2.3.1-a+b_cos-^m-c+d_cos-^n-A+B_cos-

52.1 Problem number 123

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(9A + 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(9A + 7B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} + \frac{2a(A + B) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2aB \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{9d} + \frac{10a(A + B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-75i \sqrt{2} (A + B) \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 75i \sqrt{2} (A + B) \operatorname{aweierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba \cos(dx + c)^4 + (A + B)a \cos(dx + c)^3 + Aa \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}, x\right)$$

52.2 Problem number 124

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2aB \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2a(7A + 5B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (7A + 5B) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (7A + 5B) \text{aweierstrassPInverse}(-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ba \cos(dx + c)^3 + (A + B)a \cos(dx + c)^2 + Aa \cos(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

52.3 Problem number 125

$$\int \sqrt{\cos(c + dx)} (a + a \cos(c + dx))(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aB \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2a(A + B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (A + B) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (A + B) \text{aweierstrassPInverse}(-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ba \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa\right) \sqrt{\cos(dx + c)}, x\right)$$

52.4 Problem number 126

$$\int \frac{(a + a \cos(c + dx))(A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aB \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2Ba\sqrt{\cos(dx+c)}\sin(dx+c) - i\sqrt{2}(3A+B)a\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c) + i\sin(dx+c)) + i\sqrt{2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba\cos(dx+c)^2 + (A+B)a\cos(dx+c) + Aa}{\sqrt{\cos(dx+c)}}, x\right)$$

52.5 Problem number 127

$$\int \frac{(a + a \cos(c + dx))(A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2aA \sin(dx + c)}{d\sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} (A + B)a \cos(dx + c)$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i \sqrt{2} (A + B)a \cos(dx + c)$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ba \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

52.6 Problem number 128

$$\int \frac{(a + a \cos(c + dx))(A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a(A + B) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} (A + 3B)a \cos(dx + c)^2$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i \sqrt{2} (A + 3B)a \cos(dx + c)$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ba \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

52.7 Problem number 129

$$\int \frac{(a + a \cos(c + dx))(A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(3A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2a(A + B) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a(3A + 5B) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} (A + B)a \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (A + B)a \cos(dx + c)}{5d \sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

52.8 Problem number 130

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^2(9A + 8B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^2(6A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^2(9A + 8B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\
& + \frac{2a^2(9A + 11B) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\
& + \frac{2B \left(\cos^{\frac{5}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{9d} \\
& + \frac{4a^2(6A + 5B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (6A + 5B) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (6A + 5B) a^2 \operatorname{weierstrassP}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)}{21d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^2 \cos(dx + c)^4 + (A + 2B)a^2 \cos(dx + c)^3 + (2A + B)a^2 \cos(dx + c)^2 + Aa^2 \cos(dx + c)\right) \sqrt{\cos(dx + c)}\right)$$

52.9 Problem number 131

$$\int \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^2 (A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(4A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(7A + 6B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(7A + 9B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2B \left(\cos^{\frac{3}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{7d} \\ & + \frac{4a^2(7A + 6B) \sin(dx + c) (\sqrt{\cos}(dx + c))}{21d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (7A + 6B) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (7A + 6B) a^2 \operatorname{weierstrassP}(\cos(dx + c), 0, 1) \right)}{21d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^2 \cos(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + Aa^2\right) \sqrt{\cos(dx + c)}, x\right)$$

52.10 Problem number 132

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(5A + 4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(2A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(5A + 7B) \sin(dx + c) (\sqrt{\cos}(dx + c))}{15d} \\ & + \frac{2B(a^2 + a^2 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos}(dx + c))}{5d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (2A + B)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (2A + B)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ba^2 \cos(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + Aa^2}{\sqrt{\cos(dx + c)}}, x \right)$$

52.11 Problem number 133

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^2 (3A + 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2A(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} - \frac{2a^2 (3A - B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} (3A + 2B)a^2 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} (3A + 2B)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ba^2 \cos(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + Aa^2}{\cos(dx + c)^{\frac{3}{2}}}, x \right)$$

52.12 Problem number 134

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2 A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(2A + 3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A(a^2 + a^2 \cos(dx+c)) \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2a^2(5A + 3B) \sin(dx+c)}{3d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} (2A + 3B) a^2 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - i \sqrt{2} (2A + 3B) a^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^2 \cos(dx+c)^3 + (A + 2B)a^2 \cos(dx+c)^2 + (2A + B)a^2 \cos(dx+c) + Aa^2}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

52.13 Problem number 135

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(4A + 5B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(A + 2B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(7A + 5B) \sin(dx+c)}{15d \cos(dx+c)^{\frac{3}{2}}} + \frac{2A(a^2 + a^2 \cos(dx+c)) \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} \\ & + \frac{4a^2(4A + 5B) \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A + 2B)a^2 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + 2B)a^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ba^2 \cos(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + Aa^2}{\cos(dx + c)^{\frac{7}{2}}}, x \right)$$

52.14 Problem number 136

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(3A + 4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^2(6A + 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a^2(9A + 7B) \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^2(6A + 7B) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2A(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{4a^2(3A + 4B) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (6A + 7B)a^2 \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (6A + 7B) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ba^2 \cos(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + Aa^2}{\cos(dx + c)^{\frac{9}{2}}}, x \right)$$

52.15 Problem number 137

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(17A + 15B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(121A + 105B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(17A + 15B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{20a^3(22A + 21B) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{693d} \\ & + \frac{2aB \left(\cos^{\frac{5}{2}}(dx + c)\right) (a + a \cos(dx + c))^2 \sin(dx + c)}{11d} \\ & + \frac{2(11A + 15B) \left(\cos^{\frac{5}{2}}(dx + c)\right) (a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{99d} \\ & + \frac{4a^3(121A + 105B) \sin(dx + c) (\sqrt{\cos}(dx + c))}{231d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (121 A + 105 B) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (121 A + 105 B) a^3 \right)}{231 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^3 \cos(dx + c)^5 + (A + 3B)a^3 \cos(dx + c)^4 + 3(A + B)a^3 \cos(dx + c)^3 + (3A + B)a^3 \cos(dx + c)^2 + \dots\right), dx\right)$$

52.16 Problem number 138

$$\int \sqrt{\cos(c+dx)} (a+a\cos(c+dx))^3 (A+B\cos(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(21A+17B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(13A+11B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(24A+23B) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{105d} \\ & + \frac{2aB \left(\cos^{\frac{3}{2}}(dx+c)\right) (a+a\cos(dx+c))^2 \sin(dx+c)}{9d} \\ & + \frac{2(9A+13B) \left(\cos^{\frac{3}{2}}(dx+c)\right) (a^3+a^3\cos(dx+c)) \sin(dx+c)}{63d} \\ & + \frac{4a^3(13A+11B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (13A+11B)a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 15i \sqrt{2} (13A+11B)a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)}{105d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^3 \cos(dx+c)^4 + (A+3B)a^3 \cos(dx+c)^3 + 3(A+B)a^3 \cos(dx+c)^2 + (3A+B)a^3 \cos(dx+c) + \sqrt{\cos(dx+c)}\right) dx\right)$$

52.17 Problem number 139

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(9A + 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(21A + 13B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(42A + 41B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\ & + \frac{2aB(a + a \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d} \\ & + \frac{2(7A + 11B) (a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{35d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (21A + 13B) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (21A + 13B) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{35d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^3 \cos(dx + c)^4 + (A + 3B)a^3 \cos(dx + c)^3 + 3(A + B)a^3 \cos(dx + c)^2 + (3A + B)a^3 \cos(dx + c) + Aa^3}{\sqrt{\cos(dx + c)}}\right)$$

52.18 Problem number 140

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(5A+9B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(5A+3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA(a+a\cos(dx+c))^2 \sin(dx+c)}{d \sqrt{\cos(dx+c)}} - \frac{4a^3(5A-6B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{15d} \\ & - \frac{2(5A-B)(a^3+a^3\cos(dx+c)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{5d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (5A+3B)a^3 \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} (5A+3B) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^3 \cos(dx+c)^4 + (A+3B)a^3 \cos(dx+c)^3 + 3(A+B)a^3 \cos(dx+c)^2 + (3A+B)a^3 \cos(dx+c) + Aa^3}{\cos(dx+c)^{\frac{3}{2}}}\right)$$

52.19 Problem number 141

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^3(A-B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{20a^3(A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA(a+a\cos(dx+c))^2 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2(7A+3B)(a^3+a^3\cos(dx+c)) \sin(dx+c)}{3d \sqrt{\cos(dx+c)}} \\ & - \frac{4a^3(4A+B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A+B)a^3 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} (A+B)a^3 \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ba^3 \cos(dx+c)^4 + (A+3B)a^3 \cos(dx+c)^3 + 3(A+B)a^3 \cos(dx+c)^2 + (3A+B)a^3 \cos(dx+c) + Aa^3}{\cos(dx+c)^{\frac{5}{2}}} \right)$$

52.20 Problem number 142

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^3(9A+5B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^3(3A+5B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2aA(a + a \cos(dx+c))^2 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} \\ & + \frac{2(9A+5B)(a^3 + a^3 \cos(dx+c)) \sin(dx+c)}{15d \cos(dx+c)^{\frac{3}{2}}} + \frac{4a^3(21A+20B) \sin(dx+c)}{15d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (3A+5B)a^3 \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} (3A+5B)a^3 \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ba^3 \cos(dx+c)^4 + (A+3B)a^3 \cos(dx+c)^3 + 3(A+B)a^3 \cos(dx+c)^2 + (3A+B)a^3 \cos(dx+c) + Aa^3}{\cos(dx+c)^{\frac{7}{2}}} \right)$$

52.21 Problem number 143

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(7A + 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(13A + 21B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(41A + 42B) \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} + \frac{2aA(a + a \cos(dx + c))^2 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{2(11A + 7B)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^3(7A + 9B) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (13A + 21B) a^3 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (13A + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^3 \cos(dx + c)^4 + (A + 3B)a^3 \cos(dx + c)^3 + 3(A + B)a^3 \cos(dx + c)^2 + (3A + B)a^3 \cos(dx + c) + \dots}{\cos(dx + c)^{\frac{9}{2}}}\right)$$

52.22 Problem number 144

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(17A + 21B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(11A + 13B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(23A + 24B) \sin(dx + c)}{105d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^3(11A + 13B) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2aA(a + a \cos(dx + c))^2 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(13A + 9B)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{63d \cos(dx + c)^{\frac{7}{2}}} + \frac{4a^3(17A + 21B) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (11A + 13B) a^3 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (11A + 13B) a^3 \cos(dx + c)^3 \right)}{\cos(dx + c)^{\frac{11}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^3 \cos(dx + c)^4 + (A + 3B)a^3 \cos(dx + c)^3 + 3(A + B)a^3 \cos(dx + c)^2 + (3A + B)a^3 \cos(dx + c) + Aa^3}{\cos(dx + c)^{\frac{11}{2}}}\right)$$

52.23 Problem number 145

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx))}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(5A - 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(5A - 7B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5ad} \\ & + \frac{(A - B) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{d(a + a \cos(dx + c))} + \frac{5(A - B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3ad} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6 B \cos(dx + c)^2 + 2(5A - 2B) \cos(dx + c) + 25A - 25B \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 25 \left(\sqrt{2} (iA - iB) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx + c)^3 + A \cos(dx + c)^2 \right) \sqrt{\cos(dx + c)}}{a \cos(dx + c) + a}, x \right)$$

52.24 Problem number 146

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx))}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{(3A - 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{(A - B) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \cos(dx + c))} - \frac{(3A - 5B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3ad} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(2B \cos(dx + c) - 3A + 5B) \sqrt{\cos(dx + c)} \sin(dx + c) + \left(\sqrt{2} (3iA - 5iB) \cos(dx + c) + \sqrt{2} (3iA - 5iB) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx + c)^2 + A \cos(dx + c) \right) \sqrt{\cos(dx + c)}}{a \cos(dx + c) + a}, x \right)$$

52.25 Problem number 147

$$\int \frac{\sqrt{\cos(c+dx)} (A+B \cos(c+dx))}{a+a \cos(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A-3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A-B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A-B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{d(a+a \cos(dx+c))} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A-B) \sqrt{\cos(dx+c)} \sin(dx+c) + \left(\sqrt{2}(-iA+iB) \cos(dx+c) + \sqrt{2}(-iA+iB)\right) \operatorname{weierstrassPInverse}(-4)}{d(a+a \cos(dx+c))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx+c) + A) \sqrt{\cos(dx+c)}}{a \cos(dx+c) + a}, x\right)$$

52.26 Problem number 148

$$\int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)} (a+a \cos(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A-B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A-B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{d(a+a \cos(dx+c))} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A-B)\sqrt{\cos(dx+c)}\sin(dx+c) - \left(\sqrt{2}(-iA-iB)\cos(dx+c) + \sqrt{2}(-iA-iB)\right)\text{weierstrassPInverse}(-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^2+a\cos(dx+c)},x\right)$$

52.27 Problem number 149

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A-B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \\ & - \frac{(A-B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \\ & + \frac{(3A-B)\sin(dx+c)}{ad\sqrt{\cos(dx+c)}} - \frac{(A-B)\sin(dx+c)}{d(a+a\cos(dx+c))\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2((3A-B)\cos(dx+c)+2A)\sqrt{\cos(dx+c)}\sin(dx+c) + \left(\sqrt{2}(iA-iB)\cos(dx+c)^2 + \sqrt{2}(iA-iB)\cos(dx+c)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^3+a\cos(dx+c)^2},x\right)$$

52.28 Problem number 150

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(5A - 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(5A - 3B) \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}}} - \frac{(A - B) \sin(dx + c)}{d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))} - \frac{3(A - B) \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(9(A - B) \cos(dx + c)^2 + 2(2A - 3B) \cos(dx + c) - 2A \right) \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2} (-5iA + 3iB) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^4 + a \cos(dx + c)^3}, x\right)$$

52.29 Problem number 151

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)(A + B \cos(c + dx))}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{7(5A - 8B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\
 & + \frac{5(2A - 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\
 & - \frac{7(5A - 8B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15a^2 d} + \frac{(2A - 3B) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{a^2 d (1 + \cos(dx + c))} \\
 & + \frac{(A - B) \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{3d(a + a \cos(dx + c))^2} + \frac{5(2A - 3B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3a^2 d}
 \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6 B \cos(dx + c)^3 + 2(5A - 4B) \cos(dx + c)^2 + (65A - 94B) \cos(dx + c) + 50A - 75B \right) \sqrt{\cos(dx + c)} \sin(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B \cos(dx + c)^4 + A \cos(dx + c)^3\right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x\right)$$

52.30 Problem number 152

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx))}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{(4A - 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\
 & - \frac{5(A - 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\
 & + \frac{(4A - 7B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3a^2 d (1 + \cos(dx + c))} + \frac{(A - B) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{3d(a + a \cos(dx + c))^2} \\
 & - \frac{5(A - 2B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3a^2 d}
 \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 B \cos(dx + c)^2 - (6 A - 13 B) \cos(dx + c) - 5 A + 10 B \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(\sqrt{2} (-i A + 2i B) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx + c)^3 + A \cos(dx + c)^2 \right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2}, x \right)$$

52.31 Problem number 153

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx))}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - 4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & + \frac{(2A - 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & + \frac{(A - B) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d (a + a \cos(dx + c))^2} + \frac{(2A - 5B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3a^2 d (1 + \cos(dx + c))} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 (A - 2 B) \cos(dx + c) + 2 A - 5 B \right) \sqrt{\cos(dx + c)} \sin(dx + c) + \left(\sqrt{2} (-2i A + 5i B) \cos(dx + c)^2 - 2 \sqrt{2} (2i A - 5i B) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx + c)^2 + A \cos(dx + c) \right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2}, x \right)$$

52.32 Problem number 154

$$\int \frac{\sqrt{\cos(c+dx)} (A+B \cos(c+dx))}{(a+a \cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(A+2B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{B \sin(dx+c) (\sqrt{\cos(dx+c)})}{a^2 d (1+\cos(dx+c))} + \frac{(A-B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d (a+a \cos(dx+c))^2} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3B \cos(dx+c) + A+2B) \sqrt{\cos(dx+c)} \sin(dx+c) + (\sqrt{2}(-iA-2iB) \cos(dx+c))^2 - 2\sqrt{2}(iA+2iB) \cos(dx+c)}{(a+a \cos(dx+c))^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx+c) + A) \sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2}, x\right)$$

52.33 Problem number 155

$$\int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)} (a+a \cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(2A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{A \sin(dx+c) (\sqrt{\cos(dx+c)})}{a^2 d (1+\cos(dx+c))} - \frac{(A-B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d (a+a \cos(dx+c))^2} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3A \cos(dx+c) + 4A - B) \sqrt{\cos(dx+c)} \sin(dx+c) - \left(\sqrt{2}(-2iA - iB) \cos(dx+c)^2 - 2\sqrt{2}(2iA + iB)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx+c) + A) \sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^3 + 2a^2 \cos(dx+c)^2 + a^2 \cos(dx+c)}, x\right)$$

52.34 Problem number 156

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(4A - B) \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{(5A - 2B) \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} + \frac{(4A - B) \sin(dx+c)}{a^2 d \sqrt{\cos(dx+c)}} \\ & - \frac{(5A - 2B) \sin(dx+c)}{3a^2 d (1 + \cos(dx+c)) \sqrt{\cos(dx+c)}} - \frac{(A - B) \sin(dx+c)}{3d (a + a \cos(dx+c))^2 \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(3(4A - B) \cos(dx+c)^2 + (19A - 4B) \cos(dx+c) + 6A\right) \sqrt{\cos(dx+c)} \sin(dx+c) + \left(\sqrt{2}(5iA - 2iB) \cos(dx+c)^2 - 2\sqrt{2}(2iA + iB)\right) \cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \cos(dx+c) + A) \sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^4 + 2a^2 \cos(dx+c)^3 + a^2 \cos(dx+c)^2}, x\right)$$

52.35 Problem number 157

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(7A - 4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{5(2A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{5(2A - B) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}}} - \frac{(7A - 4B) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}} (1 + \cos(dx + c))} \\ & - \frac{(A - B) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^2} - \frac{(7A - 4B) \sin(dx + c)}{a^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(7A - 4B) \cos(dx + c)^3 + (32A - 19B) \cos(dx + c)^2 + 2(4A - 3B) \cos(dx + c) - 2A \right) \sqrt{\cos(dx + c)} \sin(dx + c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^5 + 2a^2 \cos(dx + c)^4 + a^2 \cos(dx + c)^3}, x\right)$$

52.36 Problem number 158

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)(A + B \cos(c + dx))}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{7(17A - 33B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
 & + \frac{(11A - 21B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
 & - \frac{7(17A - 33B) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{30a^3 d} \\
 & + \frac{(A - B) \left(\cos^{\frac{9}{2}}(dx+c)\right) \sin(dx+c)}{5d(a + a \cos(dx+c))^3} + \frac{(7A - 12B) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{15ad(a + a \cos(dx+c))^2} \\
 & + \frac{3(11A - 21B) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{10d(a^3 + a^3 \cos(dx+c))} + \frac{(11A - 21B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{2a^3 d}
 \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 B \cos(dx+c)^4 + 4(5A - 6B) \cos(dx+c)^3 + 3(79A - 147B) \cos(dx+c)^2 + 2(188A - 357B) \cos(dx+c) \right)}{10d(a^3 + a^3 \cos(dx+c))} + \frac{(11A - 21B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{2a^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B \cos(dx+c)^5 + A \cos(dx+c)^4\right) \sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x\right)$$

52.37 Problem number 159

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx))}{(a+a \cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(7A - 17B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(13A - 33B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A - B) \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{5d (a + a \cos(dx + c))^3} + \frac{(A - 2B) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{3ad (a + a \cos(dx + c))^2} \\ & + \frac{7(7A - 17B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{30d (a^3 + a^3 \cos(dx + c))} - \frac{(13A - 33B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{6a^3 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(20 B \cos(dx + c)^3 - 3(29 A - 79 B) \cos(dx + c)^2 - 2(73 A - 188 B) \cos(dx + c) - 65 A + 165 B \right) \sqrt{\cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B \cos(dx + c)^4 + A \cos(dx + c)^3\right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3}, x\right)$$

52.38 Problem number 160

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx))}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(9A - 49B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A - 13B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A - B) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d (a + a \cos(dx + c))^3} + \frac{(3A - 8B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15ad (a + a \cos(dx + c))^2} \\ & + \frac{(3A - 13B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{6d (a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(9A - 29B) \cos(dx + c)^2 + 2(18A - 73B) \cos(dx + c) + 15A - 65B \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(\sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx + c)^3 + A \cos(dx + c)^2 \right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}, x \right)$$

52.39 Problem number 161

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx))}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(A - B) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5d (a + a \cos(dx + c))^3} + \frac{(A - 6B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15ad (a + a \cos(dx + c))^2} \\ & + \frac{(A + 9B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10d (a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(A + 9B) \cos(dx + c)^2 + 2(7A + 18B) \cos(dx + c) + 5A + 15B \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(\sqrt{2} (iA + \dots) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx + c)^2 + A \cos(dx + c) \right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}, x \right)$$

52.40 Problem number 162

$$\int \frac{\sqrt{\cos(c+dx)} (A+B \cos(c+dx))}{(a+a \cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A-B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A-B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{5d (a+a \cos(dx+c))^3} + \frac{(A+4B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{15ad (a+a \cos(dx+c))^2} \\ & - \frac{(A-B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{10d (a^3 + a^3 \cos(dx+c))} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(A-B) \cos(dx+c)^2 + 2(2A-7B) \cos(dx+c) - 5A - 5B \right) \sqrt{\cos(dx+c)} \sin(dx+c) + 5 \left(\sqrt{2} (iA + iB) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx+c) + A) \sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x\right)$$

52.41 Problem number 163

$$\int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)} (a+a \cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(9A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d(a + a \cos(dx + c))^3} - \frac{(6A - B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15ad(a + a \cos(dx + c))^2} \\ & - \frac{(9A + B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10d(a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(9A + B) \cos(dx + c)^2 + 2(33A + 2B) \cos(dx + c) + 45A - 5B \right) \sqrt{\cos(dx + c)} \sin(dx + c) + 5 \left(\sqrt{2} (3i \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^4 + 3a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + a^3 \cos(dx + c)}, x\right)$$

52.42 Problem number 164

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(49A - 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(13A - 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(49A - 9B) \sin(dx + c)}{10a^3 d \sqrt{\cos(dx + c)}} - \frac{(A - B) \sin(dx + c)}{5d(a + a \cos(dx + c))^3 \sqrt{\cos(dx + c)}} \\ & - \frac{(8A - 3B) \sin(dx + c)}{15ad(a + a \cos(dx + c))^2 \sqrt{\cos(dx + c)}} - \frac{(13A - 3B) \sin(dx + c)}{6d(a^3 + a^3 \cos(dx + c)) \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 (49 A - 9 B) \cos(dx + c)^3 + 2 (188 A - 33 B) \cos(dx + c)^2 + 5 (59 A - 9 B) \cos(dx + c) + 60 A \right) \sqrt{\cos(dx + c)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \cos(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^5 + 3 a^3 \cos(dx + c)^4 + 3 a^3 \cos(dx + c)^3 + a^3 \cos(dx + c)^2}, x \right)$$

52.43 Problem number 165

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(17A - 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(33A - 13B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(33A - 13B) \sin(dx + c)}{6a^3 d \cos(dx + c)^{\frac{3}{2}}} - \frac{(A - B) \sin(dx + c)}{5d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^3} \\ & - \frac{(2A - B) \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^2} \\ & - \frac{7(17A - 7B) \sin(dx + c)}{30d \cos(dx + c)^{\frac{3}{2}} (a^3 + a^3 \cos(dx + c))} - \frac{7(17A - 7B) \sin(dx + c)}{10a^3 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21 (17 A - 7 B) \cos(dx + c)^4 + 2 (453 A - 188 B) \cos(dx + c)^3 + 5 (139 A - 59 B) \cos(dx + c)^2 + 60 (2 A - B) \right) \sqrt{\cos(dx + c)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \cos(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^6 + 3 a^3 \cos(dx + c)^5 + 3 a^3 \cos(dx + c)^4 + a^3 \cos(dx + c)^3}, x \right)$$

52.44 Problem number 279

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(a + b \cos(c + dx))^4} dx$$

Optimal antiderivative

$$\frac{b(20A a^6 b - 35A a^4 b^3 + 28A a^2 b^5 - 8A b^7 - 8B a^7 + 8B a^5 b^2 - 7B a^3 b^4 + 2B a b^6) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{dx+c}{2}\right)}{\sqrt{a+b}}\right) - (4Ab - Ba) \operatorname{arctanh}(\sin(dx+c))}{a^5 (a-b)^{\frac{7}{2}} (a+b)^{\frac{7}{2}} d} + \frac{(6A a^6 - 65A a^4 b^2 + 68A a^2 b^4 - 24A b^6 + 26B a^5 b - 17B a^3 b^3 + 6B a b^5) \tan(dx+c)}{6a^4 (a^2 - b^2)^3 d} + \frac{b(Ab - Ba) \tan(dx+c)}{3a (a^2 - b^2) d (a + b \cos(dx+c))^3} + \frac{b(9A a^2 b - 4A b^3 - 6a^3 B + Ba b^2) \tan(dx+c)}{6a^2 (a^2 - b^2)^2 d (a + b \cos(dx+c))^2} + \frac{b(12A a^4 b - 11A a^2 b^3 + 4A b^5 - 6B a^5 + 2B a^3 b^2 - Ba b^4) \tan(dx+c)}{2a^3 (a^2 - b^2)^3 d (a + b \cos(dx+c))}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

52.45 Problem number 280

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(a + b \cos(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b^2(40Aa^6b - 84Aa^4b^3 + 69Aa^2b^5 - 20Ab^7 - 20Ba^7 + 35Ba^5b^2 - 28Ba^3b^4 + 8Bab^6) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{dx}{2} + c\right)}{\sqrt{a+b}}\right)}{a^6(a-b)^{\frac{7}{2}}(a+b)^{\frac{7}{2}}d} \\
& + \frac{(a^2A + 20Ab^2 - 8abB) \operatorname{arctanh}(\sin(dx+c))}{2a^6d} \\
& - \frac{(24Aa^6b - 146Aa^4b^3 + 167Aa^2b^5 - 60Ab^7 - 6Ba^7 + 65Ba^5b^2 - 68Ba^3b^4 + 24Bab^6) \tan(dx+c)}{6a^5(a^2-b^2)^3d} \\
& + \frac{(Aa^6 - 23Aa^4b^2 + 27Aa^2b^4 - 10Ab^6 + 12Ba^5b - 11Ba^3b^3 + 4Bab^5) \sec(dx+c) \tan(dx+c)}{2a^4(a^2-b^2)^3d} \\
& + \frac{b(Ab - Ba) \sec(dx+c) \tan(dx+c)}{3a(a^2-b^2)d(a+b \cos(dx+c))^3} \\
& + \frac{b(10Aa^2b - 5Ab^3 - 7a^3B + 2Bab^2) \sec(dx+c) \tan(dx+c)}{6a^2(a^2-b^2)^2d(a+b \cos(dx+c))^2} \\
& + \frac{b(48Aa^4b - 53Aa^2b^3 + 20Ab^5 - 27Ba^5 + 20Ba^3b^2 - 8Bab^4) \sec(dx+c) \tan(dx+c)}{6a^3(a^2-b^2)^3d(a+b \cos(dx+c))}
\end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

52.46 Problem number 296

$$\int \cos^3(c+dx) \sqrt{a+b \cos(c+dx)} (A+B \cos(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{2(36Aab - 24B a^2 - 49b^2 B) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{315b^3d} \\
 & + \frac{2(3Ab - 2Ba) \cos(dx + c) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{21b^2d} \\
 & + \frac{2B(\cos^2(dx + c)) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{9bd} \\
 & + \frac{2(24A a^2b + 75A a b^3 - 16a^3 B - 36B a b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315b^3d} \\
 & + \frac{2(24A a^3b + 57A a b^3 - 16a^4 B - 24B a^2b^2 + 147b^4 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{\frac{a + b \cos(dx + c)}{a+b}}} \\
 & - \frac{2(a^2 - b^2) (24A a^2b + 75A a b^3 - 16a^3 B - 36B a b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}}
 \end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-32i B a^5 + 48i A a^4 b - 36i B a^3 b^2 + 96i A a^2 b^3 - 39i B a b^4 - 225i A b^5) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(B \cos(dx + c)^4 + A \cos(dx + c)^3\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

52.47 Problem number 297

$$\int \cos^2(c + dx) \sqrt{a + b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(7Ab - 4Ba)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{35b^2d} + \frac{2B \cos(dx + c)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{7bd}$$

$$- \frac{2(14Aab - 8B a^2 - 25b^2B) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105b^2d}$$

$$- \frac{2(14A a^2b - 63A b^3 - 8a^3B - 19Ba b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(a^2 - b^2)(14Aab - 8B a^2 - 25b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16i Ba^4 - 28i Aa^3b + 32i Ba^2b^2 - 21i Aab^3 - 75i Bb^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b^2}{27b^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(B \cos(dx + c)^3 + A \cos(dx + c)^2\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

52.48 Problem number 298

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\frac{2B(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5bd} + \frac{2(5Ab - 2Ba) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15bd}$$

$$+ \frac{2(5Aab - 2B a^2 + 9b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(a^2 - b^2)(5Ab - 2Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-4i Ba^3 + 10i Aa^2b - 3i Bab^2 - 15i Ab^3) \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i b \sin(dx+c)}{3b} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((B \cos(dx+c)^2 + A \cos(dx+c)) \sqrt{b \cos(dx+c) + a}, x \right)$$

52.49 Problem number 299

$$\int \sqrt{a + b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\frac{2B \sin(dx+c) \sqrt{a + b \cos(dx+c)}}{3d} + \frac{2(3Ab + Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a + b \cos(dx+c)}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) bd \sqrt{\frac{a + b \cos(dx+c)}{a+b}}} + \frac{2(a^2 - b^2) B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{\frac{a + b \cos(dx+c)}{a+b}}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) bd \sqrt{a + b \cos(dx+c)}}$$

command

```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \cos(dx+c) + a} Bb^2 \sin(dx+c) + \sqrt{2} (2i Ba^2 - 3i Aab - 3i Bb^2) \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i b \sin(dx+c)}{3b} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((B \cos(dx+c) + A) \sqrt{b \cos(dx+c) + a}, x \right)$$

52.50 Problem number 304

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(18Aab - 8B a^2 - 49b^2 B)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{315b^2 d} \\ & + \frac{2(9Ab - 4Ba)(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{63b^2 d} \\ & + \frac{2B \cos(dx + c)(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{9bd} \\ & - \frac{2(18A a^2 b - 75A b^3 - 8a^3 B - 39Ba b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315b^2 d} \\ & - \frac{2(18A a^3 b - 246Aa b^3 - 8a^4 B - 33B a^2 b^2 - 147b^4 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2(a^2 - b^2)(18A a^2 b - 75A b^3 - 8a^3 B - 39Ba b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16i Ba^5 - 36i Aa^4 b + 60i Ba^3 b^2 + 33i Aa^2 b^3 - 264i Bab^4 - 225i Ab^5) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -8\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \cos(dx + c)^4 + Aa \cos(dx + c)^2 + (Ba + Ab) \cos(dx + c)^3\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

52.51 Problem number 305

$$\int \cos(c + dx)(a + b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7Ab - 2Ba)(a + b \cos(dx + c))^{3/2} \sin(dx + c)}{35bd} + \frac{2B(a + b \cos(dx + c))^{5/2} \sin(dx + c)}{7bd} \\ & + \frac{2(21Aab - 6Ba^2 + 25b^2B) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105bd} \\ & + \frac{2(21Aa^2b + 63Ab^3 - 6a^3B + 82Ba^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2(a^2 - b^2)(21Aab - 6Ba^2 + 25b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-12iBa^4 + 42iAa^3b + 11iBa^2b^2 - 126iAab^3 - 75iBb^4)\sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}\right)}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \cos(dx + c)^3 + Aa \cos(dx + c) + (Ba + Ab) \cos(dx + c)^2\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

52.52 Problem number 306

$$\int (a + b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\frac{2B(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5d} + \frac{2(5Ab + 3Ba) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15d}$$

$$+ \frac{2(20Aab + 3B a^2 + 9b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(a^2 - b^2)(5Ab + 3Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (6i Ba^3 - 5i Aa^2b - 18i Bab^2 - 15i Ab^3) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3ib \sin(dx+c)}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c)\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

52.53 Problem number 311

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2}(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(110A a^2 b - 539A b^3 - 40a^3 B - 335Ba b^2) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3465b^2 d} \\
& - \frac{2(22Aab - 8B a^2 - 81b^2 B) (a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{693b^2 d} \\
& + \frac{2(11Ab - 4Ba) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{99b^2 d} \\
& + \frac{2B \cos(dx + c) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{11bd} \\
& - \frac{2(110A a^3 b - 1254Aa b^3 - 40a^4 B - 285B a^2 b^2 - 675b^4 B) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3465b^2 d} \\
& - \frac{2(110A a^4 b - 3069A a^2 b^3 - 1617A b^5 - 40B a^5 - 255B a^3 b^2 - 3705Ba b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
& + \frac{2(a^2 - b^2) (110A a^3 b - 1254Aa b^3 - 40a^4 B - 285B a^2 b^2 - 675b^4 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (80i Ba^6 - 220i Aa^5 b + 480i Ba^4 b^2 + 1023i Aa^3 b^3 - 2535i Ba^2 b^4 - 5379i Aab^5 - 2025i Bb^6) \sqrt{b} \operatorname{weierstrassPI}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx + c)^5 + Aa^2 \cos(dx + c)^2 + (2Bab + Ab^2) \cos(dx + c)^4 + (Ba^2 + 2Aab) \cos(dx + c)^3\right) \sqrt{b}\right)$$

52.54 Problem number 312

$$\int \cos(c + dx)(a + b \cos(c + dx))^{5/2}(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(45Aab - 10B a^2 + 49b^2 B) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{315bd} \\
& + \frac{2(9Ab - 2Ba) (a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{63bd} + \frac{2B(a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9bd} \\
& + \frac{2(45A a^2 b + 75A b^3 - 10a^3 B + 114Ba b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315bd} \\
& + \frac{2(45A a^3 b + 435Aa b^3 - 10a^4 B + 279B a^2 b^2 + 147b^4 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
& - \frac{2(a^2 - b^2) (45A a^2 b + 75A b^3 - 10a^3 B + 114Ba b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-20i Ba^5 + 90i Aa^4 b + 93i Ba^3 b^2 - 345i Aa^2 b^3 - 489i Bab^4 - 225i Ab^5) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx + c)^4 + Aa^2 \cos(dx + c) + (2 Bab + Ab^2) \cos(dx + c)^3 + (Ba^2 + 2 Aab) \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}\right)$$

52.55 Problem number 313

$$\int (a + b \cos(c + dx))^{\frac{5}{2}} (A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(7Ab + 5Ba)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{35d} + \frac{2B(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7d} \\
& + \frac{2(56Aab + 15Ba^2 + 25b^2B) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105d} \\
& + \frac{2(161Aa^2b + 63Ab^3 + 15a^3B + 145Ba^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\frac{a + b \cos(dx + c)}{a+b}}} \\
& - \frac{2(a^2 - b^2)(56Aab + 15Ba^2 + 25b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (30i Ba^4 + 7i Aa^3b - 115i Ba^2b^2 - 231i Aab^3 - 75i Bb^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, 3\right)}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2Aab) \cos(dx + c)\right) \sqrt{b \cos(dx + c)}\right)$$

52.56 Problem number 319

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{\sqrt{a + b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(28Aab - 24Ba^2 - 25b^2B) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105b^3d} + \frac{2(7Ab - 6Ba) \cos(dx + c) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{35b^2d} + \frac{2B(\cos^2(dx + c)) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{7bd} + \frac{2(56Aa^2b + 63Ab^3 - 48a^3B - 44Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} + \frac{2(56Aa^3b + 49Aab^3 - 48a^4B - 32Ba^2b^2 - 25b^4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-96iBa^4 + 112iAa^3b - 52iBa^2b^2 + 84iAab^3 - 75iBb^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B \cos(dx + c)^4 + A \cos(dx + c)^3}{\sqrt{b \cos(dx + c) + a}}, x\right)$$

52.57 Problem number 320

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{\sqrt{a + b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(5Ab - 4Ba) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^2d} + \frac{2B \cos(dx + c) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{5bd} + \frac{2(10Aab - 8Ba^2 - 9b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} + \frac{2(10Aa^2b + 5Ab^3 - 8a^3B - 7Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16i Ba^3 - 20i Aa^2b + 12i Bab^2 - 15i Ab^3) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3i b^2}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{B \cos(dx+c)^3 + A \cos(dx+c)^2}{\sqrt{b \cos(dx+c) + a}}, x\right)$$

52.58 Problem number 321

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2B \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{3bd} + \frac{2(3Ab-2Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \cos(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a+b \cos(dx+c)}{a+b}}} + \frac{2(3Aab-2Ba^2-b^2B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a+b \cos(dx+c)}}$$

command

```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \cos(dx+c) + a} B b^2 \sin(dx+c) + \sqrt{2} (-4i Ba^2 + 6i Aab - 3i Bb^2) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3i b^2}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{B \cos(dx+c)^2 + A \cos(dx+c)}{\sqrt{b \cos(dx+c) + a}}, x\right)$$

52.59 Problem number 322

$$\int \frac{A + B \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\frac{a + b \cos(dx + c)}{a+b}}} + \frac{2(Ab - Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i \sqrt{2} B b^{\frac{3}{2}} \operatorname{weierstrassZeta}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}\right), \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+a}{a+b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B \cos(dx + c) + A}{\sqrt{b \cos(dx + c) + a}}, x\right)$$

52.60 Problem number 326

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2a(Ab - Ba) (\cos^2(dx + c)) \sin(dx + c)}{b(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \\
+ & \frac{2(20Aa^2b - 5Ab^3 - 24a^3B + 9Ba^2b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^3(a^2 - b^2)d} \\
- & \frac{2(5Aab - 6Ba^2 + b^2B) \cos(dx + c) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{5b^2(a^2 - b^2)d} \\
- & \frac{2(40Aa^3b - 25Aab^3 - 48a^4B + 24Ba^2b^2 + 9b^4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
+ & \frac{2(40Aa^2b + 5Ab^3 - 48a^3B - 12Ba^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(24Ba^4b^2 - 20Aa^3b^3 - 9Ba^2b^4 + 5Aab^5 - 3(Ba^2b^4 - Bb^6) \cos(dx + c)^2 + (6Ba^3b^3 - 5Aa^2b^4 - 6Bab^5 + 5 \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c)^4 + A \cos(dx + c)^3) \sqrt{b \cos(dx + c) + a}}{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}, x\right)$$

52.61 Problem number 327

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2a^2(Ab - Ba) \sin(dx + c)}{b^2(a^2 - b^2)d\sqrt{a + b \cos(dx + c)}} + \frac{2B \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3b^2d}$$

$$+ \frac{2(6Aa^2b - 3Ab^3 - 8a^3B + 5Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3(a^2 - b^2)d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$- \frac{2(6Aab - 8Ba^2 - b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(4Ba^3b^2 - 3Aa^2b^3 - Bab^4 + (Ba^2b^3 - Bb^5) \cos(dx + c)) \sqrt{b \cos(dx + c) + a} \sin(dx + c) - (\sqrt{2} (16i Ba^4b - 1$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c)^3 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c) + a}}{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}, x\right)$$

52.62 Problem number 328

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2a(Ab - Ba) \sin(dx + c)}{b(a^2 - b^2)d\sqrt{a + b \cos(dx + c)}}$$

$$+ \frac{2(Aab - 2Ba^2 + b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2(a^2 - b^2)d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(Ab - 2Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (Ba^2b^2 - Aab^3) \sqrt{b \cos(dx + c) + a} \sin(dx + c) + \left(\sqrt{2} (-4i Ba^3b + 2i Aa^2b^2 + 5i Bab^3 - 3i Ab^4) \cos(dx + c)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx + c)^2 + A \cos(dx + c) \right) \sqrt{b \cos(dx + c) + a}}{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}, x \right)$$

52.63 Problem number 329

$$\int \frac{A + B \cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(Ab - Ba) \sin(dx + c)}{(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{2(Ab - Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{a + b \cos(dx + c)}}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) b (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) bd \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (Bab^2 - Ab^3) \sqrt{b \cos(dx + c) + a} \sin(dx + c) - \left(\sqrt{2} (2i Ba^2b + i Aab^2 - 3i Bb^3) \cos(dx + c) + \sqrt{2} (2i Ba^3 + \dots)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx + c) + A \right) \sqrt{b \cos(dx + c) + a}}{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}, x \right)$$

52.64 Problem number 333

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(Ab - Ba) (\cos^3(dx + c)) \sin(dx + c)}{3b(a^2 - b^2) d (a + b \cos(dx + c))^{\frac{3}{2}}} \\ & + \frac{2a(5Aa^2b - 9Ab^3 - 8a^3B + 12Ba^2b^2) (\cos^2(dx + c)) \sin(dx + c)}{3b^2(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{2(40Aa^4b - 65Aa^2b^3 + 5Ab^5 - 64Ba^5 + 98Ba^3b^2 - 14Bab^4) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^4(a^2 - b^2)^2 d} \\ & - \frac{2(30Aa^3b - 50Aab^3 - 48a^4B + 71Ba^2b^2 - 3b^4B) \cos(dx + c) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^3(a^2 - b^2)^2 d} \\ & - \frac{2(80Aa^5b - 140Aa^3b^3 + 40Aab^5 - 128Ba^6 + 212Ba^4b^2 - 55Ba^2b^4 - 9Bb^6) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^5 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2(80Aa^4b - 80Aa^2b^3 - 5Ab^5 - 128Ba^5 + 116Ba^3b^2 + 17Bab^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^5 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c))^5 + A \cos(dx + c)^4) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

52.65 Problem number 334

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a(Ab - Ba) (\cos^2(dx + c)) \sin(dx + c)}{3b(a^2 - b^2) d (a + b \cos(dx + c))^{\frac{3}{2}}} - \frac{2a^2(3Aa^2b - 7Ab^3 - 6a^3B + 10Bab^2) \sin(dx + c)}{3b^3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

$$- \frac{2(Aab - 2Ba^2 + b^2B) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3b^3(a^2 - b^2) d}$$

$$+ \frac{2(8Aa^4b - 15Aa^2b^3 + 3Ab^5 - 16Ba^5 + 28Ba^3b^2 - 8Bab^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$- \frac{2(8Aa^3b - 9Aab^3 - 16a^4B + 16Ba^2b^2 + b^4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(8Ba^6b^2 - 4Aa^5b^3 - 13Ba^4b^4 + 8Aa^3b^5 + Ba^2b^6 + (Ba^4b^4 - 2Ba^2b^6 + Bb^8) \cos(dx + c)^2 + (10Ba^5b^3 - 5Aa^6b^2) \cos(dx + c) \right) \sqrt{a + b \cos(dx + c)} / (a + b \cos(dx + c))^{5/2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c)^4 + A \cos(dx + c)^3) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

52.66 Problem number 335

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a^2(Ab - Ba) \sin(dx + c)}{3b^2(a^2 - b^2)d(a + b \cos(dx + c))^{\frac{3}{2}}} + \frac{2a(2Aa^2b - 6Ab^3 - 5a^3B + 9Ba^2b^2) \sin(dx + c)}{3b^2(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

$$\frac{2(2Aa^3b - 6Aab^3 - 8a^4B + 15Ba^2b^2 - 3b^4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(2Aa^2b - 3Ab^3 - 8a^3B + 9Ba^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(4Ba^5b^2 - Aa^4b^3 - 8Ba^3b^4 + 5Aa^2b^5 + (5Ba^4b^3 - 2Aa^3b^4 - 9Ba^2b^5 + 6Aab^6) \cos(dx + c)) \sqrt{b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c)^3 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

52.67 Problem number 336

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a(Ab - Ba) \sin(dx + c)}{3b(a^2 - b^2)d(a + b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(Aa^2b + 3Ab^3 + 2a^3B - 6Ba^2b^2) \sin(dx + c)}{3b(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

$$\frac{2(Aa^2b + 3Ab^3 + 2a^3B - 6Ba^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(Aab + 2Ba^2 - 3b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (Ba^4b^2 + 2Aa^3b^3 - 5Ba^2b^4 + 2Aab^5 + (2Ba^3b^3 + Aa^2b^4 - 6Bab^5 + 3Ab^6) \cos(dx + c)) \sqrt{b \cos(dx + c) + a} \sin(dx + c) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \cos(dx + c)^2 + A \cos(dx + c)) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x \right)$$

52.68 Problem number 337

$$\int \frac{A + B \cos(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab - Ba) \sin(dx + c)}{3(a^2 - b^2) d(a + b \cos(dx + c))^{\frac{3}{2}}} - \frac{2(4Aab - B a^2 - 3b^2 B) \sin(dx + c)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{2(4Aab - B a^2 - 3b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{a + b \cos(dx + c)}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) b (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{2(Ab - Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) b (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (2Ba^3b^2 - 5Aa^2b^3 + 2Bab^4 + Ab^5 + (Ba^2b^3 - 4Aab^4 + 3Bb^5) \cos(dx + c)) \sqrt{b \cos(dx + c) + a} \sin(dx + c) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x \right)$$

52.69 Problem number 341

$$\int \frac{aB + bB \cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib \sin(dx+c)+2a}{3b}\right) + i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib \sin(dx+c)+2a}{3b}\right)}{bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B}{\sqrt{b \cos(dx + c) + a}}, x\right)$$

52.70 Problem number 343

$$\int \frac{aB + bB \cos(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2bB \sin(dx + c)}{(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx + c)}{a+b}}} \end{aligned}$$

command

`integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \cos(dx+c) + a} B b^2 \sin(dx+c) + \left(i \sqrt{2} B a b \cos(dx+c) + i \sqrt{2} B a^2 \right) \sqrt{b} \operatorname{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b^2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \cos(dx+c) + a} B}{b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2}, x \right)$$

52.71 Problem number 345

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))(A+B \cos(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9aA + 7bB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{10(Ab + Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(9aA + 7bB) \left(\cos^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{45d} + \frac{2(Ab + Ba) \left(\cos^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{7d} \\ & + \frac{2bB \left(\cos^{\frac{7}{2}}(dx+c) \right) \sin(dx+c)}{9d} + \frac{10(Ab + Ba) \sin(dx+c) \left(\sqrt{\cos(dx+c)} \right)}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(35 B b \cos(dx+c)^3 + 45 (Ba + Ab) \cos(dx+c)^2 + 75 Ba + 75 Ab + 7(9Aa + 7Bb) \cos(dx+c) \right) \sqrt{\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(B b \cos(dx+c)^4 + A a \cos(dx+c)^2 + (Ba + Ab) \cos(dx+c)^3 \right) \sqrt{\cos(dx+c)}, x \right)$$

52.72 Problem number 346

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7aA + 5bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ab + Ba) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2bB \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2(7aA + 5bB) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15 Bb \cos(dx + c)^2 + 35 Aa + 25 Bb + 21 (Ba + Ab) \cos(dx + c) \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \sqrt{2} (7i Aa + 5$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \cos(dx + c)^3 + Aa \cos(dx + c) + (Ba + Ab) \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}, x\right)$$

52.73 Problem number 347

$$\int \sqrt{\cos(c + dx)} (a + b \cos(c + dx))(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5aA + 3bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2bB \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2(Ab + Ba) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3Bb \cos(dx+c) + 5Ba + 5Ab) \sqrt{\cos(dx+c)} \sin(dx+c) - 5\sqrt{2}(iBa + iAb) \text{weierstrassPInverse}(-4, 0, \cos(dx+c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb \cos(dx+c)^2 + Aa + (Ba + Ab) \cos(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

52.74 Problem number 348

$$\int \frac{(a + b \cos(c + dx))(A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3aA + bB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2bB \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2Bb \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2}(-3iAa - iBb) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \dots}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Bb \cos(dx+c)^2 + Aa + (Ba + Ab) \cos(dx+c)}{\sqrt{\cos(dx+c)}}, x\right)$$

52.75 Problem number 349

$$\int \frac{(a + b \cos(c + dx))(A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(aA - bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2aA \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2Aa \sqrt{\cos(dx + c)} \sin(dx + c) + \sqrt{2}(-iBa - iAb) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)) + i \sin(dx + c)}{d \sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c)}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

52.76 Problem number 350

$$\int \frac{(a + b \cos(c + dx))(A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(aA + 3bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(Ab + Ba) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iAa - 3iBb)\cos(dx+c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + \sqrt{2}(iAa + 3iBb)\cos(dx+c)}{\cos(dx+c)^{5/2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Bb\cos(dx+c)^2 + Aa + (Ba + Ab)\cos(dx+c)}{\cos(dx+c)^{5/2}}, x\right)$$

52.77 Problem number 351

$$\int \frac{(a + b\cos(c + dx))(A + B\cos(c + dx))}{\cos^2(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(3aA + 5bB)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2(Ab + Ba)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2aA\sin(dx+c)}{5d\cos(dx+c)^{5/2}} + \frac{2(Ab + Ba)\sin(dx+c)}{3d\cos(dx+c)^{3/2}} + \frac{2(3aA + 5bB)\sin(dx+c)}{5d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(iBa + iAb)\cos(dx+c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 5\sqrt{2}(-iBa - iAb)\cos(dx+c)}{\cos(dx+c)^{7/2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Bb\cos(dx+c)^2 + Aa + (Ba + Ab)\cos(dx+c)}{\cos(dx+c)^{7/2}}, x\right)$$

52.78 Problem number 352

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^2(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9a^2A + 7Ab^2 + 14abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10(9b^2B + 11a(2Ab + Ba)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(9a^2A + 7Ab^2 + 14abB) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{2(9b^2B + 11a(2Ab + Ba)) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{77d} \\ & + \frac{2b(11Ab + 13Ba) \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{99d} \\ & + \frac{2bB \left(\cos^{\frac{7}{2}}(dx + c)\right) (a + b \cos(dx + c)) \sin(dx + c)}{11d} \\ & + \frac{10(9b^2B + 11a(2Ab + Ba)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(315 Bb^2 \cos(dx + c)^4 + 385 (2 Bab + Ab^2) \cos(dx + c)^3 + 825 Ba^2 + 1650 Aab + 675 Bb^2 + 45 (11 Ba^2 + 22 Aa) \right) \sqrt{\cos(dx + c)}}{231d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx + c)^5 + Aa^2 \cos(dx + c)^2 + (2 Bab + Ab^2) \cos(dx + c)^4 + (Ba^2 + 2 Aab) \cos(dx + c)^3\right) \sqrt{\cos(dx + c)}\right)$$

52.79 Problem number 353

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2(A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7b^2B + 9a(2Ab + Ba)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7a^2A + 5Ab^2 + 10abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7b^2B + 9a(2Ab + Ba)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{2b(9Ab + 11Ba) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\ & + \frac{2bB \left(\cos^{\frac{5}{2}}(dx + c)\right) (a + b \cos(dx + c)) \sin(dx + c)}{9d} \\ & + \frac{2(7a^2A + 5Ab^2 + 10abB) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(35 Bb^2 \cos(dx + c)^3 + 105 Aa^2 + 150 Bab + 75 Ab^2 + 45 (2 Bab + Ab^2) \cos(dx + c)^2 + 7 (9 Ba^2 + 18 Aab + 7 B$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx + c)^4 + Aa^2 \cos(dx + c) + (2 Bab + Ab^2) \cos(dx + c)^3 + (Ba^2 + 2 Aab) \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}\right)$$

52.80 Problem number 354

$$\int \sqrt{\cos(c+dx)} (a+b\cos(c+dx))^2 (A+B\cos(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5a^2A + 3Ab^2 + 6abB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5b^2B + 7a(2Ab + Ba)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(7Ab + 9Ba) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{35d} \\ & + \frac{2bB \left(\cos^{\frac{3}{2}}(dx+c)\right) (a+b\cos(dx+c)) \sin(dx+c)}{7d} \\ & + \frac{2(5b^2B + 7a(2Ab + Ba)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15 Bb^2 \cos(dx+c)^2 + 35 Ba^2 + 70 Aab + 25 Bb^2 + 21 (2 Bab + Ab^2) \cos(dx+c) \right) \sqrt{\cos(dx+c)} \sin(dx+c) - \dots}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx+c)^3 + Aa^2 + (2 Bab + Ab^2) \cos(dx+c)^2 + (Ba^2 + 2 Aab) \cos(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

52.81 Problem number 355

$$\int \frac{(a+b\cos(c+dx))^2 (A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3b^2B + 5a(2Ab + Ba)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3a^2A + Ab^2 + 2abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(5Ab + 7Ba) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \\ & + \frac{2bB(a + b \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3Bb^2 \cos(dx + c) + 10Bab + 5Ab^2) \sqrt{\cos(dx + c)} \sin(dx + c) - 5\sqrt{2}(3iAa^2 + 2iBab + iAb^2) \operatorname{weierstrassPI}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^2 \cos(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2Aab) \cos(dx + c)}{\sqrt{\cos(dx + c)}}, x\right)$$

52.82 Problem number 356

$$\int \frac{(a + b \cos(c + dx))^2 (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(a^2A - Ab^2 - 2abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(6Aab + 3Ba^2 + b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2A \sin(dx + c)}{d \sqrt{\cos(dx + c)}} + \frac{2b^2B \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i Ba^2 - 6i Aab - i Bb^2) \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i Ba^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Bb^2 \cos(dx + c)^3 + Aa^2 + (2 Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2 Aab) \cos(dx + c)}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

52.83 Problem number 357

$$\int \frac{(a + b \cos(c + dx))^2 (A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(2Aab + B a^2 - b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^2 A + 3A b^2 + 6abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2 A \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a(2Ab + Ba) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i Aa^2 - 6i Bab - 3i Ab^2) \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i Aa^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Bb^2 \cos(dx + c)^3 + Aa^2 + (2 Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2 Aab) \cos(dx + c)}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

52.84 Problem number 358

$$\int \frac{(a + b \cos(c + dx))^2 (A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3a^2 A + 5A b^2 + 10abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(2Aab + B a^2 + 3b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2 A \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2a(2Ab + Ba) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(3a^2 A + 5A b^2 + 10abB) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i B a^2 + 2i A a b + 3i B b^2) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B b^2 \cos(dx + c)^3 + A a^2 + (2 B a b + A b^2) \cos(dx + c)^2 + (B a^2 + 2 A a b) \cos(dx + c)}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

52.85 Problem number 359

$$\int \cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))^3 (A + B \cos(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(27Aa^2b + 7Ab^3 + 9a^3B + 21Bab^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(77a^3A + 165Aab^2 + 165a^2bB + 45b^3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(27Aa^2b + 7Ab^3 + 9a^3B + 21Bab^2) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} \\
& + \frac{2b(33Aab + 26Ba^2 + 9b^2B) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{77d} \\
& + \frac{2b^2(11Ab + 15Ba) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{99d} \\
& + \frac{2bB \left(\cos^{\frac{5}{2}}(dx+c)\right) (a + b \cos(dx+c))^2 \sin(dx+c)}{11d} \\
& + \frac{2(77a^3A + 165Aab^2 + 165a^2bB + 45b^3B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{231d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(315 Bb^3 \cos(dx+c)^4 + 1155 Aa^3 + 2475 Ba^2b + 2475 Aab^2 + 675 Bb^3 + 385 (3 Bab^2 + Ab^3) \cos(dx+c)^3 + 135 \right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^3 \cos(dx+c)^5 + Aa^3 \cos(dx+c) + (3 Bab^2 + Ab^3) \cos(dx+c)^4 + 3 (Ba^2b + Aab^2) \cos(dx+c)^3 + \dots\right)\right)$$

52.86 Problem number 360

$$\int \sqrt{\cos(c+dx)} (a + b \cos(c+dx))^3 (A + B \cos(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(15a^3A + 27Aab^2 + 27a^2bB + 7b^3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(21Aa^2b + 5Ab^3 + 7a^3B + 15Ba^2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2b(27Aab + 22Ba^2 + 7b^2B) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} \\
& + \frac{2b^2(9Ab + 13Ba) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{63d} \\
& + \frac{2bB \left(\cos^{\frac{3}{2}}(dx+c)\right) (a + b \cos(dx+c))^2 \sin(dx+c)}{9d} \\
& + \frac{2(21Aa^2b + 5Ab^3 + 7a^3B + 15Ba^2b^2) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(35 B b^3 \cos(dx+c)^3 + 105 B a^3 + 315 A a^2 b + 225 B a b^2 + 75 A b^3 + 45 (3 B a b^2 + A b^3) \cos(dx+c)^2 + 7 (27 B a^2 b \right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(B b^3 \cos(dx+c)^4 + A a^3 + (3 B a b^2 + A b^3) \cos(dx+c)^3 + 3 (B a^2 b + A a b^2) \cos(dx+c)^2 + (B a^3 + 3 A a b^2)\right) \sqrt{\cos(dx+c)}\right) dx$$

52.87 Problem number 361

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(15Aa^2b + 3Ab^3 + 5a^3B + 9Ba^2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^3A + 21Aa^2b^2 + 21a^2bB + 5b^3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b^2(7Ab + 11Ba) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{35d} \\ & + \frac{2b(21Aab + 18Ba^2 + 5b^2B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \\ & + \frac{2bB(a + b \cos(dx+c))^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{7d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15 B b^3 \cos(dx+c)^2 + 105 B a^2 b + 105 A a b^2 + 25 B b^3 + 21 (3 B a b^2 + A b^3) \cos(dx+c) \right) \sqrt{\cos(dx+c)} \sin(dx+c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B b^3 \cos(dx+c)^4 + A a^3 + (3 B a b^2 + A b^3) \cos(dx+c)^3 + 3 (B a^2 b + A a b^2) \cos(dx+c)^2 + (B a^3 + 3 A a^2 b)}{\sqrt{\cos(dx+c)}}\right)$$

52.88 Problem number 362

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(5a^3A - 15Aa^2b - 15a^2bB - 3b^3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(9Aa^2b + Ab^3 + 3a^3B + 3Ba^2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2b^2(5aA - bB) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2aA(a + b \cos(dx+c))^2 \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \\ & - \frac{2b(6a^2A - Ab^2 - 3abB) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(3iBa^3 + 9iAa^2b + 3iBab^2 + iAb^3)\cos(dx+c)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^3\cos(dx+c)^4 + Aa^3 + (3Bab^2 + Ab^3)\cos(dx+c)^3 + 3(Ba^2b + Aab^2)\cos(dx+c)^2 + (Ba^3 + 3Aa^2b)}{\cos(dx+c)^{\frac{3}{2}}}\right)$$

52.89 Problem number 363

$$\int \frac{(a + b\cos(c + dx))^3(A + B\cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3Aa^2b - Ab^3 + a^3B - 3Ba^2b^2)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2(a^3A + 9Aa^2b + 9a^2bB + b^3B)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2aA(a + b\cos(dx+c))^2\sin(dx+c)}{3d\cos(dx+c)^{\frac{3}{2}}} + \frac{2a^2(7Ab + 3Ba)\sin(dx+c)}{3d\sqrt{\cos(dx+c)}} \\ & - \frac{2b^2(aA - bB)\sin(dx+c)(\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iAa^3 - 9iBa^2b - 9iAab^2 - iBb^3)\cos(dx+c)^2\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^3\cos(dx+c)^4 + Aa^3 + (3Bab^2 + Ab^3)\cos(dx+c)^3 + 3(Ba^2b + Aab^2)\cos(dx+c)^2 + (Ba^3 + 3Aa^2b)}{\cos(dx+c)^{\frac{5}{2}}}\right)$$

52.90 Problem number 364

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3a^3A + 15Aab^2 + 15a^2bB - 5b^3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3Aa^2b + 3Ab^3 + a^3B + 9Bab^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(9Ab + 5Ba) \sin(dx+c)}{15d \cos^{\frac{3}{2}}(dx+c)} + \frac{2aA(a + b \cos(dx+c))^2 \sin(dx+c)}{5d \cos^{\frac{5}{2}}(dx+c)} \\ & + \frac{2a(3a^2A + 14Aab + 15abB) \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2} (iBa^3 + 3iAa^2b + 9iBab^2 + 3iAb^3) \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^3 \cos(dx+c)^4 + Aa^3 + (3Bab^2 + Ab^3) \cos(dx+c)^3 + 3(Ba^2b + Aab^2) \cos(dx+c)^2 + (Ba^3 + 3Aa^2b) \cos(dx+c) + A^2a}{\cos(dx+c)^{\frac{7}{2}}}\right)$$

52.91 Problem number 383

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)(aB + bB \cos(c + dx))}{a + b \cos(c + dx)} dx$$

Optimal antiderivative

$$\frac{6B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 B \cos(dx + c)^{\frac{3}{2}} \sin(dx + c) + 3i \sqrt{2} B \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{5 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(B \cos(dx + c)^{\frac{5}{2}}, x\right)$$

52.92 Problem number 384

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)(aB + bB \cos(c + dx))}{a + b \cos(c + dx)} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 B \sqrt{\cos(dx + c)} \sin(dx + c) - i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \text{weierstrassZeta}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(B \cos(dx + c)^{\frac{3}{2}}, x\right)$$

52.93 Problem number 385

$$\int \frac{\sqrt{\cos(c + dx)} (aB + bB \cos(c + dx))}{a + b \cos(c + dx)} dx$$

Optimal antiderivative

$$\frac{2B\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i\sqrt{2} B \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) - i\sqrt{2} B \operatorname{weierstrassZeta}(-4, 0, \cos(dx+c) - i \sin(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(B\sqrt{\cos(dx+c)}, x\right)$$

52.94 Problem number 386

$$\int \frac{aB + bB \cos(c + dx)}{\sqrt{\cos(c + dx)} (a + b \cos(c + dx))} dx$$

Optimal antiderivative

$$\frac{2B\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2} B \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i\sqrt{2} B \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B}{\sqrt{\cos(dx+c)}}, x\right)$$

52.95 Problem number 387

$$\int \frac{aB + bB \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sin(dx + c)}{d \sqrt{\cos(dx + c)}}$$

command

`integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} B \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} B \cos(dx + c)}{d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

52.96 Problem number 388

$$\int \frac{aB + bB \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}}$$

command

`integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} B \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \cos(dx + c)^2 \operatorname{weierstrassP}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3d \cos(dx + c)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

52.97 Problem number 459

$$\int (a + a \cos(c + dx))(A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(A + B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2aA \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2a(3A + 5B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2a(3A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (A + B)a \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (A + B)a \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Ba \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa \right) \sec(dx + c)^{\frac{7}{2}}, x \right)$$

52.98 Problem number 460

$$\int (a + a \cos(c + dx))(A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2a(A + B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & - \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} (A + 3B)a \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (A + 3B)a \cos(dx + c)$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ba \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa\right) \sec(dx + c)^{\frac{5}{2}}, x\right)$$

52.99 Problem number 461

$$\int (a + a \cos(c + dx))(A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2a(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} (A + B)a \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (A + B)a \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ba \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

52.100 Problem number 462

$$\int (a + a \cos(c + dx))(A + B \cos(c + dx))\sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aB \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2Ba \sqrt{\cos(dx + c)} \sin(dx + c) - i \sqrt{2} (3A + B) \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa\right) \sqrt{\sec(dx + c)}, x\right)$$

52.101 Problem number 463

$$\int \frac{(a + a \cos(c + dx))(A + B \cos(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aB \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(A + B) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(5A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} (A + B) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (A + B) \text{aweierstrassPInverse}(-$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ba \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa}{\sqrt{\sec(dx + c)}}, x\right)$$

52.102 Problem number 464

$$\int \frac{(a + a \cos(c + dx))(A + B \cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aB \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a(A + B) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(7A + 5B) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{6a(A + B) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 5B) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} (7A + 5B) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (7A + 5B) \text{aweierstrassPInverse}(-$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ba \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

52.103 Problem number 465

$$\int (a + a \cos(c + dx))^2 (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(7A + 5B) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} + \frac{2A \left(\sec^{\frac{3}{2}}(dx + c)\right) (a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{5d} \\ & + \frac{4a^2(4A + 5B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{4a^2(4A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(A + 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A + 2B) a^2 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + 2B) a^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^2 \cos(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + Aa^2\right) \sec(dx + c)^{\frac{7}{2}}, x\right)$$

52.104 Problem number 466

$$\int (a + a \cos(c + dx))^2 (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(5A + 3B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{3d} + \frac{2A(a^2 + a^2 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec}(dx + c))}{3d} \\ & - \frac{4a^2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(2A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} (2A + 3B)a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} (2A + 3B)a^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^2 \cos(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + Aa^2\right) \sec(dx + c)^{\frac{5}{2}}, x\right)$$

52.105 Problem number 467

$$\int (a + a \cos(c + dx))^2 (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2a^2(3A - B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & + \frac{4a^2 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(3A + 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} (3A + 2B)a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} (3A + 2B)a^2 \operatorname{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^2 \cos(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + Aa^2\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

52.106 Problem number 468

$$\int (a + a \cos(c + dx))^2 (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a^2(5A + 7B) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(5A + 4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(2A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (2A + B)a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (2A + B)a^2 \operatorname{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^2 \cos(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + Aa^2\right) \sqrt{\sec(dx + c)}, x\right)$$

52.107 Problem number 469

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(7A + 9B) \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{2B(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^2(7A + 6B) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(4A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(7A + 6B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (7A + 6B)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (7A + 6B)a^2 \text{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ba^2 \cos(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + Aa^2}{\sqrt{\sec(dx + c)}}, x \right)$$

52.108 Problem number 470

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(41A + 42B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{105d} \\ & + \frac{2aA \left(\sec^{\frac{3}{2}}(dx + c) \right) (a + a \sec(dx + c))^2 \sin(dx + c)}{7d} \\ & + \frac{2(11A + 7B) \left(\sec^{\frac{3}{2}}(dx + c) \right) (a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{35d} \\ & + \frac{4a^3(7A + 9B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{4a^3(7A + 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^3(13A + 21B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (13A + 21B)a^3 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (13A + 21B)a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^3 \cos(dx + c)^4 + (A + 3B)a^3 \cos(dx + c)^3 + 3(A + B)a^3 \cos(dx + c)^2 + (3A + B)a^3 \cos(dx + c) + c\right)^{\frac{9}{2}}, x\right)$$

52.109 Problem number 471

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(21A + 20B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & + \frac{2aA(a + a \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & + \frac{2(9A + 5B)(a^3 + a^3 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{4a^3(9A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(3A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (3A + 5B)a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + 5B)a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^3 \cos(dx + c)^4 + (A + 3B)a^3 \cos(dx + c)^3 + 3(A + B)a^3 \cos(dx + c)^2 + (3A + B)a^3 \cos(dx + c) + c\right)^{\frac{7}{2}}, x\right)$$

52.110 Problem number 472

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aB(a + a \sec(dx + c))^2 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{4a^3(4A + B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & + \frac{2(A - B) (a^3 + a^3 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & - \frac{4a^3(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{20a^3(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A + B) a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + B) a^3 \cos \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^3 \cos(dx + c)^4 + (A + 3B)a^3 \cos(dx + c)^3 + 3(A + B)a^3 \cos(dx + c)^2 + (3A + B)a^3 \cos(dx + c) + c\right)^{\frac{5}{2}}, x\right)$$

52.111 Problem number 473

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aB(a + a \sec(dx + c))^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(5A + 9B)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(5A - 6B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & + \frac{4a^3(5A + 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(5A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")`
 Fracas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (5A + 3B)a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (5A + 3B)a^3 \operatorname{weierstrassP} \right)$$

Fracas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^3 \cos(dx + c)^4 + (A + 3B)a^3 \cos(dx + c)^3 + 3(A + B)a^3 \cos(dx + c)^2 + (3A + B)a^3 \cos(dx + c) + c\right)^{\frac{3}{2}}, x\right)$$

52.112 Problem number 474

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aB(a + a \sec(dx + c))^2 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(7A + 11B)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{4a^3(42A + 41B) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(9A + 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(21A + 13B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (21A + 13B)a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (21A + 13B)a^3 \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Ba^3*cos(dx+c)^4+(A+3B)a^3*cos(dx+c)^3+3(A+B)a^3*cos(dx+c)^2+(3A+B)a^3*cos(dx+c)+
```

52.113 Problem number 475

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(24A + 23B) \sin(dx + c)}{105d \sec(dx + c)^{\frac{3}{2}}} + \frac{2aB(a + a \sec(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} \\ & + \frac{2(9A + 13B)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^3(13A + 11B) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(21A + 17B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(13A + 11B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (13A + 11B)a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (13A + 11B)a^3 \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral\left(\frac{Ba^3*cos(dx+c)^4+(A+3B)a^3*cos(dx+c)^3+3(A+B)a^3*cos(dx+c)^2+(3A+B)a^3*cos(dx+c)+
```

52.114 Problem number 476

$$\int \frac{(A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5A - 3B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3ad} - \frac{(A - B) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \sec(dx + c))} \\ & - \frac{3(A - B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{ad} \\ & + \frac{3(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(5A - 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-5i A + 3i B) \cos(dx + c)^2 + \sqrt{2} (-5i A + 3i B) \cos(dx + c) \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{a \cos(dx + c) + a}, x\right)$$

52.115 Problem number 477

$$\int \frac{(A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \sec(dx + c))} + \frac{(3A - B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{ad} \\ & - \frac{(3A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(iA - iB)\cos(dx + c) + \sqrt{2}(iA - iB)\right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + \left(\sqrt{2}(-\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B\cos(dx + c) + A)\sec(dx + c)^{\frac{3}{2}}}{a\cos(dx + c) + a}, x\right)$$

52.116 Problem number 478

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{a + a\cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{d(a + a \sec(dx + c))} \\ & + \frac{(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A-B)\sqrt{\cos(dx+c)}\sin(dx+c) - \left(\sqrt{2}(-iA-iB)\cos(dx+c) + \sqrt{2}(-iA-iB)\right)\text{weierstrassPInverse}(-)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}, x\right)$$

52.117 Problem number 479

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A-B)\sin(dx+c)(\sqrt{\sec(dx+c)})}{d(a+a\sec(dx+c))} \\ - & \frac{(A-3B)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad} \\ + & \frac{(A-B)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A-B)\sqrt{\cos(dx+c)}\sin(dx+c) + \left(\sqrt{2}(-iA+iB)\cos(dx+c) + \sqrt{2}(-iA+iB)\right)\text{weierstrassPInverse}(-)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}, x\right)$$

52.118 Problem number 480

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3A - 5B) \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}} + \frac{(A - B) \sin(dx + c)}{d(a + a \sec(dx + c)) \sqrt{\sec(dx + c)}} \\ & + \frac{3(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(3A - 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(3iA - 5iB) \cos(dx + c) + \sqrt{2}(3iA - 5iB)\right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \left(\sqrt{2}(3iA - 5iB) \cos(dx + c) + \sqrt{2}(3iA - 5iB)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B \cos(dx + c) + A}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}}, x\right)$$

52.119 Problem number 481

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(5A - 7B) \sin(dx + c)}{5ad \sec(dx + c)^{\frac{3}{2}}} + \frac{(A - B) \sin(dx + c)}{d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))} + \frac{5(A - B) \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}} \\ & - \frac{3(5A - 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25 \left(\sqrt{2} (i A - i B) \cos(dx + c) + \sqrt{2} (i A - i B) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{B \cos(dx + c) + A}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}}, x \right)$$

52.120 Problem number 482

$$\int \frac{(A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5A - 2B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d(1 + \sec(dx + c))} \\ & - \frac{(A - B) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{3d(a + a \sec(dx + c))^2} + \frac{(4A - B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{a^2d} \\ & - \frac{(4A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \\ & - \frac{(5A - 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (5i A - 2i B) \cos(dx + c)^2 - 2 \sqrt{2} (-5i A + 2i B) \cos(dx + c) + \sqrt{2} (5i A - 2i B) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \cos(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x \right)$$

52.121 Problem number 483

$$\int \frac{(A + B \cos(c + dx)) \sqrt{\sec(c + dx)}}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d(a + a \sec(dx + c))^2} - \frac{A \sin(dx + c) (\sqrt{\sec(dx + c)})}{a^2 d (1 + \sec(dx + c))} \\ & + \frac{A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & + \frac{(2A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-2i A - i B) \cos(dx + c)^2 - 2 \sqrt{2} (2i A + i B) \cos(dx + c) + \sqrt{2} (-2i A - i B) \right) \operatorname{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(B \cos(dx + c) + A) \sqrt{\sec(dx + c)}}{a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2}, x \right)$$

52.122 Problem number 484

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + 2B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3a^2 d (1 + \sec(dx + c))} - \frac{(A - B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d(a + a \sec(dx + c))^2} \\ & - \frac{B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & + \frac{(A + 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-iA - 2iB)\cos(dx + c)^2 - 2\sqrt{2}(iA + 2iB)\cos(dx + c) + \sqrt{2}(-iA - 2iB)\right)\text{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{B\cos(dx + c) + A}{\left(a^2\cos(dx + c)^2 + 2a^2\cos(dx + c) + a^2\right)\sqrt{\sec(dx + c)}}, x\right)$$

52.123 Problem number 485

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2A - 5B)\sin(dx + c)(\sqrt{\sec(dx + c)})}{3a^2d(1 + \sec(dx + c))} + \frac{(A - B)\sin(dx + c)(\sqrt{\sec(dx + c)})}{3d(a + a\sec(dx + c))^2} \\ & - \frac{(A - 4B)\sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^2d} \\ & + \frac{(2A - 5B)\sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^2d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-2iA + 5iB)\cos(dx + c)^2 - 2\sqrt{2}(2iA - 5iB)\cos(dx + c) + \sqrt{2}(-2iA + 5iB)\right)\text{weierstrassPInverse}(-4,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{B\cos(dx + c) + A}{\left(a^2\cos(dx + c)^2 + 2a^2\cos(dx + c) + a^2\right)\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

52.124 Problem number 486

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{5(A - 2B) \sin(dx + c)}{3a^2 d \sqrt{\sec(dx + c)}} + \frac{(4A - 7B) \sin(dx + c)}{3a^2 d (1 + \sec(dx + c)) \sqrt{\sec(dx + c)}} \\ & + \frac{(A - B) \sin(dx + c)}{3d (a + a \sec(dx + c))^2 \sqrt{\sec(dx + c)}} \\ & + \frac{(4A - 7B) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5(A - 2B) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-iA + 2iB) \cos(dx + c)^2 + 2 \sqrt{2} (-iA + 2iB) \cos(dx + c) + \sqrt{2} (-iA + 2iB) \right) \operatorname{weierstrassPInverse}(-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B \cos(dx + c) + A}{\left(a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2\right) \sec(dx + c)^{\frac{5}{2}}}, x\right)$$

52.125 Problem number 487

$$\int \frac{(A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} - \frac{(8A - 3B) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{15ad(a + a \sec(dx + c))^2} \\ & - \frac{(13A - 3B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{6d(a^3 + a^3 \sec(dx + c))} + \frac{(49A - 9B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{10a^3d} \\ & - \frac{(49A - 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & - \frac{(13A - 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-13i A + 3i B) \cos(dx + c)^3 + 3 \sqrt{2} (-13i A + 3i B) \cos(dx + c)^2 + 3 \sqrt{2} (-13i A + 3i B) \cos(dx + c) + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}, x\right)$$

52.126 Problem number 488

$$\int \frac{(A + B \cos(c + dx)) \sqrt{\sec(c + dx)}}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} - \frac{(6A - B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{15ad(a + a \sec(dx + c))^2} \\ & - \frac{(9A + B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{10d(a^3 + a^3 \sec(dx + c))} \\ & + \frac{(9A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & + \frac{(3A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (3i A + i B) \cos(dx + c)^3 + 3 \sqrt{2} (3i A + i B) \cos(dx + c)^2 + 3 \sqrt{2} (3i A + i B) \cos(dx + c) + \sqrt{2} (3i A + i B) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \cos(dx + c) + A) \sqrt{\sec(dx + c)}}{a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3}, x \right)$$

52.127 Problem number 489

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5d (a + a \sec(dx + c))^3} - \frac{(4A + B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15ad (a + a \sec(dx + c))^2} \\ & + \frac{(A + B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{6d (a^3 + a^3 \sec(dx + c))} \\ & + \frac{(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (i A + i B) \cos(dx + c)^3 + 3 \sqrt{2} (i A + i B) \cos(dx + c)^2 + 3 \sqrt{2} (i A + i B) \cos(dx + c) + \sqrt{2} (i A + i B) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{B \cos(dx + c) + A}{(a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3) \sqrt{\sec(dx + c)}}, x \right)$$

52.128 Problem number 490

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d(a + a \sec(dx + c))^3} + \frac{(2A + 3B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15ad(a + a \sec(dx + c))^2} \\ & + \frac{(A + 3B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{6d(a^3 + a^3 \sec(dx + c))} \\ & - \frac{(A + 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (i A + 3i B) \cos(dx + c)^3 + 3 \sqrt{2} (i A + 3i B) \cos(dx + c)^2 + 3 \sqrt{2} (i A + 3i B) \cos(dx + c) + \sqrt{2} (i A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B \cos(dx + c) + A}{\left(a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3\right) \sec(dx + c)^{\frac{3}{2}}, x}\right)$$

52.129 Problem number 491

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d(a + a \sec(dx + c))^3} + \frac{(3A - 8B) \sin(dx + c) (\sqrt{\sec(dx + c)})^2}{15ad(a + a \sec(dx + c))^2} \\ & + \frac{(3A - 13B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{6d(a^3 + a^3 \sec(dx + c))} \\ & - \frac{(9A - 49B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A - 13B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (3i A - 13i B) \cos(dx + c)^3 + 3 \sqrt{2} (3i A - 13i B) \cos(dx + c)^2 + 3 \sqrt{2} (3i A - 13i B) \cos(dx + c) + \sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B \cos(dx + c) + A}{\left(a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3\right) \sec(dx + c)^{\frac{5}{2}}}, x\right)$$

52.130 Problem number 492

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(13A - 33B) \sin(dx + c)}{6a^3 d \sqrt{\sec(dx + c)}} + \frac{(A - B) \sin(dx + c)}{5d(a + a \sec(dx + c))^3 \sqrt{\sec(dx + c)}} \\ & + \frac{(A - 2B) \sin(dx + c)}{3ad(a + a \sec(dx + c))^2 \sqrt{\sec(dx + c)}} + \frac{7(7A - 17B) \sin(dx + c)}{30d(a^3 + a^3 \sec(dx + c)) \sqrt{\sec(dx + c)}} \\ & + \frac{7(7A - 17B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(13A - 33B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-13i A + 33i B) \cos(dx + c)^3 + 3 \sqrt{2} (-13i A + 33i B) \cos(dx + c)^2 + 3 \sqrt{2} (-13i A + 33i B) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{B \cos(dx + c) + A}{\left(a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3 \right) \sec(dx + c)^{\frac{7}{2}}}, x \right)$$

52.131 Problem number 550

$$\int (a + b \cos(c + dx))(A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab + Ba) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2aA \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2(3aA + 5bB) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2(3aA + 5bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i Ba + i Ab) \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-i Ba - i Ab) \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(Bb \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c) \right) \sec(dx + c)^{\frac{7}{2}}, x \right)$$

52.132 Problem number 551

$$\int (a + b \cos(c + dx))(A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2(Ab + Ba) \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & - \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(aA + 3bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i Aa - 3i Bb) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i Aa + 3i Bb) \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Bb \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c) \right) \sec(dx + c)^{\frac{5}{2}}, x \right)$$

52.133 Problem number 552

$$\int (a + b \cos(c + dx))(A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & - \frac{2(aA - bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 A a \sin(dx+c)}{\sqrt{\cos(dx+c)}} + \sqrt{2} (-i B a - i A b) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2} (i B a + i A b) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb \cos(dx+c)^2 + Aa + (Ba + Ab) \cos(dx+c)\right) \sec(dx+c)^{\frac{3}{2}}, x\right)$$

52.134 Problem number 553

$$\int (a + b \cos(c + dx))(A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2bB \sin(dx+c)}{3d \sqrt{\sec(dx+c)}} + \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2(3aA + bB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 B b \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2} (-3i A a - i B b) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2} (3i A a + i B b) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb \cos(dx+c)^2 + Aa + (Ba + Ab) \cos(dx+c)\right) \sqrt{\sec(dx+c)}, x\right)$$

52.135 Problem number 554

$$\int \frac{(a + b \cos(c + dx))(A + B \cos(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bB \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(Ab + Ba) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2(5aA + 3bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i Ba + i Ab) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-i Ba - i Ab) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c)}{\sqrt{\sec(dx + c)}}, x\right)$$

52.136 Problem number 555

$$\int \frac{(a + b \cos(c + dx))(A + B \cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bB \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(Ab + Ba) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(7aA + 5bB) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{6(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7aA + 5bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(7iAa + 5iBb)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + 5\sqrt{2}(-7iAa - 5iBb)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Bb\cos(dx+c)^2 + Aa + (Ba + Ab)\cos(dx+c)}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

52.137 Problem number 556

$$\int (a + b\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(7Ab + 5Ba) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} + \frac{2aA \left(\sec^{\frac{3}{2}}(dx + c)\right) (b + a\sec(dx + c)) \sin(dx + c)}{5d} \\ & + \frac{2(3a^2A + 5b(Ab + 2Ba)) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2(3a^2A + 5b(Ab + 2Ba)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2(2Aab + Ba^2 + 3b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(iBa^2 + 2iAab + 3iBb^2)\cos(dx + c)^2\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + 5\sqrt{2}(-iBa^2 - 2iAab - 3iBb^2)\cos(dx + c)^2\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + (2Bab + Ab^2)\cos(dx+c)^2 + (Ba^2 + 2Aab)\cos(dx+c)\right)\sec(dx+c)^{\frac{7}{2}}, x\right)$$

52.138 Problem number 557

$$\int (a + b \cos(c + dx))^2 (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2a(5Ab + 3Ba) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} + \frac{2aA(b + a \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d}$$

$$- \frac{2(2Aab + B a^2 - b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

$$+ \frac{2(a^2 A + 3A b^2 + 6abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i A a^2 - 6i B a b - 3i A b^2) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i A a^2 -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(B b^2 \cos(dx + c)^3 + A a^2 + (2 B a b + A b^2) \cos(dx + c)^2 + (B a^2 + 2 A a b) \cos(dx + c)\right) \sec(dx + c)^{\frac{5}{2}}, x\right)$$

52.139 Problem number 558

$$\int (a + b \cos(c + dx))^2 (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^2 B \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2a^2 A \sin(dx + c) (\sqrt{\sec(dx + c)})}{d}$$

$$- \frac{2(a^2 A - b(Ab + 2Ba)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

$$+ \frac{2(6Aab + 3B a^2 + b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i Ba^2 - 6i Aab - i Bb^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i Ba^2 + 6i Aab +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb^2 \cos(dx + c)^3 + Aa^2 + (2 Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2 Aab) \cos(dx + c)\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

52.140 Problem number 559

$$\int (a + b \cos(c + dx))^2 (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2 B \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2b(Ab + 2Ba) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2(10Aab + 5Ba^2 + 3b^2B) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3a^2A + Ab^2 + 2abB) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (3i Aa^2 + 2i Bab + i Ab^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-3i Aa^2 - 2i B$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb^2 \cos(dx + c)^3 + Aa^2 + (2 Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2 Aab) \cos(dx + c)\right) \sqrt{\sec(dx + c)}, x\right)$$

52.141 Problem number 560

$$\int \frac{(a + b \cos(c + dx))^2 (A + B \cos(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2 B \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2b(Ab + 2Ba) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(14Aab + 7B a^2 + 5b^2 B) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(5a^2 A + 3A b^2 + 6abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(14Aab + 7B a^2 + 5b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (7i B a^2 + 14i A a b + 5i B b^2) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-7i B a^2 - 14i A a b - 5i B b^2) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B b^2 \cos(dx + c)^3 + A a^2 + (2 B a b + A b^2) \cos(dx + c)^2 + (B a^2 + 2 A a b) \cos(dx + c)}{\sqrt{\sec(dx + c)}}, x\right)$$

52.142 Problem number 561

$$\int (a + b \cos(c + dx))^3 (A + B \cos(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2a(5a^2A + 18Ab^2 + 21abB) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} \\
& + \frac{2a^2(11Ab + 7Ba) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\
& + \frac{2aA \left(\sec^{\frac{3}{2}}(dx + c)\right) (b + a \sec(dx + c))^2 \sin(dx + c)}{7d} \\
& + \frac{2(9Aa^2b + 5Ab^3 + 3a^3B + 15Bab^2) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\
& - \frac{2(9Aa^2b + 5Ab^3 + 3a^3B + 15Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(5a^3A + 21Aab^2 + 21a^2bB + 21b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (5iAa^3 + 21iBa^2b + 21iAab^2 + 21iBb^3) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^3 \cos(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \cos(dx + c)^3 + 3(Ba^2b + Aab^2) \cos(dx + c)^2 + (Ba^3 + 3Aab + Ab^3) \cos(dx + c) + c\right)^{\frac{9}{2}}, x\right)$$

52.143 Problem number 562

$$\int (a + b \cos(c + dx))^3 (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(9Ab + 5Ba) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{15d} \\ & + \frac{2a(3a^2A + 14Ab^2 + 15abB) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & + \frac{2aA(b + a \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2(3a^3A + 15Aab^2 + 15a^2bB - 5b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3Aa^2b + 3Ab^3 + a^3B + 9Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (iBa^3 + 3iAa^2b + 9iBab^2 + 3iAb^3) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left((Bb^3 \cos(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \cos(dx + c)^3 + 3(Ba^2b + Aab^2) \cos(dx + c)^2 + (Ba^3 + 3Aa^2b + Ab^3) \cos(dx + c) + c\right)^{\frac{7}{2}}, x\right)$$

52.144 Problem number 563

$$\int (a + b \cos(c + dx))^3 (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(aA - bB) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2bB(b + a \sec(dx + c))^2 \sin(dx + c)}{3d \sqrt{\sec}(dx + c)} \\ & + \frac{2a(9Aab + 3Ba^2 - 2b^2B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{3d} \\ & - \frac{2(3Aa^2b - Ab^3 + a^3B - 3Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^3A + 9Aab^2 + 9a^2bB + b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i Aa^3 - 9i Ba^2b - 9i Aab^2 - i Bb^3) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \dots$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^3 \cos(dx + c)^4 + Aa^3 + (3 Bab^2 + Ab^3) \cos(dx + c)^3 + 3(Ba^2b + Aab^2) \cos(dx + c)^2 + (Ba^3 + 3Aa^2b + 3Ac^2 + c)^{\frac{5}{2}}, x\right)$$

52.145 Problem number 564

$$\int (a + b \cos(c + dx))^3 (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bB(b + a \sec(dx + c))^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2b^2(5Ab + 9Ba) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & + \frac{2a^2(5aA - bB) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{2(5a^3A - 15Aab^2 - 15a^2bB - 3b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(9Aa^2b + Ab^3 + 3a^3B + 3Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (3i Ba^3 + 9i Aa^2b + 3i Bab^2 + i Ab^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-3i \dots$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^3 \cos(dx + c)^4 + Aa^3 + (3 Bab^2 + Ab^3) \cos(dx + c)^3 + 3(Ba^2b + Aab^2) \cos(dx + c)^2 + (Ba^3 + 3Aa^2b + 3Ac^2 + c)^{\frac{3}{2}}, x\right)$$

52.146 Problem number 565

$$\int (a + b \cos(c + dx))^3 (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(7Ab + 11Ba) \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{2bB(b + a \sec(dx + c))^2 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2b(21Aab + 18Ba^2 + 5b^2B) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(15Aa^2b + 3Ab^3 + 5a^3B + 9Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^3A + 21Aab^2 + 21a^2bB + 5b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (21i Aa^3 + 21i Ba^2b + 21i Aab^2 + 5i Bb^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^3 \cos(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \cos(dx + c)^3 + 3(Ba^2b + Aab^2) \cos(dx + c)^2 + (Ba^3 + 3Aa\right.\right.$$

52.147 Problem number 566

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(9Ab + 13Ba) \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{2b(27Aab + 22B a^2 + 7b^2 B) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2bB(b + a \sec(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2(21A a^2 b + 5A b^3 + 7a^3 B + 15Ba b^2) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(15a^3 A + 27Aa b^2 + 27a^2 b B + 7b^3 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21A a^2 b + 5A b^3 + 7a^3 B + 15Ba b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (7i Ba^3 + 21i Aa^2 b + 15i Bab^2 + 5i Ab^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^3 \cos(dx + c)^4 + Aa^3 + (3 Bab^2 + Ab^3) \cos(dx + c)^3 + 3(Ba^2 b + Aab^2) \cos(dx + c)^2 + (Ba^3 + 3Aa^2 b)}{\sqrt{\sec(dx + c)}}\right)$$

52.148 Problem number 584

$$\int \frac{(aB + bB \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)}{a + b \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} B \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3 d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(B \sec(dx + c)^{\frac{5}{2}}, x\right)$$

52.149 Problem number 585

$$\int \frac{(aB + bB \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)}{a + b \cos(c + dx)} dx$$

Optimal antiderivative

$$\frac{2B \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} - \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} B \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} B \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(B \sec(dx + c)^{\frac{3}{2}}, x\right)$$

52.150 Problem number 586

$$\int \frac{(aB + bB \cos(c + dx)) \sqrt{\sec(c + dx)}}{a + b \cos(c + dx)} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} B \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(B \sqrt{\sec(dx + c)}, x\right)$$

52.151 Problem number 587

$$\int \frac{aB + bB \cos(c + dx)}{(a + b \cos(c + dx)) \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} B \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - i \sqrt{2} B \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B}{\sqrt{\sec(dx + c)}}, x\right)$$

52.152 Problem number 588

$$\int \frac{aB + bB \cos(c + dx)}{(a + b \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2B \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2B \sqrt{\cos(dx + c)} \sin(dx + c) - i \sqrt{2} B \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

52.153 Problem number 589

$$\int \frac{aB + bB \cos(c + dx)}{(a + b \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2B \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{6B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2B \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 3i\sqrt{2} B \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))}{5d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{B}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

53 Test file number 93

Test folder name:

test_cases/4_Trig_functions/4.2_Cosine/93_4.2.4.1-a+b_cos-^m-A+B_cos+C_cos^2-

53.1 Problem number 16

$$\int (b \cos(c+dx))^{5/2} (A + C \cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(9A+7C)(b \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{45d} + \frac{2C(b \cos(dx+c))^{\frac{7}{2}} \sin(dx+c)}{9bd} \\ & + \frac{2b^2(9A+7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i\sqrt{2}(9A+7C)b^{\frac{5}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) - 3i\sqrt{2}(9A + \dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx+c)^4 + Ab^2 \cos(dx+c)^2\right) \sqrt{b \cos(dx+c)}, x\right)$$

53.2 Problem number 17

$$\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx + c))^{5/2} \sin(dx + c)}{7bd} \\ & + \frac{2b^2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2b(7A + 5C) \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} (7A + 5C) b^{3/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (7A + 5C) b^{3/2} \operatorname{weierstrassPInverse}(\dots)}{21d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)}, x\right)$$

53.3 Problem number 18

$$\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx + c))^{3/2} \sin(dx + c)}{5bd} \\ & + \frac{2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{b \cos(dx + c)} C \cos(dx + c) \sin(dx + c) + \sqrt{2} (5i A + 3i C) \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^2 + A\right) \sqrt{b \cos(dx + c)}, x\right)$$

53.4 Problem number 19

$$\int \frac{A + C \cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} + \frac{2C \sin(dx + c) \sqrt{b \cos(dx + c)}}{3bd}$$

command

`integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-3i A - i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i A + i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{b \cos(dx + c)}}{b \cos(dx + c)}, x\right)$$

53.5 Problem number 20

$$\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{bd \sqrt{b \cos(dx + c)}} - \frac{2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-iA + iC)\sqrt{b} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)^2}, x\right)$$

53.6 Problem number 21

$$\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{3bd (b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-iA - 3iC)\sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(iA + 3iC)\sqrt{b} \frac{3b^3 d \cos(dx + c)}{3b^3 d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)^3}, x\right)$$

53.7 Problem number 22

$$\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{5bd(b \cos(dx + c))^{\frac{5}{2}}} + \frac{2(3A + 5C) \sin(dx + c)}{5b^3d \sqrt{b \cos(dx + c)}} - \frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4d \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-3iA - 5iC)\sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{b \cos(dx + c)}}{b^4 \cos(dx + c)^4}, x\right)$$

53.8 Problem number 23

$$\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{7bd(b \cos(dx + c))^{\frac{7}{2}}} + \frac{2(5A + 7C) \sin(dx + c)}{21b^3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-5i A - 7i C) \sqrt{b} \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (5i A + 7i C) \sqrt{b} \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + A \right) \sqrt{b \cos(dx + c)}}{b^5 \cos(dx + c)^5}, x \right)$$

53.9 Problem number 26

$$\int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{9/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^3(5A + 7C) (b \sec(dx + c))^{3/2} \sin(dx + c)}{21d} \\ & + \frac{2b^4(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2Ab^2(b \sec(dx + c))^{5/2} \tan(dx + c)}{7d} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (5A + 7C) b^{9/2} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (5A + 7C) b^{9/2} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(Cb^4 \cos(dx + c)^2 + Ab^4 \right) \sqrt{b \sec(dx + c)} \sec(dx + c)^4, x \right)$$

53.10 Problem number 27

$$\int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\frac{2b^4(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2b^3(3A + 5C) \sin(dx + c) \sqrt{b \sec(dx + c)}}{5d} + \frac{2Ab^2(b \sec(dx + c))^{3/2} \tan(dx + c)}{5d}$$

command

```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (3A + 5C) b^{7/2} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^2 + Ab^3\right) \sqrt{b \sec(dx + c)} \sec(dx + c)^3, x\right)$$

53.11 Problem number 28

$$\int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b^2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2Ab^2 \sqrt{b \sec(dx + c)} \tan(dx + c)}{3d}$$

command

```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (A + 3C) b^{\frac{5}{2}} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (A + 3C) b^{\frac{5}{2}} \cos(dx + c)$$

$$3d \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^2 + Ab^2\right) \sqrt{b \sec(dx + c)} \sec(dx + c)^2, x\right)$$

53.12 Problem number 29

$$\int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b^2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2Ab^2 \tan(dx + c)}{d \sqrt{b \sec(dx + c)}}$$

command

`integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (A - C) b^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} (A - C)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^2 + Ab\right) \sqrt{b \sec(dx + c)} \sec(dx + c), x\right)$$

53.13 Problem number 30

$$\int (A + C \cos^2(c + dx)) \sqrt{b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2b^2 C \tan(dx + c)}{3d (b \sec(dx + c))^{\frac{3}{2}}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 C \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + \sqrt{2} (-3i A - i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

3 d

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx+c)^2 + A\right) \sqrt{b \sec(dx+c)}, x\right)$$

53.14 Problem number 31

$$\int \frac{A + C \cos^2(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(5A + 3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} + \frac{2b^2 C \tan(dx+c)}{5d (b \sec(dx+c))^{\frac{5}{2}}}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 C \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^2 \sin(dx+c) + \sqrt{2} (5i A + 3i C) \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^2 + A\right) \sqrt{b \sec(dx+c)}}{b \sec(dx+c)}, x\right)$$

53.15 Problem number 32

$$\int \frac{A + C \cos^2(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7A + 5C) \sin(dx + c)}{21bd \sqrt{b \sec(dx + c)}} \\ & + \frac{2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d} \\ & + \frac{2b^2 C \tan(dx + c)}{7d (b \sec(dx + c))^{\frac{7}{2}}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-7i A - 5i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (7i A + 5i C) \sqrt{b} \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{b \sec(dx + c)}}{b^2 \sec(dx + c)^2}, x\right)$$

53.16 Problem number 33

$$\int \frac{A + C \cos^2(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9A + 7C) \sin(dx + c)}{45bd (b \sec(dx + c))^{\frac{3}{2}}} \\ & + \frac{2(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2b^2 C \tan(dx + c)}{9d (b \sec(dx + c))^{\frac{9}{2}}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2}(-9iA - 7iC)\sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3\sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{b \sec(dx + c)}}{b^3 \sec(dx + c)^3}, x\right)$$

53.17 Problem number 37

$$\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9A + 7C)(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45bd} + \frac{2C(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^3d} \\ & + \frac{2(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2}(-9iA - 7iC)\sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3\sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^4 + A \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}, x\right)$$

53.18 Problem number 38

$$\int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^2d} \\ & + \frac{2b(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2(7A + 5C) \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-7iA - 5iC)\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(7iA + 5iC)\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{21d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^3 + A \cos(dx + c)\right) \sqrt{b \cos(dx + c)}, x\right)$$

53.19 Problem number 39

$$\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5bd} \\ & + \frac{2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{b \cos(dx+c)} C \cos(dx+c) \sin(dx+c) + \sqrt{2} (5iA + 3iC) \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx+c)^2 + A\right) \sqrt{b \cos(dx+c)}, x\right)$$

53.20 Problem number 40

$$\int \sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx)) \sec(c+dx) dx$$

Optimal antiderivative

$$\frac{2b(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}} + \frac{2C \sin(dx+c) \sqrt{b \cos(dx+c)}}{3d}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-3iA - iC) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2} (3iA + iC) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx+c)^2 + A\right) \sqrt{b \cos(dx+c)} \sec(dx+c), x\right)$$

53.21 Problem number 41

$$\int \sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx)) \sec^2(c+dx) dx$$

Optimal antiderivative

$$\frac{2Ab \sin(dx+c)}{d \sqrt{b \cos(dx+c)}} - \frac{2(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iA + iC)\sqrt{b} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{3d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^2 + A\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^2, x\right)$$

53.22 Problem number 42

$$\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{2Ab^2 \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2b(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iA - 3iC)\sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(iA + 3iC)\sqrt{b}}{3d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^2 + A\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^3, x\right)$$

53.23 Problem number 43

$$\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{2A b^3 \sin(dx+c)}{5d(b \cos(dx+c))^{\frac{5}{2}}} + \frac{2b(3A+5C) \sin(dx+c)}{5d \sqrt{b \cos(dx+c)}} - \frac{2(3A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-3iA - 5iC)\sqrt{b} \cos(dx+c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx+c)^2 + A\right) \sqrt{b \cos(dx+c)} \sec(dx+c)^4, x\right)$$

53.24 Problem number 44

$$\int \sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx)) \sec^5(c+dx) dx$$

Optimal antiderivative

$$\frac{2A b^4 \sin(dx+c)}{7d(b \cos(dx+c))^{\frac{7}{2}}} + \frac{2b^2(5A+7C) \sin(dx+c)}{21d(b \cos(dx+c))^{\frac{3}{2}}} + \frac{2b(5A+7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-5iA - 7iC)\sqrt{b} \cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2}(5iA + 7iC)\sqrt{b} \cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx+c)^2 + A\right) \sqrt{b \cos(dx+c)} \sec(dx+c)^5, x\right)$$

53.25 Problem number 45

$$\int \cos(c + dx)(b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9A + 7C)(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45d} + \frac{2C(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^2d} \\ & + \frac{2b(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i \sqrt{2} (9A + 7C)b^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 3i \sqrt{2} (9A + 7C)b^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^4 + Ab \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}, x\right)$$

53.26 Problem number 46

$$\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7bd} \\ & + \frac{2b^2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2b(7A + 5C) \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (7A + 5C)b^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (7A + 5C)b^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)}, x\right)$$

53.27 Problem number 47

$$\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\frac{2C(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5d} + \frac{2b(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{b \cos(dx + c)} Cb \cos(dx + c) \sin(dx + c) + i \sqrt{2} (5A + 3C)b^{\frac{3}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c), x\right)$$

53.28 Problem number 48

$$\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^2(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} + \frac{2bC \sin(dx + c) \sqrt{b \cos(dx + c)}}{3d}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} (3A + C) b^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (3A + C) b^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^2, x\right)$$

53.29 Problem number 49

$$\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{2A b^2 \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} - \frac{2b(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} (A - C) b^{\frac{3}{2}} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} (A - C) b^{\frac{3}{2}} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^3, x\right)$$

53.30 Problem number 50

$$\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{2A b^3 \sin(dx + c)}{3d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{2b^2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} (A + 3C) b^{\frac{3}{2}} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (A + 3C) b^{\frac{3}{2}} \cos(dx + c)}{3 d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^4, x\right)$$

53.31 Problem number 51

$$\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\frac{2A b^4 \sin(dx + c)}{5d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{2b^2(3A + 5C) \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} - \frac{2b(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} (3A + 5C) b^{\frac{3}{2}} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{3 d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^5, x\right)$$

53.32 Problem number 52

$$\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^6(c + dx) dx$$

Optimal antiderivative

$$\frac{2A b^5 \sin(dx + c)}{7d (b \cos(dx + c))^{7/2}} + \frac{2b^3(5A + 7C) \sin(dx + c)}{21d (b \cos(dx + c))^{3/2}} \\ + \frac{2b^2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (5A + 7C) b^{3/2} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (5A + 7C) b^{3/2} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^6, x\right)$$

53.33 Problem number 53

$$\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2b(9A + 7C) (b \cos(dx + c))^{3/2} \sin(dx + c)}{45d} + \frac{2C(b \cos(dx + c))^{7/2} \sin(dx + c)}{9bd} \\ + \frac{2b^2(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i \sqrt{2} (9A + 7C) b^{5/2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 3i \sqrt{2} (9A + 7C) b^{5/2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}, x\right)$$

53.34 Problem number 54

$$\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx + c))^{5/2} \sin(dx + c)}{7d} \\ & + \frac{2b^3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2b^2(7A + 5C) \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (7A + 5C) b^{5/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (7A + 5C) b^{5/2} \operatorname{weierstrassPInv}$$

21 d

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)} \sec(dx + c), x\right)$$

53.35 Problem number 55

$$\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bC(b \cos(dx + c))^{3/2} \sin(dx + c)}{5d} \\ & + \frac{2b^2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{b \cos(dx+c)} C b^2 \cos(dx+c) \sin(dx+c) + i \sqrt{2} (5A+3C) b^{\frac{5}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2 \cos(dx+c)^4 + Ab^2 \cos(dx+c)^2\right) \sqrt{b \cos(dx+c)} \sec(dx+c)^2, x\right)$$

53.36 Problem number 56

$$\int (b \cos(c+dx))^{5/2} (A + C \cos^2(c+dx)) \sec^3(c+dx) dx$$

Optimal antiderivative

$$\frac{2b^3(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}} + \frac{2b^2 C \sin(dx+c) \sqrt{b \cos(dx+c)}}{3d}$$

command

`integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} (3A+C) b^{\frac{5}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} (3A+C) b^{\frac{5}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2 \cos(dx+c)^4 + Ab^2 \cos(dx+c)^2\right) \sqrt{b \cos(dx+c)} \sec(dx+c)^3, x\right)$$

53.37 Problem number 57

$$\int (b \cos(c+dx))^{5/2} (A + C \cos^2(c+dx)) \sec^4(c+dx) dx$$

Optimal antiderivative

$$\frac{2A b^3 \sin(dx+c)}{d \sqrt{b \cos(dx+c)}} - \frac{2b^2(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}(A-C)b^{\frac{5}{2}}\cos(dx+c)\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) + i}{3d\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2\cos(dx+c)^4 + Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^4, x\right)$$

53.38 Problem number 58

$$\int (b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))\sec^5(c+dx)dx$$

Optimal antiderivative

$$\frac{2Ab^4\sin(dx+c)}{3d(b\cos(dx+c))^{\frac{3}{2}}} + \frac{2b^3(A+3C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d\sqrt{b\cos(dx+c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}(A+3C)b^{\frac{5}{2}}\cos(dx+c)^2\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}(A+3C)b^{\frac{5}{2}}\cos(dx+c)}{3d\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2\cos(dx+c)^4 + Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^5, x\right)$$

53.39 Problem number 59

$$\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^6(c + dx) dx$$

Optimal antiderivative

$$\frac{2A b^5 \sin(dx + c)}{5d (b \cos(dx + c))^{5/2}} + \frac{2b^3(3A + 5C) \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} - \frac{2b^2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (3A + 5C) b^{5/2} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^6, x\right)$$

53.40 Problem number 60

$$\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^7(c + dx) dx$$

Optimal antiderivative

$$\frac{2A b^6 \sin(dx + c)}{7d (b \cos(dx + c))^{7/2}} + \frac{2b^4(5A + 7C) \sin(dx + c)}{21d (b \cos(dx + c))^{3/2}} + \frac{2b^3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (5A + 7C)b^{\frac{5}{2}} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (5A + 7C)b^{\frac{5}{2}} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^7, x\right)$$

53.41 Problem number 61

$$\int \frac{\cos^4(c + dx) (A + C \cos^2(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(11A + 9C) (b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{77b^3d} + \frac{2C(b \cos(dx + c))^{\frac{9}{2}} \sin(dx + c)}{11b^5d} \\ & + \frac{10(11A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{10(11A + 9C) \sin(dx + c) \sqrt{b \cos(dx + c)}}{231bd} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (11iA + 9iC) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-11iA - 9iC) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^5 + A \cos(dx + c)^3\right) \sqrt{b \cos(dx + c)}}{b}, x\right)$$

53.42 Problem number 62

$$\int \frac{\cos^3(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9A+7C)(b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{45b^2d} + \frac{2C(b\cos(dx+c))^{\frac{7}{2}}\sin(dx+c)}{9b^4d} \\ & + \frac{2(9A+7C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\sqrt{b\cos(dx+c)}}{15\cos\left(\frac{dx}{2} + \frac{c}{2}\right)bd\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{2}(-9iA-7iC)\sqrt{b}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) + 3\sqrt{2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C\cos(dx+c)^4 + A\cos(dx+c)^2)\sqrt{b\cos(dx+c)}}{b}, x\right)$$

53.43 Problem number 63

$$\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{7b^3d} \\ & + \frac{2(7A+5C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})}{21\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d\sqrt{b\cos(dx+c)}} \\ & + \frac{2(7A+5C)\sin(dx+c)\sqrt{b\cos(dx+c)}}{21bd} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-7i A - 5i C) \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (7i A + 5i C) \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

21 bd

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \cos(dx + c)^3 + A \cos(dx + c)) \sqrt{b \cos(dx + c)}}{b}, x \right)$$

53.44 Problem number 64

$$\int \frac{\cos(c + dx) (A + C \cos^2(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2C(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5b^2d} + \frac{2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) \sqrt{b \cos(dx + c)}}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) bd \sqrt{\cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{b \cos(dx + c)} C \cos(dx + c) \sin(dx + c) + \sqrt{2} (5i A + 3i C) \sqrt{b} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{b}, x \right)$$

53.45 Problem number 65

$$\int \frac{A + C \cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} + \frac{2C \sin(dx + c) \sqrt{b \cos(dx + c)}}{3bd}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-3iA - iC)\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(3iA + iC)\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{b \cos(dx + c)}}{b \cos(dx + c)}, x\right)$$

53.46 Problem number 66

$$\int \frac{(A + C \cos^2(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} - \frac{2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iA + iC)\sqrt{b} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{3bd \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)}{b \cos(dx + c)}, x\right)$$

53.47 Problem number 67

$$\int \frac{(A + C \cos^2(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2Ab \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}}$$

command

`integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iA - 3iC)\sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(iA + 3iC)\sqrt{b}}{3bd \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b \cos(dx + c)}, x\right)$$

53.48 Problem number 68

$$\int \frac{(A + C \cos^2(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2Ab^2 \sin(dx + c)}{5d(b \cos(dx + c))^{\frac{5}{2}}} + \frac{2(3A + 5C) \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} - \frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-3iA - 5iC)\sqrt{b} \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^3}{b \cos(dx + c)}, x\right)$$

53.49 Problem number 69

$$\int \frac{(A + C \cos^2(c + dx)) \sec^4(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A b^3 \sin(dx + c)}{7d (b \cos(dx + c))^{\frac{7}{2}}} + \frac{2b(5A + 7C) \sin(dx + c)}{21d (b \cos(dx + c))^{\frac{3}{2}}} \\ & + \frac{2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-5iA - 7iC)\sqrt{b} \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(5iA + 7iC)\sqrt{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^4}{b \cos(dx + c)}, x\right)$$

53.50 Problem number 70

$$\int \frac{(A + C \cos^2(c + dx)) \sec^5(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2A b^4 \sin(dx + c)}{9d (b \cos(dx + c))^{\frac{9}{2}}} + \frac{2b^2(7A + 9C) \sin(dx + c)}{45d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{2(7A + 9C) \sin(dx + c)}{15d \sqrt{b \cos(dx + c)}} - \frac{2(7A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b d \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \sqrt{2} (7i A + 9i C) \sqrt{b} \cos(dx + c)^5 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^5}{b \cos(dx + c)}, x\right)$$

53.51 Problem number 71

$$\int \frac{\cos^4(c + dx) (A + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(9A + 7C) (b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45b^3d} + \frac{2C(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^5d} + \frac{2(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2}(-9iA - 7iC)\sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3\sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^4 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c)}}{b^2}, x\right)$$

53.52 Problem number 72

$$\int \frac{\cos^3(c + dx) (A + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^4d} \\ & + \frac{2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}} \\ & + \frac{2(7A + 5C) \sin(dx + c) \sqrt{b \cos(dx + c)}}{21b^2d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-7iA - 5iC)\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(7iA + 5iC)\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

$$21b^2d$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^3 + A \cos(dx + c)) \sqrt{b \cos(dx + c)}}{b^2}, x\right)$$

53.53 Problem number 73

$$\int \frac{\cos^2(c + dx) (A + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2C(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5b^3d} + \frac{2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{b \cos(dx + c)} C \cos(dx + c) \sin(dx + c) + \sqrt{2} (5i A + 3i C) \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \sqrt{b \cos(dx + c)}))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{b \cos(dx + c)}}{b^2}, x\right)$$

53.54 Problem number 74

$$\int \frac{\cos(c + dx) (A + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b d \sqrt{b \cos(dx + c)}} + \frac{2C \sin(dx + c) \sqrt{b \cos(dx + c)}}{3b^2d}$$

command

```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-3iA - iC)\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(3iA + iC)\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3b^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)}, x\right)$$

53.55 Problem number 75

$$\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{bd \sqrt{b \cos(dx + c)}} - \frac{2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iA + iC)\sqrt{b} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + \sqrt{2}(iA - iC)\sqrt{b} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{3b^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)^2}, x\right)$$

53.56 Problem number 76

$$\int \frac{(A + C \cos^2(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iA - 3iC)\sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(iA + 3iC)\sqrt{b}}{3b^2d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)}{b^2 \cos(dx + c)^2}, x\right)$$

53.57 Problem number 77

$$\int \frac{(A + C \cos^2(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2Ab \sin(dx + c)}{5d(b \cos(dx + c))^{\frac{5}{2}}} + \frac{2(3A + 5C) \sin(dx + c)}{5bd \sqrt{b \cos(dx + c)}} - \frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-3iA - 5iC)\sqrt{b}\cos(dx+c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\cos(dx+c)^2 + A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2}{b^2\cos(dx+c)^2}, x\right)$$

53.58 Problem number 78

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab^2\sin(dx+c)}{7d(b\cos(dx+c))^{\frac{7}{2}}} + \frac{2(5A+7C)\sin(dx+c)}{21d(b\cos(dx+c))^{\frac{3}{2}}} \\ & + \frac{2(5A+7C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})}{21\cos\left(\frac{dx}{2} + \frac{c}{2}\right)bd\sqrt{b\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-5iA - 7iC)\sqrt{b}\cos(dx+c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + \sqrt{2}(5iA + 7iC)\sqrt{b}\cos(dx+c)^4 \text{weierstrassPInverse}(4, 0, \cos(dx+c) + i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\cos(dx+c)^2 + A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3}{b^2\cos(dx+c)^2}, x\right)$$

53.59 Problem number 79

$$\int \frac{\cos^5(c + dx) (A + C \cos^2(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9A + 7C) (b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45b^4d} + \frac{2C(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^6d} \\ & + \frac{2(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \sqrt{2} (-9i A - 7i C) \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3 \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^4 + A \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}}{b^3}, x\right)$$

53.60 Problem number 80

$$\int \frac{\cos^4(c + dx) (A + C \cos^2(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^5d} \\ & + \frac{2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx + c)}} \\ & + \frac{2(7A + 5C) \sin(dx + c) \sqrt{b \cos(dx + c)}}{21b^3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-7i A - 5i C) \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (7i A + 5i C) \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

21 b³d

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^3 + A \cos(dx + c) \right) \sqrt{b \cos(dx + c)}}{b^3}, x \right)$$

53.61 Problem number 81

$$\int \frac{\cos^3(c + dx) (A + C \cos^2(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2C(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5b^4d} + \frac{2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) \sqrt{b \cos(dx + c)}}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) b^3 d \sqrt{\cos(dx + c)}}$$

command

`integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{b \cos(dx + c)} C \cos(dx + c) \sin(dx + c) + \sqrt{2} (5i A + 3i C) \sqrt{b} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + A \right) \sqrt{b \cos(dx + c)}}{b^3}, x \right)$$

53.62 Problem number 82

$$\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(3A+C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^2d\sqrt{b\cos(dx+c)}} + \frac{2C\sin(dx+c)\sqrt{b\cos(dx+c)}}{3b^3d}$$

command

```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-3iA-iC)\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}(3iA+iC)\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{3b^3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C\cos(dx+c)^2+A\right)\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)},x\right)$$

53.63 Problem number 83

$$\int \frac{\cos(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2A\sin(dx+c)}{b^2d\sqrt{b\cos(dx+c)}} - \frac{2(A-C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{b\cos(dx+c)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^3d\sqrt{\cos(dx+c)}}$$

command

```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-iA+iC)\sqrt{b}\cos(dx+c)\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) + \sqrt{2}(iA-iC)\sqrt{b}\cos(dx+c)\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C\cos(dx+c)^2+A\right)\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)^2},x\right)$$

53.64 Problem number 84

$$\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{3bd (b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iA - 3iC)\sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(iA + 3iC)\sqrt{b}}{3b^3 d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)^3}, x\right)$$

53.65 Problem number 85

$$\int \frac{(A + C \cos^2(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{5d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{2(3A + 5C) \sin(dx + c)}{5b^2 d \sqrt{b \cos(dx + c)}} - \frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-3iA - 5iC)\sqrt{b} \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)}{b^3 \cos(dx + c)^3}, x\right)$$

53.66 Problem number 86

$$\int \frac{(A + C \cos^2(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab \sin(dx + c)}{7d(b \cos(dx + c))^{7/2}} + \frac{2(5A + 7C) \sin(dx + c)}{21bd(b \cos(dx + c))^{3/2}} \\ & + \frac{2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-5iA - 7iC)\sqrt{b} \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(5iA + 7iC)\sqrt{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b^3 \cos(dx + c)^3}, x\right)$$

53.67 Problem number 87

$$\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{5bd(b \cos(dx + c))^{\frac{5}{2}}} + \frac{2(3A + 5C) \sin(dx + c)}{5b^3d \sqrt{b \cos(dx + c)}} - \frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4d \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-3iA - 5iC)\sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{b \cos(dx + c)}}{b^4 \cos(dx + c)^4}, x\right)$$

53.68 Problem number 88

$$\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{7bd(b \cos(dx + c))^{\frac{7}{2}}} + \frac{2(5A + 7C) \sin(dx + c)}{21b^3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4d \sqrt{b \cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-5iA - 7iC)\sqrt{b} \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(5iA + 7iC)\sqrt{b} \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{b^5 \cos(dx + c)^5}, x\right)$$

53.69 Problem number 239

$$\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9A + 7C)(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45bd} \\ & + \frac{2B(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^2d} + \frac{2C(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^3d} \\ & + \frac{10bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{10B \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \\ & + \frac{2(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-75i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 75i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^4 + B \cos(dx + c)^3 + A \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}, x\right)$$

53.70 Problem number 240

$$\int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5bd} + \frac{2C(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^2d} \\ & + \frac{2b(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2(7A + 5C) \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \\ & + \frac{6B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$5 \sqrt{2} (7i A + 5i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-7i A - 5i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)\right) \sqrt{b \cos(dx + c)}, x\right)$$

53.71 Problem number 241

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5bd} \\ & + \frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sin(dx + c) \sqrt{b \cos(dx + c)}}{3d} \\ & + \frac{2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c)}, x\right)$$

53.72 Problem number 242

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2C \sin(dx + c) \sqrt{b \cos(dx + c)}}{3d} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i A - i C) \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i A + i C) \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c)} \sec(dx + c), x\right)$$

53.73 Problem number 243

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{2Ab \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} + \frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} - \frac{2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^2, x\right)$$

53.74 Problem number 244

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{2A b^2 \sin(dx + c)}{3d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{2bB \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} + \frac{2b(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} - \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iA - 3iC)\sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(iA + 3iC)\sqrt{b}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^3, x\right)$$

53.75 Problem number 245

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab^3 \sin(dx + c)}{5d(b \cos(dx + c))^{\frac{5}{2}}} + \frac{2b^2B \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2b(3A + 5C) \sin(dx + c)}{5d\sqrt{b \cos(dx + c)}} \\ & + \frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i\sqrt{2}B\sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i\sqrt{2}B\sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^4, x\right)$$

53.76 Problem number 246

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A b^4 \sin(dx + c)}{7d (b \cos(dx + c))^{\frac{7}{2}}} + \frac{2b^3 B \sin(dx + c)}{5d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{2b^2(5A + 7C) \sin(dx + c)}{21d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{6bB \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} \\ & + \frac{2b(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{6B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$5 \sqrt{2} (5i A + 7i C) \sqrt{b} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-5i A - 7i C) \sqrt{b} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^5, x\right)$$

53.77 Problem number 247

$$\int \cos(c + dx)(b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9A + 7C) (b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45d} \\ & + \frac{2B(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7bd} + \frac{2C(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^2d} \\ & + \frac{10b^2 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{10bB \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \\ & + \frac{2b(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-75i\sqrt{2}Bb^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}Bb^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb\cos(dx+c)^4+Bb\cos(dx+c)^3+Ab\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)},x\right)$$

53.78 Problem number 248

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{5d} + \frac{2C(b\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{7bd} \\ & + \frac{2b^2(7A+5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{b\cos(dx+c)}} \\ & + \frac{2b(7A+5C)\sin(dx+c)\sqrt{b\cos(dx+c)}}{21d} \\ & + \frac{6bB\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{b\cos(dx+c)}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(7A+5C)b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(7A+5C)b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb\cos(dx+c)^3+Bb\cos(dx+c)^2+Ab\cos(dx+c)\right)\sqrt{b\cos(dx+c)},x\right)$$

53.79 Problem number 249

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx + c))^{3/2} \sin(dx + c)}{5d} \\ & + \frac{2b^2 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2bB \sin(dx + c) \sqrt{b \cos(dx + c)}}{3d} \\ & + \frac{2b(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B b^{3/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B b^{3/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c), x\right)$$

53.80 Problem number 250

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2bC \sin(dx + c) \sqrt{b \cos(dx + c)}}{3d} \\ & + \frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}(3A+C)b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+C)b^{\frac{3}{2}}\text{weierstrassPInverse}}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb*cos(dx+c)^3+Bb*cos(dx+c)^2+Ab*cos(dx+c))*sqrt(b*cos(dx+c))*sec(dx+c)^2,x)`

53.81 Problem number 251

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{2Ab^2 \sin(dx+c)}{d\sqrt{b \cos(dx+c)}} + \frac{2b^2 B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx+c)}}$$

$$- \frac{2b(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\cos(dx+c)}}$$

command

`integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)\text{weierstrassPInverse}}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb*cos(dx+c)^3+Bb*cos(dx+c)^2+Ab*cos(dx+c))*sqrt(b*cos(dx+c))*sec(dx+c)^3,x)`

53.82 Problem number 252

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab^3 \sin(dx + c)}{3d(b \cos(dx + c))^{3/2}} + \frac{2b^2B \sin(dx + c)}{d\sqrt{b \cos(dx + c)}} \\ & + \frac{2b^2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx + c)}} \\ & - \frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} (A + 3C) b^{3/2} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (A + 3C) b^{3/2} \cos(dx + c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^4, x\right)$$

53.83 Problem number 253

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab^4 \sin(dx + c)}{5d(b \cos(dx + c))^{5/2}} + \frac{2b^3B \sin(dx + c)}{3d(b \cos(dx + c))^{3/2}} + \frac{2b^2(3A + 5C) \sin(dx + c)}{5d\sqrt{b \cos(dx + c)}} \\ & + \frac{2b^2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{b \cos(dx + c)}} \\ & - \frac{2b(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} B b^{\frac{3}{2}} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B b^{\frac{3}{2}} \cos(dx + c)^3 \text{weier}$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb cos(dx + c)^3 + Bb cos(dx + c)^2 + Ab cos(dx + c)) sqrt(b cos(dx + c)) sec(dx + c)^5, x)`

53.84 Problem number 254

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^6(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A b^5 \sin(dx + c)}{7d (b \cos(dx + c))^{\frac{7}{2}}} + \frac{2b^4 B \sin(dx + c)}{5d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{2b^3 (5A + 7C) \sin(dx + c)}{21d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{6b^2 B \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} \\ & + \frac{2b^2 (5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{6bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} (5A + 7C) b^{\frac{3}{2}} \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (5A + 7C) b^{\frac{3}{2}}$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb cos(dx + c)^3 + Bb cos(dx + c)^2 + Ab cos(dx + c)) sqrt(b cos(dx + c)) sec(dx + c)^6, x)`

53.85 Problem number 255

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(9A + 7C) (b \cos(dx + c))^{3/2} \sin(dx + c)}{45d} \\ & + \frac{2B(b \cos(dx + c))^{5/2} \sin(dx + c)}{7d} + \frac{2C(b \cos(dx + c))^{7/2} \sin(dx + c)}{9bd} \\ & + \frac{10b^3 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{10b^2 B \sin(dx + c) \sqrt{b \cos(dx + c)}}{21d} \\ & + \frac{2b^2(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-75i \sqrt{2} B b^{5/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 75i \sqrt{2} B b^{5/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}, x\right)$$

53.86 Problem number 256

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bB(b \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{5d} + \frac{2C(b \cos(dx+c))^{\frac{5}{2}} \sin(dx+c)}{7d} \\ & + \frac{2b^3(7A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}} \\ & + \frac{2b^2(7A+5C) \sin(dx+c) \sqrt{b \cos(dx+c)}}{21d} \\ & + \frac{6b^2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (7A+5C)b^{\frac{5}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} (7A+5C)b^{\frac{5}{2}} \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx+c)^4 + Bb^2 \cos(dx+c)^3 + Ab^2 \cos(dx+c)^2\right) \sqrt{b \cos(dx+c)} \sec(dx+c), x\right)$$

53.87 Problem number 257

$$\int (b \cos(c+dx))^{\frac{5}{2}} (A + B \cos(c+dx) + C \cos^2(c+dx)) \sec^2(c+dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bC(b \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{5d} \\ & + \frac{2b^3B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx+c)}} \\ & + \frac{2b^2B \sin(dx+c) \sqrt{b \cos(dx+c)}}{3d} \\ & + \frac{2b^2(5A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i\sqrt{2}Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4+Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^2,x\right)$$

53.88 Problem number 258

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^3(3A+C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{b\cos(dx+c)}} \\ & + \frac{2b^2C\sin(dx+c)\sqrt{b\cos(dx+c)}}{3d} \\ & + \frac{2b^2B\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{b\cos(dx+c)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}(3A+C)b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+C)b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4+Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^3,x\right)$$

53.89 Problem number 259

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{2A b^3 \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} + \frac{2b^3 B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} - \frac{2b^2(A - C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

`integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} B b^{\frac{5}{2}} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B b^{\frac{5}{2}} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^4, x\right)$$

53.90 Problem number 260

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\frac{2A b^4 \sin(dx + c)}{3d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{2b^3 B \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} + \frac{2b^3(A + 3C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} - \frac{2b^2 B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (A + 3C) b^{\frac{5}{2}} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (A + 3C) b^{\frac{5}{2}} \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^5, x\right)$$

53.91 Problem number 261

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^6(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A b^5 \sin(dx + c)}{5d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{2b^4 B \sin(dx + c)}{3d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{2b^3 (3A + 5C) \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} \\ & + \frac{2b^3 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{2b^2 (3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B b^{\frac{5}{2}} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B b^{\frac{5}{2}} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^6, x\right)$$

53.92 Problem number 262

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^7(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab^6 \sin(dx + c)}{7d(b \cos(dx + c))^{7/2}} + \frac{2b^5 B \sin(dx + c)}{5d(b \cos(dx + c))^{5/2}} + \frac{2b^4(5A + 7C) \sin(dx + c)}{21d(b \cos(dx + c))^{3/2}} + \frac{6b^3 B \sin(dx + c)}{5d\sqrt{b \cos(dx + c)}} \\ & + \frac{2b^3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{6b^2 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$-5i \sqrt{2} (5A + 7C)b^{5/2} \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (5A + 7C)b^{5/2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^7, x\right)$$

53.93 Problem number 263

$$\int \frac{\cos^3(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(9A + 7C) (b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45b^2d} \\
& + \frac{2B(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^3d} + \frac{2C(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^4d} \\
& + \frac{10B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\
& + \frac{10B \sin(dx + c) \sqrt{b \cos(dx + c)}}{21bd} \\
& + \frac{2(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

`integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fric`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-75i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 75i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^4 + B \cos(dx + c)^3 + A \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}}{b}, x\right)$$

53.94 Problem number 264

$$\int \frac{\cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2B(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5b^2d} + \frac{2C(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^3d} \\
& + \frac{2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\
& + \frac{2(7A + 5C) \sin(dx + c) \sqrt{b \cos(dx + c)}}{21bd} \\
& + \frac{6B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

`integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$5\sqrt{2}(7iA + 5iC)\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $5\sqrt{2}(-7iA - 5iC)\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)) \sqrt{b \cos(dx + c)}}{b}, x\right)$$

53.95 Problem number 265

$$\int \frac{\cos(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5b^2d} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sin(dx + c) \sqrt{b \cos(dx + c)}}{3bd} \\ & + \frac{2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i\sqrt{2}B\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $5i\sqrt{2}B\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b}, x\right)$$

53.96 Problem number 266

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & + \frac{2C \sin(dx + c) \sqrt{b \cos(dx + c)}}{3bd} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3iA - iC) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3iA + iC) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b \cos(dx + c)}, x\right)$$

53.97 Problem number 267

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \sqrt{b} \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \sqrt{b} \cos(dx + c) \text{weiers}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A \right) \sqrt{b \cos(dx + c)} \sec(dx + c)}{b \cos(dx + c)}, x \right)$$

53.98 Problem number 268

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2B \sin(dx + c)}{d \sqrt{b \cos(dx + c)}} \\ & + \frac{2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$\sqrt{2} (-i A - 3i C) \sqrt{b} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i A + 3i C) \sqrt{b}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A \right) \sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b \cos(dx + c)}, x \right)$$

53.99 Problem number 269

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A b^2 \sin(dx + c)}{5d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{2bB \sin(dx + c)}{3d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(3A + 5C) \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^3}{b \cos(dx + c)}, x\right)$$

53.100 Problem number 270

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A b^3 \sin(dx + c)}{7d (b \cos(dx + c))^{\frac{7}{2}}} + \frac{2b^2 B \sin(dx + c)}{5d (b \cos(dx + c))^{\frac{5}{2}}} + \frac{2b(5A + 7C) \sin(dx + c)}{21d (b \cos(dx + c))^{\frac{3}{2}}} + \frac{6B \sin(dx + c)}{5d \sqrt{b \cos(dx + c)}} \\ & + \frac{2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{b \cos(dx + c)}} \\ & - \frac{6B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(5iA + 7iC)\sqrt{b}\cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 5\sqrt{2}(-5iA - 7iC)\sqrt{b}\cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^4}{b\cos(dx+c)}, x\right)$$

53.101 Problem number 271

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9A+7C)(b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{45b^3d} \\ & + \frac{2B(b\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{7b^4d} + \frac{2C(b\cos(dx+c))^{\frac{7}{2}}\sin(dx+c)}{9b^5d} \\ & + \frac{10B\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{21\cos\left(\frac{dx}{2} + \frac{c}{2}\right)bd\sqrt{b\cos(dx+c)}} \\ & + \frac{10B\sin(dx+c)\sqrt{b\cos(dx+c)}}{21b^2d} \\ & + \frac{2(9A+7C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b\cos(dx+c)}}{15\cos\left(\frac{dx}{2} + \frac{c}{2}\right)b^2d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-75i\sqrt{2}B\sqrt{b}\operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 75i\sqrt{2}C\sqrt{b}\operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C\cos(dx+c)^4 + B\cos(dx+c)^3 + A\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}}{b^2}, x\right)$$

53.102 Problem number 272

$$\int \frac{\cos^3(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5b^3d} + \frac{2C(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^4d} \\ & + \frac{2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}} \\ & + \frac{2(7A + 5C) \sin(dx + c) \sqrt{b \cos(dx + c)}}{21b^2d} \\ & + \frac{6B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (7i A + 5i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-7i A - 5i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)\right) \sqrt{b \cos(dx + c)}}{b^2}, x\right)$$

53.103 Problem number 273

$$\int \frac{\cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2C(b \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{5b^3d} \\
& + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx+c)}} \\
& + \frac{2B \sin(dx+c) \sqrt{b \cos(dx+c)}}{3b^2d} \\
& + \frac{2(5A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{\cos(dx+c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fri
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A) \sqrt{b \cos(dx+c)}}{b^2}, x\right)$$

53.104 Problem number 274

$$\int \frac{\cos(c+dx) (A + B \cos(c+dx) + C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx+c)}} \\
& + \frac{2C \sin(dx+c) \sqrt{b \cos(dx+c)}}{3b^2d} \\
& + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{\cos(dx+c)}}
\end{aligned}$$

command

`integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$\sqrt{2}(-3iA - iC)\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $\sqrt{2}(3iA + iC)\sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)}, x\right)$$

53.105 Problem number 275

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{bd \sqrt{b \cos(dx + c)}} + \frac{2B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}} - \frac{2(A - C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \sqrt{b} \cos(dx + c)$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i \sqrt{2} B \sqrt{b} \cos(dx + c)$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)^2}, x\right)$$

53.106 Problem number 276

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2B \sin(dx + c)}{bd \sqrt{b \cos(dx + c)}} \\ & + \frac{2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}} \\ & - \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`
 Fracas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iA - 3iC)\sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(iA + 3iC)\sqrt{b}}{\dots}$$

Fracas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c)} \sec(dx + c)}{b^2 \cos(dx + c)^2}, x\right)$$

53.107 Problem number 277

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab \sin(dx + c)}{5d(b \cos(dx + c))^{\frac{5}{2}}} + \frac{2B \sin(dx + c)}{3d(b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(3A + 5C) \sin(dx + c)}{5bd \sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{b \cos(dx + c)}} \\ & - \frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2}{b^2\cos(dx+c)^2},x\right)$$

53.108 Problem number 278

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab^2\sin(dx+c)}{7d(b\cos(dx+c))^{\frac{7}{2}}} + \frac{2bB\sin(dx+c)}{5d(b\cos(dx+c))^{\frac{5}{2}}} + \frac{2(5A+7C)\sin(dx+c)}{21d(b\cos(dx+c))^{\frac{3}{2}}} + \frac{6B\sin(dx+c)}{5bd\sqrt{b\cos(dx+c)}} \\ & + \frac{2(5A+7C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)bd\sqrt{b\cos(dx+c)}} \\ & - \frac{6B\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{b\cos(dx+c)}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^2d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$5\sqrt{2}(5iA+7iC)\sqrt{b}\cos(dx+c)^4 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-5iA-7iC)\sqrt{b}\cos(dx+c)^4 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3}{b^2\cos(dx+c)^2},x\right)$$

53.109 Problem number 279

$$\int \frac{\cos^5(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9A + 7C) (b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45b^4d} \\ & + \frac{2B(b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7b^5d} + \frac{2C(b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9b^6d} \\ & + \frac{10B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{b \cos(dx + c)}} \\ & + \frac{10B \sin(dx + c) \sqrt{b \cos(dx + c)}}{21b^3d} \\ & + \frac{2(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fric`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-75i \sqrt{2} B \sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $75i \sqrt{2} B \sqrt{b}$ weierstrassPInverse(-4, 0,

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^4 + B \cos(dx + c)^3 + A \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}}{b^3}, x\right)$$

53.110 Problem number 280

$$\int \frac{\cos^4(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(b \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{5b^4d} + \frac{2C(b \cos(dx+c))^{\frac{5}{2}} \sin(dx+c)}{7b^5d} \\ & + \frac{2(7A+5C) \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{b \cos(dx+c)}} \\ & + \frac{2(7A+5C) \sin(dx+c) \sqrt{b \cos(dx+c)}}{21b^3d} \\ & + \frac{6B \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (7i A + 5i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5 \sqrt{2} (-7i A - 5i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^3 + B \cos(dx+c)^2 + A \cos(dx+c)\right) \sqrt{b \cos(dx+c)}}{b^3}, x\right)$$

53.111 Problem number 281

$$\int \frac{\cos^3(c+dx) (A + B \cos(c+dx) + C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{5b^4d} \\ & + \frac{2B \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{b \cos(dx+c)}} \\ & + \frac{2B \sin(dx+c) \sqrt{b \cos(dx+c)}}{3b^3d} \\ & + \frac{2(5A+3C) \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i\sqrt{2}B\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^3},x\right)$$

53.112 Problem number 282

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3A+C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^2d\sqrt{b\cos(dx+c)}} \\ & + \frac{2C\sin(dx+c)\sqrt{b\cos(dx+c)}}{3b^3d} \\ & + \frac{2B\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{b\cos(dx+c)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^3d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-3iA-iC)\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}(3iA+iC)\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)},x\right)$$

53.113 Problem number 283

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx+c)}{b^2 d \sqrt{b \cos(dx+c)}} + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx+c)}} - \frac{2(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx+c)}}$$

command

`integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \sqrt{b} \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} B \sqrt{b} \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A) \sqrt{b \cos(dx+c)}}{b^3 \cos(dx+c)^2}, x\right)$$

53.114 Problem number 284

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx+c)}{3bd(b\cos(dx+c))^{\frac{3}{2}}} + \frac{2B \sin(dx+c)}{b^2 d \sqrt{b \cos(dx+c)}} + \frac{2(A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{b \cos(dx+c)}} - \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx+c)}}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-iA - 3iC)\sqrt{b} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(iA + 3iC)\sqrt{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)^3}, x\right)$$

53.115 Problem number 285

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \sin(dx + c)}{5d(b \cos(dx + c))^{5/2}} + \frac{2B \sin(dx + c)}{3bd(b \cos(dx + c))^{3/2}} + \frac{2(3A + 5C) \sin(dx + c)}{5b^2d\sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d\sqrt{b \cos(dx + c)}} \\ & - \frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d\sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}B\sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i\sqrt{2}B\sqrt{b} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)}{b^3 \cos(dx + c)^3}, x\right)$$

53.116 Problem number 286

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab \sin(dx + c)}{7d(b \cos(dx + c))^{7/2}} + \frac{2B \sin(dx + c)}{5d(b \cos(dx + c))^{5/2}} + \frac{2(5A + 7C) \sin(dx + c)}{21bd(b \cos(dx + c))^{3/2}} + \frac{6B \sin(dx + c)}{5b^2d\sqrt{b \cos(dx + c)}} \\ & + \frac{2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d\sqrt{b \cos(dx + c)}} \\ & - \frac{6B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d\sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(5iA + 7iC)\sqrt{b}\cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-5iA - 7iC)\sqrt{b}\cos(dx + c)^4}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b^3 \cos(dx + c)^3}, x\right)$$

53.117 Problem number 287

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \sin(dx + c)}{5bd(b \cos(dx + c))^{5/2}} + \frac{2B \sin(dx + c)}{3b^2d(b \cos(dx + c))^{3/2}} + \frac{2(3A + 5C) \sin(dx + c)}{5b^3d\sqrt{b \cos(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d\sqrt{b \cos(dx + c)}} \\ & - \frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4d\sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^4 \cos(dx + c)^4}, x \right)$$

54 Test file number 94

Test folder name:

test_cases/4_Trig_functions/4.2_Cosine/94_4.2.4.2-a+b_cos^-m-c+d_cos^-n-A+B_cos+C_cos^2-

54.1 Problem number 126

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))(A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10a(11A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(9A + 7C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} + \frac{2a(11A + 9C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{77d} \\ & + \frac{2aC \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{9d} + \frac{2aC \left(\cos^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{11d} \\ & + \frac{10a(11A + 9C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-75i\sqrt{2}(11A+9C)\text{aweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}(11A+9C)\text{aweierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ca\cos(dx+c)^5+Ca\cos(dx+c)^4+Aa\cos(dx+c)^3+Aa\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

54.2 Problem number 127

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+C\cos^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(9A+7C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(7A+5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(9A+7C)\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{45d} + \frac{2aC\left(\cos^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{7d} \\ & + \frac{2aC\left(\cos^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{9d} + \frac{2a(7A+5C)\sin(dx+c)\left(\sqrt{\cos(dx+c)}\right)}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i\sqrt{2}(7A+5C)\text{aweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}(7A+5C)\text{aweierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ca\cos(dx+c)^4+Ca\cos(dx+c)^3+Aa\cos(dx+c)^2+Aa\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

54.3 Problem number 128

$$\int \sqrt{\cos(c+dx)} (a+a\cos(c+dx)) (A+C\cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aC \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2aC \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} \\ & + \frac{2a(7A+5C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (7A+5C) \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} (7A+5C) \operatorname{aweierstrassPInv}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca \cos(dx+c)^3 + Ca \cos(dx+c)^2 + Aa \cos(dx+c) + Aa\right) \sqrt{\cos(dx+c)}, x\right)$$

54.4 Problem number 129

$$\int \frac{(a+a\cos(c+dx)) (A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aC \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2aC \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i\sqrt{2}(3A+C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(3A+C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\cos(dx+c)^3+Ca\cos(dx+c)^2+Aa\cos(dx+c)+Aa}{\sqrt{\cos(dx+c)}},x\right)$$

54.5 Problem number 130

$$\int \frac{(a+a\cos(c+dx))(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(A-C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2a(3A+C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2aA\sin(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{2aC\sin(dx+c)(\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}(3A+C)a\cos(dx+c)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+C)a\cos(dx+c)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\cos(dx+c)^3+Ca\cos(dx+c)^2+Aa\cos(dx+c)+Aa}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

54.6 Problem number 131

$$\int \frac{(a + a \cos(c + dx)) (A + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2aA \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (A + 3C)a \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (A + 3C)a \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca \cos(dx + c)^3 + Ca \cos(dx + c)^2 + Aa \cos(dx + c) + Aa}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

54.7 Problem number 132

$$\int \frac{(a + a \cos(c + dx)) (A + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2aA \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a(3A + 5C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(A+3C)a\cos(dx+c)^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+3C)a\cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\cos(dx+c)^3+Ca\cos(dx+c)^2+Aa\cos(dx+c)+Aa}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

54.8 Problem number 133

$$\int \frac{(a+a\cos(c+dx))(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(3A+5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2a(5A+7C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d}+\frac{2aA\sin(dx+c)}{7d\cos(dx+c)^{\frac{7}{2}}} \\ & +\frac{2aA\sin(dx+c)}{5d\cos(dx+c)^{\frac{5}{2}}}+\frac{2a(5A+7C)\sin(dx+c)}{21d\cos(dx+c)^{\frac{3}{2}}}+\frac{2a(3A+5C)\sin(dx+c)}{5d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(5A+7C)a\cos(dx+c)^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(5A+7C)a\cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\cos(dx+c)^3+Ca\cos(dx+c)^2+Aa\cos(dx+c)+Aa}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

54.9 Problem number 134

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2 (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(33A + 25C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(9A + 7C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{2a^2(99A + 89C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{693d} \\ & + \frac{2C \left(\cos^{\frac{5}{2}}(dx + c)\right) (a + a \cos(dx + c))^2 \sin(dx + c)}{11d} \\ & + \frac{8C \left(\cos^{\frac{5}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{99d} \\ & + \frac{8a^2(33A + 25C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{231d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(30i \sqrt{2} (33A + 25C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 30i \sqrt{2} (33A + 25C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{231d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^5 + 2Ca^2 \cos(dx + c)^4 + (A + C)a^2 \cos(dx + c)^3 + 2Aa^2 \cos(dx + c)^2 + Aa^2 \cos(dx + c) + \frac{2}{3}a^2\right) dx\right)$$

54.10 Problem number 135

$$\int \sqrt{\cos(c+dx)} (a+a\cos(c+dx))^2 (A+C\cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16a^2(3A+2C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(7A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(21A+19C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{105d} \\ & + \frac{2C \left(\cos^{\frac{3}{2}}(dx+c)\right) (a+a\cos(dx+c))^2 \sin(dx+c)}{9d} \\ & + \frac{8C \left(\cos^{\frac{3}{2}}(dx+c)\right) (a^2+a^2\cos(dx+c)) \sin(dx+c)}{63d} \\ & + \frac{4a^2(7A+5C) \sin(dx+c) (\sqrt{\cos}(dx+c))}{21d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (7A+5C)a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 15i \sqrt{2} (7A+5C)a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)}{105d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx+c)^4 + 2Ca^2 \cos(dx+c)^3 + (A+C)a^2 \cos(dx+c)^2 + 2Aa^2 \cos(dx+c) + Aa^2\right) \sqrt{\cos(dx+c)} dx\right)$$

54.11 Problem number 136

$$\int \frac{(a + a \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(7A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(35A + 33C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\ & + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d} \\ & + \frac{8C(a^2 + a^2 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{35d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10i \sqrt{2} (7A + 3C)a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} (7A + 3C)a^2 \operatorname{weierstrassP}(\cos(dx + c) + i \sin(dx + c)) \right)}{\sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2}{\sqrt{\cos(dx + c)}}, x\right)$$

54.12 Problem number 137

$$\int \frac{(a + a \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16a^2C\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{4a^2(3A+C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2A(a+a\cos(dx+c))^2\sin(dx+c)}{d\sqrt{\cos(dx+c)}} - \frac{2a^2(15A-7C)\sin(dx+c)(\sqrt{\cos(dx+c)})}{15d} \\ & - \frac{2(5A-C)(a^2+a^2\cos(dx+c))\sin(dx+c)(\sqrt{\cos(dx+c)})}{5d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(5i\sqrt{2}(3A+C)a^2\cos(dx+c)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(3A+C)a^2\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2\cos(dx+c)^4+2Ca^2\cos(dx+c)^3+(A+C)a^2\cos(dx+c)^2+2Aa^2\cos(dx+c)+Aa^2}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

54.13 Problem number 138

$$\int \frac{(a+a\cos(c+dx))^2(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^2(A-C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{8a^2(A+C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2A(a+a\cos(dx+c))^2\sin(dx+c)}{3d\cos(dx+c)^{\frac{3}{2}}} + \frac{8A(a^2+a^2\cos(dx+c))\sin(dx+c)}{3d\sqrt{\cos(dx+c)}} \\ & - \frac{2a^2(5A-C)\sin(dx+c)(\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2i \sqrt{2} (A + C)a^2 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 2i \sqrt{2} (A + C)a^2 \cos \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2}{\cos(dx + c)^{\frac{5}{2}}}, x \right)$$

54.14 Problem number 139

$$\int \frac{(a + a \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{16a^2 A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^2 (A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{8A(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a^2(17A + 15C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A + 3C)a^2 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + 3C)a^2 \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2}{\cos(dx + c)^{\frac{7}{2}}}, x \right)$$

54.15 Problem number 140

$$\int \frac{(a + a \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(3A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(33A + 35C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{8A(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^2(3A + 5C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} (3A + 7C) a^2 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} (3A + 7C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2}{\cos(dx + c)^{\frac{9}{2}}}, x\right)$$

54.16 Problem number 141

$$\int \frac{(a + a \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{16a^2(2A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a^2(19A + 21C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^2(5A + 7C) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} \\
& + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\
& + \frac{8A(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{63d \cos(dx + c)^{\frac{7}{2}}} + \frac{16a^2(2A + 3C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (5A + 7C) a^2 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 7C) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2}{\cos(dx + c)^{\frac{11}{2}}}, x\right)$$

54.17 Problem number 142

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3 (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^3(221A + 175C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(121A + 95C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(221A + 175C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{585d} \\
& + \frac{40a^3(143A + 118C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{9009d} \\
& + \frac{2C \left(\cos^{\frac{5}{2}}(dx + c)\right) (a + a \cos(dx + c))^3 \sin(dx + c)}{13d} \\
& + \frac{12C \left(\cos^{\frac{5}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{143ad} \\
& + \frac{2(143A + 145C) \left(\cos^{\frac{5}{2}}(dx + c)\right) (a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{1287d} \\
& + \frac{4a^3(121A + 95C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(195i \sqrt{2} (121A + 95C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 195i \sqrt{2} (121A + 95C) a \right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^6 + 3Ca^3 \cos(dx + c)^5 + (A + 3C)a^3 \cos(dx + c)^4 + (3A + C)a^3 \cos(dx + c)^3 + 3Aa^3\right) dx\right)$$

54.18 Problem number 143

$$\int \sqrt{\cos(c+dx)} (a+a\cos(c+dx))^3 (A+C\cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(7A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(143A+105C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^3(44A+35C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{385d} \\ & + \frac{2C \left(\cos^{\frac{3}{2}}(dx+c)\right) (a+a\cos(dx+c))^3 \sin(dx+c)}{11d} \\ & + \frac{4C \left(\cos^{\frac{3}{2}}(dx+c)\right) (a^2+a^2\cos(dx+c))^2 \sin(dx+c)}{33ad} \\ & + \frac{2(33A+35C) \left(\cos^{\frac{3}{2}}(dx+c)\right) (a^3+a^3\cos(dx+c)) \sin(dx+c)}{231d} \\ & + \frac{4a^3(143A+105C) \sin(dx+c) (\sqrt{\cos}(dx+c))}{231d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (143 A + 105 C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} (143 A + 105 C) a^3 w \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx+c)^5 + 3Ca^3 \cos(dx+c)^4 + (A+3C)a^3 \cos(dx+c)^3 + (3A+C)a^3 \cos(dx+c)^2 + 3Aa^3 \cos(dx+c) + Aa^3\right) dx\right)$$

54.19 Problem number 144

$$\int \frac{(a + a \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(27A + 17C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(21A + 11C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^3(21A + 16C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\ & + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{9d} \\ & + \frac{4C(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{21ad} \\ & + \frac{2(63A + 73C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{315d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (21A + 11C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (21A + 11C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c) + 3Aa^3}{\sqrt{\cos(dx + c)}}\right)$$

54.20 Problem number 145

$$\int \frac{(a + a \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(35A + 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} - \frac{4a^3(35A - 41C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\ & - \frac{2(7A - C) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7ad} \\ & - \frac{2(35A - 11C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{35d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (35A + 13C) a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (35A + 13C) a^3 \cos(dx + c) \right)}{\cos^{\frac{3}{2}}(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c) + a^3}{\cos^{\frac{3}{2}}(dx + c)}\right)$$

54.21 Problem number 146

$$\int \frac{(a + a \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^3(5A - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{4A(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} \\
& - \frac{8a^3(10A - 3C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \\
& - \frac{2(35A - 3C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d}
\end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (5A + 3C) a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (5A + 3C) \right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c)}{\cos(dx + c)^{\frac{5}{2}}}\right)$$

54.22 Problem number 147

$$\int \frac{(a + a \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(9A - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{4A(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{5ad \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(11A + 5C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \\ & - \frac{4a^3(21A + 5C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (3A + 5C) a^3 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + 5C) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c)}{\cos(dx + c)^{\frac{7}{2}}}\right)$$

54.23 Problem number 148

$$\int \frac{(a + a \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(13A + 35C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{12A(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{35ad \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(7A + 5C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} + \frac{8a^3(53A + 70C) \sin(dx + c)}{105d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (13A + 35C)a^3 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (13A + 35C)a^3 \cos(dx + c)^3 \right)}{\cos(dx + c)^{\frac{9}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c) + Aa^3}{\cos(dx + c)^{\frac{9}{2}}} dx \right)$$

54.24 Problem number 149

$$\int \frac{(a + a \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^3(17A + 27C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^3(11A + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{8a^3(16A + 21C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\ & + \frac{4A(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{21ad \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{2(73A + 63C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{315d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^3(17A + 27C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (11A + 21C)a^3 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (11A + 21C)a^3 \cos(dx + c)^4 \right)}{\cos(dx + c)^{\frac{11}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c) + Aa^3}{\cos(dx + c)^{\frac{11}{2}}} dx \right)$$

54.25 Problem number 150

$$\int \frac{(a + a \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\cos^{\frac{13}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(105A + 143C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^3(35A + 44C) \sin(dx + c)}{385d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^3(105A + 143C) \sin(dx + c)}{231d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{11d \cos(dx + c)^{\frac{11}{2}}} + \frac{4A(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{33ad \cos(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(35A + 33C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{231d \cos(dx + c)^{\frac{7}{2}}} + \frac{4a^3(5A + 7C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (105A + 143C) a^3 \cos(dx + c)^6 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (105A + 143C) a^3 \cos(dx + c)^5 \right)}{\cos(dx + c)^{\frac{13}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c) + a^3}{\cos(dx + c)^{\frac{13}{2}}}\right)$$

54.26 Problem number 151

$$\int \frac{\cos^{\frac{5}{2}}(c + dx) (A + C \cos^2(c + dx))}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\
 & + \frac{5(7A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\
 & - \frac{(5A + 7C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5ad} + \frac{(7A + 9C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7ad} \\
 & - \frac{(A + C) \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{d(a + a \cos(dx + c))} + \frac{5(7A + 9C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21ad}
 \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(30 C \cos(dx + c)^3 - 12 C \cos(dx + c)^2 + 2(35 A + 39 C) \cos(dx + c) + 175 A + 225 C \right) \sqrt{\cos(dx + c)} \sin(dx + c)}{d(a + a \cos(dx + c))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^4 + A \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}}{a \cos(dx + c) + a}, x\right)$$

54.27 Problem number 152

$$\int \frac{\cos^{\frac{3}{2}}(c + dx) (A + C \cos^2(c + dx))}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\
 & - \frac{(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\
 & + \frac{(5A + 7C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5ad} - \frac{(A + C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{d(a + a \cos(dx + c))} \\
 & - \frac{(3A + 5C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3ad}
 \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6 C \cos(dx + c)^2 - 4 C \cos(dx + c) - 15 A - 25 C \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(\sqrt{2} (-3i A - 5i C) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^3 + A \cos(dx + c) \right) \sqrt{\cos(dx + c)}}{a \cos(dx + c) + a}, x \right)$$

54.28 Problem number 153

$$\int \frac{\sqrt{\cos(c + dx)} (A + C \cos^2(c + dx))}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{(A + C) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \cos(dx + c))} + \frac{(3A + 5C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3ad} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(2C \cos(dx + c) + 3A + 5C) \sqrt{\cos(dx + c)} \sin(dx + c) + \left(\sqrt{2} (-3i A - 5i C) \cos(dx + c) + \sqrt{2} (-3i A - 5i C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + A \right) \sqrt{\cos(dx + c)}}{a \cos(dx + c) + a}, x \right)$$

54.29 Problem number 154

$$\int \frac{A + C \cos^2(c + dx)}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A + C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{d(a + a \cos(dx + c))} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A + C) \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2}(-iA + iC) \cos(dx + c) + \sqrt{2}(-iA + iC)\right) \operatorname{weierstrassPInverse}(-)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^2 + a \cos(dx + c)}, x\right)$$

54.30 Problem number 155

$$\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(3A + C) \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} - \frac{(A + C) \sin(dx + c)}{d(a + a \cos(dx + c)) \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2((3A + C)\cos(dx + c) + 2A)\sqrt{\cos(dx + c)}\sin(dx + c) + \left(\sqrt{2}(iA - iC)\cos(dx + c)^2 + \sqrt{2}(iA - iC)\cos(dx + c)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C\cos(dx + c)^2 + A\right)\sqrt{\cos(dx + c)}}{a\cos(dx + c)^3 + a\cos(dx + c)^2}, x\right)$$

54.31 Problem number 156

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a\cos(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A + C)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad} \\ & + \frac{(5A + 3C)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad} \\ & + \frac{(5A + 3C)\sin(dx + c)}{3ad\cos(dx + c)^{\frac{3}{2}}} - \frac{(A + C)\sin(dx + c)}{d\cos(dx + c)^{\frac{3}{2}}(a + a\cos(dx + c))} - \frac{(3A + C)\sin(dx + c)}{ad\sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3(3A + C)\cos(dx + c)^2 + 4A\cos(dx + c) - 2A\right)\sqrt{\cos(dx + c)}\sin(dx + c) - \left(\sqrt{2}(-5iA - 3iC)\cos(dx + c)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C\cos(dx + c)^2 + A\right)\sqrt{\cos(dx + c)}}{a\cos(dx + c)^4 + a\cos(dx + c)^3}, x\right)$$

54.32 Problem number 157

$$\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + a \cos(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{(7A + 5C) \sin(dx + c)}{5ad \cos(dx + c)^{\frac{5}{2}}} \\ & - \frac{(5A + 3C) \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}}} - \frac{(A + C) \sin(dx + c)}{d \cos(dx + c)^{\frac{5}{2}} (a + a \cos(dx + c))} + \frac{3(7A + 5C) \sin(dx + c)}{5ad \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(9(7A + 5C) \cos(dx + c)^3 + 2(19A + 15C) \cos(dx + c)^2 - 4A \cos(dx + c) + 6A \right) \sqrt{\cos(dx + c)} \sin(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^5 + a \cos(dx + c)^4}, x\right)$$

54.33 Problem number 158

$$\int \frac{\cos^{\frac{5}{2}}(c + dx) (A + C \cos^2(c + dx))}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(5A + 14C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{4(5A + 14C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15a^2 d} - \frac{(A + 3C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{a^2 d (1 + \cos(dx + c))} \\ & - \frac{(A + C) \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{3d (a + a \cos(dx + c))^2} - \frac{5(A + 3C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3a^2 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 C \cos(dx + c)^3 - 8 C \cos(dx + c)^2 - 2(15 A + 47 C) \cos(dx + c) - 25 A - 75 C \right) \sqrt{\cos(dx + c)} \sin(dx + c) -}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^4 + A \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2}, x\right)$$

54.34 Problem number 159

$$\int \frac{\cos^{\frac{3}{2}}(c + dx) (A + C \cos^2(c + dx))}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2(A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{(A + 7C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3a^2 d (1 + \cos(dx + c))} - \frac{(A + C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{3d (a + a \cos(dx + c))^2} \\ & + \frac{2(A + 5C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3a^2 d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 C \cos(dx + c)^2 + (3 A + 13 C) \cos(dx + c) + 2 A + 10 C \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 2 \left(\sqrt{2} (i A + 5i C) \cos(dx + c) \right)}{a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^3 + A \cos(dx + c) \right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2}, x \right)$$

54.35 Problem number 160

$$\int \frac{\sqrt{\cos(c + dx)} (A + C \cos^2(c + dx))}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & + \frac{(A - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & - \frac{(A + C) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d (a + a \cos(dx + c))^2} + \frac{(A - 5C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3a^2 d (1 + \cos(dx + c))} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(6C \cos(dx + c) - A + 5C) \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2} (-i A + 5i C) \cos(dx + c)^2 - 2 \sqrt{2} (i A - 5i C) \right)}{a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + A \right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2}, x \right)$$

54.36 Problem number 161

$$\int \frac{A + C \cos^2(c + dx)}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2(A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{(A - C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{a^2 d (1 + \cos(dx + c))} - \frac{(A + C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d (a + a \cos(dx + c))^2} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3(A - C) \cos(dx + c) + 4A - 2C) \sqrt{\cos(dx + c)} \sin(dx + c) + 2 \left(\sqrt{2} (iA + iC) \cos(dx + c)^2 + 2 \sqrt{2} (iA -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^3 + 2a^2 \cos(dx + c)^2 + a^2 \cos(dx + c)}, x\right)$$

54.37 Problem number 162

$$\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx) (a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{(5A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} + \frac{4A \sin(dx + c)}{a^2 d \sqrt{\cos(dx + c)}} \\ & - \frac{(5A - C) \sin(dx + c)}{3a^2 d (1 + \cos(dx + c)) \sqrt{\cos(dx + c)}} - \frac{(A + C) \sin(dx + c)}{3d (a + a \cos(dx + c))^2 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 A \cos(dx + c)^2 + (19 A + C) \cos(dx + c) + 6 A \right) \sqrt{\cos(dx + c)} \sin(dx + c) + \left(\sqrt{2} (5i A - i C) \cos(dx + c) \right)^5}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + A \right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^4 + 2 a^2 \cos(dx + c)^3 + a^2 \cos(dx + c)^2}, x \right)$$

54.38 Problem number 163

$$\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(7A + C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & + \frac{2(5A + C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & + \frac{2(5A + C) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}}} - \frac{(7A + C) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}} (1 + \cos(dx + c))} \\ & - \frac{(A + C) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^2} - \frac{(7A + C) \sin(dx + c)}{a^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(7A + C) \cos(dx + c)^3 + 4(8A + C) \cos(dx + c)^2 + 8A \cos(dx + c) - 2A \right) \sqrt{\cos(dx + c)} \sin(dx + c) + 2}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + A \right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^5 + 2 a^2 \cos(dx + c)^4 + a^2 \cos(dx + c)^3}, x \right)$$

54.39 Problem number 164

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+C\cos^2(c+dx))}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(7A+33C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{10\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3d} \\ & - \frac{(13A+63C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3d} \\ & + \frac{7(7A+33C)\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{30a^3d} - \frac{(A+C)\left(\cos^{\frac{9}{2}}(dx+c)\right)\sin(dx+c)}{5d(a+a\cos(dx+c))^3} \\ & - \frac{2(A+6C)\left(\cos^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{15ad(a+a\cos(dx+c))^2} - \frac{(13A+63C)\left(\cos^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{10d(a^3+a^3\cos(dx+c))} \\ & - \frac{(13A+63C)\sin(dx+c)\sqrt{\cos(dx+c)}}{6a^3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12C\cos(dx+c)^4-24C\cos(dx+c)^3-3(29A+147C)\cos(dx+c)^2-2(73A+357C)\cos(dx+c)-65A\right)}{10d(a+a\cos(dx+c))^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C\cos(dx+c)^5+A\cos(dx+c)^3\right)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^3+3a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3},x\right)$$

54.40 Problem number 165

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+C\cos^2(c+dx))}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(9A+119C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A+11C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A+C) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{5d(a+a\cos(dx+c))^3} - \frac{2C \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{3ad(a+a\cos(dx+c))^2} \\ & - \frac{(9A+119C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{30d(a^3+a^3\cos(dx+c))} + \frac{(A+11C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{2a^3 d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(20 C \cos(dx+c)^3 + 3(9A+79C) \cos(dx+c)^2 + 4(9A+94C) \cos(dx+c) + 15A + 165C \right) \sqrt{\cos(dx+c)} \operatorname{sn}\left(\sqrt{\cos(dx+c)} x, \sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^4 + A \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x\right)$$

54.41 Problem number 166

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+C\cos^2(c+dx))}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - 49C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A - 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A + C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d(a + a \cos(dx + c))^3} + \frac{2(A - 4C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15ad(a + a \cos(dx + c))^2} \\ & + \frac{(A - 13C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{6d(a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(A - 29C) \cos(dx + c)^2 + 2(7A - 73C) \cos(dx + c) + 5A - 65C \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(\sqrt{2} (i A$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^3 + A \cos(dx + c)\right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}, x\right)$$

54.42 Problem number 167

$$\int \frac{\sqrt{\cos(c + dx)} (A + C \cos^2(c + dx))}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A + C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d(a + a \cos(dx + c))^3} + \frac{2(2A - 3C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15ad(a + a \cos(dx + c))^2} \\ & - \frac{(A - 9C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10d(a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(A - 9C) \cos(dx + c)^2 + 4(A - 9C) \cos(dx + c) - 5A - 15C \right) \sqrt{\cos(dx + c)} \sin(dx + c) + 5 \left(\sqrt{2} (iA + \dots) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + A \right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}, x \right)$$

54.43 Problem number 168

$$\int \frac{A + C \cos^2(c + dx)}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(9A - C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(3A + C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & - \frac{(A + C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d (a + a \cos(dx + c))^3} - \frac{2(3A - 2C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15ad (a + a \cos(dx + c))^2} \\ & - \frac{(9A - C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10d (a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(9A - C) \cos(dx + c)^2 + 2(33A - 7C) \cos(dx + c) + 45A - 5C \right) \sqrt{\cos(dx + c)} \sin(dx + c) + 5 \left(\sqrt{2} (3i \dots) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + A \right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^4 + 3a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + a^3 \cos(dx + c)}, x \right)$$

54.44 Problem number 169

$$\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(49A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(13A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(49A - C) \sin(dx + c)}{10a^3 d \sqrt{\cos(dx + c)}} - \frac{(A + C) \sin(dx + c)}{5d(a + a \cos(dx + c))^3 \sqrt{\cos(dx + c)}} \\ & - \frac{2(4A - C) \sin(dx + c)}{15ad(a + a \cos(dx + c))^2 \sqrt{\cos(dx + c)}} - \frac{(13A - C) \sin(dx + c)}{6d(a^3 + a^3 \cos(dx + c)) \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(49A - C) \cos(dx + c)^3 + 4(94A - C) \cos(dx + c)^2 + 5(59A + C) \cos(dx + c) + 60A \right) \sqrt{\cos(dx + c)} \sin(dx + c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^5 + 3a^3 \cos(dx + c)^4 + 3a^3 \cos(dx + c)^3 + a^3 \cos(dx + c)^2}, x\right)$$

54.45 Problem number 170

$$\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(119A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(11A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} + \frac{(11A + C) \sin(dx + c)}{2a^3 d \cos(dx + c)^{\frac{3}{2}}} \\ & - \frac{(A + C) \sin(dx + c)}{5d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^3} - \frac{2A \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^2} \\ & - \frac{(119A + 9C) \sin(dx + c)}{30d \cos(dx + c)^{\frac{3}{2}} (a^3 + a^3 \cos(dx + c))} - \frac{(119A + 9C) \sin(dx + c)}{10a^3 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 (119 A + 9 C) \cos(dx + c)^4 + 6 (151 A + 11 C) \cos(dx + c)^3 + 5 (139 A + 9 C) \cos(dx + c)^2 + 120 A \cos(dx + c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^6 + 3 a^3 \cos(dx + c)^5 + 3 a^3 \cos(dx + c)^4 + a^3 \cos(dx + c)^3}, x\right)$$

54.46 Problem number 283

$$\int \cos^{\frac{3}{2}}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2C \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{10C \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15 C \cos(dx + c)^2 + 21 B \cos(dx + c) + 25 C \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 25i \sqrt{2} C \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c), x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^3 + B \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}, x\right)$$

54.47 Problem number 284

$$\int \sqrt{\cos(c + dx)} (B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6C \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2B \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 (3 C \cos(dx + c) + 5 B) \sqrt{\cos(dx + c)} \sin(dx + c) - 5i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c), x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

54.48 Problem number 285

$$\int \frac{B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2C \sqrt{\cos(dx + c)} \sin(dx + c) - i \sqrt{2} C \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} C \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c) + B\right) \sqrt{\cos(dx + c)}, x\right)$$

54.49 Problem number 286

$$\int \frac{B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{3/2}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C \cos(dx + c) + B}{\sqrt{\cos(dx + c)}}, x\right)$$

54.50 Problem number 287

$$\int \frac{B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} C \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} C \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C \cos(dx + c) + B}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

54.51 Problem number 288

$$\int \frac{B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2C \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} B \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} B \cos(dx+c)^2 \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx+c) + B}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

54.52 Problem number 289

$$\int \frac{B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{6B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2C \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{6B \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} C \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} C \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C \cos(dx + c) + B}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

54.53 Problem number 425

$$\int \cos^{\frac{3}{2}}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2C \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2(7A + 5C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$2 \left(15 C \cos(dx + c)^2 + 21 B \cos(dx + c) + 35 A + 25 C\right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \sqrt{2} (7i A + 5i C) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

54.54 Problem number 426

$$\int \sqrt{\cos(c+dx)} (A + B \cos(c+dx) + C \cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5A + 3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2B \sin(dx+c) (\sqrt{\cos}(dx+c))}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3C \cos(dx+c) + 5B) \sqrt{\cos(dx+c)} \sin(dx+c) - 5i \sqrt{2} B \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx+c)^2 + B \cos(dx+c) + A\right) \sqrt{\cos(dx+c)}, x\right)$$

54.55 Problem number 427

$$\int \frac{A + B \cos(c+dx) + C \cos^2(c+dx)}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3A + C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \sin(dx+c) (\sqrt{\cos}(dx+c))}{3d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2C\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}(-3iA-iC)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{\sqrt{\cos(dx+c)}},x\right)$$

54.56 Problem number 428

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(A-C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2B\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} +\frac{2A\sin(dx+c)}{d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i\sqrt{2}B\cos(dx+c)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}B\cos(dx+c)\text{weierstrassPI}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

54.57 Problem number 429

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2B \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i A - 3i C) \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2} (i A + 3i C) \cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx+c)^2 + B \cos(dx+c) + A}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

54.58 Problem number 430

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(3A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2B \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2(3A+5C) \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}B\cos(dx+c)^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\cos(dx+c)^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

54.59 Problem number 431

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx)+C\cos^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(9A+7B+7C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{10a(11A+11B+9C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{231\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(9A+7B+7C)\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{45d} \\ & + \frac{2a(11A+11B+9C)\left(\cos^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{77d} \\ & + \frac{2a(B+C)\left(\cos^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{9d} + \frac{2aC\left(\cos^{\frac{9}{2}}(dx+c)\right)\sin(dx+c)}{11d} \\ & + \frac{10a(11A+11B+9C)\sin(dx+c)\left(\sqrt{\cos(dx+c)}\right)}{231d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-75i\sqrt{2}(11A+11B+9C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}(11A+11B+9C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ca\cos(dx+c)^5+(B+C)a\cos(dx+c)^4+(A+B)a\cos(dx+c)^3+Aa\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

54.60 Problem number 432

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))(A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(9A + 9B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(9A + 9B + 7C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{2a(B + C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{2aC \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{9d} \\ & + \frac{2a(7A + 5B + 5C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-15i \sqrt{2} (7A + 5B + 5C) a \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} (7A + 5B + 5C) a \sqrt{\cos(dx + c)}}{21d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca \cos(dx + c)^4 + (B + C)a \cos(dx + c)^3 + (A + B)a \cos(dx + c)^2 + Aa \cos(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

54.61 Problem number 433

$$\int \sqrt{\cos(c + dx)} (a + a \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5A + 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 7B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(B + C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2aC \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2a(7A + 7B + 5C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{21d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fri`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$-5i \sqrt{2} (7A + 7B + 5C) \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (7A + 7B + 5C) \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca \cos^3(dx + c) + (B + C)a \cos^2(dx + c) + (A + B)a \cos(dx + c) + Aa\right) \sqrt{\cos(dx + c)}, x\right)$$

54.62 Problem number 434

$$\int \frac{(a + a \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5A + 5B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3A + B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aC \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2a(B + C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{3d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$-5i\sqrt{2}(3A+B+C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(3A+B+C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\cos(dx+c)^3+(B+C)a\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa}{\sqrt{\cos(dx+c)}},x\right)$$

54.63 Problem number 435

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(A-B-C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(3A+3B+C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2aA\sin(dx+c)}{d\sqrt{\cos(dx+c)}} + \frac{2aC\sin(dx+c)(\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$-i\sqrt{2}(3A+3B+C)a\cos(dx+c)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+3B+C)a\cos(dx+c)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\cos(dx+c)^3+(B+C)a\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

54.64 Problem number 436

$$\int \frac{(a + a \cos(c + dx))(A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(A + B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a(A + B) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (A + 3B + 3C)a \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (A + 3B + 3C)a \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca \cos(dx + c)^3 + (B + C)a \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

54.65 Problem number 437

$$\int \frac{(a + a \cos(c + dx))(A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(3A + 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2a(A + B) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a(3A + 5B + 5C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (A + B + 3C)a \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (A + B + 3C)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca \cos(dx + c)^3 + (B + C)a \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

54.66 Problem number 438

$$\int \frac{(a + a \cos(c + dx))(A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(3A + 3B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A + 7B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2aA \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{2a(A + B) \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2a(5A + 7B + 7C) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a(3A + 3B + 5C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (5A + 7B + 7C)a \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (5A + 7B + 7C)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca \cos(dx + c)^3 + (B + C)a \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa}{\cos(dx + c)^{\frac{9}{2}}}, x\right)$$

54.67 Problem number 439

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(9A + 8B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(66A + 55B + 50C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(9A + 8B + 7C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{2a^2(99A + 121B + 89C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{693d} \\ & + \frac{2C \left(\cos^{\frac{5}{2}}(dx + c)\right) (a + a \cos(dx + c))^2 \sin(dx + c)}{11d} \\ & + \frac{2(11B + 4C) \left(\cos^{\frac{5}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{99d} \\ & + \frac{4a^2(66A + 55B + 50C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{231d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (66A + 55B + 50C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (66A + 55B + 50C) a^2 \right)}{231d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^5 + (B + 2C)a^2 \cos(dx + c)^4 + (A + 2B + C)a^2 \cos(dx + c)^3 + (2A + B)a^2 \cos(dx + c)^2 + (A + B)a^2 \cos(dx + c) + a^2\right) dx\right)$$

54.68 Problem number 440

$$\int \sqrt{\cos(c+dx)} (a+a\cos(c+dx))^2 (A+B\cos(c+dx)+C\cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(12A+9B+8C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(7A+6B+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(21A+27B+19C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{105d} \\ & + \frac{2C \left(\cos^{\frac{3}{2}}(dx+c)\right) (a+a\cos(dx+c))^2 \sin(dx+c)}{9d} \\ & + \frac{2(9B+4C) \left(\cos^{\frac{3}{2}}(dx+c)\right) (a^2+a^2\cos(dx+c)) \sin(dx+c)}{63d} \\ & + \frac{4a^2(7A+6B+5C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (7A+6B+5C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 15i \sqrt{2} (7A+6B+5C) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx+c)^4 + (B+2C)a^2 \cos(dx+c)^3 + (A+2B+C)a^2 \cos(dx+c)^2 + (2A+B)a^2 \cos(dx+c) + \dots\right)\right)$$

54.69 Problem number 441

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(5A + 4B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(14A + 7B + 6C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(35A + 49B + 33C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\ & + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d} \\ & + \frac{2(7B + 4C) (a^2 + a^2 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{35d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (14A + 7B + 6C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (14A + 7B + 6C) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c)}{\sqrt{\cos(dx + c)}}\right)$$

54.70 Problem number 442

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(5B + 4C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(3A + 2B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} - \frac{2a^2(15A - 5B - 7C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \\ & - \frac{2(5A - C) (a^2 + a^2 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (3A + 2B + C) a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c)}{\cos(dx + c)^{\frac{3}{2}}}\right)$$

54.71 Problem number 443

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(2A + 3B + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(4A + 3B) (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{3d \sqrt{\cos(dx + c)}} \\ & - \frac{2a^2(5A + 3B - C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} (2A + 3B + 2C)a^2 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} (2A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c)}{\cos(dx + c)^{\frac{5}{2}}} \right)$$

54.72 Problem number 444

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^2(4A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^2(A + 2B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2(4A + 5B)(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2a^2(17A + 25B + 15C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A + 2B + 3C)a^2 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c)}{\cos(dx + c)^{\frac{7}{2}}} \right)$$

54.73 Problem number 445

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(3A + 4B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(6A + 7B + 14C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(33A + 49B + 35C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{2(4A + 7B)(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^2(3A + 4B + 5C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (6A + 7B + 14C) a^2 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (6A + 7B + 14C) a^2 \cos(dx + c)^3 \right)}{\cos(dx + c)^{\frac{9}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c)}{\cos(dx + c)^{\frac{9}{2}}}\right)$$

54.74 Problem number 446

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^2(8A + 9B + 12C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^2(5A + 6B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a^2(19A + 27B + 21C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{5}{2}}} \\
& + \frac{4a^2(5A + 6B + 7C) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\
& + \frac{2(4A + 9B)(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{63d \cos(dx + c)^{\frac{7}{2}}} + \frac{4a^2(8A + 9B + 12C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$\frac{2 \left(15i \sqrt{2} (5A + 6B + 7C) a^2 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 6B + 7C) a^2 \cos(dx + c)^5 \right)}{105d \cos(dx + c)^{\frac{5}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c)}{\cos(dx + c)^{\frac{11}{2}}}\right) dx$$

54.75 Problem number 447

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^3(221A + 195B + 175C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(121A + 105B + 95C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(221A + 195B + 175C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{585d} \\
& + \frac{20a^3(286A + 273B + 236C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{9009d} \\
& + \frac{2C \left(\cos^{\frac{5}{2}}(dx+c)\right) (a + a \cos(dx+c))^3 \sin(dx+c)}{13d} \\
& + \frac{2(13B + 6C) \left(\cos^{\frac{5}{2}}(dx+c)\right) (a^2 + a^2 \cos(dx+c))^2 \sin(dx+c)}{143ad} \\
& + \frac{2(143A + 195B + 145C) \left(\cos^{\frac{5}{2}}(dx+c)\right) (a^3 + a^3 \cos(dx+c)) \sin(dx+c)}{1287d} \\
& + \frac{4a^3(121A + 105B + 95C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{231d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(195i \sqrt{2} (121 A + 105 B + 95 C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 195i \sqrt{2} (121 A + 105 B + 95 C) a^3 \right)}{231d}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((C*a^3*cos(dx+c)^6 + (B+3*C)*a^3*cos(dx+c)^5 + (A+3*B+3*C)*a^3*cos(dx+c)^4 + (3*A+3*B+C)*a^3*cos(dx+c)^3)
```

54.76 Problem number 448

$$\int \sqrt{\cos(c+dx)} (a+a\cos(c+dx))^3 (A+B\cos(c+dx)+C\cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(21A+17B+15C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(143A+121B+105C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(264A+253B+210C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{1155d} \\ & + \frac{2C \left(\cos^{\frac{3}{2}}(dx+c)\right) (a+a\cos(dx+c))^3 \sin(dx+c)}{11d} \\ & + \frac{2(11B+6C) \left(\cos^{\frac{3}{2}}(dx+c)\right) (a^2+a^2\cos(dx+c))^2 \sin(dx+c)}{99ad} \\ & + \frac{2(99A+143B+105C) \left(\cos^{\frac{3}{2}}(dx+c)\right) (a^3+a^3\cos(dx+c)) \sin(dx+c)}{693d} \\ & + \frac{4a^3(143A+121B+105C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{231d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (143 A + 121 B + 105 C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 15i \sqrt{2} (143 A + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx+c)^5 + (B+3C)a^3 \cos(dx+c)^4 + (A+3B+3C)a^3 \cos(dx+c)^3 + (3A+3B+C)a^3 \cos(dx+c)^2 + \dots\right)\right)$$

54.77 Problem number 449

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(27A + 21B + 17C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(21A + 13B + 11C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(42A + 41B + 32C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\ & + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{9d} \\ & + \frac{2(3B + 2C) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{21ad} \\ & + \frac{2(63A + 99B + 73C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{315d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (21A + 13B + 11C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (21A + 13B + 11C) a^3 \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2}{\sqrt{\cos(dx + c)}}\right)$$

54.78 Problem number 450

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(5A + 9B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(35A + 21B + 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\ & - \frac{4a^3(35A - 42B - 41C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\ & - \frac{2(7A - C) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7ad} \\ & - \frac{2(35A - 7B - 11C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{35d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (35A + 21B + 13C) a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2}{\cos(dx + c)^{\frac{3}{2}}}\right)$$

54.79 Problem number 451

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(5A - 5B - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(5A + 5B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(2A + B)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} \\ & - \frac{4a^3(20A + 5B - 6C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \\ & - \frac{2(35A + 15B - 3C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm='f`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (5A + 5B + 3C) a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (5A \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2}{\cos(dx + c)^{\frac{5}{2}}}\right)$$

54.80 Problem number 452

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{4a^3(9A + 5B - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(3A + 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2(6A + 5B)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{15ad \cos(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(33A + 35B + 15C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \\
& - \frac{4a^3(21A + 20B + 5C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d}
\end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="f`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (3A + 5B + 5C) a^3 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + 5B + 5C) a^3 \cos(dx + c)^3 \right)}{\cos(dx + c)^{\frac{7}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2}{\cos(dx + c)^{\frac{7}{2}}}\right)$$

54.81 Problem number 453

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{4a^3(7A + 9B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(13A + 21B + 35C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{2(6A + 7B)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{35ad \cos(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(7A + 9B + 5C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} \\
& + \frac{4a^3(106A + 147B + 140C) \sin(dx + c)}{105d \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (13A + 21B + 35C) a^3 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2}{\cos(dx + c)^{\frac{9}{2}}}\right)$$

54.82 Problem number 454

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{4a^3(17A + 21B + 27C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(11A + 13B + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(32A + 41B + 42C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\
& + \frac{2(2A + 3B)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{21ad \cos(dx + c)^{\frac{7}{2}}} \\
& + \frac{2(73A + 99B + 63C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{315d \cos(dx + c)^{\frac{5}{2}}} \\
& + \frac{4a^3(17A + 21B + 27C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (11A + 13B + 21C) a^3 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2}{\cos(dx + c)^{\frac{11}{2}}}\right) dx$$

54.83 Problem number 455

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{13}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{4a^3(15A + 17B + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(105A + 121B + 143C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(210A + 253B + 264C) \sin(dx + c)}{1155d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^3(105A + 121B + 143C) \sin(dx + c)}{231d \cos(dx + c)^{\frac{3}{2}}} \\
& + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c)}{11d \cos(dx + c)^{\frac{11}{2}}} + \frac{2(6A + 11B)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{99ad \cos(dx + c)^{\frac{9}{2}}} \\
& + \frac{2(105A + 143B + 99C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{693d \cos(dx + c)^{\frac{7}{2}}} \\
& + \frac{4a^3(15A + 17B + 21C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algorithm="
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (105A + 121B + 143C) a^3 \cos(dx + c)^6 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15 \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2}{\cos(dx + c)^{\frac{13}{2}}}\right)$$

54.84 Problem number 456

$$\int \frac{\cos^{\frac{5}{2}}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{3(5A - 7B + 7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\
& + \frac{5(7A - 7B + 9C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\
& - \frac{(5A - 7B + 7C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5ad} \\
& + \frac{(7A - 7B + 9C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7ad} \\
& - \frac{(A - B + C) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{d(a + a \cos(dx+c))} \\
& + \frac{5(7A - 7B + 9C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21ad}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="fri
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(30 C \cos(dx+c)^3 + 6(7B - 2C) \cos(dx+c)^2 + 2(35A - 14B + 39C) \cos(dx+c) + 175A - 175B + 225C \right)}{a^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^4 + B \cos(dx+c)^3 + A \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}}{a \cos(dx+c) + a}, x\right)$$

54.85 Problem number 457

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A + B \cos(c+dx) + C \cos^2(c+dx))}{a + a \cos(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(5A - 5B + 7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(3A - 5B + 5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(5A - 5B + 7C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5ad} - \frac{(A - B + C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{d(a + a \cos(dx+c))} \\ & - \frac{(3A - 5B + 5C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3ad} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6C \cos(dx+c)^2 + 2(5B - 2C) \cos(dx+c) - 15A + 25B - 25C \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 5 \left(\sqrt{2} (-3i \dots) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^3 + B \cos(dx+c)^2 + A \cos(dx+c)\right) \sqrt{\cos(dx+c)}}{a \cos(dx+c) + a}, x\right)$$

54.86 Problem number 458

$$\int \frac{\sqrt{\cos(c+dx)} (A + B \cos(c+dx) + C \cos^2(c+dx))}{a + a \cos(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - 3B + 3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(3A - 3B + 5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A - B + C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{d(a + a \cos(dx+c))} + \frac{(3A - 3B + 5C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3ad} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fri
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(2C \cos(dx+c) + 3A - 3B + 5C) \sqrt{\cos(dx+c)} \sin(dx+c) + \left(\sqrt{2}(-3iA + 3iB - 5iC) \cos(dx+c) + \sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C \cos(dx+c)^2 + B \cos(dx+c) + A\right) \sqrt{\cos(dx+c)}}{a \cos(dx+c) + a}, x\right)$$

54.87 Problem number 459

$$\int \frac{A + B \cos(c+dx) + C \cos^2(c+dx)}{\sqrt{\cos(c+dx)} (a + a \cos(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B + 3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A + B - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A - B + C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{d(a + a \cos(dx+c))} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fri
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(A - B + C) \sqrt{\cos(dx+c)} \sin(dx+c) - \left(\sqrt{2}(-iA - iB + iC) \cos(dx+c) + \sqrt{2}(-iA - iB + iC)\right) \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C \cos(dx+c)^2 + B \cos(dx+c) + A\right) \sqrt{\cos(dx+c)}}{a \cos(dx+c)^2 + a \cos(dx+c)}, x\right)$$

54.88 Problem number 460

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A - B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A - B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(3A - B + C) \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} - \frac{(A - B + C) \sin(dx + c)}{d(a + a \cos(dx + c)) \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$2((3A - B + C) \cos(dx + c) + 2A) \sqrt{\cos(dx + c)} \sin(dx + c) + \left(\sqrt{2}(iA - iB - iC) \cos(dx + c)^2 + \sqrt{2}(iA -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^3 + a \cos(dx + c)^2}, x\right)$$

54.89 Problem number 461

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A - 3B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(5A - 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(5A - 3B + 3C) \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}}} - \frac{(A - B + C) \sin(dx + c)}{d \cos(dx + c)^{\frac{3}{2}}(a + a \cos(dx + c))} \\ & - \frac{(3A - 3B + C) \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(3A - 3B + C) \cos(dx + c)^2 + 2(2A - 3B) \cos(dx + c) - 2A \right) \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2} (-5i A \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A \right) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^4 + a \cos(dx + c)^3}, x \right)$$

54.90 Problem number 462

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + a \cos(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3(7A - 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{(5A - 5B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{(7A - 5B + 5C) \sin(dx + c)}{5ad \cos(dx + c)^{\frac{5}{2}}} - \frac{(5A - 5B + 3C) \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}}} \\ & - \frac{(A - B + C) \sin(dx + c)}{d \cos(dx + c)^{\frac{5}{2}} (a + a \cos(dx + c))} + \frac{3(7A - 5B + 5C) \sin(dx + c)}{5ad \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(9(7A - 5B + 5C) \cos(dx + c)^3 + 2(19A - 10B + 15C) \cos(dx + c)^2 - 2(2A - 5B) \cos(dx + c) + 6A \right) \sqrt{\cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A \right) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^5 + a \cos(dx + c)^4}, x \right)$$

54.91 Problem number 463

$$\int \frac{\cos^{\frac{5}{2}}(c+dx) (A+B\cos(c+dx)+C\cos^2(c+dx))}{(a+a\cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(20A-35B+56C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5(A-2B+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(20A-35B+56C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15a^2 d} \\ & - \frac{(A-2B+3C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{a^2 d (1+\cos(dx+c))} - \frac{(A-B+C) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{3d (a+a\cos(dx+c))^2} \\ & - \frac{5(A-2B+3C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a^2 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6C \cos(dx+c)^3 + 2(5B-4C) \cos(dx+c)^2 - (30A-65B+94C) \cos(dx+c) - 25A + 50B - 75C \right) \sqrt{\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^4 + B \cos(dx+c)^3 + A \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2}, x\right)$$

54.92 Problem number 464

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A+B\cos(c+dx)+C\cos^2(c+dx))}{(a+a\cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A-4B+7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(2A-5B+10C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{(A-4B+7C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3a^2 d (1+\cos(dx+c))} - \frac{(A-B+C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{3d (a+a\cos(dx+c))^2} \\ & + \frac{(2A-5B+10C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a^2 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2,x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2C \cos(dx+c)^2 + (3A-6B+13C) \cos(dx+c) + 2A-5B+10C \right) \sqrt{\cos(dx+c)} \sin(dx+c) + \left(\sqrt{2} (-2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^3 + B \cos(dx+c)^2 + A \cos(dx+c)\right) \sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2}, x\right)$$

54.93 Problem number 465

$$\int \frac{\sqrt{\cos(c+dx)} (A+B\cos(c+dx)+C\cos^2(c+dx))}{(a+a\cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(B - 4C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(A + 2B - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{(A - B + C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d(a + a \cos(dx + c))^2} + \frac{(A + 2B - 5C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{3a^2 d (1 + \cos(dx + c))} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3(B - 2C) \cos(dx + c) + A + 2B - 5C) \sqrt{\cos(dx + c)} \sin(dx + c) + \left(\sqrt{2}(-iA - 2iB + 5iC) \cos(dx + c)\right)^2}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x\right)$$

54.94 Problem number 466

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(2A + B + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{(A - C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{a^2 d (1 + \cos(dx + c))} - \frac{(A - B + C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{3d(a + a \cos(dx + c))^2} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="f`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3(A-C)\cos(dx+c)+4A-B-2C)\sqrt{\cos(dx+c)}\sin(dx+c)-\left(\sqrt{2}(-2iA-iB-2iC)\cos(dx+c)\right)^2}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^3+2a^2\cos(dx+c)^2+a^2\cos(dx+c)},x\right)$$

54.95 Problem number 467

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(4A-B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d} \\ & - \frac{(5A-2B-C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d} \\ & + \frac{(4A-B)\sin(dx+c)}{a^2d\sqrt{\cos(dx+c)}} - \frac{(5A-2B-C)\sin(dx+c)}{3a^2d(1+\cos(dx+c))\sqrt{\cos(dx+c)}} \\ & - \frac{(A-B+C)\sin(dx+c)}{3d(a+a\cos(dx+c))^2\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="f`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3(4A-B)\cos(dx+c)^2+(19A-4B+C)\cos(dx+c)+6A\right)\sqrt{\cos(dx+c)}\sin(dx+c)+\left(\sqrt{2}(5iA-2iB-2iC)\cos(dx+c)\right)^2}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^4+2a^2\cos(dx+c)^3+a^2\cos(dx+c)^2},x\right)$$

54.96 Problem number 468

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(7A - 4B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(10A - 5B + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(10A - 5B + 2C) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}}} - \frac{(7A - 4B + C) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}} (1 + \cos(dx + c))} \\ & - \frac{(A - B + C) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^2} - \frac{(7A - 4B + C) \sin(dx + c)}{a^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(7A - 4B + C) \cos(dx + c)^3 + (32A - 19B + 4C) \cos(dx + c)^2 + 2(4A - 3B) \cos(dx + c) - 2A \right) \sqrt{\cos(dx + c)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^5 + 2a^2 \cos(dx + c)^4 + a^2 \cos(dx + c)^3}, x\right)$$

54.97 Problem number 469

$$\int \frac{\cos^{\frac{7}{2}}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{7(7A - 17B + 33C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
& - \frac{(13A - 33B + 63C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
& + \frac{7(7A - 17B + 33C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{30a^3 d} \\
& - \frac{(A - B + C) \left(\cos^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{5d (a + a \cos(dx + c))^3} \\
& - \frac{(2A - 7B + 12C) \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{15ad (a + a \cos(dx + c))^2} \\
& - \frac{(13A - 33B + 63C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{10d (a^3 + a^3 \cos(dx + c))} \\
& - \frac{(13A - 33B + 63C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{6a^3 d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 C \cos(dx + c)^4 + 4 (5 B - 6 C) \cos(dx + c)^3 - 3 (29 A - 79 B + 147 C) \cos(dx + c)^2 - 2 (73 A - 188 B + 35 C) \cos(dx + c) + 2 A \right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3}, x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^5 + B \cos(dx + c)^4 + A \cos(dx + c)^3\right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3}, x\right)$$

54.98 Problem number 470

$$\int \frac{\cos^{\frac{5}{2}}(c+dx) (A+B\cos(c+dx)+C\cos^2(c+dx))}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(9A - 49B + 119C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A - 13B + 33C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - B + C) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{5d(a+a\cos(dx+c))^3} + \frac{(B - 2C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{3ad(a+a\cos(dx+c))^2} \\ & - \frac{(9A - 49B + 119C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{30d(a^3 + a^3 \cos(dx+c))} \\ & + \frac{(3A - 13B + 33C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{6a^3 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(20 C \cos(dx+c)^3 + 3(9A - 29B + 79C) \cos(dx+c)^2 + 2(18A - 73B + 188C) \cos(dx+c) + 15A - 65B + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^4 + B \cos(dx+c)^3 + A \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x\right)$$

54.99 Problem number 471

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A+B\cos(c+dx)+C\cos^2(c+dx))}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A+9B-49C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A+3B-13C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A-B+C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{5d(a+a\cos(dx+c))^3} + \frac{(2A+3B-8C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15ad(a+a\cos(dx+c))^2} \\ & + \frac{(A+3B-13C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{6d(a^3+a^3\cos(dx+c))} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(A+9B-29C) \cos(dx+c)^2 + 2(7A+18B-73C) \cos(dx+c) + 5A+15B-65C \right) \sqrt{\cos(dx+c)} \sin(dx+c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^3 + B \cos(dx+c)^2 + A \cos(dx+c)\right) \sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x\right)$$

54.100 Problem number 472

$$\int \frac{\sqrt{\cos(c+dx)} (A+B\cos(c+dx)+C\cos^2(c+dx))}{(a+a\cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - B + C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d(a + a \cos(dx + c))^3} + \frac{(4A + B - 6C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15ad(a + a \cos(dx + c))^2} \\ & - \frac{(A - B - 9C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10d(a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="f`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$2 \left(3(A - B - 9C) \cos(dx + c)^2 + 2(2A - 7B - 18C) \cos(dx + c) - 5A - 5B - 15C \right) \sqrt{\cos(dx + c)} \sin(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}, x\right)$$

54.101 Problem number 473

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(9A + B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A + B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - B + C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d(a + a \cos(dx + c))^3} - \frac{(6A - B - 4C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15ad(a + a \cos(dx + c))^2} \\ & - \frac{(9A + B - C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10d(a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(9A + B - C) \cos(dx + c)^2 + 2(33A + 2B - 7C) \cos(dx + c) + 45A - 5B - 5C \right) \sqrt{\cos(dx + c)} \sin(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A \right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^4 + 3a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + a^3 \cos(dx + c)}, x \right)$$

54.102 Problem number 474

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(49A - 9B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & - \frac{(13A - 3B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(49A - 9B - C) \sin(dx + c)}{10a^3 d \sqrt{\cos(dx + c)}} - \frac{(A - B + C) \sin(dx + c)}{5d(a + a \cos(dx + c))^3 \sqrt{\cos(dx + c)}} \\ & - \frac{(8A - 3B - 2C) \sin(dx + c)}{15ad(a + a \cos(dx + c))^2 \sqrt{\cos(dx + c)}} - \frac{(13A - 3B - C) \sin(dx + c)}{6d(a^3 + a^3 \cos(dx + c)) \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(49A - 9B - C) \cos(dx + c)^3 + 2(188A - 33B - 2C) \cos(dx + c)^2 + 5(59A - 9B + C) \cos(dx + c) + 60A \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A \right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^5 + 3a^3 \cos(dx + c)^4 + 3a^3 \cos(dx + c)^3 + a^3 \cos(dx + c)^2}, x \right)$$

54.103 Problem number 475

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(119A - 49B + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(33A - 13B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(33A - 13B + 3C) \sin(dx + c)}{6a^3 d \cos(dx + c)^{\frac{3}{2}}} - \frac{(A - B + C) \sin(dx + c)}{5d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^3} \\ & - \frac{(2A - B) \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^2} \\ & - \frac{(119A - 49B + 9C) \sin(dx + c)}{30d \cos(dx + c)^{\frac{3}{2}} (a^3 + a^3 \cos(dx + c))} - \frac{(119A - 49B + 9C) \sin(dx + c)}{10a^3 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(119A - 49B + 9C) \cos(dx + c)^4 + 2(453A - 188B + 33C) \cos(dx + c)^3 + 5(139A - 59B + 9C) \cos(dx + c)^2 \right)}{10a^3 d \sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^6 + 3a^3 \cos(dx + c)^5 + 3a^3 \cos(dx + c)^4 + a^3 \cos(dx + c)^3}, x\right)$$

54.104 Problem number 623

$$\int \cos^2(c + dx) \sqrt{a + b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(24a^2C + 7b^2(9A + 7C)) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{315b^3d} \\ & - \frac{4aC \cos(dx + c) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{21b^2d} \\ & + \frac{2C(\cos^2(dx + c)) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{9bd} \\ & - \frac{4a(21Ab^2 + 8a^2C + 18b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315b^3d} \\ & - \frac{2(16a^4C + 6a^2b^2(7A + 4C) - 21b^4(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{4a(a^2 - b^2) (21Ab^2 + 8a^2C + 18b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-32iCa^5 - 12i(7A + 3C)a^3b^2 - 3i(21A + 13C)ab^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b}{27b^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^4 + A \cos(dx + c)^2\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.105 Problem number 624

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{8aC(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{35b^2d} + \frac{2C \cos(dx + c) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{7bd} \\ & + \frac{2(8a^2C + 5b^2(7A + 5C)) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105b^2d} \\ & + \frac{2a(35Ab^2 + 8a^2C + 19b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2(a^2 - b^2) (35Ab^2 + 8a^2C + 25b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16i C a^4 + 2i (35 A + 16 C) a^2 b^2 - 15i (7 A + 5 C) b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx + c)}{27b^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^3 + A \cos(dx + c)\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.106 Problem number 625

$$\int \sqrt{a + b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2C(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5bd} - \frac{4aC \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15bd}$$

$$\frac{2(2a^2C - 3b^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{4a(a^2 - b^2) C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-4i C a^3 - 3i (5A + C) a b^2) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx + c) + 3i b \sin(dx + c) + 2a}{3b}\right) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^2 + A\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.107 Problem number 630

$$\int \cos^2(c + dx) (a + b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{4a(33Ab^2 + 8a^2C + 34b^2C)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{1155b^3d} \\
& + \frac{2(8a^2C + 3b^2(11A + 9C))(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{231b^3d} \\
& - \frac{4aC \cos(dx + c)(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{33b^2d} \\
& + \frac{2C(\cos^2(dx + c))(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{11bd} \\
& - \frac{2(16a^4C + 6a^2b^2(11A + 8C) - 25b^4(11A + 9C)) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{1155b^3d} \\
& - \frac{4a(8a^4C + 3a^2b^2(11A + 6C) - b^4(451A + 348C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{1155 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
& + \frac{2(a^2 - b^2)(16a^4C + 6a^2b^2(11A + 8C) - 25b^4(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{1155 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-32iCa^6 - 12i(11A + 5C)a^4b^2 + i(121A + 123C)a^2b^4 - 75i(11A + 9C)b^6)\sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - b^2)}{3}, x\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^5 + Ca \cos(dx + c)^4 + Ab \cos(dx + c)^3 + Aa \cos(dx + c)^2\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.108 Problem number 631

$$\int \cos(c + dx)(a + b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(8a^2C + 7b^2(9A + 7C)) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{315b^2d} \\
& - \frac{8aC(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{63b^2d} + \frac{2C \cos(dx + c) (a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{9bd} \\
& + \frac{2a(63Ab^2 + 8a^2C + 39b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315b^2d} \\
& + \frac{2(8a^4C + 21b^4(9A + 7C) + 3a^2b^2(21A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}} \\
& - \frac{2a(a^2 - b^2) (63Ab^2 + 8a^2C + 39b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2}(-8iCa^5 - 3i(21A + 10C)a^3b^2 + 3i(63A + 44C)ab^4)\sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^4 + Ca \cos(dx + c)^3 + Ab \cos(dx + c)^2 + Aa \cos(dx + c)\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.109 Problem number 632

$$\int (a + b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{-\frac{4aC(a+b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{35bd} + \frac{2C(a+b\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{7bd}}{2(6a^2C-5b^2(7A+5C))\sin(dx+c)\sqrt{a+b\cos(dx+c)}} + \frac{4a(70Ab^2-3a^2C+41b^2C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{105bd}}{105\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^2d\sqrt{\frac{a+b\cos(dx+c)}{a+b}}} + \frac{2(a^2-b^2)(35Ab^2-6a^2C+25b^2C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{105\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^2d\sqrt{a+b\cos(dx+c)}}$$

command

```
integrate((a+b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-12iCa^4 - i(35A - 11C)a^2b^2 - 15i(7A + 5C)b^4)\sqrt{b}\operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)}{a+b}\right)}{105\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^2d\sqrt{a+b\cos(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb\cos(dx+c)^3 + Ca\cos(dx+c)^2 + Ab\cos(dx+c) + Aa\right)\sqrt{b\cos(dx+c)+a}, x\right)$$

54.110 Problem number 638

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(240a^4C - 539b^4(13A + 11C) + 10a^2b^2(143A + 124C)) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45045b^3d} \\
& - \frac{4a(143Ab^2 + 24a^2C + 166b^2C) (a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{9009b^3d} \\
& + \frac{2(24a^2C + 11b^2(13A + 11C)) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{1287b^3d} \\
& - \frac{12aC \cos(dx + c) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{143b^2d} \\
& + \frac{2C(\cos^2(dx + c)) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{13bd} \\
& - \frac{4a(120a^4C + 5a^2b^2(143A + 94C) - 3b^4(2717A + 2174C)) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{45045b^3d} \\
& - \frac{2(240a^6C - 1617b^6(13A + 11C) + 10a^4b^2(143A + 76C) - 3a^2b^4(13299A + 10223C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{\frac{a + b \cos(dx + c)}{a + b}}\right)}{45045 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}} \\
& + \frac{4a(a^2 - b^2) (120a^4C + 5a^2b^2(143A + 94C) - 3b^4(2717A + 2174C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{\frac{a + b \cos(dx + c)}{a + b}}\right)}{45045 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-480i Ca^7 - 20i(143A + 67C)a^5b^2 + 3i(4433A + 3761C)a^3b^4 - 3i(23309A + 18973C)ab^6 \right) \sqrt{b} \operatorname{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^6 + 2Cab \cos(dx + c)^5 + 2Aab \cos(dx + c)^3 + Aa^2 \cos(dx + c)^2 + (Ca^2 + Ab^2) \cos(dx + c)\right), dx\right)$$

54.111 Problem number 639

$$\int \cos(c + dx)(a + b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(99Ab^2 + 8a^2C + 67b^2C)(a + b \cos(dx + c))^{3/2} \sin(dx + c)}{693b^2d} \\ & + \frac{2(8a^2C + 9b^2(11A + 9C))(a + b \cos(dx + c))^{5/2} \sin(dx + c)}{693b^2d} \\ & - \frac{8aC(a + b \cos(dx + c))^{7/2} \sin(dx + c)}{99b^2d} + \frac{2C \cos(dx + c)(a + b \cos(dx + c))^{7/2} \sin(dx + c)}{11bd} \\ & + \frac{2(8a^4C + 15b^4(11A + 9C) + 3a^2b^2(33A + 19C)) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{693b^2d} \\ & + \frac{2a(8a^4C + 3a^2b^2(33A + 17C) + 3b^4(319A + 247C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{693 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{2(a^2 - b^2)(8a^4C + 15b^4(11A + 9C) + 3a^2b^2(33A + 19C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{693 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16iCa^6 + 6i(33A + 16C)a^4b^2 - 3i(253A + 169C)a^2b^4 - 45i(11A + 9C)b^6) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^5 + 2Cab \cos(dx + c)^4 + 2Aab \cos(dx + c)^2 + Aa^2 \cos(dx + c) + (Ca^2 + Ab^2) \cos(dx + c)\right) dx\right)$$

54.112 Problem number 640

$$\int (a + b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(10a^2C - 7b^2(9A + 7C)) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{315bd} \\ & - \frac{4aC(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{63bd} + \frac{2C(a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9bd} \\ & + \frac{4a(84Ab^2 - 5a^2C + 57b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315bd} \\ & - \frac{2(10a^4C - 21b^4(9A + 7C) - 3a^2b^2(161A + 93C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a+b}}} \\ & - \frac{4a(a^2 - b^2) (84Ab^2 - 5a^2C + 57b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \left(-20i Ca^5 + 3i(7A + 31C)a^3b^2 - 3i(231A + 163C)ab^4\right) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + 2Cab \cos(dx + c)^3 + 2Aab \cos(dx + c) + Aa^2 + (Ca^2 + Ab^2) \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}\right)$$

54.113 Problem number 646

$$\int (a + b \cos(c + dx))^{3/2} (a^2 - b^2 \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{4ab(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{35d} - \frac{2b(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7d}$$

$$+ \frac{2b(41a^2 - 25b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105d}$$

$$+ \frac{4a(73a^2 - 41b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$- \frac{2(41a^4 - 66a^2b^2 + 25b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((a+b*cos(d*x+c))^(3/2)*(a^2-b^2*cos(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-23i a^4 - 116i a^2 b^2 + 75i b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx + c) + 3i b \sin(dx + c) + 2a}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(b^3 \cos(dx + c)^3 + ab^2 \cos(dx + c)^2 - a^2 b \cos(dx + c) - a^3\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.114 Problem number 647

$$\int \sqrt{a + b \cos(c + dx)} (a^2 - b^2 \cos^2(c + dx)) dx$$

Optimal antiderivative

$$- \frac{2b(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5d} + \frac{4ab \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15d}$$

$$+ \frac{2(17a^2 - 9b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$- \frac{4a(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((a^2-b^2*cos(d*x+c)^2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-11i a^3 + 3i ab^2) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3i b \sin(dx+c)+2a}{3b}\right) + \sqrt{2} (11i a^3 - 3i ab^2) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)-3i b \sin(dx+c)+2a}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(b^2 \cos(dx+c)^2 - a^2\right) \sqrt{b \cos(dx+c) + a}, x\right)$$

54.115 Problem number 648

$$\int \frac{\cos^3(c+dx) (A+C \cos^2(c+dx))}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(42Ab^2 + 32a^2C + 31b^2C) \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{315b^4d} \\ & + \frac{2(48a^2C + 7b^2(9A+7C)) \cos(dx+c) \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{315b^3d} \\ & - \frac{16aC(\cos^2(dx+c)) \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{63b^2d} \\ & + \frac{2C(\cos^3(dx+c)) \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{9bd} \\ & + \frac{2(128a^4C + 21b^4(9A+7C) + 12a^2b^2(14A+9C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^5 d \sqrt{\frac{a+b \cos(dx+c)}{a+b}}} \\ & - \frac{2a(128a^4C + 4a^2b^2(42A+19C) + 3b^4(49A+37C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^5 d \sqrt{a+b \cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\sqrt{2} (-128i Ca^5 - 12i(14A+5C)a^3b^2 - 3i(42A+31C)ab^4) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3i b \sin(dx+c)+2a}{3b}\right) + 2\sqrt{2} (128i Ca^5 + 12i(14A+5C)a^3b^2 + 3i(42A+31C)ab^4) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)-3i b \sin(dx+c)+2a}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C \cos(dx+c)^5 + A \cos(dx+c)^3}{\sqrt{b \cos(dx+c) + a}}, x\right)$$

54.116 Problem number 649

$$\int \frac{\cos^2(c+dx) (A+C \cos^2(c+dx))}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(24a^2C + 5b^2(7A + 5C)) \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{105b^3d} \\ & - \frac{12aC \cos(dx+c) \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{35b^2d} \\ & + \frac{2C(\cos^2(dx+c)) \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{7bd} \\ & - \frac{4a(35Ab^2 + 24a^2C + 22b^2C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \cos(dx+c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{\frac{a+b \cos(dx+c)}{a+b}}} \\ & + \frac{2(48a^4C + 5b^4(7A + 5C) + 2a^2b^2(35A + 16C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \cos(dx+c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a+b \cos(dx+c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-96iCa^4 - 4i(35A + 13C)a^2b^2 - 15i(7A + 5C)b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b^2c}{27b^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx+c)^4 + A \cos(dx+c)^2}{\sqrt{b \cos(dx+c) + a}}, x\right)$$

54.117 Problem number 650

$$\int \frac{\cos(c+dx)(A+C\cos^2(c+dx))}{\sqrt{a+b\cos(c+dx)}} dx$$

Optimal antiderivative

$$\frac{8aC \sin(dx+c) \sqrt{a+b\cos(dx+c)}}{15b^2d} + \frac{2C \cos(dx+c) \sin(dx+c) \sqrt{a+b\cos(dx+c)}}{5bd}$$

$$+ \frac{2(8a^2C + 3b^2(5A + 3C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b\cos(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a+b\cos(dx+c)}{a+b}}}$$

$$+ \frac{2a(15Ab^2 + 8a^2C + 7b^2C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a+b\cos(dx+c)}}$$

command

`integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\sqrt{2}(-8iCa^3 - 3i(5A+2C)ab^2)\sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2}{3b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C\cos(dx+c)^3 + A\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

54.118 Problem number 651

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2C \sin(dx+c) \sqrt{a+b\cos(dx+c)}}{3bd}$$

$$+ \frac{4aC \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b\cos(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a+b\cos(dx+c)}{a+b}}}$$

$$+ \frac{2(2a^2C + b^2(3A + C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a+b\cos(dx+c)}}$$

command

`integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-6i\sqrt{2}Cab^{\frac{3}{2}}\text{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)}{a+b\cos(dx+c)}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C\cos(dx+c)^2+A}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

54.119 Problem number 656

$$\int \frac{\cos^3(c+dx)(A+C\cos^2(c+dx))}{(a+b\cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(Ab^2+a^2C)(\cos^3(dx+c))\sin(dx+c)}{b(a^2-b^2)d\sqrt{a+b\cos(dx+c)}} \\ & +\frac{2(2a^2b^2(70A-31C)+192a^4C-5b^4(7A+5C))\sin(dx+c)\sqrt{a+b\cos(dx+c)}}{105b^4(a^2-b^2)d} \\ & -\frac{2a(35Ab^2+48a^2C-13b^2C)\cos(dx+c)\sin(dx+c)\sqrt{a+b\cos(dx+c)}}{35b^3(a^2-b^2)d} \\ & +\frac{2(7Ab^2+8a^2C-b^2C)(\cos^2(dx+c))\sin(dx+c)\sqrt{a+b\cos(dx+c)}}{7b^2(a^2-b^2)d} \\ & -\frac{2a(4a^2b^2(70A-43C)+384a^4C-b^4(175A+107C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)}{105\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^5(a^2-b^2)d\sqrt{\frac{a+b\cos(dx+c)}{a+b}}} \\ & +\frac{2(384a^4C+5b^4(7A+5C)+4a^2b^2(70A+29C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{105\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^5d\sqrt{a+b\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(192 C a^5 b^2 + 2 (70 A - 31 C) a^3 b^4 - 5 (7 A + 5 C) a b^6 + 15 (C a^2 b^5 - C b^7) \cos(dx + c)^3 - 24 (C a^3 b^4 - C a b^6) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \cos(dx + c)^5 + A \cos(dx + c)^3) \sqrt{b \cos(dx + c) + a}}{b^2 \cos(dx + c)^2 + 2 ab \cos(dx + c) + a^2}, x \right)$$

54.120 Problem number 657

$$\int \frac{\cos^2(c + dx) (A + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(A b^2 + a^2 C) (\cos^2(dx + c)) \sin(dx + c)}{b(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \\ & - \frac{2a(5A b^2 + 8a^2 C - 3b^2 C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{5b^3(a^2 - b^2) d} \\ & + \frac{2(5A b^2 + 6a^2 C - b^2 C) \cos(dx + c) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{5b^2(a^2 - b^2) d} \\ & + \frac{2(2a^2 b^2(5A - 4C) + 16a^4 C - b^4(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{a + b \cos(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{4a(5A b^2 + 2(4a^2 + b^2) C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(8 C a^4 b^2 + (5 A - 3 C) a^2 b^4 - (C a^2 b^4 - C b^6) \cos(dx + c)^2 + 2 (C a^3 b^3 - C a b^5) \cos(dx + c) \right) \sqrt{b \cos(dx + c) + a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \cos(dx + c)^4 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c) + a}}{b^2 \cos(dx + c)^2 + 2 ab \cos(dx + c) + a^2}, x \right)$$

54.121 Problem number 658

$$\int \frac{\cos(c+dx)(A+C\cos^2(c+dx))}{(a+b\cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2a(Ab^2+a^2C)\sin(dx+c)}{b^2(a^2-b^2)d\sqrt{a+b\cos(dx+c)}} + \frac{2C\sin(dx+c)\sqrt{a+b\cos(dx+c)}}{3b^2d}$$

$$- \frac{2a(3Ab^2+8a^2C-5b^2C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^3(a^2-b^2)d\sqrt{\frac{a+b\cos(dx+c)}{a+b}}}$$

$$+ \frac{2(3Ab^2+(8a^2+b^2)C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^3d\sqrt{a+b\cos(dx+c)}}$$

command

```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(4Ca^3b^2+(3A-C)ab^4+(Ca^2b^3-Cb^5)\cos(dx+c))\sqrt{b\cos(dx+c)+a}\sin(dx+c)-\left(\sqrt{2}(16iCa^4b+2i\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C\cos(dx+c)^3+A\cos(dx+c))\sqrt{b\cos(dx+c)+a}}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2},x\right)$$

54.122 Problem number 659

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Ab^2 + a^2C) \sin(dx + c)}{b(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \\
& + \frac{2(Ab^2 + 2a^2C - b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
& - \frac{4aC \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(Ca^2b^2 + Ab^4) \sqrt{b \cos(dx + c) + a} \sin(dx + c) + \left(\sqrt{2}(-4iCa^3b + i(A + 5C)ab^3) \cos(dx + c) + \sqrt{2}(-4iC\right)}{$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c) + a}}{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}, x\right)$$

54.123 Problem number 663

$$\int \frac{\cos^3(c + dx) (A + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(Ab^2 + a^2C) (\cos^3(dx + c)) \sin(dx + c)}{3b(a^2 - b^2) d (a + b \cos(dx + c))^{\frac{3}{2}}} \\
& + \frac{4(3Ab^4 - a^2b^2(A - 6C) - 4a^4C) (\cos^2(dx + c)) \sin(dx + c)}{3b^2(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\
& - \frac{4a(a^2b^2(10A - 49C) - b^4(20A - 7C) + 32a^4C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^4(a^2 - b^2)^2 d} \\
& + \frac{2(a^2b^2(15A - 71C) - b^4(35A - 3C) + 48a^4C) \cos(dx + c) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^3(a^2 - b^2)^2 d} \\
& + \frac{2(4a^4b^2(10A - 53C) - 5a^2b^4(15A - 11C) + 128a^6C + 3b^6(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^5 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
& - \frac{2a(4a^2b^2(10A - 29C) + 128a^4C - b^4(45A + 17C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^5 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^5 + A \cos(dx + c)^3) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

54.124 Problem number 664

$$\int \frac{\cos^2(c + dx) (A + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab^2 + a^2C) (\cos^2(dx + c)) \sin(dx + c)}{3b(a^2 - b^2)d(a + b \cos(dx + c))^{\frac{3}{2}}} - \frac{4a(2Ab^4 - 3a^4C + 5a^2b^2C) \sin(dx + c)}{3b^3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} + \frac{2(Ab^2 + 2a^2C - b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3b^3(a^2 - b^2)d}$$

$$\frac{4a(a^2b^2(A - 14C) - b^4(3A - 4C) + 8a^4C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$\frac{2(2a^2b^2(A - 8C) + 16a^4C - b^4(3A + C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(8Ca^6b^2 + (A - 13C)a^4b^4 - (5A - C)a^2b^6 + (Ca^4b^4 - 2Ca^2b^6 + Cb^8) \cos(dx + c)^2 + 2(5Ca^5b^3 + (A - 8C)a^4b^2 \cos(dx + c) - 2a^3b^4 \cos^2(dx + c) + Ab^5) \sqrt{a + b \cos(dx + c)} \right)}{3b^3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^4 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

54.125 Problem number 665

$$\int \frac{\cos(c + dx) (A + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a(Ab^2 + a^2C) \sin(dx + c)}{3b^2(a^2 - b^2)d(a + b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(3Ab^4 - 5a^4C + a^2b^2(A + 9C)) \sin(dx + c)}{3b^2(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

$$\frac{2(3b^4(A - C) - 8a^4C + a^2b^2(A + 15C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$\frac{2a(Ab^2 - 8a^2C + 9b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(4Ca^5b^2 - 2(A+4C)a^3b^4 - 2Aab^6 + (5Ca^4b^3 - (A+9C)a^2b^5 - 3Ab^7)\cos(dx+c))\sqrt{b\cos(dx+c)+a}\sin(dx+c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\cos(dx+c)^3 + A\cos(dx+c))\sqrt{b\cos(dx+c)+a}}{b^3\cos(dx+c)^3 + 3ab^2\cos(dx+c)^2 + 3a^2b\cos(dx+c) + a^3}, x\right)$$

54.126 Problem number 666

$$\int \frac{A + C \cos^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab^2 + a^2C)\sin(dx+c)}{3b(a^2 - b^2)d(a + b\cos(dx+c))^{3/2}} - \frac{4a(2Ab^2 - a^2C + 3b^2C)\sin(dx+c)}{3b(a^2 - b^2)^2 d\sqrt{a + b\cos(dx+c)}} + \frac{4a(2Ab^2 - (a^2 - 3b^2)C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a + b\cos(dx+c)}}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)b^2(a^2 - b^2)^2 d\sqrt{\frac{a + b\cos(dx+c)}{a+b}}} + \frac{2(Ab^2 - 2a^2C + 3b^2C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a + b\cos(dx+c)}{a+b}}}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)b^2(a^2 - b^2)d\sqrt{a + b\cos(dx+c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(Ca^4b^2 - 5(A+C)a^2b^4 + Ab^6 + 2(Ca^3b^3 - (2A+3C)ab^5)\cos(dx+c))\sqrt{b\cos(dx+c)+a}\sin(dx+c) + (\dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\cos(dx+c)^2 + A)\sqrt{b\cos(dx+c)+a}}{b^3\cos(dx+c)^3 + 3ab^2\cos(dx+c)^2 + 3a^2b\cos(dx+c) + a^3}, x\right)$$

54.127 Problem number 669

$$\int \frac{A + C \cos^2(c + dx)}{(a + b \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(Ab^2 + a^2C) \sin(dx + c)}{5b(a^2 - b^2)d(a + b \cos(dx + c))^{5/2}} - \frac{4a(4Ab^2 - a^2C + 5b^2C) \sin(dx + c)}{15b(a^2 - b^2)^2 d(a + b \cos(dx + c))^{3/2}} \\ & + \frac{2(2a^4C - 3b^4(3A + 5C) - a^2b^2(23A + 19C)) \sin(dx + c)}{15b(a^2 - b^2)^3 d \sqrt{a + b \cos(dx + c)}} \\ & - \frac{2(2a^4C - 3b^4(3A + 5C) - a^2b^2(23A + 19C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2)^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{4a(4Ab^2 - (a^2 - 5b^2)C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c) + a}}{b^4 \cos(dx + c)^4 + 4ab^3 \cos(dx + c)^3 + 6a^2b^2 \cos(dx + c)^2 + 4a^3b \cos(dx + c) + a^4}, x\right)$$

54.128 Problem number 670

$$\int \frac{a^2 - b^2 \cos^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2b \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{3d} + \frac{4a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \cos(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a+b \cos(dx+c)}{a+b}}} + \frac{2(a^2-b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a+b \cos(dx+c)}}$$

command

```
integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6i \sqrt{2} ab^{\frac{3}{2}} \operatorname{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}\right), \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\sqrt{b \cos(dx+c)+a} (b \cos(dx+c)-a), x\right)$$

54.129 Problem number 671

$$\int \frac{a^2 - b^2 \cos^2(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b \cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a+b \cos(dx+c)}{a+b}}} + \frac{4a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a+b \cos(dx+c)}}$$

command

```
integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} a \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3ib \sin(dx+c) + 2a}{3b} \right) + 5i \sqrt{2} a \sqrt{b} \text{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{b \cos(dx+c) - a}{\sqrt{b \cos(dx+c) + a}}, x \right)$$

54.130 Problem number 672

$$\int \frac{a^2 - b^2 \cos^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4ab \sin(dx+c)}{(a^2 - b^2) d \sqrt{a + b \cos(dx+c)}} \\ & + \frac{4a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a + b \cos(dx+c)}}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx+c)}{a+b}}} \\ & - \frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{\frac{a + b \cos(dx+c)}{a+b}}}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d \sqrt{a + b \cos(dx+c)}} \end{aligned}$$

command

`integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 \sqrt{b \cos(dx+c) + a} ab^2 \sin(dx+c) + \left(\sqrt{2} (-i a^2 b + 3i b^3) \cos(dx+c) + \sqrt{2} (-i a^3 + 3i ab^2) \right) \sqrt{b} \text{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{b \cos(dx+c) + a} (b \cos(dx+c) - a)}{b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2}, x \right)$$

54.131 Problem number 673

$$\int \frac{a^2 - b^2 \cos^2(c + dx)}{(a + b \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4ab \sin(dx + c)}{3(a^2 - b^2) d (a + b \cos(dx + c))^{\frac{3}{2}}} - \frac{2b(5a^2 + 3b^2) \sin(dx + c)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{2(5a^2 + 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{4a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

`integrate((a^2-b^2*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(7a^3b^2 + ab^4 + (5a^2b^3 + 3b^5) \cos(dx + c)) \sqrt{b \cos(dx + c) + a} \sin(dx + c) - \left(\sqrt{2} (i a^3 b^2 - 9i a b^4) \cos(dx + c) + \dots\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{b \cos(dx + c) + a} (b \cos(dx + c) - a)}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

54.132 Problem number 674

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx)) (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10b(11A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(9A + 7C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} + \frac{2b(11A + 9C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{77d} \\ & + \frac{2aC \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{9d} + \frac{2bC \left(\cos^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{11d} \\ & + \frac{10b(11A + 9C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-75i \sqrt{2} (11A + 9C) b \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 75i \sqrt{2} (11A + 9C) b \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^5 + Ca \cos(dx + c)^4 + Ab \cos(dx + c)^3 + Aa \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}, x\right)$$

54.133 Problem number 675

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))(A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(9A + 7C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} + \frac{2aC \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2bC \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{9d} + \frac{2a(7A + 5C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i\sqrt{2}(7A+5C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}(7A+5C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb\cos(dx+c)^4+Ca\cos(dx+c)^3+Ab\cos(dx+c)^2+Aa\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

54.134 Problem number 676

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+C\cos^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5A+3C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2b(7A+5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2aC\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{5d} + \frac{2bC\left(\cos^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{7d} \\ & + \frac{2b(7A+5C)\sin(dx+c)\left(\sqrt{\cos(dx+c)}\right)}{21d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(7A+5C)b\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(7A+5C)b\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb\cos(dx+c)^3+Ca\cos(dx+c)^2+Ab\cos(dx+c)+Aa\right)\sqrt{\cos(dx+c)},x\right)$$

54.135 Problem number 677

$$\int \frac{(a + b \cos(c + dx)) (A + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2bC \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2aC \sin(dx + c) (\sqrt{\cos}(dx + c))}{3d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} (3A + C) \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (3A + C) \operatorname{aweierstrassPInverse}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb \cos(dx + c)^3 + Ca \cos(dx + c)^2 + Ab \cos(dx + c) + Aa}{\sqrt{\cos(dx + c)}}, x\right)$$

54.136 Problem number 678

$$\int \frac{(a + b \cos(c + dx)) (A + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{d \sqrt{\cos(dx + c)}} + \frac{2bC \sin(dx + c) (\sqrt{\cos}(dx + c))}{3d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}(3A+C)b\cos(dx+c)$ weierstrassPInverse(-4, 0, $\cos(dx+c) + i\sin(dx+c)$) $+ i\sqrt{2}(3A+C)b\cos(dx+c)$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb\cos(dx+c)^3 + Ca\cos(dx+c)^2 + Ab\cos(dx+c) + Aa}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

54.137 Problem number 679

$$\int \frac{(a+b\cos(c+dx))(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(A-C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A+3C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA\sin(dx+c)}{3d\cos(dx+c)^{\frac{3}{2}}} + \frac{2Ab\sin(dx+c)}{d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}(A+3C)a\cos(dx+c)^2$ weierstrassPInverse(-4, 0, $\cos(dx+c) + i\sin(dx+c)$) $+ i\sqrt{2}(A+3C)a\cos(dx+c)$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb\cos(dx+c)^3 + Ca\cos(dx+c)^2 + Ab\cos(dx+c) + Aa}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

54.138 Problem number 680

$$\int \frac{(a + b \cos(c + dx)) (A + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2Ab \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a(3A + 5C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (A + 3C)b \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (A + 3C)b \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb \cos(dx + c)^3 + Ca \cos(dx + c)^2 + Ab \cos(dx + c) + Aa}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

54.139 Problem number 681

$$\int \frac{(a + b \cos(c + dx)) (A + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2aA \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{2Ab \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2a(5A + 7C) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} + \frac{2b(3A + 5C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(5A+7C)a\cos(dx+c)^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(5A+7C)a\cos(dx+c)^4$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb\cos(dx+c)^3+Ca\cos(dx+c)^2+Ab\cos(dx+c)+Aa}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

54.140 Problem number 682

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+C\cos^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4ab(9A+7C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(11a^2(7A+5C)+5b^2(11A+9C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{231\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{4ab(9A+7C)\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{45d} \\ & + \frac{2(4a^2C+b^2(11A+9C))\left(\cos^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{77d} \\ & + \frac{8abC\left(\cos^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{99d} + \frac{2C\left(\cos^{\frac{5}{2}}(dx+c)\right)(a+b\cos(dx+c))^2\sin(dx+c)}{11d} \\ & + \frac{2(11a^2(7A+5C)+5b^2(11A+9C))\sin(dx+c)\left(\sqrt{\cos(dx+c)}\right)}{231d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$462i\sqrt{2}(9A+7C)ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-462i\sqrt{2}(9A+7C)ab\cos(dx+c)^4$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^5+2Cab\cos(dx+c)^4+2Aab\cos(dx+c)^2+Aa^2\cos(dx+c)+(Ca^2+Ab^2)\cos(dx+c)\right),x\right)$$

54.141 Problem number 683

$$\int \sqrt{\cos(c+dx)} (a+b\cos(c+dx))^2 (A+C\cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3a^2(5A+3C)+b^2(9A+7C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab(7A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(4a^2C+b^2(9A+7C)) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} \\ & + \frac{8abC \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{63d} + \frac{2C \left(\cos^{\frac{3}{2}}(dx+c)\right) (a+b\cos(dx+c))^2 \sin(dx+c)}{9d} \\ & + \frac{4ab(7A+5C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-30i\sqrt{2}(7A+5C)ab\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+30i\sqrt{2}(7A+5C)ab\operatorname{weierstrassP}(\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2\cos(dx+c)^4+2Cab\cos(dx+c)^3+2Aab\cos(dx+c)+Aa^2+(Ca^2+Ab^2)\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}\right)$$

54.142 Problem number 684

$$\int \frac{(a+b\cos(c+dx))^2 (A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4ab(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7a^2(3A + C) + b^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8abC \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2(4a^2C + b^2(7A + 5C)) \sin(dx + c) (\sqrt{\cos}(dx + c))}{21d} \\ & + \frac{2C(a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos}(dx + c))}{7d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$42i \sqrt{2} (5A + 3C) ab \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 42i \sqrt{2} (5A + 3C) ab \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2 \cos(dx + c)^4 + 2Cab \cos(dx + c)^3 + 2Aab \cos(dx + c) + Aa^2 + (Ca^2 + Ab^2) \cos(dx + c)^2}{\sqrt{\cos(dx + c)}}, x\right)$$

54.143 Problem number 685

$$\int \frac{(a + b \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5a^2(A - C) - b^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2b^2(5A - C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2A(a + b \cos(dx + c))^2 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\ & - \frac{4ab(3A - C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{3d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-10i\sqrt{2}(3A+C)ab\cos(dx+c)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}(3A+C)ab\cos(dx+c)}{\cos(dx+c)^{3/2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2\cos(dx+c)^4+2Cab\cos(dx+c)^3+2Aab\cos(dx+c)+Aa^2+(Ca^2+Ab^2)\cos(dx+c)^2}{\cos(dx+c)^{3/2}},x\right)$$

54.144 Problem number 686

$$\int \frac{(a+b\cos(c+dx))^2(A+C\cos^2(c+dx))}{\cos^{5/2}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4ab(A-C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2(b^2(3A+C)+a^2(A+3C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2A(a+b\cos(dx+c))^2\sin(dx+c)}{3d\cos(dx+c)^{3/2}}+\frac{8aAb\sin(dx+c)}{3d\sqrt{\cos(dx+c)}} \\ & -\frac{2b^2(A-C)\sin(dx+c)(\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-6i\sqrt{2}(A-C)ab\cos(dx+c)^2\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+6i\sqrt{2}(A-C)ab\cos(dx+c)^2\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))}{\cos(dx+c)^{5/2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2\cos(dx+c)^4+2Cab\cos(dx+c)^3+2Aab\cos(dx+c)+Aa^2+(Ca^2+Ab^2)\cos(dx+c)^2}{\cos(dx+c)^{5/2}},x\right)$$

54.145 Problem number 687

$$\int \frac{(a + b \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(5b^2(A - C) + a^2(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{8aAb \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2A(a + b \cos(dx + c))^2 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2(4Ab^2 + a^2(3A + 5C)) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-10i \sqrt{2} (A + 3C) ab \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} (A + 3C) ab$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2 \cos(dx + c)^4 + 2Cab \cos(dx + c)^3 + 2Aab \cos(dx + c) + Aa^2 + (Ca^2 + Ab^2) \cos(dx + c)^2}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

54.146 Problem number 688

$$\int \frac{(a + b \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4ab(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7b^2(A + 3C) + a^2(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8aAb \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{2(4Ab^2 + a^2(5A + 7C)) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2A(a + b \cos(dx + c))^2 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{4ab(3A + 5C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-42i \sqrt{2} (3A + 5C)ab \cos(dx + c)^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Cb^2 \cos(dx + c)^4 + 2Cab \cos(dx + c)^3 + 2Aab \cos(dx + c) + Aa^2 + (Ca^2 + Ab^2) \cos(dx + c)^2}{\cos(dx + c)^{\frac{9}{2}}}, x \right)$$

54.147 Problem number 689

$$\int \sqrt{\cos(c + dx)} (a + b \cos(c + dx))^3 (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(a^2(5A + 3C) + b^2(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2b(33a^2(7A + 5C) + 5b^2(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{231 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(99Ab^2 + 8a^2C + 77b^2C) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{165d} \\ & + \frac{2b(8a^2C + 3b^2(11A + 9C)) \left(\cos^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{231d} \\ & + \frac{4aC \left(\cos^{\frac{3}{2}}(dx + c) \right) (a + b \cos(dx + c))^2 \sin(dx + c)}{33d} \\ & + \frac{2C \left(\cos^{\frac{3}{2}}(dx + c) \right) (a + b \cos(dx + c))^3 \sin(dx + c)}{11d} \\ & + \frac{2b(33a^2(7A + 5C) + 5b^2(11A + 9C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(105 C b^3 \cos(dx + c)^4 + 385 C a b^2 \cos(dx + c)^3 + 165 (7 A + 5 C) a^2 b + 25 (11 A + 9 C) b^3 + 15 (33 C a^2 b + (11 A + 9 C) a^3) \cos(dx + c)^3 + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C b^3 \cos(dx + c)^5 + 3 C a b^2 \cos(dx + c)^4 + 3 A a^2 b \cos(dx + c) + A a^3 + (3 C a^2 b + A b^3) \cos(dx + c)^3 + \dots}{\sqrt{\cos(dx + c)}} \right)$$

54.148 Problem number 690

$$\int \frac{(a + b \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(9a^2(5A + 3C) + b^2(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(7a^2(3A + C) + 3b^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2b(24a^2C + 7b^2(9A + 7C)) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{315d} \\ & + \frac{2a(63A b^2 + 8a^2C + 45b^2C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{63d} \\ & + \frac{4aC(a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \\ & + \frac{2C(a + b \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{9d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(35 C b^3 \cos(dx + c)^3 + 135 C a b^2 \cos(dx + c)^2 + 105 C a^3 + 45 (7 A + 5 C) a b^2 + 7 (27 C a^2 b + (9 A + 7 C) b^3) \cos(dx + c)^3 + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C b^3 \cos(dx + c)^5 + 3 C a b^2 \cos(dx + c)^4 + 3 A a^2 b \cos(dx + c) + A a^3 + (3 C a^2 b + A b^3) \cos(dx + c)^3 + \dots}{\sqrt{\cos(dx + c)}} \right)$$

54.149 Problem number 691

$$\int \frac{(a + b \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(5a^2(A - C) - 3b^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(21a^2(3A + C) + b^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2ab^2(35A - 11C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{35d} + \frac{2A(a + b \cos(dx + c))^3 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\ & - \frac{2b(6a^2(7A - 3C) - b^2(7A + 5C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \\ & - \frac{2b(7A - C) (a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$- \frac{5 \sqrt{2} (21i(3A + C)a^2b + i(7A + 5C)b^3) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx + c)^5 + 3Cab^2 \cos(dx + c)^4 + 3Aa^2b \cos(dx + c) + Aa^3 + (3Ca^2b + Ab^3) \cos(dx + c)^3 + C}{\cos(dx + c)^{\frac{3}{2}}}\right)$$

54.150 Problem number 692

$$\int \frac{(a + b \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b(15a^2(A-C) - b^2(5A+3C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3b^2(3A+C) + a^2(A+3C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2b^3(35A-3C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15d} + \frac{2A(a+b\cos(dx+c))^3 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} \\ & + \frac{4Ab(a+b\cos(dx+c))^2 \sin(dx+c)}{d \sqrt{\cos(dx+c)}} - \frac{2ab^2(5A-C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2} \left(i(A+3C)a^3 + 3i(3A+C)ab^2 \right) \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx+c)^5 + 3Cab^2 \cos(dx+c)^4 + 3Aa^2b \cos(dx+c) + Aa^3 + (3Ca^2b + Ab^3) \cos(dx+c)^3 + (C)}{\cos(dx+c)^{\frac{5}{2}}}\right)$$

54.151 Problem number 693

$$\int \frac{(a+b\cos(c+dx))^3 (A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(15b^2(A-C) + a^2(3A+5C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(b^2(3A+C) + 3a^2(A+3C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4Ab(a+b\cos(dx+c))^2 \sin(dx+c)}{5d \cos(dx+c)^{\frac{3}{2}}} + \frac{2A(a+b\cos(dx+c))^3 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} \\ & + \frac{2a(8Ab^2 + a^2(3A+5C)) \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} - \frac{2b^3(9A-5C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{15d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (3i(A+3C)a^2b + i(3A+C)b^3) \cos(dx+c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Cb^3 \cos(dx+c)^5 + 3Cab^2 \cos(dx+c)^4 + 3Aa^2b \cos(dx+c) + Aa^3 + (3Ca^2b + Ab^3) \cos(dx+c)^3 + (C}{\cos(dx+c)^{\frac{7}{2}}} \right)$$

54.152 Problem number 694

$$\int \frac{(a+b \cos(c+dx))^3 (A+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(5b^2(A-C) + 3a^2(3A+5C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(21b^2(A+3C) + a^2(5A+7C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(24Ab^2 + 5a^2(5A+7C)) \sin(dx+c)}{105d \cos(dx+c)^{\frac{3}{2}}} + \frac{12Ab(a+b \cos(dx+c))^2 \sin(dx+c)}{35d \cos(dx+c)^{\frac{5}{2}}} \\ & + \frac{2A(a+b \cos(dx+c))^3 \sin(dx+c)}{7d \cos(dx+c)^{\frac{7}{2}}} + \frac{6b(8Ab^2 + 7a^2(3A+5C)) \sin(dx+c)}{35d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (i(5A+7C)a^3 + 21i(A+3C)ab^2) \cos(dx+c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Cb^3 \cos(dx+c)^5 + 3Cab^2 \cos(dx+c)^4 + 3Aa^2b \cos(dx+c) + Aa^3 + (3Ca^2b + Ab^3) \cos(dx+c)^3 + (C}{\cos(dx+c)^{\frac{9}{2}}} \right)$$

54.153 Problem number 695

$$\int \frac{(a + b \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(9b^2(3A + 5C) + a^2(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(7b^2(A + 3C) + 3a^2(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(24Ab^2 + 7a^2(7A + 9C)) \sin(dx + c)}{315d \cos(dx + c)^{\frac{5}{2}}} + \frac{2b(8Ab^2 + 9a^2(5A + 7C)) \sin(dx + c)}{63d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{4Ab(a + b \cos(dx + c))^2 \sin(dx + c)}{21d \cos(dx + c)^{\frac{7}{2}}} + \frac{2A(a + b \cos(dx + c))^3 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\ & + \frac{2a(9b^2(3A + 5C) + a^2(7A + 9C)) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (3i(5A + 7C)a^2b + 7i(A + 3C)b^3) \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx + c)^5 + 3Cab^2 \cos(dx + c)^4 + 3Aa^2b \cos(dx + c) + Aa^3 + (3Ca^2b + Ab^3) \cos(dx + c)^3 + (C)}{\cos(dx + c)^{\frac{11}{2}}}\right)$$

54.154 Problem number 696

$$\int \sqrt{\cos(c + dx)} (a + b \cos(c + dx))^4 (A + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(39a^4(5A + 3C) + 78a^2b^2(9A + 7C) + 7b^4(13A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{8ab(11a^2(7A + 5C) + 5b^2(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(192a^4C + 77b^4(13A + 11C) + 11a^2b^2(637A + 491C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{6435d} \\
& + \frac{4ab(1573Ab^2 + 96a^2C + 1259b^2C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{9009d} \\
& + \frac{2(48a^2C + 11b^2(13A + 11C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + b \cos(dx + c))^2 \sin(dx + c)}{1287d} \\
& + \frac{16aC \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + b \cos(dx + c))^3 \sin(dx + c)}{143d} \\
& + \frac{2C \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + b \cos(dx + c))^4 \sin(dx + c)}{13d} \\
& + \frac{8ab(11a^2(7A + 5C) + 5b^2(11A + 9C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3465 C b^4 \cos(dx + c)^5 + 16380 C a b^3 \cos(dx + c)^4 + 8580 (7 A + 5 C) a^3 b + 3900 (11 A + 9 C) a b^3 + 385 (78 C a^2 b^2 + A b^4) \cos(dx + c)^4 + \dots \right)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C b^4 \cos(dx + c)^6 + 4 C a b^3 \cos(dx + c)^5 + 4 A a^3 b \cos(dx + c) + A a^4 + (6 C a^2 b^2 + A b^4) \cos(dx + c)^4 + \dots\right), dx\right)$$

54.155 Problem number 697

$$\int \frac{(a + b \cos(c + dx))^4 (A + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8ab(3a^2(5A + 3C) + b^2(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(77a^4(3A + C) + 66a^2b^2(7A + 5C) + 5b^4(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab(891Ab^2 + 96a^2C + 673b^2C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3465d} \\ & + \frac{2(64a^4C + 15b^4(11A + 9C) + 9a^2b^2(143A + 101C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{693d} \\ & + \frac{2(16a^2C + 3b^2(11A + 9C)) (a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d} \\ & + \frac{16aC(a + b \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{99d} \\ & + \frac{2C(a + b \cos(dx + c))^4 \sin(dx + c) (\sqrt{\cos(dx + c)})}{11d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(315 C b^4 \cos(dx + c)^4 + 1540 C a b^3 \cos(dx + c)^3 + 1155 C a^4 + 990 (7 A + 5 C) a^2 b^2 + 75 (11 A + 9 C) b^4 + 45 (66$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C b^4 \cos(dx + c)^6 + 4 C a b^3 \cos(dx + c)^5 + 4 A a^3 b \cos(dx + c) + A a^4 + (6 C a^2 b^2 + A b^4) \cos(dx + c)^4 + 4$$

54.156 Problem number 698

$$\int \frac{(a + b \cos(c + dx))^4 (A + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(15a^4(A - C) - 18a^2b^2(5A + 3C) - b^4(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab(7a^2(3A + C) + b^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2b^2(3a^2(105A - 41C) - 7b^2(9A + 7C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\ & + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\ & - \frac{4ab(a^2(63A - 31C) - 6b^2(7A + 5C)) \sin(dx + c) (\sqrt{\cos}(dx + c))}{63d} \\ & - \frac{2ab(21A - 5C) (a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos}(dx + c))}{21d} \\ & - \frac{2b(9A - C) (a + b \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos}(dx + c))}{9d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{60 \sqrt{2} (7i(3A + C)a^3b + i(7A + 5C)ab^3) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{\cos(dx + c)^{\frac{3}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + 4Ab^3 \cos(dx + c)^3 + 4Aab^2 \cos(dx + c)^2 + 4Aa^2b \cos(dx + c) + Aa^3}{\cos(dx + c)^{\frac{3}{2}}}\right)$$

54.157 Problem number 699

$$\int \frac{(a + b \cos(c + dx))^4 (A + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8ab(5a^2(A - C) - b^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(42a^2b^2(3A + C) + 7a^4(A + 3C) + b^4(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{4ab^3(175A - 27C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{3d \cos^{\frac{3}{2}}(dx + c)} + \frac{16Ab(a + b \cos(dx + c))^3 \sin(dx + c)}{3d \sqrt{\cos(dx + c)}} \\ & - \frac{2b^2(3a^2(49A - 13C) - b^2(7A + 5C)) \sin(dx + c) (\sqrt{\cos}(dx + c))}{21d} \\ & - \frac{2b^2(21A - C) (a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos}(dx + c))}{7d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (7i(A + 3C)a^4 + 42i(3A + C)a^2b^2 + i(7A + 5C)b^4) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + 4Aa^3b \cos(dx + c)^3 + 4Aa^2b^2 \cos(dx + c)^2 + 4Aab^3 \cos(dx + c) + Aa^4}{\cos(dx + c)^{\frac{5}{2}}}\right)$$

54.158 Problem number 700

$$\int \frac{(a + b \cos(c + dx))^4 (A + C \cos^2(c + dx))}{\cos^{7/2}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(30a^2b^2(A - C) - b^4(5A + 3C) + a^4(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab(b^2(3A + C) + a^2(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2b^2(b^2(59A - 3C) + 3a^2(3A + 5C)) \left(\cos^{3/2}(dx + c)\right) \sin(dx + c)}{15d} \\ & + \frac{16Ab(a + b \cos(dx + c))^3 \sin(dx + c)}{15d \cos(dx + c)^{3/2}} + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{5d \cos(dx + c)^{5/2}} \\ & + \frac{2(16Ab^2 + a^2(3A + 5C)) (a + b \cos(dx + c))^2 \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \\ & - \frac{4ab(2b^2(33A - 5C) + 3a^2(3A + 5C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{20 \sqrt{2} (i(A + 3C)a^3b + i(3A + C)ab^3) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \dots}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + \dots}{\cos(dx + c)^{7/2}}\right)$$

54.159 Problem number 701

$$\int \frac{(a + b \cos(c + dx))^4 (A + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{8ab(5b^2(A - C) + a^2(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7b^4(3A + C) + 42a^2b^2(A + 3C) + a^4(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(48Ab^2 + 5a^2(5A + 7C)) (a + b \cos(dx + c))^2 \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{16Ab(a + b \cos(dx + c))^3 \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{4ab(96Ab^2 + a^2(101A + 175C)) \sin(dx + c)}{105d \sqrt{\cos(dx + c)}} \\ & - \frac{2b^2(b^2(87A - 35C) + 5a^2(5A + 7C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i(5A + 7C)a^4 + 42i(A + 3C)a^2b^2 + 7i(3A + C)b^4) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + 4a^2b^2 \cos(dx + c)^3 + 4Ab^2 \cos(dx + c)^2 + 4a^2b \cos(dx + c) + Ab^2}{\cos(dx + c)^{\frac{9}{2}}}\right)$$

54.160 Problem number 702

$$\int \frac{(a + b \cos(c + dx))^4 (A + C \cos^2(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(15b^4(A - C) + 18a^2b^2(3A + 5C) + a^4(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab(7b^2(A + 3C) + a^2(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab(32Ab^2 + a^2(101A + 147C)) \sin(dx + c)}{315d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(48Ab^2 + 7a^2(7A + 9C)) (a + b \cos(dx + c))^2 \sin(dx + c)}{315d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{16Ab(a + b \cos(dx + c))^3 \sin(dx + c)}{63d \cos(dx + c)^{\frac{7}{2}}} + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(192Ab^4 + 21a^4(7A + 9C) + 7a^2b^2(155A + 261C)) \sin(dx + c)}{315d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{60 \sqrt{2} (i(5A + 7C)a^3b + 7i(A + 3C)ab^3) \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + 4Ab^3 \cos(dx + c)^3 + 4Aab^2 \cos(dx + c)^2 + 4Aa^2b \cos(dx + c) + Aa^3}{\cos(dx + c)^{\frac{11}{2}}}\right)$$

54.161 Problem number 703

$$\int \frac{(a + b \cos(c + dx))^4 (A + C \cos^2(c + dx))}{\cos^{\frac{13}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8ab(3b^2(3A + 5C) + a^2(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(77b^4(A + 3C) + 66a^2b^2(5A + 7C) + 5a^4(9A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab(96Ab^2 + a^2(673A + 891C)) \sin(dx + c)}{3465d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(64Ab^4 + 15a^4(9A + 11C) + 9a^2b^2(101A + 143C)) \sin(dx + c)}{693d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(16Ab^2 + 3a^2(9A + 11C)) (a + b \cos(dx + c))^2 \sin(dx + c)}{231d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{16Ab(a + b \cos(dx + c))^3 \sin(dx + c)}{99d \cos(dx + c)^{\frac{9}{2}}} + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{11d \cos(dx + c)^{\frac{11}{2}}} \\ & + \frac{8ab(3b^2(3A + 5C) + a^2(7A + 9C)) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (5i(9A + 11C)a^4 + 66i(5A + 7C)a^2b^2 + 77i(A + 3C)b^4) \cos(dx + c)^6 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{15 \sqrt{2} (5i(9A + 11C)a^4 + 66i(5A + 7C)a^2b^2 + 77i(A + 3C)b^4) \cos(dx + c)^6 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + 4Ab^3 \cos(dx + c)^3 + 4Aab^2 \cos(dx + c)^2 + 4Aa^2b \cos(dx + c) + Aa^3}{\cos(dx + c)^{\frac{13}{2}}}\right)$$

54.162 Problem number 814

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} (B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(7bB - 4aC)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{35b^2d} + \frac{2C \cos(dx + c)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{7bd}$$

$$- \frac{2(14abB - 8a^2C - 25b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105b^2d}$$

$$- \frac{2(14a^2bB - 63b^3B - 8a^3C - 19Ca b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(a^2 - b^2)(14abB - 8a^2C - 25b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16i C a^4 - 28i B a^3 b + 32i C a^2 b^2 - 21i B a b^3 - 75i C b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b}{27b^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^3 + B \cos(dx + c)^2\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.163 Problem number 815

$$\int \sqrt{a + b \cos(c + dx)} (B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2C(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5bd} + \frac{2(5bB - 2aC) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15bd}$$

$$+ \frac{2(5abB - 2a^2C + 9b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}$$

$$+ \frac{2(a^2 - b^2)(5bB - 2aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-4iCa^3 + 10iBa^2b - 3iCab^2 - 15iBb^3)\sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i b \sin(dx+c)}{3b}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.164 Problem number 816

$$\int \sqrt{a + b \cos(c + dx)} (B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\frac{2C \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3d}$$

$$+ \frac{2(3bB + aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}$$

$$+ \frac{2(a^2 - b^2)C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \cos(dx+c) + a} C b^2 \sin(dx+c) + \sqrt{2} (2i C a^2 - 3i B a b - 3i C b^2) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^2-3b^2)}{3b^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx+c)^2 + B \cos(dx+c)\right) \sqrt{b \cos(dx+c) + a} \sec(dx+c), x\right)$$

54.165 Problem number 820

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2} (B\cos(c+dx)+C\cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(18abB - 8a^2C - 49b^2C)(a+b\cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{315b^2d} \\ & + \frac{2(9bB - 4aC)(a+b\cos(dx+c))^{\frac{5}{2}} \sin(dx+c)}{63b^2d} \\ & + \frac{2C \cos(dx+c)(a+b\cos(dx+c))^{\frac{5}{2}} \sin(dx+c)}{9bd} \\ & - \frac{2(18a^2bB - 75b^3B - 8a^3C - 39Ca b^2) \sin(dx+c) \sqrt{a+b\cos(dx+c)}}{315b^2d} \\ & - \frac{2(18a^3bB - 246ab^3B - 8a^4C - 33a^2b^2C - 147Cb^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a+b\cos(dx+c)}{a+b}}} \\ & + \frac{2(a^2 - b^2)(18a^2bB - 75b^3B - 8a^3C - 39Ca b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b\cos(dx+c)}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a+b\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16i C a^5 - 36i B a^4 b + 60i C a^3 b^2 + 33i B a^2 b^3 - 264i C a b^4 - 225i B b^5) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^2-3b^2)}{3b^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb \cos(dx+c)^4 + Ba \cos(dx+c)^2 + (Ca + Bb) \cos(dx+c)^3\right) \sqrt{b \cos(dx+c) + a}, x\right)$$

54.166 Problem number 821

$$\int (a + b \cos(c + dx))^{3/2} (B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7bB - 2aC)(a + b \cos(dx + c))^{3/2} \sin(dx + c)}{35bd} + \frac{2C(a + b \cos(dx + c))^{5/2} \sin(dx + c)}{7bd} \\ & + \frac{2(21abB - 6a^2C + 25b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105bd} \\ & + \frac{2(21a^2bB + 63b^3B - 6a^3C + 82Ca b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a+b}}} \\ & + \frac{2(a^2 - b^2)(21abB - 6a^2C + 25b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-12iCa^4 + 42iBa^3b + 11iCa^2b^2 - 126iBab^3 - 75iCb^4)\sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}\right)}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + Ba \cos(dx + c) + (Ca + Bb) \cos(dx + c)^2\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.167 Problem number 822

$$\int (a + b \cos(c + dx))^{3/2} (B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\frac{2C(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5d} + \frac{2(5bB + 3aC) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15d}$$

$$+ \frac{2(20abB + 3a^2C + 9b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}$$

$$+ \frac{2(a^2 - b^2)(5bB + 3aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate((a+b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (6i Ca^3 - 5i Ba^2b - 18i Cab^2 - 15i Bb^3) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i b \sin(dx+c)}{3b}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + Ba \cos(dx + c) + (Ca + Bb) \cos(dx + c)^2\right) \sqrt{b \cos(dx + c) + a} \sec(dx + c), x\right)$$

54.168 Problem number 827

$$\int \cos(c + dx)(a + b \cos(c + dx))^{5/2} (B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(110a^2bB - 539b^3B - 40a^3C - 335Ca b^2) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3465b^2d} \\
& - \frac{2(22abB - 8a^2C - 81b^2C) (a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{693b^2d} \\
& + \frac{2(11bB - 4aC) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{99b^2d} \\
& + \frac{2C \cos(dx + c) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{11bd} \\
& - \frac{2(110a^3bB - 1254a b^3B - 40a^4C - 285a^2b^2C - 675C b^4) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3465b^2d} \\
& - \frac{2(110a^4bB - 3069a^2b^3B - 1617b^5B - 40a^5C - 255a^3b^2C - 3705Ca b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}\right)\right)}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
& + \frac{2(a^2 - b^2) (110a^3bB - 1254a b^3B - 40a^4C - 285a^2b^2C - 675C b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (80i Ca^6 - 220i Ba^5b + 480i Ca^4b^2 + 1023i Ba^3b^3 - 2535i Ca^2b^4 - 5379i Bab^5 - 2025i Cb^6) \sqrt{b} \operatorname{weierstrassPI}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^5 + Ba^2 \cos(dx + c)^2 + (2Cab + Bb^2) \cos(dx + c)^4 + (Ca^2 + 2Bab) \cos(dx + c)^3\right) \sqrt{b}\right)$$

54.169 Problem number 828

$$\int (a + b \cos(c + dx))^{5/2} (B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(45abB - 10a^2C + 49b^2C)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{315bd} \\
& + \frac{2(9bB - 2aC)(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{63bd} + \frac{2C(a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{9bd} \\
& + \frac{2(45a^2bB + 75b^3B - 10a^3C + 114Ca b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315bd} \\
& + \frac{2(45a^3bB + 435a b^3B - 10a^4C + 279a^2b^2C + 147C b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
& - \frac{2(a^2 - b^2)(45a^2bB + 75b^3B - 10a^3C + 114Ca b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-20iCa^5 + 90iBa^4b + 93iCa^3b^2 - 345iBa^2b^3 - 489iCab^4 - 225iBb^5)\sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}\right)}{315bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ba^2 \cos(dx + c) + (2Cab + Bb^2) \cos(dx + c)^3 + (Ca^2 + 2Bab) \cos(dx + c)^2\right) \sqrt{b} \sec(dx + c)\right)$$

54.170 Problem number 829

$$\int (a + b \cos(c + dx))^{5/2} (B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\frac{2(7bB + 5aC)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{35d} + \frac{2C(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7d}$$

$$+ \frac{2(56abB + 15a^2C + 25b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105d}$$

$$+ \frac{2(161a^2bB + 63b^3B + 15a^3C + 145Ca b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(a^2 - b^2)(56abB + 15a^2C + 25b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((a+b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")`
 Fracas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (30i C a^4 + 7i B a^3 b - 115i C a^2 b^2 - 231i B a b^3 - 75i C b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, 3\right)$$

Fracas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ba^2 \cos(dx + c) + (2Cab + Bb^2) \cos(dx + c)^3 + (Ca^2 + 2Bab) \cos(dx + c)^2\right) \sqrt{a + b \cos(dx + c)} + c, x\right)$$

54.171 Problem number 835

$$\int \frac{\cos(c + dx) (B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{a + b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(5bB - 4aC) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^2d} + \frac{2C \cos(dx + c) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{5bd}$$

$$+ \frac{2(10abB - 8a^2C - 9b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(10a^2bB + 5b^3B - 8a^3C - 7Ca b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16i Ca^3 - 20i Ba^2b + 12i Cab^2 - 15i Bb^3) \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib^2}{3b} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C \cos(dx+c)^3 + B \cos(dx+c)^2}{\sqrt{b \cos(dx+c) + a}}, x \right)$$

54.172 Problem number 836

$$\int \frac{B \cos(c+dx) + C \cos^2(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C \sin(dx+c) \sqrt{a+b \cos(dx+c)}}{3bd} \\ & + \frac{2(3bB - 2aC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a+b \cos(dx+c)}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) b^2 d \sqrt{\frac{a+b \cos(dx+c)}{a+b}}} \\ & - \frac{2(3abB - 2a^2C - b^2C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{\frac{a+b \cos(dx+c)}{a+b}}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) b^2 d \sqrt{a+b \cos(dx+c)}} \end{aligned}$$

command

`integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \cos(dx+c) + a} C b^2 \sin(dx+c) + \sqrt{2} (-4i Ca^2 + 6i Bab - 3i Cb^2) \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib^2}{3b} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C \cos(dx+c)^2 + B \cos(dx+c)}{\sqrt{b \cos(dx+c) + a}}, x \right)$$

54.173 Problem number 837

$$\int \frac{(B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}$$

$$+ \frac{2(bB - aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3i \sqrt{2} C b^{\frac{3}{2}} \operatorname{weierstrassZeta}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}\right), \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+1}{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c)) \sec(dx + c)}{\sqrt{b \cos(dx + c) + a}}, x\right)$$

54.174 Problem number 841

$$\int \frac{\cos^2(c + dx) (B \cos(c + dx) + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2a(bB - aC) (\cos^2(dx + c)) \sin(dx + c)}{b(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \\
+ & \frac{2(20a^2bB - 5b^3B - 24a^3C + 9Ca b^2) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^3(a^2 - b^2) d} \\
- & \frac{2(5abB - 6a^2C + b^2C) \cos(dx + c) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{5b^2(a^2 - b^2) d} \\
- & \frac{2(40a^3bB - 25a b^3B - 48a^4C + 24a^2b^2C + 9Cb^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
+ & \frac{2(40a^2bB + 5b^3B - 48a^3C - 12Ca b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(24 C a^4 b^2 - 20 B a^3 b^3 - 9 C a^2 b^4 + 5 B a b^5 - 3 (C a^2 b^4 - C b^6) \cos(dx + c)^2 + (6 C a^3 b^3 - 5 B a^2 b^4 - 6 C a b^5 + 5 B b^6) \sin(dx + c) \right)}{15 b^4 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^4 + B \cos(dx + c)^3\right) \sqrt{b \cos(dx + c) + a}}{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}, x\right)$$

54.175 Problem number 842

$$\int \frac{\cos(c + dx) (B \cos(c + dx) + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2a^2(bB - aC) \sin(dx + c)}{b^2(a^2 - b^2)d\sqrt{a + b \cos(dx + c)}} + \frac{2C \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3b^2d}$$

$$+ \frac{2(6a^2bB - 3b^3B - 8a^3C + 5Ca b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3(a^2 - b^2)d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(6abB - 8a^2C - b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(4Ca^3b^2 - 3Ba^2b^3 - Cab^4 + (Ca^2b^3 - Cb^5) \cos(dx + c)) \sqrt{b \cos(dx + c) + a} \sin(dx + c) - \left(\sqrt{2} (16i Ca^4b - 1\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^3 + B \cos(dx + c)^2) \sqrt{b \cos(dx + c) + a}}{b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2}, x\right)$$

54.176 Problem number 843

$$\int \frac{B \cos(c + dx) + C \cos^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2a(bB - aC) \sin(dx + c)}{b(a^2 - b^2)d\sqrt{a + b \cos(dx + c)}}$$

$$+ \frac{2(abB - 2a^2C + b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2(a^2 - b^2)d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(bB - 2aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(Ca^2b^2 - Bab^3)\sqrt{b\cos(dx+c)+a}\sin(dx+c) + \left(\sqrt{2}(-4iCa^3b + 2iBa^2b^2 + 5iCab^3 - 3iBb^4)\cos(dx+c)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}}{b^2\cos(dx+c)^2 + 2ab\cos(dx+c) + a^2}, x\right)$$

54.177 Problem number 844

$$\int \frac{(B\cos(c+dx) + C\cos^2(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(bB - aC)\sin(dx+c)}{(a^2 - b^2)d\sqrt{a+b\cos(dx+c)}} \\ & + \frac{2(bB - aC)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)b(a^2 - b^2)d\sqrt{\frac{a+b\cos(dx+c)}{a+b}}} \\ & + \frac{2C\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)bd\sqrt{a+b\cos(dx+c)}} \end{aligned}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(Cab^2 - Bb^3)\sqrt{b\cos(dx+c)+a}\sin(dx+c) - \left(\sqrt{2}(2iCa^2b + iBab^2 - 3iCb^3)\cos(dx+c) + \sqrt{2}(2iCa^3 + \dots)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C\cos(dx+c)^2 + B\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)}{b^2\cos(dx+c)^2 + 2ab\cos(dx+c) + a^2}, x\right)$$

54.178 Problem number 847

$$\int \frac{\cos^2(c + dx) (B \cos(c + dx) + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a(bB - aC) (\cos^2(dx + c)) \sin(dx + c)}{3b(a^2 - b^2)d(a + b \cos(dx + c))^{3/2}} - \frac{2a^2(3a^2bB - 7b^3B - 6a^3C + 10Cab^2) \sin(dx + c)}{3b^3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

$$- \frac{2(abB - 2a^2C + b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3b^3(a^2 - b^2)d}$$

$$+ \frac{2(8a^4bB - 15a^2b^3B + 3b^5B - 16a^5C + 28a^3b^2C - 8Cab^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a + b \cos(dx + c)}{a + b}}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$- \frac{2(8a^3bB - 9ab^3B - 16a^4C + 16a^2b^2C + Cb^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(8Ca^6b^2 - 4Ba^5b^3 - 13Ca^4b^4 + 8Ba^3b^5 + Ca^2b^6 + (Ca^4b^4 - 2Ca^2b^6 + Cb^8) \cos(dx + c)^2 + (10Ca^5b^3 - 5Ba^4b^4) \cos(dx + c) \right) \sqrt{a + b \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^4 + B \cos(dx + c)^3) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

54.179 Problem number 848

$$\int \frac{\cos(c + dx) (B \cos(c + dx) + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a^2(bB - aC) \sin(dx + c)}{3b^2(a^2 - b^2)d(a + b \cos(dx + c))^{\frac{3}{2}}} + \frac{2a(2a^2bB - 6b^3B - 5a^3C + 9Ca b^2) \sin(dx + c)}{3b^2(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

$$\frac{2(2a^3bB - 6a b^3B - 8a^4C + 15a^2b^2C - 3Cb^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(2a^2bB - 3b^3B - 8a^3C + 9Ca b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`
 Fracas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(4Ca^5b^2 - Ba^4b^3 - 8Ca^3b^4 + 5Ba^2b^5 + (5Ca^4b^3 - 2Ba^3b^4 - 9Ca^2b^5 + 6Bab^6) \cos(dx + c)) \sqrt{b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

Fracas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^3 + B \cos(dx + c)^2) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

54.180 Problem number 849

$$\int \frac{B \cos(c + dx) + C \cos^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a(bB - aC) \sin(dx + c)}{3b(a^2 - b^2)d(a + b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(a^2bB + 3b^3B + 2a^3C - 6Ca b^2) \sin(dx + c)}{3b(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

$$\frac{2(a^2bB + 3b^3B + 2a^3C - 6Ca b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$+ \frac{2(abB + 2a^2C - 3b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (Ca^4b^2 + 2Ba^3b^3 - 5Ca^2b^4 + 2Bab^5 + (2Ca^3b^3 + Ba^2b^4 - 6Cab^5 + 3Bb^6) \cos(dx + c)) \sqrt{b \cos(dx + c) + a} s$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c)) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x \right)$$

54.181 Problem number 850

$$\int \frac{(B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bB - aC) \sin(dx + c)}{3(a^2 - b^2) d (a + b \cos(dx + c))^{\frac{3}{2}}} - \frac{2(4abB - a^2C - 3b^2C) \sin(dx + c)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{2(4abB - a^2C - 3b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{a + b \cos(dx + c)}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) b (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{2(bB - aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a + b}} \right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) b (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (2Ca^3b^2 - 5Ba^2b^3 + 2Cab^4 + Bb^5 + (Ca^2b^3 - 4Bab^4 + 3Cb^5) \cos(dx + c)) \sqrt{b \cos(dx + c) + a} \sin(dx + c) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c)) \sqrt{b \cos(dx + c) + a} \sec(dx + c)}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x \right)$$

54.182 Problem number 853

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(B\cos(c+dx)+C\cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9Ba+7bC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{10(bB+aC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(9Ba+7bC)\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{45d} + \frac{2(bB+aC)\left(\cos^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{7d} \\ & + \frac{2bC\left(\cos^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{9d} + \frac{10(bB+aC)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{21d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(35Cb\cos(dx+c)^3+45(Ca+Bb)\cos(dx+c)^2+75Ca+75Bb+7(9Ba+7Cb)\cos(dx+c)\right)\sqrt{\cos(dx+c)}}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb\cos(dx+c)^4+Ba\cos(dx+c)^2+(Ca+Bb)\cos(dx+c)^3\right)\sqrt{\cos(dx+c)},x\right)$$

54.183 Problem number 854

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(B\cos(c+dx)+C\cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6(bB+aC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(7Ba+5bC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(bB+aC)\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{5d} + \frac{2bC\left(\cos^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{7d} \\ & + \frac{2(7Ba+5bC)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{21d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15 C b \cos(dx + c)^2 + 35 B a + 25 C b + 21 (C a + B b) \cos(dx + c) \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \sqrt{2} (7i B a + 5i C b)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ba \cos(dx + c) + (Ca + Bb) \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}, x\right)$$

54.184 Problem number 855

$$\int \frac{(a + b \cos(c + dx)) (B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5Ba + 3bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(bB + aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2bC \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2(bB + aC) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 C b \cos(dx + c) + 5 C a + 5 B b \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \sqrt{2} (i C a + i B b) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb \cos(dx + c)^2 + Ba + (Ca + Bb) \cos(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

54.185 Problem number 856

$$\int \frac{(a + b \cos(c + dx)) (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bB + aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3Ba + bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2bC \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2Cb\sqrt{\cos(dx+c)}\sin(dx+c) + \sqrt{2}(-3iBa - iCb)\operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + v}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb \cos(dx + c)^2 + Ba + (Ca + Bb) \cos(dx + c)}{\sqrt{\cos(dx + c)}}, x\right)$$

54.186 Problem number 857

$$\int \frac{(a + b \cos(c + dx)) (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(Ba - bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(bB + aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2aB \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2Ba\sqrt{\cos(dx+c)}\sin(dx+c) + \sqrt{2}(-iCa - iBb)\cos(dx+c)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c) + i\sin(dx+c))}{\cos(dx+c)^{\frac{5}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb\cos(dx+c)^2 + Ba + (Ca + Bb)\cos(dx+c)}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

54.187 Problem number 858

$$\int \frac{(a + b\cos(c + dx))(B\cos(c + dx) + C\cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(bB + aC)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2(Ba + 3bC)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2aB\sin(dx+c)}{3d\cos(dx+c)^{\frac{3}{2}}} + \frac{2(bB + aC)\sin(dx+c)}{d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-iBa - 3iCb)\cos(dx+c)^2\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c) + i\sin(dx+c)) + \sqrt{2}(iBa + 3iCb)\cos(dx+c)}{\cos(dx+c)^{\frac{7}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb\cos(dx+c)^2 + Ba + (Ca + Bb)\cos(dx+c)}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

54.188 Problem number 859

$$\int \frac{(a + b \cos(c + dx)) (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(3Ba + 5bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(bB + aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aB \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2(bB + aC) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(3Ba + 5bC) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (iCa + iBb) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-iCa - iBb) \cos(dx + c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb \cos(dx + c)^2 + Ba + (Ca + Bb) \cos(dx + c)}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

54.189 Problem number 860

$$\int \cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))^2 (B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(9B a^2 + 7b^2 B + 14abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{10(9b^2 C + 11a(2bB + aC)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(9B a^2 + 7b^2 B + 14abC) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} \\
& + \frac{2(9b^2 C + 11a(2bB + aC)) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{77d} \\
& + \frac{2b(11bB + 13aC) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{99d} \\
& + \frac{2bC \left(\cos^{\frac{7}{2}}(dx+c)\right) (a + b \cos(dx+c)) \sin(dx+c)}{11d} \\
& + \frac{10(9b^2 C + 11a(2bB + aC)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{231d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(315 C b^2 \cos(dx+c)^4 + 385 (2 Cab + B b^2) \cos(dx+c)^3 + 825 C a^2 + 1650 B a b + 675 C b^2 + 45 (11 C a^2 + 22 B a b) \right) \sqrt{\cos(dx+c)}}{231 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C b^2 \cos(dx+c)^5 + B a^2 \cos(dx+c)^2 + (2 Cab + B b^2) \cos(dx+c)^4 + (C a^2 + 2 B a b) \cos(dx+c)^3\right) \sqrt{\cos(dx+c)}\right) dx$$

54.190 Problem number 861

$$\int \sqrt{\cos(c+dx)} (a + b \cos(c+dx))^2 (B \cos(c+dx) + C \cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(7b^2C + 9a(2bB + aC)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(7Ba^2 + 5b^2B + 10abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(7b^2C + 9a(2bB + aC)) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} \\
& + \frac{2b(9bB + 11aC) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{63d} \\
& + \frac{2bC \left(\cos^{\frac{5}{2}}(dx+c)\right) (a + b \cos(dx+c)) \sin(dx+c)}{9d} \\
& + \frac{2(7Ba^2 + 5b^2B + 10abC) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(35 C b^2 \cos(dx+c)^3 + 105 B a^2 + 150 C a b + 75 B b^2 + 45 (2 C a b + B b^2) \cos(dx+c)^2 + 7 (9 C a^2 + 18 B a b + 7 C b^2) \cos(dx+c) + 2 a^2 b \right) \sqrt{\cos(dx+c)}}{21 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C b^2 \cos(dx+c)^4 + B a^2 \cos(dx+c) + (2 C a b + B b^2) \cos(dx+c)^3 + (C a^2 + 2 B a b) \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}\right)$$

54.191 Problem number 862

$$\int \frac{(a + b \cos(c + dx))^2 (B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5B a^2 + 3b^2 B + 6abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5b^2 C + 7a(2bB + aC)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(7bB + 9aC) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{35d} \\ & + \frac{2bC \left(\cos^{\frac{3}{2}}(dx+c)\right) (a + b \cos(dx+c)) \sin(dx+c)}{7d} \\ & + \frac{2(5b^2 C + 7a(2bB + aC)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15 C b^2 \cos(dx+c)^2 + 35 C a^2 + 70 B a b + 25 C b^2 + 21 (2 C a b + B b^2) \cos(dx+c) \right) \sqrt{\cos(dx+c)} \sin(dx+c)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C b^2 \cos(dx+c)^3 + B a^2 + (2 C a b + B b^2) \cos(dx+c)^2 + (C a^2 + 2 B a b) \cos(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

54.192 Problem number 863

$$\int \frac{(a + b \cos(c + dx))^2 (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3b^2 C + 5a(2bB + aC)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3B a^2 + b^2 B + 2abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(5bB + 7aC) \sin(dx+c) (\sqrt{\cos(dx+c)})}{15d} \\ & + \frac{2bC(a + b \cos(dx+c)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{5d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$2(3Cb^2 \cos(dx+c) + 10Cab + 5Bb^2) \sqrt{\cos(dx+c)} \sin(dx+c) - 5\sqrt{2}(3iBa^2 + 2iCab + iBb^2) \text{weierstrassP}$

Fricas 1.3.7 via sagemath 9.3 output

$\text{integral}\left(\frac{Cb^2 \cos(dx+c)^3 + Ba^2 + (2Cab + Bb^2) \cos(dx+c)^2 + (Ca^2 + 2Bab) \cos(dx+c)}{\sqrt{\cos(dx+c)}}, x\right)$

54.193 Problem number 864

$$\int \frac{(a + b \cos(c + dx))^2 (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(Ba^2 - b^2B - 2abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(6abB + 3a^2C + b^2C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2B \sin(dx+c)}{d \sqrt{\cos(dx+c)}} + \frac{2b^2C \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$\sqrt{2}(-3iCa^2 - 6iBab - iCb^2) \cos(dx+c) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2}(3iCa^2$

Fricas 1.3.7 via sagemath 9.3 output

$\text{integral}\left(\frac{Cb^2 \cos(dx+c)^3 + Ba^2 + (2Cab + Bb^2) \cos(dx+c)^2 + (Ca^2 + 2Bab) \cos(dx+c)}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$

54.194 Problem number 865

$$\int \frac{(a + b \cos(c + dx))^2 (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(2abB + a^2C - b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ba^2 + 3b^2B + 6abC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2B \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a(2bB + aC) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$\frac{\sqrt{2}(-iBa^2 - 6iCab - 3iBb^2) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(iBa^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2 \cos(dx + c)^3 + Ba^2 + (2Cab + Bb^2) \cos(dx + c)^2 + (Ca^2 + 2Bab) \cos(dx + c)}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

54.195 Problem number 866

$$\int \frac{(a + b \cos(c + dx))^2 (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3Ba^2 + 5b^2B + 10abC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(2abB + a^2C + 3b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2B \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2a(2bB + aC) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(3Ba^2 + 5b^2B + 10abC) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (iCa^2 + 2iBab + 3iCb^2) \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2} (-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb^2 \cos(dx + c)^3 + Ba^2 + (2Cab + Bb^2) \cos(dx + c)^2 + (Ca^2 + 2Bab) \cos(dx + c)}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

54.196 Problem number 867

$$\int \frac{(a + b \cos(c + dx))^2 (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(6abB + 3a^2C + 5b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5Ba^2 + 7b^2B + 14abC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2B \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{2a(2bB + aC) \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(5Ba^2 + 7b^2B + 14abC) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(6abB + 3a^2C + 5b^2C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (5iBa^2 + 14iCab + 7iBb^2) \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2} (-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb^2 \cos(dx + c)^3 + Ba^2 + (2Cab + Bb^2) \cos(dx + c)^2 + (Ca^2 + 2Bab) \cos(dx + c)}{\cos(dx + c)^{\frac{9}{2}}}, x\right)$$

54.197 Problem number 868

$$\int \sqrt{\cos(c+dx)} (a+b\cos(c+dx))^3 (B\cos(c+dx)+C\cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(27a^2bB + 7b^3B + 9a^3C + 21Ca b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(77a^3B + 165Ba b^2 + 165a^2bC + 45C b^3) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(27a^2bB + 7b^3B + 9a^3C + 21Ca b^2) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} \\ & + \frac{2b(33abB + 26a^2C + 9b^2C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{77d} \\ & + \frac{2b^2(11bB + 15aC) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{99d} \\ & + \frac{2bC \left(\cos^{\frac{5}{2}}(dx+c)\right) (a+b\cos(dx+c))^2 \sin(dx+c)}{11d} \\ & + \frac{2(77a^3B + 165Ba b^2 + 165a^2bC + 45C b^3) \sin(dx+c) (\sqrt{\cos(dx+c)})}{231d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(315 C b^3 \cos(dx+c)^4 + 1155 B a^3 + 2475 C a^2 b + 2475 B a b^2 + 675 C b^3 + 385 (3 C a b^2 + B b^3) \cos(dx+c)^3 + 135 \right)}{231 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C b^3 \cos(dx+c)^5 + B a^3 \cos(dx+c) + (3 C a b^2 + B b^3) \cos(dx+c)^4 + 3 (C a^2 b + B a b^2) \cos(dx+c)^3 + \dots\right)\right)$$

54.198 Problem number 869

$$\int \frac{(a + b \cos(c + dx))^3 (B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(15a^3B + 27Ba^2b + 27a^2bC + 7C^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^2bB + 5b^3B + 7a^3C + 15Ca^2b) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(27abB + 22a^2C + 7b^2C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{2b^2(9bB + 13aC) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\ & + \frac{2bC \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + b \cos(dx + c))^2 \sin(dx + c)}{9d} \\ & + \frac{2(21a^2bB + 5b^3B + 7a^3C + 15Ca^2b) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(35 C b^3 \cos(dx + c)^3 + 105 C a^3 + 315 B a^2 b + 225 C a b^2 + 75 B b^3 + 45 (3 C a b^2 + B b^3) \cos(dx + c)^2 + 7 (27 C a^2 b + 5 b^3 B + 7 a^3 C + 15 C a^2 b) \sin(dx + c) \sqrt{\cos(dx + c)} \right)}{21 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C b^3 \cos(dx + c)^4 + B a^3 + (3 C a b^2 + B b^3) \cos(dx + c)^3 + 3 (C a^2 b + B a b^2) \cos(dx + c)^2 + (C a^3 + 3 B a^2 b + 7 a^3 C + 15 C a^2 b) \sin(dx + c) \sqrt{\cos(dx + c)}\right), dx\right)$$

54.199 Problem number 870

$$\int \frac{(a + b \cos(c + dx))^3 (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(15a^2bB + 3b^3B + 5a^3C + 9Ca b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^3B + 21Ba b^2 + 21a^2bC + 5C b^3) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b^2(7bB + 11aC) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2b(21abB + 18a^2C + 5b^2C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \\ & + \frac{2bC(a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15 C b^3 \cos(dx + c)^2 + 105 C a^2 b + 105 B a b^2 + 25 C b^3 + 21 (3 C a b^2 + B b^3) \cos(dx + c) \right) \sqrt{\cos(dx + c)} \sin(dx + c)}{\cos^{\frac{3}{2}}(c + dx)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C b^3 \cos(dx + c)^4 + B a^3 + (3 C a b^2 + B b^3) \cos(dx + c)^3 + 3 (C a^2 b + B a b^2) \cos(dx + c)^2 + (C a^3 + 3 B a^2 b)}{\sqrt{\cos(dx + c)}}\right)$$

54.200 Problem number 871

$$\int \frac{(a + b \cos(c + dx))^3 (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(5a^3B - 15Ba^2b^2 - 15a^2bC - 3Cb^3) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(9a^2bB + b^3B + 3a^3C + 3Ca^2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2b^2(5Ba - bC) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2aB(a + b \cos(dx+c))^2 \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \\ & - \frac{2b(6Ba^2 - b^2B - 3abC) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (3iCa^3 + 9iBa^2b + 3iCab^2 + iBb^3) \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx+c)^4 + Ba^3 + (3Cab^2 + Bb^3) \cos(dx+c)^3 + 3(Ca^2b + Bab^2) \cos(dx+c)^2 + (Ca^3 + 3Ba^2b) \cos(dx+c) + a^3}{\cos(dx+c)^{\frac{3}{2}}}\right)$$

54.201 Problem number 872

$$\int \frac{(a + b \cos(c + dx))^3 (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(3a^2bB - b^3B + a^3C - 3Ca^2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^3B + 9Ba^2b^2 + 9a^2bC + Cb^3) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aB(a + b \cos(dx+c))^2 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2a^2(7bB + 3aC) \sin(dx+c)}{3d \sqrt{\cos(dx+c)}} \\ & - \frac{2b^2(Ba - bC) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i Ba^3 - 9i Ca^2b - 9i Bab^2 - i Cb^3) \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Cb^3 \cos(dx + c)^4 + Ba^3 + (3Cab^2 + Bb^3) \cos(dx + c)^3 + 3(Ca^2b + Bab^2) \cos(dx + c)^2 + (Ca^3 + 3Ba^2b)}{\cos(dx + c)^{\frac{5}{2}}} \right)$$

54.202 Problem number 873

$$\int \frac{(a + b \cos(c + dx))^3 (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(3a^3B + 15Ba^2b + 15a^2bC - 5Cb^3) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(3a^2bB + 3b^3B + a^3C + 9Ca^2b^2) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a^2(9bB + 5aC) \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} + \frac{2aB(a + b \cos(dx + c))^2 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{2a(3Ba^2 + 14b^2B + 15abC) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i Ca^3 + 3i Ba^2b + 9i Cab^2 + 3i Bb^3) \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Cb^3 \cos(dx + c)^4 + Ba^3 + (3Cab^2 + Bb^3) \cos(dx + c)^3 + 3(Ca^2b + Bab^2) \cos(dx + c)^2 + (Ca^3 + 3Ba^2b)}{\cos(dx + c)^{\frac{7}{2}}} \right)$$

54.203 Problem number 874

$$\int \frac{(a + b \cos(c + dx))^3 (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(9a^2bB + 5b^3B + 3a^3C + 15Ca b^2) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5a^3B + 21Ba b^2 + 21a^2bC + 21C b^3) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(11bB + 7aC) \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{2a(5B a^2 + 18b^2B + 21abC) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2aB(a + b \cos(dx + c))^2 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{2(9a^2bB + 5b^3B + 3a^3C + 15Ca b^2) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="fr`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (5i Ba^3 + 21i Ca^2b + 21i Bab^2 + 21i Cb^3) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx + c)^4 + Ba^3 + (3Cab^2 + Bb^3) \cos(dx + c)^3 + 3(Ca^2b + Bab^2) \cos(dx + c)^2 + (Ca^3 + 3Ba^2) \cos(dx + c) + B}{\cos(dx + c)^{\frac{9}{2}}}\right)$$

54.204 Problem number 875

$$\int \frac{(a + b \cos(c + dx))^3 (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{13}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(7a^3B + 27Ba^2b^2 + 27a^2bC + 15Cb^3) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(15a^2bB + 7b^3B + 5a^3C + 21Cab^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a^2(13bB + 9aC) \sin(dx+c)}{63d \cos(dx+c)^{\frac{7}{2}}} + \frac{2a(7Ba^2 + 22b^2B + 27abC) \sin(dx+c)}{45d \cos(dx+c)^{\frac{5}{2}}} \\
& + \frac{2(15a^2bB + 7b^3B + 5a^3C + 21Cab^2) \sin(dx+c)}{21d \cos(dx+c)^{\frac{3}{2}}} + \frac{2aB(a + b \cos(dx+c))^2 \sin(dx+c)}{9d \cos(dx+c)^{\frac{9}{2}}} \\
& + \frac{2(7a^3B + 27Ba^2b^2 + 27a^2bC + 15Cb^3) \sin(dx+c)}{15d \sqrt{\cos(dx+c)}}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algorithm="fr
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (5i Ca^3 + 15i Ba^2b + 21i Cab^2 + 7i Bb^3) \cos(dx+c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx+c)^4 + Ba^3 + (3Cab^2 + Bb^3) \cos(dx+c)^3 + 3(Ca^2b + Bab^2) \cos(dx+c)^2 + (Ca^3 + 3Ba^2b) \cos(dx+c) + a^3}{\cos(dx+c)^{\frac{11}{2}}}\right) dx$$

54.205 Problem number 1000

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx)}{(a + b \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& b(12Ab^6 - 12a^5bB + 15a^3b^3B - 6ab^5B - a^2b^4(29A - 2C) + 5a^4b^2(4A - C) + 6a^6C) \arctan \left(\frac{\sqrt{a-b} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a+b}} \right) \\
& - \frac{a^5(a-b)^{\frac{5}{2}}(a+b)^{\frac{5}{2}}d}{(12Ab^2 - 6abB + a^2(A + 2C)) \operatorname{arctanh}(\sin(dx + c))} \\
& + \frac{(12Ab^5 - 2Ba^5 + 11Ba^3b^2 - 6Ba^4b + a^4b(6A - 5C) - a^2b^3(21A - 2C)) \tan(dx + c)}{2a^5d} \\
& - \frac{(6Ab^4 + 6a^3bB - 3ab^3B + a^4(A - 4C) - a^2b^2(10A - C)) \sec(dx + c) \tan(dx + c)}{2a^4(a^2 - b^2)^2d} \\
& + \frac{(Ab^2 - a(bB - aC)) \sec(dx + c) \tan(dx + c)}{2a^3(a^2 - b^2)^2d} \\
& + \frac{2a(a^2 - b^2)d(a + b \cos(dx + c))^2}{(7a^2Ab^2 - 4Ab^4 - 5a^3bB + 2ab^3B + 3a^4C) \sec(dx + c) \tan(dx + c)} \\
& + \frac{(7a^2Ab^2 - 4Ab^4 - 5a^3bB + 2ab^3B + 3a^4C) \sec(dx + c) \tan(dx + c)}{2a^2(a^2 - b^2)^2d(a + b \cos(dx + c))}
\end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

54.206 Problem number 1007

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx)}{(a + b \cos(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(35a^4Ab^4 - 28a^2Ab^6 + 8Ab^8 + 8a^7bB - 8a^5b^3B + 7a^3b^5B - 2ab^7B - 2a^8C - a^6b^2(20A + 3C)) \arctan\left(\frac{\sqrt{a-b}}{\sqrt{a+b}}\right)}{a^5(a-b)^{\frac{7}{2}}(a+b)^{\frac{7}{2}}d} \\
& - \frac{(4Ab - Ba) \operatorname{arctanh}(\sin(dx + c))}{a^5d} \\
& + \frac{(68a^2Ab^4 - 24Ab^6 + 26a^5bB - 17a^3b^3B + 6ab^5B + a^6(6A - 11C) - a^4b^2(65A + 4C)) \tan(dx + c)}{6a^4(a^2 - b^2)^3d} \\
& + \frac{(Ab^2 - a(bB - aC)) \tan(dx + c)}{3a(a^2 - b^2)d(a + b \cos(dx + c))^3} \\
& - \frac{(4Ab^4 + 6a^3bB - ab^3B - 3a^4C - a^2b^2(9A + 2C)) \tan(dx + c)}{6a^2(a^2 - b^2)^2d(a + b \cos(dx + c))^2} \\
& - \frac{(11a^2Ab^4 - 4Ab^6 + 6a^5bB - 2a^3b^3B + ab^5B - 2a^6C - 3a^4b^2(4A + C)) \tan(dx + c)}{2a^3(a^2 - b^2)^3d(a + b \cos(dx + c))}
\end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

54.207 Problem number 1014

$$\int \cos^2(c + dx) \sqrt{a + b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(63Ab^2 - 36abB + 24a^2C + 49b^2C)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{315b^3d} \\
& + \frac{2(3bB - 2aC) \cos(dx + c)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{21b^2d} \\
& + \frac{2C(\cos^2(dx + c))(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{9bd} \\
& + \frac{2(24a^2bB + 75b^3B - 16a^3C - 6ab^2(7A + 6C)) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315b^3d} \\
& + \frac{2(24a^3bB + 57ab^3B - 16a^4C - 6a^2b^2(7A + 4C) + 21b^4(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
& - \frac{2(a^2 - b^2)(24a^2bB + 75b^3B - 16a^3C - 6ab^2(7A + 6C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a + b \cos(dx + c)}{a + b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm='f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-32iCa^5 + 48iBa^4b - 12i(7A + 3C)a^3b^2 + 96iBa^2b^3 - 3i(21A + 13C)ab^4 - 225iBb^5) \sqrt{b} \operatorname{weierstrassP}(\dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^4 + B \cos(dx + c)^3 + A \cos(dx + c)^2\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.208 Problem number 1015

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(7bB - 4aC)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{35b^2d} + \frac{2C \cos(dx + c)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{7bd}$$

$$+ \frac{2(35Ab^2 - 14abB + 8a^2C + 25b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105b^2d}$$

$$\frac{2(14a^2bB - 63b^3B - 8a^3C - ab^2(35A + 19C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$\frac{2(a^2 - b^2)(35Ab^2 - 14abB + 8a^2C + 25b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16iCa^4 - 28iBa^3b + 2i(35A + 16C)a^2b^2 - 21iBab^3 - 15i(7A + 5C)b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b)}{3b^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.209 Problem number 1016

$$\int \sqrt{a + b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2C(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5bd} + \frac{2(5bB - 2aC) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15bd}$$

$$+ \frac{2(3b^2(5A + 3C) + a(5bB - 2aC)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$\frac{2(a^2 - b^2)(5bB - 2aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{a + b \cos(dx + c)}}$$

command

```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-4i Ca^3 + 10i Ba^2b - 3i (5A + C)ab^2 - 15i Bb^3) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx)}{27b^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.210 Problem number 1021

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(88a^2bB + 539b^3B - 48a^3C - 6ab^2(33A + 34C)) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3465b^3d} \\ & + \frac{2(99Ab^2 - 44abB + 24a^2C + 81b^2C) (a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{693b^3d} \\ & + \frac{2(11bB - 6aC) \cos(dx + c) (a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{99b^2d} \\ & + \frac{2C(\cos^2(dx + c)) (a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{11bd} \\ & + \frac{2(88a^3bB + 429ab^3B - 48a^4C - 18a^2b^2(11A + 8C) + 75b^4(11A + 9C)) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3465b^3d} \\ & + \frac{2(88a^4bB + 363a^2b^3B + 1617b^5B - 48a^5C - 18a^3b^2(11A + 6C) + 6ab^4(451A + 348C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{2(a^2 - b^2) (88a^3bB + 429ab^3B - 48a^4C - 18a^2b^2(11A + 8C) + 75b^4(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-96i Ca^6 + 176i Ba^5b - 36i (11A + 5C)a^4b^2 + 660i Ba^3b^3 + 3i (121A + 123C)a^2b^4 - 2904i Bab^5 - 225i (11A + 5C)ab^6 + 315b^7) \sqrt{b \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb \cos(dx + c)^5 + (Ca + Bb) \cos(dx + c)^4 + Aa \cos(dx + c)^2 + (Ba + Ab) \cos(dx + c)^3\right) \sqrt{b \cos(dx + c)}\right)$$

54.211 Problem number 1022

$$\int \cos(c + dx)(a + b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(63Ab^2 - 18abB + 8a^2C + 49b^2C)(a + b \cos(dx + c))^{3/2} \sin(dx + c)}{315b^2d} \\ & + \frac{2(9bB - 4aC)(a + b \cos(dx + c))^{5/2} \sin(dx + c)}{63b^2d} \\ & + \frac{2C \cos(dx + c)(a + b \cos(dx + c))^{5/2} \sin(dx + c)}{9bd} \\ & - \frac{2(18a^2bB - 75b^3B - 8a^3C - 3ab^2(21A + 13C)) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315b^2d} \\ & - \frac{2(18a^3bB - 246ab^3B - 8a^4C - 21b^4(9A + 7C) - 3a^2b^2(21A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2(a^2 - b^2)(18a^2bB - 75b^3B - 8a^3C - 3ab^2(21A + 13C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a + b \cos(dx + c)}{a + b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (16i Ca^5 - 36i Ba^4b + 6i (21A + 10C)a^3b^2 + 33i Ba^2b^3 - 6i (63A + 44C)ab^4 - 225i Bb^5) \sqrt{b} \text{weierstrassPInv}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb \cos(dx + c)^4 + (Ca + Bb) \cos(dx + c)^3 + Aa \cos(dx + c) + (Ba + Ab) \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}\right)$$

54.212 Problem number 1023

$$\int (a + b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7bB - 2aC)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{35bd} + \frac{2C(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7bd} \\ & + \frac{2(35Ab^2 + 21abB - 6a^2C + 25b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105bd} \\ & + \frac{2(21a^2bB + 63b^3B - 6a^3C + 2ab^2(70A + 41C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{2(a^2 - b^2)(35Ab^2 + 21abB - 6a^2C + 25b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-12iCa^4 + 42iBa^3b - i(35A - 11C)a^2b^2 - 126iBab^3 - 15i(7A + 5C)b^4)\sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}\right)}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + (Ca + Bb) \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c)\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.213 Problem number 1029

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(520a^3bB + 4355ab^3B - 240a^4C + 539b^4(13A + 11C) - 10a^2b^2(143A + 124C)) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{45045b^3d} \\
& + \frac{2(104a^2bB + 1053b^3B - 48a^3C - 2ab^2(143A + 166C)) (a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{9009b^3d} \\
& + \frac{2(143Ab^2 - 52abB + 24a^2C + 121b^2C) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{1287b^3d} \\
& + \frac{2(13bB - 6aC) \cos(dx + c) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{143b^2d} \\
& + \frac{2C(\cos^2(dx + c)) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{13bd} \\
& + \frac{2(520a^4bB + 3705a^2b^3B + 8775b^5B - 240a^5C - 10a^3b^2(143A + 94C) + 6ab^4(2717A + 2174C)) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{45045b^3d} \\
& + \frac{2(520a^5bB + 3315a^3b^3B + 48165ab^5B - 240a^6C + 1617b^6(13A + 11C) - 10a^4b^2(143A + 76C) + 3a^2b^4(13299A + 10449B - 10449C)) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}}{45045 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}} \\
& - \frac{2(a^2 - b^2) (520a^4bB + 3705a^2b^3B + 8775b^5B - 240a^5C - 10a^3b^2(143A + 94C) + 6ab^4(2717A + 2174C)) \sqrt{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}}{45045 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

`integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm='fricas')`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-480i Ca^7 + 1040i Ba^6b - 20i(143A + 67C)a^5b^2 + 6240i Ba^4b^3 + 3i(4433A + 3761C)a^3b^4 - 32955i Ba^2b^5)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2 \cos(dx + c)^6 + (2Cab + Bb^2) \cos(dx + c)^5 + Aa^2 \cos(dx + c)^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx + c)\right), dx\right)$$

54.214 Problem number 1030

$$\int \cos(c + dx)(a + b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(110a^2bB - 539b^3B - 40a^3C - 5ab^2(99A + 67C)) (a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3465b^2d} \\ & + \frac{2(99Ab^2 - 22abB + 8a^2C + 81b^2C) (a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{693b^2d} \\ & + \frac{2(11bB - 4aC) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{99b^2d} \\ & + \frac{2C \cos(dx + c) (a + b \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{11bd} \\ & - \frac{2(110a^3bB - 1254ab^3B - 40a^4C - 75b^4(11A + 9C) - 15a^2b^2(33A + 19C)) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3465b^2d} \\ & - \frac{2(110a^4bB - 3069a^2b^3B - 1617b^5B - 40a^5C - 15a^3b^2(33A + 17C) - 15ab^4(319A + 247C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2(a^2 - b^2) (110a^3bB - 1254ab^3B - 40a^4C - 75b^4(11A + 9C) - 15a^2b^2(33A + 19C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{Elliptic}}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (80i Ca^6 - 220i Ba^5b + 30i (33A + 16C)a^4b^2 + 1023i Ba^3b^3 - 15i (253A + 169C)a^2b^4 - 5379i Bab^5 - 225i ($$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2 \cos(dx + c)^5 + (2Cab + Bb^2) \cos(dx + c)^4 + Aa^2 \cos(dx + c) + (Ca^2 + 2Bab + Ab^2) \cos(dx + c)\right)^3\right)$$

54.215 Problem number 1031

$$\int (a + b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(63Ab^2 + 45abB - 10a^2C + 49b^2C)(a + b \cos(dx + c))^{3/2} \sin(dx + c)}{315bd} \\ & + \frac{2(9bB - 2aC)(a + b \cos(dx + c))^{5/2} \sin(dx + c)}{63bd} + \frac{2C(a + b \cos(dx + c))^{7/2} \sin(dx + c)}{9bd} \\ & + \frac{2(45a^2bB + 75b^3B - 10a^3C + 6ab^2(28A + 19C)) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{315bd} \\ & + \frac{2(45a^3bB + 435ab^3B - 10a^4C + 21b^4(9A + 7C) + 3a^2b^2(161A + 93C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}\right)\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{2(a^2 - b^2)(45a^2bB + 75b^3B - 10a^3C + 6ab^2(28A + 19C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-20i Ca^5 + 90i Ba^4b + 3i(7A + 31C)a^3b^2 - 345i Ba^2b^3 - 3i(231A + 163C)ab^4 - 225i Bb^5) \sqrt{b} \operatorname{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + (2Cab + Bb^2) \cos(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2AbC) \cos(dx + c) + Cb^2\right)\right)$$

54.216 Problem number 1038

$$\int (a + b \cos(c + dx))^{3/2} (abB - a^2C + b^2B \cos(c + dx) + b^2C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2b(7bB - 2aC)(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{35d} + \frac{2bC(a + b \cos(dx + c))^{\frac{5}{2}} \sin(dx + c)}{7d}$$

$$+ \frac{2b(56abB - 41a^2C + 25b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{105d}$$

$$+ \frac{2(161a^2bB + 63b^3B - 146a^3C + 82Ca^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$- \frac{2(a^2 - b^2)(56abB - 41a^2C + 25b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((a+b*cos(d*x+c))^(3/2)*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (23i Ca^4 + 7i Ba^3b + 116i Ca^2b^2 - 231i Bab^3 - 75i Cb^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3}{27b^3}\right)}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^3 - Ca^3 + Ba^2b + (Cab^2 + Bb^3) \cos(dx + c)^2 - (Ca^2b - 2Bab^2) \cos(dx + c)\right) \sqrt{b \cos(dx + c)}\right)$$

54.217 Problem number 1039

$$\int \sqrt{a + b \cos(c + dx)} (abB - a^2C + b^2B \cos(c + dx) + b^2C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2bC(a + b \cos(dx + c))^{\frac{3}{2}} \sin(dx + c)}{5d} + \frac{2b(5bB - 2aC) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15d}$$

$$+ \frac{2(20abB - 17a^2C + 9b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$- \frac{2(a^2 - b^2)(5bB - 2aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((a+b*cos(d*x+c))^(1/2)*(a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (11i Ca^3 - 5i Ba^2b - 3i Cab^2 - 15i Bb^3) \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib \sin(dx+c)}{3b} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(Cb^2 \cos(dx+c)^2 + Bb^2 \cos(dx+c) - Ca^2 + Bab \right) \sqrt{b \cos(dx+c) + a}, x \right)$$

54.218 Problem number 1040

$$\int \frac{\cos^2(c+dx) (A + B \cos(c+dx) + C \cos^2(c+dx))}{\sqrt{a + b \cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(35Ab^2 - 28abB + 24a^2C + 25b^2C) \sin(dx+c) \sqrt{a + b \cos(dx+c)}}{105b^3d} \\ & + \frac{2(7bB - 6aC) \cos(dx+c) \sin(dx+c) \sqrt{a + b \cos(dx+c)}}{35b^2d} \\ & + \frac{2C(\cos^2(dx+c)) \sin(dx+c) \sqrt{a + b \cos(dx+c)}}{7bd} \\ & + \frac{2(56a^2bB + 63b^3B - 48a^3C - 2ab^2(35A + 22C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right)}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{\frac{a + b \cos(dx+c)}{a+b}}} \\ & + \frac{2(56a^3bB + 49ab^3B - 48a^4C - 5b^4(7A + 5C) - 2a^2b^2(35A + 16C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{\frac{b}{a+b}} \right)}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-96i Ca^4 + 112i Ba^3b - 4i (35A + 13C)a^2b^2 + 84i Bab^3 - 15i (7A + 5C)b^4) \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3ib \sin(dx+c)}{3b} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C \cos(dx+c)^4 + B \cos(dx+c)^3 + A \cos(dx+c)^2}{\sqrt{b \cos(dx+c) + a}}, x \right)$$

54.219 Problem number 1041

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{a+b\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5bB-4aC)\sin(dx+c)\sqrt{a+b\cos(dx+c)}}{15b^2d} + \frac{2C\cos(dx+c)\sin(dx+c)\sqrt{a+b\cos(dx+c)}}{5bd} \\ & + \frac{2(15Ab^2-10abB+8a^2C+9b^2C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^3d\sqrt{\frac{a+b\cos(dx+c)}{a+b}}} \\ & + \frac{2(10a^2bB+5b^3B-8a^3C-ab^2(15A+7C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^3d\sqrt{a+b\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(16iCa^3-20iBa^2b+6i(5A+2C)ab^2-15iBb^3)\sqrt{b}\operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)}{27b^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C\cos(dx+c)^3+B\cos(dx+c)^2+A\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}},x\right)$$

54.220 Problem number 1042

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C\sin(dx+c)\sqrt{a+b\cos(dx+c)}}{3bd} \\ & + \frac{2(3bB-2aC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^2d\sqrt{\frac{a+b\cos(dx+c)}{a+b}}} \\ & + \frac{2(3Ab^2-3abB+2a^2C+b^2C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b^2d\sqrt{a+b\cos(dx+c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \cos(dx + c) + a} C b^2 \sin(dx + c) + \sqrt{2} (-4i C a^2 + 6i B a b - 3i (3A + C) b^2) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\sqrt{b \cos(dx + c) + a}}, x\right)$$

54.221 Problem number 1047

$$\int \frac{\cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) (\cos^2(dx + c)) \sin(dx + c)}{b(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{2(20a^2bB - 5b^3B - 3ab^2(5A - 3C) - 24a^3C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^3(a^2 - b^2) d} \\ & + \frac{2(5Ab^2 - 5abB + 6a^2C - b^2C) \cos(dx + c) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{5b^2(a^2 - b^2) d} \\ & - \frac{2(40a^3bB - 25ab^3B - 6a^2b^2(5A - 4C) - 48a^4C + 3b^4(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & + \frac{2(40a^2bB + 5b^3B - 48a^3C - 6ab^2(5A + 2C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(24 C a^4 b^2 - 20 B a^3 b^3 + 3 (5 A - 3 C) a^2 b^4 + 5 B a b^5 - 3 (C a^2 b^4 - C b^6) \cos(dx + c) \right)^2 + (6 C a^3 b^3 - 5 B a^2 b^4 - 6$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx+c)^4 + B \cos(dx+c)^3 + A \cos(dx+c)^2 \right) \sqrt{b \cos(dx+c) + a}}{b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2}, x \right)$$

54.222 Problem number 1048

$$\int \frac{\cos(c+dx) (A + B \cos(c+dx) + C \cos^2(c+dx))}{(a + b \cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(Ab^2 - a(bB - aC)) \sin(dx+c)}{b^2(a^2 - b^2)d\sqrt{a+b\cos(dx+c)}} + \frac{2C \sin(dx+c) \sqrt{a+b\cos(dx+c)}}{3b^2d} \\ & + \frac{2(6a^2bB - 3b^3B - ab^2(3A - 5C) - 8a^3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a+b\cos(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3(a^2 - b^2)d\sqrt{\frac{a+b\cos(dx+c)}{a+b}}} \\ & + \frac{2(3Ab^2 - 6abB + 8a^2C + b^2C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d\sqrt{a+b\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(4Ca^3b^2 - 3Ba^2b^3 + (3A - C)ab^4 + (Ca^2b^3 - Cb^5) \cos(dx+c)) \sqrt{b \cos(dx+c) + a} \sin(dx+c) - \left(\sqrt{2} (16i \dots) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx+c)^3 + B \cos(dx+c)^2 + A \cos(dx+c) \right) \sqrt{b \cos(dx+c) + a}}{b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2}, x \right)$$

54.223 Problem number 1049

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab^2 - a(bB - aC)) \sin(dx + c)}{b(a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} + \frac{2(Ab^2 - abB + 2a^2C - b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2) d \sqrt{\frac{a + b \cos(dx + c)}{a+b}}} + \frac{2(bB - 2aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(Ca^2b^2 - Bab^3 + Ab^4) \sqrt{b \cos(dx + c) + a} \sin(dx + c) + \left(\sqrt{2}(-4iCa^3b + 2iBa^2b^2 + i(A + 5C)ab^3 - 3iBb^4)\right) \sqrt{a + b \cos(dx + c)}}{b^2 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c) + a}}{b^2 \cos^2(dx + c) + 2ab \cos(dx + c) + a^2}, x\right)$$

54.224 Problem number 1053

$$\int \frac{\cos^3(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{2(Ab^2 - a(bB - aC)) (\cos^3(dx + c)) \sin(dx + c)}{3b(a^2 - b^2) d (a + b \cos(dx + c))^{\frac{3}{2}}} \\
 + & \frac{2(6Ab^4 + 5a^3bB - 9ab^3B - 2a^2b^2(A - 6C) - 8a^4C) (\cos^2(dx + c)) \sin(dx + c)}{3b^2(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\
 + & \frac{2(40a^4bB - 65a^2b^3B + 5b^5B - 2a^3b^2(10A - 49C) + 2ab^4(20A - 7C) - 64a^5C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^4(a^2 - b^2)^2 d} \\
 - & \frac{2(30a^3bB - 50ab^3B - a^2b^2(15A - 71C) + b^4(35A - 3C) - 48a^4C) \cos(dx + c) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{15b^3(a^2 - b^2)^2 d} \\
 - & \frac{2(80a^5bB - 140a^3b^3B + 40ab^5B - 4a^4b^2(10A - 53C) + 5a^2b^4(15A - 11C) - 128a^6C - 3b^6(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{a + b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^5 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
 + & \frac{2(80a^4bB - 80a^2b^3B - 5b^5B - 4a^3b^2(10A - 29C) - 128a^5C + ab^4(45A + 17C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^5 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}
 \end{aligned}$$

command

`integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^5 + B \cos(dx + c)^4 + A \cos(dx + c)^3) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

54.225 Problem number 1054

$$\int \frac{\cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Ab^2 - a(bB - aC)) (\cos^2(dx + c)) \sin(dx + c)}{3b(a^2 - b^2) d(a + b \cos(dx + c))^{\frac{3}{2}}} \\
& - \frac{2a(4Ab^4 + a(3a^2bB - 7b^3B - 6a^3C + 10Ca b^2)) \sin(dx + c)}{3b^3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\
& + \frac{2(Ab^2 - abB + 2a^2C - b^2C) \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3b^3(a^2 - b^2) d} \\
& + \frac{2(8a^4bB - 15a^2b^3B + 3b^5B - 2a^3b^2(A - 14C) + 2ab^4(3A - 4C) - 16a^5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\
& - \frac{2(8a^3bB - 9ab^3B - 2a^2b^2(A - 8C) - 16a^4C + b^4(3A + C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm='f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^4 + B \cos(dx + c)^3 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

54.226 Problem number 1055

$$\int \frac{\cos(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a(Ab^2 - a(bB - aC)) \sin(dx + c)}{3b^2(a^2 - b^2)d(a + b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(3Ab^4 + 2a^3bB - 6ab^3B - 5a^4C + a^2b^2(A + 9C)) \sin(dx + c)}{3b^2(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

$$\frac{2(2a^3bB - 6ab^3B + 3b^4(A - C) - 8a^4C + a^2b^2(A + 15C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a + b \cos(dx + c)}{a + b}}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$\frac{2(2a^2bB - 3b^3B - 8a^3C + ab^2(A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(4Ca^5b^2 - Ba^4b^3 - 2(A + 4C)a^3b^4 + 5Ba^2b^5 - 2Aab^6 + (5Ca^4b^3 - 2Ba^3b^4 - (A + 9C)a^2b^5 + 6Bab^6 - 3Aa^2b^3)) \sin(dx + c)}{3b^2(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)) \sqrt{b \cos(dx + c) + a}}{b^3 \cos(dx + c)^3 + 3ab^2 \cos(dx + c)^2 + 3a^2b \cos(dx + c) + a^3}, x\right)$$

54.227 Problem number 1056

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab^2 - a(bB - aC)) \sin(dx + c)}{3b(a^2 - b^2)d(a + b \cos(dx + c))^{\frac{3}{2}}} + \frac{2(a^2bB + 3b^3B + 2a^3C - 2ab^2(2A + 3C)) \sin(dx + c)}{3b(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}}$$

$$\frac{2(a^2bB + 3b^3B + 2a^3C - 2ab^2(2A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}$$

$$\frac{2(Ab^2 - abB - 2a^2C + 3b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (Ca^4b^2 + 2Ba^3b^3 - 5(A+C)a^2b^4 + 2Bab^5 + Ab^6 + (2Ca^3b^3 + Ba^2b^4 - 2(2A+3C)ab^5 + 3Bb^6) \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A) \sqrt{b \cos(dx+c) + a}}{b^3 \cos(dx+c)^3 + 3ab^2 \cos(dx+c)^2 + 3a^2b \cos(dx+c) + a^3}, x \right)$$

54.228 Problem number 1060

$$\int \frac{A + B \cos(c+dx) + C \cos^2(c+dx)}{(a + b \cos(c+dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) \sin(dx+c)}{5b(a^2 - b^2) d(a + b \cos(dx+c))^{\frac{5}{2}}} + \frac{2(3a^2bB + 5b^3B + 2a^3C - 2ab^2(4A + 5C)) \sin(dx+c)}{15b(a^2 - b^2)^2 d(a + b \cos(dx+c))^{\frac{3}{2}}} \\ & + \frac{2(3a^3bB + 29ab^3B + 2a^4C - 3b^4(3A + 5C) - a^2b^2(23A + 19C)) \sin(dx+c)}{15b(a^2 - b^2)^3 d\sqrt{a + b \cos(dx+c)}} \\ & - \frac{2(3a^3bB + 29ab^3B + 2a^4C - 3b^4(3A + 5C) - a^2b^2(23A + 19C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2)^3 d\sqrt{\frac{a + b \cos(dx+c)}{a+b}}} \\ & + \frac{2(3a^2bB + 5b^3B + 2a^3C - 2ab^2(4A + 5C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a+b}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 (a^2 - b^2)^2 d\sqrt{a + b \cos(dx+c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A) \sqrt{b \cos(dx+c) + a}}{b^4 \cos(dx+c)^4 + 4ab^3 \cos(dx+c)^3 + 6a^2b^2 \cos(dx+c)^2 + 4a^3b \cos(dx+c) + a^4}, x \right)$$

54.229 Problem number 1061

$$\int \frac{abB - a^2C + b^2B \cos(c + dx) + b^2C \cos^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2bC \sin(dx + c) \sqrt{a + b \cos(dx + c)}}{3d} + \frac{2(3bB - 2aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} - \frac{2(a^2 - b^2) C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \sqrt{b \cos(dx + c) + a} C b^2 \sin(dx + c) + \sqrt{2} (5i C a^2 - 3i B a b - 3i C b^2) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^2 - 3b^2)}{3b^2}, x\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c) - Ca + Bb\right) \sqrt{b \cos(dx + c) + a}, x\right)$$

54.230 Problem number 1062

$$\int \frac{abB - a^2C + b^2B \cos(c + dx) + b^2C \cos^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} + \frac{2(bB - 2aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \cos(dx + c)}}$$

command

`integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i\sqrt{2}Cb^{\frac{3}{2}}\text{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+a^2}{b^2}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb\cos(dx+c)-Ca+Bb}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

54.231 Problem number 1063

$$\int \frac{abB - a^2C + b^2B\cos(c+dx) + b^2C\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b(bB-2aC)\sin(dx+c)}{(a^2-b^2)d\sqrt{a+b\cos(dx+c)}} \\ & + \frac{2(bB-2aC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cos(dx+c)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)(a^2-b^2)d\sqrt{\frac{a+b\cos(dx+c)}{a+b}}} \\ & + \frac{2C\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{a+b}}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{a+b\cos(dx+c)}} \end{aligned}$$

command

`integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(2Cab^2-Bb^3)\sqrt{b\cos(dx+c)+a}\sin(dx+c)-\left(\sqrt{2}(iCa^2b+iBab^2-3iCb^3)\cos(dx+c)+\sqrt{2}(iCa^3+iBab^2)\right)\sqrt{a+b\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(Cb\cos(dx+c)-Ca+Bb)\sqrt{b\cos(dx+c)+a}}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}, x\right)$$

54.232 Problem number 1064

$$\int \frac{abB - a^2C + b^2B \cos(c + dx) + b^2C \cos^2(c + dx)}{(a + b \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(bB - 2aC) \sin(dx + c)}{3(a^2 - b^2)d(a + b \cos(dx + c))^{3/2}} - \frac{2b(4abB - 5a^2C - 3b^2C) \sin(dx + c)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(dx + c)}} \\ & + \frac{2(4abB - 5a^2C - 3b^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{a + b \cos(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2 - b^2)^2 d \sqrt{\frac{a + b \cos(dx + c)}{a + b}}} \\ & - \frac{2(bB - 2aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b}{a + b}}\right) \sqrt{\frac{a + b \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2 - b^2) d \sqrt{a + b \cos(dx + c)}} \end{aligned}$$

command

`integrate((a*b*B-a^2*C+b^2*B*cos(d*x+c)+b^2*C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(7Ca^3b^2 - 5Ba^2b^3 + Cab^4 + Bb^5 + (5Ca^2b^3 - 4Bab^4 + 3Cb^5) \cos(dx + c)) \sqrt{b \cos(dx + c) + a} \sin(dx + c) + \dots}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Cb \cos(dx + c) - Ca + Bb) \sqrt{b \cos(dx + c) + a}}{b^3 \cos^3(dx + c) + 3ab^2 \cos^2(dx + c) + 3a^2b \cos(dx + c) + a^3}, x\right)$$

54.233 Problem number 1065

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))(A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(9Ab + 9Ba + 7bC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7aA + 5bB + 5aC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(9Ab + 9Ba + 7bC) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} \\ & + \frac{2(bB + aC) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} + \frac{2bC \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{9d} \\ & + \frac{2(7aA + 5bB + 5aC) \sin(dx+c) \left(\sqrt{\cos(dx+c)}\right)}{21d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$2 \left(35 Cb \cos(dx+c)^3 + 45 (Ca + Bb) \cos(dx+c)^2 + 15 (7A + 5C)a + 75 Bb + 7 (9Ba + (9A + 7C)b) \cos(dx+c) \right) \sqrt{\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx+c)^4 + (Ca + Bb) \cos(dx+c)^3 + Aa \cos(dx+c) + (Ba + Ab) \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}\right)$$

54.234 Problem number 1066

$$\int \sqrt{\cos(c+dx)} (a + b \cos(c+dx)) (A + B \cos(c+dx) + C \cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5aA + 3bB + 3aC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7Ab + 7Ba + 5bC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(bB + aC) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2bC \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} \\ & + \frac{2(7Ab + 7Ba + 5bC) \sin(dx+c) \left(\sqrt{\cos(dx+c)}\right)}{21d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15 C b \cos(dx + c)^2 + 35 B a + 5 (7 A + 5 C) b + 21 (C a + B b) \cos(dx + c) \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \sqrt{2} (7$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C b \cos(dx + c)^3 + (C a + B b) \cos(dx + c)^2 + A a + (B a + A b) \cos(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

54.235 Problem number 1067

$$\int \frac{(a + b \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5Ab + 5Ba + 3bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(bB + a(3A + C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2bC \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2(bB + aC) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 C b \cos(dx + c) + 5 C a + 5 B b \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \sqrt{2} (i (3 A + C) a + i B b) \text{weierstrassPInverse}(-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C b \cos(dx + c)^3 + (C a + B b) \cos(dx + c)^2 + A a + (B a + A b) \cos(dx + c)}{\sqrt{\cos(dx + c)}}, x\right)$$

54.236 Problem number 1068

$$\int \frac{(a + b \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bB - a(A - C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3Ab + 3Ba + bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{d \sqrt{\cos(dx + c)}} + \frac{2bC \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i Ba - i(3A + C)b) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i Ba + i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb \cos(dx + c)^3 + (Ca + Bb) \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c)}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

54.237 Problem number 1069

$$\int \frac{(a + b \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(Ab + Ba - bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3bB + a(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(Ab + Ba) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-i(A+3C)a-3iBb)\cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + \sqrt{2}(i(A+3C)b + \dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb\cos(dx+c)^3 + (Ca+Bb)\cos(dx+c)^2 + Aa + (Ba+Ab)\cos(dx+c)}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

54.238 Problem number 1070

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(3aA+5bB+5aC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2(Ab+Ba+3bC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2aA\sin(dx+c)}{5d\cos(dx+c)^{\frac{5}{2}}}+\frac{2(Ab+Ba)\sin(dx+c)}{3d\cos(dx+c)^{\frac{3}{2}}}+\frac{2(3aA+5bB+5aC)\sin(dx+c)}{5d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(iBa+i(A+3C)b)\cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5\sqrt{2}(-iBa + \dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb\cos(dx+c)^3 + (Ca+Bb)\cos(dx+c)^2 + Aa + (Ba+Ab)\cos(dx+c)}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

54.239 Problem number 1071

$$\int \frac{(a + b \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(3Ab + 3Ba + 5bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5aA + 7bB + 7aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{2(Ab + Ba) \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(5aA + 7bB + 7aC) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(3Ab + 3Ba + 5bC) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i (5 A + 7 C) a + 7 i B b) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-i ($$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb \cos(dx + c)^3 + (Ca + Bb) \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c)}{\cos(dx + c)^{\frac{9}{2}}}, x\right)$$

54.240 Problem number 1072

$$\int \cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(18Aab + 9B a^2 + 7b^2 B + 14abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(110abB + 11a^2(7A + 5C) + 5b^2(11A + 9C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(18Aab + 9B a^2 + 7b^2 B + 14abC) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} \\
& + \frac{2(11A b^2 + 22abB + 4a^2 C + 9b^2 C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{77d} \\
& + \frac{2b(11bB + 4aC) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{99d} \\
& + \frac{2C \left(\cos^{\frac{5}{2}}(dx+c)\right) (a + b \cos(dx+c))^2 \sin(dx+c)}{11d} \\
& + \frac{2(110abB + 11a^2(7A + 5C) + 5b^2(11A + 9C)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{231d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(315 C b^2 \cos(dx+c)^4 + 385 (2 Cab + B b^2) \cos(dx+c)^3 + 165 (7 A + 5 C) a^2 + 1650 B a b + 75 (11 A + 9 C) b^2 + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Cb^2 cos(dx+c)^5 + (2Cab + Bb^2) cos(dx+c)^4 + Aa^2 cos(dx+c) + (Ca^2 + 2Bab + Ab^2) cos(dx+c)^3
```

54.241 Problem number 1073

$$\int \sqrt{\cos(c+dx)} (a + b \cos(c+dx))^2 (A + B \cos(c+dx) + C \cos^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(18abB + 3a^2(5A + 3C) + b^2(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(14Aab + 7B a^2 + 5b^2B + 10abC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(9A b^2 + 18abB + 4a^2C + 7b^2C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\
& + \frac{2b(9bB + 4aC) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\
& + \frac{2C \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + b \cos(dx + c))^2 \sin(dx + c)}{9d} \\
& + \frac{2(14Aab + 7B a^2 + 5b^2B + 10abC) \sin(dx + c) (\sqrt{\cos}(dx + c))}{21d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(35 C b^2 \cos(dx + c)^3 + 105 B a^2 + 30 (7 A + 5 C) a b + 75 B b^2 + 45 (2 C a b + B b^2) \cos(dx + c)^2 + 7 (9 C a^2 + 18 B a b + 5 C b^2) \cos(dx + c) + A a^2 + B a b + C b^2 \right)}{15 d \cos\left(\frac{dx}{2} + \frac{c}{2}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C b^2 \cos(dx + c)^4 + (2 C a b + B b^2) \cos(dx + c)^3 + A a^2 + (C a^2 + 2 B a b + A b^2) \cos(dx + c)^2 + (B a^2 + 2 A a b + C b^2) \cos(dx + c) + A a^2 + B a b + C b^2\right) \sqrt{\cos(dx + c)}\right) dx$$

54.242 Problem number 1074

$$\int \frac{(a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(10Aab + 5B a^2 + 3b^2B + 6abC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(14abB + 7a^2(3A + C) + b^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(7bB + 4aC) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2(7A b^2 + 14abB + 4a^2C + 5b^2C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{21d} \\ & + \frac{2C(a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos}(dx + c))}{7d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15 C b^2 \cos(dx + c)^2 + 35 C a^2 + 70 B a b + 5 (7 A + 5 C) b^2 + 21 (2 C a b + B b^2) \cos(dx + c) \right) \sqrt{\cos(dx + c)} \sin(dx + c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C b^2 \cos(dx + c)^4 + (2 C a b + B b^2) \cos(dx + c)^3 + A a^2 + (C a^2 + 2 B a b + A b^2) \cos(dx + c)^2 + (B a^2 + 2 A a b + C b^2) \cos(dx + c) + A^2}{\sqrt{\cos(dx + c)}}\right)$$

54.243 Problem number 1075

$$\int \frac{(a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(10abB - 5a^2(A - C) + b^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3B a^2 + b^2B + 2ab(3A + C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2b^2(5A - C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2A(a + b \cos(dx + c))^2 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\ & - \frac{2b(6aA - bB - 2aC) \sin(dx + c) (\sqrt{\cos}(dx + c))}{3d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(3iBa^2 + 2i(3A+C)ab + iBb^2)\cos(dx+c)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2\cos(dx+c)^4+(2Cab+Bb^2)\cos(dx+c)^3+Aa^2+(Ca^2+2Bab+Ab^2)\cos(dx+c)^2+(Ba^2+2}{\cos(dx+c)^{\frac{3}{2}}}\right)$$

54.244 Problem number 1076

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(Ba^2-b^2B+2ab(A-C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2(6abB+b^2(3A+C)+a^2(A+3C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2A(a+b\cos(dx+c))^2\sin(dx+c)}{3d\cos(dx+c)^{\frac{3}{2}}}+\frac{2a(4Ab+3Ba)\sin(dx+c)}{3d\sqrt{\cos(dx+c)}} \\ & -\frac{2b^2(A-C)\sin(dx+c)(\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-i(A+3C)a^2-6iBab-i(3A+C)b^2)\cos(dx+c)^2\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2\cos(dx+c)^4+(2Cab+Bb^2)\cos(dx+c)^3+Aa^2+(Ca^2+2Bab+Ab^2)\cos(dx+c)^2+(Ba^2+2}{\cos(dx+c)^{\frac{5}{2}}}\right)$$

54.245 Problem number 1077

$$\int \frac{(a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(10abB + 5b^2(A - C) + a^2(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ba^2 + 3b^2B + 2ab(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(4Ab + 5Ba) \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} + \frac{2A(a + b \cos(dx + c))^2 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(4Ab^2 + 10abB + a^2(3A + 5C)) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i Ba^2 + 2i (A + 3C)ab + 3i Bb^2) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2 \cos(dx + c)^4 + (2Cab + Bb^2) \cos(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2AbC) \cos(dx + c) + C^2}{\cos(dx + c)^{\frac{7}{2}}}\right)$$

54.246 Problem number 1078

$$\int \frac{(a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(6Aab + 3B a^2 + 5b^2 B + 10abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(14abB + 7b^2(A+3C) + a^2(5A+7C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(4Ab + 7Ba) \sin(dx+c)}{35d \cos(dx+c)^{\frac{5}{2}}} + \frac{2(4A b^2 + 14abB + a^2(5A+7C)) \sin(dx+c)}{21d \cos(dx+c)^{\frac{3}{2}}} \\ & + \frac{2A(a+b \cos(dx+c))^2 \sin(dx+c)}{7d \cos(dx+c)^{\frac{7}{2}}} + \frac{2(6Aab + 3B a^2 + 5b^2 B + 10abC) \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i(5A+7C)a^2 + 14i Bab + 7i(A+3C)b^2) \cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2 \cos(dx+c)^4 + (2Cab + Bb^2) \cos(dx+c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx+c)^2 + (Ba^2 + 2AbC) \cos(dx+c) + C^2}{\cos(dx+c)^{\frac{9}{2}}}\right)$$

54.247 Problem number 1079

$$\int \frac{(a+b \cos(c+dx))^2 (A+B \cos(c+dx) + C \cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(18abB + 3b^2(3A+5C) + a^2(7A+9C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(10Aab + 5B a^2 + 7b^2 B + 14abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(4Ab + 9Ba) \sin(dx+c)}{63d \cos(dx+c)^{\frac{7}{2}}} + \frac{2(4A b^2 + 18abB + a^2(7A+9C)) \sin(dx+c)}{45d \cos(dx+c)^{\frac{5}{2}}} \\ & + \frac{2(10Aab + 5B a^2 + 7b^2 B + 14abC) \sin(dx+c)}{21d \cos(dx+c)^{\frac{3}{2}}} + \frac{2A(a+b \cos(dx+c))^2 \sin(dx+c)}{9d \cos(dx+c)^{\frac{9}{2}}} \\ & + \frac{2(18abB + 3b^2(3A+5C) + a^2(7A+9C)) \sin(dx+c)}{15d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (5i Ba^2 + 2i (5A + 7C)ab + 7i Bb^2) \cos(dx + c)^5 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Cb^2 \cos(dx + c)^4 + (2Cab + Bb^2) \cos(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2$$

54.248 Problem number 1080

$$\int \sqrt{\cos(c + dx)} (a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(27a^2bB + 7b^3B + 3a^3(5A + 3C) + 3ab^2(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(77a^3B + 165Bab^2 + 33a^2b(7A + 5C) + 5b^3(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{231 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(242a^2bB + 77b^3B + 24a^3C + 33ab^2(9A + 7C)) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{495d} \\ & + \frac{2b(99Ab^2 + 143abB + 24a^2C + 81b^2C) \left(\cos^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{693d} \\ & + \frac{2(11bB + 6aC) \left(\cos^{\frac{3}{2}}(dx + c) \right) (a + b \cos(dx + c))^2 \sin(dx + c)}{99d} \\ & + \frac{2C \left(\cos^{\frac{3}{2}}(dx + c) \right) (a + b \cos(dx + c))^3 \sin(dx + c)}{11d} \\ & + \frac{2(77a^3B + 165Bab^2 + 33a^2b(7A + 5C) + 5b^3(11A + 9C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(315 C b^3 \cos(dx + c)^4 + 1155 B a^3 + 495 (7 A + 5 C) a^2 b + 2475 B a b^2 + 75 (11 A + 9 C) b^3 + 385 (3 C a b^2 + B b^3) \right) \cos(dx + c)^3 + (C a^3 + (3 C a^2 b + 3 B a b^2 + A b^3) \cos(dx + c)^2 + (3 C a b^2 + B b^3) \cos(dx + c) + C b^3) \cos(dx + c) + C b^3$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(C b^3 \cos(dx + c)^5 + (3 C a b^2 + B b^3) \cos(dx + c)^4 + A a^3 + (3 C a^2 b + 3 B a b^2 + A b^3) \cos(dx + c)^3 + (C a^3 + (3 C a^2 b + 3 B a b^2 + A b^3) \cos(dx + c)^2 + (3 C a b^2 + B b^3) \cos(dx + c) + C b^3) \cos(dx + c) + C b^3 \right) \sqrt{\cos(dx + c)} \right) dx$$

54.249 Problem number 1081

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(15a^3B + 27Ba^2b + 9a^2b(5A + 3C) + b^3(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^2bB + 5b^3B + 7a^3(3A + C) + 3ab^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(63A^2b^2 + 99abB + 24a^2C + 49b^2C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\ & + \frac{2(54a^2bB + 15b^3B + 8a^3C + 9ab^2(7A + 5C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{63d} \\ & + \frac{2(3bB + 2aC) (a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \\ & + \frac{2C(a + b \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{9d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="f")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(35 C b^3 \cos(dx + c)^3 + 105 C a^3 + 315 B a^2 b + 45 (7 A + 5 C) a b^2 + 75 B b^3 + 45 (3 C a b^2 + B b^3) \cos(dx + c)^2 + 7 (3 C a b^2 + B b^3) \cos(dx + c) + C b^3 \right) \sqrt{\cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C b^3 \cos(dx + c)^5 + (3 C a b^2 + B b^3) \cos(dx + c)^4 + A a^3 + (3 C a^2 b + 3 B a b^2 + A b^3) \cos(dx + c)^3 + (C a^3 + (3 C a^2 b + 3 B a b^2 + A b^3) \cos(dx + c)^2 + (3 C a b^2 + B b^3) \cos(dx + c) + C b^3) \cos(dx + c) + C b^3}{\sqrt{\cos(dx + c)}} \right) dx$$

54.250 Problem number 1082

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(15a^2bB + 3b^3B - 5a^3(A - C) + 3ab^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^3B + 21Ba^2b + 21a^2b(3A + C) + b^3(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2b^2(35aA - 7bB - 11aC) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{35d} + \frac{2A(a + b \cos(dx + c))^3 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\ & + \frac{2b(21abB - 6a^2(7A - 3C) + b^2(7A + 5C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \\ & - \frac{2b(7A - C) (a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (21i Ba^3 + 21i(3A + C)a^2b + 21i Bab^2 + i(7A + 5C)b^3) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{\cos(dx + c)^{\frac{3}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^3 + 3Ab^2) \cos(dx + c)^2 + (3Aab + 3Bab) \cos(dx + c) + Aa^2 + Ab^2}{\cos(dx + c)^{\frac{3}{2}}}\right)$$

54.251 Problem number 1083

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(5a^3B - 15Ba b^2 + 15a^2b(A - C) - b^3(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
+ & \frac{2(9a^2bB + b^3B + 3a b^2(3A + C) + a^3(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
- & \frac{2b^2(35Ab + 15Ba - 3bC) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\
+ & \frac{2A(a + b \cos(dx + c))^3 \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(2Ab + Ba) (a + b \cos(dx + c))^2 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\
- & \frac{2b(6B a^2 - b^2B + 3ab(5A - C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d}
\end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i(A + 3C)a^3 + 9iBa^2b + 3i(3A + C)ab^2 + iBb^3) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)) + \dots}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^3 + \dots)}{\cos(dx + c)^{\frac{5}{2}}}\right)$$

54.252 Problem number 1084

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(15a^2bB - 5b^3B + 15ab^2(A - C) + a^3(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(a^3B + 9Ba b^2 + b^3(3A + C) + 3a^2b(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(6Ab + 5Ba)(a + b \cos(dx + c))^2 \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} + \frac{2A(a + b \cos(dx + c))^3 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} \\
& + \frac{2a(24Ab^2 + 35abB + 3a^2(3A + 5C)) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \\
& - \frac{2b^2(9Ab + 5Ba - 5bC) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i Ba^3 + 3i (A + 3C) a^2 b + 9i Bab^2 + i (3A + C) b^3) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)) + \dots}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^3 + \dots)}{\cos(dx + c)^{\frac{7}{2}}}\right)$$

54.253 Problem number 1085

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(3a^3B + 15Ba^2b + 5b^3(A - C) + 3a^2b(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(21a^2bB + 21b^3B + 21ab^2(A + 3C) + a^3(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a(24Ab^2 + 63abB + 5a^2(5A + 7C)) \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(6Ab + 7Ba)(a + b \cos(dx + c))^2 \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{2A(a + b \cos(dx + c))^3 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\
& + \frac{2(24Ab^3 + 21a^3B + 98Ba^2b + 21a^2b(3A + 5C)) \sin(dx + c)}{35d \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i(5A + 7C)a^3 + 21iBa^2b + 21i(A + 3C)ab^2 + 21iBb^3) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{\cos(dx + c)^{\frac{9}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^3 + 3Aab^2 + 3Bab^2) \cos(dx + c)^2 + (3Aab + 3Bab) \cos(dx + c) + Aa + Bb}{\cos(dx + c)^{\frac{9}{2}}}\right) dx$$

54.254 Problem number 1086

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(27a^2bB + 15b^3B + 9ab^2(3A + 5C) + a^3(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(5a^3B + 21Ba^2b + 7b^3(A + 3C) + 3a^2b(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a(24Ab^2 + 99abB + 7a^2(7A + 9C)) \sin(dx + c)}{315d \cos(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(8Ab^3 + 15a^3B + 54Ba^2b + 9a^2b(5A + 7C)) \sin(dx + c)}{63d \cos(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(2Ab + 3Ba)(a + b \cos(dx + c))^2 \sin(dx + c)}{21d \cos(dx + c)^{\frac{7}{2}}} + \frac{2A(a + b \cos(dx + c))^3 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\
& + \frac{2(27a^2bB + 15b^3B + 9ab^2(3A + 5C) + a^3(7A + 9C)) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (5i Ba^3 + 3i (5A + 7C)a^2b + 21i Bab^2 + 7i (A + 3C)b^3) \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^3 + 3Aab^2 + 3Bab^2 + Ab^3) \cos(dx + c)^2 + (3Aab^2 + 3Bab^2 + Ab^3) \cos(dx + c) + Aa^3}{\cos(dx + c)^{\frac{11}{2}}}\right)$$

54.255 Problem number 1087

$$\int \sqrt{\cos(c + dx)} (a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(468a^3bB + 364a^2b^2B + 39a^4(5A + 3C) + 78a^2b^2(9A + 7C) + 7b^4(13A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
+ & \frac{2(77a^4B + 330B a^2b^2 + 45b^4B + 44a^3b(7A + 5C) + 20a^2b^3(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
+ & \frac{2(3458a^3bB + 4004a^2b^2B + 192a^4C + 77b^4(13A + 11C) + 11a^2b^2(637A + 491C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{6435d} \\
+ & \frac{2b(2171a^2bB + 1053b^3B + 192a^3C + 2a^2b^2(1573A + 1259C)) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{9009d} \\
+ & \frac{2(143Ab^2 + 221abB + 48a^2C + 121b^2C) \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + b \cos(dx + c))^2 \sin(dx + c)}{1287d} \\
+ & \frac{2(13bB + 8aC) \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + b \cos(dx + c))^3 \sin(dx + c)}{143d} \\
+ & \frac{2C \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + b \cos(dx + c))^4 \sin(dx + c)}{13d} \\
+ & \frac{2(77a^4B + 330B a^2b^2 + 45b^4B + 44a^3b(7A + 5C) + 20a^2b^3(11A + 9C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm='fricas')
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3465 C b^4 \cos(dx + c)^5 + 15015 B a^4 + 8580 (7 A + 5 C) a^3 b + 64350 B a^2 b^2 + 3900 (11 A + 9 C) a b^3 + 8775 B b^4 \right)}{231 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C b^4 \cos(dx + c)^6 + (4 C a b^3 + B b^4) \cos(dx + c)^5 + A a^4 + (6 C a^2 b^2 + 4 B a b^3 + A b^4) \cos(dx + c)^4 + 2 (2 A a^3 b + 4 B a^2 b^2 + 3 C a b^3) \cos(dx + c)^3 + (A a^2 b^2 + 2 B a b^3 + C b^4) \cos(dx + c)^2 + (A a b^3 + 2 B a^2 b^2 + C b^4) \cos(dx + c) + A a^4 + B a^3 b + C a^2 b^2\right), dx\right)$$

54.256 Problem number 1088

$$\int \frac{(a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(15a^4B + 54Ba^2b^2 + 7b^4B + 12a^3b(5A + 3C) + 4ab^3(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(308a^3bB + 220ab^3B + 77a^4(3A + C) + 66a^2b^2(7A + 5C) + 5b^4(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(1353a^2bB + 539b^3B + 192a^3C + 2ab^2(891A + 673C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3465d} \\ & + \frac{2(682a^3bB + 660ab^3B + 64a^4C + 15b^4(11A + 9C) + 9a^2b^2(143A + 101C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{693d} \\ & + \frac{2(33Ab^2 + 55abB + 16a^2C + 27b^2C) (a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d} \\ & + \frac{2(11bB + 8aC) (a + b \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{99d} \\ & + \frac{2C(a + b \cos(dx + c))^4 \sin(dx + c) (\sqrt{\cos(dx + c)})}{11d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(315 C b^4 \cos(dx + c)^4 + 1155 C a^4 + 4620 B a^3 b + 990 (7 A + 5 C) a^2 b^2 + 3300 B a b^3 + 75 (11 A + 9 C) b^4 + 385 (4 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C b^4 \cos(dx + c)^6 + (4 C a b^3 + B b^4) \cos(dx + c)^5 + A a^4 + (6 C a^2 b^2 + 4 B a b^3 + A b^4) \cos(dx + c)^4 + 2(2 C a b^3 + B b^4) \cos(dx + c)^3 + (A a^3 + 3 C a^2 b + 3 A a b^2 + B b^3) \cos(dx + c)^2 + 2(2 C a^2 b + 3 A a b^2 + B b^3) \cos(dx + c) + A a^2 + 2 C a b + B b^2}{\sqrt{\cos(dx + c)}}, dx\right)$$

54.257 Problem number 1089

$$\int \frac{(a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(60a^3bB + 36ab^3B - 15a^4(A - C) + 18a^2b^2(5A + 3C) + b^4(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^4B + 42Ba^2b^2 + 5b^4B + 28a^3b(3A + C) + 4ab^3(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b^2(162abB - a^2(315A - 123C) + 7b^2(9A + 7C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\ & + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\ & + \frac{2b(117a^2bB + 15b^3B - a^3(126A - 62C) + 12ab^2(7A + 5C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{63d} \\ & - \frac{2b(21aA - 3bB - 5aC) (a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \\ & - \frac{2b(9A - C) (a + b \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{9d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (21i Ba^4 + 28i (3A + C)a^3b + 42i Ba^2b^2 + 4i (7A + 5C)ab^3 + 5i Bb^4) \cos(dx + c) \operatorname{weierstrassPInverse}(\dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(2a^3b + 2a^2b^2 + Ab^3) \cos(dx + c)^3 + (2a^2b^2 + 2ab^3 + Ab^4) \cos(dx + c)^2 + (2ab^3 + Ab^4) \cos(dx + c) + Ab^4}{\cos^{\frac{3}{2}}(c + dx)}\right)$$

54.258 Problem number 1090

$$\int \frac{(a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5a^4B - 30Ba^2b^2 - 3b^4B + 20a^3b(A - C) - 4ab^3(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(84a^3bB + 28ab^3B + 42a^2b^2(3A + C) + 7a^4(A + 3C) + b^4(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2b^2(350Aab + 105Ba^2 - 21b^2B - 54abC) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{3d \cos^{\frac{3}{2}}(dx + c)} + \frac{2(8Ab + 3Ba)(a + b \cos(dx + c))^3 \sin(dx + c)}{3d \sqrt{\cos(dx + c)}} \\ & - \frac{2b(42a^3B - 28Ba^2b^2 + 3a^2b(49A - 13C) - b^3(7A + 5C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \\ & - \frac{2b(21Ab + 7Ba - bC)(a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (7i(A + 3C)a^4 + 84iBa^3b + 42i(3A + C)a^2b^2 + 28iBab^3 + i(7A + 5C)b^4) \cos(dx + c)^2 \operatorname{weierstrassP}\left(\frac{dx + c}{2}, \sqrt{2}\right)}{\cos(dx + c)^{\frac{5}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(2Ab^3 + 3Ba^2b) \cos(dx + c)^3 + (3Aa^2 + 6Ab^2) \cos(dx + c)^2 + 2Ab \cos(dx + c) + A}{\cos(dx + c)^{\frac{5}{2}}}, dx\right)$$

54.259 Problem number 1091

$$\int \frac{(a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{7/2}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(20a^3bB - 20ab^3B + 30a^2b^2(A - C) - b^4(5A + 3C) + a^4(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^4B + 18Ba^2b^2 + b^4B + 4ab^3(3A + C) + 4a^3b(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2b^2(50abB + b^2(59A - 3C) + 3a^2(3A + 5C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\ & + \frac{2(8Ab + 5Ba)(a + b \cos(dx + c))^3 \sin(dx + c)}{15d \cos^{\frac{3}{2}}(dx + c)} + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{5d \cos^{\frac{5}{2}}(dx + c)} \\ & + \frac{2(16Ab^2 + 15abB + a^2(3A + 5C))(a + b \cos(dx + c))^2 \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \\ & - \frac{2b(105a^2bB - 5b^3B + 4ab^2(33A - 5C) + 6a^3(3A + 5C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i Ba^4 + 4i(A + 3C)a^3b + 18i Ba^2b^2 + 4i(3A + C)ab^3 + i Bb^4) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(2\right)}{\dots}$$

54.260 Problem number 1092

$$\int \frac{(a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(3a^4B + 30Ba^2b^2 - 5b^4B + 20ab^3(A - C) + 4a^3b(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(28a^3bB + 84ab^3B + 7b^4(3A + C) + 42a^2b^2(A + 3C) + a^4(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(48Ab^2 + 77abB + 5a^2(5A + 7C)) (a + b \cos(dx + c))^2 \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(8Ab + 7Ba) (a + b \cos(dx + c))^3 \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{2a(192Ab^3 + 63a^3B + 413Ba b^2 + a^2(202Ab + 350bC)) \sin(dx + c)}{105d \sqrt{\cos(dx + c)}} \\ & - \frac{2b^2(98abB + b^2(87A - 35C) + 5a^2(5A + 7C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i(5A + 7C)a^4 + 28iBa^3b + 42i(A + 3C)a^2b^2 + 84iBab^3 + 7i(3A + C)b^4) \cos(dx + c)^4 \operatorname{weierstrassP}(\dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(2a^3b + 3Ab^2) \cos(dx + c)^3 + (3a^2b^2 + 2Ab^3) \cos(dx + c)^2 + 2ab^3 \cos(dx + c) + b^4}{\cos^{\frac{9}{2}}(c + dx)}\right)$$

54.261 Problem number 1093

$$\int \frac{(a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(36a^3bB + 60ab^3B + 15b^4(A - C) + 18a^2b^2(3A + 5C) + a^4(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5a^4B + 42Ba^2b^2 + 21b^4B + 28ab^3(A + 3C) + 4a^3b(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(64Ab^3 + 75a^3B + 261Bab^2 + a^2(202Ab + 294bC)) \sin(dx + c)}{315d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(48Ab^2 + 117abB + 7a^2(7A + 9C)) (a + b \cos(dx + c))^2 \sin(dx + c)}{315d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(8Ab + 9Ba) (a + b \cos(dx + c))^3 \sin(dx + c)}{63d \cos(dx + c)^{\frac{7}{2}}} + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(192Ab^4 + 756a^3bB + 1098ab^3B + 21a^4(7A + 9C) + 7a^2b^2(155A + 261C)) \sin(dx + c)}{315d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (5i Ba^4 + 4i (5A + 7C)a^3b + 42i Ba^2b^2 + 28i (A + 3C)ab^3 + 21i Bb^4) \cos(dx + c)^5 \operatorname{weierstrassPInverse}(\dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(2\cos(dx + c)^3 + \dots)}{\dots}\right)$$

54.262 Problem number 1094

$$\int \frac{(a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{13}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(7a^4B + 54Ba^2b^2 + 15b^4B + 12ab^3(3A + 5C) + 4a^3b(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(220a^3bB + 308ab^3B + 77b^4(A + 3C) + 66a^2b^2(5A + 7C) + 5a^4(9A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(192Ab^3 + 539a^3B + 1353Ba^2b^2 + 2a^2b(673A + 891C)) \sin(dx + c)}{3465d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(64Ab^4 + 660a^3bB + 682ab^3B + 15a^4(9A + 11C) + 9a^2b^2(101A + 143C)) \sin(dx + c)}{693d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(16Ab^2 + 55abB + 3a^2(9A + 11C)) (a + b \cos(dx + c))^2 \sin(dx + c)}{231d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{2(8Ab + 11Ba) (a + b \cos(dx + c))^3 \sin(dx + c)}{99d \cos(dx + c)^{\frac{9}{2}}} + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c)}{11d \cos(dx + c)^{\frac{11}{2}}} \\ & + \frac{2(7a^4B + 54Ba^2b^2 + 15b^4B + 12ab^3(3A + 5C) + 4a^3b(7A + 9C)) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algorithm="Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$- \frac{15 \sqrt{2} (5i(9A + 11C)a^4 + 220iBa^3b + 66i(5A + 7C)a^2b^2 + 308iBab^3 + 77i(A + 3C)b^4) \cos(dx + c)^6 \operatorname{weierstrassE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15d \sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(2a^3b(7A + 9C) + 12ab^3(3A + 5C) + 2a^2b^2(5A + 7C) + 220a^3bB + 308ab^3B + 77b^4(A + 3C) + 66a^2b^2(5A + 7C) + 5a^4(9A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2(7a^4B + 54Ba^2b^2 + 15b^4B + 12ab^3(3A + 5C) + 4a^3b(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\cos^{\frac{13}{2}}(c + dx)} dx\right)$$

54.263 Problem number 1160

$$\int (a + a \cos(c + dx)) (A + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5A + 7C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} + \frac{2aA \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2aA \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{2a(3A + 5C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2a(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (5A + 7C)a \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (5A + 7C)a \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Ca \cos(dx + c)^3 + Ca \cos(dx + c)^2 + Aa \cos(dx + c) + Aa \right) \sec(dx + c)^{\frac{9}{2}}, x \right)$$

54.264 Problem number 1161

$$\int (a + a \cos(c + dx)) (A + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2aA \left(\sec^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{5d} \\ & + \frac{2a(3A+5C) \sin(dx+c) (\sqrt{\sec}(dx+c))}{5d} \\ & - \frac{2a(3A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (A+3C)a \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} (A+3C)a \cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Ca \cos(dx+c)^3 + Ca \cos(dx+c)^2 + Aa \cos(dx+c) + Aa \right) \sec(dx+c)^{\frac{7}{2}}, x \right)$$

54.265 Problem number 1162

$$\int (a + a \cos(c+dx)) (A + C \cos^2(c+dx)) \sec^{\frac{5}{2}}(c+dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2aA \sin(dx+c) (\sqrt{\sec}(dx+c))}{d} \\ & - \frac{2a(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i\sqrt{2}(A+3C)a\cos(dx+c)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(A+3C)a\cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca\cos(dx+c)^3+Ca\cos(dx+c)^2+Aa\cos(dx+c)+Aa\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

54.266 Problem number 1163

$$\int (a+a\cos(c+dx))(A+C\cos^2(c+dx))\sec^{\frac{3}{2}}(c+dx)dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aC\sin(dx+c)}{3d\sqrt{\sec(dx+c)}} + \frac{2aA\sin(dx+c)(\sqrt{\sec(dx+c)})}{d} \\ & - \frac{2a(A-C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(3A+C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i\sqrt{2}(3A+C)a\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+C)a\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca\cos(dx+c)^3+Ca\cos(dx+c)^2+Aa\cos(dx+c)+Aa\right)\sec(dx+c)^{\frac{3}{2}},x\right)$$

54.267 Problem number 1164

$$\int (a + a \cos(c + dx)) (A + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aC \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2aC \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (3A + C) \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (3A + C) \operatorname{aweierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca \cos(dx + c)^3 + Ca \cos(dx + c)^2 + Aa \cos(dx + c) + Aa\right) \sqrt{\sec(dx + c)}, x\right)$$

54.268 Problem number 1165

$$\int \frac{(a + a \cos(c + dx)) (A + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aC \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2aC \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(7A + 5C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i\sqrt{2}(7A+5C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(7A+5C)a\text{weierstrassPInv}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\cos(dx+c)^3+Ca\cos(dx+c)^2+Aa\cos(dx+c)+Aa}{\sqrt{\sec(dx+c)}},x\right)$$

54.269 Problem number 1166

$$\int \frac{(a+a\cos(c+dx))(A+C\cos^2(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aC\sin(dx+c)}{9d\sec(dx+c)^{\frac{7}{2}}} + \frac{2aC\sin(dx+c)}{7d\sec(dx+c)^{\frac{5}{2}}} + \frac{2a(9A+7C)\sin(dx+c)}{45d\sec(dx+c)^{\frac{3}{2}}} + \frac{2a(7A+5C)\sin(dx+c)}{21d\sqrt{\sec(dx+c)}} \\ & + \frac{2a(9A+7C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(7A+5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-15i\sqrt{2}(7A+5C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}(7A+5C)a\text{weierstrassPInv}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\cos(dx+c)^3+Ca\cos(dx+c)^2+Aa\cos(dx+c)+Aa}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

54.270 Problem number 1167

$$\int (a + a \cos(c + dx))^2 (A + C \cos^2(c + dx)) \sec^{\frac{11}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(5A + 7C) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} + \frac{2a^2(19A + 21C) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{8A(a^2 + a^2 \cos(dx + c)) \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\ & + \frac{2A(a + a \cos(dx + c))^2 \left(\sec^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{9d} \\ & + \frac{16a^2(2A + 3C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{16a^2(2A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (5A + 7C) a^2 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 7C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2\right) \sec(dx + c)^{\frac{11}{2}}, x\right)$$

54.271 Problem number 1168

$$\int (a + a \cos(c + dx))^2 (A + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(33A + 35C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{105d} \\ & + \frac{8A(a^2 + a^2 \cos(dx + c)) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{35d} \\ & + \frac{2A(a + a \cos(dx + c))^2 \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{4a^2(3A + 5C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{4a^2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(3A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} (3A + 7C) a^2 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} (3A + 7C) a^2 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right) \sec(dx + c)^{\frac{9}{2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2\right) \sec(dx + c)^{\frac{9}{2}}, x\right)$$

54.272 Problem number 1169

$$\int (a + a \cos(c + dx))^2 (A + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8A(a^2 + a^2 \cos(dx + c)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\ & + \frac{2A(a + a \cos(dx + c))^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2a^2(17A + 15C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15d} \\ & - \frac{16a^2 A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A + 3C) a^2 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + 3C) a^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2\right) \sec(dx + c)^{\frac{7}{2}}, x\right)$$

54.273 Problem number 1170

$$\int (a + a \cos(c + dx))^2 (A + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \cos(dx + c))^2 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} - \frac{2a^2(5A - C) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{8A(a^2 + a^2 \cos(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & - \frac{4a^2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2i \sqrt{2} (A + C) a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 2i \sqrt{2} (A + C) a^2 \cos \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2\right) \sec(dx + c)^{\frac{5}{2}}, x\right)$$

54.274 Problem number 1171

$$\int (a + a \cos(c + dx))^2 (A + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(15A - 7C) \sin(dx + c)}{15d\sqrt{\sec(dx + c)}} - \frac{2(5A - C)(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{5d\sqrt{\sec(dx + c)}} \\ & + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & + \frac{16a^2 C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (3A + C)a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + C)a^2 \operatorname{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

54.275 Problem number 1172

$$\int (a + a \cos(c + dx))^2 (A + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(35A + 33C) \sin(dx + c)}{105d\sqrt{\sec(dx + c)}} + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{7d\sqrt{\sec(dx + c)}} \\ & + \frac{8C(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{35d\sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(7A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} (7A + 3C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} (7A + 3C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2) \sqrt{\sec(dx + c)}}{\sqrt{\sec(dx + c)}}, x \right)$$

54.276 Problem number 1173

$$\int \frac{(a + a \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(21A + 19C) \sin(dx + c)}{105d \sec(dx + c)^{\frac{3}{2}}} + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{8C(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{63d \sec(dx + c)^{\frac{3}{2}}} + \frac{4a^2(7A + 5C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{16a^2(3A + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (7A + 5C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (7A + 5C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2) \sqrt{\sec(dx + c)}}{\sqrt{\sec(dx + c)}}, x \right)$$

54.277 Problem number 1174

$$\int \frac{(a + a \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(99A + 89C) \sin(dx + c)}{693d \sec(dx + c)^{\frac{5}{2}}} + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{11d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{8C(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{99d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^2(9A + 7C) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{8a^2(33A + 25C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(33A + 25C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(30i \sqrt{2} (33A + 25C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 30i \sqrt{2} (33A + 25C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \cos(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 + (A + C)a^2 \cos(dx + c)^2 + 2Aa^2 \cos(dx + c) + Aa^2}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

54.278 Problem number 1175

$$\int (a + a \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sec^{\frac{13}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^3(105A + 143C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{231d} + \frac{8a^3(35A + 44C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{385d} \\
& + \frac{2(35A + 33C) (a^3 + a^3 \cos(dx + c)) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{231d} \\
& + \frac{4A(a^2 + a^2 \cos(dx + c))^2 \left(\sec^{\frac{9}{2}}(dx + c) \right) \sin(dx + c)}{33ad} \\
& + \frac{2A(a + a \cos(dx + c))^3 \left(\sec^{\frac{11}{2}}(dx + c) \right) \sin(dx + c)}{11d} \\
& + \frac{4a^3(5A + 7C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\
& - \frac{4a^3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(105A + 143C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (105A + 143C) a^3 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (105A + 143C) a^3 \cos(dx + c)^5 \right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c) + c\right)^{\frac{13}{2}}, x\right)$$

54.279 Problem number 1176

$$\int (a + a \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sec^{\frac{11}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^3(16A + 21C) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{2(73A + 63C) (a^3 + a^3 \cos(dx + c)) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\ & + \frac{4A(a^2 + a^2 \cos(dx + c))^2 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{21ad} \\ & + \frac{2A(a + a \cos(dx + c))^3 \left(\sec^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{9d} \\ & + \frac{4a^3(17A + 27C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{4a^3(17A + 27C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(11A + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (11A + 21C) a^3 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (11A + 21C) a^3 \cos(dx + c)^4 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c) + c\right)^{\frac{11}{2}}, x\right)$$

54.280 Problem number 1177

$$\int (a + a \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7A + 5C) (a^3 + a^3 \cos(dx + c)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\ & + \frac{12A(a^2 + a^2 \cos(dx + c))^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35ad} \\ & + \frac{2A(a + a \cos(dx + c))^3 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{8a^3(53A + 70C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{105d} \\ & - \frac{4a^3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(13A + 35C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (13A + 35C) a^3 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (13A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 + c\right)^{\frac{9}{2}}, x\right)$$

54.281 Problem number 1178

$$\int (a + a \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4A(a^2 + a^2 \cos(dx + c))^2 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5ad} \\ & + \frac{2A(a + a \cos(dx + c))^3 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} - \frac{4a^3(21A + 5C) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & + \frac{2(11A + 5C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{4a^3(9A - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (3A + 5C) a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + 5C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 + c\right)^{\frac{7}{2}}, x\right)$$

54.282 Problem number 1179

$$\int (a + a \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \cos(dx + c))^3 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} - \frac{8a^3(10A - 3C) \sin(dx + c)}{15d\sqrt{\sec(dx + c)}} \\ & - \frac{2(35A - 3C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{15d\sqrt{\sec(dx + c)}} \\ & + \frac{4A(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{ad} \\ & - \frac{4a^3(5A - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (5A + 3C) a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (5A + 3C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 + c\right)^{\frac{5}{2}}, x\right)$$

54.283 Problem number 1180

$$\int (a + a \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(35A - 41C) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} - \frac{2(7A - C) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{7ad \sqrt{\sec(dx + c)}} \\ & - \frac{2(35A - 11C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{35d \sqrt{\sec(dx + c)}} \\ & + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & + \frac{4a^3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(35A + 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (35A + 13C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (35A + 13C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c) + c\right)^{\frac{3}{2}}, x\right)$$

54.284 Problem number 1181

$$\int (a + a \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^3(21A + 16C) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{9d \sqrt{\sec(dx + c)}} \\ & + \frac{4C(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{21ad \sqrt{\sec(dx + c)}} + \frac{2(63A + 73C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{315d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(27A + 17C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(21A + 11C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (21A + 11C)a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (21A + 11C)a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c) + Aa^3\right) \sqrt{\sec(c + dx)} dx\right)$$

54.285 Problem number 1182

$$\int \frac{(a + a \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{8a^3(44A + 35C) \sin(dx + c)}{385d \sec(dx + c)^{\frac{3}{2}}} + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{11d \sec(dx + c)^{\frac{3}{2}}} \\
 & + \frac{4C(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{33ad \sec(dx + c)^{\frac{3}{2}}} \\
 & + \frac{2(33A + 35C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{231d \sec(dx + c)^{\frac{3}{2}}} + \frac{4a^3(143A + 105C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\
 & + \frac{4a^3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
 & + \frac{4a^3(143A + 105C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
 \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (143A + 105C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (143A + 105C) a^3 v \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c)}{\sqrt{\sec(dx + c)}}\right)$$

54.286 Problem number 1183

$$\int \frac{(a + a \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{40a^3(143A + 118C) \sin(dx + c)}{9009d \sec(dx + c)^{\frac{5}{2}}} + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{13d \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{12C(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{143ad \sec(dx + c)^{\frac{5}{2}}} + \frac{2(143A + 145C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{1287d \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{4a^3(221A + 175C) \sin(dx + c)}{585d \sec(dx + c)^{\frac{3}{2}}} + \frac{4a^3(121A + 95C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\
& + \frac{4a^3(221A + 175C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(121A + 95C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(195i \sqrt{2} (121A + 95C)a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 195i \sqrt{2} (121A + 95C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 + (3A + C)a^3 \cos(dx + c)^2 + 3Aa^3 \cos(dx + c)}{\sec(dx + c)^{\frac{3}{2}}}\right)$$

54.287 Problem number 1184

$$\int \frac{(A + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5A + 3C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3ad} + \frac{(7A + 5C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5ad} \\ & - \frac{(A + C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \cos(dx + c))} + \frac{3(7A + 5C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5ad} \\ & - \frac{3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-5iA - 3iC) \cos(dx + c)^3 + \sqrt{2} (-5iA - 3iC) \cos(dx + c)^2 \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \cos(dx + c)^2 + A \right) \sec(dx + c)^{\frac{7}{2}}}{a \cos(dx + c) + a}, x \right)$$

54.288 Problem number 1185

$$\int \frac{(A + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5A + 3C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3ad} - \frac{(A + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \cos(dx + c))} \\ & - \frac{(3A + C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{ad} \\ & + \frac{(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-5iA - 3iC)\cos(dx+c)^2 + \sqrt{2}(-5iA - 3iC)\cos(dx+c)\right)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\cos(dx+c)^2 + A)\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c) + a}, x\right)$$

54.289 Problem number 1186

$$\int \frac{(A + C\cos^2(c + dx))\sec^{\frac{3}{2}}(c + dx)}{a + a\cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A + C)\sin(dx+c)(\sqrt{\sec(dx+c)})}{ad} - \frac{(A + C)\sin(dx+c)(\sqrt{\sec(dx+c)})}{d(a + a\cos(dx+c))} \\ & - \frac{(3A + C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad} \\ & - \frac{(A - C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(iA - iC)\cos(dx+c) + \sqrt{2}(iA - iC)\right)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + \left(\sqrt{2}(-5iA - 3iC)\cos(dx+c)^2 + \sqrt{2}(-5iA - 3iC)\cos(dx+c)\right)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\cos(dx+c)^2 + A)\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c) + a}, x\right)$$

54.290 Problem number 1187

$$\int \frac{(A + C \cos^2(c + dx)) \sqrt{\sec(c + dx)}}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + C) \sin(dx + c)}{d(a + a \cos(dx + c)) \sqrt{\sec(dx + c)}} \\ & + \frac{(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A + C) \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2}(-iA + iC) \cos(dx + c) + \sqrt{2}(-iA + iC)\right) \operatorname{weierstrassPInverse}(-)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + A\right) \sqrt{\sec(dx + c)}}{a \cos(dx + c) + a}, x\right)$$

54.291 Problem number 1188

$$\int \frac{A + C \cos^2(c + dx)}{(a + a \cos(c + dx)) \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + C) \sin(dx + c)}{d(a + a \cos(dx + c)) \sec(dx + c)^{\frac{3}{2}}} + \frac{(3A + 5C) \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}} \\ & - \frac{(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-3iA - 5iC)\cos(dx + c) + \sqrt{2}(-3iA - 5iC)\right)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C\cos(dx + c)^2 + A}{(a\cos(dx + c) + a)\sqrt{\sec(dx + c)}}, x\right)$$

54.292 Problem number 1189

$$\int \frac{A + C\cos^2(c + dx)}{(a + a\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A + C)\sin(dx + c)}{d(a + a\cos(dx + c))\sec(dx + c)^{\frac{5}{2}}} + \frac{(5A + 7C)\sin(dx + c)}{5ad\sec(dx + c)^{\frac{3}{2}}} - \frac{(3A + 5C)\sin(dx + c)}{3ad\sqrt{\sec(dx + c)}} \\ & + \frac{3(5A + 7C)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad} \\ & - \frac{(3A + 5C)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\left(\sqrt{2}(-3iA - 5iC)\cos(dx + c) + \sqrt{2}(-3iA - 5iC)\right)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C\cos(dx + c)^2 + A}{(a\cos(dx + c) + a)\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

54.293 Problem number 1190

$$\int \frac{A + C \cos^2(c + dx)}{(a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + C) \sin(dx + c)}{d(a + a \cos(dx + c)) \sec(dx + c)^{\frac{7}{2}}} + \frac{(7A + 9C) \sin(dx + c)}{7ad \sec(dx + c)^{\frac{5}{2}}} \\ & - \frac{(5A + 7C) \sin(dx + c)}{5ad \sec(dx + c)^{\frac{3}{2}}} + \frac{5(7A + 9C) \sin(dx + c)}{21ad \sqrt{\sec(dx + c)}} \\ & - \frac{3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5(7A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{25 \left(\sqrt{2} (7i A + 9i C) \cos(dx + c) + \sqrt{2} (7i A + 9i C) \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \dots}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx + c)^2 + A}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}}, x\right)$$

54.294 Problem number 1191

$$\int \frac{(A + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5A + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d} - \frac{(7A + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d(1 + \cos(dx + c))} \\ & - \frac{(A + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d(a + a \cos(dx + c))^2} - \frac{(7A + C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{a^2d} \\ & + \frac{(7A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \\ & + \frac{2(5A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (5iA + iC) \cos(dx + c)^3 + 2\sqrt{2} (5iA + iC) \cos(dx + c)^2 + \sqrt{2} (5iA + iC) \cos(dx + c) \right) \operatorname{weierstrassPI}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^{\frac{5}{2}}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x \right)$$

54.295 Problem number 1192

$$\int \frac{(A + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4A \sin(dx + c) (\sqrt{\sec(dx + c)})}{a^2d} - \frac{(5A - C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3a^2d(1 + \cos(dx + c))} \\ & - \frac{(A + C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d(a + a \cos(dx + c))^2} \\ & - \frac{4A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \\ & - \frac{(5A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (5i A - i C) \cos(dx + c)^2 - 2\sqrt{2} (-5i A + i C) \cos(dx + c) + \sqrt{2} (5i A - i C)\right) \text{weierstrassPInverse}(-4, 0, \cos$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^{\frac{3}{2}}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x\right)$$

54.296 Problem number 1193

$$\int \frac{(A + C \cos^2(c + dx)) \sqrt{\sec(c + dx)}}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A - C) \sin(dx + c)}{a^2 d (1 + \cos(dx + c)) \sqrt{\sec(dx + c)}} - \frac{(A + C) \sin(dx + c)}{3d (a + a \cos(dx + c))^2 \sqrt{\sec(dx + c)}} \\ & + \frac{(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2(A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (i A + i C) \cos(dx + c)^2 + 2\sqrt{2} (i A + i C) \cos(dx + c) + \sqrt{2} (i A + i C)\right) \text{weierstrassPInverse}(-4, 0, \cos$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{\sec(dx + c)}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x\right)$$

54.297 Problem number 1194

$$\int \frac{A + C \cos^2(c + dx)}{(a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A + C) \sin(dx + c)}{3d(a + a \cos(dx + c))^2 \sec(dx + c)^{\frac{3}{2}}} + \frac{(A - 5C) \sin(dx + c)}{3a^2d(1 + \cos(dx + c)) \sqrt{\sec(dx + c)}} \\ & + \frac{4C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & + \frac{(A - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-iA + 5iC) \cos(dx + c)^2 - 2\sqrt{2}(iA - 5iC) \cos(dx + c) + \sqrt{2}(-iA + 5iC)\right) \operatorname{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx + c)^2 + A}{\left(a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2\right) \sqrt{\sec(dx + c)}}, x\right)$$

54.298 Problem number 1195

$$\int \frac{A + C \cos^2(c + dx)}{(a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + C) \sin(dx + c)}{3d(a + a \cos(dx + c))^2 \sec(dx + c)^{\frac{5}{2}}} \\ & - \frac{(A + 7C) \sin(dx + c)}{3a^2 d (1 + \cos(dx + c)) \sec(dx + c)^{\frac{3}{2}}} + \frac{2(A + 5C) \sin(dx + c)}{3a^2 d \sqrt{\sec(dx + c)}} \\ & - \frac{(A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2(A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (i A + 5i C) \cos(dx + c)^2 + 2 \sqrt{2} (i A + 5i C) \cos(dx + c) + \sqrt{2} (i A + 5i C) \right) \operatorname{weierstrassPInverse}(-4, 0, \dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx + c)^2 + A}{\left(a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2\right) \sec(dx + c)^{\frac{3}{2}}}, x\right)$$

54.299 Problem number 1196

$$\int \frac{A + C \cos^2(c + dx)}{(a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + C) \sin(dx + c)}{3d(a + a \cos(dx + c))^2 \sec(dx + c)^{\frac{7}{2}}} - \frac{(A + 3C) \sin(dx + c)}{a^2 d (1 + \cos(dx + c)) \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{4(5A + 14C) \sin(dx + c)}{15a^2 d \sec(dx + c)^{\frac{3}{2}}} - \frac{5(A + 3C) \sin(dx + c)}{3a^2 d \sqrt{\sec(dx + c)}} \\ & + \frac{4(5A + 14C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25 \left(\sqrt{2} (-i A - 3i C) \cos(dx + c)^2 + 2 \sqrt{2} (-i A - 3i C) \cos(dx + c) + \sqrt{2} (-i A - 3i C) \right) \text{weierstrassPInverse}(\dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C \cos(dx + c)^2 + A}{\left(a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2 \right) \sec(dx + c)^{\frac{5}{2}}}, x \right)$$

54.300 Problem number 1197

$$\int \frac{(A + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(11A + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{2a^3 d} - \frac{(A + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5d (a + a \cos(dx + c))^3} \\ & - \frac{2A \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3ad (a + a \cos(dx + c))^2} - \frac{(119A + 9C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{30d (a^3 + a^3 \cos(dx + c))} \\ & - \frac{(119A + 9C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{10a^3 d} \\ & + \frac{(119A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(11A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \left(\sqrt{2} (11i A + i C) \cos(dx + c)^4 + 3 \sqrt{2} (11i A + i C) \cos(dx + c)^3 + 3 \sqrt{2} (11i A + i C) \cos(dx + c)^2 + \sqrt{2} (11i A + i C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \cos(dx+c)^2 + A) \sec(dx+c)^{\frac{5}{2}}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x \right)$$

54.301 Problem number 1198

$$\int \frac{(A + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(49A - C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{10a^3d} - \frac{(A + C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d(a + a \cos(dx+c))^3} \\ & - \frac{2(4A - C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{15ad(a + a \cos(dx+c))^2} - \frac{(13A - C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{6d(a^3 + a^3 \cos(dx+c))} \\ & - \frac{(49A - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & - \frac{(13A - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-13iA + iC) \cos(dx+c)^3 + 3 \sqrt{2} (-13iA + iC) \cos(dx+c)^2 + 3 \sqrt{2} (-13iA + iC) \cos(dx+c) + \sqrt{2} (-13iA + iC) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \cos(dx+c)^2 + A) \sec(dx+c)^{\frac{3}{2}}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x \right)$$

54.302 Problem number 1199

$$\int \frac{(A + C \cos^2(c + dx)) \sqrt{\sec(c + dx)}}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + C) \sin(dx + c)}{5d (a + a \cos(dx + c))^3 \sqrt{\sec(dx + c)}} - \frac{2(3A - 2C) \sin(dx + c)}{15ad (a + a \cos(dx + c))^2 \sqrt{\sec(dx + c)}} \\ & - \frac{(9A - C) \sin(dx + c)}{10d (a^3 + a^3 \cos(dx + c)) \sqrt{\sec(dx + c)}} \\ & + \frac{(9A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (3i A + i C) \cos(dx + c)^3 + 3 \sqrt{2} (3i A + i C) \cos(dx + c)^2 + 3 \sqrt{2} (3i A + i C) \cos(dx + c) + \sqrt{2} (3i A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{\sec(dx + c)}}{a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3}, x\right)$$

54.303 Problem number 1200

$$\int \frac{A + C \cos^2(c + dx)}{(a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{(A+C)\sin(dx+c)}{5d(a+a\cos(dx+c))^3\sec(dx+c)^{\frac{3}{2}}} + \frac{2(2A-3C)\sin(dx+c)}{15ad(a+a\cos(dx+c))^2\sqrt{\sec(dx+c)}} \\
 & - \frac{(A-9C)\sin(dx+c)}{10d(a^3+a^3\cos(dx+c))\sqrt{\sec(dx+c)}} \\
 & + \frac{(A-9C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{10\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3d} \\
 & + \frac{(A+3C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3d}
 \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\left(\sqrt{2}(iA+3iC)\cos(dx+c)^3+3\sqrt{2}(iA+3iC)\cos(dx+c)^2+3\sqrt{2}(iA+3iC)\cos(dx+c)+\sqrt{2}(iA+3iC)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C\cos(dx+c)^2+A}{\left(a^3\cos(dx+c)^3+3a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3\right)\sqrt{\sec(dx+c)}},x\right)$$

54.304 Problem number 1201

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{(A+C)\sin(dx+c)}{5d(a+a\cos(dx+c))^3\sec(dx+c)^{\frac{5}{2}}} + \frac{2(A-4C)\sin(dx+c)}{15ad(a+a\cos(dx+c))^2\sec(dx+c)^{\frac{3}{2}}} \\
 & + \frac{(A-13C)\sin(dx+c)}{6d(a^3+a^3\cos(dx+c))\sqrt{\sec(dx+c)}} \\
 & - \frac{(A-49C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{10\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3d} \\
 & + \frac{(A-13C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3d}
 \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (i A - 13i C) \cos(dx + c)^3 + 3 \sqrt{2} (i A - 13i C) \cos(dx + c)^2 + 3 \sqrt{2} (i A - 13i C) \cos(dx + c) + \sqrt{2} (i A - 13i C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C \cos(dx + c)^2 + A}{\left(a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3 \right) \sec(dx + c)^{\frac{3}{2}}}, x \right)$$

54.305 Problem number 1202

$$\int \frac{A + C \cos^2(c + dx)}{(a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + C) \sin(dx + c)}{5d(a + a \cos(dx + c))^3 \sec(dx + c)^{\frac{7}{2}}} - \frac{2C \sin(dx + c)}{3ad(a + a \cos(dx + c))^2 \sec(dx + c)^{\frac{5}{2}}} \\ & - \frac{(9A + 119C) \sin(dx + c)}{30d(a^3 + a^3 \cos(dx + c)) \sec(dx + c)^{\frac{3}{2}}} + \frac{(A + 11C) \sin(dx + c)}{2a^3 d \sqrt{\sec(dx + c)}} \\ & + \frac{(9A + 119C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + 11C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

`integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \left(\sqrt{2} (i A + 11i C) \cos(dx + c)^3 + 3 \sqrt{2} (i A + 11i C) \cos(dx + c)^2 + 3 \sqrt{2} (i A + 11i C) \cos(dx + c) + \sqrt{2} (i A + 11i C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C \cos(dx + c)^2 + A}{\left(a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3 \right) \sec(dx + c)^{\frac{5}{2}}}, x \right)$$

54.306 Problem number 1203

$$\int \frac{A + C \cos^2(c + dx)}{(a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A + C) \sin(dx + c)}{5d(a + a \cos(dx + c))^3 \sec(dx + c)^{\frac{9}{2}}} - \frac{2(A + 6C) \sin(dx + c)}{15ad(a + a \cos(dx + c))^2 \sec(dx + c)^{\frac{7}{2}}} \\ & -\frac{(13A + 63C) \sin(dx + c)}{10d(a^3 + a^3 \cos(dx + c)) \sec(dx + c)^{\frac{5}{2}}} + \frac{7(7A + 33C) \sin(dx + c)}{30a^3d \sec(dx + c)^{\frac{3}{2}}} - \frac{(13A + 63C) \sin(dx + c)}{6a^3d \sqrt{\sec(dx + c)}} \\ & + \frac{7(7A + 33C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & - \frac{(13A + 63C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \left(\sqrt{2} (-13i A - 63i C) \cos(dx + c)^3 + 3 \sqrt{2} (-13i A - 63i C) \cos(dx + c)^2 + 3 \sqrt{2} (-13i A - 63i C) \cos(dx + c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx + c)^2 + A}{(a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3) \sec(dx + c)^{\frac{7}{2}}, x\right)$$

54.307 Problem number 1253

$$\int (B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2B \left(\sec^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{5d} \\ & + \frac{6B \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d} \\ & - \frac{6B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} C \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} C \cos(dx+c)^2 \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(C \cos(dx+c)^2 + B \cos(dx+c) \right) \sec(dx+c)^{\frac{9}{2}}, x \right)$$

54.308 Problem number 1254

$$\int (B \cos(c+dx) + C \cos^2(c+dx)) \sec^{\frac{7}{2}}(c+dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2C \sin(dx+c) (\sqrt{\sec(dx+c)})}{d} \\ & - \frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) \sec(dx + c)^{\frac{7}{2}}, x\right)$$

54.309 Problem number 1255

$$\int (B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} C \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} C \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) \sec(dx + c)^{\frac{5}{2}}, x\right)$$

54.310 Problem number 1256

$$\int (B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} B \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} B \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx+c)^2 + B \cos(dx+c)\right) \sec(dx+c)^{\frac{3}{2}}, x\right)$$

54.311 Problem number 1257

$$\int (B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2C \sin(dx+c)}{3d \sqrt{\sec(dx+c)}} + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$2C\sqrt{\cos(dx+c)}\sin(dx+c) - i\sqrt{2}C\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}C\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C\cos(dx+c)^2 + B\cos(dx+c)\right)\sqrt{\sec(dx+c)}, x\right)$$

54.312 Problem number 1258

$$\int \frac{B\cos(c+dx) + C\cos^2(c+dx)}{\sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C\sin(dx+c)}{5d\sec(dx+c)^{\frac{3}{2}}} + \frac{2B\sin(dx+c)}{3d\sqrt{\sec(dx+c)}} \\ & + \frac{6C\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos(dx+c)}\right)\left(\sqrt{\sec(dx+c)}\right)}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2B\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos(dx+c)}\right)\left(\sqrt{\sec(dx+c)}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \end{aligned}$$

command

`integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C\cos(dx+c)^2 + B\cos(dx+c)}{\sqrt{\sec(dx+c)}}, x\right)$$

54.313 Problem number 1259

$$\int \frac{B \cos(c + dx) + C \cos^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2B \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{10C \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{6B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-25i \sqrt{2} C \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} C \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c)}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

54.314 Problem number 1260

$$\int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} + \frac{2A \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2(3A + 5C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i\sqrt{2} B \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i\sqrt{2} B \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sec(dx + c)^{\frac{7}{2}}, x\right)$$

54.315 Problem number 1261

$$\int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2B \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & - \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$\sqrt{2} (-iA - 3iC) \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (iA + 3iC) \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sec(dx + c)^{\frac{5}{2}}, x\right)$$

54.316 Problem number 1262

$$\int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \operatorname{BweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \operatorname{BweierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

54.317 Problem number 1263

$$\int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2C\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}(-3iA-iC)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sqrt{\sec(dx+c)},x\right)$$

54.318 Problem number 1264

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C\sin(dx+c)}{5d\sec(dx+c)^{\frac{3}{2}}} + \frac{2B\sin(dx+c)}{3d\sqrt{\sec(dx+c)}} \\ & + \frac{2(5A+3C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2B\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{\sqrt{\sec(dx+c)}},x\right)$$

54.319 Problem number 1265

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2B \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(7A + 5C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{6B \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (7i A + 5i C) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-7i A - 5i C) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

54.320 Problem number 1266

$$\int (a + a \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5A + 7B + 7C) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} + \frac{2a(A + B) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2aA \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{2a(3A + 3B + 5C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{2a(3A + 3B + 5C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A + 7B + 7C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(5A+7B+7C)a\cos(dx+c)^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(5A+7$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ca\cos(dx+c)^3+(B+C)a\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa\right)\sec(dx+c)^{\frac{9}{2}},x\right)$$

54.321 Problem number 1267

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^{\frac{7}{2}}(c+dx)dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(A+B)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3d} + \frac{2aA\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{5d} \\ & + \frac{2a(3A+5B+5C)\sin(dx+c)(\sqrt{\sec}(dx+c))}{5d} \\ & - \frac{2a(3A+5B+5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos}(dx+c))(\sqrt{\sec}(dx+c))}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(A+B+3C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos}(dx+c))(\sqrt{\sec}(dx+c))}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(A+B+3C)a\cos(dx+c)^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+B+3$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ca\cos(dx+c)^3+(B+C)a\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa\right)\sec(dx+c)^{\frac{7}{2}},x\right)$$

54.322 Problem number 1268

$$\int (a + a \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2aA \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2a(A + B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{d}$$

$$- \frac{2a(A + B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}$$

$$+ \frac{2a(A + 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (A + 3B + 3C) a \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (A + 3B + 3C) a \sin(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Ca \cos(dx + c)^3 + (B + C) a \cos(dx + c)^2 + (A + B) a \cos(dx + c) + Aa \right) \sec(dx + c)^{\frac{5}{2}}, x \right)$$

54.323 Problem number 1269

$$\int (a + a \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2aC \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2aA \sin(dx + c) (\sqrt{\sec(dx + c)})}{d}$$

$$- \frac{2a(A - B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}$$

$$+ \frac{2a(3A + 3B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} (3A + 3B + C) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (3A + 3B + C) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$\text{integral}\left(\left(Ca \cos(dx + c)^3 + (B + C)a \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$

54.324 Problem number 1270

$$\int (a + a \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aC \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(B + C) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(5A + 5B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3A + B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} (3A + B + C) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (3A + B + C) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$\text{integral}\left(\left(Ca \cos(dx + c)^3 + (B + C)a \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa\right) \sqrt{\sec(dx + c)}, x\right)$

54.325 Problem number 1271

$$\int \frac{(a + a \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aC \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a(B + C) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(7A + 7B + 5C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(5A + 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 7B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (7A + 7B + 5C) \operatorname{aweberstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (7A + 7B + 5C) \operatorname{aweberstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca \cos(dx + c)^3 + (B + C)a \cos(dx + c)^2 + (A + B)a \cos(dx + c) + Aa}{\sqrt{\sec(dx + c)}}, x\right)$$

54.326 Problem number 1272

$$\int \frac{(a + a \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aC \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2a(B + C) \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2a(9A + 9B + 7C) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(7A + 5B + 5C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(9A + 9B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-15i\sqrt{2}(7A+5B+5C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}(7A+5B+5C)av$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\cos(dx+c)^3+(B+C)a\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

54.327 Problem number 1273

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^{\frac{11}{2}}(c+dx)dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(5A+6B+7C)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{21d} \\ & + \frac{2a^2(19A+27B+21C)\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{105d} \\ & + \frac{2(4A+9B)(a^2+a^2\cos(dx+c))\left(\sec^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{63d} \\ & + \frac{2A(a+a\cos(dx+c))^2\left(\sec^{\frac{9}{2}}(dx+c)\right)\sin(dx+c)}{9d} \\ & + \frac{4a^2(8A+9B+12C)\sin(dx+c)(\sqrt{\sec(dx+c)})}{15d} \\ & - \frac{4a^2(8A+9B+12C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{4a^2(5A+6B+7C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (5A + 6B + 7C)a^2 \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 6B + 7C)a^2 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 6B + 7C)a^2 \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 6B + 7C)a^2 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 6B + 7C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + c\right)^{\frac{11}{2}}, x\right)$$

54.328 Problem number 1274

$$\int (a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(33A + 49B + 35C) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{2(4A + 7B) (a^2 + a^2 \cos(dx + c)) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2A(a + a \cos(dx + c))^2 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{4a^2(3A + 4B + 5C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{4a^2(3A + 4B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(6A + 7B + 14C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="f")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (6A + 7B + 14C)a^2 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (6A + 7B + 14C)a^2 \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (6A + 7B + 14C)a^2 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (6A + 7B + 14C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + c\right)^{\frac{9}{2}}, x\right)$$

54.329 Problem number 1275

$$\int (a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(4A + 5B) (a^2 + a^2 \cos(dx + c)) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{15d} \\ & + \frac{2A(a + a \cos(dx + c))^2 \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2a^2(17A + 25B + 15C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15d} \\ & - \frac{4a^2(4A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(A + 2B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm='f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A + 2B + 3C) a^2 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + c\right)^{\frac{7}{2}}, x\right)$$

54.330 Problem number 1276

$$\int (a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \cos(dx + c))^2 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} - \frac{2a^2(5A + 3B - C) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2(4A + 3B)(a^2 + a^2 \cos(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & - \frac{4a^2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(2A + 3B + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="f`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} (2A + 3B + 2C) a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} (2A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + c\right)^{\frac{5}{2}}, x\right)$$

54.331 Problem number 1277

$$\int (a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(15A - 5B - 7C) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} - \frac{2(5A - C)(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{5d \sqrt{\sec(dx + c)}} \\ & + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & + \frac{4a^2(5B + 4C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(3A + 2B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (3A + 2B + C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + 2B + C) a^2 \sqrt{\sec(dx + c)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c) + c\right)^{\frac{3}{2}}, x\right)$$

54.332 Problem number 1278

$$\int (a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(35A + 49B + 33C) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{7d \sqrt{\sec(dx + c)}} \\ & + \frac{2(7B + 4C)(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{35d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(5A + 4B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(14A + 7B + 6C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (14A + 7B + 6C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (14A + 7B + 6C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ca^2*cos(dx+c)^4+(B+2C)a^2*cos(dx+c)^3+(A+2B+C)a^2*cos(dx+c)^2+(2A+B)a^2*cos(dx+c)`

54.333 Problem number 1279

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(21A + 27B + 19C) \sin(dx + c)}{105d \sec(dx + c)^{\frac{3}{2}}} + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(9B + 4C)(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{63d \sec(dx + c)^{\frac{3}{2}}} + \frac{4a^2(7A + 6B + 5C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(12A + 9B + 8C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(7A + 6B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (7A + 6B + 5C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (7A + 6B + 5C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ca^2*cos(dx+c)^4+(B+2C)a^2*cos(dx+c)^3+(A+2B+C)a^2*cos(dx+c)^2+(2A+B)a^2*cos(dx+c)`

54.334 Problem number 1280

$$\int \frac{(a + a \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(99A + 121B + 89C) \sin(dx + c)}{693d \sec(dx + c)^{\frac{5}{2}}} + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{11d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(11B + 4C)(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{99d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{4a^2(9A + 8B + 7C) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{4a^2(66A + 55B + 50C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(9A + 8B + 7C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(66A + 55B + 50C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (66A + 55B + 50C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (66A + 55B + 50C) a^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \cos(dx + c)^4 + (B + 2C)a^2 \cos(dx + c)^3 + (A + 2B + C)a^2 \cos(dx + c)^2 + (2A + B)a^2 \cos(dx + c)}{\sec(dx + c)^{\frac{3}{2}}}\right)$$

54.335 Problem number 1281

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{13}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(105A + 121B + 143C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{231d} \\ & + \frac{4a^3(210A + 253B + 264C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{1155d} \\ & + \frac{2(105A + 143B + 99C) (a^3 + a^3 \cos(dx + c)) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{693d} \\ & + \frac{2(6A + 11B) (a^2 + a^2 \cos(dx + c))^2 \left(\sec^{\frac{9}{2}}(dx + c) \right) \sin(dx + c)}{99ad} \\ & + \frac{2A(a + a \cos(dx + c))^3 \left(\sec^{\frac{11}{2}}(dx + c) \right) \sin(dx + c)}{11d} \\ & + \frac{4a^3(15A + 17B + 21C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{4a^3(15A + 17B + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(105A + 121B + 143C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, algorithm="`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$2 \left(15i \sqrt{2} (105A + 121B + 143C) a^3 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2 + c\right)^{\frac{13}{2}}, x\right)$$

54.336 Problem number 1282

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{11}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(32A + 41B + 42C) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{2(73A + 99B + 63C) (a^3 + a^3 \cos(dx + c)) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\ & + \frac{2(2A + 3B) (a^2 + a^2 \cos(dx + c))^2 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{21ad} \\ & + \frac{2A(a + a \cos(dx + c))^3 \left(\sec^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{9d} \\ & + \frac{4a^3(17A + 21B + 27C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{4a^3(17A + 21B + 27C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(11A + 13B + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algorithm="`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (11A + 13B + 21C) a^3 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2 + c\right)^{\frac{11}{2}}, x\right)$$

54.337 Problem number 1283

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7A + 9B + 5C) (a^3 + a^3 \cos(dx + c)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\ & + \frac{2(6A + 7B) (a^2 + a^2 \cos(dx + c))^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35ad} \\ & + \frac{2A(a + a \cos(dx + c))^3 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{4a^3(106A + 147B + 140C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{105d} \\ & - \frac{4a^3(7A + 9B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(13A + 21B + 35C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="f`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (13A + 21B + 35C) a^3 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2 + c\right)^{\frac{9}{2}}, x\right)$$

54.338 Problem number 1284

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(6A + 5B) (a^2 + a^2 \cos(dx + c))^2 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15ad} \\ & + \frac{2A(a + a \cos(dx + c))^3 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} - \frac{4a^3(21A + 20B + 5C) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & + \frac{2(33A + 35B + 15C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{4a^3(9A + 5B - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(3A + 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (3A + 5B + 5C) a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + 5B + 5C) a^3 \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2 + c\right)^{\frac{7}{2}}, x\right)$$

54.339 Problem number 1285

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \cos(dx + c))^3 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} - \frac{4a^3(20A + 5B - 6C) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & - \frac{2(35A + 15B - 3C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & + \frac{2(2A + B) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c) (\sqrt{\sec}(dx + c))}{ad} \\ & - \frac{4a^3(5A - 5B - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(5A + 5B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (5A + 5B + 3C) a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (5A \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2 + c\right)^{\frac{5}{2}}, x\right)$$

54.340 Problem number 1286

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(35A - 42B - 41C) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} - \frac{2(7A - C) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{7ad \sqrt{\sec(dx + c)}} \\ & - \frac{2(35A - 7B - 11C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{35d \sqrt{\sec(dx + c)}} \\ & + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & + \frac{4a^3(5A + 9B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(35A + 21B + 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (35A + 21B + 13C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (35A + 21B + 13C) a^3 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2 + C a^3 \cos(dx + c) + C a^3\right), x\right)$$

54.341 Problem number 1287

$$\int (a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(42A + 41B + 32C) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{9d \sqrt{\sec(dx + c)}} \\ & + \frac{2(3B + 2C) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{21ad \sqrt{\sec(dx + c)}} \\ & + \frac{2(63A + 99B + 73C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{315d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(27A + 21B + 17C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(21A + 13B + 11C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (21A + 13B + 11C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (21A + 13B + 11C) a^3 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C a^3 \cos(dx + c)^5 + (B + 3C) a^3 \cos(dx + c)^4 + (A + 3B + 3C) a^3 \cos(dx + c)^3 + (3A + 3B + C) a^3 \cos(dx + c)^2 + (2A + 2B + C) a^3 \cos(dx + c) + A a^3\right) \sqrt{\sec(c + dx)} dx\right)$$

54.342 Problem number 1288

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^3(264A + 253B + 210C) \sin(dx + c)}{1155d \sec(dx + c)^{\frac{3}{2}}} + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{11d \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(11B + 6C)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{99ad \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(99A + 143B + 105C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{693d \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{4a^3(143A + 121B + 105C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\
& + \frac{4a^3(21A + 17B + 15C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(143A + 121B + 105C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="f")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (143A + 121B + 105C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (143A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2}{\sqrt{\sec(dx + c)}}\right)$$

54.343 Problem number 1289

$$\int \frac{(a + a \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{20a^3(286A + 273B + 236C) \sin(dx + c)}{9009d \sec(dx + c)^{\frac{5}{2}}} + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{13d \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(13B + 6C)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{143ad \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(143A + 195B + 145C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{1287d \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{4a^3(221A + 195B + 175C) \sin(dx + c)}{585d \sec(dx + c)^{\frac{3}{2}}} + \frac{4a^3(121A + 105B + 95C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\
& + \frac{4a^3(221A + 195B + 175C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(121A + 105B + 95C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(195i \sqrt{2} (121A + 105B + 95C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 195i \sqrt{2} (121A \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \cos(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)^3 + (3A + 3B + C)a^3 \cos(dx + c)^2}{\sec(dx + c)^{\frac{3}{2}}}\right)$$

54.344 Problem number 1290

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5A - 5B + 3C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3ad} + \frac{(7A - 5B + 5C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5ad} \\ & - \frac{(A - B + C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \cos(dx + c))} + \frac{3(7A - 5B + 5C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5ad} \\ & - \frac{3(7A - 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{(5A - 5B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-5iA + 5iB - 3iC) \cos(dx + c)^3 + \sqrt{2} (-5iA + 5iB - 3iC) \cos(dx + c)^2 \right) \operatorname{weierstrassPInverse}(-4, 0)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A \right) \sec(dx + c)^{\frac{7}{2}}}{a \cos(dx + c) + a}, x \right)$$

54.345 Problem number 1291

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5A - 3B + 3C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3ad} - \frac{(A - B + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \cos(dx + c))} \\ & - \frac{(3A - 3B + C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{ad} \\ & + \frac{(3A - 3B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{(5A - 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-5iA + 3iB - 3iC)\cos(dx + c)^2 + \sqrt{2}(-5iA + 3iB - 3iC)\cos(dx + c)\right)\text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(dx + c)^{\frac{5}{2}}}{a\cos(dx + c) + a}, x\right)$$

54.346 Problem number 1292

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^{\frac{3}{2}}(c + dx)}{a + a\cos(c + dx)} dx$$

Optimal antiderivative

$$\frac{(3A - B + C)\sin(dx + c)(\sqrt{\sec(dx + c)})}{ad} - \frac{(A - B + C)\sin(dx + c)(\sqrt{\sec(dx + c)})}{d(a + a\cos(dx + c))}$$

$$\frac{(3A - B + C)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad}$$

$$\frac{(A - B - C)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(iA - iB - iC)\cos(dx + c) + \sqrt{2}(iA - iB - iC)\right)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(dx + c)^{\frac{3}{2}}}{a\cos(dx + c) + a}, x\right)$$

54.347 Problem number 1293

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)}}{a + a \cos(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \sin(dx + c)}{d(a + a \cos(dx + c)) \sqrt{\sec(dx + c)}} \\ & + \frac{(A - B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A + B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A - B + C) \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2}(-iA - iB + iC) \cos(dx + c) + \sqrt{2}(-iA - iB + iC)\right) \operatorname{weierstrassE}\left(\frac{dx}{2} + \frac{c}{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(\left(C \cos(dx + c)\right)^2 + B \cos(dx + c) + A\right) \sqrt{\sec(dx + c)}}{a \cos(dx + c) + a}, x\right)$$

54.348 Problem number 1294

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx)) \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \sin(dx + c)}{d(a + a \cos(dx + c)) \sec(dx + c)^{\frac{3}{2}}} + \frac{(3A - 3B + 5C) \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}} \\ & - \frac{(A - 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(3A - 3B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-3iA + 3iB - 5iC) \cos(dx + c) + \sqrt{2}(-3iA + 3iB - 5iC)\right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(a \cos(dx + c) + a) \sqrt{\sec(dx + c)}}, x\right)$$

54.349 Problem number 1295

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A - B + C) \sin(dx + c)}{d(a + a \cos(dx + c)) \sec(dx + c)^{\frac{5}{2}}} + \frac{(5A - 5B + 7C) \sin(dx + c)}{5ad \sec(dx + c)^{\frac{3}{2}}} - \frac{(3A - 5B + 5C) \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}} \\ & + \frac{3(5A - 5B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(3A - 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2}(-3iA + 5iB - 5iC) \cos(dx + c) + \sqrt{2}(-3iA + 5iB - 5iC)\right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}}, x\right)$$

54.350 Problem number 1296

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \sin(dx + c)}{d(a + a \cos(dx + c)) \sec(dx + c)^{\frac{7}{2}}} + \frac{(7A - 7B + 9C) \sin(dx + c)}{7ad \sec(dx + c)^{\frac{5}{2}}} \\ & - \frac{(5A - 7B + 7C) \sin(dx + c)}{5ad \sec(dx + c)^{\frac{3}{2}}} + \frac{5(7A - 7B + 9C) \sin(dx + c)}{21ad \sqrt{\sec(dx + c)}} \\ & - \frac{3(5A - 7B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5(7A - 7B + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fri`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25 \left(\sqrt{2} (7i A - 7i B + 9i C) \cos(dx + c) + \sqrt{2} (7i A - 7i B + 9i C) \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}}, x\right)$$

54.351 Problem number 1297

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(10A - 5B + 2C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d} - \frac{(7A - 4B + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d(1 + \cos(dx + c))} \\ & - \frac{(A - B + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d(a + a \cos(dx + c))^2} - \frac{(7A - 4B + C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{a^2d} \\ & + \frac{(7A - 4B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & + \frac{(10A - 5B + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-10i A + 5i B - 2i C) \cos(dx + c)^3 - 2 \sqrt{2} (10i A - 5i B + 2i C) \cos(dx + c)^2 + \sqrt{2} (-10i A + 5i B - 2i C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sec(dx + c)^{\frac{5}{2}}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x\right)$$

54.352 Problem number 1298

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(4A - B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{a^2d} - \frac{(5A - 2B - C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3a^2d(1 + \cos(dx + c))} \\ & - \frac{(A - B + C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d(a + a \cos(dx + c))^2} \\ & - \frac{(4A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & - \frac{(5A - 2B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (5i A - 2i B - i C) \cos(dx + c)^2 - 2\sqrt{2} (-5i A + 2i B + i C) \cos(dx + c) + \sqrt{2} (5i A - 2i B - i C)\right) \text{weierstr}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sec(dx + c)^{\frac{3}{2}}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x\right)$$

54.353 Problem number 1299

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)}}{(a + a \cos(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A - C) \sin(dx + c)}{a^2 d (1 + \cos(dx + c)) \sqrt{\sec(dx + c)}} - \frac{(A - B + C) \sin(dx + c)}{3d (a + a \cos(dx + c))^2 \sqrt{\sec(dx + c)}} \\ & + \frac{(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(2A + B + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-2i A - i B - 2i C) \cos(dx + c)^2 - 2\sqrt{2} (2i A + i B + 2i C) \cos(dx + c) + \sqrt{2} (-2i A - i B - 2i C)\right) \text{weierstr}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{\sec(dx + c)}}{a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2}, x\right)$$

54.354 Problem number 1300

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \sin(dx + c)}{3d(a + a \cos(dx + c))^2 \sec(dx + c)^{\frac{3}{2}}} + \frac{(A + 2B - 5C) \sin(dx + c)}{3a^2 d (1 + \cos(dx + c)) \sqrt{\sec(dx + c)}} \\ & - \frac{(B - 4C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(A + 2B - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-iA - 2iB + 5iC) \cos(dx + c)^2 - 2\sqrt{2}(iA + 2iB - 5iC) \cos(dx + c) + \sqrt{2}(-iA - 2iB + 5iC)\right) \operatorname{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\left(a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2\right) \sqrt{\sec(dx + c)}}, x\right)$$

54.355 Problem number 1301

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \sin(dx + c)}{3d(a + a \cos(dx + c))^2 \sec(dx + c)^{\frac{5}{2}}} \\ & - \frac{(A - 4B + 7C) \sin(dx + c)}{3a^2d(1 + \cos(dx + c)) \sec(dx + c)^{\frac{3}{2}}} + \frac{(2A - 5B + 10C) \sin(dx + c)}{3a^2d \sqrt{\sec(dx + c)}} \\ & - \frac{(A - 4B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & + \frac{(2A - 5B + 10C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-2iA + 5iB - 10iC) \cos(dx + c)^2 - 2\sqrt{2}(2iA - 5iB + 10iC) \cos(dx + c) + \sqrt{2}(-2iA + 5iB - 10iC)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(a^2 \cos(dx + c)^2 + 2a^2 \cos(dx + c) + a^2) \sec(dx + c)^{\frac{3}{2}}}, x\right)$$

54.356 Problem number 1302

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \sin(dx + c)}{3d(a + a \cos(dx + c))^2 \sec(dx + c)^{\frac{7}{2}}} - \frac{(A - 2B + 3C) \sin(dx + c)}{a^2d(1 + \cos(dx + c)) \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{(20A - 35B + 56C) \sin(dx + c)}{15a^2d \sec(dx + c)^{\frac{3}{2}}} - \frac{5(A - 2B + 3C) \sin(dx + c)}{3a^2d \sqrt{\sec(dx + c)}} \\ & + \frac{(20A - 35B + 56C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & - \frac{5(A - 2B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25 \left(\sqrt{2} (-i A + 2i B - 3i C) \cos(dx + c)^2 + 2 \sqrt{2} (-i A + 2i B - 3i C) \cos(dx + c) + \sqrt{2} (-i A + 2i B - 3i C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\left(a^2 \cos(dx + c)^2 + 2 a^2 \cos(dx + c) + a^2 \right) \sec(dx + c)^{\frac{5}{2}}}, x \right)$$

54.357 Problem number 1303

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(33A - 13B + 3C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{6a^3 d} - \frac{(A - B + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5d (a + a \cos(dx + c))^3} \\ & - \frac{(2A - B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3ad (a + a \cos(dx + c))^2} - \frac{(119A - 49B + 9C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{30d (a^3 + a^3 \cos(dx + c))} \\ & - \frac{(119A - 49B + 9C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{10a^3 d} \\ & + \frac{(119A - 49B + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(33A - 13B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (33i A - 13i B + 3i C) \cos(dx + c)^4 + 3 \sqrt{2} (33i A - 13i B + 3i C) \cos(dx + c)^3 + 3 \sqrt{2} (33i A - 13i B + \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx+c)^2 + B \cos(dx+c) + A \right) \sec(dx+c)^{\frac{5}{2}}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x \right)$$

54.358 Problem number 1304

$$\int \frac{(A + B \cos(c+dx) + C \cos^2(c+dx)) \sec^{\frac{3}{2}}(c+dx)}{(a + a \cos(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(49A - 9B - C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{10a^3d} - \frac{(A - B + C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d(a + a \cos(dx+c))^3} \\ & - \frac{(8A - 3B - 2C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{15ad(a + a \cos(dx+c))^2} - \frac{(13A - 3B - C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{6d(a^3 + a^3 \cos(dx+c))} \\ & - \frac{(49A - 9B - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & - \frac{(13A - 3B - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-13iA + 3iB + iC) \cos(dx+c)^3 + 3 \sqrt{2} (-13iA + 3iB + iC) \cos(dx+c)^2 + 3 \sqrt{2} (-13iA + 3iB + \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx+c)^2 + B \cos(dx+c) + A \right) \sec(dx+c)^{\frac{3}{2}}}{a^3 \cos(dx+c)^3 + 3a^3 \cos(dx+c)^2 + 3a^3 \cos(dx+c) + a^3}, x \right)$$

54.359 Problem number 1305

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)}}{(a + a \cos(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \sin(dx + c)}{5d (a + a \cos(dx + c))^3 \sqrt{\sec(dx + c)}} - \frac{(6A - B - 4C) \sin(dx + c)}{15ad (a + a \cos(dx + c))^2 \sqrt{\sec(dx + c)}} \\ & - \frac{(9A + B - C) \sin(dx + c)}{10d (a^3 + a^3 \cos(dx + c)) \sqrt{\sec(dx + c)}} \\ & + \frac{(9A + B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A + B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (3iA + iB + iC) \cos(dx + c)^3 + 3 \sqrt{2} (3iA + iB + iC) \cos(dx + c)^2 + 3 \sqrt{2} (3iA + iB + iC) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{\sec(dx + c)}}{a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3}, x\right)$$

54.360 Problem number 1306

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A - B + C) \sin(dx + c)}{5d(a + a \cos(dx + c))^3 \sec(dx + c)^{\frac{3}{2}}} + \frac{(4A + B - 6C) \sin(dx + c)}{15ad(a + a \cos(dx + c))^2 \sqrt{\sec(dx + c)}} \\ & -\frac{(A - B - 9C) \sin(dx + c)}{10d(a^3 + a^3 \cos(dx + c)) \sqrt{\sec(dx + c)}} \\ & + \frac{(A - B - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (iA + iB + 3iC) \cos(dx + c)^3 + 3 \sqrt{2} (iA + iB + 3iC) \cos(dx + c)^2 + 3 \sqrt{2} (iA + iB + 3iC) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3) \sqrt{\sec(dx + c)}}, x\right)$$

54.361 Problem number 1307

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A - B + C) \sin(dx + c)}{5d(a + a \cos(dx + c))^3 \sec(dx + c)^{\frac{5}{2}}} + \frac{(2A + 3B - 8C) \sin(dx + c)}{15ad(a + a \cos(dx + c))^2 \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{(A + 3B - 13C) \sin(dx + c)}{6d(a^3 + a^3 \cos(dx + c)) \sqrt{\sec(dx + c)}} \\ & -\frac{(A + 9B - 49C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + 3B - 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (i A + 3i B - 13i C) \cos(dx + c)^3 + 3 \sqrt{2} (i A + 3i B - 13i C) \cos(dx + c)^2 + 3 \sqrt{2} (i A + 3i B - 13i C) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\left(a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3 \right) \sec(dx + c)^{\frac{3}{2}}}, x \right)$$

54.362 Problem number 1308

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \sin(dx + c)}{5d(a + a \cos(dx + c))^3 \sec(dx + c)^{\frac{7}{2}}} + \frac{(B - 2C) \sin(dx + c)}{3ad(a + a \cos(dx + c))^2 \sec(dx + c)^{\frac{5}{2}}} \\ & - \frac{(9A - 49B + 119C) \sin(dx + c)}{30d(a^3 + a^3 \cos(dx + c)) \sec(dx + c)^{\frac{3}{2}}} + \frac{(3A - 13B + 33C) \sin(dx + c)}{6a^3d \sqrt{\sec(dx + c)}} \\ & - \frac{(9A - 49B + 119C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & + \frac{(3A - 13B + 33C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (3i A - 13i B + 33i C) \cos(dx + c)^3 + 3 \sqrt{2} (3i A - 13i B + 33i C) \cos(dx + c)^2 + 3 \sqrt{2} (3i A - 13i B + 33i C) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\left(a^3 \cos(dx + c)^3 + 3 a^3 \cos(dx + c)^2 + 3 a^3 \cos(dx + c) + a^3 \right) \sec(dx + c)^{\frac{5}{2}}}, x \right)$$

54.363 Problem number 1309

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \sin(dx + c)}{5d(a + a \cos(dx + c))^3 \sec(dx + c)^{\frac{9}{2}}} - \frac{(2A - 7B + 12C) \sin(dx + c)}{15ad(a + a \cos(dx + c))^2 \sec(dx + c)^{\frac{7}{2}}} \\ & - \frac{(13A - 33B + 63C) \sin(dx + c)}{10d(a^3 + a^3 \cos(dx + c)) \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{7(7A - 17B + 33C) \sin(dx + c)}{30a^3 d \sec(dx + c)^{\frac{3}{2}}} - \frac{(13A - 33B + 63C) \sin(dx + c)}{6a^3 d \sqrt{\sec(dx + c)}} \\ & + \frac{7(7A - 17B + 33C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(13A - 33B + 63C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="f
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output
```

$$5 \left(\sqrt{2} (-13i A + 33i B - 63i C) \cos(dx + c)^3 + 3 \sqrt{2} (-13i A + 33i B - 63i C) \cos(dx + c)^2 + 3 \sqrt{2} (-13i A + \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\left(a^3 \cos(dx + c)^3 + 3a^3 \cos(dx + c)^2 + 3a^3 \cos(dx + c) + a^3\right) \sec(dx + c)^{\frac{7}{2}}, x}\right)$$

54.364 Problem number 1359

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \sin(dx + c)}{4d(a + a \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^{\frac{7}{2}}} - \frac{(3A - 11B + 19C) \sin(dx + c)}{16ad(a + a \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{(7A - 15B + 31C) \sin(dx + c)}{16a^2d \sec(dx + c)^{\frac{3}{2}} \sqrt{a + a \cos(dx + c)}} - \frac{(11A - 35B + 63C) \sin(dx + c)}{16a^2d \sqrt{a + a \cos(dx + c)} \sqrt{\sec(dx + c)}} \\ & + \frac{(8A - 20B + 39C) \arcsin\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{4a^{\frac{5}{2}}d} \\ & - \frac{(43A - 115B + 219C) \arctan\left(\frac{\sin(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{\cos(dx + c)} \sqrt{a + a \cos(dx + c)}}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)}) \sqrt{2}}{32a^{\frac{5}{2}}d} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$\sqrt{2} \left((43A - 115B + 219C) \cos(dx + c)^3 + 3(43A - 115B + 219C) \cos(dx + c)^2 + 3(43A - 115B + 219C) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

54.365 Problem number 1360

$$\int (a + b \cos(c + dx)) (A + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5A + 7C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} + \frac{2Ab \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2aA \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{2b(3A + 5C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2b(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (5A + 7C)a \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (5A + 7C)a \cos(dx + c)^3$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Cb \cos(dx + c)^3 + Ca \cos(dx + c)^2 + Ab \cos(dx + c) + Aa \right) \sec(dx + c)^{\frac{9}{2}}, x \right)$$

54.366 Problem number 1361

$$\int (a + b \cos(c + dx)) (A + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2aA \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2a(3A + 5C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2a(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2b(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i\sqrt{2}(A+3C)b\cos(dx+c)^2$ weierstrassPInverse(-4, 0, $\cos(dx+c) + i\sin(dx+c)$) + $5i\sqrt{2}(A+3C)b\cos(dx+c)$

Fricas 1.3.7 via sagemath 9.3 output

integral($(Cb\cos(dx+c)^3 + Ca\cos(dx+c)^2 + Ab\cos(dx+c) + Aa)\sec(dx+c)^{\frac{7}{2}}, x$)

54.367 Problem number 1362

$$\int (a + b \cos(c + dx)) (A + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2aA \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2Ab \sin(dx+c) (\sqrt{\sec(dx+c)})}{d}$$

$$- \frac{2b(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

$$+ \frac{2a(A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}(A+3C)a\cos(dx+c)$ weierstrassPInverse(-4, 0, $\cos(dx+c) + i\sin(dx+c)$) + $i\sqrt{2}(A+3C)a\cos(dx+c)$

Fricas 1.3.7 via sagemath 9.3 output

integral($(Cb\cos(dx+c)^3 + Ca\cos(dx+c)^2 + Ab\cos(dx+c) + Aa)\sec(dx+c)^{\frac{5}{2}}, x$)

54.368 Problem number 1363

$$\int (a + b \cos(c + dx)) (A + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2bC \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2aA \sin(dx + c) (\sqrt{\sec(dx + c)})}{d}$$

$$- \frac{2a(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

$$+ \frac{2b(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (3A + C) \operatorname{bweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (3A + C) \operatorname{bweierstrassPInverse}(-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + Ca \cos(dx + c)^2 + Ab \cos(dx + c) + Aa\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

54.369 Problem number 1364

$$\int (a + b \cos(c + dx)) (A + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2bC \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2aC \sin(dx + c)}{3d \sqrt{\sec(dx + c)}}$$

$$+ \frac{2b(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

$$+ \frac{2a(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (3A + C) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (3A + C) \text{aweierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ca \cos(dx + c)^2 + Ab \cos(dx + c) + Aa\right) \sqrt{\sec(dx + c)}, x\right)$$

54.370 Problem number 1365

$$\int \frac{(a + b \cos(c + dx))(A + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bC \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2aC \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2b(7A + 5C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (7A + 5C) \text{bweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (7A + 5C) \text{bweierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb \cos(dx + c)^3 + Ca \cos(dx + c)^2 + Ab \cos(dx + c) + Aa}{\sqrt{\sec(dx + c)}}, x\right)$$

54.371 Problem number 1366

$$\int \frac{(a + b \cos(c + dx)) (A + C \cos^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bC \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2aC \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2b(9A + 7C) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(7A + 5C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2b(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-15i \sqrt{2} (7A + 5C) \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} (7A + 5C) \operatorname{aweierstrassP}$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb \cos(dx + c)^3 + Ca \cos(dx + c)^2 + Ab \cos(dx + c) + Aa}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

54.372 Problem number 1367

$$\int (a + b \cos(c + dx))^2 (A + C \cos^2(c + dx)) \sec^{\frac{11}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4ab(5A + 7C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} + \frac{2(4Ab^2 + a^2(7A + 9C)) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{45d} \\
& + \frac{8aAb \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{63d} + \frac{2A(a + b \cos(dx + c))^2 \left(\sec^{\frac{9}{2}}(dx + c) \right) \sin(dx + c)}{9d} \\
& + \frac{2(3b^2(3A + 5C) + a^2(7A + 9C)) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15d} \\
& - \frac{2(3b^2(3A + 5C) + a^2(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\
& + \frac{4ab(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-30i \sqrt{2} (5A + 7C) ab \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 30i \sqrt{2} (5A + 7C)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Cb^2 \cos(dx + c)^4 + 2Cab \cos(dx + c)^3 + 2Aab \cos(dx + c) + Aa^2 + (Ca^2 + Ab^2) \cos(dx + c)^2 \right) \sec(dx + c)^{\frac{11}{2}}, x \right)$$

54.373 Problem number 1368

$$\int (a + b \cos(c + dx))^2 (A + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(4Ab^2 + a^2(5A + 7C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} + \frac{8aAb \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2A(a + b \cos(dx + c))^2 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{4ab(3A + 5C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{4ab(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7b^2(A + 3C) + a^2(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-42i \sqrt{2} (3A + 5C)ab \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + 2Cab \cos(dx + c)^3 + 2Aab \cos(dx + c) + Aa^2 + (Ca^2 + Ab^2) \cos(dx + c)^2\right) \sec(dx + c)^{\frac{9}{2}}, x\right)$$

54.374 Problem number 1369

$$\int (a + b \cos(c + dx))^2 (A + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8aAb \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} + \frac{2A(a + b \cos(dx + c))^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2(4Ab^2 + a^2(3A + 5C)) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2(5b^2(A - C) + a^2(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-10i\sqrt{2}(A+3C)ab\cos(dx+c)^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}(A+3C)ab\cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4+2Cab\cos(dx+c)^3+2Aab\cos(dx+c)+Aa^2+(Ca^2+Ab^2)\cos(dx+c)^2\right)\sec(dx+c)^{\frac{7}{2}},x\right)$$

54.375 Problem number 1370

$$\int (a+b\cos(c+dx))^2(A+C\cos^2(c+dx))\sec^{\frac{5}{2}}(c+dx)dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a+b\cos(dx+c))^2\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3d} \\ & - \frac{2b^2(A-C)\sin(dx+c)}{3d\sqrt{\sec(dx+c)}} + \frac{8aAb\sin(dx+c)(\sqrt{\sec(dx+c)})}{3d} \\ & - \frac{4ab(A-C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(b^2(3A+C)+a^2(A+3C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-6i\sqrt{2}(A-C)ab\cos(dx+c)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+6i\sqrt{2}(A-C)ab\cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4+2Cab\cos(dx+c)^3+2Aab\cos(dx+c)+Aa^2+(Ca^2+Ab^2)\cos(dx+c)^2\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

54.376 Problem number 1371

$$\int (a + b \cos(c + dx))^2 (A + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(5A - C) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} - \frac{4ab(3A - C) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2A(a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2(5a^2(A - C) - b^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-10i \sqrt{2} (3A + C) \operatorname{abweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} (3A + C) \operatorname{abweierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + 2Cab \cos(dx + c)^3 + 2Aab \cos(dx + c) + Aa^2 + (Ca^2 + Ab^2) \cos(dx + c)^2\right) \sec(dx + c)^{\frac{3}{2}}, x\right)$$

54.377 Problem number 1372

$$\int (a + b \cos(c + dx))^2 (A + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8abC \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(4a^2C + b^2(7A + 5C)) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} + \frac{2C(a + b \cos(dx + c))^2 \sin(dx + c)}{7d \sqrt{\sec(dx + c)}} \\ & + \frac{4ab(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7a^2(3A + C) + b^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$42i \sqrt{2} (5A + 3C)ab\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 42i \sqrt{2} (5A + 3C)ab\text{weierstrassP}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

integral($\left((Cb^2 \cos(dx + c)^4 + 2Cab \cos(dx + c)^3 + 2Aab \cos(dx + c) + Aa^2 + (Ca^2 + Ab^2) \cos(dx + c)^2 \right) \sqrt{\sec(dx + c)}$, x)

54.378 Problem number 1373

$$\int \frac{(a + b \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8abC \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(4a^2C + b^2(9A + 7C)) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2C(a + b \cos(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{3}{2}}} + \frac{4ab(7A + 5C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(3a^2(5A + 3C) + b^2(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-30i \sqrt{2} (7A + 5C)ab\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 30i \sqrt{2} (7A + 5C)ab\text{weierstrassP}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

integral($\left(\frac{Cb^2 \cos(dx + c)^4 + 2Cab \cos(dx + c)^3 + 2Aab \cos(dx + c) + Aa^2 + (Ca^2 + Ab^2) \cos(dx + c)^2}{\sqrt{\sec(dx + c)}} \right)$, x)

54.379 Problem number 1374

$$\int \frac{(a + b \cos(c + dx))^2 (A + C \cos^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8abC \sin(dx + c)}{99d \sec(dx + c)^{\frac{7}{2}}} + \frac{2(4a^2C + b^2(11A + 9C)) \sin(dx + c)}{77d \sec(dx + c)^{\frac{5}{2}}} + \frac{2C(a + b \cos(dx + c))^2 \sin(dx + c)}{11d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{4ab(9A + 7C) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(11a^2(7A + 5C) + 5b^2(11A + 9C)) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{4ab(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(11a^2(7A + 5C) + 5b^2(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$462i \sqrt{2} (9A + 7C) ab \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 462i \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2 \cos(dx + c)^4 + 2Cab \cos(dx + c)^3 + 2Aab \cos(dx + c) + Aa^2 + (Ca^2 + Ab^2) \cos(dx + c)^2}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

54.380 Problem number 1375

$$\int (a + b \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sec^{\frac{11}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b(8Ab^2 + 9a^2(5A + 7C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\
& + \frac{2a(24Ab^2 + 7a^2(7A + 9C)) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\
& + \frac{4Ab(a + b \cos(dx + c))^2 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{21d} \\
& + \frac{2A(a + b \cos(dx + c))^3 \left(\sec^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{9d} \\
& + \frac{2a(9b^2(3A + 5C) + a^2(7A + 9C)) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15d} \\
& - \frac{2a(9b^2(3A + 5C) + a^2(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2b(7b^2(A + 3C) + 3a^2(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (3i(5A + 7C)a^2b + 7i(A + 3C)b^3) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{-}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^5 + 3Cab^2 \cos(dx + c)^4 + 3Aa^2b \cos(dx + c) + Aa^3 + (3Ca^2b + Ab^3) \cos(dx + c)^3 + (a + c)^{\frac{11}{2}}, x\right)
\right)$$

54.381 Problem number 1376

$$\int (a + b \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(24Ab^2 + 5a^2(5A + 7C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{12Ab(a + b \cos(dx + c))^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2A(a + b \cos(dx + c))^3 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{6b(8Ab^2 + 7a^2(3A + 5C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{35d} \\ & - \frac{2b(5b^2(A - C) + 3a^2(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(21b^2(A + 3C) + a^2(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2} (i(5A + 7C)a^3 + 21i(A + 3C)ab^2) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^5 + 3Cab^2 \cos(dx + c)^4 + 3Aa^2b \cos(dx + c) + Aa^3 + (3Ca^2b + Ab^3) \cos(dx + c)^3 + (a + c)^{\frac{9}{2}}, x\right)$$

54.382 Problem number 1377

$$\int (a + b \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4Ab(a + b \cos(dx + c))^2 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2A(a + b \cos(dx + c))^3 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & - \frac{2b^3(9A - 5C) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} + \frac{2a(8Ab^2 + a^2(3A + 5C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{2a(15b^2(A - C) + a^2(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(b^2(3A + C) + 3a^2(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (3i(A + 3C)a^2b + i(3A + C)b^3) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^5 + 3Cab^2 \cos(dx + c)^4 + 3Aa^2b \cos(dx + c) + Aa^3 + (3Ca^2b + Ab^3) \cos(dx + c)^3 + (a + c)^{\frac{7}{2}}, x\right)$$

54.383 Problem number 1378

$$\int (a + b \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b^3(35A - 3C) \sin(dx + c)}{15d \sec(dx + c)^{\frac{3}{2}}} + \frac{2A(a + b \cos(dx + c))^3 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} \\ & - \frac{2a b^2(5A - C) \sin(dx + c)}{d \sqrt{\sec(dx + c)}} + \frac{4Ab(a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2b(15a^2(A - C) - b^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3b^2(3A + C) + a^2(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i(A + 3C)a^3 + 3i(3A + C)ab^2) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^5 + 3Cab^2 \cos(dx + c)^4 + 3Aa^2b \cos(dx + c) + Aa^3 + (3Ca^2b + Ab^3) \cos(dx + c)^3 + (a + c)^{\frac{5}{2}}, x\right)$$

54.384 Problem number 1379

$$\int (a + b \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a b^2 (35A - 11C) \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} - \frac{2b(6a^2(7A - 3C) - b^2(7A + 5C)) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & - \frac{2b(7A - C)(a + b \cos(dx + c))^2 \sin(dx + c)}{7d \sqrt{\sec(dx + c)}} \\ & + \frac{2A(a + b \cos(dx + c))^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2a(5a^2(A - C) - 3b^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(21a^2(3A + C) + b^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (21i(3A + C)a^2b + i(7A + 5C)b^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-21i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^5 + 3Cab^2 \cos(dx + c)^4 + 3Aa^2b \cos(dx + c) + Aa^3 + (3Ca^2b + Ab^3) \cos(dx + c)^3 + (a + c)^{\frac{3}{2}}, x\right)\right)$$

54.385 Problem number 1380

$$\int (a + b \cos(c + dx))^3 (A + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(24a^2C + 7b^2(9A + 7C)) \sin(dx + c)}{315d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(63Ab^2 + 8a^2C + 45b^2C) \sin(dx + c)}{63d \sqrt{\sec(dx + c)}} \\ & + \frac{4aC(a + b \cos(dx + c))^2 \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} + \frac{2C(a + b \cos(dx + c))^3 \sin(dx + c)}{9d \sqrt{\sec(dx + c)}} \\ & + \frac{2b(9a^2(5A + 3C) + b^2(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7a^2(3A + C) + 3b^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (7i (3A + C)a^3 + 3i (7A + 5C)ab^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \sqrt{2} (-7$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb^3 cos(dx + c)^5 + 3Cab^2 cos(dx + c)^4 + 3Aa^2b cos(dx + c) + Aa^3 + (3Ca^2b + Ab^3) cos(dx + c)^3 + (C`

54.386 Problem number 1381

$$\int \frac{(a + b \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(8a^2C + 3b^2(11A + 9C)) \sin(dx + c)}{231d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a(99Ab^2 + 8a^2C + 77b^2C) \sin(dx + c)}{165d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{4aC(a + b \cos(dx + c))^2 \sin(dx + c)}{33d \sec(dx + c)^{\frac{3}{2}}} + \frac{2C(a + b \cos(dx + c))^3 \sin(dx + c)}{11d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2b(33a^2(7A + 5C) + 5b^2(11A + 9C)) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(a^2(5A + 3C) + b^2(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(33a^2(7A + 5C) + 5b^2(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (33i (7A + 5C)a^2b + 5i (11A + 9C)b^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb^3 cos(dx + c)^5 + 3Cab^2 cos(dx + c)^4 + 3Aa^2b cos(dx + c) + Aa^3 + (3Ca^2b + Ab^3) cos(dx + c)^3 + (C`

54.387 Problem number 1382

$$\int \frac{(a + b \cos(c + dx))^3 (A + C \cos^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(24a^2C + 11b^2(13A + 11C)) \sin(dx + c)}{1287d \sec(dx + c)^{\frac{7}{2}}} + \frac{6a(143Ab^2 + 8a^2C + 117b^2C) \sin(dx + c)}{1001d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{12aC(a + b \cos(dx + c))^2 \sin(dx + c)}{143d \sec(dx + c)^{\frac{5}{2}}} + \frac{2C(a + b \cos(dx + c))^3 \sin(dx + c)}{13d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2b(39a^2(9A + 7C) + 7b^2(13A + 11C)) \sin(dx + c)}{585d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2a(11a^2(7A + 5C) + 15b^2(11A + 9C)) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{2b(39a^2(9A + 7C) + 7b^2(13A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(11a^2(7A + 5C) + 15b^2(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{195 \sqrt{2} (11i(7A + 5C)a^3 + 15i(11A + 9C)ab^2) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 195}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx + c)^5 + 3Cab^2 \cos(dx + c)^4 + 3Aa^2b \cos(dx + c) + Aa^3 + (3Ca^2b + Ab^3) \cos(dx + c)^3 + (C}{\sec(dx + c)^{\frac{3}{2}}}\right)$$

54.388 Problem number 1383

$$\int (a + b \cos(c + dx))^4 (A + C \cos^2(c + dx)) \sec^{\frac{13}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(64Ab^4 + 15a^4(9A + 11C) + 9a^2b^2(101A + 143C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{693d} \\ & + \frac{4ab(96Ab^2 + a^2(673A + 891C)) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{3465d} \\ & + \frac{2(16Ab^2 + 3a^2(9A + 11C)) (a + b \cos(dx + c))^2 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{231d} \\ & + \frac{16Ab(a + b \cos(dx + c))^3 \left(\sec^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{99d} \\ & + \frac{2A(a + b \cos(dx + c))^4 \left(\sec^{\frac{11}{2}}(dx + c)\right) \sin(dx + c)}{11d} \\ & + \frac{8ab(3b^2(3A + 5C) + a^2(7A + 9C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{8ab(3b^2(3A + 5C) + a^2(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(77b^4(A + 3C) + 66a^2b^2(5A + 7C) + 5a^4(9A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (5i(9A + 11C)a^4 + 66i(5A + 7C)a^2b^2 + 77i(A + 3C)b^4) \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + \dots + c\right)^{\frac{13}{2}}, x\right)$$

54.389 Problem number 1384

$$\int (a + b \cos(c + dx))^4 (A + C \cos^2(c + dx)) \sec^{\frac{11}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4ab(32Ab^2 + a^2(101A + 147C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\ & + \frac{2(48Ab^2 + 7a^2(7A + 9C)) (a + b \cos(dx + c))^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\ & + \frac{16Ab(a + b \cos(dx + c))^3 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\ & + \frac{2A(a + b \cos(dx + c))^4 \left(\sec^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{9d} \\ & + \frac{2(192Ab^4 + 21a^4(7A + 9C) + 7a^2b^2(155A + 261C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{315d} \\ & - \frac{2(15b^4(A - C) + 18a^2b^2(3A + 5C) + a^4(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab(7b^2(A + 3C) + a^2(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$60 \sqrt{2} (i(5A + 7C)a^3b + 7i(A + 3C)ab^3) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left((Cb^4 \cos(dx + c))^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + \dots + c\right)^{\frac{11}{2}}, x\right)$$

54.390 Problem number 1385

$$\int (a + b \cos(c + dx))^4 (A + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(48Ab^2 + 5a^2(5A + 7C)) (a + b \cos(dx + c))^2 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{16Ab(a + b \cos(dx + c))^3 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2A(a + b \cos(dx + c))^4 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & - \frac{2b^2(b^2(87A - 35C) + 5a^2(5A + 7C)) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} \\ & + \frac{4ab(96Ab^2 + a^2(101A + 175C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{105d} \\ & - \frac{8ab(5b^2(A - C) + a^2(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7b^4(3A + C) + 42a^2b^2(A + 3C) + a^4(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i(5A + 7C)a^4 + 42i(A + 3C)a^2b^2 + 7i(3A + C)b^4) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + \dots + c\right)^{\frac{9}{2}}, x\right)$$

54.391 Problem number 1386

$$\int (a + b \cos(c + dx))^4 (A + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b^2(b^2(59A - 3C) + 3a^2(3A + 5C)) \sin(dx + c)}{15d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{16Ab(a + b \cos(dx + c))^3 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\ & + \frac{2A(a + b \cos(dx + c))^4 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & - \frac{4ab(2b^2(33A - 5C) + 3a^2(3A + 5C)) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & + \frac{2(16Ab^2 + a^2(3A + 5C)) (a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{2(30a^2b^2(A - C) - b^4(5A + 3C) + a^4(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab(b^2(3A + C) + a^2(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$20 \sqrt{2} (i(A + 3C)a^3b + i(3A + C)ab^3) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + \right. \right. \\ \left. \left. + c\right)^{\frac{7}{2}}, x\right)$$

54.392 Problem number 1387

$$\int (a + b \cos(c + dx))^4 (A + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a b^3 (175A - 27C) \sin(dx + c)}{105d \sec(dx + c)^{\frac{3}{2}}} + \frac{2A(a + b \cos(dx + c))^4 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} \\ & - \frac{2b^2(3a^2(49A - 13C) - b^2(7A + 5C)) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & - \frac{2b^2(21A - C)(a + b \cos(dx + c))^2 \sin(dx + c)}{7d \sqrt{\sec(dx + c)}} \\ & + \frac{16Ab(a + b \cos(dx + c))^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & - \frac{8ab(5a^2(A - C) - b^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(42a^2b^2(3A + C) + 7a^4(A + 3C) + b^4(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (7i(A + 3C)a^4 + 42i(3A + C)a^2b^2 + i(7A + 5C)b^4) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + \dots + c\right)^{\frac{5}{2}}, x\right)$$

54.393 Problem number 1388

$$\int (a + b \cos(c + dx))^4 (A + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(3a^2(105A - 41C) - 7b^2(9A + 7C)) \sin(dx + c)}{315d \sec(dx + c)^{\frac{3}{2}}} \\ & - \frac{4ab(a^2(63A - 31C) - 6b^2(7A + 5C)) \sin(dx + c)}{63d \sqrt{\sec(dx + c)}} \\ & - \frac{2ab(21A - 5C)(a + b \cos(dx + c))^2 \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} - \frac{2b(9A - C)(a + b \cos(dx + c))^3 \sin(dx + c)}{9d \sqrt{\sec(dx + c)}} \\ & + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2(15a^4(A - C) - 18a^2b^2(5A + 3C) - b^4(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab(7a^2(3A + C) + b^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$60 \sqrt{2} (7i(3A + C)a^3b + i(7A + 5C)ab^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 60 \sqrt{2} (-7$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + \dots + c\right)^{\frac{3}{2}}, x\right)$$

54.394 Problem number 1389

$$\int (a + b \cos(c + dx))^4 (A + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4ab(891Ab^2 + 96a^2C + 673b^2C) \sin(dx + c)}{3465d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(64a^4C + 15b^4(11A + 9C) + 9a^2b^2(143A + 101C)) \sin(dx + c)}{693d \sqrt{\sec(dx + c)}} \\ & + \frac{2(16a^2C + 3b^2(11A + 9C)) (a + b \cos(dx + c))^2 \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{16aC(a + b \cos(dx + c))^3 \sin(dx + c)}{99d \sqrt{\sec(dx + c)}} + \frac{2C(a + b \cos(dx + c))^4 \sin(dx + c)}{11d \sqrt{\sec(dx + c)}} \\ & + \frac{8ab(3a^2(5A + 3C) + b^2(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(77a^4(3A + C) + 66a^2b^2(7A + 5C) + 5b^4(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (77i(3A + C)a^4 + 66i(7A + 5C)a^2b^2 + 5i(11A + 9C)b^4) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + \dots\right)\right)$$

54.395 Problem number 1390

$$\int \frac{(a + b \cos(c + dx))^4 (A + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4ab(1573Ab^2 + 96a^2C + 1259b^2C) \sin(dx + c)}{9009d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(192a^4C + 77b^4(13A + 11C) + 11a^2b^2(637A + 491C)) \sin(dx + c)}{6435d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(48a^2C + 11b^2(13A + 11C)) (a + b \cos(dx + c))^2 \sin(dx + c)}{1287d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{16aC(a + b \cos(dx + c))^3 \sin(dx + c)}{143d \sec(dx + c)^{\frac{3}{2}}} + \frac{2C(a + b \cos(dx + c))^4 \sin(dx + c)}{13d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{8ab(11a^2(7A + 5C) + 5b^2(11A + 9C)) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{2(39a^4(5A + 3C) + 78a^2b^2(9A + 7C) + 7b^4(13A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\sec(dx + c)})}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab(11a^2(7A + 5C) + 5b^2(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$780 \sqrt{2} (11i(7A + 5C)a^3b + 5i(11A + 9C)ab^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 780$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + 4Cab^3 \cos(dx + c)^5 + 4Aa^3b \cos(dx + c) + Aa^4 + (6Ca^2b^2 + Ab^4) \cos(dx + c)^4 + 4Ab^3 \cos(dx + c)^3 + 4Aa^2b \cos(dx + c)^2 + 4Aab \cos(dx + c) + Aa^3}{\sqrt{\sec(dx + c)}}\right)$$

54.396 Problem number 1451

$$\int (a + b \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5aA + 7bB + 7aC) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} + \frac{2(Ab + Ba) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2aA \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{2(3Ab + 3Ba + 5bC) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2(3Ab + 3Ba + 5bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5aA + 7bB + 7aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i (5 A + 7 C) a + 7 i B b) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-i ($$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + (Ca + Bb) \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c)\right) \sec(dx + c)^{\frac{9}{2}}, x\right)$$

54.397 Problem number 1452

$$\int (a + b \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2(Ab + Ba) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2aA \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d}$$

$$+ \frac{2(3aA + 5bB + 5aC) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d}$$

$$- \frac{2(3aA + 5bB + 5aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}$$

$$+ \frac{2(Ab + Ba + 3bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm="fric"`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i Ba + i (A + 3C)b) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-i Ba$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Cb \cos(dx + c)^3 + (Ca + Bb) \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c) \right) \sec(dx + c)^{\frac{7}{2}}, x \right)$$

54.398 Problem number 1453

$$\int (a + b \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2aA \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2(Ab + Ba) \sin(dx + c) (\sqrt{\sec}(dx + c))}{d}$$

$$- \frac{2(Ab + Ba - bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}$$

$$+ \frac{2(3bB + a(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-i(A+3C)a-3iBb)\cos(dx+c)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}(i(A+3C)a+3iBb)\cos(dx+c)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb\cos(dx+c)^3+(Ca+Bb)\cos(dx+c)^2+Ca+(Ba+Ab)\cos(dx+c)\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

54.399 Problem number 1454

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^{\frac{3}{2}}(c+dx)dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bC\sin(dx+c)}{3d\sqrt{\sec(dx+c)}} + \frac{2aA\sin(dx+c)(\sqrt{\sec(dx+c)})}{d} \\ & + \frac{2(bB-a(A-C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(3Ab+3Ba+bC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-3iBa-i(3A+C)b)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}(3iBa+i(3A+C)b)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb\cos(dx+c)^3+(Ca+Bb)\cos(dx+c)^2+Ca+(Ba+Ab)\cos(dx+c)\right)\sec(dx+c)^{\frac{3}{2}},x\right)$$

54.400 Problem number 1455

$$\int (a + b \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bC \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(bB + aC) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2(5Ab + 5Ba + 3bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(bB + a(3A + C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$5 \sqrt{2} (i(3A + C)a + iBb) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-i(3A + C)a - iBb)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^3 + (Ca + Bb) \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c)\right) \sqrt{\sec(dx + c)}, x\right)$$

54.401 Problem number 1456

$$\int \frac{(a + b \cos(c + dx)) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bC \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(bB + aC) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(7Ab + 7Ba + 5bC) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(5aA + 3bB + 3aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7Ab + 7Ba + 5bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$5\sqrt{2}(7iBa + i(7A + 5C)b)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-7iBa - i(7A + 5C)b)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb \cos(dx + c)^3 + (Ca + Bb) \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c)}{\sqrt{\sec(dx + c)}}, x\right)$$

54.402 Problem number 1457

$$\int \frac{(a + b \cos(c + dx))(A + B \cos(c + dx) + C \cos^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bC \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2(bB + aC) \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(9Ab + 9Ba + 7bC) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(7aA + 5bB + 5aC) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(9Ab + 9Ba + 7bC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7aA + 5bB + 5aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$15\sqrt{2}(i(7A + 5C)a + 5iBb)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15\sqrt{2}(-i(7A + 5C)a - 5iBb)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb \cos(dx + c)^3 + (Ca + Bb) \cos(dx + c)^2 + Aa + (Ba + Ab) \cos(dx + c)}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

54.403 Problem number 1458

$$\int (a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{11}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(10Aab + 5B a^2 + 7b^2 B + 14abC) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} \\ & + \frac{2(4A b^2 + 18abB + a^2(7A + 9C)) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{45d} \\ & + \frac{2a(4Ab + 9Ba) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{63d} \\ & + \frac{2A(a + b \cos(dx + c))^2 \left(\sec^{\frac{9}{2}}(dx + c) \right) \sin(dx + c)}{9d} \\ & + \frac{2(18abB + 3b^2(3A + 5C) + a^2(7A + 9C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{2(18abB + 3b^2(3A + 5C) + a^2(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(10Aab + 5B a^2 + 7b^2 B + 14abC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algorithm="`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (5i B a^2 + 2i (5 A + 7 C) a b + 7i B b^2) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left((C b^2 \cos(dx + c)^4 + (2 C a b + B b^2) \cos(dx + c)^3 + A a^2 + (C a^2 + 2 B a b + A b^2) \cos(dx + c)^2 + (B a^2 + 2 a b C + c) \frac{11}{2}, x\right)\right)$$

54.404 Problem number 1459

$$\int (a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(4Ab^2 + 14abB + a^2(5A + 7C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} \\ & + \frac{2a(4Ab + 7Ba) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2A(a + b \cos(dx + c))^2 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2(6Aab + 3B a^2 + 5b^2B + 10abC) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2(6Aab + 3B a^2 + 5b^2B + 10abC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(14abB + 7b^2(A + 3C) + a^2(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i(5A + 7C)a^2 + 14iBab + 7i(A + 3C)b^2) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + (2Cab + Bb^2) \cos(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2Abc + c^2) \cos(dx + c) + c^2\right), x\right)$$

54.405 Problem number 1460

$$\int (a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(4Ab + 5Ba) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} + \frac{2A(a + b \cos(dx + c))^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2(4Ab^2 + 10abB + a^2(3A + 5C)) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2(10abB + 5b^2(A - C) + a^2(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ba^2 + 3b^2B + 2ab(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (iBa^2 + 2i(A + 3C)ab + 3iBb^2) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + (2Cab + Bb^2) \cos(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2Abc + Bb^2) \cos(dx + c) + c\right)^{\frac{7}{2}}, x\right)$$

54.406 Problem number 1461

$$\int (a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2A(a + b \cos(dx + c))^2 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} - \frac{2b^2(A - C) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2a(4Ab + 3Ba) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d}$$

$$- \frac{2(Ba^2 - b^2B + 2ab(A - C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

$$+ \frac{2(6abB + b^2(3A + C) + a^2(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i(A + 3C)a^2 - 6iBab - i(3A + C)b^2) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^4 + (2Cab + Bb^2) \cos(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2Abc + c^2) \cos(dx + c) + c^{\frac{5}{2}}\right), x\right)$$

54.407 Problem number 1462

$$\int (a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\frac{2b^2(5A - C) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} - \frac{2b(6aA - bB - 2aC) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}}$$

$$+ \frac{2A(a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d}$$

$$+ \frac{2(10abB - 5a^2(A - C) + b^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

$$+ \frac{2(3Ba^2 + b^2B + 2ab(3A + C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (3iBa^2 + 2i(3A+C)ab + iBb^2)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 5\sqrt{2} (-3iBa$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2 \cos(dx+c)^4 + (2Cab + Bb^2) \cos(dx+c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx+c)^2 + (Ba^2 + 2\right.\right. \\ \left.\left.+ c\right)^{\frac{3}{2}}, x\right)$$

54.408 Problem number 1463

$$\int (a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2b(7bB + 4aC) \sin(dx+c)}{35d \sec(dx+c)^{\frac{3}{2}}} + \frac{2(7Ab^2 + 14abB + 4a^2C + 5b^2C) \sin(dx+c)}{21d \sqrt{\sec(dx+c)}} \\ + \frac{2C(a + b \cos(dx+c))^2 \sin(dx+c)}{7d \sqrt{\sec(dx+c)}} \\ + \frac{2(10Aab + 5Ba^2 + 3b^2B + 6abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ + \frac{2(14abB + 7a^2(3A+C) + b^2(7A+5C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (7i(3A+C)a^2 + 14iBab + i(7A+5C)b^2)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 5\sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2 \cos(dx+c)^4 + (2Cab + Bb^2) \cos(dx+c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx+c)^2 + (Ba^2 + 2\right.\right.$$

54.409 Problem number 1464

$$\int \frac{(a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(9bB + 4aC) \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(9Ab^2 + 18abB + 4a^2C + 7b^2C) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2C(a + b \cos(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(14Aab + 7Ba^2 + 5b^2B + 10abC) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(18abB + 3a^2(5A + 3C) + b^2(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(14Aab + 7Ba^2 + 5b^2B + 10abC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (7i Ba^2 + 2i (7A + 5C)ab + 5i Bb^2) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \sqrt{2} (-7$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2 \cos(dx + c)^4 + (2Cab + Bb^2) \cos(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2$$

54.410 Problem number 1465

$$\int \frac{(a + b \cos(c + dx))^2 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b(11bB + 4aC) \sin(dx + c)}{99d \sec(dx + c)^{\frac{7}{2}}} + \frac{2(11Ab^2 + 22abB + 4a^2C + 9b^2C) \sin(dx + c)}{77d \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{2C(a + b \cos(dx + c))^2 \sin(dx + c)}{11d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(18Aab + 9Ba^2 + 7b^2B + 14abC) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(110abB + 11a^2(7A + 5C) + 5b^2(11A + 9C)) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\
& + \frac{2(18Aab + 9Ba^2 + 7b^2B + 14abC) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(110abB + 11a^2(7A + 5C) + 5b^2(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm='f')

```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (11i(7A + 5C)a^2 + 110iBab + 5i(11A + 9C)b^2) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2 \cos(dx + c)^4 + (2Cab + Bb^2) \cos(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \cos(dx + c)^2 + (Ba^2 + 2AbC) \cos(dx + c) + C^2}{\sec(dx + c)^{\frac{3}{2}}}\right)$$

54.411 Problem number 1466

$$\int (a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{11}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(8Ab^3 + 15a^3B + 54Bab^2 + 9a^2b(5A + 7C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\
& + \frac{2a(24Ab^2 + 99abB + 7a^2(7A + 9C)) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\
& + \frac{2(2Ab + 3Ba)(a + b \cos(dx + c))^2 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{21d} \\
& + \frac{2A(a + b \cos(dx + c))^3 \left(\sec^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{9d} \\
& + \frac{2(27a^2bB + 15b^3B + 9ab^2(3A + 5C) + a^3(7A + 9C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\
& - \frac{2(27a^2bB + 15b^3B + 9ab^2(3A + 5C) + a^3(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(5a^3B + 21Bab^2 + 7b^3(A + 3C) + 3a^2b(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algorithm="")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (5i Ba^3 + 3i (5A + 7C)a^2b + 21i Bab^2 + 7i (A + 3C)b^3) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^2b + 3Ab^2) \cos(dx + c)^2 + (Ca^2b + 3Ab^2) \cos(dx + c) + c\right)^{\frac{11}{2}}, x\right)$$

54.412 Problem number 1467

$$\int (a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(24Ab^2 + 63abB + 5a^2(5A + 7C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{2(6Ab + 7Ba)(a + b \cos(dx + c))^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2A(a + b \cos(dx + c))^3 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2(24A^3b^3 + 21a^3B + 98Ba^2b^2 + 21a^2b(3A + 5C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{35d} \\ & - \frac{2(3a^3B + 15Ba^2b^2 + 5b^3(A - C) + 3a^2b(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^2bB + 21b^3B + 21a^2b^2(A + 3C) + a^3(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm='fricas')`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i(5A + 7C)a^3 + 21iBa^2b + 21i(A + 3C)ab^2 + 21iBb^3) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)^2, x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^2b + 3Ab^2) \cos(dx + c)^2 + Cb^2\right), x\right)$$

54.413 Problem number 1468

$$\int (a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(6Ab + 5Ba)(a + b \cos(dx + c))^2 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\ & + \frac{2A(a + b \cos(dx + c))^3 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} - \frac{2b^2(9Ab + 5Ba - 5bC) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(24Ab^2 + 35abB + 3a^2(3A + 5C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{2(15a^2bB - 5b^3B + 15ab^2(A - C) + a^3(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^3B + 9Ba b^2 + b^3(3A + C) + 3a^2b(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm='f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i Ba^3 + 3i(A + 3C)a^2b + 9i Bab^2 + i(3A + C)b^3) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca + c)^{\frac{7}{2}}, x\right)$$

54.414 Problem number 1469

$$\int (a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b^2(35Ab + 15Ba - 3bC) \sin(dx + c)}{15d \sec(dx + c)^{\frac{3}{2}}} + \frac{2A(a + b \cos(dx + c))^3 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} \\ & - \frac{2b(6Ba^2 - b^2B + 3ab(5A - C)) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2(2Ab + Ba)(a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2(5a^3B - 15Bab^2 + 15a^2b(A - C) - b^3(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(9a^2bB + b^3B + 3ab^2(3A + C) + a^3(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm="f")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i(A + 3C)a^3 + 9iBa^2b + 3i(3A + C)ab^2 + iBb^3) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + \dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^2b + b^3) \cos(dx + c)^2 + (Ca^2b + b^3) \cos(dx + c) + c\right)^{\frac{5}{2}}, x\right)$$

54.415 Problem number 1470

$$\int (a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b^2(35aA - 7bB - 11aC) \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{2b(21abB - 6a^2(7A - 3C) + b^2(7A + 5C)) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & - \frac{2b(7A - C) (a + b \cos(dx + c))^2 \sin(dx + c)}{7d \sqrt{\sec(dx + c)}} \\ & + \frac{2A(a + b \cos(dx + c))^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & + \frac{2(15a^2bB + 3b^3B - 5a^3(A - C) + 3ab^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^3B + 21Ba^2b^2 + 21a^2b(3A + C) + b^3(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (21i Ba^3 + 21i(3A + C)a^2b + 21i Bab^2 + i(7A + 5C)b^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^2b + b^3) \cos(dx + c)^2 + a^3\right), x\right)$$

54.416 Problem number 1471

$$\int (a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b(63Ab^2 + 99abB + 24a^2C + 49b^2C) \sin(dx + c)}{315d \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(54a^2bB + 15b^3B + 8a^3C + 9ab^2(7A + 5C)) \sin(dx + c)}{63d \sqrt{\sec(dx + c)}} \\
& + \frac{2(3bB + 2aC)(a + b \cos(dx + c))^2 \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} + \frac{2C(a + b \cos(dx + c))^3 \sin(dx + c)}{9d \sqrt{\sec(dx + c)}} \\
& + \frac{2(15a^3B + 27Ba^2b + 9a^2b(5A + 3C) + b^3(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(21a^2bB + 5b^3B + 7a^3(3A + C) + 3ab^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (7i(3A + C)a^3 + 21iBa^2b + 3i(7A + 5C)ab^2 + 5iBb^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^2b + 3Ab^2) \cos(dx + c)^2 + (Cab + 3Ab) \cos(dx + c) + Aa\right) \sqrt{\sec(dx + c)}\right) dx$$

54.417 Problem number 1472

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b(99Ab^2 + 143abB + 24a^2C + 81b^2C) \sin(dx + c)}{693d \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(242a^2bB + 77b^3B + 24a^3C + 33ab^2(9A + 7C)) \sin(dx + c)}{495d \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(11bB + 6aC)(a + b \cos(dx + c))^2 \sin(dx + c)}{99d \sec(dx + c)^{\frac{3}{2}}} + \frac{2C(a + b \cos(dx + c))^3 \sin(dx + c)}{11d \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(77a^3B + 165Ba b^2 + 33a^2b(7A + 5C) + 5b^3(11A + 9C)) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\
& + \frac{2(27a^2bB + 7b^3B + 3a^3(5A + 3C) + 3ab^2(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(77a^3B + 165Ba b^2 + 33a^2b(7A + 5C) + 5b^3(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm='f')

```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (77i Ba^3 + 33i (7A + 5C)a^2b + 165i Bab^2 + 5i (11A + 9C)b^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + \dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^3 + 3Aab^2 + 3Bab^2) \cos(dx + c)^2 + (3Aab + 3Bab) \cos(dx + c) + Aa + Bb}{\sqrt{\sec(dx + c)}}\right)$$

54.418 Problem number 1473

$$\int \frac{(a + b \cos(c + dx))^3 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b(143Ab^2 + 195abB + 24a^2C + 121b^2C) \sin(dx + c)}{1287d \sec(dx + c)^{\frac{7}{2}}} \\
& + \frac{2(338a^2bB + 117b^3B + 24a^3C + 39ab^2(11A + 9C)) \sin(dx + c)}{1001d \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(13bB + 6aC)(a + b \cos(dx + c))^2 \sin(dx + c)}{143d \sec(dx + c)^{\frac{5}{2}}} + \frac{2C(a + b \cos(dx + c))^3 \sin(dx + c)}{13d \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(117a^3B + 273Ba^2b + 39a^2b(9A + 7C) + 7b^3(13A + 11C)) \sin(dx + c)}{585d \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(165a^2bB + 45b^3B + 11a^3(7A + 5C) + 15ab^2(11A + 9C)) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\
& + \frac{2(117a^3B + 273Ba^2b + 39a^2b(9A + 7C) + 7b^3(13A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(165a^2bB + 45b^3B + 11a^3(7A + 5C) + 15ab^2(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$195 \sqrt{2} (11i(7A + 5C)a^3 + 165iBa^2b + 15i(11A + 9C)ab^2 + 45iBb^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \cos(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 + (Ca^3 + 3Aab^2 + 3Bab^2 + Ab^3) \cos(dx + c)^2 + (3Aab^2 + 3Bab^2 + Ab^3) \cos(dx + c) + Aa^3}{\sec(dx + c)^{\frac{3}{2}}}\right)$$

54.419 Problem number 1474

$$\int (a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{13}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(64A b^4 + 660a^3 b B + 682a b^3 B + 15a^4(9A + 11C) + 9a^2 b^2(101A + 143C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{693d} \\
& + \frac{2a(192A b^3 + 539a^3 B + 1353Ba b^2 + 2a^2 b(673A + 891C)) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{3465d} \\
& + \frac{2(16A b^2 + 55ab B + 3a^2(9A + 11C)) (a + b \cos(dx + c))^2 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{231d} \\
& + \frac{2(8Ab + 11Ba) (a + b \cos(dx + c))^3 \left(\sec^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{99d} \\
& + \frac{2A(a + b \cos(dx + c))^4 \left(\sec^{\frac{11}{2}}(dx + c)\right) \sin(dx + c)}{11d} \\
& + \frac{2(7a^4 B + 54B a^2 b^2 + 15b^4 B + 12a b^3(3A + 5C) + 4a^3 b(7A + 9C)) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15d} \\
& - \frac{2(7a^4 B + 54B a^2 b^2 + 15b^4 B + 12a b^3(3A + 5C) + 4a^3 b(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(220a^3 b B + 308a b^3 B + 77b^4(A + 3C) + 66a^2 b^2(5A + 7C) + 5a^4(9A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(13/2),x, algorithm="
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (5i(9A + 11C)a^4 + 220iBa^3b + 66i(5A + 7C)a^2b^2 + 308iBab^3 + 77i(A + 3C)b^4) \cos(dx + c)^5 \operatorname{weierst}}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(2a^3b + 2ab^3) \cos(dx + c)^3 + 2a^2b^2 \cos(dx + c)^2 + 2ab^2 \cos(dx + c) + b^2\right)^{\frac{13}{2}}, x\right)$$

54.420 Problem number 1475

$$\int (a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{11}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(64Ab^3 + 75a^3B + 261Ba^2b^2 + a^2(202Ab + 294bC)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\ & + \frac{2(48Ab^2 + 117abB + 7a^2(7A + 9C)) (a + b \cos(dx + c))^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\ & + \frac{2(8Ab + 9Ba) (a + b \cos(dx + c))^3 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\ & + \frac{2A(a + b \cos(dx + c))^4 \left(\sec^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{9d} \\ & + \frac{2(192Ab^4 + 756a^3bB + 1098ab^3B + 21a^4(7A + 9C) + 7a^2b^2(155A + 261C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{315d} \\ & - \frac{2(36a^3bB + 60a^2b^2B + 15b^4(A - C) + 18a^2b^2(3A + 5C) + a^4(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5a^4B + 42Ba^2b^2 + 21b^4B + 28ab^3(A + 3C) + 4a^3b(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(11/2),x, algorithm="`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (5i Ba^4 + 4i (5A + 7C)a^3b + 42i Ba^2b^2 + 28i (A + 3C)ab^3 + 21i Bb^4) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(\dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left((Cb^4 \cos(dx + c))^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(Ca^2b^2 + 4Cab^3 + Ab^4) \cos(dx + c)^3 + 2(Cab^3 + Ab^4) \cos(dx + c)^2 + 2(Ca^2b^2 + 4Cab^3 + Ab^4) \cos(dx + c) + 2(Cab^3 + Ab^4)\right), x\right)$$

54.421 Problem number 1476

$$\int (a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{9}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(48Ab^2 + 77abB + 5a^2(5A + 7C)) (a + b \cos(dx + c))^2 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{2(8Ab + 7Ba) (a + b \cos(dx + c))^3 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2A(a + b \cos(dx + c))^4 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & - \frac{2b^2(98abB + b^2(87A - 35C) + 5a^2(5A + 7C)) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(192Ab^3 + 63a^3B + 413Ba b^2 + a^2(202Ab + 350bC)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{105d} \\ & - \frac{2(3a^4B + 30B a^2b^2 - 5b^4B + 20a b^3(A - C) + 4a^3b(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(28a^3bB + 84a b^3B + 7b^4(3A + C) + 42a^2b^2(A + 3C) + a^4(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(9/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i(5A + 7C)a^4 + 28iBa^3b + 42i(A + 3C)a^2b^2 + 84iBab^3 + 7i(3A + C)b^4) \cos(dx + c)^3 \operatorname{weierstrassP}(\dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(2a^3b + 3Ab^2) \cos(dx + c)^3 + (3a^2b^2 + 2Ab^3) \cos(dx + c)^2 + 2a^2b \cos(dx + c) + c\right)^{\frac{9}{2}}, x\right)$$

54.422 Problem number 1477

$$\int (a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(50abB + b^2(59A - 3C) + 3a^2(3A + 5C)) \sin(dx + c)}{15d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(8Ab + 5Ba)(a + b \cos(dx + c))^3 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\ & + \frac{2A(a + b \cos(dx + c))^4 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & - \frac{2b(105a^2bB - 5b^3B + 4ab^2(33A - 5C) + 6a^3(3A + 5C)) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & + \frac{2(16Ab^2 + 15abB + a^2(3A + 5C))(a + b \cos(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{2(20a^3bB - 20ab^3B + 30a^2b^2(A - C) - b^4(5A + 3C) + a^4(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^4B + 18Ba^2b^2 + b^4B + 4ab^3(3A + C) + 4a^3b(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(7/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (iBa^4 + 4i(A + 3C)a^3b + 18iBa^2b^2 + 4i(3A + C)ab^3 + iBb^4) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, c$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(Ca^3b + Ab^3) \cos(dx + c)^3 + (2Aab^2 + 2Bab^2) \cos(dx + c)^2 + (2Aab + 2Bab) \cos(dx + c) + a^2 + b^2\right), x\right)$$

54.423 Problem number 1478

$$\int (a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(350Aab + 105B a^2 - 21b^2B - 54abC) \sin(dx + c)}{105d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2A(a + b \cos(dx + c))^4 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} \\ & - \frac{2b(42a^3B - 28Ba b^2 + 3a^2b(49A - 13C) - b^3(7A + 5C)) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & - \frac{2b(21Ab + 7Ba - bC) (a + b \cos(dx + c))^2 \sin(dx + c)}{7d \sqrt{\sec(dx + c)}} \\ & + \frac{2(8Ab + 3Ba) (a + b \cos(dx + c))^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & - \frac{2(5a^4B - 30B a^2b^2 - 3b^4B + 20a^3b(A - C) - 4a b^3(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(84a^3bB + 28a b^3B + 42a^2b^2(3A + C) + 7a^4(A + 3C) + b^4(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(5/2),x, algorithm='fricas')
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output
```

$$5 \sqrt{2} (7i(A + 3C)a^4 + 84iBa^3b + 42i(3A + C)a^2b^2 + 28iBab^3 + i(7A + 5C)b^4) \cos(dx + c) \operatorname{weierstrassPInverse}\left(\frac{\cos(dx + c)}{2} + \frac{1}{2}, \sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(2Ab^2 + 3Aa^2) \cos(dx + c)^3 + (2Aa^2 + 2Ab^2) \cos(dx + c)^2 + 2Aa \cos(dx + c) + A\right) \sec^{\frac{5}{2}}(c + dx), x\right)$$

54.424 Problem number 1479

$$\int (a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(162abB - a^2(315A - 123C) + 7b^2(9A + 7C)) \sin(dx + c)}{315d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2b(117a^2bB + 15b^3B - a^3(126A - 62C) + 12ab^2(7A + 5C)) \sin(dx + c)}{63d \sqrt{\sec(dx + c)}} \\ & - \frac{2b(21aA - 3bB - 5aC) (a + b \cos(dx + c))^2 \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & - \frac{2b(9A - C) (a + b \cos(dx + c))^3 \sin(dx + c)}{9d \sqrt{\sec(dx + c)}} \\ & + \frac{2A(a + b \cos(dx + c))^4 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & + \frac{2(60a^3bB + 36ab^3B - 15a^4(A - C) + 18a^2b^2(5A + 3C) + b^4(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^4B + 42Ba^2b^2 + 5b^4B + 28a^3b(3A + C) + 4ab^3(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (21i Ba^4 + 28i (3A + C)a^3b + 42i Ba^2b^2 + 4i (7A + 5C)ab^3 + 5i Bb^4) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(a + b \cos(dx + c))^{\frac{3}{2}}, x\right)\right)$$

54.425 Problem number 1480

$$\int (a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx)) \sqrt{\sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(1353a^2bB + 539b^3B + 192a^3C + 2ab^2(891A + 673C)) \sin(dx + c)}{3465d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(682a^3bB + 660ab^3B + 64a^4C + 15b^4(11A + 9C) + 9a^2b^2(143A + 101C)) \sin(dx + c)}{693d \sqrt{\sec(dx + c)}} \\ & + \frac{2(33Ab^2 + 55abB + 16a^2C + 27b^2C) (a + b \cos(dx + c))^2 \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{2(11bB + 8aC) (a + b \cos(dx + c))^3 \sin(dx + c)}{99d \sqrt{\sec(dx + c)}} + \frac{2C(a + b \cos(dx + c))^4 \sin(dx + c)}{11d \sqrt{\sec(dx + c)}} \\ & + \frac{2(15a^4B + 54Ba^2b^2 + 7b^4B + 12a^3b(5A + 3C) + 4ab^3(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(308a^3bB + 220ab^3B + 77a^4(3A + C) + 66a^2b^2(7A + 5C) + 5b^4(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (77i(3A + C)a^4 + 308iBa^3b + 66i(7A + 5C)a^2b^2 + 220iBab^3 + 5i(11A + 9C)b^4) \operatorname{weierstrassPInverse}(\dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(2Ab^3 + 3Aab^2 + 2A^2b) \cos(dx + c)^3 + (2A^2b^2 + 2Aab) \cos(dx + c)^2 + 2A^2b^2 \cos(dx + c) + A^2\right) \sqrt{\sec(dx + c)} dx\right)$$

54.426 Problem number 1481

$$\int \frac{(a + b \cos(c + dx))^4 (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(2171a^2bB + 1053b^3B + 192a^3C + 2ab^2(1573A + 1259C)) \sin(dx + c)}{9009d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(3458a^3bB + 4004ab^3B + 192a^4C + 77b^4(13A + 11C) + 11a^2b^2(637A + 491C)) \sin(dx + c)}{6435d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(143Ab^2 + 221abB + 48a^2C + 121b^2C) (a + b \cos(dx + c))^2 \sin(dx + c)}{1287d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(13bB + 8aC) (a + b \cos(dx + c))^3 \sin(dx + c)}{143d \sec(dx + c)^{\frac{3}{2}}} + \frac{2C(a + b \cos(dx + c))^4 \sin(dx + c)}{13d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(77a^4B + 330Ba^2b^2 + 45b^4B + 44a^3b(7A + 5C) + 20ab^3(11A + 9C)) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{2(468a^3bB + 364ab^3B + 39a^4(5A + 3C) + 78a^2b^2(9A + 7C) + 7b^4(13A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(77a^4B + 330Ba^2b^2 + 45b^4B + 44a^3b(7A + 5C) + 20ab^3(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output
```

$$195 \sqrt{2} (77i Ba^4 + 44i (7A + 5C)a^3b + 330i Ba^2b^2 + 20i (11A + 9C)ab^3 + 45i Bb^4) \operatorname{weierstrassPInverse}(-4, 0, \dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \cos(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^4 + 2(2ab^3 + 3Aa^2b) \cos(dx + c)^3 + (Aa^2 + 2Ab^2) \cos(dx + c)^2 + 2Ab \cos(dx + c) + A^2}{\sqrt{\sec(dx + c)}}, dx\right)$$

55 Test file number 95

Test folder name:

test_cases/4_Trig_functions/4.2_Cosine/95_4.2.7-d_trig-^m-a+b-c_cos-ⁿ-^p

55.1 Problem number 61

$$\int \frac{1}{\sqrt{1 + \cos^2(x)}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{\frac{1}{2} - \frac{\cos(2x)}{2}} \operatorname{EllipticF}(\cos(x), i)}{\sin(x)}$$

command

`integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & \sqrt{2\sqrt{2}-3} (2i\sqrt{2}+3i) \operatorname{ellipticF}\left(\sqrt{2\sqrt{2}-3}(\cos(x)+i\sin(x)), 12\sqrt{2}+17\right) \\ & + \sqrt{2\sqrt{2}-3} (-2i\sqrt{2}-3i) \operatorname{ellipticF}\left(\sqrt{2\sqrt{2}-3}(\cos(x)-i\sin(x)), 12\sqrt{2}+17\right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{\cos(x)^2+1}}, x\right)$$

55.2 Problem number 62

$$\int \frac{1}{\sqrt{-1 - \cos^2(x)}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{\frac{1}{2} - \frac{\cos(2x)}{2}} \operatorname{EllipticF}(\cos(x), i) \sqrt{1 + \cos^2(x)}}{\sin(x) \sqrt{-1 - (\cos^2(x))}}$$

command

`integrate(1/(-1-cos(x)^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2\sqrt{2} + 3 \right) \sqrt{2\sqrt{2} - 3} \operatorname{ellipticF} \left(\sqrt{2\sqrt{2} - 3} e^{ix}, 12\sqrt{2} + 17 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{2}{\sqrt{e^{4ix} + 6e^{2ix} + 1}}, x \right)$$

55.3 Problem number 63

$$\int \frac{1}{\sqrt{a + b \cos^2(x)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{2} - \frac{\cos(2x)}{2}} \operatorname{EllipticF} \left(\cos(x), \sqrt{-\frac{b}{a}} \right) \sqrt{1 + \frac{b \cos^2(x)}{a}}}{\sin(x) \sqrt{a + b \cos^2(x)}}$$

command

`integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-2ib \sqrt{\frac{a^2 + ab}{b^2}} - 2ia - ib \right) \sqrt{b} \sqrt{\frac{2b \sqrt{\frac{a^2 + ab}{b^2}} - 2a - b}{b}} \operatorname{ellipticF} \left(\sqrt{\frac{2b \sqrt{\frac{a^2 + ab}{b^2}} - 2a - b}{b}} (\cos(x) + i) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{b \cos(x)^2 + a}}, x \right)$$

55.4 Problem number 75

$$\int \frac{1}{a + b \cos^6(x)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\cot(x)\sqrt{a^{\frac{1}{3}} + b^{\frac{1}{3}}}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}}\sqrt{a^{\frac{1}{3}} + b^{\frac{1}{3}}}} - \frac{\arctan\left(\frac{\cot(x)\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{1}{3}}b^{\frac{1}{3}}}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}}\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{1}{3}}b^{\frac{1}{3}}}}$$

$$- \frac{\arctan\left(\frac{\cot(x)\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{2}{3}}b^{\frac{1}{3}}}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}}\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{2}{3}}b^{\frac{1}{3}}}}$$

command

```
integrate(1/(a+b*cos(x)^6),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

55.5 Problem number 76

$$\int \frac{1}{a + b \cos^8(x)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\cot(x)\sqrt{(-a)^{\frac{1}{4}} - b^{\frac{1}{4}}}}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} - b^{\frac{1}{4}}}} + \frac{\arctan\left(\frac{\cot(x)\sqrt{(-a)^{\frac{1}{4}} - ib^{\frac{1}{4}}}}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} - ib^{\frac{1}{4}}}}$$

$$+ \frac{\arctan\left(\frac{\cot(x)\sqrt{(-a)^{\frac{1}{4}} + ib^{\frac{1}{4}}}}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} + ib^{\frac{1}{4}}}} + \frac{\arctan\left(\frac{\cot(x)\sqrt{(-a)^{\frac{1}{4}} + b^{\frac{1}{4}}}}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} + b^{\frac{1}{4}}}}$$

command

```
integrate(1/(a+b*cos(x)^8),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

55.6 Problem number 78

$$\int \frac{1}{a - b \cos^6(x)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\cot(x)\sqrt{a^{\frac{1}{3}} - b^{\frac{1}{3}}}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}}\sqrt{a^{\frac{1}{3}} - b^{\frac{1}{3}}}} - \frac{\arctan\left(\frac{\cot(x)\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{1}{3}}b^{\frac{1}{3}}}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}}\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{1}{3}}b^{\frac{1}{3}}}} - \frac{\arctan\left(\frac{\cot(x)\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{2}{3}}b^{\frac{1}{3}}}}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}}\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{2}{3}}b^{\frac{1}{3}}}}$$

command

```
integrate(1/(a-b*cos(x)^6),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

55.7 Problem number 79

$$\int \frac{1}{a - b \cos^8(x)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\cot(x)\sqrt{a^{\frac{1}{4}} - b^{\frac{1}{4}}}}{a^{\frac{1}{8}}}\right)}{4a^{\frac{7}{8}}\sqrt{a^{\frac{1}{4}} - b^{\frac{1}{4}}}} - \frac{\arctan\left(\frac{\cot(x)\sqrt{a^{\frac{1}{4}} - ib^{\frac{1}{4}}}}{a^{\frac{1}{8}}}\right)}{4a^{\frac{7}{8}}\sqrt{a^{\frac{1}{4}} - ib^{\frac{1}{4}}}}$$

$$- \frac{\arctan\left(\frac{\cot(x)\sqrt{a^{\frac{1}{4}} + ib^{\frac{1}{4}}}}{a^{\frac{1}{8}}}\right)}{4a^{\frac{7}{8}}\sqrt{a^{\frac{1}{4}} + ib^{\frac{1}{4}}}} - \frac{\arctan\left(\frac{\cot(x)\sqrt{a^{\frac{1}{4}} + b^{\frac{1}{4}}}}{a^{\frac{1}{8}}}\right)}{4a^{\frac{7}{8}}\sqrt{a^{\frac{1}{4}} + b^{\frac{1}{4}}}}$$

command

```
integrate(1/(a-b*cos(x)^8),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

56 Test file number 96

Test folder name:

```
test_cases/4_Trig_functions/4.2_Cosine/96_4.2.8-a+b*cos-^m-c+d_trig-^n
```

56.1 Problem number 9

$$\int \frac{(c + d \sec(e + fx))^4}{a + b \cos(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^3(4ac - bd) \operatorname{arctanh}(\sin(fx + e))}{2a^2 f} \\ & + \frac{d(2ac - bd)(2a^2 c^2 - 2abcd + b^2 d^2) \operatorname{arctanh}(\sin(fx + e))}{a^4 f} \\ & + \frac{2(ac - bd)^4 \operatorname{arctan}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{a+b}}\right)}{a^4 f \sqrt{a-b} \sqrt{a+b}} \\ & + \frac{d^4 \tan(fx + e)}{af} + \frac{d^2(6a^2 c^2 - 4abcd + b^2 d^2) \tan(fx + e)}{a^3 f} \\ & + \frac{d^3(4ac - bd) \sec(fx + e) \tan(fx + e)}{2a^2 f} + \frac{d^4(\tan^3(fx + e))}{3af} \end{aligned}$$

command

`integrate((c+d*sec(f*x+e))^4/(a+b*cos(f*x+e)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{6(a^4 c^4 - 4a^3 b c^3 d + 6a^2 b^2 c^2 d^2 - 4ab^3 c d^3 + b^4 d^4) \sqrt{-a^2 + b^2} \cos(fx + e)^3 \log\left(\frac{2ab \cos(fx + e) + (2a^2 - b^2) \cos(fx + e)}{b^2 \cos(fx + e)}\right)}{\dots} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

56.2 Problem number 14

$$\int \frac{1}{(a + b \cos(e + fx))(c + d \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2d(2ac^2 - ad^2 - bcd) \operatorname{arctanh}\left(\frac{\sqrt{c-d} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{c+d}}\right)}{(c-d)^{\frac{3}{2}} (c+d)^{\frac{3}{2}} (ac-bd)^2 f} \\ & + \frac{d^2 \sin(fx + e)}{(ac - bd)(c^2 - d^2) f (d + c \cos(fx + e))} + \frac{2a^2 \operatorname{arctan}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{a+b}}\right)}{(ac - bd)^2 f \sqrt{a-b} \sqrt{a+b}} \end{aligned}$$

command

```
integrate(1/(a+b*cos(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

57 Test file number 97

Test folder name:

test_cases/4_Trig_functions/4.2_Cosine/97_4.2.9_trig^m-a+b_cos^n+c_cos^-2_n-^p

57.1 Problem number 8

$$\int \frac{\csc^2(x)}{a + b \cos(x) + c \cos^2(x)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sin(x)}{2(a+b+c)(1-\cos(x))} + \frac{\sin(x)}{2(a-b+c)(1+\cos(x))} \\ & - \frac{2bc \arctan\left(\frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}}\right) \left(1 + \frac{b^2-2c(a+c)}{b\sqrt{-4ac+b^2}}\right)}{(a-b+c)(a+b+c)\sqrt{b-2c-\sqrt{-4ac+b^2}}\sqrt{b+2c-\sqrt{-4ac+b^2}}} \\ & - \frac{2bc \arctan\left(\frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}}\right) \left(1 + \frac{-b^2+2c(a+c)}{b\sqrt{-4ac+b^2}}\right)}{(a-b+c)(a+b+c)\sqrt{b-2c+\sqrt{-4ac+b^2}}\sqrt{b+2c+\sqrt{-4ac+b^2}}} \end{aligned}$$

command

```
integrate(csc(x)^2/(a+b*cos(x)+c*cos(x)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

57.2 Problem number 18

$$\int \frac{\sec(x)}{a + b \cos(x) + c \cos^2(x)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(\sin(x))}{a} - \frac{2c \operatorname{arctan}\left(\frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}}\right) \left(1 + \frac{b}{\sqrt{-4ac+b^2}}\right)}{a \sqrt{b-2c-\sqrt{-4ac+b^2}} \sqrt{b+2c-\sqrt{-4ac+b^2}}} - \frac{2c \operatorname{arctan}\left(\frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}}\right) \left(1 - \frac{b}{\sqrt{-4ac+b^2}}\right)}{a \sqrt{b-2c+\sqrt{-4ac+b^2}} \sqrt{b+2c+\sqrt{-4ac+b^2}}}$$

command

```
integrate(sec(x)/(a+b*cos(x)+c*cos(x)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

58 Test file number 98

Test folder name:

test_cases/4_Trig_functions/4.3_Tangent/98_4.3.0-a_trg-^m-b_tan-^n

58.1 Problem number 61

$$\int \csc(a + bx) \sqrt{d \tan(a + bx)} dx$$

Optimal antiderivative

$$\frac{\csc(bx + a) \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin(2bx + 2a)}\right) \sqrt{d \tan(bx + a)}}{\sin\left(a + \frac{\pi}{4} + bx\right) b}$$

command

`integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{i d} \operatorname{ellipticF}(\cos (b x+a)+i \sin (b x+a),-1)+\sqrt{-i d} \operatorname{ellipticF}(\cos (b x+a)-i \sin (b x+a),-1)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \tan (b x+a)} \operatorname{csc}(b x+a), x\right)$$

58.2 Problem number 62

$$\int \operatorname{csc}^3(a+b x) \sqrt{d \tan (a+b x)} dx$$

Optimal antiderivative

$$\frac{\frac{2 d \operatorname{csc}(b x+a)}{3 b \sqrt{d \tan (b x+a)}}}{2 \operatorname{csc}(b x+a) \sqrt{\frac{1}{2}+\frac{\sin (2 b x+2 a)}{2}} \operatorname{EllipticF}\left(\cos \left(a+\frac{\pi}{4}+b x\right), \sqrt{2}\right)\left(\sqrt{\sin (2 b x+2 a)}\right) \sqrt{d \tan (b x+a)}}{3 \sin \left(a+\frac{\pi}{4}+b x\right) b}$$

command

`integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\left(\cos (b x+a)^2-1\right) \sqrt{i d} \operatorname{ellipticF}(\cos (b x+a)+i \sin (b x+a),-1)+\left(\cos (b x+a)^2-1\right) \sqrt{-i d} \operatorname{ellipticF}(\cos (b x+a)-i \sin (b x+a),-1)\right)}{3\left(b \cos (b x+a)^2-b\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \tan (b x+a)} \operatorname{csc}(b x+a)^3, x\right)$$

58.3 Problem number 63

$$\int \csc^5(a + bx) \sqrt{d \tan(a + bx)} dx$$

Optimal antiderivative

$$\frac{4d \csc(bx + a)}{7b \sqrt{d \tan(bx + a)}} - \frac{2d(\csc^3(bx + a))}{7b \sqrt{d \tan(bx + a)}} - \frac{4 \csc(bx + a) \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin(2bx + 2a)}\right) \sqrt{d \tan(bx + a)}}{7 \sin\left(a + \frac{\pi}{4} + bx\right) b}$$

command

```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \left(\cos(bx + a)^4 - 2 \cos(bx + a)^2 + 1 \right) \sqrt{i d} \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + 2 \left(\cos(bx + a)^4 - 2 \cos(bx + a)^2 + 1 \right) \sqrt{i d} \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1) \right)}{7 \left(b \cos(bx + a) + d \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \tan(bx + a)} \csc(bx + a)^5, x\right)$$

58.4 Problem number 80

$$\int \csc(a + bx) (d \tan(a + bx))^{5/2} dx$$

Optimal antiderivative

$$\frac{d^2 \csc(bx + a) \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin(2bx + 2a)}\right) \sqrt{d \tan(bx + a)}}{3 \sin\left(a + \frac{\pi}{4} + bx\right) b} + \frac{2d \csc(bx + a) (d \tan(bx + a))^{3/2}}{3b}$$

command

```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{i d} d^2 \cos (b x+a) \operatorname{ellipticF}(\cos (b x+a)+i \sin (b x+a),-1)+\sqrt{-i d} d^2 \cos (b x+a) \operatorname{ellipticF}(\cos (b x+a)-i \sin (b x+a),-1)}{3 b \cos (b x+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \tan (b x+a)} d^2 \csc (b x+a) \tan (b x+a)^2, x\right)$$

58.5 Problem number 81

$$\int \csc ^3(a+b x)(d \tan (a+b x))^{5 / 2} d x$$

Optimal antiderivative

$$\frac{2 d^2 \csc (b x+a) \sqrt{\frac{1}{2}+\frac{\sin (2 b x+2 a)}{2}} \operatorname{EllipticF}\left(\cos \left(a+\frac{\pi}{4}+b x\right), \sqrt{2}\right)\left(\sqrt{\sin (2 b x+2 a)}\right) \sqrt{d \tan (b x+a)}}{3 \sin \left(a+\frac{\pi}{4}+b x\right) b} + \frac{2 d \csc (b x+a)(d \tan (b x+a))^{\frac{3}{2}}}{3 b}$$

command

`integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{i d} d^2 \cos (b x+a) \operatorname{ellipticF}(\cos (b x+a)+i \sin (b x+a),-1)+\sqrt{-i d} d^2 \cos (b x+a) \operatorname{ellipticF}(\cos (b x+a)-i \sin (b x+a),-1)\right)}{3 b \cos (b x+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \tan (b x+a)} d^2 \csc (b x+a)^3 \tan (b x+a)^2, x\right)$$

58.6 Problem number 82

$$\int \csc^5(a + bx)(d \tan(a + bx))^{5/2} dx$$

Optimal antiderivative

$$\frac{4d^3 \csc(bx + a)}{3b \sqrt{d \tan(bx + a)}} - \frac{4d^2 \csc(bx + a) \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin(2bx + 2a)}\right) \sqrt{d \tan(bx + a)}}{3 \sin\left(a + \frac{\pi}{4} + bx\right) b} + \frac{2d(\csc^3(bx + a))(d \tan(bx + a))^{\frac{3}{2}}}{3b}$$

command

```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \left(d^2 \cos(bx + a)^3 - d^2 \cos(bx + a) \right) \sqrt{i d} \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + 2 \left(d^2 \cos(bx + a)^3 - \right. \right.}{3 \left(b \cos(bx + a) \right)^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \tan(bx + a)} d^2 \csc(bx + a)^5 \tan(bx + a)^2, x\right)$$

58.7 Problem number 83

$$\int \csc^7(a + bx)(d \tan(a + bx))^{5/2} dx$$

Optimal antiderivative

$$\frac{40d^3 \csc(bx + a)}{21b \sqrt{d \tan(bx + a)}} - \frac{20d^3(\csc^3(bx + a))}{21b \sqrt{d \tan(bx + a)}} - \frac{40d^2 \csc(bx + a) \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin(2bx + 2a)}\right) \sqrt{d \tan(bx + a)}}{21 \sin\left(a + \frac{\pi}{4} + bx\right) b} + \frac{2d(\csc^5(bx + a))(d \tan(bx + a))^{\frac{3}{2}}}{3b}$$

command

```
integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(20 \left(d^2 \cos (bx + a)^5 - 2 d^2 \cos (bx + a)^3 + d^2 \cos (bx + a) \right) \sqrt{i d} \operatorname{ellipticF}(\cos (bx + a) + i \sin (bx + a), -1) + \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \tan (bx + a)} d^2 \csc (bx + a)^7 \tan (bx + a)^2, x\right)$$

58.8 Problem number 101

$$\int \frac{\csc(a + bx)}{(d \tan(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \csc (bx + a)}{3bd \sqrt{d \tan (bx + a)}} + \frac{\csc (bx + a) \sqrt{\frac{1}{2} + \frac{\sin (2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos \left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin (2bx + 2a)}\right) \sqrt{d \tan (bx + a)}}{3 \sin \left(a + \frac{\pi}{4} + bx\right) b d^2}$$

command

```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\cos (bx + a)^2 - 1\right) \sqrt{i d} \operatorname{ellipticF}(\cos (bx + a) + i \sin (bx + a), -1) + \left(\cos (bx + a)^2 - 1\right) \sqrt{-i d} \operatorname{ellipticF}(\cos (bx + a) - i \sin (bx + a), -1)}{3 \left(bd^2 \cos (bx + a)^2 - bd^2\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \tan (bx + a)} \csc (bx + a)}{d^2 \tan (bx + a)^2}, x\right)$$

58.9 Problem number 102

$$\int \frac{\csc^3(a + bx)}{(d \tan(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \csc(bx + a)}{21bd\sqrt{d \tan(bx + a)}} - \frac{2(\csc^3(bx + a))}{7bd\sqrt{d \tan(bx + a)}} + \frac{2 \csc(bx + a) \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \left(\sqrt{\sin(2bx + 2a)}\right) \sqrt{d \tan(bx + a)}}{21 \sin\left(a + \frac{\pi}{4} + bx\right) b d^2}$$

command

```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\cos(bx + a)^4 - 2 \cos(bx + a)^2 + 1 \right) \sqrt{i d} \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + \left(\cos(bx + a)^4 - 2 \cos(bx + a)^2 + 1 \right) \sqrt{-i d} \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1) \right) \sqrt{d \tan(bx + a)} \\ \hline 21 \left(b d^2 \cos(bx + a)^4 - 2 b d^2 \cos(bx + a)^2 + b d^2 \right) \sqrt{d \tan(bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \tan(bx + a)} \csc(bx + a)^3}{d^2 \tan(bx + a)^2}, x\right)$$

58.10 Problem number 115

$$\int (a \sin(e + fx))^{3/2} \sqrt{b \tan(e + fx)} dx$$

Optimal antiderivative

$$-\frac{2b(a \sin(fx + e))^{3/2}}{3f\sqrt{b \tan(fx + e)}} + \frac{4a^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) \left(\sqrt{\cos(fx + e)}\right) \sqrt{b \tan(fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a \sin(fx + e)}}$$

command

`integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{a \sin(fx + e)} a \sqrt{\frac{b \sin(fx + e)}{\cos(fx + e)}} \cos(fx + e) - \sqrt{2} \sqrt{-ab} \operatorname{aweierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) \right) / 3f$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)} a \sin(fx + e), x\right)$$

58.11 Problem number 117

$$\int \frac{\sqrt{b \tan(e + fx)}}{\sqrt{a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)} \sqrt{b \tan(fx + e)})}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{a \sin(fx + e)}}$$

command

`integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-ab} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + \sqrt{2} \sqrt{-ab} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e))}{af}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}}{a \sin(fx + e)}, x\right)$$

58.12 Problem number 119

$$\int \frac{\sqrt{b \tan(e + fx)}}{(a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{b}{a^2 f \sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}} + \frac{\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)} \sqrt{b \tan(fx + e)})}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^2 f \sqrt{a \sin(fx + e)}}$$

command

```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{2} \cos(fx + e)^2 - \sqrt{2}\right) \sqrt{-ab} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + \left(\sqrt{2} \cos(fx + e)^2 - \sqrt{2}\right) \sqrt{-ab} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e))}{2 \left(a^3 f \cos\left(\frac{fx}{2} + \frac{e}{2}\right) \sqrt{a \sin(fx + e)}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}}{\left(a^3 \cos(fx + e)^2 - a^3\right) \sin(fx + e)}, x\right)$$

58.13 Problem number 120

$$\int (a \sin(e + fx))^{5/2} (b \tan(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{24a^2b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) \sqrt{a \sin(fx + e)}}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \tan(fx + e)}} - \frac{2b(a \sin(fx + e))^{5/2} \sqrt{b \tan(fx + e)}}{5f} + \frac{12a^2b \sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}}{5f}$$

command

```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6 \sqrt{2} \sqrt{-ab} a^2 b \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 6 \sqrt{2} \sqrt{-ab} b \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(a^2 b \cos(fx + e)^2 - a^2 b\right) \sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)} \tan(fx + e), x\right)$$

58.14 Problem number 122

$$\int \sqrt{a \sin(e + fx)} (b \tan(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{4b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) \sqrt{a \sin(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \tan(fx + e)}} + \frac{2b \sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}}{f}$$

command

```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} \sqrt{-ab} b \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + \sqrt{2} \sqrt{-ab} b \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)} b \tan(fx + e), x\right)$$

58.15 Problem number 124

$$\int \frac{(b \tan(e + fx))^{3/2}}{(a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) \sqrt{a \sin(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^2 f \sqrt{\cos(fx + e)} \sqrt{b \tan(fx + e)}} \\ & + \frac{2b \sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}}{a^2 f} \end{aligned}$$

command

```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-ab} b \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + \sqrt{2} \sqrt{-ab} b \operatorname{weierstrassZeta}(-4, 0, \cos(fx + e) - i \sin(fx + e))}{a^2 f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)} b \tan(fx + e)}{a^2 \cos(fx + e)^2 - a^2}, x\right)$$

58.16 Problem number 126

$$\int \frac{(a \sin(e + fx))^{9/2}}{\sqrt{b \tan(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^4 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) \sqrt{a \sin(fx + e)}}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \tan(fx + e)}} \\ & - \frac{4a^2 b (a \sin(fx + e))^{\frac{5}{2}}}{15 f (b \tan(fx + e))^{\frac{3}{2}}} - \frac{2b (a \sin(fx + e))^{\frac{9}{2}}}{9 f (b \tan(fx + e))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6 \sqrt{2} \sqrt{-ab} a^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 6 \sqrt{2} \sqrt{-ab} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(a^4 \cos(fx + e)^4 - 2 a^4 \cos(fx + e)^2 + a^4 \right) \sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}}{b \tan(fx + e)}, x \right)$$

58.17 Problem number 128

$$\int \frac{(a \sin(e + fx))^{5/2}}{\sqrt{b \tan(e + fx)}} dx$$

Optimal antiderivative

$$\frac{4a^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right) \sqrt{a \sin(fx + e)}}{5 \cos \left(\frac{fx}{2} + \frac{e}{2} \right) f \sqrt{\cos(fx + e)} \sqrt{b \tan(fx + e)}} - \frac{2b(a \sin(fx + e))^{5/2}}{5f (b \tan(fx + e))^{3/2}}$$

command

`integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{a \sin(fx + e)} a^2 \sqrt{\frac{b \sin(fx + e)}{\cos(fx + e)}} \cos(fx + e)^2 + \sqrt{2} \sqrt{-ab} a^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\left(a^2 \cos(fx + e)^2 - a^2 \right) \sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}}{b \tan(fx + e)}, x \right)$$

58.18 Problem number 130

$$\int \frac{\sqrt{a \sin(e + fx)}}{\sqrt{b \tan(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) \sqrt{a \sin(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{b \tan(fx + e)}}$$

command

```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-ab} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + \sqrt{2} \sqrt{-ab} \operatorname{weierstrassZeta}(-4, 0, \cos(fx + e) - i \sin(fx + e))}{bf}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}}{b \tan(fx + e)}, x\right)$$

58.19 Problem number 132

$$\int \frac{1}{(a \sin(e + fx))^{3/2} \sqrt{b \tan(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) \sqrt{a \sin(fx + e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^2 f \sqrt{\cos(fx + e)} \sqrt{b \tan(fx + e)}} - \frac{b \sqrt{a \sin(fx + e)}}{a^2 f (b \tan(fx + e))^{3/2}}$$

command

```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{a \sin(fx + e)} \sqrt{\frac{b \sin(fx + e)}{\cos(fx + e)}} \cos(fx + e)^2 + \left(\sqrt{2} \cos(fx + e)^2 - \sqrt{2}\right) \sqrt{-ab} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + \left(\sqrt{2} \cos(fx + e)^2 - \sqrt{2}\right) \sqrt{-ab} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{a \sin (fx+e)} \sqrt{b \tan (fx+e)}}{\left(a^2 b \cos (fx+e)^2 - a^2 b\right) \tan (fx+e)}, x \right)$$

58.20 Problem number 139

$$\int \frac{(a \sin (e+fx))^{11/2}}{(b \tan (e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4a^4(a \sin (fx+e))^{\frac{3}{2}}}{77bf \sqrt{b \tan (fx+e)}} - \frac{2a^2(a \sin (fx+e))^{\frac{7}{2}}}{77bf \sqrt{b \tan (fx+e)}} + \frac{2(a \sin (fx+e))^{\frac{11}{2}}}{11bf \sqrt{b \tan (fx+e)}} \\ & + \frac{8a^6 \sqrt{\frac{\cos (fx+e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos (fx+e)}) \sqrt{b \tan (fx+e)}}{77 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f \sqrt{a \sin (fx+e)}} \end{aligned}$$

command

```
integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \sqrt{2} \sqrt{-ab} a^5 \operatorname{weierstrassPInverse}(-4, 0, \cos (fx+e) + i \sin (fx+e)) + 2 \sqrt{2} \sqrt{-ab} a^5 \operatorname{weierstrassPInverse}(- \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(a^5 \cos (fx+e)^4 - 2 a^5 \cos (fx+e)^2 + a^5\right) \sqrt{a \sin (fx+e)} \sqrt{b \tan (fx+e)} \sin (fx+e)}{b^2 \tan (fx+e)^2}, x \right)$$

58.21 Problem number 140

$$\int \frac{(a \sin (e+fx))^{7/2}}{(b \tan (e+fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2a^2(a \sin (fx+e))^{\frac{3}{2}}}{21bf \sqrt{b \tan (fx+e)}}+\frac{2(a \sin (fx+e))^{\frac{7}{2}}}{7bf \sqrt{b \tan (fx+e)}} \\ +\frac{4a^4 \sqrt{\frac{\cos (fx+e)}{2}+\frac{1}{2}} \operatorname{EllipticF}\left(\sin \left(\frac{fx}{2}+\frac{e}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos (fx+e)}\right) \sqrt{b \tan (fx+e)}}{21 \cos \left(\frac{fx}{2}+\frac{e}{2}\right) b^2 f \sqrt{a \sin (fx+e)}}$$

command

```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\sqrt{2} \sqrt{-ab} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos (fx+e)+i \sin (fx+e))+\sqrt{2} \sqrt{-ab} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos (fx+e)-i \sin (fx+e))\right) \\ \frac{1}{21 b^2 f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(a^3 \cos (fx+e)^2-a^3\right) \sqrt{a \sin (fx+e)} \sqrt{b \tan (fx+e)} \sin (fx+e)}{b^2 \tan (fx+e)^2}, x\right)$$

58.22 Problem number 141

$$\int \frac{(a \sin (e+fx))^{3/2}}{(b \tan (e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(a \sin (fx+e))^{\frac{3}{2}}}{3bf \sqrt{b \tan (fx+e)}} \\ +\frac{2a^2 \sqrt{\frac{\cos (fx+e)}{2}+\frac{1}{2}} \operatorname{EllipticF}\left(\sin \left(\frac{fx}{2}+\frac{e}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos (fx+e)}\right) \sqrt{b \tan (fx+e)}}{3 \cos \left(\frac{fx}{2}+\frac{e}{2}\right) b^2 f \sqrt{a \sin (fx+e)}}$$

command

```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{a \sin (fx+e)} a \sqrt{\frac{b \sin (fx+e)}{\cos (fx+e)}} \cos (fx+e) + \sqrt{2} \sqrt{-ab} \operatorname{weierstrassPInverse}(-4, 0, \cos (fx+e) + i \sin (fx+e))}{3 b^2 f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \sin (fx+e)} \sqrt{b \tan (fx+e)} a \sin (fx+e)}{b^2 \tan (fx+e)^2}, x\right)$$

58.23 Problem number 142

$$\int \frac{1}{\sqrt{a \sin (e+fx)} (b \tan (e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{bf \sqrt{a \sin (fx+e)} \sqrt{b \tan (fx+e)}} \frac{\sqrt{\frac{\cos (fx+e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos (fx+e)}) \sqrt{b \tan (fx+e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) b^2 f \sqrt{a \sin (fx+e)}}$$

command

`integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{2} \cos (fx+e)^2 - \sqrt{2}\right) \sqrt{-ab} \operatorname{weierstrassPInverse}(-4, 0, \cos (fx+e) + i \sin (fx+e)) + \left(\sqrt{2} \cos (fx+e)\right)^2}{2\left(ab^2 f \cos (fx+e)\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \sin (fx+e)} \sqrt{b \tan (fx+e)}}{ab^2 \sin (fx+e) \tan (fx+e)^2}, x\right)$$

58.24 Problem number 143

$$\int \frac{1}{(a \sin(e + fx))^{5/2} (b \tan(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{1}{3bf (a \sin(fx + e))^{\frac{5}{2}} \sqrt{b \tan(fx + e)}} + \frac{1}{6a^2bf \sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}} \\ & - \frac{\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)} \sqrt{b \tan(fx + e)})}{6 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^2b^2f \sqrt{a \sin(fx + e)}} \end{aligned}$$

command

`integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} \cos(fx + e)^4 - 2\sqrt{2} \cos(fx + e)^2 + \sqrt{2}\right) \sqrt{-ab} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}}{\left(a^3b^2 \cos(fx + e)^2 - a^3b^2\right) \sin(fx + e) \tan(fx + e)^2}, x\right)$$

58.25 Problem number 144

$$\int \frac{1}{(a \sin(e + fx))^{9/2} (b \tan(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{1}{5bf (a \sin(fx + e))^{\frac{9}{2}} \sqrt{b \tan(fx + e)}} + \frac{1}{30a^2bf (a \sin(fx + e))^{\frac{5}{2}} \sqrt{b \tan(fx + e)}} \\ & + \frac{1}{12a^4bf \sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}} \\ & - \frac{\sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)} \sqrt{b \tan(fx + e)})}{12 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) a^4b^2f \sqrt{a \sin(fx + e)}} \end{aligned}$$

command

```
integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} \cos(fx + e)^6 - 3\sqrt{2} \cos(fx + e)^4 + 3\sqrt{2} \cos(fx + e)^2 - \sqrt{2} \right) \sqrt{-ab} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{a \sin(fx + e)} \sqrt{b \tan(fx + e)}}{\left(a^5 b^2 \cos(fx + e)^4 - 2 a^5 b^2 \cos(fx + e)^2 + a^5 b^2 \right) \sin(fx + e) \tan(fx + e)^2}, x \right)$$

58.26 Problem number 241

$$\int \sec^5(a + bx) (d \tan(a + bx))^{3/2} dx$$

Optimal antiderivative

$$\frac{4d^2 \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF} \left(\cos \left(a + \frac{\pi}{4} + bx \right), \sqrt{2} \right) \sec(bx + a) \left(\sqrt{\sin(2bx + 2a)} \right)}{77 \sin \left(a + \frac{\pi}{4} + bx \right) b \sqrt{d \tan(bx + a)}} - \frac{4d \sec(bx + a) \sqrt{d \tan(bx + a)}}{77b} - \frac{2d(\sec^3(bx + a)) \sqrt{d \tan(bx + a)}}{77b} + \frac{2d(\sec^5(bx + a)) \sqrt{d \tan(bx + a)}}{11b}$$

command

```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \sqrt{i d} d \cos(bx + a)^5 \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + 2 \sqrt{-i d} d \cos(bx + a)^5 \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1) \right) \sqrt{d \tan(bx + a)}$$

$$77 b \cos(bx + a)^5$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{d \tan(bx + a)} d \sec(bx + a)^5 \tan(bx + a), x \right)$$

58.27 Problem number 242

$$\int \sec^3(a + bx)(d \tan(a + bx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2d^2 \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \sec(bx + a) \left(\sqrt{\sin(2bx + 2a)}\right)}{21 \sin\left(a + \frac{\pi}{4} + bx\right) b \sqrt{d \tan(bx + a)}} - \frac{2d \sec(bx + a) \sqrt{d \tan(bx + a)}}{21b} + \frac{2d(\sec^3(bx + a)) \sqrt{d \tan(bx + a)}}{7b}$$

command

```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{i d} d \cos(bx + a)^3 \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + \sqrt{-i d} d \cos(bx + a)^3 \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1) \right)}{21 b \cos(bx + a)^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \tan(bx + a)} d \sec(bx + a)^3 \tan(bx + a), x\right)$$

58.28 Problem number 243

$$\int \sec(a + bx)(d \tan(a + bx))^{3/2} dx$$

Optimal antiderivative

$$\frac{d^2 \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \sec(bx + a) \left(\sqrt{\sin(2bx + 2a)}\right)}{3 \sin\left(a + \frac{\pi}{4} + bx\right) b \sqrt{d \tan(bx + a)}} + \frac{2d \sec(bx + a) \sqrt{d \tan(bx + a)}}{3b}$$

command

```
integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{i d} d \cos (b x+a) \operatorname{ellipticF}(\cos (b x+a)+i \sin (b x+a),-1)+\sqrt{-i d} d \cos (b x+a) \operatorname{ellipticF}(\cos (b x+a)-i \sin (b x+a),-1)}{3 b \cos (b x+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \tan (b x+a)} d \sec (b x+a) \tan (b x+a), x\right)$$

58.29 Problem number 253

$$\int \frac{\sec ^5(e+f x)}{\sqrt{d \tan (e+f x)}} d x$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4 \sqrt{\frac{1}{2}+\frac{\sin (2 f x+2 e)}{2}} \operatorname{EllipticF}\left(\cos \left(e+\frac{\pi}{4}+f x\right), \sqrt{2}\right) \sec (f x+e)\left(\sqrt{\sin (2 f x+2 e)}\right)}{7 \sin \left(e+\frac{\pi}{4}+f x\right) f \sqrt{d \tan (f x+e)}} \\ & +\frac{4 \sec (f x+e) \sqrt{d \tan (f x+e)}}{7 d f}+\frac{2\left(\sec ^3(f x+e)\right) \sqrt{d \tan (f x+e)}}{7 d f} \end{aligned}$$

command

`integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2 \sqrt{i d} \cos (f x+e)^3 \operatorname{ellipticF}(\cos (f x+e)+i \sin (f x+e),-1)+2 \sqrt{-i d} \cos (f x+e)^3 \operatorname{ellipticF}(\cos (f x+e)-i \sin (f x+e),-1)\right)}{7 d f \cos (f x+e)^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \tan (f x+e)} \sec (f x+e)^5}{d \tan (f x+e)}, x\right)$$

58.30 Problem number 254

$$\int \frac{\sec^3(e + fx)}{\sqrt{d \tan(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(2fx + 2e)}{2}} \operatorname{EllipticF}\left(\cos\left(e + \frac{\pi}{4} + fx\right), \sqrt{2}\right) \sec(fx + e) \left(\sqrt{\sin(2fx + 2e)}\right)}{3 \sin\left(e + \frac{\pi}{4} + fx\right) f \sqrt{d \tan(fx + e)}} + \frac{2 \sec(fx + e) \sqrt{d \tan(fx + e)}}{3df}$$

command

```
integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{id} \cos(fx + e) \operatorname{ellipticF}(\cos(fx + e) + i \sin(fx + e), -1) + \sqrt{-id} \cos(fx + e) \operatorname{ellipticF}(\cos(fx + e) - i \sin(fx + e), -1) \right)}{3df \cos(fx + e)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \tan(fx + e)} \sec(fx + e)^3}{d \tan(fx + e)}, x\right)$$

58.31 Problem number 255

$$\int \frac{\sec(e + fx)}{\sqrt{d \tan(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{2} + \frac{\sin(2fx + 2e)}{2}} \operatorname{EllipticF}\left(\cos\left(e + \frac{\pi}{4} + fx\right), \sqrt{2}\right) \sec(fx + e) \left(\sqrt{\sin(2fx + 2e)}\right)}{\sin\left(e + \frac{\pi}{4} + fx\right) f \sqrt{d \tan(fx + e)}}$$

command

```
integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{i d} \operatorname{ellipticF}(\cos (f x+e)+i \sin (f x+e),-1)+\sqrt{-i d} \operatorname{ellipticF}(\cos (f x+e)-i \sin (f x+e),-1)}{d f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \tan (f x+e)} \sec (f x+e)}{d \tan (f x+e)}, x\right)$$

58.32 Problem number 269

$$\int \frac{\sec (a+b x)}{(d \tan (a+b x))^{5 / 2}} d x$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{2}+\frac{\sin (2 b x+2 a)}{2}} \operatorname{EllipticF}\left(\cos \left(a+\frac{\pi}{4}+b x\right), \sqrt{2}\right) \sec (b x+a)\left(\sqrt{\sin (2 b x+2 a)}\right)}{3 \sin \left(a+\frac{\pi}{4}+b x\right) b d^2 \sqrt{d \tan (b x+a)}} - \frac{2 \sec (b x+a)}{3 b d(d \tan (b x+a))^{\frac{3}{2}}}$$

command

`integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\cos (b x+a)^2-1\right) \sqrt{i d} \operatorname{ellipticF}(\cos (b x+a)+i \sin (b x+a),-1)+\left(\cos (b x+a)^2-1\right) \sqrt{-i d} \operatorname{ellipticF}(\cos (b x+a)-i \sin (b x+a),-1)}{3\left(b d^3 \cos (b x+a)^2-b d^3\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \tan (b x+a)} \sec (b x+a)}{d^3 \tan (b x+a)^3}, x\right)$$

58.33 Problem number 292

$$\int (d \sec(e + fx))^{3/2} \sqrt{b \tan(e + fx)} dx$$

Optimal antiderivative

$$\frac{d^2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{b \tan(fx + e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \sec(fx + e)} \sqrt{\sin(fx + e)}} + \frac{d^2 (b \tan(fx + e))^{\frac{3}{2}}}{bf \sqrt{d \sec(fx + e)}}$$

command

`integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 d \sqrt{\frac{b \sin(fx + e)}{\cos(fx + e)}} \sqrt{\frac{d}{\cos(fx + e)}} \sin(fx + e) - i \sqrt{-2i bd} \operatorname{dweierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)} d \sec(fx + e), x\right)$$

58.34 Problem number 294

$$\int \frac{\sqrt{b \tan(e + fx)}}{\sqrt{d \sec(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{b \tan(fx + e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \sec(fx + e)} \sqrt{\sin(fx + e)}}$$

command

`integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{-2i bd} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e))) - i \sqrt{2i bd} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) - i \sin(fx + e)))}{df}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)}}{d \sec(fx + e)}, x\right)$$

58.35 Problem number 296

$$\int \frac{\sqrt{b \tan(e + fx)}}{(d \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{4\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{b \tan(fx + e)}}{5 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{d \sec(fx + e)} \sqrt{\sin(fx + e)}} + \frac{2(b \tan(fx + e))^{\frac{3}{2}}}{5bf (d \sec(fx + e))^{\frac{5}{2}}}$$

command

`integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{\frac{b \sin(fx + e)}{\cos(fx + e)}} \sqrt{\frac{d}{\cos(fx + e)}} \cos(fx + e)^2 \sin(fx + e) + i \sqrt{-2ibd} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(fx + e)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)}}{d^3 \sec(fx + e)^3}, x\right)$$

58.36 Problem number 298

$$\int \frac{\sqrt{b \tan(e + fx)}}{(d \sec(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$-\frac{8\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{b \tan(fx + e)}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^4 f \sqrt{d \sec(fx + e)} \sqrt{\sin(fx + e)}} + \frac{2(b \tan(fx + e))^{\frac{3}{2}}}{9bf (d \sec(fx + e))^{\frac{9}{2}}} + \frac{4(b \tan(fx + e))^{\frac{3}{2}}}{15bd^2 f (d \sec(fx + e))^{\frac{5}{2}}}$$

command

`integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left((5 \cos (fx + e)^4 + 6 \cos (fx + e)^2) \sqrt{\frac{b \sin (fx + e)}{\cos (fx + e)}} \sqrt{\frac{d}{\cos (fx + e)}} \sin (fx + e) + 6i \sqrt{-2i bd} \operatorname{weierstrassZeta} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{d \sec (fx + e)} \sqrt{b \tan (fx + e)}}{d^5 \sec (fx + e)^5}, x \right)$$

58.37 Problem number 299

$$\int (d \sec (e + fx))^{5/2} (b \tan (e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{b^2 d^2 \sqrt{\frac{1}{2} + \frac{\sin (fx + e)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right), \sqrt{2} \right) \sqrt{d \sec (fx + e)} \left(\sqrt{\sin (fx + e)} \right)}{6 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) f \sqrt{b \tan (fx + e)}} + \frac{b (d \sec (fx + e))^{5/2} \sqrt{b \tan (fx + e)}}{3f} - \frac{b d^2 \sqrt{d \sec (fx + e)} \sqrt{b \tan (fx + e)}}{6f}$$

command

```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{-2i bd} bd^2 \cos (fx + e)^2 \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e)) + \sqrt{2i bd} bd^2 \cos (fx + e)^2 \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) - i \sin (fx + e))$$

12 f cos

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{d \sec (fx + e)} \sqrt{b \tan (fx + e)} bd^2 \sec (fx + e)^2 \tan (fx + e), x \right)$$

58.38 Problem number 301

$$\int \sqrt{d \sec(e + fx)} (b \tan(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{b^2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \sec(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{b \tan(fx + e)}} + \frac{b \sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)}}{f}$$

command

```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-2i bd} \operatorname{bweierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)) + \sqrt{2i bd} \operatorname{bweierstrassPInverse}(4, 0, \cos(fx + e) - i \sin(fx + e))}{2f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)} b \tan(fx + e), x\right)$$

58.39 Problem number 303

$$\int \frac{(b \tan(e + fx))^{3/2}}{(d \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \sec(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{b \tan(fx + e)}} - \frac{2b \sqrt{b \tan(fx + e)}}{3f (d \sec(fx + e))^{\frac{3}{2}}}$$

command

```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2b \sqrt{\frac{b \sin (fx + e)}{\cos (fx + e)}} \sqrt{\frac{d}{\cos (fx + e)}} \cos (fx + e)^2 - \sqrt{-2i bd} \operatorname{bweierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e))}{3 d^2 f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec (fx + e)} \sqrt{b \tan (fx + e)} b \tan (fx + e)}{d^2 \sec (fx + e)^2}, x\right)$$

58.40 Problem number 305

$$\int \frac{(b \tan (e + fx))^{3/2}}{(d \sec (e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{4b^2 \sqrt{\frac{1}{2} + \frac{\sin (fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \sec (fx + e)} \left(\sqrt{\sin (fx + e)}\right)}{21 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^4 f \sqrt{b \tan (fx + e)}} - \frac{2b \sqrt{b \tan (fx + e)}}{7f (d \sec (fx + e))^{\frac{7}{2}}} + \frac{2b \sqrt{b \tan (fx + e)}}{21d^2 f (d \sec (fx + e))^{\frac{3}{2}}}$$

command

`integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{-2i bd} \operatorname{bweierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e)) + \sqrt{2i bd} \operatorname{bweierstrassPInverse}(4, 0, \cos (fx + e) - i \sin (fx + e)) \right)}{21 d^4 f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec (fx + e)} \sqrt{b \tan (fx + e)} b \tan (fx + e)}{d^4 \sec (fx + e)^4}, x\right)$$

58.41 Problem number 308

$$\int (d \sec(e + fx))^{3/2} (b \tan(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{b^2 d^2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{b \tan(fx + e)}}{2 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \sec(fx + e)} \sqrt{\sin(fx + e)}} + \frac{b(d \sec(fx + e))^{\frac{3}{2}} (b \tan(fx + e))^{\frac{3}{2}}}{3f} - \frac{b d^2 (b \tan(fx + e))^{\frac{3}{2}}}{2f \sqrt{d \sec(fx + e)}}$$

command

```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i \sqrt{-2i b d} b^2 d \cos(fx + e)^2 \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e))) - 3i \sqrt{2i b d} b^2 d \sin(fx + e)^2 \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) - i \sin(fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)} b^2 d \sec(fx + e) \tan(fx + e)^2, x\right)$$

58.42 Problem number 310

$$\int \frac{(b \tan(e + fx))^{5/2}}{\sqrt{d \sec(e + fx)}} dx$$

Optimal antiderivative

$$\frac{3b^2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{b \tan(fx + e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{d \sec(fx + e)} \sqrt{\sin(fx + e)}} + \frac{b(b \tan(fx + e))^{\frac{3}{2}}}{f \sqrt{d \sec(fx + e)}}$$

command

```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2b^2 \sqrt{\frac{b \sin(fx + e)}{\cos(fx + e)}} \sqrt{\frac{d}{\cos(fx + e)}} \sin(fx + e) - 3i \sqrt{-2ibd} b^2 \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)} b^2 \tan(fx + e)^2}{d \sec(fx + e)}, x\right)$$

58.43 Problem number 312

$$\int \frac{(b \tan(e + fx))^{5/2}}{(d \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{6b^2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{b \tan(fx + e)}}{5 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{d \sec(fx + e)} \sqrt{\sin(fx + e)}} - \frac{2b(b \tan(fx + e))^{3/2}}{5f (d \sec(fx + e))^{5/2}}$$

command

`integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2b^2 \sqrt{\frac{b \sin(fx + e)}{\cos(fx + e)}} \sqrt{\frac{d}{\cos(fx + e)}} \cos(fx + e)^2 \sin(fx + e) - 3i \sqrt{-2ibd} b^2 \text{weierstrassZeta}(4, 0, \text{weierstrassP$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)} b^2 \tan(fx + e)^2}{d^3 \sec(fx + e)^3}, x\right)$$

58.44 Problem number 314

$$\int \frac{(b \tan(e + fx))^{5/2}}{(d \sec(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{4b^2 \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{b \tan(fx+e)}}{15 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^4 f \sqrt{d \sec(fx+e)} \sqrt{\sin(fx+e)}} - \frac{2b(b \tan(fx+e))^{\frac{3}{2}}}{9f(d \sec(fx+e))^{\frac{9}{2}}} + \frac{2b(b \tan(fx+e))^{\frac{3}{2}}}{15d^2 f (d \sec(fx+e))^{\frac{5}{2}}}$$

command

`integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(-3i \sqrt{-2i bd} b^2 \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx+e) + i \sin(fx+e))) + 3i \sqrt{2i bd} b^2 \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx+e) - i \sin(fx+e))) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec(fx+e)} \sqrt{b \tan(fx+e)} b^2 \tan(fx+e)^2}{d^5 \sec(fx+e)^5}, x\right)$$

58.45 Problem number 316

$$\int \frac{(d \sec(e+fx))^{5/2}}{\sqrt{b \tan(e+fx)}} dx$$

Optimal antiderivative

$$\frac{d^2 \sqrt{\frac{1}{2} + \frac{\sin(fx+e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \sec(fx+e)} \left(\sqrt{\sin(fx+e)}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{b \tan(fx+e)}} + \frac{d^2 \sqrt{d \sec(fx+e)} \sqrt{b \tan(fx+e)}}{bf}$$

command

`integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{-2i bd} d^2 \operatorname{weierstrassPInverse}(4, 0, \cos(fx+e) + i \sin(fx+e)) + \sqrt{2i bd} d^2 \operatorname{weierstrassPInverse}(4, 0, \cos(fx+e) - i \sin(fx+e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)} d^2 \sec(fx + e)^2}{b \tan(fx + e)}, x\right)$$

58.46 Problem number 318

$$\int \frac{\sqrt{d \sec(e + fx)}}{\sqrt{b \tan(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \sec(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) f \sqrt{b \tan(fx + e)}}$$

command

`integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-2ibd} \text{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)) + \sqrt{2ibd} \text{weierstrassPInverse}(4, 0, \cos(fx + e) - i \sin(fx + e))}{bf}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)}}{b \tan(fx + e)}, x\right)$$

58.47 Problem number 320

$$\int \frac{1}{(d \sec(e + fx))^{3/2} \sqrt{b \tan(e + fx)}} dx$$

Optimal antiderivative

$$\frac{4\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \text{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \sec(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) d^2 f \sqrt{b \tan(fx + e)}} + \frac{2\sqrt{b \tan(fx + e)}}{3bf (d \sec(fx + e))^{3/2}}$$

command

```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{\frac{b \sin(fx + e)}{\cos(fx + e)}} \sqrt{\frac{d}{\cos(fx + e)}} \cos(fx + e)^2 + \sqrt{-2ibd} \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)) \right) / 3bd^2f$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)}}{bd^2 \sec(fx + e)^2 \tan(fx + e)}, x\right)$$

58.48 Problem number 323

$$\int \frac{(d \sec(e + fx))^{3/2}}{(b \tan(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2d^2}{bf \sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)}} \\ & + \frac{2d^2 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{b \tan(fx + e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) b^2 f \sqrt{d \sec(fx + e)} \sqrt{\sin(fx + e)}} \end{aligned}$$

command

```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2d \sqrt{\frac{b \sin(fx + e)}{\cos(fx + e)}} \sqrt{\frac{d}{\cos(fx + e)}} \cos(fx + e)^2 + i \sqrt{-2ibd} d \sin(fx + e) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e) + i \sin(fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)} d \sec(fx + e)}{b^2 \tan(fx + e)^2}, x\right)$$

58.49 Problem number 325

$$\int \frac{1}{\sqrt{d \sec(e + fx)} (b \tan(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2}{bf \sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)}} \\ & + \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{b \tan(fx + e)}}{\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) b^2 f \sqrt{d \sec(fx + e)} \sqrt{\sin(fx + e)}} \end{aligned}$$

command

`integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{\frac{b \sin(fx + e)}{\cos(fx + e)}} \sqrt{\frac{d}{\cos(fx + e)}} \cos(fx + e)^2 + i \sqrt{-2i bd} \sin(fx + e) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInv} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)}}{b^2 d \sec(fx + e) \tan(fx + e)^2}, x\right)$$

58.50 Problem number 327

$$\int \frac{1}{(d \sec(e + fx))^{5/2} (b \tan(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2}{bf (d \sec(fx + e))^{\frac{5}{2}} \sqrt{b \tan(fx + e)}} \\ & + \frac{24 \sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{b \tan(fx + e)}}{5 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) b^2 d^2 f \sqrt{d \sec(fx + e)} \sqrt{\sin(fx + e)}} \\ & - \frac{12 (b \tan(fx + e))^{\frac{3}{2}}}{5 b^3 f (d \sec(fx + e))^{\frac{5}{2}}} \end{aligned}$$

command

```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6i \sqrt{-2i bd} \sin (fx + e) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e))) - 6i \sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{d \sec (fx + e)} \sqrt{b \tan (fx + e)}}{b^2 d^3 \sec (fx + e)^3 \tan (fx + e)^2}, x \right)$$

58.51 Problem number 329

$$\int \frac{(d \sec (e + fx))^{5/2}}{(b \tan (e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2d^2 \sqrt{\frac{1}{2} + \frac{\sin (fx + e)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right), \sqrt{2} \right) \sqrt{d \sec (fx + e)} \left(\sqrt{\sin (fx + e)} \right)}{3 \sin \left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2} \right) b^2 f \sqrt{b \tan (fx + e)}} - \frac{2d^2 \sqrt{d \sec (fx + e)}}{3bf (b \tan (fx + e))^{\frac{3}{2}}}$$

command

```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2d^2 \sqrt{\frac{b \sin (fx + e)}{\cos (fx + e)}} \sqrt{\frac{d}{\cos (fx + e)}} \cos (fx + e)^2 + \left(d^2 \cos (fx + e)^2 - d^2 \right) \sqrt{-2i bd} \operatorname{weierstrassPInverse}(4, 0, \cos (fx + e))$$

$$3 \left(b^3 f \cos (fx + e) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{d \sec (fx + e)} \sqrt{b \tan (fx + e)} d^2 \sec (fx + e)^2}{b^3 \tan (fx + e)^3}, x \right)$$

58.52 Problem number 331

$$\int \frac{\sqrt{d \sec(e + fx)}}{(b \tan(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{4\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \sec(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) b^2 f \sqrt{b \tan(fx + e)}} - \frac{2\sqrt{d \sec(fx + e)}}{3bf (b \tan(fx + e))^{\frac{3}{2}}}$$

command

`integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{\frac{b \sin(fx + e)}{\cos(fx + e)}} \sqrt{\frac{d}{\cos(fx + e)}} \cos(fx + e)^2 - \sqrt{-2ibd} \left(\cos(fx + e)^2 - 1 \right) \operatorname{weierstrassPInverse}(4, 0, \cos(fx + e)) \right) \frac{1}{3 \left(b^3 f \cos(fx + e) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec(fx + e)} \sqrt{b \tan(fx + e)}}{b^3 \tan(fx + e)^3}, x\right)$$

58.53 Problem number 333

$$\int \frac{1}{(d \sec(e + fx))^{3/2} (b \tan(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{8\sqrt{\frac{1}{2} + \frac{\sin(fx + e)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right), \sqrt{2}\right) \sqrt{d \sec(fx + e)} \left(\sqrt{\sin(fx + e)}\right)}{3 \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) b^2 d^2 f \sqrt{b \tan(fx + e)}} - \frac{4\sqrt{b \tan(fx + e)}}{3b^3 f (d \sec(fx + e))^{\frac{3}{2}}} - \frac{2}{3bf (d \sec(fx + e))^{\frac{3}{2}} (b \tan(fx + e))^{\frac{3}{2}}}$$

command

```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \sqrt{-2i bd} \left(\cos (fx + e)^2 - 1 \right) \text{weierstrassPInverse}(4, 0, \cos (fx + e) + i \sin (fx + e)) + 2 \sqrt{2i bd} \left(\cos (fx + e) + i \sin (fx + e) \right) \right) \frac{1}{3 \left(b^3 d^2 \sec^2 (fx + e) \tan^3 (fx + e) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{d \sec (fx + e)} \sqrt{b \tan (fx + e)}}{b^3 d^2 \sec (fx + e)^2 \tan (fx + e)^3}, x \right)$$

59 Test file number 101

Test folder name:

test_cases/4_Trig_functions/4.3_Tangent/101_4.3.1.2-d_sec-^m-a+b_tan-^n

59.1 Problem number 185

$$\int (e \sec (c + dx))^{7/2} (a + ia \tan (c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ia(e \sec (dx + c))^{7/2}}{7d} + \frac{2ae(e \sec (dx + c))^{5/2} \sin (dx + c)}{5d} \\ & - \frac{6ae^4 \sqrt{\frac{\cos (dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d \sqrt{\cos (dx + c)} \sqrt{e \sec (dx + c)}} \\ & + \frac{6ae^3 \sin (dx + c) \sqrt{e \sec (dx + c)}}{5d} \end{aligned}$$

command

```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\frac{\sqrt{2} \left(21i ae^{(7i dx + 7i c + \frac{7}{2})} + 77i ae^{(5i dx + 5i c + \frac{7}{2})} + 23i ae^{(3i dx + 3i c + \frac{7}{2})} + 7i ae^{(i dx + i c + \frac{7}{2})} \right) e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 21 \left(i \sqrt{2} ae^{\frac{7}{2}} + i \sqrt{2} a \right) \right)$$

35 (de^{(6i dx + 6i c)} + 3

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \left(-42i a e^3 e^{(7i dx + 7i c)} - 154i a e^3 e^{(5i dx + 5i c)} - 46i a e^3 e^{(3i dx + 3i c)} - 14i a e^3 e^{(i dx + i c)} \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}$$

$$35 \left(d e^{(6i dx + 6i c)} + 3 d e^{(4i dx + 4i c)} \right)$$

59.2 Problem number 186

$$\int (e \sec(c + dx))^{5/2} (a + ia \tan(c + dx)) dx$$

Optimal antiderivative

$$\frac{2ia(e \sec(dx + c))^{\frac{5}{2}}}{5d} + \frac{2ae(e \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3d}$$

$$+ \frac{2a e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\frac{\sqrt{2} \left(-5i a e^{\frac{5}{2}} + 5i a e^{(4i dx + 4i c + \frac{5}{2})} - 12i a e^{(2i dx + 2i c + \frac{5}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 5 \left(i \sqrt{2} a e^{\frac{5}{2}} + i \sqrt{2} a e^{(4i dx + 4i c + \frac{5}{2})} + 2i \sqrt{2} a e^{(2i dx + 2i c + \frac{5}{2})} \right) \right)$$

$$15 \left(d e^{(4i dx + 4i c)} + 2 d e^{(2i dx + 2i c)} + d \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \left(-10i a e^2 e^{(4i dx + 4i c)} + 24i a e^2 e^{(2i dx + 2i c)} + 10i a e^2 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + 15 \left(d e^{(4i dx + 4i c)} + 2 d e^{(2i dx + 2i c)} + d \right)$$

$$15 \left(d e^{(4i dx + 4i c)} + 2 d e^{(2i dx + 2i c)} + d \right)$$

59.3 Problem number 187

$$\int (e \sec(c + dx))^{3/2} (a + ia \tan(c + dx)) dx$$

Optimal antiderivative

$$\frac{2ia(e \sec(dx + c))^{3/2}}{3d} - \frac{2a e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} + \frac{2ae \sin(dx + c) \sqrt{e \sec(dx + c)}}{d}$$

command

```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(3i a e^{(3i dx + 3i c + \frac{3}{2})} + i a e^{(i dx + i c + \frac{3}{2})} \right) e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 3 \left(i \sqrt{2} a e^{\frac{3}{2}} + i \sqrt{2} a e^{(2i dx + 2i c + \frac{3}{2})} \right) \operatorname{weierstrassZeta}(-4, 0) \right)}{3 (d e^{(2i dx + 2i c)} + d)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(-6i a e e^{(3i dx + 3i c)} - 2i a e e^{(i dx + i c)} \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)} + 3 (d e^{(2i dx + 2i c)} + d) \operatorname{integral} \left(\frac{i \sqrt{2} a e \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}}}{3 (d e^{(2i dx + 2i c)} + d)} \right)}{3 (d e^{(2i dx + 2i c)} + d)}$$

59.4 Problem number 188

$$\int \sqrt{e \sec(c + dx)} (a + ia \tan(c + dx)) dx$$

Optimal antiderivative

$$\frac{2ia \sqrt{e \sec(dx + c)}}{d} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} a e^{\frac{1}{2}} \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) - \frac{i \sqrt{2} a e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c + \frac{1}{2}\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{2i \sqrt{2} a \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)} + \text{dintegral} \left(-\frac{i \sqrt{2} a \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(-\frac{1}{2} i dx - \frac{1}{2} i c\right)}}{d}, x \right)}{d}$$

59.5 Problem number 189

$$\int \frac{a + ia \tan(c + dx)}{\sqrt{e \sec(c + dx)}} dx$$

Optimal antiderivative

$$-\frac{2ia}{d \sqrt{e \sec(dx + c)}} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}}$$

command

```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2i \sqrt{2} a e^{\left(-\frac{1}{2}\right)} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(-2i a e^{(2i dx + 2i c)} - 2i a \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)} + (d e e^{(i dx + i c)} - d e) \text{integral} \left(\frac{\sqrt{2} \left(-i a e^{(2i dx + 2i c)} - 2i a e^{(i dx + i c)} \right)}{d e e^{(3i dx + 3i c)}}, x \right)}{d e e^{(i dx + i c)} - d e}$$

59.6 Problem number 190

$$\int \frac{a + ia \tan(c + dx)}{(e \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ia}{3d(e \sec(dx + c))^{\frac{3}{2}}} + \frac{2a \sin(dx + c)}{3de \sqrt{e \sec(dx + c)}} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^2} \end{aligned}$$

command

```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-2i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, e^{(i dx + ic)}) + \frac{\sqrt{2} (-i a e^{(2i dx + 2i c)} - i a) e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}}\right) e^{-\left(\frac{3}{2}\right)}}{3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{3 d e^2 \operatorname{integral}\left(-\frac{i \sqrt{2} a \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(-\frac{1}{2} i dx - \frac{1}{2} i c\right)}}{3 d e^2}, x\right) + \sqrt{2} (-i a e^{(2i dx + 2i c)} - i a) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)}}{3 d e^2}$$

59.7 Problem number 191

$$\int \frac{a + ia \tan(c + dx)}{(e \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ia}{5d(e \sec(dx + c))^{\frac{5}{2}}} + \frac{2a \sin(dx + c)}{5de (e \sec(dx + c))^{\frac{3}{2}}} + \frac{6a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^2 \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} \end{aligned}$$

command

```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(12i\sqrt{2}ae^{(idx+ic)}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,e^{(idx+ic)}))\right) + \frac{\sqrt{2}(-iae^{(4idx+4ic)}+4iae^{(2idx+2ic)})}{\sqrt{e^{(2idx+2ic)}+1}}}{10d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2}(-iae^{(5idx+5ic)}+iae^{(4idx+4ic)}-8iae^{(3idx+3ic)}-4iae^{(2idx+2ic)}-7iae^{(idx+ic)}-5ia)\sqrt{\frac{e}{e^{(2idx+2ic)}+1}}e^{(\frac{1}{2}i)}}{10(de^3e^{(2idx+2ic)})}$$

59.8 Problem number 192

$$\int \frac{a + ia \tan(c + dx)}{(e \sec(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} &-\frac{2ia}{7d(e \sec(dx+c))^{\frac{7}{2}}} + \frac{2a \sin(dx+c)}{7de(e \sec(dx+c))^{\frac{5}{2}}} + \frac{10a \sin(dx+c)}{21de^3 \sqrt{e \sec(dx+c)}} \\ &+ \frac{10a \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{e \sec(dx+c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^4} \end{aligned}$$

command

```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-40i\sqrt{2}ae^{(2idx+2ic)}\text{weierstrassPInverse}(-4,0,e^{(idx+ic)})\right) + \frac{\sqrt{2}(-3iae^{(6idx+6ic)}-19iae^{(4idx+4ic)}-9iae^{(2idx+2ic)}+7ia)}{\sqrt{e^{(2idx+2ic)}+1}}}{84d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(84de^4e^{(2idx+2ic)}\text{integral}\left(-\frac{5i\sqrt{2}a\sqrt{\frac{e}{e^{(2idx+2ic)}+1}}e^{(-\frac{1}{2}idx-\frac{1}{2}ic)}}{21de^4},x\right) + \sqrt{2}(-3iae^{(6idx+6ic)}-19iae^{(4idx+4ic)})\right)}{84de^4}$$

59.9 Problem number 193

$$\int (e \sec(c + dx))^{3/2} (a + ia \tan(c + dx))^2 dx$$

Optimal antiderivative

$$\frac{14ia^2 (e \sec(dx + c))^{3/2}}{15d} - \frac{14a^2 e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} \\ + \frac{14a^2 e \sin(dx + c) \sqrt{e \sec(dx + c)}}{5d} + \frac{2i (e \sec(dx + c))^{3/2} (a^2 + ia^2 \tan(dx + c))}{5d}$$

command

`integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(21ia^2 e^{(5i dx + 5i c + \frac{3}{2})} + 16ia^2 e^{(3i dx + 3i c + \frac{3}{2})} + 7ia^2 e^{(i dx + i c + \frac{3}{2})} \right) e^{(\frac{1}{2}i dx + \frac{1}{2}i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 21 \left(i \sqrt{2} a^2 e^{\frac{3}{2}} + i \sqrt{2} a^2 e^{(4i dx + 4i c + \frac{3}{2})} \right) \right)}{15 \left(de^{(4i dx + 4i c)} + 2 de^{(2i dx + 2i c)} + d \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(-42ia^2 e^{(5i dx + 5i c)} - 32ia^2 e^{(3i dx + 3i c)} - 14ia^2 e^{(i dx + i c)} \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(\frac{1}{2}i dx + \frac{1}{2}i c)} + 15 \left(de^{(4i dx + 4i c)} + 2 de^{(2i dx + 2i c)} + d \right)}{15 \left(de^{(4i dx + 4i c)} + 2 de^{(2i dx + 2i c)} + d \right)}$$

59.10 Problem number 194

$$\int \sqrt{e \sec(c + dx)} (a + ia \tan(c + dx))^2 dx$$

Optimal antiderivative

$$\frac{10ia^2 \sqrt{e \sec(dx + c)}}{3d} + \frac{10a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ + \frac{2i \sqrt{e \sec(dx + c)} (a^2 + ia^2 \tan(dx + c))}{3d}$$

command

`integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(-5i a^2 e^{\frac{1}{2}} - 7i a^2 e^{(2i dx + 2i c + \frac{1}{2})} \right) e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c \right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 5 \left(i \sqrt{2} a^2 e^{\frac{1}{2}} + i \sqrt{2} a^2 e^{(2i dx + 2i c + \frac{1}{2})} \right) \text{weierstrassPInverse}(-4, \dots) \right)}{3 (de^{(2i dx + 2i c)} + d)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} (14i a^2 e^{(2i dx + 2i c)} + 10i a^2) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c \right)} + 3 (de^{(2i dx + 2i c)} + d) \text{integral} \left(-\frac{5i \sqrt{2} a^2 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}}}{3} \right)}{3 (de^{(2i dx + 2i c)} + d)}$$

59.11 Problem number 195

$$\int \frac{(a + ia \tan(c + dx))^2}{\sqrt{e \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{6a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} - \frac{6a^2 \sin(dx + c) \sqrt{e \sec(dx + c)}}{de} - \frac{4i(a^2 + ia^2 \tan(dx + c))}{d \sqrt{e \sec(dx + c)}}$$

command

`integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-3i \sqrt{2} a^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) - \frac{i \sqrt{2} a^2 e^{\left(\frac{3}{2} i dx + \frac{3}{2} i c \right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right) e^{(-\frac{1}{2})}}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} (-4i a^2 e^{(2i dx + 2i c)} - 2i a^2 e^{(i dx + i c)} - 6i a^2) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c \right)} + (dee^{(i dx + i c)} - de) \text{integral} \left(\frac{\sqrt{2} (-\dots)}{\dots} \right)}{dee^{(i dx + i c)} - de}$$

59.12 Problem number 196

$$\int \frac{(a + ia \tan(c + dx))^2}{(e \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^2} - \frac{4i(a^2 + ia^2 \tan(dx + c))}{3d (e \sec(dx + c))^{\frac{3}{2}}}$$

command

```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} (i a^2 e^{(2i dx + 2i c)} + i a^2) e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right) e^{(-\frac{3}{2})}}{3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{3 d e^2 \operatorname{integral}\left(\frac{i \sqrt{2} a^2 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2} i dx - \frac{1}{2} i c)}}{3 d e^2}, x\right) + \sqrt{2} (-2i a^2 e^{(2i dx + 2i c)} - 2i a^2) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(\frac{1}{2} i dx)}}{3 d e^2}$$

59.13 Problem number 197

$$\int \frac{(a + ia \tan(c + dx))^2}{(e \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \frac{4i(a^2 + ia^2 \tan(dx + c))}{5d (e \sec(dx + c))^{\frac{5}{2}}}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^2 \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}}$$

command

```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(2i \sqrt{2} a^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \frac{\sqrt{2} (-i a^2 e^{(3i dx + 3i c)} - i a^2 e^{(i dx + i c)}) e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{5 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} (-i a^2 e^{(4i dx + 4i c)} + i a^2 e^{(3i dx + 3i c)} - 3i a^2 e^{(2i dx + 2i c)} + i a^2 e^{(i dx + i c)} - 2i a^2) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(\frac{1}{2} i dx + \frac{1}{2} i c)} + 5 (de^3 e^{(i dx + i c)} - d)}{5 (de^3 e^{(i dx + i c)} - d)}$$

59.14 Problem number 198

$$\int \frac{(a + ia \tan(c + dx))^2}{(e \sec(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sin(dx + c)}{7d e^3 \sqrt{e \sec(dx + c)}} + \frac{2a^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4} - \frac{4i(a^2 + ia^2 \tan(dx + c))}{7d (e \sec(dx + c))^{\frac{7}{2}}}$$

command

`integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-4i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} (-i a^2 e^{(4i dx + 4i c)} - 4i a^2 e^{(2i dx + 2i c)} - 3i a^2) e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right) e^{(-\frac{7}{2} i c)}}{14 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{14 d e^4 \text{integral}\left(-\frac{i \sqrt{2} a^2 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2} i dx - \frac{1}{2} i c)}}{7 d e^4}, x\right) + \sqrt{2} (-i a^2 e^{(4i dx + 4i c)} - 4i a^2 e^{(2i dx + 2i c)} - 3i a^2) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}}}{14 d e^4}$$

59.15 Problem number 199

$$\int \frac{(a + ia \tan(c + dx))^2}{(e \sec(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sin(dx + c)}{9d e^3 (e \sec(dx + c))^{\frac{3}{2}}} + \frac{2a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4 \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} - \frac{4i(a^2 + ia^2 \tan(dx + c))}{9d (e \sec(dx + c))^{\frac{9}{2}}}$$

command

`integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(24i \sqrt{2} a^2 e^{(i dx + i c)} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \frac{\sqrt{2} (-i a^2 e^{(6i dx + 6i c)} - 5i a^2 e^{(4i dx + 4i c)} - 5i a^2 e^{(2i dx + 2i c)} - 1)}{\sqrt{e^{(2i dx + 2i c)}}}\right)}{36 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} (-i a^2 e^{(7i dx + 7i c)} + i a^2 e^{(6i dx + 6i c)} - 5i a^2 e^{(5i dx + 5i c)} + 5i a^2 e^{(4i dx + 4i c)} - 19i a^2 e^{(3i dx + 3i c)} - 5i a^2 e^{(2i dx + 2i c)} - 1)$$

59.16 Problem number 200

$$\int \frac{(a + ia \tan(c + dx))^2}{(e \sec(c + dx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sin(dx + c)}{11d e^3 (e \sec(dx + c))^{\frac{5}{2}}} + \frac{10a^2 \sin(dx + c)}{33d e^5 \sqrt{e \sec(dx + c)}} + \frac{10a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)})}{33 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^6} - \frac{4i(a^2 + ia^2 \tan(dx + c))}{11d (e \sec(dx + c))^{\frac{11}{2}}}$$

command

```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-80i\sqrt{2}a^2e^{(2i dx+2i c)}\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)})+\frac{\sqrt{2}(-3ia^2e^{(8i dx+8i c)}-18ia^2e^{(6i dx+6i c)}-56ia^2e^{(4i dx+4i c)})}{\sqrt{e^{(2i dx+2i c)}+1}}\right)}{264d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(264de^6e^{(2i dx+2i c)}\text{integral}\left(-\frac{5i\sqrt{2}a^2\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)}}{33de^6},x\right)+\sqrt{2}(-3ia^2e^{(8i dx+8i c)}-18ia^2e^{(6i dx+6i c)})\right)}{264de^6}$$

59.17 Problem number 201

$$\int (e \sec(c + dx))^{7/2} (a + ia \tan(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10ia^3(e \sec(dx+c))^{\frac{7}{2}}}{21d} + \frac{2a^3e(e \sec(dx+c))^{\frac{5}{2}} \sin(dx+c)}{3d} \\ & - \frac{2a^3e^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)}} \\ & + \frac{2a^3e^3 \sin(dx+c) \sqrt{e \sec(dx+c)}}{d} + \frac{2ia(e \sec(dx+c))^{\frac{7}{2}} (a + ia \tan(dx+c))^2}{11d} \\ & + \frac{10i(e \sec(dx+c))^{\frac{7}{2}} (a^3 + ia^3 \tan(dx+c))}{33d} \end{aligned}$$

command

```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} \left(231ia^3e^{(11i dx+11i c+\frac{7}{2})} + 1309ia^3e^{(9i dx+9i c+\frac{7}{2})} + 946ia^3e^{(7i dx+7i c+\frac{7}{2})} + 870ia^3e^{(5i dx+5i c+\frac{7}{2})} + 407ia^3e^{(3i dx+3i c+\frac{7}{2})} + 77ia^3e^{(i dx+i c+\frac{7}{2})} \right) \right)}{\sqrt{e^{(2i dx+2i c)}+1}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \left(-462i a^3 e^3 e^{(11i dx + 11i c)} - 2618i a^3 e^3 e^{(9i dx + 9i c)} - 1892i a^3 e^3 e^{(7i dx + 7i c)} - 1740i a^3 e^3 e^{(5i dx + 5i c)} - 814i a^3 e^3 e^{(3i dx + 3i c)} \right)$$

59.18 Problem number 202

$$\int (e \sec(c + dx))^{5/2} (a + ia \tan(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{26ia^3(e \sec(dx + c))^{5/2}}{35d} + \frac{26a^3e(e \sec(dx + c))^{3/2} \sin(dx + c)}{21d} \\ & + \frac{26a^3e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2ia(e \sec(dx + c))^{5/2} (a + ia \tan(dx + c))^2}{9d} + \frac{26i(e \sec(dx + c))^{5/2} (a^3 + ia^3 \tan(dx + c))}{63d} \end{aligned}$$

command

```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\frac{\sqrt{2} \left(-195i a^3 e^{5/2} + 195i a^3 e^{(8i dx + 8i c + 5/2)} - 1158i a^3 e^{(6i dx + 6i c + 5/2)} - 1456i a^3 e^{(4i dx + 4i c + 5/2)} - 858i a^3 e^{(2i dx + 2i c + 5/2)} \right) e^{(1/2 i dx + 1/2 i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 1 \right)$$

315 (de^(8i dx + 8i c))

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \left(-390i a^3 e^2 e^{(8i dx + 8i c)} + 2316i a^3 e^2 e^{(6i dx + 6i c)} + 2912i a^3 e^2 e^{(4i dx + 4i c)} + 1716i a^3 e^2 e^{(2i dx + 2i c)} + 390i a^3 e^2 \right) \sqrt{\frac{e^{(2i dx + 2i c)}}{e^{(2i dx + 2i c)} + 1}}$$

315 (de^(8i dx + 8i c))

59.19 Problem number 203

$$\int (e \sec(c + dx))^{3/2} (a + ia \tan(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{22ia^3(e \sec(dx + c))^{\frac{3}{2}}}{15d} - \frac{22a^3e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} \\ & + \frac{22a^3e \sin(dx + c) \sqrt{e \sec(dx + c)}}{5d} + \frac{2ia(e \sec(dx + c))^{\frac{3}{2}} (a + ia \tan(dx + c))^2}{7d} \\ & + \frac{22i(e \sec(dx + c))^{\frac{3}{2}} (a^3 + ia^3 \tan(dx + c))}{35d} \end{aligned}$$

command

`integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(231i a^3 e^{(7i dx + 7i c + \frac{3}{2})} + 287i a^3 e^{(5i dx + 5i c + \frac{3}{2})} + 253i a^3 e^{(3i dx + 3i c + \frac{3}{2})} + 77i a^3 e^{(i dx + i c + \frac{3}{2})} \right) e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 231 \left(i \sqrt{2} a^3 e^{\frac{3}{2}} \right)}{105 (de^{(6i dx + 6i c)} + 1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(-462i a^3 e^{(7i dx + 7i c)} - 574i a^3 e^{(5i dx + 5i c)} - 506i a^3 e^{(3i dx + 3i c)} - 154i a^3 e^{(i dx + i c)} \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{105 (de^{(6i dx + 6i c)} + 3 de^{(4i dx + 4i c)})}$$

59.20 Problem number 204

$$\int \sqrt{e \sec(c + dx)} (a + ia \tan(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6ia^3 \sqrt{e \sec(dx + c)}}{d} \\ & + \frac{6a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2ia \sqrt{e \sec(dx + c)} (a + ia \tan(dx + c))^2}{5d} + \frac{6i \sqrt{e \sec(dx + c)} (a^3 + ia^3 \tan(dx + c))}{5d} \end{aligned}$$

command

`integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(-15i a^3 e^{\frac{1}{2}} - 25i a^3 e^{(4i dx + 4i c + \frac{1}{2})} - 36i a^3 e^{(2i dx + 2i c + \frac{1}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 15 \left(i \sqrt{2} a^3 e^{\frac{1}{2}} + i \sqrt{2} a^3 e^{(4i dx + 4i c + \frac{1}{2})} + 2i \right) \right)}{5 \left(de^{(4i dx + 4i c)} + 2 de^{(2i dx + 2i c)} + d \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(50i a^3 e^{(4i dx + 4i c)} + 72i a^3 e^{(2i dx + 2i c)} + 30i a^3 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + 5 \left(de^{(4i dx + 4i c)} + 2 de^{(2i dx + 2i c)} \right)}{5 \left(de^{(4i dx + 4i c)} + 2 de^{(2i dx + 2i c)} + d \right)}$$

59.21 Problem number 205

$$\int \frac{(a + ia \tan(c + dx))^3}{\sqrt{e \sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{26ia^3}{3d\sqrt{e \sec(dx+c)}} + \frac{14a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)}} \\ & -\frac{6a^3 \tan(dx+c)}{d\sqrt{e \sec(dx+c)}} - \frac{2ia^3 (\tan^2(dx+c))}{3d\sqrt{e \sec(dx+c)}} \end{aligned}$$

command

`integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(-9i a^3 e^{(3i dx + 3i c)} - 7i a^3 e^{(i dx + i c)} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 21 \left(-i \sqrt{2} a^3 e^{(2i dx + 2i c)} - i \sqrt{2} a^3 \right) \right) \operatorname{weierstrassZeta}\left(-4, 0, \operatorname{weierstrassZeta}\right)}{3 \left(de^{\frac{1}{2}} + de^{(2i dx + 2i c + \frac{1}{2})} \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(-24i a^3 e^{(4i dx + 4i c)} - 18i a^3 e^{(3i dx + 3i c)} - 70i a^3 e^{(2i dx + 2i c)} - 14i a^3 e^{(i dx + i c)} - 42i a^3 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{3 \left(de^{(3i dx + 3i c)} + de^{(i dx + i c)} + d \right)}$$

59.22 Problem number 206

$$\int \frac{(a + ia \tan(c + dx))^3}{(e \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{10ia^3 \sqrt{e \sec(dx + c)}}{3de^2} - \frac{10a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^2} - \frac{4ia(a + ia \tan(dx + c))^2}{3d(e \sec(dx + c))^{\frac{3}{2}}}$$

command

`integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-5i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} (2i a^3 e^{(2i dx + 2i c)} + 5i a^3) e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right) e^{(-\frac{3}{2})}}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{3de^2 \operatorname{integral}\left(\frac{5i \sqrt{2} a^3 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2} i dx - \frac{1}{2} i c)}}{3de^2}, x\right) + \sqrt{2} (-4i a^3 e^{(2i dx + 2i c)} - 10i a^3) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{3de^2}$$

59.23 Problem number 207

$$\int \frac{(a + ia \tan(c + dx))^3}{(e \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{6ia^3}{5de^2 \sqrt{e \sec(dx + c)}} - \frac{6a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^2 \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} - \frac{4ia(a + ia \tan(dx + c))^2}{5d(e \sec(dx + c))^{\frac{5}{2}}}$$

command

`integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3i \sqrt{2} a^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \frac{\sqrt{2} (i a^3 e^{(3i dx + 3i c)} + i a^3 e^{(i dx + i c)}) e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{5 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} (-2i a^3 e^{(4i dx + 4i c)} + 2i a^3 e^{(3i dx + 3i c)} + 4i a^3 e^{(2i dx + 2i c)} + 2i a^3 e^{(i dx + i c)} + 6i a^3) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{5 (de^3 e^{(i dx + i c)}}$$

59.24 Problem number 208

$$\int \frac{(a + ia \tan(c + dx))^3}{(e \sec(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4} - \frac{2i(a + ia \tan(dx + c))^3}{7d (e \sec(dx + c))^{7/2}} - \frac{4i(a^3 + ia^3 \tan(dx + c))}{21d e^2 (e \sec(dx + c))^{3/2}}$$

command

`integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(2i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} (-3i a^3 e^{(4i dx + 4i c)} - 5i a^3 e^{(2i dx + 2i c)} - 2i a^3) e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right) e^{(-\frac{7}{2})}}{21 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{21 d e^4 \text{integral}\left(\frac{i \sqrt{2} a^3 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2} i dx - \frac{1}{2} i c)}}{21 d e^4}, x\right) + \sqrt{2} (-3i a^3 e^{(4i dx + 4i c)} - 5i a^3 e^{(2i dx + 2i c)} - 2i a^3) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}}}{21 d e^4}$$

59.25 Problem number 209

$$\int \frac{(a + ia \tan(c + dx))^3}{(e \sec(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4 \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} - \frac{2i(a + ia \tan(dx + c))^3}{9d (e \sec(dx + c))^{\frac{9}{2}}} - \frac{4i(a^3 + ia^3 \tan(dx + c))}{15d e^2 (e \sec(dx + c))^{\frac{5}{2}}}$$

command

`integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(12i \sqrt{2} a^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \frac{\sqrt{2} (-5i a^3 e^{(5i dx + 5i c)} - 16i a^3 e^{(3i dx + 3i c)} - 11i)}{\sqrt{e^{(2i dx + 2i c)} + 1}}\right)}{90 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} (-5i a^3 e^{(6i dx + 6i c)} + 5i a^3 e^{(5i dx + 5i c)} - 16i a^3 e^{(4i dx + 4i c)} + 16i a^3 e^{(3i dx + 3i c)} - 23i a^3 e^{(2i dx + 2i c)} + 11i a^3 e^{(i dx + i c)})$$

59.26 Problem number 210

$$\int \frac{(a + ia \tan(c + dx))^3}{(e \sec(c + dx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{10a^3 \sin(dx + c)}{77d e^5 \sqrt{e \sec(dx + c)}} + \frac{10a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^6} - \frac{2i(a + ia \tan(dx + c))^3}{11d (e \sec(dx + c))^{\frac{11}{2}}} - \frac{20i(a^3 + ia^3 \tan(dx + c))}{77d e^2 (e \sec(dx + c))^{\frac{7}{2}}}$$

command

```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-40i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)}) + \frac{\sqrt{2}(-7i a^3 e^{(6i dx+6i c)} - 31i a^3 e^{(4i dx+4i c)} - 61i a^3 e^{(2i dx+2i c)} - 37i a^3) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx+2i c)} + 1}}\right)}{308 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{308 d e^6 \text{integral} \left(-\frac{5i\sqrt{2}a^3 \sqrt{\frac{e}{e^{(2i dx+2i c)} + 1}} e^{\left(-\frac{1}{2}i dx - \frac{1}{2}i c\right)}}{77 d e^6}, x \right) + \sqrt{2}(-7i a^3 e^{(6i dx+6i c)} - 31i a^3 e^{(4i dx+4i c)} - 61i a^3 e^{(2i dx+2i c)} - 37i a^3)}{308 d e^6}$$

59.27 Problem number 211

$$\int \frac{(a + ia \tan(c + dx))^3}{(e \sec(c + dx))^{13/2}} dx$$

Optimal antiderivative

$$\frac{14a^3 \sin(dx + c)}{117d e^5 (e \sec(dx + c))^{\frac{3}{2}}} + \frac{14a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{39 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^6 \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} - \frac{2i(a + ia \tan(dx + c))^3}{13d (e \sec(dx + c))^{\frac{13}{2}}} - \frac{28i(a^3 + ia^3 \tan(dx + c))}{117d e^2 (e \sec(dx + c))^{\frac{9}{2}}}$$

command

```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(336i\sqrt{2}a^3 e^{(i dx+i c)} \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)})) + \frac{\sqrt{2}(-9i a^3 e^{(8i dx+8i c)} - 50i a^3 e^{(6i dx+6i c)} - 124i a^3 e^{(4i dx+4i c)} - 124i a^3 e^{(2i dx+2i c)} - 37i a^3)}{\sqrt{e^{(2i dx+2i c)} + 1}}\right)}{936 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2}(-9i a^3 e^{(9i dx+9i c)} + 9i a^3 e^{(8i dx+8i c)} - 50i a^3 e^{(7i dx+7i c)} + 50i a^3 e^{(6i dx+6i c)} - 124i a^3 e^{(5i dx+5i c)} + 124i a^3 e^{(4i dx+4i c)} - 124i a^3 e^{(2i dx+2i c)} - 37i a^3)$$

59.28 Problem number 212

$$\int \frac{(a + ia \tan(c + dx))^3}{(e \sec(c + dx))^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6a^3 \sin(dx + c)}{55d e^5 (e \sec(dx + c))^{5/2}} + \frac{2a^3 \sin(dx + c)}{11d e^7 \sqrt{e \sec(dx + c)}} \\ & + \frac{2a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{11 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^8} \\ & - \frac{2i(a + ia \tan(dx + c))^3}{15d (e \sec(dx + c))^{15/2}} - \frac{12i(a^3 + ia^3 \tan(dx + c))}{55d e^2 (e \sec(dx + c))^{11/2}} \end{aligned}$$

command

`integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-480i \sqrt{2} a^3 e^{(2i dx + 2i c)} \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \sqrt{2} (-11i a^3 e^{(10i dx + 10i c)} - 73i a^3 e^{(8i dx + 8i c)} - 218i a^3 e^{(6i dx + 6i c)} - 11i a^3 e^{(4i dx + 4i c)} - 11i a^3 e^{(2i dx + 2i c)} - 11i a^3)\right)}{\sqrt{e^{(2i dx + 2i c)}}} \cdot 2640 d$$

Fricas 1.3.7 via sagemath 9.3 output

$$\left(2640 d e^8 e^{(2i dx + 2i c)} \operatorname{integral}\left(-\frac{i \sqrt{2} a^3 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2} i dx - \frac{1}{2} i c)}}{11 d e^8}, x\right) + \sqrt{2} (-11i a^3 e^{(10i dx + 10i c)} - 73i a^3 e^{(8i dx + 8i c)} - 218i a^3 e^{(6i dx + 6i c)} - 11i a^3 e^{(4i dx + 4i c)} - 11i a^3 e^{(2i dx + 2i c)} - 11i a^3)\right)$$

59.29 Problem number 213

$$\int (e \sec(c + dx))^{3/2} (a + ia \tan(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{22ia^4 (e \sec(dx + c))^{3/2}}{9d} - \frac{22a^4 e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} \\ & + \frac{22a^4 e \sin(dx + c) \sqrt{e \sec(dx + c)}}{3d} + \frac{2ia (e \sec(dx + c))^{3/2} (a + ia \tan(dx + c))^3}{9d} \\ & + \frac{10i (e \sec(dx + c))^{3/2} (a^2 + ia^2 \tan(dx + c))^2}{21d} + \frac{22i (e \sec(dx + c))^{3/2} (a^4 + ia^4 \tan(dx + c))}{21d} \end{aligned}$$

command

```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\frac{\sqrt{2} \left(231i a^4 e^{(9i dx + 9i c + \frac{3}{2})} + 406i a^4 e^{(7i dx + 7i c + \frac{3}{2})} + 540i a^4 e^{(5i dx + 5i c + \frac{3}{2})} + 330i a^4 e^{(3i dx + 3i c + \frac{3}{2})} + 77i a^4 e^{(i dx + i c + \frac{3}{2})} \right) e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \left(-462i a^4 e e^{(9i dx + 9i c)} - 812i a^4 e e^{(7i dx + 7i c)} - 1080i a^4 e e^{(5i dx + 5i c)} - 660i a^4 e e^{(3i dx + 3i c)} - 154i a^4 e e^{(i dx + i c)} \right) \sqrt{e^{(2i dx + 2i c)} + 1}$$

$$63 \left(d e^{(8i dx + 8i c)} \right)$$

59.30 Problem number 214

$$\int \sqrt{e \sec(c + dx)} (a + ia \tan(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{78ia^4 \sqrt{e \sec(dx + c)}}{7d} \\ & + \frac{78a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2ia \sqrt{e \sec(dx + c)} (a + ia \tan(dx + c))^3}{7d} + \frac{26i \sqrt{e \sec(dx + c)} (a^2 + ia^2 \tan(dx + c))^2}{35d} \\ & + \frac{78i \sqrt{e \sec(dx + c)} (a^4 + ia^4 \tan(dx + c))}{35d} \end{aligned}$$

command

```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\frac{\sqrt{2} \left(-195i a^4 e^{\frac{1}{2}} - 365i a^4 e^{(6i dx + 6i c + \frac{1}{2})} - 793i a^4 e^{(4i dx + 4i c + \frac{1}{2})} - 663i a^4 e^{(2i dx + 2i c + \frac{1}{2})} \right) e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 195 \left(i \sqrt{2} a^4 e^{\frac{1}{2}} + i \sqrt{e^{(2i dx + 2i c)} + 1} \right) \right)$$

$$35 \left(d e^{(6i dx + 6i c)} + 3 d e^{(4i dx + 4i c)} + 3 d e^{(2i dx + 2i c)} + d \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} (730i a^4 e^{(6i dx+6i c)} + 1586i a^4 e^{(4i dx+4i c)} + 1326i a^4 e^{(2i dx+2i c)} + 390i a^4) \sqrt{\frac{e}{e^{(2i dx+2i c)} + 1}} e^{(\frac{1}{2}i dx + \frac{1}{2}i c)} + 35 (d$$

$$35 (de^{(6i dx+6i c)} + 3 de^{(4i dx+4i c)})$$

59.31 Problem number 215

$$\int \frac{(a + ia \tan(c + dx))^4}{\sqrt{e \sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{154ia^4(e \sec(dx + c))^{\frac{3}{2}}}{15de^2} + \frac{154a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} \\ & - \frac{154a^4 \sin(dx + c) \sqrt{e \sec(dx + c)}}{5de} - \frac{4ia(a + ia \tan(dx + c))^3}{d \sqrt{e \sec(dx + c)}} \\ & - \frac{22i(e \sec(dx + c))^{\frac{3}{2}} (a^4 + ia^4 \tan(dx + c))}{5de^2} \end{aligned}$$

command

`integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\frac{\sqrt{2} (-111i a^4 e^{(5i dx+5i c)} - 176i a^4 e^{(3i dx+3i c)} - 77i a^4 e^{(i dx+i c)}) e^{(\frac{1}{2}i dx + \frac{1}{2}i c)}}{\sqrt{e^{(2i dx+2i c)} + 1}} + 231 \left(-i \sqrt{2} a^4 e^{(4i dx+4i c)} - 2i \sqrt{2} a^4 e^{(2i dx+2i c)} \right) \right)$$

$$15 \left(de^{\frac{1}{2}} + de^{(4i dx+4i c + \frac{1}{2})} + 2 de^{(2i dx+2i c)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} (-240i a^4 e^{(6i dx+6i c)} - 222i a^4 e^{(5i dx+5i c)} - 1034i a^4 e^{(4i dx+4i c)} - 352i a^4 e^{(3i dx+3i c)} - 1232i a^4 e^{(2i dx+2i c)} - 154$$

59.32 Problem number 216

$$\int \frac{(a + ia \tan(c + dx))^4}{(e \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{10ia^4 \sqrt{e \sec(dx + c)}}{de^2} - \frac{10a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^2} - \frac{4ia(a + ia \tan(dx + c))^3}{3d(e \sec(dx + c))^{\frac{3}{2}}} - \frac{2i \sqrt{e \sec(dx + c)} (a^4 + ia^4 \tan(dx + c))}{de^2}$$

command

`integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} (4i a^4 e^{(4i dx + 4i c)} + 21i a^4 e^{(2i dx + 2i c)} + 15i a^4) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 15 \left(-i \sqrt{2} a^4 e^{(2i dx + 2i c)} - i \sqrt{2} a^4 \right) \operatorname{weierstrassPInverse} \right)}{3 \left(de^{\frac{3}{2}} + de^{(2i dx + 2i c + \frac{3}{2})} \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(-8i a^4 e^{(4i dx + 4i c)} - 42i a^4 e^{(2i dx + 2i c)} - 30i a^4 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + 3 \left(de^2 e^{(2i dx + 2i c)} + de^2 \right) \operatorname{integrate}}{3 \left(de^2 e^{(2i dx + 2i c)} + de^2 \right)}$$

59.33 Problem number 217

$$\int \frac{(a + ia \tan(c + dx))^4}{(e \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{42a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^2 \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} + \frac{42a^4 \sin(dx + c) \sqrt{e \sec(dx + c)}}{5de^3} - \frac{4ia(a + ia \tan(dx + c))^3}{5d(e \sec(dx + c))^{\frac{5}{2}}} + \frac{28i(a^4 + ia^4 \tan(dx + c))}{5de^2 \sqrt{e \sec(dx + c)}}$$

command

`integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21i \sqrt{2} a^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \frac{\sqrt{2} (2i a^4 e^{(3i dx + 3i c)} + 7i a^4 e^{(i dx + i c)}) e^{(\frac{1}{2}i dx + \frac{1}{2}i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{5d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} (-4i a^4 e^{(4i dx + 4i c)} + 4i a^4 e^{(3i dx + 3i c)} + 28i a^4 e^{(2i dx + 2i c)} + 14i a^4 e^{(i dx + i c)} + 42i a^4) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(\frac{1}{2}i dx + \frac{1}{2}i c)}}{5 (de^3 e^{(i dx + i c)})}$$

59.34 Problem number 218

$$\int \frac{(a + ia \tan(c + dx))^4}{(e \sec(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{10a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4} - \frac{4ia(a + ia \tan(dx + c))^3}{7d (e \sec(dx + c))^{\frac{7}{2}}} + \frac{20i(a^4 + ia^4 \tan(dx + c))}{21d e^2 (e \sec(dx + c))^{\frac{3}{2}}}$$

command

`integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} (3i a^4 e^{(4i dx + 4i c)} - 2i a^4 e^{(2i dx + 2i c)} - 5i a^4) e^{(\frac{1}{2}i dx + \frac{1}{2}i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{21d} e^{(-\frac{7}{2}i dx - \frac{7}{2}i c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{21 d e^4 \text{integral}\left(-\frac{5i \sqrt{2} a^4 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)}}{21 d e^4}, x\right) + \sqrt{2} (-6i a^4 e^{(4i dx + 4i c)} + 4i a^4 e^{(2i dx + 2i c)} + 10i a^4)}{21 d e^4}$$

59.35 Problem number 219

$$\int \frac{(a + ia \tan(c + dx))^4}{(e \sec(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^4 \sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)}} - \frac{4ia(a + ia \tan(dx+c))^3}{9d (e \sec(dx+c))^{\frac{9}{2}}} + \frac{4i(a^4 + ia^4 \tan(dx+c))}{15d e^2 (e \sec(dx+c))^{\frac{5}{2}}}$$

command

`integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-6i \sqrt{2} a^4 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, e^{i dx+i c}))\right) + \frac{\sqrt{2} (-5i a^4 e^{(5i dx+5i c)} - 7i a^4 e^{(3i dx+3i c)} - 2i a^4 e^{(i dx+i c)})}{\sqrt{e^{(2i dx+2i c)} + 1}}}{45 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} (-5i a^4 e^{(6i dx+6i c)} + 5i a^4 e^{(5i dx+5i c)} - 7i a^4 e^{(4i dx+4i c)} + 7i a^4 e^{(3i dx+3i c)} + 4i a^4 e^{(2i dx+2i c)} + 2i a^4 e^{(i dx+i c)} + 6i a^4)$$

59.36 Problem number 220

$$\int \frac{(a + ia \tan(c + dx))^4}{(e \sec(c + dx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{2a^4 \sin(dx+c)}{77d e^5 \sqrt{e \sec(dx+c)}} - \frac{2a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{e \sec(dx+c)}}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^6} - \frac{4ia(a + ia \tan(dx+c))^3}{11d (e \sec(dx+c))^{\frac{11}{2}}} + \frac{4i(a^4 + ia^4 \tan(dx+c))}{77d e^2 (e \sec(dx+c))^{\frac{7}{2}}}$$

command

```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(4i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)}) + \frac{\sqrt{2}(-7i a^4 e^{(6i dx+6i c)} - 20i a^4 e^{(4i dx+4i c)} - 17i a^4 e^{(2i dx+2i c)} - 4i a^4) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx+2i c)} + 1}}\right)}{154d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{154de^6\text{integral}\left(\frac{i\sqrt{2}a^4\sqrt{\frac{e}{e^{(2i dx+2i c)} + 1}}e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)}}{77de^6},x\right) + \sqrt{2}(-7i a^4 e^{(6i dx+6i c)} - 20i a^4 e^{(4i dx+4i c)} - 17i a^4 e^{(2i dx+2i c)} - 4i a^4)}{154de^6}$$

59.37 Problem number 221

$$\int \frac{(a + ia \tan(c + dx))^4}{(e \sec(c + dx))^{13/2}} dx$$

Optimal antiderivative

$$\frac{2a^4 \sin(dx + c)}{117de^5 (e \sec(dx + c))^{\frac{3}{2}}} + \frac{2a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{39 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) de^6 \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} - \frac{4ia(a + ia \tan(dx + c))^3}{13d (e \sec(dx + c))^{\frac{13}{2}}} - \frac{4i(a^4 + ia^4 \tan(dx + c))}{117de^2 (e \sec(dx + c))^{\frac{9}{2}}}$$

command

```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(24i\sqrt{2}a^4\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)})) + \frac{\sqrt{2}(-9i a^4 e^{(7i dx+7i c)} - 37i a^4 e^{(5i dx+5i c)} - 59i a^4 e^{(3i dx+3i c)} - 37i a^4 e^{(i dx+i c)})}{\sqrt{e^{(2i dx+2i c)} + 1}}\right)}{468d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2}(-9i a^4 e^{(8i dx+8i c)} + 9i a^4 e^{(7i dx+7i c)} - 37i a^4 e^{(6i dx+6i c)} + 37i a^4 e^{(5i dx+5i c)} - 59i a^4 e^{(4i dx+4i c)} + 59i a^4 e^{(3i dx+3i c)} - 37i a^4 e^{(i dx+i c)})$$

59.38 Problem number 222

$$\int \frac{(a + ia \tan(c + dx))^4}{(e \sec(c + dx))^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^4 \sin(dx + c)}{55d e^5 (e \sec(dx + c))^{5/2}} + \frac{2a^4 \sin(dx + c)}{33d e^7 \sqrt{e \sec(dx + c)}} \\ & + \frac{2a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{33 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d e^8} \\ & - \frac{4ia(a + ia \tan(dx + c))^3}{15d (e \sec(dx + c))^{15/2}} - \frac{4i(a^4 + ia^4 \tan(dx + c))}{55d e^2 (e \sec(dx + c))^{11/2}} \end{aligned}$$

command

```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-80i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \sqrt{2} (-11i a^4 e^{(8i dx + 8i c)} - 58i a^4 e^{(6i dx + 6i c)} - 128i a^4 e^{(4i dx + 4i c)} - 166i a^4 e^{(2i dx + 2i c)} + 1)\right)}{1320 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{1320 d e^8 \operatorname{integral}\left(-\frac{i \sqrt{2} a^4 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)}}{33 d e^8}, x\right) + \sqrt{2} (-11i a^4 e^{(8i dx + 8i c)} - 58i a^4 e^{(6i dx + 6i c)} - 128i a^4 e^{(4i dx + 4i c)} - 166i a^4 e^{(2i dx + 2i c)} + 1)}{1320 d e^8}$$

59.39 Problem number 223

$$\int \frac{(e \sec(c + dx))^{11/2}}{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ie^2(e \sec(dx + c))^{7/2}}{7ad} + \frac{2e^3(e \sec(dx + c))^{5/2} \sin(dx + c)}{5ad} \\ & - \frac{6e^6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} + \frac{6e^5 \sin(dx + c) \sqrt{e \sec(dx + c)}}{5ad} \end{aligned}$$

command

```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\frac{\sqrt{2} \left(21i e^{(7i dx + 7i c + \frac{11}{2})} + 77i e^{(5i dx + 5i c + \frac{11}{2})} + 103i e^{(3i dx + 3i c + \frac{11}{2})} + 7i e^{(i dx + i c + \frac{11}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 21 \left(i \sqrt{2} e^{\frac{11}{2}} + i \sqrt{2} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} \right) \right) \frac{1}{35 \left(ade^{(6i dx + 6i c)} + 3 ad \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \left(-42i e^5 e^{(7i dx + 7i c)} - 154i e^5 e^{(5i dx + 5i c)} - 206i e^5 e^{(3i dx + 3i c)} - 14i e^5 e^{(i dx + i c)} \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + \frac{1}{35 \left(ade^{(6i dx + 6i c)} + 3 ade^{(4i dx + 4i c)} + ad \right)}$$

59.40 Problem number 224

$$\int \frac{(e \sec(c + dx))^{9/2}}{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ie^2(e \sec(dx + c))^{\frac{5}{2}}}{5ad} + \frac{2e^3(e \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3ad} \\ & + \frac{2e^4 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\frac{\sqrt{2} \left(-5i e^{\frac{9}{2}} + 5i e^{(4i dx + 4i c + \frac{9}{2})} + 12i e^{(2i dx + 2i c + \frac{9}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 5 \left(i \sqrt{2} e^{\frac{9}{2}} + i \sqrt{2} e^{(4i dx + 4i c + \frac{9}{2})} + 2i \sqrt{2} e^{(2i dx + 2i c + \frac{9}{2})} \right) \right) \frac{1}{15 \left(ade^{(4i dx + 4i c)} + 2 ade^{(2i dx + 2i c)} + ad \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \left(-10i e^4 e^{(4i dx + 4i c)} - 24i e^4 e^{(2i dx + 2i c)} + 10i e^4 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + \frac{15 \left(ade^{(4i dx + 4i c)} + 2 ade^{(2i dx + 2i c)} + ad \right)}{15 \left(ade^{(4i dx + 4i c)} + 2 ade^{(2i dx + 2i c)} + ad \right)}$$

59.41 Problem number 225

$$\int \frac{(e \sec(c + dx))^{7/2}}{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$\frac{-\frac{2ie^2(e \sec(dx + c))^{\frac{3}{2}}}{3ad} - \frac{2e^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}}}{+ \frac{2e^3 \sin(dx + c) \sqrt{e \sec(dx + c)}}{ad}}$$

command

```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(3i e^{(3i dx + 3i c + \frac{7}{2})} + 5i e^{(i dx + i c + \frac{7}{2})} \right) e^{(\frac{1}{2}i dx + \frac{1}{2}i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 3 \left(i \sqrt{2} e^{\frac{7}{2}} + i \sqrt{2} e^{(2i dx + 2i c + \frac{7}{2})} \right) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassZeta}(-4, 0, \dots))) \dots)) \dots \right)}{3 (ade^{(2i dx + 2i c)} + ad)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(-6i e^3 e^{(3i dx + 3i c)} - 10i e^3 e^{(i dx + i c)} \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(\frac{1}{2}i dx + \frac{1}{2}i c)} + 3 (ade^{(2i dx + 2i c)} + ad) \operatorname{integral} \left(\frac{i \sqrt{2} e^3 \sqrt{\dots}}{\dots} \right)}{3 (ade^{(2i dx + 2i c)} + ad)}$$

59.42 Problem number 226

$$\int \frac{(e \sec(c + dx))^{5/2}}{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$\frac{-\frac{2ie^2 \sqrt{e \sec(dx + c)}}{ad} + \frac{2e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad}}$$

command

`integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} e^{\frac{5}{2}} \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{i \sqrt{2} e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c + \frac{5}{2}\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{-2i \sqrt{2} e^2 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)} + ad \text{integral} \left(-\frac{i \sqrt{2} e^2 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(-\frac{1}{2} i dx - \frac{1}{2} i c\right)}}{ad}, x \right)}{ad}$$

59.43 Problem number 227

$$\int \frac{(e \sec(c + dx))^{3/2}}{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$\frac{2ie^2}{ad \sqrt{e \sec(dx + c)}} + \frac{2e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}}$$

command

`integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-i \sqrt{2} e^{(i dx + i c + \frac{3}{2})} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \frac{\sqrt{2} \left(-i e^{\frac{3}{2}} - i e^{(2i dx + 2i c + \frac{3}{2})} \right)}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(ade^{(i dx + i c)} \text{integral} \left(-\frac{i \sqrt{2} e \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)}}{ad}, x \right) + \sqrt{2} (2i e e^{(2i dx + 2i c)} + 2i e) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)} \right)}{ad}$$

59.44 Problem number 228

$$\int \frac{\sqrt{e \sec(c + dx)}}{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{2i \sqrt{e \sec(dx+c)}}{3d(a + ia \tan(dx+c))}$$

command

`integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-2i \sqrt{2} e^{(2i dx + 2i c + \frac{1}{2})} \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} \left(i e^{\frac{1}{2}} + i e^{(2i dx + 2i c + \frac{1}{2})}\right) e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}}\right) e^{(-2i dx - 2i c)}}{3 ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(3 ad e^{(2i dx + 2i c)} \operatorname{integral}\left(-\frac{i \sqrt{2} \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2} i dx - \frac{1}{2} i c)}}{3 ad}, x\right) + \sqrt{2} \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} (i e^{(2i dx + 2i c)} + i) e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)}\right)}{3 ad}$$

59.45 Problem number 229

$$\int \frac{1}{\sqrt{e \sec(c + dx)} (a + ia \tan(c + dx))} dx$$

Optimal antiderivative

$$\frac{6\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)}} + \frac{2i}{5d \sqrt{e \sec(dx+c)} (a + ia \tan(dx+c))}$$

command

`integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(12i\sqrt{2}e^{(3i dx+3i c)}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)})) + \sqrt{2}\frac{(7ie^{(4i dx+4i c)}+8ie^{(2i dx+2i c)}+i)}{\sqrt{e^{(2i dx+2i c)}+1}}\right)}{10ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}\left(-5ie^{(5i dx+5i c)}-7ie^{(4i dx+4i c)}-4ie^{(3i dx+3i c)}-8ie^{(2i dx+2i c)}+ie^{(i dx+i c)}-i\right)e^{\left(\frac{1}{2}i dx+\frac{1}{2}i c\right)}$$

$$10(ad e e^{(4i dx+4i c)})$$

59.46 Problem number 230

$$\int \frac{1}{(e \sec(c+dx))^{3/2}(a+ia \tan(c+dx))} dx$$

Optimal antiderivative

$$\frac{10 \sin(dx+c)}{21ade\sqrt{e \sec(dx+c)}} + \frac{10\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{e \sec(dx+c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad e^2} + \frac{2i}{7d(e \sec(dx+c))^{\frac{3}{2}}(a+ia \tan(dx+c))}$$

command

`integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-40i\sqrt{2}e^{(4i dx+4i c)}\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)}) + \sqrt{2}\frac{(-7ie^{(6i dx+6i c)}+9ie^{(4i dx+4i c)}+19ie^{(2i dx+2i c)}+3i)e^{\left(\frac{1}{2}i dx\right)}}{\sqrt{e^{(2i dx+2i c)}+1}}\right)}{84ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\left(84ade^2e^{(4i dx+4i c)}\text{integral}\left(-\frac{5i\sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}e^{\left(-\frac{1}{2}i dx-\frac{1}{2}i c\right)}}{21ade^2},x\right) + \sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}(-7ie^{(6i dx+6i c)})\right)$$

$$84ade^2$$

59.47 Problem number 231

$$\int \frac{1}{(e \sec(c + dx))^{5/2} (a + ia \tan(c + dx))} dx$$

Optimal antiderivative

$$\frac{14 \sin(dx + c)}{45ade (e \sec(dx + c))^{3/2}} + \frac{14 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad e^2 \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} + \frac{2i}{9d (e \sec(dx + c))^{5/2} (a + ia \tan(dx + c))}$$

command

`integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(336i \sqrt{2} e^{(5i dx + 5i c)} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \frac{\sqrt{2} (-9i e^{(8i dx + 8i c)} + 174i e^{(6i dx + 6i c)} - 124i e^{(5i dx + 5i c)} - 212i e^{(4i dx + 4i c)} - 9i e^{(3i dx + 3i c)} - 9i e^{(2i dx + 2i c)} - 9i e^{(i dx + i c)} - 9i)}{360 ad} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} (-9i e^{(9i dx + 9i c)} + 9i e^{(8i dx + 8i c)} - 162i e^{(7i dx + 7i c)} - 174i e^{(6i dx + 6i c)} - 124i e^{(5i dx + 5i c)} - 212i e^{(4i dx + 4i c)} - 9i e^{(3i dx + 3i c)} - 9i e^{(2i dx + 2i c)} - 9i e^{(i dx + i c)} - 9i)$$

59.48 Problem number 232

$$\int \frac{1}{(e \sec(c + dx))^{7/2} (a + ia \tan(c + dx))} dx$$

Optimal antiderivative

$$\frac{18 \sin(dx + c)}{77ade (e \sec(dx + c))^{5/2}} + \frac{30 \sin(dx + c)}{77ad e^3 \sqrt{e \sec(dx + c)}} + \frac{30 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)})}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad e^4} + \frac{2i}{11d (e \sec(dx + c))^{7/2} (a + ia \tan(dx + c))}$$

command

`integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-480i\sqrt{2}e^{(6i dx+6i c)}\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)})+\sqrt{2}\frac{(-11ie^{(10i dx+10i c)}-121ie^{(8i dx+8i c)}+70ie^{(6i dx+6i c)}+226e^{(4i dx+4i c)})}{\sqrt{e^{(2i dx+2i c)}+1}}\right)}{1232ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\left(1232ade^4e^{(6i dx+6i c)}\text{integral}\left(-\frac{15i\sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)}}{77ade^4},x\right)+\sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}(-11ie^{(10i dx+10i c)}-121ie^{(8i dx+8i c)}+70ie^{(6i dx+6i c)}+226e^{(4i dx+4i c)})\right)$$

1232ade

59.49 Problem number 233

$$\int \frac{(e \sec(c + dx))^{15/2}}{(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{22e^5(e \sec(dx + c))^{\frac{5}{2}} \sin(dx + c)}{45a^2d} + \frac{22e^3(e \sec(dx + c))^{\frac{9}{2}} \sin(dx + c)}{63a^2d} \\ & - \frac{22e^8 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} \\ & + \frac{22e^7 \sin(dx + c) \sqrt{e \sec(dx + c)}}{15a^2d} - \frac{4ie^2(e \sec(dx + c))^{\frac{11}{2}}}{7d(a^2 + ia^2 \tan(dx + c))} \end{aligned}$$

command

`integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{2}\left(231ie^{(9i dx+9i c+\frac{15}{2})}+1078ie^{(7i dx+7i c+\frac{15}{2})}+1980ie^{(5i dx+5i c+\frac{15}{2})}+1770ie^{(3i dx+3i c+\frac{15}{2})}+77ie^{(i dx+i c+\frac{15}{2})}\right)e^{(\frac{1}{2}i dx+\frac{1}{2}i c)}\right)}{\sqrt{e^{(2i dx+2i c)}+1}}$$

315(a^2de^{(8i dx+8i c)})

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2}\left(-462ie^7e^{(9i dx+9i c)}-2156ie^7e^{(7i dx+7i c)}-3960ie^7e^{(5i dx+5i c)}-3540ie^7e^{(3i dx+3i c)}-154ie^7e^{(i dx+i c)}\right)\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}$$

315(a^2de^{(8i dx+8i c)})

59.50 Problem number 234

$$\int \frac{(e \sec(c + dx))^{13/2}}{(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6e^5(e \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{7a^2d} + \frac{18e^3(e \sec(dx + c))^{\frac{7}{2}} \sin(dx + c)}{35a^2d} \\ & + \frac{6e^6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & - \frac{4ie^2(e \sec(dx + c))^{\frac{9}{2}}}{5d(a^2 + ia^2 \tan(dx + c))} \end{aligned}$$

command

`integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(-15i e^{\frac{13}{2}} + 15i e^{(6i dx + 6i c + \frac{13}{2})} + 51i e^{(4i dx + 4i c + \frac{13}{2})} + 61i e^{(2i dx + 2i c + \frac{13}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 15 \left(i \sqrt{2} e^{\frac{13}{2}} + i \sqrt{2} e^{(6i dx + 6i c)} \right) \right)}{35 (a^2 d e^{(6i dx + 6i c)} + 3 a^2 d e^{(4i dx + 4i c)} + 3 a^2 d e^{(2i dx + 2i c)} + 3 a^2 d)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(-30i e^6 e^{(6i dx + 6i c)} - 102i e^6 e^{(4i dx + 4i c)} - 122i e^6 e^{(2i dx + 2i c)} + 30i e^6 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + 35 (a^2 d e^{(6i dx + 6i c)} + 3 a^2 d e^{(4i dx + 4i c)} + 3 a^2 d e^{(2i dx + 2i c)} + 3 a^2 d)}{35 (a^2 d e^{(6i dx + 6i c)} + 3 a^2 d e^{(4i dx + 4i c)} + 3 a^2 d e^{(2i dx + 2i c)} + 3 a^2 d)}$$

59.51 Problem number 235

$$\int \frac{(e \sec(c + dx))^{11/2}}{(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14e^3(e \sec(dx + c))^{\frac{5}{2}} \sin(dx + c)}{15a^2d} - \frac{14e^6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} \\ & + \frac{14e^5 \sin(dx + c) \sqrt{e \sec(dx + c)}}{5a^2d} - \frac{4ie^2(e \sec(dx + c))^{\frac{7}{2}}}{3d(a^2 + ia^2 \tan(dx + c))} \end{aligned}$$

command

`integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(21i e^{(5i dx + 5i c + \frac{11}{2})} + 56i e^{(3i dx + 3i c + \frac{11}{2})} + 47i e^{(i dx + i c + \frac{11}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 21 \left(i \sqrt{2} e^{\frac{11}{2}} + i \sqrt{2} e^{(4i dx + 4i c + \frac{11}{2})} + 2i \sqrt{2} e^{(2i dx + 2i c + \frac{11}{2})} \right) \right)}{15 \left(a^2 d e^{(4i dx + 4i c)} + 2 a^2 d e^{(2i dx + 2i c)} + a^2 d \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(-42i e^5 e^{(5i dx + 5i c)} - 112i e^5 e^{(3i dx + 3i c)} - 94i e^5 e^{(i dx + i c)} \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + 15 \left(a^2 d e^{(4i dx + 4i c)} + 2 a^2 d e^{(2i dx + 2i c)} + a^2 d \right)}{15 \left(a^2 d e^{(4i dx + 4i c)} + 2 a^2 d e^{(2i dx + 2i c)} + a^2 d \right)}$$

59.52 Problem number 236

$$\int \frac{(e \sec(c + dx))^{9/2}}{(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{10e^3 (e \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3a^2 d} + \frac{10e^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} - \frac{4ie^2 (e \sec(dx + c))^{\frac{5}{2}}}{d(a^2 + ia^2 \tan(dx + c))}$$

command

`integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(7i e^{\frac{9}{2}} + 5i e^{(2i dx + 2i c + \frac{9}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 5 \left(i \sqrt{2} e^{\frac{9}{2}} + i \sqrt{2} e^{(2i dx + 2i c + \frac{9}{2})} \right) \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) \right)}{3 \left(a^2 d e^{(2i dx + 2i c)} + a^2 d \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(-10i e^4 e^{(2i dx + 2i c)} - 14i e^4 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + 3 \left(a^2 d e^{(2i dx + 2i c)} + a^2 d \right) \operatorname{integral} \left(-\frac{5i \sqrt{2} e^4 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}}}{3 \left(a^2 d e^{(2i dx + 2i c)} + a^2 d \right)} \right)}{3 \left(a^2 d e^{(2i dx + 2i c)} + a^2 d \right)}$$

59.53 Problem number 237

$$\int \frac{(e \sec(c + dx))^{7/2}}{(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{6e^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} - \frac{6e^3 \sin(dx + c) \sqrt{e \sec(dx + c)}}{a^2 d} + \frac{4ie^2 (e \sec(dx + c))^{\frac{3}{2}}}{d (a^2 + ia^2 \tan(dx + c))}$$

command

`integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-3i \sqrt{2} e^{(i dx + i c + \frac{7}{2})} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \frac{\sqrt{2} \left(-2i e^{\frac{7}{2}} - 3i e^{(2i dx + 2i c + \frac{7}{2})} \right)}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{a^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(a^2 d e^{(i dx + i c)} \operatorname{integral} \left(-\frac{3i \sqrt{2} e^3 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{a^2 d}, x \right) + \sqrt{2} \left(6i e^3 e^{(2i dx + 2i c)} + 4i e^3 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} \right)}{a^2 d}$$

59.54 Problem number 238

$$\int \frac{(e \sec(c + dx))^{5/2}}{(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{2e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} + \frac{4ie^2 \sqrt{e \sec(dx + c)}}{3d(a^2 + ia^2 \tan(dx + c))}$$

command

`integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-i \sqrt{2} e^{(2i dx + 2i c + \frac{5}{2})} \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} \left(-i e^{\frac{5}{2}} - i e^{(2i dx + 2i c + \frac{5}{2})} \right) e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right) e^{(-2i dx - 2i c)}}{3 a^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(3 a^2 d e^{(2i dx + 2i c)} \operatorname{integral} \left(\frac{i \sqrt{2} e^2 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2} i dx - \frac{1}{2} i c)}}{3 a^2 d}, x \right) + \sqrt{2} (2i e^2 e^{(2i dx + 2i c)} + 2i e^2) \sqrt{\frac{e}{e^{(2i dx + 2i c)}}} \right)}{3 a^2 d}$$

59.55 Problem number 239

$$\int \frac{(e \sec(c + dx))^{3/2}}{(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{2e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} + \frac{4ie^2}{5d \sqrt{e \sec(dx + c)} (a^2 + ia^2 \tan(dx + c))}$$

command

`integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(2i\sqrt{2}e^{(3i dx+3i c+\frac{3}{2})}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)})) + \frac{\sqrt{2}\left(i e^{\frac{3}{2}}+2i e^{(4i dx+4i c+\frac{3}{2})}+3i e^{(2i dx+2i c+\frac{3}{2})}\right)}{\sqrt{e^{(2i dx+2i c)}+1}}\right)}{5a^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(5a^2de^{(3i dx+3i c)}\text{integral}\left(-\frac{i\sqrt{2}e\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}e^{(\frac{1}{2}i dx+\frac{1}{2}i c)}}{5a^2d},x\right) + \sqrt{2}(2iee^{(4i dx+4i c)}+3iee^{(2i dx+2i c)}+ie)\right)}{5a^2d}$$

59.56 Problem number 240

$$\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\frac{2e \sin(dx+c)}{7a^2d\sqrt{e \sec(dx+c)}} + \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{e \sec(dx+c)}}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} + \frac{4ie^2}{7d(e \sec(dx+c))^{\frac{3}{2}}(a^2+ia^2 \tan(dx+c))}$$

command

`integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-4i\sqrt{2}e^{(4i dx+4i c+\frac{1}{2})}\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)}) + \frac{\sqrt{2}\left(i e^{\frac{1}{2}}+3i e^{(4i dx+4i c+\frac{1}{2})}+4i e^{(2i dx+2i c+\frac{1}{2})}\right)e^{(\frac{1}{2}i dx+\frac{1}{2}i c)}}{\sqrt{e^{(2i dx+2i c)}+1}}\right)}{14a^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(14a^2de^{(4i dx+4i c)}\text{integral}\left(-\frac{i\sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)}}{7a^2d},x\right) + \sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}(3ie^{(4i dx+4i c)}+4ie^{(2i dx+2i c)}+ie)\right)}{14a^2d}$$

59.57 Problem number 241

$$\int \frac{1}{\sqrt{e \sec(c+dx)} (a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\frac{2e \sin(dx+c)}{9a^2 d (e \sec(dx+c))^{\frac{3}{2}}} + \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)}} + \frac{4ie^2}{9d (e \sec(dx+c))^{\frac{5}{2}} (a^2 + ia^2 \tan(dx+c))}$$

command

`integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(24i \sqrt{2} e^{(5i dx+5i c)} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx+i c)})) + \frac{\sqrt{2} (15i e^{(6i dx+6i c)} + 19i e^{(4i dx+4i c)})}{\sqrt{e^{(2i dx+2i c)}}}\right)}{36 a^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \sqrt{\frac{e}{e^{(2i dx+2i c)} + 1}} \left(-9i e^{(7i dx+7i c)} - 15i e^{(6i dx+6i c)} - 5i e^{(5i dx+5i c)} - 19i e^{(4i dx+4i c)} + 5i e^{(3i dx+3i c)} - 5i e^{(2i dx+2i c)}\right)$$

36

59.58 Problem number 242

$$\int \frac{1}{(e \sec(c+dx))^{3/2} (a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\frac{2e \sin(dx+c)}{11a^2 d (e \sec(dx+c))^{\frac{5}{2}}} + \frac{10 \sin(dx+c)}{33a^2 d e \sqrt{e \sec(dx+c)}} + \frac{10\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)})}{33 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d e^2} + \frac{4ie^2}{11d (e \sec(dx+c))^{\frac{7}{2}} (a^2 + ia^2 \tan(dx+c))}$$

command

`integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-80i\sqrt{2}e^{(6i dx+6i c)}\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)})+\sqrt{2}\frac{(-11ie^{(8i dx+8i c)}+30ie^{(6i dx+6i c)}+56ie^{(4i dx+4i c)}+18ie^{(2i dx+2i c)})}{\sqrt{e^{(2i dx+2i c)}+1}}\right)}{264a^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(264a^2de^2e^{(6i dx+6i c)}\text{integral}\left(-\frac{5i\sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)}}{33a^2de^2},x\right)+\sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}(-11ie^{(8i dx+8i c)}+30ie^{(6i dx+6i c)}+56ie^{(4i dx+4i c)}+18ie^{(2i dx+2i c)})\right)}{264a^2de^2}$$

59.59 Problem number 243

$$\int \frac{1}{(e \sec(c+dx))^{5/2}(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e \sin(dx+c)}{13a^2d(e \sec(dx+c))^{\frac{7}{2}}} + \frac{14 \sin(dx+c)}{65a^2de(e \sec(dx+c))^{\frac{3}{2}}} \\ & + \frac{42\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{65 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d e^2 \sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)}} \\ & + \frac{4ie^2}{13d(e \sec(dx+c))^{\frac{9}{2}}(a^2+ia^2 \tan(dx+c))} \end{aligned}$$

command

`integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(672i\sqrt{2}e^{(7i dx+7i c)}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)}))\right)+\sqrt{2}\frac{(-13ie^{(10i dx+10i c)}+373ie^{(8i dx+8i c)}-299ie^{(6i dx+6i c)}-373ie^{(4i dx+4i c)}-198ie^{(2i dx+2i c)})}{\sqrt{e^{(2i dx+2i c)}+1}}}{1040a^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}(-13ie^{(11i dx+11i c)}+13ie^{(10i dx+10i c)}-299ie^{(9i dx+9i c)}-373ie^{(8i dx+8i c)}-198ie^{(7i dx+7i c)})$$

59.60 Problem number 244

$$\int \frac{1}{(e \sec(c + dx))^{7/2} (a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e \sin(dx + c)}{15a^2 d (e \sec(dx + c))^{\frac{9}{2}}} + \frac{6 \sin(dx + c)}{35a^2 d e (e \sec(dx + c))^{\frac{5}{2}}} + \frac{2 \sin(dx + c)}{7a^2 d e^3 \sqrt{e \sec(dx + c)}} \\ & + \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d e^4} \\ & + \frac{4ie^2}{15d (e \sec(dx + c))^{\frac{11}{2}} (a^2 + ia^2 \tan(dx + c))} \end{aligned}$$

command

`integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-960i \sqrt{2} e^{(8i dx + 8i c)} \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} (-15i e^{(12i dx + 12i c)} - 200i e^{(10i dx + 10i c)} + 245i e^{(8i dx + 8i c)} + \dots)}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right) / 3360 a^2 d$$

Fricas 1.3.7 via sagemath 9.3 output

$$\left(3360 a^2 d e^4 e^{(8i dx + 8i c)} \operatorname{integral}\left(-\frac{i \sqrt{2} \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)}}{7 a^2 d e^4}, x \right) + \sqrt{2} \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} (-15i e^{(12i dx + 12i c)} - 200i e^{(10i dx + 10i c)} + 245i e^{(8i dx + 8i c)} + \dots) \right) / 3360 a^2 d$$

59.61 Problem number 245

$$\int \frac{(e \sec(c + dx))^{15/2}}{(a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{22ie^4 (e \sec(dx + c))^{\frac{7}{2}}}{21a^3 d} + \frac{22e^5 (e \sec(dx + c))^{\frac{5}{2}} \sin(dx + c)}{15a^3 d} \\ & - \frac{22e^8 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} \\ & + \frac{22e^7 \sin(dx + c) \sqrt{e \sec(dx + c)}}{5a^3 d} - \frac{4ie^2 (e \sec(dx + c))^{\frac{11}{2}}}{3ad (a + ia \tan(dx + c))^2} \end{aligned}$$

command

```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(231i e^{(7i dx + 7i c + \frac{15}{2})} + 847i e^{(5i dx + 5i c + \frac{15}{2})} + 1133i e^{(3i dx + 3i c + \frac{15}{2})} + 637i e^{(i dx + i c + \frac{15}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 231 \left(i \sqrt{2} e^{\frac{15}{2}} + i \right) \right)}{105 \left(a^3 d e^{(6i dx + 6i c)} + 3 a^3 d \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(-462i e^{7e^{(7i dx + 7i c)}} - 1694i e^{7e^{(5i dx + 5i c)}} - 2266i e^{7e^{(3i dx + 3i c)}} - 1274i e^{7e^{(i dx + i c)}} \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{105 \left(a^3 d e^{(6i dx + 6i c)} + 3 a^3 d \right)}$$

59.62 Problem number 246

$$\int \frac{(e \sec(c + dx))^{13/2}}{(a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{18ie^4(e \sec(dx + c))^{5/2}}{5a^3d} + \frac{6e^5(e \sec(dx + c))^{3/2} \sin(dx + c)}{a^3d} \\ & + \frac{6e^6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & - \frac{4ie^2(e \sec(dx + c))^{9/2}}{ad(a + ia \tan(dx + c))^2} \end{aligned}$$

command

```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(25i e^{\frac{13}{2}} + 15i e^{(4i dx + 4i c + \frac{13}{2})} + 36i e^{(2i dx + 2i c + \frac{13}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 15 \left(i \sqrt{2} e^{\frac{13}{2}} + i \sqrt{2} e^{(4i dx + 4i c + \frac{13}{2})} + 2i \sqrt{2} e^{(2i dx + 2i c + \frac{13}{2})} \right) \right)}{5 \left(a^3 d e^{(4i dx + 4i c)} + 2 a^3 d e^{(2i dx + 2i c)} + a^3 d \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \left(-30i e^6 e^{(4i dx + 4i c)} - 72i e^6 e^{(2i dx + 2i c)} - 50i e^6 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + 5 \left(a^3 d e^{(4i dx + 4i c)} + 2 a^3 d e^{(2i dx + 2i c)} \right)$$

$$5 \left(a^3 d e^{(4i dx + 4i c)} + 2 a^3 d e^{(2i dx + 2i c)} + a^3 d \right)$$

59.63 Problem number 247

$$\int \frac{(e \sec(c + dx))^{11/2}}{(a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{14ie^4(e \sec(dx + c))^{\frac{3}{2}}}{3a^3d} + \frac{14e^6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} - \frac{14e^5 \sin(dx + c) \sqrt{e \sec(dx + c)}}{a^3 d} + \frac{4ie^2(e \sec(dx + c))^{\frac{7}{2}}}{ad(a + ia \tan(dx + c))^2}$$

command

```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(-12i e^{\frac{11}{2}} - 21i e^{(4i dx + 4i c + \frac{11}{2})} - 35i e^{(2i dx + 2i c + \frac{11}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 21 \left(-i \sqrt{2} e^{(3i dx + 3i c + \frac{11}{2})} - i \sqrt{2} e^{(i dx + i c + \frac{11}{2})} \right) \right)}{3 \left(a^3 d e^{(3i dx + 3i c)} + a^3 d e^{(i dx + i c)} \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \left(42i e^5 e^{(4i dx + 4i c)} + 70i e^5 e^{(2i dx + 2i c)} + 24i e^5 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + 3 \left(a^3 d e^{(3i dx + 3i c)} + a^3 d e^{(i dx + i c)} \right)$$

$$3 \left(a^3 d e^{(3i dx + 3i c)} + a^3 d e^{(i dx + i c)} \right)$$

59.64 Problem number 248

$$\int \frac{(e \sec(c + dx))^{9/2}}{(a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{10ie^4 \sqrt{e \sec(dx + c)}}{3a^3d} - \frac{10e^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} + \frac{4ie^2 (e \sec(dx + c))^{\frac{5}{2}}}{3ad (a + ia \tan(dx + c))^2}$$

command

`integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-5i \sqrt{2} e^{(2i dx + 2i c + \frac{9}{2})} \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} \left(-2i e^{\frac{9}{2}} - 5i e^{(2i dx + 2i c + \frac{9}{2})} \right) e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right) e^{(-2i dx)}}{3 a^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(3 a^3 d e^{(2i dx + 2i c)} \operatorname{integral}\left(\frac{5i \sqrt{2} e^4 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2} i dx - \frac{1}{2} i c)}}{3 a^3 d}, x\right) + \sqrt{2} (10i e^4 e^{(2i dx + 2i c)} + 4i e^4) \sqrt{\frac{e}{e^{(2i dx + 2i c)}}} \right)}{3 a^3 d}$$

59.65 Problem number 249

$$\int \frac{(e \sec(c + dx))^{7/2}}{(a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{6ie^4}{5a^3d \sqrt{e \sec(dx + c)}} - \frac{6e^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} + \frac{4ie^2 (e \sec(dx + c))^{\frac{3}{2}}}{5ad (a + ia \tan(dx + c))^2}$$

command

`integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3i \sqrt{2} e^{(3i dx + 3i c + \frac{7}{2})} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \frac{\sqrt{2} \left(-i e^{\frac{7}{2}} + 3i e^{(4i dx + 4i c + \frac{7}{2})} \right)}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{5 a^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(5 a^3 d e^{(3i dx + 3i c)} \text{integral} \left(\frac{3i \sqrt{2} e^3 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{5 a^3 d}, x \right) + \sqrt{2} \left(-6i e^3 e^{(4i dx + 4i c)} - 4i e^3 e^{(2i dx + 2i c)} + \right) \right)}{5 a^3 d}$$

59.66 Problem number 250

$$\int \frac{(e \sec(c + dx))^{5/2}}{(a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{2e^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) \sqrt{e \sec(dx+c)}}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} + \frac{4ie^2 \sqrt{e \sec(dx+c)}}{7ad (a + ia \tan(dx+c))^2} - \frac{2ie^2 \sqrt{e \sec(dx+c)}}{21d (a^3 + ia^3 \tan(dx+c))}$$

command

`integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(2i \sqrt{2} e^{(4i dx + 4i c + \frac{5}{2})} \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} \left(3i e^{\frac{5}{2}} + 2i e^{(4i dx + 4i c + \frac{5}{2})} + 5i e^{(2i dx + 2i c + \frac{5}{2})} \right) e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{21 a^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(21 a^3 d e^{(4i dx + 4i c)} \text{integral} \left(\frac{i \sqrt{2} e^2 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2} i dx - \frac{1}{2} i c)}}{21 a^3 d}, x \right) + \sqrt{2} \left(2i e^2 e^{(4i dx + 4i c)} + 5i e^2 e^{(2i dx + 2i c)} + \right) \right)}{21 a^3 d}$$

59.67 Problem number 251

$$\int \frac{(e \sec(c + dx))^{3/2}}{(a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{2e^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)}} + \frac{4ie^2}{9ad \sqrt{e \sec(dx+c)} (a + ia \tan(dx+c))^2} + \frac{2ie^2}{45d \sqrt{e \sec(dx+c)} (a^3 + ia^3 \tan(dx+c))}$$

command

`integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(12i \sqrt{2} e^{(5i dx + 5i c + \frac{3}{2})} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \frac{\sqrt{2} \left(5i e^{\frac{3}{2}} + 12i e^{(6i dx + 6i c + \frac{3}{2})}\right)}{90 a^3 d}\right)}{90 a^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(90 a^3 d e^{(5i dx + 5i c)} \operatorname{integral}\left(-\frac{i \sqrt{2} e \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}}}{15 a^3 d} e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}, x\right) + \sqrt{2} (12i e e^{(6i dx + 6i c)} + 23i e e^{(4i dx + 4i c)} + \dots)\right)}{90 a^3 d}$$

59.68 Problem number 252

$$\int \frac{\sqrt{e \sec(c + dx)}}{(a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{10e \sin(dx+c)}{77a^3 d \sqrt{e \sec(dx+c)}} + \frac{10 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{e \sec(dx+c)}}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} + \frac{2i \sqrt{e \sec(dx+c)}}{11d (a + ia \tan(dx+c))^3} + \frac{20ie^2}{77d (e \sec(dx+c))^{\frac{3}{2}} (a^3 + ia^3 \tan(dx+c))}$$

command

```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-40i\sqrt{2}e^{(6i dx+6i c+\frac{1}{2})}\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)})+\frac{\sqrt{2}\left(7ie^{\frac{1}{2}}+37ie^{(6i dx+6i c+\frac{1}{2})}+61ie^{(4i dx+4i c+\frac{1}{2})}+31ie^{(2i dx+2i c)}\right)}{\sqrt{e^{(2i dx+2i c)}+1}}\right)}{308a^3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(308a^3de^{(6i dx+6i c)}\text{integral}\left(-\frac{5i\sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)}}{77a^3d},x\right)+\sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}(37ie^{(6i dx+6i c)}-\dots)\right)}{308a^3d}$$

59.69 Problem number 253

$$\int \frac{1}{\sqrt{e \sec(c+dx)} (a+ia \tan(c+dx))^3} dx$$

Optimal antiderivative

$$\frac{14e \sin(dx+c)}{117a^3d(e \sec(dx+c))^{\frac{3}{2}}} + \frac{14\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{39 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d \sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)}} + \frac{2i}{13d \sqrt{e \sec(dx+c)} (a+ia \tan(dx+c))^3} + \frac{28ie^2}{117d(e \sec(dx+c))^{\frac{5}{2}} (a^3+ia^3 \tan(dx+c))}$$

command

```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(336i\sqrt{2}e^{(7i dx+7i c)}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)}))\right)+\frac{\sqrt{2}\left(219ie^{(8i dx+8i c)}+302ie^{(6i dx+6i c)}+\dots\right)}{\sqrt{e^{(2i dx+2i c)}+1}}}{936a^3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2}\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}\left(-117ie^{(9i dx+9i c)}-219ie^{(8i dx+8i c)}-34ie^{(7i dx+7i c)}-302ie^{(6i dx+6i c)}+124ie^{(5i dx+5i c)}-\dots\right)$$

59.70 Problem number 254

$$\int \frac{1}{(e \sec(c + dx))^{3/2} (a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6e \sin(dx + c)}{55a^3 d (e \sec(dx + c))^{5/2}} + \frac{2 \sin(dx + c)}{11a^3 d e \sqrt{e \sec(dx + c)}} \\ & + \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{11 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d e^2} \\ & + \frac{2i}{15d (e \sec(dx + c))^{3/2} (a + ia \tan(dx + c))^3} + \frac{12ie^2}{55d (e \sec(dx + c))^{7/2} (a^3 + ia^3 \tan(dx + c))} \end{aligned}$$

command

`integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-480i \sqrt{2} e^{(8i dx + 8i c)} \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} (-55i e^{(10i dx + 10i c)} + 235i e^{(8i dx + 8i c)} + 446i e^{(6i dx + 6i c)} + 21i e^{(4i dx + 4i c)} + 5i e^{(2i dx + 2i c)} + 1)}{\sqrt{e^{(2i dx + 2i c)} + 1}}\right)}{2640 a^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(2640 a^3 d e^2 e^{(8i dx + 8i c)} \operatorname{integral}\left(-\frac{i \sqrt{2} \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)}}{11 a^3 d e^2}, x\right) + \sqrt{2} \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} (-55i e^{(10i dx + 10i c)} + 235i e^{(8i dx + 8i c)} + 446i e^{(6i dx + 6i c)} + 21i e^{(4i dx + 4i c)} + 5i e^{(2i dx + 2i c)} + 1)\right)}{2640 a^3 d}$$

59.71 Problem number 255

$$\int \frac{(e \sec(c + dx))^{15/2}}{(a + ia \tan(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{154e^5 (e \sec(dx + c))^{5/2} \sin(dx + c)}{15a^4 d} \\ & + \frac{154e^8 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} \\ & - \frac{154e^7 \sin(dx + c) \sqrt{e \sec(dx + c)}}{5a^4 d} + \frac{4ie^2 (e \sec(dx + c))^{11/2}}{ad (a + ia \tan(dx + c))^3} + \frac{44ie^4 (e \sec(dx + c))^{7/2}}{3d (a^4 + ia^4 \tan(dx + c))} \end{aligned}$$

command

```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(-120i e^{\frac{15}{2}} - 231i e^{(6i dx + 6i c + \frac{15}{2})} - 616i e^{(4i dx + 4i c + \frac{15}{2})} - 517i e^{(2i dx + 2i c + \frac{15}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 231 \left(-i \sqrt{2} e^{(5i dx + 5i c + \frac{15}{2})} \right) \right)}{15 \left(a^4 d e^{(5i dx + 5i c)} + 2 a^4 d e^{(3i dx + 3i c)} \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{2} \left(462i e^7 e^{(6i dx + 6i c)} + 1232i e^7 e^{(4i dx + 4i c)} + 1034i e^7 e^{(2i dx + 2i c)} + 240i e^7 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + 15 \left(a^4 d e^{(5i dx + 5i c)} + 2 a^4 d e^{(3i dx + 3i c)} \right)}$$

59.72 Problem number 256

$$\int \frac{(e \sec(c + dx))^{13/2}}{(a + ia \tan(c + dx))^4} dx$$

Optimal antiderivative

$$\frac{10e^5 (e \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{a^4 d} - \frac{10e^6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} + \frac{4ie^2 (e \sec(dx + c))^{\frac{9}{2}}}{3ad (a + ia \tan(dx + c))^3} + \frac{12ie^4 (e \sec(dx + c))^{\frac{5}{2}}}{d (a^4 + ia^4 \tan(dx + c))}$$

command

```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\frac{\sqrt{2} \left(-4i e^{\frac{13}{2}} - 15i e^{(4i dx + 4i c + \frac{13}{2})} - 21i e^{(2i dx + 2i c + \frac{13}{2})} \right) e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} + 15 \left(-i \sqrt{2} e^{(4i dx + 4i c + \frac{13}{2})} - i \sqrt{2} e^{(2i dx + 2i c + \frac{13}{2})} \right) \right)}{3 \left(a^4 d e^{(4i dx + 4i c)} + a^4 d e^{(2i dx + 2i c)} \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{2} \left(30i e^6 e^{(4i dx + 4i c)} + 42i e^6 e^{(2i dx + 2i c)} + 8i e^6 \right) \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + 3 \left(a^4 d e^{(4i dx + 4i c)} + a^4 d e^{(2i dx + 2i c)} \right)$$

$$3 \left(a^4 d e^{(4i dx + 4i c)} + a^4 d e^{(2i dx + 2i c)} \right)$$

59.73 Problem number 257

$$\int \frac{(e \sec(c + dx))^{11/2}}{(a + ia \tan(c + dx))^4} dx$$

Optimal antiderivative

$$\frac{42e^6 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)}} + \frac{42e^5 \sin(dx+c) \sqrt{e \sec(dx+c)}}{5a^4 d} + \frac{4ie^2 (e \sec(dx+c))^{\frac{7}{2}}}{5ad (a + ia \tan(dx+c))^3} - \frac{28ie^4 (e \sec(dx+c))^{\frac{3}{2}}}{5d (a^4 + ia^4 \tan(dx+c))}$$

command

`integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21i \sqrt{2} e^{(3i dx + 3i c + \frac{11}{2})} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \frac{\sqrt{2} \left(-2i e^{\frac{11}{2}} + 21i e^{(4i dx + 4i c)} \right)}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{5a^4 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(5a^4 d e^{(3i dx + 3i c)} \operatorname{integral} \left(\frac{21i \sqrt{2} e^5 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)}}{5a^4 d}, x \right) + \sqrt{2} \left(-42i e^5 e^{(4i dx + 4i c)} - 28i e^5 e^{(2i dx + 2i c)} \right) \right)}{5a^4 d}$$

59.74 Problem number 258

$$\int \frac{(e \sec(c + dx))^{9/2}}{(a + ia \tan(c + dx))^4} dx$$

Optimal antiderivative

$$\frac{10e^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{e \sec(dx+c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} + \frac{4ie^2 (e \sec(dx+c))^{\frac{5}{2}}}{7ad (a + ia \tan(dx+c))^3} - \frac{20ie^4 \sqrt{e \sec(dx+c)}}{21d (a^4 + ia^4 \tan(dx+c))}$$

command

`integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} e^{(4i dx + 4i c + \frac{9}{2})} \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} \left(-3i e^{\frac{9}{2}} + 5i e^{(4i dx + 4i c + \frac{9}{2})} + 2i e^{(2i dx + 2i c + \frac{9}{2})} \right) e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)}}{\sqrt{e^{(2i dx + 2i c)} + 1}} \right)}{21 a^4 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(21 a^4 d e^{(4i dx + 4i c)} \operatorname{integral} \left(-\frac{5i \sqrt{2} e^4 \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(-\frac{1}{2} i dx - \frac{1}{2} i c)}}{21 a^4 d}, x \right) + \sqrt{2} \left(-10i e^4 e^{(4i dx + 4i c)} - 4i e^4 e^{(2i dx + 2i c)} \right) \right)}{21 a^4 d}$$

59.75 Problem number 259

$$\int \frac{(e \sec(c + dx))^{7/2}}{(a + ia \tan(c + dx))^4} dx$$

Optimal antiderivative

$$\frac{2e^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx+c)} \sqrt{e \sec(dx+c)}} + \frac{4ie^2 (e \sec(dx+c))^{\frac{3}{2}}}{9ad (a + ia \tan(dx+c))^3} - \frac{4ie^4}{15d \sqrt{e \sec(dx+c)} (a^4 + ia^4 \tan(dx+c))}$$

command

```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-6i\sqrt{2}e^{(5i dx+5i c+\frac{7}{2})}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)}))\right) + \frac{\sqrt{2}\left(5ie^{\frac{7}{2}}-6ie^{(6i dx+6i c+\frac{7}{2})}\right)}{45a^4d}}{45a^4d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(45a^4de^{(5i dx+5i c)}\text{integral}\left(\frac{i\sqrt{2}e^3\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}e^{(\frac{1}{2}i dx+\frac{1}{2}i c)}}{15a^4d},x\right) + \sqrt{2}\left(-6ie^3e^{(6i dx+6i c)}-4ie^3e^{(4i dx+4i c)}\right)\right)}{45a^4d}$$

59.76 Problem number 260

$$\int \frac{(e \sec(c + dx))^{5/2}}{(a + ia \tan(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2e^3 \sin(dx + c)}{77a^4d\sqrt{e \sec(dx + c)}} \\ & -\frac{2e^2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{77 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4d} \\ & + \frac{4ie^2\sqrt{e \sec(dx + c)}}{11ad(a + ia \tan(dx + c))^3} - \frac{4ie^4}{77d(e \sec(dx + c))^{\frac{3}{2}}(a^4 + ia^4 \tan(dx + c))} \end{aligned}$$

command

```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(4i\sqrt{2}e^{(6i dx+6i c+\frac{5}{2})}\text{weierstrassPInverse}(-4,0,e^{(i dx+i c)})\right) + \frac{\sqrt{2}\left(7ie^{\frac{5}{2}}+4ie^{(6i dx+6i c+\frac{5}{2})}+17ie^{(4i dx+4i c+\frac{5}{2})}+20ie^{(2i dx+2i c)}\right)}{\sqrt{e^{(2i dx+2i c)}+1}}}{154a^4d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(154a^4de^{(6i dx+6i c)}\text{integral}\left(\frac{i\sqrt{2}e^2\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)}}{77a^4d},x\right) + \sqrt{2}\left(4ie^2e^{(6i dx+6i c)}+17ie^2e^{(4i dx+4i c)}\right)\right)}{154a^4d}$$

59.77 Problem number 261

$$\int \frac{(e \sec(c + dx))^{3/2}}{(a + ia \tan(c + dx))^4} dx$$

Optimal antiderivative

$$\frac{2e^3 \sin(dx + c)}{117a^4 d (e \sec(dx + c))^{\frac{3}{2}}} + \frac{2e^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{39 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx + c)} \sqrt{e \sec(dx + c)}} + \frac{4ie^2}{13ad \sqrt{e \sec(dx + c)} (a + ia \tan(dx + c))^3} + \frac{4ie^4}{117d (e \sec(dx + c))^{\frac{5}{2}} (a^4 + ia^4 \tan(dx + c))}$$

command

`integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(24i \sqrt{2} e^{(7i dx + 7i c + \frac{3}{2})} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)})) + \sqrt{2} \left(9i e^{\frac{3}{2}} + 24i e^{(8i dx + 8i c + \frac{3}{2})}\right)\right)}{468 a^4 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(468 a^4 d e^{(7i dx + 7i c)} \operatorname{integral}\left(-\frac{i \sqrt{2} e \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} e^{(\frac{1}{2} i dx + \frac{1}{2} i c)}}{39 a^4 d}, x\right) + \sqrt{2} (24i e e^{(8i dx + 8i c)} + 55i e e^{(6i dx + 6i c)})\right)}{468 a^4 d}$$

59.78 Problem number 262

$$\int \frac{\sqrt{e \sec(c + dx)}}{(a + ia \tan(c + dx))^4} dx$$

Optimal antiderivative

$$\frac{2e \sin(dx + c)}{33a^4 d \sqrt{e \sec(dx + c)}} + \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{e \sec(dx + c)}}{33 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} + \frac{2i \sqrt{e \sec(dx + c)}}{15d (a + ia \tan(dx + c))^4} + \frac{14i \sqrt{e \sec(dx + c)}}{165ad (a + ia \tan(dx + c))^3} + \frac{4ie^2}{33d (e \sec(dx + c))^{\frac{3}{2}} (a^4 + ia^4 \tan(dx + c))}$$

command

```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-80i \sqrt{2} e^{(8i dx + 8i c + \frac{1}{2})} \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \frac{\sqrt{2} \left(11i e^{\frac{1}{2}} + 85i e^{(8i dx + 8i c + \frac{1}{2})} + 166i e^{(6i dx + 6i c + \frac{1}{2})} + 128i e^{(4i dx + 4i c + \frac{1}{2})} \right)}{\sqrt{e^{(2i dx + 2i c)}}} \right) / 1320 a^4 d$$

Fricas 1.3.7 via sagemath 9.3 output

$$\left(1320 a^4 d e^{(8i dx + 8i c)} \text{integral} \left(-\frac{i \sqrt{2} \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}}} {33 a^4 d} e^{(-\frac{1}{2} i dx - \frac{1}{2} i c)}, x \right) + \sqrt{2} \sqrt{\frac{e}{e^{(2i dx + 2i c)} + 1}} (85i e^{(8i dx + 8i c)}) \right) / 1320 a^4 d$$

59.79 Problem number 578

$$\int (d \sec(e + fx))^{7/2} (a + b \tan(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(d \sec(fx + e))^{7/2}}{7f} + \frac{2ad(d \sec(fx + e))^{5/2} \sin(fx + e)}{5f} \\ & - \frac{6a d^4 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{fx}{2} + \frac{e}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{fx}{2} + \frac{e}{2} \right) f \sqrt{\cos(fx + e)} \sqrt{d \sec(fx + e)}} \\ & + \frac{6a d^3 \sin(fx + e) \sqrt{d \sec(fx + e)}}{5f} \end{aligned}$$

command

```
integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-21i \sqrt{2} a d^{7/2} \cos(fx + e)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 21i \sqrt{2} a d^{7/2} \sin(fx + e)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(b d^3 \sec(fx + e)^3 \tan(fx + e) + a d^3 \sec(fx + e)^3 \right) \sqrt{d \sec(fx + e)}, x \right)$$

59.80 Problem number 579

$$\int (d \sec(e + fx))^{5/2} (a + b \tan(e + fx)) dx$$

Optimal antiderivative

$$\frac{2b(d \sec(fx + e))^{5/2}}{5f} + \frac{2ad(d \sec(fx + e))^{3/2} \sin(fx + e)}{3f} + \frac{2a d^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{d \sec(fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f}$$

command

```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} a d^{5/2} \cos(fx + e)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + 5i \sqrt{2} a d^{5/2} \cos(fx + e)^2 \operatorname{weier}$$

15 f cos(f

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(bd^2 \sec(fx + e)^2 \tan(fx + e) + ad^2 \sec(fx + e)^2\right) \sqrt{d \sec(fx + e)}, x\right)$$

59.81 Problem number 580

$$\int (d \sec(e + fx))^{3/2} (a + b \tan(e + fx)) dx$$

Optimal antiderivative

$$\frac{2b(d \sec(fx + e))^{3/2}}{3f} - \frac{2a d^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{d \sec(fx + e)}} + \frac{2ad \sin(fx + e) \sqrt{d \sec(fx + e)}}{f}$$

command

```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} a d^{\frac{3}{2}} \cos (fx + e) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e))) + 3i \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bd \sec (fx + e) \tan (fx + e) + ad \sec (fx + e)) \sqrt{d \sec (fx + e)}, x\right)$$

59.82 Problem number 581

$$\int \sqrt{d \sec (e + fx)} (a + b \tan (e + fx)) dx$$

Optimal antiderivative

$$\frac{2b \sqrt{d \sec (fx + e)}}{f} + \frac{2a \sqrt{\frac{\cos (fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin \left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos (fx + e)}) \sqrt{d \sec (fx + e)}}{\cos \left(\frac{fx}{2} + \frac{e}{2}\right) f}$$

command

```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} a \sqrt{d} \operatorname{weierstrassPInverse}(-4, 0, \cos (fx + e) + i \sin (fx + e)) + i \sqrt{2} a \sqrt{d} \operatorname{weierstrassPInverse}(-4, 0, \cos ($$

f

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \sec (fx + e)} (b \tan (fx + e) + a), x\right)$$

59.83 Problem number 582

$$\int \frac{a + b \tan(e + fx)}{\sqrt{d \sec(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{2b}{f \sqrt{d \sec(fx + e)}} + \frac{2a \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{d \sec(fx + e)}}$$

command

```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$i \sqrt{2} a \sqrt{d}$ weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(fx + e) + i sin(fx + e))) - $i \sqrt{2} a \sqrt{d}$ weierstrass

df

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec(fx + e)} (b \tan(fx + e) + a)}{d \sec(fx + e)}, x\right)$$

59.84 Problem number 583

$$\int \frac{a + b \tan(e + fx)}{(d \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2b}{3f (d \sec(fx + e))^{3/2}} + \frac{2a \sin(fx + e)}{3df \sqrt{d \sec(fx + e)}} + \frac{2a \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{d \sec(fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) d^2 f}$$

command

```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} a \sqrt{d} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + i \sqrt{2} a \sqrt{d} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e))$$

 $3 d^2 f$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec(fx + e)} (b \tan(fx + e) + a)}{d^2 \sec(fx + e)^2}, x\right)$$

59.85 Problem number 584

$$\int \frac{a + b \tan(e + fx)}{(d \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b}{5f(d \sec(fx + e))^{5/2}} + \frac{2a \sin(fx + e)}{5df(d \sec(fx + e))^{3/2}} \\ & + \frac{6a \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) d^2 f \sqrt{\cos(fx + e)} \sqrt{d \sec(fx + e)}} \end{aligned}$$

command

`integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i \sqrt{2} a \sqrt{d} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) - 3i \sqrt{2} a \sqrt{d} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) - i \sin(fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec(fx + e)} (b \tan(fx + e) + a)}{d^3 \sec(fx + e)^3}, x\right)$$

59.86 Problem number 585

$$\int \frac{a + b \tan(e + fx)}{(d \sec(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b}{7f(d \sec(fx + e))^{7/2}} + \frac{2a \sin(fx + e)}{7df(d \sec(fx + e))^{5/2}} + \frac{10a \sin(fx + e)}{21d^3 f \sqrt{d \sec(fx + e)}} \\ & + \frac{10a \sqrt{\frac{\cos(fx + e)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{d \sec(fx + e)}}{21 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) d^4 f} \end{aligned}$$

command

```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} a \sqrt{d} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + 5i \sqrt{2} a \sqrt{d} \operatorname{weierstrassPInverse}(-4, 0, \cos$$

2

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \sec(fx + e)} (b \tan(fx + e) + a)}{d^4 \sec(fx + e)^4}, x\right)$$

59.87 Problem number 586

$$\int (d \sec(e + fx))^{5/2} (a + b \tan(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{18ab(d \sec(fx + e))^{5/2}}{35f} + \frac{2(7a^2 - 2b^2) d(d \sec(fx + e))^{3/2} \sin(fx + e)}{21f} \\ & + \frac{2(7a^2 - 2b^2) d^2 \sqrt{\frac{\cos(fx + e)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{d \sec(fx + e)}}{21 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f} \\ & + \frac{2b(d \sec(fx + e))^{5/2} (a + b \tan(fx + e))}{7f} \end{aligned}$$

command

```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (7a^2 - 2b^2) d^{\frac{5}{2}} \cos(fx + e)^3 \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + 5i \sqrt{2} (7a^2 - 2b^2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 d^2 \sec(fx + e)^2 \tan(fx + e)^2 + 2abd^2 \sec(fx + e)^2 \tan(fx + e) + a^2 d^2 \sec(fx + e)^2\right) \sqrt{d \sec(fx + e)}, x\right)$$

59.88 Problem number 587

$$\int (d \sec(e + fx))^{3/2} (a + b \tan(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{14ab(d \sec(fx + e))^{\frac{3}{2}}}{15f} - \frac{2(5a^2 - 2b^2) d^2 \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{d \sec(fx + e)}} + \frac{2(5a^2 - 2b^2) d \sin(fx + e) \sqrt{d \sec(fx + e)}}{5f} + \frac{2b(d \sec(fx + e))^{\frac{3}{2}} (a + b \tan(fx + e))}{5f}$$

command

```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} (5a^2 - 2b^2) d^{\frac{3}{2}} \cos(fx + e)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 d \sec(fx + e) \tan(fx + e)^2 + 2abd \sec(fx + e) \tan(fx + e) + a^2 d \sec(fx + e)\right) \sqrt{d \sec(fx + e)}, x\right)$$

59.89 Problem number 588

$$\int \sqrt{d \sec(e + fx)} (a + b \tan(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10ab\sqrt{d \sec(fx + e)}}{3f} \\ & + \frac{2(3a^2 - 2b^2) \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{d \sec(fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) f} \\ & + \frac{2b\sqrt{d \sec(fx + e)} (a + b \tan(fx + e))}{3f} \end{aligned}$$

command

`integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i a^2 + 2i b^2) \sqrt{d} \cos(fx + e) \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + \sqrt{2} (3i a^2 - 2i b^2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \tan(fx + e)^2 + 2ab \tan(fx + e) + a^2\right) \sqrt{d \sec(fx + e)}, x\right)$$

59.90 Problem number 589

$$\int \frac{(a + b \tan(e + fx))^2}{\sqrt{d \sec(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6ab}{f \sqrt{d \sec(fx + e)}} + \frac{2(a^2 - 2b^2) \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) f \sqrt{\cos(fx + e)} \sqrt{d \sec(fx + e)}} \\ & + \frac{2b(a + b \tan(fx + e))}{f \sqrt{d \sec(fx + e)}} \end{aligned}$$

command

```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (i a^2 - 2i b^2) \sqrt{d} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + \sqrt{2} (-i a^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b^2 \tan(fx + e)^2 + 2ab \tan(fx + e) + a^2) \sqrt{d \sec(fx + e)}}{d \sec(fx + e)}, x \right)$$

59.91 Problem number 590

$$\int \frac{(a + b \tan(e + fx))^2}{(d \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ab}{3f(d \sec(fx + e))^{\frac{3}{2}}} + \frac{2(a^2 + 2b^2) \sin(fx + e)}{3df \sqrt{d \sec(fx + e)}} \\ & + \frac{2(a^2 + 2b^2) \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)} \sqrt{d \sec(fx + e)}}{3 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) d^2 f} \\ & - \frac{2b(a + b \tan(fx + e))}{f(d \sec(fx + e))^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i a^2 - 2i b^2) \sqrt{d} \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + \sqrt{2} (i a^2 + 2i b^2) \sqrt{d} \text{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b^2 \tan(fx + e)^2 + 2ab \tan(fx + e) + a^2) \sqrt{d \sec(fx + e)}}{d^2 \sec(fx + e)^2}, x \right)$$

59.92 Problem number 591

$$\int \frac{(a + b \tan(e + fx))^2}{(d \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ab}{15f(d \sec(fx + e))^{\frac{5}{2}}} + \frac{2(3a^2 + 2b^2) \sin(fx + e)}{15df(d \sec(fx + e))^{\frac{3}{2}}} \\ & + \frac{2(3a^2 + 2b^2) \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) d^2 f \sqrt{\cos(fx + e)} \sqrt{d \sec(fx + e)}} - \frac{2b(a + b \tan(fx + e))}{3f(d \sec(fx + e))^{\frac{5}{2}}} \end{aligned}$$

command

`integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (3i a^2 + 2i b^2) \sqrt{d} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + \sqrt{2} (-3i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2 \tan(fx + e)^2 + 2ab \tan(fx + e) + a^2) \sqrt{d \sec(fx + e)}}{d^3 \sec(fx + e)^3}, x\right)$$

59.93 Problem number 592

$$\int \frac{(a + b \tan(e + fx))^2}{(d \sec(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6ab}{35f(d \sec(fx + e))^{\frac{7}{2}}} + \frac{2(5a^2 + 2b^2) \sin(fx + e)}{35df(d \sec(fx + e))^{\frac{5}{2}}} + \frac{2(5a^2 + 2b^2) \sin(fx + e)}{21d^3 f \sqrt{d \sec(fx + e)}} \\ & + \frac{2(5a^2 + 2b^2) \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right) (\sqrt{\cos(fx + e)}) \sqrt{d \sec(fx + e)}}{21 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) d^4 f} \\ & - \frac{2b(a + b \tan(fx + e))}{5f(d \sec(fx + e))^{\frac{7}{2}}} \end{aligned}$$

command

```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-5i a^2 - 2i b^2) \sqrt{d} \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + \sqrt{2} (5i a^2 + 2i b^2) \sqrt{d} \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b^2 \tan(fx + e)^2 + 2ab \tan(fx + e) + a^2) \sqrt{d \sec(fx + e)}}{d^4 \sec(fx + e)^4}, x \right)$$

59.94 Problem number 593

$$\int \frac{(a + b \tan(e + fx))^2}{(d \sec(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{10ab}{63f(d \sec(fx + e))^{9/2}} + \frac{2(7a^2 + 2b^2) \sin(fx + e)}{63df(d \sec(fx + e))^{7/2}} + \frac{2(7a^2 + 2b^2) \sin(fx + e)}{45d^3f(d \sec(fx + e))^{5/2}} \\ & + \frac{2(7a^2 + 2b^2) \sqrt{\frac{\cos(fx + e)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{fx}{2} + \frac{e}{2}\right) d^4 f \sqrt{\cos(fx + e)} \sqrt{d \sec(fx + e)}} - \frac{2b(a + b \tan(fx + e))}{7f(d \sec(fx + e))^{9/2}} \end{aligned}$$

command

```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \sqrt{2} (-7i a^2 - 2i b^2) \sqrt{d} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 3 \sqrt{2} (7i a^2 + 2i b^2) \sqrt{d} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b^2 \tan(fx + e)^2 + 2ab \tan(fx + e) + a^2) \sqrt{d \sec(fx + e)}}{d^5 \sec(fx + e)^5}, x \right)$$

59.95 Problem number 594

$$\int (d \sec(e + fx))^{5/2} (a + b \tan(e + fx))^3 dx$$

Optimal antiderivative

$$\frac{a(7a^2 - 6b^2) d^2 \sqrt{2} \sqrt{\frac{1 + \sqrt{1 + \tan^2(fx + e)}}{\sqrt{1 + \tan^2(fx + e)}}} \operatorname{EllipticF}\left(\sin\left(\frac{\arctan(\tan(fx + e))}{2}\right), \sqrt{2}\right) \sqrt{d \sec(fx + e)}}{21 \cos\left(\frac{\arctan(\tan(fx + e))}{2}\right) f (\sec^2(fx + e))^{\frac{1}{4}}}$$

$$+ \frac{2a(7a^2 - 6b^2) d^2 \sqrt{d \sec(fx + e)} \tan(fx + e)}{21f}$$

$$+ \frac{2b d^2 (\sec^2(fx + e)) \sqrt{d \sec(fx + e)} (a + b \tan(fx + e))^2}{9f}$$

$$+ \frac{2b d^2 (\sec^2(fx + e)) \sqrt{d \sec(fx + e)} (154a^2 - 28b^2 + 65ab \tan(fx + e))}{315f}$$

command

`integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i \sqrt{2} (7a^3 - 6ab^2) d^{\frac{5}{2}} \cos(fx + e)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + 15i \sqrt{2} (7a^3 -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^3 d^2 \sec(fx + e)^2 \tan(fx + e)^3 + 3ab^2 d^2 \sec(fx + e)^2 \tan(fx + e)^2 + 3a^2 b d^2 \sec(fx + e)^2 \tan(fx + e)\right)\right)$$

59.96 Problem number 595

$$\int (d \sec(e + fx))^{3/2} (a + b \tan(e + fx))^3 dx$$

Optimal antiderivative

$$\frac{a(5a^2 - 6b^2) \sqrt{2} \sqrt{\frac{1 + \sqrt{1 + \tan^2(fx + e)}}{\sqrt{1 + \tan^2(fx + e)}}} \operatorname{EllipticE}\left(\sin\left(\frac{\arctan(\tan(fx + e))}{2}\right), \sqrt{2}\right) (d \sec(fx + e))^{\frac{3}{2}}}{5 \cos\left(\frac{\arctan(\tan(fx + e))}{2}\right) f (\sec^2(fx + e))^{\frac{3}{4}}}$$

$$+ \frac{2a(5a^2 - 6b^2) \cos(fx + e) (d \sec(fx + e))^{\frac{3}{2}} \sin(fx + e)}{5f}$$

$$+ \frac{2b(d \sec(fx + e))^{\frac{3}{2}} (a + b \tan(fx + e))^2}{7f} + \frac{2b(d \sec(fx + e))^{\frac{3}{2}} (90a^2 - 20b^2 + 33ab \tan(fx + e))}{105f}$$

command

```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-21i\sqrt{2}(5a^3 - 6ab^2)d^{\frac{3}{2}}\cos(fx + e)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^3 d \sec(fx + e) \tan(fx + e)^3 + 3ab^2 d \sec(fx + e) \tan(fx + e)^2 + 3a^2 b d \sec(fx + e) \tan(fx + e) + a^3 d\right) \sqrt{d \sec(fx + e)}, x\right)$$

59.97 Problem number 596

$$\int \sqrt{d \sec(e + fx)} (a + b \tan(e + fx))^3 dx$$

Optimal antiderivative

$$\frac{a(a^2 - 2b^2)\sqrt{2} \sqrt{\frac{1 + \sqrt{1 + \tan^2(fx + e)}}{\sqrt{1 + \tan^2(fx + e)}}} \text{EllipticF}\left(\sin\left(\frac{\arctan(\tan(fx + e))}{2}\right), \sqrt{2}\right) \sqrt{d \sec(fx + e)}}{\cos\left(\frac{\arctan(\tan(fx + e))}{2}\right) f (\sec^2(fx + e))^{\frac{1}{4}}}$$

$$+ \frac{2b\sqrt{d \sec(fx + e)} (a + b \tan(fx + e))^2}{5f} + \frac{2b\sqrt{d \sec(fx + e)} (14a^2 - 4b^2 + 3ab \tan(fx + e))}{5f}$$

command

```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(ia^3 - 2iab^2)\sqrt{d}\cos(fx + e)^2 \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + 5\sqrt{2}(-ia^3 + 2iab^2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^3 \tan(fx + e)^3 + 3ab^2 \tan(fx + e)^2 + 3a^2 b \tan(fx + e) + a^3\right) \sqrt{d \sec(fx + e)}, x\right)$$

59.98 Problem number 597

$$\int \frac{(a + b \tan(e + fx))^3}{\sqrt{d \sec(e + fx)}} dx$$

Optimal antiderivative

$$\frac{a(a^2 - 6b^2) \sqrt{2} \sqrt{\frac{1 + \sqrt{1 + \tan^2(fx + e)}}{\sqrt{1 + \tan^2(fx + e)}}} \operatorname{EllipticE}\left(\sin\left(\frac{\arctan(\tan(fx + e))}{2}\right), \sqrt{2}\right) (\sec^2(fx + e))^{\frac{1}{4}}}{\cos\left(\frac{\arctan(\tan(fx + e))}{2}\right) f \sqrt{d \sec(fx + e)}} - \frac{2a(a^2 - 6b^2) \tan(fx + e)}{f \sqrt{d \sec(fx + e)}} - \frac{2(b - a \tan(fx + e))(a + b \tan(fx + e))^2}{f \sqrt{d \sec(fx + e)}} - \frac{2b(\sec^2(fx + e))(6a^2 - 4b^2 + 3ab \tan(fx + e))}{3f \sqrt{d \sec(fx + e)}}$$

command

`integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \sqrt{2} (-i a^3 + 6i ab^2) \sqrt{d} \cos(fx + e) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e)) + i \sin(fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^3 \tan(fx + e)^3 + 3ab^2 \tan(fx + e)^2 + 3a^2b \tan(fx + e) + a^3) \sqrt{d \sec(fx + e)}}{d \sec(fx + e)}, x\right)$$

59.99 Problem number 598

$$\int \frac{(a + b \tan(e + fx))^3}{(d \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a(a^2 + 6b^2) \sqrt{2} \sqrt{\frac{1 + \sqrt{1 + \tan^2(fx + e)}}{\sqrt{1 + \tan^2(fx + e)}}} \operatorname{EllipticF}\left(\sin\left(\frac{\arctan(\tan(fx + e))}{2}\right), \sqrt{2}\right) (\sec^2(fx + e))^{\frac{3}{4}}}{3 \cos\left(\frac{\arctan(\tan(fx + e))}{2}\right) f (d \sec(fx + e))^{\frac{3}{2}}} - \frac{2(b - a \tan(fx + e))(a + b \tan(fx + e))^2}{3f (d \sec(fx + e))^{\frac{3}{2}}} - \frac{2b(\sec^2(fx + e))(2a^2 - 4b^2 + ab \tan(fx + e))}{3f (d \sec(fx + e))^{\frac{3}{2}}}$$

command

`integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i a^3 - 6i ab^2) \sqrt{d} \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + \sqrt{2} (i a^3 + 6i ab^2) \sqrt{d} \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b^3 \tan(fx + e)^3 + 3ab^2 \tan(fx + e)^2 + 3a^2b \tan(fx + e) + a^3) \sqrt{d \sec(fx + e)}}{d^2 \sec(fx + e)^2}, x \right)$$

59.100 Problem number 599

$$\int \frac{(a + b \tan(e + fx))^3}{(d \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{3a(a^2 + 2b^2) \sqrt{2} \sqrt{\frac{1 + \sqrt{1 + \tan^2(fx + e)}}{\sqrt{1 + \tan^2(fx + e)}}} \text{EllipticE} \left(\sin \left(\frac{\arctan(\tan(fx + e))}{2} \right), \sqrt{2} \right) (\sec^2(fx + e))^{\frac{1}{4}}}{5 \cos \left(\frac{\arctan(\tan(fx + e))}{2} \right) d^2 f \sqrt{d \sec(fx + e)}} - \frac{6a(a^2 + 2b^2) \tan(fx + e)}{5d^2 f \sqrt{d \sec(fx + e)}} - \frac{2(\cos^2(fx + e)) (b - a \tan(fx + e)) (a + b \tan(fx + e))^2}{5d^2 f \sqrt{d \sec(fx + e)}} - \frac{2(2b(a^2 + 2b^2) - a(3a^2 + 5b^2) \tan(fx + e))}{5d^2 f \sqrt{d \sec(fx + e)}}$$

command

`integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \sqrt{2} (-i a^3 - 2i ab^2) \sqrt{d} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 3 \sqrt{2} (i a^3 + 2i ab^2) \sqrt{d} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(b^3 \tan(fx + e)^3 + 3ab^2 \tan(fx + e)^2 + 3a^2b \tan(fx + e) + a^3) \sqrt{d \sec(fx + e)}}{d^3 \sec(fx + e)^3}, x \right)$$

59.101 Problem number 600

$$\int \frac{(a + b \tan(e + fx))^3}{(d \sec(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{a(5a^2 + 6b^2) \sqrt{2} \sqrt{\frac{1 + \sqrt{1 + \tan^2(fx + e)}}{\sqrt{1 + \tan^2(fx + e)}}} \operatorname{EllipticF}\left(\sin\left(\frac{\arctan(\tan(fx + e))}{2}\right), \sqrt{2}\right) (\sec^2(fx + e))^{\frac{3}{4}}}{21 \cos\left(\frac{\arctan(\tan(fx + e))}{2}\right) d^2 f (d \sec(fx + e))^{\frac{3}{2}}}$$

$$\frac{2(\cos^2(fx + e))(b - a \tan(fx + e))(a + b \tan(fx + e))^2}{7d^2 f (d \sec(fx + e))^{\frac{3}{2}}}$$

$$\frac{2(2b(3a^2 + 2b^2) - a(5a^2 + 3b^2) \tan(fx + e))}{21d^2 f (d \sec(fx + e))^{\frac{3}{2}}}$$

command

`integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-5i a^3 - 6i ab^2) \sqrt{d} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + \sqrt{2}(5i a^3 + 6i ab^2) \sqrt{d} \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^3 \tan(fx + e))^3 + 3ab^2 \tan(fx + e)^2 + 3a^2b \tan(fx + e) + a^3}{d^4 \sec(fx + e)^4} \sqrt{d \sec(fx + e)}, x\right)$$

59.102 Problem number 601

$$\int \frac{(a + b \tan(e + fx))^3}{(d \sec(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{a(7a^2 + 6b^2) \sqrt{2} \sqrt{\frac{1 + \sqrt{1 + \tan^2(fx + e)}}{\sqrt{1 + \tan^2(fx + e)}}} \operatorname{EllipticE}\left(\sin\left(\frac{\arctan(\tan(fx + e))}{2}\right), \sqrt{2}\right) (\sec^2(fx + e))^{\frac{1}{4}}}{15 \cos\left(\frac{\arctan(\tan(fx + e))}{2}\right) d^4 f \sqrt{d \sec(fx + e)}}$$

$$\frac{2(\cos^4(fx + e))(b - a \tan(fx + e))(a + b \tan(fx + e))^2}{9d^4 f \sqrt{d \sec(fx + e)}}$$

$$\frac{2(\cos^2(fx + e))(2b(5a^2 + 2b^2) - a(7a^2 + b^2) \tan(fx + e))}{45d^4 f \sqrt{d \sec(fx + e)}}$$

command

`integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2}(-7ia^3 - 6iab^2)\sqrt{d} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e))) + 3\sqrt{2} \operatorname{weierstrassP}(-4, 0, \cos(fx + e) + i \sin(fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^3 \tan(fx + e)^3 + 3ab^2 \tan(fx + e)^2 + 3a^2b \tan(fx + e) + a^3) \sqrt{d \sec(fx + e)}}{d^5 \sec(fx + e)^5}, x\right)$$

59.103 Problem number 602

$$\int \frac{(a + b \tan(e + fx))^3}{(d \sec(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{5a(3a^2 + 2b^2) \sqrt{2} \sqrt{\frac{1 + \sqrt{1 + \tan^2(fx + e)}}{\sqrt{1 + \tan^2(fx + e)}}} \operatorname{EllipticF}\left(\sin\left(\frac{\arctan(\tan(fx + e))}{2}\right), \sqrt{2}\right) (\sec^2(fx + e))^{\frac{3}{4}}}{77 \cos\left(\frac{\arctan(\tan(fx + e))}{2}\right) d^4 f (d \sec(fx + e))^{\frac{3}{2}}}$$

$$+ \frac{10a(3a^2 + 2b^2) \tan(fx + e)}{77 d^4 f (d \sec(fx + e))^{\frac{3}{2}}} - \frac{2(\cos^4(fx + e)) (b - a \tan(fx + e)) (a + b \tan(fx + e))^2}{11 d^4 f (d \sec(fx + e))^{\frac{3}{2}}}$$

$$- \frac{2(\cos^2(fx + e)) (2b(7a^2 + 2b^2) - a(9a^2 - b^2) \tan(fx + e))}{77 d^4 f (d \sec(fx + e))^{\frac{3}{2}}}$$

command

`integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(3ia^3 + 2iab^2)\sqrt{d} \operatorname{weierstrassPInverse}(-4, 0, \cos(fx + e) + i \sin(fx + e)) + 5\sqrt{2}(-3ia^3 - 2iab^2)\sqrt{d} \operatorname{weierstrassP}(-4, 0, \cos(fx + e) + i \sin(fx + e))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^3 \tan(fx + e)^3 + 3ab^2 \tan(fx + e)^2 + 3a^2b \tan(fx + e) + a^3) \sqrt{d \sec(fx + e)}}{d^6 \sec(fx + e)^6}, x\right)$$

59.104 Problem number 655

$$\int (e \cos(c + dx))^{7/2} (a + ia \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ia(e \cos(dx + c))^{7/2}}{7d} \\ & + \frac{10a(e \cos(dx + c))^{7/2} \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \cos(dx + c)^{7/2}} \\ & + \frac{2a(e \cos(dx + c))^{7/2} \tan(dx + c)}{7d} + \frac{10a(e \cos(dx + c))^{7/2} (\sec^2(dx + c)) \tan(dx + c)}{21d} \end{aligned}$$

command

`integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-20i \sqrt{2} a e^{(i dx + i c + \frac{7}{2})} \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) + \sqrt{\frac{1}{2}} \left(7i a e^{\frac{7}{2}} - 3i a e^{(4i dx + 4i c + \frac{7}{2})} - 16i a e^{(2i dx + 2i c)}\right)\right)}{42 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(\sqrt{\frac{1}{2}} \left(-3i a e^3 e^{(4i dx + 4i c)} - 16i a e^3 e^{(2i dx + 2i c)} + 7i a e^3\right) \sqrt{e e^{(2i dx + 2i c)} + e} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)} + 42 d e^{(i dx + i c)} \operatorname{integral}\left(-\right)\right)}{42 d}$$

59.105 Problem number 656

$$\int (e \cos(c + dx))^{5/2} (a + ia \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ia(e \cos(dx + c))^{5/2}}{5d} \\ & + \frac{6a(e \cos(dx + c))^{5/2} \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \cos(dx + c)^{5/2}} \\ & + \frac{2a(e \cos(dx + c))^{5/2} \tan(dx + c)}{5d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6i\sqrt{2}ae^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,e^{(idx+ic)})) + \sqrt{\frac{1}{2}}\left(5iae^{\frac{5}{2}} - iae^{(2idx+2ic+\frac{5}{2})}\right)\sqrt{e^{(2idx+2ic)}}}{5d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{\frac{1}{2}}\left(-iae^2e^{(3idx+3ic)} + iae^2e^{(2idx+2ic)} - 7iae^2e^{(idx+ic)} - 5iae^2\right)\sqrt{ee^{(2idx+2ic)} + e}e^{(-\frac{1}{2}idx-\frac{1}{2}ic)} + 5\left(de^{(idx+ic)} - d\right)}{5\left(de^{(idx+ic)} - d\right)}$$

59.106 Problem number 657

$$\int (e \cos(c + dx))^{3/2} (a + ia \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ia(e \cos(dx + c))^{\frac{3}{2}}}{3d} \\ & + \frac{2a(e \cos(dx + c))^{\frac{3}{2}} \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2a(e \cos(dx + c))^{\frac{3}{2}} \tan(dx + c)}{3d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(i\sqrt{2}ae^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,e^{(idx+ic)}) + i\sqrt{\frac{1}{2}}a\sqrt{e^{(2idx+2ic)}+1}e^{(\frac{1}{2}idx+\frac{1}{2}ic+\frac{3}{2})}\right)}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{-2i\sqrt{\frac{1}{2}}\sqrt{ee^{(2idx+2ic)}+e}ae^{(\frac{1}{2}idx+\frac{1}{2}ic)} + 3d\text{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{ee^{(2idx+2ic)}+e}ae^{(-\frac{1}{2}idx-\frac{1}{2}ic)}}{3(de^{(2idx+2ic)}+d)},x\right)}{3d}$$

59.107 Problem number 658

$$\int \sqrt{e \cos(c + dx)} (a + ia \tan(c + dx)) dx$$

Optimal antiderivative

$$-\frac{2ia \sqrt{e \cos(dx + c)}}{d} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)}}$$

command

```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2i \sqrt{2} a e^{\frac{1}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$-4i \sqrt{\frac{1}{2}} \sqrt{e e^{(2i dx + 2i c)} + e} a e^{(\frac{1}{2} i dx + \frac{1}{2} i c)} + (d e^{(i dx + i c)} - d) \operatorname{integral}\left(\frac{\sqrt{\frac{1}{2}} (-2i a e^{(2i dx + 2i c)} - 4i a e^{(i dx + i c)} - 2i a) \sqrt{e e^{(2i dx + 2i c)}}}{d e^{(4i dx + 4i c)} - 2 d e^{(3i dx + 3i c)} + 2 d e^{(2i dx + 2i c)}}\right)$$

$$d e^{(i dx + i c)} - d$$

59.108 Problem number 659

$$\int \frac{a + ia \tan(c + dx)}{\sqrt{e \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2ia}{d \sqrt{e \cos(dx + c)}} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{e \cos(dx + c)}}$$

command

```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-2i \sqrt{\frac{1}{2}} a \sqrt{e^{(2i dx + 2i c)} + 1} e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + \left(i \sqrt{2} a e^{(2i dx + 2i c)} + i \sqrt{2} a \right) \text{weierstrassPInverse}(-4, 0, e^{(i dx + i c)}) \right)}{de^{\frac{1}{2}} + de^{(2i dx + 2i c + \frac{1}{2})}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{4i \sqrt{\frac{1}{2}} \sqrt{e e^{(2i dx + 2i c)} + e} a e^{\left(\frac{1}{2}i dx + \frac{1}{2}i c\right)} + (de e^{(2i dx + 2i c)} + de) \text{integral} \left(-\frac{2i \sqrt{\frac{1}{2}} \sqrt{e e^{(2i dx + 2i c)} + e} a e^{\left(-\frac{1}{2}i dx - \frac{1}{2}i c\right)}}{de e^{(2i dx + 2i c)} + de}, x \right)}{de e^{(2i dx + 2i c)} + de}$$

59.109 Problem number 660

$$\int \frac{a + ia \tan(c + dx)}{(e \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2ia}{3d (e \cos(dx + c))^{\frac{3}{2}}} - \frac{2a \left(\cos^{\frac{3}{2}}(dx + c) \right) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d (e \cos(dx + c))^{\frac{3}{2}}} + \frac{2a \sin(dx + c)}{de \sqrt{e \cos(dx + c)}}$$

command

`integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{\frac{1}{2}} \left(3i a e^{(4i dx + 4i c)} + i a e^{(2i dx + 2i c)} \right) \sqrt{e^{(2i dx + 2i c)} + 1} e^{\left(-\frac{1}{2}i dx - \frac{1}{2}i c\right)} + 3 \left(i \sqrt{2} a e^{(4i dx + 4i c)} + 2i \sqrt{2} a e^{(2i dx + 2i c)} \right) \right)}{3 \left(de^{\frac{3}{2}} + de^{(4i dx + 4i c + \frac{3}{2})} + 2 de^{(2i dx + 2i c)} \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{\frac{1}{2}} \left(-12i a e^{(4i dx + 4i c)} - 4i a e^{(2i dx + 2i c)} \right) \sqrt{e e^{(2i dx + 2i c)} + e} e^{\left(-\frac{1}{2}i dx - \frac{1}{2}i c\right)} + 3 \left(de^2 e^{(4i dx + 4i c)} + 2 de^2 e^{(2i dx + 2i c)} + de^2 \right)}{3 \left(de^2 e^{(4i dx + 4i c)} + 2 de^2 e^{(2i dx + 2i c)} + de^2 \right)}$$

59.110 Problem number 661

$$\int \frac{a + ia \tan(c + dx)}{(e \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2ia}{5d(e \cos(dx + c))^{5/2}} + \frac{2a \left(\cos^{5/2}(dx + c) \right) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d(e \cos(dx + c))^{5/2}} + \frac{2a \cos(dx + c) \sin(dx + c)}{3d(e \cos(dx + c))^{5/2}}$$

command

`integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{\frac{1}{2}} (5i a e^{(5i dx + 5i c)} - 12i a e^{(3i dx + 3i c)} - 5i a e^{(i dx + i c)}) \sqrt{e^{(2i dx + 2i c)} + 1} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)} + 5 (i \sqrt{2} a e^{(6i dx + 6i c)} + 3 a e^{(4i dx + 4i c)}) \right)}{15 \left(d e^{\frac{5}{2}} + d e^{(6i dx + 6i c + \frac{5}{2})} + 3 d e^{(4i dx + 4i c + \frac{5}{2})} \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{\frac{1}{2}} (-20i a e^{(5i dx + 5i c)} + 48i a e^{(3i dx + 3i c)} + 20i a e^{(i dx + i c)}) \sqrt{e^{(2i dx + 2i c)} + 1} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)} + 15 (d e^3 e^{(6i dx + 6i c)} + 3 d e^3 e^{(4i dx + 4i c)})}{15 (d e^3 e^{(6i dx + 6i c)} + 3 d e^3 e^{(4i dx + 4i c)} + 3 d e^{\frac{5}{2}})}$$

59.111 Problem number 662

$$\int \frac{a + ia \tan(c + dx)}{(e \cos(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2ia}{7d(e \cos(dx + c))^{7/2}} - \frac{6a \left(\cos^{7/2}(dx + c) \right) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d(e \cos(dx + c))^{7/2}} + \frac{2a \cos(dx + c) \sin(dx + c)}{5d(e \cos(dx + c))^{7/2}} + \frac{6a \cos^3(dx + c) \sin(dx + c)}{5d(e \cos(dx + c))^{7/2}}$$

command

`integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{\frac{1}{2}} \left(21i a e^{(8i dx+8i c)} + 77i a e^{(6i dx+6i c)} + 23i a e^{(4i dx+4i c)} + 7i a e^{(2i dx+2i c)} \right) \sqrt{e^{(2i dx+2i c)} + 1} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)} \right)}{35 \left(d e^{\frac{7}{2}} + d e^{(8i dx+8i c)} \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{\frac{1}{2}} \left(-84i a e^{(8i dx+8i c)} - 308i a e^{(6i dx+6i c)} - 92i a e^{(4i dx+4i c)} - 28i a e^{(2i dx+2i c)} \right) \sqrt{e^{(2i dx+2i c)} + 1} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)} + \dots}{35 \left(d e^4 e^{(8i dx+8i c)} + 4 d e^4 e^{(6i dx+6i c)} \right)}$$

59.112 Problem number 663

$$\int \frac{(e \cos(c + dx))^{7/2}}{(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{2(e \cos(dx + c))^{\frac{7}{2}} \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \cos(dx + c)^{\frac{7}{2}}} + \frac{2 \cos(dx + c) (e \cos(dx + c))^{\frac{7}{2}} \sin(dx + c)}{15a^2 d} + \frac{6(e \cos(dx + c))^{\frac{7}{2}} \tan(dx + c)}{35a^2 d} + \frac{2(e \cos(dx + c))^{\frac{7}{2}} (\sec^2(dx + c)) \tan(dx + c)}{7a^2 d} + \frac{4i(\cos^2(dx + c)) (e \cos(dx + c))^{\frac{7}{2}}}{15d(a^2 + ia^2 \tan(dx + c))}$$

command

`integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{\frac{1}{2}} \left(7i e^{\frac{7}{2}} - 15i e^{(10i dx+10i c+\frac{7}{2})} - 185i e^{(8i dx+8i c+\frac{7}{2})} + 430i e^{(6i dx+6i c+\frac{7}{2})} + 162i e^{(4i dx+4i c+\frac{7}{2})} + 49i e^{(2i dx+2i c+\frac{7}{2})} \right) \right)$$

1680 a

Fricas 1.3.7 via sagemath 9.3 output

$$\left(1680 a^2 d e^{(7i dx+7i c)} \operatorname{integral} \left(-\frac{2i \sqrt{\frac{1}{2}} \sqrt{e^{(2i dx+2i c)} + 1} e^{3e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)}}}{7(a^2 d e^{(2i dx+2i c)} + a^2 d)}, x \right) + \sqrt{\frac{1}{2}} \left(-15i e^3 e^{(10i dx+10i c)} - 185i e^{(8i dx+8i c)} + \dots \right) \right)$$

59.113 Problem number 664

$$\int \frac{(e \cos(c + dx))^{5/2}}{(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{42(e \cos(dx + c))^{5/2} \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{65 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \cos(dx + c)^{5/2}} + \frac{2 \cos(dx + c) (e \cos(dx + c))^{5/2} \sin(dx + c)}{13a^2 d} + \frac{14(e \cos(dx + c))^{5/2} \tan(dx + c)}{65a^2 d} + \frac{4i(\cos^2(dx + c)) (e \cos(dx + c))^{5/2}}{13d(a^2 + ia^2 \tan(dx + c))}$$

command

`integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{\frac{1}{2}} \left(5i e^{5/2} - 13i e^{(8i dx + 8i c + 5/2)} + 386i e^{(6i dx + 6i c + 5/2)} + 88i e^{(4i dx + 4i c + 5/2)} + 30i e^{(2i dx + 2i c + 5/2)}\right) \sqrt{e^{(2i dx + 2i c)} + 1} e^{5/2}\right)$$

520 a

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{\frac{1}{2}} \left(-13i e^2 e^{(9i dx + 9i c)} + 13i e^2 e^{(8i dx + 8i c)} - 286i e^2 e^{(7i dx + 7i c)} - 386i e^2 e^{(6i dx + 6i c)} + 88i e^2 e^{(5i dx + 5i c)} - 88i e^2 e^{(4i dx + 4i c)}\right)$$

59.114 Problem number 665

$$\int \frac{(e \cos(c + dx))^{3/2}}{(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{10(e \cos(dx + c))^{3/2} \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{33 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \cos(dx + c)^{3/2}} + \frac{2 \cos(dx + c) (e \cos(dx + c))^{3/2} \sin(dx + c)}{11a^2 d} + \frac{10(e \cos(dx + c))^{3/2} \tan(dx + c)}{33a^2 d} + \frac{4i(\cos^2(dx + c)) (e \cos(dx + c))^{3/2}}{11d(a^2 + ia^2 \tan(dx + c))}$$

command

`integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{\frac{1}{2}}\left(3ie^{\frac{3}{2}} - 11ie^{(6i dx+6i c+\frac{3}{2})} + 41ie^{(4i dx+4i c+\frac{3}{2})} + 15ie^{(2i dx+2i c+\frac{3}{2})}\right)\sqrt{e^{(2i dx+2i c)}+1}e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)} - 40i\sqrt{2}\right)}{132a^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(132a^2de^{(5i dx+5i c)}\text{integral}\left(-\frac{10i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i dx+2i c)}+e}ee^{(-\frac{1}{2}i dx-\frac{1}{2}i c)}}{33(a^2de^{(2i dx+2i c)}+a^2d)}, x\right) + \sqrt{\frac{1}{2}}(-11iee^{(6i dx+6i c)} + 41iee^{(4i dx+4i c)} + 15iee^{(2i dx+2i c)})\sqrt{e^{(2i dx+2i c)}+1}e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)} - 40i\sqrt{2}\right)}{132a^2d}$$

59.115 Problem number 666

$$\int \frac{\sqrt{e \cos(c+dx)}}{(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{e \cos(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx+c)}} + \frac{2i\sqrt{e \cos(dx+c)}}{9d(a+ia \tan(dx+c))^2} + \frac{2i\sqrt{e \cos(dx+c)}}{9d(a^2+ia^2 \tan(dx+c))}$$

command

`integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{\frac{1}{2}}\left(ie^{\frac{1}{2}} + 15ie^{(4i dx+4i c+\frac{1}{2})} + 41ie^{(2i dx+2i c+\frac{1}{2})}\right)\sqrt{e^{(2i dx+2i c)}+1}e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)} + 12i\sqrt{2}e^{(4i dx+4i c+\frac{1}{2})}\text{weierstrass}\right)}{18a^2d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{\frac{1}{2}}\sqrt{ee^{(2i dx+2i c)}+e}(-9ie^{(5i dx+5i c)} - 15ie^{(4i dx+4i c)} + 41ie^{(3i dx+3i c)} - 41ie^{(2i dx+2i c)} + ie^{(i dx+i c)} - i)e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)}$$

18(a^2de^{(5i dx+5i c)} - 15a^2de^{(4i dx+4i c)} + 41a^2de^{(3i dx+3i c)} - 41a^2de^{(2i dx+2i c)} + a^2de^{(i dx+i c)} - ia^2d)

59.116 Problem number 667

$$\int \frac{1}{\sqrt{e \cos(c + dx)} (a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)})}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{e \cos(dx+c)}} + \frac{2i}{7d \sqrt{e \cos(dx+c)} (a^2 + ia^2 \tan(dx+c))} + \frac{2i}{7d \sqrt{e \cos(dx+c)} (a + ia \tan(dx+c))^2}$$

command

`integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{\frac{1}{2}} \sqrt{e^{(2i dx+2i c)} + 1} (3i e^{(2i dx+2i c)} + i) e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)} - 2i \sqrt{2} e^{(3i dx+3i c)} \operatorname{weierstrassPInverse}(-4, 0, e^{(i dx+i c)})\right)}{7 a^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(7 a^2 d e e^{(3i dx+3i c)} \operatorname{integral}\left(-\frac{2i \sqrt{\frac{1}{2}} \sqrt{e e^{(2i dx+2i c)} + e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)}}}{7 (a^2 d e e^{(2i dx+2i c)} + a^2 d e)}, x\right) + \sqrt{\frac{1}{2}} \sqrt{e e^{(2i dx+2i c)} + e^{(i dx+i c)}} (3i e^{(2i dx+2i c)})\right)}{7 a^2 d e}$$

59.117 Problem number 668

$$\int \frac{1}{(e \cos(c + dx))^{3/2} (a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{2\left(\cos^{\frac{3}{2}}(dx+c)\right) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d (e \cos(dx+c))^{\frac{3}{2}}} + \frac{4i(\cos^2(dx+c))}{5d (e \cos(dx+c))^{\frac{3}{2}} (a^2 + ia^2 \tan(dx+c))}$$

command

`integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{\frac{1}{2}} \sqrt{e^{(2i dx+2i c)} + 1} (-2i e^{(2i dx+2i c)} - i) e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)} - i \sqrt{2} e^{(2i dx+2i c)} \text{weierstrassZeta}(-4, 0, \text{weierstrass} \right)}{5 a^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(5 a^2 d e^2 e^{(2i dx+2i c)} \text{integral} \left(-\frac{2i \sqrt{\frac{1}{2}} \sqrt{e e^{(2i dx+2i c)} + e} e^{(\frac{1}{2}i dx + \frac{1}{2}i c)}}{5 (a^2 d e^2 e^{(2i dx+2i c)} + a^2 d e^2)}, x \right) + \sqrt{\frac{1}{2}} \sqrt{e e^{(2i dx+2i c)} + e} (4i e^{(2i dx+2i c)} \right)}{5 a^2 d e^2}$$

59.118 Problem number 669

$$\int \frac{1}{(e \cos(c + dx))^{5/2} (a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{2 \left(\cos^{\frac{5}{2}}(dx + c) \right) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d (e \cos(dx + c))^{\frac{5}{2}}} + \frac{4i (\cos^2(dx + c))}{3d (e \cos(dx + c))^{\frac{5}{2}} (a^2 + ia^2 \tan(dx + c))}$$

command

`integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(-i \sqrt{2} e^{(i dx+i c)} \text{weierstrassPInverse}(-4, 0, e^{(i dx+i c)}) - 2i \sqrt{\frac{1}{2}} \sqrt{e^{(2i dx+2i c)} + 1} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)} \right) e^{(-i dx - i c - \frac{5}{2})}}{3 a^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(3 a^2 d e^3 e^{(i dx+i c)} \text{integral} \left(\frac{2i \sqrt{\frac{1}{2}} \sqrt{e e^{(2i dx+2i c)} + e} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)}}{3 (a^2 d e^3 e^{(2i dx+2i c)} + a^2 d e^3)}, x \right) + 4i \sqrt{\frac{1}{2}} \sqrt{e e^{(2i dx+2i c)} + e} e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)} \right)}{3 a^2 d e^3}$$

59.119 Problem number 670

$$\int \frac{1}{(e \cos(c + dx))^{7/2} (a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{6 \left(\cos^{\frac{7}{2}}(dx + c) \right) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d (e \cos(dx + c))^{\frac{7}{2}}} - \frac{6(\cos^3(dx + c)) \sin(dx + c)}{a^2 d (e \cos(dx + c))^{\frac{7}{2}}} + \frac{4i(\cos^2(dx + c))}{d (e \cos(dx + c))^{\frac{7}{2}} (a^2 + ia^2 \tan(dx + c))}$$

command

`integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{\frac{1}{2}} \sqrt{e^{(2i dx + 2i c)} + 1} (-3i e^{(2i dx + 2i c)} - 2i) e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)} + 3 \left(-i \sqrt{2} e^{(2i dx + 2i c)} - i \sqrt{2} \right) \operatorname{weierstrassZeta} \left(\frac{2i dx + 2i c}{2} \right) \right)}{a^2 d e^{\frac{7}{2}} + a^2 d e^{(2i dx + 2i c + \frac{7}{2})}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{\frac{1}{2}} \sqrt{e^{(2i dx + 2i c)} + e} (12i e^{(2i dx + 2i c)} + 8i) e^{(-\frac{1}{2}i dx - \frac{1}{2}i c)} + (a^2 d e^4 e^{(2i dx + 2i c)} + a^2 d e^4) \operatorname{integral} \left(-\frac{6i \sqrt{\frac{1}{2}} \sqrt{e e^{(2i dx + 2i c)}}}{a^2 d e^4 e^{(2i dx + 2i c)}} \right)}{a^2 d e^4 e^{(2i dx + 2i c)} + a^2 d e^4}$$

59.120 Problem number 671

$$\int \frac{1}{(e \cos(c + dx))^{9/2} (a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{10 \left(\cos^{\frac{9}{2}}(dx + c) \right) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d (e \cos(dx + c))^{\frac{9}{2}}} + \frac{10(\cos^3(dx + c)) \sin(dx + c)}{3a^2 d (e \cos(dx + c))^{\frac{9}{2}}} - \frac{4i(\cos^2(dx + c))}{d (e \cos(dx + c))^{\frac{9}{2}} (a^2 + ia^2 \tan(dx + c))}$$

command

```
integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{\frac{1}{2}} (5i e^{(3i dx+3i c)} + 7i e^{(i dx+i c)}) \sqrt{e^{(2i dx+2i c)} + 1} e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)} + 5 \left(i \sqrt{2} e^{(4i dx+4i c)} + 2i \sqrt{2} e^{(2i dx+2i c)} \right) \right)}{3 \left(a^2 d e^{\frac{9}{2}} + a^2 d e^{(4i dx+4i c+\frac{9}{2})} + 2 a^2 d e^{(2i dx+2i c+\frac{9}{2})} \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{\frac{1}{2}} \sqrt{e^{(2i dx+2i c)} + 1} (-20i e^{(3i dx+3i c)} - 28i e^{(i dx+i c)}) e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)} + 3 (a^2 d e^5 e^{(4i dx+4i c)} + 2 a^2 d e^5 e^{(2i dx+2i c)} + a^2 d e^5)}{3 (a^2 d e^5 e^{(4i dx+4i c)} + 2 a^2 d e^5 e^{(2i dx+2i c)} + a^2 d e^5)}$$

59.121 Problem number 672

$$\int \frac{1}{(e \cos(c + dx))^{11/2} (a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{14 \left(\cos^{\frac{11}{2}}(dx + c) \right) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d (e \cos(dx + c))^{\frac{11}{2}}} + \frac{14 (\cos^3(dx + c) \sin(dx + c))}{15 a^2 d (e \cos(dx + c))^{\frac{11}{2}}} + \frac{14 (\cos^5(dx + c) \sin(dx + c))}{5 a^2 d (e \cos(dx + c))^{\frac{11}{2}}} - \frac{4i (\cos^2(dx + c))}{3 d (e \cos(dx + c))^{\frac{11}{2}} (a^2 + ia^2 \tan(dx + c))}$$

command

```
integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{\frac{1}{2}} (21i e^{(6i dx+6i c)} + 56i e^{(4i dx+4i c)} + 47i e^{(2i dx+2i c)}) \sqrt{e^{(2i dx+2i c)} + 1} e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)} + 21 \left(i \sqrt{2} e^{(6i dx+6i c)} \right) \right)}{15 \left(a^2 d e^{\frac{11}{2}} + a^2 d e^{(6i dx+6i c+\frac{11}{2})} + 3 a^2 d e^{(4i dx+4i c+\frac{11}{2})} + 2 a^2 d e^{(2i dx+2i c+\frac{11}{2})} \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\sqrt{\frac{1}{2}} \sqrt{e^{(2i dx+2i c)} + e^{(-84i e^{(6i dx+6i c)} - 224i e^{(4i dx+4i c)} - 188i e^{(2i dx+2i c)})e^{(-\frac{1}{2}i dx-\frac{1}{2}i c)} + 15 (a^2 d e^6 e^{(6i dx+6i c)} + 3 a^2 d e^6 e^{(4i dx+4i c)} - 15 (a^2 d e^6 e^{(6i dx+6i c)} + 3 a^2 d e^6 e^{(4i dx+4i c)} -$$

60 Test file number 103

Test folder name:

test_cases/4_Trig_functions/4.3_Tangent/103_4.3.2.1-a+b_tan-^m-c+d_tan-^n

60.1 Problem number 526

$$\int \cot^4(c + dx)(a + b \tan(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{i(-ib + a)^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right)}{d} \\ & + \frac{i(ib + a)^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right)}{d} \\ & + \frac{5b(8a^2 - b^2) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{a}}\right)}{8d\sqrt{a}} \\ & + \frac{(8a^2 - 11b^2) \cot(dx + c) \sqrt{a + b \tan(dx + c)}}{8d} \\ & - \frac{13ab(\cot^2(dx + c)) \sqrt{a + b \tan(dx + c)}}{12d} - \frac{a^2(\cot^3(dx + c)) \sqrt{a + b \tan(dx + c)}}{3d} \end{aligned}$$

command

```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.2 Problem number 552

$$\int \frac{\cot(c + dx)}{(a + b \tan(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{a}}\right)}{a^{5/2} d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right)}{(-ib + a)^{5/2} d} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right)}{(ib + a)^{5/2} d} + \frac{2b^2(3a^2 + b^2)}{a^2(a^2 + b^2)^2 d \sqrt{a + b \tan(dx + c)}} \\ & + \frac{2b^2}{3a(a^2 + b^2) d (a + b \tan(dx + c))^{3/2}} \end{aligned}$$

command

```
integrate(cot(d*x+c)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.3 Problem number 553

$$\int \frac{\cot^2(c + dx)}{(a + b \tan(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5b \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{a}}\right)}{a^{7/2} d} + \frac{i \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right)}{(-ib + a)^{5/2} d} \\ & - \frac{i \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right)}{(ib + a)^{5/2} d} - \frac{b(a^4 + 10a^2b^2 + 5b^4)}{a^3(a^2 + b^2)^2 d \sqrt{a + b \tan(dx + c)}} \\ & - \frac{b(3a^2 + 5b^2)}{3a^2(a^2 + b^2) d (a + b \tan(dx + c))^{3/2}} - \frac{\cot(dx + c)}{ad(a + b \tan(dx + c))^{3/2}} \end{aligned}$$

command

`integrate(cot(d*x+c)^2/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.4 Problem number 554

$$\int \frac{1}{(a + b \tan(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{i \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right)}{(-ib + a)^{\frac{7}{2}} d} + \frac{i \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right)}{(ib + a)^{\frac{7}{2}} d} \\ & -\frac{2b(3a^2 - b^2)}{(a^2 + b^2)^3 d \sqrt{a + b \tan(dx + c)}} - \frac{2b}{5(a^2 + b^2) d (a + b \tan(dx + c))^{\frac{5}{2}}} \\ & -\frac{4ab}{3(a^2 + b^2)^2 d (a + b \tan(dx + c))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate(1/(a+b*tan(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.5 Problem number 597

$$\int \frac{1}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b^{\frac{5}{2}} (7a^2 + 3b^2) \arctan \left(\frac{\sqrt{b} \left(\sqrt{\tan(dx+c)} \right)}{\sqrt{a}} \right)}{a^{\frac{5}{2}} (a^2 + b^2)^2 d} \\
& - \frac{(a^2 + 2ab - b^2) \arctan \left(-1 + \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) \right) \sqrt{2}}{2 (a^2 + b^2)^2 d} \\
& - \frac{(a^2 + 2ab - b^2) \arctan \left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) \right) \sqrt{2}}{2 (a^2 + b^2)^2 d} \\
& - \frac{(a^2 - 2ab - b^2) \ln \left(1 - \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) + \tan(dx+c) \right) \sqrt{2}}{4 (a^2 + b^2)^2 d} \\
& + \frac{(a^2 - 2ab - b^2) \ln \left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) + \tan(dx+c) \right) \sqrt{2}}{4 (a^2 + b^2)^2 d} \\
& + \frac{-2a^2 - 3b^2}{a^2 (a^2 + b^2) d \sqrt{\tan(dx+c)}} + \frac{b^2}{a (a^2 + b^2) d \sqrt{\tan(dx+c)} (a + b \tan(dx+c))}
\end{aligned}$$

command

```
integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.6 Problem number 598

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b^{\frac{7}{2}}(9a^2 + 5b^2) \arctan\left(\frac{\sqrt{b}(\sqrt{\tan(dx+c)})}{\sqrt{a}}\right)}{a^{\frac{7}{2}}(a^2 + b^2)^2 d} \\
& - \frac{(a^2 - 2ab - b^2) \arctan\left(-1 + \sqrt{2}(\sqrt{\tan(dx+c)})\right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& - \frac{(a^2 - 2ab - b^2) \arctan\left(1 + \sqrt{2}(\sqrt{\tan(dx+c)})\right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& + \frac{(a^2 + 2ab - b^2) \ln\left(1 - \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& - \frac{(a^2 + 2ab - b^2) \ln\left(1 + \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& + \frac{b(4a^2 + 5b^2)}{a^3(a^2 + b^2)d\sqrt{\tan(dx+c)}} + \frac{-2a^2 - 5b^2}{3a^2(a^2 + b^2)d\tan(dx+c)^{\frac{3}{2}}} \\
& + \frac{b^2}{a(a^2 + b^2)d\tan(dx+c)^{\frac{3}{2}}(a + b\tan(dx+c))}
\end{aligned}$$

command

```
integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.7 Problem number 599

$$\int \frac{\tan^{\frac{11}{2}}(c+dx)}{(a+b\tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{a^{\frac{7}{2}}(35a^4 + 102a^2b^2 + 99b^4) \arctan\left(\frac{\sqrt{b}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a}}\right)}{4b^{\frac{9}{2}}(a^2 + b^2)^3 d} \\
& - \frac{(a+b)(a^2 - 4ab + b^2) \arctan\left(-1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& - \frac{(a+b)(a^2 - 4ab + b^2) \arctan\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& + \frac{(a-b)(a^2 + 4ab + b^2) \ln\left(1 - \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& - \frac{(a-b)(a^2 + 4ab + b^2) \ln\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& - \frac{a(35a^4 + 67a^2b^2 + 24b^4)\left(\sqrt{\tan(dx+c)}\right)}{4b^4(a^2 + b^2)^2 d} + \frac{(35a^4 + 67a^2b^2 + 8b^4)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{12b^3(a^2 + b^2)^2 d} \\
& - \frac{a^2\left(\tan^{\frac{7}{2}}(dx+c)\right)}{2b(a^2 + b^2)d(a + b \tan(dx+c))^2} - \frac{a^2(7a^2 + 15b^2)\left(\tan^{\frac{5}{2}}(dx+c)\right)}{4b^2(a^2 + b^2)^2 d(a + b \tan(dx+c))}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(11/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.8 Problem number 600

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+b \tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{a^{\frac{5}{2}}(15a^4 + 46a^2b^2 + 63b^4) \arctan\left(\frac{\sqrt{b}(\sqrt{\tan(dx+c)})}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}(a^2 + b^2)^3 d} \\
& + \frac{(a-b)(a^2 + 4ab + b^2) \arctan\left(-1 + \sqrt{2}(\sqrt{\tan(dx+c)})\right) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& + \frac{(a-b)(a^2 + 4ab + b^2) \arctan\left(1 + \sqrt{2}(\sqrt{\tan(dx+c)})\right) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& + \frac{(a+b)(a^2 - 4ab + b^2) \ln\left(1 - \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& - \frac{(a+b)(a^2 - 4ab + b^2) \ln\left(1 + \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& + \frac{(15a^4 + 31a^2b^2 + 8b^4)(\sqrt{\tan(dx+c)})}{4b^3(a^2 + b^2)^2 d} \\
& - \frac{a^2(\tan^{\frac{5}{2}}(dx+c))}{2b(a^2 + b^2)d(a + b \tan(dx+c))^2} - \frac{a^2(5a^2 + 13b^2)(\tan^{\frac{3}{2}}(dx+c))}{4b^2(a^2 + b^2)^2 d(a + b \tan(dx+c))}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(9/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.9 Problem number 601

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+b \tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{a^{\frac{3}{2}}(3a^4 + 6a^2b^2 + 35b^4) \arctan\left(\frac{\sqrt{b}(\sqrt{\tan(dx+c)})}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}(a^2 + b^2)^3 d} \\
& + \frac{(a+b)(a^2 - 4ab + b^2) \arctan(-1 + \sqrt{2}(\sqrt{\tan(dx+c)})) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& + \frac{(a+b)(a^2 - 4ab + b^2) \arctan(1 + \sqrt{2}(\sqrt{\tan(dx+c)})) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& - \frac{(a-b)(a^2 + 4ab + b^2) \ln\left(1 - \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& + \frac{(a-b)(a^2 + 4ab + b^2) \ln\left(1 + \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& - \frac{a^2(\tan^{\frac{3}{2}}(dx+c))}{2b(a^2 + b^2)d(a + b \tan(dx+c))^2} - \frac{a^2(3a^2 + 11b^2)(\sqrt{\tan(dx+c)})}{4b^2(a^2 + b^2)^2 d(a + b \tan(dx+c))}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.10 Problem number 602

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+b \tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(a-b)(a^2+4ab+b^2) \arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2+b^2)^3 d} \\
& - \frac{(a-b)(a^2+4ab+b^2) \arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2+b^2)^3 d} \\
& - \frac{(a+b)(a^2-4ab+b^2) \ln\left(1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{4(a^2+b^2)^3 d} \\
& + \frac{(a+b)(a^2-4ab+b^2) \ln\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{4(a^2+b^2)^3 d} \\
& + \frac{(a^4+18a^2b^2-15b^4) \arctan\left(\frac{\sqrt{b}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a}}\right) \sqrt{a}}{4b^{\frac{3}{2}}(a^2+b^2)^3 d} \\
& - \frac{a^2\left(\sqrt{\tan(dx+c)}\right)}{2b(a^2+b^2)d(a+b\tan(dx+c))^2} + \frac{a(a^2+9b^2)\left(\sqrt{\tan(dx+c)}\right)}{4b(a^2+b^2)^2d(a+b\tan(dx+c))}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.11 Problem number 603

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(a+b)(a^2-4ab+b^2) \arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2+b^2)^3 d} \\
& - \frac{(a+b)(a^2-4ab+b^2) \arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2+b^2)^3 d} \\
& + \frac{(a-b)(a^2+4ab+b^2) \ln\left(1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{4(a^2+b^2)^3 d} \\
& - \frac{(a-b)(a^2+4ab+b^2) \ln\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{4(a^2+b^2)^3 d} \\
& + \frac{(3a^4-26a^2b^2+3b^4) \arctan\left(\frac{\sqrt{b}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a}}\right)}{4(a^2+b^2)^3 d \sqrt{a} \sqrt{b}} \\
& + \frac{a\left(\sqrt{\tan(dx+c)}\right)}{2(a^2+b^2) d (a+b \tan(dx+c))^2} + \frac{(3a^2-5b^2)\left(\sqrt{\tan(dx+c)}\right)}{4(a^2+b^2)^2 d (a+b \tan(dx+c))}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.12 Problem number 604

$$\int \frac{\sqrt{\tan(c+dx)}}{(a+b \tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(a-b)(a^2+4ab+b^2) \arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2+b^2)^3 d} \\
& + \frac{(a-b)(a^2+4ab+b^2) \arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2+b^2)^3 d} \\
& + \frac{(a+b)(a^2-4ab+b^2) \ln\left(1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{4(a^2+b^2)^3 d} \\
& - \frac{(a+b)(a^2-4ab+b^2) \ln\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{4(a^2+b^2)^3 d} \\
& - \frac{(15a^4-18a^2b^2-b^4) \arctan\left(\frac{\sqrt{b}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a}}\right) \sqrt{b}}{4a^{\frac{3}{2}}(a^2+b^2)^3 d} \\
& - \frac{b\left(\sqrt{\tan(dx+c)}\right)}{2(a^2+b^2)d(a+b\tan(dx+c))^2} - \frac{b(7a^2-b^2)\left(\sqrt{\tan(dx+c)}\right)}{4a(a^2+b^2)^2 d(a+b\tan(dx+c))}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.13 Problem number 605

$$\int \frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b^{\frac{3}{2}}(35a^4 + 6a^2b^2 + 3b^4) \arctan\left(\frac{\sqrt{b}(\sqrt{\tan(dx+c)})}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}(a^2 + b^2)^3 d} \\
& + \frac{(a+b)(a^2 - 4ab + b^2) \arctan\left(-1 + \sqrt{2}(\sqrt{\tan(dx+c)})\right) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& + \frac{(a+b)(a^2 - 4ab + b^2) \arctan\left(1 + \sqrt{2}(\sqrt{\tan(dx+c)})\right) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& - \frac{(a-b)(a^2 + 4ab + b^2) \ln\left(1 - \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& + \frac{(a-b)(a^2 + 4ab + b^2) \ln\left(1 + \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& + \frac{b^2(\sqrt{\tan(dx+c)})}{2a(a^2 + b^2)d(a + b \tan(dx+c))^2} + \frac{b^2(11a^2 + 3b^2)(\sqrt{\tan(dx+c)})}{4a^2(a^2 + b^2)^2 d(a + b \tan(dx+c))}
\end{aligned}$$

command

```
integrate(1/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.14 Problem number 606

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b^{\frac{5}{2}} (63a^4 + 46a^2b^2 + 15b^4) \arctan \left(\frac{\sqrt{b} \left(\sqrt{\tan(dx+c)} \right)}{\sqrt{a}} \right)}{4a^{\frac{7}{2}} (a^2 + b^2)^3 d} \\
& - \frac{(a-b)(a^2 + 4ab + b^2) \arctan \left(-1 + \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) \right) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& - \frac{(a-b)(a^2 + 4ab + b^2) \arctan \left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) \right) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& - \frac{(a+b)(a^2 - 4ab + b^2) \ln \left(1 - \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) + \tan(dx+c) \right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& + \frac{(a+b)(a^2 - 4ab + b^2) \ln \left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) + \tan(dx+c) \right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& + \frac{-8a^4 - 31a^2b^2 - 15b^4}{4a^3(a^2 + b^2)^2 d \sqrt{\tan(dx+c)}} + \frac{b^2}{2a(a^2 + b^2) d \sqrt{\tan(dx+c)} (a + b \tan(dx+c))^2} \\
& + \frac{b^2(13a^2 + 5b^2)}{4a^2(a^2 + b^2)^2 d \sqrt{\tan(dx+c)} (a + b \tan(dx+c))}
\end{aligned}$$

command

```
integrate(1/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.15 Problem number 607

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b^{\frac{7}{2}}(99a^4 + 102a^2b^2 + 35b^4) \arctan\left(\frac{\sqrt{b}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a}}\right)}{4a^{\frac{9}{2}}(a^2 + b^2)^3 d} \\
& - \frac{(a+b)(a^2 - 4ab + b^2) \arctan\left(-1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& - \frac{(a+b)(a^2 - 4ab + b^2) \arctan\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)^3 d} \\
& + \frac{(a-b)(a^2 + 4ab + b^2) \ln\left(1 - \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& - \frac{(a-b)(a^2 + 4ab + b^2) \ln\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^3 d} \\
& + \frac{b(24a^4 + 67a^2b^2 + 35b^4)}{4a^4(a^2 + b^2)^2 d \sqrt{\tan(dx+c)}} + \frac{-8a^4 - 67a^2b^2 - 35b^4}{12a^3(a^2 + b^2)^2 d \tan(dx+c)^{\frac{3}{2}}} \\
& + \frac{b^2}{2a(a^2 + b^2) d \tan(dx+c)^{\frac{3}{2}} (a + b \tan(dx+c))^2} \\
& + \frac{b^2(15a^2 + 7b^2)}{4a^2(a^2 + b^2)^2 d \tan(dx+c)^{\frac{3}{2}} (a + b \tan(dx+c))}
\end{aligned}$$

command

```
integrate(1/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.16 Problem number 671

$$\int \frac{\tan^{\frac{5}{3}}(c+dx)}{a+b \tan(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} + \frac{b \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} \\
& + \frac{b \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{(a^2+b^2)d} - \frac{3a^{\frac{5}{3}} \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2b^{\frac{2}{3}}(a^2+b^2)d} - \frac{a \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{2(a^2+b^2)d} \\
& + \frac{a^{\frac{5}{3}} \ln(a + b \tan(dx+c))}{2b^{\frac{2}{3}}(a^2+b^2)d} + \frac{a \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{4(a^2+b^2)d} \\
& - \frac{a^{\frac{5}{3}} \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{b^{\frac{2}{3}}(a^2+b^2)d} + \frac{a \arctan\left(\frac{\left(1 - 2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right) \sqrt{3}}{2(a^2+b^2)d} \\
& + \frac{b \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right) \sqrt{3}}{4(a^2+b^2)d} \\
& - \frac{b \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right) \sqrt{3}}{4(a^2+b^2)d}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.17 Problem number 672

$$\int \frac{\sqrt[3]{\tan(c+dx)}}{a+b \tan(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} + \frac{b \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} \\
& + \frac{b \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{(a^2+b^2)d} - \frac{3a^{\frac{1}{3}}b^{\frac{2}{3}} \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} \\
& - \frac{a \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{2(a^2+b^2)d} + \frac{a^{\frac{1}{3}}b^{\frac{2}{3}} \ln(a + b \tan(dx+c))}{2(a^2+b^2)d} \\
& + \frac{a \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{4(a^2+b^2)d} \\
& + \frac{a^{\frac{1}{3}}b^{\frac{2}{3}} \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{(a^2+b^2)d} \\
& - \frac{a \arctan\left(\frac{\left(1 - 2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{2(a^2+b^2)d} \\
& - \frac{b \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{4(a^2+b^2)d} \\
& + \frac{b \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{4(a^2+b^2)d}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.18 Problem number 673

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)}(a+b\tan(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{b \arctan \left(-\sqrt{3} + 2 \left(\tan^{\frac{1}{3}}(dx+c) \right) \right)}{2(a^2+b^2)d} - \frac{b \arctan \left(\sqrt{3} + 2 \left(\tan^{\frac{1}{3}}(dx+c) \right) \right)}{2(a^2+b^2)d} \\
& - \frac{b \arctan \left(\tan^{\frac{1}{3}}(dx+c) \right)}{(a^2+b^2)d} - \frac{3b^{\frac{4}{3}} \ln \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} \left(\tan^{\frac{1}{3}}(dx+c) \right) \right)}{2a^{\frac{1}{3}}(a^2+b^2)d} \\
& + \frac{a \ln \left(1 + \tan^{\frac{2}{3}}(dx+c) \right)}{2(a^2+b^2)d} + \frac{b^{\frac{4}{3}} \ln \left(a + b \tan(dx+c) \right)}{2a^{\frac{1}{3}}(a^2+b^2)d} \\
& - \frac{a \ln \left(1 - \left(\tan^{\frac{2}{3}}(dx+c) \right) + \tan^{\frac{4}{3}}(dx+c) \right)}{4(a^2+b^2)d} \\
& - \frac{b^{\frac{4}{3}} \arctan \left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}} \left(\tan^{\frac{1}{3}}(dx+c) \right) \right) \sqrt{3}}{3a^{\frac{1}{3}}} \right) \sqrt{3}}{a^{\frac{1}{3}}(a^2+b^2)d} - \frac{a \arctan \left(\frac{\left(1 - 2 \left(\tan^{\frac{2}{3}}(dx+c) \right) \right) \sqrt{3}}{3} \right) \sqrt{3}}{2(a^2+b^2)d} \\
& - \frac{b \ln \left(1 - \sqrt{3} \left(\tan^{\frac{1}{3}}(dx+c) \right) + \tan^{\frac{2}{3}}(dx+c) \right) \sqrt{3}}{4(a^2+b^2)d} \\
& + \frac{b \ln \left(1 + \sqrt{3} \left(\tan^{\frac{1}{3}}(dx+c) \right) + \tan^{\frac{2}{3}}(dx+c) \right) \sqrt{3}}{4(a^2+b^2)d}
\end{aligned}$$

command

```
integrate(1/tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.19 Problem number 674

$$\int \frac{1}{\tan^{\frac{5}{3}}(c+dx)(a+b \tan(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{b \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} - \frac{b \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} \\
& - \frac{b \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{(a^2+b^2)d} - \frac{3b^{\frac{8}{3}} \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2a^{\frac{5}{3}}(a^2+b^2)d} \\
& + \frac{a \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{2(a^2+b^2)d} + \frac{b^{\frac{8}{3}} \ln(a + b \tan(dx+c))}{2a^{\frac{5}{3}}(a^2+b^2)d} \\
& - \frac{a \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{4(a^2+b^2)d} \\
& + \frac{b^{\frac{8}{3}} \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{a^{\frac{5}{3}}(a^2+b^2)d} + \frac{a \arctan\left(\frac{\left(1 - 2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{2(a^2+b^2)d} \\
& + \frac{b \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{4(a^2+b^2)d} \\
& - \frac{b \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{4(a^2+b^2)d} \\
& - \frac{3a}{2(a^2+b^2)d \tan(dx+c)^{\frac{2}{3}}} - \frac{3b^2}{2a(a^2+b^2)d \tan(dx+c)^{\frac{2}{3}}}
\end{aligned}$$

command

```
integrate(1/tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.20 Problem number 689

$$\int (a + b \tan(c + dx))^{4/3} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{(-ib+a)^{\frac{4}{3}}x}{4} - \frac{(ib+a)^{\frac{4}{3}}x}{4} + \frac{i(-ib+a)^{\frac{4}{3}}\ln(\cos(dx+c))}{4d} \\
& - \frac{i(ib+a)^{\frac{4}{3}}\ln(\cos(dx+c))}{4d} + \frac{3i(-ib+a)^{\frac{4}{3}}\ln\left((-ib+a)^{\frac{1}{3}} - (a+b\tan(dx+c))^{\frac{1}{3}}\right)}{4d} \\
& - \frac{3i(ib+a)^{\frac{4}{3}}\ln\left((ib+a)^{\frac{1}{3}} - (a+b\tan(dx+c))^{\frac{1}{3}}\right)}{4d} \\
& - \frac{i(-ib+a)^{\frac{4}{3}}\arctan\left(\frac{\left(1+\frac{2(a+b\tan(dx+c))^{\frac{1}{3}}}{(-ib+a)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2d} \\
& + \frac{i(ib+a)^{\frac{4}{3}}\arctan\left(\frac{\left(1+\frac{2(a+b\tan(dx+c))^{\frac{1}{3}}}{(ib+a)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2d} + \frac{3b(a+b\tan(dx+c))^{\frac{1}{3}}}{d}
\end{aligned}$$

command

```
integrate((a+b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.21 Problem number 693

$$\int \frac{1}{(a+b\tan(c+dx))^{2/3}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{x}{4(a - \sqrt{-b^2})^{\frac{2}{3}}} - \frac{b \ln(\cos(dx + c))}{4d(a - \sqrt{-b^2})^{\frac{2}{3}} \sqrt{-b^2}} \\
& \frac{3b \ln\left(\left(a - \sqrt{-b^2}\right)^{\frac{1}{3}} - (a + b \tan(dx + c))^{\frac{1}{3}}\right)}{4d(a - \sqrt{-b^2})^{\frac{2}{3}} \sqrt{-b^2}} \\
& b \arctan\left(\frac{\left(1 + \frac{2(a + b \tan(dx + c))^{\frac{1}{3}}}{(a - \sqrt{-b^2})^{\frac{1}{3}}}\right) \sqrt{3}}{3}\right) \sqrt{3} \\
& + \frac{x}{2d(a - \sqrt{-b^2})^{\frac{2}{3}} \sqrt{-b^2}} - \frac{x}{4(a + \sqrt{-b^2})^{\frac{2}{3}}} \\
& + \frac{b \ln(\cos(dx + c))}{4d\sqrt{-b^2} (a + \sqrt{-b^2})^{\frac{2}{3}}} + \frac{3b \ln\left(\left(a + \sqrt{-b^2}\right)^{\frac{1}{3}} - (a + b \tan(dx + c))^{\frac{1}{3}}\right)}{4d\sqrt{-b^2} (a + \sqrt{-b^2})^{\frac{2}{3}}} \\
& b \arctan\left(\frac{\left(1 + \frac{2(a + b \tan(dx + c))^{\frac{1}{3}}}{(a + \sqrt{-b^2})^{\frac{1}{3}}}\right) \sqrt{3}}{3}\right) \sqrt{3} \\
& - \frac{x}{2d\sqrt{-b^2} (a + \sqrt{-b^2})^{\frac{2}{3}}}
\end{aligned}$$

command

```
integrate(1/(a+b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.22 Problem number 1230

$$\int (a + b \tan(e + fx))^2 \sqrt{c + d \tan(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{i(-ib + a)^2 \operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{-id + c}}\right) \sqrt{-id + c}}{f} \\ & + \frac{i(ib + a)^2 \operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{id + c}}\right) \sqrt{id + c}}{f} \\ & + \frac{4ab \sqrt{c + d \tan(fx + e)}}{f} + \frac{2b^2(c + d \tan(fx + e))^{\frac{3}{2}}}{3df} \end{aligned}$$

command

```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.23 Problem number 1232

$$\int \frac{\sqrt{c + d \tan(e + fx)}}{a + b \tan(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{-id + c}}\right) \sqrt{-id + c}}{(ia + b) f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{id + c}}\right) \sqrt{id + c}}{(ia - b) f} \\ & - \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{c + d \tan(fx + e)}}{\sqrt{-ad + bc}}\right) \sqrt{b} \sqrt{-ad + bc}}{(a^2 + b^2) f} \end{aligned}$$

command

```
integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.24 Problem number 1233

$$\int \frac{\sqrt{c + d \tan(e + fx)}}{(a + b \tan(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{i \operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{-id + c}}\right) \sqrt{-id + c}}{(-ib + a)^2 f} \\ & + \frac{i \operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{id + c}}\right) \sqrt{id + c}}{(ib + a)^2 f} \\ & - \frac{(-3a^2d + 4abc + b^2d) \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{c + d \tan(fx + e)}}{\sqrt{-ad + bc}}\right) \sqrt{b}}{(a^2 + b^2)^2 f \sqrt{-ad + bc}} \\ & - \frac{b \sqrt{c + d \tan(fx + e)}}{(a^2 + b^2) f (a + b \tan(fx + e))} \end{aligned}$$

command

```
integrate((c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.25 Problem number 1251

$$\int \frac{1}{(a + b \tan(e + fx)) \sqrt{c + d \tan(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{-id + c}}\right)}{(ia + b) f \sqrt{-id + c}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{id + c}}\right)}{(ia - b) f \sqrt{id + c}} \\ & - \frac{2b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{c + d \tan(fx + e)}}{\sqrt{-ad + bc}}\right)}{(a^2 + b^2) f \sqrt{-ad + bc}} \end{aligned}$$

command

```
integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.26 Problem number 1252

$$\int \frac{1}{(a + b \tan(e + fx))^2 \sqrt{c + d \tan(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{b^{\frac{3}{2}} (-5a^2d + 4abc - b^2d) \operatorname{arctanh} \left(\frac{\sqrt{b} \sqrt{c + d \tan(fx + e)}}{\sqrt{-ad + bc}} \right)}{(a^2 + b^2)^2 (-ad + bc)^{\frac{3}{2}} f} \\ & - \frac{i \operatorname{arctanh} \left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{-id + c}} \right)}{(-ib + a)^2 f \sqrt{-id + c}} + \frac{i \operatorname{arctanh} \left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{id + c}} \right)}{(ib + a)^2 f \sqrt{id + c}} \\ & - \frac{b^2 \sqrt{c + d \tan(fx + e)}}{(a^2 + b^2) (-ad + bc) f (a + b \tan(fx + e))} \end{aligned}$$

command

```
integrate(1/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.27 Problem number 1255

$$\int \frac{(a + b \tan(e + fx))^2}{(c + d \tan(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{i(-ib + a)^2 \operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{-id + c}}\right)}{(-id + c)^{\frac{3}{2}} f} \\ & + \frac{i(ib + a)^2 \operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{id + c}}\right)}{(id + c)^{\frac{3}{2}} f} - \frac{2(-ad + bc)^2}{d(c^2 + d^2) f \sqrt{c + d \tan(fx + e)}} \end{aligned}$$

command

```
integrate((a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.28 Problem number 1256

$$\int \frac{a + b \tan(e + fx)}{(c + d \tan(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(ia + b) \operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{-id + c}}\right)}{(-id + c)^{\frac{3}{2}} f} \\ & + \frac{(ia - b) \operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{id + c}}\right)}{(id + c)^{\frac{3}{2}} f} + \frac{-2ad + 2bc}{(c^2 + d^2) f \sqrt{c + d \tan(fx + e)}} \end{aligned}$$

command

```
integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.29 Problem number 1257

$$\int \frac{1}{(a + b \tan(e + fx))(c + d \tan(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{-id + c}}\right)}{(ia + b)(-id + c)^{\frac{3}{2}} f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{id + c}}\right)}{(ia - b)(id + c)^{\frac{3}{2}} f} - \frac{2b^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{c + d \tan(fx + e)}}{\sqrt{-ad + bc}}\right)}{(a^2 + b^2)(-ad + bc)^{\frac{3}{2}} f} + \frac{2d^2}{(-ad + bc)(c^2 + d^2) f \sqrt{c + d \tan(fx + e)}}$$

command

`integrate(1/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

60.30 Problem number 1262

$$\int \frac{a + b \tan(e + fx)}{(c + d \tan(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{(ia + b) \operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{-id + c}}\right)}{(-id + c)^{\frac{5}{2}} f} + \frac{(ia - b) \operatorname{arctanh}\left(\frac{\sqrt{c + d \tan(fx + e)}}{\sqrt{id + c}}\right)}{(id + c)^{\frac{5}{2}} f} - \frac{2(2acd - b(c^2 - d^2))}{(c^2 + d^2)^2 f \sqrt{c + d \tan(fx + e)}} + \frac{-\frac{2ad}{3} + \frac{2bc}{3}}{(c^2 + d^2) f (c + d \tan(fx + e))^{\frac{3}{2}}}$$

command

`integrate((a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61 Test file number 104

Test folder name:

test_cases/4_Trig_functions/4.3_Tangent/104_4.3.3.1-a+b_tan^m-c+d_tanⁿ-A+B_tan-

61.1 Problem number 321

$$\int \cot(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{a}}\right) \sqrt{a}}{d} \\ & + \frac{(-iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right) \sqrt{-ib + a}}{d} \\ & + \frac{(iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right) \sqrt{ib + a}}{d} \end{aligned}$$

command

```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.2 Problem number 322

$$\int \cot^2(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(Ab + 2Ba) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{a}}\right)}{d\sqrt{a}} \\ & + \frac{(iA + B) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right) \sqrt{-ib + a}}{d} \\ & - \frac{(iA - B) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right) \sqrt{ib + a}}{d} - \frac{A \cot(dx + c) \sqrt{a + b \tan(dx + c)}}{d} \end{aligned}$$

command

```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.3 Problem number 323

$$\int \cot^3(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(8a^2A + Ab^2 - 4abB) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}d} \\ & - \frac{(-iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right) \sqrt{-ib + a}}{d} \\ & - \frac{(iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right) \sqrt{ib + a}}{d} \\ & - \frac{(Ab + 4Ba) \cot(dx + c) \sqrt{a + b \tan(dx + c)}}{4ad} - \frac{A(\cot^2(dx + c)) \sqrt{a + b \tan(dx + c)}}{2d} \end{aligned}$$

command

```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.4 Problem number 324

$$\int \cot^4(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(8Aa^2b - Ab^3 + 16a^3B + 2Ba^2b^2) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}d} \\ & - \frac{(iA + B) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right) \sqrt{-ib + a}}{d} \\ & + \frac{(iA - B) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right) \sqrt{ib + a}}{d} \\ & + \frac{(8a^2A + Ab^2 - 2abB) \cot(dx + c) \sqrt{a + b \tan(dx + c)}}{8a^2d} \\ & - \frac{(Ab + 6Ba) (\cot^2(dx + c)) \sqrt{a + b \tan(dx + c)}}{12ad} \\ & - \frac{A(\cot^3(dx + c)) \sqrt{a + b \tan(dx + c)}}{3d} \end{aligned}$$

command

```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.5 Problem number 347

$$\int \frac{\cot(c + dx)(A + B \tan(c + dx))}{\sqrt{a + b \tan(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2A \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{a}}\right)}{d\sqrt{a}} + \frac{(-iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right)}{d\sqrt{-ib + a}} \\ & + \frac{(iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right)}{d\sqrt{ib + a}} \end{aligned}$$

command

```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.6 Problem number 348

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(Ab - 2Ba) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}d} + \frac{(iA + B) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{-ib+a}}\right)}{d\sqrt{-ib+a}} \\ & - \frac{(iA - B) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{ib+a}}\right)}{d\sqrt{ib+a}} - \frac{A \cot(dx+c) \sqrt{a+b\tan(dx+c)}}{ad} \end{aligned}$$

command

```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.7 Problem number 349

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(8a^2A - 3Ab^2 + 4abB) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}d} \\ & - \frac{(-iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{-ib+a}}\right)}{d\sqrt{-ib+a}} - \frac{(iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{ib+a}}\right)}{d\sqrt{ib+a}} \\ & + \frac{(3Ab - 4Ba) \cot(dx+c) \sqrt{a+b\tan(dx+c)}}{4a^2d} - \frac{A(\cot^2(dx+c)) \sqrt{a+b\tan(dx+c)}}{2ad} \end{aligned}$$

command

```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.8 Problem number 350

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-iB+A) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{-ib+a}}\right)}{(-ib+a)^{\frac{3}{2}}d} + \frac{(iB+A) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{ib+a}}\right)}{(ib+a)^{\frac{3}{2}}d} \\ & + \frac{2(6Aa^2b+3Ab^3-8a^3B-5Ba^2b^2)\sqrt{a+b\tan(dx+c)}}{3b^3(a^2+b^2)d} \\ & - \frac{2(3Aab-4Ba^2-b^2B)\sqrt{a+b\tan(dx+c)}\tan(dx+c)}{3b^2(a^2+b^2)d} \\ & + \frac{2a(Ab-Ba)(\tan^2(dx+c))}{b(a^2+b^2)d\sqrt{a+b\tan(dx+c)}} \end{aligned}$$

command

```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.9 Problem number 351

$$\int \frac{\tan^2(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{(iA + B) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right)}{(-ib + a)^{\frac{3}{2}} d} - \frac{(iA - B) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right)}{(ib + a)^{\frac{3}{2}} d} - \frac{2a^2(Ab - Ba)}{b^2(a^2 + b^2)d\sqrt{a + b \tan(dx + c)}} + \frac{2B\sqrt{a + b \tan(dx + c)}}{b^2 d}$$

command

```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.10 Problem number 352

$$\int \frac{\tan(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(-iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right)}{(-ib + a)^{\frac{3}{2}} d} - \frac{(iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right)}{(ib + a)^{\frac{3}{2}} d} + \frac{2a(Ab - Ba)}{b(a^2 + b^2)d\sqrt{a + b \tan(dx + c)}}$$

command

```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.11 Problem number 353

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(iA + B) \operatorname{arctanh} \left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}} \right)}{(-ib + a)^{\frac{3}{2}} d} \\ & + \frac{(iA - B) \operatorname{arctanh} \left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}} \right)}{(ib + a)^{\frac{3}{2}} d} - \frac{2(Ab - Ba)}{(a^2 + b^2) d \sqrt{a + b \tan(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.12 Problem number 354

$$\int \frac{\cot(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2A \operatorname{arctanh} \left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{a}} \right)}{a^{\frac{3}{2}} d} + \frac{(-iB + A) \operatorname{arctanh} \left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}} \right)}{(-ib + a)^{\frac{3}{2}} d} \\ & + \frac{(iB + A) \operatorname{arctanh} \left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}} \right)}{(ib + a)^{\frac{3}{2}} d} + \frac{2b(Ab - Ba)}{a(a^2 + b^2) d \sqrt{a + b \tan(dx + c)}} \end{aligned}$$

command

```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.13 Problem number 355

$$\int \frac{\cot^2(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3Ab - 2Ba) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{a}}\right)}{a^{5/2} d} \\ & + \frac{(iA + B) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right)}{(-ib + a)^{3/2} d} - \frac{(iA - B) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right)}{(ib + a)^{3/2} d} \\ & - \frac{b(a^2 A + 3A b^2 - 2abB)}{a^2 (a^2 + b^2) d \sqrt{a + b \tan(dx + c)}} - \frac{A \cot(dx + c)}{ad \sqrt{a + b \tan(dx + c)}} \end{aligned}$$

command

`integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.14 Problem number 356

$$\int \frac{\cot^3(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(8a^2 A - 15A b^2 + 12abB) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{a}}\right)}{4a^{7/2} d} \\ & - \frac{(-iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right)}{(-ib + a)^{3/2} d} \\ & - \frac{(iB + A) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right)}{(ib + a)^{3/2} d} + \frac{b(7A a^2 b + 15A b^3 - 4a^3 B - 12Ba b^2)}{4a^3 (a^2 + b^2) d \sqrt{a + b \tan(dx + c)}} \\ & + \frac{(5Ab - 4Ba) \cot(dx + c)}{4a^2 d \sqrt{a + b \tan(dx + c)}} - \frac{A(\cot^2(dx + c))}{2ad \sqrt{a + b \tan(dx + c)}} \end{aligned}$$

command

```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.15 Problem number 357

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{-ib+a}}\right)}{(-ib+a)^{5/2}d} + \frac{(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{ib+a}}\right)}{(ib+a)^{5/2}d} \\ & + \frac{2(8Aa^4b+17Aa^2b^3+3Ab^5-16Ba^5-30Ba^3b^2-8Bab^4)\sqrt{a+b\tan(dx+c)}}{3b^4(a^2+b^2)^2d} \\ & - \frac{2(4Aa^3b+10Aab^3-8a^4B-15Ba^2b^2-b^4B)\sqrt{a+b\tan(dx+c)}\tan(dx+c)}{3b^3(a^2+b^2)^2d} \\ & + \frac{2a(Aa^2b+3Ab^3-2a^3B-4Bab^2)(\tan^2(dx+c))}{b^2(a^2+b^2)^2d\sqrt{a+b\tan(dx+c)}} \\ & + \frac{2a(Ab-Ba)(\tan^3(dx+c))}{3b(a^2+b^2)d(a+b\tan(dx+c))^{3/2}} \end{aligned}$$

command

```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.16 Problem number 358

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-iB+A) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{-ib+a}}\right)}{(-ib+a)^{\frac{5}{2}}d} \\ & + \frac{(iB+A) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{ib+a}}\right)}{(ib+a)^{\frac{5}{2}}d} - \frac{2a^2(Aa^2b+7Ab^3-4a^3B-10Bab^2)}{3b^3(a^2+b^2)^2d\sqrt{a+b\tan(dx+c)}} \\ & - \frac{2(Aab-4Ba^2-3b^2B)\sqrt{a+b\tan(dx+c)}}{3b^3(a^2+b^2)d} + \frac{2a(Ab-Ba)(\tan^2(dx+c))}{3b(a^2+b^2)d(a+b\tan(dx+c))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.17 Problem number 359

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(iA+B) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{-ib+a}}\right)}{(-ib+a)^{\frac{5}{2}}d} - \frac{(iA-B) \operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{ib+a}}\right)}{(ib+a)^{\frac{5}{2}}d} \\ & + \frac{2a(2Ab^3-a(a^2+3b^2)B)}{b^2(a^2+b^2)^2d\sqrt{a+b\tan(dx+c)}} - \frac{2a^2(Ab-Ba)}{3b^2(a^2+b^2)d(a+b\tan(dx+c))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.18 Problem number 360

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-iB+A)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{-ib+a}}\right)}{(-ib+a)^{\frac{5}{2}}d} - \frac{(iB+A)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{ib+a}}\right)}{(ib+a)^{\frac{5}{2}}d} \\ & + \frac{2a^2A-2Ab^2+4abB}{(a^2+b^2)^2d\sqrt{a+b\tan(dx+c)}} + \frac{2a(Ab-Ba)}{3b(a^2+b^2)d(a+b\tan(dx+c))^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.19 Problem number 361

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{-ib+a}}\right)}{(-ib+a)^{\frac{5}{2}}d} + \frac{(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(dx+c)}}{\sqrt{ib+a}}\right)}{(ib+a)^{\frac{5}{2}}d} \\ & - \frac{2(2Aab-Ba^2+b^2B)}{(a^2+b^2)^2d\sqrt{a+b\tan(dx+c)}} - \frac{2(Ab-Ba)}{3(a^2+b^2)d(a+b\tan(dx+c))^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.20 Problem number 385

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \arctan\left(-1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2d} \\ & - \frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \arctan\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2d} \\ & - \frac{(2ab(A-B) + a^2(A+B) - b^2(A+B)) \ln\left(1 - \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4d} \\ & + \frac{(2ab(A-B) + a^2(A+B) - b^2(A+B)) \ln\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4d} \\ & - \frac{2(2Aab + B a^2 - b^2 B) \left(\sqrt{\tan(dx+c)}\right)}{d} + \frac{2(a^2 A - A b^2 - 2abB) \left(\tan^{\frac{3}{2}}(dx+c)\right)}{3d} \\ & + \frac{2(2Aab + B a^2 - b^2 B) \left(\tan^{\frac{5}{2}}(dx+c)\right)}{5d} + \frac{2b(9Ab + 11Ba) \left(\tan^{\frac{7}{2}}(dx+c)\right)}{63d} \\ & + \frac{2bB \left(\tan^{\frac{7}{2}}(dx+c)\right) (a + b \tan(dx+c))}{9d} \end{aligned}$$

command

```
integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.21 Problem number 386

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\
& - \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\
& + \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\
& - \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\
& + \frac{2(a^2A - Ab^2 - 2abB) \left(\sqrt{\tan(dx + c)}\right)}{d} + \frac{2(2Aab + Ba^2 - b^2B) \left(\tan^{\frac{3}{2}}(dx + c)\right)}{3d} \\
& + \frac{2b(7Ab + 9Ba) \left(\tan^{\frac{5}{2}}(dx + c)\right)}{35d} + \frac{2bB \left(\tan^{\frac{5}{2}}(dx + c)\right) (a + b \tan(dx + c))}{7d}
\end{aligned}$$

command

`integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")`
 Fracas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fracas 1.3.7 via sagemath 9.3 output

Timed out

61.22 Problem number 387

$$\int \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^2 (A + B \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\
& + \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\
& + \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\
& - \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\
& + \frac{2(2Aab + Ba^2 - b^2B) \left(\sqrt{\tan(dx + c)}\right)}{d} + \frac{2b(5Ab + 7Ba) \left(\tan^{\frac{3}{2}}(dx + c)\right)}{15d} \\
& + \frac{2bB \left(\tan^{\frac{3}{2}}(dx + c)\right) (a + b \tan(dx + c))}{5d}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.23 Problem number 388

$$\int \frac{(a + b \tan(c + dx))^2 (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\ & + \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\ & - \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\ & + \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\ & + \frac{2b(3Ab + 5Ba) \left(\sqrt{\tan(dx + c)}\right)}{3d} + \frac{2bB \left(\sqrt{\tan(dx + c)}\right) (a + b \tan(dx + c))}{3d} \end{aligned}$$

command

```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.24 Problem number 389

$$\int \frac{(a + b \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\ & - \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\ & - \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\ & + \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\ & - \frac{2a^2A}{d\sqrt{\tan(dx + c)}} + \frac{2b^2B\left(\sqrt{\tan(dx + c)}\right)}{d} \end{aligned}$$

command

```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.25 Problem number 390

$$\int \frac{(a + b \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\
& - \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\
& + \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\
& - \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\
& - \frac{2a(2Ab + Ba)}{d\sqrt{\tan(dx + c)}} - \frac{2a^2A}{3d \tan(dx + c)^{\frac{3}{2}}}
\end{aligned}$$

command

```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.26 Problem number 391

$$\int \frac{(a + b \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\
& + \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2d} \\
& + \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\
& - \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4d} \\
& + \frac{2a^2A - 2Ab^2 - 4abB}{d\sqrt{\tan(dx + c)}} - \frac{2a^2A}{5d \tan(dx + c)^{\frac{5}{2}}} - \frac{2a(2Ab + Ba)}{3d \tan(dx + c)^{\frac{3}{2}}}
\end{aligned}$$

command

```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.27 Problem number 398

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^{\frac{5}{2}}(Ab - Ba) \arctan\left(\frac{\sqrt{b}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a}}\right)}{b^{\frac{5}{2}}(a^2 + b^2)d} \\ & - \frac{(a(A - B) + b(A + B)) \arctan\left(-1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)d} \\ & - \frac{(a(A - B) + b(A + B)) \arctan\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)d} \\ & + \frac{(b(A - B) - a(A + B)) \ln\left(1 - \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)d} \\ & - \frac{(b(A - B) - a(A + B)) \ln\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)d} \\ & + \frac{2(Ab - Ba)\left(\sqrt{\tan(dx+c)}\right)}{b^2d} + \frac{2B\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3bd} \end{aligned}$$

command

```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.28 Problem number 399

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^{\frac{3}{2}}(Ab - Ba) \arctan\left(\frac{\sqrt{b}(\sqrt{\tan(dx+c)})}{\sqrt{a}}\right)}{b^{\frac{3}{2}}(a^2 + b^2)d} \\ & + \frac{(b(A - B) - a(A + B)) \arctan\left(-1 + \sqrt{2}(\sqrt{\tan(dx+c)})\right) \sqrt{2}}{2(a^2 + b^2)d} \\ & + \frac{(b(A - B) - a(A + B)) \arctan\left(1 + \sqrt{2}(\sqrt{\tan(dx+c)})\right) \sqrt{2}}{2(a^2 + b^2)d} \\ & + \frac{(a(A - B) + b(A + B)) \ln\left(1 - \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)d} \\ & - \frac{(a(A - B) + b(A + B)) \ln\left(1 + \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)d} \\ & + \frac{2B(\sqrt{\tan(dx+c)})}{bd} \end{aligned}$$

command

```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.29 Problem number 400

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(a(A - B) + b(A + B)) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)d} \\
& + \frac{(a(A - B) + b(A + B)) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)d} \\
& - \frac{(b(A - B) - a(A + B)) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4(a^2 + b^2)d} \\
& + \frac{(b(A - B) - a(A + B)) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4(a^2 + b^2)d} \\
& - \frac{2(Ab - Ba) \arctan\left(\frac{\sqrt{b} \left(\sqrt{\tan(dx + c)}\right)}{\sqrt{a}}\right) \sqrt{a}}{(a^2 + b^2)d\sqrt{b}}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.30 Problem number 401

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(b(A - B) - a(A + B)) \arctan \left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)} \right) \right) \sqrt{2}}{2(a^2 + b^2)d} \\
& - \frac{(b(A - B) - a(A + B)) \arctan \left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)} \right) \right) \sqrt{2}}{2(a^2 + b^2)d} \\
& - \frac{(a(A - B) + b(A + B)) \ln \left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)} \right) + \tan(dx + c) \right) \sqrt{2}}{4(a^2 + b^2)d} \\
& + \frac{(a(A - B) + b(A + B)) \ln \left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)} \right) + \tan(dx + c) \right) \sqrt{2}}{4(a^2 + b^2)d} \\
& + \frac{2(Ab - Ba) \arctan \left(\frac{\sqrt{b} \left(\sqrt{\tan(dx + c)} \right)}{\sqrt{a}} \right) \sqrt{b}}{(a^2 + b^2)d\sqrt{a}}
\end{aligned}$$

command

```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.31 Problem number 402

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2b^{\frac{3}{2}}(Ab - Ba) \arctan\left(\frac{\sqrt{b}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a}}\right)}{a^{\frac{3}{2}}(a^2 + b^2)d} \\
& - \frac{(a(A - B) + b(A + B)) \arctan\left(-1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)d} \\
& - \frac{(a(A - B) + b(A + B)) \arctan\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)d} \\
& + \frac{(b(A - B) - a(A + B)) \ln\left(1 - \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)d} \\
& - \frac{(b(A - B) - a(A + B)) \ln\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)d} \\
& - \frac{2A}{ad\sqrt{\tan(dx+c)}}
\end{aligned}$$

command

```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.32 Problem number 403

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b^{\frac{5}{2}}(Ab - Ba) \arctan\left(\frac{\sqrt{b}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a}}\right)}{a^{\frac{5}{2}}(a^2 + b^2)d} \\
& + \frac{(b(A - B) - a(A + B)) \arctan\left(-1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)d} \\
& + \frac{(b(A - B) - a(A + B)) \arctan\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)d} \\
& + \frac{(a(A - B) + b(A + B)) \ln\left(1 - \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)d} \\
& - \frac{(a(A - B) + b(A + B)) \ln\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)d} \\
& + \frac{2Ab - 2Ba}{a^2d\sqrt{\tan(dx+c)}} - \frac{2A}{3ad\tan(dx+c)^{\frac{3}{2}}}
\end{aligned}$$

command

```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.33 Problem number 404

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{a^{\frac{3}{2}}(A a^2 b + 5A b^3 - 3a^3 B - 7B a b^2) \arctan\left(\frac{\sqrt{b}(\sqrt{\tan(dx+c)})}{\sqrt{a}}\right)}{b^{\frac{5}{2}}(a^2 + b^2)^2 d} \\
& - \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \arctan\left(-1 + \sqrt{2}(\sqrt{\tan(dx+c)})\right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& - \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \arctan\left(1 + \sqrt{2}(\sqrt{\tan(dx+c)})\right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& + \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \ln\left(1 - \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& - \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \ln\left(1 + \sqrt{2}(\sqrt{\tan(dx+c)}) + \tan(dx+c)\right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& - \frac{(Aab - 3B a^2 - 2b^2 B) (\sqrt{\tan(dx+c)})}{b^2(a^2 + b^2) d} + \frac{a(Ab - Ba) (\tan^{\frac{3}{2}}(dx+c))}{b(a^2 + b^2) d (a + b \tan(dx+c))}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.34 Problem number 405

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& + \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& + \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& - \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& + \frac{(Aa^2b - 3Ab^3 + a^3B + 5Ba^2b) \arctan\left(\frac{\sqrt{b} \left(\sqrt{\tan(dx + c)}\right)}{\sqrt{a}}\right) \sqrt{a}}{b^{\frac{3}{2}} (a^2 + b^2)^2 d} \\
& + \frac{a(Ab - Ba) \left(\sqrt{\tan(dx + c)}\right)}{b(a^2 + b^2) d (a + b \tan(dx + c))}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.35 Problem number 406

$$\int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& + \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& - \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& + \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& - \frac{(3Aa^2b - Ab^3 - a^3B + 3Ba^2b^2) \arctan\left(\frac{\sqrt{b} \left(\sqrt{\tan(dx + c)}\right)}{\sqrt{a}}\right)}{(a^2 + b^2)^2 d \sqrt{a} \sqrt{b}} - \frac{(Ab - Ba) \left(\sqrt{\tan(dx + c)}\right)}{(a^2 + b^2) d (a + b \tan(dx + c))}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.36 Problem number 407

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& - \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& - \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& + \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& + \frac{(5Aa^2b + Ab^3 - 3a^3B + Ba^2b) \arctan\left(\frac{\sqrt{b} \left(\sqrt{\tan(dx + c)}\right)}{\sqrt{a}}\right) \sqrt{b}}{a^{\frac{3}{2}} (a^2 + b^2)^2 d} \\
& + \frac{b(Ab - Ba) \left(\sqrt{\tan(dx + c)}\right)}{a(a^2 + b^2) d (a + b \tan(dx + c))}
\end{aligned}$$

command

```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.37 Problem number 408

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b^{\frac{3}{2}} (7A a^2 b + 3A b^3 - 5a^3 B - Ba b^2) \arctan \left(\frac{\sqrt{b} \left(\sqrt{\tan(dx+c)} \right)}{\sqrt{a}} \right)}{a^{\frac{5}{2}} (a^2 + b^2)^2 d} \\
& - \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \arctan \left(-1 + \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) \right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& - \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \arctan \left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) \right) \sqrt{2}}{2(a^2 + b^2)^2 d} \\
& + \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \ln \left(1 - \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) + \tan(dx+c) \right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& - \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \ln \left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)} \right) + \tan(dx+c) \right) \sqrt{2}}{4(a^2 + b^2)^2 d} \\
& + \frac{-2a^2 A - 3A b^2 + abB}{a^2 (a^2 + b^2) d \sqrt{\tan(dx+c)}} + \frac{b(Ab - Ba)}{a (a^2 + b^2) d \sqrt{\tan(dx+c)} (a + b \tan(dx+c))}
\end{aligned}$$

command

```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

61.38 Problem number 474

$$\int \sqrt[3]{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(-ib + a)^{\frac{1}{3}} (-iB + A) x}{4} - \frac{(ib + a)^{\frac{1}{3}} (iB + A) x}{4} \\
& - \frac{(ib + a)^{\frac{1}{3}} (iA - B) \ln(\cos(dx + c))}{4d} + \frac{(-ib + a)^{\frac{1}{3}} (iA + B) \ln(\cos(dx + c))}{4d} \\
& + \frac{3(-ib + a)^{\frac{1}{3}} (iA + B) \ln\left(\left(-ib + a\right)^{\frac{1}{3}} - (a + b \tan(dx + c))^{\frac{1}{3}}\right)}{4d} \\
& - \frac{3(ib + a)^{\frac{1}{3}} (iA - B) \ln\left(\left(ib + a\right)^{\frac{1}{3}} - (a + b \tan(dx + c))^{\frac{1}{3}}\right)}{4d} \\
& - \frac{(-ib + a)^{\frac{1}{3}} (iA + B) \arctan\left(\frac{\left(1 + \frac{2(a + b \tan(dx + c))^{\frac{1}{3}}}{(-ib + a)^{\frac{1}{3}}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{2d} \\
& + \frac{(ib + a)^{\frac{1}{3}} (iA - B) \arctan\left(\frac{\left(1 + \frac{2(a + b \tan(dx + c))^{\frac{1}{3}}}{(ib + a)^{\frac{1}{3}}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{2d} + \frac{3B(a + b \tan(dx + c))^{\frac{1}{3}}}{d}
\end{aligned}$$

command

```
integrate((a+b*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

62 Test file number 106

Test folder name:

test_cases/4_Trig_functions/4.3_Tangent/106_4.3.7-d_trig-[^]m-a+b-c_tan-[^]n-[^]p

62.1 Problem number 150

$$\int \frac{\csc^4(e + fx)}{(a + b \tan^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{8(a-2b)b \tan(fx+e)}{3a^4 f \sqrt{a+b(\tan^2(fx+e))}} - \frac{(a-2b) \cot(fx+e)}{a^2 f (a+b(\tan^2(fx+e)))^{3/2}} \\ & - \frac{\cot^3(fx+e)}{3af(a+b(\tan^2(fx+e)))^{3/2}} - \frac{4(a-2b)b \tan(fx+e)}{3a^3 f (a+b(\tan^2(fx+e)))^{3/2}} \end{aligned}$$

command

```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(2(a^3 - 9a^2b + 16ab^2 - 8b^3) \cos(fx+e)^7 - 3(a^3 - 10a^2b + 24ab^2 - 16b^3) \cos(fx+e)^5 - 12(a^2b - 4ab^2 + 4b^3) \cos(fx+e)^3 + 3(a^2b - 4ab^2 + 4b^3) \cos(fx+e) - 3(a^2b - 4ab^2 + 4b^3)\right) \cos(fx+e)^7 - 3\left((a^6 - 2a^5b + a^4b^2) f \cos(fx+e)^6 - a^4b^2 f - (a^6 - 4a^5b + 3a^4b^2) f \cos(fx+e)^4 + 3(a^6 - 4a^5b + 3a^4b^2) f \cos(fx+e)^2 - a^4b^2 f\right) \cos(fx+e)^5 - 12\left((a^6 - 4a^5b + 3a^4b^2) f \cos(fx+e)^3 - a^4b^2 f\right) \cos(fx+e)^3 + 3\left((a^6 - 4a^5b + 3a^4b^2) f \cos(fx+e) - a^4b^2 f\right) \cos(fx+e) - 3(a^2b - 4ab^2 + 4b^3)}{3\left((a^6 - 2a^5b + a^4b^2) f \cos(fx+e)^6 - a^4b^2 f - (a^6 - 4a^5b + 3a^4b^2) f \cos(fx+e)^4 + 3(a^6 - 4a^5b + 3a^4b^2) f \cos(fx+e)^2 - a^4b^2 f\right) \cos(fx+e)^5 - 12\left((a^6 - 4a^5b + 3a^4b^2) f \cos(fx+e)^3 - a^4b^2 f\right) \cos(fx+e)^3 + 3\left((a^6 - 4a^5b + 3a^4b^2) f \cos(fx+e) - a^4b^2 f\right) \cos(fx+e) - 3(a^2b - 4ab^2 + 4b^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

63 Test file number 115

Test folder name:

test_cases/4_Trig_functions/4.5_Secant/115_4.5.0-a_sec~m-b_trg~n

63.1 Problem number 9

$$\int \sec^{\frac{7}{2}}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2\left(\sec^{\frac{5}{2}}(bx+a)\right) \sin(bx+a)}{5b} + \frac{6 \sin(bx+a) (\sqrt{\sec(bx+a)})}{5b} \\ & - \frac{6 \sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx+a)}) (\sqrt{\sec(bx+a)})}{5 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b} \end{aligned}$$

command

```
integrate(sec(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i\sqrt{2}\cos(bx+a)^2\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+3i\sqrt{2}\cos$$

5 b cos

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sec(bx+a)^{\frac{7}{2}},x\right)$$

63.2 Problem number 10

$$\int \sec^{\frac{5}{2}}(a+bx) dx$$

Optimal antiderivative

$$\frac{2\left(\sec^{\frac{3}{2}}(bx+a)\right)\sin(bx+a)}{3b} + \frac{2\sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx+a)}) (\sqrt{\sec(bx+a)})}{3\cos\left(\frac{a}{2} + \frac{bx}{2}\right)b}$$

command

```
integrate(sec(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i\sqrt{2}\cos(bx+a)\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\cos(bx+a)\text{weierstrassPInverse}$$

3 b cos(bx+a)

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sec(bx+a)^{\frac{5}{2}},x\right)$$

63.3 Problem number 11

$$\int \sec^{\frac{3}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{2 \sin(bx + a) (\sqrt{\sec(bx + a)})}{b} - \frac{2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)}) (\sqrt{\sec(bx + a)})}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(sec(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) + i \sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \cos(bx + a) - i \sin(bx + a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sec(bx + a)^{\frac{3}{2}}, x\right)$$

63.4 Problem number 12

$$\int \sqrt{\sec(a + bx)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)}) (\sqrt{\sec(bx + a)})}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(sec(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) + i \sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{\sec(bx + a)}, x\right)$$

63.5 Problem number 13

$$\int \frac{1}{\sqrt{\sec(a+bx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx+a)}) (\sqrt{\sec(bx+a)})}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

`integrate(1/sec(b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i\sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) + i \sin(bx+a))) - i\sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \cos(bx+a) - i \sin(bx+a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{\sec(bx+a)}}, x\right)$$

63.6 Problem number 14

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{2 \sin(bx+a)}{3b\sqrt{\sec(bx+a)}} + \frac{2\sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx+a)}) (\sqrt{\sec(bx+a)})}{3 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

`integrate(1/sec(b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{\cos(bx+a)} \sin(bx+a) - i\sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) + i \sin(bx+a)) + i\sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) - i \sin(bx+a))}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sec^{\frac{3}{2}}(bx+a)}, x\right)$$

63.7 Problem number 15

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{2 \sin(bx+a)}{5b \sec(bx+a)^{\frac{3}{2}}} + \frac{6 \sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx+a)}) (\sqrt{\sec(bx+a)})}{5 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/sec(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \cos(bx+a)^{\frac{3}{2}} \sin(bx+a) + 3i \sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) + i \sin(bx+a)))}{5b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sec(bx+a)^{\frac{5}{2}}}, x\right)$$

63.8 Problem number 16

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{2 \sin(bx+a)}{7b \sec(bx+a)^{\frac{5}{2}}} + \frac{10 \sin(bx+a)}{21b \sqrt{\sec(bx+a)}} + \frac{10 \sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx+a)}) (\sqrt{\sec(bx+a)})}{21 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/sec(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \cos(bx+a)^3 + 5 \cos(bx+a) \right) \sin(bx+a)}{\sqrt{\cos(bx+a)}} - 5i \sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) + i \sin(bx+a)) + 5i \sqrt{2} \operatorname{weiers}$$

21 b

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sec(bx+a)^{\frac{7}{2}}}, x\right)$$

63.9 Problem number 17

$$\int (c \sec(a + bx))^{7/2} dx$$

Optimal antiderivative

$$\frac{2c(c \sec(bx+a))^{\frac{5}{2}} \sin(bx+a)}{5b} - \frac{6c^4 \sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos(bx+a)} \sqrt{c \sec(bx+a)}} + \frac{6c^3 \sin(bx+a) \sqrt{c \sec(bx+a)}}{5b}$$

command

`integrate((c*sec(b*x+a))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} c^{\frac{7}{2}} \cos(bx+a)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx+a) + i \sin(bx+a))) + 3i \sqrt{2} c$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{c \sec(bx+a)} c^3 \sec(bx+a)^3, x\right)$$

63.10 Problem number 18

$$\int (c \sec(a + bx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2c(c \sec(bx + a))^{\frac{3}{2}} \sin(bx + a)}{3b} + \frac{2c^2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)}) \sqrt{c \sec(bx + a)}}{3 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate((c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} c^{\frac{5}{2}} \cos(bx + a) \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) + i \sqrt{2} c^{\frac{5}{2}} \cos(bx + a) \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a))}{3b \cos(bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{c \sec(bx + a)} c^2 \sec(bx + a)^2, x\right)$$

63.11 Problem number 19

$$\int (c \sec(a + bx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{2c^2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos(bx + a)} \sqrt{c \sec(bx + a)}} + \frac{2c \sin(bx + a) \sqrt{c \sec(bx + a)}}{b}$$

command

```
integrate((c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} c^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a))) + i \sqrt{2} c^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) - i \sin(bx + a)))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{c \sec(bx + a)} c \sec(bx + a), x\right)$$

63.12 Problem number 20

$$\int \sqrt{c \sec(a + bx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx+a)}) \sqrt{c \sec(bx+a)}}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate((c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{c}\operatorname{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\sqrt{c}\operatorname{weierstrassPInverse}(-4,0,\cos(bx+a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{c \sec(bx+a)}, x\right)$$

63.13 Problem number 21

$$\int \frac{1}{\sqrt{c \sec(a + bx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{a}{2} + \frac{bx}{2}\right) b \sqrt{\cos(bx+a)} \sqrt{c \sec(bx+a)}}$$

command

```
integrate(1/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i\sqrt{2}\sqrt{c}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))-i\sqrt{2}\sqrt{c}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))}{bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \sec(bx+a)}}{c \sec(bx+a)}, x\right)$$

63.14 Problem number 22

$$\int \frac{1}{(c \sec(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(bx + a)}{3bc \sqrt{c \sec(bx + a)}} + \frac{2 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos(bx + a)}) \sqrt{c \sec(bx + a)}}{3 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) bc^2}$$

command

```
integrate(1/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{\frac{c}{\cos(bx + a)}} \cos(bx + a) \sin(bx + a) - i \sqrt{2} \sqrt{c} \operatorname{weierstrassPInverse}(-4, 0, \cos(bx + a) + i \sin(bx + a)) + i \sqrt{2} \sqrt{c}}{3bc^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \sec(bx + a)}}{c^2 \sec(bx + a)^2}, x\right)$$

63.15 Problem number 23

$$\int \frac{1}{(c \sec(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(bx + a)}{5bc (c \sec(bx + a))^{3/2}} + \frac{6 \sqrt{\frac{\cos(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) bc^2 \sqrt{\cos(bx + a)} \sqrt{c \sec(bx + a)}}$$

command

```
integrate(1/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{\frac{c}{\cos (bx+a)}} \cos (bx+a)^2 \sin (bx+a) + 3i \sqrt{2} \sqrt{c} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a)))$$

5b

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \sec (bx+a)}}{c^3 \sec (bx+a)^3}, x\right)$$

63.16 Problem number 24

$$\int \frac{1}{(c \sec (a+bx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin (bx+a)}{7bc(c \sec (bx+a))^{5/2}} + \frac{10 \sin (bx+a)}{21bc^3 \sqrt{c \sec (bx+a)}} + \frac{10 \sqrt{\frac{\cos (bx+a)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\cos (bx+a)}) \sqrt{c \sec (bx+a)}}{21 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) bc^4}$$

command

```
integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \cos (bx+a)^3 + 5 \cos (bx+a)\right) \sqrt{\frac{c}{\cos (bx+a)}} \sin (bx+a) - 5i \sqrt{2} \sqrt{c} \operatorname{weierstrassPInverse}(-4, 0, \cos (bx+a))$$

21bc⁴

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \sec (bx+a)}}{c^4 \sec (bx+a)^4}, x\right)$$

63.17 Problem number 55

$$\int (a \sec^3(x))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{154a^2 \left(\cos^{\frac{3}{2}}(x)\right) \sqrt{\frac{\cos(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a(\sec^3(x))}}{195 \cos\left(\frac{x}{2}\right)} \\ & + \frac{154a^2 \cos(x) \sin(x) \sqrt{a(\sec^3(x))}}{195} + \frac{154a^2 \sqrt{a(\sec^3(x))} \tan(x)}{585} \\ & + \frac{22a^2 (\sec^2(x)) \sqrt{a(\sec^3(x))} \tan(x)}{117} + \frac{2a^2 (\sec^4(x)) \sqrt{a(\sec^3(x))} \tan(x)}{13} \end{aligned}$$

command

`integrate((a*sec(x)^3)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231i \sqrt{2} a^{\frac{5}{2}} \cos(x)^5 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(x) + i \sin(x))) - 231i \sqrt{2} a^{\frac{5}{2}} \cos(x)^5 \operatorname{we}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \sec(x)^3} a^2 \sec(x)^6, x\right)$$

63.18 Problem number 56

$$\int (a \sec^3(x))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10a \left(\cos^{\frac{3}{2}}(x)\right) \sqrt{\frac{\cos(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a(\sec^3(x))}}{21 \cos\left(\frac{x}{2}\right)} \\ & + \frac{10a \sin(x) \sqrt{a(\sec^3(x))}}{21} + \frac{2a \sec(x) \sqrt{a(\sec^3(x))} \tan(x)}{7} \end{aligned}$$

command

`integrate((a*sec(x)^3)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5i\sqrt{2}a^{\frac{3}{2}}\cos(x)^2\operatorname{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x)) - 5i\sqrt{2}a^{\frac{3}{2}}\cos(x)^2\operatorname{weierstrassPInverse}(-4,0,\cos(x)-i\sin(x))}{21\cos(x)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a\sec(x)^3}a\sec(x)^3,x\right)$$

63.19 Problem number 57

$$\int \sqrt{a\sec^3(x)} dx$$

Optimal antiderivative

$$\frac{2\left(\cos^{\frac{3}{2}}(x)\right)\sqrt{\frac{\cos(x)}{2} + \frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{x}{2}\right),\sqrt{2}\right)\sqrt{a(\sec^3(x))}}{\cos\left(\frac{x}{2}\right)} + 2\cos(x)\sin(x)\sqrt{a(\sec^3(x))}$$

command

`integrate((a*sec(x)^3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & 2\sqrt{\frac{a}{\cos(x)^3}}\cos(x)\sin(x) \\ & + i\sqrt{2}\sqrt{a}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x))) \\ & - i\sqrt{2}\sqrt{a}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(x)-i\sin(x))) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a\sec(x)^3},x\right)$$

63.20 Problem number 58

$$\int \frac{1}{\sqrt{a \sec^3(x)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{x}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{x}{2}\right) \cos(x)^{\frac{3}{2}} \sqrt{a(\sec^3(x))}} + \frac{2 \tan(x)}{3\sqrt{a(\sec^3(x))}}$$

command

```
integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{\frac{a}{\cos(x)^3}} \cos(x)^2 \sin(x) + i\sqrt{2}\sqrt{a} \operatorname{weierstrassPInverse}(-4, 0, \cos(x) + i \sin(x)) - i\sqrt{2}\sqrt{a} \operatorname{weierstrassPInverse}(-4, 0, \cos(x) - i \sin(x))}{3a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \sec(x)^3}}{a \sec(x)^3}, x\right)$$

63.21 Problem number 59

$$\int \frac{1}{(a \sec^3(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{14\sqrt{\frac{\cos(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{x}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{x}{2}\right) a \cos(x)^{\frac{3}{2}} \sqrt{a(\sec^3(x))}} + \frac{14 \sin(x)}{45a \sqrt{a(\sec^3(x))}} + \frac{2(\cos^2(x)) \sin(x)}{9a \sqrt{a(\sec^3(x))}}$$

command

```
integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(5 \cos(x)^5 + 7 \cos(x)^3\right) \sqrt{\frac{a}{\cos(x)^3}} \sin(x) - 21i\sqrt{2}\sqrt{a} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(x) + i \sin(x))) + 21i\sqrt{2}\sqrt{a} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(x) - i \sin(x)))}{45a^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \sec(x)^3}}{a^2 \sec(x)^6}, x\right)$$

63.22 Problem number 60

$$\int \frac{1}{(a \sec^3(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{26 \sqrt{\frac{\cos(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{x}{2}\right), \sqrt{2}\right)}{77 \cos\left(\frac{x}{2}\right) a^2 \cos(x)^{\frac{3}{2}} \sqrt{a(\sec^3(x))}} + \frac{78 \cos(x) \sin(x)}{385 a^2 \sqrt{a(\sec^3(x))}}$$

$$+ \frac{26(\cos^3(x) \sin(x))}{165 a^2 \sqrt{a(\sec^3(x))}} + \frac{2(\cos^5(x) \sin(x))}{15 a^2 \sqrt{a(\sec^3(x))}} + \frac{26 \tan(x)}{77 a^2 \sqrt{a(\sec^3(x))}}$$

command

```
integrate(1/(a*sec(x)^3)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(77 \cos(x)^8 + 91 \cos(x)^6 + 117 \cos(x)^4 + 195 \cos(x)^2 \right) \sqrt{\frac{a}{\cos(x)^3}} \sin(x) + 195i \sqrt{2} \sqrt{a} \operatorname{weierstrassPInverse}(-$$

1155 a³

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \sec(x)^3}}{a^3 \sec(x)^9}, x\right)$$

63.23 Problem number 70

$$\int \sec^4(c + dx) \sqrt{b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{10(b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{21bd} + \frac{2(b \sec(dx + c))^{\frac{7}{2}} \sin(dx + c)}{7b^3d}$$

$$+ \frac{10 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \text{weiers}$$

21 d cos(dx +

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \sec(dx + c)} \sec(dx + c)^4, x\right)$$

63.24 Problem number 71

$$\int \sec^3(c + dx) \sqrt{b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(b \sec(dx + c))^{\frac{5}{2}} \sin(dx + c)}{5b^2 d} - \frac{6b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{6 \sin(dx + c) \sqrt{b \sec(dx + c)}}{5d}$$

command

`integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} \sqrt{b} \cos(dx + c)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \sec(dx + c)} \sec(dx + c)^3, x\right)$$

63.25 Problem number 72

$$\int \sec^2(c + dx) \sqrt{b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3bd} + \frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{3d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} \sec(dx + c)^2, x\right)$$

63.26 Problem number 73

$$\int \sec(c + dx) \sqrt{b \sec(c + dx)} dx$$

Optimal antiderivative

$$-\frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2 \sin(dx + c) \sqrt{b \sec(dx + c)}}{d}$$

command

```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} \sec(dx + c), x\right)$$

63.27 Problem number 74

$$\int \sqrt{b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx-c)+i\sin(dx-c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx+c)}, x\right)$$

63.28 Problem number 75

$$\int \cos(c + dx) \sqrt{b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2b\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}}$$

command

```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i\sqrt{2}\sqrt{b}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\sqrt{b}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx-c)+i\sin(dx-c)))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx+c)} \cos(dx+c), x\right)$$

63.29 Problem number 76

$$\int \cos^2(c + dx) \sqrt{b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2b \sin(dx + c)}{3d \sqrt{b \sec(dx + c)}} + \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{\frac{b}{\cos(dx + c)}} \cos(dx + c) \sin(dx + c) - i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

$$3d$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} \cos(dx + c)^2, x\right)$$

63.30 Problem number 77

$$\int \cos^3(c + dx) \sqrt{b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx + c)}{5d (b \sec(dx + c))^{\frac{3}{2}}} + \frac{6b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{\frac{b}{\cos(dx + c)}} \cos(dx + c)^2 \sin(dx + c) + 3i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 3i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))$$

$$5d$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} \cos(dx + c)^3, x\right)$$

63.31 Problem number 78

$$\int \cos^4(c + dx) \sqrt{b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2b^3 \sin(dx + c)}{7d (b \sec(dx + c))^{\frac{5}{2}}} + \frac{10b \sin(dx + c)}{21d \sqrt{b \sec(dx + c)}} + \frac{10 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \cos(dx + c)^3 + 5 \cos(dx + c) \right) \sqrt{\frac{b}{\cos(dx + c)}} \sin(dx + c) - 5i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

21 d

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} \cos(dx + c)^4, x\right)$$

63.32 Problem number 79

$$\int \cos^5(c + dx) \sqrt{b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2b^4 \sin(dx + c)}{9d (b \sec(dx + c))^{\frac{7}{2}}} + \frac{14b^2 \sin(dx + c)}{45d (b \sec(dx + c))^{\frac{5}{2}}} + \frac{14b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \cos(dx + c)^4 + 7 \cos(dx + c)^2 \right) \sqrt{\frac{b}{\cos(dx + c)}} \sin(dx + c) + 21i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassP}(\cos(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} \cos(dx + c)^5, x\right)$$

63.33 Problem number 80

$$\int \sec^3(c + dx)(b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{10(b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{21d} + \frac{2(b \sec(dx + c))^{\frac{7}{2}} \sin(dx + c)}{7b^2d} \\ + \frac{10b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} b^{\frac{3}{2}} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} b^{\frac{3}{2}} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

$$21 d \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b \sec(dx + c)^4, x\right)$$

63.34 Problem number 81

$$\int \sec^2(c + dx)(b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2(b \sec(dx + c))^{\frac{5}{2}} \sin(dx + c)}{5bd} - \frac{6b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} \\ + \frac{6b \sin(dx + c) \sqrt{b \sec(dx + c)}}{5d}$$

command

```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} b^{\frac{3}{2}} \cos(dx+c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 3i \sqrt{2} b$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx+c)} b \sec(dx+c)^3, x\right)$$

63.35 Problem number 82

$$\int \sec(c+dx)(b \sec(c+dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2(b \sec(dx+c))^{\frac{3}{2}} \sin(dx+c)}{3d} + \frac{2b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} b^{\frac{3}{2}} \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} b^{\frac{3}{2}} \cos(dx+c) \operatorname{weierstrassP}$$

$$3 d \cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx+c)} b \sec(dx+c)^2, x\right)$$

63.36 Problem number 83

$$\int (b \sec(c+dx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{2b^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} + \frac{2b \sin(dx+c) \sqrt{b \sec(dx+c)}}{d}$$

command

```
integrate((b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} b^{\frac{3}{2}} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} b^{\frac{3}{2}} \text{weierstrassZeta}$$

d

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \sec(dx + c)} b \sec(dx + c), x\right)$$

63.37 Problem number 84

$$\int \cos(c + dx) (b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} b^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} b^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

d

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b \sec(dx + c)} b \cos(dx + c) \sec(dx + c), x\right)$$

63.38 Problem number 85

$$\int \cos^2(c + dx)(b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

`integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}(0, -4, 0, \cos(dx + c) + i \sin(dx + c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b \cos(dx + c)^2 \sec(dx + c), x\right)$$

63.39 Problem number 86

$$\int \cos^3(c + dx)(b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx + c)}{3d \sqrt{b \sec(dx + c)}} + \frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2b \sqrt{\frac{b}{\cos(dx + c)}} \cos(dx + c) \sin(dx + c) - i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassPInverse}(0, -4, 0, \cos(dx + c) + i \sin(dx + c))}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b \cos(dx + c)^3 \sec(dx + c), x\right)$$

63.40 Problem number 87

$$\int \cos^4(c + dx)(b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b^3 \sin(dx + c)}{5d (b \sec(dx + c))^{3/2}} + \frac{6b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2b \sqrt{\frac{b}{\cos(dx + c)}} \cos(dx + c)^2 \sin(dx + c) + 3i \sqrt{2} b^{3/2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b \cos(dx + c)^4 \sec(dx + c), x\right)$$

63.41 Problem number 88

$$\int \cos^5(c + dx)(b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b^4 \sin(dx + c)}{7d (b \sec(dx + c))^{5/2}} + \frac{10b^2 \sin(dx + c)}{21d \sqrt{b \sec(dx + c)}} + \frac{10b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} b^{3/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} b^{3/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

21 d

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b \cos(dx + c)^5 \sec(dx + c), x\right)$$

63.42 Problem number 89

$$\int \cos^6(c + dx)(b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b^5 \sin(dx + c)}{9d(b \sec(dx + c))^{7/2}} + \frac{14b^3 \sin(dx + c)}{45d(b \sec(dx + c))^{5/2}} + \frac{14b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i \sqrt{2} b^{3/2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2} b^{3/2} \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b \cos(dx + c)^6 \sec(dx + c), x\right)$$

63.43 Problem number 90

$$\int \sec^2(c + dx)(b \sec(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{10b(b \sec(dx + c))^{3/2} \sin(dx + c)}{21d} + \frac{2(b \sec(dx + c))^{7/2} \sin(dx + c)}{7bd} + \frac{10b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} b^{5/2} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} b^{5/2} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

21 d cos(dx + c)

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b^2 \sec(dx + c)^4, x\right)$$

63.44 Problem number 91

$$\int \sec(c + dx)(b \sec(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2(b \sec(dx + c))^{5/2} \sin(dx + c)}{5d} - \frac{6b^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{6b^2 \sin(dx + c) \sqrt{b \sec(dx + c)}}{5d}$$

command

```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} b^{5/2} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2} b$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b^2 \sec(dx + c)^3, x\right)$$

63.45 Problem number 92

$$\int (b \sec(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b(b \sec(dx + c))^{3/2} \sin(dx + c)}{3d} + \frac{2b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b^{\frac{5}{2}} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} b^{\frac{5}{2}} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3 d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b^2 \sec(dx + c)^2, x\right)$$

63.46 Problem number 93

$$\int \cos(c + dx) (b \sec(c + dx))^{5/2} dx$$

Optimal antiderivative

$$-\frac{2b^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2b^2 \sin(dx + c) \sqrt{b \sec(dx + c)}}{d}$$

command

`integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b^{\frac{5}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} b^{\frac{5}{2}} \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b^2 \cos(dx + c) \sec(dx + c)^2, x\right)$$

63.47 Problem number 94

$$\int \cos^2(c + dx) (b \sec(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b\sec(dx+c)}b^2\cos(dx+c)^2\sec(dx+c)^2,x\right)$$

63.48 Problem number 95

$$\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}dx$$

Optimal antiderivative

$$\frac{2b^3\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d\sqrt{\cos(dx+c)}\sqrt{b\sec(dx+c)}}$$

command

```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\cos(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{b\sec(dx+c)}b^2\cos(dx+c)^3\sec(dx+c)^2,x\right)$$

63.49 Problem number 96

$$\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}dx$$

Optimal antiderivative

$$\frac{2b^3 \sin(dx+c)}{3d\sqrt{b \sec(dx+c)}} + \frac{2b^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2b^2 \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) - i \sqrt{2} b^{\frac{5}{2}} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx+c)} b^2 \cos(dx+c)^4 \sec(dx+c)^2, x\right)$$

63.50 Problem number 97

$$\int \cos^5(c+dx)(b \sec(c+dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b^4 \sin(dx+c)}{5d(b \sec(dx+c))^{\frac{3}{2}}} + \frac{6b^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}}$$

command

`integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2b^2 \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^2 \sin(dx+c) + 3i \sqrt{2} b^{\frac{5}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))}{5d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx+c)} b^2 \cos(dx+c)^5 \sec(dx+c)^2, x\right)$$

63.51 Problem number 98

$$\int \cos^6(c + dx)(b \sec(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b^5 \sin(dx + c)}{7d(b \sec(dx + c))^{5/2}} + \frac{10b^3 \sin(dx + c)}{21d\sqrt{b \sec(dx + c)}} + \frac{10b^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} b^{5/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} b^{5/2} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{21d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b^2 \cos(dx + c)^6 \sec(dx + c)^2, x\right)$$

63.52 Problem number 99

$$\int \cos^7(c + dx)(b \sec(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b^6 \sin(dx + c)}{9d(b \sec(dx + c))^{7/2}} + \frac{14b^4 \sin(dx + c)}{45d(b \sec(dx + c))^{3/2}} + \frac{14b^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21i \sqrt{2} b^{5/2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 21i \sqrt{2} b^{5/2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b^2 \cos(dx + c)^7 \sec(dx + c)^2, x\right)$$

63.53 Problem number 100

$$\int (b \sec(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\frac{2b(b \sec(dx + c))^{\frac{5}{2}} \sin(dx + c)}{5d} - \frac{6b^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{6b^3 \sin(dx + c) \sqrt{b \sec(dx + c)}}{5d}$$

command

```
integrate((b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} b^{\frac{7}{2}} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2} b$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c)} b^3 \sec(dx + c)^3, x\right)$$

63.54 Problem number 101

$$\int \frac{\sec^5(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{10(b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{21b^2d} + \frac{2(b \sec(dx + c))^{\frac{7}{2}} \sin(dx + c)}{7b^4d} + \frac{10 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd}$$

command

```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} \sqrt{b} \cos(dx + c)^3 \text{weiers}$$

$$21 bd \cos(dx +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \sec(dx + c)} \sec(dx + c)^4}{b}, x\right)$$

63.55 Problem number 102

$$\int \frac{\sec^4(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(b \sec(dx + c))^{\frac{5}{2}} \sin(dx + c)}{5b^3 d} - \frac{6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{6 \sin(dx + c) \sqrt{b \sec(dx + c)}}{5bd}$$

command

```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} \sqrt{b} \cos(dx + c)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \sec(dx + c)} \sec(dx + c)^3}{b}, x\right)$$

63.56 Problem number 103

$$\int \frac{\sec^3(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3b^2d} + \frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd}$$

command

```
integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \cos(dx + c) \operatorname{weierstrassP}(\dots) + 3bd \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \sec(dx + c)^2}{b}, x\right)$$

63.57 Problem number 104

$$\int \frac{\sec^2(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2 \sin(dx + c) \sqrt{b \sec(dx + c)}}{bd}$$

command

```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))$$

bd

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \sec(dx + c)}{b}, x\right)$$

63.58 Problem number 105

$$\int \frac{\sec(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd}$$

command

```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

bd

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b}, x\right)$$

63.59 Problem number 106

$$\int \frac{1}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate(1/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b \sec(dx + c)}, x\right)$$

63.60 Problem number 107

$$\int \frac{\cos(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c)}{3d \sqrt{b \sec(dx + c)}} + \frac{2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd}$$

command

```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{\frac{b}{\cos(dx + c)}} \cos(dx + c) \sin(dx + c) - i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3bd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \cos(dx + c)}{b \sec(dx + c)}, x\right)$$

63.61 Problem number 108

$$\int \frac{\cos^2(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2b \sin(dx + c)}{5d (b \sec(dx + c))^{\frac{3}{2}}} + \frac{6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{\frac{b}{\cos(dx + c)}} \cos(dx + c)^2 \sin(dx + c) + 3i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)))$$

5b

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \cos(dx + c)^2}{b \sec(dx + c)}, x\right)$$

63.62 Problem number 109

$$\int \frac{\cos^3(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx + c)}{7d (b \sec(dx + c))^{\frac{5}{2}}} + \frac{10 \sin(dx + c)}{21d \sqrt{b \sec(dx + c)}} + \frac{10 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd}$$

command

```
integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \cos(dx + c)^3 + 5 \cos(dx + c) \right) \sqrt{\frac{b}{\cos(dx + c)}} \sin(dx + c) - 5i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

21 bd

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \sec(dx + c)} \cos(dx + c)^3}{b \sec(dx + c)}, x \right)$$

63.63 Problem number 110

$$\int \frac{\cos^4(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2b^3 \sin(dx + c)}{9d (b \sec(dx + c))^{\frac{7}{2}}} + \frac{14b \sin(dx + c)}{45d (b \sec(dx + c))^{\frac{3}{2}}} + \frac{14 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \cos(dx + c)^4 + 7 \cos(dx + c)^2 \right) \sqrt{\frac{b}{\cos(dx + c)}} \sin(dx + c) + 21i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassP}(\dots))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \sec(dx + c)} \cos(dx + c)^4}{b \sec(dx + c)}, x \right)$$

63.64 Problem number 111

$$\int \frac{\sec^6(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{10(b \sec(dx+c))^{\frac{3}{2}} \sin(dx+c)}{21b^3d} + \frac{2(b \sec(dx+c))^{\frac{7}{2}} \sin(dx+c)}{7b^5d}$$

$$+ \frac{10 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d}$$

command

```
integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} \sqrt{b} \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} \sqrt{b} \cos(dx+c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))$$

$$21 b^2 d \cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)^4}{b^2}, x\right)$$

63.65 Problem number 112

$$\int \frac{\sec^5(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(b \sec(dx+c))^{\frac{5}{2}} \sin(dx+c)}{5b^4d} - \frac{6 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}}$$

$$+ \frac{6 \sin(dx+c) \sqrt{b \sec(dx+c)}}{5b^2d}$$

command

```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} \sqrt{b} \cos(dx+c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 3i \sqrt{2} \sqrt{b} \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)^3}{b^2}, x\right)$$

63.66 Problem number 113

$$\int \frac{\sec^4(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3b^3d} + \frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d}$$

command

```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \cos(dx + c) \operatorname{weierstrassP}(\dots)$$

$$3b^2d \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \sec^2(dx + c)}{b^2}, x\right)$$

63.67 Problem number 114

$$\int \frac{\sec^3(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2 \sin(dx + c) \sqrt{b \sec(dx + c)}}{b^2d}$$

command

```
integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

 $b^2 d$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \sec(dx + c)}{b^2}, x\right)$$

63.68 Problem number 115

$$\int \frac{\sec^2(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d}$$

command

```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

 $b^2 d$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b^2}, x\right)$$

63.69 Problem number 116

$$\int \frac{\sec(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{b^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b^2 \sec(dx + c)}, x\right)$$

63.70 Problem number 117

$$\int \frac{1}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2 \sin(dx + c)}{3bd \sqrt{b \sec(dx + c)}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d}$$

command

```
integrate(1/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{\frac{b}{\cos(dx + c)}} \cos(dx + c) \sin(dx + c) - i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{3 b^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b^2 \sec(dx + c)^2}, x\right)$$

63.71 Problem number 118

$$\int \frac{\cos(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c)}{5d (b \sec(dx + c))^{3/2}} + \frac{6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

`integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{\frac{b}{\cos(dx + c)}} \cos(dx + c)^2 \sin(dx + c) + 3i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)))$$

5b

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \cos(dx + c)}{b^2 \sec(dx + c)^2}, x\right)$$

63.72 Problem number 119

$$\int \frac{\cos^2(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b \sin(dx + c)}{7d (b \sec(dx + c))^{5/2}} + \frac{10 \sin(dx + c)}{21bd \sqrt{b \sec(dx + c)}} + \frac{10 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d}$$

command

`integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \cos(dx + c)^3 + 5 \cos(dx + c) \right) \sqrt{\frac{b}{\cos(dx + c)}} \sin(dx + c) - 5i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

$$21 b^2 d$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \sec(dx + c)} \cos(dx + c)^2}{b^2 \sec(dx + c)^2}, x \right)$$

63.73 Problem number 120

$$\int \frac{\cos^3(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx + c)}{9d (b \sec(dx + c))^{7/2}} + \frac{14 \sin(dx + c)}{45d (b \sec(dx + c))^{3/2}} + \frac{14 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) b d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \cos(dx + c)^4 + 7 \cos(dx + c)^2 \right) \sqrt{\frac{b}{\cos(dx + c)}} \sin(dx + c) + 21i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassP}(\dots))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \sec(dx + c)} \cos(dx + c)^3}{b^2 \sec(dx + c)^2}, x \right)$$

63.74 Problem number 121

$$\int \frac{\sec^7(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{10(b \sec(dx+c))^{\frac{3}{2}} \sin(dx+c)}{21b^4d} + \frac{2(b \sec(dx+c))^{\frac{7}{2}} \sin(dx+c)}{7b^6d}$$

$$+ \frac{10 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d}$$

command

```
integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} \sqrt{b} \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} \sqrt{b} \cos(dx+c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)))$$

$$21 b^3 d \cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)^4}{b^3}, x\right)$$

63.75 Problem number 122

$$\int \frac{\sec^6(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(b \sec(dx+c))^{\frac{5}{2}} \sin(dx+c)}{5b^5d} - \frac{6 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}}$$

$$+ \frac{6 \sin(dx+c) \sqrt{b \sec(dx+c)}}{5b^3d}$$

command

```
integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i \sqrt{2} \sqrt{b} \cos(dx+c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 3i \sqrt{2} \sqrt{b} \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)^3}{b^3}, x\right)$$

63.76 Problem number 123

$$\int \frac{\sec^5(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{3b^4d} + \frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d}$$

command

```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \cos(dx + c) \operatorname{weierstrassP}(\dots) + \frac{3b^3d \cos(dx + c)}{3b^3d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \sec^2(dx + c)}{b^3}, x\right)$$

63.77 Problem number 124

$$\int \frac{\sec^4(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2 \sin(dx + c) \sqrt{b \sec(dx + c)}}{b^3d}$$

command

```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

 $b^3 d$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \sec(dx + c)}{b^3}, x\right)$$

63.78 Problem number 125

$$\int \frac{\sec^3(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d}$$

command

`integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

 $b^3 d$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b^3}, x\right)$$

63.79 Problem number 126

$$\int \frac{\sec^2(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i\sqrt{2}\sqrt{b}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - i\sqrt{2}\sqrt{b}\operatorname{weierstrassZeta}(-4,0,\cos(dx+c)-i\sin(dx+c))}{b^3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(dx+c)}}{b^3\sec(dx+c)},x\right)$$

63.80 Problem number 127

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2\sin(dx+c)}{3b^2d\sqrt{b\sec(dx+c)}} + \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b\sec(dx+c)}}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)b^3d}$$

command

```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) - i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{3b^3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(dx+c)}}{b^3\sec(dx+c)^2},x\right)$$

63.81 Problem number 128

$$\int \frac{1}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c)}{5bd (b \sec(dx + c))^{3/2}} + \frac{6 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

`integrate(1/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{\frac{b}{\cos(dx + c)}} \cos(dx + c)^2 \sin(dx + c) + 3i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)))$$

5b

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b^3 \sec(dx + c)^3}, x\right)$$

63.82 Problem number 129

$$\int \frac{\cos(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c)}{7d (b \sec(dx + c))^{5/2}} + \frac{10 \sin(dx + c)}{21b^2 d \sqrt{b \sec(dx + c)}} + \frac{10 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d}$$

command

`integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \cos(dx + c)^3 + 5 \cos(dx + c) \right) \sqrt{\frac{b}{\cos(dx + c)}} \sin(dx + c) - 5i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{21 b^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \cos(dx + c)}{b^3 \sec(dx + c)^3}, x\right)$$

63.83 Problem number 130

$$\int \frac{\cos^2(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2b \sin(dx + c)}{9d (b \sec(dx + c))^{7/2}} + \frac{14 \sin(dx + c)}{45bd (b \sec(dx + c))^{3/2}} + \frac{14 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

`integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5 \cos(dx + c)^4 + 7 \cos(dx + c)^2 \right) \sqrt{\frac{b}{\cos(dx + c)}} \sin(dx + c) + 21i \sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassP}(\dots))}{21 b^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \cos(dx + c)^2}{b^3 \sec(dx + c)^3}, x\right)$$

63.84 Problem number 131

$$\int \frac{1}{(b \sec(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c)}{7bd (b \sec(dx + c))^{5/2}} + \frac{10 \sin(dx + c)}{21b^3 d \sqrt{b \sec(dx + c)}} + \frac{10 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d}$$

command

```
integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \cos(dx + c)^3 + 5 \cos(dx + c) \right) \sqrt{\frac{b}{\cos(dx + c)}} \sin(dx + c) - 5i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

$$21 b^4 d$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b^4 \sec(dx + c)^4}, x\right)$$

63.85 Problem number 230

$$\int (d \csc(a + bx))^{9/2} \sqrt{c \sec(a + bx)} dx$$

Optimal antiderivative

$$\frac{4c d^3 (d \csc(bx + a))^{\frac{3}{2}}}{7b \sqrt{c \sec(bx + a)}} - \frac{2cd (d \csc(bx + a))^{\frac{7}{2}}}{7b \sqrt{c \sec(bx + a)}}$$

$$\frac{4d^4 \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} \left(\sqrt{\sin(2bx + 2a)}\right)}{7 \sin\left(a + \frac{\pi}{4} + bx\right) b}$$

command

```
integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(i d^4 \cos(bx + a)^2 - i d^4 \right) \sqrt{-4i cd} \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) \sin(bx + a) + \left(-i d^4 \cos(bx + a) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} d^4 \csc(bx + a)^4, x\right)$$

63.86 Problem number 232

$$\int (d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)} dx$$

Optimal antiderivative

$$\frac{2cd(d \csc(bx + a))^{\frac{3}{2}}}{3b\sqrt{c \sec(bx + a)}} - \frac{2d^2 \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} \left(\sqrt{\sin(2bx + 2a)}\right)}{3 \sin\left(a + \frac{\pi}{4} + bx\right) b}$$

command

```
integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{-4i cd} d^2 \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) \sin(bx + a) + i \sqrt{4i cd} d^2 \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1)}{3 b \sin(bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} d^2 \csc(bx + a)^2, x\right)$$

63.87 Problem number 234

$$\int \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} \left(\sqrt{\sin(2bx + 2a)}\right)}{\sin\left(a + \frac{\pi}{4} + bx\right) b}$$

command

```
integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{-4i cd} \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + i \sqrt{4i cd} \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1)}{2 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)}, x\right)$$

63.88 Problem number 246

$$\int (d \csc(a + bx))^{9/2} (c \sec(a + bx))^{5/2} dx$$

Optimal antiderivative

$$\frac{20c d^3 (d \csc(bx + a))^{\frac{3}{2}} (c \sec(bx + a))^{\frac{3}{2}}}{21b} - \frac{2cd (d \csc(bx + a))^{\frac{7}{2}} (c \sec(bx + a))^{\frac{3}{2}}}{7b} + \frac{40c d^5 (c \sec(bx + a))^{\frac{3}{2}}}{21b \sqrt{d \csc(bx + a)}}$$

$$\frac{40c^2 d^4 \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} \left(\sqrt{\sin(2bx + 2a)}\right)}{21 \sin\left(a + \frac{\pi}{4} + bx\right) b}$$

command

```
integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10 \left(i c^2 d^4 \cos(bx + a)^3 - i c^2 d^4 \cos(bx + a) \right) \sqrt{-4i cd} \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) \sin(bx + a) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} c^2 d^4 \csc(bx + a)^4 \sec(bx + a)^2, x\right)$$

63.89 Problem number 248

$$\int (d \csc(a + bx))^{5/2} (c \sec(a + bx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2cd (d \csc(bx + a))^{\frac{3}{2}} (c \sec(bx + a))^{\frac{3}{2}}}{3b} + \frac{4c d^3 (c \sec(bx + a))^{\frac{3}{2}}}{3b \sqrt{d \csc(bx + a)}}$$

$$\frac{4c^2 d^2 \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} \left(\sqrt{\sin(2bx + 2a)}\right)}{3 \sin\left(a + \frac{\pi}{4} + bx\right) b}$$

command

```
integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{-4i cd} c^2 d^2 \cos (bx + a) \operatorname{ellipticF}(\cos (bx + a) + i \sin (bx + a), -1) \sin (bx + a) - i \sqrt{4i cd} c^2 d^2 \cos (bx + a) \right) \\ \hline 3 b \cos (bx + a)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc (bx + a)} \sqrt{c \sec (bx + a)} c^2 d^2 \csc (bx + a)^2 \sec (bx + a)^2, x\right)$$

63.90 Problem number 250

$$\int \sqrt{d \csc (a + bx)} (c \sec (a + bx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2cd(c \sec (bx + a))^{3/2}}{3b \sqrt{d \csc (bx + a)}} \\ \frac{2c^2 \sqrt{\frac{1}{2} + \frac{\sin (2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos \left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \sqrt{d \csc (bx + a)} \sqrt{c \sec (bx + a)} \left(\sqrt{\sin (2bx + 2a)}\right)}{3 \sin \left(a + \frac{\pi}{4} + bx\right) b}$$

command

```
integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{-4i cd} c^2 \cos (bx + a) \operatorname{ellipticF}(\cos (bx + a) + i \sin (bx + a), -1) + i \sqrt{4i cd} c^2 \cos (bx + a) \operatorname{ellipticF}(\cos (bx + a) - i \sin (bx + a), -1) \\ \hline 3 b \cos (bx + a)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{d \csc (bx + a)} \sqrt{c \sec (bx + a)} c^2 \sec (bx + a)^2, x\right)$$

63.91 Problem number 252

$$\int \frac{(c \sec(a + bx))^{5/2}}{(d \csc(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2c(c \sec(bx + a))^{\frac{3}{2}}}{3bd\sqrt{d \csc(bx + a)}} + \frac{c^2 \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} \left(\sqrt{\sin(2bx + 2a)}\right)}{3 \sin\left(a + \frac{\pi}{4} + bx\right) b d^2}$$

command

```
integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{-4i cd} c^2 \cos(bx + a) \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) - i \sqrt{4i cd} c^2 \cos(bx + a) \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1)}{6 b d^2 \cos(bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} c^2 \sec(bx + a)^2}{d^2 \csc(bx + a)^2}, x\right)$$

63.92 Problem number 263

$$\int \frac{(d \csc(a + bx))^{9/2}}{(c \sec(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2d^3(d \csc(bx + a))^{\frac{3}{2}}}{21bc\sqrt{c \sec(bx + a)}} - \frac{2d(d \csc(bx + a))^{\frac{7}{2}}}{7bc\sqrt{c \sec(bx + a)}} + \frac{2d^4 \sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right) \sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} \left(\sqrt{\sin(2bx + 2a)}\right)}{21 \sin\left(a + \frac{\pi}{4} + bx\right) b c^2}$$

command

`integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(i d^4 \cos (b x+a)^2-i d^4\right) \sqrt{-4 i c d} \operatorname{ellipticF}\left(\cos (b x+a)+i \sin (b x+a),-1\right) \sin (b x+a)+\left(-i d^4 \cos (b x+a)^2+i d^4\right) \sqrt{-4 i c d} \operatorname{ellipticF}\left(\cos (b x+a)-i \sin (b x+a),-1\right) \sin (b x+a)}{21 b c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \csc (b x+a)} \sqrt{c \sec (b x+a)} d^4 \csc (b x+a)^4}{c^2 \sec (b x+a)^2}, x\right)$$

63.93 Problem number 265

$$\int \frac{(d \csc (a+b x))^{5/2}}{(c \sec (a+b x))^{3/2}} d x$$

Optimal antiderivative

$$\frac{2 d(d \csc (b x+a))^{\frac{3}{2}}}{3 b c \sqrt{c \sec (b x+a)}} + \frac{d^2 \sqrt{\frac{1}{2} + \frac{\sin (2 b x+2 a)}{2}} \operatorname{EllipticF}\left(\cos \left(a+\frac{\pi}{4}+b x\right), \sqrt{2}\right) \sqrt{d \csc (b x+a)} \sqrt{c \sec (b x+a)} \left(\sqrt{\sin (2 b x+2 a)}\right)}{3 \sin \left(a+\frac{\pi}{4}+b x\right) b c^2}$$

command

`integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{i \sqrt{-4 i c d} d^2 \operatorname{ellipticF}\left(\cos (b x+a)+i \sin (b x+a),-1\right) \sin (b x+a)-i \sqrt{4 i c d} d^2 \operatorname{ellipticF}\left(\cos (b x+a)-i \sin (b x+a),-1\right) \sin (b x+a)}{6 b c^2 \sin (b x+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{d \csc (b x+a)} \sqrt{c \sec (b x+a)} d^2 \csc (b x+a)^2}{c^2 \sec (b x+a)^2}, x\right)$$

64 Test file number 118

Test folder name:

test_cases/4_Trig_functions/4.5_Secant/118_4.5.1.2-d_sec-^n-a+b_sec-^m

64.1 Problem number 165

$$\int \sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2a \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{6a \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{6a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} a \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} a \cos(dx + c)^2 \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(a \sec(dx + c)^3 + a \sec(dx + c)^2 \right) \sqrt{\sec(dx + c)}, x \right)$$

64.2 Problem number 166

$$\int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2a \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & - \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} a \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \cos(dx + c) \operatorname{weierstrassPInv}$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(a \sec(dx + c)^2 + a \sec(dx + c) \right) \sqrt{\sec(dx + c)}, x \right)$$

64.3 Problem number 167

$$\int \sqrt{\sec(c + dx)} (a + a \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & - \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c)) -$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a \sec(dx + c) + a\right) \sqrt{\sec(dx + c)}, x\right)$$

64.4 Problem number 168

$$\int \frac{a + a \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c)) -$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a \sec(dx + c) + a}{\sqrt{\sec(dx + c)}}, x\right)$$

64.5 Problem number 169

$$\int \frac{a + a \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2a \sqrt{\cos(dx + c)} \sin(dx + c) - i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} \operatorname{aweierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a \sec(dx + c) + a}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

64.6 Problem number 170

$$\int \frac{a + a \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{6a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a \sec(dx + c) + a}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

64.7 Problem number 171

$$\int \frac{a + a \sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{10a \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{6a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-25i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a \sec(dx + c) + a}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

64.8 Problem number 172

$$\int \sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^2 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{4a^2 \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2a^2 \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{12a^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{12a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} a^2 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} a^2 \cos(dx + c)^3 \operatorname{wei} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^2 \sec(dx + c)^4 + 2a^2 \sec(dx + c)^3 + a^2 \sec(dx + c)^2\right) \sqrt{\sec(dx + c)}, x\right)$$

64.9 Problem number 173

$$\int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2 \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2a^2 \left(\sec^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{5d} \\ & + \frac{16a^2 \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d} \\ & - \frac{16a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^2 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} a^2 \cos(dx+c)^2 \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(a^2 \sec(dx+c)^3 + 2a^2 \sec(dx+c)^2 + a^2 \sec(dx+c) \right) \sqrt{\sec(dx+c)}, x \right)$$

64.10 Problem number 174

$$\int \sqrt{\sec(c+dx)} (a + a \sec(c+dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{4a^2 \sin(dx+c) (\sqrt{\sec(dx+c)})}{d} \\ & - \frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{8a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2i \sqrt{2} a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 2i \sqrt{2} a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2\right) \sqrt{\sec(dx + c)}, x\right)$$

64.11 Problem number 175

$$\int \frac{(a + a \sec(c + dx))^2}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} + \frac{4a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

d

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}{\sqrt{\sec(dx + c)}}, x\right)$$

64.12 Problem number 176

$$\int \frac{(a + a \sec(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(a^2 \sqrt{\cos(dx + c)} \sin(dx + c) - 2i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

64.13 Problem number 177

$$\int \frac{(a + a \sec(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{4a^2 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{16a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}{\sec(dx + c)^{\frac{5}{2}}}, x \right)$$

64.14 Problem number 178

$$\int \frac{(a + a \sec(c + dx))^2}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{8a^2 \sin(dx + c)}{7d \sqrt{\sec(dx + c)}} \\ & + \frac{12a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{8a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{7 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}{\sec(dx + c)^{\frac{7}{2}}}, x \right)$$

64.15 Problem number 179

$$\int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{52a^3 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} + \frac{6a^3 \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2a^3 \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{28a^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{28a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{52a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(65i \sqrt{2} a^3 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 65i \sqrt{2} a^3 \cos(dx + c)^3 \operatorname{wei} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^3 \sec(dx + c)^4 + 3a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + a^3 \sec(dx + c)\right) \sqrt{\sec(dx + c)}, x\right)$$

64.16 Problem number 180

$$\int \sqrt{\sec(c + dx)} (a + a \sec(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{d} + \frac{2a^3 \left(\sec^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{5d} \\ & + \frac{36a^3 \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d} \\ & - \frac{36a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} a^3 \cos(dx+c)^2 \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(a^3 \sec(dx+c)^3 + 3a^3 \sec(dx+c)^2 + 3a^3 \sec(dx+c) + a^3 \right) \sqrt{\sec(dx+c)}, x \right)$$

64.17 Problem number 181

$$\int \frac{(a + a \sec(c + dx))^3}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{6a^3 \sin(dx+c) (\sqrt{\sec(dx+c)})}{d} \\ & - \frac{4a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{20a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}{\sqrt{\sec(dx + c)}}, x \right)$$

64.18 Problem number 182

$$\int \frac{(a + a \sec(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2a^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & + \frac{4a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{20a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}{\sec(dx + c)^{\frac{3}{2}}}, x \right)$$

64.19 Problem number 183

$$\int \frac{(a + a \sec(c + dx))^3}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a^3 \sin(dx + c)}{d \sqrt{\sec(dx + c)}} \\ & + \frac{36a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

64.20 Problem number 184

$$\int \frac{(a + a \sec(c + dx))^3}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{6a^3 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{52a^3 \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{28a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{52a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(65i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 65i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}{\sec(dx + c)^{\frac{7}{2}}}, x \right)$$

64.21 Problem number 185

$$\int \frac{(a + a \sec(c + dx))^3}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{6a^3 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{68a^3 \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{44a^3 \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{68a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{44a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(165i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 165i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}{\sec(dx + c)^{\frac{9}{2}}}, x \right)$$

64.22 Problem number 186

$$\int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32a^4 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{122a^4 \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{45d} \\ & + \frac{8a^4 \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{2a^4 \left(\sec^{\frac{9}{2}}(dx + c) \right) \sin(dx + c)}{9d} \\ & + \frac{152a^4 \sin(dx + c) (\sqrt{\sec}(dx + c))}{15d} \\ & - \frac{152a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{32a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{7 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(360i \sqrt{2} a^4 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 360i \sqrt{2} a^4 \cos(dx + c)^4 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(a^4 \sec(dx + c)^5 + 4a^4 \sec(dx + c)^4 + 6a^4 \sec(dx + c)^3 + 4a^4 \sec(dx + c)^2 + a^4 \sec(dx + c) \right) \sqrt{\sec(dx + c)} \right)$$

64.23 Problem number 187

$$\int \sqrt{\sec(c + dx)} (a + a \sec(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{94a^4 \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{21d} + \frac{8a^4 \left(\sec^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{5d} \\ & + \frac{2a^4 \left(\sec^{\frac{7}{2}}(dx+c) \right) \sin(dx+c)}{7d} + \frac{64a^4 \sin(dx+c) (\sqrt{\sec}(dx+c))}{5d} \\ & - \frac{64a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{136a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(170i \sqrt{2} a^4 \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 170i \sqrt{2} a^4 \cos(dx+c)^3 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(a^4 \sec(dx+c)^4 + 4a^4 \sec(dx+c)^3 + 6a^4 \sec(dx+c)^2 + 4a^4 \sec(dx+c) + a^4 \right) \sqrt{\sec(dx+c)}, x \right)$$

64.24 Problem number 188

$$\int \frac{(a + a \sec(c + dx))^4}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^4 \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2a^4 \left(\sec^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{5d} \\ & + \frac{66a^4 \sin(dx+c) (\sqrt{\sec}(dx+c))}{5d} \\ & - \frac{56a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{32a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(40i \sqrt{2} a^4 \cos(dx+c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 40i \sqrt{2} a^4 \cos(dx+c)^2 \text{wei} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^4 \sec(dx+c)^4 + 4a^4 \sec(dx+c)^3 + 6a^4 \sec(dx+c)^2 + 4a^4 \sec(dx+c) + a^4}{\sqrt{\sec(dx+c)}}, x \right)$$

64.25 Problem number 189

$$\int \frac{(a + a \sec(c + dx))^4}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^4 \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2a^4 \sin(dx+c)}{3d \sqrt{\sec(dx+c)}} + \frac{8a^4 \sin(dx+c) (\sqrt{\sec(dx+c)})}{d} \\ & + \frac{40a^4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} a^4 \cos(dx+c) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 10i \sqrt{2} a^4 \cos(dx+c) \text{wei} \right)$$

$3d \cos(dx+c)$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^4 \sec(dx+c)^4 + 4a^4 \sec(dx+c)^3 + 6a^4 \sec(dx+c)^2 + 4a^4 \sec(dx+c) + a^4}{\sec(dx+c)^{\frac{3}{2}}}, x \right)$$

64.26 Problem number 190

$$\int \frac{(a + a \sec(c + dx))^4}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^4 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{8a^4 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2a^4 \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & + \frac{56a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{32a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(40i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 40i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^4 \sec(dx + c)^4 + 4a^4 \sec(dx + c)^3 + 6a^4 \sec(dx + c)^2 + 4a^4 \sec(dx + c) + a^4}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

64.27 Problem number 191

$$\int \frac{(a + a \sec(c + dx))^4}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^4 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{8a^4 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{94a^4 \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{64a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{136a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(170i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 170i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^4 \sec(dx + c)^4 + 4 a^4 \sec(dx + c)^3 + 6 a^4 \sec(dx + c)^2 + 4 a^4 \sec(dx + c) + a^4}{\sec(dx + c)^{\frac{7}{2}}}, x \right)$$

64.28 Problem number 192

$$\int \frac{(a + a \sec(c + dx))^4}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^4 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{8a^4 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{122a^4 \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{32a^4 \sin(dx + c)}{7d \sqrt{\sec(dx + c)}} \\ & + \frac{152a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{32a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{7 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(360i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 360i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^4 \sec(dx + c)^4 + 4 a^4 \sec(dx + c)^3 + 6 a^4 \sec(dx + c)^2 + 4 a^4 \sec(dx + c) + a^4}{\sec(dx + c)^{\frac{9}{2}}}, x \right)$$

64.29 Problem number 193

$$\int \frac{(a + a \sec(c + dx))^4}{\sec^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^4 \sin(dx + c)}{11d \sec(dx + c)^{\frac{9}{2}}} + \frac{8a^4 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{150a^4 \sin(dx + c)}{77d \sec(dx + c)^{\frac{5}{2}}} + \frac{128a^4 \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{904a^4 \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{128a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{904a^4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3390i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 3390i \sqrt{2} a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^4 \sec(dx + c)^4 + 4a^4 \sec(dx + c)^3 + 6a^4 \sec(dx + c)^2 + 4a^4 \sec(dx + c) + a^4}{\sec(dx + c)^{\frac{11}{2}}}, x\right)$$

64.30 Problem number 194

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5(\sec^{\frac{3}{2}}(dx + c)) \sin(dx + c)}{3ad} - \frac{(\sec^{\frac{5}{2}}(dx + c)) \sin(dx + c)}{d(a + a \sec(dx + c))} - \frac{3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{ad} \\ & + \frac{3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx+c)^2 + i \sqrt{2} \cos(dx+c) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5 \left(-i \sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sec(dx+c)^{\frac{7}{2}}}{a \sec(dx+c) + a}, x \right)$$

64.31 Problem number 195

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{d(a+a \sec(dx+c))} + \frac{3 \sin(dx+c) (\sqrt{\sec(dx+c)})}{ad} \\ & - \frac{3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(i \sqrt{2} \cos(dx+c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \left(-i \sqrt{2} \cos(dx+c) - i \sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sec(dx+c)^{\frac{5}{2}}}{a \sec(dx+c) + a}, x \right)$$

64.32 Problem number 196

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sin(dx + c) (\sqrt{\sec(dx + c)})}{d(a + a \sec(dx + c))} \\ & + \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(-i\sqrt{2} \cos(dx + c) - i\sqrt{2}) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + (i\sqrt{2} \cos(dx + c) + i\sqrt{2})}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(dx + c)^{\frac{3}{2}}}{a \sec(dx + c) + a}, x\right)$$

64.33 Problem number 197

$$\int \frac{\sqrt{\sec(c + dx)}}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sin(dx + c) (\sqrt{\sec(dx + c)})}{d(a + a \sec(dx + c))} \\ & - \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a},x\right)$$

64.34 Problem number 198

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sin(dx+c)(\sqrt{\sec(dx+c)})}{d(a+a\sec(dx+c))} \\ & + \frac{3\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \\ & - \frac{\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \end{aligned}$$

command

```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)^2+a\sec(dx+c)},x\right)$$

64.35 Problem number 199

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5 \sin(dx+c)}{3ad \sqrt{\sec(dx+c)}} - \frac{\sin(dx+c)}{d(a+a\sec(dx+c)) \sqrt{\sec(dx+c)}} \\ & - \frac{3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx+c) + i \sqrt{2} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5 \left(-i \sqrt{2} \cos(dx+c) - \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a \sec(dx+c)^3 + a \sec(dx+c)^2}, x\right)$$

64.36 Problem number 200

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7 \sin(dx+c)}{5ad \sec(dx+c)^{\frac{3}{2}}} - \frac{\sin(dx+c)}{d \sec(dx+c)^{\frac{3}{2}} (a+a\sec(dx+c))} - \frac{5 \sin(dx+c)}{3ad \sqrt{\sec(dx+c)}} \\ & + \frac{21 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{5 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25 \left(-i \sqrt{2} \cos(dx + c) - i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25 \left(i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\sec(dx + c)}}{a \sec(dx + c)^4 + a \sec(dx + c)^3}, x \right)$$

64.37 Problem number 201

$$\int \frac{\sec^{\frac{9}{2}}(c + dx)}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10 \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d} - \frac{7 \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d (1 + \sec(dx + c))} \\ & - \frac{\left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{3d (a + a \sec(dx + c))^2} - \frac{7 \sin(dx + c) (\sqrt{\sec(dx + c)})}{a^2d} \\ & + \frac{7 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \\ & + \frac{10 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$10 \left(i \sqrt{2} \cos(dx + c)^3 + 2i \sqrt{2} \cos(dx + c)^2 + i \sqrt{2} \cos(dx + c) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10 \left(-i \sqrt{2} \cos(dx + c)^3 - 2i \sqrt{2} \cos(dx + c)^2 - i \sqrt{2} \cos(dx + c) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sec(dx + c)^{\frac{9}{2}}}{a^2 \sec(dx + c)^2 + 2a^2 \sec(dx + c) + a^2}, x \right)$$

64.38 Problem number 202

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5\left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3a^2d(1 + \sec(dx + c))} - \frac{\left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{3d(a + a \sec(dx + c))^2} \\ & + \frac{4 \sin(dx + c) (\sqrt{\sec(dx + c)})}{a^2d} \\ & - \frac{4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & - \frac{5 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(-i \sqrt{2} \cos(dx + c)^2 - 2i \sqrt{2} \cos(dx + c) - i \sqrt{2} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(dx + c)^{\frac{7}{2}}}{a^2 \sec(dx + c)^2 + 2a^2 \sec(dx + c) + a^2}, x\right)$$

64.39 Problem number 203

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3d(a+a\sec(dx+c))^2} - \frac{\sin(dx+c)(\sqrt{\sec(dx+c)})}{a^2d(1+\sec(dx+c))} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & + \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(i\sqrt{2}\cos(dx+c)^2 + 2i\sqrt{2}\cos(dx+c) + i\sqrt{2}\right)\operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(dx+c)^{\frac{5}{2}}}{a^2\sec(dx+c)^2 + 2a^2\sec(dx+c) + a^2}, x\right)$$

64.40 Problem number 204

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3d(a+a\sec(dx+c))^2} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i \sqrt{2} \cos(dx+c)^2 - 2i \sqrt{2} \cos(dx+c) - i \sqrt{2}\right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + (i}{6 \left(a^2 d \cos(dx+c)\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{3}{2}}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

64.41 Problem number 205

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3d(a+a\sec(dx+c))^2} + \frac{\sin(dx+c) (\sqrt{\sec(dx+c)})}{a^2 d (1+\sec(dx+c))} \\ & -\frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & +\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} \cos(dx+c)^2 + 2i \sqrt{2} \cos(dx+c) + i \sqrt{2}\right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 2}{6 \left(a^2 d \cos(dx+c)\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

64.42 Problem number 206

$$\int \frac{1}{\sqrt{\sec(c+dx)} (a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5 \sin(dx+c) (\sqrt{\sec(dx+c)})}{3a^2d(1+\sec(dx+c))} - \frac{\sin(dx+c) (\sqrt{\sec(dx+c)})}{3d(a+a\sec(dx+c))^2} \\ & + \frac{4\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & - \frac{5\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

`integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(-i \sqrt{2} \cos(dx+c)^2 - 2i \sqrt{2} \cos(dx+c) - i \sqrt{2} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a^2 \sec(dx+c)^3 + 2a^2 \sec(dx+c)^2 + a^2 \sec(dx+c)}, x\right)$$

64.43 Problem number 207

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10 \sin(dx+c)}{3a^2d\sqrt{\sec(dx+c)}} - \frac{7 \sin(dx+c)}{3a^2d(1+\sec(dx+c))\sqrt{\sec(dx+c)}} \\ & - \frac{\sin(dx+c)}{3d(a+a\sec(dx+c))^2\sqrt{\sec(dx+c)}} \\ & - \frac{7\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & + \frac{10\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$10 \left(i \sqrt{2} \cos(dx + c)^2 + 2i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^4 + 2a^2 \sec(dx + c)^3 + a^2 \sec(dx + c)^2}, x \right)$$

64.44 Problem number 208

$$\int \frac{1}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{56 \sin(dx + c)}{15a^2 d \sec(dx + c)^{\frac{3}{2}}} - \frac{3 \sin(dx + c)}{a^2 d \sec(dx + c)^{\frac{3}{2}} (1 + \sec(dx + c))} \\ & - \frac{\sin(dx + c)}{3d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))^2} - \frac{5 \sin(dx + c)}{a^2 d \sqrt{\sec(dx + c)}} \\ & + \frac{56 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & - \frac{5 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \end{aligned}$$

command

```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$75 \left(-i \sqrt{2} \cos(dx + c)^2 - 2i \sqrt{2} \cos(dx + c) - i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^5 + 2a^2 \sec(dx + c)^4 + a^2 \sec(dx + c)^3}, x \right)$$

64.45 Problem number 209

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{11 \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{2a^3d} - \frac{\left(\sec^{\frac{9}{2}}(dx+c) \right) \sin(dx+c)}{5d(a+a\sec(dx+c))^3} \\ & - \frac{2 \left(\sec^{\frac{7}{2}}(dx+c) \right) \sin(dx+c)}{3ad(a+a\sec(dx+c))^2} - \frac{119 \left(\sec^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{30d(a^3+a^3\sec(dx+c))} \\ & - \frac{119 \sin(dx+c) (\sqrt{\sec}(dx+c))}{10a^3d} \\ & + \frac{119 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & + \frac{11 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{165 \left(i \sqrt{2} \cos(dx+c)^4 + 3i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c)^2 + i \sqrt{2} \cos(dx+c) \right) \operatorname{weierstrassPInverse}(-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(dx+c)^{\frac{11}{2}}}{a^3 \sec(dx+c)^3 + 3a^3 \sec(dx+c)^2 + 3a^3 \sec(dx+c) + a^3}, x\right)$$

64.46 Problem number 210

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\left(\sec^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{5d(a+a\sec(dx+c))^3} - \frac{8\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{15ad(a+a\sec(dx+c))^2} \\ & - \frac{13\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{6d(a^3+a^3\sec(dx+c))} + \frac{49\sin(dx+c)(\sqrt{\sec(dx+c)})}{10a^3d} \\ & - \frac{49\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{10\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^3d} \\ & - \frac{13\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{6\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^3d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$65 \left(-i \sqrt{2} \cos(dx+c)^3 - 3i \sqrt{2} \cos(dx+c)^2 - 3i \sqrt{2} \cos(dx+c) - i \sqrt{2} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(dx+c)^{\frac{9}{2}}}{a^3\sec(dx+c)^3 + 3a^3\sec(dx+c)^2 + 3a^3\sec(dx+c) + a^3}, x\right)$$

64.47 Problem number 211

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\left(\sec^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{5d(a+a \sec(dx+c))^3} - \frac{2\left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5ad(a+a \sec(dx+c))^2} \\ & - \frac{9 \sin(dx+c) (\sqrt{\sec(dx+c)})}{10d(a^3+a^3 \sec(dx+c))} \\ & + \frac{9 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c)^2 + 3i \sqrt{2} \cos(dx+c) + i \sqrt{2} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(dx+c)^{\frac{7}{2}}}{a^3 \sec(dx+c)^3 + 3a^3 \sec(dx+c)^2 + 3a^3 \sec(dx+c) + a^3}, x\right)$$

64.48 Problem number 212

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a \sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d(a+a \sec(dx+c))^3} - \frac{4 \sin(dx+c) (\sqrt{\sec(dx+c)})}{15ad(a+a \sec(dx+c))^2} \\ & + \frac{\sin(dx+c) (\sqrt{\sec(dx+c)})}{6d(a^3+a^3 \sec(dx+c))} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx + c)^3 + 3i \sqrt{2} \cos(dx + c)^2 + 3i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sec(dx + c)^{\frac{5}{2}}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x \right)$$

64.49 Problem number 213

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(\sec^{\frac{3}{2}}(dx + c)) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} - \frac{\sin(dx + c) (\sqrt{\sec(dx + c)})}{15ad(a + a \sec(dx + c))^2} + \frac{\sin(dx + c) (\sqrt{\sec(dx + c)})}{6d(a^3 + a^3 \sec(dx + c))} \\ & - \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx + c)^3 + 3i \sqrt{2} \cos(dx + c)^2 + 3i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sec(dx + c)^{\frac{3}{2}}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x \right)$$

64.50 Problem number 214

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(\sec^{\frac{3}{2}}(dx+c)) \sin(dx+c)}{5d(a+a\sec(dx+c))^3} + \frac{2 \sin(dx+c) (\sqrt{\sec(dx+c)})}{5ad(a+a\sec(dx+c))^2} \\ & + \frac{\sin(dx+c) (\sqrt{\sec(dx+c)})}{2d(a^3+a^3\sec(dx+c))} \\ & - \frac{9\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c)^2 + 3i \sqrt{2} \cos(dx+c) + i \sqrt{2} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a^3 \sec(dx+c)^3 + 3a^3 \sec(dx+c)^2 + 3a^3 \sec(dx+c) + a^3}, x\right)$$

64.51 Problem number 215

$$\int \frac{1}{\sqrt{\sec(c+dx)} (a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sin(dx+c)(\sqrt{\sec(dx+c)})}{5d(a+a\sec(dx+c))^3} - \frac{8\sin(dx+c)(\sqrt{\sec(dx+c)})}{15ad(a+a\sec(dx+c))^2} \\ & - \frac{13\sin(dx+c)(\sqrt{\sec(dx+c)})}{6d(a^3+a^3\sec(dx+c))} \\ & + \frac{49\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{13\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{6\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

`integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$65 \left(-i \sqrt{2} \cos(dx+c)^3 - 3i \sqrt{2} \cos(dx+c)^2 - 3i \sqrt{2} \cos(dx+c) - i \sqrt{2} \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a^3 \sec(dx+c)^4 + 3a^3 \sec(dx+c)^3 + 3a^3 \sec(dx+c)^2 + a^3 \sec(dx+c)}, x\right)$$

64.52 Problem number 216

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{11\sin(dx+c)}{2a^3 d \sqrt{\sec(dx+c)}} - \frac{\sin(dx+c)}{5d(a+a\sec(dx+c))^3 \sqrt{\sec(dx+c)}} \\ & - \frac{2\sin(dx+c)}{3ad(a+a\sec(dx+c))^2 \sqrt{\sec(dx+c)}} - \frac{119\sin(dx+c)}{30d(a^3+a^3\sec(dx+c)) \sqrt{\sec(dx+c)}} \\ & + \frac{119\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{11\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{2\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$165 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c)^2 + 3i \sqrt{2} \cos(dx+c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\sec(dx+c)}}{a^3 \sec(dx+c)^5 + 3a^3 \sec(dx+c)^4 + 3a^3 \sec(dx+c)^3 + a^3 \sec(dx+c)^2}, x \right)$$

64.53 Problem number 217

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{77 \sin(dx+c)}{10a^3 d \sec(dx+c)^{\frac{3}{2}}} - \frac{\sin(dx+c)}{5d \sec(dx+c)^{\frac{3}{2}} (a+a\sec(dx+c))^3} \\ & - \frac{4 \sin(dx+c)}{5ad \sec(dx+c)^{\frac{3}{2}} (a+a\sec(dx+c))^2} \\ & - \frac{63 \sin(dx+c)}{10d \sec(dx+c)^{\frac{3}{2}} (a^3+a^3 \sec(dx+c))} - \frac{21 \sin(dx+c)}{2a^3 d \sqrt{\sec(dx+c)}} \\ & + \frac{231 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & - \frac{21 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \end{aligned}$$

command

```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$105 \left(-i \sqrt{2} \cos(dx+c)^3 - 3i \sqrt{2} \cos(dx+c)^2 - 3i \sqrt{2} \cos(dx+c) - i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a^3 \sec(dx+c)^6 + 3a^3 \sec(dx+c)^5 + 3a^3 \sec(dx+c)^4 + a^3 \sec(dx+c)^3}, x\right)$$

64.54 Problem number 351

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} \\ & + \frac{2a \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} + \frac{10a \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15 a \cos(dx+c)^2 + 21 a \cos(dx+c) + 25 a \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 25i \sqrt{2} \text{aweierstrassPInverse}(-4, 0, c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(a \cos(dx+c)^3 \sec(dx+c) + a \cos(dx+c)^3\right) \sqrt{\cos(dx+c)}, x\right)$$

64.55 Problem number 352

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx)) dx$$

Optimal antiderivative

$$\frac{6a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2a \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3a \cos(dx+c) + 5a) \sqrt{\cos(dx+c)} \sin(dx+c) - 5i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a \cos(dx+c)^2 \sec(dx+c) + a \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}, x\right)$$

64.56 Problem number 353

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx)) dx$$

Optimal antiderivative

$$\frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2a \sqrt{\cos(dx+c)} \sin(dx+c) - i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a \cos(dx+c) \sec(dx+c) + a \cos(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

64.57 Problem number 354

$$\int \sqrt{\cos(c+dx)} (a + a \sec(c+dx)) dx$$

Optimal antiderivative

$$\frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx+c) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((a \sec(dx+c) + a) \sqrt{\cos(dx+c)}, x\right)$$

64.58 Problem number 355

$$\int \frac{a + a \sec(c+dx)}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$-\frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sin(dx+c)}{d \sqrt{\cos(dx+c)}}$$

command

```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} a \cos(dx + c)$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i \sqrt{2} a \cos(dx + c)$ weierstrassPInv

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{a \sec(dx + c) + a}{\sqrt{\cos(dx + c)}}, x\right)$$

64.59 Problem number 356

$$\int \frac{a + a \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} a \cos(dx + c)^2$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $i \sqrt{2} a \cos(dx + c)^2$ weierstrassPI

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{a \sec(dx + c) + a}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

64.60 Problem number 357

$$\int \frac{a + a \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{6a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2a \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{6a \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} a \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} a \cos(dx+c)^3 \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a \sec(dx+c) + a}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

64.61 Problem number 358

$$\int \frac{a + a \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{6a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sin(dx+c)}{7d \cos(dx+c)^{\frac{7}{2}}} \\ & + \frac{2a \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{10a \sin(dx+c)}{21d \cos(dx+c)^{\frac{3}{2}}} + \frac{6a \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-25i \sqrt{2} a \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} a \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{a \sec(dx + c) + a}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

64.62 Problem number 359

$$\int \cos^{\frac{9}{2}}(c + dx)(a + a \sec(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32a^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{20a^2 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{32a^2 \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} + \frac{4a^2 \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2a^2 \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{9d} + \frac{20a^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$2 \left(75i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 75i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(a^2 \cos(dx + c)^4 \sec(dx + c)^2 + 2a^2 \cos(dx + c)^4 \sec(dx + c) + a^2 \cos(dx + c)^4\right) \sqrt{\cos(dx + c)}, x\right)$$

64.63 Problem number 360

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2 dx$$

Optimal antiderivative

$$\frac{12a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{8a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{4a^2 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2a^2 \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} + \frac{8a^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{7d}$$

command

`integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 10i \sqrt{2} a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^2 \cos(dx+c)^3 \sec(dx+c)^2 + 2a^2 \cos(dx+c)^3 \sec(dx+c) + a^2 \cos(dx+c)^3\right) \sqrt{\cos(dx+c)}, x\right)$$

64.64 Problem number 361

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2 dx$$

Optimal antiderivative

$$\frac{16a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a^2 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{4a^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right) \sqrt{\cos(dx + c)}, x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(a^2 \cos(dx + c)^2 \sec(dx + c)^2 + 2 a^2 \cos(dx + c)^2 \sec(dx + c) + a^2 \cos(dx + c)^2 \right) \sqrt{\cos(dx + c)}, x \right)$$

64.65 Problem number 362

$$\int \cos^{\frac{3}{2}}(c + dx) (a + a \sec(c + dx))^2 dx$$

Optimal antiderivative

$$\frac{4a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} + \frac{8a^2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} + \frac{2a^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(a^2 \sqrt{\cos(dx + c)} \sin(dx + c) - 2i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right) \sqrt{\cos(dx + c)}, x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(a^2 \cos(dx + c) \sec(dx + c)^2 + 2 a^2 \cos(dx + c) \sec(dx + c) + a^2 \cos(dx + c) \right) \sqrt{\cos(dx + c)}, x \right)$$

64.66 Problem number 363

$$\int \sqrt{\cos(c+dx)} (a+a\sec(c+dx))^2 dx$$

Optimal antiderivative

$$\frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a^2 \sin(dx+c)}{d \sqrt{\cos(dx+c)}}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} a^2 \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - i \sqrt{2} a^2 \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) \right)}{d \cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2\right) \sqrt{\cos(dx+c)}, x\right)$$

64.67 Problem number 364

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a^2 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{4a^2 \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2i \sqrt{2} a^2 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 2i \sqrt{2} a^2 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) \right)}{d \cos(dx+c)^{\frac{3}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}{\sqrt{\cos(dx+c)}}, x\right)$$

64.68 Problem number 365

$$\int \frac{(a + a \sec(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{16a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{4a^2 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{16a^2 \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} a^2 \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} a^2 \cos(dx+c)^3 \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

64.69 Problem number 366

$$\int \frac{(a + a \sec(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{12a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{7 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2 \sin(dx+c)}{7d \cos(dx+c)^{\frac{7}{2}}} + \frac{4a^2 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{8a^2 \sin(dx+c)}{7d \cos(dx+c)^{\frac{3}{2}}} + \frac{12a^2 \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} a^2 \cos(dx+c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 10i \sqrt{2} a^2 \cos(dx+c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}{\cos(dx+c)^{\frac{5}{2}}}, x \right)$$

64.70 Problem number 367

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a \sec(c+dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{68a^3 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{44a^3 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{68a^3 \left(\cos^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{45d} + \frac{6a^3 \left(\cos^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{7d} \\ & + \frac{2a^3 \left(\cos^{\frac{7}{2}}(dx+c) \right) \sin(dx+c)}{9d} + \frac{44a^3 \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(165i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 165i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(a^3 \cos(dx+c)^4 \sec(dx+c)^3 + 3a^3 \cos(dx+c)^4 \sec(dx+c)^2 + 3a^3 \cos(dx+c)^4 \sec(dx+c) + a^3 \cos(dx+c)^4 \right), x \right)$$

64.71 Problem number 368

$$\int \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^3 dx$$

Optimal antiderivative

$$\frac{28a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{52a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{6a^3 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2a^3 \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} + \frac{52a^3 \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d}$$

command

`integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(65i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 65i \sqrt{2} a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^3 \cos(dx+c)^3 \sec(dx+c)^3 + 3a^3 \cos(dx+c)^3 \sec(dx+c)^2 + 3a^3 \cos(dx+c)^3 \sec(dx+c) + a^3 \cos(dx+c)^3\right) dx\right)$$

64.72 Problem number 369

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^3 dx$$

Optimal antiderivative

$$\frac{36a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{4a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a^3 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2a^3 \sin(dx+c) (\sqrt{\cos(dx+c)})}{d}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(a^3 \cos(dx + c)^2 \sec(dx + c)^3 + 3 a^3 \cos(dx + c)^2 \sec(dx + c)^2 + 3 a^3 \cos(dx + c)^2 \sec(dx + c) + a^3 \cos(dx + c) \right) dx \right)$$

64.73 Problem number 370

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{20a^3 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a^3 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} + \frac{2a^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(a^3 \cos(dx + c) \sec(dx + c)^3 + 3 a^3 \cos(dx + c) \sec(dx + c)^2 + 3 a^3 \cos(dx + c) \sec(dx + c) + a^3 \cos(dx + c) \right) dx \right)$$

64.74 Problem number 371

$$\int \sqrt{\cos(c+dx)} (a + a \sec(c+dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{20a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^3 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{6a^3 \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} a^3 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} a^3 \cos(dx+c)^2 \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^3 \sec(dx+c)^3 + 3a^3 \sec(dx+c)^2 + 3a^3 \sec(dx+c) + a^3\right) \sqrt{\cos(dx+c)}, x\right)$$

64.75 Problem number 372

$$\int \frac{(a + a \sec(c+dx))^3}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{36a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^3 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2a^3 \sin(dx+c)}{d \cos(dx+c)^{\frac{3}{2}}} + \frac{36a^3 \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} a^3 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \cos(dx + c)^3 \text{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}{\sqrt{\cos(dx + c)}}, x \right)$$

64.76 Problem number 373

$$\int \frac{(a + a \sec(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{28a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{52a^3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a^3 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{6a^3 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{52a^3 \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} + \frac{28a^3 \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(65i \sqrt{2} a^3 \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 65i \sqrt{2} a^3 \cos(dx + c)^4 \text{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}{\cos(dx + c)^{\frac{3}{2}}}, x \right)$$

64.77 Problem number 374

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{21\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{5\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{7\left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5ad} \\ & - \frac{\left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{d(a+a\sec(dx+c))} - \frac{5\sin(dx+c)(\sqrt{\cos(dx+c)})}{3ad} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(6\cos(dx+c)^2 - 4\cos(dx+c) - 25\right)\sqrt{\cos(dx+c)}\sin(dx+c) - 25\left(-i\sqrt{2}\cos(dx+c) - i\sqrt{2}\right)\text{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}, x\right)$$

64.78 Problem number 375

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5\sin(dx+c)(\sqrt{\cos(dx+c)})}{3ad} - \frac{\sin(dx+c)(\sqrt{\cos(dx+c)})}{d(a+a\sec(dx+c))} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(2 \cos(dx + c) + 5)\sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx + c)^{\frac{3}{2}}}{a \sec(dx + c) + a}, x \right)$$

64.79 Problem number 376

$$\int \frac{\sqrt{\cos(c + dx)}}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} - \frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} - \frac{\sin(dx + c)}{d(a + a \sec(dx + c)) \sqrt{\cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(i \sqrt{2} \cos(dx + c) + i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \left(-i \sqrt{2} \cos(dx + c) - i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\cos(dx + c)}}{a \sec(dx + c) + a}, x \right)$$

64.80 Problem number 377

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{\sin(dx+c)}{d(a+a\sec(dx+c))\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-i\sqrt{2}\cos(dx+c) - i\sqrt{2}\right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + \left(i\sqrt{2}\cos(dx+c) + i\sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)\sec(dx+c) + a\cos(dx+c)}, x\right)$$

64.81 Problem number 378

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} - \frac{\sin(dx+c)}{d(a+a\sec(dx+c))\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)^2\sec(dx+c)+a\cos(dx+c)^2},x\right)$$

64.82 Problem number 379

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \\ & -\frac{\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \\ & -\frac{\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}(a+a\sec(dx+c))}+\frac{3\sin(dx+c)}{ad\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c)+\left(i\sqrt{2}\cos(dx+c)^2+i\sqrt{2}\cos(dx+c)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)^3\sec(dx+c)+a\cos(dx+c)^3},x\right)$$

64.83 Problem number 380

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{5\sin(dx+c)}{3ad\cos(dx+c)^{\frac{3}{2}}} \\ & - \frac{\sin(dx+c)}{d\cos(dx+c)^{\frac{5}{2}}(a+a\sec(dx+c))} - \frac{3\sin(dx+c)}{ad\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(9\cos(dx+c)^2 + 4\cos(dx+c) - 2\right)\sqrt{\cos(dx+c)}\sin(dx+c) + 5\left(i\sqrt{2}\cos(dx+c)^3 + i\sqrt{2}\cos(dx+c)\right)^2}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)^4\sec(dx+c) + a\cos(dx+c)^4}, x\right)$$

64.84 Problem number 381

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{56 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{56 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15 a^2 d} - \frac{3 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{a^2 d (1 + \sec(dx+c))} \\ & - \frac{\left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3d (a + a \sec(dx+c))^2} - \frac{5 \sin(dx+c) (\sqrt{\cos}(dx+c))}{a^2 d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 \cos(dx+c)^3 - 8 \cos(dx+c)^2 - 94 \cos(dx+c) - 75\right) \sqrt{\cos(dx+c)} \sin(dx+c) - 75 \left(-i \sqrt{2} \cos(dx+c)\right)^2}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(dx+c)^{\frac{5}{2}}}{a^2 \sec(dx+c)^2 + 2 a^2 \sec(dx+c) + a^2}, x\right)$$

64.85 Problem number 382

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{7 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{10 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} + \frac{10 \sin(dx+c) (\sqrt{\cos}(dx+c))}{3 a^2 d} \\ & - \frac{7 \sin(dx+c) (\sqrt{\cos}(dx+c))}{3 a^2 d (1 + \sec(dx+c))} - \frac{\sin(dx+c) (\sqrt{\cos}(dx+c))}{3d (a + a \sec(dx+c))^2} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \cos(dx + c)^2 + 13 \cos(dx + c) + 10 \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 10 \left(i \sqrt{2} \cos(dx + c)^2 + 2i \sqrt{2} \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx + c)^{\frac{3}{2}}}{a^2 \sec(dx + c)^2 + 2a^2 \sec(dx + c) + a^2}, x \right)$$

64.86 Problem number 383

$$\int \frac{\sqrt{\cos(c + dx)}}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & - \frac{5 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & - \frac{5 \sin(dx + c)}{3a^2 d (1 + \sec(dx + c)) \sqrt{\cos(dx + c)}} - \frac{\sin(dx + c)}{3d (a + a \sec(dx + c))^2 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(6 \cos(dx + c) + 5) \sqrt{\cos(dx + c)} \sin(dx + c) + 5 \left(-i \sqrt{2} \cos(dx + c)^2 - 2i \sqrt{2} \cos(dx + c) - i \sqrt{2} \right) \text{weierst}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\cos(dx + c)}}{a^2 \sec(dx + c)^2 + 2a^2 \sec(dx + c) + a^2}, x \right)$$

64.87 Problem number 384

$$\int \frac{1}{\sqrt{\cos(c+dx)} (a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{\sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}} (a+a \sec(dx+c))^2} + \frac{\sin(dx+c)}{a^2 d (1+\sec(dx+c)) \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(3 \cos(dx+c) + 2)\sqrt{\cos(dx+c)} \sin(dx+c) - 2\left(i\sqrt{2} \cos(dx+c)^2 + 2i\sqrt{2} \cos(dx+c) + i\sqrt{2}\right) \text{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2 \cos(dx+c) \sec(dx+c)^2 + 2a^2 \cos(dx+c) \sec(dx+c) + a^2 \cos(dx+c)}, x\right)$$

64.88 Problem number 385

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} + \frac{\sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}} (a+a \sec(dx+c))^2}$$

command

`integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i\sqrt{2}\cos(dx+c)^2 - 2i\sqrt{2}\cos(dx+c) - i\sqrt{2}\right)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + (i}{6\left(a^2d\cos(dx+c)\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^2\sec(dx+c)^2 + 2a^2\cos(dx+c)^2\sec(dx+c) + a^2\cos(dx+c)^2}, x\right)$$

64.89 Problem number 386

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{\sin(dx+c)}{3d\cos(dx+c)^{\frac{3}{2}}(a+a\sec(dx+c))^2} - \frac{\sin(dx+c)}{a^2 d(1+\sec(dx+c))\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3\cos(dx+c)+4)\sqrt{\cos(dx+c)}\sin(dx+c) + 2\left(i\sqrt{2}\cos(dx+c)^2 + 2i\sqrt{2}\cos(dx+c) + i\sqrt{2}\right)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))}{6\left(a^2d\cos(dx+c)\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^3\sec(dx+c)^2 + 2a^2\cos(dx+c)^3\sec(dx+c) + a^2\cos(dx+c)^3}, x\right)$$

64.90 Problem number 387

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5\sin(dx+c)}{3a^2 d \cos(dx+c)^{\frac{3}{2}} (1+\sec(dx+c))} \\ & - \frac{\sin(dx+c)}{3d \cos(dx+c)^{\frac{5}{2}} (a+a\sec(dx+c))^2} + \frac{4\sin(dx+c)}{a^2 d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(12\cos(dx+c)^2 + 19\cos(dx+c) + 6\right)\sqrt{\cos(dx+c)}\sin(dx+c) - 5\left(-i\sqrt{2}\cos(dx+c)^3 - 2i\sqrt{2}\cos(dx+c)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^4 \sec(dx+c)^2 + 2a^2 \cos(dx+c)^4 \sec(dx+c) + a^2 \cos(dx+c)^4}, x\right)$$

64.91 Problem number 388

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\frac{7\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} + \frac{10\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} + \frac{10\sin(dx+c)}{3a^2 d \cos(dx+c)^{\frac{3}{2}}} - \frac{7\sin(dx+c)}{3a^2 d \cos(dx+c)^{\frac{5}{2}} (1 + \sec(dx+c))} - \frac{\sin(dx+c)}{3d \cos(dx+c)^{\frac{7}{2}} (a + a \sec(dx+c))^2} - \frac{7\sin(dx+c)}{a^2 d \sqrt{\cos(dx+c)}}$$

command

`integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(21\cos(dx+c)^3 + 32\cos(dx+c)^2 + 8\cos(dx+c) - 2\right)\sqrt{\cos(dx+c)}\sin(dx+c) + 10\left(i\sqrt{2}\cos(dx+c)\right)^4}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^5\sec(dx+c)^2 + 2a^2\cos(dx+c)^5\sec(dx+c) + a^2\cos(dx+c)^5}, x\right)$$

64.92 Problem number 389

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\frac{231\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} - \frac{21\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} + \frac{77\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{10a^3 d} - \frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{5d(a+a\sec(dx+c))^3} - \frac{4\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{5ad(a+a\sec(dx+c))^2} - \frac{63\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{10d(a^3+a^3\sec(dx+c))} - \frac{21\sin(dx+c)\left(\sqrt{\cos(dx+c)}\right)}{2a^3 d}$$

command

```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(4 \cos(dx + c)^4 - 8 \cos(dx + c)^3 - 147 \cos(dx + c)^2 - 238 \cos(dx + c) - 105 \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 10$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx + c)^{\frac{5}{2}}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x \right)$$

64.93 Problem number 390

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{119 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{11 \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{11 \sin(dx + c) (\sqrt{\cos(dx + c)})}{2a^3 d} - \frac{\sin(dx + c) (\sqrt{\cos(dx + c)})}{5d (a + a \sec(dx + c))^3} \\ & - \frac{2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{3ad (a + a \sec(dx + c))^2} - \frac{119 \sin(dx + c) (\sqrt{\cos(dx + c)})}{30d (a^3 + a^3 \sec(dx + c))} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(20 \cos(dx + c)^3 + 237 \cos(dx + c)^2 + 376 \cos(dx + c) + 165 \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 165 \left(i \sqrt{2} \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cos(dx + c)^{\frac{3}{2}}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x \right)$$

64.94 Problem number 391

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{49 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{13 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{\sin(dx+c)}{5d(a+a\sec(dx+c))^3 \sqrt{\cos(dx+c)}} - \frac{8 \sin(dx+c)}{15ad(a+a\sec(dx+c))^2 \sqrt{\cos(dx+c)}} \\ & - \frac{13 \sin(dx+c)}{6d(a^3+a^3\sec(dx+c)) \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(87 \cos(dx+c)^2 + 146 \cos(dx+c) + 65 \right) \sqrt{\cos(dx+c)} \sin(dx+c) + 65 \left(-i \sqrt{2} \cos(dx+c)^3 - 3i \sqrt{2} \cos(dx+c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3 \sec(dx+c)^3 + 3a^3 \sec(dx+c)^2 + 3a^3 \sec(dx+c) + a^3}, x\right)$$

64.95 Problem number 392

$$\int \frac{1}{\sqrt{\cos(c+dx)} (a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} - \frac{\sin(dx+c)}{5d \cos(dx+c)^{\frac{3}{2}} (a+a\sec(dx+c))^3} \\ & + \frac{2 \sin(dx+c)}{5ad(a+a\sec(dx+c))^2 \sqrt{\cos(dx+c)}} + \frac{\sin(dx+c)}{2d(a^3+a^3\sec(dx+c)) \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(9 \cos(dx+c)^2 + 12 \cos(dx+c) + 5 \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 5 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\cos(dx+c)}}{a^3 \cos(dx+c) \sec(dx+c)^3 + 3 a^3 \cos(dx+c) \sec(dx+c)^2 + 3 a^3 \cos(dx+c) \sec(dx+c) + a^3 \cos(dx+c)} dx \right)$$

64.96 Problem number 393

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} + \frac{\sin(dx+c)}{5d \cos(dx+c)^{\frac{3}{2}} (a+a \sec(dx+c))^3} \\ & - \frac{\sin(dx+c)}{15ad (a+a \sec(dx+c))^2 \sqrt{\cos(dx+c)}} + \frac{\sin(dx+c)}{6d (a^3 + a^3 \sec(dx+c)) \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \cos(dx+c)^2 + 14 \cos(dx+c) + 5 \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 5 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^2 \sec(dx+c)^3 + 3 a^3 \cos(dx+c)^2 \sec(dx+c)^2 + 3 a^3 \cos(dx+c)^2 \sec(dx+c) + a^3 \cos(dx+c)} dx \right)$$

64.97 Problem number 394

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} - \frac{\sin(dx+c)}{5d \cos(dx+c)^{\frac{3}{2}} (a+a\sec(dx+c))^3} \\ & - \frac{4 \sin(dx+c)}{15ad (a+a\sec(dx+c))^2 \sqrt{\cos(dx+c)}} + \frac{\sin(dx+c)}{6d (a^3 + a^3 \sec(dx+c)) \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \cos(dx+c)^2 + 4 \cos(dx+c) - 5 \right) \sqrt{\cos(dx+c)} \sin(dx+c) + 5 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^3 \sec(dx+c)^3 + 3a^3 \cos(dx+c)^3 \sec(dx+c)^2 + 3a^3 \cos(dx+c)^3 \sec(dx+c) + a^3 \cos(dx+c)}\right)$$

64.98 Problem number 395

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} - \frac{\sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}} (a+a\sec(dx+c))^3} \\ & - \frac{2 \sin(dx+c)}{5ad \cos(dx+c)^{\frac{3}{2}} (a+a\sec(dx+c))^2} - \frac{9 \sin(dx+c)}{10d (a^3 + a^3 \sec(dx+c)) \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(9 \cos(dx+c)^2 + 22 \cos(dx+c) + 15 \right) \sqrt{\cos(dx+c)} \sin(dx+c) + 5 \left(i \sqrt{2} \cos(dx+c)^3 + 3i \sqrt{2} \cos(dx+c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^4 \sec(dx+c)^3 + 3a^3 \cos(dx+c)^4 \sec(dx+c)^2 + 3a^3 \cos(dx+c)^4 \sec(dx+c) + a^3 \cos(dx+c)} \right)$$

64.99 Problem number 396

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a \sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{49 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & - \frac{13 \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & - \frac{\sin(dx+c)}{5d \cos(dx+c)^{\frac{7}{2}} (a+a \sec(dx+c))^3} - \frac{8 \sin(dx+c)}{15ad \cos(dx+c)^{\frac{5}{2}} (a+a \sec(dx+c))^2} \\ & - \frac{13 \sin(dx+c)}{6d \cos(dx+c)^{\frac{3}{2}} (a^3+a^3 \sec(dx+c))} + \frac{49 \sin(dx+c)}{10a^3 d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(147 \cos(dx+c)^3 + 376 \cos(dx+c)^2 + 295 \cos(dx+c) + 60 \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 65 \left(-i \sqrt{2} \cos(dx+c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^5 \sec(dx+c)^3 + 3a^3 \cos(dx+c)^5 \sec(dx+c)^2 + 3a^3 \cos(dx+c)^5 \sec(dx+c) + a^3 \cos(dx+c)} \right)$$

64.100 Problem number 397

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{119 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{11 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} + \frac{11 \sin(dx+c)}{2a^3 d \cos(dx+c)^{\frac{3}{2}}} \\ & - \frac{\sin(dx+c)}{5d \cos(dx+c)^{\frac{9}{2}} (a+a\sec(dx+c))^3} - \frac{2 \sin(dx+c)}{3ad \cos(dx+c)^{\frac{7}{2}} (a+a\sec(dx+c))^2} \\ & - \frac{119 \sin(dx+c)}{30d \cos(dx+c)^{\frac{5}{2}} (a^3 + a^3 \sec(dx+c))} - \frac{119 \sin(dx+c)}{10a^3 d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(357 \cos(dx+c)^4 + 906 \cos(dx+c)^3 + 695 \cos(dx+c)^2 + 120 \cos(dx+c) - 20 \right) \sqrt{\cos(dx+c)} \sin(dx+c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3 \cos(dx+c)^6 \sec(dx+c)^3 + 3a^3 \cos(dx+c)^6 \sec(dx+c)^2 + 3a^3 \cos(dx+c)^6 \sec(dx+c) + a^3 \cos(dx+c)}\right)$$

64.101 Problem number 579

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2b \left(\sec^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{5d} \\ & + \frac{6b \sin(dx+c) (\sqrt{\sec}(dx+c)})}{5d} \\ & - \frac{6b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} a \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} a \cos(dx+c)^2 \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(b \sec(dx+c)^3 + a \sec(dx+c)^2 \right) \sqrt{\sec(dx+c)}, x \right)$$

64.102 Problem number 580

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2a \sin(dx+c) (\sqrt{\sec}(dx+c))}{d} \\ & - \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} b \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} b \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b \sec(dx + c)^2 + a \sec(dx + c)\right) \sqrt{\sec(dx + c)}, x\right)$$

64.103 Problem number 581

$$\int \sqrt{\sec(c + dx)} (a + b \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} a \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b \sec(dx + c) + a\right) \sqrt{\sec(dx + c)}, x\right)$$

64.104 Problem number 582

$$\int \frac{a + b \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} b \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} b \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b \sec(dx + c) + a}{\sqrt{\sec(dx + c)}}, x\right)$$

64.105 Problem number 583

$$\int \frac{a + b \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2a \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2a\sqrt{\cos(dx+c)}\sin(dx+c) - i\sqrt{2}\operatorname{aweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}\operatorname{aweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

64.106 Problem number 584

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a\sin(dx+c)}{5d\sec(dx+c)^{\frac{3}{2}}} + \frac{2b\sin(dx+c)}{3d\sqrt{\sec(dx+c)}} \\ & + \frac{6a\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2b\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}\operatorname{bweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}\operatorname{bweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

64.107 Problem number 585

$$\int \frac{a + b \sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2b \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{10a \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{6b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-25i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b \sec(dx + c) + a}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

64.108 Problem number 586

$$\int \sec^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7a^2 + 5b^2) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} + \frac{4ab \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2b^2 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{12ab \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{12ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-126i \sqrt{2} ab \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 126i \sqrt{2} ab \cos(dx + c)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 126i \sqrt{2} ab \cos(dx + c) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 126i \sqrt{2} ab \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 \sec(dx + c)^4 + 2ab \sec(dx + c)^3 + a^2 \sec(dx + c)^2\right) \sqrt{\sec(dx + c)}, x\right)$$

64.109 Problem number 587

$$\int \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4ab \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} + \frac{2b^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2(5a^2 + 3b^2) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{2(5a^2 + 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-10i \sqrt{2} ab \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} ab \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} ab \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} ab \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 \sec(dx + c)^3 + 2ab \sec(dx + c)^2 + a^2 \sec(dx + c)\right) \sqrt{\sec(dx + c)}, x\right)$$

64.110 Problem number 588

$$\int \sqrt{\sec(c+dx)} (a+b\sec(c+dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2 \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{4ab \sin(dx+c) (\sqrt{\sec}(dx+c))}{d} \\ & - \frac{4ab \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3a^2 + b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-6i \sqrt{2} ab \cos(dx+c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 6i \sqrt{2} ab$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \sec(dx+c)^2 + 2ab \sec(dx+c) + a^2\right) \sqrt{\sec(dx+c)}, x\right)$$

64.111 Problem number 589

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2 \sin(dx+c) (\sqrt{\sec}(dx+c))}{d} \\ & + \frac{2(a^2 - b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2i \sqrt{2} \operatorname{abweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2i \sqrt{2} \operatorname{abweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}{\sqrt{\sec(dx + c)}}, x\right)$$

64.112 Problem number 590

$$\int \frac{(a + b \sec(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^2 + 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2a^2 \sqrt{\cos(dx + c)} \sin(dx + c) + 6i \sqrt{2} \operatorname{abweierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

64.113 Problem number 591

$$\int \frac{(a + b \sec(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{4ab \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2(3a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-10i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

64.114 Problem number 592

$$\int \frac{(a + b \sec(c + dx))^2}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{4ab \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(5a^2 + 7b^2) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{12ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5a^2 + 7b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$126i \sqrt{2} ab \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 126i \sqrt{2} ab \text{weierstrass}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

64.115 Problem number 593

$$\int \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(21a^2 + 5b^2) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} + \frac{32ab^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2b^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) (a + b \sec(dx + c)) \sin(dx + c)}{7d} + \frac{2a(5a^2 + 9b^2) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{2a(5a^2 + 9b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(21a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$5 \sqrt{2} (21i a^2 b + 5i b^3) \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-21i a^2 b -$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^3 \sec(dx + c)^4 + 3ab^2 \sec(dx + c)^3 + 3a^2 b \sec(dx + c)^2 + a^3 \sec(dx + c)\right) \sqrt{\sec(dx + c)}, x\right)$$

64.116 Problem number 594

$$\int \sqrt{\sec(c+dx)} (a+b\sec(c+dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8ab^2 \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2b^2 \left(\sec^{\frac{3}{2}}(dx+c)\right) (a+b\sec(dx+c)) \sin(dx+c)}{5d} \\ & + \frac{6b(5a^2+b^2) \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d} \\ & - \frac{6b(5a^2+b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(a^2+b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (i a^3 + i a b^2) \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5\sqrt{2} (-i a^3 - i a b^2) \cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3\right) \sqrt{\sec(dx+c)}, x\right)$$

64.117 Problem number 595

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16ab^2 \sin(dx+c) (\sqrt{\sec(dx+c)})}{3d} + \frac{2b^2(a+b\sec(dx+c)) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3d} \\ & + \frac{2a(a^2-3b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(9a^2+b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-9i a^2 b - i b^3) \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (9i a^2 b + i b^3) \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}{\sqrt{\sec(dx + c)}}, x\right)$$

64.118 Problem number 596

$$\int \frac{(a + b \sec(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(a + b \sec(dx + c)) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} - \frac{2b(a^2 - 3b^2) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & + \frac{2b(3a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(a^2 + 9b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i a^3 - 9i ab^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i a^3 + 9i ab^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

64.119 Problem number 597

$$\int \frac{(a + b \sec(c + dx))^3}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(a + b \sec(dx + c)) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{8a^2b \sin(dx + c)}{5d \sqrt{\sec(dx + c)}} \\ & + \frac{6a(a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i a^2 b + i b^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-i a^2 b - i b^3) \operatorname{weierstrassPInv}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

64.120 Problem number 598

$$\int \frac{(a + b \sec(c + dx))^3}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32a^2b \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a^2(a + b \sec(dx + c)) \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a(5a^2 + 21b^2) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2b(9a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5a^2 + 21b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(5ia^3 + 21iab^2)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-5ia^3 - 21iab^2)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

64.121 Problem number 599

$$\int \frac{(a + b \sec(c + dx))^3}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{40a^2b \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a(7a^2 + 27b^2) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2a^2(a + b \sec(dx + c)) \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2b(15a^2 + 7b^2) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(7a^2 + 27b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(15a^2 + 7b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15\sqrt{2}(15ia^2b + 7ib^3)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15\sqrt{2}(-15ia^2b - 7ib^3)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}{\sec(dx + c)^{\frac{9}{2}}}, x\right)$$

64.122 Problem number 600

$$\int \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8ab(7a^2 + 5b^2) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} + \frac{14b^2(7a^2 + b^2) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{44ab^3 \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{63d} + \frac{2b^2 \left(\sec^{\frac{5}{2}}(dx + c)\right) (a + b \sec(dx + c))^2 \sin(dx + c)}{9d} \\ & + \frac{2(15a^4 + 54a^2b^2 + 7b^4) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15d} \\ & - \frac{2(15a^4 + 54a^2b^2 + 7b^4) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab(7a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$60 \sqrt{2} (7i a^3 b + 5i ab^3) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 60 \sqrt{2} (-7i a^3 b -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^4 \sec(dx + c)^5 + 4ab^3 \sec(dx + c)^4 + 6a^2b^2 \sec(dx + c)^3 + 4a^3b \sec(dx + c)^2 + a^4 \sec(dx + c)\right) \sqrt{\sec(dx + c)}\right)$$

64.123 Problem number 601

$$\int \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(39a^2 + 5b^2) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} + \frac{36ab^3 \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2b^2 \left(\sec^{\frac{3}{2}}(dx + c)\right) (a + b \sec(dx + c))^2 \sin(dx + c)}{7d} \\ & + \frac{8ab(5a^2 + 3b^2) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{8ab(5a^2 + 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^4 + 42a^2b^2 + 5b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(21ia^4 + 42ia^2b^2 + 5ib^4) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^4 \sec(dx + c)^4 + 4ab^3 \sec(dx + c)^3 + 6a^2b^2 \sec(dx + c)^2 + 4a^3b \sec(dx + c) + a^4\right) \sqrt{\sec(dx + c)}, x\right)$$

64.124 Problem number 602

$$\int \frac{(a + b \sec(c + dx))^4}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{28ab^3 \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} + \frac{2b^2(29a^2 + 3b^2) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & + \frac{2b^2(a + b \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & + \frac{2(5a^4 - 30a^2b^2 - 3b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab(3a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$20 \sqrt{2} (3i a^3 b + i a b^3) \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 20 \sqrt{2} (-3i a^3 b -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^4 \sec(dx + c)^4 + 4 ab^3 \sec(dx + c)^3 + 6 a^2 b^2 \sec(dx + c)^2 + 4 a^3 b \sec(dx + c) + a^4}{\sqrt{\sec(dx + c)}}, x\right)$$

64.125 Problem number 603

$$\int \frac{(a + b \sec(c + dx))^4}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b^2(a^2 - b^2) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} + \frac{2a^2(a + b \sec(dx + c))^2 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & -\frac{4ab(a^2 - 6b^2) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & + \frac{8ab(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^4 + 18a^2b^2 + b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i a^4 - 18i a^2 b^2 - i b^4) \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i a^4 + 18i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^4 \sec(dx + c)^4 + 4 ab^3 \sec(dx + c)^3 + 6 a^2 b^2 \sec(dx + c)^2 + 4 a^3 b \sec(dx + c) + a^4}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

64.126 Problem number 604

$$\int \frac{(a + b \sec(c + dx))^4}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(a + b \sec(dx + c))^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{28a^3b \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & - \frac{2b^2(a^2 - 5b^2) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & + \frac{2(3a^4 + 30a^2b^2 - 5b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab(a^2 + 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$20 \sqrt{2} (i a^3 b + 3i a b^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 20 \sqrt{2} (-i a^3 b - 3i a b^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b^4 \sec(dx + c)^4 + 4 ab^3 \sec(dx + c)^3 + 6 a^2 b^2 \sec(dx + c)^2 + 4 a^3 b \sec(dx + c) + a^4}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

64.127 Problem number 605

$$\int \frac{(a + b \sec(c + dx))^4}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{36a^3b \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a^2(a + b \sec(dx + c))^2 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a^2(5a^2 + 39b^2) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{8ab(3a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5a^4 + 42a^2b^2 + 21b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (5i a^4 + 42i a^2 b^2 + 21i b^4) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2} (-5i a^4 - 42i a^2 b^2 - 21i b^4) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^4 \sec(dx + c)^4 + 4 ab^3 \sec(dx + c)^3 + 6 a^2 b^2 \sec(dx + c)^2 + 4 a^3 b \sec(dx + c) + a^4}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

64.128 Problem number 606

$$\int \frac{(a + b \sec(c + dx))^4}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{44a^3 b \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{14a^2(a^2 + 7b^2) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2a^2(a + b \sec(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{8ab(5a^2 + 7b^2) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(7a^4 + 54a^2 b^2 + 15b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8ab(5a^2 + 7b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$60\sqrt{2} (5i a^3 b + 7i ab^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 60\sqrt{2} (-5i a^3 b - 7i ab^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^4 \sec(dx + c)^4 + 4 ab^3 \sec(dx + c)^3 + 6 a^2 b^2 \sec(dx + c)^2 + 4 a^3 b \sec(dx + c) + a^4}{\sec(dx + c)^{\frac{9}{2}}}, x\right)$$

64.129 Problem number 607

$$\int \frac{(a + b \sec(c + dx))^4}{\sec^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{52a^3b \sin(dx + c)}{99d \sec(dx + c)^{\frac{7}{2}}} + \frac{2a^2(9a^2 + 59b^2) \sin(dx + c)}{77d \sec(dx + c)^{\frac{5}{2}}} + \frac{8ab(7a^2 + 9b^2) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2a^2(a + b \sec(dx + c))^2 \sin(dx + c)}{11d \sec(dx + c)^{\frac{9}{2}}} + \frac{2(45a^4 + 330a^2b^2 + 77b^4) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{8ab(7a^2 + 9b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(45a^4 + 330a^2b^2 + 77b^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (45i a^4 + 330i a^2 b^2 + 77i b^4) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \sqrt{2} (-45i a^4 - \dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b^4 \sec(dx + c)^4 + 4ab^3 \sec(dx + c)^3 + 6a^2b^2 \sec(dx + c)^2 + 4a^3b \sec(dx + c) + a^4}{\sec(dx + c)^{\frac{11}{2}}}, x\right)$$

64.130 Problem number 630

$$\int \frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) \sqrt{a + b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}}$$

command

```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{a} b \text{weierstrassPInverse} \left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a \cos(dx+c)+3ia \sin(dx+c)+2b}{3a} \right) + i \sqrt{2} \sqrt{a} b \text{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \sec(dx+c) + a}}{\sqrt{\sec(dx+c)}}, x \right)$$

64.131 Problem number 631

$$\int \frac{\sqrt{a + b \sec(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(a^2 - b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{a}{a+b}} \right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad \sqrt{a+b \sec(dx+c)}} + \frac{2 \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{3d \sqrt{\sec(dx+c)}} + \frac{2b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{a}{a+b}} \right) \sqrt{a+b \sec(dx+c)}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6a^2 \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + 3i \sqrt{2} a^{\frac{3}{2}} b \text{weierstrassZeta} \left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \text{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{\frac{3}{2}}}, x \right)$$

64.132 Problem number 632

$$\int \frac{\sqrt{a + b \sec(c + dx)}}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4b(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}} (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2 \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2b \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15ad \sqrt{\sec(dx + c)}} \\ & + \frac{2(9a^2 - 2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i a^2 b - 4i b^3) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2 b - 8b^3)}{27a^3}, \frac{3a \cos(dx+c) + 3i a \sin(dx+c) + 2b}{3a}\right) + \sqrt{2} (3i a^2 b$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

64.133 Problem number 633

$$\int \frac{\sqrt{a + b \sec(c + dx)}}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(25a^4 - 17a^2b^2 - 8b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a+b \sec(dx+c)}} + \frac{2 \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7d \sec(dx+c)^{\frac{5}{2}}} + \frac{2b \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{35ad \sec(dx+c)^{\frac{3}{2}}} + \frac{2(25a^2 - 4b^2) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{105a^2 d \sqrt{\sec(dx+c)}} + \frac{2b(19a^2 + 8b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

`integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-75i a^4 + 32i a^2 b^2 + 16i b^4) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

64.134 Problem number 637

$$\int \frac{(a+b \sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\frac{2(a^2 - b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a+b \sec(dx+c)}} + \frac{2a \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{3d \sqrt{\sec(dx+c)}} + \frac{8b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

`integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a^2 \sqrt{\frac{a \cos(dx+c)+b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + 12i \sqrt{2} a^{\frac{3}{2}} b \text{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}\right), \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(b \sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

64.135 Problem number 638

$$\int \frac{(a+b \sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(a^2-b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2a \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5d \sec(dx+c)^{\frac{3}{2}}} + \frac{4b \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5d \sqrt{\sec(dx+c)}} \\ & + \frac{2(3a^2+b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{2} (3i a^2 b - i b^3) \sqrt{a} \text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a \cos(dx+c)+3i a \sin(dx+c)+2b}{3a}\right) + 2 \sqrt{2} (-3i a^2 b + i b^3) \sqrt{a} \text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a \cos(dx+c)+3i a \sin(dx+c)+2b}{3a}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(b \sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

64.136 Problem number 639

$$\int \frac{(a + b \sec(c + dx))^{3/2}}{\sec^{7/2}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(25a^4 - 31a^2b^2 + 6b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2a \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7d \sec(dx+c)^{5/2}} + \frac{16b \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{35d \sec(dx+c)^{3/2}} \\ & + \frac{2(25a^2 + 3b^2) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{105ad \sqrt{\sec(dx+c)}} \\ & + \frac{4b(41a^2 - 3b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-75i a^4 + 11i a^2 b^2 - 12i b^4) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2b - 8b^3)}{27a^3}, \frac{3a \cos(dx+c) + 3i a \sin(dx+c) + 2b}{3a}\right) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b \sec(dx+c) + a)^{3/2}}{\sec(dx+c)^{7/2}}, x\right)$$

64.137 Problem number 644

$$\int \frac{(a + b \sec(c + dx))^{5/2}}{\sec^{5/2}(c + dx)} dx$$

Optimal antiderivative

$$\frac{16b(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}} (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a + b \sec(dx + c)}} + \frac{2a^2 \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{5d \sec(dx + c)^{3/2}} + \frac{22ab \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15d \sqrt{\sec(dx + c)}}$$

$$+ \frac{2(9a^2 + 23b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}}$$

command

`integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-33i a^2 b + i b^3) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2 b - 8b^3)}{27a^3}, \frac{3a \cos(dx + c) + 3i a \sin(dx + c) + 2b}{3a}\right) + \sqrt{2} (33i a^2 b - i b^3) \sqrt{a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{5/2}}, x\right)$$

64.138 Problem number 645

$$\int \frac{(a + b \sec(c + dx))^{5/2}}{\sec^{7/2}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5a^4 - 2a^2b^2 - 3b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2a^2 \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7d \sec(dx+c)^{\frac{5}{2}}} + \frac{6ab \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7d \sec(dx+c)^{\frac{3}{2}}} \\ & + \frac{2(5a^2 + 9b^2) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{21d \sqrt{\sec(dx+c)}} \\ & + \frac{2b(29a^2 + 3b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-15i a^4 - 23i a^2 b^2 + 6i b^4) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b^2 \sec(dx+c)^2 + 2ab \sec(dx+c) + a^2) \sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

64.139 Problem number 646

$$\int \frac{(a+b \sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{4b(57a^4 - 62a^2b^2 + 5b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a+b \sec(dx+c)}} \\
 & + \frac{2a^2 \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{9d \sec(dx+c)^{\frac{7}{2}}} + \frac{38ab \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{63d \sec(dx+c)^{\frac{5}{2}}} \\
 & + \frac{2(49a^2 + 75b^2) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{315d \sec(dx+c)^{\frac{3}{2}}} \\
 & + \frac{2b(163a^2 + 5b^2) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{315ad \sqrt{\sec(dx+c)}} \\
 & + \frac{2(147a^4 + 279a^2b^2 - 10b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}
 \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-489i a^4 b + 93i a^2 b^3 - 20i b^5) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(b^2 \sec(dx+c)^2 + 2ab \sec(dx+c) + a^2\right) \sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{\frac{9}{2}}}, x\right)$$

64.140 Problem number 650

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{a+b \sec(dx+c)}}$$

command

`integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}\sqrt{a}\operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right) + i\sqrt{2}\sqrt{a}\operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)}{ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

64.141 Problem number 651

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2b\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad\sqrt{a+b\sec(dx+c)}} + \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{a}{a+b}}\right) \sqrt{a+b\sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad\sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

`integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2i\sqrt{2}\sqrt{a}b\operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right) - 2i\sqrt{2}\sqrt{a}b\operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)}{ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b\sec(dx+c)^2+a\sec(dx+c)}, x\right)$$

64.142 Problem number 652

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2(a^2 + 2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a+b \sec(dx+c)}} + \frac{2 \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{3ad \sqrt{\sec(dx+c)}} - \frac{4b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

`integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a^2 \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - 6i \sqrt{2} a^{\frac{3}{2}} b \operatorname{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}\right), \operatorname{weiers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b \sec(dx+c)^3 + a \sec(dx+c)^2}, x\right)$$

64.143 Problem number 653

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(7a^2 + 8b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2 \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5ad \sec(dx+c)^{\frac{3}{2}}} - \frac{8b \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{15a^2 d \sqrt{\sec(dx+c)}} \\ & + \frac{2(9a^2 + 8b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output
```

$$4\sqrt{2}(-3ia^2b - 4ib^3)\sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right) + 4\sqrt{2}(\dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b \sec(dx+c)^4 + a \sec(dx+c)^3}, x\right)$$

64.144 Problem number 656

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \sin(dx+c) (\sqrt{\sec(dx+c)})}{(a^2 - b^2) d \sqrt{a+b \sec(dx+c)}} \\ & - \frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2 - b^2) d \sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a^2 \sqrt{\frac{a \cos(dx+c)+b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \sqrt{2} (-i a b \cos(dx+c) - i b^2) \sqrt{a} \text{weierstrassPInverse}\left(-\frac{4}{3} \left(\frac{a \cos(dx+c)+b}{\cos(dx+c)}\right)^3, \frac{a \cos(dx+c)+b}{\cos(dx+c)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)+a} \sec(dx+c)^{\frac{3}{2}}}{b^2 \sec(dx+c)^2 + 2 a b \sec(dx+c) + a^2}, x\right)$$

64.145 Problem number 657

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b \sin(dx+c) (\sqrt{\sec(dx+c)})}{(a^2 - b^2) d \sqrt{a + b \sec(dx+c)}} \\ & + \frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a d \sqrt{a + b \sec(dx+c)}} \\ & + \frac{2b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a (a^2 - b^2) d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a^2 b \sqrt{\frac{a \cos(dx+c)+b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2} (3i a^2 b - 2i b^3 + (3i a^3 - 2i a b^2) \cos(dx+c)) \sqrt{a} \text{weierstrassPInverse}\left(-\frac{4}{3} \left(\frac{a \cos(dx+c)+b}{\cos(dx+c)}\right)^3, \frac{a \cos(dx+c)+b}{\cos(dx+c)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)+a} \sqrt{\sec(dx+c)}}{b^2 \sec(dx+c)^2 + 2 a b \sec(dx+c) + a^2}, x\right)$$

64.146 Problem number 658

$$\int \frac{1}{\sqrt{\sec(c+dx)} (a+b\sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx+c) (\sqrt{\sec(dx+c)})}{a(a^2-b^2) d \sqrt{a+b\sec(dx+c)}} \\ - \frac{4b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a+b\sec(dx+c)}} \\ + \frac{2(a^2-2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b\sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2-b^2) d \sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

`integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a^2 b^2 \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \sqrt{2} (-5i a^2 b^2 + 4i b^4 + (-5i a^3 b + 4i a b^3) \cos(dx+c)) \sqrt{\sec(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(dx+c)+a} \sqrt{\sec(dx+c)}}{b^2 \sec^3(dx+c) + 2ab\sec^2(dx+c) + a^2 \sec(dx+c)}, x\right)$$

64.147 Problem number 659

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx+c)}{a(a^2-b^2)d\sqrt{\sec(dx+c)}\sqrt{a+b\sec(dx+c)}} + \frac{2(a^2+8b^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{2}\sqrt{\frac{a}{a+b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{a+b}}(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3d\sqrt{a+b\sec(dx+c)}} + \frac{2(a^2-4b^2)\sin(dx+c)\sqrt{a+b\sec(dx+c)}}{3a^2(a^2-b^2)d\sqrt{\sec(dx+c)}} - \frac{2b(5a^2-8b^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{2}\sqrt{\frac{a}{a+b}}\right)\sqrt{a+b\sec(dx+c)}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3(a^2-b^2)d\sqrt{\frac{b+a\cos(dx+c)}{a+b}}\sqrt{\sec(dx+c)}}$$

command

```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (3i a^4 b + 16i a^2 b^3 - 16i b^5 + (3i a^5 + 16i a^3 b^2 - 16i a b^4) \cos(dx+c)) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}\right),$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^2\sec(dx+c)^4+2ab\sec(dx+c)^3+a^2\sec(dx+c)^2}, x\right)$$

64.148 Problem number 660

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx+c)}{a(a^2-b^2)d \sec(dx+c)^{\frac{3}{2}} \sqrt{a+b \sec(dx+c)}} + \frac{8b(a^2+4b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{a+b \sec(dx+c)}} + \frac{2(a^2-6b^2) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5a^2(a^2-b^2)d \sec(dx+c)^{\frac{3}{2}}} - \frac{2b(3a^2-8b^2) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5a^3(a^2-b^2)d \sqrt{\sec(dx+c)}} + \frac{2(3a^4+8a^2b^2-16b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2-b^2) d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

`integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-9i a^4 b^2 - 28i a^2 b^4 + 32i b^6 + (-9i a^5 b - 28i a^3 b^3 + 32i a b^5) \cos(dx+c)) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-3a^2)}{3a^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c)+a} \sqrt{\sec(dx+c)}}{b^2 \sec(dx+c)^5 + 2ab \sec(dx+c)^4 + a^2 \sec(dx+c)^3}, x\right)$$

64.149 Problem number 663

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \sin(dx+c) (\sqrt{\sec(dx+c)})}{3b(a^2-b^2)d(a+b \sec(dx+c))^{\frac{3}{2}}} + \frac{2a(a^2-5b^2) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3b(a^2-b^2)^2 d \sqrt{a+b \sec(dx+c)}} + \frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2-b^2) d \sqrt{a+b \sec(dx+c)}} + \frac{8b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2-b^2)^2 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-3i a^2 b^2 - i b^4 + (-3i a^4 - i a^2 b^2) \cos(dx + c)^2 - 2(3i a^3 b + i a b^3) \cos(dx + c) \right) \sqrt{a} \text{weierstrassPInverse} \left(-\frac{4}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \sec(dx + c) + a} \sec(dx + c)^{\frac{5}{2}}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2 b \sec(dx + c) + a^3}, x \right)$$

64.150 Problem number 664

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a \sin(dx + c) (\sqrt{\sec(dx + c)})}{3(a^2 - b^2) d(a + b \sec(dx + c))^{\frac{3}{2}}} + \frac{4(a^2 + b^2) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3(a^2 - b^2)^2 d \sqrt{a + b \sec(dx + c)}}$$

$$\frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{a}{a + b}} \right) \sqrt{\frac{b + a \cos(dx + c)}{a + b}} (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a (a^2 - b^2) d \sqrt{a + b \sec(dx + c)}}$$

$$\frac{2(3a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{a}{a + b}} \right) \sqrt{a + b \sec(dx + c)}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}}$$

command

```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\sqrt{2} \left(-3i a^2 b^3 + i b^5 + (-3i a^4 b + i a^2 b^3) \cos(dx + c)^2 + 2(-3i a^3 b^2 + i a b^4) \cos(dx + c) \right) \sqrt{a} \text{weierstrassPInverse} \left(-\frac{4}{3} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)+a} \sec(dx+c)^{\frac{3}{2}}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3}, x\right)$$

64.151 Problem number 665

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b \sin(dx+c) (\sqrt{\sec(dx+c)})}{3(a^2-b^2)d(a+b \sec(dx+c))^{\frac{3}{2}}} - \frac{2b(5a^2-b^2) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3a(a^2-b^2)^2 d \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2(3a^2-2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2-b^2) d \sqrt{a+b \sec(dx+c)}} \\ & + \frac{4b(3a^2-b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2-b^2)^2 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-9i a^4 b^2 + 9i a^2 b^4 - 4i b^6 + (-9i a^6 + 9i a^4 b^2 - 4i a^2 b^4) \cos(dx+c)^2 - 2(9i a^5 b - 9i a^3 b^3 + 4i a b^5) \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)+a} \sqrt{\sec(dx+c)}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3}, x\right)$$

64.152 Problem number 666

$$\int \frac{1}{\sqrt{\sec(c+dx)} (a+b\sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx+c) (\sqrt{\sec(dx+c)})}{3a(a^2-b^2)d(a+b\sec(dx+c))^{\frac{3}{2}}} + \frac{8b^2(2a^2-b^2) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3a^2(a^2-b^2)^2 d \sqrt{a+b\sec(dx+c)}}$$

$$- \frac{2b(9a^2-8b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2-b^2) d \sqrt{a+b\sec(dx+c)}}$$

$$+ \frac{2(3a^4-15a^2b^2+8b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b\sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2-b^2)^2 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

`integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4\sqrt{2} \left(-6i a^4 b^3 + 9i a^2 b^5 - 4i b^7 + (-6i a^6 b + 9i a^4 b^3 - 4i a^2 b^5) \cos(dx+c)^2 + 2(-6i a^5 b^2 + 9i a^3 b^4 - 4i a b^6) \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(dx+c)+a} \sqrt{\sec(dx+c)}}{b^3 \sec(dx+c)^4 + 3ab^2 \sec(dx+c)^3 + 3a^2b \sec(dx+c)^2 + a^3 \sec(dx+c)}, x\right)$$

64.153 Problem number 667

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{2b^2 \sin(dx+c)}{3a(a^2-b^2)d(a+b\sec(dx+c))^{\frac{3}{2}}\sqrt{\sec(dx+c)}} \\
 & + \frac{4b^2(5a^2-3b^2)\sin(dx+c)}{3a^2(a^2-b^2)^2d\sqrt{\sec(dx+c)}\sqrt{a+b\sec(dx+c)}} \\
 & + \frac{2(a^4+16a^2b^2-16b^4)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{a}{a+b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{a+b}}(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^4(a^2-b^2)d\sqrt{a+b\sec(dx+c)}} \\
 & + \frac{2(a^4-13a^2b^2+8b^4)\sin(dx+c)\sqrt{a+b\sec(dx+c)}}{3a^3(a^2-b^2)^2d\sqrt{\sec(dx+c)}} \\
 & - \frac{8b(2a^4-7a^2b^2+4b^4)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{a}{a+b}}\right)\sqrt{a+b\sec(dx+c)}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^4(a^2-b^2)^2d\sqrt{\frac{b+a\cos(dx+c)}{a+b}}\sqrt{\sec(dx+c)}}
 \end{aligned}$$

command

`integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-3i a^6 b^2 - 37i a^4 b^4 + 68i a^2 b^6 - 32i b^8 + (-3i a^8 - 37i a^6 b^2 + 68i a^4 b^4 - 32i a^2 b^6) \cos(dx+c)^2 - 2(3i a^7 b + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^5+3ab^2\sec(dx+c)^4+3a^2b\sec^2(dx+c)^3+a^3\sec(dx+c)^2},x\right)$$

64.154 Problem number 668

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{\frac{5}{2}}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b^2 \sin(dx+c)}{3a(a^2-b^2)d \sec(dx+c)^{\frac{3}{2}}(a+b \sec(dx+c))^{\frac{3}{2}}} \\
& + \frac{8b^2(3a^2-2b^2)\sin(dx+c)}{3a^2(a^2-b^2)^2 d \sec(dx+c)^{\frac{3}{2}} \sqrt{a+b \sec(dx+c)}} \\
& - \frac{2b(17a^4+116a^2b^2-128b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^5 (a^2-b^2) d \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2(3a^4-71a^2b^2+48b^4)\sin(dx+c) \sqrt{a+b \sec(dx+c)}}{15a^3(a^2-b^2)^2 d \sec(dx+c)^{\frac{3}{2}}} \\
& - \frac{4b(7a^4-49a^2b^2+32b^4)\sin(dx+c) \sqrt{a+b \sec(dx+c)}}{15a^4(a^2-b^2)^2 d \sqrt{\sec(dx+c)}} \\
& + \frac{2(9a^6+55a^4b^2-212a^2b^4+128b^6) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^5 (a^2-b^2)^2 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}
\end{aligned}$$

command

```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2}\left(-21ia^6b^3-121ia^4b^5+260ia^2b^7-128ib^9+(-21ia^8b-121ia^6b^3+260ia^4b^5-128ia^2b^7)\cos(dx+c)\right)^2}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c)+a} \sqrt{\sec(dx+c)}}{b^3 \sec(dx+c)^6 + 3ab^2 \sec(dx+c)^5 + 3a^2b \sec(dx+c)^4 + a^3 \sec(dx+c)^3}, x\right)$$

64.155 Problem number 669

$$\int \frac{1}{\sqrt{\sec(c+dx)} \sqrt{2+3\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{3\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{5}}{5}\right) \sqrt{3+2\cos(dx+c)} (\sqrt{\sec(dx+c)}) \sqrt{5}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{2+3\sec(dx+c)}} + \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{5}}{5}\right) \sqrt{5} \sqrt{2+3\sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{3+2\cos(dx+c)} \sqrt{\sec(dx+c)}}$$

command

`integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$i \operatorname{weierstrassPInverse}(8, -4, \cos(dx+c) + i \sin(dx+c) + 1) - i \operatorname{weierstrassPInverse}(8, -4, \cos(dx+c) - i \sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{3\sec(dx+c)+2} \sqrt{\sec(dx+c)}}{3\sec(dx+c)^2+2\sec(dx+c)}, x\right)$$

64.156 Problem number 670

$$\int \frac{1}{\sqrt{\sec(c+dx)} \sqrt{-2+3\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{3\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), 2i\right) \sqrt{3-2\cos(dx+c)} (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{-2+3\sec(dx+c)}} - \frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), 2i\right) \sqrt{-2+3\sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{3-2\cos(dx+c)} \sqrt{\sec(dx+c)}}$$

command

`integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

- weierstrassPInverse(8, 4, cos(dx + c) + i sin(dx + c) - 1) + weierstrassPInverse(8, 4, cos(dx + c) - i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{3 \sec(dx + c) - 2} \sqrt{\sec(dx + c)}}{3 \sec(dx + c)^2 - 2 \sec(dx + c)}, x\right)$$

64.157 Problem number 671

$$\int \frac{1}{\sqrt{2 - 3 \sec(c + dx)} \sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), 2i\right) \sqrt{2 - 3 \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{3 - 2 \cos(dx + c)} \sqrt{\sec(dx + c)}} + \frac{3 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), 2i\right) \sqrt{3 - 2 \cos(dx + c)} (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{2 - 3 \sec(dx + c)}}$$

command

`integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

-i weierstrassPInverse(8, 4, cos(dx + c) + i sin(dx + c) - 1) + i weierstrassPInverse(8, 4, cos(dx + c) - i sin(dx + c))

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-3 \sec(dx + c) + 2} \sqrt{\sec(dx + c)}}{3 \sec(dx + c)^2 - 2 \sec(dx + c)}, x\right)$$

64.158 Problem number 672

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)} \sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{5}}{5}\right) \sqrt{5} \sqrt{-2-3\sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{3+2\cos(dx+c)} \sqrt{\sec(dx+c)}} + \frac{3\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{5}}{5}\right) \sqrt{3+2\cos(dx+c)} (\sqrt{\sec(dx+c)}) \sqrt{5}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{-2-3\sec(dx+c)}}$$

command

`integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

weierstrassPInverse(8, -4, cos(dx+c) + i sin(dx+c) + 1) + weierstrassPInverse(8, -4, cos(dx+c) - i sin(dx+c) + 1)

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-3\sec(dx+c)-2} \sqrt{\sec(dx+c)}}{3\sec(dx+c)^2 + 2\sec(dx+c)}, x\right)$$

64.159 Problem number 673

$$\int \frac{1}{\sqrt{\sec(c+dx)} \sqrt{3+2\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{4\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{\sqrt{30}}{5}\right) \sqrt{2+3\cos(dx+c)} (\sqrt{\sec(dx+c)}) \sqrt{5}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{3+2\sec(dx+c)}} + \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{\sqrt{30}}{5}\right) \sqrt{5} \sqrt{3+2\sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{2+3\cos(dx+c)} \sqrt{\sec(dx+c)}}$$

command

`integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$4i \sqrt{6}$ weierstrassPInverse $\left(-\frac{44}{27}, \frac{784}{729}, \cos(dx+c) + i \sin(dx+c) + \frac{4}{9}\right) - 4i \sqrt{6}$ weierstrassPInverse $\left(-\frac{44}{27}, \frac{784}{729}, \cos(dx+c) + i \sin(dx+c) - \frac{4}{9}\right)$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{2 \sec(dx+c)+3} \sqrt{\sec(dx+c)}}{2 \sec(dx+c)^2 + 3 \sec(dx+c)}, x\right)$$

64.160 Problem number 674

$$\int \frac{1}{\sqrt{3-2 \sec(c+dx)} \sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{6}\right) \sqrt{3-2 \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{-2+3 \cos(dx+c)} \sqrt{\sec(dx+c)}} + \frac{4 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{6}\right) \sqrt{-2+3 \cos(dx+c)} (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{3-2 \sec(dx+c)}}$$

command

`integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-4i \sqrt{6}$ weierstrassPInverse $\left(-\frac{44}{27}, -\frac{784}{729}, \cos(dx+c) + i \sin(dx+c) - \frac{4}{9}\right) + 4i \sqrt{6}$ weierstrassPInverse $\left(-\frac{44}{27}, -\frac{784}{729}, \cos(dx+c) + i \sin(dx+c) + \frac{4}{9}\right)$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{-2 \sec(dx+c)+3} \sqrt{\sec(dx+c)}}{2 \sec(dx+c)^2 - 3 \sec(dx+c)}, x\right)$$

64.161 Problem number 675

$$\int \frac{1}{\sqrt{\sec(c+dx)} \sqrt{-3+2\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{4\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{\sqrt{30}}{5}\right) \sqrt{2-3\cos(dx+c)} (\sqrt{\sec(dx+c)}) \sqrt{5}}{15 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{-3+2\sec(dx+c)}} + \frac{2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{\sqrt{30}}{5}\right) \sqrt{5} \sqrt{-3+2\sec(dx+c)}}{3 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{2-3\cos(dx+c)} \sqrt{\sec(dx+c)}}$$

command

`integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4\sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, -\frac{784}{729}, \cos(dx+c) + i \sin(dx+c) - \frac{4}{9}\right) + 4\sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, -\frac{784}{729}, \cos(dx+c) + i \sin(dx+c) - \frac{4}{9}\right)}{15 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{-3+2\sec(dx+c)} \sqrt{\sec(dx+c)}} + \frac{2\sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, -\frac{784}{729}, \cos(dx+c) + i \sin(dx+c) - \frac{4}{9}\right) \sqrt{5} \sqrt{-3+2\sec(dx+c)}}{3 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{2-3\cos(dx+c)} \sqrt{\sec(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{2\sec(dx+c)-3} \sqrt{\sec(dx+c)}}{2\sec(dx+c)^2 - 3\sec(dx+c)}, x\right)$$

64.162 Problem number 676

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)} \sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{6}\right) \sqrt{-3-2\sec(dx+c)}}{3 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{-2-3\cos(dx+c)} \sqrt{\sec(dx+c)}} + \frac{4\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{6}\right) \sqrt{-2-3\cos(dx+c)} (\sqrt{\sec(dx+c)})}{3 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{-3-2\sec(dx+c)}}$$

command

`integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4\sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, \frac{784}{729}, \cos(dx+c) + i \sin(dx+c) + \frac{4}{9}\right) + 4\sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, \frac{784}{729}, \cos(dx+c) - i \sin(dx+c) + \frac{4}{9}\right)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2 \sec(dx+c) - 3} \sqrt{\sec(dx+c)}}{2 \sec(dx+c)^2 + 3 \sec(dx+c)}, x\right)$$

64.163 Problem number 677

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{5}}{5}\right) \sqrt{3+2\cos(dx+c)} (\sqrt{\sec(dx+c)} \sqrt{5})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{2+3\sec(dx+c)}}$$

command

`integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \operatorname{weierstrassPInverse}(8, -4, \cos(dx+c) + i \sin(dx+c) + 1) + i \operatorname{weierstrassPInverse}(8, -4, \cos(dx+c) - i \sin(dx+c) + 1)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sec(dx+c)}}{\sqrt{3 \sec(dx+c) + 2}}, x\right)$$

64.164 Problem number 678

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), 2i\right) \sqrt{3-2\cos(dx+c)} (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{-2+3\sec(dx+c)}}$$

command

```
integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\operatorname{weierstrassPInverse}(8, 4, \cos(dx+c) + i \sin(dx+c) - 1) + \operatorname{weierstrassPInverse}(8, 4, \cos(dx+c) - i \sin(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sec(dx+c)}}{\sqrt{3\sec(dx+c)-2}}, x\right)$$

64.165 Problem number 679

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), 2i\right) \sqrt{3-2\cos(dx+c)} (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{2-3\sec(dx+c)}}$$

command

```
integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \operatorname{weierstrassPInverse}(8, 4, \cos(dx+c) + i \sin(dx+c) - 1) + i \operatorname{weierstrassPInverse}(8, 4, \cos(dx+c) - i \sin(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-3\sec(dx+c)+2} \sqrt{\sec(dx+c)}}{3\sec(dx+c)-2}, x\right)$$

64.166 Problem number 680

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{5}}{5}\right) \sqrt{3+2\cos(dx+c)} (\sqrt{\sec(dx+c)}) \sqrt{5}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{-2-3\sec(dx+c)}}$$

command

`integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\operatorname{weierstrassPInverse}(8, -4, \cos(dx+c) + i \sin(dx+c) + 1) + \operatorname{weierstrassPInverse}(8, -4, \cos(dx+c) - i \sin(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}{3\sec(dx+c)+2}, x\right)$$

64.167 Problem number 681

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{\sqrt{30}}{5}\right) \sqrt{2+3\cos(dx+c)} (\sqrt{\sec(dx+c)}) \sqrt{5}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{3+2\sec(dx+c)}}$$

command

`integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, \frac{784}{729}, \cos(dx+c) + i \sin(dx+c) + \frac{4}{9}\right) + i\sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, \frac{784}{729}, \cos(dx+c) - i \sin(dx+c) + \frac{4}{9}\right)}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sec(dx+c)}}{\sqrt{2\sec(dx+c)+3}}, x\right)$$

64.168 Problem number 682

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{6}\right) \sqrt{-2+3\cos(dx+c)} (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{3-2\sec(dx+c)}}$$

command

`integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, -\frac{784}{729}, \cos(dx+c) + i\sin(dx+c) - \frac{4}{9}\right) + i\sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, -\frac{784}{729}, \cos(dx+c) + i\sin(dx+c) - \frac{4}{9}\right)}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}{2\sec(dx+c)-3}, x\right)$$

64.169 Problem number 683

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{\sqrt{30}}{5}\right) \sqrt{2-3\cos(dx+c)} (\sqrt{\sec(dx+c)}) \sqrt{5}}{5\sin\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{-3+2\sec(dx+c)}}$$

command

`integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, -\frac{784}{729}, \cos(dx+c) + i\sin(dx+c) - \frac{4}{9}\right) + \sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, -\frac{784}{729}, \cos(dx+c) + i\sin(dx+c) - \frac{4}{9}\right)}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sec(dx+c)}}{\sqrt{2\sec(dx+c)-3}}, x\right)$$

64.170 Problem number 684

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{6}\right) \sqrt{-2-3\cos(dx+c)} (\sqrt{\sec(dx+c)})}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right) d\sqrt{-3-2\sec(dx+c)}}$$

command

```
integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, \frac{784}{729}, \cos(dx+c) + i \sin(dx+c) + \frac{4}{9}\right) + \sqrt{6} \operatorname{weierstrassPInverse}\left(-\frac{44}{27}, \frac{784}{729}, \cos(dx+c) + i \sin(dx+c) - \frac{4}{9}\right)}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{-2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}{2\sec(dx+c)+3}, x\right)$$

64.171 Problem number 794

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{14a\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{2}\right)}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{10b\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{14a\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{45d} + \frac{2b\left(\cos^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{7d} \\ & + \frac{2a\left(\cos^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{9d} + \frac{10b\sin(dx+c)(\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(35 a \cos(dx + c)^3 + 45 b \cos(dx + c)^2 + 49 a \cos(dx + c) + 75 b \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 75i \sqrt{2} b \text{weierstrassPInverse}(-4, 0, c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b \cos(dx + c)^4 \sec(dx + c) + a \cos(dx + c)^4\right) \sqrt{\cos(dx + c)}, x\right)$$

64.172 Problem number 795

$$\int \cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6b \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{10a \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2b \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} \\ & + \frac{2a \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{10a \sin(dx + c) \left(\sqrt{\cos(dx + c)}\right)}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15 a \cos(dx + c)^2 + 21 b \cos(dx + c) + 25 a \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 25i \sqrt{2} a \text{weierstrassPInverse}(-4, 0, c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b \cos(dx + c)^3 \sec(dx + c) + a \cos(dx + c)^3\right) \sqrt{\cos(dx + c)}, x\right)$$

64.173 Problem number 796

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2b \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3a \cos(dx+c) + 5b) \sqrt{\cos(dx+c)} \sin(dx+c) - 5i \sqrt{2} b \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b \cos(dx+c)^2 \sec(dx+c) + a \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}, x\right)$$

64.174 Problem number 797

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$2a\sqrt{\cos(dx+c)}\sin(dx+c) - i\sqrt{2}\text{aweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}\text{aweierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$\text{integral}\left(\left(b\cos(dx+c)\sec(dx+c) + a\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$

64.175 Problem number 798

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))dx$$

Optimal antiderivative

$$\frac{2a\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} + \frac{2b\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d}$$

command

`integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}\text{bweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}\text{bweierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$\text{integral}\left(\left(b\sec(dx+c) + a\right)\sqrt{\cos(dx+c)},x\right)$

64.176 Problem number 799

$$\int \frac{a + b \sec(c + dx)}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2b \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} a \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \cos(dx + c) \operatorname{weierstrassPInv}$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b \sec(dx + c) + a}{\sqrt{\cos(dx + c)}}, x\right)$$

64.177 Problem number 800

$$\int \frac{a + b \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2b \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} b \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} b \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b \sec(dx + c) + a}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

64.178 Problem number 801

$$\int \frac{a + b \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2a \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{6b \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} a \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} a \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b \sec(dx + c) + a}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

64.179 Problem number 802

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7a^2+9b^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{20ab\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(7a^2+9b^2)\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{45d} + \frac{4ab\left(\cos^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{7d} \\ & + \frac{2a^2\left(\cos^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{9d} + \frac{20ab\sin(dx+c)\left(\sqrt{\cos(dx+c)}\right)}{21d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-150i\sqrt{2}\operatorname{abweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+150i\sqrt{2}\operatorname{abweierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2\cos(dx+c)^4\sec(dx+c)^2+2ab\cos(dx+c)^4\sec(dx+c)+a^2\cos(dx+c)^4\right)\sqrt{\cos(dx+c)},x\right)$$

64.180 Problem number 803

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{12ab\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(5a^2+7b^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{4ab\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{5d} + \frac{2a^2\left(\cos^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{7d} \\ & + \frac{2(5a^2+7b^2)\sin(dx+c)\left(\sqrt{\cos(dx+c)}\right)}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$126i \sqrt{2} ab \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 126i \sqrt{2} ab \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)), x$

Fricas 1.3.7 via sagemath 9.3 output

$\text{integral}\left(\left(b^2 \cos(dx + c)^3 \sec(dx + c)^2 + 2ab \cos(dx + c)^3 \sec(dx + c) + a^2 \cos(dx + c)^3\right) \sqrt{\cos(dx + c)}, x\right)$

64.181 Problem number 804

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2 \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{4ab \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-10i \sqrt{2} ab \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} ab \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)), x$

Fricas 1.3.7 via sagemath 9.3 output

$\text{integral}\left(\left(b^2 \cos(dx + c)^2 \sec(dx + c)^2 + 2ab \cos(dx + c)^2 \sec(dx + c) + a^2 \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}, x\right)$

64.182 Problem number 805

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^2 dx$$

Optimal antiderivative

$$\frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2(a^2 + 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2a^2 \sqrt{\cos(dx + c)} \sin(dx + c) + 6i \sqrt{2} ab \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)), x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx + c) \sec(dx + c)^2 + 2ab \cos(dx + c) \sec(dx + c) + a^2 \cos(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

64.183 Problem number 806

$$\int \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^2 dx$$

Optimal antiderivative

$$\frac{2(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2b^2 \sin(dx + c)}{d \sqrt{\cos(dx + c)}}$$

command

`integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-2i \sqrt{2} ab \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2i \sqrt{2} ab \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2\right) \sqrt{\cos(dx + c)}, x\right)$$

64.184 Problem number 807

$$\int \frac{(a + b \sec(c + dx))^2}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b^2 \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{4ab \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-6i \sqrt{2} ab \cos(dx + c)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 6i \sqrt{2} ab \cos(dx + c)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}{\sqrt{\cos(dx + c)}}, x\right)$$

64.185 Problem number 808

$$\int \frac{(a + b \sec(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(5a^2 + 3b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b^2 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{4ab \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(5a^2 + 3b^2) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-10i \sqrt{2} ab \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} ab \cos(dx + c)^3 \operatorname{weierst}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

64.186 Problem number 809

$$\int \frac{(a + b \sec(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{12ab \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2b^2 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{4ab \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2(7a^2 + 5b^2) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} + \frac{12ab \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-126i \sqrt{2} ab \cos(dx+c)^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 126i \sqrt{2} ab \cos(dx+c)^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))) + 126i \sqrt{2} ab \cos(dx+c)^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 126i \sqrt{2} ab \cos(dx+c)^4 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^2 \sec(dx+c)^2 + 2ab \sec(dx+c) + a^2}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

64.187 Problem number 810

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b \sec(c+dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(7a^2 + 27b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(15a^2 + 7b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7a^2 + 27b^2) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} + \frac{40a^2b \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{63d} \\ & + \frac{2a^2 \left(\cos^{\frac{7}{2}}(dx+c)\right) (a+b \sec(dx+c)) \sin(dx+c)}{9d} \\ & + \frac{2b(15a^2 + 7b^2) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(35 a^3 \cos(dx+c)^3 + 135 a^2 b \cos(dx+c)^2 + 225 a^2 b + 105 b^3 + 7 (7 a^3 + 27 ab^2) \cos(dx+c) \right) \sqrt{\cos(dx+c)} \sin(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^3 \cos(dx+c)^4 \sec(dx+c)^3 + 3ab^2 \cos(dx+c)^4 \sec(dx+c)^2 + 3a^2b \cos(dx+c)^4 \sec(dx+c) + a^3 \cos(dx+c)^4\right), x\right)$$

64.188 Problem number 811

$$\int \cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(9a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5a^2 + 21b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{32a^2b \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{35d} + \frac{2a^2 \left(\cos^{\frac{5}{2}}(dx + c)\right) (a + b \sec(dx + c)) \sin(dx + c)}{7d} \\ & + \frac{2a(5a^2 + 21b^2) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15 a^3 \cos(dx + c)^2 + 63 a^2 b \cos(dx + c) + 25 a^3 + 105 ab^2 \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \sqrt{2} (5i a^3 + 21i ab^2) \sqrt{\cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^3 \cos(dx + c)^3 \sec(dx + c)^3 + 3 ab^2 \cos(dx + c)^3 \sec(dx + c)^2 + 3 a^2 b \cos(dx + c)^3 \sec(dx + c) + a^3 \cos(dx + c)\right) dx\right)$$

64.189 Problem number 812

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6a(a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2 \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + b \sec(dx + c)) \sin(dx + c)}{5d} + \frac{8a^2 b \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 (a^3 \cos(dx + c) + 5 a^2 b) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \sqrt{2} (i a^2 b + i b^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c)) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^3 \cos(dx + c)^2 \sec(dx + c)^3 + 3 ab^2 \cos(dx + c)^2 \sec(dx + c)^2 + 3 a^2 b \cos(dx + c)^2 \sec(dx + c) + a^3 \cos(dx + c)\right) dx\right)$$

64.190 Problem number 813

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(3a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(a^2 + 9b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & - \frac{2b(a^2 - 3b^2) \sin(dx + c)}{3d \sqrt{\cos(dx + c)}} + \frac{2a^2(a + b \sec(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i a^3 - 9i ab^2) \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i a^3 + 9i ab^2) \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^3 \cos(dx + c) \sec(dx + c)^3 + 3 ab^2 \cos(dx + c) \sec(dx + c)^2 + 3 a^2 b \cos(dx + c) \sec(dx + c) + a^3 \cos(dx + c)\right) dx\right)$$

64.191 Problem number 814

$$\int \sqrt{\cos(c+dx)} (a+b \sec(c+dx))^3 dx$$

Optimal antiderivative

$$\frac{2a(a^2 - 3b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2b(9a^2 + b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{16a^2 b^2 \sin(dx+c)}{3d \sqrt{\cos(dx+c)}} + \frac{2b^2(a+b \sec(dx+c)) \sin(dx+c)}{3d \sqrt{\cos(dx+c)}}$$

command

`integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-9i a^2 b - i b^3) \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2} (9i a^2 b + i b^3) \cos(dx+c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2 b \sec(dx+c) + a^3\right) \sqrt{\cos(dx+c)}, x\right)$$

64.192 Problem number 815

$$\int \frac{(a+b \sec(c+dx))^3}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\frac{6b(5a^2 + b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a(a^2 + b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{8a^2 b^2 \sin(dx+c)}{5d \cos(dx+c)^{\frac{3}{2}}} + \frac{2b^2(a+b \sec(dx+c)) \sin(dx+c)}{5d \cos(dx+c)^{\frac{3}{2}}} + \frac{6b(5a^2 + b^2) \sin(dx+c)}{5d \sqrt{\cos(dx+c)}}$$

command

```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (i a^3 + i a b^2) \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2} (-i a^3 - i a b^2) c$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}{\sqrt{\cos(dx + c)}}, x\right)$$

64.193 Problem number 816

$$\int \frac{(a + b \sec(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5a^2 + 9b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(21a^2 + 5b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{32a^2 b^2 \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{2b(21a^2 + 5b^2) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2b^2(a + b \sec(dx + c)) \sin(dx + c)}{7d \cos(dx + c)^{\frac{5}{2}}} + \frac{2a(5a^2 + 9b^2) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (21i a^2 b + 5i b^3) \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2} (-21i a^2 b -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

64.194 Problem number 837

$$\int \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4b(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2\left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{5d} \\ & + \frac{2b \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{15ad} \\ & + \frac{2(9a^2 - 2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(3a^3 \cos(dx + c) + a^2b) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + \sqrt{2}(-3ia^2b - 4ib^3) \sqrt{a} \operatorname{weierstrassF}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c) + a} \cos(dx + c)^{\frac{5}{2}}, x\right)$$

64.195 Problem number 838

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2 \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{3d} + \frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a^2 \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + 3i \sqrt{2} a^{\frac{3}{2}} b \operatorname{weierstrassZeta}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2b - 8b^3)}{27a^3}\right), \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c) + a} \cos(dx + c)^{\frac{3}{2}}, x\right)$$

64.196 Problem number 839

$$\int \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{a} b \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2b - 8b^3)}{27a^3}, \frac{3a \cos(dx + c) + 3i a \sin(dx + c) + 2b}{3a}\right) + i \sqrt{2} \sqrt{a} b \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}, x\right)$$

64.197 Problem number 842

$$\int \cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(25a^4 - 31a^2b^2 + 6b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\ & + \frac{16b \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{35d} \\ & + \frac{2a \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7d} \\ & + \frac{2(25a^2 + 3b^2) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{105ad} \\ & + \frac{4b(41a^2 - 3b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(15 a^4 \cos(dx+c)^2 + 24 a^3 b \cos(dx+c) + 25 a^4 + 3 a^2 b^2 \right) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b \cos(dx+c)^3 \sec(dx+c) + a \cos(dx+c)^3\right) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}, x\right)$$

64.198 Problem number 843

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2a\left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{5d} \\ & + \frac{4b \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{5d} \\ & + \frac{2(3a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b + a \cos(dx + c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(a^3 \cos(dx + c) + 2a^2b) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) - 2\sqrt{2} (3ia^2b - ib^3) \sqrt{a} \operatorname{weierstrassPI}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b \cos(dx + c)^2 \sec(dx + c) + a \cos(dx + c)^2\right) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}, x\right)$$

64.199 Problem number 844

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2a \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{3d} \\
& + \frac{8b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a^2 \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + 12i \sqrt{2} a^{\frac{3}{2}} b \operatorname{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}\right), \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((b \cos(dx+c) \sec(dx+c) + a \cos(dx+c)) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}, x\right)$$

64.200 Problem number 848

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b \sec(c+dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{4b(57a^4 - 62a^2b^2 + 5b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
 & + \frac{2(49a^2 + 75b^2) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{315d} \\
 & + \frac{38ab \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{63d} \\
 & + \frac{2a^2 \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{9d} \\
 & + \frac{2b(163a^2 + 5b^2) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{315ad} \\
 & + \frac{2(147a^4 + 279a^2b^2 - 10b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}
 \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(35 a^5 \cos(dx+c)^3 + 95 a^4 b \cos(dx+c)^2 + 163 a^4 b + 5 a^2 b^3 + (49 a^5 + 75 a^3 b^2) \cos(dx+c) \right) \sqrt{\frac{a \cos(dx+c) + \cos(dx+c)}{\cos(dx+c)}}}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx+c)^4 \sec(dx+c)^2 + 2 ab \cos(dx+c)^4 \sec(dx+c) + a^2 \cos(dx+c)^4\right) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}\right)$$

64.201 Problem number 849

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(5a^4 - 2a^2b^2 - 3b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{6ab \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7d} \\
& + \frac{2a^2 \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7d} \\
& + \frac{2(5a^2 + 9b^2) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{21d} \\
& + \frac{2b(29a^2 + 3b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(3a^4 \cos(dx+c)^2 + 9a^3b \cos(dx+c) + 5a^4 + 9a^2b^2 \right) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2} (-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx+c)^3 \sec(dx+c)^2 + 2ab \cos(dx+c)^3 \sec(dx+c) + a^2 \cos(dx+c)^3\right) \sqrt{b \sec(dx+c) + a} \sqrt{c} \right)$$

64.202 Problem number 850

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{16b(a^2 - b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} + \frac{2a^2 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5d} + \frac{22ab \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{15d} + \frac{2(9a^2 + 23b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(3a^3 \cos(dx+c) + 11a^2b) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2} (-33i a^2b + i b^3) \sqrt{a} \operatorname{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cos(dx+c)^2 \sec(dx+c)^2 + 2ab \cos(dx+c)^2 \sec(dx+c) + a^2 \cos(dx+c)^2\right) \sqrt{b \sec(dx+c) + a} \sqrt{c}\right)$$

64.203 Problem number 855

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2b(7a^2 + 8b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} + \frac{2 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5ad} - \frac{8b \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{15a^2d} + \frac{2(9a^2 + 8b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}$$

command

```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (3 a^3 \cos(dx + c) - 4 a^2 b) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) - 4 \sqrt{2} (-3i a^2 b - 4i b^3) \sqrt{a} \text{weierstrassZeta}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2b - 8b^3)}{27a^3}\right), \text{weierstrassZeta}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\cos(dx + c)^{\frac{5}{2}}}{\sqrt{b \sec(dx + c) + a}}, x\right)$$

64.204 Problem number 856

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(a^2 + 2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2 \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{3ad} - \frac{4b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}$$

command

```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a^2 \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) - 6i \sqrt{2} a^{\frac{3}{2}} b \text{weierstrassZeta}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2b - 8b^3)}{27a^3}\right), \text{weierstrassZeta}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\cos(dx + c)^{\frac{3}{2}}}{\sqrt{b \sec(dx + c) + a}}, x\right)$$

64.205 Problem number 857

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2b\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad\sqrt{\cos(dx+c)}\sqrt{a+b\sec(dx+c)}} + \frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)})\sqrt{a+b\sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad\sqrt{\frac{b+a\cos(dx+c)}{a+b}}}$$

command

```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2i\sqrt{2}\sqrt{a}b\operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right) - 2i\sqrt{2}\sqrt{a}b\operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

64.206 Problem number 858

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d\sqrt{\cos(dx+c)}\sqrt{a+b\sec(dx+c)}}$$

command

```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a \cos(dx+c)+3ia \sin(dx+c)+2b}{3a}\right) + i \sqrt{2} \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a \cos(dx+c)+3ia \sin(dx+c)+2b}{3a}\right)}{ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c)+a} \sqrt{\cos(dx+c)}}{b \cos(dx+c) \sec(dx+c) + a \cos(dx+c)}, x\right)$$

64.207 Problem number 862

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2 \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{a(a^2-b^2)d\sqrt{a+b \sec(dx+c)}} \\ & - \frac{8b(a^2+4b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2(a^2-6b^2) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5a^2(a^2-b^2)d} \\ & - \frac{2b(3a^2-8b^2) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{5a^3(a^2-b^2)d} \\ & + \frac{2(3a^4+8a^2b^2-16b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2-b^2) d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(3a^4b^2 - 8a^2b^4 - (a^6 - a^4b^2) \cos(dx+c)^2 + 2(a^5b - a^3b^3) \cos(dx+c)\right) \sqrt{\frac{a \cos(dx+c)+b}{\cos(dx+c)}} \sqrt{\cos(dx+c)}}{ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c)+a} \cos(dx+c)^{\frac{5}{2}}}{b^2 \sec(dx+c)^2 + 2ab \sec(dx+c) + a^2}, x\right)$$

64.208 Problem number 863

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{a(a^2-b^2)d\sqrt{a+b\sec(dx+c)}} \\ & + \frac{2(a^2+8b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx+c)} \sqrt{a+b\sec(dx+c)}} \\ & + \frac{2(a^2-4b^2) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b\sec(dx+c)}}{3a^2(a^2-b^2)d} \\ & - \frac{2b(5a^2-8b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b\sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2-b^2) d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(a^4b - 4a^2b^3 + (a^5 - a^3b^2) \cos(dx+c)) \sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \left(\sqrt{2} (3ia^5 + 16ia^3b)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(dx+c)+a} \cos(dx+c)^{\frac{3}{2}}}{b^2 \sec(dx+c)^2 + 2ab\sec(dx+c) + a^2}, x\right)$$

64.209 Problem number 864

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx + c)}{a(a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} - \frac{4b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2(a^2 - 2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}$$

command

`integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a^2 b^2 \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2} (-5i a^3 b + 4i a b^3) \cos(dx + c) + \sqrt{2} (-5i a^2 b^2 + 4i a b^3) \sin(dx + c)\right) \sqrt{\cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^2 \sec^2(dx + c) + 2ab \sec(dx + c) + a^2}, x\right)$$

64.210 Problem number 865

$$\int \frac{1}{\sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b \sin(dx + c)}{(a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2 \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}$$

command

`integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a^2 b \sqrt{\frac{a \cos(dx+c)+b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \left(\sqrt{2} (3i a^3 - 2i ab^2) \cos(dx+c) + \sqrt{2} (3i a^2 b - 2i b^3) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \sec(dx+c)+a} \sqrt{\cos(dx+c)}}{b^2 \cos(dx+c) \sec(dx+c)^2 + 2ab \cos(dx+c) \sec(dx+c) + a^2 \cos(dx+c)}, x \right)$$

64.211 Problem number 866

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2a \sin(dx+c)}{(a^2-b^2) d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\ 2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}} \right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)} \\ \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2-b^2) d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}$$

command

`integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a^2 \sqrt{\frac{a \cos(dx+c)+b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \left(-i \sqrt{2} ab \cos(dx+c) - i \sqrt{2} b^2 \right) \sqrt{a} \text{weierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \sec(dx+c)+a} \sqrt{\cos(dx+c)}}{b^2 \cos(dx+c)^2 \sec(dx+c)^2 + 2ab \cos(dx+c)^2 \sec(dx+c) + a^2 \cos(dx+c)^2}, x \right)$$

64.212 Problem number 869

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2b^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a(a^2-b^2)d(a+b\sec(dx+c))^{\frac{3}{2}}} + \frac{4b^2(5a^2-3b^2) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a^2(a^2-b^2)^2 d \sqrt{a+b\sec(dx+c)}}$$

$$+ \frac{2(a^4+16a^2b^2-16b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2-b^2) d \sqrt{\cos(dx+c)} \sqrt{a+b\sec(dx+c)}}$$

$$+ \frac{2(a^4-13a^2b^2+8b^4) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b\sec(dx+c)}}{3a^3(a^2-b^2)^2 d}$$

$$- \frac{8b(2a^4-7a^2b^2+4b^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b\sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2-b^2)^2 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}$$

command

`integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(a^6 b^2 - 13 a^4 b^4 + 8 a^2 b^6 + (a^8 - 2 a^6 b^2 + a^4 b^4) \cos(dx+c)^2 + 2 (a^7 b - 8 a^5 b^3 + 5 a^3 b^5) \cos(dx+c) \right) \sqrt{\frac{a \cos(dx+c)}{\cos(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(dx+c)+a} \cos(dx+c)^{\frac{3}{2}}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3}, x\right)$$

64.213 Problem number 870

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b^2 \sin(dx+c)}{3a(a^2-b^2)d(a+b\sec(dx+c))^{\frac{3}{2}}\sqrt{\cos(dx+c)}} \\
& + \frac{8b^2(2a^2-b^2)\sin(dx+c)}{3a^2(a^2-b^2)^2 d\sqrt{\cos(dx+c)}\sqrt{a+b\sec(dx+c)}} \\
& - \frac{2b(9a^2-8b^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{a}{a+b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3(a^2-b^2)d\sqrt{\cos(dx+c)}\sqrt{a+b\sec(dx+c)}} \\
& + \frac{2(3a^4-15a^2b^2+8b^4)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{a}{a+b}}\right)(\sqrt{\cos(dx+c)}\sqrt{a+b\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3(a^2-b^2)^2 d\sqrt{\frac{b+a\cos(dx+c)}{a+b}}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(8a^4b^3-4a^2b^5+(9a^5b^2-5a^3b^4)\cos(dx+c))\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-4(\sqrt{2}(-6i\sqrt{a+b}))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\sec(dx+c)^3+3ab^2\sec(dx+c)^2+3a^2b\sec(dx+c)+a^3},x\right)$$

64.214 Problem number 871

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b \sin(dx+c)}{3(a^2-b^2)d(a+b \sec(dx+c))^{\frac{3}{2}} \sqrt{\cos(dx+c)}} \\
& - \frac{2b(5a^2-b^2) \sin(dx+c)}{3a(a^2-b^2)^2 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2(3a^2-2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2-b^2) d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{4b(3a^2-b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2-b^2)^2 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}
\end{aligned}$$

command

```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(5a^4b^2 - a^2b^4 + 2(3a^5b - a^3b^3) \cos(dx+c)) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - (\sqrt{2} (-9i a^6))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}}{b^3 \cos(dx+c) \sec(dx+c)^3 + 3ab^2 \cos(dx+c) \sec(dx+c)^2 + 3a^2b \cos(dx+c) \sec(dx+c) + a^3 \cos(dx+c)}\right)$$

64.215 Problem number 872

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a \sin(dx + c)}{3(a^2 - b^2) d(a + b \sec(dx + c))^{\frac{3}{2}} \sqrt{\cos(dx + c)}} + \frac{4(a^2 + b^2) \sin(dx + c)}{3(a^2 - b^2)^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2b \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a(a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} - \frac{2(3a^2 + b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a(a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}$$

command

```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(2a^4b + 2a^2b^3 + (3a^5 + a^3b^2) \cos(dx + c)) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) - 2(\sqrt{2}(-3ia^4b +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^3 \cos(dx + c)^2 \sec(dx + c)^3 + 3ab^2 \cos(dx + c)^2 \sec(dx + c)^2 + 3a^2b \cos(dx + c)^2 \sec(dx + c) + a^3 \cos(dx + c)}\right)$$

64.216 Problem number 873

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2a^2 \sin(dx+c)}{3b(a^2-b^2)d(a+b \sec(dx+c))^{\frac{3}{2}} \sqrt{\cos(dx+c)}} \\
& + \frac{2a(a^2-5b^2) \sin(dx+c)}{3b(a^2-b^2)^2 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2-b^2) d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{8b \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2-b^2)^2 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}
\end{aligned}$$

command

```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(4a^3b \cos(dx+c) - a^4 + 5a^2b^2) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \left(\sqrt{2}(-3ia^4 - ia^2b^2) \cos(dx+c)\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) (a^2-b^2)^2 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}}{b^3 \cos(dx+c)^3 \sec(dx+c)^3 + 3ab^2 \cos(dx+c)^3 \sec(dx+c)^2 + 3a^2b \cos(dx+c)^3 \sec(dx+c) + a^3 \cos(dx+c)} dx\right)$$

65 Test file number 119

Test folder name:

test_cases/4_Trig_functions/4.5_Secant/119_4.5.1.3-d_sinⁿ-a+b_sec^m

65.1 Problem number 108

$$\int (a + a \sec(c + dx))(e \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{a e^{\frac{5}{2}} \arctan\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{d} + \frac{a e^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{d} \\
 & -\frac{2ae(e \sin(dx+c))^{\frac{3}{2}}}{3d} - \frac{2ae \cos(dx+c)(e \sin(dx+c))^{\frac{3}{2}}}{5d} \\
 & -\frac{6a e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx+c)}}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\sin(dx+c)}}
 \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$36i \sqrt{2} \sqrt{-i} a e^{\frac{5}{2}} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c))) - 36i \sqrt{2} \sqrt{i} a e^{\frac{5}{2}} \operatorname{wei}$

Fricas 1.3.7 via sagemath 9.3 output

$\operatorname{integral}\left(-\left(ae^2 \cos(dx+c)^2 - ae^2 + \left(ae^2 \cos(dx+c)^2 - ae^2\right) \sec(dx+c)\right) \sqrt{e \sin(dx+c)}, x\right)$

65.2 Problem number 109

$$\int (a + a \sec(c + dx))(e \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{a e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{d} + \frac{a e^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{d} \\
 & -\frac{2a e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx+c)}\right)}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{e \sin(dx+c)}} \\
 & -\frac{2ae \sqrt{e \sin(dx+c)}}{d} - \frac{2ae \cos(dx+c) \sqrt{e \sin(dx+c)}}{3d}
 \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4\sqrt{2}\sqrt{-i}ae^{\frac{3}{2}}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+4\sqrt{2}\sqrt{i}ae^{\frac{3}{2}}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((ae\sec(dx+c)+ae)\sqrt{e\sin(dx+c)}\sin(dx+c),x\right)$$

65.3 Problem number 110

$$\int (a + a\sec(c + dx))\sqrt{e\sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\sqrt{e}}{d} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\sqrt{e}}{d} \\ & -\frac{2a\sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right),\sqrt{2}\right)\sqrt{e\sin(dx+c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right)d\sqrt{\sin(dx+c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4i\sqrt{2}\sqrt{-i}ae^{\frac{1}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-4i\sqrt{2}\sqrt{i}ae^{\frac{1}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((a\sec(dx+c)+a)\sqrt{e\sin(dx+c)},x\right)$$

65.4 Problem number 111

$$\int \frac{a + a \sec(c + dx)}{\sqrt{e \sin(c + dx)}} dx$$

Optimal antiderivative

$$\frac{a \arctan\left(\frac{\sqrt{e \sin(dx + c)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{a \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx + c)}}{\sqrt{e}}\right)}{d\sqrt{e}} - \frac{2a\sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d\sqrt{e \sin(dx + c)}}$$

command

`integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(4\sqrt{2}\sqrt{-i} \operatorname{aweierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + 4\sqrt{2}\sqrt{i} \operatorname{aweierstrassPInverse}(4, 0, \cos(dx + c) - i \sin(dx + c))\right) \sqrt{e \sin(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(a \sec(dx + c) + a)\sqrt{e \sin(dx + c)}}{e \sin(dx + c)}, x\right)$$

65.5 Problem number 112

$$\int \frac{a + a \sec(c + dx)}{(e \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a \arctan\left(\frac{\sqrt{e \sin(dx + c)}}{\sqrt{e}}\right)}{de^{\frac{3}{2}}} + \frac{a \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx + c)}}{\sqrt{e}}\right)}{de^{\frac{3}{2}}} - \frac{2a}{de\sqrt{e \sin(dx + c)}} - \frac{2a \cos(dx + c)}{de\sqrt{e \sin(dx + c)}} + \frac{2a\sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) de^2 \sqrt{\sin(dx + c)}}$$

command

```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-4i\sqrt{2}\sqrt{-i}a\sin(dx+c)\operatorname{weierstrassZeta}(4,0,\operatorname{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))\right)+4i\sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}{e^2\cos(dx+c)^2-e^2},x\right)$$

65.6 Problem number 113

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{de^{\frac{5}{2}}} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{de^{\frac{5}{2}}} \\ & - \frac{2a}{3de(e\sin(dx+c))^{\frac{3}{2}}} - \frac{2a\cos(dx+c)}{3de(e\sin(dx+c))^{\frac{3}{2}}} \\ & - \frac{2a\sqrt{\frac{1}{2}+\frac{\sin(dx+c)}{2}}}{3\sin\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)de^2\sqrt{e\sin(dx+c)}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right),\sqrt{2}\right)\left(\sqrt{\sin(dx+c)}\right) \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4\sqrt{-i}\left(\sqrt{2}a\cos(dx+c)-\sqrt{2}a\right)\operatorname{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+4\sqrt{i}\left(\sqrt{2}a\cos(dx+c)+\sqrt{2}a\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}{(e^3\cos(dx+c)^2-e^3)\sin(dx+c)},x\right)$$

65.7 Problem number 114

$$\int (a + a \sec(c + dx))^2 (e \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^2 e^{5/2} \arctan\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{d} + \frac{2a^2 e^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{d} \\ & -\frac{4a^2 e (e \sin(dx+c))^{3/2}}{3d} - \frac{2a^2 e \cos(dx+c) (e \sin(dx+c))^{3/2}}{5d} \\ & + \frac{a^2 e \sec(dx+c) (e \sin(dx+c))^{3/2}}{d} \\ & + \frac{9a^2 e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx+c)}}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\sin(dx+c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-27i \sqrt{2} \sqrt{-i} a^2 \cos(dx+c) e^{5/2} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c))) + 27i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(a^2 e^2 \cos(dx+c)^2 - a^2 e^2 + \left(a^2 e^2 \cos(dx+c)^2 - a^2 e^2\right) \sec(dx+c)^2 + 2\left(a^2 e^2 \cos(dx+c)^2 - a^2 e^2\right) \sec(dx+c)\right) \sqrt{e \sin(dx+c)}\right)$$

65.8 Problem number 115

$$\int (a + a \sec(c + dx))^2 (e \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 e^{3/2} \arctan\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{d} + \frac{2a^2 e^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{d} \\ & + \frac{a^2 e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx+c)}\right)}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{e \sin(dx+c)}} \\ & - \frac{4a^2 e \sqrt{e \sin(dx+c)}}{d} - \frac{2a^2 e \cos(dx+c) \sqrt{e \sin(dx+c)}}{3d} \\ & + \frac{a^2 e \sec(dx+c) \sqrt{e \sin(dx+c)}}{d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \sqrt{-i} a^2 \cos(dx + c) e^{\frac{3}{2}} \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} \sqrt{i} a^2 \cos(dx + c) e^{\frac{3}{2}} \text{wei}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(a^2 e \sec(dx + c)^2 + 2 a^2 e \sec(dx + c) + a^2 e\right) \sqrt{e \sin(dx + c)} \sin(dx + c), x\right)$$

65.9 Problem number 116

$$\int (a + a \sec(c + dx))^2 \sqrt{e \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2 \sec(dx + c) (e \sin(dx + c))^{\frac{3}{2}}}{de} - \frac{2a^2 \arctan\left(\frac{\sqrt{e \sin(dx + c)}}{\sqrt{e}}\right) \sqrt{e}}{d} \\ & + \frac{2a^2 \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx + c)}}{\sqrt{e}}\right) \sqrt{e}}{d} \\ & - \frac{a^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{\sin(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$i \sqrt{2} \sqrt{-i} a^2 \cos(dx + c) e^{\frac{1}{2}} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) - i \sqrt{2} \sqrt{i} a^2 \cos(dx + c) e^{\frac{1}{2}} \text{wei}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2\right) \sqrt{e \sin(dx + c)}, x\right)$$

65.10 Problem number 117

$$\int \frac{(a + a \sec(c + dx))^2}{\sqrt{e \sin(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \arctan\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a^2 \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{d\sqrt{e}} \\ & - \frac{3a^2 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx+c)}\right)}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d\sqrt{e \sin(dx+c)}} \\ & + \frac{a^2 \sec(dx+c) \sqrt{e \sin(dx+c)}}{de} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(3 \sqrt{2} \sqrt{-i} a^2 \cos(dx+c) \operatorname{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c)) + 3 \sqrt{2} \sqrt{i} a^2 \cos(dx+c) \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2\right) \sqrt{e \sin(dx+c)}}{e \sin(dx+c)}, x\right)$$

65.11 Problem number 118

$$\int \frac{(a + a \sec(c + dx))^2}{(e \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a^2 \arctan\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{de^{\frac{3}{2}}} + \frac{2a^2 \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{de^{\frac{3}{2}}} \\ & + \frac{3a^2 \sec(dx+c) (e \sin(dx+c))^{\frac{3}{2}}}{de^3} - \frac{4a^2}{de \sqrt{e \sin(dx+c)}} \\ & - \frac{2a^2 \cos(dx+c)}{de \sqrt{e \sin(dx+c)}} - \frac{2a^2 \sec(dx+c)}{de \sqrt{e \sin(dx+c)}} \\ & + \frac{5a^2 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx+c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) de^2 \sqrt{\sin(dx+c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(-5i \sqrt{2} \sqrt{-i} a^2 \cos(dx+c) \sin(dx+c) \text{weierstrassZeta}(4,0, \text{weierstrassPInverse}(4,0, \cos(dx+c) + i \sin(dx+c))) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\left(a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2 \right) \sqrt{e \sin(dx+c)}}{e^2 \cos(dx+c)^2 - e^2}, x \right)$$

65.12 Problem number 119

$$\int \frac{(a + a \sec(c + dx))^2}{(e \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \arctan\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{de^{\frac{5}{2}}} + \frac{2a^2 \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{de^{\frac{5}{2}}} \\ & - \frac{4a^2}{3de(e \sin(dx+c))^{\frac{3}{2}}} - \frac{2a^2 \cos(dx+c)}{3de(e \sin(dx+c))^{\frac{3}{2}}} - \frac{2a^2 \sec(dx+c)}{3de(e \sin(dx+c))^{\frac{3}{2}}} \\ & - \frac{7a^2 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx+c)}\right)}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) de^2 \sqrt{e \sin(dx+c)}} \\ & + \frac{5a^2 \sec(dx+c) \sqrt{e \sin(dx+c)}}{3de^3} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$7 \sqrt{-i} \left(\sqrt{2} a^2 \cos(dx+c)^2 - \sqrt{2} a^2 \cos(dx+c) \right) \text{weierstrassPInverse}(4,0, \cos(dx+c) + i \sin(dx+c)) + 7 \sqrt{i} \left(\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\left(a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2 \right) \sqrt{e \sin(dx+c)}}{\left(e^3 \cos(dx+c)^2 - e^3 \right) \sin(dx+c)}, x \right)$$

65.13 Problem number 120

$$\int \frac{(e \sin(c + dx))^{7/2}}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e(e \sin(dx + c))^{\frac{5}{2}}}{5ad} \\ & + \frac{4e^4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{21 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) ad \sqrt{e \sin(dx + c)}} \\ & - \frac{2e^3 \cos(dx + c) \sqrt{e \sin(dx + c)}}{21ad} + \frac{2e^3 (\cos^3(dx + c)) \sqrt{e \sin(dx + c)}}{7ad} \end{aligned}$$

command

```
integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \sqrt{2} \sqrt{-i} e^{\frac{7}{2}} \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} \sqrt{i} e^{\frac{7}{2}} \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\left(e^3 \cos(dx + c)^2 - e^3\right) \sqrt{e \sin(dx + c)} \sin(dx + c)}{a \sec(dx + c) + a}, x\right)$$

65.14 Problem number 121

$$\int \frac{(e \sin(c + dx))^{5/2}}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e(e \sin(dx + c))^{\frac{3}{2}}}{3ad} - \frac{2e \cos(dx + c) (e \sin(dx + c))^{\frac{3}{2}}}{5ad} \\ & + \frac{4e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) ad \sqrt{\sin(dx + c)}} \end{aligned}$$

command

`integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3i \sqrt{2} \sqrt{-i} e^{\frac{5}{2}} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) - 3i \sqrt{2} \sqrt{i} e^{\frac{5}{2}} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx + c) - i \sin(dx + c))) \right)}{3ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\left(e^2 \cos(dx + c)^2 - e^2 \right) \sqrt{e \sin(dx + c)}}{a \sec(dx + c) + a}, x \right)$$

65.15 Problem number 122

$$\int \frac{(e \sin(c + dx))^{3/2}}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{4e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right) \left(\sqrt{\sin(dx + c)} \right)}{3 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) ad \sqrt{e \sin(dx + c)}} + \frac{2e \sqrt{e \sin(dx + c)}}{ad} - \frac{2e \cos(dx + c) \sqrt{e \sin(dx + c)}}{3ad}$$

command

`integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} \sqrt{-i} e^{\frac{3}{2}} \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} \sqrt{i} e^{\frac{3}{2}} \text{weierstrassPInverse}(4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{3ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \sin(dx + c)} e \sin(dx + c)}{a \sec(dx + c) + a}, x \right)$$

65.16 Problem number 123

$$\int \frac{\sqrt{e \sin(c + dx)}}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\frac{-\frac{2e}{ad\sqrt{e \sin(dx+c)}} + \frac{2e \cos(dx+c)}{ad\sqrt{e \sin(dx+c)}}}{4\sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \frac{\text{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx+c)}}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) ad\sqrt{\sin(dx+c)}}$$

command

```
integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(e^{\frac{1}{2}} \sin(dx+c)^{\frac{3}{2}} + \sqrt{-i} \left(-i \sqrt{2} \cos(dx+c) e^{\frac{1}{2}} - i \sqrt{2} e^{\frac{1}{2}} \right) \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{e \sin(dx+c)}}{a \sec(dx+c) + a}, x\right)$$

65.17 Problem number 124

$$\int \frac{1}{(a + a \sec(c + dx)) \sqrt{e \sin(c + dx)}} dx$$

Optimal antiderivative

$$\frac{-\frac{2e}{3ad(e \sin(dx+c))^{\frac{3}{2}}} + \frac{2e \cos(dx+c)}{3ad(e \sin(dx+c))^{\frac{3}{2}}}}{4\sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \frac{\text{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx+c)}\right)}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) ad\sqrt{e \sin(dx+c)}}$$

command

```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{-i} \left(\sqrt{2} \cos(dx+c) + \sqrt{2} \right) \text{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{i} \left(\sqrt{2} \cos(dx+c) + \sqrt{2} \right) \right)}{3 \left(ad \cos(dx+c) e^{\frac{1}{2}} + ade^{\frac{1}{2}} \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \sin(dx+c)}}{(ae \sec(dx+c) + ae) \sin(dx+c)}, x \right)$$

65.18 Problem number 125

$$\int \frac{1}{(a + a \sec(c + dx))(e \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2e}{5ad(e \sin(dx+c))^{\frac{5}{2}}} + \frac{2e \cos(dx+c)}{5ad(e \sin(dx+c))^{\frac{5}{2}}} - \frac{4 \cos(dx+c)}{5ade \sqrt{e \sin(dx+c)}} \\ & + \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \text{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right) \sqrt{e \sin(dx+c)}}{5 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) ad e^2 \sqrt{\sin(dx+c)}} \end{aligned}$$

command

`integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{-i} \left(i \sqrt{2} \cos(dx+c) + i \sqrt{2} \right) \sin(dx+c) \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c))) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{e \sin(dx+c)}}{ae^2 \cos(dx+c)^2 - ae^2 + (ae^2 \cos(dx+c)^2 - ae^2) \sec(dx+c)}, x \right)$$

65.19 Problem number 126

$$\int \frac{1}{(a + a \sec(c + dx))(e \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2e}{7ad(e \sin(dx + c))^{7/2}} + \frac{2e \cos(dx + c)}{7ad(e \sin(dx + c))^{7/2}} - \frac{4 \cos(dx + c)}{21ade(e \sin(dx + c))^{3/2}} \\ & - \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{21 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) ad e^2 \sqrt{e \sin(dx + c)}} \end{aligned}$$

command

`integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{-i} \left(\sqrt{2} \cos(dx + c)^3 + \sqrt{2} \cos(dx + c)^2 - \sqrt{2} \cos(dx + c) - \sqrt{2} \right) \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{e \sin(dx + c)}}{\left(ae^3 \cos(dx + c)^2 - ae^3 + \left(ae^3 \cos(dx + c)^2 - ae^3\right) \sec(dx + c)\right) \sin(dx + c)}, x\right)$$

65.20 Problem number 127

$$\int \frac{(e \sin(c + dx))^{7/2}}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4e(e \sin(dx + c))^{5/2}}{5a^2d} \\ & - \frac{52e^4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{21 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) a^2d \sqrt{e \sin(dx + c)}} \\ & - \frac{4e^3 \sqrt{e \sin(dx + c)}}{a^2d} + \frac{26e^3 \cos(dx + c) \sqrt{e \sin(dx + c)}}{21a^2d} \\ & + \frac{2e^3 (\cos^3(dx + c)) \sqrt{e \sin(dx + c)}}{7a^2d} \end{aligned}$$

command

```
integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(65 \sqrt{2} \sqrt{-i} e^{\frac{7}{2}} \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + 65 \sqrt{2} \sqrt{i} e^{\frac{7}{2}} \text{weierstrassPInverse}(4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\left(e^3 \cos(dx + c)^2 - e^3 \right) \sqrt{e \sin(dx + c)} \sin(dx + c)}{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}, x \right)$$

65.21 Problem number 128

$$\int \frac{(e \sin(c + dx))^{5/2}}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4e(e \sin(dx + c))^{\frac{3}{2}}}{3a^2d} - \frac{12e \cos(dx + c) (e \sin(dx + c))^{\frac{3}{2}}}{5a^2d} \\ & + \frac{4e^3}{a^2d \sqrt{e \sin(dx + c)}} - \frac{2e^3 \cos(dx + c)}{a^2d \sqrt{e \sin(dx + c)}} - \frac{2e^3 (\cos^3(dx + c))}{a^2d \sqrt{e \sin(dx + c)}} \\ & + \frac{44e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right) \sqrt{e \sin(dx + c)}}{5 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) a^2d \sqrt{\sin(dx + c)}} \end{aligned}$$

command

```
integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(3 \cos(dx + c)^2 e^{\frac{5}{2}} - 7 \cos(dx + c) e^{\frac{5}{2}} - 40 e^{\frac{5}{2}} \right) \sin(dx + c)^{\frac{3}{2}} + 33 \sqrt{-i} \left(i \sqrt{2} \cos(dx + c) e^{\frac{5}{2}} + i \sqrt{2} e^{\frac{5}{2}} \right) \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + 33 \sqrt{i} \left(-i \sqrt{2} \cos(dx + c) e^{\frac{5}{2}} - i \sqrt{2} e^{\frac{5}{2}} \right) \text{weierstrassPInverse}(4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\left(e^2 \cos(dx + c)^2 - e^2 \right) \sqrt{e \sin(dx + c)}}{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}, x \right)$$

65.22 Problem number 129

$$\int \frac{(e \sin(c + dx))^{3/2}}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4e^3}{3a^2d(e \sin(dx + c))^{\frac{3}{2}}} - \frac{2e^3 \cos(dx + c)}{3a^2d(e \sin(dx + c))^{\frac{3}{2}}} - \frac{2e^3(\cos^3(dx + c))}{3a^2d(e \sin(dx + c))^{\frac{3}{2}}} \\ & + \frac{4e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx + c)}\right)}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) a^2 d \sqrt{e \sin(dx + c)}} \\ & + \frac{4e \sqrt{e \sin(dx + c)}}{a^2 d} - \frac{4e \cos(dx + c) \sqrt{e \sin(dx + c)}}{3a^2 d} \end{aligned}$$

command

`integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \sqrt{-i} \left(\sqrt{2} \cos(dx + c) e^{\frac{3}{2}} + \sqrt{2} e^{\frac{3}{2}} \right) \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + 3 \sqrt{i} \left(\sqrt{2} \cos \right)}{\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \sin(dx + c)} e \sin(dx + c)}{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}, x\right)$$

65.23 Problem number 130

$$\int \frac{\sqrt{e \sin(c + dx)}}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4e^3}{5a^2d(e \sin(dx + c))^{\frac{5}{2}}} - \frac{2e^3 \cos(dx + c)}{5a^2d(e \sin(dx + c))^{\frac{5}{2}}} - \frac{2e^3(\cos^3(dx + c))}{5a^2d(e \sin(dx + c))^{\frac{5}{2}}} \\ & - \frac{4e}{a^2d \sqrt{e \sin(dx + c)}} + \frac{16e \cos(dx + c)}{5a^2d \sqrt{e \sin(dx + c)}} \\ & - \frac{28 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx + c)}}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) a^2 d \sqrt{\sin(dx + c)}} \end{aligned}$$

command

`integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(9 \cos(dx+c) e^{\frac{1}{2}} + 8 e^{\frac{1}{2}} \right) \sin(dx+c)^{\frac{3}{2}} + 7 \sqrt{-i} \left(-i \sqrt{2} \cos(dx+c)^2 e^{\frac{1}{2}} - 2i \sqrt{2} \cos(dx+c) e^{\frac{1}{2}} - i \sqrt{2} e^{\frac{1}{2}} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \sin(dx+c)}}{a^2 \sec(dx+c)^2 + 2 a^2 \sec(dx+c) + a^2}, x \right)$$

65.24 Problem number 131

$$\int \frac{1}{(a + a \sec(c + dx))^2 \sqrt{e \sin(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4e^3}{7a^2d(e \sin(dx+c))^{\frac{7}{2}}} - \frac{2e^3 \cos(dx+c)}{7a^2d(e \sin(dx+c))^{\frac{7}{2}}} - \frac{2e^3(\cos^3(dx+c))}{7a^2d(e \sin(dx+c))^{\frac{7}{2}}} \\ & - \frac{4e}{3a^2d(e \sin(dx+c))^{\frac{3}{2}}} + \frac{16e \cos(dx+c)}{21a^2d(e \sin(dx+c))^{\frac{3}{2}}} \\ & - \frac{20 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \text{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right) \left(\sqrt{\sin(dx+c)} \right)}{21 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) a^2 d \sqrt{e \sin(dx+c)}} \end{aligned}$$

command

`integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \sqrt{-i} \left(\sqrt{2} \cos(dx+c)^2 + 2 \sqrt{2} \cos(dx+c) + \sqrt{2} \right) \text{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c)) + \right. \\ \left. 21 \left(a^2 d \cos \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \sin(dx+c)}}{\left(a^2 e \sec(dx+c)^2 + 2 a^2 e \sec(dx+c) + a^2 e \right) \sin(dx+c)}, x \right)$$

65.25 Problem number 132

$$\int \frac{1}{(a + a \sec(c + dx))^2 (e \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4e^3}{9a^2d(e \sin(dx+c))^{\frac{9}{2}}} - \frac{2e^3 \cos(dx+c)}{9a^2d(e \sin(dx+c))^{\frac{9}{2}}} - \frac{2e^3(\cos^3(dx+c))}{9a^2d(e \sin(dx+c))^{\frac{9}{2}}} \\ & - \frac{4e}{5a^2d(e \sin(dx+c))^{\frac{5}{2}}} + \frac{16e \cos(dx+c)}{45a^2d(e \sin(dx+c))^{\frac{5}{2}}} - \frac{4 \cos(dx+c)}{15a^2de \sqrt{e \sin(dx+c)}} \\ & + \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \sin(dx+c)}}{15 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) a^2 d e^2 \sqrt{\sin(dx+c)}} \end{aligned}$$

command

`integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{-i} \left(i \sqrt{2} \cos(dx+c)^2 + 2i \sqrt{2} \cos(dx+c) + i \sqrt{2} \right) \sin(dx+c) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(\dots)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{e \sin(dx+c)}}{a^2 e^2 \cos(dx+c)^2 - a^2 e^2 + \left(a^2 e^2 \cos(dx+c)^2 - a^2 e^2\right) \sec(dx+c)^2 + 2 \left(a^2 e^2 \cos(dx+c)^2 - a^2 e^2\right) \sec(dx+c)}\right)$$

65.26 Problem number 133

$$\int \frac{1}{(a + a \sec(c + dx))^2 (e \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4e^3}{11a^2d(e \sin(dx+c))^{\frac{11}{2}}} - \frac{2e^3 \cos(dx+c)}{11a^2d(e \sin(dx+c))^{\frac{11}{2}}} - \frac{2e^3(\cos^3(dx+c))}{11a^2d(e \sin(dx+c))^{\frac{11}{2}}} \\ & - \frac{4e}{7a^2d(e \sin(dx+c))^{\frac{7}{2}}} + \frac{16e \cos(dx+c)}{77a^2d(e \sin(dx+c))^{\frac{7}{2}}} - \frac{4 \cos(dx+c)}{231a^2de(e \sin(dx+c))^{\frac{3}{2}}} \\ & - \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \left(\sqrt{\sin(dx+c)}\right)}{231 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) a^2 d e^2 \sqrt{e \sin(dx+c)}} \end{aligned}$$

command

```
integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{-i} \left(\sqrt{2} \cos(dx+c)^4 + 2\sqrt{2} \cos(dx+c)^3 - 2\sqrt{2} \cos(dx+c) - \sqrt{2} \right) \text{weierstrassPInverse}(4, 0, \cos(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\sqrt{e \sin(dx+c)}}{\left(a^2 e^3 \cos(dx+c)^2 - a^2 e^3 + \left(a^2 e^3 \cos(dx+c)^2 - a^2 e^3 \right) \sec(dx+c)^2 + 2 \left(a^2 e^3 \cos(dx+c)^2 - a^2 e^3 \right) \right)} \right)$$

65.27 Problem number 282

$$\int (e \csc(c+dx))^{3/2} (a+a \sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2ae \sqrt{e \csc(dx+c)}}{d} - \frac{2ae \cos(dx+c) \sqrt{e \csc(dx+c)}}{d} \\ & - \frac{ae \arctan(\sqrt{\sin(dx+c)}) \sqrt{e \csc(dx+c)} (\sqrt{\sin(dx+c)})}{d} \\ & + \frac{ae \operatorname{arctanh}(\sqrt{\sin(dx+c)}) \sqrt{e \csc(dx+c)} (\sqrt{\sin(dx+c)})}{d} \\ & + \frac{2ae \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \csc(dx+c)} (\sqrt{\sin(dx+c)})}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d} \end{aligned}$$

command

```
integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \sqrt{2i} ae^{\frac{3}{2}} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c))) + 4 \sqrt{-2i} ae^{\frac{3}{2}} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx+c) - i \sin(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((ae \csc(dx+c) \sec(dx+c) + ae \csc(dx+c)) \sqrt{e \csc(dx+c)}, x \right)$$

65.28 Problem number 284

$$\int \frac{a + a \sec(c + dx)}{\sqrt{e \csc(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a \arctan\left(\sqrt{\sin(dx+c)}\right)}{d \sqrt{e \csc(dx+c)} \sqrt{\sin(dx+c)}} + \frac{a \operatorname{arctanh}\left(\sqrt{\sin(dx+c)}\right)}{d \sqrt{e \csc(dx+c)} \sqrt{\sin(dx+c)}} \\ & - \frac{2a \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d \sqrt{e \csc(dx+c)} \sqrt{\sin(dx+c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(4 \sqrt{2i} \operatorname{aweierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c))) + 4 \sqrt{-2i} \operatorname{aweierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx+c) - i \sin(dx+c))) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \csc(dx+c)} (a \sec(dx+c) + a)}{e \csc(dx+c)}, x\right)$$

65.29 Problem number 286

$$\int \frac{a + a \sec(c + dx)}{(e \csc(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a \sin(dx+c)}{3d e^2 \sqrt{e \csc(dx+c)}} - \frac{2a \cos(dx+c) \sin(dx+c)}{5d e^2 \sqrt{e \csc(dx+c)}} \\ & - \frac{a \arctan\left(\sqrt{\sin(dx+c)}\right)}{d e^2 \sqrt{e \csc(dx+c)} \sqrt{\sin(dx+c)}} + \frac{a \operatorname{arctanh}\left(\sqrt{\sin(dx+c)}\right)}{d e^2 \sqrt{e \csc(dx+c)} \sqrt{\sin(dx+c)}} \\ & - \frac{6a \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d e^2 \sqrt{e \csc(dx+c)} \sqrt{\sin(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(36 \sqrt{2i} \operatorname{aweierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) + 36 \sqrt{-2i} \operatorname{aweierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) - i \sin(dx + c))) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \csc(dx + c)} (a \sec(dx + c) + a)}{e^3 \csc(dx + c)^3}, x\right)$$

65.30 Problem number 288

$$\int (e \csc(c + dx))^{3/2} (a + a \sec(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^2 e \sqrt{e \csc(dx + c)}}{d} - \frac{2a^2 e \cos(dx + c) \sqrt{e \csc(dx + c)}}{d} - \frac{2a^2 e \sec(dx + c) \sqrt{e \csc(dx + c)}}{d} \\ & - \frac{2a^2 e \arctan\left(\sqrt{\sin(dx + c)}\right) \sqrt{e \csc(dx + c)} \left(\sqrt{\sin(dx + c)}\right)}{d} \\ & + \frac{2a^2 e \operatorname{arctanh}\left(\sqrt{\sin(dx + c)}\right) \sqrt{e \csc(dx + c)} \left(\sqrt{\sin(dx + c)}\right)}{d} \\ & + \frac{5a^2 e \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \csc(dx + c)} \left(\sqrt{\sin(dx + c)}\right)}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d} \\ & + \frac{3a^2 e \sin(dx + c) \sqrt{e \csc(dx + c)} \tan(dx + c)}{d} \end{aligned}$$

command

```
integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2i} a^2 \cos(dx + c) e^{\frac{3}{2}} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) + 5 \sqrt{-2i} a^2 \cos(dx + c) e^{\frac{3}{2}} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) - i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(a^2 e \csc(dx + c) \sec(dx + c)^2 + 2 a^2 e \csc(dx + c) \sec(dx + c) + a^2 e \csc(dx + c)\right) \sqrt{e \csc(dx + c)}, x\right)$$

65.31 Problem number 290

$$\int \frac{(a + a \sec(c + dx))^2}{\sqrt{e \csc(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 \arctan\left(\sqrt{\sin(dx+c)}\right)}{d\sqrt{e \csc(dx+c)}\sqrt{\sin(dx+c)}} + \frac{2a^2 \operatorname{arctanh}\left(\sqrt{\sin(dx+c)}\right)}{d\sqrt{e \csc(dx+c)}\sqrt{\sin(dx+c)}} \\ & - \frac{a^2 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d\sqrt{e \csc(dx+c)}\sqrt{\sin(dx+c)}} + \frac{a^2 \tan(dx+c)}{d\sqrt{e \csc(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2i} a^2 \cos(dx+c) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c))) + \sqrt{-2i} a^2 \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2\right) \sqrt{e \csc(dx+c)}}{e \csc(dx+c)}, x\right)$$

65.32 Problem number 292

$$\int \frac{(a + a \sec(c + dx))^2}{(e \csc(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4a^2 \sin(dx+c)}{3de^2 \sqrt{e \csc(dx+c)}} - \frac{2a^2 \cos(dx+c) \sin(dx+c)}{5de^2 \sqrt{e \csc(dx+c)}} \\ & - \frac{2a^2 \arctan\left(\sqrt{\sin(dx+c)}\right)}{de^2 \sqrt{e \csc(dx+c)}\sqrt{\sin(dx+c)}} + \frac{2a^2 \operatorname{arctanh}\left(\sqrt{\sin(dx+c)}\right)}{de^2 \sqrt{e \csc(dx+c)}\sqrt{\sin(dx+c)}} \\ & + \frac{9a^2 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) de^2 \sqrt{e \csc(dx+c)}\sqrt{\sin(dx+c)}} + \frac{a^2 \tan(dx+c)}{de^2 \sqrt{e \csc(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(27 \sqrt{2i} a^2 \cos(dx + c) \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) + 27 \sqrt{-2i} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2 \right) \sqrt{e \csc(dx + c)}}{e^3 \csc(dx + c)^3}, x \right)$$

65.33 Problem number 293

$$\int \frac{(e \csc(c + dx))^{5/2}}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4e^2 \cot(dx + c) \sqrt{e \csc(dx + c)}}{21ad} + \frac{2e^2 \cot(dx + c) (\csc^2(dx + c)) \sqrt{e \csc(dx + c)}}{7ad} \\ & - \frac{2e^2 (\csc^3(dx + c)) \sqrt{e \csc(dx + c)}}{7ad} \\ & - \frac{4e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \text{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right) \sqrt{e \csc(dx + c)} \left(\sqrt{\sin(dx + c)} \right)}{21 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) ad} \end{aligned}$$

command

```
integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2i} \left(i \cos(dx + c) e^{\frac{5}{2}} + i e^{\frac{5}{2}} \right) \sin(dx + c) \text{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{-2i} \right) -$$

$$21 (ad \cos ($$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \csc(dx + c)} e^2 \csc(dx + c)^2}{a \sec(dx + c) + a}, x \right)$$

65.34 Problem number 294

$$\int \frac{(e \csc(c + dx))^{3/2}}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4e \cos(dx + c) \sqrt{e \csc(dx + c)}}{5ad} + \frac{2e \cot(dx + c) \csc(dx + c) \sqrt{e \csc(dx + c)}}{5ad} \\ & - \frac{2e(\csc^2(dx + c)) \sqrt{e \csc(dx + c)}}{5ad} \\ & + \frac{4e \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \csc(dx + c)} \left(\sqrt{\sin(dx + c)}\right)}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) ad} \end{aligned}$$

command

`integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2i} \left(\cos(dx + c) e^{\frac{3}{2}} + e^{\frac{3}{2}} \right) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) + \sqrt{-} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \csc(dx + c)} e \csc(dx + c)}{a \sec(dx + c) + a}, x\right)$$

65.35 Problem number 295

$$\int \frac{\sqrt{e \csc(c + dx)}}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \cot(dx + c) \sqrt{e \csc(dx + c)}}{3ad} - \frac{2 \csc(dx + c) \sqrt{e \csc(dx + c)}}{3ad} \\ & - \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \csc(dx + c)} \left(\sqrt{\sin(dx + c)}\right)}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) ad} \end{aligned}$$

command

`integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2i} \left(i \cos(dx+c) e^{\frac{1}{2}} + i e^{\frac{1}{2}} \right) \text{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{-2i} \left(-i \cos(dx+c) + i \sin(dx+c) \right) \right)}{3(ad \cos(dx+c) + ad)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \csc(dx+c)}}{a \sec(dx+c) + a}, x \right)$$

65.36 Problem number 296

$$\int \frac{1}{\sqrt{e \csc(c+dx)} (a + a \sec(c+dx))} dx$$

Optimal antiderivative

$$\frac{2 \cot(dx+c)}{ad \sqrt{e \csc(dx+c)}} - \frac{2 \csc(dx+c)}{ad \sqrt{e \csc(dx+c)}} - \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \text{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right)}{\sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) ad \sqrt{e \csc(dx+c)} \sqrt{\sin(dx+c)}}$$

command

`integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2i} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c))) + \sqrt{-2i} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c))) \right)}{ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \csc(dx+c)}}{ae \csc(dx+c) \sec(dx+c) + ae \csc(dx+c)}, x \right)$$

65.37 Problem number 297

$$\int \frac{1}{(e \csc(c + dx))^{3/2} (a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\frac{2}{ade \sqrt{e \csc(dx + c)}} - \frac{2 \cos(dx + c)}{3ade \sqrt{e \csc(dx + c)}} + \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) ade \sqrt{e \csc(dx + c)} \sqrt{\sin(dx + c)}}$$

command

```
integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((\cos(dx + c) - 3) \sqrt{\sin(dx + c)} - i \sqrt{2i} \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{-2i} \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) \right)}{3 ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \csc(dx + c)}}{ae^2 \csc(dx + c)^2 \sec(dx + c) + ae^2 \csc(dx + c)^2}, x\right)$$

65.38 Problem number 298

$$\int \frac{1}{(e \csc(c + dx))^{5/2} (a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\frac{2 \sin(dx + c)}{3ad e^2 \sqrt{e \csc(dx + c)}} - \frac{2 \cos(dx + c) \sin(dx + c)}{5ad e^2 \sqrt{e \csc(dx + c)}} + \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) ad e^2 \sqrt{e \csc(dx + c)} \sqrt{\sin(dx + c)}}$$

command

```
integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) + 3 \sqrt{-2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) - i \sin(dx + c))) \right)$$

15 ad

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{e \csc(dx + c)}}{ae^3 \csc(dx + c)^3 \sec(dx + c) + ae^3 \csc(dx + c)^3}, x \right)$$

65.39 Problem number 299

$$\int \frac{1}{(e \csc(c + dx))^{7/2} (a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cos(dx + c)}{21ad e^3 \sqrt{e \csc(dx + c)}} + \frac{2(\cos^3(dx + c))}{7ad e^3 \sqrt{e \csc(dx + c)}} + \frac{2(\sin^2(dx + c))}{5ad e^3 \sqrt{e \csc(dx + c)}} \\ & + 4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right) \\ & + \frac{21 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) ad e^3 \sqrt{e \csc(dx + c)} \sqrt{\sin(dx + c)}}{21 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) ad e^3 \sqrt{e \csc(dx + c)} \sqrt{\sin(dx + c)}} \end{aligned}$$

command

```
integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(15 \cos(dx + c)^3 - 21 \cos(dx + c)^2 - 5 \cos(dx + c) + 21 \right) \sqrt{\sin(dx + c)} + 5i \sqrt{2i} \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

105 ad

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{e \csc(dx + c)}}{ae^4 \csc(dx + c)^4 \sec(dx + c) + ae^4 \csc(dx + c)^4}, x \right)$$

65.40 Problem number 300

$$\int \frac{(e \csc(c + dx))^{5/2}}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{4e^2 \cot(dx+c) \sqrt{e \csc(dx+c)}}{231a^2d} + \frac{16e^2 \cot(dx+c) (\csc^2(dx+c)) \sqrt{e \csc(dx+c)}}{77a^2d} \\
 & - \frac{2e^2 (\cot^3(dx+c)) (\csc^2(dx+c)) \sqrt{e \csc(dx+c)}}{11a^2d} - \frac{4e^2 (\csc^3(dx+c)) \sqrt{e \csc(dx+c)}}{7a^2d} \\
 & - \frac{2e^2 \cot(dx+c) (\csc^4(dx+c)) \sqrt{e \csc(dx+c)}}{11a^2d} + \frac{4e^2 (\csc^5(dx+c)) \sqrt{e \csc(dx+c)}}{11a^2d} \\
 & - \frac{4e^2 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \csc(dx+c)} \left(\sqrt{\sin(dx+c)}\right)}{231 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) a^2d}
 \end{aligned}$$

command

```
integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2i} \left(i \cos(dx+c)^2 e^{\frac{5}{2}} + 2i \cos(dx+c) e^{\frac{5}{2}} + i e^{\frac{5}{2}} \right) \sin(dx+c) \operatorname{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \csc(dx+c)} e^2 \csc(dx+c)^2}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

65.41 Problem number 301

$$\int \frac{(e \csc(c+dx))^{3/2}}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{4e \cos(dx+c) \sqrt{e \csc(dx+c)}}{15a^2d} + \frac{16e \cot(dx+c) \csc(dx+c) \sqrt{e \csc(dx+c)}}{45a^2d} \\
 & - \frac{2e (\cot^3(dx+c)) \csc(dx+c) \sqrt{e \csc(dx+c)}}{9a^2d} - \frac{4e (\csc^2(dx+c)) \sqrt{e \csc(dx+c)}}{5a^2d} \\
 & - \frac{2e \cot(dx+c) (\csc^3(dx+c)) \sqrt{e \csc(dx+c)}}{9a^2d} + \frac{4e (\csc^4(dx+c)) \sqrt{e \csc(dx+c)}}{9a^2d} \\
 & + \frac{4e \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right) \sqrt{e \csc(dx+c)} \left(\sqrt{\sin(dx+c)}\right)}{15 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) a^2d}
 \end{aligned}$$

command

`integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{2i} \left(\cos(dx+c)^2 e^{\frac{3}{2}} + 2 \cos(dx+c) e^{\frac{3}{2}} + e^{\frac{3}{2}} \right) \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx+c))) \right) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \csc(dx+c)} e \csc(dx+c)}{a^2 \sec(dx+c)^2 + 2 a^2 \sec(dx+c) + a^2}, x \right)$$

65.42 Problem number 302

$$\int \frac{\sqrt{e \csc(c+dx)}}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16 \cot(dx+c) \sqrt{e \csc(dx+c)}}{21a^2d} - \frac{2(\cot^3(dx+c)) \sqrt{e \csc(dx+c)}}{7a^2d} \\ & - \frac{4 \csc(dx+c) \sqrt{e \csc(dx+c)}}{3a^2d} - \frac{2 \cot(dx+c) (\csc^2(dx+c)) \sqrt{e \csc(dx+c)}}{7a^2d} \\ & + \frac{4(\csc^3(dx+c)) \sqrt{e \csc(dx+c)}}{7a^2d} \\ & - \frac{20 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \text{EllipticF} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right) \sqrt{e \csc(dx+c)} \left(\sqrt{\sin(dx+c)} \right)}{21 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) a^2d} \end{aligned}$$

command

`integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \sqrt{2i} \left(i \cos(dx+c)^2 e^{\frac{1}{2}} + 2i \cos(dx+c) e^{\frac{1}{2}} + i e^{\frac{1}{2}} \right) \text{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c)) \right) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \csc(dx+c)}}{a^2 \sec(dx+c)^2 + 2 a^2 \sec(dx+c) + a^2}, x \right)$$

65.43 Problem number 303

$$\int \frac{1}{\sqrt{e \csc(c + dx)} (a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16 \cot(dx + c)}{5a^2 d \sqrt{e \csc(dx + c)}} - \frac{2(\cot^3(dx + c))}{5a^2 d \sqrt{e \csc(dx + c)}} - \frac{4 \csc(dx + c)}{a^2 d \sqrt{e \csc(dx + c)}} \\ & - \frac{2 \cot(dx + c) (\csc^2(dx + c))}{5a^2 d \sqrt{e \csc(dx + c)}} + \frac{4(\csc^3(dx + c))}{5a^2 d \sqrt{e \csc(dx + c)}} \\ & - \frac{28 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) a^2 d \sqrt{e \csc(dx + c)} \sqrt{\sin(dx + c)}} \end{aligned}$$

command

`integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(7 \sqrt{2i} (\cos(dx + c) + 1) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))) + 7 \sqrt{-2i} \right) \\ \hline 5 \left(a^2 d \cos(d \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \csc(dx + c)}}{a^2 e \csc(dx + c) \sec(dx + c)^2 + 2 a^2 e \csc(dx + c) \sec(dx + c) + a^2 e \csc(dx + c)}, x\right)$$

65.44 Problem number 304

$$\int \frac{1}{(e \csc(c + dx))^{3/2} (a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4}{a^2 d e \sqrt{e \csc(dx + c)}} - \frac{4 \cos(dx + c)}{3a^2 d e \sqrt{e \csc(dx + c)}} - \frac{2 \cos(dx + c) (\cot^2(dx + c))}{3a^2 d e \sqrt{e \csc(dx + c)}} \\ & - \frac{2 \cot(dx + c) \csc(dx + c)}{3a^2 d e \sqrt{e \csc(dx + c)}} + \frac{4(\csc^2(dx + c))}{3a^2 d e \sqrt{e \csc(dx + c)}} \\ & + \frac{4 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) a^2 d e \sqrt{e \csc(dx + c)} \sqrt{\sin(dx + c)}} \end{aligned}$$

command

```
integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \sqrt{2i} (-i \cos(dx+c) - i) \text{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c)) + 3 \sqrt{-2i} (i \cos(dx+c) + i \sin(dx+c)) \right)}{3 \left(a^2 d \cos(dx+c) + \dots \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \csc(dx+c)}}{a^2 e^2 \csc(dx+c)^2 \sec(dx+c)^2 + 2 a^2 e^2 \csc(dx+c)^2 \sec(dx+c) + a^2 e^2 \csc(dx+c)^2}, x \right)$$

65.45 Problem number 305

$$\int \frac{1}{(e \csc(c+dx))^{5/2} (a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cot(dx+c)}{a^2 d e^2 \sqrt{e \csc(dx+c)}} - \frac{2(\cos^2(dx+c)) \cot(dx+c)}{a^2 d e^2 \sqrt{e \csc(dx+c)}} + \frac{4 \csc(dx+c)}{a^2 d e^2 \sqrt{e \csc(dx+c)}} \\ & + \frac{4 \sin(dx+c)}{3 a^2 d e^2 \sqrt{e \csc(dx+c)}} - \frac{12 \cos(dx+c) \sin(dx+c)}{5 a^2 d e^2 \sqrt{e \csc(dx+c)}} \\ & + \frac{44 \sqrt{\frac{1}{2} + \frac{\sin(dx+c)}{2}} \text{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right)}{5 \sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) a^2 d e^2 \sqrt{e \csc(dx+c)} \sqrt{\sin(dx+c)}} \end{aligned}$$

command

```
integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(33 \sqrt{2i} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx+c) + i \sin(dx+c))) + 33 \sqrt{-2i} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx+c) - i \sin(dx+c))) \right)}{15 a^2 d \cos(dx+c) + \dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{e \csc(dx+c)}}{a^2 e^3 \csc(dx+c)^3 \sec(dx+c)^2 + 2 a^2 e^3 \csc(dx+c)^3 \sec(dx+c) + a^2 e^3 \csc(dx+c)^3}, x \right)$$

65.46 Problem number 306

$$\int \frac{1}{(e \csc(c + dx))^{7/2} (a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4}{a^2 d e^3 \sqrt{e \csc(dx + c)}} + \frac{26 \cos(dx + c)}{21 a^2 d e^3 \sqrt{e \csc(dx + c)}} + \frac{2(\cos^3(dx + c))}{7 a^2 d e^3 \sqrt{e \csc(dx + c)}} \\ & + \frac{4(\sin^2(dx + c))}{5 a^2 d e^3 \sqrt{e \csc(dx + c)}} - \frac{52 \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right)}{21 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) a^2 d e^3 \sqrt{e \csc(dx + c)} \sqrt{\sin(dx + c)}} \end{aligned}$$

command

```
integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(15 \cos(dx + c)^3 - 42 \cos(dx + c)^2 + 65 \cos(dx + c) - 168 \right) \sqrt{\sin(dx + c)} - 65i \sqrt{2i} \operatorname{weierstrassPInverse}(4, \dots) \right) / 105 a^2 d$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{e \csc(dx + c)}}{a^2 e^4 \csc(dx + c)^4 \sec(dx + c)^2 + 2 a^2 e^4 \csc(dx + c)^4 \sec(dx + c) + a^2 e^4 \csc(dx + c)^4}, x\right)$$

66 Test file number 122

Test folder name:

test_cases/4_Trig_functions/4.5_Secant/122_4.5.2.3-g_sec~p-a+b_sec~m-c+d_sec~n

66.1 Problem number 277

$$\int \frac{(g \sec(e + fx))^{3/2}}{\sqrt{a + b \sec(e + fx)} (c + c \sec(e + fx))} dx$$

Optimal antiderivative

$$\frac{g(b+a \cos (fx+e)) \sin (fx+e) \sqrt{g \sec (fx+e)}}{(a-b) f(c+c \cos (fx+e)) \sqrt{a+b \sec (fx+e)}} + \frac{g(b+a \cos (fx+e)) \sqrt{\frac{\cos (fx+e)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{g \sec (fx+e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) (a-b) c f \sqrt{\frac{b+a \cos (fx+e)}{a+b}} \sqrt{a+b \sec (fx+e)}} + \frac{g \sqrt{\frac{\cos (fx+e)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{fx}{2} + \frac{e}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos (fx+e)}{a+b}} \sqrt{g \sec (fx+e)}}{\cos\left(\frac{fx}{2} + \frac{e}{2}\right) c f \sqrt{a+b \sec (fx+e)}}$$

command

```
integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 a g \sqrt{\frac{a \cos (fx+e)+b}{\cos (fx+e)}} \sqrt{\frac{g}{\cos (fx+e)}} \cos (fx+e) \sin (fx+e) + \sqrt{2} (i(3 a-2 b) g \cos (fx+e) + i(3 a-2 b))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sec (fx+e)+a} \sqrt{g \sec (fx+e)} g \sec (fx+e)}{bc \sec (fx+e)^2+(a+b) c \sec (fx+e)+ac}, x\right)$$

67 Test file number 123

Test folder name:

test_cases/4_Trig_functions/4.5_Secant/123_4.5.3.1-a+b_sec~m-d_sec~n-A+B_sec-

67.1 Problem number 1

$$\int (b \sec (c+dx))^{5/2} (A+B \sec (c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ab(b \sec(dx+c))^{\frac{3}{2}} \sin(dx+c)}{3d} + \frac{2B(b \sec(dx+c))^{\frac{5}{2}} \sin(dx+c)}{5d} \\ & - \frac{6b^3B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} \\ & + \frac{6b^2B \sin(dx+c) \sqrt{b \sec(dx+c)}}{5d} \\ & + \frac{2Ab^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} Ab^{\frac{5}{2}} \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} Ab^{\frac{5}{2}} \cos(dx+c)^2 \operatorname{weier}$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \sec(dx+c)^3 + Ab^2 \sec(dx+c)^2\right) \sqrt{b \sec(dx+c)}, x\right)$$

67.2 Problem number 2

$$\int (b \sec(c+dx))^{3/2} (A + B \sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(b \sec(dx+c))^{\frac{3}{2}} \sin(dx+c)}{3d} - \frac{2Ab^2 \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} \\ & + \frac{2Ab \sin(dx+c) \sqrt{b \sec(dx+c)}}{d} \\ & + \frac{2bB \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B b^{\frac{3}{2}} \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B b^{\frac{3}{2}} \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb \sec(dx + c)^2 + Ab \sec(dx + c)\right) \sqrt{b \sec(dx + c)}, x\right)$$

67.3 Problem number 3

$$\int \sqrt{b \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2B \sin(dx + c) \sqrt{b \sec(dx + c)}}{d} \\ & + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} A \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((B \sec(dx + c) + A) \sqrt{b \sec(dx + c)}, x\right)$$

67.4 Problem number 4

$$\int \frac{A + B \sec(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd}$$

command

`integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c)}}{b \sec(dx + c)}, x\right)$$

67.5 Problem number 5

$$\int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2A \sin(dx + c)}{3bd \sqrt{b \sec(dx + c)}} + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d}$$

command

`integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2A\sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) - i\sqrt{2}A\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c)}}{b^2 \sec(dx+c)^2}, x\right)$$

67.6 Problem number 6

$$\int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A \sin(dx+c)}{5bd(b \sec(dx+c))^{\frac{3}{2}}} + \frac{2B \sin(dx+c)}{3b^2d\sqrt{b \sec(dx+c)}} \\ & + \frac{6A\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2d\sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} \\ & + \frac{2B\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}B\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i\sqrt{2}B\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c)}}{b^3 \sec(dx+c)^3}, x\right)$$

67.7 Problem number 179

$$\int \sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(7A + 5B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} + \frac{2a(A + B) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2aB \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{6a(A + B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{6a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(7A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (7A + 5B)a \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (7A + 5B)a \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Ba \sec(dx + c)^4 + (A + B)a \sec(dx + c)^3 + Aa \sec(dx + c)^2 \right) \sqrt{\sec(dx + c)}, x \right)$$

67.8 Problem number 180

$$\int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(A+B)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3d} + \frac{2aB\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{5d} \\ & + \frac{2a(5A+3B)\sin(dx+c)\left(\sqrt{\sec(dx+c)}\right)}{5d} \\ & - \frac{2a(5A+3B)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos(dx+c)}\right)\left(\sqrt{\sec(dx+c)}\right)}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2a(A+B)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos(dx+c)}\right)\left(\sqrt{\sec(dx+c)}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(A+B)a\cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 5i\sqrt{2}(A+B)a\cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba\sec(dx+c)^3 + (A+B)a\sec(dx+c)^2 + Aa\sec(dx+c)\right)\sqrt{\sec(dx+c)}, x\right)$$

67.9 Problem number 181

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))(A+B\sec(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aB\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3d} + \frac{2a(A+B)\sin(dx+c)\left(\sqrt{\sec(dx+c)}\right)}{d} \\ & - \frac{2a(A+B)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos(dx+c)}\right)\left(\sqrt{\sec(dx+c)}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2a(3A+B)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos(dx+c)}\right)\left(\sqrt{\sec(dx+c)}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}(3A+B)a\cos(dx+c)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+B)a\cos(dx+c)$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ba\sec(dx+c)^2+(A+B)a\sec(dx+c)+Aa\right)\sqrt{\sec(dx+c)},x\right)$$

67.10 Problem number 182

$$\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aB\sin(dx+c)(\sqrt{\sec(dx+c)})}{d} \\ & + \frac{2a(A-B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(A+B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}(A+B)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(A+B)a\text{weierstrassPInverse}(-4,$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ba\sec(dx+c)^2+(A+B)a\sec(dx+c)+Aa}{\sqrt{\sec(dx+c)}},x\right)$$

67.11 Problem number 183

$$\int \frac{(a + a \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2Aa \sqrt{\cos(dx + c)} \sin(dx + c) - i \sqrt{2} (A + 3B) \operatorname{awierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

67.12 Problem number 184

$$\int \frac{(a + a \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(A + B) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(3A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} (A + B) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (A + B) \text{aweierstrassPInverse}(-$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ba \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

67.13 Problem number 185

$$\int \frac{(a + a \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a(A + B) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(5A + 7B) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{6a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A + 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} (5A + 7B) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (5A + 7B) \text{aweierstrassPInverse}(-$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ba \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

67.14 Problem number 186

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(7A+6B)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{21d} + \frac{2a^2(7A+9B)\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{35d} \\ & + \frac{2B\left(\sec^{\frac{5}{2}}(dx+c)\right)(a^2+a^2\sec(dx+c))\sin(dx+c)}{7d} \\ & + \frac{4a^2(4A+3B)\sin(dx+c)(\sqrt{\sec}(dx+c))}{5d} \\ & - \frac{4a^2(4A+3B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos}(dx+c))(\sqrt{\sec}(dx+c))}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{4a^2(7A+6B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos}(dx+c))(\sqrt{\sec}(dx+c))}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(5i\sqrt{2}(7A+6B)a^2\cos(dx+c)^3\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(7A+6B)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^2\sec(dx+c)^4+(A+2B)a^2\sec(dx+c)^3+(2A+B)a^2\sec(dx+c)^2+Aa^2\sec(dx+c)\right)\sqrt{\sec(dx+c)}\right)$$

67.15 Problem number 187

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2(A+B\sec(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(5A+7B) \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15d} + \frac{2B \left(\sec^{\frac{3}{2}}(dx+c)\right) (a^2 + a^2 \sec(dx+c)) \sin(dx+c)}{5d} \\ & + \frac{4a^2(5A+4B) \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d} \\ & - \frac{4a^2(5A+4B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(2A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (2A+B)a^2 \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} (2A+B)a^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^2 \sec(dx+c)^3 + (A+2B)a^2 \sec(dx+c)^2 + (2A+B)a^2 \sec(dx+c) + Aa^2\right) \sqrt{\sec(dx+c)}, x\right)$$

67.16 Problem number 188

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(3A+5B) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3d} + \frac{2B(a^2 + a^2 \sec(dx+c)) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3d} \\ & - \frac{4a^2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(3A+2B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} (3A + 2B)a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} (3A + 2B)a^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\sqrt{\sec(dx + c)}}, x \right)$$

67.17 Problem number 189

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} - \frac{2a^2(A - 3B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & + \frac{4a^2 A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^2(2A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} (2A + 3B)a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} (2A + 3B)a^2 \operatorname{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{3}{2}}}, x \right)$$

67.18 Problem number 190

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a^2(7A + 5B) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(4A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(A + 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A + 2B) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + 2B) a^2 \operatorname{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

67.19 Problem number 191

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(9A + 7B) \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{2A(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^2(6A + 7B) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(3A + 4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(6A + 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (6A + 7B)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (6A + 7B)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{7}{2}}}, x \right)$$

67.20 Problem number 192

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(11A + 9B) \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^2(8A + 9B) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2A(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{4a^2(5A + 6B) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(8A + 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(5A + 6B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (5A + 6B)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 6B)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{9}{2}}}, x \right)$$

67.21 Problem number 193

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(13A+11B)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{21d} + \frac{4a^3(24A+23B)\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{105d} \\ & + \frac{2aB\left(\sec^{\frac{5}{2}}(dx+c)\right)(a+a\sec(dx+c))^2\sin(dx+c)}{9d} \\ & + \frac{2(9A+13B)\left(\sec^{\frac{5}{2}}(dx+c)\right)(a^3+a^3\sec(dx+c))\sin(dx+c)}{63d} \\ & + \frac{4a^3(21A+17B)\sin(dx+c)(\sqrt{\sec(dx+c)})}{15d} \\ & - \frac{4a^3(21A+17B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{4a^3(13A+11B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (13A+11B)a^3 \cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 15i \sqrt{2} (13A \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^3 \sec(dx+c)^5 + (A+3B)a^3 \sec(dx+c)^4 + 3(A+B)a^3 \sec(dx+c)^3 + (3A+B)a^3 \sec(dx+c)^2 + \right.$$

67.22 Problem number 194

$$\int \sqrt{\sec(c+dx)} (a+a\sec(c+dx))^3 (A+B\sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(42A+41B) \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{105d} \\ & + \frac{2aB \left(\sec^{\frac{3}{2}}(dx+c)\right) (a+a\sec(dx+c))^2 \sin(dx+c)}{7d} \\ & + \frac{2(7A+11B) \left(\sec^{\frac{3}{2}}(dx+c)\right) (a^3+a^3\sec(dx+c)) \sin(dx+c)}{35d} \\ & + \frac{4a^3(9A+7B) \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d} \\ & - \frac{4a^3(9A+7B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(21A+13B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (21A+13B)a^3 \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} (21A+ \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^3 \sec(dx+c)^4 + (A+3B)a^3 \sec(dx+c)^3 + 3(A+B)a^3 \sec(dx+c)^2 + (3A+B)a^3 \sec(dx+c) + \dots\right)\right)$$

67.23 Problem number 195

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(20A + 21B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & + \frac{2aB(a + a \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & + \frac{2(5A + 9B) (a^3 + a^3 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{4a^3(5A + 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(5A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (5A + 3B) a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (5A + 3B) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^3 \sec(dx + c)^4 + (A + 3B)a^3 \sec(dx + c)^3 + 3(A + B)a^3 \sec(dx + c)^2 + (3A + B)a^3 \sec(dx + c) + Aa^3}{\sqrt{\sec(dx + c)}}\right)$$

67.24 Problem number 196

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA(a + a \sec(dx + c))^2 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{4a^3(A + 4B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & - \frac{2(A - B)(a^3 + a^3 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & + \frac{4a^3(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{20a^3(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A + B)a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + B)a^3 \cos \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^3 \sec(dx + c)^4 + (A + 3B)a^3 \sec(dx + c)^3 + 3(A + B)a^3 \sec(dx + c)^2 + (3A + B)a^3 \sec(dx + c) + A}{\sec(dx + c)^{\frac{3}{2}}}\right)$$

67.25 Problem number 197

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA(a + a \sec(dx + c))^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(9A + 5B)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & - \frac{4a^3(6A - 5B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & + \frac{4a^3(9A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(3A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (3A + 5B)a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + 5B)a^3 \text{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ba^3 \sec(dx + c)^4 + (A + 3B)a^3 \sec(dx + c)^3 + 3(A + B)a^3 \sec(dx + c)^2 + (3A + B)a^3 \sec(dx + c) + Aa^3}{\sec(dx + c)^{\frac{5}{2}}} \right)$$

67.26 Problem number 198

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA(a + a \sec(dx + c))^2 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(11A + 7B)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{4a^3(41A + 42B) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(7A + 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(13A + 21B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (13A + 21B)a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (13A + 21B)a^3 \text{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ba^3 \sec(dx + c)^4 + (A + 3B)a^3 \sec(dx + c)^3 + 3(A + B)a^3 \sec(dx + c)^2 + (3A + B)a^3 \sec(dx + c) + Aa^3}{\sec(dx + c)^{\frac{7}{2}}} \right)$$

67.27 Problem number 199

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(23A + 24B) \sin(dx + c)}{105d \sec(dx + c)^{\frac{3}{2}}} + \frac{2aA(a + a \sec(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} \\ & + \frac{2(13A + 9B)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^3(11A + 13B) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(17A + 21B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(11A + 13B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (11A + 13B) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (11A + 13B) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^3 \sec(dx + c)^4 + (A + 3B)a^3 \sec(dx + c)^3 + 3(A + B)a^3 \sec(dx + c)^2 + (3A + B)a^3 \sec(dx + c) + Aa^3}{\sec(dx + c)^{\frac{9}{2}}}\right)$$

67.28 Problem number 200

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{20a^3(21A + 22B) \sin(dx + c)}{693d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^3(15A + 17B) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2aA(a + a \sec(dx + c))^2 \sin(dx + c)}{11d \sec(dx + c)^{\frac{9}{2}}} \\
& + \frac{2(15A + 11B)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{99d \sec(dx + c)^{\frac{7}{2}}} + \frac{4a^3(105A + 121B) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\
& + \frac{4a^3(15A + 17B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(105A + 121B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (105A + 121B) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (105A + 121B) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^3 \sec(dx + c)^4 + (A + 3B)a^3 \sec(dx + c)^3 + 3(A + B)a^3 \sec(dx + c)^2 + (3A + B)a^3 \sec(dx + c) + Aa^3}{\sec(dx + c)^{\frac{11}{2}}}\right)$$

67.29 Problem number 201

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)(A + B \sec(c + dx))}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5(A-B)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3ad} - \frac{(5A-7B)\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{5ad} \\ & + \frac{(A-B)\left(\sec^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{d(a+a\sec(dx+c))} - \frac{3(5A-7B)\sin(dx+c)(\sqrt{\sec(dx+c)})}{5ad} \\ & + \frac{3(5A-7B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \\ & + \frac{5(A-B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \end{aligned}$$

command

`integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25\left(\sqrt{2}(iA-iB)\cos(dx+c)^3+\sqrt{2}(iA-iB)\cos(dx+c)^2\right)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B\sec(dx+c)^4+A\sec(dx+c)^3\right)\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a},x\right)$$

67.30 Problem number 202

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A-5B)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3ad} \\ & + \frac{(A-B)\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{d(a+a\sec(dx+c))} + \frac{3(A-B)\sin(dx+c)(\sqrt{\sec(dx+c)})}{ad} \\ & - \frac{3(A-B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \\ & - \frac{(3A-5B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (3i A - 5i B) \cos(dx + c)^2 + \sqrt{2} (3i A - 5i B) \cos(dx + c)\right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(B \sec(dx + c)^3 + A \sec(dx + c)^2\right) \sqrt{\sec(dx + c)}}{a \sec(dx + c) + a}, x\right)$$

67.31 Problem number 203

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{d(a + a \sec(dx + c))} - \frac{(A - 3B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{ad} \\ & + \frac{(A - 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-i A + i B) \cos(dx + c) + \sqrt{2} (-i A + i B)\right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \left(\sqrt{2} \cos(dx + c)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(B \sec(dx + c)^2 + A \sec(dx + c)\right) \sqrt{\sec(dx + c)}}{a \sec(dx + c) + a}, x\right)$$

67.32 Problem number 204

$$\int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A-B) \sin(dx+c) (\sqrt{\sec(dx+c)})}{d(a+a \sec(dx+c))} \\ & - \frac{(A-B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A-B) \sqrt{\cos(dx+c)} \sin(dx+c) + \left(\sqrt{2}(-iA-iB) \cos(dx+c) + \sqrt{2}(-iA-iB)\right) \operatorname{weierstrassPInverse}(-4$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx+c) + A) \sqrt{\sec(dx+c)}}{a \sec(dx+c) + a}, x\right)$$

67.33 Problem number 205

$$\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+a \sec(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A-B) \sin(dx+c) (\sqrt{\sec(dx+c)})}{d(a+a \sec(dx+c))} \\ & + \frac{(3A-B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A-B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A-B)\sqrt{\cos(dx+c)}\sin(dx+c) - \left(\sqrt{2}(iA-iB)\cos(dx+c) + \sqrt{2}(iA-iB)\right)\text{weierstrassPInverse}(-4, \dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a\sec(dx+c)^2+a\sec(dx+c)}, x\right)$$

67.34 Problem number 206

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} dx$$

Optimal antiderivative

$$\frac{(5A-3B)\sin(dx+c)}{3ad\sqrt{\sec(dx+c)}} - \frac{(A-B)\sin(dx+c)}{d(a+a\sec(dx+c))\sqrt{\sec(dx+c)}}$$

$$\frac{3(A-B)\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad}$$

$$+ \frac{(5A-3B)\sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad}$$

command

```
integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-5iA+3iB)\cos(dx+c) + \sqrt{2}(-5iA+3iB)\right)\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + \dots$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a\sec(dx+c)^3+a\sec(dx+c)^2}, x\right)$$

67.35 Problem number 207

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(7A - 5B) \sin(dx + c)}{5ad \sec(dx + c)^{\frac{3}{2}}} - \frac{(A - B) \sin(dx + c)}{d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))} - \frac{5(A - B) \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}} \\ & + \frac{3(7A - 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{5(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25 \left(\sqrt{2} (-iA + iB) \cos(dx + c) + \sqrt{2} (-iA + iB) \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a \sec(dx + c)^4 + a \sec(dx + c)^3}, x\right)$$

67.36 Problem number 208

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(9A - 7B) \sin(dx + c)}{7ad \sec(dx + c)^{\frac{5}{2}}} - \frac{7(A - B) \sin(dx + c)}{5ad \sec(dx + c)^{\frac{3}{2}}} \\ & - \frac{(A - B) \sin(dx + c)}{d \sec(dx + c)^{\frac{5}{2}} (a + a \sec(dx + c))} + \frac{5(9A - 7B) \sin(dx + c)}{21ad \sqrt{\sec(dx + c)}} \\ & - \frac{21(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5(9A - 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25 \left(\sqrt{2} (9i A - 7i B) \cos(dx + c) + \sqrt{2} (9i A - 7i B) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a \sec(dx + c)^5 + a \sec(dx + c)^4}, x \right)$$

67.37 Problem number 209

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{5(A - 2B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d} + \frac{(4A - 7B) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d(1 + \sec(dx + c))} \\ & + \frac{(A - B) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{3d(a + a \sec(dx + c))^2} + \frac{(4A - 7B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{a^2d} \\ & - \frac{(4A - 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \\ & - \frac{5(A - 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-i A + 2i B) \cos(dx + c)^3 + 2 \sqrt{2} (-i A + 2i B) \cos(dx + c)^2 + \sqrt{2} (-i A + 2i B) \cos(dx + c) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \sec(dx + c)^4 + A \sec(dx + c)^3) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}, x \right)$$

67.38 Problem number 210

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2A-5B)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3a^2d(1+\sec(dx+c))} + \frac{(A-B)\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{3d(a+a\sec(dx+c))^2} \\ & - \frac{(A-4B)\sin(dx+c)(\sqrt{\sec(dx+c)})}{a^2d} \\ & + \frac{(A-4B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d} \\ & + \frac{(2A-5B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{2}(-2iA+5iB)\cos(dx+c)^2-2\sqrt{2}(2iA-5iB)\cos(dx+c)+\sqrt{2}(-2iA+5iB)\right)\operatorname{weierstrassPInverse}(-4$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B\sec(dx+c)^3+A\sec(dx+c)^2\right)\sqrt{\sec(dx+c)}}{a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2},x\right)$$

67.39 Problem number 211

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d(a + a \sec(dx + c))^2} - \frac{B \sin(dx + c) (\sqrt{\sec(dx + c)})}{a^2 d (1 + \sec(dx + c))} \\ & + \frac{B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(A + 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-i A - 2i B) \cos(dx + c)^2 - 2 \sqrt{2} (i A + 2i B) \cos(dx + c) + \sqrt{2} (-i A - 2i B) \right) \operatorname{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B \sec(dx + c)^2 + A \sec(dx + c)\right) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}, x\right)$$

67.40 Problem number 212

$$\int \frac{\sqrt{\sec(c + dx)} (A + B \sec(c + dx))}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2A + B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3a^2 d (1 + \sec(dx + c))} + \frac{(A - B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d(a + a \sec(dx + c))^2} \\ & - \frac{A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(2A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-2iA - iB)\cos(dx + c)^2 - 2\sqrt{2}(2iA + iB)\cos(dx + c) + \sqrt{2}(-2iA - iB)\right)\text{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B\sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a^2\sec(dx + c)^2 + 2a^2\sec(dx + c) + a^2}, x\right)$$

67.41 Problem number 213

$$\int \frac{A + B\sec(c + dx)}{\sqrt{\sec(c + dx)}(a + a\sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(5A - 2B)\sin(dx + c)(\sqrt{\sec(dx + c)})}{3a^2d(1 + \sec(dx + c))} - \frac{(A - B)\sin(dx + c)(\sqrt{\sec(dx + c)})}{3d(a + a\sec(dx + c))^2} \\ & + \frac{(4A - B)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^2d} \\ & - \frac{(5A - 2B)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^2d} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(5iA - 2iB)\cos(dx + c)^2 - 2\sqrt{2}(-5iA + 2iB)\cos(dx + c) + \sqrt{2}(5iA - 2iB)\right)\text{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B\sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a^2\sec(dx + c)^3 + 2a^2\sec(dx + c)^2 + a^2\sec(dx + c)}, x\right)$$

67.42 Problem number 214

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5(2A - B) \sin(dx + c)}{3a^2 d \sqrt{\sec(dx + c)}} - \frac{(7A - 4B) \sin(dx + c)}{3a^2 d (1 + \sec(dx + c)) \sqrt{\sec(dx + c)}} \\ & - \frac{(A - B) \sin(dx + c)}{3d (a + a \sec(dx + c))^2 \sqrt{\sec(dx + c)}} \\ & - \frac{(7A - 4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{5(2A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (2iA - iB) \cos(dx + c)^2 + 2\sqrt{2} (2iA - iB) \cos(dx + c) + \sqrt{2} (2iA - iB) \right) \operatorname{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^4 + 2a^2 \sec(dx + c)^3 + a^2 \sec(dx + c)^2}, x\right)$$

67.43 Problem number 215

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{7(8A - 5B) \sin(dx + c)}{15a^2 d \sec(dx + c)^{\frac{3}{2}} (A - B) \sin(dx + c)} - \frac{(3A - 2B) \sin(dx + c)}{a^2 d \sec(dx + c)^{\frac{3}{2}} (1 + \sec(dx + c))} - \frac{5(3A - 2B) \sin(dx + c)}{3d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))^2} - \frac{3a^2 d \sqrt{\sec(dx + c)}}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d}$$

$$+ \frac{7(8A - 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d}$$

$$- \frac{5(3A - 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d}$$

command

```
integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25 \left(\sqrt{2} (-3i A + 2i B) \cos(dx + c)^2 + 2 \sqrt{2} (-3i A + 2i B) \cos(dx + c) + \sqrt{2} (-3i A + 2i B) \right) \operatorname{weierstrassPInve}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^5 + 2a^2 \sec(dx + c)^4 + a^2 \sec(dx + c)^3}, x\right)$$

67.44 Problem number 216

$$\int \frac{\sec^{\frac{9}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(13A - 33B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{6a^3d} + \frac{(A - B) \left(\sec^{\frac{9}{2}}(dx + c) \right) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} \\
& + \frac{(A - 2B) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{3ad(a + a \sec(dx + c))^2} + \frac{7(7A - 17B) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{30d(a^3 + a^3 \sec(dx + c))} \\
& + \frac{7(7A - 17B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{10a^3d} \\
& - \frac{7(7A - 17B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\
& - \frac{(13A - 33B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d}
\end{aligned}$$

command

```
integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-13i A + 33i B) \cos(dx + c)^4 + 3 \sqrt{2} (-13i A + 33i B) \cos(dx + c)^3 + 3 \sqrt{2} (-13i A + 33i B) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B \sec(dx + c)^5 + A \sec(dx + c)^4\right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x\right)$$

67.45 Problem number 217

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} + \frac{(3A - 8B) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{15ad(a + a \sec(dx + c))^2} \\ & + \frac{(3A - 13B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{6d(a^3 + a^3 \sec(dx + c))} - \frac{(9A - 49B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{10a^3d} \\ & + \frac{(9A - 49B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & + \frac{(3A - 13B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (3i A - 13i B) \cos(dx + c)^3 + 3 \sqrt{2} (3i A - 13i B) \cos(dx + c)^2 + 3 \sqrt{2} (3i A - 13i B) \cos(dx + c) + \sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B \sec(dx + c)^4 + A \sec(dx + c)^3\right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x\right)$$

67.46 Problem number 218

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} + \frac{(A - 6B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{15ad(a + a \sec(dx + c))^2} \\ & - \frac{(A + 9B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{10d(a^3 + a^3 \sec(dx + c))} \\ & + \frac{(A + 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & + \frac{(A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (i A + 3i B) \cos(dx + c)^3 + 3 \sqrt{2} (i A + 3i B) \cos(dx + c)^2 + 3 \sqrt{2} (i A + 3i B) \cos(dx + c) + \sqrt{2} (i A + 3i B) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \sec(dx + c)^3 + A \sec(dx + c)^2 \right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x \right)$$

67.47 Problem number 219

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} - \frac{(A + 4B) \sin(dx + c) (\sqrt{\sec(dx + c)})^2}{15ad(a + a \sec(dx + c))^2} \\ & + \frac{(A + B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{6d(a^3 + a^3 \sec(dx + c))} \\ & - \frac{(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (i A + i B) \cos(dx + c)^3 + 3 \sqrt{2} (i A + i B) \cos(dx + c)^2 + 3 \sqrt{2} (i A + i B) \cos(dx + c) + \sqrt{2} (i A + i B) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \sec(dx + c)^2 + A \sec(dx + c) \right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x \right)$$

67.48 Problem number 220

$$\int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A-B) \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d (a+a \sec(dx+c))^3} + \frac{(3A+2B) \sin(dx+c) (\sqrt{\sec(dx+c)})}{15ad (a+a \sec(dx+c))^2} \\ & + \frac{(3A+B) \sin(dx+c) (\sqrt{\sec(dx+c)})}{6d (a^3+a^3 \sec(dx+c))} \\ & - \frac{(9A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (3i A + i B) \cos(dx+c)^3 + 3 \sqrt{2} (3i A + i B) \cos(dx+c)^2 + 3 \sqrt{2} (3i A + i B) \cos(dx+c) + \sqrt{2} (3i A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx+c) + A) \sqrt{\sec(dx+c)}}{a^3 \sec(dx+c)^3 + 3 a^3 \sec(dx+c)^2 + 3 a^3 \sec(dx+c) + a^3}, x\right)$$

67.49 Problem number 221

$$\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+a \sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d (a + a \sec(dx + c))^3} - \frac{(8A - 3B) \sin(dx + c) (\sqrt{\sec(dx + c)})^2}{15ad (a + a \sec(dx + c))^2} \\ & - \frac{(13A - 3B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{6d (a^3 + a^3 \sec(dx + c))} \\ & + \frac{(49A - 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(13A - 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-13i A + 3i B) \cos(dx + c)^3 + 3 \sqrt{2} (-13i A + 3i B) \cos(dx + c)^2 + 3 \sqrt{2} (-13i A + 3i B) \cos(dx + c) \right) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^4 + 3 a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + a^3 \sec(dx + c)}, x\right)$$

67.50 Problem number 222

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(33A - 13B) \sin(dx + c)}{6a^3 d \sqrt{\sec(dx + c)}} - \frac{(A - B) \sin(dx + c)}{5d (a + a \sec(dx + c))^3 \sqrt{\sec(dx + c)}} \\ & - \frac{(2A - B) \sin(dx + c)}{3ad (a + a \sec(dx + c))^2 \sqrt{\sec(dx + c)}} - \frac{7(17A - 7B) \sin(dx + c)}{30d (a^3 + a^3 \sec(dx + c)) \sqrt{\sec(dx + c)}} \\ & + \frac{7(17A - 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(33A - 13B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (33i A - 13i B) \cos(dx + c)^3 + 3 \sqrt{2} (33i A - 13i B) \cos(dx + c)^2 + 3 \sqrt{2} (33i A - 13i B) \cos(dx + c) + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^5 + 3 a^3 \sec(dx + c)^4 + 3 a^3 \sec(dx + c)^3 + a^3 \sec(dx + c)^2}, x \right)$$

67.51 Problem number 223

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(33A - 17B) \sin(dx + c)}{30a^3 d \sec(dx + c)^{\frac{3}{2}}} - \frac{(A - B) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))^3} \\ & - \frac{(12A - 7B) \sin(dx + c)}{15ad \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))^2} \\ & - \frac{3(21A - 11B) \sin(dx + c)}{10d \sec(dx + c)^{\frac{3}{2}} (a^3 + a^3 \sec(dx + c))} - \frac{(21A - 11B) \sin(dx + c)}{2a^3 d \sqrt{\sec(dx + c)}} \\ & + \frac{7(33A - 17B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & - \frac{(21A - 11B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \left(\sqrt{2} (-21i A + 11i B) \cos(dx + c)^3 + 3 \sqrt{2} (-21i A + 11i B) \cos(dx + c)^2 + 3 \sqrt{2} (-21i A + 11i B) \cos(dx + c) + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \sec(dx+c) + A) \sqrt{\sec(dx+c)}}{a^3 \sec(dx+c)^6 + 3 a^3 \sec(dx+c)^5 + 3 a^3 \sec(dx+c)^4 + a^3 \sec(dx+c)^3}, x\right)$$

67.52 Problem number 394

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))(A+B \sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab+Ba) \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3d} + \frac{2bB \left(\sec^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{5d} \\ & + \frac{2(5aA+3bB) \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d} \\ & - \frac{2(5aA+3bB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ab+Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i Ba + i Ab) \cos(dx+c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5 \sqrt{2} (-i Ba - i Ab) \cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb \sec(dx+c)^3 + Aa \sec(dx+c) + (Ba + Ab) \sec(dx+c)^2\right) \sqrt{\sec(dx+c)}, x\right)$$

67.53 Problem number 395

$$\int \sqrt{\sec(c+dx)} (a+b\sec(c+dx))(A+B\sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bB \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2(Ab+Ba) \sin(dx+c) (\sqrt{\sec(dx+c)})}{d} \\ & - \frac{2(Ab+Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(3aA+bB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3iAa - iBb) \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2} (3iAa + iBb) \cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Bb \sec^2(dx+c) + Aa + (Ba + Ab) \sec(dx+c) \right) \sqrt{\sec(dx+c)}, x \right)$$

67.54 Problem number 396

$$\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bB \sin(dx+c) (\sqrt{\sec(dx+c)})}{d} \\ & + \frac{2(aA-bB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(Ab+Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 B b \sin(dx+c)}{\sqrt{\cos(dx+c)}} + \sqrt{2} (-i B a - i A b) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2} (i B a + i A b) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{B b \sec(dx+c)^2 + A a + (B a + A b) \sec(dx+c)}{\sqrt{\sec(dx+c)}}, x\right)$$

67.55 Problem number 397

$$\int \frac{(a + b \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx+c)}{3d \sqrt{\sec(dx+c)}} \\ & + \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(aA + 3bB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 A a \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2} (-i A a - 3i B b) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2} (i A a + 3i B b) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{B b \sec(dx+c)^2 + A a + (B a + A b) \sec(dx+c)}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

67.56 Problem number 398

$$\int \frac{(a + b \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(Ab + Ba) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2(3aA + 5bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i Ba + i Ab) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-i Ba - i Ab) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

67.57 Problem number 399

$$\int \frac{(a + b \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(Ab + Ba) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(5aA + 7bB) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{6(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5aA + 7bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(5iAa + 7iBb)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-5iAa - 7iBb)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

67.58 Problem number 400

$$\int \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5b^2B + 7a(2Ab + Ba)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} \\ & + \frac{2b(7Ab + 9Ba) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2bB \left(\sec^{\frac{5}{2}}(dx + c)\right) (a + b \sec(dx + c)) \sin(dx + c)}{7d} \\ & + \frac{2(5a^2A + 3Ab^2 + 6abB) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2(5a^2A + 3Ab^2 + 6abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5b^2B + 7a(2Ab + Ba)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(7iBa^2 + 14iAab + 5iBb^2) \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-7iBa^2 - 14iAab - 5iBb^2) \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

integral($(Bb^2 \sec(dx + c)^4 + Aa^2 \sec(dx + c) + (2 Bab + Ab^2) \sec(dx + c)^3 + (Ba^2 + 2 Aab) \sec(dx + c)^2) \sqrt{\sec(dx + c)}$)

67.59 Problem number 401

$$\int \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2 (A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(5Ab + 7Ba) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} + \frac{2bB \left(\sec^{\frac{3}{2}}(dx + c)\right) (a + b \sec(dx + c)) \sin(dx + c)}{5d} \\ & + \frac{2(3b^2B + 5a(2Ab + Ba)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{2(3b^2B + 5a(2Ab + Ba)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3a^2A + Ab^2 + 2abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (3i Aa^2 + 2i Bab + i Ab^2) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-$$

Fricas 1.3.7 via sagemath 9.3 output

integral($(Bb^2 \sec(dx + c)^3 + Aa^2 + (2 Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2 Aab) \sec(dx + c)) \sqrt{\sec(dx + c)}$, x)

67.60 Problem number 402

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(3Ab + 5Ba) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} + \frac{2bB(a + b \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & + \frac{2(a^2A - Ab^2 - 2abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(6Aab + 3Ba^2 + b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i Ba^2 - 6i Aab - i Bb^2) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i Ba^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)}{\sqrt{\sec(dx + c)}}, x\right)$$

67.61 Problem number 403

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2A \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2b^2B \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & - \frac{2(b^2B - a(2Ab + Ba)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^2A + 3Ab^2 + 6abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i Aa^2 - 6i Bab - 3i Ab^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i Aa^2 + 6i Bab + 3i Ab^2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2 Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2 Aab) \sec(dx + c)}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

67.62 Problem number 404

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 A \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(2Ab + Ba) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2(3a^2 A + 5Ab^2 + 10abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(2Aab + B a^2 + 3b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i Ba^2 + 2i Aab + 3i Bb^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-i Ba^2 - 2i Aab - 3i Bb^2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2 Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2 Aab) \sec(dx + c)}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

67.63 Problem number 405

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 A \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a(2Ab + Ba) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(5a^2 A + 7A b^2 + 14abB) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(6Aab + 3B a^2 + 5b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5a^2 A + 7A b^2 + 14abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (5i Aa^2 + 14i Bab + 7i Ab^2) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-5i Aa^2 - 14i Bab - 7i Ab^2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2 Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2 Aab) \sec(dx + c)}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

67.64 Problem number 406

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2 A \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2a(2Ab + Ba) \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(7a^2 A + 9A b^2 + 18abB) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(7b^2 B + 5a(2Ab + Ba)) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(7a^2 A + 9A b^2 + 18abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7b^2 B + 5a(2Ab + Ba)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15\sqrt{2} (5i Ba^2 + 10i Aab + 7i Bb^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15\sqrt{2} (-5i Ba^2 -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2 Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2 Aab) \sec(dx + c)}{\sec(dx + c)^{\frac{9}{2}}}, x\right)$$

67.65 Problem number 407

$$\int \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(21Aa^2b + 5Ab^3 + 7a^3B + 15Ba^2b^2) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} \\ & + \frac{2b(27Aab + 22Ba^2 + 7b^2B) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{2b^2(9Ab + 13Ba) \left(\sec^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\ & + \frac{2bB \left(\sec^{\frac{5}{2}}(dx + c)\right) (a + b \sec(dx + c))^2 \sin(dx + c)}{9d} \\ & + \frac{2(15a^3A + 27Aa^2b + 27a^2bB + 7b^3B) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{2(15a^3A + 27Aa^2b + 27a^2bB + 7b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21Aa^2b + 5Ab^3 + 7a^3B + 15Ba^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15\sqrt{2} (7i Ba^3 + 21i Aa^2b + 15i Bab^2 + 5i Ab^3) \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb^3 \sec(dx + c)^5 + Aa^3 \sec(dx + c) + (3 Bab^2 + Ab^3) \sec(dx + c)^4 + 3 (Ba^2b + Aab^2) \sec(dx + c)^3 + \dots\right)\right)$$

67.66 Problem number 408

$$\int \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^3 (A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(21Aab + 18B a^2 + 5b^2B) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{21d} \\ & + \frac{2b^2(7Ab + 11Ba) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2bB \left(\sec^{\frac{3}{2}}(dx + c)\right) (a + b \sec(dx + c))^2 \sin(dx + c)}{7d} \\ & + \frac{2(15A a^2b + 3A b^3 + 5a^3B + 9Ba b^2) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2(15A a^2b + 3A b^3 + 5a^3B + 9Ba b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^3A + 21Aa b^2 + 21a^2bB + 5b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (21i Aa^3 + 21i Ba^2b + 21i Aab^2 + 5i Bb^3) \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb^3 \sec(dx + c)^4 + Aa^3 + (3 Bab^2 + Ab^3) \sec(dx + c)^3 + 3 (Ba^2b + Aab^2) \sec(dx + c)^2 + (Ba^3 + 3Aa^2b + \dots)\right)\right)$$

67.67 Problem number 409

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(5Ab + 9Ba) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{15d} \\ & + \frac{2b(15Aab + 14Ba^2 + 3b^2B) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & + \frac{2bB(a + b \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & + \frac{2(5a^3A - 15Aab^2 - 15a^2bB - 3b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(9Aa^2b + Ab^3 + 3a^3B + 3Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (3i Ba^3 + 9i Aa^2b + 3i Bab^2 + i Ab^3) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^3 \sec(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx + c)^3 + 3(Ba^2b + Aab^2) \sec(dx + c)^2 + (Ba^3 + 3Aa^2b) \sec(dx + c)}{\sqrt{\sec(dx + c)}}\right)$$

67.68 Problem number 410

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b^2(aA - bB) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d} + \frac{2aA(a + b \sec(dx + c))^2 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & - \frac{2b(2a^2A - 3Ab^2 - 9abB) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & + \frac{2(3Aa^2b - Ab^3 + a^3B - 3Ba^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^3A + 9Aab^2 + 9a^2bB + b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i Aa^3 - 9i Ba^2b - 9i Aab^2 - i Bb^3) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \dots$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^3 \sec(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx + c)^3 + 3(Ba^2b + Aab^2) \sec(dx + c)^2 + (Ba^3 + 3Aa^2b)}{\sec(dx + c)^{\frac{3}{2}}}\right)$$

67.69 Problem number 411

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA(a + b \sec(dx + c))^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a^2(9Ab + 5Ba) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & - \frac{2b^2(aA - 5bB) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & + \frac{2(3a^3A + 15Aa^2b + 15a^2bB - 5b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3Aa^2b + 3Ab^3 + a^3B + 9Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (iBa^3 + 3iAa^2b + 9iBab^2 + 3iAb^3) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5\sqrt{2} (-iE$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Bb^3 \sec(dx+c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx+c)^3 + 3(Ba^2b + Aab^2) \sec(dx+c)^2 + (Ba^3 + 3Aa^2}{\sec(dx+c)^{\frac{5}{2}}}$$

67.70 Problem number 412

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(11Ab + 7Ba) \sin(dx+c)}{35d \sec(dx+c)^{\frac{3}{2}}} + \frac{2aA(a + b \sec(dx+c))^2 \sin(dx+c)}{7d \sec(dx+c)^{\frac{5}{2}}} \\ & + \frac{2a(5a^2A + 18Ab^2 + 21abB) \sin(dx+c)}{21d \sqrt{\sec(dx+c)}} \\ & + \frac{2(9Aa^2b + 5Ab^3 + 3a^3B + 15Ba^2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5a^3A + 21Aa^2b^2 + 21a^2bB + 21b^3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (5iAa^3 + 21iBa^2b + 21iAab^2 + 21iBb^3) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5\sqrt{2} (-iE$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Bb^3 \sec(dx+c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx+c)^3 + 3(Ba^2b + Aab^2) \sec(dx+c)^2 + (Ba^3 + 3Aa^2}{\sec(dx+c)^{\frac{7}{2}}}$$

67.71 Problem number 413

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(13Ab + 9Ba) \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a(7a^2A + 22Ab^2 + 27abB) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2aA(a + b \sec(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2(15Aa^2b + 7Ab^3 + 5a^3B + 21Bab^2) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(7a^3A + 27Aab^2 + 27a^2bB + 15b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(15Aa^2b + 7Ab^3 + 5a^3B + 21Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (5i Ba^3 + 15i Aa^2b + 21i Bab^2 + 7i Ab^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^3 \sec(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx + c)^3 + 3(Ba^2b + Aab^2) \sec(dx + c)^2 + (Ba^3 + 3Aa^2b) \sec(dx + c) + A^2}{\sec(dx + c)^{\frac{9}{2}}}\right)$$

67.72 Problem number 414

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(15Ab + 11Ba) \sin(dx + c)}{99d \sec(dx + c)^{\frac{7}{2}}} + \frac{2a(9a^2A + 26Ab^2 + 33abB) \sin(dx + c)}{77d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(21Aa^2b + 9Ab^3 + 7a^3B + 27Ba^2b^2) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{2aA(a + b \sec(dx + c))^2 \sin(dx + c)}{11d \sec(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(45a^3A + 165Aab^2 + 165a^2bB + 77b^3B) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{2(21Aa^2b + 9Ab^3 + 7a^3B + 27Ba^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(45a^3A + 165Aab^2 + 165a^2bB + 77b^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (45i Aa^3 + 165i Ba^2b + 165i Aab^2 + 77i Bb^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^3 \sec(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx + c)^3 + 3(Ba^2b + Aab^2) \sec(dx + c)^2 + (Ba^3 + 3Aa^2b) \sec(dx + c) + Aa}{\sec(dx + c)^{\frac{11}{2}}}\right)$$

67.73 Problem number 438

$$\int \frac{\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a^2 - b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a + b}} (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2A \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2(Ab + 3Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) \sqrt{a + b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 A a^2 \sqrt{\frac{a \cos(dx+c)+b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2} (-3i A a^2 - 3i B a b + 2i A b^2) \sqrt{a} \operatorname{weierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx+c) + A) \sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

67.74 Problem number 439

$$\int \frac{\sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(a^2 - b^2) (2Ab - 5Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2A \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5d \sec(dx+c)^{\frac{3}{2}}} + \frac{2(Ab + 5Ba) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{15ad \sqrt{\sec(dx+c)}} \\ & + \frac{2(9a^2A - 2Ab^2 + 5abB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-15i B a^3 - 3i A a^2 b + 10i B a b^2 - 4i A b^3) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a \cos(dx+c)+3i a s}{3a}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx+c) + A) \sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

67.75 Problem number 440

$$\int \frac{\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(a^2 - b^2) (25a^2A + 8Ab^2 - 14abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2A \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(Ab + 7Ba) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{35ad \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(25a^2A - 4Ab^2 + 7abB) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{105a^2d \sqrt{\sec(dx + c)}} \\ & + \frac{2(19Aa^2b + 8Ab^3 + 63a^3B - 14Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-75i Aa^4 - 21i Ba^3b + 32i Aa^2b^2 - 28i Bab^3 + 16i Ab^4) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2b - 8b^3)}{27a^3}, \frac{3}{27a^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

67.76 Problem number 445

$$\int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sec^{5/2}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(a^2 - b^2) (3Ab + 5Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}} (\sqrt{\sec} \\ & + \frac{2aA \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{5d \sec(dx + c)^{3/2}} + \frac{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{a + b \sec(dx + c)}}{15d \sqrt{\sec(dx + c)}} + \frac{2(6Ab + 5Ba) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15d \sqrt{\sec(dx + c)}} \\ & + \frac{2(9a^2A + 3Ab^2 + 20abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-15i Ba^3 - 18i Aa^2b - 5i Bab^2 + 6i Ab^3) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2b - 8b^3)}{27a^3}, \frac{3a \cos(dx+c) + 3i a s}{3a}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{5/2}}, x\right)$$

67.77 Problem number 446

$$\int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sec^{7/2}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2) (25a^2A - 6Ab^2 + 21abB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2aA \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7d \sec(dx+c)^{\frac{5}{2}}} + \frac{2(8Ab + 7Ba) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{35d \sec(dx+c)^{\frac{3}{2}}} \\
& + \frac{2(25a^2A + 3Ab^2 + 42abB) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{105ad \sqrt{\sec(dx+c)}} \\
& + \frac{2(82Aa^2b - 6Ab^3 + 63a^3B + 21Ba^2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}
\end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-75i Aa^4 - 126i Ba^3b + 11i Aa^2b^2 + 42i Bab^3 - 12i Ab^4) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}\right),$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(Bb \sec(dx+c)^2 + Aa + (Ba + Ab) \sec(dx+c)\right) \sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

67.78 Problem number 447

$$\int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2) (39A a^2 b + 8A b^3 + 75a^3 B - 18Ba b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b}{a+b}}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a + b \sec(dx + c)}} \\
& + \frac{2aA \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2(10Ab + 9Ba) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{63d \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(49a^2 A + 3A b^2 + 72abB) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{315ad \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(88A a^2 b - 4A b^3 + 75a^3 B + 9Ba b^2) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{315a^2 d \sqrt{\sec(dx + c)}} \\
& + \frac{2(147A a^4 + 33a^2 A b^2 + 8A b^4 + 246a^3 b B - 18a b^3 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}}
\end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-225i Ba^5 - 264i Aa^4 b + 33i Ba^3 b^2 + 60i Aa^2 b^3 - 36i Bab^4 + 16i Ab^5 \right) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)\right) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{9}{2}}}, x\right)$$

67.79 Problem number 453

$$\int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA(a+b\sec(dx+c))^{\frac{3}{2}}\sin(dx+c)}{7d\sec(dx+c)^{\frac{5}{2}}} \\ & + \frac{2(a^2-b^2)(25a^2A+15Ab^2+56abB)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{a}{a+b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{105\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad\sqrt{a+b\sec(dx+c)}} \\ & + \frac{2a(10Ab+7Ba)\sin(dx+c)\sqrt{a+b\sec(dx+c)}}{35d\sec(dx+c)^{\frac{3}{2}}} \\ & + \frac{2(25a^2A+45Ab^2+77abB)\sin(dx+c)\sqrt{a+b\sec(dx+c)}}{105d\sqrt{\sec(dx+c)}} \\ & + \frac{2(145Aa^2b+15Ab^3+63a^3B+161Bab^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{a}{a+b}}\right)\sqrt{a+b}}{105\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad\sqrt{\frac{b+a\cos(dx+c)}{a+b}}\sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-75iAa^4-231iBa^3b-115iAa^2b^2+7iBab^3+30iAb^4)\sqrt{a}\operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3}\right),$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(Bb^2\sec(dx+c)^3+Aa^2+(2Bab+Ab^2)\sec(dx+c)^2+(Ba^2+2Aab)\sec(dx+c)\right)\sqrt{b\sec(dx+c)}}{\sec(dx+c)^{\frac{7}{2}}}\right)$$

67.80 Problem number 454

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2aA(a+b\sec(dx+c))^{\frac{3}{2}}\sin(dx+c)}{9d\sec(dx+c)^{\frac{7}{2}}} \\
& + \frac{2(a^2-b^2)(114Aa^2b-10Ab^3+75a^3B+45Ba^2b^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{a}{a+b}}\right)}{315\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d\sqrt{a+b\sec(dx+c)}} \\
& + \frac{2a(4Ab+3Ba)\sin(dx+c)\sqrt{a+b\sec(dx+c)}}{21d\sec(dx+c)^{\frac{5}{2}}} \\
& + \frac{2(49a^2A+75Ab^2+135abB)\sin(dx+c)\sqrt{a+b\sec(dx+c)}}{315d\sec(dx+c)^{\frac{3}{2}}} \\
& + \frac{2(163Aa^2b+5Ab^3+75a^3B+135Ba^2b^2)\sin(dx+c)\sqrt{a+b\sec(dx+c)}}{315ad\sqrt{\sec(dx+c)}} \\
& + \frac{2(147Aa^4+279a^2Ab^2-10Ab^4+435a^3bB+45ab^3B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{a}{a+b}}\right)}{315\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d\sqrt{\frac{b+a\cos(dx+c)}{a+b}}\sqrt{\sec(dx+c)}}
\end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}\left(-225iBa^5-489iAa^4b-345iBa^3b^2+93iAa^2b^3+90iBab^4-20iAb^5\right)\sqrt{a}\operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(Bb^2\sec(dx+c)^3+Aa^2+(2Bab+Ab^2)\sec(dx+c)^2+(Ba^2+2Aab)\sec(dx+c)\right)\sqrt{b\sec(dx+c)}}{\sec(dx+c)^{\frac{9}{2}}}\right)$$

67.81 Problem number 455

$$\int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^{11/2}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA(a + b \sec(dx + c))^{3/2} \sin(dx + c)}{11d \sec(dx + c)^{9/2}} \\ & + \frac{2(a^2 - b^2)(675Aa^4 + 285a^2Ab^2 + 40Ab^4 + 1254a^3bB - 110ab^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2a(14Ab + 11Ba) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{99d \sec(dx + c)^{7/2}} \\ & + \frac{2(81a^2A + 113Ab^2 + 209abB) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{693d \sec(dx + c)^{5/2}} \\ & + \frac{2(1145Aa^2b + 15Ab^3 + 539a^3B + 825Ba^2b^2) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3465ad \sec(dx + c)^{3/2}} \\ & + \frac{2(675Aa^4 + 1025a^2Ab^2 - 20Ab^4 + 1793a^3bB + 55ab^3B) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3465a^2d \sqrt{\sec(dx + c)}} \\ & + \frac{2(3705Aa^4b + 255Aa^2b^3 + 40Ab^5 + 1617Ba^5 + 3069Ba^3b^2 - 110Bab^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-2025i Aa^6 - 5379i Ba^5b - 2535i Aa^4b^2 + 1023i Ba^3b^3 + 480i Aa^2b^4 - 220i Bab^5 + 80i Ab^6) \sqrt{a} \operatorname{weierstrassE}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Bb^2 \sec(dx + c))^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)}{\sec(dx + c)^{11/2}} \sqrt{b \sec(dx + c)} + \dots\right)$$

67.82 Problem number 459

$$\int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}} (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{a + b \sec(dx + c)}} + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}}$$

command

`integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3i \sqrt{2} A a^{\frac{3}{2}} \operatorname{weierstrassZeta}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2b - 8b^3)}{27a^3}, \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2b - 8b^3)}{27a^3}, \frac{3a \cos(dx + c) + 3}{3}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b \sec(dx + c)^2 + a \sec(dx + c)}, x\right)$$

67.83 Problem number 460

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(a^2A + 2Ab^2 - 3abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}} (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a + b \sec(dx + c)}} + \frac{2A \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3ad \sqrt{\sec(dx + c)}} - \frac{2(2Ab - 3Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}}$$

command

`integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 A a^2 \sqrt{\frac{a \cos(dx+c)+b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2} (-3i A a^2 + 6i B a b - 4i A b^2) \sqrt{a} \operatorname{weierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx+c)+A) \sqrt{b \sec(dx+c)+a} \sqrt{\sec(dx+c)}}{b \sec(dx+c)^3 + a \sec(dx+c)^2}, x\right)$$

67.84 Problem number 461

$$\int \frac{A + B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7Aa^2b + 8Ab^3 - 5a^3B - 10Bab^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2A \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5ad \sec(dx+c)^{\frac{3}{2}}} - \frac{2(4Ab - 5Ba) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{15a^2 d \sqrt{\sec(dx+c)}} \\ & + \frac{2(9a^2A + 8Ab^2 - 10abB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-15i B a^3 + 12i A a^2 b - 20i B a b^2 + 16i A b^3) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a \cos(dx+c)+3b}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx+c)+A) \sqrt{b \sec(dx+c)+a} \sqrt{\sec(dx+c)}}{b \sec(dx+c)^4 + a \sec(dx+c)^3}, x\right)$$

67.85 Problem number 464

$$\int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab - Ba) \sin(dx+c) (\sqrt{\sec(dx+c)})}{(a^2 - b^2) d \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2(Ab - Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a (a^2 - b^2) d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (Ba^3 - Aa^2b) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \sqrt{2} (3i Aa^2b - i Bab^2 - 2i Ab^3 + (3i Aa^3 - i B$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx+c) + A) \sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b^2 \sec(dx+c)^2 + 2ab \sec(dx+c) + a^2}, x\right)$$

67.86 Problem number 465

$$\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+b \sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b(Ab - Ba) \sin(dx + c) (\sqrt{\sec(dx + c)})}{a(a^2 - b^2) d \sqrt{a + b \sec(dx + c)}} + \frac{2(2Ab - Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}} (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a + b \sec(dx + c)}} + \frac{2(a^2A - 2Ab^2 + abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}}$$

command

`integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(Ba^3b - Aa^2b^2) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + \sqrt{2} (3iBa^3b - 5iAa^2b^2 - 2iBab^3 + 4iAb^4)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^2 \sec(dx + c)^3 + 2ab \sec(dx + c)^2 + a^2 \sec(dx + c)}, x\right)$$

67.87 Problem number 466

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b(Ab - Ba) \sin(dx + c)}{a(a^2 - b^2) d \sqrt{\sec(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2(a^2A + 8Ab^2 - 6abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}} (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a + b \sec(dx + c)}} + \frac{2(a^2A - 4Ab^2 + 3abB) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3a^2 (a^2 - b^2) d \sqrt{\sec(dx + c)}} + \frac{2(5Aa^2b - 8Ab^3 - 3a^3B + 6Ba^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}}$$

command

```
integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (3i Aa^4b - 15i Ba^3b^2 + 16i Aa^2b^3 + 12i Bab^4 - 16i Ab^5 + (3i Aa^5 - 15i Ba^4b + 16i Aa^3b^2 + 12i Ba^2b^3 - 16i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \sec(dx+c) + A) \sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b^2 \sec(dx+c)^4 + 2ab \sec(dx+c)^3 + a^2 \sec(dx+c)^2}, x \right)$$

67.88 Problem number 467

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b(Ab - Ba) \sin(dx + c)}{a(a^2 - b^2) d \sec(dx + c)^{\frac{3}{2}} \sqrt{a + b \sec(dx + c)}}$$

$$- \frac{2(12Aa^2b + 48Ab^3 - 5a^3B - 40Ba^2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a-b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{a + b \sec(dx + c)}}$$

$$+ \frac{2(a^2A - 6Aa^2b + 5abB) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{5a^2(a^2 - b^2) d \sec(dx + c)^{\frac{3}{2}}}$$

$$- \frac{2(9Aa^2b - 24Aa^3b^3 - 5a^3B + 20Ba^2b^2) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15a^3(a^2 - b^2) d \sqrt{\sec(dx + c)}}$$

$$+ \frac{2(9Aa^4 + 24a^2Ab^2 - 48Ab^4 - 25a^3bB + 40a^2b^3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4(a^2 - b^2) d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx + c)}}$$

command

```
integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (15i Ba^5b - 27i Aa^4b^2 + 80i Ba^3b^3 - 84i Aa^2b^4 - 80i Bab^5 + 96i Ab^6 + (15i Ba^6 - 27i Aa^5b + 80i Ba^4b^2 - 80i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \sec(dx+c) + A) \sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b^2 \sec(dx+c)^5 + 2ab \sec(dx+c)^4 + a^2 \sec(dx+c)^3}, x \right)$$

67.89 Problem number 469

$$\int \frac{\sec^{3/2}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(Ab - Ba) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3b(a^2 - b^2) d(a+b\sec(dx+c))^{3/2}} \\ & + \frac{2(2Aa^2b + 2Ab^3 + a^3B - 5Ba^2b^2) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3b(a^2 - b^2)^2 d\sqrt{a+b\sec(dx+c)}} \\ & - \frac{2(Ab - Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a(a^2 - b^2) d\sqrt{a+b\sec(dx+c)}} \\ & - \frac{2(3a^2A + Ab^2 - 4abB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b\sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a(a^2 - b^2)^2 d\sqrt{\frac{b+a\cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i Ba^3b^2 + 6i Aa^2b^3 - i Bab^4 - 2i Ab^5 + (-3i Ba^5 + 6i Aa^4b - i Ba^3b^2 - 2i Aa^2b^3) \cos(dx+c)^2 - 2(3i Ba^6 - 27i Aa^5b + 80i Ba^4b^2 - 80i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \sec(dx+c)^2 + A \sec(dx+c)) \sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3}, x \right)$$

67.90 Problem number 470

$$\int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab - Ba) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3(a^2 - b^2) d(a + b \sec(dx + c))^{\frac{3}{2}}} \\ & - \frac{2(5Aa^2b - Ab^3 - 2a^3B - 2Bab^2) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3a(a^2 - b^2)^2 d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(3a^2A - 2Ab^2 - abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}} (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2) d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(6Aa^2b - 2Ab^3 - 3a^3B - Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-9i Aa^4b^2 + 6i Ba^3b^3 + 9i Aa^2b^4 - 2i Bab^5 - 4i Ab^6 + (-9i Aa^6 + 6i Ba^5b + 9i Aa^4b^2 - 2i Ba^3b^3 - 4i Aa^2b^4) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x\right)$$

67.91 Problem number 471

$$\int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(Ab - Ba) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3a(a^2 - b^2) d(a + b \sec(dx + c))^{3/2}} \\ & + \frac{2b(8Aa^2b - 4Ab^3 - 5a^3B + Ba^2b^2) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3a^2(a^2 - b^2)^2 d \sqrt{a + b \sec(dx + c)}} \\ & - \frac{2(9Aa^2b - 8Ab^3 - 3a^3B + 2Ba^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2) d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(3Aa^4 - 15a^2Ab^2 + 8Ab^4 + 6a^3bB - 2ab^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-9i Ba^5b^2 + 24i Aa^4b^3 + 9i Ba^3b^4 - 36i Aa^2b^5 - 4i Bab^6 + 16i Ab^7 + (-9i Ba^7 + 24i Aa^6b + 9i Ba^5b^2 - 36i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^3 \sec(dx + c)^4 + 3ab^2 \sec(dx + c)^3 + 3a^2b \sec(dx + c)^2 + a^3 \sec(dx + c)}, x\right)$$

67.92 Problem number 472

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(Ab - Ba) \sin(dx + c)}{3a(a^2 - b^2) d(a + b \sec(dx + c))^{\frac{3}{2}} \sqrt{\sec(dx + c)}} \\ & + \frac{2b(10Aa^2b - 6Ab^3 - 7a^3B + 3Ba^2b^2) \sin(dx + c)}{3a^2(a^2 - b^2)^2 d \sqrt{\sec(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(Aa^4 + 16a^2Ab^2 - 16Ab^4 - 9a^3bB + 8ab^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) \sqrt{\frac{b}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2 - b^2) d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(Aa^4 - 13a^2Ab^2 + 8Ab^4 + 8a^3bB - 4ab^3B) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3a^3(a^2 - b^2)^2 d \sqrt{\sec(dx + c)}} \\ & - \frac{2(8Aa^4b - 28Aa^2b^3 + 16Ab^5 - 3Ba^5 + 15Ba^3b^2 - 8Bab^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-3i Aa^6b^2 + 24i Ba^5b^3 - 37i Aa^4b^4 - 36i Ba^3b^5 + 68i Aa^2b^6 + 16i Bab^7 - 32i Ab^8 + (-3i Aa^8 + 24i Ba^7b - \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^3 \sec(dx + c)^5 + 3ab^2 \sec(dx + c)^4 + 3a^2b \sec(dx + c)^3 + a^3 \sec(dx + c)^2}, x\right)$$

67.93 Problem number 473

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(Ab - Ba) \sin(dx + c)}{3a(a^2 - b^2) d \sec(dx + c)^{\frac{3}{2}} (a + b \sec(dx + c))^{\frac{3}{2}}} \\ + & \frac{2b(12Aa^2b - 8Ab^3 - 9a^3B + 5Bab^2) \sin(dx + c)}{3a^2(a^2 - b^2)^2 d \sec(dx + c)^{\frac{3}{2}} \sqrt{a + b \sec(dx + c)}} \\ - & \frac{2(17Aa^4b + 116Aa^2b^3 - 128Ab^5 - 5Ba^5 - 80Ba^3b^2 + 80Bab^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^5 (a^2 - b^2) d \sqrt{a + b \sec(dx + c)}} \\ + & \frac{2(3Aa^4 - 71a^2Ab^2 + 48Ab^4 + 50a^3bB - 30ab^3B) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15a^3(a^2 - b^2)^2 d \sec(dx + c)^{\frac{3}{2}}} \\ - & \frac{2(14Aa^4b - 98Aa^2b^3 + 64Ab^5 - 5Ba^5 + 65Ba^3b^2 - 40Bab^4) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15a^4(a^2 - b^2)^2 d \sqrt{\sec(dx + c)}} \\ + & \frac{2(9Aa^6 + 55Aa^4b^2 - 212a^2Ab^4 + 128Ab^6 - 40a^5bB + 140a^3b^3B - 80ab^5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^5 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^3 \sec(dx + c)^6 + 3ab^2 \sec(dx + c)^5 + 3a^2b \sec(dx + c)^4 + a^3 \sec(dx + c)^3}, x\right)$$

67.94 Problem number 483

$$\int \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A + 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2aA \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2a(5A + 7B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (5A + 7B) \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (5A + 7B) \operatorname{aweierstrassPInverse}(\dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba \cos(dx + c)^3 \sec(dx + c)^2 + (A + B)a \cos(dx + c)^3 \sec(dx + c) + Aa \cos(dx + c)^3\right) \sqrt{\cos(dx + c)}, x\right)$$

67.95 Problem number 484

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(3A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2a(A + B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i\sqrt{2}(A+B)\text{awierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+B)\text{awierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{\sqrt{\cos(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ba\cos(dx+c)^2\sec(dx+c)^2+(A+B)a\cos(dx+c)^2\sec(dx+c)+Aa\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

67.96 Problem number 485

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(A+B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(A+3B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2aA\sin(dx+c)\left(\sqrt{\cos(dx+c)}\right)}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2Aa\sqrt{\cos(dx+c)}\sin(dx+c)-i\sqrt{2}(A+3B)\text{awierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(A+3B)\text{awierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{\sqrt{\cos(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ba\cos(dx+c)\sec(dx+c)^2+(A+B)a\cos(dx+c)\sec(dx+c)+Aa\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

67.97 Problem number 486

$$\int \sqrt{\cos(c+dx)} (a+a \sec(c+dx))(A+B \sec(c+dx)) dx$$

Optimal antiderivative

$$\frac{2a(A-B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a(A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2aB \sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

command

```
integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}(A+B)a \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i\sqrt{2}(A+B)a \cos(dx+c)}{d\sqrt{\cos(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba \sec(dx+c)^2 + (A+B)a \sec(dx+c) + Aa\right) \sqrt{\cos(dx+c)}, x\right)$$

67.98 Problem number 487

$$\int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2a(A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a(3A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2aB \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2a(A+B) \sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

command

```
integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i\sqrt{2}(3A+B)a\cos(dx+c)^2 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+B)a\cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ba\sec(dx+c)^2+(A+B)a\sec(dx+c)+Aa}{\sqrt{\cos(dx+c)}},x\right)$$

67.99 Problem number 488

$$\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(5A+3B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2a(A+B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2aB\sin(dx+c)}{5d\cos(dx+c)^{\frac{5}{2}}}+\frac{2a(A+B)\sin(dx+c)}{3d\cos(dx+c)^{\frac{3}{2}}}+\frac{2a(5A+3B)\sin(dx+c)}{5d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(A+B)a\cos(dx+c)^3 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+B)a\cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ba\sec(dx+c)^2+(A+B)a\sec(dx+c)+Aa}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

67.100 Problem number 489

$$\int \cos^{\frac{9}{2}}(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(8A + 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(5A + 6B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(8A + 9B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{2a^2(11A + 9B) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\ & + \frac{2A \left(\cos^{\frac{5}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{9d} \\ & + \frac{4a^2(5A + 6B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (5A + 6B) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 6B) a^2 \operatorname{weierstrassP}(\cos(dx + c) + i \sin(dx + c)) \right)}{21d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^2 \cos(dx + c)^4 \sec(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^4 \sec(dx + c)^2 + (2A + B)a^2 \cos(dx + c)^4 \sec(dx + c)\right), dx\right)$$

67.101 Problem number 490

$$\int \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(3A + 4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(6A + 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(9A + 7B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{35d} \\ & + \frac{2A \left(\cos^{\frac{3}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{7d} \\ & + \frac{4a^2(6A + 7B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (6A + 7B) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (6A + 7B) a^2 \operatorname{weierstrassP}(\cos(dx + c) + i \sin(dx + c)) \right)}{21d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^2 \cos(dx + c)^3 \sec(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^3 \sec(dx + c)^2 + (2A + B)a^2 \cos(dx + c)^3 \sec(dx + c)\right), dx\right)$$

67.102 Problem number 491

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(4A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(A + 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(7A + 5B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \\ & + \frac{2A(a^2 + a^2 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (A + 2B) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + 2B) a^2 \operatorname{weierstrassP} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ba^2 \cos(dx + c)^2 \sec(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 \sec(dx + c)^2 + (2A + B)a^2 \cos(dx + c)^2 \sec(dx + c)\right) dx\right)$$

67.103 Problem number 492

$$\int \cos^{\frac{3}{2}}(c + dx) (a + a \sec(c + dx))^2 (A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2 A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(2A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} + \frac{2a^2(A - 3B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} (2A + 3B)a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} (2A + 3B)a^2 \right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ba^2 cos(dx + c) sec(dx + c)^3 + (A + 2B)a^2 cos(dx + c) sec(dx + c)^2 + (2A + B)a^2 cos(dx + c) sec(dx + c)), x)`

67.104 Problem number 493

$$\int \sqrt{\cos(c + dx)} (a + a \sec(c + dx))^2 (A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^2 B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(3A + 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a^2(3A + 5B) \sin(dx + c)}{3d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} (3A + 2B)a^2 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} (3A + 2B)a^2 \right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ba^2 sec(dx + c)^3 + (A + 2B)a^2 sec(dx + c)^2 + (2A + B)a^2 sec(dx + c) + Aa^2) sqrt(cos(dx + c)), x)`

67.105 Problem number 494

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^2(5A + 4B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(2A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(5A + 7B) \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} + \frac{2B(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{4a^2(5A + 4B) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (2A + B)a^2 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (2A + B)a^2 \right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\sqrt{\cos(dx + c)}}, x\right)$$

67.106 Problem number 495

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^2(4A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(7A + 6B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(7A + 9B) \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^2(7A + 6B) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2B(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{4a^2(4A + 3B) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (7A + 6B)a^2 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (7A + 6B) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

67.107 Problem number 496

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)(A + B \sec(c + dx))}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(7A - 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{5(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(7A - 5B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5ad} \\ & - \frac{(A - B) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{d(a + a \cos(dx + c))} - \frac{5(A - B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3ad} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6 A \cos(dx + c)^2 - 2(2A - 5B) \cos(dx + c) - 25A + 25B \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 25 \left(\sqrt{2} (-iA + iB) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2 \right) \sqrt{\cos(dx + c)}}{a \sec(dx + c) + a}, x \right)$$

67.108 Problem number 497

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3(A - B) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{(5A - 3B) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{(A - B) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \cos(dx + c))} + \frac{(5A - 3B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3ad} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(2A \cos(dx + c) + 5A - 3B) \sqrt{\cos(dx + c)} \sin(dx + c) + \left(\sqrt{2} (-5iA + 3iB) \cos(dx + c) + \sqrt{2} (-5iA + 3iB) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c) \right) \sqrt{\cos(dx + c)}}{a \sec(dx + c) + a}, x \right)$$

67.109 Problem number 498

$$\int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal antiderivative

$$\frac{(3A-B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} - \frac{(A-B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} - \frac{(A-B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{d(a+a \cos(dx+c))}$$

command

```
integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A-B) \sqrt{\cos(dx+c)} \sin(dx+c) - \left(\sqrt{2}(iA-iB) \cos(dx+c) + \sqrt{2}(iA-iB)\right) \operatorname{weierstrassPInverse}(-4, \dots)}{d(a+a \cos(dx+c))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx+c) + A) \sqrt{\cos(dx+c)}}{a \sec(dx+c) + a}, x\right)$$

67.110 Problem number 499

$$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+a \sec(c+dx))} dx$$

Optimal antiderivative

$$\frac{(A-B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{(A+B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{(A-B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{d(a+a \cos(dx+c))}$$

command

```
integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(A - B)\sqrt{\cos(dx + c)} \sin(dx + c) + \left(\sqrt{2}(-iA - iB)\cos(dx + c) + \sqrt{2}(-iA - iB)\right)\text{weierstrassPInverse}(-4$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{a \cos(dx + c) \sec(dx + c) + a \cos(dx + c)}, x\right)$$

67.111 Problem number 500

$$\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A - 3B) \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} + \frac{(A - B) \sin(dx + c)}{d(a + a \cos(dx + c)) \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2((A - 3B)\cos(dx + c) - 2B)\sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2}(-iA + iB)\cos(dx + c)^2 + \sqrt{2}(-iA + iB)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{a \cos(dx + c)^2 \sec(dx + c) + a \cos(dx + c)^2}, x\right)$$

67.112 Problem number 501

$$\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(3A - 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(3A - 5B) \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}}} + \frac{(A - B) \sin(dx + c)}{d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))} + \frac{3(A - B) \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(9(A - B) \cos(dx + c)^2 + 2(3A - 2B) \cos(dx + c) + 2B \right) \sqrt{\cos(dx + c)} \sin(dx + c) + \left(\sqrt{2} (3iA - 5iB) \cos \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^3 \sec(dx + c) + a \cos(dx + c)^3}, x\right)$$

67.113 Problem number 502

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(8A - 5B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5(3A - 2B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{7(8A - 5B) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15a^2 d} - \frac{(3A - 2B) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{a^2 d (1 + \cos(dx+c))} \\ & - \frac{(A - B) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{3d(a + a \cos(dx+c))^2} - \frac{5(3A - 2B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a^2 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6A \cos(dx+c)^3 - 2(4A - 5B) \cos(dx+c)^2 - (94A - 65B) \cos(dx+c) - 75A + 50B \right) \sqrt{\cos(dx+c)} \sin(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B \cos(dx+c)^2 \sec(dx+c) + A \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

67.114 Problem number 503

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(7A - 4B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{5(2A - B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{(7A - 4B) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3a^2 d (1 + \cos(dx+c))} - \frac{(A - B) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{3d(a + a \cos(dx+c))^2} \\ & + \frac{5(2A - B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a^2 d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 A \cos(dx + c)^2 + (13 A - 6 B) \cos(dx + c) + 10 A - 5 B \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(\sqrt{2} (2i A - i B) \cos(dx + c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)) \sqrt{\cos(dx + c)}}{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}, x \right)$$

67.115 Problem number 504

$$\int \frac{\sqrt{\cos(c + dx)} (A + B \sec(c + dx))}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(4A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & - \frac{(5A - 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & - \frac{(A - B) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d(a + a \cos(dx + c))^2} - \frac{(5A - 2B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3a^2 d (1 + \cos(dx + c))} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 (2 A - B) \cos(dx + c) + 5 A - 2 B \right) \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2} (5i A - 2i B) \cos(dx + c) \right)^2 - 2 \sqrt{2} \left(\dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}, x \right)$$

67.116 Problem number 505

$$\int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(2A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{A \sin(dx + c) (\sqrt{\cos(dx + c)})}{a^2 d (1 + \cos(dx + c))} - \frac{(A - B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d (a + a \cos(dx + c))^2} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(3A \cos(dx + c) + 2A + B) \sqrt{\cos(dx + c)} \sin(dx + c) + \left(\sqrt{2}(-2iA - iB) \cos(dx + c)^2 - 2\sqrt{2}(2iA + iB) \cos(dx + c)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c) \sec(dx + c)^2 + 2a^2 \cos(dx + c) \sec(dx + c) + a^2 \cos(dx + c)}, x\right)$$

67.117 Problem number 506

$$\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx) (a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(A + 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{B \sin(dx + c) (\sqrt{\cos(dx + c)})}{a^2 d (1 + \cos(dx + c))} + \frac{(A - B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d (a + a \cos(dx + c))^2} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3B \cos(dx+c) - A + 4B) \sqrt{\cos(dx+c)} \sin(dx+c) - \left(\sqrt{2}(-iA - 2iB) \cos(dx+c)^2 - 2\sqrt{2}(iA + 2iB)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \sec(dx+c) + A) \sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^2 \sec(dx+c)^2 + 2a^2 \cos(dx+c)^2 \sec(dx+c) + a^2 \cos(dx+c)^2}, x\right)$$

67.118 Problem number 507

$$\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - 4B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(2A - 5B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} - \frac{(A - 4B) \sin(dx+c)}{a^2 d \sqrt{\cos(dx+c)}} \\ & + \frac{(2A - 5B) \sin(dx+c)}{3a^2 d (1 + \cos(dx+c)) \sqrt{\cos(dx+c)}} + \frac{(A - B) \sin(dx+c)}{3d (a + a \cos(dx+c))^2 \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(3(A - 4B) \cos(dx+c)^2 + (4A - 19B) \cos(dx+c) - 6B\right) \sqrt{\cos(dx+c)} \sin(dx+c) - \left(\sqrt{2}(-2iA + 5iB)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \sec(dx+c) + A) \sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^3 \sec(dx+c)^2 + 2a^2 \cos(dx+c)^3 \sec(dx+c) + a^2 \cos(dx+c)^3}, x\right)$$

67.119 Problem number 508

$$\int \frac{A + B \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(4A - 7B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5(A - 2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5(A - 2B) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}}} + \frac{(4A - 7B) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}} (1 + \cos(dx + c))} \\ & + \frac{(A - B) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^2} + \frac{(4A - 7B) \sin(dx + c)}{a^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(4A - 7B) \cos(dx + c)^3 + (19A - 32B) \cos(dx + c)^2 + 2(3A - 4B) \cos(dx + c) + 2B \right) \sqrt{\cos(dx + c)} \sin(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^4 \sec(dx + c)^2 + 2a^2 \cos(dx + c)^4 \sec(dx + c) + a^2 \cos(dx + c)^4}, x\right)$$

67.120 Problem number 509

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(17A - 7B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(33A - 13B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - B) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{5d(a + a \cos(dx+c))^3} - \frac{(2A - B) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{3ad(a + a \cos(dx+c))^2} \\ & - \frac{7(17A - 7B) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{30d(a^3 + a^3 \cos(dx+c))} + \frac{(33A - 13B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{6a^3 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(20 A \cos(dx+c)^3 + 3(79 A - 29 B) \cos(dx+c)^2 + 2(188 A - 73 B) \cos(dx+c) + 165 A - 65 B \right) \sqrt{\cos(dx+c)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx+c) \sec(dx+c) + A \cos(dx+c)) \sqrt{\cos(dx+c)}}{a^3 \sec(dx+c)^3 + 3a^3 \sec(dx+c)^2 + 3a^3 \sec(dx+c) + a^3}, x\right)$$

67.121 Problem number 510

$$\int \frac{\sqrt{\cos(c+dx)} (A + B \sec(c+dx))}{(a + a \sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(49A - 9B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(13A - 3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - B) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{5d(a + a \cos(dx+c))^3} - \frac{(8A - 3B) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15ad(a + a \cos(dx+c))^2} \\ & - \frac{(13A - 3B) \sin(dx+c) (\sqrt{\cos(dx+c)})}{6d(a^3 + a^3 \cos(dx+c))} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 (29 A - 9 B) \cos(dx + c)^2 + 2 (73 A - 18 B) \cos(dx + c) + 65 A - 15 B \right) \sqrt{\cos(dx + c)} \sin(dx + c) + 5 \left(\sqrt{\cos(dx + c)} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x \right)$$

67.122 Problem number 511

$$\int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(9A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(3A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & - \frac{(A - B) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5d (a + a \cos(dx + c))^3} - \frac{(6A - B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15ad (a + a \cos(dx + c))^2} \\ & + \frac{(9A + B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10d (a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 (9 A + B) \cos(dx + c)^2 + 2 (18 A + 7 B) \cos(dx + c) + 15 A + 5 B \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(\sqrt{2} (3i A + \dots) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c) \sec(dx + c)^3 + 3 a^3 \cos(dx + c) \sec(dx + c)^2 + 3 a^3 \cos(dx + c) \sec(dx + c) + a^3 \cos(dx + c)}, x \right)$$

67.123 Problem number 512

$$\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d (a + a \cos(dx + c))^3} \\ & + \frac{(4A + B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15ad (a + a \cos(dx + c))^2} + \frac{(A - B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10d (a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(A - B) \cos(dx + c)^2 + 2(7A - 2B) \cos(dx + c) + 5A + 5B \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(\sqrt{2} (iA + iB) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^2 \sec(dx + c)^3 + 3a^3 \cos(dx + c)^2 \sec(dx + c)^2 + 3a^3 \cos(dx + c)^2 \sec(dx + c) + a^3 \cos(dx + c)}\right)$$

67.124 Problem number 513

$$\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + 9B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + 3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A - B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d (a + a \cos(dx + c))^3} + \frac{(A - 6B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15ad (a + a \cos(dx + c))^2} \\ & - \frac{(A + 9B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10d (a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(A + 9B) \cos(dx + c)^2 + 2(2A + 33B) \cos(dx + c) - 5A + 45B \right) \sqrt{\cos(dx + c)} \sin(dx + c) + 5 \left(\sqrt{2} (i A \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 \sec(dx + c)^3 + 3 a^3 \cos(dx + c)^3 \sec(dx + c)^2 + 3 a^3 \cos(dx + c)^3 \sec(dx + c) + a^3 \cos(dx + c)}\right)$$

67.125 Problem number 514

$$\int \frac{A + B \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(9A - 49B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A - 13B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(9A - 49B) \sin(dx + c)}{10a^3 d \sqrt{\cos(dx + c)}} + \frac{(A - B) \sin(dx + c)}{5d (a + a \cos(dx + c))^3 \sqrt{\cos(dx + c)}} \\ & + \frac{(3A - 8B) \sin(dx + c)}{15ad (a + a \cos(dx + c))^2 \sqrt{\cos(dx + c)}} + \frac{(3A - 13B) \sin(dx + c)}{6d (a^3 + a^3 \cos(dx + c)) \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(9A - 49B) \cos(dx + c)^3 + 2(33A - 188B) \cos(dx + c)^2 + 5(9A - 59B) \cos(dx + c) - 60B \right) \sqrt{\cos(dx + c)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^4 \sec(dx + c)^3 + 3a^3 \cos(dx + c)^4 \sec(dx + c)^2 + 3a^3 \cos(dx + c)^4 \sec(dx + c) + a^3 \cos(dx + c)} \right)$$

67.126 Problem number 562

$$\int \cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(5aA + 7bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(Ab + Ba) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5d} + \frac{2aA \left(\cos^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{7d} \\ & + \frac{2(5aA + 7bB) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15Aa \cos(dx + c)^2 + 25Aa + 35Bb + 21(Ba + Ab) \cos(dx + c) \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5\sqrt{2} (5iAa + \dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(Bb \cos(dx + c)^3 \sec(dx + c)^2 + Aa \cos(dx + c)^3 + (Ba + Ab) \cos(dx + c)^3 \sec(dx + c) \right) \sqrt{\cos(dx + c)} \right)$$

67.127 Problem number 563

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3aA + 5bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2(Ab + Ba) \sin(dx + c) (\sqrt{\cos}(dx + c))}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(3Aa \cos(dx + c) + 5Ba + 5Ab) \sqrt{\cos(dx + c)} \sin(dx + c) - 5\sqrt{2} (iBa + iAb) \operatorname{weierstrassPInverse}(-4, 0, \cos$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \cos(dx + c)^2 \sec(dx + c)^2 + Aa \cos(dx + c)^2 + (Ba + Ab) \cos(dx + c)^2 \sec(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

67.128 Problem number 564

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(aA + 3bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \sin(dx + c) (\sqrt{\cos}(dx + c))}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 A a \sqrt{\cos (d x+c)} \sin (d x+c)+\sqrt{2}(-i A a-3 i B b) \operatorname{weierstrassPInverse}(-4,0, \cos (d x+c)+i \sin (d x+c))+\sqrt{2}(-i A a-3 i B b) \operatorname{weierstrassPInverse}(-4,0, \cos (d x+c)+i \sin (d x+c))}{\cos (d x+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(B b \cos (d x+c) \sec (d x+c)^2+A a \cos (d x+c)+(B a+A b) \cos (d x+c) \sec (d x+c)\right) \sqrt{\cos (d x+c)}, x\right)$$

67.129 Problem number 565

$$\int \sqrt{\cos (c+d x)}(a+b \sec (c+d x))(A+B \sec (c+d x)) d x$$

Optimal antiderivative

$$\frac{2(a A-b B) \sqrt{\frac{\cos (d x+c)}{2}+\frac{1}{2}} \operatorname{EllipticE}\left(\sin \left(\frac{d x}{2}+\frac{c}{2}\right), \sqrt{2}\right)}{\cos \left(\frac{d x}{2}+\frac{c}{2}\right) d} + \frac{2(A b+B a) \sqrt{\frac{\cos (d x+c)}{2}+\frac{1}{2}} \operatorname{EllipticF}\left(\sin \left(\frac{d x}{2}+\frac{c}{2}\right), \sqrt{2}\right)}{\cos \left(\frac{d x}{2}+\frac{c}{2}\right) d} + \frac{2 b B \sin (d x+c)}{d \sqrt{\cos (d x+c)}}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 B b \sqrt{\cos (d x+c)} \sin (d x+c)+\sqrt{2}(-i B a-i A b) \cos (d x+c) \operatorname{weierstrassPInverse}(-4,0, \cos (d x+c)+i \sin (d x+c))+\sqrt{2}(-i B a-i A b) \cos (d x+c) \operatorname{weierstrassPInverse}(-4,0, \cos (d x+c)+i \sin (d x+c))}{\cos (d x+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(B b \sec (d x+c)^2+A a+(B a+A b) \sec (d x+c)\right) \sqrt{\cos (d x+c)}, x\right)$$

67.130 Problem number 566

$$\int \frac{(a + b \sec(c + dx))(A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3aA + bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2bB \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(Ab + Ba) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i Aa - i Bb) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i Aa + i Bb) \cos$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)}{\sqrt{\cos(dx + c)}}, x\right)$$

67.131 Problem number 567

$$\int \frac{(a + b \sec(c + dx))(A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(5aA + 3bB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ab + Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2bB \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2(Ab + Ba) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(5aA + 3bB) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(iBa + iAb)\cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + 5\sqrt{2}(-iBa - iAb)\cos(dx + c)^3$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Bb\sec(dx + c)^2 + Aa + (Ba + Ab)\sec(dx + c)}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

67.132 Problem number 568

$$\int \cos^{\frac{7}{2}}(c + dx)(a + b\sec(c + dx))^2(A + B\sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(6Aab + 3Ba^2 + 5b^2B)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2(5a^2A + 7b(Ab + 2Ba))\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2a(9Ab + 7Ba)\left(\cos^{\frac{3}{2}}(dx + c)\right)\sin(dx + c)}{35d} \\ & + \frac{2aA\left(\cos^{\frac{3}{2}}(dx + c)\right)(b + a\cos(dx + c))\sin(dx + c)}{7d} \\ & + \frac{2(5a^2A + 7b(Ab + 2Ba))\sin(dx + c)(\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(15Aa^2\cos(dx + c)^2 + 25Aa^2 + 70Bab + 35Ab^2 + 21(Ba^2 + 2Aab)\cos(dx + c)\right)\sqrt{\cos(dx + c)}\sin(dx + c) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Bb^2\cos(dx + c)^3\sec(dx + c)^3 + Aa^2\cos(dx + c)^3 + (2Bab + Ab^2)\cos(dx + c)^3\sec(dx + c)^2 + (Ba^2 +$$

67.133 Problem number 569

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3a^2A + 5b(Ab + 2Ba)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(2Aab + B a^2 + 3b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7Ab + 5Ba) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \\ & + \frac{2aA(b + a \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3Aa^2 \cos(dx + c) + 5Ba^2 + 10Aab) \sqrt{\cos(dx + c)} \sin(dx + c) - 5\sqrt{2}(iBa^2 + 2iAab + 3iBb^2) \operatorname{weierstrassP}}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx + c)^2 \sec(dx + c)^3 + Aa^2 \cos(dx + c)^2 + (2Bab + Ab^2) \cos(dx + c)^2 \sec(dx + c)^2 + (Ba^2 + \dots\right)\right)$$

67.134 Problem number 570

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(2Aab + B a^2 - b^2B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^2A + 3A b^2 + 6abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b^2B \sin(dx + c)}{d \sqrt{\cos(dx + c)}} + \frac{2a^2A \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-i A a^2 - 6i B a b - 3i A b^2) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (i A a^2 - 6i B a b - 3i A b^2) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx + c) \sec(dx + c)^3 + Aa^2 \cos(dx + c) + (2Bab + Ab^2) \cos(dx + c) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

67.135 Problem number 571

$$\int \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^2 (A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(a^2 A - A b^2 - 2abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(6Aab + 3B a^2 + b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b^2 B \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2b(Ab + 2Ba) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-3i B a^2 - 6i A a b - i B b^2) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i B a^2 + 6i A a b + i B b^2) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \sec(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

67.136 Problem number 572

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(10Aab + 5B a^2 + 3b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3a^2 A + A b^2 + 2abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b^2 B \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2b(Ab + 2Ba) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(10Aab + 5B a^2 + 3b^2 B) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (3i Aa^2 + 2i Bab + i Ab^2) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)}{\sqrt{\cos(dx + c)}}, x\right)$$

67.137 Problem number 573

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(5a^2 A + 3A b^2 + 6abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(14Aab + 7B a^2 + 5b^2 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b^2 B \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{2b(Ab + 2Ba) \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(14Aab + 7B a^2 + 5b^2 B) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(5a^2 A + 3A b^2 + 6abB) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(7iBa^2 + 14iAab + 5iBb^2)\cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 5\sqrt{2}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Bb^2 \sec(dx+c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx+c)^2 + (Ba^2 + 2Aab) \sec(dx+c)}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

67.138 Problem number 594

$$\int \cos^{\frac{7}{2}}(c+dx) \sqrt{a+b\sec(c+dx)} (A+B\sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(a^2 - b^2)(25a^2A + 8Ab^2 - 14abB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx+c)} \sqrt{a+b\sec(dx+c)}} \\ & + \frac{2(Ab + 7Ba) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b\sec(dx+c)}}{35ad} \\ & + \frac{2A \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b\sec(dx+c)}}{7d} \\ & + \frac{2(25a^2A - 4Ab^2 + 7abB) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b\sec(dx+c)}}{105a^2d} \\ & + \frac{2(19Aa^2b + 8Ab^3 + 63a^3B - 14Ba^2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)})}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(15Aa^4 \cos(dx+c)^2 + 25Aa^4 + 7Ba^3b - 4Aa^2b^2 + 3(7Ba^4 + Aa^3b) \cos(dx+c)\right) \sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}} \sqrt{\cos(dx+c)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left((B\cos(dx+c)^3 \sec(dx+c) + A\cos(dx+c)^3) \sqrt{b\sec(dx+c)+a} \sqrt{\cos(dx+c)}\right), x\right)$$

67.139 Problem number 595

$$\int \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(a^2 - b^2) (2Ab - 5Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2A \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{5d} \\ & + \frac{2(Ab + 5Ba) \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{15ad} \\ & + \frac{2(9a^2A - 2Ab^2 + 5abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (3 A a^3 \cos(dx + c) + 5 B a^3 + A a^2 b) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + \sqrt{2} (-15i B a^3 - 3i A a^2 b)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2\right) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}, x\right)$$

67.140 Problem number 596

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2A(a^2 - b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2A \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{3d} \\
& + \frac{2(Ab + 3Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 Aa^2 \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2} (-3i Aa^2 - 3i Bab + 2i Ab^2) \sqrt{a} \operatorname{weierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((B \cos(dx+c) \sec(dx+c) + A \cos(dx+c)) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}, x\right)$$

67.141 Problem number 600

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2) (39A a^2 b + 8A b^3 + 75a^3 B - 18Ba b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b}{a+b}}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\
& + \frac{2(49a^2 A + 3A b^2 + 72abB) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{315ad} \\
& + \frac{2(10Ab + 9Ba) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{63d} \\
& + \frac{2aA \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{9d} \\
& + \frac{2(88A a^2 b - 4A b^3 + 75a^3 B + 9Ba b^2) \sin(dx + c) (\sqrt{\cos}(dx + c)) \sqrt{a + b \sec(dx + c)}}{315a^2 d} \\
& + \frac{2(147A a^4 + 33a^2 A b^2 + 8A b^4 + 246a^3 b B - 18a b^3 B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{a}{a+b}}}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(35 A a^5 \cos(dx + c)^3 + 75 B a^5 + 88 A a^4 b + 9 B a^3 b^2 - 4 A a^2 b^3 + 5 (9 B a^5 + 10 A a^4 b) \cos(dx + c)^2 + (49 A a^5 + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \cos(dx + c)^4 \sec(dx + c)^2 + Aa \cos(dx + c)^4 + (Ba + Ab) \cos(dx + c)^4 \sec(dx + c)\right) \sqrt{b \sec(dx + c)}\right)$$

67.142 Problem number 601

$$\int \cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2)(25a^2A - 6Ab^2 + 21abB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2(8Ab + 7Ba) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{35d} \\
& + \frac{2aA \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7d} \\
& + \frac{2(25a^2A + 3Ab^2 + 42abB) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{105ad} \\
& + \frac{2(82Aa^2b - 6Ab^3 + 63a^3B + 21Bab^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)})}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(15Aa^4 \cos(dx+c)^2 + 25Aa^4 + 42Ba^3b + 3Aa^2b^2 + 3(7Ba^4 + 8Aa^3b) \cos(dx+c) \right) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \cos(dx+c)^3 \sec(dx+c)^2 + Aa \cos(dx+c)^3 + (Ba + Ab) \cos(dx+c)^3 \sec(dx+c)\right) \sqrt{b \sec(dx+c)}\right)$$

67.143 Problem number 602

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2)(3Ab + 5Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\
& + \frac{2aA \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{5d} \\
& + \frac{2(6Ab + 5Ba) \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{15d} \\
& + \frac{2(9a^2A + 3Ab^2 + 20abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(3Aa^3 \cos(dx + c) + 5Ba^3 + 6Aa^2b) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + \sqrt{2}(-15iBa^3 - 18iAa^3)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \cos(dx + c)^2 \sec(dx + c)^2 + Aa \cos(dx + c)^2 + (Ba + Ab) \cos(dx + c)^2 \sec(dx + c)\right) \sqrt{b \sec(dx + c)}\right)$$

67.144 Problem number 607

$$\int \cos^{\frac{11}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2aA \left(\cos^{\frac{9}{2}}(dx+c) \right) (a+b \sec(dx+c))^{\frac{3}{2}} \sin(dx+c)}{11d} \\
& + \frac{2(a^2-b^2) (675Aa^4 + 285a^2Ab^2 + 40Ab^4 + 1254a^3bB - 110ab^3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2(1145Aa^2b + 15Ab^3 + 539a^3B + 825Ba^2b^2) \left(\cos^{\frac{3}{2}}(dx+c) \right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{3465ad} \\
& + \frac{2(81a^2A + 113Ab^2 + 209abB) \left(\cos^{\frac{5}{2}}(dx+c) \right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{693d} \\
& + \frac{2a(14Ab + 11Ba) \left(\cos^{\frac{7}{2}}(dx+c) \right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{99d} \\
& + \frac{2(675Aa^4 + 1025a^2Ab^2 - 20Ab^4 + 1793a^3bB + 55ab^3B) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{3465a^2d} \\
& + \frac{2(3705Aa^4b + 255Aa^2b^3 + 40Ab^5 + 1617Ba^5 + 3069Ba^3b^2 - 110Bab^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(315 Aa^6 \cos(dx+c)^4 + 675 Aa^6 + 1793 Ba^5b + 1025 Aa^4b^2 + 55 Ba^3b^3 - 20 Aa^2b^4 + 35 (11 Ba^6 + 23 Aa^5b) \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Bb^2 \cos(dx+c)^5 \sec(dx+c)^3 + Aa^2 \cos(dx+c)^5 + (2Bab + Ab^2) \cos(dx+c)^5 \sec(dx+c)^2 + (Ba^2 + \dots \right) \right)$$

67.145 Problem number 608

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA\left(\cos^{\frac{7}{2}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{3}{2}}\sin(dx+c)}{9d} \\ & + \frac{2(a^2-b^2)(114Aa^2b-10Ab^3+75a^3B+45Bab^2)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{a}{a+b}}\right)}{315\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d\sqrt{\cos(dx+c)}\sqrt{a+b\sec(dx+c)}} \\ & + \frac{2(49a^2A+75Aab+135abB)\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)\sqrt{a+b\sec(dx+c)}}{315d} \\ & + \frac{2a(4Ab+3Ba)\left(\cos^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)\sqrt{a+b\sec(dx+c)}}{21d} \\ & + \frac{2(163Aa^2b+5Ab^3+75a^3B+135Bab^2)\sin(dx+c)(\sqrt{\cos(dx+c)})\sqrt{a+b\sec(dx+c)}}{315ad} \\ & + \frac{2(147Aa^4+279a^2Ab^2-10Ab^4+435a^3bB+45ab^3B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\sqrt{\frac{a}{a+b}}\right)}{315\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d\sqrt{\frac{b+a\cos(dx+c)}{a+b}}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6\left(35Aa^5\cos(dx+c)^3+75Ba^5+163Aa^4b+135Ba^3b^2+5Aa^2b^3+5(9Ba^5+19Aa^4b)\cos(dx+c)^2+(49Aa^5\right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2\cos(dx+c)^4\sec(dx+c)^3+Aa^2\cos(dx+c)^4+(2Bab+Ab^2)\cos(dx+c)^4\sec(dx+c)^2+(Ba^2+\right.$$

67.146 Problem number 609

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \left(\cos^{\frac{5}{2}}(dx+c) \right) (a+b\sec(dx+c))^{\frac{3}{2}} \sin(dx+c)}{7d} \\ & + \frac{2(a^2-b^2)(25a^2A+15Ab^2+56abB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx+c)} \sqrt{a+b\sec(dx+c)}} \\ & + \frac{2a(10Ab+7Ba) \left(\cos^{\frac{3}{2}}(dx+c) \right) \sin(dx+c) \sqrt{a+b\sec(dx+c)}}{35d} \\ & + \frac{2(25a^2A+45Ab^2+77abB) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b\sec(dx+c)}}{105d} \\ & + \frac{2(145Aa^2b+15Ab^3+63a^3B+161Ba^2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)})}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b+a\cos(dx+c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(15Aa^4 \cos(dx+c)^2 + 25Aa^4 + 77Ba^3b + 45Aa^2b^2 + 3(7Ba^4 + 15Aa^3b) \cos(dx+c) \right) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \cos(dx+c)^3 \sec(dx+c)^3 + Aa^2 \cos(dx+c)^3 + (2Bab + Ab^2) \cos(dx+c)^3 \sec(dx+c)^2 + (Ba^2 + \dots\right)$$

67.147 Problem number 615

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(7Aa^2b + 8Ab^3 - 5a^3B - 10Bab^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx+c)} \sqrt{a+b\sec(dx+c)}} \\ & + \frac{2A\left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b\sec(dx+c)}}{5ad} \\ & - \frac{2(4Ab - 5Ba) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b\sec(dx+c)}}{15a^2d} \\ & + \frac{2(9a^2A + 8Ab^2 - 10abB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b\sec(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(3Aa^3 \cos(dx+c) + 5Ba^3 - 4Aa^2b) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2}(-15iBa^3 + 12iAa^2b)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(B \cos(dx+c)^2 \sec(dx+c) + A \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}}{\sqrt{b \sec(dx+c) + a}}, x\right)$$

67.148 Problem number 616

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2(a^2A + 2Ab^2 - 3abB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} + \frac{2A \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{3ad}$$

$$- \frac{2(2Ab - 3Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6Aa^2 \sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2} (-3iAa^2 + 6iBab - 4iAb^2) \sqrt{a} \operatorname{weierstrassPInverse}(\dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cos(dx+c) \sec(dx+c) + A \cos(dx+c)) \sqrt{\cos(dx+c)}}{\sqrt{b \sec(dx+c) + a}}, x\right)$$

67.149 Problem number 617

$$\int \frac{\sqrt{\cos(c+dx)} (A + B \sec(c+dx))}{\sqrt{a + b \sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} + \frac{2A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}$$

command

`integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$3i\sqrt{2} Aa^{\frac{3}{2}} \text{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}\right), \text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3}{3}\right)$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(B \sec(dx+c) + A) \sqrt{\cos(dx+c)}}{\sqrt{b \sec(dx+c) + a}}, x\right)$$

67.150 Problem number 621

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{\frac{3}{2}}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(Ab - Ba) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{a(a^2 - b^2) d \sqrt{a + b \sec(dx+c)}} \\ & - \frac{2(12Aa^2b + 48Ab^3 - 5a^3B - 40Bab^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx+c)} \sqrt{a + b \sec(dx+c)}} \\ & + \frac{2(a^2A - 6Ab^2 + 5abB) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a + b \sec(dx+c)}}{5a^2(a^2 - b^2) d} \\ & - \frac{2(9Aa^2b - 24Aa^3b^3 - 5a^3B + 20Bab^2) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a + b \sec(dx+c)}}{15a^3(a^2 - b^2) d} \\ & + \frac{2(9Aa^4 + 24a^2Ab^2 - 48Ab^4 - 25a^3bB + 40ab^3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2 - b^2) d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(5Ba^5b - 9Aa^4b^2 - 20Ba^3b^3 + 24Aa^2b^4 + 3(Aa^6 - Aa^4b^2) \cos(dx+c)^2 + (5Ba^6 - 6Aa^5b - 5Ba^4b^2 + 6Aa^3b^3)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx+c)^2 \sec(dx+c) + A \cos(dx+c)^2 \right) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}}{b^2 \sec(dx+c)^2 + 2ab \sec(dx+c) + a^2}, x \right)$$

67.151 Problem number 622

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(Ab - Ba) \sin(dx+c) (\sqrt{\cos(dx+c)})}{a(a^2 - b^2) d \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2(a^2A + 8Ab^2 - 6abB) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}} \right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2(a^2A - 4Ab^2 + 3abB) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{3a^2(a^2 - b^2) d} \\ & - \frac{2(5Aa^2b - 8Ab^3 - 3a^3B + 6Ba^2b^2) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}} \right) (\sqrt{\cos(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2) d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(Aa^4b + 3Ba^3b^2 - 4Aa^2b^3 + (Aa^5 - Aa^3b^2) \cos(dx+c)) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \left(\sqrt{\cos(dx+c)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(B \cos(dx+c) \sec(dx+c) + A \cos(dx+c) \right) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}}{b^2 \sec(dx+c)^2 + 2ab \sec(dx+c) + a^2}, x \right)$$

67.152 Problem number 623

$$\int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b(Ab - Ba) \sin(dx + c)}{a(a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} - \frac{2(2Ab - Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2(a^2A - 2Ab^2 + abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}$$

command

`integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (Ba^3b - Aa^2b^2) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + \left(\sqrt{2} (3i Ba^4 - 5i Aa^3b - 2i Ba^2b^2 + 4i Aab^2)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}, x\right)$$

67.153 Problem number 624

$$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+b \sec(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba) \sin(dx + c)}{(a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2(Ab - Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}$$

command

```
integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 (Ba^3 - Aa^2b) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2} (3i Aa^3 - i Ba^2b - 2i Aab^2) \cos(dx + c) + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^2 \cos(dx + c) \sec(dx + c)^2 + 2ab \cos(dx + c) \sec(dx + c) + a^2 \cos(dx + c)}, x \right)$$

67.154 Problem number 628

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^{\frac{5}{2}}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(Ab - Ba) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a(a^2 - b^2) d (a + b \sec(dx + c))^{\frac{3}{2}}} \\ & + \frac{2b(12Aa^2b - 8Ab^3 - 9a^3B + 5Ba^2b^2) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a^2(a^2 - b^2)^2 d \sqrt{a + b \sec(dx + c)}} \\ & - \frac{2(17Aa^4b + 116Aa^2b^3 - 128Ab^5 - 5Ba^5 - 80Ba^3b^2 + 80Bab^4) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \dots \right)}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^5 (a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(3Aa^4 - 71a^2Ab^2 + 48Ab^4 + 50a^3bB - 30ab^3B) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15a^3(a^2 - b^2)^2 d} \\ & - \frac{2(14Aa^4b - 98Aa^2b^3 + 64Ab^5 - 5Ba^5 + 65Ba^3b^2 - 40Bab^4) \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{15a^4(a^2 - b^2)^2 d} \\ & + \frac{2(9Aa^6 + 55Aa^4b^2 - 212a^2Ab^4 + 128Ab^6 - 40a^5bB + 140a^3b^3B - 80ab^5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \dots \right)}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^5 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \cos(dx+c)^2 \sec(dx+c) + A \cos(dx+c)^2) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3}, x \right)$$

67.155 Problem number 629

$$\int \frac{\cos^{3/2}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(Ab - Ba) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a(a^2 - b^2) d(a+b \sec(dx+c))^{3/2}} \\ & + \frac{2b(10Aa^2b - 6Ab^3 - 7a^3B + 3Ba^2b^2) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a^2(a^2 - b^2)^2 d \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2(Aa^4 + 16a^2Ab^2 - 16Ab^4 - 9a^3bB + 8ab^3B) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2 - b^2) d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2(Aa^4 - 13a^2Ab^2 + 8Ab^4 + 8a^3bB - 4ab^3B) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{3a^3(a^2 - b^2)^2 d} \\ & - \frac{2(8Aa^4b - 28Aa^2b^3 + 16Ab^5 - 3Ba^5 + 15Ba^3b^2 - 8Bab^4) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2 - b^2)^2 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(Aa^6b^2 + 8Ba^5b^3 - 13Aa^4b^4 - 4Ba^3b^5 + 8Aa^2b^6 + (Aa^8 - 2Aa^6b^2 + Aa^4b^4) \cos(dx+c)^2 + (2Aa^7b + 9Ba^6b^2) \cos(dx+c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(B \cos(dx+c) \sec(dx+c) + A \cos(dx+c)) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3}, x \right)$$

67.156 Problem number 630

$$\int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2b(Ab - Ba) \sin(dx + c)}{3a(a^2 - b^2) d(a + b \sec(dx + c))^{\frac{3}{2}} \sqrt{\cos(dx + c)}} + \frac{2b(8Aa^2b - 4Ab^3 - 5a^3B + Ba^2b^2) \sin(dx + c)}{3a^2(a^2 - b^2)^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} - \frac{2(9Aa^2b - 8Ab^3 - 3a^3B + 2Ba^2b^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2(3Aa^4 - 15a^2Ab^2 + 8Ab^4 + 6a^3bB - 2ab^3B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}$$

command

`integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(5Ba^5b^2 - 8Aa^4b^3 - Ba^3b^4 + 4Aa^2b^5 + (6Ba^6b - 9Aa^5b^2 - 2Ba^4b^3 + 5Aa^3b^4) \cos(dx + c)) \sqrt{\frac{a \cos(dx + c)}{\cos(dx + c)}}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x\right)$$

67.157 Problem number 631

$$\int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Ab - Ba) \sin(dx + c)}{3(a^2 - b^2) d(a + b \sec(dx + c))^{\frac{3}{2}} \sqrt{\cos(dx + c)}} \\
& - \frac{2(5Aa^2b - Ab^3 - 2a^3B - 2Bab^2) \sin(dx + c)}{3a(a^2 - b^2)^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\
& + \frac{2(3a^2A - 2Ab^2 - abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\
& + \frac{2(6Aa^2b - 2Ab^3 - 3a^3B - Bab^2) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}
\end{aligned}$$

command

```
integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(2Ba^5b - 5Aa^4b^2 + 2Ba^3b^3 + Aa^2b^4 + (3Ba^6 - 6Aa^5b + Ba^4b^2 + 2Aa^3b^3) \cos(dx + c)) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^3 \cos(dx + c) \sec(dx + c)^3 + 3ab^2 \cos(dx + c) \sec(dx + c)^2 + 3a^2b \cos(dx + c) \sec(dx + c) + a^3 \cos(dx + c)}\right)$$

67.158 Problem number 632

$$\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a(Ab - Ba) \sin(dx + c)}{3b(a^2 - b^2) d(a + b \sec(dx + c))^{\frac{3}{2}} \sqrt{\cos(dx + c)}} + \frac{2(2Aa^2b + 2Ab^3 + a^3B - 5Ba^2b^2) \sin(dx + c)}{3b(a^2 - b^2)^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} - \frac{2(Ab - Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a(a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} - \frac{2(3a^2A + Ab^2 - 4abB) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a(a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}$$

command

```
integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(Ba^5 + 2Aa^4b - 5Ba^3b^2 + 2Aa^2b^3 + (3Aa^5 - 4Ba^4b + Aa^3b^2) \cos(dx + c)) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^3 \cos(dx + c)^2 \sec(dx + c)^3 + 3ab^2 \cos(dx + c)^2 \sec(dx + c)^2 + 3a^2b \cos(dx + c)^2 \sec(dx + c) + a^3 \cos(dx + c)}\right)$$

68 Test file number 124

Test folder name:

test_cases/4_Trig_functions/4.5_Secant/124_4.5.4.1-a+b_sec~m-A+B_sec+C_sec^2-

68.1 Problem number 16

$$\int (b \sec(c + dx))^{5/2} (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2b(7A + 5C) (b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{21d} + \frac{2b^2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C(b \sec(dx + c))^{\frac{5}{2}} \tan(dx + c)}{7d}$$

command

```
integrate((b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (7A + 5C) b^{\frac{5}{2}} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (7A + 5C) b^{\frac{5}{2}} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \sec(dx + c)^4 + Ab^2 \sec(dx + c)^2\right) \sqrt{b \sec(dx + c)}, x\right)$$

68.2 Problem number 17

$$\int (b \sec(c + dx))^{3/2} (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2b^2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2b(5A + 3C) \sin(dx + c) \sqrt{b \sec(dx + c)}}{5d} + \frac{2C(b \sec(dx + c))^{\frac{3}{2}} \tan(dx + c)}{5d}$$

command

```
integrate((b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} (5A + 3C) b^{\frac{3}{2}} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} (5A + 3C) b^{\frac{3}{2}} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \sec(dx + c)^3 + Ab \sec(dx + c)\right) \sqrt{b \sec(dx + c)}, x\right)$$

68.3 Problem number 18

$$\int \sqrt{b \sec(c + dx)} (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sqrt{b \sec(dx + c)} \tan(dx + c)}{3d}$$

command

```
integrate((b*sec(d*x+c))^(1/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-3i A - i C) \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i A + i C) \sqrt{b} \cos(dx + c)}{3 d \cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \sec(dx + c)^2 + A\right) \sqrt{b \sec(dx + c)}, x\right)$$

68.4 Problem number 19

$$\int \frac{A + C \sec^2(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) + \frac{2C \tan(dx + c)}{d \sqrt{b \sec(dx + c)}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}}$$

command

```
integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (i A - i C) \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + \sqrt{2} (-i A + i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \sec(dx + c)^2 + A \right) \sqrt{b \sec(dx + c)}}{b \sec(dx + c)}, x \right)$$

68.5 Problem number 20

$$\int \frac{A + C \sec^2(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) b^2 d} + \frac{2A \tan(dx + c)}{3d (b \sec(dx + c))^{\frac{3}{2}}}$$

command

`integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2A \sqrt{\frac{b}{\cos(dx + c)}} \cos(dx + c) \sin(dx + c) + \sqrt{2} (-i A - 3i C) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

$3b^2d$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \sec(dx + c)^2 + A \right) \sqrt{b \sec(dx + c)}}{b^2 \sec(dx + c)^2}, x \right)$$

68.6 Problem number 21

$$\int \frac{A + C \sec^2(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2A \tan(dx + c)}{5d (b \sec(dx + c))^{\frac{5}{2}}}$$

command

```
integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2A \sqrt{\frac{b}{\cos(dx + c)}} \cos(dx + c)^2 \sin(dx + c) + \sqrt{2} (3iA + 5iC) \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \frac{b}{\cos(dx + c)}))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + A\right) \sqrt{b \sec(dx + c)}}{b^3 \sec(dx + c)^3}, x\right)$$

68.7 Problem number 22

$$\int \frac{A + C \sec^2(c + dx)}{(b \sec(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(5A + 7C) \sin(dx + c)}{21b^3 d \sqrt{b \sec(dx + c)}} + \frac{2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d} + \frac{2A \tan(dx + c)}{7d (b \sec(dx + c))^{\frac{7}{2}}}$$

command

`integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-5iA - 7iC)\sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(5iA + 7iC)\sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \sec(dx + c)^2 + A) \sqrt{b \sec(dx + c)}}{b^4 \sec(dx + c)^4}, x\right)$$

68.8 Problem number 23

$$\int \frac{A + C \sec^2(c + dx)}{(b \sec(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2(7A + 9C) \sin(dx + c)}{45b^3d (b \sec(dx + c))^{\frac{3}{2}}} + \frac{2(7A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2A \tan(dx + c)}{9d (b \sec(dx + c))^{\frac{9}{2}}}$$

command

`integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3\sqrt{2}(-7iA - 9iC)\sqrt{b} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3\sqrt{2}(7iA + 9iC)\sqrt{b} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \sec(dx + c)^2 + A) \sqrt{b \sec(dx + c)}}{b^5 \sec(dx + c)^5}, x\right)$$

68.9 Problem number 47

$$\int (b \sec(c + dx))^{3/2} (B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(b \sec(dx + c))^{3/2} \sin(dx + c)}{3d} + \frac{2C(b \sec(dx + c))^{5/2} \sin(dx + c)}{5bd} \\ & - \frac{6b^2C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} \\ & + \frac{6bC \sin(dx + c) \sqrt{b \sec(dx + c)}}{5d} \\ & + \frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B b^{3/2} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B b^{3/2} \cos(dx + c)^2 \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \sec(dx + c)^3 + Bb \sec(dx + c)^2\right) \sqrt{b \sec(dx + c)}, x\right)$$

68.10 Problem number 48

$$\int \sqrt{b \sec(c + dx)} (B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C(b \sec(dx + c))^{3/2} \sin(dx + c)}{3bd} - \frac{2bB \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} \\ & + \frac{2B \sin(dx + c) \sqrt{b \sec(dx + c)}}{d} \\ & + \frac{2C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((b*sec(d*x+c))^(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} C \sqrt{b} \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} C \sqrt{b} \cos(dx + c) \text{weier}$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \sec(dx + c)^2 + B \sec(dx + c)\right) \sqrt{b \sec(dx + c)}, x\right)$$

68.11 Problem number 49

$$\int \frac{B \sec(c + dx) + C \sec^2(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} + \frac{2C \sin(dx + c) \sqrt{b \sec(dx + c)}}{bd} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd} \end{aligned}$$

command

```
integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \sec(dx + c) + B) \sqrt{b \sec(dx + c)}}{b}, x\right)$$

68.12 Problem number 50

$$\int \frac{B \sec(c + dx) + C \sec^2(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} + \frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d}$$

command

```
integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} C \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} C \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx+c) + B) \sqrt{b \sec(dx+c)}}{b^2 \sec(dx+c)}, x\right)$$

68.13 Problem number 51

$$\int \frac{B \sec(c + dx) + C \sec^2(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2B \sin(dx+c)}{3b^2 d \sqrt{b \sec(dx+c)}} + \frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d}$$

command

`integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2B\sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) - i\sqrt{2}B\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx+c) + B)\sqrt{b \sec(dx+c)}}{b^3 \sec(dx+c)^2}, x\right)$$

68.14 Problem number 52

$$\int \frac{B \sec(c+dx) + C \sec^2(c+dx)}{(b \sec(c+dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \sin(dx+c)}{5b^2d (b \sec(dx+c))^{\frac{3}{2}}} + \frac{2C \sin(dx+c)}{3b^3d \sqrt{b \sec(dx+c)}} \\ & + \frac{6B \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} \\ & + \frac{2C \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4d} \end{aligned}$$

command

`integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}C\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i\sqrt{2}C\sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx+c) + B)\sqrt{b \sec(dx+c)}}{b^4 \sec(dx+c)^3}, x\right)$$

68.15 Problem number 53

$$\int \frac{B \sec(c + dx) + C \sec^2(c + dx)}{(b \sec(c + dx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \sin(dx + c)}{7b^2d (b \sec(dx + c))^{5/2}} + \frac{2C \sin(dx + c)}{5b^3d (b \sec(dx + c))^{3/2}} + \frac{10B \sin(dx + c)}{21b^4d \sqrt{b \sec(dx + c)}} \\ & + \frac{6C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} \\ & + \frac{10B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^5d} \end{aligned}$$

command

```
integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-25i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c) + B) \sqrt{b \sec(dx + c)}}{b^5 \sec(dx + c)^4}, x\right)$$

68.16 Problem number 65

$$\int (b \sec(c + dx))^{3/2} (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B(b \sec(dx+c))^{\frac{3}{2}} \sin(dx+c)}{3d} \\ & - \frac{2b^2(5A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} \\ & + \frac{2b(5A+3C) \sin(dx+c) \sqrt{b \sec(dx+c)}}{5d} \\ & + \frac{2bB \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(b \sec(dx+c))^{\frac{3}{2}} \tan(dx+c)}{5d} \end{aligned}$$

command

```
integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B b^{\frac{3}{2}} \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} B b^{\frac{3}{2}} \cos(dx+c)^2 \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \sec(dx+c)^3 + Bb \sec(dx+c)^2 + Ab \sec(dx+c)\right) \sqrt{b \sec(dx+c)}, x\right)$$

68.17 Problem number 66

$$\int \sqrt{b \sec(c+dx)} (A + B \sec(c+dx) + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2bB \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} + \frac{2B \sin(dx+c) \sqrt{b \sec(dx+c)}}{d} \\ & + \frac{2(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \sqrt{b \sec(dx+c)} \tan(dx+c)}{3d} \end{aligned}$$

command

```
integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i A - i C) \sqrt{b} \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i A + i C) \sqrt{b} \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \sec(dx + c)^2 + B \sec(dx + c) + A\right) \sqrt{b \sec(dx + c)}, x\right)$$

68.18 Problem number 67

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd} \\ & + \frac{2C \tan(dx + c)}{d \sqrt{b \sec(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A\right) \sqrt{b \sec(dx + c)}}{b \sec(dx + c)}, x\right)$$

68.19 Problem number 68

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{(b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) bd \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} + \frac{2(A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d} + \frac{2A \tan(dx+c)}{3d (b \sec(dx+c))^{3/2}}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2A \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + \sqrt{2} (-iA - 3iC) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx+c)^2 + B \sec(dx+c) + A) \sqrt{b \sec(dx+c)}}{b^2 \sec(dx+c)^2}, x\right)$$

68.20 Problem number 69

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{(b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2B \sin(dx+c)}{3b^2 d \sqrt{b \sec(dx+c)}} + \frac{2(3A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cos(dx+c)} \sqrt{b \sec(dx+c)}} + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) \sqrt{b \sec(dx+c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d} + \frac{2A \tan(dx+c)}{5d (b \sec(dx+c))^{5/2}}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i \sqrt{2} B \sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $5i \sqrt{2} B \sqrt{b}$ weierstrassPInverse(-4, 0, co

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c)}}{b^3 \sec(dx + c)^3}, x \right)$$

68.21 Problem number 70

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{(b \sec(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \sin(dx + c)}{5b^2 d (b \sec(dx + c))^{3/2}} + \frac{2(5A + 7C) \sin(dx + c)}{21b^3 d \sqrt{b \sec(dx + c)}} \\ & + \frac{6B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^3 d \sqrt{\cos(dx + c)} \sqrt{b \sec(dx + c)}} \\ & + \frac{2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) \sqrt{b \sec(dx + c)}}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d} \\ & + \frac{2A \tan(dx + c)}{7d (b \sec(dx + c))^{7/2}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$5 \sqrt{2} (5i A + 7i C) \sqrt{b}$ weierstrassPInverse(-4, 0, cos(dx + c) + i sin(dx + c)) + $5 \sqrt{2} (-5i A - 7i C) \sqrt{b}$ weierst

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c)}}{b^4 \sec(dx + c)^4}, x \right)$$

69 Test file number 125

Test folder name:

test_cases/4_Trig_functions/4.5_Secant/125_4.5.4.2-a+b_sec^m-d_sec^n-A+B_sec+C_sec^2-

69.1 Problem number 207

$$\int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx)) (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(7A + 5C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} + \frac{2aC \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2aC \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{2a(5A + 3C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2a(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (7A + 5C)a \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (7A + 5C)a \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Ca \sec(dx + c)^4 + Ca \sec(dx + c)^3 + Aa \sec(dx + c)^2 + Aa \sec(dx + c) \right) \sqrt{\sec(dx + c)}, x \right)$$

69.2 Problem number 208

$$\int \sqrt{\sec(c+dx)} (a+a \sec(c+dx)) (A+C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aC \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2aC \left(\sec^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{5d} \\ & + \frac{2a(5A+3C) \sin(dx+c) (\sqrt{\sec}(dx+c))}{5d} \\ & - \frac{2a(5A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (3A+C)a \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} (3A+C)a \cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Ca \sec(dx+c)^3 + Ca \sec(dx+c)^2 + Aa \sec(dx+c) + Aa \right) \sqrt{\sec(dx+c)}, x \right)$$

69.3 Problem number 209

$$\int \frac{(a+a \sec(c+dx)) (A+C \sec^2(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aC \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d} + \frac{2aC \sin(dx+c) (\sqrt{\sec}(dx+c))}{d} \\ & + \frac{2a(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}(3A+C)a\cos(dx+c)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+C)a\cos(dx-$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\sec(dx+c)^3+Ca\sec(dx+c)^2+Aa\sec(dx+c)+Aa}{\sqrt{\sec(dx+c)}},x\right)$$

69.4 Problem number 210

$$\int \frac{(a+a\sec(c+dx))(A+C\sec^2(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA\sin(dx+c)}{3d\sqrt{\sec(dx+c)}} + \frac{2aC\sin(dx+c)(\sqrt{\sec(dx+c)})}{d} \\ & + \frac{2a(A-C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2a(A+3C)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}(A+3C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(A+3C)a\text{weierstrassPInverse}(-$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\sec(dx+c)^3+Ca\sec(dx+c)^2+Aa\sec(dx+c)+Aa}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

69.5 Problem number 211

$$\int \frac{(a + a \sec(c + dx)) (A + C \sec^2(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2aA \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (A + 3C) \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (A + 3C) \operatorname{aweierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca \sec(dx + c)^3 + Ca \sec(dx + c)^2 + Aa \sec(dx + c) + Aa}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

69.6 Problem number 212

$$\int \frac{(a + a \sec(c + dx)) (A + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2aA \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(5A + 7C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(5A+7C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(5A+7C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\sec(dx+c)^3+Ca\sec(dx+c)^2+Aa\sec(dx+c)+Aa}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

69.7 Problem number 213

$$\int \frac{(a+a\sec(c+dx))(A+C\sec^2(c+dx))}{\sec^{\frac{9}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA\sin(dx+c)}{9d\sec(dx+c)^{\frac{7}{2}}} + \frac{2aA\sin(dx+c)}{7d\sec(dx+c)^{\frac{5}{2}}} + \frac{2a(7A+9C)\sin(dx+c)}{45d\sec(dx+c)^{\frac{3}{2}}} + \frac{2a(5A+7C)\sin(dx+c)}{21d\sqrt{\sec(dx+c)}} \\ & + \frac{2a(7A+9C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(5A+7C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i\sqrt{2}(5A+7C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}(5A+7C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\sec(dx+c)^3+Ca\sec(dx+c)^2+Aa\sec(dx+c)+Aa}{\sec(dx+c)^{\frac{9}{2}}},x\right)$$

69.8 Problem number 214

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(7A+5C)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{21d} + \frac{2a^2(21A+19C)\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{105d} \\ & + \frac{2C\left(\sec^{\frac{5}{2}}(dx+c)\right)(a+a\sec(dx+c))^2\sin(dx+c)}{9d} \\ & + \frac{8C\left(\sec^{\frac{5}{2}}(dx+c)\right)(a^2+a^2\sec(dx+c))\sin(dx+c)}{63d} \\ & + \frac{16a^2(3A+2C)\sin(dx+c)(\sqrt{\sec}(dx+c))}{15d} \\ & - \frac{16a^2(3A+2C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos}(dx+c))(\sqrt{\sec}(dx+c))}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{4a^2(7A+5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos}(dx+c))(\sqrt{\sec}(dx+c))}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(15i\sqrt{2}(7A+5C)a^2\cos(dx+c)^4\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15i\sqrt{2}(7A+5C)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2\sec(dx+c)^5+2Ca^2\sec(dx+c)^4+(A+C)a^2\sec(dx+c)^3+2Aa^2\sec(dx+c)^2+Aa^2\sec(dx+c)\right)\right)$$

69.9 Problem number 215

$$\int \sqrt{\sec(c+dx)} (a+a\sec(c+dx))^2 (A+C\sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(35A+33C) \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{105d} \\ & + \frac{2C \left(\sec^{\frac{3}{2}}(dx+c)\right) (a+a\sec(dx+c))^2 \sin(dx+c)}{7d} \\ & + \frac{8C \left(\sec^{\frac{3}{2}}(dx+c)\right) (a^2+a^2\sec(dx+c)) \sin(dx+c)}{35d} \\ & + \frac{4a^2(5A+3C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{5d} \\ & - \frac{4a^2(5A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(7A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} (7A+3C)a^2 \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 10i \sqrt{2} (7A+3C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \sec(dx+c)^4 + 2Ca^2 \sec(dx+c)^3 + (A+C)a^2 \sec(dx+c)^2 + 2Aa^2 \sec(dx+c) + Aa^2\right) \sqrt{\sec(dx+c)}\right)$$

69.10 Problem number 216

$$\int \frac{(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(15A + 17C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & + \frac{2C(a + a \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & + \frac{8C(a^2 + a^2 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{16a^2C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output
```

$$2 \left(5i \sqrt{2} (3A + C)a^2 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + C)a^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \sec(dx + c)^4 + 2Ca^2 \sec(dx + c)^3 + (A + C)a^2 \sec(dx + c)^2 + 2Aa^2 \sec(dx + c) + Aa^2}{\sqrt{\sec(dx + c)}}, x\right)$$

69.11 Problem number 217

$$\int \frac{(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^2 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} - \frac{2a^2(A - 5C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & - \frac{2(A - C)(a^2 + a^2 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & + \frac{4a^2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2i \sqrt{2} (A + C)a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 2i \sqrt{2} (A + C)a^2 \cos \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \sec(dx + c)^4 + 2Ca^2 \sec(dx + c)^3 + (A + C)a^2 \sec(dx + c)^2 + 2Aa^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

69.12 Problem number 218

$$\int \frac{(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{8A(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & - \frac{2a^2(7A - 15C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & + \frac{16a^2 A \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A + 3C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + 3C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ca^2 \sec(dx + c)^4 + 2Ca^2 \sec(dx + c)^3 + (A + C)a^2 \sec(dx + c)^2 + 2Aa^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{5}{2}}}, x \right)$$

69.13 Problem number 219

$$\int \frac{(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^2 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{8A(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a^2(33A + 35C) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(3A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(10i \sqrt{2} (3A + 7C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} (3A + 7C)a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ca^2 \sec(dx + c)^4 + 2Ca^2 \sec(dx + c)^3 + (A + C)a^2 \sec(dx + c)^2 + 2Aa^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{7}{2}}}, x \right)$$

69.14 Problem number 220

$$\int \frac{(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(19A + 21C) \sin(dx + c)}{105d \sec(dx + c)^{\frac{3}{2}}} + \frac{2A(a + a \sec(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} \\ & + \frac{8A(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^2(5A + 7C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{16a^2(2A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (5A + 7C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 7C) a^2 \operatorname{weierstrassP} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \sec(dx + c)^4 + 2Ca^2 \sec(dx + c)^3 + (A + C)a^2 \sec(dx + c)^2 + 2Aa^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{9}{2}}}, x\right)$$

69.15 Problem number 221

$$\int \frac{(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(89A + 99C) \sin(dx + c)}{693d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^2(7A + 9C) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{2A(a + a \sec(dx + c))^2 \sin(dx + c)}{11d \sec(dx + c)^{\frac{9}{2}}} \\ & + \frac{8A(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{99d \sec(dx + c)^{\frac{7}{2}}} + \frac{8a^2(25A + 33C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(7A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(25A + 33C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(30i \sqrt{2} (25A + 33C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 30i \sqrt{2} (25A + 33C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \sec(dx + c)^4 + 2Ca^2 \sec(dx + c)^3 + (A + C)a^2 \sec(dx + c)^2 + 2Aa^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{11}{2}}}, x\right)$$

69.16 Problem number 222

$$\int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^3(143A + 105C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{231d} + \frac{8a^3(44A + 35C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{385d} \\
& + \frac{2C \left(\sec^{\frac{5}{2}}(dx + c) \right) (a + a \sec(dx + c))^3 \sin(dx + c)}{11d} \\
& + \frac{4C \left(\sec^{\frac{5}{2}}(dx + c) \right) (a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{33ad} \\
& + \frac{2(33A + 35C) \left(\sec^{\frac{5}{2}}(dx + c) \right) (a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{231d} \\
& + \frac{4a^3(7A + 5C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\
& - \frac{4a^3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\
& + \frac{4a^3(143A + 105C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{231 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}
\end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (143A + 105C) a^3 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (143A + 105C) a^3 \sin(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)}{231 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(C a^3 \sec(dx + c)^6 + 3 C a^3 \sec(dx + c)^5 + (A + 3 C) a^3 \sec(dx + c)^4 + (3 A + C) a^3 \sec(dx + c)^3 + 3 A a^3 \sec(dx + c)^2 \right) dx \right)$$

69.17 Problem number 223

$$\int \sqrt{\sec(c + dx)} (a + a \sec(c + dx))^3 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{8a^3(21A + 16C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{105d} \\
 & + \frac{2C \left(\sec^{\frac{3}{2}}(dx + c) \right) (a + a \sec(dx + c))^3 \sin(dx + c)}{9d} \\
 & + \frac{4C \left(\sec^{\frac{3}{2}}(dx + c) \right) (a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{21ad} \\
 & + \frac{2(63A + 73C) \left(\sec^{\frac{3}{2}}(dx + c) \right) (a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{315d} \\
 & + \frac{4a^3(27A + 17C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15d} \\
 & - \frac{4a^3(27A + 17C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\
 & + \frac{4a^3(21A + 11C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}
 \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (21A + 11C) a^3 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (21A + 11C) a^3 \sin(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(C a^3 \sec(dx + c)^5 + 3 C a^3 \sec(dx + c)^4 + (A + 3 C) a^3 \sec(dx + c)^3 + (3 A + C) a^3 \sec(dx + c)^2 + 3 A a^3 \sec(dx + c) + A^2 \right) \sqrt{\sec(dx + c)} \right) dx$$

69.18 Problem number 224

$$\int \frac{(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{8a^3(70A + 53C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{105d} \\
& + \frac{2C(a + a \sec(dx + c))^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{7d} \\
& + \frac{12C(a^2 + a^2 \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{35ad} \\
& + \frac{2(5A + 7C) (a^3 + a^3 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\
& - \frac{4a^3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(35A + 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (35A + 13C) a^3 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (35A + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + 3Ca^3 \sec(dx + c)^4 + (A + 3C)a^3 \sec(dx + c)^3 + (3A + C)a^3 \sec(dx + c)^2 + 3Aa^3 \sec(dx + c)}{\sqrt{\sec(dx + c)}}\right)$$

69.19 Problem number 225

$$\int \frac{(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{3d\sqrt{\sec(dx + c)}} + \frac{4a^3(5A + 21C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\
& - \frac{2(5A - 3C) (a^2 + a^2 \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{15ad} \\
& - \frac{2(5A - 9C) (a^3 + a^3 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\
& + \frac{4a^3(5A - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (5A + 3C) a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (5A + 3C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + 3Ca^3 \sec(dx + c)^4 + (A + 3C)a^3 \sec(dx + c)^3 + (3A + C)a^3 \sec(dx + c)^2 + 3Aa^3 \sec(dx + c)}{\sec(dx + c)^{\frac{3}{2}}}\right)$$

69.20 Problem number 226

$$\int \frac{(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{4A(a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{5ad \sqrt{\sec(dx + c)}} \\ & - \frac{8a^3(3A - 10C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{2(9A - 5C) (a^3 + a^3 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & + \frac{4a^3(9A - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (3A + 5C)a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + 5C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + 3Ca^3 \sec(dx + c)^4 + (A + 3C)a^3 \sec(dx + c)^3 + (3A + C)a^3 \sec(dx + c)^2 + 3Aa^3 \sec(dx + c)}{\sec(dx + c)^{\frac{5}{2}}}\right)$$

69.21 Problem number 227

$$\int \frac{(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{12A(a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{35ad \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(7A + 5C) (a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} - \frac{4a^3(41A - 35C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{105d} \\ & + \frac{4a^3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(13A + 35C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (13A + 35C)a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (13A + 35C)a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ca^3 \sec(dx + c)^5 + 3Ca^3 \sec(dx + c)^4 + (A + 3C)a^3 \sec(dx + c)^3 + (3A + C)a^3 \sec(dx + c)^2 + 3Aa^3 \sec(dx + c)}{\sec(dx + c)^{\frac{7}{2}}} \right)$$

69.22 Problem number 228

$$\int \frac{(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{4A(a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{21ad \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(73A + 63C)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{315d \sec(dx + c)^{\frac{3}{2}}} + \frac{8a^3(16A + 21C) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(17A + 27C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(11A + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (11A + 21C)a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (11A + 21C)a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ca^3 \sec(dx + c)^5 + 3Ca^3 \sec(dx + c)^4 + (A + 3C)a^3 \sec(dx + c)^3 + (3A + C)a^3 \sec(dx + c)^2 + 3Aa^3 \sec(dx + c)}{\sec(dx + c)^{\frac{9}{2}}} \right)$$

69.23 Problem number 229

$$\int \frac{(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^3(35A + 44C) \sin(dx + c)}{385d \sec(dx + c)^{\frac{3}{2}}} + \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{11d \sec(dx + c)^{\frac{9}{2}}} \\ & + \frac{4A(a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{33ad \sec(dx + c)^{\frac{7}{2}}} \\ & + \frac{2(35A + 33C)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{231d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^3(105A + 143C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(105A + 143C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (105A + 143C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (105A + 143C) a^3 \sqrt{\sec(dx + c)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + 3Ca^3 \sec(dx + c)^4 + (A + 3C)a^3 \sec(dx + c)^3 + (3A + C)a^3 \sec(dx + c)^2 + 3Aa^3 \sec(dx + c)}{\sec(dx + c)^{\frac{11}{2}}}\right)$$

69.24 Problem number 230

$$\int \frac{(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx))}{\sec^{\frac{13}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{40a^3(118A + 143C) \sin(dx + c)}{9009d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^3(175A + 221C) \sin(dx + c)}{585d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{13d \sec(dx + c)^{\frac{11}{2}}} + \frac{12A(a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{143ad \sec(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(145A + 143C) (a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{1287d \sec(dx + c)^{\frac{7}{2}}} + \frac{4a^3(95A + 121C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(175A + 221C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(95A + 121C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(195i \sqrt{2} (95A + 121C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 195i \sqrt{2} (95A + 121C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + 3Ca^3 \sec(dx + c)^4 + (A + 3C)a^3 \sec(dx + c)^3 + (3A + C)a^3 \sec(dx + c)^2 + 3Aa^3 \sec(dx + c)}{\sec(dx + c)^{\frac{13}{2}}}\right)$$

69.25 Problem number 231

$$\int \frac{\sec^{\frac{5}{2}}(c+dx) (A+C \sec^2(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3A+5C) \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3ad} + \frac{(5A+7C) \left(\sec^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{5ad} \\ & - \frac{(A+C) \left(\sec^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{d(a+a \sec(dx+c))} + \frac{3(5A+7C) \sin(dx+c) (\sqrt{\sec}(dx+c))}{5ad} \\ & - \frac{3(5A+7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(3A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-3iA - 5iC) \cos(dx+c)^3 + \sqrt{2} (-3iA - 5iC) \cos(dx+c)^2 \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx+c)^4 + A \sec(dx+c)^2\right) \sqrt{\sec(dx+c)}}{a \sec(dx+c) + a}, x\right)$$

69.26 Problem number 232

$$\int \frac{\sec^{\frac{3}{2}}(c+dx) (A+C \sec^2(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A + 5C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3ad} - \frac{(A + C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \sec(dx + c))} \\ & - \frac{(A + 3C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{ad} \\ & + \frac{(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-3iA - 5iC) \cos(dx + c)^2 + \sqrt{2} (-3iA - 5iC) \cos(dx + c) \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \sec(dx + c)^3 + A \sec(dx + c) \right) \sqrt{\sec(dx + c)}}{a \sec(dx + c) + a}, x \right)$$

69.27 Problem number 233

$$\int \frac{\sqrt{\sec(c + dx)} (A + C \sec^2(c + dx))}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \sec(dx + c))} + \frac{(A + 3C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{ad} \\ & - \frac{(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-iA+iC)\cos(dx+c)+\sqrt{2}(-iA+iC)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(\sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C\sec(dx+c)^2+A\right)\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a},x\right)$$

69.28 Problem number 234

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A+C)\sin(dx+c)(\sqrt{\sec(dx+c)})}{d(a+a\sec(dx+c))} \\ & +\frac{(3A+C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \\ & -\frac{(A-C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)ad} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A+C)\sqrt{\cos(dx+c)}\sin(dx+c)-\left(\sqrt{2}(iA-iC)\cos(dx+c)+\sqrt{2}(iA-iC)\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C\sec(dx+c)^2+A\right)\sqrt{\sec(dx+c)}}{a\sec(dx+c)^2+a\sec(dx+c)},x\right)$$

69.29 Problem number 235

$$\int \frac{A + C \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5A + 3C) \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}} - \frac{(A + C) \sin(dx + c)}{d(a + a \sec(dx + c)) \sqrt{\sec(dx + c)}} \\ & - \frac{(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-5iA - 3iC) \cos(dx + c) + \sqrt{2}(-5iA - 3iC)\right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + A\right) \sqrt{\sec(dx + c)}}{a \sec(dx + c)^3 + a \sec(dx + c)^2}, x\right)$$

69.30 Problem number 236

$$\int \frac{A + C \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(7A + 5C) \sin(dx + c)}{5ad \sec(dx + c)^{\frac{3}{2}}} - \frac{(A + C) \sin(dx + c)}{d \sec(dx + c)^{\frac{3}{2}}(a + a \sec(dx + c))} - \frac{(5A + 3C) \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}} \\ & + \frac{3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-5i A - 3i C) \cos(dx + c) + \sqrt{2} (-5i A - 3i C) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^2 + A \right) \sqrt{\sec(dx + c)}}{a \sec(dx + c)^4 + a \sec(dx + c)^3}, x \right)$$

69.31 Problem number 237

$$\int \frac{\sec^{\frac{5}{2}}(c + dx) (A + C \sec^2(c + dx))}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(A + 5C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d} - \frac{(A + 7C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d (1 + \sec(dx + c))} \\ & - \frac{(A + C) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{3d (a + a \sec(dx + c))^2} - \frac{(A + 7C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{a^2d} \\ & + \frac{(A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \\ & + \frac{2(A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (i A + 5i C) \cos(dx + c)^3 + 2 \sqrt{2} (i A + 5i C) \cos(dx + c)^2 + \sqrt{2} (i A + 5i C) \cos(dx + c) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^4 + A \sec(dx + c)^2 \right) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}, x \right)$$

69.32 Problem number 238

$$\int \frac{\sec^{\frac{3}{2}}(c+dx) (A+C \sec^2(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A-5C) \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3a^2d(1+\sec(dx+c))} \\ & - \frac{(A+C) \left(\sec^{\frac{5}{2}}(dx+c) \right) \sin(dx+c)}{3d(a+a \sec(dx+c))^2} + \frac{4C \sin(dx+c) (\sqrt{\sec}(dx+c))}{a^2d} \\ & - \frac{4C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \\ & + \frac{(A-5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-iA + 5iC) \cos(dx+c)^2 - 2\sqrt{2} (iA - 5iC) \cos(dx+c) + \sqrt{2} (-iA + 5iC) \right) \operatorname{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \sec(dx+c)^3 + A \sec(dx+c) \right) \sqrt{\sec(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x \right)$$

69.33 Problem number 239

$$\int \frac{\sqrt{\sec(c+dx)} (A+C \sec^2(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A+C) \left(\sec^{\frac{3}{2}}(dx+c) \right) \sin(dx+c)}{3d(a+a \sec(dx+c))^2} + \frac{(A-C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{a^2 d (1+\sec(dx+c))} \\ & - \frac{(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & + \frac{2(A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (iA + iC) \cos(dx+c)^2 + 2\sqrt{2} (iA + iC) \cos(dx+c) + \sqrt{2} (iA + iC) \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \sec^2(dx+c) + A \right) \sqrt{\sec(dx+c)}}{a^2 \sec^2(dx+c) + 2a^2 \sec(dx+c) + a^2}, x \right)$$

69.34 Problem number 240

$$\int \frac{A + C \sec^2(c + dx)}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(5A-C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3a^2 d (1+\sec(dx+c))} - \frac{(A+C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3d(a+a \sec(dx+c))^2} \\ & + \frac{4A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & - \frac{(5A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (5i A - i C) \cos(dx + c)^2 - 2\sqrt{2} (-5i A + i C) \cos(dx + c) + \sqrt{2} (5i A - i C)\right) \text{weierstrassPInverse}(-4, 0, \dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \sec(dx + c)^2 + A) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^3 + 2a^2 \sec(dx + c)^2 + a^2 \sec(dx + c)}, x\right)$$

69.35 Problem number 241

$$\int \frac{A + C \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5A + C) \sin(dx + c)}{3a^2 d \sqrt{\sec(dx + c)}} - \frac{(7A + C) \sin(dx + c)}{3a^2 d (1 + \sec(dx + c)) \sqrt{\sec(dx + c)}} \\ & - \frac{(A + C) \sin(dx + c)}{3d (a + a \sec(dx + c))^2 \sqrt{\sec(dx + c)}} \\ & - \frac{(7A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2(5A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\sqrt{2} (5i A + i C) \cos(dx + c)^2 + 2\sqrt{2} (5i A + i C) \cos(dx + c) + \sqrt{2} (5i A + i C)\right) \text{weierstrassPInverse}(-4, 0, \dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \sec(dx + c)^2 + A) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^4 + 2a^2 \sec(dx + c)^3 + a^2 \sec(dx + c)^2}, x\right)$$

69.36 Problem number 242

$$\int \frac{A + C \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(14A + 5C) \sin(dx + c)}{15a^2d \sec(dx + c)^{\frac{3}{2}}} - \frac{(3A + C) \sin(dx + c)}{a^2d \sec(dx + c)^{\frac{3}{2}} (1 + \sec(dx + c))} \\ & - \frac{(A + C) \sin(dx + c)}{3d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))^2} - \frac{5(3A + C) \sin(dx + c)}{3a^2d \sqrt{\sec(dx + c)}} \\ & + \frac{4(14A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & - \frac{5(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{25 \left(\sqrt{2} (-3iA - iC) \cos(dx + c)^2 + 2\sqrt{2} (-3iA - iC) \cos(dx + c) + \sqrt{2} (-3iA - iC) \right) \operatorname{weierstrassPInverse}(\dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + A) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^5 + 2a^2 \sec(dx + c)^4 + a^2 \sec(dx + c)^3}, x\right)$$

69.37 Problem number 243

$$\int \frac{\sec^{\frac{7}{2}}(c + dx) (A + C \sec^2(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(A + 11C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{2a^3d} - \frac{(A + C) \left(\sec^{\frac{9}{2}}(dx + c) \right) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} \\
& - \frac{2C \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{3ad(a + a \sec(dx + c))^2} - \frac{(9A + 119C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{30d(a^3 + a^3 \sec(dx + c))} \\
& - \frac{(9A + 119C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{10a^3d} \\
& + \frac{(9A + 119C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3d} \\
& + \frac{(A + 11C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3d}
\end{aligned}$$

command

```
integrate(sec(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \left(\sqrt{2} (iA + 11iC) \cos(dx + c)^4 + 3 \sqrt{2} (iA + 11iC) \cos(dx + c)^3 + 3 \sqrt{2} (iA + 11iC) \cos(dx + c)^2 + \sqrt{2} (iA + 11iC) \cos(dx + c) \right)}{10a^3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \sec(dx + c)^5 + A \sec(dx + c)^3 \right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x \right)$$

69.38 Problem number 244

$$\int \frac{\sec^{\frac{5}{2}}(c + dx) (A + C \sec^2(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + C) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{5d (a + a \sec(dx + c))^3} + \frac{2(A - 4C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{15ad (a + a \sec(dx + c))^2} \\ & + \frac{(A - 13C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{6d (a^3 + a^3 \sec(dx + c))} - \frac{(A - 49C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{10a^3d} \\ & + \frac{(A - 49C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3d} \\ & + \frac{(A - 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (iA - 13iC) \cos(dx + c)^3 + 3 \sqrt{2} (iA - 13iC) \cos(dx + c)^2 + 3 \sqrt{2} (iA - 13iC) \cos(dx + c) + \sqrt{2} (iA - 13iC) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \sec(dx + c)^4 + A \sec(dx + c)^2 \right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x \right)$$

69.39 Problem number 245

$$\int \frac{\sec^{\frac{3}{2}}(c + dx) (A + C \sec^2(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d (a + a \sec(dx + c))^3} + \frac{2(2A - 3C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{15ad (a + a \sec(dx + c))^2} \\ & + \frac{(A - 9C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{10d (a^3 + a^3 \sec(dx + c))} \\ & - \frac{(A - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3d} \\ & + \frac{(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (i A + 3i C) \cos(dx + c)^3 + 3 \sqrt{2} (i A + 3i C) \cos(dx + c)^2 + 3 \sqrt{2} (i A + 3i C) \cos(dx + c) + \sqrt{2} (i A + 3i C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^3 + A \sec(dx + c) \right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x \right)$$

69.40 Problem number 246

$$\int \frac{\sqrt{\sec(c + dx)} (A + C \sec^2(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5d (a + a \sec(dx + c))^3} + \frac{2(3A - 2C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15ad (a + a \sec(dx + c))^2} \\ & + \frac{(3A + C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{6d (a^3 + a^3 \sec(dx + c))} \\ & - \frac{(9A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \end{aligned}$$

command

```
integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (3i A + i C) \cos(dx + c)^3 + 3 \sqrt{2} (3i A + i C) \cos(dx + c)^2 + 3 \sqrt{2} (3i A + i C) \cos(dx + c) + \sqrt{2} (3i A + i C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^2 + A \right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x \right)$$

69.41 Problem number 247

$$\int \frac{A + C \sec^2(c + dx)}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d (a + a \sec(dx + c))^3} - \frac{2(4A - C) \sin(dx + c) (\sqrt{\sec(dx + c)})^2}{15ad (a + a \sec(dx + c))^2} \\ & - \frac{(13A - C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{6d (a^3 + a^3 \sec(dx + c))} \\ & + \frac{(49A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(13A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

```
integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-13i A + i C) \cos(dx + c)^3 + 3 \sqrt{2} (-13i A + i C) \cos(dx + c)^2 + 3 \sqrt{2} (-13i A + i C) \cos(dx + c) + \sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + A) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^4 + 3 a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + a^3 \sec(dx + c)}, x\right)$$

69.42 Problem number 248

$$\int \frac{A + C \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{(11A + C) \sin(dx + c)}{2a^3 d \sqrt{\sec(dx + c)}} - \frac{(A + C) \sin(dx + c)}{5d (a + a \sec(dx + c))^3 \sqrt{\sec(dx + c)}} - \frac{2A \sin(dx + c)}{3ad (a + a \sec(dx + c))^2 \sqrt{\sec(dx + c)}} - \frac{(119A + 9C) \sin(dx + c)}{30d (a^3 + a^3 \sec(dx + c)) \sqrt{\sec(dx + c)}}$$

$$\frac{(119A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d}$$

$$+ \frac{(11A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d}$$

command

```
integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \left(\sqrt{2} (11i A + i C) \cos(dx + c)^3 + 3 \sqrt{2} (11i A + i C) \cos(dx + c)^2 + 3 \sqrt{2} (11i A + i C) \cos(dx + c) + \sqrt{2} (11i A + i C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + A) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^5 + 3 a^3 \sec(dx + c)^4 + 3 a^3 \sec(dx + c)^3 + a^3 \sec(dx + c)^2}, x\right)$$

69.43 Problem number 249

$$\int \frac{A + C \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{7(33A + 7C) \sin(dx + c)}{30a^3 d \sec(dx + c)^{\frac{3}{2}}} - \frac{(A + C) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))^3} \\
& - \frac{2(6A + C) \sin(dx + c)}{15ad \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))^2} \\
& - \frac{(63A + 13C) \sin(dx + c)}{10d \sec(dx + c)^{\frac{3}{2}} (a^3 + a^3 \sec(dx + c))} - \frac{(63A + 13C) \sin(dx + c)}{6a^3 d \sqrt{\sec(dx + c)}} \\
& + \frac{7(33A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
& + \frac{(63A + 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d}
\end{aligned}$$

command

```
integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-63i A - 13i C) \cos(dx + c)^3 + 3 \sqrt{2} (-63i A - 13i C) \cos(dx + c)^2 + 3 \sqrt{2} (-63i A - 13i C) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + A) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^6 + 3 a^3 \sec(dx + c)^5 + 3 a^3 \sec(dx + c)^4 + a^3 \sec(dx + c)^3}, x\right)$$

69.44 Problem number 534

$$\int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(7A + 7B + 5C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} + \frac{2a(B + C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2aC \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{2a(5A + 3B + 3C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2a(5A + 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(7A + 7B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (7A + 7B + 5C)a \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (7A + 7B + 5C)a \sin(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Ca \sec^4(dx + c) + (B + C)a \sec^3(dx + c) + (A + B)a \sec^2(dx + c) + Aa \sec(dx + c) \right) \sqrt{\sec(dx + c)}, x \right)$$

69.45 Problem number 535

$$\int \sqrt{\sec(c + dx)} (a + a \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(B + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2aC \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2a(5A + 5B + 3C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\ & - \frac{2a(5A + 5B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2a(3A + B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-5i\sqrt{2}(3A+B+C)a\cos(dx+c)$ weierstrassPInverse(-4, 0, cos(dx+c) + i sin(dx+c)) + $5i\sqrt{2}(3A+B+C)a\sin(dx+c)$

Fricas 1.3.7 via sagemath 9.3 output

integral($\left(\frac{Ca\sec^3(dx+c) + (B+C)a\sec^2(dx+c) + (A+B)a\sec(dx+c) + Aa}{\sqrt{\sec(dx+c)}}\right), x$)

69.46 Problem number 536

$$\int \frac{(a + a \sec(c + dx))(A + B \sec(c + dx) + C \sec^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aC \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3d} + \frac{2a(B+C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{d} \\ & + \frac{2a(A-B-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3A+3B+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i\sqrt{2}(3A+3B+C)a\cos(dx+c)$ weierstrassPInverse(-4, 0, cos(dx+c) + i sin(dx+c)) + $i\sqrt{2}(3A+3B+C)a\sin(dx+c)$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca\sec^3(dx+c) + (B+C)a\sec^2(dx+c) + (A+B)a\sec(dx+c) + Aa}{\sqrt{\sec(dx+c)}}, x\right)$$

69.47 Problem number 537

$$\int \frac{(a + a \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2aC \sin(dx + c) (\sqrt{\sec(dx + c)})}{d} \\ & + \frac{2a(A + B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} (A + 3B + 3C) a \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (A + 3B + 3C) a \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca \sec(dx + c)^3 + (B + C)a \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

69.48 Problem number 538

$$\int \frac{(a + a \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(A + B) \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(3A + 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm="fri
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (A + B + 3C) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (A + B + 3C) \text{aweierstra}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca \sec(dx + c)^3 + (B + C)a \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

69.49 Problem number 539

$$\int \frac{(a + a \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2a(A + B) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(5A + 7B + 7C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(3A + 3B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A + 7B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm="fri
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (5A + 7B + 7C) \text{aweierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (5A + 7B + 7C) \text{aweier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca \sec(dx + c)^3 + (B + C)a \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

69.50 Problem number 540

$$\int \frac{(a + a \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2a(A + B) \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2a(7A + 9B + 9C) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{2a(5A + 5B + 7C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2a(7A + 9B + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A + 5B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i \sqrt{2} (5A + 5B + 7C) a \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} (5A + 5B + 7C) a$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca \sec(dx + c)^3 + (B + C)a \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa}{\sec(dx + c)^{\frac{9}{2}}}, x\right)$$

69.51 Problem number 541

$$\int \sec^{\frac{3}{2}}(c + dx) (a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^2(7A + 6B + 5C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} \\
& + \frac{2a^2(21A + 27B + 19C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{105d} \\
& + \frac{2C \left(\sec^{\frac{5}{2}}(dx + c) \right) (a + a \sec(dx + c))^2 \sin(dx + c)}{9d} \\
& + \frac{2(9B + 4C) \left(\sec^{\frac{5}{2}}(dx + c) \right) (a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{63d} \\
& + \frac{4a^2(12A + 9B + 8C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15d} \\
& - \frac{4a^2(12A + 9B + 8C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\
& + \frac{4a^2(7A + 6B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}
\end{aligned}$$

command

```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (7A + 6B + 5C) a^2 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (7A + 6B + 5C) a^2 \sin(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)}{105d}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Ca^2 sec(dx + c)^5 + (B + 2C)a^2 sec(dx + c)^4 + (A + 2B + C)a^2 sec(dx + c)^3 + (2A + B)a^2 sec(dx + c)^2 + (A + B)a^2 sec(dx + c) + A)a^2 dx
```

69.52 Problem number 542

$$\int \sqrt{\sec(c + dx)} (a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2a^2(35A + 49B + 33C) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\
& + \frac{2C \left(\sec^{\frac{3}{2}}(dx + c)\right) (a + a \sec(dx + c))^2 \sin(dx + c)}{7d} \\
& + \frac{2(7B + 4C) \left(\sec^{\frac{3}{2}}(dx + c)\right) (a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{35d} \\
& + \frac{4a^2(5A + 4B + 3C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\
& + \frac{4a^2(5A + 4B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^2(14A + 7B + 6C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (14A + 7B + 6C) a^2 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (14A + 7B + 6C) a^2 \sin(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)}{105d}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(((Ca^2 sec(dx + c)^4 + (B + 2C)a^2 sec(dx + c)^3 + (A + 2B + C)a^2 sec(dx + c)^2 + (2A + B)a^2 sec(dx + c)
```

69.53 Problem number 543

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(15A + 25B + 17C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & + \frac{2C(a + a \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & + \frac{2(5B + 4C) (a^2 + a^2 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{4a^2(5B + 4C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(3A + 2B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (3A + 2B + C)a^2 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A - \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \sec(dx + c)^4 + (B + 2C)a^2 \sec(dx + c)^3 + (A + 2B + C)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c)}{\sqrt{\sec(dx + c)}}\right)$$

69.54 Problem number 544

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^2 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} - \frac{2a^2(A - 3B - 5C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & - \frac{2(A - C) (a^2 + a^2 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3d} \\ & + \frac{4a^2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(2A + 3B + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(i \sqrt{2} (2A + 3B + 2C)a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} (2A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ca^2 \sec(dx + c)^4 + (B + 2C)a^2 \sec(dx + c)^3 + (A + 2B + C)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c)}{\sec(dx + c)^{\frac{3}{2}}} \right)$$

69.55 Problem number 545

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2A(a + a \sec(dx + c))^2 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(4A + 5B)(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}}$$

$$- \frac{2a^2(7A + 5B - 15C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d}$$

$$+ \frac{4a^2(4A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}$$

$$+ \frac{4a^2(A + 2B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (A + 2B + 3C)a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + 2B + 3C)a^2 \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ca^2 \sec(dx + c)^4 + (B + 2C)a^2 \sec(dx + c)^3 + (A + 2B + C)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c)}{\sec(dx + c)^{\frac{5}{2}}} \right)$$

69.56 Problem number 546

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^2 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(4A + 7B)(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2a^2(33A + 49B + 35C) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^2(3A + 4B + 5C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(6A + 7B + 14C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (6A + 7B + 14C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (6A + 7B + 14C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \sec(dx + c)^4 + (B + 2C)a^2 \sec(dx + c)^3 + (A + 2B + C)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c)}{\sec(dx + c)^{\frac{7}{2}}}\right)$$

69.57 Problem number 547

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2a^2(19A + 27B + 21C) \sin(dx + c)}{105d \sec(dx + c)^{\frac{3}{2}}} + \frac{2A(a + a \sec(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} \\
& + \frac{2(4A + 9B)(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^2(5A + 6B + 7C) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\
& + \frac{4a^2(8A + 9B + 12C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^2(5A + 6B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (5A + 6B + 7C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 6B + 7C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \sec(dx + c)^4 + (B + 2C)a^2 \sec(dx + c)^3 + (A + 2B + C)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c)}{\sec(dx + c)^{\frac{9}{2}}}\right)$$

69.58 Problem number 548

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2a^2(89A + 121B + 99C) \sin(dx + c)}{693d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^2(7A + 8B + 9C) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2A(a + a \sec(dx + c))^2 \sin(dx + c)}{11d \sec(dx + c)^{\frac{9}{2}}} \\
& + \frac{2(4A + 11B)(a^2 + a^2 \sec(dx + c)) \sin(dx + c)}{99d \sec(dx + c)^{\frac{7}{2}}} + \frac{4a^2(50A + 55B + 66C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\
& + \frac{4a^2(7A + 8B + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^2(50A + 55B + 66C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algorithm="`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (50A + 55B + 66C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (50A + 55B + 66C) a^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \sec(dx + c)^4 + (B + 2C)a^2 \sec(dx + c)^3 + (A + 2B + C)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c)}{\sec(dx + c)^{\frac{11}{2}}}\right)$$

69.59 Problem number 549

$$\int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^3(143A + 121B + 105C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{231d} \\
& + \frac{4a^3(264A + 253B + 210C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{1155d} \\
& + \frac{2C \left(\sec^{\frac{5}{2}}(dx + c) \right) (a + a \sec(dx + c))^3 \sin(dx + c)}{11d} \\
& + \frac{2(11B + 6C) \left(\sec^{\frac{5}{2}}(dx + c) \right) (a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{99ad} \\
& + \frac{2(99A + 143B + 105C) \left(\sec^{\frac{5}{2}}(dx + c) \right) (a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{693d} \\
& + \frac{4a^3(21A + 17B + 15C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15d} \\
& - \frac{4a^3(21A + 17B + 15C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\
& + \frac{4a^3(143A + 121B + 105C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{231 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}
\end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (143A + 121B + 105C) a^3 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ca^3 sec(dx + c)^6 + (B + 3C)a^3 sec(dx + c)^5 + (A + 3B + 3C)a^3 sec(dx + c)^4 + (3A + 3B + C)a^3 sec(dx + c)^3), dx)`

69.60 Problem number 550

$$\int \sqrt{\sec(c+dx)} (a+a\sec(c+dx))^3 (A+B\sec(c+dx)+C\sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(42A+41B+32C) \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{105d} \\ & + \frac{2C \left(\sec^{\frac{3}{2}}(dx+c)\right) (a+a\sec(dx+c))^3 \sin(dx+c)}{9d} \\ & + \frac{2(3B+2C) \left(\sec^{\frac{3}{2}}(dx+c)\right) (a^2+a^2\sec(dx+c))^2 \sin(dx+c)}{21ad} \\ & + \frac{2(63A+99B+73C) \left(\sec^{\frac{3}{2}}(dx+c)\right) (a^3+a^3\sec(dx+c)) \sin(dx+c)}{315d} \\ & + \frac{4a^3(27A+21B+17C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{15d} \\ & - \frac{4a^3(27A+21B+17C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(21A+13B+11C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (21A+13B+11C) a^3 \cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 15i \sqrt{\sec(dx+c)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \sec(dx+c)^5 + (B+3C)a^3 \sec(dx+c)^4 + (A+3B+3C)a^3 \sec(dx+c)^3 + (3A+3B+C)a^3 \sec(dx+c)^2 + (2A+2B+C)a^3 \sec(dx+c) + Ca^3\right) dx\right)$$

69.61 Problem number 551

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(140A + 147B + 106C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{105d} \\ & + \frac{2C(a + a \sec(dx + c))^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{7d} \\ & + \frac{2(7B + 6C) (a^2 + a^2 \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{35ad} \\ & + \frac{2(5A + 9B + 7C) (a^3 + a^3 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{4a^3(5A + 9B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(35A + 21B + 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm='f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (35A + 21B + 13C) a^3 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + (B + 3C)a^3 \sec(dx + c)^4 + (A + 3B + 3C)a^3 \sec(dx + c)^3 + (3A + 3B + C)a^3 \sec(dx + c)^2}{\sqrt{\sec(dx + c)}}\right)$$

69.62 Problem number 552

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{4a^3(5A + 20B + 21C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{2(5A - 3C) (a^2 + a^2 \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{15ad} \\ & - \frac{2(5A - 5B - 9C) (a^3 + a^3 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & + \frac{4a^3(5A - 5B - 9C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(5A + 5B + 3C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (5A + 5B + 3C) a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (5A + 5B + 3C) a^3 \sin(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + (B + 3C)a^3 \sec(dx + c)^4 + (A + 3B + 3C)a^3 \sec(dx + c)^3 + (3A + 3B + C)a^3 \sec(dx + c)^2}{\sec(dx + c)^{\frac{3}{2}}}\right)$$

69.63 Problem number 553

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(6A + 5B)(a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{15ad \sqrt{\sec(dx + c)}} \\ & - \frac{4a^3(6A - 5B - 20C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{2(9A + 5B - 5C)(a^3 + a^3 \sec(dx + c)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & + \frac{4a^3(9A + 5B - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(3A + 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (3A + 5B + 5C) a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + (B + 3C)a^3 \sec(dx + c)^4 + (A + 3B + 3C)a^3 \sec(dx + c)^3 + (3A + 3B + C)a^3 \sec(dx + c)^2}{\sec(dx + c)^{\frac{5}{2}}}\right)$$

69.64 Problem number 554

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(6A + 7B)(a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{35ad \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(7A + 9B + 5C)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ & - \frac{4a^3(41A + 42B - 35C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{105d} \\ & + \frac{4a^3(7A + 9B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(13A + 21B + 35C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (13A + 21B + 35C)a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (13A + 21B + 35C)a^3 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + (B + 3C)a^3 \sec(dx + c)^4 + (A + 3B + 3C)a^3 \sec(dx + c)^3 + (3A + 3B + C)a^3 \sec(dx + c)^2}{\sec(dx + c)^{\frac{7}{2}}}\right)$$

69.65 Problem number 555

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2(2A + 3B)(a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{21ad \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(73A + 99B + 63C)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{315d \sec(dx + c)^{\frac{3}{2}}} + \frac{4a^3(32A + 41B + 42C) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(17A + 21B + 27C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(11A + 13B + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (11A + 13B + 21C)a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (11A + 13B + 21C)a^3 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ca^3 \sec(dx + c)^5 + (B + 3C)a^3 \sec(dx + c)^4 + (A + 3B + 3C)a^3 \sec(dx + c)^3 + (3A + 3B + C)a^3 \sec(dx + c)^2}{\sec(dx + c)^{\frac{9}{2}}} dx \right)$$

69.66 Problem number 556

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(210A + 253B + 264C) \sin(dx + c)}{1155d \sec(dx + c)^{\frac{3}{2}}} + \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{11d \sec(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(6A + 11B)(a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{99ad \sec(dx + c)^{\frac{7}{2}}} \\ & + \frac{2(105A + 143B + 99C)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{693d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{4a^3(105A + 121B + 143C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(15A + 17B + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(105A + 121B + 143C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (105A + 121B + 143C)a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (105A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ca^3 \sec(dx + c)^5 + (B + 3C)a^3 \sec(dx + c)^4 + (A + 3B + 3C)a^3 \sec(dx + c)^3 + (3A + 3B + C)a^3 \sec(dx + c)^2}{\sec(dx + c)^{\frac{11}{2}}} \right)$$

69.67 Problem number 557

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{13}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{20a^3(236A + 273B + 286C) \sin(dx + c)}{9009d \sec(dx + c)^{\frac{5}{2}}} + \frac{4a^3(175A + 195B + 221C) \sin(dx + c)}{585d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2A(a + a \sec(dx + c))^3 \sin(dx + c)}{13d \sec(dx + c)^{\frac{11}{2}}} + \frac{2(6A + 13B)(a^2 + a^2 \sec(dx + c))^2 \sin(dx + c)}{143ad \sec(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(145A + 195B + 143C)(a^3 + a^3 \sec(dx + c)) \sin(dx + c)}{1287d \sec(dx + c)^{\frac{7}{2}}} \\ & + \frac{4a^3(95A + 105B + 121C) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{4a^3(175A + 195B + 221C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(95A + 105B + 121C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x, algorithm="`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(195i \sqrt{2} (95A + 105B + 121C)a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 195i \sqrt{2} (95A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Ca^3 \sec(dx + c)^5 + (B + 3C)a^3 \sec(dx + c)^4 + (A + 3B + 3C)a^3 \sec(dx + c)^3 + (3A + 3B + C)a^3 \sec(dx + c)^2}{\sec(dx + c)^{\frac{13}{2}}} \right)$$

69.68 Problem number 558

$$\int \frac{\sec^{\frac{5}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A-5B+5C) \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3ad} + \frac{(5A-5B+7C) \left(\sec^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{5ad} \\ & - \frac{(A-B+C) \left(\sec^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{d(a+a \sec(dx+c))} + \frac{3(5A-5B+7C) \sin(dx+c) (\sqrt{\sec}(dx+c))}{5ad} \\ & - \frac{3(5A-5B+7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(3A-5B+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-3iA + 5iB - 5iC) \cos(dx+c)^3 + \sqrt{2} (-3iA + 5iB - 5iC) \cos(dx+c)^2 \right) \operatorname{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx+c)^4 + B \sec(dx+c)^3 + A \sec(dx+c)^2\right) \sqrt{\sec(dx+c)}}{a \sec(dx+c) + a}, x\right)$$

69.69 Problem number 559

$$\int \frac{\sec^{\frac{3}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A - 3B + 5C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3ad} - \frac{(A - B + C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \sec(dx + c))} \\ & - \frac{(A - 3B + 3C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{ad} \\ & + \frac{(A - 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(3A - 3B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-3iA + 3iB - 5iC) \cos(dx + c)^2 + \sqrt{2} (-3iA + 3iB - 5iC) \cos(dx + c) \right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^3 + B \sec(dx + c)^2 + A \sec(dx + c)\right) \sqrt{\sec(dx + c)}}{a \sec(dx + c) + a}, x\right)$$

69.70 Problem number 560

$$\int \frac{\sqrt{\sec(c + dx)} (A + B \sec(c + dx) + C \sec^2(c + dx))}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \sec(dx + c))} + \frac{(A - B + 3C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{ad} \\ & - \frac{(A - B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A + B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-iA - iB + iC)\cos(dx + c) + \sqrt{2}(-iA - iB + iC)\right)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\sec(dx + c)^2 + B\sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a\sec(dx + c) + a}, x\right)$$

69.71 Problem number 561

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{\sqrt{\sec(c + dx)}(a + a\sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B + C)\sin(dx + c)\sqrt{\sec(dx + c)}}{d(a + a\sec(dx + c))} \\ + & \frac{(3A - B + C)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad} \\ - & \frac{(A - B - C)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)ad} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(A - B + C)\sqrt{\cos(dx + c)}\sin(dx + c) - \left(\sqrt{2}(iA - iB - iC)\cos(dx + c) + \sqrt{2}(iA - iB - iC)\right)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\sec(dx + c)^2 + B\sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a\sec(dx + c)^2 + a\sec(dx + c)}, x\right)$$

69.72 Problem number 562

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\frac{(5A - 3B + 3C) \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}} - \frac{(A - B + C) \sin(dx + c)}{d(a + a \sec(dx + c)) \sqrt{\sec(dx + c)}}$$

$$- \frac{(3A - 3B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad}$$

$$+ \frac{(5A - 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$\left(\sqrt{2}(-5iA + 3iB - 3iC) \cos(dx + c) + \sqrt{2}(-5iA + 3iB - 3iC)\right) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i s$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A\right) \sqrt{\sec(dx + c)}}{a \sec(dx + c)^3 + a \sec(dx + c)^2}, x\right)$$

69.73 Problem number 563

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\frac{(7A - 5B + 5C) \sin(dx + c)}{5ad \sec(dx + c)^{\frac{3}{2}}} - \frac{(A - B + C) \sin(dx + c)}{d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))} - \frac{(5A - 5B + 3C) \sin(dx + c)}{3ad \sqrt{\sec(dx + c)}}$$

$$+ \frac{3(7A - 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad}$$

$$- \frac{(5A - 5B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fri`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-5i A + 5i B - 3i C) \cos(dx + c) + \sqrt{2} (-5i A + 5i B - 3i C) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A \right) \sqrt{\sec(dx + c)}}{a \sec(dx + c)^4 + a \sec(dx + c)^3}, x \right)$$

69.74 Problem number 564

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(9A - 7B + 7C) \sin(dx + c)}{7ad \sec(dx + c)^{\frac{5}{2}}} - \frac{(7A - 7B + 5C) \sin(dx + c)}{5ad \sec(dx + c)^{\frac{3}{2}}} \\ & - \frac{(A - B + C) \sin(dx + c)}{d \sec(dx + c)^{\frac{5}{2}} (a + a \sec(dx + c))} + \frac{5(9A - 7B + 7C) \sin(dx + c)}{21ad \sqrt{\sec(dx + c)}} \\ & - \frac{3(7A - 7B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{5(9A - 7B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fri`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25 \left(\sqrt{2} (9i A - 7i B + 7i C) \cos(dx + c) + \sqrt{2} (9i A - 7i B + 7i C) \right) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A \right) \sqrt{\sec(dx + c)}}{a \sec(dx + c)^5 + a \sec(dx + c)^4}, x \right)$$

69.75 Problem number 565

$$\int \frac{\sec^{\frac{5}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2A-5B+10C) \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3a^2d} - \frac{(A-4B+7C) \left(\sec^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{3a^2d(1+\sec(dx+c))} \\ & - \frac{(A-B+C) \left(\sec^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{3d(a+a \sec(dx+c))^2} - \frac{(A-4B+7C) \sin(dx+c) (\sqrt{\sec(dx+c)})}{a^2d} \\ & + \frac{(A-4B+7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \\ & + \frac{(2A-5B+10C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-2iA+5iB-10iC)\cos(dx+c)^3 - 2\sqrt{2}(2iA-5iB+10iC)\cos(dx+c)^2 + \sqrt{2}(-2iA+5iB-10iC)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx+c)^4 + B \sec(dx+c)^3 + A \sec(dx+c)^2\right) \sqrt{\sec(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

69.76 Problem number 566

$$\int \frac{\sec^{\frac{3}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + 2B - 5C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3a^2d(1 + \sec(dx + c))} \\ & - \frac{(A - B + C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{3d(a + a \sec(dx + c))^2} - \frac{(B - 4C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{a^2d} \\ & + \frac{(B - 4C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \\ & + \frac{(A + 2B - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-iA - 2iB + 5iC) \cos(dx + c)^2 - 2\sqrt{2} (iA + 2iB - 5iC) \cos(dx + c) + \sqrt{2} (-iA - 2iB + 5iC) \right) \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \sec(dx + c)^3 + B \sec(dx + c)^2 + A \sec(dx + c) \right) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^2 + 2a^2 \sec(dx + c) + a^2}, x \right)$$

69.77 Problem number 567

$$\int \frac{\sqrt{\sec(c + dx)} (A + B \sec(c + dx) + C \sec^2(c + dx))}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d(a + a \sec(dx + c))^2} + \frac{(A - C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{a^2d(1 + \sec(dx + c))} \\ & - \frac{(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \\ & + \frac{(2A + B + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2d} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-2iA - iB - 2iC)\cos(dx + c)^2 - 2\sqrt{2}(2iA + iB + 2iC)\cos(dx + c) + \sqrt{2}(-2iA - iB - 2iC)\right)\text{weierstr}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\sec(dx+c)^2 + B\sec(dx+c) + A)\sqrt{\sec(dx+c)}}{a^2\sec(dx+c)^2 + 2a^2\sec(dx+c) + a^2}, x\right)$$

69.78 Problem number 568

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{\sqrt{\sec(c + dx)}(a + a\sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5A - 2B - C)\sin(dx + c)(\sqrt{\sec(dx + c)})}{3a^2d(1 + \sec(dx + c))} - \frac{(A - B + C)\sin(dx + c)(\sqrt{\sec(dx + c)})}{3d(a + a\sec(dx + c))^2} \\ & + \frac{(4A - B)\sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2}\text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^2d} \\ & - \frac{(5A - 2B - C)\sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2}\text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)(\sqrt{\cos(dx + c)})(\sqrt{\sec(dx + c)})}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^2d} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(5iA - 2iB - iC)\cos(dx + c)^2 - 2\sqrt{2}(-5iA + 2iB + iC)\cos(dx + c) + \sqrt{2}(5iA - 2iB - iC)\right)\text{weierstr}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\sec(dx+c)^2 + B\sec(dx+c) + A)\sqrt{\sec(dx+c)}}{a^2\sec(dx+c)^3 + 2a^2\sec(dx+c)^2 + a^2\sec(dx+c)}, x\right)$$

69.79 Problem number 569

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(10A - 5B + 2C) \sin(dx + c)}{3a^2 d \sqrt{\sec(dx + c)}} - \frac{(7A - 4B + C) \sin(dx + c)}{3a^2 d (1 + \sec(dx + c)) \sqrt{\sec(dx + c)}} \\ & - \frac{(A - B + C) \sin(dx + c)}{3d (a + a \sec(dx + c))^2 \sqrt{\sec(dx + c)}} \\ & - \frac{(7A - 4B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(10A - 5B + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-10i A + 5i B - 2i C) \cos(dx + c)^2 - 2 \sqrt{2} (10i A - 5i B + 2i C) \cos(dx + c) + \sqrt{2} (-10i A + 5i B - 2i C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^4 + 2 a^2 \sec(dx + c)^3 + a^2 \sec(dx + c)^2}, x\right)$$

69.80 Problem number 570

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{(56A - 35B + 20C) \sin(dx + c)}{15a^2d \sec(dx + c)^{\frac{3}{2}}} - \frac{(3A - 2B + C) \sin(dx + c)}{a^2d \sec(dx + c)^{\frac{3}{2}} (1 + \sec(dx + c))}$$

$$- \frac{(A - B + C) \sin(dx + c)}{3d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))^2} - \frac{5(3A - 2B + C) \sin(dx + c)}{3a^2d \sqrt{\sec(dx + c)}}$$

$$+ \frac{(56A - 35B + 20C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d}$$

$$- \frac{5(3A - 2B + C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2d}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$25 \left(\sqrt{2} (-3iA + 2iB - iC) \cos(dx + c)^2 + 2\sqrt{2} (-3iA + 2iB - iC) \cos(dx + c) + \sqrt{2} (-3iA + 2iB - iC) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^5 + 2a^2 \sec(dx + c)^4 + a^2 \sec(dx + c)^3}, x\right)$$

69.81 Problem number 571

$$\int \frac{\sec^{\frac{7}{2}}(c + dx) (A + B \sec(c + dx) + C \sec^2(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{(3A - 13B + 33C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{6a^3d} - \frac{(A - B + C) \left(\sec^{\frac{9}{2}}(dx + c) \right) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} \\
 & + \frac{(B - 2C) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{3ad(a + a \sec(dx + c))^2} - \frac{(9A - 49B + 119C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{30d(a^3 + a^3 \sec(dx + c))} \\
 & - \frac{(9A - 49B + 119C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{10a^3d} \\
 & + \frac{(9A - 49B + 119C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\
 & + \frac{(3A - 13B + 33C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d}
 \end{aligned}$$

command

`integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \left(\sqrt{2} (3i A - 13i B + 33i C) \cos(dx + c)^4 + 3 \sqrt{2} (3i A - 13i B + 33i C) \cos(dx + c)^3 + 3 \sqrt{2} (3i A - 13i B + 33i C) \cos(dx + c)^2 + 3 \sqrt{2} (3i A - 13i B + 33i C) \cos(dx + c) + 3 \sqrt{2} (3i A - 13i B + 33i C) \right)}{10 a^3 d \cos\left(\frac{dx}{2} + \frac{c}{2}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^5 + B \sec(dx + c)^4 + A \sec(dx + c)^3\right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x\right)$$

69.82 Problem number 572

$$\int \frac{\sec^{\frac{5}{2}}(c + dx) (A + B \sec(c + dx) + C \sec^2(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A - B + C) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} + \frac{(2A + 3B - 8C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{15ad(a + a \sec(dx + c))^2} \\ & + \frac{(A + 3B - 13C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{6d(a^3 + a^3 \sec(dx + c))} - \frac{(A + 9B - 49C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{10a^3d} \\ & + \frac{(A + 9B - 49C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & + \frac{(A + 3B - 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="f`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$5 \left(\sqrt{2} (iA + 3iB - 13iC) \cos(dx + c)^3 + 3\sqrt{2} (iA + 3iB - 13iC) \cos(dx + c)^2 + 3\sqrt{2} (iA + 3iB - 13iC) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^4 + B \sec(dx + c)^3 + A \sec(dx + c)^2\right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x\right)$$

69.83 Problem number 573

$$\int \frac{\sec^{\frac{3}{2}}(c + dx) (A + B \sec(c + dx) + C \sec^2(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A - B + C) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} + \frac{(4A + B - 6C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{15ad(a + a \sec(dx + c))^2} \\ & + \frac{(A - B - 9C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{10d(a^3 + a^3 \sec(dx + c))} \\ & - \frac{(A - B - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \\ & + \frac{(A + B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (i A + i B + 3i C) \cos(dx + c)^3 + 3 \sqrt{2} (i A + i B + 3i C) \cos(dx + c)^2 + 3 \sqrt{2} (i A + i B + 3i C) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^3 + B \sec(dx + c)^2 + A \sec(dx + c) \right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x \right)$$

69.84 Problem number 574

$$\int \frac{\sqrt{\sec(c + dx)} (A + B \sec(c + dx) + C \sec^2(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5d(a + a \sec(dx + c))^3} + \frac{(6A - B - 4C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15ad(a + a \sec(dx + c))^2} \\ & + \frac{(3A + B + C) \sin(dx + c) (\sqrt{\sec}(dx + c))}{6d(a^3 + a^3 \sec(dx + c))} \\ & - \frac{(9A + B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{10 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \\ & + \frac{(3A + B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{6 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (3i A + i B + i C) \cos(dx + c)^3 + 3 \sqrt{2} (3i A + i B + i C) \cos(dx + c)^2 + 3 \sqrt{2} (3i A + i B + i C) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A \right) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x \right)$$

69.85 Problem number 575

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d (a + a \sec(dx + c))^3} - \frac{(8A - 3B - 2C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15ad (a + a \sec(dx + c))^2} \\ & - \frac{(13A - 3B - C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{6d (a^3 + a^3 \sec(dx + c))} \\ & + \frac{(49A - 9B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(13A - 3B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-13i A + 3i B + i C) \cos(dx + c)^3 + 3 \sqrt{2} (-13i A + 3i B + i C) \cos(dx + c)^2 + 3 \sqrt{2} (-13i A + 3i B + \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^4 + 3 a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + a^3 \sec(dx + c)}, x\right)$$

69.86 Problem number 576

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(33A - 13B + 3C) \sin(dx + c)}{6a^3 d \sqrt{\sec(dx + c)}} - \frac{(A - B + C) \sin(dx + c)}{5d (a + a \sec(dx + c))^3 \sqrt{\sec(dx + c)}} \\
& - \frac{(2A - B) \sin(dx + c)}{3ad (a + a \sec(dx + c))^2 \sqrt{\sec(dx + c)}} - \frac{(119A - 49B + 9C) \sin(dx + c)}{30d (a^3 + a^3 \sec(dx + c)) \sqrt{\sec(dx + c)}} \\
& - \frac{(119A - 49B + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
& + \frac{(33A - 13B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d}
\end{aligned}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (33i A - 13i B + 3i C) \cos(dx + c)^3 + 3 \sqrt{2} (33i A - 13i B + 3i C) \cos(dx + c)^2 + 3 \sqrt{2} (33i A - 13i B +
\right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^5 + 3 a^3 \sec(dx + c)^4 + 3 a^3 \sec(dx + c)^3 + a^3 \sec(dx + c)^2}, x\right)$$

69.87 Problem number 577

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{7(33A - 17B + 7C) \sin(dx + c)}{30a^3 d \sec(dx + c)^{\frac{3}{2}}} - \frac{(A - B + C) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))^3} \\
& - \frac{(12A - 7B + 2C) \sin(dx + c)}{15ad \sec(dx + c)^{\frac{3}{2}} (a + a \sec(dx + c))^2} \\
& - \frac{(63A - 33B + 13C) \sin(dx + c)}{10d \sec(dx + c)^{\frac{3}{2}} (a^3 + a^3 \sec(dx + c))} - \frac{(63A - 33B + 13C) \sin(dx + c)}{6a^3 d \sqrt{\sec(dx + c)}} \\
& + \frac{7(33A - 17B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
& - \frac{(63A - 33B + 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d}
\end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(\sqrt{2} (-63i A + 33i B - 13i C) \cos(dx + c)^3 + 3 \sqrt{2} (-63i A + 33i B - 13i C) \cos(dx + c)^2 + 3 \sqrt{2} (-63i A + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^6 + 3 a^3 \sec(dx + c)^5 + 3 a^3 \sec(dx + c)^4 + a^3 \sec(dx + c)^3}, x\right)$$

69.88 Problem number 981

$$\int \sec^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(7aA + 5bB + 5aC) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} \\
& + \frac{2(9Ab + 9Ba + 7Cb) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{45d} + \frac{2(bB + aC) \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} \\
& + \frac{2bC \left(\sec^{\frac{9}{2}}(dx + c) \right) \sin(dx + c)}{9d} + \frac{2(9Ab + 9Ba + 7Cb) \sin(dx + c) (\sqrt{\sec}(dx + c))}{15d} \\
& - \frac{2(9Ab + 9Ba + 7Cb) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{15 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\
& + \frac{2(7aA + 5bB + 5aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}
\end{aligned}$$

command

`integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$15 \sqrt{2} (i(7A + 5C)a + 5iBb) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \sqrt{2} (-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Cb \sec(dx + c)^5 + (Ca + Bb) \sec(dx + c)^4 + Aa \sec(dx + c)^2 + (Ba + Ab) \sec(dx + c)^3 \right) \sqrt{\sec(dx + c)} \right)$$

69.89 Problem number 982

$$\int \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(7Ab + 7Ba + 5Cb) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{21d} + \frac{2(bB + aC) \left(\sec^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\
& + \frac{2bC \left(\sec^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{2(5aA + 3bB + 3aC) \sin(dx + c) (\sqrt{\sec}(dx + c))}{5d} \\
& - \frac{2(5aA + 3bB + 3aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\
& + \frac{2(7Ab + 7Ba + 5Cb) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d}
\end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(7iBa + i(7A + 5C)b)\cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + 5\sqrt{2}(-7i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb\sec(dx + c)^4 + (Ca + Bb)\sec(dx + c)^3 + Aa\sec(dx + c) + (Ba + Ab)\sec(dx + c)^2\right)\sqrt{\sec(dx + c)}, x\right)$$

69.90 Problem number 983

$$\int \sqrt{\sec(c + dx)} (a + b\sec(c + dx)) (A + B\sec(c + dx) + C\sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bB + aC)\left(\sec^{\frac{3}{2}}(dx + c)\right)\sin(dx + c)}{3d} + \frac{2bC\left(\sec^{\frac{5}{2}}(dx + c)\right)\sin(dx + c)}{5d} \\ & + \frac{2(5Ab + 5Ba + 3Cb)\sin(dx + c)\left(\sqrt{\sec(dx + c)}\right)}{5d} \\ & - \frac{2(5Ab + 5Ba + 3Cb)\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos(dx + c)}\right)\left(\sqrt{\sec(dx + c)}\right)}{5\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{2(bB + a(3A + C))\sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\left(\sqrt{\cos(dx + c)}\right)\left(\sqrt{\sec(dx + c)}\right)}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(i(3A + C)a + iBb)\cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + 5\sqrt{2}(-i(3A$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb\sec(dx + c)^3 + (Ca + Bb)\sec(dx + c)^2 + Aa + (Ba + Ab)\sec(dx + c)\right)\sqrt{\sec(dx + c)}, x\right)$$

69.91 Problem number 984

$$\int \frac{(a + b \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2bC \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{3d} + \frac{2(bB + aC) \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & - \frac{2(bB - a(A - C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(3Ab + 3Ba + Cb) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`
 Fracas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i Ba - i(3A + C)b) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i Ba + i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Cb \sec(dx + c)^3 + (Ca + Bb) \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)}{\sqrt{\sec(dx + c)}}, x \right)$$

69.92 Problem number 985

$$\int \frac{(a + b \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} + \frac{2bC \sin(dx + c) (\sqrt{\sec}(dx + c))}{d} \\ & + \frac{2(Ab + Ba - Cb) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2(3bB + a(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(dx + c)) (\sqrt{\sec}(dx + c))}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}(-i(A+3C)a-3iBb)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}(i(A+3C)a+3iBb)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb\sec(dx+c)^3+(Ca+Bb)\sec(dx+c)^2+Ca+(Ba+Ab)\sec(dx+c)}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

69.93 Problem number 986

$$\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx)+C\sec^2(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA\sin(dx+c)}{5d\sec(dx+c)^{\frac{3}{2}}} + \frac{2(Ab+Ba)\sin(dx+c)}{3d\sqrt{\sec(dx+c)}} \\ & + \frac{2(3aA+5bB+5aC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(Ab+Ba+3Cb)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(iBa+i(A+3C)b)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-iBa-i(A+3C)b)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb\sec(dx+c)^3+(Ca+Bb)\sec(dx+c)^2+Ca+(Ba+Ab)\sec(dx+c)}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

69.94 Problem number 987

$$\int \frac{(a + b \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(Ab + Ba) \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(5aA + 7bB + 7aC) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(3Ab + 3Ba + 5Cb) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5aA + 7bB + 7aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$5 \sqrt{2} (i (5 A + 7 C) a + 7 i B b) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-i (5 A + 7 C) a -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb \sec(dx + c)^3 + (Ca + Bb) \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

69.95 Problem number 988

$$\int \frac{(a + b \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2(Ab + Ba) \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(7aA + 9bB + 9aC) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(5Ab + 5Ba + 7Cb) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(7aA + 9bB + 9aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5Ab + 5Ba + 7Cb) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$15\sqrt{2}(5iBa + i(5A + 7C)b)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + 15\sqrt{2}(-5iBa - i(5A + 7C)b)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb\sec(dx+c)^3 + (Ca+Bb)\sec(dx+c)^2 + Aa + (Ba+Ab)\sec(dx+c)}{\sec(dx+c)^{\frac{9}{2}}}, x\right)$$

69.96 Problem number 989

$$\int \frac{(a + b\sec(c + dx))(A + B\sec(c + dx) + C\sec^2(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aA\sin(dx+c)}{11d\sec(dx+c)^{\frac{9}{2}}} + \frac{2(Ab+Ba)\sin(dx+c)}{9d\sec(dx+c)^{\frac{7}{2}}} + \frac{2(9aA+11bB+11aC)\sin(dx+c)}{77d\sec(dx+c)^{\frac{5}{2}}} \\ & + \frac{2(7Ab+7Ba+9Cb)\sin(dx+c)}{45d\sec(dx+c)^{\frac{3}{2}}} + \frac{10(9aA+11bB+11aC)\sin(dx+c)}{231d\sqrt{\sec(dx+c)}} \\ & + \frac{2(7Ab+7Ba+9Cb)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{15\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \\ & + \frac{10(9aA+11bB+11aC)\sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{231\cos\left(\frac{dx}{2} + \frac{c}{2}\right)d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$75\sqrt{2}(i(9A + 11C)a + 11iBb)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + 75\sqrt{2}(-i(9A + 11C)a - 11iBb)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb\sec(dx+c)^3 + (Ca+Bb)\sec(dx+c)^2 + Aa + (Ba+Ab)\sec(dx+c)}{\sec(dx+c)^{\frac{11}{2}}}, x\right)$$

69.97 Problem number 990

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx)+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(14Aab+7Ba^2+5b^2B+10abC)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{21d} \\ & + \frac{2(9Ab^2+18abB+4a^2C+7b^2C)\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{45d} \\ & + \frac{2b(9bB+4aC)\left(\sec^{\frac{7}{2}}(dx+c)\right)\sin(dx+c)}{63d} \\ & + \frac{2C\left(\sec^{\frac{5}{2}}(dx+c)\right)(a+b\sec(dx+c))^2\sin(dx+c)}{9d} \\ & + \frac{2(18abB+3a^2(5A+3C)+b^2(9A+7C))\sin(dx+c)(\sqrt{\sec}(dx+c))}{15d} \\ & - \frac{2(18abB+3a^2(5A+3C)+b^2(9A+7C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos}(dx+c))}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(14Aab+7Ba^2+5b^2B+10abC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos}(dx+c))(\sqrt{\sec}(dx+c))}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15\sqrt{2}(7iBa^2+2i(7A+5C)ab+5iBb^2)\cos(dx+c)^4\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2\sec(dx+c)^5+(2Cab+Bb^2)\sec(dx+c)^4+Aa^2\sec(dx+c)+(Ca^2+2Bab+Ab^2)\sec(dx+c)\right)^3\right)$$

69.98 Problem number 991

$$\int \sqrt{\sec(c+dx)} (a+b\sec(c+dx))^2 (A+B\sec(c+dx)+C\sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7Ab^2 + 14abB + 4a^2C + 5b^2C) \left(\sec^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{21d} \\ & + \frac{2b(7bB + 4aC) \left(\sec^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{35d} \\ & + \frac{2C \left(\sec^{\frac{3}{2}}(dx+c)\right) (a+b\sec(dx+c))^2 \sin(dx+c)}{7d} \\ & + \frac{2(10Aab + 5Ba^2 + 3b^2B + 6abC) \sin(dx+c) (\sqrt{\sec}(dx+c))}{5d} \\ & - \frac{2(10Aab + 5Ba^2 + 3b^2B + 6abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(14abB + 7a^2(3A+C) + b^2(7A+5C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx+c)) (\sqrt{\sec}(dx+c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm='fricas')`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (7i(3A+C)a^2 + 14iBab + i(7A+5C)b^2) \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \sec(dx+c)^4 + (2Cab + Bb^2) \sec(dx+c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \sec(dx+c)^2 + (Ba^2 + 2AbC) \sec(dx+c) + Ca^3\right), dx\right)$$

69.99 Problem number 992

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(5bB + 4aC) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{15d} \\ & + \frac{2(5A b^2 + 10abB + 4a^2C + 3b^2C) \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & + \frac{2C(a + b \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{5d} \\ & - \frac{2(10abB - 5a^2(A - C) + b^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3B a^2 + b^2B + 2ab(3A + C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (3i Ba^2 + 2i(3A + C)ab + i Bb^2) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2 \sec(dx + c)^4 + (2Cab + Bb^2) \sec(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2$$

69.100 Problem number 993

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b^2(A-C)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3d} + \frac{2A(a+b\sec(dx+c))^2\sin(dx+c)}{3d\sqrt{\sec(dx+c)}} \\ & + \frac{2b(3bB-2a(A-3C))\sin(dx+c)(\sqrt{\sec(dx+c)})}{3d} \\ & + \frac{2(Ba^2-b^2B+2ab(A-C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(6abB+b^2(3A+C)+a^2(A+3C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2}\left(-i(A+3C)a^2-6iBab-i(3A+C)b^2\right)\cos(dx+c)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2\sec(dx+c)^4+(2Cab+Bb^2)\sec(dx+c)^3+Aa^2+(Ca^2+2Bab+Ab^2)\sec(dx+c)^2+(Ba^2+2AbC)\sec(dx+c)+A^2}{\sec(dx+c)^{\frac{3}{2}}}\right)$$

69.101 Problem number 994

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx)+C\sec^2(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2A(a+b\sec(dx+c))^2\sin(dx+c)}{5d\sec(dx+c)^{\frac{3}{2}}} + \frac{2a(4Ab+5Ba)\sin(dx+c)}{15d\sqrt{\sec(dx+c)}} \\ & - \frac{2b^2(A-5C)\sin(dx+c)(\sqrt{\sec(dx+c)})}{5d} \\ & + \frac{2(10abB+5b^2(A-C)+a^2(3A+5C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2(Ba^2+3b^2B+2ab(A+3C))\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\sqrt{\cos(dx+c)})(\sqrt{\sec(dx+c)})}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (iBa^2 + 2i(A+3C)ab + 3iBb^2) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5\sqrt{2} (-iBa^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Cb^2 \sec(dx+c)^4 + (2Cab + Bb^2) \sec(dx+c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \sec(dx+c)^2 + (Ba^2 + 2$$

69.102 Problem number 995

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(4Ab + 7Ba) \sin(dx+c)}{35d \sec(dx+c)^{\frac{3}{2}}} + \frac{2A(a + b \sec(dx+c))^2 \sin(dx+c)}{7d \sec(dx+c)^{\frac{5}{2}}} \\ & + \frac{2(4Ab^2 + 14abB + a^2(5A + 7C)) \sin(dx+c)}{21d \sqrt{\sec(dx+c)}} \\ & + \frac{2(6Aab + 3Ba^2 + 5b^2B + 10abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(14abB + 7b^2(A + 3C) + a^2(5A + 7C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx+c)}) (\sqrt{\sec(dx+c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (i(5A+7C)a^2 + 14iBab + 7i(A+3C)b^2) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5\sqrt{2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Cb^2 \sec(dx+c)^4 + (2Cab + Bb^2) \sec(dx+c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \sec(dx+c)^2 + (Ba^2 + 2$$

69.103 Problem number 996

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(4Ab + 9Ba) \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{2(4Ab^2 + 18abB + a^2(7A + 9C)) \sin(dx + c)}{45d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2A(a + b \sec(dx + c))^2 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2(10Aab + 5Ba^2 + 7b^2B + 14abC) \sin(dx + c)}{21d \sqrt{\sec(dx + c)}} \\ & + \frac{2(18abB + 3b^2(3A + 5C) + a^2(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(10Aab + 5Ba^2 + 7b^2B + 14abC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)}) (\sqrt{\sec(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (5i Ba^2 + 2i (5A + 7C)ab + 7i Bb^2) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \sqrt{2} (-5$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2 \sec(dx + c)^4 + (2Cab + Bb^2) \sec(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2$$

69.104 Problem number 997

$$\int \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(54a^2bB + 15b^3B + 8a^3C + 9ab^2(7A + 5C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{63d} \\
& + \frac{2b(63Ab^2 + 99abB + 24a^2C + 49b^2C) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\
& + \frac{2(3bB + 2aC) \left(\sec^{\frac{3}{2}}(dx + c)\right) (a + b \sec(dx + c))^2 \sin(dx + c)}{21d} \\
& + \frac{2C \left(\sec^{\frac{3}{2}}(dx + c)\right) (a + b \sec(dx + c))^3 \sin(dx + c)}{9d} \\
& + \frac{2(15a^3B + 27Ba^2b^2 + 9a^2b(5A + 3C) + b^3(9A + 7C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\
& - \frac{2(15a^3B + 27Ba^2b^2 + 9a^2b(5A + 3C) + b^3(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(21a^2bB + 5b^3B + 7a^3(3A + C) + 3ab^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (7i(3A + C)a^3 + 21iBa^2b + 3i(7A + 5C)ab^2 + 5iBb^3) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \sec(dx + c)^5 + (3Cab^2 + Bb^3) \sec(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \sec(dx + c)^3 + (Ca^2b + Ab^2) \sec(dx + c)^2 + (Ca^2 + Ab^2) \sec(dx + c) + Aa + Ab\right) \sqrt{\sec(c + dx)}\right) dx$$

69.105 Problem number 998

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b(35Ab^2 + 63abB + 24a^2C + 25b^2C) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\
& + \frac{2(98a^2bB + 21b^3B + 24a^3C + 21ab^2(5A + 3C)) \sin(dx + c) (\sqrt{\sec}(dx + c))}{35d} \\
& + \frac{2(7bB + 6aC) (a + b \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec}(dx + c))}{35d} \\
& + \frac{2C(a + b \sec(dx + c))^3 \sin(dx + c) (\sqrt{\sec}(dx + c))}{7d} \\
& - \frac{2(15a^2bB + 3b^3B - 5a^3(A - C) + 3ab^2(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c))}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(21a^3B + 21Ba^2b + 21a^2b(3A + C) + b^3(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos}(dx + c))}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="f")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (21i Ba^3 + 21i(3A + C)a^2b + 21i Bab^2 + i(7A + 5C)b^3) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \sec(dx + c)^5 + (3Cab^2 + Bb^3) \sec(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \sec(dx + c)^3 + (Ca^3 + 3Ab^2) \sec(dx + c)^2 + (3Aab + 3Bb^2) \sec(dx + c) + A^2a + 2ABb + 3C^2c}{\sqrt{\sec(dx + c)}}\right) dx$$

69.106 Problem number 999

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2b^2(5aA - 5bB - 9aC) \left(\sec^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{15d} + \frac{2A(a + b \sec(dx + c))^3 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\
& + \frac{2b(45abB - a^2(10A - 42C) + 3b^2(5A + 3C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\
& - \frac{2b(5A - 3C) (a + b \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\
& + \frac{2(5a^3B - 15Ba^2b + 15a^2b(A - C) - b^3(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(9a^2bB + b^3B + 3a^2b(3A + C) + a^3(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i(A + 3C)a^3 + 9iBa^2b + 3i(3A + C)ab^2 + iBb^3) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \sec(dx + c)^5 + (3Cab^2 + Bb^3) \sec(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \sec(dx + c)^3 + (Ca^3}{\sec(dx + c)^{\frac{3}{2}}}\right)$$

69.107 Problem number 1000

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2b^2(9Ab + 5Ba - 5Cb) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\
+ & \frac{2A(a + b \sec(dx + c))^3 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(6Ab + 5Ba)(a + b \sec(dx + c))^2 \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\
- & \frac{2b(10B a^2 - 15b^2 B + 3ab(7A - 15C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\
+ & \frac{2(15a^2 b B - 5b^3 B + 15a b^2(A - C) + a^3(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
+ & \frac{2(a^3 B + 9B a b^2 + b^3(3A + C) + 3a^2 b(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}
\end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (i B a^3 + 3i (A + 3C) a^2 b + 9i B a b^2 + i (3A + C) b^3) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)) + \dots$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C b^3 \sec(dx + c)^5 + (3 C a b^2 + B b^3) \sec(dx + c)^4 + A a^3 + (3 C a^2 b + 3 B a b^2 + A b^3) \sec(dx + c)^3 + (C a^3 + 3 A a^2 b + 3 B a b^2 + A b^3) \sec(dx + c)^2 + (C a^2 b + 3 A a b^2 + A b^3) \sec(dx + c) + C a b^2 + A b^3}{\sec(dx + c)^{\frac{5}{2}}}\right)$$

69.108 Problem number 1001

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(6Ab + 7Ba)(a + b \sec(dx + c))^2 \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{2A(a + b \sec(dx + c))^3 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2a(24Ab^2 + 63abB + 5a^2(5A + 7C)) \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} \\ & - \frac{2b^2(11Ab + 7Ba - 35Cb) \sin(dx + c) (\sqrt{\sec(dx + c)})}{35d} \\ & + \frac{2(3a^3B + 15Bab^2 + 5b^3(A - C) + 3a^2b(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^2bB + 21b^3B + 21ab^2(A + 3C) + a^3(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2} (i(5A + 7C)a^3 + 21iBa^2b + 21i(A + 3C)ab^2 + 21iBb^3) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \sec(dx + c)^5 + (3Cab^2 + Bb^3) \sec(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \sec(dx + c)^3 + (Ca^3 + 3Aab^2 + 3Bab^2) \sec(dx + c)^2 + (3Aab + 3Bab) \sec(dx + c) + Aa + Ba}{\sec(dx + c)^{\frac{7}{2}}}\right)$$

69.109 Problem number 1002

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(24Aab^2 + 99abB + 7a^2(7A + 9C)) \sin(dx + c)}{315d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(2Ab + 3Ba)(a + b \sec(dx + c))^2 \sin(dx + c)}{21d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2A(a + b \sec(dx + c))^3 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2(8Ab^3 + 15a^3B + 54Bab^2 + 9a^2b(5A + 7C)) \sin(dx + c)}{63d \sqrt{\sec(dx + c)}} \\ & + \frac{2(27a^2bB + 15b^3B + 9ab^2(3A + 5C) + a^3(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5a^3B + 21Bab^2 + 7b^3(A + 3C) + 3a^2b(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15\sqrt{2} (5iBa^3 + 3i(5A + 7C)a^2b + 21iBab^2 + 7i(A + 3C)b^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Cb^3 \sec(dx + c)^5 + (3Cab^2 + Bb^3) \sec(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \sec(dx + c)^3 + (Ca^3}{\sec(dx + c)^{\frac{9}{2}}}$$

69.110 Problem number 1003

$$\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(24Ab^2 + 143abB + 9a^2(9A + 11C)) \sin(dx + c)}{693d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(24Ab^3 + 77a^3B + 242Bab^2 + 33a^2b(7A + 9C)) \sin(dx + c)}{495d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(6Ab + 11Ba)(a + b \sec(dx + c))^2 \sin(dx + c)}{99d \sec(dx + c)^{\frac{7}{2}}} + \frac{2A(a + b \sec(dx + c))^3 \sin(dx + c)}{11d \sec(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(165a^2bB + 77b^3B + 33a^2b^2(5A + 7C) + 5a^3(9A + 11C)) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{2(7a^3B + 27Bab^2 + 3b^3(3A + 5C) + 3a^2b(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(165a^2bB + 77b^3B + 33a^2b^2(5A + 7C) + 5a^3(9A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sqrt{\cos(dx + c)})}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (5i (9A + 11C)a^3 + 165i Ba^2b + 33i (5A + 7C)ab^2 + 77i Bb^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{Cb^3 \sec(dx + c)^5 + (3Cab^2 + Bb^3) \sec(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \sec(dx + c)^3 + (Ca^3}{\sec(dx + c)^{\frac{11}{2}}}$$

69.111 Problem number 1004

$$\int \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(682a^3bB + 660ab^3B + 64a^4C + 15b^4(11A + 9C) + 9a^2b^2(143A + 101C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{693d} \\ & + \frac{2b(1353a^2bB + 539b^3B + 192a^3C + 2ab^2(891A + 673C)) \left(\sec^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{3465d} \\ & + \frac{2(33A b^2 + 55abB + 16a^2C + 27b^2C) \left(\sec^{\frac{3}{2}}(dx + c)\right) (a + b \sec(dx + c))^2 \sin(dx + c)}{231d} \\ & + \frac{2(11bB + 8aC) \left(\sec^{\frac{3}{2}}(dx + c)\right) (a + b \sec(dx + c))^3 \sin(dx + c)}{99d} \\ & + \frac{2C \left(\sec^{\frac{3}{2}}(dx + c)\right) (a + b \sec(dx + c))^4 \sin(dx + c)}{11d} \\ & + \frac{2(15a^4B + 54B a^2b^2 + 7b^4B + 12a^3b(5A + 3C) + 4a b^3(9A + 7C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ & - \frac{2(15a^4B + 54B a^2b^2 + 7b^4B + 12a^3b(5A + 3C) + 4a b^3(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(308a^3bB + 220ab^3B + 77a^4(3A + C) + 66a^2b^2(7A + 5C) + 5b^4(11A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fr"`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (77i (3A + C)a^4 + 308i Ba^3b + 66i (7A + 5C)a^2b^2 + 220i Bab^3 + 5i (11A + 9C)b^4) \cos(dx + c)^5 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + c)$$

Fricas 1.3.7 via sagemath 9.3 output

integral($(Cb^4 \sec(dx + c)^6 + (4Cab^3 + Bb^4) \sec(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \sec(dx + c)^4 + 2(2$

69.112 Problem number 1005

$$\int \frac{(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(261a^2bB + 75b^3B + 64a^3C + 2ab^2(147A + 101C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\ & + \frac{2(1098a^3bB + 756ab^3B + 192a^4C + 21b^4(9A + 7C) + 7a^2b^2(261A + 155C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{315d} \\ & + \frac{2(63Ab^2 + 117abB + 48a^2C + 49b^2C) (a + b \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{315d} \\ & + \frac{2(9bB + 8aC) (a + b \sec(dx + c))^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{63d} \\ & + \frac{2C(a + b \sec(dx + c))^4 \sin(dx + c) (\sqrt{\sec(dx + c)})}{9d} \\ & - \frac{2(60a^3bB + 36ab^3B - 15a^4(A - C) + 18a^2b^2(5A + 3C) + b^4(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^4B + 42Ba^2b^2 + 5b^4B + 28a^3b(3A + C) + 4ab^3(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{\sec(dx + c)}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (21i Ba^4 + 28i (3A + C)a^3b + 42i Ba^2b^2 + 4i (7A + 5C)ab^3 + 5i Bb^4) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(\dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

integral($\frac{Cb^4 \sec(dx + c)^6 + (4Cab^3 + Bb^4) \sec(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \sec(dx + c)^4 + 2(2$

69.113 Problem number 1006

$$\int \frac{(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(98abB - a^2(35A - 87C) + 5b^2(7A + 5C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{2A(a + b \sec(dx + c))^4 \sin(dx + c)}{3d \sqrt{\sec(dx + c)}} \\ & + \frac{2b(609a^2bB + 63b^3B - a^3(70A - 366C) + 84ab^2(5A + 3C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{105d} \\ & - \frac{2b(35aA - 21bB - 39aC) (a + b \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{105d} \\ & - \frac{2b(7A - 3C) (a + b \sec(dx + c))^3 \sin(dx + c) (\sqrt{\sec(dx + c)})}{21d} \\ & + \frac{2(5a^4B - 30Ba^2b^2 - 3b^4B + 20a^3b(A - C) - 4ab^3(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(84a^3bB + 28ab^3B + 42a^2b^2(3A + C) + 7a^4(A + 3C) + b^4(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm='fricas')`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (7i(A + 3C)a^4 + 84iBa^3b + 42i(3A + C)a^2b^2 + 28iBab^3 + i(7A + 5C)b^4) \cos(dx + c)^3 \operatorname{weierstrassPInv}(\dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \sec(dx + c)^6 + (4Cab^3 + Bb^4) \sec(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \sec(dx + c)^4 + 2(2a^3b + 2Ab^2) \sec(dx + c)^3 + (4a^2b + 4Ab) \sec(dx + c)^2 + 2a \sec(dx + c) + 2}{\sec(dx + c)^{\frac{3}{2}}}, dx\right)$$

69.114 Problem number 1007

$$\int \frac{(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(5B a^2 - 5b^2 B + 14ab(A - C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\ + & \frac{2A(a + b \sec(dx + c))^4 \sin(dx + c)}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(8Ab + 5Ba)(a + b \sec(dx + c))^3 \sin(dx + c)}{15d \sqrt{\sec(dx + c)}} \\ - & \frac{2b(10a^3 B - 60Ba b^2 + a^2 b(31A - 87C) - 3b^3(5A + 3C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ - & \frac{2b(11Ab + 5Ba - 3Cb)(a + b \sec(dx + c))^2 \sin(dx + c) (\sqrt{\sec(dx + c)})}{15d} \\ + & \frac{2(20a^3 b B - 20a b^3 B + 30a^2 b^2(A - C) - b^4(5A + 3C) + a^4(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ + & \frac{2(a^4 B + 18B a^2 b^2 + b^4 B + 4a b^3(3A + C) + 4a^3 b(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i B a^4 + 4i (A + 3 C) a^3 b + 18i B a^2 b^2 + 4i (3 A + C) a b^3 + i B b^4) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, c)}{15d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C b^4 \sec(dx + c)^6 + (4 C a b^3 + B b^4) \sec(dx + c)^5 + A a^4 + (6 C a^2 b^2 + 4 B a b^3 + A b^4) \sec(dx + c)^4 + 2 (2 C a b^2 + B a^2) \sec(dx + c)^3 + (A + B) \sec(dx + c)^2 + A}{\sec(dx + c)^5}, dx\right)$$

69.115 Problem number 1008

$$\int \frac{(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b^2(98abB + b^2(87A - 35C) + 5a^2(5A + 7C)) \left(\sec^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ + & \frac{2(8Ab + 7Ba)(a + b \sec(dx + c))^3 \sin(dx + c)}{35d \sec(dx + c)^{\frac{3}{2}}} + \frac{2A(a + b \sec(dx + c))^4 \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\ + & \frac{2(48Ab^2 + 77abB + 5a^2(5A + 7C))(a + b \sec(dx + c))^2 \sin(dx + c)}{105d \sqrt{\sec(dx + c)}} \\ - & \frac{2b(217a^2bB - 105b^3B + 12a^2b^2(19A - 35C) + 10a^3(5A + 7C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{105d} \\ + & \frac{2(3a^4B + 30Ba^2b^2 - 5b^4B + 20ab^3(A - C) + 4a^3b(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ + & \frac{2(28a^3bB + 84ab^3B + 7b^4(3A + C) + 42a^2b^2(A + 3C) + a^4(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2} \left(i(5A + 7C)a^4 + 28iBa^3b + 42i(A + 3C)a^2b^2 + 84iBab^3 + 7i(3A + C)b^4 \right) \cos(dx + c) \operatorname{weierstrassPInverse}\left(\frac{\cos(dx + c)}{2} + \frac{1}{2}\right)}{105d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \sec(dx + c)^6 + (4Cab^3 + Bb^4) \sec(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \sec(dx + c)^4 + 2(2a^3b^2 + 2a^2b^3 + Ab^4) \sec(dx + c)^3 + (2a^2b^3 + 2ab^4) \sec(dx + c)^2 + (2ab^4) \sec(dx + c) + a^4}{\sec^{\frac{7}{2}}(dx + c)}, dx\right)$$

69.116 Problem number 1009

$$\int \frac{(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(48Ab^2 + 117abB + 7a^2(7A + 9C)) (a + b \sec(dx + c))^2 \sin(dx + c)}{315d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(8Ab + 9Ba) (a + b \sec(dx + c))^3 \sin(dx + c)}{63d \sec(dx + c)^{\frac{5}{2}}} + \frac{2A(a + b \sec(dx + c))^4 \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} \\ & + \frac{2a(64Ab^3 + 75a^3B + 261Ba^2b^2 + a^2(202Ab + 294Cb)) \sin(dx + c)}{315d \sqrt{\sec(dx + c)}} \\ & - \frac{2b^2(162abB + 3b^2(41A - 105C) + 7a^2(7A + 9C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{315d} \\ & + \frac{2(36a^3bB + 60a^3b^3B + 15b^4(A - C) + 18a^2b^2(3A + 5C) + a^4(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5a^4B + 42Ba^2b^2 + 21b^4B + 28ab^3(A + 3C) + 4a^3b(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (5i Ba^4 + 4i (5A + 7C)a^3b + 42i Ba^2b^2 + 28i (A + 3C)ab^3 + 21i Bb^4) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{\sec(dx + c)^{\frac{9}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \sec(dx + c)^6 + (4Cab^3 + Bb^4) \sec(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \sec(dx + c)^4 + 2(2a^3b^2 + 2a^2b^3 + Ab^4) \sec(dx + c)^3 + 2(2a^2b^3 + 2ab^4) \sec(dx + c)^2 + 2ab^4 \sec(dx + c) + 2b^4}{\sec(dx + c)^{\frac{9}{2}}}\right)$$

69.117 Problem number 1010

$$\int \frac{(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(192Ab^3 + 539a^3B + 1353Bab^2 + 2a^2b(673A + 891C)) \sin(dx + c)}{3465d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(16Ab^2 + 55abB + 3a^2(9A + 11C)) (a + b \sec(dx + c))^2 \sin(dx + c)}{231d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(8Ab + 11Ba) (a + b \sec(dx + c))^3 \sin(dx + c)}{99d \sec(dx + c)^{\frac{7}{2}}} + \frac{2A(a + b \sec(dx + c))^4 \sin(dx + c)}{11d \sec(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(64Ab^4 + 660a^3bB + 682ab^3B + 15a^4(9A + 11C) + 9a^2b^2(101A + 143C)) \sin(dx + c)}{693d \sqrt{\sec(dx + c)}} \\ & + \frac{2(7a^4B + 54Ba^2b^2 + 15b^4B + 12ab^3(3A + 5C) + 4a^3b(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(220a^3bB + 308ab^3B + 77b^4(A + 3C) + 66a^2b^2(5A + 7C) + 5a^4(9A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algorithm="
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$15 \sqrt{2} (5i(9A + 11C)a^4 + 220iBa^3b + 66i(5A + 7C)a^2b^2 + 308iBab^3 + 77i(A + 3C)b^4) \operatorname{weierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \sec(dx + c)^6 + (4Cab^3 + Bb^4) \sec(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) \sec(dx + c)^4 + 2(2$$

sec

69.118 Problem number 1011

$$\int \frac{(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{13}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(192A b^3 + 1053a^3 B + 2171Ba b^2 + a^2(2518Ab + 3146Cb)) \sin(dx + c)}{9009d \sec(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(192A b^4 + 4004a^3 b B + 3458a b^3 B + 77a^4(11A + 13C) + 11a^2 b^2(491A + 637C)) \sin(dx + c)}{6435d \sec(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(48A b^2 + 221abB + 11a^2(11A + 13C)) (a + b \sec(dx + c))^2 \sin(dx + c)}{1287d \sec(dx + c)^{\frac{7}{2}}} \\ & + \frac{2(8Ab + 13Ba) (a + b \sec(dx + c))^3 \sin(dx + c)}{143d \sec(dx + c)^{\frac{9}{2}}} + \frac{2A(a + b \sec(dx + c))^4 \sin(dx + c)}{13d \sec(dx + c)^{\frac{11}{2}}} \\ & + \frac{2(45a^4 B + 330B a^2 b^2 + 77b^4 B + 44a b^3(5A + 7C) + 20a^3 b(9A + 11C)) \sin(dx + c)}{231d \sqrt{\sec(dx + c)}} \\ & + \frac{2(364a^3 b B + 468a b^3 B + 39b^4(3A + 5C) + 78a^2 b^2(7A + 9C) + a^4(77A + 91C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(45a^4 B + 330B a^2 b^2 + 77b^4 B + 44a b^3(5A + 7C) + 20a^3 b(9A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x, algorithm="`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$195 \sqrt{2} (45i Ba^4 + 20i(9A + 11C)a^3 b + 330i Ba^2 b^2 + 44i(5A + 7C)ab^3 + 77i Bb^4) \operatorname{weierstrassPInverse}(-4, 0,$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^4 \sec(dx + c)^6 + (4Cab^3 + Bb^4) \sec(dx + c)^5 + Aa^4 + (6Ca^2 b^2 + 4Bab^3 + Ab^4) \sec(dx + c)^4 + 2(2$$

`sec`

69.119 Problem number 1035

$$\int \frac{\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(a^2 - b^2)(2Ab - 5Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}} (\sqrt{\sec(dx + c)})^{\frac{3}{2}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a + b \sec(dx + c)}} + \frac{2A \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{5d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(Ab + 5Ba) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15ad \sqrt{\sec(dx + c)}}$$

$$\frac{2(2Ab^2 - 5abB - 3a^2(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-15i Ba^3 - 3i(A + 5C)a^2b + 10i Bab^2 - 4i Ab^3) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2 - 4b^2)}{3a^2}, \frac{8(9a^2b - 8b^3)}{27a^3}, \frac{3a \cos(dx + c)}{27a^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

69.120 Problem number 1036

$$\int \frac{\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2) (25a^2A + 8Ab^2 - 14abB + 35a^2C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b}{b+a}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2A \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7d \sec(dx+c)^{\frac{5}{2}}} + \frac{2(Ab + 7Ba) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{35ad \sec(dx+c)^{\frac{3}{2}}} \\
& - \frac{2(4Ab^2 - 7abB - 5a^2(5A + 7C)) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{105a^2 d \sqrt{\sec(dx+c)}} \\
& + \frac{2(8Ab^3 + 63a^3B - 14Ba^2b^2 + a^2b(19A + 35C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}
\end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$\sqrt{2} \left(-15i(5A + 7C)a^4 - 21iBa^3b + 2i(16A + 35C)a^2b^2 - 28iBab^3 + 16iAb^4 \right) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx+c)^2 + B \sec(dx+c) + A\right) \sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

69.121 Problem number 1037

$$\int \frac{\sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx) + C \sec^2(c+dx))}{\sec^{\frac{9}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2) (16A b^3 - 75a^3 B - 24Ba b^2 + 6a^2 b(6A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{a + b \sec(dx + c)}} \\
& + \frac{2A \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{9d \sec(dx + c)^{\frac{7}{2}}} + \frac{2(Ab + 9Ba) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{63ad \sec(dx + c)^{\frac{5}{2}}} \\
& - \frac{2(6A b^2 - 9abB - 7a^2(7A + 9C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{315a^2 d \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(8A b^3 + 75a^3 B - 12Ba b^2 + a^2 b(13A + 21C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{315a^3 d \sqrt{\sec(dx + c)}} \\
& - \frac{2(16A b^4 - 57a^3 b B - 24a b^3 B + 6a^2 b^2(4A + 7C) - 21a^4(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{\frac{a}{a+b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}}
\end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-225i Ba^5 - 3i(13A + 21C)a^4 b + 96i Ba^3 b^2 - 12i(3A + 7C)a^2 b^3 + 48i Bab^4 - 32i Ab^5) \sqrt{a} \operatorname{weierstrassP}\left(\frac{dx + c}{2}, \frac{a}{2}, \frac{b}{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{9}{2}}}, x\right)$$

69.122 Problem number 1043

$$\int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2A(a + b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{7d \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(a^2 - b^2)(25a^2A - 6Ab^2 + 21abB + 35a^2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{b - a \sec(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a + b \sec(dx + c)}} \\
& + \frac{2(3Ab + 7Ba) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{35d \sec(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(3Ab^2 + 42abB + 5a^2(5A + 7C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{105ad \sqrt{\sec(dx + c)}} \\
& - \frac{2(6Ab^3 - 63a^3B - 21Ba^2b - 2a^2b(41A + 70C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{b + a \sec(dx + c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}}
\end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm="maxima")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-15i(5A + 7C)a^4 - 126iBa^3b + i(11A - 35C)a^2b^2 + 42iBab^3 - 12iAb^4 \right) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2 + b^2)}{3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(Cb \sec(dx + c)^3 + (Ca + Bb) \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)\right) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

69.123 Problem number 1044

$$\int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2A(a + b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{9d \sec(dx + c)^{\frac{7}{2}}} \\
& + \frac{2(a^2 - b^2)(8Ab^3 + 75a^3B - 18Ba^2b^2 + a^2(39Ab + 63Cb)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a+b}{2}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a + b \sec(dx + c)}} \\
& + \frac{2(Ab + 3Ba) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{21d \sec(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(3Ab^2 + 72abB + 7a^2(7A + 9C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{315ad \sec(dx + c)^{\frac{3}{2}}} \\
& - \frac{2(4Ab^3 - 75a^3B - 9Ba^2b^2 - 2a^2b(44A + 63C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{315a^2d \sqrt{\sec(dx + c)}} \\
& + \frac{2(8Ab^4 + 246a^3bB - 18ab^3B + 21a^4(7A + 9C) + 3a^2b^2(11A + 21C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{\frac{a+b}{2}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}}
\end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="Fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-225i Ba^5 - 6i(44A + 63C)a^4b + 33i Ba^3b^2 + 6i(10A + 21C)a^2b^3 - 36i Bab^4 + 16i Ab^5) \sqrt{a} \operatorname{weierstrassPI}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Cb \sec(dx + c)^3 + (Ca + Bb) \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{9}{2}}}, x\right)$$

69.124 Problem number 1050

$$\int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{9/2}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5Ab + 9Ba)(a + b \sec(dx + c))^{3/2} \sin(dx + c)}{63d \sec(dx + c)^{5/2}} + \frac{2A(a + b \sec(dx + c))^{5/2} \sin(dx + c)}{9d \sec(dx + c)^{7/2}} \\ & - \frac{2(a^2 - b^2)(10Ab^3 - 75a^3B - 45Bab^2 - 6a^2b(19A + 28C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(15Ab^2 + 90abB + 7a^2(7A + 9C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{315d \sec(dx + c)^{3/2}} \\ & + \frac{2(5Ab^3 + 75a^3B + 135Bab^2 + a^2b(163A + 231C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{315ad \sqrt{\sec(dx + c)}} \\ & - \frac{2(10Ab^4 - 435a^3bB - 45ab^3B - 21a^4(7A + 9C) - 3a^2b^2(93A + 161C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-225i Ba^5 - 3i(163A + 231C)a^4b - 345i Ba^3b^2 + 3i(31A + 7C)a^2b^3 + 90i Bab^4 - 20i Ab^5) \sqrt{a} \operatorname{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Cb^2 \sec(dx + c))^4 + (2Cab + Bb^2) \sec(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2AbC) \sec(dx + c) + C^2}{\sec(dx + c)^{9/2}}\right)$$

69.125 Problem number 1051

$$\int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sec^{11/2}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5Ab + 11Ba)(a + b \sec(dx + c))^{3/2} \sin(dx + c)}{99d \sec(dx + c)^{7/2}} + \frac{2A(a + b \sec(dx + c))^{5/2} \sin(dx + c)}{11d \sec(dx + c)^{9/2}} \\ & + \frac{2(a^2 - b^2)(40Ab^4 + 1254a^3bB - 110ab^3B + 75a^4(9A + 11C) + 15a^2b^2(19A + 33C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\frac{dx + c}{2}, \frac{1}{2}\right)}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(5Ab^2 + 44abB + 3a^2(9A + 11C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{231d \sec(dx + c)^{5/2}} \\ & + \frac{2(15Ab^3 + 539a^3B + 825Ba b^2 + 5a^2b(229A + 297C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3465ad \sec(dx + c)^{3/2}} \\ & - \frac{2(20Ab^4 - 1793a^3bB - 55ab^3B - 75a^4(9A + 11C) - 5a^2b^2(205A + 297C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3465a^2d \sqrt{\sec(dx + c)}} \\ & + \frac{2(40Ab^5 + 1617Ba^5 + 3069Ba^3b^2 - 110Bab^4 + 15a^2b^3(17A + 33C) + 15a^4b(247A + 319C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algorithm="maxima")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-225i(9A + 11C)a^6 - 5379iBa^5b - 15i(169A + 253C)a^4b^2 + 1023iBa^3b^3 + 30i(16A + 33C)a^2b^4 - 220i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(Cb^2 \sec(dx + c))^4 + (2Cab + Bb^2) \sec(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2AbC) \sec(dx + c) + C^2}{\sec(dx + c)^{11/2}}\right)$$

69.126 Problem number 1055

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(2Ab^2 - 3abB + a^2(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{a + b \sec(dx + c)}} + \frac{2A \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3ad \sqrt{\sec(dx + c)}} - \frac{2(2Ab - 3Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$6 Aa^2 \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + \sqrt{2} (-3i(A + 3C)a^2 + 6iBab - 4iAb^2) \sqrt{a} \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b \sec(dx + c)^3 + a \sec(dx + c)^2}, x\right)$$

69.127 Problem number 1056

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2(8Ab^3 - 5a^3B - 10Ba^2b^2 + a^2b(7A + 15C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{b+c}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a+b \sec(dx+c)}} + \frac{2A \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5ad \sec(dx+c)^{\frac{3}{2}}} - \frac{2(4Ab - 5Ba) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{15a^2 d \sqrt{\sec(dx+c)}} + \frac{2(8Ab^2 - 10abB + 3a^2(3A + 5C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="Fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-15i Ba^3 + 6i(2A + 5C)a^2b - 20i Bab^2 + 16i Ab^3) \sqrt{a} \operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a \cos(dx+c)}{27a^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx+c)^2 + B \sec(dx+c) + A) \sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b \sec(dx+c)^4 + a \sec(dx+c)^3}, x\right)$$

69.128 Problem number 1057

$$\int \frac{A + B \sec(c+dx) + C \sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2(48Ab^4 - 49a^3bB - 56a^2b^3B + 5a^4(5A + 7C) + 2a^2b^2(16A + 35C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{b+c}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{a+b \sec(dx+c)}} + \frac{2A \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7ad \sec(dx+c)^{\frac{5}{2}}} - \frac{2(6Ab - 7Ba) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{35a^2 d \sec(dx+c)^{\frac{3}{2}}} + \frac{2(24Ab^2 - 28abB + 5a^2(5A + 7C)) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{105a^3 d \sqrt{\sec(dx+c)}} + \frac{2(48Ab^3 - 63a^3B - 56Ba^2b^2 + a^2(44Ab + 70Cb)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b \sec(dx+c)}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-15i (5A + 7C)a^4 + 84i Ba^3b - 4i (13A + 35C)a^2b^2 + 112i Bab^3 - 96i Ab^4 \right) \sqrt{a} \operatorname{weierstrassPInverse} \left(-\frac{4(3a^2 + b^2)}{a}, x \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \sec(dx+c)^2 + B \sec(dx+c) + A \right) \sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b \sec(dx+c)^5 + a \sec(dx+c)^4}, x \right)$$

69.129 Problem number 1061

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab^2 - a(bB - aC)) \sin(dx+c) (\sqrt{\sec(dx+c)})}{a(a^2 - b^2) d \sqrt{a + b \sec(dx+c)}} \\ - \frac{2(2Ab - Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{a}{a+b}} \right) \sqrt{\frac{b + a \cos(dx+c)}{a+b}} (\sqrt{\sec(dx+c)})}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d \sqrt{a + b \sec(dx+c)}} \\ - \frac{2(2Ab^2 - abB - a^2(A - C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{a}{a+b}} \right) \sqrt{a + b \sec(dx+c)}}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(Ca^4 - Ba^3b + Aa^2b^2) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \sqrt{2} (3i Ba^3b - i(5A + C)a^2b^2 - 2i Ba^2b^2)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\left(C \sec(dx+c)^2 + B \sec(dx+c) + A \right) \sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b^2 \sec(dx+c)^3 + 2ab \sec(dx+c)^2 + a^2 \sec(dx+c)}, x \right)$$

69.130 Problem number 1062

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) \sin(dx + c)}{a(a^2 - b^2) d \sqrt{\sec(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(8Ab^2 - 6abB + a^2(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{a + b \sec(dx + c)}} \\ & - \frac{2(4Ab^2 - 3abB - a^2(A - 3C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3a^2(a^2 - b^2) d \sqrt{\sec(dx + c)}} \\ & + \frac{2(8Ab^3 + 3a^3B - 6Bab^2 - a^2(5Ab - 3Cb)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a + b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx + c)}{a+b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (3i(A + 3C)a^4b - 15iBa^3b^2 + 2i(8A - 3C)a^2b^3 + 12iBab^4 - 16iAb^5 + (3i(A + 3C)a^5 - 15iBa^4b + 2i($$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^2 \sec(dx + c)^4 + 2ab \sec(dx + c)^3 + a^2 \sec(dx + c)^2}, x\right)$$

69.131 Problem number 1063

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab^2 - a(bB - aC)) \sin(dx + c)}{a(a^2 - b^2) d \sec(dx + c)^{\frac{3}{2}} \sqrt{a + b \sec(dx + c)}} - \frac{2(48Ab^3 - 5a^3B - 40Bab^2 + 6a^2b(2A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{a + b \sec(dx + c)}} - \frac{2(6Ab^2 - 5abB - a^2(A - 5C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{5a^2(a^2 - b^2) d \sec(dx + c)^{\frac{3}{2}}} + \frac{2(24Ab^3 + 5a^3B - 20Bab^2 - a^2(9Ab - 15Cb)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15a^3(a^2 - b^2) d \sqrt{\sec(dx + c)}} - \frac{2(48Ab^4 + 25a^3bB - 40ab^3B - 6a^2b^2(4A - 5C) - 3a^4(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (15i Ba^5b - 3i(9A + 25C)a^4b^2 + 80i Ba^3b^3 - 12i(7A - 5C)a^2b^4 - 80i Bab^5 + 96i Ab^6 + (15i Ba^6 - 3i(9A + 25C)a^5b - 80i Ba^3b^3 + 12i(7A - 5C)a^2b^4 - 80i Bab^5 + 96i Ab^6 - (15i Ba^6 - 3i(9A + 25C)a^5b - 80i Ba^3b^3 + 12i(7A - 5C)a^2b^4 - 80i Bab^5 + 96i Ab^6)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^2 \sec(dx + c)^5 + 2ab \sec(dx + c)^4 + a^2 \sec(dx + c)^3}, x\right)$$

69.132 Problem number 1066

$$\int \frac{\sqrt{\sec(c+dx)} (A + B \sec(c+dx) + C \sec^2(c+dx))}{(a + b \sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3b(a^2 - b^2) d(a + b \sec(dx+c))^{\frac{3}{2}}} \\ + & \frac{2(Ab^4 + 2a^3bB + 2ab^3B + a^4C - 5a^2b^2(A+C)) \sin(dx+c) (\sqrt{\sec(dx+c)})}{3ab(a^2 - b^2)^2 d\sqrt{a + b \sec(dx+c)}} \\ & - \frac{2(2Ab^2 + abB - a^2(3A+C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2) d\sqrt{a + b \sec(dx+c)}} \\ & - \frac{2(2Ab^3 + 3a^3B + Ba^2b - 2a^2b(3A+2C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{a+b}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2)^2 d\sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sqrt{\sec(dx+c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-3i(3A+C)a^4b^2 + 6iBa^3b^3 + i(9A-C)a^2b^4 - 2iBab^5 - 4iAb^6 + (-3i(3A+C)a^6 + 6iBa^5b + i(9A - \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx+c)^2 + B \sec(dx+c) + A) \sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3}, x\right)$$

69.133 Problem number 1067

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3a(a^2 - b^2) d(a + b \sec(dx + c))^{\frac{3}{2}}} \\ & - \frac{2(4Ab^4 + 5a^3bB - ab^3B - 2a^4C - 2a^2b^2(4A + C)) \sin(dx + c) (\sqrt{\sec(dx + c)})}{3a^2(a^2 - b^2)^2 d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(8Ab^3 + 3a^3B - 2Ba^2b - a^2b(9A + C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2) d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(8Ab^4 + 6a^3bB - 2ab^3B + 3a^4(A - C) - a^2b^2(15A + C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{b + a \cos(dx + c)}{a + b}}\right) \sqrt{\sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(-9i Ba^5b^2 + 6i(4A + C)a^4b^3 + 9i Ba^3b^4 - 2i(18A + C)a^2b^5 - 4i Bab^6 + 16i Ab^7 + (-9i Ba^7 + 6i(4A + C)a^6b) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^3 \sec(dx + c)^4 + 3ab^2 \sec(dx + c)^3 + 3a^2b \sec(dx + c)^2 + a^3 \sec(dx + c)}, x\right)$$

69.134 Problem number 1068

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) \sin(dx + c)}{3a(a^2 - b^2) d(a + b \sec(dx + c))^{\frac{3}{2}} \sqrt{\sec(dx + c)}} \\ & + \frac{2(10a^2Ab^2 - 6Ab^4 - 7a^3bB + 3ab^3B + 4a^4C) \sin(dx + c)}{3a^2(a^2 - b^2)^2 d \sqrt{\sec(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & - \frac{2(16Ab^4 + 9a^3bB - 8ab^3B - 2a^2b^2(8A - C) - a^4(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2 - b^2) d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(8Ab^4 + 8a^3bB - 4ab^3B + a^4(A - 5C) - a^2b^2(13A - C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3a^3(a^2 - b^2)^2 d \sqrt{\sec(dx + c)}} \\ & - \frac{2(16Ab^5 - 3Ba^5 + 15Ba^3b^2 - 8Bab^4 - 2a^2b^3(14A - C) + a^4(8Ab - 6Cb)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}} \sqrt{\sec(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^3 \sec(dx + c)^5 + 3ab^2 \sec(dx + c)^4 + 3a^2b \sec(dx + c)^3 + a^3 \sec(dx + c)^2}, x\right)$$

69.135 Problem number 1069

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Ab^2 - a(bB - aC)) \sin(dx + c)}{3a(a^2 - b^2) d \sec(dx + c)^{\frac{3}{2}} (a + b \sec(dx + c))^{\frac{3}{2}}} \\
& - \frac{2(8Ab^4 + 9a^3bB - 5ab^3B - 2a^2b^2(6A - C) - 6a^4C) \sin(dx + c)}{3a^2(a^2 - b^2)^2 d \sec(dx + c)^{\frac{3}{2}} \sqrt{a + b \sec(dx + c)}} \\
& + \frac{2(128Ab^5 + 5Ba^5 + 80Ba^3b^2 - 80Bab^4 - 4a^2b^3(29A - 10C) - a^4b(17A + 45C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^5 (a^2 - b^2) d \sqrt{a + b \sec(dx + c)}} \\
& + \frac{2(48Ab^4 + 50a^3bB - 30ab^3B + a^4(3A - 35C) - a^2b^2(71A - 15C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15a^3(a^2 - b^2)^2 d \sec(dx + c)^{\frac{3}{2}}} \\
& - \frac{2(64Ab^5 - 5Ba^5 + 65Ba^3b^2 - 40Bab^4 + 2a^4b(7A - 20C) - 2a^2b^3(49A - 10C)) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15a^4(a^2 - b^2)^2 d \sqrt{\sec(dx + c)}} \\
& + \frac{2(128Ab^6 - 40a^5bB + 140a^3b^3B - 80ab^5B + 5a^4b^2(11A - 15C) - 4a^2b^4(53A - 10C) + 3a^6(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^5 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}
\end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^3 \sec(dx + c)^6 + 3ab^2 \sec(dx + c)^5 + 3a^2b \sec(dx + c)^4 + a^3 \sec(dx + c)^3}, x\right)$$

69.136 Problem number 1075

$$\int \cos^{\frac{9}{2}}(c + dx) (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(7A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(7A + 9C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} + \frac{2A \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{9d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 A \cos(dx + c)^3 + (7 A + 9 C) \cos(dx + c) \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 3 \sqrt{2} (-7i A - 9i C) \text{weierstrassZeta}(x)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^4 \sec(dx + c)^2 + A \cos(dx + c)^4\right) \sqrt{\cos(dx + c)}, x\right)$$

69.137 Problem number 1076

$$\int \cos^{\frac{7}{2}}(c + dx) (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2A \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{2(5A + 7C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d}$$

command

```
integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 A \cos(dx + c)^2 + 5 A + 7 C \right) \sqrt{\cos(dx + c)} \sin(dx + c) + \sqrt{2} (-5i A - 7i C) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

21 d

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx + c)^3 \sec(dx + c)^2 + A \cos(dx + c)^3\right) \sqrt{\cos(dx + c)}, x\right)$$

69.138 Problem number 1077

$$\int \cos^{\frac{5}{2}}(c + dx) (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2A \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2A \cos(dx + c)^{\frac{3}{2}} \sin(dx + c) + \sqrt{2} (3iA + 5iC) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i))}{5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^2 \sec(dx + c)^2 + A \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}, x\right)$$

69.139 Problem number 1078

$$\int \cos^{\frac{3}{2}}(c + dx) (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2A \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2A \sqrt{\cos(dx + c)} \sin(dx + c) + \sqrt{2} (-iA - 3iC) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c) \sec(dx + c)^2 + A \cos(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

69.140 Problem number 1079

$$\int \sqrt{\cos(c+dx)} (A + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\frac{2(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sin(dx+c)}{d \sqrt{\cos(dx+c)}}$$

command

```
integrate(cos(d*x+c)^(1/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (i A - i C) \cos(dx+c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + \sqrt{2}}{d \sqrt{\cos(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \sec(dx+c)^2 + A\right) \sqrt{\cos(dx+c)}, x\right)$$

69.141 Problem number 1080

$$\int \frac{A + C \sec^2(c+dx)}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}}$$

command

```
integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-3i A - i C) \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2} (3i A + i C) \cos(dx+c)}{3d \cos(dx+c)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \sec(dx+c)^2 + A}{\sqrt{\cos(dx+c)}}, x\right)$$

69.142 Problem number 1081

$$\int \frac{A + C \sec^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2(5A + 3C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}}$$

command

```
integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-5i A - 3i C) \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \sec(dx + c)^2 + A}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

69.143 Problem number 1082

$$\int \frac{A + C \sec^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{2(7A + 5C) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}}$$

command

```
integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-7i A - 5i C) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (7i A + 5i C) \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \sec(dx + c)^2 + A}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

69.144 Problem number 1083

$$\int \cos^{\frac{9}{2}}(c + dx)(a + a \sec(c + dx))(A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(7A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 9C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} + \frac{2aA \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2aA \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{9d} + \frac{2a(5A + 7C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-15i \sqrt{2} (5A + 7C) a \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} (5A + 7C) a \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca \cos(dx + c)^4 \sec(dx + c)^3 + Ca \cos(dx + c)^4 \sec(dx + c)^2 + Aa \cos(dx + c)^4 \sec(dx + c) + Aa \cos(dx + c)^4 \sec(dx + c)\right), x\right)$$

69.145 Problem number 1084

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))(A+C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(3A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A+7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2aA \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} \\ & + \frac{2a(5A+7C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} (5A+7C) \operatorname{aweierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5i \sqrt{2} (5A+7C) \operatorname{aweierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca \cos(dx+c)^3 \sec(dx+c)^3 + Ca \cos(dx+c)^3 \sec(dx+c)^2 + Aa \cos(dx+c)^3 \sec(dx+c) + Aa \cos(dx+c)^3\right), dx\right)$$

69.146 Problem number 1085

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))(A+C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(3A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2aA \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i\sqrt{2}(A+3C)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+3C)a\text{weierstrassPInverse}(\dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ca\cos(dx+c)^2\sec(dx+c)^3+Ca\cos(dx+c)^2\sec(dx+c)^2+Aa\cos(dx+c)^2\sec(dx+c)+Aa\cos(dx+c)\right)\right)$$

69.147 Problem number 1086

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))(A+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(A-C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(A+3C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2aC\sin(dx+c)}{d\sqrt{\cos(dx+c)}} + \frac{2aA\sin(dx+c)(\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}(A+3C)a\cos(dx+c)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(A+3C)a\cos(dx+c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ca\cos(dx+c)\sec(dx+c)^3+Ca\cos(dx+c)\sec(dx+c)^2+Aa\cos(dx+c)\sec(dx+c)+Aa\cos(dx+c)\right)\right)$$

69.148 Problem number 1087

$$\int \sqrt{\cos(c+dx)} (a+a \sec(c+dx)) (A+C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\frac{2a(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2aC \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2aC \sin(dx+c)}{d \sqrt{\cos(dx+c)}}$$

command

`integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} (3A+C)a \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} (3A+C)a \cos(dx+c)}{d \sqrt{\cos(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca \sec(dx+c)^3 + Ca \sec(dx+c)^2 + Aa \sec(dx+c) + Aa\right) \sqrt{\cos(dx+c)}, x\right)$$

69.149 Problem number 1088

$$\int \frac{(a+a \sec(c+dx)) (A+C \sec^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$-\frac{2a(5A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2a(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2aC \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2aC \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2a(5A+3C) \sin(dx+c)}{5d \sqrt{\cos(dx+c)}}$$

command

```
integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(3A+C)a\cos(dx+c)^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(3A+C)a\cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\sec(dx+c)^3+Ca\sec(dx+c)^2+Aa\sec(dx+c)+Aa}{\sqrt{\cos(dx+c)}},x\right)$$

69.150 Problem number 1089

$$\int \frac{(a+a\sec(c+dx))(A+C\sec^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a(5A+3C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2a(7A+5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d}+\frac{2aC\sin(dx+c)}{7d\cos(dx+c)^{\frac{7}{2}}} \\ & +\frac{2aC\sin(dx+c)}{5d\cos(dx+c)^{\frac{5}{2}}}+\frac{2a(7A+5C)\sin(dx+c)}{21d\cos(dx+c)^{\frac{3}{2}}}+\frac{2a(5A+3C)\sin(dx+c)}{5d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(7A+5C)a\cos(dx+c)^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(7A+5C)a\cos(dx+c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Ca\sec(dx+c)^3+Ca\sec(dx+c)^2+Aa\sec(dx+c)+Aa}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

69.151 Problem number 1090

$$\int \cos^{\frac{11}{2}}(c + dx)(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(7A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(25A + 33C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(7A + 9C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{2a^2(89A + 99C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{693d} \\ & + \frac{2A \left(\cos^{\frac{5}{2}}(dx + c)\right) (a + a \cos(dx + c))^2 \sin(dx + c)}{11d} \\ & + \frac{8A \left(\cos^{\frac{5}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{99d} \\ & + \frac{8a^2(25A + 33C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(30i \sqrt{2} (25A + 33C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 30i \sqrt{2} (25A + 33C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{231d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^5 \sec(dx + c)^4 + 2Ca^2 \cos(dx + c)^5 \sec(dx + c)^3 + (A + C)a^2 \cos(dx + c)^5 \sec(dx + c)\right) dx\right)$$

69.152 Problem number 1091

$$\int \cos^{\frac{9}{2}}(c + dx)(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16a^2(2A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(19A + 21C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\ & + \frac{2A \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + a \cos(dx + c))^2 \sin(dx + c)}{9d} \\ & + \frac{8A \left(\cos^{\frac{3}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{63d} \\ & + \frac{4a^2(5A + 7C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (5A + 7C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 7C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{105d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^4 \sec(dx + c)^4 + 2Ca^2 \cos(dx + c)^4 \sec(dx + c)^3 + (A + C)a^2 \cos(dx + c)^4 \sec(dx + c)\right) dx\right)$$

69.153 Problem number 1092

$$\int \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(3A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(33A + 35C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\ & + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d} \\ & + \frac{8A(a^2 + a^2 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{35d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10i \sqrt{2} (3A + 7C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} (3A + 7C) a^2 \operatorname{weierstrassP}(\cos(dx + c) + i \sin(dx + c)) \right)}{105d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx + c)^3 \sec(dx + c)^4 + 2Ca^2 \cos(dx + c)^3 \sec(dx + c)^3 + (A + C)a^2 \cos(dx + c)^3 \sec(dx + c)\right) dx\right)$$

69.154 Problem number 1093

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16a^2 A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(a+a\cos(dx+c))^2 \sin(dx+c)}{d \sqrt{\cos(dx+c)}} + \frac{2a^2(7A-15C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{15d} \\ & + \frac{2(A-5C)(a^2+a^2\cos(dx+c)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{5d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (A+3C)a^2 \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} (A+3C)a^2 \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \cos(dx+c)^2 \sec(dx+c)^4 + 2Ca^2 \cos(dx+c)^2 \sec(dx+c)^3 + (A+C)a^2 \cos(dx+c)^2 \sec(dx+c)\right) dx\right)$$

69.155 Problem number 1094

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2 (A+C\sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(a+a\cos(dx+c))^2 \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{8C(a^2+a^2\cos(dx+c)) \sin(dx+c)}{3d \sqrt{\cos(dx+c)}} \\ & + \frac{2a^2(A-5C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2i \sqrt{2} (A + C) a^2 \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 2i \sqrt{2} (A + C) a^2 \cos \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(C a^2 \cos(dx + c) \sec(dx + c)^4 + 2 C a^2 \cos(dx + c) \sec(dx + c)^3 + (A + C) a^2 \cos(dx + c) \sec(dx + c)^2 + \right. \right.$$

69.156 Problem number 1095

$$\int \sqrt{\cos(c + dx)} (a + a \sec(c + dx))^2 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{16a^2 C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{4a^2 (3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{2C (a + a \cos(dx + c))^2 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{8C (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a^2 (15A + 17C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (3A + C) a^2 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + C) a^2 \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(C a^2 \sec(dx + c)^4 + 2 C a^2 \sec(dx + c)^3 + (A + C) a^2 \sec(dx + c)^2 + 2 A a^2 \sec(dx + c) + A a^2 \right) \sqrt{\cos(dx + c)} \right.$$

69.157 Problem number 1096

$$\int \frac{(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^2(7A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(35A + 33C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{8C(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^2(5A + 3C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10i \sqrt{2} (7A + 3C) a^2 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} (7A + 3C) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \sec(dx + c)^4 + 2Ca^2 \sec(dx + c)^3 + (A + C)a^2 \sec(dx + c)^2 + 2Aa^2 \sec(dx + c) + Aa^2}{\sqrt{\cos(dx + c)}}, x\right)$$

69.158 Problem number 1097

$$\int \frac{(a + a \sec(c + dx))^2 (A + C \sec^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{16a^2(3A + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^2(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a^2(21A + 19C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^2(7A + 5C) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} \\
& + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\
& + \frac{8C(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{63d \cos(dx + c)^{\frac{7}{2}}} + \frac{16a^2(3A + 2C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (7A + 5C) a^2 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (7A + 5C) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \sec(dx + c)^4 + 2Ca^2 \sec(dx + c)^3 + (A + C)a^2 \sec(dx + c)^2 + 2Aa^2 \sec(dx + c) + Aa^2}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

69.159 Problem number 1098

$$\int \cos^{\frac{13}{2}}(c + dx) (a + a \sec(c + dx))^3 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^3(175A + 221C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(95A + 121C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(175A + 221C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{585d} \\
& + \frac{40a^3(118A + 143C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{9009d} \\
& + \frac{2A \left(\cos^{\frac{5}{2}}(dx + c)\right) (a + a \cos(dx + c))^3 \sin(dx + c)}{13d} \\
& + \frac{12A \left(\cos^{\frac{5}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{143ad} \\
& + \frac{2(145A + 143C) \left(\cos^{\frac{5}{2}}(dx + c)\right) (a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{1287d} \\
& + \frac{4a^3(95A + 121C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(195i \sqrt{2} (95A + 121C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 195i \sqrt{2} (95A + 121C) a \right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^6 \sec(dx + c)^5 + 3Ca^3 \cos(dx + c)^6 \sec(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^6 \sec(dx + c)\right) dx\right)$$

69.160 Problem number 1099

$$\int \cos^{\frac{11}{2}}(c + dx)(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(105A + 143C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^3(35A + 44C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{385d} \\ & + \frac{2A \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + a \cos(dx + c))^3 \sin(dx + c)}{11d} \\ & + \frac{4A \left(\cos^{\frac{3}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{33ad} \\ & + \frac{2(35A + 33C) \left(\cos^{\frac{3}{2}}(dx + c)\right) (a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{231d} \\ & + \frac{4a^3(105A + 143C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{231d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (105 A + 143 C) a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (105 A + 143 C) a^3 w \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C a^3 \cos(dx + c)^5 \sec(dx + c)^5 + 3 C a^3 \cos(dx + c)^5 \sec(dx + c)^4 + (A + 3 C) a^3 \cos(dx + c)^5 \sec(dx + c)^3 + \dots\right)\right)$$

69.161 Problem number 1100

$$\int \cos^{\frac{9}{2}}(c + dx)(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(17A + 27C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(11A + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^3(16A + 21C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{105d} \\ & + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos}(dx + c))}{9d} \\ & + \frac{4A(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos}(dx + c))}{21ad} \\ & + \frac{2(73A + 63C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos}(dx + c))}{315d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output
```

$$\frac{2 \left(15i \sqrt{2} (11A + 21C)a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (11A + 21C)a^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{315d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^4 \sec(dx + c)^5 + 3Ca^3 \cos(dx + c)^4 \sec(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^4 \sec(dx + c)^3\right), dx\right)$$

69.162 Problem number 1101

$$\int \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(13A + 35C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} + \frac{4a^3(41A - 35C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\
& + \frac{2(A - 7C) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7ad} \\
& + \frac{2(11A - 35C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{35d}
\end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (13A + 35C) a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (13A + 35C) a^3 \right)}{105d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^3 \sec(dx + c)^5 + 3Ca^3 \cos(dx + c)^3 \sec(dx + c)^4 + (A + 3C)a^3 \cos(dx + c)^3 \sec(dx + c)^3\right) dx\right)$$

69.163 Problem number 1102

$$\int \cos^{\frac{5}{2}}(c + dx) (a + a \sec(c + dx))^3 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^3(9A - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{4C(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} \\
& + \frac{8a^3(3A - 10C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \\
& + \frac{2(3A - 35C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (3A + 5C)a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + 5C) \right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Ca^3 cos(dx + c)^2 sec(dx + c)^5 + 3Ca^3 cos(dx + c)^2 sec(dx + c)^4 + (A + 3C)a^3 cos(dx + c)^2 sec(dx + c)^3), dx)
```

69.164 Problem number 1103

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(5A - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{4C(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{5ad \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(5A + 11C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \\ & - \frac{4a^3(5A + 21C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (5A + 3C)a^3 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (5A + 3C) \right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Ca^3 cos(dx + c) sec(dx + c)^5 + 3Ca^3 cos(dx + c) sec(dx + c)^4 + (A + 3C)a^3 cos(dx + c) sec(dx + c)^3), dx)
```

69.165 Problem number 1104

$$\int \sqrt{\cos(c+dx)} (a+a \sec(c+dx))^3 (A+C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^3(5A+7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(35A+13C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(a+a \cos(dx+c))^3 \sin(dx+c)}{7d \cos(dx+c)^{\frac{7}{2}}} + \frac{12C(a^2+a^2 \cos(dx+c))^2 \sin(dx+c)}{35ad \cos(dx+c)^{\frac{5}{2}}} \\ & + \frac{2(5A+7C)(a^3+a^3 \cos(dx+c)) \sin(dx+c)}{15d \cos(dx+c)^{\frac{3}{2}}} + \frac{8a^3(70A+53C) \sin(dx+c)}{105d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5i \sqrt{2} (35A+13C)a^3 \cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} (35A+13C)a^3 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Ca^3 sec(dx+c)^5 + 3Ca^3 sec(dx+c)^4 + (A+3C)a^3 sec(dx+c)^3 + (3A+C)a^3 sec(dx+c)^2 + 3Aa^3 sec(dx+c)), dx)
```

69.166 Problem number 1105

$$\int \frac{(a+a \sec(c+dx))^3 (A+C \sec^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{4a^3(27A + 17C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^3(21A + 11C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{8a^3(21A + 16C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\
& + \frac{4C(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{21ad \cos(dx + c)^{\frac{7}{2}}} \\
& + \frac{2(63A + 73C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{315d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^3(27A + 17C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (21A + 11C) a^3 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (21A + 11C) a^3 \sin(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) \right)}{15d \sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + 3Ca^3 \sec(dx + c)^4 + (A + 3C)a^3 \sec(dx + c)^3 + (3A + C)a^3 \sec(dx + c)^2 + 3Aa^3 \sec(dx + c)}{\sqrt{\cos(dx + c)}}\right)$$

69.167 Problem number 1106

$$\int \frac{(a + a \sec(c + dx))^3 (A + C \sec^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(143A + 105C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^3(44A + 35C) \sin(dx + c)}{385d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^3(143A + 105C) \sin(dx + c)}{231d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{11d \cos(dx + c)^{\frac{11}{2}}} + \frac{4C(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{33ad \cos(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(33A + 35C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{231d \cos(dx + c)^{\frac{7}{2}}} + \frac{4a^3(7A + 5C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (143A + 105C) a^3 \cos(dx + c)^6 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (143A + 105C) a^3 \cos(dx + c)^5 \right)}{\cos(dx + c)^{\frac{3}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + 3Ca^3 \sec(dx + c)^4 + (A + 3C)a^3 \sec(dx + c)^3 + (3A + C)a^3 \sec(dx + c)^2 + 3Aa^3 \sec(dx + c)}{\cos(dx + c)^{\frac{3}{2}}}\right)$$

69.168 Problem number 1107

$$\int \frac{\cos^{\frac{7}{2}}(c + dx) (A + C \sec^2(c + dx))}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{5(9A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(7A + 5C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5ad} + \frac{(9A + 7C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7ad} \\ & - \frac{(A + C) \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{d(a + a \cos(dx + c))} + \frac{5(9A + 7C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21ad} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(30 A \cos(dx + c)^3 - 12 A \cos(dx + c)^2 + 2(39 A + 35 C) \cos(dx + c) + 225 A + 175 C \right) \sqrt{\cos(dx + c)} \sin(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^3 \sec(dx + c)^2 + A \cos(dx + c)^3 \right) \sqrt{\cos(dx + c)}}{a \sec(dx + c) + a}, x \right)$$

69.169 Problem number 1108

$$\int \frac{\cos^{\frac{5}{2}}(c + dx) (A + C \sec^2(c + dx))}{a + a \sec(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(7A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{(7A + 5C) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5ad} - \frac{(A + C) \left(\cos^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{d(a + a \cos(dx + c))} \\ & - \frac{(5A + 3C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3ad} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6 A \cos(dx + c)^2 - 4 A \cos(dx + c) - 25 A - 15 C \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(\sqrt{2} (-5i A - 3i C) \cos(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 \sec(dx + c)^2 + A \cos(dx + c)^2 \right) \sqrt{\cos(dx + c)}}{a \sec(dx + c) + a}, x \right)$$

69.170 Problem number 1109

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A+C \sec^2(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(5A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A+C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{d(a+a \cos(dx+c))} + \frac{(5A+3C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3ad} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(2A \cos(dx+c) + 5A + 3C) \sqrt{\cos(dx+c)} \sin(dx+c) + \left(\sqrt{2}(-5iA - 3iC) \cos(dx+c) + \sqrt{2}(-5iA - 3iC)\right)}{3ad}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c) \sec^2(dx+c) + A \cos(dx+c)\right) \sqrt{\cos(dx+c)}}{a \sec(dx+c) + a}, x\right)$$

69.171 Problem number 1110

$$\int \frac{\sqrt{\cos(c+dx)} (A+C \sec^2(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A+C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{d(a+a \cos(dx+c))} \end{aligned}$$

command

```
integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(A+C)\sqrt{\cos(dx+c)}\sin(dx+c) - \left(\sqrt{2}(iA-iC)\cos(dx+c) + \sqrt{2}(iA-iC)\right)\text{weierstrassPInverse}(-4, \dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C\sec(dx+c)^2 + A\right)\sqrt{\cos(dx+c)}}{a\sec(dx+c) + a}, x\right)$$

69.172 Problem number 1111

$$\int \frac{A + C \sec^2(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A + 3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A + 3C) \sin(dx+c)}{ad\sqrt{\cos(dx+c)}} - \frac{(A + C) \sin(dx+c)}{d(a + a \cos(dx+c))\sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2((A + 3C)\cos(dx+c) + 2C)\sqrt{\cos(dx+c)}\sin(dx+c) + \left(\sqrt{2}(-iA+iC)\cos(dx+c)^2 + \sqrt{2}(-iA+iC)\cos(dx+c)\right)\text{weierstrassPInverse}(-4, \dots)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\left(C\sec(dx+c)^2 + A\right)\sqrt{\cos(dx+c)}}{a\cos(dx+c)\sec(dx+c) + a\cos(dx+c)}, x\right)$$

69.173 Problem number 1112

$$\int \frac{A + C \sec^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(3A + 5C) \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}}} - \frac{(A + C) \sin(dx + c)}{d \cos(dx + c)^{\frac{3}{2}}(a + a \cos(dx + c))} - \frac{(A + 3C) \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(A + 3C) \cos(dx + c)^2 + 4C \cos(dx + c) - 2C \right) \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2} (-3iA - 5iC) \cos(dx + c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + A\right) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^2 \sec(dx + c) + a \cos(dx + c)^2}, x\right)$$

69.174 Problem number 1113

$$\int \frac{A + C \sec^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3(5A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(3A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} + \frac{(5A + 7C) \sin(dx + c)}{5ad \cos(dx + c)^{\frac{5}{2}}} \\ & - \frac{(3A + 5C) \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}}} - \frac{(A + C) \sin(dx + c)}{d \cos(dx + c)^{\frac{5}{2}}(a + a \cos(dx + c))} + \frac{3(5A + 7C) \sin(dx + c)}{5ad \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(9(5A + 7C) \cos(dx + c)^3 + 2(15A + 19C) \cos(dx + c)^2 - 4C \cos(dx + c) + 6C \right) \sqrt{\cos(dx + c)} \sin(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^2 + A \right) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^3 \sec(dx + c) + a \cos(dx + c)^3}, x \right)$$

69.175 Problem number 1114

$$\int \frac{\cos^{\frac{5}{2}}(c + dx) (A + C \sec^2(c + dx))}{(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(14A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & - \frac{5(3A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d} \\ & + \frac{4(14A + 5C) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{15a^2 d} - \frac{(3A + C) \left(\cos^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{a^2 d (1 + \cos(dx + c))} \\ & - \frac{(A + C) \left(\cos^{\frac{7}{2}}(dx + c) \right) \sin(dx + c)}{3d (a + a \cos(dx + c))^2} - \frac{5(3A + C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3a^2 d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6A \cos(dx + c)^3 - 8A \cos(dx + c)^2 - 2(47A + 15C) \cos(dx + c) - 75A - 25C \right) \sqrt{\cos(dx + c)} \sin(dx + c) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx + c)^2 \sec(dx + c)^2 + A \cos(dx + c)^2 \right) \sqrt{\cos(dx + c)}}{a^2 \sec(dx + c)^2 + 2a^2 \sec(dx + c) + a^2}, x \right)$$

69.176 Problem number 1115

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A+C \sec^2(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(7A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2(5A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{(7A+C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3a^2 d (1+\cos(dx+c))} - \frac{(A+C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{3d (a+a \cos(dx+c))^2} \\ & + \frac{2(5A+C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a^2 d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2A \cos(dx+c)^2 + (13A+3C) \cos(dx+c) + 10A+2C \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 2 \left(\sqrt{2} (5iA+iC) \cos \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c) \sec(dx+c)^2 + A \cos(dx+c)\right) \sqrt{\cos(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

69.177 Problem number 1116

$$\int \frac{\sqrt{\cos(c+dx)} (A+C \sec^2(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\frac{4A \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d}$$

$$- \frac{(5A - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d}$$

$$- \frac{(A + C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3d(a + a \cos(dx+c))^2} - \frac{(5A - C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a^2 d (1 + \cos(dx+c))}$$

command

`integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(6A \cos(dx+c) + 5A - C) \sqrt{\cos(dx+c)} \sin(dx+c) - \left(\sqrt{2}(5iA - iC) \cos(dx+c)^2 - 2\sqrt{2}(-5iA + iC)\right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx+c)^2 + A\right) \sqrt{\cos(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

69.178 Problem number 1117

$$\int \frac{A + C \sec^2(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{(A - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d}$$

$$+ \frac{2(A + C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d}$$

$$+ \frac{(A - C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{a^2 d (1 + \cos(dx+c))} - \frac{(A + C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d(a + a \cos(dx+c))^2}$$

command

`integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(3(A-C)\cos(dx+c)+2A-4C)\sqrt{\cos(dx+c)}\sin(dx+c)-2\left(\sqrt{2}(iA+iC)\cos(dx+c)^2+2\sqrt{2}(iA+C)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\sec(dx+c)^2+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)\sec(dx+c)^2+2a^2\cos(dx+c)\sec(dx+c)+a^2\cos(dx+c)},x\right)$$

69.179 Problem number 1118

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4C\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d} \\ & +\frac{(A-5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d}+\frac{4C\sin(dx+c)}{a^2d\sqrt{\cos(dx+c)}} \\ & +\frac{(A-5C)\sin(dx+c)}{3a^2d(1+\cos(dx+c))\sqrt{\cos(dx+c)}}-\frac{(A+C)\sin(dx+c)}{3d(a+a\cos(dx+c))^2\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(12C\cos(dx+c)^2+(A+19C)\cos(dx+c)+6C\right)\sqrt{\cos(dx+c)}\sin(dx+c)+\left(\sqrt{2}(-iA+5iC)\cos(dx+c)+\sqrt{2}(iA+C)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\sec(dx+c)^2+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^2\sec(dx+c)^2+2a^2\cos(dx+c)^2\sec(dx+c)+a^2\cos(dx+c)^2},x\right)$$

69.180 Problem number 1119

$$\int \frac{A + C \sec^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2(A + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{2(A + 5C) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}}} - \frac{(A + 7C) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}} (1 + \cos(dx + c))} \\ & - \frac{(A + C) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^2} - \frac{(A + 7C) \sin(dx + c)}{a^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(A + 7C) \cos(dx + c)^3 + 4(A + 8C) \cos(dx + c)^2 + 8C \cos(dx + c) - 2C \right) \sqrt{\cos(dx + c)} \sin(dx + c) + 2}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + A\right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^3 \sec(dx + c)^2 + 2a^2 \cos(dx + c)^3 \sec(dx + c) + a^2 \cos(dx + c)^3}, x\right)$$

69.181 Problem number 1120

$$\int \frac{\cos^{\frac{5}{2}}(c + dx) (A + C \sec^2(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{7(33A + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
& - \frac{(63A + 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
& + \frac{7(33A + 7C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{30a^3 d} - \frac{(A + C) \left(\cos^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{5d (a + a \cos(dx + c))^3} \\
& - \frac{2(6A + C) \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{15ad (a + a \cos(dx + c))^2} - \frac{(63A + 13C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{10d (a^3 + a^3 \cos(dx + c))} \\
& - \frac{(63A + 13C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{6a^3 d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 A \cos(dx + c)^4 - 24 A \cos(dx + c)^3 - 3 (147 A + 29 C) \cos(dx + c)^2 - 2 (357 A + 73 C) \cos(dx + c) - 315 A \right)}{6a^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 \sec(dx + c)^2 + A \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x\right)$$

69.182 Problem number 1121

$$\int \frac{\cos^{\frac{3}{2}}(c + dx) (A + C \sec^2(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(119A + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(11A + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A + C) \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{5d(a + a \cos(dx + c))^3} - \frac{2A \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{3ad(a + a \cos(dx + c))^2} \\ & - \frac{(119A + 9C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{30d(a^3 + a^3 \cos(dx + c))} + \frac{(11A + C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{2a^3 d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(20 A \cos(dx + c)^3 + 3(79 A + 9 C) \cos(dx + c)^2 + 4(94 A + 9 C) \cos(dx + c) + 165 A + 15 C \right) \sqrt{\cos(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c) \sec(dx + c)^2 + A \cos(dx + c)\right) \sqrt{\cos(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x\right)$$

69.183 Problem number 1122

$$\int \frac{\sqrt{\cos(c + dx)} (A + C \sec^2(c + dx))}{(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(49A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(13A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A + C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{5d(a + a \cos(dx + c))^3} - \frac{2(4A - C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15ad(a + a \cos(dx + c))^2} \\ & - \frac{(13A - C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{6d(a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

```
integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 (29 A - C) \cos(dx + c)^2 + 2 (73 A - 7 C) \cos(dx + c) + 65 A - 5 C \right) \sqrt{\cos(dx + c)} \sin(dx + c) + 5 \left(\sqrt{2} (-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \sec(dx + c)^2 + A) \sqrt{\cos(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x \right)$$

69.184 Problem number 1123

$$\int \frac{A + C \sec^2(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(9A - C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A + C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A + C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d (a + a \cos(dx + c))^3} - \frac{2(3A - 2C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15ad (a + a \cos(dx + c))^2} \\ & + \frac{(9A - C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10d (a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

```
integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 (9 A - C) \cos(dx + c)^2 + 4 (9 A - C) \cos(dx + c) + 15 A + 5 C \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(\sqrt{2} (3i A +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{(C \sec(dx + c)^2 + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c) \sec(dx + c)^3 + 3 a^3 \cos(dx + c) \sec(dx + c)^2 + 3 a^3 \cos(dx + c) \sec(dx + c) + a^3 \cos(dx + c)}, x \right)$$

69.185 Problem number 1124

$$\int \frac{A + C \sec^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A + C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d (a + a \cos(dx + c))^3} + \frac{2(2A - 3C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15ad (a + a \cos(dx + c))^2} \\ & + \frac{(A - 9C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10d (a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(A - 9C) \cos(dx + c)^2 + 2(7A - 33C) \cos(dx + c) + 5A - 45C \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(\sqrt{2} (iA + \dots) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^2 \sec(dx + c)^3 + 3a^3 \cos(dx + c)^2 \sec(dx + c)^2 + 3a^3 \cos(dx + c)^2 \sec(dx + c) + a^3 \cos(dx + c)}\right)$$

69.186 Problem number 1125

$$\int \frac{A + C \sec^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - 49C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A - 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - 49C) \sin(dx + c)}{10a^3 d \sqrt{\cos(dx + c)}} - \frac{(A + C) \sin(dx + c)}{5d (a + a \cos(dx + c))^3 \sqrt{\cos(dx + c)}} \\ & + \frac{2(A - 4C) \sin(dx + c)}{15ad (a + a \cos(dx + c))^2 \sqrt{\cos(dx + c)}} + \frac{(A - 13C) \sin(dx + c)}{6d (a^3 + a^3 \cos(dx + c)) \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(A - 49C) \cos(dx + c)^3 + 4(A - 94C) \cos(dx + c)^2 - 5(A + 59C) \cos(dx + c) - 60C \right) \sqrt{\cos(dx + c)} \sin(dx + c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + A\right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 \sec(dx + c)^3 + 3a^3 \cos(dx + c)^3 \sec(dx + c)^2 + 3a^3 \cos(dx + c)^3 \sec(dx + c) + a^3 \cos(dx + c)}\right)$$

69.187 Problem number 1126

$$\int \frac{A + C \sec^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(9A + 119C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + 11C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} + \frac{(A + 11C) \sin(dx + c)}{2a^3 d \cos(dx + c)^{\frac{3}{2}}} \\ & - \frac{(A + C) \sin(dx + c)}{5d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^3} - \frac{2C \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^2} \\ & - \frac{(9A + 119C) \sin(dx + c)}{30d \cos(dx + c)^{\frac{3}{2}} (a^3 + a^3 \cos(dx + c))} - \frac{(9A + 119C) \sin(dx + c)}{10a^3 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 (9 A + 119 C) \cos(dx + c)^4 + 6 (11 A + 151 C) \cos(dx + c)^3 + 5 (9 A + 139 C) \cos(dx + c)^2 + 120 C \cos(dx + c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^2 + A \right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^4 \sec(dx + c)^3 + 3 a^3 \cos(dx + c)^4 \sec(dx + c)^2 + 3 a^3 \cos(dx + c)^4 \sec(dx + c) + a^3 \cos(dx + c)}, x \right)$$

69.188 Problem number 1173

$$\int \cos^{\frac{9}{2}}(c + dx) (B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} \\ & + \frac{10B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{21 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) d} + \frac{2C \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{5d} \\ & + \frac{2B \left(\cos^{\frac{5}{2}}(dx + c) \right) \sin(dx + c)}{7d} + \frac{10B \sin(dx + c) \left(\sqrt{\cos(dx + c)} \right)}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15 B \cos(dx + c)^2 + 21 C \cos(dx + c) + 25 B \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 25i \sqrt{2} B \text{weierstrassPInverse}(-4, C, B, dx + c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\left(C \cos(dx + c)^4 \sec(dx + c)^2 + B \cos(dx + c)^4 \sec(dx + c) \right) \sqrt{\cos(dx + c)}, x \right)$$

69.189 Problem number 1174

$$\int \cos^{\frac{7}{2}}(c + dx) (B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2C \sin(dx+c) (\sqrt{\cos}(dx+c))}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3B \cos(dx+c) + 5C) \sqrt{\cos(dx+c)} \sin(dx+c) - 5i \sqrt{2} C \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx+c)^3 \sec(dx+c)^2 + B \cos(dx+c)^3 \sec(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

69.190 Problem number 1175

$$\int \cos^{\frac{5}{2}}(c + dx) (B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sin(dx+c) (\sqrt{\cos}(dx+c))}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 B \sqrt{\cos(dx+c)} \sin(dx+c) - i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx+c)^2 \sec(dx+c)^2 + B \cos(dx+c)^2 \sec(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

69.191 Problem number 1176

$$\int \cos^{\frac{3}{2}}(c+dx) (B \sec(c+dx) + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

`integrate(cos(d*x+c)^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} C \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} C \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx+c) \sec(dx+c)^2 + B \cos(dx+c) \sec(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

69.192 Problem number 1177

$$\int \sqrt{\cos(c+dx)} (B \sec(c+dx) + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} B \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \sec(dx+c)^2 + B \sec(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

69.193 Problem number 1178

$$\int \frac{B \sec(c+dx) + C \sec^2(c+dx)}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2B \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} C \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} C \cos(dx + c)^2 \text{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C \sec(dx + c)^2 + B \sec(dx + c)}{\sqrt{\cos(dx + c)}}, x\right)$$

69.194 Problem number 1179

$$\int \frac{B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6C \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2B \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{6C \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \cos(dx + c)^3 \text{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C \sec(dx + c)^2 + B \sec(dx + c)}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

69.195 Problem number 1180

$$\int \cos^{\frac{7}{2}}(c + dx) (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{6B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5A+7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2A \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} \\ & + \frac{2(5A+7C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15 A \cos(dx+c)^2 + 21 B \cos(dx+c) + 25 A + 35 C \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 5 \sqrt{2} (5i A + 7i C) \operatorname{weierstra}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx+c)^3 \sec(dx+c)^2 + B \cos(dx+c)^3 \sec(dx+c) + A \cos(dx+c)^3\right) \sqrt{\cos(dx+c)}, x\right)$$

69.196 Problem number 1181

$$\int \cos^{\frac{5}{2}}(c + dx) (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3A+5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2B \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3A \cos(dx+c) + 5B) \sqrt{\cos(dx+c)} \sin(dx+c) - 5i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx+c)^2 \sec(dx+c)^2 + B \cos(dx+c)^2 \sec(dx+c) + A \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}, x\right)$$

69.197 Problem number 1182

$$\int \cos^{\frac{3}{2}}(c+dx) (A + B \sec(c+dx) + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(A+3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2A \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2A \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2} (-iA - 3iC) \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2}}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C \cos(dx+c) \sec(dx+c)^2 + B \cos(dx+c) \sec(dx+c) + A \cos(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

69.198 Problem number 1183

$$\int \sqrt{\cos(c+dx)} (A + B \sec(c+dx) + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\frac{2(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sin(dx+c)}{d \sqrt{\cos(dx+c)}}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$-i \sqrt{2} B \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + i \sqrt{2} B \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \sec(dx+c)^2 + B \sec(dx+c) + A\right) \sqrt{\cos(dx+c)}, x\right)$$

69.199 Problem number 1184

$$\int \frac{A + B \sec(c+dx) + C \sec^2(c+dx)}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3A+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2B \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-3i A - i C) \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2} (3i A + i C) \cos(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C \sec(dx + c)^2 + B \sec(dx + c) + A}{\sqrt{\cos(dx + c)}}, x\right)$$

69.200 Problem number 1185

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(5A + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2B \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2B \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(5A + 3C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i \sqrt{2} B \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{C \sec(dx + c)^2 + B \sec(dx + c) + A}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

69.201 Problem number 1186

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{6B \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7A + 5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2C \sin(dx+c)}{7d \cos(dx+c)^{\frac{7}{2}}} \\ & + \frac{2B \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2(7A + 5C) \sin(dx+c)}{21d \cos(dx+c)^{\frac{3}{2}}} + \frac{6B \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (7i A + 5i C) \cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5 \sqrt{2} (-7i A - 5i C) \cos(dx+c)^{\frac{3}{2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{C \sec(dx+c)^2 + B \sec(dx+c) + A}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

69.202 Problem number 1187

$$\int \cos^{\frac{9}{2}}(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(7A + 9B + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A + 5B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 9B + 9C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{2a(A + B) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{2aA \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{9d} \\ & + \frac{2a(5A + 5B + 7C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$-15i \sqrt{2} (5A + 5B + 7C) a \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15i \sqrt{2} (5A + 5B + 7C) a$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ca cos(dx + c)^4 sec(dx + c)^3 + (B + C)a cos(dx + c)^4 sec(dx + c)^2 + (A + B)a cos(dx + c)^4 sec(dx + c)`

69.203 Problem number 1188

$$\int \cos^{\frac{7}{2}}(c + dx) (a + a \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(3A + 3B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(5A + 7B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + B) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2aA \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} \\ & + \frac{2a(5A + 7B + 7C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(5A+7B+7C)\text{aweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(5A+7B+7C)\text{aweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ca\cos(dx+c)^3\sec(dx+c)^3+(B+C)a\cos(dx+c)^3\sec(dx+c)^2+(A+B)a\cos(dx+c)^3\sec(dx+c)\right)dx\right)$$

69.204 Problem number 1189

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx)+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(3A+5B+5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2a(A+B+3C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2aA\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{5d} + \frac{2a(A+B)\sin(dx+c)\left(\sqrt{\cos(dx+c)}\right)}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-5i\sqrt{2}(A+B+3C)\text{aweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+B+3C)\text{aweierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Ca\cos(dx+c)^2\sec(dx+c)^3+(B+C)a\cos(dx+c)^2\sec(dx+c)^2+(A+B)a\cos(dx+c)^2\sec(dx+c)\right)dx\right)$$

69.205 Problem number 1190

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(A + B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(A + 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aC \sin(dx + c)}{d \sqrt{\cos(dx + c)}} + \frac{2aA \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} (A + 3B + 3C)a \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} (A + 3B + 3C)a \cos(dx + c)}{3d \sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca \cos(dx + c) \sec(dx + c)^3 + (B + C)a \cos(dx + c) \sec(dx + c)^2 + (A + B)a \cos(dx + c) \sec(dx + c) - \frac{2aC \sin(dx + c)}{d \sqrt{\cos(dx + c)}} - \frac{2aA \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d}\right) dx\right)$$

69.206 Problem number 1191

$$\int \sqrt{\cos(c + dx)} (a + a \sec(c + dx))(A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(A - B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3A + 3B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aC \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a(B + C) \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fri
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2}(3A+3B+C)a\cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + i\sqrt{2}(3A+3B+}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca\sec(dx+c)^3 + (B+C)a\sec(dx+c)^2 + (A+B)a\sec(dx+c) + Aa\right)\sqrt{\cos(dx+c)}, x\right)$$

69.207 Problem number 1192

$$\int \frac{(a + a \sec(c + dx))(A + B \sec(c + dx) + C \sec^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5A + 5B + 3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(3A + B + C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aC \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2a(B+C) \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2a(5A+5B+3C) \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fri
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i\sqrt{2}(3A+B+C)a\cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 5i\sqrt{2}(3A+B+}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca\sec(dx+c)^3 + (B+C)a\sec(dx+c)^2 + (A+B)a\sec(dx+c) + Aa}{\sqrt{\cos(dx+c)}}, x\right)$$

69.208 Problem number 1193

$$\int \frac{(a + a \sec(c + dx)) (A + B \sec(c + dx) + C \sec^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(5A + 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(7A + 7B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} + \frac{2aC \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{2a(B + C) \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2a(7A + 7B + 5C) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} + \frac{2a(5A + 3B + 3C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-5i \sqrt{2} (7A + 7B + 5C)a \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} (7A + 7B + 5C)a \sin(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{5d \sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca \sec(dx + c)^3 + (B + C)a \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

69.209 Problem number 1194

$$\int \cos^{\frac{11}{2}}(c + dx)(a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^2(7A + 8B + 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^2(50A + 55B + 66C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^2(7A + 8B + 9C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\
& + \frac{2a^2(89A + 121B + 99C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{693d} \\
& + \frac{2A \left(\cos^{\frac{5}{2}}(dx + c)\right) (a + a \cos(dx + c))^2 \sin(dx + c)}{11d} \\
& + \frac{2(4A + 11B) \left(\cos^{\frac{5}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{99d} \\
& + \frac{4a^2(50A + 55B + 66C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (50A + 55B + 66C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (50A + 55B + 66C) a^2 \right)}{231d}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Ca^2 cos(dx + c)^5 sec(dx + c)^4 + (B + 2C)a^2 cos(dx + c)^5 sec(dx + c)^3 + (A + 2B + C)a^2 cos(dx + c)^5), dx)
```

69.210 Problem number 1195

$$\int \cos^{\frac{9}{2}}(c + dx) (a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4a^2(8A + 9B + 12C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^2(5A + 6B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a^2(19A + 27B + 21C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\
& + \frac{2A \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + a \cos(dx + c))^2 \sin(dx + c)}{9d} \\
& + \frac{2(4A + 9B) \left(\cos^{\frac{3}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{63d} \\
& + \frac{4a^2(5A + 6B + 7C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (5A + 6B + 7C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (5A + 6B + 7C) \right)}{21d}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Ca^2 cos(dx + c)^4 sec(dx + c)^4 + (B + 2C)a^2 cos(dx + c)^4 sec(dx + c)^3 + (A + 2B + C)a^2 cos(dx + c)^2 sec(dx + c)^2), dx)
```

69.211 Problem number 1196

$$\int \cos^{\frac{7}{2}}(c + dx) (a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(3A + 4B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(6A + 7B + 14C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(33A + 49B + 35C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\ & + \frac{2A(a + a \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d} \\ & + \frac{2(4A + 7B)(a^2 + a^2 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{35d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (6A + 7B + 14C) a^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (6A + 7B + 14C) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ca^2 cos(dx + c)^3 sec(dx + c)^4 + (B + 2C)a^2 cos(dx + c)^3 sec(dx + c)^3 + (A + 2B + C)a^2 cos(dx + c)^2 sec(dx + c)^2), x)`

69.212 Problem number 1197

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(4A + 5B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(A + 2B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} + \frac{2a^2(7A + 5B - 15C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \\ & + \frac{2(A - 5C)(a^2 + a^2 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{5d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (A + 2B + 3C)a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (A + 2B + 3C)a^2 \right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ca^2 cos(dx + c)^2 sec(dx + c)^4 + (B + 2C)a^2 cos(dx + c)^2 sec(dx + c)^3 + (A + 2B + C)a^2 cos(dx + c)^2 sec(dx + c)^2), x)`

69.213 Problem number 1198

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(2A + 3B + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(3B + 4C)(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{3d \sqrt{\cos(dx + c)}} \\ & + \frac{2a^2(A - 3B - 5C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(i \sqrt{2} (2A + 3B + 2C)a^2 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} (2A + 3B + 2C)a^2 \right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ca^2 cos(dx + c) sec(dx + c)^4 + (B + 2C)a^2 cos(dx + c) sec(dx + c)^3 + (A + 2B + C)a^2 cos(dx + c) sec(dx + c)^2), x)`

69.214 Problem number 1199

$$\int \sqrt{\cos(c+dx)} (a+a \sec(c+dx))^2 (A+B \sec(c+dx)+C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(5B+4C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(3A+2B+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(a+a \cos(dx+c))^2 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2(5B+4C)(a^2+a^2 \cos(dx+c)) \sin(dx+c)}{15d \cos(dx+c)^{\frac{3}{2}}} \\ & + \frac{2a^2(15A+25B+17C) \sin(dx+c)}{15d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (3A+2B+C)a^2 \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 5i \sqrt{2} (3A + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^2 \sec(dx+c)^4 + (B+2C)a^2 \sec(dx+c)^3 + (A+2B+C)a^2 \sec(dx+c)^2 + (2A+B)a^2 \sec(dx+c) + \dots\right) \sqrt{\cos(c+dx)} dx\right)$$

69.215 Problem number 1200

$$\int \frac{(a+a \sec(c+dx))^2 (A+B \sec(c+dx)+C \sec^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(5A + 4B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(14A + 7B + 6C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(35A + 49B + 33C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{2(7B + 4C)(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{4a^2(5A + 4B + 3C) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (14A + 7B + 6C) a^2 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (14A + 7B + 6C) a^2 \right)}{105d \cos(dx + c)^{\frac{3}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^2 \sec(dx + c)^4 + (B + 2C)a^2 \sec(dx + c)^3 + (A + 2B + C)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c)}{\sqrt{\cos(dx + c)}}\right)$$

69.216 Problem number 1201

$$\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^2(12A + 9B + 8C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^2(7A + 6B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(21A + 27B + 19C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{4a^2(7A + 6B + 5C) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} + \frac{2C(a + a \cos(dx + c))^2 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(9B + 4C)(a^2 + a^2 \cos(dx + c)) \sin(dx + c)}{63d \cos(dx + c)^{\frac{7}{2}}} + \frac{4a^2(12A + 9B + 8C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(15i \sqrt{2} (7A + 6B + 5C)a^2 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (7$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{Ca^2 \sec(dx + c)^4 + (B + 2C)a^2 \sec(dx + c)^3 + (A + 2B + C)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c)}{\cos(dx + c)^{\frac{3}{2}}} \right)$$

69.217 Problem number 1202

$$\int \cos^{\frac{11}{2}}(c + dx)(a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(15A + 17B + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(105A + 121B + 143C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(210A + 253B + 264C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{1155d} \\ & + \frac{2A \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + a \cos(dx + c))^3 \sin(dx + c)}{11d} \\ & + \frac{2(6A + 11B) \left(\cos^{\frac{3}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{99ad} \\ & + \frac{2(105A + 143B + 99C) \left(\cos^{\frac{3}{2}}(dx + c)\right) (a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{693d} \\ & + \frac{4a^3(105A + 121B + 143C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (105 A + 121 B + 143 C) a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (105 A + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C a^3 \cos(dx + c)^5 \sec(dx + c)^5 + (B + 3C) a^3 \cos(dx + c)^5 \sec(dx + c)^4 + (A + 3B + 3C) a^3 \cos(dx + c)^4 \sec(dx + c)^5\right) dx\right)$$

69.218 Problem number 1203

$$\int \cos^{\frac{9}{2}}(c + dx) (a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(17A + 21B + 27C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(11A + 13B + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(32A + 41B + 42C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\ & + \frac{2A(a + a \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{9d} \\ & + \frac{2(2A + 3B) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{21ad} \\ & + \frac{2(73A + 99B + 63C) (a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{315d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (11 A + 13 B + 21 C) a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} (11 A + 13 B + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(C a^3 \cos(dx + c)^4 \sec(dx + c)^5 + (B + 3C) a^3 \cos(dx + c)^4 \sec(dx + c)^4 + (A + 3B + 3C) a^3 \cos(dx + c)^4 \sec(dx + c)^5\right) dx\right)$$

69.219 Problem number 1204

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx)+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(7A+9B+5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{4a^3(13A+21B+35C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2C(a+a\cos(dx+c))^3\sin(dx+c)}{d\sqrt{\cos(dx+c)}} \\ & + \frac{4a^3(41A+42B-35C)\sin(dx+c)(\sqrt{\cos(dx+c)})}{105d} \\ & + \frac{2(A-7C)(a^2+a^2\cos(dx+c))^2\sin(dx+c)(\sqrt{\cos(dx+c)})}{7ad} \\ & + \frac{2(11A+7B-35C)(a^3+a^3\cos(dx+c))\sin(dx+c)(\sqrt{\cos(dx+c)})}{35d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(5i\sqrt{2}(13A+21B+35C)a^3\cos(dx+c)\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(1\right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ca^3cos(dx+c)^3sec(dx+c)^5+(B+3C)a^3cos(dx+c)^3sec(dx+c)^4+(A+3B+3C)a^3cos(dx+c)`

69.220 Problem number 1205

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx)+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(9A + 5B - 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(3A + 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(B + 2C)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} \\ & + \frac{4a^3(6A - 5B - 20C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \\ & + \frac{2(3A - 15B - 35C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (3A + 5B + 5C) a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (3A + 5B + 5C) a^3 \cos(dx + c) \right)}{15d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^3 \cos(dx + c)^2 \sec(dx + c)^5 + (B + 3C)a^3 \cos(dx + c)^2 \sec(dx + c)^4 + (A + 3B + 3C)a^3 \cos(dx + c)\right) dx\right)$$

69.221 Problem number 1206

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3(5A - 5B - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(5A + 5B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} + \frac{2(5B + 6C)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{15ad \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2(15A + 35B + 33C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \\ & - \frac{4a^3(5A + 20B + 21C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (5A + 5B + 3C)a^3 \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (5A + 5B + 3C)a^3 \cos(dx + c)^3 \right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ca^3 cos(dx + c) sec(dx + c)^5 + (B + 3C)a^3 cos(dx + c) sec(dx + c)^4 + (A + 3B + 3C)a^3 cos(dx + c) sec(dx + c)^3), x)`

69.222 Problem number 1207

$$\int \sqrt{\cos(c + dx)} (a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^3(5A + 9B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(35A + 21B + 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{2(7B + 6C)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{35ad \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{2(5A + 9B + 7C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{4a^3(140A + 147B + 106C) \sin(dx + c)}{105d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5i \sqrt{2} (35A + 21B + 13C)a^3 \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} (35A + 21B + 13C)a^3 \cos(dx + c)^4 \right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Ca^3 sec(dx + c)^5 + (B + 3C)a^3 sec(dx + c)^4 + (A + 3B + 3C)a^3 sec(dx + c)^3 + (3A + 3B + C)a^3 sec(dx + c)^2), x)`

69.223 Problem number 1208

$$\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^3(27A + 21B + 17C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(21A + 13B + 11C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^3(42A + 41B + 32C) \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} + \frac{2C(a + a \cos(dx + c))^3 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\ & + \frac{2(3B + 2C)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{21ad \cos(dx + c)^{\frac{7}{2}}} \\ & + \frac{2(63A + 99B + 73C)(a^3 + a^3 \cos(dx + c)) \sin(dx + c)}{315d \cos(dx + c)^{\frac{5}{2}}} \\ & + \frac{4a^3(27A + 21B + 17C) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15i \sqrt{2} (21A + 13B + 11C) a^3 \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 15i \sqrt{2} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^3 \sec(dx + c)^5 + (B + 3C)a^3 \sec(dx + c)^4 + (A + 3B + 3C)a^3 \sec(dx + c)^3 + (3A + 3B + C)a^3 \sec(dx + c)^2}{\sqrt{\cos(dx + c)}}\right)$$

69.224 Problem number 1209

$$\int \cos^{\frac{13}{2}}(c + dx)(a + a \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^4(185A + 208B + 247C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{195 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^4(100A + 113B + 132C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{4a^4(5255A + 6019B + 6721C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15015d} \\ & + \frac{2a(8A + 13B) \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + a \cos(dx + c))^3 \sin(dx + c)}{143d} \\ & + \frac{2A \left(\cos^{\frac{3}{2}}(dx + c)\right) (a + a \cos(dx + c))^4 \sin(dx + c)}{13d} \\ & + \frac{2(13A + 17B + 11C) \left(\cos^{\frac{3}{2}}(dx + c)\right) (a^2 + a^2 \cos(dx + c))^2 \sin(dx + c)}{99d} \\ & + \frac{4(1355A + 1612B + 1573C) \left(\cos^{\frac{3}{2}}(dx + c)\right) (a^4 + a^4 \cos(dx + c)) \sin(dx + c)}{9009d} \\ & + \frac{8a^4(100A + 113B + 132C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$\frac{2 \left(390i \sqrt{2} (100 A + 113 B + 132 C) a^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 390i \sqrt{2} (100 A \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^4 \cos(dx + c)^6 \sec(dx + c)^6 + (B + 4C)a^4 \cos(dx + c)^6 \sec(dx + c)^5 + (A + 4B + 6C)a^4 \cos(dx + c)^6 \sec(dx + c)^4 + \dots\right), dx\right)$$

69.225 Problem number 1210

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx)+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^4(16A+19B+24C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{8a^4(113A+132B+187C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{231\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{4a^4(667A+803B+913C)\sin(dx+c)(\sqrt{\cos(dx+c)})}{1155d} \\ & + \frac{2a(8A+11B)(a+a\cos(dx+c))^3\sin(dx+c)(\sqrt{\cos(dx+c)})}{99d} \\ & + \frac{2A(a+a\cos(dx+c))^4\sin(dx+c)(\sqrt{\cos(dx+c)})}{11d} \\ & + \frac{2(43A+55B+33C)(a^2+a^2\cos(dx+c))^2\sin(dx+c)(\sqrt{\cos(dx+c)})}{231d} \\ & + \frac{4(769A+946B+891C)(a^4+a^4\cos(dx+c))\sin(dx+c)(\sqrt{\cos(dx+c)})}{3465d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$\frac{2\left(30i\sqrt{2}(113A+132B+187C)a^4\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-30i\sqrt{2}(113A+\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^4\cos(dx+c)^5\sec(dx+c)^6+(B+4C)a^4\cos(dx+c)^5\sec(dx+c)^5+(A+4B+6C)a^4\cos(dx+c)\right)\right)$$

69.226 Problem number 1211

$$\int \cos^{\frac{9}{2}}(c + dx)(a + a \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^4(19A + 24B + 21C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{8a^4(12A + 17B + 28C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2C(a + a \cos(dx + c))^4 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\ & + \frac{4a^4(73A + 83B + 7C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d} \\ & + \frac{2a(A - 9C)(a + a \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{9d} \\ & + \frac{2(5A + 3B - 21C)(a^2 + a^2 \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \\ & + \frac{4(86A + 81B - 126C)(a^4 + a^4 \cos(dx + c)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{315d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(30i \sqrt{2} (12A + 17B + 28C)a^4 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 30i \sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^4 \cos(dx + c)^4 \sec(dx + c)^6 + (B + 4C)a^4 \cos(dx + c)^4 \sec(dx + c)^5 + (A + 4B + 6C)a^4 \cos(dx + c)\right)\right)$$

69.227 Problem number 1212

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx)+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a^4(8A+7B)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{8a^4(17A+28B+35C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & + \frac{2C(a+a\cos(dx+c))^4\sin(dx+c)}{3d\cos(dx+c)^{\frac{3}{2}}} + \frac{2a(3B+8C)(a+a\cos(dx+c))^3\sin(dx+c)}{3d\sqrt{\cos(dx+c)}} \\ & + \frac{4a^4(83A+7B-175C)\sin(dx+c)(\sqrt{\cos(dx+c)})}{105d} \\ & + \frac{2(A-7B-21C)(a^2+a^2\cos(dx+c))^2\sin(dx+c)(\sqrt{\cos(dx+c)})}{7d} \\ & + \frac{4(27A-42B-175C)(a^4+a^4\cos(dx+c))\sin(dx+c)(\sqrt{\cos(dx+c)})}{105d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(10i\sqrt{2}(17A+28B+35C)a^4\cos(dx+c)^2\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}\right)}{105d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^4\cos(dx+c)^3\sec(dx+c)^6+(B+4C)a^4\cos(dx+c)^3\sec(dx+c)^5+(A+4B+6C)a^4\cos(dx+c)^2\sec(dx+c)^4\right)\right)$$

69.228 Problem number 1213

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx)+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{56a^4(A-C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{8a^4(4A+5B+4C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a(5B+8C)(a+a\cos(dx+c))^3 \sin(dx+c)}{15d \cos(dx+c)^{\frac{3}{2}}} + \frac{2C(a+a\cos(dx+c))^4 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} \\
& + \frac{2(5A+15B+19C)(a^2+a^2\cos(dx+c))^2 \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \\
& + \frac{4a^4(A-25B-41C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{15d} \\
& - \frac{4(6A+25B+34C)(a^4+a^4\cos(dx+c)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{15d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10i \sqrt{2} (4A + 5B + 4C) a^4 \cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 10i \sqrt{2} (4 \right)}{15d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^4 \cos(dx+c)^2 \sec(dx+c)^6 + (B+4C)a^4 \cos(dx+c)^2 \sec(dx+c)^5 + (A+4B+6C)a^4 \cos(dx+c)\right) dx\right)$$

69.229 Problem number 1214

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4 (A+B\sec(c+dx)+C\sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{8a^4(7B+8C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{8a^4(35A+28B+17C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a(7B+8C)(a+a\cos(dx+c))^3 \sin(dx+c)}{35d \cos(dx+c)^{\frac{5}{2}}} + \frac{2C(a+a\cos(dx+c))^4 \sin(dx+c)}{7d \cos(dx+c)^{\frac{7}{2}}} \\
& + \frac{2(35A+77B+73C)(a^2+a^2\cos(dx+c))^2 \sin(dx+c)}{105d \cos(dx+c)^{\frac{3}{2}}} \\
& + \frac{4(175A+238B+197C)(a^4+a^4\cos(dx+c)) \sin(dx+c)}{105d \sqrt{\cos(dx+c)}} \\
& - \frac{4a^4(175A+287B+253C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{105d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10i \sqrt{2} (35A + 28B + 17C) a^4 \cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 10i \sqrt{2} \right)}{105d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Ca^4 \cos(dx+c) \sec(dx+c)^6 + (B+4C)a^4 \cos(dx+c) \sec(dx+c)^5 + (A+4B+6C)a^4 \cos(dx+c)\right) dx\right)$$

69.230 Problem number 1215

$$\int \sqrt{\cos(c+dx)} (a+a\sec(c+dx))^4 (A+B\sec(c+dx)+C\sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{8a^4(21A + 24B + 19C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{8a^4(28A + 17B + 12C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a(9B + 8C)(a + a \cos(dx+c))^3 \sin(dx+c)}{63d \cos(dx+c)^{\frac{7}{2}}} + \frac{2C(a + a \cos(dx+c))^4 \sin(dx+c)}{9d \cos(dx+c)^{\frac{9}{2}}} \\
& + \frac{2(63A + 117B + 97C)(a^2 + a^2 \cos(dx+c))^2 \sin(dx+c)}{315d \cos(dx+c)^{\frac{5}{2}}} \\
& + \frac{4(21A + 24B + 19C)(a^4 + a^4 \cos(dx+c)) \sin(dx+c)}{45d \cos(dx+c)^{\frac{3}{2}}} \\
& + \frac{4a^4(287A + 253B + 193C) \sin(dx+c)}{105d \sqrt{\cos(dx+c)}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(30i \sqrt{2} (28A + 17B + 12C) a^4 \cos(dx+c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 30i \sqrt{2} \right)}{105d \sqrt{\cos(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Ca^4 sec(dx+c)^6 + (B+4C)a^4 sec(dx+c)^5 + (A+4B+6C)a^4 sec(dx+c)^4 + 2(2A+3B+2C)a^4
```

69.231 Problem number 1216

$$\int \frac{(a + a \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{8a^4(24A + 19B + 16C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{8a^4(187A + 132B + 113C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{4a^4(913A + 803B + 667C) \sin(dx+c)}{1155d \cos(dx+c)^{\frac{3}{2}}} \\
& + \frac{2a(11B + 8C)(a + a \cos(dx+c))^3 \sin(dx+c)}{99d \cos(dx+c)^{\frac{9}{2}}} + \frac{2C(a + a \cos(dx+c))^4 \sin(dx+c)}{11d \cos(dx+c)^{\frac{11}{2}}} \\
& + \frac{2(33A + 55B + 43C)(a^2 + a^2 \cos(dx+c))^2 \sin(dx+c)}{231d \cos(dx+c)^{\frac{7}{2}}} \\
& + \frac{4(891A + 946B + 769C)(a^4 + a^4 \cos(dx+c)) \sin(dx+c)}{3465d \cos(dx+c)^{\frac{5}{2}}} \\
& + \frac{8a^4(24A + 19B + 16C) \sin(dx+c)}{15d \sqrt{\cos(dx+c)}}
\end{aligned}$$

command

`integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="f`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(30i \sqrt{2} (187A + 132B + 113C) a^4 \cos(dx+c)^6 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 30 \right)}{15d \sqrt{\cos(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Ca^4 \sec(dx+c)^6 + (B+4C)a^4 \sec(dx+c)^5 + (A+4B+6C)a^4 \sec(dx+c)^4 + 2(2A+3B+2C)a^4}{\sqrt{\cos(dx+c)}}\right)$$

69.232 Problem number 1217

$$\int \frac{\cos^{\frac{7}{2}}(c+dx) (A + B \sec(c+dx) + C \sec^2(c+dx))}{a + a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{3(7A - 7B + 5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\
 & + \frac{5(9A - 7B + 7C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\
 & - \frac{(7A - 7B + 5C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5ad} \\
 & + \frac{(9A - 7B + 7C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7ad} \\
 & - \frac{(A - B + C) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{d(a + a \cos(dx+c))} \\
 & + \frac{5(9A - 7B + 7C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21ad}
 \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(30 A \cos(dx+c)^3 - 6(2A - 7B) \cos(dx+c)^2 + 2(39A - 14B + 35C) \cos(dx+c) + 225A - 175B + 175C \right)}{a \sec(dx+c) + a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^3 \sec(dx+c)^2 + B \cos(dx+c)^3 \sec(dx+c) + A \cos(dx+c)^3\right) \sqrt{\cos(dx+c)}}{a \sec(dx+c) + a}, x\right)$$

69.233 Problem number 1218

$$\int \frac{\cos^{\frac{5}{2}}(c+dx) (A + B \sec(c+dx) + C \sec^2(c+dx))}{a + a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(7A - 5B + 5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(5A - 5B + 3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(7A - 5B + 5C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5ad} - \frac{(A - B + C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{d(a + a \cos(dx+c))} \\ & - \frac{(5A - 5B + 3C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3ad} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 A \cos(dx+c)^2 - 2(2A - 5B) \cos(dx+c) - 25A + 25B - 15C \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 5 \left(\sqrt{2} (-5i A \right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^2 \sec(dx+c)^2 + B \cos(dx+c)^2 \sec(dx+c) + A \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}}{a \sec(dx+c) + a}, x\right)$$

69.234 Problem number 1219

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A + B \sec(c+dx) + C \sec^2(c+dx))}{a + a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A - 3B + C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(5A - 3B + 3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & - \frac{(A - B + C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{d(a + a \cos(dx+c))} + \frac{(5A - 3B + 3C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3ad} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(2A \cos(dx+c) + 5A - 3B + 3C) \sqrt{\cos(dx+c)} \sin(dx+c) + \left(\sqrt{2}(-5iA + 3iB - 3iC) \cos(dx+c) + \sqrt{2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx+c) \sec(dx+c)^2 + B \cos(dx+c) \sec(dx+c) + A \cos(dx+c) \right) \sqrt{\cos(dx+c)}}{a \sec(dx+c) + a}, x \right)$$

69.235 Problem number 1220

$$\int \frac{\sqrt{\cos(c+dx)} (A + B \sec(c+dx) + C \sec^2(c+dx))}{a + a \sec(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A - B + C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{(A - B - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{(A - B + C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{d(a + a \cos(dx+c))} \end{aligned}$$

command

`integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(A - B + C) \sqrt{\cos(dx+c)} \sin(dx+c) - \left(\sqrt{2}(iA - iB - iC) \cos(dx+c) + \sqrt{2}(iA - iB - iC)\right) \text{weierstr}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx+c)^2 + B \sec(dx+c) + A \right) \sqrt{\cos(dx+c)}}{a \sec(dx+c) + a}, x \right)$$

69.236 Problem number 1221

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A + B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(A - B + 3C) \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} - \frac{(A - B + C) \sin(dx + c)}{d(a + a \cos(dx + c)) \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2((A - B + 3C) \cos(dx + c) + 2C) \sqrt{\cos(dx + c)} \sin(dx + c) + (\sqrt{2}(-iA - iB + iC) \cos(dx + c)^2 + \sqrt{2}(-i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a \cos(dx + c) \sec(dx + c) + a \cos(dx + c)}, x\right)$$

69.237 Problem number 1222

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - 3B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(3A - 3B + 5C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) ad} \\ & + \frac{(3A - 3B + 5C) \sin(dx + c)}{3ad \cos^{\frac{3}{2}}(dx + c)} - \frac{(A - B + C) \sin(dx + c)}{d \cos^{\frac{3}{2}}(dx + c) (a + a \cos(dx + c))} \\ & - \frac{(A - 3B + 3C) \sin(dx + c)}{ad \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$\frac{2 \left(3(A - 3B + 3C) \cos(dx + c)^2 - 2(3B - 2C) \cos(dx + c) - 2C \right) \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2}(-3iA + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A \right) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^2 \sec(dx + c) + a \cos(dx + c)^2}, x \right)$$

69.238 Problem number 1223

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3(5A - 5B + 7C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{5 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & - \frac{(3A - 5B + 5C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right)}{3 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) ad} \\ & + \frac{(5A - 5B + 7C) \sin(dx + c)}{5ad \cos(dx + c)^{\frac{5}{2}}} - \frac{(3A - 5B + 5C) \sin(dx + c)}{3ad \cos(dx + c)^{\frac{3}{2}}} \\ & - \frac{(A - B + C) \sin(dx + c)}{d \cos(dx + c)^{\frac{5}{2}} (a + a \cos(dx + c))} + \frac{3(5A - 5B + 7C) \sin(dx + c)}{5ad \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$\frac{2 \left(9(5A - 5B + 7C) \cos(dx + c)^3 + 2(15A - 10B + 19C) \cos(dx + c)^2 + 2(5B - 2C) \cos(dx + c) + 6C \right) \sqrt{\cos(dx + c)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A \right) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^3 \sec(dx + c) + a \cos(dx + c)^3}, x \right)$$

69.239 Problem number 1224

$$\int \frac{\cos^{\frac{7}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{7(11A - 8B + 5C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{5(30A - 21B + 14C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{7(11A - 8B + 5C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15a^2 d} \\ & + \frac{(30A - 21B + 14C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7a^2 d} \\ & - \frac{(11A - 8B + 5C) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{3a^2 d (1 + \cos(dx+c))} \\ & - \frac{(A - B + C) \left(\cos^{\frac{9}{2}}(dx+c)\right) \sin(dx+c)}{3d (a + a \cos(dx+c))^2} \\ & + \frac{5(30A - 21B + 14C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21a^2 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(30 A \cos(dx+c)^4 - 6(4A - 7B) \cos(dx+c)^3 + 2(61A - 28B + 35C) \cos(dx+c)^2 + (961A - 658B + 455C) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^3 \sec(dx+c)^2 + B \cos(dx+c)^3 \sec(dx+c) + A \cos(dx+c)^3\right) \sqrt{\cos(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

69.240 Problem number 1225

$$\int \frac{\cos^{\frac{5}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(56A - 35B + 20C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{5(3A - 2B + C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(56A - 35B + 20C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15a^2 d} \\ & - \frac{(3A - 2B + C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{a^2 d (1 + \cos(dx+c))} - \frac{(A - B + C) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{3d (a + a \cos(dx+c))^2} \\ & - \frac{5(3A - 2B + C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a^2 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(6A \cos(dx+c)^3 - 2(4A - 5B) \cos(dx+c)^2 - (94A - 65B + 30C) \cos(dx+c) - 75A + 50B - 25C \right) \sqrt{\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^2 \sec(dx+c)^2 + B \cos(dx+c)^2 \sec(dx+c) + A \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

69.241 Problem number 1226

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(7A-4B+C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(10A-5B+2C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & - \frac{(7A-4B+C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{3a^2 d (1+\cos(dx+c))} - \frac{(A-B+C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{3d (a+a \cos(dx+c))^2} \\ & + \frac{(10A-5B+2C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a^2 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="f`

`Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output`

$$2 \left(2A \cos(dx+c)^2 + (13A-6B+3C) \cos(dx+c) + 10A-5B+2C \right) \sqrt{\cos(dx+c)} \sin(dx+c) + \left(\sqrt{2} (-1 \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c) \sec(dx+c)^2 + B \cos(dx+c) \sec(dx+c) + A \cos(dx+c)\right) \sqrt{\cos(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

69.242 Problem number 1227

$$\int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx) + C \sec^2(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal antiderivative

$$\frac{(4A - B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d}$$

$$- \frac{(5A - 2B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d}$$

$$- \frac{(A - B + C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3d(a + a \cos(dx + c))^2} - \frac{(5A - 2B - C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{3a^2 d (1 + \cos(dx + c))}$$

command

`integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3(2A - B) \cos(dx + c) + 5A - 2B - C) \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2}(5iA - 2iB - iC) \cos(dx + c)\right)^2}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A\right) \sqrt{\cos(dx + c)}}{a^2 \sec(dx + c)^2 + 2a^2 \sec(dx + c) + a^2}, x\right)$$

69.243 Problem number 1228

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d}$$

$$+ \frac{(2A + B + 2C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d}$$

$$+ \frac{(A - C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{a^2 d (1 + \cos(dx + c))} - \frac{(A - B + C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{3d(a + a \cos(dx + c))^2}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(3(A-C)\cos(dx+c)+2A+B-4C)\sqrt{\cos(dx+c)}\sin(dx+c)+\left(\sqrt{2}(-2iA-iB-2iC)\cos(dx+c)\right)^2-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\sec(dx+c)^2+B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)\sec(dx+c)^2+2a^2\cos(dx+c)\sec(dx+c)+a^2\cos(dx+c)},x\right)$$

69.244 Problem number 1229

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(B-4C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d} \\ & + \frac{(A+2B-5C)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d} - \frac{(B-4C)\sin(dx+c)}{a^2d\sqrt{\cos(dx+c)}} \\ & + \frac{(A+2B-5C)\sin(dx+c)}{3a^2d(1+\cos(dx+c))\sqrt{\cos(dx+c)}} - \frac{(A-B+C)\sin(dx+c)}{3d(a+a\cos(dx+c))^2\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(3(B-4C)\cos(dx+c)^2-(A-4B+19C)\cos(dx+c)-6C\right)\sqrt{\cos(dx+c)}\sin(dx+c)-\left(\sqrt{2}(-iA-2iB-2iC)\cos(dx+c)\right)^2-$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C\sec(dx+c)^2+B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^2\sec(dx+c)^2+2a^2\cos(dx+c)^2\sec(dx+c)+a^2\cos(dx+c)^2},x\right)$$

69.245 Problem number 1230

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - 4B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(2A - 5B + 10C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\ & + \frac{(2A - 5B + 10C) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}}} - \frac{(A - 4B + 7C) \sin(dx + c)}{3a^2 d \cos(dx + c)^{\frac{3}{2}} (1 + \cos(dx + c))} \\ & - \frac{(A - B + C) \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}} (a + a \cos(dx + c))^2} - \frac{(A - 4B + 7C) \sin(dx + c)}{a^2 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(A - 4B + 7C) \cos(dx + c)^3 + (4A - 19B + 32C) \cos(dx + c)^2 - 2(3B - 4C) \cos(dx + c) - 2C \right) \sqrt{\cos(dx + c)}}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A\right) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^3 \sec(dx + c)^2 + 2a^2 \cos(dx + c)^3 \sec(dx + c) + a^2 \cos(dx + c)^3}, x\right)$$

69.246 Problem number 1231

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{(20A - 35B + 56C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\
& - \frac{5(A - 2B + 3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d} \\
& + \frac{(20A - 35B + 56C) \sin(dx+c)}{15a^2 d \cos(dx+c)^{\frac{5}{2}}} - \frac{5(A - 2B + 3C) \sin(dx+c)}{3a^2 d \cos(dx+c)^{\frac{3}{2}}} \\
& - \frac{(A - 2B + 3C) \sin(dx+c)}{a^2 d \cos(dx+c)^{\frac{5}{2}} (1 + \cos(dx+c))} \\
& - \frac{(A - B + C) \sin(dx+c)}{3d \cos(dx+c)^{\frac{5}{2}} (a + a \cos(dx+c))^2} + \frac{(20A - 35B + 56C) \sin(dx+c)}{5a^2 d \sqrt{\cos(dx+c)}}
\end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(20A - 35B + 56C) \cos(dx+c)^4 + (95A - 160B + 261C) \cos(dx+c)^3 + 2(15A - 20B + 37C) \cos(dx+c) \right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx+c)^2 + B \sec(dx+c) + A\right) \sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^4 \sec(dx+c)^2 + 2a^2 \cos(dx+c)^4 \sec(dx+c) + a^2 \cos(dx+c)^4}, x\right)$$

69.247 Problem number 1232

$$\int \frac{\cos^{\frac{5}{2}}(c+dx) (A + B \sec(c+dx) + C \sec^2(c+dx))}{(a + a \sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{7(33A - 17B + 7C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
& - \frac{(63A - 33B + 13C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\
& + \frac{7(33A - 17B + 7C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{30a^3 d} \\
& - \frac{(A - B + C) \left(\cos^{\frac{9}{2}}(dx + c)\right) \sin(dx + c)}{5d (a + a \cos(dx + c))^3} \\
& - \frac{(12A - 7B + 2C) \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{15ad (a + a \cos(dx + c))^2} \\
& - \frac{(63A - 33B + 13C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{10d (a^3 + a^3 \cos(dx + c))} \\
& - \frac{(63A - 33B + 13C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{6a^3 d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(12 A \cos(dx + c)^4 - 4(6 A - 5 B) \cos(dx + c)^3 - 3(147 A - 79 B + 29 C) \cos(dx + c)^2 - 2(357 A - 188 B + 73 C) \cos(dx + c) - 2 C \right) \sqrt{\cos(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 \sec(dx + c)^2 + B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2\right) \sqrt{\cos(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x\right)$$

69.248 Problem number 1233

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(119A - 49B + 9C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(33A - 13B + 3C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - B + C) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{5d (a + a \cos(dx+c))^3} - \frac{(2A - B) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{3ad (a + a \cos(dx+c))^2} \\ & - \frac{(119A - 49B + 9C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{30d (a^3 + a^3 \cos(dx+c))} \\ & + \frac{(33A - 13B + 3C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{6a^3 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(20 A \cos(dx+c)^3 + 3 (79 A - 29 B + 9 C) \cos(dx+c)^2 + 2 (188 A - 73 B + 18 C) \cos(dx+c) + 165 A - 65 B \right)}{6 a^3 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c) \sec(dx+c)^2 + B \cos(dx+c) \sec(dx+c) + A \cos(dx+c)\right) \sqrt{\cos(dx+c)}}{a^3 \sec(dx+c)^3 + 3 a^3 \sec(dx+c)^2 + 3 a^3 \sec(dx+c) + a^3}, x\right)$$

69.249 Problem number 1234

$$\int \frac{\sqrt{\cos(c+dx)} (A + B \sec(c+dx) + C \sec^2(c+dx))}{(a + a \sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(49A - 9B - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(13A - 3B - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - B + C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{5d(a + a \cos(dx+c))^3} - \frac{(8A - 3B - 2C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{15ad(a + a \cos(dx+c))^2} \\ & - \frac{(13A - 3B - C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{6d(a^3 + a^3 \cos(dx+c))} \end{aligned}$$

command

`integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(29A - 9B - C) \cos(dx+c)^2 + 2(73A - 18B - 7C) \cos(dx+c) + 65A - 15B - 5C \right) \sqrt{\cos(dx+c)} \sin(dx+c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx+c)^2 + B \sec(dx+c) + A\right) \sqrt{\cos(dx+c)}}{a^3 \sec(dx+c)^3 + 3a^3 \sec(dx+c)^2 + 3a^3 \sec(dx+c) + a^3}, x\right)$$

69.250 Problem number 1235

$$\int \frac{A + B \sec(c+dx) + C \sec^2(c+dx)}{\sqrt{\cos(c+dx)} (a + a \sec(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(9A + B - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A + B + C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - B + C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d(a + a \cos(dx + c))^3} - \frac{(6A - B - 4C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{15ad(a + a \cos(dx + c))^2} \\ & + \frac{(9A + B - C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{10d(a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(9A + B - C) \cos(dx + c)^2 + 2(18A + 7B - 2C) \cos(dx + c) + 15A + 5B + 5C \right) \sqrt{\cos(dx + c)} \sin(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A\right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c) \sec(dx + c)^3 + 3a^3 \cos(dx + c) \sec(dx + c)^2 + 3a^3 \cos(dx + c) \sec(dx + c) + a^3 \cos(dx + c)}\right)$$

69.251 Problem number 1236

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B - 9C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A - B + C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{5d(a + a \cos(dx + c))^3} + \frac{(4A + B - 6C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{15ad(a + a \cos(dx + c))^2} \\ & + \frac{(A - B - 9C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{10d(a^3 + a^3 \cos(dx + c))} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(A - B - 9C) \cos(dx + c)^2 + 2(7A - 2B - 33C) \cos(dx + c) + 5A + 5B - 45C \right) \sqrt{\cos(dx + c)} \sin(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A \right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^2 \sec(dx + c)^3 + 3a^3 \cos(dx + c)^2 \sec(dx + c)^2 + 3a^3 \cos(dx + c)^2 \sec(dx + c) + a^3 \cos(dx + c)} \right)$$

69.252 Problem number 1237

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + 9B - 49C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(A + 3B - 13C) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & - \frac{(A + 9B - 49C) \sin(dx + c)}{10a^3 d \sqrt{\cos(dx + c)}} - \frac{(A - B + C) \sin(dx + c)}{5d(a + a \cos(dx + c))^3 \sqrt{\cos(dx + c)}} \\ & + \frac{(2A + 3B - 8C) \sin(dx + c)}{15ad(a + a \cos(dx + c))^2 \sqrt{\cos(dx + c)}} + \frac{(A + 3B - 13C) \sin(dx + c)}{6d(a^3 + a^3 \cos(dx + c)) \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3(A + 9B - 49C) \cos(dx + c)^3 + 2(2A + 33B - 188C) \cos(dx + c)^2 - 5(A - 9B + 59C) \cos(dx + c) - 6 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A \right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 \sec(dx + c)^3 + 3a^3 \cos(dx + c)^3 \sec(dx + c)^2 + 3a^3 \cos(dx + c)^3 \sec(dx + c) + a^3 \cos(dx + c)} \right)$$

69.253 Problem number 1238

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{7/2}(c + dx)(a + a \sec(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(9A - 49B + 119C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A - 13B + 33C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d} \\ & + \frac{(3A - 13B + 33C) \sin(dx + c)}{6a^3 d \cos(dx + c)^{3/2}} - \frac{(A - B + C) \sin(dx + c)}{5d \cos(dx + c)^{3/2} (a + a \cos(dx + c))^3} \\ & + \frac{(B - 2C) \sin(dx + c)}{3ad \cos(dx + c)^{3/2} (a + a \cos(dx + c))^2} \\ & - \frac{(9A - 49B + 119C) \sin(dx + c)}{30d \cos(dx + c)^{3/2} (a^3 + a^3 \cos(dx + c))} - \frac{(9A - 49B + 119C) \sin(dx + c)}{10a^3 d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3(9A - 49B + 119C) \cos(dx + c)^4 + 2(33A - 188B + 453C) \cos(dx + c)^3 + 5(9A - 59B + 139C) \cos(dx + c)^2 \right)}{10a^3 d \sqrt{\cos(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A\right) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^4 \sec(dx + c)^3 + 3a^3 \cos(dx + c)^4 \sec(dx + c)^2 + 3a^3 \cos(dx + c)^4 \sec(dx + c) + a^3 \cos(dx + c)^4}, dx\right)$$

69.254 Problem number 1239

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{(a+a \sec(c+dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(176A - 57B + 8C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & + \frac{(339A - 108B + 17C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{42 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & - \frac{(43A - 15B + C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{42a^4 d (1 + \cos(dx+c))^2} \\ & - \frac{(176A - 57B + 8C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{30a^4 d (1 + \cos(dx+c))} \\ & - \frac{(A - B + C) \left(\cos^{\frac{9}{2}}(dx+c)\right) \sin(dx+c)}{7d (a + a \cos(dx+c))^4} \\ & - \frac{(13A - 6B - C) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{35ad (a + a \cos(dx+c))^3} \\ & + \frac{(339A - 108B + 17C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{42a^4 d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(140 A \cos(dx+c)^4 + 7(368 A - 111 B + 24 C) \cos(dx+c)^3 + (6259 A - 1968 B + 337 C) \cos(dx+c)^2 + (5548 \right)}{a^4 \sec(dx+c)^4 + 4 a^4 \sec(dx+c)^3 + 6 a^4 \sec(dx+c)^2 + 4 a^4 \sec(dx+c) + a^4}, x$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c) \sec(dx+c)^2 + B \cos(dx+c) \sec(dx+c) + A \cos(dx+c)\right) \sqrt{\cos(dx+c)}}{a^4 \sec(dx+c)^4 + 4 a^4 \sec(dx+c)^3 + 6 a^4 \sec(dx+c)^2 + 4 a^4 \sec(dx+c) + a^4}, x\right)$$

69.255 Problem number 1240

$$\int \frac{\sqrt{\cos(c+dx)} (A + B \sec(c+dx) + C \sec^2(c+dx))}{(a + a \sec(c+dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(57A - 8B - C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & - \frac{(108A - 17B - 4C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{42 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & - \frac{(141A - 29B - 13C) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{210 a^4 d (1 + \cos(dx+c))^2} \\ & - \frac{(A - B + C) \left(\cos^{\frac{7}{2}}(dx+c)\right) \sin(dx+c)}{7d (a + a \cos(dx+c))^4} \\ & - \frac{(11A - 4B - 3C) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{35ad (a + a \cos(dx+c))^3} \\ & - \frac{(108A - 17B - 4C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{42a^4 d (1 + \cos(dx+c))} \end{aligned}$$

command

`integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21 (37A - 8B - C) \cos(dx+c)^3 + (1968A - 337B - 104C) \cos(dx+c)^2 + (1761A - 284B - 73C) \cos(dx+c) + a^4 \right)}{42a^4 d (1 + \cos(dx+c))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx+c)^2 + B \sec(dx+c) + A) \sqrt{\cos(dx+c)}}{a^4 \sec(dx+c)^4 + 4a^4 \sec(dx+c)^3 + 6a^4 \sec(dx+c)^2 + 4a^4 \sec(dx+c) + a^4}, x\right)$$

69.256 Problem number 1241

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(8A + B) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & + \frac{(17A + 4B + 3C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{42 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & - \frac{(A - B + C) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d (a + a \cos(dx + c))^4} \\ & - \frac{(9A - 2B - 5C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{35ad (a + a \cos(dx + c))^3} \\ & - \frac{(83A + B - 15C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{210a^4 d (1 + \cos(dx + c))^2} + \frac{(8A + B) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10a^4 d (1 + \cos(dx + c))} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^4,x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21 (8A + B) \cos(dx + c)^3 + (337A + 104B + 15C) \cos(dx + c)^2 + (284A + 73B + 60C) \cos(dx + c) + 85A \right)}{10a^4 d (1 + \cos(dx + c))^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A\right) \sqrt{\cos(dx + c)}}{a^4 \cos(dx + c) \sec(dx + c)^4 + 4a^4 \cos(dx + c) \sec(dx + c)^3 + 6a^4 \cos(dx + c) \sec(dx + c)^2 + 4a^4 \cos(dx + c)}\right)$$

69.257 Problem number 1242

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & + \frac{(4A + 3B + 4C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{42 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & - \frac{(A - B + C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{7d (a + a \cos(dx + c))^4} \\ & + \frac{(41A + 15B - C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{210a^4 d (1 + \cos(dx + c))^2} \\ & + \frac{(A - C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{10a^4 d (1 + \cos(dx + c))} - \frac{(A - C) \sin(dx + c) (\sqrt{\cos}(dx + c))}{5ad (a + a \cos(dx + c))^3} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^4,x, algorithm='f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(21 (A - C) \cos(dx + c)^3 + (104A + 15B - 64C) \cos(dx + c)^2 + (73A + 60B - 53C) \cos(dx + c) + 20A + 15C \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \sec(dx + c)^2 + B \sec(dx + c) + A\right) \sqrt{\cos(dx + c)}}{a^4 \cos(dx + c)^2 \sec(dx + c)^4 + 4a^4 \cos(dx + c)^2 \sec(dx + c)^3 + 6a^4 \cos(dx + c)^2 \sec(dx + c)^2 + 4a^4 \cos(dx + c)^2}\right)$$

69.258 Problem number 1243

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(B + 8C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & + \frac{(3A + 4B + 17C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{42 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & + \frac{(15A - B - 83C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{210 a^4 d (1 + \cos(dx + c))^2} \\ & - \frac{(B + 8C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{10 a^4 d (1 + \cos(dx + c))} - \frac{(A - B + C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d (a + a \cos(dx + c))^4} \\ & + \frac{(5A + 2B - 9C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{35ad (a + a \cos(dx + c))^3} \end{aligned}$$

command

```
integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^4,x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21 (B + 8C) \cos(dx + c)^3 - (15A - 64B - 587C) \cos(dx + c)^2 - (60A - 53B - 724C) \cos(dx + c) - 15A \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^4 \cos(dx + c)^3 \sec(dx + c)^4 + 4a^4 \cos(dx + c)^3 \sec(dx + c)^3 + 6a^4 \cos(dx + c)^3 \sec(dx + c)^2 + 4a^4 \cos(dx + c)^2 \sec(dx + c) + 4a^4 \cos(dx + c)}\right)$$

69.259 Problem number 1244

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\cos^{7/2}(c + dx)(a + a \sec(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + 8B - 57C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & + \frac{(4A + 17B - 108C) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{42 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d} \\ & - \frac{(A + 8B - 57C) \sin(dx + c)}{10a^4 d \sqrt{\cos(dx + c)}} + \frac{(13A + 29B - 141C) \sin(dx + c)}{210a^4 d (1 + \cos(dx + c))^2 \sqrt{\cos(dx + c)}} \\ & + \frac{(4A + 17B - 108C) \sin(dx + c)}{42a^4 d (1 + \cos(dx + c)) \sqrt{\cos(dx + c)}} - \frac{(A - B + C) \sin(dx + c)}{7d (a + a \cos(dx + c))^4 \sqrt{\cos(dx + c)}} \\ & + \frac{(3A + 4B - 11C) \sin(dx + c)}{35ad (a + a \cos(dx + c))^3 \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^4,x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(21 (A + 8B - 57C) \cos(dx + c)^4 + (64A + 587B - 4248C) \cos(dx + c)^3 + (53A + 724B - 5421C) \cos(dx + c)^2 + 4a^4 \cos(dx + c) \right)}{a^4 \cos(dx + c)^4 \sec(dx + c)^4 + 4a^4 \cos(dx + c)^4 \sec(dx + c)^3 + 6a^4 \cos(dx + c)^4 \sec(dx + c)^2 + 4a^4 \cos(dx + c)^4 \sec(dx + c) + 4a^4 \cos(dx + c)^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^4 \cos(dx + c)^4 \sec(dx + c)^4 + 4a^4 \cos(dx + c)^4 \sec(dx + c)^3 + 6a^4 \cos(dx + c)^4 \sec(dx + c)^2 + 4a^4 \cos(dx + c)^4 \sec(dx + c) + 4a^4 \cos(dx + c)^4}, dx\right)$$

69.260 Problem number 1292

$$\int \cos^{\frac{9}{2}}(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7aA + 9bB + 9aC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5Ab + 5Ba + 7Cb) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(7aA + 9bB + 9aC) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{45d} \\ & + \frac{2(Ab + Ba) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{7d} + \frac{2aA \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c)}{9d} \\ & + \frac{2(5Ab + 5Ba + 7Cb) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(35 Aa \cos(dx + c)^3 + 45 (Ba + Ab) \cos(dx + c)^2 + 75 Ba + 15 (5 A + 7 C)b + 7 ((7 A + 9 C)a + 9 Bb) \cos(dx + c) \right)}{21d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \cos(dx + c)^4 \sec(dx + c)^3 + (Ca + Bb) \cos(dx + c)^4 \sec(dx + c)^2 + Aa \cos(dx + c)^4 + (Ba + Ab) \cos(dx + c)^3\right), dx\right)$$

69.261 Problem number 1293

$$\int \cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3Ab + 3Ba + 5Cb) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5aA + 7bB + 7aC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ab + Ba) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2aA \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{7d} \\ & + \frac{2(5aA + 7bB + 7aC) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15 A a \cos(dx+c)^2 + 5 (5 A + 7 C) a + 35 B b + 21 (B a + A b) \cos(dx+c) \right) \sqrt{\cos(dx+c)} \sin(dx+c) - 5 \sqrt{2} (aA + bB + cC) \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) + 21 (aA + bB + cC) \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 d}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Cb*cos(dx+c)^3*sec(dx+c)^3 + (Ca+Bb)*cos(dx+c)^3*sec(dx+c)^2 + Aa*cos(dx+c)^3 + (Ba+Ab)*cos(dx+c)^2*sec(dx+c)), dx)
```

69.262 Problem number 1294

$$\int \cos^{\frac{5}{2}}(c+dx) (a+b \sec(c+dx)) (A+B \sec(c+dx) + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3aA + 5bB + 5aC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(Ab + Ba + 3Cb) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2aA \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{5d} + \frac{2(Ab + Ba) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(3Aa \cos(dx+c) + 5Ba + 5Ab) \sqrt{\cos(dx+c)} \sin(dx+c) - 5\sqrt{2}(iBa + i(A+3C)b) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))}{\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb cos(dx+c)^2 sec(dx+c)^3 + (Ca+Bb) cos(dx+c)^2 sec(dx+c)^2 + Aa cos(dx+c)^2 + (Ba+Ab) cos(dx+c)), x)`

69.263 Problem number 1295

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))(A+B \sec(c+dx)+C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab+Ba-Cb) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3bB+a(A+3C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2bC \sin(dx+c)}{d \sqrt{\cos(dx+c)}} + \frac{2aA \sin(dx+c) (\sqrt{\cos(dx+c)})}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2}(-i(A+3C)a - 3iBb) \cos(dx+c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2}(i(A+3C)a + 3iBb) \sin(dx+c)}{\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb cos(dx+c) sec(dx+c)^3 + (Ca+Bb) cos(dx+c) sec(dx+c)^2 + Aa cos(dx+c) + (Ba+Ab) cos(dx+c)), x)`

69.264 Problem number 1296

$$\int \sqrt{\cos(c+dx)} (a+b\sec(c+dx)) (A+B\sec(c+dx)+C\sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(bB - a(A - C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3Ab + 3Ba + Cb) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2bC \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2(bB + aC) \sin(dx+c)}{d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-3iBa - i(3A + C)b) \cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + \sqrt{2} (3iBa + i(3A + C)b) \cos(dx+c)}{d \sqrt{\cos(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb \sec(dx+c)^3 + (Ca + Bb) \sec(dx+c)^2 + Aa + (Ba + Ab) \sec(dx+c)\right) \sqrt{\cos(dx+c)}, x\right)$$

69.265 Problem number 1297

$$\int \frac{(a+b\sec(c+dx)) (A+B\sec(c+dx)+C\sec^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5Ab + 5Ba + 3Cb) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(bB + a(3A + C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2bC \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} + \frac{2(bB + aC) \sin(dx+c)}{3d \cos(dx+c)^{\frac{3}{2}}} + \frac{2(5Ab + 5Ba + 3Cb) \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(i(3A+C)a+iBb)\cos(dx+c)^3 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-i(3A$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb\sec(dx+c)^3+(Ca+Bb)\sec(dx+c)^2+Ca+(Ba+Ab)\sec(dx+c)}{\sqrt{\cos(dx+c)}},x\right)$$

69.266 Problem number 1298

$$\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx)+C\sec^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(5aA+3bB+3aC)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticE}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2(7Ab+7Ba+5Cb)\sqrt{\frac{\cos(dx+c)}{2}+\frac{1}{2}}\text{EllipticF}\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{21\cos\left(\frac{dx}{2}+\frac{c}{2}\right)d} \\ & +\frac{2bC\sin(dx+c)}{7d\cos(dx+c)^{\frac{7}{2}}}+\frac{2(bB+aC)\sin(dx+c)}{5d\cos(dx+c)^{\frac{5}{2}}} \\ & +\frac{2(7Ab+7Ba+5Cb)\sin(dx+c)}{21d\cos(dx+c)^{\frac{3}{2}}}+\frac{2(5aA+3bB+3aC)\sin(dx+c)}{5d\sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fri`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(7iBa+i(7A+5C)b)\cos(dx+c)^4 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-7i$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{Cb\sec(dx+c)^3+(Ca+Bb)\sec(dx+c)^2+Ca+(Ba+Ab)\sec(dx+c)}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

69.267 Problem number 1299

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx)+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(18abB + 3b^2(3A + 5C) + a^2(7A + 9C)) \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(10Aab + 5B a^2 + 7b^2B + 14abC) \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(4A b^2 + 18abB + a^2(7A + 9C)) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{45d} \\ & + \frac{2a(4Ab + 9Ba) \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c)}{63d} \\ & + \frac{2A \left(\cos^{\frac{3}{2}}(dx+c)\right) (b+a\cos(dx+c))^2 \sin(dx+c)}{9d} \\ & + \frac{2(10Aab + 5B a^2 + 7b^2B + 14abC) \sin(dx+c) (\sqrt{\cos}(dx+c))}{21d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(35 A a^2 \cos(dx+c)^3 + 75 B a^2 + 30 (5 A + 7 C) a b + 105 B b^2 + 45 (B a^2 + 2 A a b) \cos(dx+c)^2 + 7 ((7 A + 9 C) a \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C b^2 \cos(dx+c)^4 \sec(dx+c)^4 + (2 C a b + B b^2) \cos(dx+c)^4 \sec(dx+c)^3 + A a^2 \cos(dx+c)^4 + (C a^2 + \right.$$

69.268 Problem number 1300

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx)+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(6Aab + 3B a^2 + 5b^2B + 10abC) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(14abB + 7b^2(A+3C) + a^2(5A+7C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a(4Ab + 7Ba) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{35d} \\ & + \frac{2(4A b^2 + 14abB + a^2(5A+7C)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \\ & + \frac{2A(b + a \cos(dx+c))^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{7d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(15 A a^2 \cos(dx+c)^2 + 5 (5 A + 7 C) a^2 + 70 B a b + 35 A b^2 + 21 (B a^2 + 2 A a b) \cos(dx+c) \right) \sqrt{\cos(dx+c)} \sin(dx+c)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C b^2 \cos(dx+c)^3 \sec(dx+c)^4 + (2 C a b + B b^2) \cos(dx+c)^3 \sec(dx+c)^3 + A a^2 \cos(dx+c)^3 + (C a^2 + \dots\right)\right)$$

69.269 Problem number 1301

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx)+C\sec^2(c+dx))dx$$

Optimal antiderivative

$$\frac{2(10abB + 5b^2(A - C) + a^2(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

$$+ \frac{2(Ba^2 + 3b^2B + 2ab(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

$$+ \frac{2a^2(A - 5C) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{5d} + \frac{2C(b + a \cos(dx + c))^2 \sin(dx + c)}{d \sqrt{\cos(dx + c)}}$$

$$+ \frac{2a(2Ab + Ba - 6Cb) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2} (iBa^2 + 2i(A + 3C)ab + 3iBb^2) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^2 \cos(dx + c)^2 \sec(dx + c)^4 + (2Cab + Bb^2) \cos(dx + c)^2 \sec(dx + c)^3 + Aa^2 \cos(dx + c)^2 + (Ca^2 + \dots)\right) dx\right)$$

69.270 Problem number 1302

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(Ba^2 - b^2B + 2ab(A - C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

$$+ \frac{2(6abB + b^2(3A + C) + a^2(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

$$+ \frac{2C(b + a \cos(dx + c))^2 \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2b(3bB + 4aC) \sin(dx + c)}{3d \sqrt{\cos(dx + c)}}$$

$$+ \frac{2a^2(A - C) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} (-i(A+3C)a^2 - 6iBab - i(3A+C)b^2) \cos(dx+c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2 \cos(dx+c) \sec(dx+c)^4 + (2Cab + Bb^2) \cos(dx+c) \sec(dx+c)^3 + Aa^2 \cos(dx+c) + (Ca^2 + 2Bab + Ab^2) \sec(dx+c)^2 + (Ba^2 + 2Cab + Bb^2) \sec(dx+c) + Ca^2 + 2Bab + Ab^2\right) dx\right)$$

69.271 Problem number 1303

$$\int \sqrt{\cos(c+dx)} (a+b \sec(c+dx))^2 (A+B \sec(c+dx) + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(10abB - 5a^2(A-C) + b^2(5A+3C)) \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(3Ba^2 + b^2B + 2ab(3A+C)) \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(5bB + 4aC) \sin(dx+c)}{15d \cos(dx+c)^{\frac{3}{2}}} + \frac{2C(b+a \cos(dx+c))^2 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} \\ & + \frac{2(5Ab^2 + 10abB + 4a^2C + 3b^2C) \sin(dx+c)}{5d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \sqrt{2} (3iBa^2 + 2i(3A+C)ab + iBb^2) \cos(dx+c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) + 5$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(Cb^2 \sec(dx+c)^4 + (2Cab + Bb^2) \sec(dx+c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \sec(dx+c)^2 + (Ba^2 + 2Cab + Bb^2) \sec(dx+c) + Ca^2 + 2Bab + Ab^2\right) dx\right)$$

69.272 Problem number 1304

$$\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx) + C \sec^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(10Aab + 5B a^2 + 3b^2B + 6abC) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(14abB + 7a^2(3A + C) + b^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(7bB + 4aC) \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{2(7A b^2 + 14abB + 4a^2C + 5b^2C) \sin(dx + c)}{21d \cos(dx + c)^{\frac{3}{2}}} \\ & + \frac{2C(b + a \cos(dx + c))^2 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} + \frac{2(10Aab + 5B a^2 + 3b^2B + 6abC) \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (7i (3A + C)a^2 + 14i Bab + i (7A + 5C)b^2) \cos(dx + c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)) + i \sin(dx + c)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^2 \sec(dx + c)^4 + (2Cab + Bb^2) \sec(dx + c)^3 + Aa^2 + (Ca^2 + 2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2AbC) \sec(dx + c) + C}{\sqrt{\cos(dx + c)}}\right)$$

69.273 Problem number 1305

$$\int \cos^{\frac{11}{2}}(c + dx)(a + b \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(7a^3B + 27Ba^2b^2 + 3b^3(3A + 5C) + 3a^2b(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
+ & \frac{2(165a^2bB + 77b^3B + 33a^2b^2(5A + 7C) + 5a^3(9A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
+ & \frac{2(24A^3b^3 + 77a^3B + 242Ba^2b^2 + 33a^2b(7A + 9C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{495d} \\
+ & \frac{2a(24A^2b^2 + 143abB + 9a^2(9A + 11C)) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c)}{693d} \\
+ & \frac{2(6Ab + 11Ba) \left(\cos^{\frac{3}{2}}(dx + c)\right) (b + a \cos(dx + c))^2 \sin(dx + c)}{99d} \\
+ & \frac{2A \left(\cos^{\frac{3}{2}}(dx + c)\right) (b + a \cos(dx + c))^3 \sin(dx + c)}{11d} \\
+ & \frac{2(165a^2bB + 77b^3B + 33a^2b^2(5A + 7C) + 5a^3(9A + 11C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{231d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(315 A a^3 \cos(dx + c)^4 + 75 (9 A + 11 C) a^3 + 2475 B a^2 b + 495 (5 A + 7 C) a b^2 + 1155 B b^3 + 385 (B a^3 + 3 A a^2 b) \right)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Cb^3 cos(dx + c)^5 sec(dx + c)^5 + (3Cab^2 + Bb^3) cos(dx + c)^5 sec(dx + c)^4 + Aa^3 cos(dx + c)^5 + (3Ca^2
```

69.274 Problem number 1306

$$\int \cos^{\frac{9}{2}}(c + dx) (a + b \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(27a^2bB + 15b^3B + 9ab^2(3A + 5C) + a^3(7A + 9C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(5a^3B + 21Ba^2b + 7b^3(A + 3C) + 3a^2b(5A + 7C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a(24Ab^2 + 99abB + 7a^2(7A + 9C)) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c)}{315d} \\
& + \frac{2(8Ab^3 + 15a^3B + 54Ba^2b + 9a^2b(5A + 7C)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{63d} \\
& + \frac{2(2Ab + 3Ba) (b + a \cos(dx+c))^2 \sin(dx+c) (\sqrt{\cos(dx+c)})}{21d} \\
& + \frac{2A(b + a \cos(dx+c))^3 \sin(dx+c) (\sqrt{\cos(dx+c)})}{9d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(35 A a^3 \cos(dx+c)^3 + 75 B a^3 + 45 (5 A + 7 C) a^2 b + 315 B a b^2 + 105 A b^3 + 45 (B a^3 + 3 A a^2 b) \cos(dx+c)^2 + 7 \right)}{63 d}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Cb^3 cos(dx+c)^4 sec(dx+c)^5 + (3Cab^2 + Bb^3) cos(dx+c)^4 sec(dx+c)^4 + Aa^3 cos(dx+c)^4 + (3Ca^2
```

69.275 Problem number 1307

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^3 (A+B \sec(c+dx)+C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3a^3B + 15Ba^2b^2 + 5b^3(A - C) + 3a^2b(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^2bB + 21b^3B + 21ab^2(A + 3C) + a^3(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(11Ab + 7Ba - 35Cb) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{35d} + \frac{2C(b + a \cos(dx + c))^3 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\ & + \frac{2a(21abB + 6b^2(3A - 7C) + a^2(5A + 7C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \\ & + \frac{2a(A - 7C) (b + a \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i (5 A + 7 C) a^3 + 21 i B a^2 b + 21 i (A + 3 C) a b^2 + 21 i B b^3) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \cos(dx + c)^3 \sec(dx + c)^5 + (3Cab^2 + Bb^3) \cos(dx + c)^3 \sec(dx + c)^4 + Aa^3 \cos(dx + c)^3 + (3Ca^2\right.\right.$$

69.276 Problem number 1308

$$\int \cos^{\frac{5}{2}}(c + dx) (a + b \sec(c + dx))^3 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(15a^2bB - 5b^3B + 15ab^2(A - C) + a^3(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(a^3B + 9Ba^2b^2 + b^3(3A + C) + 3a^2b(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2a^2(3aA - 15bB - 35aC) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\ & + \frac{2C(b + a \cos(dx + c))^3 \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(bB + 2aC) (b + a \cos(dx + c))^2 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\ & + \frac{2a(Ba^2 - 6b^2B + 3ab(A - 5C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{3d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(iBa^3 + 3i(A+3C)a^2b + 9iBab^2 + i(3A+C)b^3) \cos(dx+c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) +$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb^3 cos(dx+c)^2 sec(dx+c)^5 + (3Cab^2 + Bb^3) cos(dx+c)^2 sec(dx+c)^4 + Aa^3 cos(dx+c)^2 + (3Ca^2b +`

69.277 Problem number 1309

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx)+C\sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5a^3B - 15Ba^2b + 15a^2b(A-C) - b^3(5A+3C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(9a^2bB + b^3B + 3a^2b(3A+C) + a^3(A+3C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(5bB + 6aC)(b+a\cos(dx+c))^2 \sin(dx+c)}{15d \cos(dx+c)^{\frac{3}{2}}} + \frac{2C(b+a\cos(dx+c))^3 \sin(dx+c)}{5d \cos(dx+c)^{\frac{5}{2}}} \\ & + \frac{2b(15A^2b^2 + 35abB + 24a^2C + 9b^2C) \sin(dx+c)}{15d \sqrt{\cos(dx+c)}} \\ & + \frac{2a^2(5aA - 5bB - 9aC) \sin(dx+c) (\sqrt{\cos(dx+c)})}{15d} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5\sqrt{2}(i(A+3C)a^3 + 9iBa^2b + 3i(3A+C)ab^2 + iBb^3) \cos(dx+c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) +$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb^3 cos(dx+c) sec(dx+c)^5 + (3Cab^2 + Bb^3) cos(dx+c) sec(dx+c)^4 + Aa^3 cos(dx+c) + (3Ca^2b +`

69.278 Problem number 1310

$$\int \sqrt{\cos(c+dx)} (a+b\sec(c+dx))^3 (A+B\sec(c+dx)+C\sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(15a^2bB + 3b^3B - 5a^3(A - C) + 3ab^2(5A + 3C)) \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2(21a^3B + 21Ba^2b + 21a^2b(3A + C) + b^3(7A + 5C)) \sqrt{\frac{\cos(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ & + \frac{2b(35A^2b^2 + 63abB + 24a^2C + 25b^2C) \sin(dx+c)}{105d \cos(dx+c)^{\frac{3}{2}}} \\ & + \frac{2(7bB + 6aC)(b+a\cos(dx+c))^2 \sin(dx+c)}{35d \cos(dx+c)^{\frac{5}{2}}} + \frac{2C(b+a\cos(dx+c))^3 \sin(dx+c)}{7d \cos(dx+c)^{\frac{7}{2}}} \\ & + \frac{2(98a^2bB + 21b^3B + 24a^3C + 21ab^2(5A + 3C)) \sin(dx+c)}{35d \sqrt{\cos(dx+c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(21iBa^3 + 21i(3A+C)a^2b + 21iBab^2 + i(7A+5C)b^3) \cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^3 \sec(dx+c)^5 + (3Cab^2 + Bb^3) \sec(dx+c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \sec(dx+c)^3 + (Ca^2b + 3Ab^2) \sec(dx+c)^2 + (Ca^2b + 3Ab^2) \sec(dx+c) + Ca^2b + 3Ab^2\right) \sqrt{\cos(dx+c)}\right) dx$$

69.279 Problem number 1311

$$\int \frac{(a+b\sec(c+dx))^3 (A+B\sec(c+dx)+C\sec^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(15a^3B + 27Ba^2b^2 + 9a^2b(5A + 3C) + b^3(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ + & \frac{2(21a^2bB + 5b^3B + 7a^3(3A + C) + 3ab^2(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\ + & \frac{2b(63Ab^2 + 99abB + 24a^2C + 49b^2C) \sin(dx + c)}{315d \cos(dx + c)^{\frac{5}{2}}} \\ + & \frac{2(54a^2bB + 15b^3B + 8a^3C + 9ab^2(7A + 5C)) \sin(dx + c)}{63d \cos(dx + c)^{\frac{3}{2}}} \\ + & \frac{2(3bB + 2aC)(b + a \cos(dx + c))^2 \sin(dx + c)}{21d \cos(dx + c)^{\frac{7}{2}}} + \frac{2C(b + a \cos(dx + c))^3 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\ + & \frac{2(15a^3B + 27Ba^2b^2 + 9a^2b(5A + 3C) + b^3(9A + 7C)) \sin(dx + c)}{15d \sqrt{\cos(dx + c)}} \end{aligned}$$

command

`integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="f`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (7i(3A + C)a^3 + 21iBa^2b + 3i(7A + 5C)ab^2 + 5iBb^3) \cos(dx + c)^5 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb^3 \sec(dx + c)^5 + (3Cab^2 + Bb^3) \sec(dx + c)^4 + Aa^3 + (3Ca^2b + 3Bab^2 + Ab^3) \sec(dx + c)^3 + (Ca^3 + 3Aab^2 + 3Bab^2) \sec(dx + c)^2 + (3Aa^2b + 3Bab^2 + Ab^3) \sec(dx + c) + Ca^3}{\sqrt{\cos(dx + c)}}\right)$$

69.280 Problem number 1312

$$\int \cos^{\frac{11}{2}}(c + dx)(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(7a^4B + 54Ba^2b^2 + 15b^4B + 12ab^3(3A + 5C) + 4a^3b(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \nu\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(220a^3bB + 308ab^3B + 77b^4(A + 3C) + 66a^2b^2(5A + 7C) + 5a^4(9A + 11C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \nu\right)}{231 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a(192Ab^3 + 539a^3B + 1353Bab^2 + 2a^2b(673A + 891C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{3465d} \\
& + \frac{2(64Ab^4 + 660a^3bB + 682ab^3B + 15a^4(9A + 11C) + 9a^2b^2(101A + 143C)) \sin(dx + c) (\sqrt{\cos}(dx + c))}{693d} \\
& + \frac{2(16Ab^2 + 55abB + 3a^2(9A + 11C)) (b + a \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos}(dx + c))}{231d} \\
& + \frac{2(8Ab + 11Ba) (b + a \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos}(dx + c))}{99d} \\
& + \frac{2A(b + a \cos(dx + c))^4 \sin(dx + c) (\sqrt{\cos}(dx + c))}{11d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(315 Aa^4 \cos(dx + c)^4 + 75 (9A + 11C)a^4 + 3300 Ba^3b + 990 (5A + 7C)a^2b^2 + 4620 Bab^3 + 1155 Ab^4 + 385 (B^2 + C^2)b^4 \right)}{11d}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Cb^4 cos(dx + c)^5 sec(dx + c)^6 + (4Cab^3 + Bb^4) cos(dx + c)^5 sec(dx + c)^5 + Aa^4 cos(dx + c)^5 + (6Ca^3b + 3Aab^2) cos(dx + c)^4 sec(dx + c)^4 + (4Aab^2 + 3Aab^2) cos(dx + c)^4 sec(dx + c)^3 + (4Aab^2 + 3Aab^2) cos(dx + c)^3 sec(dx + c)^2 + (4Aab^2 + 3Aab^2) cos(dx + c)^2 sec(dx + c) + (4Aab^2 + 3Aab^2) cos(dx + c) sec(dx + c) + (4Aab^2 + 3Aab^2) sec(dx + c) + (4Aab^2 + 3Aab^2)) dx
```

69.281 Problem number 1313

$$\int \cos^{\frac{9}{2}}(c + dx)(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(36a^3bB + 60ab^3B + 15b^4(A - C) + 18a^2b^2(3A + 5C) + a^4(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(5a^4B + 42Ba^2b^2 + 21b^4B + 28ab^3(A + 3C) + 4a^3b(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a^2(162abB + 3b^2(41A - 105C) + 7a^2(7A + 9C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{315d} \\
& + \frac{2C(b + a \cos(dx + c))^4 \sin(dx + c)}{d \sqrt{\cos(dx + c)}} \\
& + \frac{2a(15a^3B + 117Ba^2b^2 + 2b^3(31A - 63C) + 12a^2b(5A + 7C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{63d} \\
& + \frac{2a(5Ab + 3Ba - 21Cb) (b + a \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \\
& + \frac{2a(A - 9C) (b + a \cos(dx + c))^3 \sin(dx + c) (\sqrt{\cos(dx + c)})}{9d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm='f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (5i Ba^4 + 4i (5A + 7C)a^3b + 42i Ba^2b^2 + 28i (A + 3C)ab^3 + 21i Bb^4) \cos(dx + c) \operatorname{weierstrassPInverse}(-)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^4 \sec(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^4 \sec(dx + c)^5 + Aa^4 \cos(dx + c)^4 + (6Ca^3b + 4a^2b^2) \cos(dx + c)^3 \sec(dx + c)^4 + (4a^2b^3 + 3a^3b^2) \cos(dx + c)^3 \sec(dx + c)^3 + (3a^2b^4 + 2a^3b^3) \cos(dx + c)^3 \sec(dx + c)^2 + (2a^2b^4 + a^3b^3) \cos(dx + c)^3 \sec(dx + c) + (a^2b^4 + a^3b^3) \cos(dx + c)^3 + (a^2b^4 + a^3b^3) \cos(dx + c)^2 + (a^2b^4 + a^3b^3) \cos(dx + c) + a^2b^4 + a^3b^3\right) dx$$

69.282 Problem number 1314

$$\int \cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(3a^4B + 30Ba^2b^2 - 5b^4B + 20ab^3(A - C) + 4a^3b(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(28a^3bB + 84ab^3B + 7b^4(3A + C) + 42a^2b^2(A + 3C) + a^4(5A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2a^2(54Aab + 21Ba^2 - 105b^2B - 350abC) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{105d} \\
& + \frac{2C(b + a \cos(dx + c))^4 \sin(dx + c)}{3d \cos(dx + c)^{\frac{3}{2}}} + \frac{2(3bB + 8aC)(b + a \cos(dx + c))^3 \sin(dx + c)}{3d \sqrt{\cos(dx + c)}} \\
& + \frac{2a(28a^2bB - 42b^3B + 3ab^2(13A - 49C) + a^3(5A + 7C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{21d} \\
& + \frac{2a(aA - 7bB - 21aC)(b + a \cos(dx + c))^2 \sin(dx + c) (\sqrt{\cos(dx + c)})}{7d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm='f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i(5A + 7C)a^4 + 28iBa^3b + 42i(A + 3C)a^2b^2 + 84iBab^3 + 7i(3A + C)b^4) \cos(dx + c)^2 \operatorname{weierstrassP}(\dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^3 \sec(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^3 \sec(dx + c)^5 + Aa^4 \cos(dx + c)^3 + (6Ca^3b + 3Aab^2) \cos(dx + c)^2 \sec(dx + c)^4 + (3A^2b + 3Aab) \cos(dx + c)^2 \sec(dx + c)^3 + (3A^2b + 3Aab) \cos(dx + c) \sec(dx + c)^2 + 3A^2b \cos(dx + c) \sec(dx + c) + 3A^2b\right) dx\right)$$

69.283 Problem number 1315

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(20a^3bB - 20ab^3B + 30a^2b^2(A - C) - b^4(5A + 3C) + a^4(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(a^4B + 18Ba^2b^2 + b^4B + 4ab^3(3A + C) + 4a^3b(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& - \frac{2a^2(50abB - a^2(3A - 59C) + 3b^2(5A + 3C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c)}{15d} \\
& + \frac{2(5bB + 8aC)(b + a \cos(dx + c))^3 \sin(dx + c)}{15d \cos(dx + c)^{\frac{3}{2}}} + \frac{2C(b + a \cos(dx + c))^4 \sin(dx + c)}{5d \cos(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(5Ab^2 + 15abB + 16a^2C + 3b^2C)(b + a \cos(dx + c))^2 \sin(dx + c)}{5d \sqrt{\cos(dx + c)}} \\
& + \frac{2a(5a^3B - 105Ba^2b + 4a^2b(5A - 33C) - 6b^3(5A + 3C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{15d}
\end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i Ba^4 + 4i (A + 3C) a^3 b + 18i Ba^2 b^2 + 4i (3A + C) a b^3 + i B b^4) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, c)}{15d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx + c)^2 \sec(dx + c)^6 + (4Cab^3 + Bb^4) \cos(dx + c)^2 \sec(dx + c)^5 + Aa^4 \cos(dx + c)^2 + (6Ca^3b + 3a^2b^2) \cos(dx + c) \sec(dx + c)^4 + 3a^2b \sec(dx + c)^3 + a^2 \sec(dx + c)^2 + a \sec(dx + c)\right), dx\right)$$

69.284 Problem number 1316

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^4 (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(5a^4B - 30Ba^2b^2 - 3b^4B + 20a^3b(A - C) - 4ab^3(5A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(84a^3bB + 28ab^3B + 42a^2b^2(3A + C) + 7a^4(A + 3C) + b^4(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(35Ab^2 + 77abB + 48a^2C + 25b^2C)(b + a \cos(dx + c))^2 \sin(dx + c)}{105d \cos(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(7bB + 8aC)(b + a \cos(dx + c))^3 \sin(dx + c)}{35d \cos(dx + c)^{\frac{5}{2}}} + \frac{2C(b + a \cos(dx + c))^4 \sin(dx + c)}{7d \cos(dx + c)^{\frac{7}{2}}} \\
& + \frac{2b(413a^2bB + 63b^3B + 192a^3C + 2ab^2(175A + 101C)) \sin(dx + c)}{105d \sqrt{\cos(dx + c)}} \\
& - \frac{2a^2(98abB - a^2(35A - 87C) + 5b^2(7A + 5C)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{105d}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\sqrt{2}(7i(A+3C)a^4 + 84iBa^3b + 42i(3A+C)a^2b^2 + 28iBab^3 + i(7A+5C)b^4) \cos(dx+c)^4 \operatorname{weierstrassP}(\operatorname{sn}(dx+c, \sqrt{2}), \operatorname{sn}(dx+c, \sqrt{2}), \operatorname{sn}(dx+c, \sqrt{2}))}{105d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Cb^4 \cos(dx+c) \sec(dx+c)^6 + (4Cab^3 + Bb^4) \cos(dx+c) \sec(dx+c)^5 + Aa^4 \cos(dx+c) + (6Ca^2b^2\right) dx$$

69.285 Problem number 1317

$$\int \sqrt{\cos(c+dx)} (a + b \sec(c+dx))^4 (A + B \sec(c+dx) + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(60a^3bB + 36ab^3B - 15a^4(A - C) + 18a^2b^2(5A + 3C) + b^4(9A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2(21a^4B + 42Ba^2b^2 + 5b^4B + 28a^3b(3A + C) + 4ab^3(7A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) d} \\
& + \frac{2b(261a^2bB + 75b^3B + 64a^3C + 2ab^2(147A + 101C)) \sin(dx + c)}{315d \cos(dx + c)^{\frac{3}{2}}} \\
& + \frac{2(63A^2b^2 + 117abB + 48a^2C + 49b^2C) (b + a \cos(dx + c))^2 \sin(dx + c)}{315d \cos(dx + c)^{\frac{5}{2}}} \\
& + \frac{2(9bB + 8aC) (b + a \cos(dx + c))^3 \sin(dx + c)}{63d \cos(dx + c)^{\frac{7}{2}}} + \frac{2C(b + a \cos(dx + c))^4 \sin(dx + c)}{9d \cos(dx + c)^{\frac{9}{2}}} \\
& + \frac{2(1098a^3bB + 756ab^3B + 192a^4C + 21b^4(9A + 7C) + 7a^2b^2(261A + 155C)) \sin(dx + c)}{315d \sqrt{\cos(dx + c)}}
\end{aligned}$$

command

```
integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="f")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15 \sqrt{2} (21i Ba^4 + 28i (3A + C)a^3b + 42i Ba^2b^2 + 4i (7A + 5C)ab^3 + 5i Bb^4) \cos(dx + c)^5 \operatorname{weierstrassPInverse}(\dots)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Cb^4 sec(dx + c)^6 + (4Cab^3 + Bb^4) sec(dx + c)^5 + Aa^4 + (6Ca^2b^2 + 4Bab^3 + Ab^4) sec(dx + c)^4 + 2(2
```

69.286 Problem number 1334

$$\int \cos^{\frac{9}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2) (16Ab^3 - 75a^3B - 24Bab^2 + 6a^2b(6A + 7C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\
& - \frac{2(6Ab^2 - 9abB - 7a^2(7A + 9C)) \left(\cos^{\frac{3}{2}}(dx + c)\right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{315a^2d} \\
& + \frac{2(Ab + 9Ba) \left(\cos^{\frac{5}{2}}(dx + c)\right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{63ad} \\
& + \frac{2A \left(\cos^{\frac{7}{2}}(dx + c)\right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{9d} \\
& + \frac{2(8Ab^3 + 75a^3B - 12Bab^2 + a^2b(13A + 21C)) \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{315a^3d} \\
& - \frac{2(16Ab^4 - 57a^3bB - 24ab^3B + 6a^2b^2(4A + 7C) - 21a^4(7A + 9C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{315 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}
\end{aligned}$$

command

`integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(35 A a^5 \cos(dx + c)^3 + 75 B a^5 + (13 A + 21 C) a^4 b - 12 B a^3 b^2 + 8 A a^2 b^3 + 5 (9 B a^5 + A a^4 b) \cos(dx + c)^2 + (7 A a^5 + 13 A a^4 b + 75 B a^5 + 13 A a^4 b + 21 C a^4 b) \cos(dx + c) + 7 A a^5 + 13 A a^4 b + 75 B a^5 + 13 A a^4 b + 21 C a^4 b \right) \sqrt{a + b \sec(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx + c)^4 \sec(dx + c)^2 + B \cos(dx + c)^4 \sec(dx + c) + A \cos(dx + c)^4\right) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}\right)$$

69.287 Problem number 1335

$$\int \cos^{\frac{7}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2) (25a^2A + 8Ab^2 - 14abB + 35a^2C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{\cos(dx+c)}}}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2(Ab + 7Ba) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{35ad} \\
& + \frac{2A \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7d} \\
& - \frac{2(4Ab^2 - 7abB - 5a^2(5A + 7C)) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{105a^2d} \\
& + \frac{2(8Ab^3 + 63a^3B - 14Ba^2b^2 + a^2b(19A + 35C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\frac{b+a \cos(dx+c)}{\cos(dx+c)}})}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(15 A a^4 \cos(dx+c)^2 + 5 (5 A + 7 C) a^4 + 7 B a^3 b - 4 A a^2 b^2 + 3 (7 B a^4 + A a^3 b) \cos(dx+c) \right) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}}}{\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(C \cos(dx+c)^3 \sec(dx+c)^2 + B \cos(dx+c)^3 \sec(dx+c) + A \cos(dx+c)^3\right) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}\right) dx$$

69.288 Problem number 1336

$$\int \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx) + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(a^2 - b^2)(2Ab - 5Ba) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2A \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5d} \\
& + \frac{2(Ab + 5Ba) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{15ad} \\
& - \frac{2(2Ab^2 - 5abB - 3a^2(3A + 5C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b+a\cos(dx+c)}{a+b}}}
\end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(3Aa^3 \cos(dx+c) + 5Ba^3 + Aa^2b) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2}(-15iBa^3 - 3i(A +$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((C cos(dx+c)^2 sec(dx+c)^2 + B cos(dx+c)^2 sec(dx+c) + A cos(dx+c)^2) sqrt(b sec(dx+c) + a) sqrt(cos`

69.289 Problem number 1341

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b \sec(c+dx))^{3/2} (A+B \sec(c+dx) + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2A \left(\cos^{\frac{7}{2}}(dx+c) \right) (a+b \sec(dx+c))^{\frac{3}{2}} \sin(dx+c)}{9d} \\
& + \frac{2(a^2-b^2)(8Ab^3+75a^3B-18Ba^2b^2+a^2(39Ab+63Cb)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \right)}{315 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2(3Ab^2+72abB+7a^2(7A+9C)) \left(\cos^{\frac{3}{2}}(dx+c) \right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{315ad} \\
& + \frac{2(Ab+3Ba) \left(\cos^{\frac{5}{2}}(dx+c) \right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{21d} \\
& - \frac{2(4Ab^3-75a^3B-9Ba^2b^2-2a^2b(44A+63C)) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{315a^2d} \\
& + \frac{2(8Ab^4+246a^3bB-18ab^3B+21a^4(7A+9C)+3a^2b^2(11A+21C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \right)}{315 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^3 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}
\end{aligned}$$

command

`integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(35 A a^5 \cos(dx+c)^3 + 75 B a^5 + 2(44 A + 63 C) a^4 b + 9 B a^3 b^2 - 4 A a^2 b^3 + 5(9 B a^5 + 10 A a^4 b) \cos(dx+c)^2 + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb cos(dx+c)^4 sec(dx+c)^3 + (Ca+Bb) cos(dx+c)^4 sec(dx+c)^2 + Aa cos(dx+c)^4 + (Ba+Ab) cos(dx+c)^3), dx)`

69.290 Problem number 1342

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^{\frac{3}{2}} (A+B \sec(c+dx)+C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2A \left(\cos^{\frac{5}{2}}(dx+c) \right) (a+b \sec(dx+c))^{\frac{3}{2}} \sin(dx+c)}{7d} \\
& + \frac{2(a^2-b^2) (25a^2A - 6Ab^2 + 21abB + 35a^2C) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{a}{a+b}} \right) \sqrt{b-a}}{105 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\
& + \frac{2(3Ab + 7Ba) \left(\cos^{\frac{3}{2}}(dx+c) \right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{35d} \\
& + \frac{2(3Ab^2 + 42abB + 5a^2(5A + 7C)) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{105ad} \\
& - \frac{2(6Ab^3 - 63a^3B - 21Bab^2 - 2a^2b(41A + 70C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{a}{a+b}} \right) \sqrt{b-a}}{105 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}
\end{aligned}$$

command

`integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(15 Aa^4 \cos(dx+c)^2 + 5(5A+7C)a^4 + 42Ba^3b + 3Aa^2b^2 + 3(7Ba^4 + 8Aa^3b) \cos(dx+c) \right) \sqrt{\frac{a \cos(dx+c)}{\cos(dx+c)}}}{\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral((Cb cos(dx+c)^3 sec(dx+c)^3 + (Ca+Bb) cos(dx+c)^3 sec(dx+c)^2 + Aa cos(dx+c)^3 + (Ba+Ab) cos(dx+c)^2), dx)`

69.291 Problem number 1348

$$\int \cos^{\frac{11}{2}}(c+dx) (a+b \sec(c+dx))^{5/2} (A+B \sec(c+dx) + C \sec^2(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(5Ab + 11Ba) \left(\cos^{\frac{7}{2}}(dx + c) \right) (a + b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{99d} \\
& + \frac{2A \left(\cos^{\frac{9}{2}}(dx + c) \right) (a + b \sec(dx + c))^{\frac{5}{2}} \sin(dx + c)}{11d} \\
& + \frac{2(a^2 - b^2) (40Ab^4 + 1254a^3bB - 110ab^3B + 75a^4(9A + 11C) + 15a^2b^2(19A + 33C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\frac{dx + c}{2}\right)}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\
& + \frac{2(15Ab^3 + 539a^3B + 825Bab^2 + 5a^2b(229A + 297C)) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{3465ad} \\
& + \frac{2(5Ab^2 + 44abB + 3a^2(9A + 11C)) \left(\cos^{\frac{5}{2}}(dx + c) \right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{231d} \\
& - \frac{2(20Ab^4 - 1793a^3bB - 55ab^3B - 75a^4(9A + 11C) - 5a^2b^2(205A + 297C)) \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{3465a^2d} \\
& + \frac{2(40Ab^5 + 1617Ba^5 + 3069Ba^3b^2 - 110Bab^4 + 15a^2b^3(17A + 33C) + 15a^4b(247A + 319C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\frac{dx + c}{2}\right)}{3465 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}
\end{aligned}$$

command

```
integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(315 Aa^6 \cos(dx + c)^4 + 75 (9A + 11C)a^6 + 1793 Ba^5b + 5 (205A + 297C)a^4b^2 + 55 Ba^3b^3 - 20 Aa^2b^4 + 35 (11A + 11C)ab^5 + 35 Aa^6 \cos^2(dx + c) + 70 Aa^5b \cos(dx + c) + 35 Aa^4b^2 \cos^2(dx + c) + 70 Aa^3b^3 \cos(dx + c) + 35 Aa^2b^4 \cos^2(dx + c) + 70 Aab^5 \cos(dx + c) + 35 Aa^6 \cos^4(dx + c) + 70 Aa^5b \cos^3(dx + c) + 35 Aa^4b^2 \cos^2(dx + c) + 70 Aa^3b^3 \cos(dx + c) + 35 Aa^2b^4 \cos^2(dx + c) + 70 Aab^5 \cos(dx + c) + 35 Aa^6 \cos^4(dx + c) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral((Cb^2*cos(dx+c)^5*sec(dx+c)^4 + (2Cab+Bb^2)*cos(dx+c)^5*sec(dx+c)^3 + Aa^2*cos(dx+c)^5 + (Ca^2 + 2Cab+Bb^2)*cos(dx+c)^4*sec(dx+c)^3 + 5a^2b^2*cos(dx+c)^4*sec(dx+c)^2 + 5a^2b*cos(dx+c)^3*sec(dx+c)^2 + 5a^2*cos(dx+c)^2*sec(dx+c)^2 + 5a*cos(dx+c)*sec(dx+c)^2 + 5*cos(dx+c)*sec(dx+c)^2), dx)
```

69.292 Problem number 1349

$$\int \cos^{\frac{9}{2}}(c + dx)(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx) + C \sec^2(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5Ab + 9Ba) \left(\cos^{\frac{5}{2}}(dx + c) \right) (a + b \sec(dx + c))^{\frac{3}{2}} \sin(dx + c)}{63d} \\ & + \frac{2A \left(\cos^{\frac{7}{2}}(dx + c) \right) (a + b \sec(dx + c))^{\frac{5}{2}} \sin(dx + c)}{9d} \\ & - \frac{2(a^2 - b^2) (10Ab^3 - 75a^3B - 45Bab^2 - 6a^2b(19A + 28C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \right)}{315 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(15Ab^2 + 90abB + 7a^2(7A + 9C)) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{315d} \\ & + \frac{2(5Ab^3 + 75a^3B + 135Bab^2 + a^2b(163A + 231C)) \sin(dx + c) (\sqrt{\cos}(dx + c)) \sqrt{a + b \sec(dx + c)}}{315ad} \\ & - \frac{2(10Ab^4 - 435a^3bB - 45ab^3B - 21a^4(7A + 9C) - 3a^2b^2(93A + 161C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \right)}{315 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6 \left(35 A a^5 \cos(dx + c)^3 + 75 B a^5 + (163 A + 231 C) a^4 b + 135 B a^3 b^2 + 5 A a^2 b^3 + 5 (9 B a^5 + 19 A a^4 b) \cos(dx + c) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(C b^2 \cos(dx + c)^4 \sec(dx + c)^4 + (2 C a b + B b^2) \cos(dx + c)^4 \sec(dx + c)^3 + A a^2 \cos(dx + c)^4 + (C a^2 + \dots \right) dx \right)$$

69.293 Problem number 1356

$$\int \frac{\cos^{\frac{7}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2(48Ab^4 - 49a^3bB - 56ab^3B + 5a^4(5A+7C) + 2a^2b^2(16A+35C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} - \frac{2(6Ab - 7Ba) \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{35a^2d} + \frac{2A \left(\cos^{\frac{5}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{7ad} + \frac{2(24Ab^2 - 28abB + 5a^2(5A+7C)) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{105a^3d} - \frac{2(48Ab^3 - 63a^3B - 56Ba^2b + a^2(44Ab + 70Cb)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right)}{105 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}$$

command

`integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(15 A a^4 \cos(dx+c)^2 + 5 (5 A + 7 C) a^4 - 28 B a^3 b + 24 A a^2 b^2 + 3 (7 B a^4 - 6 A a^3 b) \cos(dx+c) \right) \sqrt{\frac{a \cos(dx+c)}{\cos(dx+c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^3 \sec(dx+c)^2 + B \cos(dx+c)^3 \sec(dx+c) + A \cos(dx+c)^3\right) \sqrt{\cos(dx+c)}}{\sqrt{b \sec(dx+c) + a}}, x\right)$$

69.294 Problem number 1357

$$\int \frac{\cos^{\frac{5}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(8Ab^3 - 5a^3B - 10Ba^2b + a^2b(7A + 15C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{b+c}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx+c)} \sqrt{a+b \sec(dx+c)}} \\ & + \frac{2A \left(\cos^{\frac{3}{2}}(dx+c)\right) \sin(dx+c) \sqrt{a+b \sec(dx+c)}}{5ad} \\ & - \frac{2(4Ab - 5Ba) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a+b \sec(dx+c)}}{15a^2d} \\ & + \frac{2(8Ab^2 - 10abB + 3a^2(3A + 5C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx+c)})}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(3Aa^3 \cos(dx+c) + 5Ba^3 - 4Aa^2b) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \sqrt{2} (-15iBa^3 + 6i(2Aa^3 - 5Ba^3 - 4Aa^2b)) \sqrt{\frac{a \cos(dx+c) + b}{\cos(dx+c)}} \sqrt{\cos(dx+c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx+c)^2 \sec(dx+c)^2 + B \cos(dx+c)^2 \sec(dx+c) + A \cos(dx+c)^2\right) \sqrt{\cos(dx+c)}}{\sqrt{b \sec(dx+c) + a}}, x\right)$$

69.295 Problem number 1358

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A+B \sec(c+dx) + C \sec^2(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal antiderivative

$$\frac{2(2Ab^2 - 3abB + a^2(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} + \frac{2A \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{3ad} - \frac{2(2Ab - 3Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="Fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 A a^2 \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + \sqrt{2} (-3i(A + 3C)a^2 + 6i Bab - 4i Ab^2) \sqrt{a} \operatorname{weierstrassP}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c) \sec(dx + c)^2 + B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)\right) \sqrt{\cos(dx + c)}}{\sqrt{b \sec(dx + c) + a}}, x\right)$$

69.296 Problem number 1363

$$\int \frac{\cos^{\frac{5}{2}}(c + dx) (A + B \sec(c + dx) + C \sec^2(c + dx))}{(a + b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(Ab^2 - a(bB - aC)) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c)}{a(a^2 - b^2) d \sqrt{a + b \sec(dx + c)}} \\
& - \frac{2(48Ab^3 - 5a^3B - 40Bab^2 + 6a^2b(2A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b}{a+b}}}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\
& - \frac{2(6Ab^2 - 5abB - a^2(A - 5C)) \left(\cos^{\frac{3}{2}}(dx + c) \right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{5a^2(a^2 - b^2) d} \\
& + \frac{2(24Ab^3 + 5a^3B - 20Bab^2 - a^2(9Ab - 15Cb)) \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{15a^3(a^2 - b^2) d} \\
& - \frac{2(48Ab^4 + 25a^3bB - 40ab^3B - 6a^2b^2(4A - 5C) - 3a^4(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}
\end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6 \left(5Ba^5b - 3(3A - 5C)a^4b^2 - 20Ba^3b^3 + 24Aa^2b^4 + 3(Aa^6 - Aa^4b^2) \cos(dx + c)^2 + (5Ba^6 - 6Aa^5b - 5Ba^4b^2) \sin(dx + c)^2 \right) \sqrt{a + b \sec(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\left(C \cos(dx + c)^2 \sec(dx + c)^2 + B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2\right) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}\right)$$

69.297 Problem number 1364

$$\int \frac{\cos^{\frac{3}{2}}(c + dx) (A + B \sec(c + dx) + C \sec^2(c + dx))}{(a + b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) \sin(dx + c) (\sqrt{\cos(dx + c)})}{a(a^2 - b^2) d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(8Ab^2 - 6abB + a^2(A + 3C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & - \frac{2(4Ab^2 - 3abB - a^2(A - 3C)) \sin(dx + c) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{3a^2(a^2 - b^2) d} \\ & + \frac{2(8Ab^3 + 3a^3B - 6Ba^2b^2 - a^2(5Ab - 3Cb)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output")`

$$6((A - 3C)a^4b + 3Ba^3b^2 - 4Aa^2b^3 + (Aa^5 - Aa^3b^2) \cos(dx + c)) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \cos(dx + c) \sec(dx + c))^2 + B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2} \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)} dx\right)$$

69.298 Problem number 1365

$$\int \frac{\sqrt{\cos(c + dx)} (A + B \sec(c + dx) + C \sec^2(c + dx))}{(a + b \sec(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) \sin(dx + c)}{a(a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & - \frac{2(2Ab - Ba) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a+b}}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & - \frac{2(2Ab^2 - abB - a^2(A - C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) (\sqrt{\cos(dx + c)}) \sqrt{a + b \sec(dx + c)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2) d \sqrt{\frac{b + a \cos(dx + c)}{a+b}}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(Ca^4 - Ba^3b + Aa^2b^2) \sqrt{\frac{a \cos(dx + c) + b}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) - \left(\sqrt{2} (3iBa^4 - i(5A + C)a^3b - 2iBa^2b^2 + i(5A + C)a^2b - 2iBa^2b^2 + i(5A + C)a^2b - 2iBa^2b^2)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}, x\right)$$

69.299 Problem number 1368

$$\int \frac{\cos^{5/2}(c + dx) (A + B \sec(c + dx) + C \sec^2(c + dx))}{(a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) \left(\cos^{3/2}(dx + c)\right) \sin(dx + c)}{3a(a^2 - b^2) d (a + b \sec(dx + c))^{3/2}} \\ & - \frac{2(8Ab^4 + 9a^3bB - 5ab^3B - 2a^2b^2(6A - C) - 6a^4C) \left(\cos^{3/2}(dx + c)\right) \sin(dx + c)}{3a^2(a^2 - b^2)^2 d \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(128Ab^5 + 5Ba^5 + 80Ba^3b^2 - 80Bab^4 - 4a^2b^3(29A - 10C) - a^4b(17A + 45C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\frac{\cos(dx + c)}{2} + \frac{1}{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^5 (a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(48Ab^4 + 50a^3bB - 30ab^3B + a^4(3A - 35C) - a^2b^2(71A - 15C)) \left(\cos^{3/2}(dx + c)\right) \sin(dx + c) \sqrt{a + b \sec(dx + c)}}{15a^3(a^2 - b^2)^2 d} \\ & - \frac{2(64Ab^5 - 5Ba^5 + 65Ba^3b^2 - 40Bab^4 + 2a^4b(7A - 20C) - 2a^2b^3(49A - 10C)) \sin(dx + c) (\sqrt{\cos(dx + c)})^2}{15a^4(a^2 - b^2)^2 d} \\ & + \frac{2(128Ab^6 - 40a^5bB + 140a^3b^3B - 80a^2b^5B + 5a^4b^2(11A - 15C) - 4a^2b^4(53A - 10C) + 3a^6(3A + 5C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\frac{\cos(dx + c)}{2} + \frac{1}{2}\right)}{15 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^5 (a^2 - b^2)^2 d \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx+c)^2 \sec(dx+c)^2 + B \cos(dx+c)^2 \sec(dx+c) + A \cos(dx+c)^2 \right) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3} \right)$$

69.300 Problem number 1369

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A + B \sec(c+dx) + C \sec^2(c+dx))}{(a + b \sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a(a^2 - b^2) d(a + b \sec(dx+c))^{\frac{3}{2}}} \\ & + \frac{2(10a^2Ab^2 - 6Ab^4 - 7a^3bB + 3ab^3B + 4a^4C) \sin(dx+c) (\sqrt{\cos(dx+c)})}{3a^2(a^2 - b^2)^2 d \sqrt{a + b \sec(dx+c)}} \\ & - \frac{2(16Ab^4 + 9a^3bB - 8ab^3B - 2a^2b^2(8A - C) - a^4(A + 3C)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2 - b^2) d \sqrt{\cos(dx+c)} \sqrt{a + b \sec(dx+c)}} \\ & + \frac{2(8Ab^4 + 8a^3bB - 4ab^3B + a^4(A - 5C) - a^2b^2(13A - C)) \sin(dx+c) (\sqrt{\cos(dx+c)}) \sqrt{a + b \sec(dx+c)}}{3a^3(a^2 - b^2)^2 d} \\ & - \frac{2(16Ab^5 - 3Ba^5 + 15Ba^3b^2 - 8Ba^4 - 2a^2b^3(14A - C) + a^4(8Ab - 6Cb)) \sqrt{\frac{\cos(dx+c)}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^4 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx+c)}{a + b}}} \end{aligned}$$

command

`integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\left(C \cos(dx+c) \sec(dx+c)^2 + B \cos(dx+c) \sec(dx+c) + A \cos(dx+c) \right) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3} \right)$$

69.301 Problem number 1370

$$\int \frac{\sqrt{\cos(c+dx)} (A + B \sec(c+dx) + C \sec^2(c+dx))}{(a + b \sec(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) \sin(dx + c)}{3a(a^2 - b^2) d(a + b \sec(dx + c))^{\frac{3}{2}} \sqrt{\cos(dx + c)}} \\ & - \frac{2(4Ab^4 + 5a^3bB - ab^3B - 2a^4C - 2a^2b^2(4A + C)) \sin(dx + c)}{3a^2(a^2 - b^2)^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(8Ab^3 + 3a^3B - 2Ba^2b^2 - a^2b(9A + C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ & + \frac{2(8Ab^4 + 6a^3bB - 2ab^3B + 3a^4(A - C) - a^2b^2(15A + C)) \sqrt{\frac{\cos(dx + c)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a+b}}\right) \sqrt{\frac{b+a \cos(dx+c)}{a+b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 (a^2 - b^2)^2 d \sqrt{\frac{b+a \cos(dx+c)}{a+b}}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(2Ca^6b - 5Ba^5b^2 + 2(4A + C)a^4b^3 + Ba^3b^4 - 4Aa^2b^5 + (3Ca^7 - 6Ba^6b + (9A + C)a^5b^2 + 2Ba^4b^3 - 5Aa^3b^4))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x\right)$$

69.302 Problem number 1371

$$\int \frac{A + B \sec(c + dx) + C \sec^2(c + dx)}{\sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab^2 - a(bB - aC)) \sin(dx + c)}{3b(a^2 - b^2) d(a + b \sec(dx + c))^{\frac{3}{2}} \sqrt{\cos(dx + c)}} \\ + & \frac{2(Ab^4 + 2a^3bB + 2ab^3B + a^4C - 5a^2b^2(A + C)) \sin(dx + c)}{3ab(a^2 - b^2)^2 d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ - & \frac{2(2Ab^2 + abB - a^2(3A + C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) \sqrt{\frac{b + a \cos(dx + c)}{a + b}}}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2) d \sqrt{\cos(dx + c)} \sqrt{a + b \sec(dx + c)}} \\ - & \frac{2(2Ab^3 + 3a^3B + Ba^2b - 2a^2b(3A + 2C)) \sqrt{\frac{\cos(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \sqrt{\frac{a}{a + b}}\right) (\sqrt{\cos(dx + c)})}{3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos(dx + c)}{a + b}}} \end{aligned}$$

command

`integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`
 Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(Ca^6 + 2Ba^5b - 5(A + C)a^4b^2 + 2Ba^3b^3 + Aa^2b^4 + (3Ba^6 - 2(3A + 2C)a^5b + Ba^4b^2 + 2Aa^3b^3) \cos(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(C \sec(dx + c)^2 + B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^3 \cos(dx + c) \sec(dx + c)^3 + 3ab^2 \cos(dx + c) \sec(dx + c)^2 + 3a^2b \cos(dx + c) \sec(dx + c) + a^3 \cos(dx + c)}\right)$$

70 Test file number 126

Test folder name:

test_cases/4_Trig_functions/4.5_Secant/126_4.5.7-d_trig-^m-a+b-c_sec-ⁿ-^p

70.1 Problem number 259

$$\int \frac{\sec(e + fx)}{\sqrt{a + b \sec^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{EllipticF}\left(\sin(fx + e), \sqrt{\frac{a}{a+b}}\right) \sqrt{1 - \frac{a(\sin^2(fx + e))}{a+b}}}{f \sqrt{2 \cos(2fx + 2e) + 2} \sqrt{(\sec^2(fx + e))(a + b - a(\sin^2(fx + e)))}}$$

command

```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(2i a^{\frac{3}{2}} \sqrt{\frac{ab+b^2}{a^2}} + \sqrt{a}(ia + 2ib)\right) \sqrt{\frac{2a \sqrt{\frac{ab+b^2}{a^2}} - a - 2b}{a}} \operatorname{ellipticF}\left(\sqrt{\frac{2a \sqrt{\frac{ab+b^2}{a^2}} - a - 2b}{a}} (\cos(fx + e)), \sqrt{\frac{ab+b^2}{a^2}}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec(fx + e)}{\sqrt{b \sec^2(fx + e) + a}}, x\right)$$

71 Test file number 127

Test folder name:

test_cases/4_Trig_functions/4.6_Cosecant/127_4.6.0-a_csc-^m-b_trg-ⁿ

71.1 Problem number 9

$$\int \csc^{\frac{7}{2}}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cos (bx+a)\left(\csc^{\frac{5}{2}}(bx+a)\right)}{5b}-\frac{6 \cos (bx+a)\left(\sqrt{\csc (bx+a)}\right)}{5b} \\ & +\frac{6 \sqrt{\frac{1}{2}+\frac{\sin (bx+a)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{a}{2}+\frac{\pi}{4}+\frac{bx}{2}\right), \sqrt{2}\right)\left(\sqrt{\csc (bx+a)}\right)\left(\sqrt{\sin (bx+a)}\right)}{5 \sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{bx}{2}\right) b} \end{aligned}$$

command

```
integrate(csc(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \sqrt{2i}\left(\cos (bx+a)^2-1\right) \operatorname{weierstrassZeta}(4,0, \operatorname{weierstrassPInverse}(4,0, \cos (bx+a)+i \sin (bx+a))) + 3 \sqrt{-2i}}{5(b \csc (bx+a))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\csc (bx+a)^{\frac{7}{2}}, x\right)$$

71.2 Problem number 10

$$\int \csc^{\frac{5}{2}}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cos (bx+a)\left(\csc^{\frac{3}{2}}(bx+a)\right)}{3b} \\ & -\frac{2 \sqrt{\frac{1}{2}+\frac{\sin (bx+a)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{a}{2}+\frac{\pi}{4}+\frac{bx}{2}\right), \sqrt{2}\right)\left(\sqrt{\csc (bx+a)}\right)\left(\sqrt{\sin (bx+a)}\right)}{3 \sin \left(\frac{a}{2}+\frac{\pi}{4}+\frac{bx}{2}\right) b} \end{aligned}$$

command

```
integrate(csc(b*x+a)^(5/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2i} \sin (bx + a) \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + i \sin (bx + a)) + i \sqrt{-2i} \sin (bx + a) \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) - i \sin (bx + a))}{3 b \sin (bx + a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\operatorname{csc}(bx + a)^{\frac{5}{2}}, x\right)$$

71.3 Problem number 11

$$\int \operatorname{csc}^{\frac{3}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{-\frac{2 \cos (bx + a) (\sqrt{\operatorname{csc}}(bx + a))}{b} + 2 \sqrt{\frac{1}{2} + \frac{\sin (bx + a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\operatorname{csc}}(bx + a)) (\sqrt{\sin}(bx + a))}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

`integrate(csc(b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + i \sin (bx + a))) + \sqrt{-2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) - i \sin (bx + a)))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\operatorname{csc}(bx + a)^{\frac{3}{2}}, x\right)$$

71.4 Problem number 12

$$\int \sqrt{\csc(a + bx)} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\csc}(bx + a)) (\sqrt{\sin}(bx + a))}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(csc(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a)) + i\sqrt{-2i} \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) - i \sin(bx + a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{\csc(bx + a)}, x\right)$$

71.5 Problem number 13

$$\int \frac{1}{\sqrt{\csc(a + bx)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\csc}(bx + a)) (\sqrt{\sin}(bx + a))}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a))) + \sqrt{-2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) - i \sin(bx + a)))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{\csc(bx + a)}}, x\right)$$

71.6 Problem number 14

$$\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{\frac{2 \cos (bx+a)}{3b \sqrt{\csc (bx+a)}} - 2 \sqrt{\frac{1}{2} + \frac{\sin (bx+a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\csc (bx+a)}) (\sqrt{\sin (bx+a)})}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/csc(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \cos (bx+a) \sqrt{\sin (bx+a)} + i \sqrt{2i} \operatorname{weierstrassPInverse}(4, 0, \cos (bx+a) + i \sin (bx+a)) - i \sqrt{-2i} \operatorname{weierstrassPInverse}(4, 0, \cos (bx+a) - i \sin (bx+a))}{3 b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\csc (bx+a)^{\frac{3}{2}}}, x\right)$$

71.7 Problem number 15

$$\int \frac{1}{\csc^{\frac{5}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{\frac{2 \cos (bx+a)}{5b \csc (bx+a)^{\frac{3}{2}}} - 6 \sqrt{\frac{1}{2} + \frac{\sin (bx+a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\csc (bx+a)}) (\sqrt{\sin (bx+a)})}{5 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/csc(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \sqrt{2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + i \sin (bx + a))) + 3 \sqrt{-2i} \operatorname{weierstrassZeta}(4, 0,$$

 $5b$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\csc (bx + a)^{\frac{5}{2}}}, x\right)$$

71.8 Problem number 16

$$\int \frac{1}{\csc^{\frac{7}{2}}(a + bx)} dx$$

Optimal antiderivative

$$\frac{-\frac{2 \cos (bx + a)}{7b \csc (bx + a)^{\frac{5}{2}}} - \frac{10 \cos (bx + a)}{21b \sqrt{\csc (bx + a)}}}{10 \sqrt{\frac{1}{2} + \frac{\sin (bx + a)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) (\sqrt{\csc (bx + a)}) (\sqrt{\sin (bx + a)})} \frac{1}{21 \sin \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

`integrate(1/csc(b*x+a)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \cos (bx + a)^3 - 8 \cos (bx + a)\right) \sqrt{\sin (bx + a)} - 5i \sqrt{2i} \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + i \sin (bx + a))$$

 $21b$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\csc (bx + a)^{\frac{7}{2}}}, x\right)$$

71.9 Problem number 17

$$\int (c \csc(a + bx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2c \cos(bx + a) (c \csc(bx + a))^{\frac{5}{2}}}{5b} - \frac{6c^3 \cos(bx + a) \sqrt{c \csc(bx + a)}}{5b} \\ & + \frac{6c^4 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b \sqrt{c \csc(bx + a)} \sqrt{\sin(bx + a)}} \end{aligned}$$

command

```
integrate((c*csc(b*x+a))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(c^3 \cos(bx + a)^2 - c^3 \right) \sqrt{2ic} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a))) + 3 \left($$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{c \csc(bx + a)} c^3 \csc(bx + a)^3, x\right)$$

71.10 Problem number 18

$$\int (c \csc(a + bx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2c \cos(bx + a) (c \csc(bx + a))^{\frac{3}{2}}}{3b} \\ & - \frac{2c^2 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \csc(bx + a)} \left(\sqrt{\sin(bx + a)}\right)}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b} \end{aligned}$$

command

```
integrate((c*csc(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2i c} c^2 \sin (bx+a) \text{weierstrassPInverse}(4,0, \cos (bx+a)+i \sin (bx+a))+i \sqrt{-2i c} c^2 \sin (bx+a) \text{weierstrassPInverse}(4,0, \cos (bx+a)-i \sin (bx+a))}{3 b \sin (bx+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{c \csc (bx+a)} c^2 \csc (bx+a)^2, x\right)$$

71.11 Problem number 19

$$\int (c \csc (a+bx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{2 c \cos (bx+a) \sqrt{c \csc (bx+a)}}{b} + \frac{2 c^2 \sqrt{\frac{1}{2} + \frac{\sin (bx+a)}{2}} \text{EllipticE}\left(\cos \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{\sin \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b \sqrt{c \csc (bx+a)} \sqrt{\sin (bx+a)}}$$

command

`integrate((c*csc(b*x+a))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 c \sqrt{\frac{c}{\sin (bx+a)}} \cos (bx+a) + \sqrt{2i c} c \text{weierstrassZeta}(4,0, \text{weierstrassPInverse}(4,0, \cos (bx+a)+i \sin (bx+a)))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{c \csc (bx+a)} c \csc (bx+a), x\right)$$

71.12 Problem number 20

$$\int \sqrt{c \csc (a+bx)} dx$$

Optimal antiderivative

$$-\frac{2 \sqrt{\frac{1}{2} + \frac{\sin (bx+a)}{2}} \text{EllipticF}\left(\cos \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \csc (bx+a)} \left(\sqrt{\sin (bx+a)}\right)}{\sin \left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b}$$

command

```
integrate((c*csc(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2ic}\operatorname{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{-2ic}\operatorname{weierstrassPInverse}(4,0,\cos(bx+a)-i\sin(bx+a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{c\csc(bx+a)},x\right)$$

71.13 Problem number 21

$$\int \frac{1}{\sqrt{c\csc(a+bx)}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{1}{2}+\frac{\sin(bx+a)}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{a}{2}+\frac{\pi}{4}+\frac{bx}{2}\right),\sqrt{2}\right)}{\sin\left(\frac{a}{2}+\frac{\pi}{4}+\frac{bx}{2}\right)b\sqrt{c\csc(bx+a)}\sqrt{\sin(bx+a)}}$$

command

```
integrate(1/(c*csc(b*x+a))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2ic}\operatorname{weierstrassZeta}(4,0,\operatorname{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a)))+\sqrt{-2ic}\operatorname{weierstrassZeta}(4,0,\operatorname{weierstrassPInverse}(4,0,\cos(bx+a)-i\sin(bx+a)))}{bc}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c\csc(bx+a)}}{c\csc(bx+a)},x\right)$$

71.14 Problem number 22

$$\int \frac{1}{(c \csc(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2 \cos(bx + a)}{3bc \sqrt{c \csc(bx + a)}} - 2 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \csc(bx + a)} \left(\sqrt{\sin(bx + a)}\right)}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) bc^2}$$

command

```
integrate(1/(c*csc(b*x+a))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{\frac{c}{\sin(bx + a)}} \cos(bx + a) \sin(bx + a) + i \sqrt{2ic} \operatorname{weierstrassPInverse}(4, 0, \cos(bx + a) + i \sin(bx + a)) - i \sqrt{-}}{3bc^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \csc(bx + a)}}{c^2 \csc(bx + a)^2}, x\right)$$

71.15 Problem number 23

$$\int \frac{1}{(c \csc(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2 \cos(bx + a)}{5bc (c \csc(bx + a))^{3/2}} - \frac{6 \sqrt{\frac{1}{2} + \frac{\sin(bx + a)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) bc^2 \sqrt{c \csc(bx + a)} \sqrt{\sin(bx + a)}}$$

command

```
integrate(1/(c*csc(b*x+a))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\cos (bx + a)^3 - \cos (bx + a) \right) \sqrt{\frac{c}{\sin (bx + a)}} + 3 \sqrt{2i c} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + c))}{5 bc^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \csc (bx + a)}}{c^3 \csc (bx + a)^3}, x\right)$$

71.16 Problem number 24

$$\int \frac{1}{(c \csc(a + bx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{-\frac{2 \cos (bx + a)}{7bc (c \csc (bx + a))^{\frac{5}{2}}} - \frac{10 \cos (bx + a)}{21b c^3 \sqrt{c \csc (bx + a)}}}{10 \sqrt{\frac{1}{2} + \frac{\sin (bx + a)}{2}}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{c \csc (bx + a)} \left(\sqrt{\sin (bx + a)}\right)}{21 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{bx}{2}\right) b c^4}$$

command

```
integrate(1/(c*csc(b*x+a))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \cos (bx + a)^3 - 8 \cos (bx + a) \right) \sqrt{\frac{c}{\sin (bx + a)}} \sin (bx + a) - 5i \sqrt{2i c} \operatorname{weierstrassPInverse}(4, 0, \cos (bx + a) + c)}{21 bc^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c \csc (bx + a)}}{c^4 \csc (bx + a)^4}, x\right)$$

71.17 Problem number 55

$$\int (a \csc^3(x))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{154a^2 \cot(x) \sqrt{a(\csc^3(x))}}{585} - \frac{22a^2 \cot(x) (\csc^2(x)) \sqrt{a(\csc^3(x))}}{117} \\ & - \frac{2a^2 \cot(x) (\csc^4(x)) \sqrt{a(\csc^3(x))}}{13} - \frac{154a^2 \cos(x) \sin(x) \sqrt{a(\csc^3(x))}}{195} \\ & + \frac{154a^2 \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right) \left(\sin^{\frac{3}{2}}(x)\right) \sqrt{a(\csc^3(x))}}{195 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right)} \end{aligned}$$

command

```
integrate((a*csc(x)^3)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$231 \left(a^2 \cos(x)^4 - 2a^2 \cos(x)^2 + a^2 \right) \sqrt{2ia} \sin(x) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(x) + i \sin(x)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \csc(x)^3} a^2 \csc(x)^6, x\right)$$

71.18 Problem number 56

$$\int (a \csc^3(x))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{10a \cos(x) \sqrt{a(\csc^3(x))}}{21} - \frac{2a \cot(x) \csc(x) \sqrt{a(\csc^3(x))}}{7} \\ & - \frac{10a \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right) \left(\sin^{\frac{3}{2}}(x)\right) \sqrt{a(\csc^3(x))}}{21 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right)} \end{aligned}$$

command

`integrate((a*csc(x)^3)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(i a \cos(x)^2 - i a \right) \sqrt{2i a} \operatorname{weierstrassPInverse}(4, 0, \cos(x) + i \sin(x)) + 5 \left(-i a \cos(x)^2 + i a \right) \sqrt{-2i a} \operatorname{weierstrassPInverse}(4, 0, \cos(x) - i \sin(x))$$

$$21 \left(\cos(x)^2 - 1 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \csc(x)^3} a \csc(x)^3, x\right)$$

71.19 Problem number 57

$$\int \sqrt{a \csc^3(x)} dx$$

Optimal antiderivative

$$\begin{aligned} & -2 \cos(x) \sin(x) \sqrt{a (\csc^3(x))} \\ & + \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right) \left(\sin^{\frac{3}{2}}(x)\right) \sqrt{a (\csc^3(x))}}{\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)} \end{aligned}$$

command

`integrate((a*csc(x)^3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -2 \sqrt{-\frac{a}{(\cos(x)^2 - 1) \sin(x)}} \cos(x) \sin(x) \\ & - \sqrt{2i a} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(x) + i \sin(x))) \\ & - \sqrt{-2i a} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(x) - i \sin(x))) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \csc(x)^3}, x\right)$$

71.20 Problem number 58

$$\int \frac{1}{\sqrt{a \csc^3(x)}} dx$$

Optimal antiderivative

$$-\frac{2 \cot(x)}{3 \sqrt{a (\csc^3(x))}} - \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \sin(x)^{\frac{3}{2}} \sqrt{a (\csc^3(x))}}$$

command

```
integrate(1/(a*csc(x)^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\cos(x)^3 - \cos(x) \right) \sqrt{-\frac{a}{(\cos(x)^2 - 1) \sin(x)}} - i \sqrt{2i a} \operatorname{weierstrassPInverse}(4, 0, \cos(x) + i \sin(x)) + i \sqrt{-2i a}}{3 a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \csc(x)^3}}{a \csc(x)^3}, x\right)$$

71.21 Problem number 59

$$\int \frac{1}{(a \csc^3(x))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{14 \cos(x)}{45 a \sqrt{a (\csc^3(x))}} - \frac{14 \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right)}{15 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) a \sin(x)^{\frac{3}{2}} \sqrt{a (\csc^3(x))}} - \frac{2 \cos(x) (\sin^2(x))}{9 a \sqrt{a (\csc^3(x))}}$$

command

```
integrate(1/(a*csc(x)^3)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \cos(x)^5 - 17 \cos(x)^3 + 12 \cos(x) \right) \sqrt{-\frac{a}{(\cos(x)^2 - 1) \sin(x)}} \sin(x) - 21 \sqrt{2i a} \operatorname{weierstrassZeta}(4, 0, \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{a \csc(x)^3}}{a^2 \csc(x)^6}, x \right)$$

71.22 Problem number 60

$$\int \frac{1}{(a \csc^3(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{26 \cot(x)}{77a^2 \sqrt{a (\csc^3(x))}} - \frac{26 \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{\pi}{4} + \frac{x}{2} \right), \sqrt{2} \right)}{77 \sin \left(\frac{\pi}{4} + \frac{x}{2} \right) a^2 \sin(x)^{\frac{3}{2}} \sqrt{a (\csc^3(x))}} \\ & - \frac{78 \cos(x) \sin(x)}{385a^2 \sqrt{a (\csc^3(x))}} - \frac{26 \cos(x) (\sin^3(x))}{165a^2 \sqrt{a (\csc^3(x))}} - \frac{2 \cos(x) (\sin^5(x))}{15a^2 \sqrt{a (\csc^3(x))}} \end{aligned}$$

command

```
integrate(1/(a*csc(x)^3)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(77 \cos(x)^9 - 399 \cos(x)^7 + 852 \cos(x)^5 - 1010 \cos(x)^3 + 480 \cos(x) \right) \sqrt{-\frac{a}{(\cos(x)^2 - 1) \sin(x)}} + 195i \sqrt{2}$$

1155 a

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{a \csc(x)^3}}{a^3 \csc(x)^9}, x \right)$$

72 Test file number 133

Test folder name:

test_cases/4_Trig_functions/4.6_Cosecant/133_4.6.4.2-a+b_csc-^m-d_csc-^n-A+B_csc+C_csc^2-

72.1 Problem number 1

$$\int \frac{(a + b \csc(x)) (A + B \csc(x) + C \csc^2(x))}{\sqrt{\csc(x)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2bC \cos(x) \left(\csc^{\frac{3}{2}}(x)\right)}{3} - 2(bB + aC) \cos(x) \left(\sqrt{\csc(x)}\right) \\ & + \frac{2(bB - a(A - C)) \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right) \left(\sqrt{\csc(x)}\right) \left(\sqrt{\sin(x)}\right)}{\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)} \\ & - \frac{2(3Ab + 3Ba + Cb) \sqrt{\frac{1}{2} + \frac{\sin(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{x}{2}\right), \sqrt{2}\right) \left(\sqrt{\csc(x)}\right) \left(\sqrt{\sin(x)}\right)}{3 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right)} \end{aligned}$$

command

```
integrate((a+b*csc(x))*(A+B*csc(x)+C*csc(x)^2)/csc(x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$\sqrt{2i} (-3iBa - i(3A + C)b) \sin(x) \operatorname{weierstrassPInverse}(4, 0, \cos(x) + i \sin(x)) + \sqrt{-2i} (3iBa + i(3A + C)b) \sin(x)$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{Cb \csc(x)^3 + (Ca + Bb) \csc(x)^2 + Aa + (Ba + Ab) \csc(x)}{\sqrt{\csc(x)}}, x\right)$$

73 Test file number 135

Test folder name:

test_cases/4_Trig_functions/4.7_Miscellaneous/135_4.7.1-c_trig-^m-d_trig-ⁿ

73.1 Problem number 87

$$\int \frac{\sin^2(a + bx)}{\sin^{\frac{5}{2}}(2a + 2bx)} dx$$

Optimal antiderivative

$$-\frac{\sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right)}{6 \sin\left(a + \frac{\pi}{4} + bx\right) b} + \frac{\sin^2(bx + a)}{3b \sin(2bx + 2a)^{\frac{3}{2}}}$$

command

```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2i} \cos(bx + a)^2 \operatorname{ellipticF}(\cos(bx + a) + i \sin(bx + a), -1) + \sqrt{-2i} \cos(bx + a)^2 \operatorname{ellipticF}(\cos(bx + a) - i \sin(bx + a), -1)}{12 b \cos(bx + a)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(bx + a)^2 - 1}{(\cos(2bx + 2a)^2 - 1) \sqrt{\sin(2bx + 2a)}}, x\right)$$

73.2 Problem number 110

$$\int \frac{\csc^2(a + bx)}{\sqrt{\sin(2a + 2bx)}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{\frac{1}{2} + \frac{\sin(2bx + 2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right)}{3 \sin\left(a + \frac{\pi}{4} + bx\right) b} - \frac{(\csc^2(bx + a)) (\sqrt{\sin(2bx + 2a)})}{3b}$$

command

`integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2i} \left(\cos (bx+a)^2 - 1 \right) \text{ellipticF}(\cos (bx+a) + i \sin (bx+a), -1) + \sqrt{-2i} \left(\cos (bx+a)^2 - 1 \right) \text{ellipticF}(\cos (bx+a) - i \sin (bx+a), -1)}{3 \left(b \cos (bx+a)^2 - b \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\csc (bx+a)^2}{\sqrt{\sin (2bx+2a)}}, x \right)$$

73.3 Problem number 112

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{10 \sqrt{\frac{1}{2} + \frac{\sin(2bx+2a)}{2}} \text{EllipticF} \left(\cos \left(a + \frac{\pi}{4} + bx \right), \sqrt{2} \right)}{21 \sin \left(a + \frac{\pi}{4} + bx \right) b} \\ & - \frac{10 \cos(2bx+2a)}{21b \sin(2bx+2a)^{\frac{3}{2}}} - \frac{\csc^2(bx+a)}{7b \sin(2bx+2a)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{20 \sqrt{2i} \left(\cos (bx+a)^6 - 2 \cos (bx+a)^4 + \cos (bx+a)^2 \right) \text{ellipticF}(\cos (bx+a) + i \sin (bx+a), -1) + 20 \sqrt{-2i} \left(\cos (bx+a)^6 - 2 \cos (bx+a)^4 + \cos (bx+a)^2 \right) \text{ellipticF}(\cos (bx+a) - i \sin (bx+a), -1)}{8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(- \frac{\csc (bx+a)^2}{\left(\cos (2bx+2a)^2 - 1 \right) \sqrt{\sin (2bx+2a)}}, x \right)$$

73.4 Problem number 114

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)} dx$$

Optimal antiderivative

$$\frac{30 \sqrt{\frac{1}{2} + \frac{\sin(2bx+2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right)}{77 \sin\left(a + \frac{\pi}{4} + bx\right) b} - \frac{18 \cos(2bx+2a)}{77b \sin(2bx+2a)^{\frac{7}{2}}} - \frac{\csc^2(bx+a)}{11b \sin(2bx+2a)^{\frac{7}{2}}} - \frac{30 \cos(2bx+2a)}{77b \sin(2bx+2a)^{\frac{3}{2}}}$$

command

```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{240 \sqrt{2} i \left(\cos(bx+a)^{10} - 3 \cos(bx+a)^8 + 3 \cos(bx+a)^6 - \cos(bx+a)^4 \right) \operatorname{ellipticF}(\cos(bx+a) + i \sin(bx+a))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\csc(bx+a)^2}{\left(\cos(2bx+2a)^4 - 2 \cos(2bx+2a)^2 + 1\right) \sqrt{\sin(2bx+2a)}}, x\right)$$

73.5 Problem number 175

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{2} + \frac{\sin(2bx+2a)}{2}} \operatorname{EllipticF}\left(\cos\left(a + \frac{\pi}{4} + bx\right), \sqrt{2}\right)}{6 \sin\left(a + \frac{\pi}{4} + bx\right) b} - \frac{\cos^2(bx+a)}{3b \sin(2bx+2a)^{\frac{3}{2}}}$$

command

```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2i} \left(\cos (bx+a)^2 - 1 \right) \text{ellipticF}(\cos (bx+a) + i \sin (bx+a), -1) + \sqrt{-2i} \left(\cos (bx+a)^2 - 1 \right) \text{ellipticF}(\cos (bx+a) - i \sin (bx+a), -1)}{12 \left(b \cos (bx+a)^2 - b \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\cos (bx+a)^2}{\left(\cos (2bx+2a)^2 - 1 \right) \sqrt{\sin (2bx+2a)}}, x \right)$$

74 Test file number 139

Test folder name:

test_cases/4_Trig_functions/4.7_Miscellaneous/139_4.7.5_x^m_trig-a+b_log-c_x^n~p

74.1 Problem number 55

$$\int \frac{\sqrt{\sin (a+b \log (cx^n))}}{x} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{1}{2} + \frac{\sin (a+b \ln (cx^n))}{2}} \text{EllipticE} \left(\cos \left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln (cx^n)}{2} \right), \sqrt{2} \right)}{\sin \left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln (cx^n)}{2} \right) bn}$$

command

```
integrate(sin(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$i \sqrt{2} \sqrt{-i} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos (bn \log (x) + b \log (c) + a) + i \sin (bn \log (x) + b \log (c) + a)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\sin (b \log (cx^n) + a)}}{x}, x \right)$$

74.2 Problem number 60

$$\int \frac{\sin^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(a + b \ln(cx^n))}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right) bn} - \frac{2 \cos(a + b \ln(cx^n)) \left(\sqrt{\sin(a + b \ln(cx^n))}\right)}{3bn}$$

command

```
integrate(sin(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-i} \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a) + i \sin(bn \log(x) + b \log(c) + a)) + \sqrt{2} \sqrt{i} \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a) - i \sin(bn \log(x) + b \log(c) + a))}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sin(b \log(cx^n) + a)^{\frac{3}{2}}}{x}, x\right)$$

74.3 Problem number 64

$$\int \frac{1}{x \sqrt{\sin(a + b \log(cx^n))}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(a + b \ln(cx^n))}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(1/x/sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-i} \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a) + i \sin(bn \log(x) + b \log(c) + a)) + \sqrt{2} \sqrt{i} \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a) - i \sin(bn \log(x) + b \log(c) + a))}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \sqrt{\sin(b \log(cx^n) + a)}}, x\right)$$

74.4 Problem number 66

$$\int \frac{1}{x \sin^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(a + b \ln(cx^n))}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right) bn} - \frac{2 \cos(a + b \ln(cx^n))}{bn \sqrt{\sin(a + b \ln(cx^n))}}$$

command

```
integrate(1/x/sin(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \sqrt{-i} \sin(bn \log(x) + b \log(c) + a) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{\sin(b \log(cx^n) + a)}}{x \cos(b \log(cx^n) + a)^2 - x}, x\right)$$

74.5 Problem number 68

$$\int \frac{1}{x \sin^{\frac{5}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{1}{2} + \frac{\sin(a + b \ln(cx^n))}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right) bn} - \frac{2 \cos(a + b \ln(cx^n))}{3bn \sin(a + b \ln(cx^n))^{\frac{3}{2}}}$$

command

```
integrate(1/x/sin(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} \sqrt{-i} \cos(bn \log(x) + b \log(c) + a)^2 - \sqrt{2} \sqrt{-i}\right) \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a) + i)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{1}{\left(x \cos(b \log(cx^n) + a)^2 - x\right) \sqrt{\sin(b \log(cx^n) + a)}}, x\right)$$

74.6 Problem number 111

$$\int \frac{\sqrt{\cos(a + b \log(cx^n))}}{x} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(a + b \ln(cx^n))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(cos(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$i \sqrt{2}$ weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(bn log(x) + b log(c) + a) + i sin(bn log(x) + b log(c) -

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\cos(b \log(cx^n) + a)}}{x}, x\right)$$

74.7 Problem number 113

$$\int \frac{\cos^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(a + b \ln(cx^n))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn} + \frac{2 \sin(a + b \ln(cx^n)) (\sqrt{\cos(a + b \ln(cx^n))})}{3bn}$$

command

```
integrate(cos(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$2 \sqrt{\cos(bn \log(x) + b \log(c) + a)} \sin(bn \log(x) + b \log(c) + a) - i \sqrt{2}$ weierstrassPInverse(-4, 0, cos(bn log(x) +

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(b \log(cx^n) + a)^{\frac{3}{2}}}{x}, x\right)$$

74.8 Problem number 115

$$\int \frac{\cos^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{6\sqrt{\frac{\cos(a + b \ln(cx^n))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn} + \frac{2\left(\cos^{\frac{3}{2}}(a + b \ln(cx^n))\right) \sin(a + b \ln(cx^n))}{5bn}$$

command

```
integrate(cos(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \cos(bn \log(x) + b \log(c) + a)^{\frac{3}{2}} \sin(bn \log(x) + b \log(c) + a) + 3i \sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(bn \log(x) + b \log(c) + a))}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(b \log(cx^n) + a)^{\frac{5}{2}}}{x}, x\right)$$

74.9 Problem number 117

$$\int \frac{1}{x \sqrt{\cos(a + b \log(cx^n))}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(a + b \ln(cx^n))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(1/x/cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bn \log(x) + b \log(c) + a)) + i \sin(bn \log(x) + b \log(c) + a) + i \sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(bn \log(x) + b \log(c) + a))}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \sqrt{\cos(b \log(cx^n) + a)}}, x\right)$$

74.10 Problem number 119

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(a + b \ln(cx^n))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn} + \frac{2 \sin(a + b \ln(cx^n))}{bn \sqrt{\cos(a + b \ln(cx^n))}}$$

command

```
integrate(1/x/cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \cos(bn \log(x) + b \log(c) + a) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bn \log(x) + b \log(c) + a)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \cos(b \log(cx^n) + a)^{\frac{3}{2}}}, x\right)$$

74.11 Problem number 121

$$\int \frac{1}{x \cos^{\frac{5}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(a + b \ln(cx^n))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn} + \frac{2 \sin(a + b \ln(cx^n))}{3bn \cos(a + b \ln(cx^n))^{\frac{3}{2}}}$$

command

```
integrate(1/x/cos(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \cos(bn \log(x) + b \log(c) + a)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(bn \log(x) + b \log(c) + a) + i \sin(bn \log(x) + b \log(c) + a))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \cos(b \log(cx^n) + a)^{\frac{5}{2}}}, x\right)$$

74.12 Problem number 267

$$\int \frac{\sqrt{\sec(a + b \log(cx^n))}}{x} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(a + b \ln(cx^n))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) (\sqrt{\cos(a + b \ln(cx^n))}) (\sqrt{\sec(a + b \ln(cx^n))})}{\cos\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(sec(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cos(bn \log(x) + b \log(c) + a) + i \sin(bn \log(x) + b \log(c) + a)) + i\sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \cos(bn \log(x) + b \log(c) + a) + i \sin(bn \log(x) + b \log(c) + a))}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sec(b \log(cx^n) + a)}}{x}, x\right)$$

74.13 Problem number 269

$$\int \frac{\sec^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2 \sin(a + b \ln(cx^n)) (\sqrt{\sec(a + b \ln(cx^n))})}{bn} - \frac{2\sqrt{\frac{\cos(a + b \ln(cx^n))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) (\sqrt{\cos(a + b \ln(cx^n))}) (\sqrt{\sec(a + b \ln(cx^n))})}{\cos\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(sec(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i\sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(bn \log(x) + b \log(c) + a) + i \sin(bn \log(x) + b \log(c) + a)))}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sec^{\frac{3}{2}}(b \log(cx^n) + a)}{x}, x\right)$$

74.14 Problem number 271

$$\int \frac{\sec^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2 \left(\sec^{\frac{3}{2}}(a + b \ln(cx^n)) \right) \sin(a + b \ln(cx^n))}{3bn} + \frac{2 \sqrt{\frac{\cos(a + b \ln(cx^n))}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(\frac{a}{2} + \frac{b \ln(cx^n)}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(a + b \ln(cx^n))) (\sqrt{\sec}(a + b \ln(cx^n)))}{3 \cos \left(\frac{a}{2} + \frac{b \ln(cx^n)}{2} \right) bn}$$

command

```
integrate(sec(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2} \cos(bn \log(x) + b \log(c) + a) \operatorname{weierstrassPInverse}(-4, 0, \cos(bn \log(x) + b \log(c) + a) + i \sin(bn \log(x) + b \log(c) + a))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sec(b \log(cx^n) + a)^{\frac{5}{2}}}{x}, x \right)$$

74.15 Problem number 273

$$\int \frac{1}{x \sqrt{\sec(a + b \log(cx^n))}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(a + b \ln(cx^n))}{2}} + \frac{1}{2} \operatorname{EllipticE} \left(\sin \left(\frac{a}{2} + \frac{b \ln(cx^n)}{2} \right), \sqrt{2} \right) (\sqrt{\cos}(a + b \ln(cx^n))) (\sqrt{\sec}(a + b \ln(cx^n)))}{\cos \left(\frac{a}{2} + \frac{b \ln(cx^n)}{2} \right) bn}$$

command

```
integrate(1/x/sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$i\sqrt{2}$ weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(bn log(x) + b log(c) + a) + i sin(bn log(x) + b log(c) -

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{1}{x\sqrt{\sec(b\log(cx^n)+a)}}, x\right)$$

74.16 Problem number 275

$$\int \frac{1}{x \sec^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{2 \sin(a + b \ln(cx^n))}{3bn \sqrt{\sec(a + b \ln(cx^n))}} + \frac{2 \sqrt{\frac{\cos(a + b \ln(cx^n))}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) (\sqrt{\cos(a + b \ln(cx^n))}) (\sqrt{\sec(a + b \ln(cx^n))})}{3 \cos\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

`integrate(1/x/sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$2 \sqrt{\cos(bn \log(x) + b \log(c) + a)} \sin(bn \log(x) + b \log(c) + a) - i \sqrt{2}$ weierstrassPInverse(-4, 0, cos(bn log(x) +

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{1}{x \sec(b \log(cx^n) + a)^{\frac{3}{2}}}, x\right)$$

74.17 Problem number 277

$$\int \frac{1}{x \sec^{\frac{5}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{2 \sin(a + b \ln(cx^n))}{5bn \sec(a + b \ln(cx^n))^{\frac{3}{2}}} + \frac{6 \sqrt{\frac{\cos(a + b \ln(cx^n))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) (\sqrt{\cos(a + b \ln(cx^n))}) (\sqrt{\sec(a + b \ln(cx^n))})}{5 \cos\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

`integrate(1/x/sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \cos(bn \log(x) + b \log(c) + a)^{\frac{3}{2}} \sin(bn \log(x) + b \log(c) + a) + 3i \sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \sec(b \log(cx^n) + a)^{\frac{5}{2}}}, x\right)$$

74.18 Problem number 309

$$\int \frac{\sqrt{\csc(a + b \log(cx^n))}}{x} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{1}{2} + \frac{\sin(a + b \ln(cx^n))}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) (\sqrt{\csc(a + b \ln(cx^n))}) (\sqrt{\sin(a + b \ln(cx^n))})}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

`integrate(csc(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{2i} \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a) + i \sin(bn \log(x) + b \log(c) + a)) + i \sqrt{-2i} \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a) - i \sin(bn \log(x) + b \log(c) + a))}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\csc(b \log(cx^n) + a)}}{x}, x\right)$$

74.19 Problem number 311

$$\int \frac{\csc^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2 \cos(a + b \ln(cx^n)) (\sqrt{\csc}(a + b \ln(cx^n)))}{bn} + \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(a + b \ln(cx^n))}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) (\sqrt{\csc}(a + b \ln(cx^n))) (\sqrt{\sin}(a + b \ln(cx^n)))}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(csc(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a) + i \sin(bn \log(x) + b \log(c) + a)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\csc(b \log(cx^n) + a)^{\frac{3}{2}}}{x}, x\right)$$

74.20 Problem number 313

$$\int \frac{\csc^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2 \cos(a + b \ln(cx^n)) \left(\csc^{\frac{3}{2}}(a + b \ln(cx^n))\right)}{3bn} - \frac{2 \sqrt{\frac{1}{2} + \frac{\sin(a + b \ln(cx^n))}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) (\sqrt{\csc}(a + b \ln(cx^n))) (\sqrt{\sin}(a + b \ln(cx^n)))}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(csc(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \sqrt{2i} \sin(bn \log(x) + b \log(c) + a) \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a) + i \sin(bn \log(x) + b \log(c) + a))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{csc}(b \log(cx^n) + a)^{\frac{5}{2}}}{x}, x\right)$$

74.21 Problem number 315

$$\int \frac{1}{x \sqrt{\operatorname{csc}(a + b \log(cx^n))}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{1}{2} + \frac{\sin(a + b \ln(cx^n))}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) (\sqrt{\operatorname{csc}(a + b \ln(cx^n))}) (\sqrt{\sin(a + b \ln(cx^n))})}{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

`integrate(1/x/csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a) + i \sin(bn \log(x) + b \log(c) + a)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \sqrt{\operatorname{csc}(b \log(cx^n) + a)}}, x\right)$$

74.22 Problem number 317

$$\int \frac{1}{x \operatorname{csc}^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{\frac{2 \cos(a + b \ln(cx^n))}{3bn \sqrt{\csc(a + b \ln(cx^n))}}{2 \sqrt{\frac{1}{2} + \frac{\sin(a + b \ln(cx^n))}{2}}} \operatorname{EllipticF}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) (\sqrt{\csc(a + b \ln(cx^n))}) (\sqrt{\sin(a + b \ln(cx^n))})}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(1/x/csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \cos(bn \log(x) + b \log(c) + a) \sqrt{\sin(bn \log(x) + b \log(c) + a)} + i \sqrt{2i} \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a))}{3 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right) bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \csc(b \log(cx^n) + a)^{\frac{3}{2}}}, x\right)$$

74.23 Problem number 319

$$\int \frac{1}{x \csc^{\frac{5}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{\frac{2 \cos(a + b \ln(cx^n))}{5bn \csc(a + b \ln(cx^n))^{\frac{3}{2}}}}{6 \sqrt{\frac{1}{2} + \frac{\sin(a + b \ln(cx^n))}{2}}} \operatorname{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) (\sqrt{\csc(a + b \ln(cx^n))}) (\sqrt{\sin(a + b \ln(cx^n))})}{5 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(1/x/csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \sqrt{2i} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(bn \log(x) + b \log(c) + a)) + i \sin(bn \log(x) + b \log(c) + a))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \csc(b \log(cx^n) + a)^{\frac{5}{2}}}, x\right)$$

75 Test file number 141

Test folder name:

test_cases/4_Trig_functions/4.7_Miscellaneous/141_4.7.7_Trig_functions

75.1 Problem number 232

$$\int (a \cos(c + dx) + b \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx + c) - a \sin(dx + c))(a \cos(dx + c) + b \sin(dx + c))^{5/2}}{7d} - \frac{10(a^2 + b^2)(b \cos(dx + c) - a \sin(dx + c)) \sqrt{a \cos(dx + c) + b \sin(dx + c)}}{21d} + \frac{10(a^2 + b^2)^2 \sqrt{\frac{\cos(c + dx - \arctan(a, b))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right), \sqrt{2}\right) \sqrt{\frac{a \cos(dx + c) + b \sin(dx + c)}{\sqrt{a^2 + b^2}}}}{21 \cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right) d \sqrt{a \cos(dx + c) + b \sin(dx + c)}}$$

command

`integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \sqrt{2} (i a^3 - a^2 b + i a b^2 - b^3) \sqrt{a - i b} \operatorname{weierstrassPInverse}\left(-\frac{4(a^2 + 2i a b - b^2)}{a^2 + b^2}, 0, \cos(dx + c) + i \sin(dx + c)\right) + 5}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(3 a b^2 \cos(dx + c) + (a^3 - 3 a b^2) \cos(dx + c)^3 + (b^3 + (3 a^2 b - b^3) \cos(dx + c)^2) \sin(dx + c)\right) \sqrt{a \cos(dx + c) + b \sin(dx + c)}\right)$$

75.2 Problem number 233

$$\int (a \cos(c + dx) + b \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx + c) - a \sin(dx + c))(a \cos(dx + c) + b \sin(dx + c))^{3/2}}{5d} + \frac{6(a^2 + b^2) \sqrt{\frac{\cos(c + dx - \arctan(a, b))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right), \sqrt{2}\right) \sqrt{a \cos(dx + c) + b \sin(dx + c)}}{5 \cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right) d \sqrt{\frac{a \cos(dx + c) + b \sin(dx + c)}{\sqrt{a^2 + b^2}}}}$$

command

```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\sqrt{2}(-ia^2 - ib^2)\sqrt{a - ib} \operatorname{weierstrassZeta}\left(-\frac{4(a^2 + 2iab - b^2)}{a^2 + b^2}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4(a^2 + 2iab - b^2)}{a^2 + b^2}, 0, \cos(dx + c)\right), x\right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(2ab \cos(dx + c) \sin(dx + c) + (a^2 - b^2) \cos(dx + c)^2 + b^2\right) \sqrt{a \cos(dx + c) + b \sin(dx + c)}, x\right)$$

75.3 Problem number 234

$$\int (a \cos(c + dx) + b \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx + c) - a \sin(dx + c)) \sqrt{a \cos(dx + c) + b \sin(dx + c)}}{3d} + \frac{2(a^2 + b^2) \sqrt{\frac{\cos(c + dx - \arctan(a, b))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right), \sqrt{2}\right) \sqrt{\frac{a \cos(dx + c) + b \sin(dx + c)}{\sqrt{a^2 + b^2}}}}{3 \cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right) d \sqrt{a \cos(dx + c) + b \sin(dx + c)}}$$

command

```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{a - ib} (-ia + b) \operatorname{weierstrassPInverse}\left(-\frac{4(a^2 + 2iab - b^2)}{a^2 + b^2}, 0, \cos(dx + c) + i \sin(dx + c)\right) + \sqrt{2} \sqrt{a + ib} (ia + b) \operatorname{weierstrassPInverse}\left(-\frac{4(a^2 + 2iab - b^2)}{a^2 + b^2}, 0, \cos(dx + c) - i \sin(dx + c)\right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((a \cos(dx + c) + b \sin(dx + c))^{\frac{3}{2}}, x\right)$$

75.4 Problem number 235

$$\int \sqrt{a \cos(c + dx) + b \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(c + dx - \arctan(a, b))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right), \sqrt{2}\right) \sqrt{a \cos(dx + c) + b \sin(dx + c)}}{\cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right) d \sqrt{\frac{a \cos(dx + c) + b \sin(dx + c)}{\sqrt{a^2 + b^2}}}}$$

command

`integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$i \sqrt{2} \sqrt{a - ib} \operatorname{weierstrassZeta}\left(-\frac{4(a^2 + 2iab - b^2)}{a^2 + b^2}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4(a^2 + 2iab - b^2)}{a^2 + b^2}, 0, \cos(dx + c) + i \sin(dx + c)\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \cos(dx + c) + b \sin(dx + c)}, x\right)$$

75.5 Problem number 236

$$\int \frac{1}{\sqrt{a \cos(c + dx) + b \sin(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(c + dx - \arctan(a, b))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right), \sqrt{2}\right) \sqrt{\frac{a \cos(dx + c) + b \sin(dx + c)}{\sqrt{a^2 + b^2}}}}{\cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right) d \sqrt{a \cos(dx + c) + b \sin(dx + c)}}$$

command

`integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{a - ib} (-ia + b) \operatorname{weierstrassPInverse}\left(-\frac{4(a^2 + 2iab - b^2)}{a^2 + b^2}, 0, \cos(dx + c) + i \sin(dx + c)\right) + \sqrt{2} \sqrt{a + ib} (ia + b) \operatorname{weierstrassPInverse}\left(-\frac{4(a^2 + 2iab - b^2)}{a^2 + b^2}, 0, \cos(dx + c) + i \sin(dx + c)\right)}{(a^2 + b^2)d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{a \cos(dx + c) + b \sin(dx + c)}}, x\right)$$

75.6 Problem number 237

$$\int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx + c) - a \sin(dx + c))}{(a^2 + b^2) d \sqrt{a \cos(dx + c) + b \sin(dx + c)}} - \frac{2 \sqrt{\frac{\cos(c + dx - \arctan(a, b))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right), \sqrt{2}\right) \sqrt{a \cos(dx + c) + b \sin(dx + c)}}{\cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right) (a^2 + b^2) d \sqrt{\frac{a \cos(dx + c) + b \sin(dx + c)}{\sqrt{a^2 + b^2}}}}$$

command

`integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(-i \sqrt{2} a \cos(dx + c) - i \sqrt{2} b \sin(dx + c)) \sqrt{a - ib} \operatorname{weierstrassZeta}\left(-\frac{4(a^2 + 2i ab - b^2)}{a^2 + b^2}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4}{a^2 + b^2}, x\right)\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \cos(dx + c) + b \sin(dx + c)}}{2 ab \cos(dx + c) \sin(dx + c) + (a^2 - b^2) \cos(dx + c)^2 + b^2}, x\right)$$

75.7 Problem number 238

$$\int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx + c) - a \sin(dx + c))}{3(a^2 + b^2) d (a \cos(dx + c) + b \sin(dx + c))^{3/2}} + \frac{2 \sqrt{\frac{\cos(c + dx - \arctan(a, b))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right), \sqrt{2}\right) \sqrt{\frac{a \cos(dx + c) + b \sin(dx + c)}{\sqrt{a^2 + b^2}}}}{3 \cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2}\right) (a^2 + b^2) d \sqrt{a \cos(dx + c) + b \sin(dx + c)}}$$

command

`integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (-i a^3 + a^2 b + i a b^2 - b^3) \cos(dx + c)^2 - 2 \sqrt{2} (i a^2 b - a b^2) \cos(dx + c) \sin(dx + c) + \sqrt{2} (-i a b^2 + b^3) \right) \sqrt{a \cos(dx + c) + b \sin(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{a \cos(dx + c) + b \sin(dx + c)}}{3 a b^2 \cos(dx + c) + (a^3 - 3 a b^2) \cos(dx + c)^3 + (b^3 + (3 a^2 b - b^3) \cos(dx + c)^2) \sin(dx + c)}, x \right)$$

75.8 Problem number 239

$$\int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(b \cos(dx + c) - a \sin(dx + c))}{5(a^2 + b^2) d (a \cos(dx + c) + b \sin(dx + c))^{5/2}} - \frac{6(b \cos(dx + c) - a \sin(dx + c))}{5(a^2 + b^2)^2 d \sqrt{a \cos(dx + c) + b \sin(dx + c)}} \\ - \frac{6 \sqrt{\frac{\cos(c + dx - \arctan(a, b))}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2} \right), \sqrt{2} \right) \sqrt{a \cos(dx + c) + b \sin(dx + c)}}{5 \cos \left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(a, b)}{2} \right) (a^2 + b^2)^2 d \sqrt{\frac{a \cos(dx + c) + b \sin(dx + c)}{\sqrt{a^2 + b^2}}}}$$

command

`integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(-3i \sqrt{2} a b^2 \cos(dx + c) + \sqrt{2} (-i a^3 + 3i a b^2) \cos(dx + c)^3 + (-i \sqrt{2} b^3 + \sqrt{2} (-3i a^2 b + i b^3) \cos(dx + c)^2) \right) \sqrt{a \cos(dx + c) + b \sin(dx + c)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{a \cos(dx + c) + b \sin(dx + c)}}{(a^4 - 6 a^2 b^2 + b^4) \cos(dx + c)^4 + b^4 + 2(3 a^2 b^2 - b^4) \cos(dx + c)^2 + 4(a b^3 \cos(dx + c) + (a^3 b - a b^3) \cos(dx + c)^2)}, x \right)$$

75.9 Problem number 240

$$\int (2 \cos(c + dx) + 3 \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\frac{130 \cdot 13^{\frac{3}{4}} \sqrt{\frac{\cos(c + dx - \arctan(\frac{3}{2}))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right), \sqrt{2}\right)}{21 \cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right) d} - \frac{2(3 \cos(dx + c) - 2 \sin(dx + c))(2 \cos(dx + c) + 3 \sin(dx + c))^{\frac{5}{2}}}{7d} - \frac{130(3 \cos(dx + c) - 2 \sin(dx + c)) \sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}}{21d}$$

command

```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(130i + 195) \sqrt{3i + 2} \sqrt{2} \operatorname{weierstrassPInverse}\left(\frac{48}{13}i + \frac{20}{13}, 0, \cos(dx + c) - i \sin(dx + c)\right) - (130i - 195) \sqrt{2} \sqrt{-3i}}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(46 \cos(dx + c)^3 - 9 \left(\cos(dx + c)^2 + 3\right) \sin(dx + c) - 54 \cos(dx + c)\right) \sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}\right)$$

75.10 Problem number 241

$$\int (2 \cos(c + dx) + 3 \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{78 \cdot 13^{\frac{1}{4}} \sqrt{\frac{\cos(c + dx - \arctan(\frac{3}{2}))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right), \sqrt{2}\right)}{5 \cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right) d} - \frac{2(3 \cos(dx + c) - 2 \sin(dx + c))(2 \cos(dx + c) + 3 \sin(dx + c))^{\frac{3}{2}}}{5d}$$

command

`integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-39i \sqrt{3i+2} \sqrt{2} \text{weierstrassZeta}\left(\frac{48}{13}i + \frac{20}{13}, 0, \text{weierstrassPInverse}\left(\frac{48}{13}i + \frac{20}{13}, 0, \cos(dx+c) - i \sin(dx+c)\right)\right) + 39$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\left(5 \cos(dx+c)^2 - 12 \cos(dx+c) \sin(dx+c) - 9\right) \sqrt{2 \cos(dx+c) + 3 \sin(dx+c)}, x\right)$$

75.11 Problem number 242

$$\int (2 \cos(c+dx) + 3 \sin(c+dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2 \cdot 13^{\frac{3}{4}} \sqrt{\frac{\cos\left(\frac{c+dx - \arctan\left(\frac{3}{2}\right)}{2}\right) + \frac{1}{2}}}{2} \text{EllipticF}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan\left(\frac{3}{2}\right)}{2}\right), \sqrt{2}\right)}{3 \cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan\left(\frac{3}{2}\right)}{2}\right) d} - \frac{2(3 \cos(dx+c) - 2 \sin(dx+c)) \sqrt{2 \cos(dx+c) + 3 \sin(dx+c)}}{3d}$$

command

`integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$(2i+3) \sqrt{3i+2} \sqrt{2} \text{weierstrassPInverse}\left(\frac{48}{13}i + \frac{20}{13}, 0, \cos(dx+c) - i \sin(dx+c)\right) - (2i-3) \sqrt{2} \sqrt{-3i+2} \text{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(2 \cos(dx+c) + 3 \sin(dx+c)\right)^{\frac{3}{2}}, x\right)$$

75.12 Problem number 243

$$\int \sqrt{2 \cos(c + dx) + 3 \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{2 \cdot 13^{\frac{1}{4}} \sqrt{\frac{\cos(c + dx - \arctan(\frac{3}{2}))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right), \sqrt{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right) d}$$

command

```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \sqrt{3i + 2} \sqrt{2} \operatorname{weierstrassZeta}\left(\frac{48}{13}i + \frac{20}{13}, 0, \operatorname{weierstrassPInverse}\left(\frac{48}{13}i + \frac{20}{13}, 0, \cos(dx + c) - i \sin(dx + c)\right)\right) + i \sqrt{2}}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}, x\right)$$

75.13 Problem number 244

$$\int \frac{1}{\sqrt{2 \cos(c + dx) + 3 \sin(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2 \cdot 13^{\frac{3}{4}} \sqrt{\frac{\cos(c + dx - \arctan(\frac{3}{2}))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right), \sqrt{2}\right)}{13 \cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right) d}$$

command

```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(2i + 3) \sqrt{3i + 2} \sqrt{2} \operatorname{weierstrassPInverse}\left(\frac{48}{13}i + \frac{20}{13}, 0, \cos(dx + c) - i \sin(dx + c)\right) - (2i - 3) \sqrt{2} \sqrt{-3i + 2} \operatorname{weierstrassPInverse}\left(\frac{48}{13}i + \frac{20}{13}, 0, \cos(dx + c) - i \sin(dx + c)\right)}{13 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}}, x\right)$$

75.14 Problem number 245

$$\int \frac{1}{(2 \cos(c + dx) + 3 \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \cdot 13^{\frac{1}{4}} \sqrt{\frac{\cos(c + dx - \arctan(\frac{3}{2}))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right), \sqrt{2}\right)}{13 \cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right) d} - \frac{2(3 \cos(dx + c) - 2 \sin(dx + c))}{13d \sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}}$$

command

`integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{3i + 2} \left(2i \sqrt{2} \cos(dx + c) + 3i \sqrt{2} \sin(dx + c) \right) \operatorname{weierstrassZeta}\left(\frac{48}{13}i + \frac{20}{13}, 0, \operatorname{weierstrassPInverse}\left(\frac{48}{13}i + \frac{20}{13}, 0, c\right)\right)}{13d \sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}}{5 \cos(dx + c)^2 - 12 \cos(dx + c) \sin(dx + c) - 9}, x\right)$$

75.15 Problem number 246

$$\int \frac{1}{(2 \cos(c + dx) + 3 \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \cdot 13^{\frac{3}{4}} \sqrt{\frac{\cos(c + dx - \arctan(\frac{3}{2}))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right), \sqrt{2}\right)}{507 \cos\left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2}\right) d} - \frac{2(3 \cos(dx + c) - 2 \sin(dx + c))}{39d (2 \cos(dx + c) + 3 \sin(dx + c))^{\frac{3}{2}}}$$

command

`integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{3i+2} \left(-(10i+15) \sqrt{2} \cos(dx+c)^2 + (24i+36) \sqrt{2} \cos(dx+c) \sin(dx+c) + (18i+27) \sqrt{2} \right)}{\text{weierstrass}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{2 \cos(dx+c) + 3 \sin(dx+c)}}{46 \cos(dx+c)^3 - 9 (\cos(dx+c)^2 + 3) \sin(dx+c) - 54 \cos(dx+c)}, x \right)$$

75.16 Problem number 247

$$\int \frac{1}{(2 \cos(c+dx) + 3 \sin(c+dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{6 \cdot 13^{\frac{1}{4}} \sqrt{\frac{\cos(c+dx - \arctan(\frac{3}{2}))}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2} \right), \sqrt{2} \right)}{845 \cos \left(\frac{c}{2} + \frac{dx}{2} - \frac{\arctan(\frac{3}{2})}{2} \right) d} - \frac{2(3 \cos(dx+c) - 2 \sin(dx+c))}{65d (2 \cos(dx+c) + 3 \sin(dx+c))^{\frac{5}{2}}} - \frac{6(3 \cos(dx+c) - 2 \sin(dx+c))}{845d \sqrt{2 \cos(dx+c) + 3 \sin(dx+c)}}$$

command

`integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \sqrt{3i+2} \left(46i \sqrt{2} \cos(dx+c)^3 + 9 \left(-i \sqrt{2} \cos(dx+c)^2 - 3i \sqrt{2} \right) \sin(dx+c) - 54i \sqrt{2} \cos(dx+c) \right)}{\text{weierstrass}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{2 \cos(dx+c) + 3 \sin(dx+c)}}{119 \cos(dx+c)^4 - 54 \cos(dx+c)^2 + 24 (5 \cos(dx+c)^3 - 9 \cos(dx+c)) \sin(dx+c) - 81}, x \right)$$

75.17 Problem number 403

$$\int (2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5 \cos(ex + d) - 3 \sin(ex + d))(2 + 3 \cos(ex + d) + 5 \sin(ex + d))^{3/2}}{5e} \\ & - \frac{32(5 \cos(ex + d) - 3 \sin(ex + d)) \sqrt{2 + 3 \cos(ex + d) + 5 \sin(ex + d)}}{15e} \\ & + \frac{64 \sqrt{\frac{\cos(ex - \arctan(\frac{5}{3}) + d)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right), \frac{\sqrt{510 - 30\sqrt{34}}}{15}\right)}{\cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right) e \sqrt{2 + \sqrt{34}}} \\ & + \frac{796 \sqrt{\frac{\cos(ex - \arctan(\frac{5}{3}) + d)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right), \frac{\sqrt{510 - 30\sqrt{34}}}{15}\right) \sqrt{2 + \sqrt{34}}}{15 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right) e} \end{aligned}$$

command

```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2}{765} \left(-(1677i + 2795) \sqrt{5i + 3} \sqrt{2} \operatorname{weierstrassPInverse}\left(\frac{860}{289}i + \frac{1376}{867}, -\frac{5480}{132651}i - \frac{12056}{14739}, \cos(xe + d) - i \sin(xe + d)\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\left(16 \cos(ex + d)^2 - 10(3 \cos(ex + d) + 2) \sin(ex + d) - 12 \cos(ex + d) - 29\right) \sqrt{3 \cos(ex + d) + 5 \sin(ex + d)}\right)$$

75.18 Problem number 404

$$\int (2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2} dx$$

Optimal antiderivative

$$\frac{2(5 \cos(ex + d) - 3 \sin(ex + d)) \sqrt{2 + 3 \cos(ex + d) + 5 \sin(ex + d)}}{3e} + \frac{20 \sqrt{\frac{\cos(ex - \arctan(\frac{5}{3}) + d)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right), \frac{\sqrt{510 - 30\sqrt{34}}}{15}\right)}{\cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right) e \sqrt{2 + \sqrt{34}}} + \frac{16 \sqrt{\frac{\cos(ex - \arctan(\frac{5}{3}) + d)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right), \frac{\sqrt{510 - 30\sqrt{34}}}{15}\right) \sqrt{2 + \sqrt{34}}}{3 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right) e}$$

command

```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{153} \left((159i + 265) \sqrt{5i + 3} \sqrt{2} \operatorname{weierstrassPInverse}\left(\frac{860}{289}i + \frac{1376}{867}, -\frac{5480}{132651}i - \frac{12056}{14739}, \cos(xe + d) - i \sin(xe + d)\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{\frac{3}{2}}, x\right)$$

75.19 Problem number 405

$$\int \sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(ex - \arctan(\frac{5}{3}) + d)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right), \frac{\sqrt{510 - 30\sqrt{34}}}{15}\right) \sqrt{2 + \sqrt{34}}}{\cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right) e}$$

command

```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{51} \left((3i + 5) \sqrt{5i + 3} \sqrt{2} \operatorname{weierstrassPInverse}\left(\frac{860}{289}i + \frac{1376}{867}, -\frac{5480}{132651}i - \frac{12056}{14739}, \cos(xe + d) - i \sin(xe + d)\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{3 \cos(ex + d) + 5 \sin(ex + d) + 2}, x\right)$$

75.20 Problem number 406

$$\int \frac{1}{\sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos\left(\frac{ex - \arctan\left(\frac{5}{3}\right) + d\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan\left(\frac{5}{3}\right)}{2}\right), \frac{\sqrt{510 - 30\sqrt{34}}}{15}\right)}{\cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan\left(\frac{5}{3}\right)}{2}\right) e \sqrt{2 + \sqrt{34}}}$$

command

```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{34} \left((3i + 5) \sqrt{5i + 3} \sqrt{2} \operatorname{weierstrassPInverse}\left(\frac{860}{289}i + \frac{1376}{867}, -\frac{5480}{132651}i - \frac{12056}{14739}, \cos(xe + d) - i \sin(xe + d) - \frac{1}{5}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{3 \cos(ex + d) + 5 \sin(ex + d) + 2}}, x\right)$$

75.21 Problem number 407

$$\int \frac{1}{(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{-5 \cos(ex + d) + 3 \sin(ex + d)}{15e \sqrt{2 + 3 \cos(ex + d) + 5 \sin(ex + d)}} \sqrt{\frac{\cos\left(\frac{ex - \arctan\left(\frac{5}{3}\right) + d\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan\left(\frac{5}{3}\right)}{2}\right), \frac{\sqrt{510 - 30\sqrt{34}}}{15}\right) \sqrt{2 + \sqrt{34}}}{15 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan\left(\frac{5}{3}\right)}{2}\right) e}$$

command

```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{5i+3} \left(-(9i+15) \sqrt{2} \cos(xe+d) - (15i+25) \sqrt{2} \sin(xe+d) - (6i+10) \sqrt{2} \right) \text{weierstrassPInverse}\left(\frac{860}{289}i + \right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3 \cos(ex+d) + 5 \sin(ex+d) + 2}}{16 \cos(ex+d)^2 - 10(3 \cos(ex+d) + 2) \sin(ex+d) - 12 \cos(ex+d) - 29}, x\right)$$

75.22 Problem number 408

$$\int \frac{1}{(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{-5 \cos(ex+d) + 3 \sin(ex+d)}{45e(2 + 3 \cos(ex+d) + 5 \sin(ex+d))^{3/2}} + \frac{\frac{4 \cos(ex+d)}{135} - \frac{4 \sin(ex+d)}{225}}{e \sqrt{2 + 3 \cos(ex+d) + 5 \sin(ex+d)}} \\ & + \frac{\sqrt{\frac{\cos(ex - \arctan(\frac{5}{3}) + d)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right), \frac{\sqrt{510 - 30\sqrt{34}}}{15}\right)}{45 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right) e \sqrt{2 + \sqrt{34}}} \\ & + \frac{4 \sqrt{\frac{\cos(ex - \arctan(\frac{5}{3}) + d)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right), \frac{\sqrt{510 - 30\sqrt{34}}}{15}\right) \sqrt{2 + \sqrt{34}}}{675 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right) e} \end{aligned}$$

command

`integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{53 \sqrt{5i+3} \left((48i+80) \sqrt{2} \cos(xe+d)^2 + 10 \left(-(9i+15) \sqrt{2} \cos(xe+d) - (6i+10) \sqrt{2} \right) \sin(xe+d) - (36i+20) \sqrt{2} \right)}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{\sqrt{3 \cos(ex+d) + 5 \sin(ex+d) + 2}}{198 \cos(ex+d)^3 + 96 \cos(ex+d)^2 - 5 \left(2 \cos(ex+d)^2 + 36 \cos(ex+d) + 37 \right) \sin(ex+d) - 261 \cos(ex+d)}, x\right)$$

75.23 Problem number 409

$$\int \frac{1}{(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{7/2}} dx$$

Optimal antiderivative

$$\frac{-5 \cos(ex + d) + 3 \sin(ex + d)}{75e(2 + 3 \cos(ex + d) + 5 \sin(ex + d))^{\frac{5}{2}}} + \frac{\frac{8 \cos(ex+d)}{675} - \frac{8 \sin(ex+d)}{1125}}{e(2 + 3 \cos(ex + d) + 5 \sin(ex + d))^{\frac{3}{2}}}$$

$$\frac{199(5 \cos(ex + d) - 3 \sin(ex + d))}{101250e \sqrt{2 + 3 \cos(ex + d) + 5 \sin(ex + d)}}$$

$$8 \sqrt{\frac{\cos(ex - \arctan(\frac{5}{3}) + d)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right), \frac{\sqrt{510 - 30\sqrt{34}}}{15}\right)$$

$$\frac{3375 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right) e \sqrt{2 + \sqrt{34}}}{199 \sqrt{\frac{\cos(ex - \arctan(\frac{5}{3}) + d)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right), \frac{\sqrt{510 - 30\sqrt{34}}}{15}\right) \sqrt{2 + \sqrt{34}}}$$

$$\frac{101250 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right) e}{101250 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(\frac{5}{3})}{2}\right) e}$$

command

```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$559 \sqrt{5i + 3} \left(-(594i + 990) \sqrt{2} \cos(xe + d)^3 - (288i + 480) \sqrt{2} \cos(xe + d)^2 + 5 \left((6i + 10) \sqrt{2} \cos(xe + d) \right)^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\sqrt{3 \cos(ex + d) + 5 \sin(ex + d) + 2}}{644 \cos(ex + d)^4 + 1584 \cos(ex + d)^3 + 284 \cos(ex + d)^2 + 20 \left(48 \cos(ex + d)^3 - 4 \cos(ex + d)^2 - 1\right)}\right)$$

75.24 Problem number 410

$$\int (a + b \cos(d + ex) + c \sin(d + ex))^{5/2} dx$$

Optimal antiderivative

$$\frac{2(c \cos(ex + d) - b \sin(ex + d))(a + b \cos(ex + d) + c \sin(ex + d))^{3/2}}{5e} - \frac{16(ac \cos(ex + d) - ab \sin(ex + d)) \sqrt{a + b \cos(ex + d) + c \sin(ex + d)}}{15e}$$

$$+ \frac{2(23a^2 + 9b^2 + 9c^2) \sqrt{\frac{\cos(d + ex - \arctan(b, c))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right)}{15 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right) e \sqrt{\frac{a + b \cos(ex + d) + c \sin(ex + d)}{a + \sqrt{b^2 + c^2}}}}$$

$$- \frac{16a(a^2 - b^2 - c^2) \sqrt{\frac{\cos(d + ex - \arctan(b, c))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right)}{15 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right) e \sqrt{a + b \cos(ex + d) + c \sin(ex + d)}}$$

command

```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(2ab \cos(ex + d) + (b^2 - c^2) \cos^2(ex + d) + a^2 + c^2 + 2(bc \cos(ex + d) + ac) \sin(ex + d)\right) \sqrt{b \cos(ex + d)}\right)$$

75.25 Problem number 411

$$\int (a + b \cos(d + ex) + c \sin(d + ex))^{3/2} dx$$

Optimal antiderivative

$$\frac{2(c \cos(ex + d) - b \sin(ex + d)) \sqrt{a + b \cos(ex + d) + c \sin(ex + d)}}{3e} + \frac{8a \sqrt{\frac{\cos(d + ex - \arctan(b, c))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right) \sqrt{a + b \cos(ex + d)}}{3 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right) e \sqrt{\frac{a + b \cos(ex + d) + c \sin(ex + d)}{a + \sqrt{b^2 + c^2}}}} + \frac{2(a^2 - b^2 - c^2) \sqrt{\frac{\cos(d + ex - \arctan(b, c))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right)}{3 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right) e \sqrt{a + b \cos(ex + d) + c \sin(ex + d)}}$$

command

```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((b \cos(ex + d) + c \sin(ex + d) + a)^{\frac{3}{2}}, x\right)$$

75.26 Problem number 412

$$\int \sqrt{a + b \cos(d + ex) + c \sin(d + ex)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(d + ex - \arctan(b, c))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right) \sqrt{a + b \cos(ex + d)}}{\cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right) e \sqrt{\frac{a + b \cos(ex + d) + c \sin(ex + d)}{a + \sqrt{b^2 + c^2}}}}$$

command

```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (i a b + a c) \sqrt{b + i c} \operatorname{weierstrassPInverse}\left(\frac{4(4 a^2 b^2 - 3 b^4 - 4 a^2 c^2 + 6 i b c^3 + 3 c^4 - 2 i (4 a^2 b - 3 b^3) c)}{3(b^4 + 2 b^2 c^2 + c^4)}, -\frac{8(8 a^3 b^3 - 9 a b^5 + 27 a b c^4 - 9 i a c^5)}{3(b^4 + 2 b^2 c^2 + c^4)}\right), -\frac{8(8 a^3 b^3 - 9 a b^5 + 27 a b c^4 - 9 i a c^5)}{3(b^4 + 2 b^2 c^2 + c^4)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cos(ex + d) + c \sin(ex + d) + a}, x\right)$$

75.27 Problem number 413

$$\int \frac{1}{\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(d + ex - \arctan(b, c))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right) \sqrt{a + b \cos(ex + d) + c \sin(ex + d)}}{\cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right) e \sqrt{a + b \cos(ex + d) + c \sin(ex + d)}}$$

command

`integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} \sqrt{b + i c} (i b + c) \operatorname{weierstrassPInverse}\left(\frac{4(4 a^2 b^2 - 3 b^4 - 4 a^2 c^2 + 6 i b c^3 + 3 c^4 - 2 i (4 a^2 b - 3 b^3) c)}{3(b^4 + 2 b^2 c^2 + c^4)}, -\frac{8(8 a^3 b^3 - 9 a b^5 + 27 a b c^4 - 9 i a c^5)}{3(b^4 + 2 b^2 c^2 + c^4)}\right), -\frac{8(8 a^3 b^3 - 9 a b^5 + 27 a b c^4 - 9 i a c^5)}{3(b^4 + 2 b^2 c^2 + c^4)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{b \cos(ex + d) + c \sin(ex + d) + a}}, x\right)$$

75.28 Problem number 414

$$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2c \cos(ex + d) - 2b \sin(ex + d)}{(a^2 - b^2 - c^2) e \sqrt{a + b \cos(ex + d) + c \sin(ex + d)}} + \frac{2\sqrt{\frac{\cos(d + ex - \arctan(b, c))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right) \sqrt{a + b \cos(ex + d) + c \sin(ex + d)}}{\cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right) (a^2 - b^2 - c^2) e \sqrt{\frac{a + b \cos(ex + d) + c \sin(ex + d)}{a + \sqrt{b^2 + c^2}}}}$$

command

```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \cos(ex+d) + c \sin(ex+d) + a}}{2ab \cos(ex+d) + (b^2 - c^2) \cos^2(ex+d) + a^2 + c^2 + 2(bc \cos(ex+d) + ac) \sin(ex+d)}, x \right)$$

75.29 Problem number 415

$$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\frac{2c \cos(ex+d)}{3} - \frac{2b \sin(ex+d)}{3}}{(a^2 - b^2 - c^2) e (a + b \cos(ex+d) + c \sin(ex+d))^{3/2}} \\ & + \frac{\frac{8ac \cos(ex+d)}{3} - \frac{8ab \sin(ex+d)}{3}}{(a^2 - b^2 - c^2)^2 e \sqrt{a + b \cos(ex+d) + c \sin(ex+d)}} \\ & + \frac{8a \sqrt{\frac{\cos(d + ex - \arctan(b, c))}{2}} + \frac{1}{2} \text{EllipticE} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}} \right) \sqrt{a + b \cos(ex+d)}}{3 \cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2} \right) (a^2 - b^2 - c^2)^2 e \sqrt{\frac{a + b \cos(ex+d) + c \sin(ex+d)}{a + \sqrt{b^2 + c^2}}}} \\ & - \frac{2 \sqrt{\frac{\cos(d + ex - \arctan(b, c))}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}} \right) \sqrt{a + b \cos(ex+d)}}{3 \cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2} \right) (a^2 - b^2 - c^2) e \sqrt{a + b \cos(ex+d) + c \sin(ex+d)}} \end{aligned}$$

command

```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \cos(ex+d) + c \sin(ex+d) + a}}{(b^3 - 3bc^2) \cos^3(ex+d) + a^3 + 3ac^2 + 3(ab^2 - ac^2) \cos^2(ex+d) + 3(a^2b + bc^2) \cos(ex+d) + (6abc \cos^2(ex+d) + a^2c + b^3 - 3bc^2)}$$

75.30 Problem number 416

$$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\frac{2c \cos(ex+d)}{5} - \frac{2b \sin(ex+d)}{5}}{(a^2 - b^2 - c^2) e (a + b \cos(ex + d) + c \sin(ex + d))^{5/2}} \\ & + \frac{\frac{16ac \cos(ex+d)}{15} - \frac{16ab \sin(ex+d)}{15}}{(a^2 - b^2 - c^2)^2 e (a + b \cos(ex + d) + c \sin(ex + d))^{3/2}} \\ & + \frac{\frac{2c(23a^2+9b^2+9c^2) \cos(ex+d)}{15} - \frac{2b(23a^2+9b^2+9c^2) \sin(ex+d)}{15}}{(a^2 - b^2 - c^2)^3 e \sqrt{a + b \cos(ex + d) + c \sin(ex + d)}} \\ & + \frac{2(23a^2 + 9b^2 + 9c^2) \sqrt{\frac{\cos(d + ex - \arctan(b, c))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right)}{15 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right) (a^2 - b^2 - c^2)^3 e \sqrt{\frac{a + b \cos(ex + d) + c \sin(ex + d)}{a + \sqrt{b^2 + c^2}}}} \\ & - \frac{16a \sqrt{\frac{\cos(d + ex - \arctan(b, c))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right) \sqrt{a + b \cos(ex + d) + c \sin(ex + d)}}{15 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(b, c)}{2}\right) (a^2 - b^2 - c^2)^2 e \sqrt{a + b \cos(ex + d) + c \sin(ex + d)}} \end{aligned}$$

command

```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{(b^4 - 6b^2c^2 + c^4) \cos^4(ex + d) + a^4 + 6a^2c^2 + c^4 + 4(ab^3 - 3abc^2) \cos^3(ex + d) + 2(3a^2b^2 - c^4 - 3(a^2b^2 - c^4) \cos^2(ex + d) + a^2c^2 + c^4) \cos(ex + d) + a^4 + 6a^2c^2 + c^4}, dx\right)$$

75.31 Problem number 448

$$\int \frac{(a + b \sec(d + ex) + c \tan(d + ex))^{3/2}}{\sec^{3/2}(d + ex)} dx$$

Optimal antiderivative

$$\frac{2(c \cos(ex + d) - a \sin(ex + d))(a + b \sec(ex + d) + c \tan(ex + d))^{3/2}}{3e \sec(ex + d)^{3/2} (b + a \cos(ex + d) + c \sin(ex + d))}$$

$$+ \frac{8b \sqrt{\frac{\cos(d + ex - \arctan(a, c))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}}\right) (a + b \sec(ex + d))}{3 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2}\right) e \sec(ex + d)^{3/2} (b + a \cos(ex + d) + c \sin(ex + d)) \sqrt{\frac{b + a \cos(ex + d) + c \sin(ex + d)}{b + \sqrt{a^2 + c^2}}}}$$

$$+ \frac{2(a^2 - b^2 + c^2) \sqrt{\frac{\cos(d + ex - \arctan(a, c))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}}\right)}{3 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2}\right) e \sec(ex + d)^{3/2} (b + a \cos(ex + d) + c \sin(ex + d))}$$

command

```
integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(b \sec(ex + d) + c \tan(ex + d) + a)^{3/2}}{\sec(ex + d)^{3/2}}, x\right)$$

75.32 Problem number 449

$$\int \frac{\sqrt{a + b \sec(d + ex) + c \tan(d + ex)}}{\sqrt{\sec(d + ex)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(d + ex - \arctan(a, c))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}}\right) \sqrt{a + b \sec(ex + d)}}{\cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2}\right) e \sqrt{\sec(ex + d)} \sqrt{\frac{b + a \cos(ex + d) + c \sin(ex + d)}{b + \sqrt{a^2 + c^2}}}}$$

command

`integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left((-i ab + bc) \sqrt{2a - 2ic} \operatorname{weierstrassPInverse} \left(-\frac{4(3a^4 - 4a^2b^2 + 4b^2c^2 + 6iac^3 - 3c^4 + 2i(3a^3 - 4ab^2)c)}{3(a^4 + 2a^2c^2 + c^4)}, \frac{8(9a^5b - 8a^3b^3 - 27abc^4 - 9i b^2c^3)}{3(a^4 + 2a^2c^2 + c^4)} \right), \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \sec(ex + d) + c \tan(ex + d) + a}}{\sqrt{\sec(ex + d)}}, x \right)$$

75.33 Problem number 450

$$\int \frac{\sqrt{\sec(d + ex)}}{\sqrt{a + b \sec(d + ex) + c \tan(d + ex)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(d + ex - \arctan(a, c))}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}} \right) (\sqrt{\sec}(ex + d) + a)}{\cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2} \right) e \sqrt{a + b \sec(ex + d) + c \tan(ex + d)}}$$

command

`integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2a - 2ic} (-ia + c) \operatorname{weierstrassPInverse} \left(-\frac{4(3a^4 - 4a^2b^2 + 4b^2c^2 + 6iac^3 - 3c^4 + 2i(3a^3 - 4ab^2)c)}{3(a^4 + 2a^2c^2 + c^4)}, \frac{8(9a^5b - 8a^3b^3 - 27abc^4 - 9i b^2c^3)}{3(a^4 + 2a^2c^2 + c^4)} \right), \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{\sec(ex + d)}}{\sqrt{b \sec(ex + d) + c \tan(ex + d) + a}}, x \right)$$

75.34 Problem number 451

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2\left(\sec^{\frac{3}{2}}(ex+d)\right)\left(c\cos(ex+d)-a\sin(ex+d)\right)\left(b+a\cos(ex+d)+c\sin(ex+d)\right)}{(a^2-b^2+c^2)e\left(a+b\sec(ex+d)+c\tan(ex+d)\right)^{\frac{3}{2}}}$$

$$\frac{2\sqrt{\frac{\cos(d+ex-\arctan(a,c))}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan(a,c)}{2}\right),\sqrt{2}\sqrt{\frac{\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}}\right)\left(\sec^{\frac{3}{2}}(ex+d)\right)}{\cos\left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan(a,c)}{2}\right)\left(a^2-b^2+c^2\right)e\sqrt{\frac{b+a\cos(ex+d)+c\sin(ex+d)}{b+\sqrt{a^2+c^2}}}\left(a+b\sec(ex+d)\right)}$$

command

`integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sec^{\frac{3}{2}}(ex+d)}{b^2\sec^2(ex+d)+c^2\tan^2(ex+d)+2ab\sec(ex+d)+a^2+2(bc\sec(ex+d)+ac)\tan(ex+d)},x\right)$$

75.35 Problem number 452

$$\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2 \left(\sec^{\frac{5}{2}}(ex+d) \right) (c \cos(ex+d) - a \sin(ex+d)) (b + a \cos(ex+d) + c \sin(ex+d))}{3 (a^2 - b^2 + c^2) e (a + b \sec(ex+d) + c \tan(ex+d))^{\frac{5}{2}}} \\
& + \frac{8 \left(\sec^{\frac{5}{2}}(ex+d) \right) (bc \cos(ex+d) - ab \sin(ex+d)) (b + a \cos(ex+d) + c \sin(ex+d))^2}{3 (a^2 - b^2 + c^2)^2 e (a + b \sec(ex+d) + c \tan(ex+d))^{\frac{5}{2}}} \\
& + \frac{8b \sqrt{\frac{\cos(d+ex - \arctan(a,c))}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a,c)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2+c^2}}{b + \sqrt{a^2+c^2}}} \right) \left(\sec^{\frac{5}{2}}(ex+d) \right)}{3 \cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a,c)}{2} \right) (a^2 - b^2 + c^2)^2 e \sqrt{\frac{b + a \cos(ex+d) + c \sin(ex+d)}{b + \sqrt{a^2+c^2}}} (a + b \sec(ex+d))} \\
& + \frac{2 \sqrt{\frac{\cos(d+ex - \arctan(a,c))}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a,c)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2+c^2}}{b + \sqrt{a^2+c^2}}} \right) \left(\sec^{\frac{5}{2}}(ex+d) \right)}{3 \cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a,c)}{2} \right) (a^2 - b^2 + c^2) e (a + b \sec(ex+d))}
\end{aligned}$$

command

```
integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \sec(ex+d) + c \tan(ex+d) + a}}{b^3 \sec^3(ex+d) + c^3 \tan^3(ex+d) + 3ab^2 \sec^2(ex+d) + 3a^2b \sec(ex+d) + a^3 + 3(bc^2 \sec(ex+d) + a^2c \tan(ex+d))} \right)$$

75.36 Problem number 453

$$\int \cos^{\frac{3}{2}}(d+ex)(a + b \sec(d+ex) + c \tan(d+ex))^{3/2} dx$$

Optimal antiderivative

$$\frac{2 \left(\cos^{\frac{3}{2}}(ex + d) \right) (c \cos(ex + d) - a \sin(ex + d)) (a + b \sec(ex + d) + c \tan(ex + d))^{\frac{3}{2}}}{3e(b + a \cos(ex + d) + c \sin(ex + d))} +$$

$$\frac{8b \left(\cos^{\frac{3}{2}}(ex + d) \right) \sqrt{\frac{\cos(d + ex - \arctan(a, c))}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}} \right)}{3 \cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2} \right) e (b + a \cos(ex + d) + c \sin(ex + d)) \sqrt{\frac{b + a \cos(ex + d) + c \sin(ex + d)}{b + \sqrt{a^2 + c^2}}}}$$

$$+ \frac{2(a^2 - b^2 + c^2) \left(\cos^{\frac{3}{2}}(ex + d) \right) \sqrt{\frac{\cos(d + ex - \arctan(a, c))}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}} \right)}{3 \cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2} \right) e (b + a \cos(ex + d) + c \sin(ex + d)) \sqrt{\frac{b + a \cos(ex + d) + c \sin(ex + d)}{b + \sqrt{a^2 + c^2}}}}$$

command

```
integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

```
integral((b*cos(ex+d)*sec(ex+d)+c*cos(ex+d)*tan(ex+d)+a*cos(ex+d))*sqrt(b*sec(ex+d)+c*tan(ex+d)),x)
```

75.37 Problem number 454

$$\int \sqrt{\cos(d + ex)} \sqrt{a + b \sec(d + ex) + c \tan(d + ex)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(d + ex - \arctan(a, c))}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}} \right) (\sqrt{\cos}(ex + d))}{\cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a, c)}{2} \right) e \sqrt{\frac{b + a \cos(ex + d) + c \sin(ex + d)}{b + \sqrt{a^2 + c^2}}}}$$

command

```
integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}(-iab+bc)\sqrt{a-ic}\operatorname{weierstrassPInverse}\left(-\frac{4(3a^4-4a^2b^2+4b^2c^2+6iac^3-3c^4+2i(3a^3-4ab^2)c)}{3(a^4+2a^2c^2+c^4)}, \frac{8(9a^5b-8a^3b^3-27abc^4-9i(a^4b-2ab^2c-2a^2bc^2-2b^3c-2abc^3))}{3(a^4+2a^2c^2+c^4)}\right), \frac{8(9a^5b-8a^3b^3-27abc^4-9i(a^4b-2ab^2c-2a^2bc^2-2b^3c-2abc^3))}{3(a^4+2a^2c^2+c^4)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)}, x\right)$$

75.38 Problem number 455

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{\frac{\cos(d+ex-\arctan(a,c))}{2}+\frac{1}{2}}\operatorname{EllipticF}\left(\sin\left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan(a,c)}{2}\right), \sqrt{2}\sqrt{\frac{\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}}\right)\sqrt{\frac{b+a\cos(ex+d)}{b}}}{\cos\left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan(a,c)}{2}\right)e\sqrt{\cos(ex+d)}\sqrt{a+b\sec(ex+d)+c\tan(ex+d)}}$$

command

`integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2}\sqrt{a-ic}(-ia+c)\operatorname{weierstrassPInverse}\left(-\frac{4(3a^4-4a^2b^2+4b^2c^2+6iac^3-3c^4+2i(3a^3-4ab^2)c)}{3(a^4+2a^2c^2+c^4)}, \frac{8(9a^5b-8a^3b^3-27abc^4-9i(a^4b-2ab^2c-2a^2bc^2-2b^3c-2abc^3))}{3(a^4+2a^2c^2+c^4)}\right), \frac{8(9a^5b-8a^3b^3-27abc^4-9i(a^4b-2ab^2c-2a^2bc^2-2b^3c-2abc^3))}{3(a^4+2a^2c^2+c^4)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)}}{b\cos(ex+d)\sec(ex+d)+c\cos(ex+d)\tan(ex+d)+a\cos(ex+d)}, x\right)$$

75.39 Problem number 456

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(c\cos(ex+d)-a\sin(ex+d))(b+a\cos(ex+d)+c\sin(ex+d))}{(a^2-b^2+c^2)e\cos(ex+d)^{\frac{3}{2}}(a+b\sec(ex+d)+c\tan(ex+d))^{\frac{3}{2}}}$$

$$\frac{2\sqrt{\frac{\cos(d+ex-\arctan(a,c))}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan(a,c)}{2}\right), \sqrt{2}\sqrt{\frac{\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}}\right)(b+a\cos(ex+d))}{\cos\left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan(a,c)}{2}\right)(a^2-b^2+c^2)e\cos(ex+d)^{\frac{3}{2}}\sqrt{\frac{b+a\cos(ex+d)+c\sin(ex+d)}{b+\sqrt{a^2+c^2}}}(a+b\sec(ex+d))}$$

command

```
integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \sec(ex+d) + c \tan(ex+d) + a} \sqrt{\cos(ex+d)}}{b^2 \cos^2(ex+d) \sec^2(ex+d) + c^2 \cos^2(ex+d) \tan^2(ex+d) + 2ab \cos^2(ex+d) \sec(ex+d) + a^2 \cos^2(ex+d)} \right)$$

75.40 Problem number 457

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{5}{2}}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(c \cos(ex+d) - a \sin(ex+d))(b + a \cos(ex+d) + c \sin(ex+d))}{3(a^2 - b^2 + c^2) e \cos(ex+d)^{\frac{5}{2}} (a + b \sec(ex+d) + c \tan(ex+d))^{\frac{5}{2}}} \\ & + \frac{8(bc \cos(ex+d) - ab \sin(ex+d))(b + a \cos(ex+d) + c \sin(ex+d))^2}{3(a^2 - b^2 + c^2)^2 e \cos(ex+d)^{\frac{5}{2}} (a + b \sec(ex+d) + c \tan(ex+d))^{\frac{5}{2}}} \\ & + \frac{8b \sqrt{\frac{\cos(d+ex - \arctan(a,c))}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a,c)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}} \right) (b + a \cos(ex+d))}{3 \cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a,c)}{2} \right) (a^2 - b^2 + c^2)^2 e \cos(ex+d)^{\frac{5}{2}} \sqrt{\frac{b + a \cos(ex+d) + c \sin(ex+d)}{b + \sqrt{a^2 + c^2}}} (a + b \sec(ex+d))} \\ & + \frac{2 \sqrt{\frac{\cos(d+ex - \arctan(a,c))}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a,c)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}} \right) (b + a \cos(ex+d))}{3 \cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(a,c)}{2} \right) (a^2 - b^2 + c^2) e \cos(ex+d)^{\frac{5}{2}} (a + b \sec(ex+d))} \end{aligned}$$

command

```
integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{b^3 \cos^3(ex+d) \sec^3(ex+d) + c^3 \cos^3(ex+d) \tan^3(ex+d) + 3ab^2 \cos^3(ex+d) \sec^2(ex+d) + 3a^2b \cos^3(ex+d) \tan^2(ex+d)} \right)$$

75.41 Problem number 462

$$\int \frac{(a + c \cot(d + ex) + b \csc(d + ex))^{3/2}}{\csc^{\frac{3}{2}}(d + ex)} dx$$

Optimal antiderivative

$$\frac{2(a + c \cot(ex + d) + b \csc(ex + d))^{\frac{3}{2}} (a \cos(ex + d) - c \sin(ex + d))}{3e \csc(ex + d)^{\frac{3}{2}} (b + c \cos(ex + d) + a \sin(ex + d))}$$

$$+ \frac{8b(a + c \cot(ex + d) + b \csc(ex + d))^{\frac{3}{2}} \sqrt{\frac{\cos(d + ex - \arctan(c, a))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right)\right)}{3 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right) e \csc(ex + d)^{\frac{3}{2}} (b + c \cos(ex + d) + a \sin(ex + d)) \sqrt{\frac{b + c \cos(ex + d) + a \sin(ex + d)}{b + \sqrt{a^2 + c^2}}}}$$

$$+ \frac{2(a^2 - b^2 + c^2) (a + c \cot(ex + d) + b \csc(ex + d))^{\frac{3}{2}} \sqrt{\frac{\cos(d + ex - \arctan(c, a))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right)\right)}{3 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right) e \csc(ex + d)^{\frac{3}{2}} (b + c \cos(ex + d) + a \sin(ex + d)) \sqrt{\frac{b + c \cos(ex + d) + a \sin(ex + d)}{b + \sqrt{a^2 + c^2}}}}$$

command

```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(c \cot(ex + d) + b \csc(ex + d) + a)^{\frac{3}{2}}}{\csc(ex + d)^{\frac{3}{2}}}, x\right)$$

75.42 Problem number 463

$$\int \frac{\sqrt{a + c \cot(d + ex) + b \csc(d + ex)}}{\sqrt{\csc(d + ex)}} dx$$

Optimal antiderivative

$$2 \sqrt{\frac{\cos(d + ex - \arctan(c, a))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}}\right) \sqrt{a + c \cot(ex + d)}$$

$$+ \frac{\cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right) e \sqrt{\csc(ex + d)} \sqrt{\frac{b + c \cos(ex + d) + a \sin(ex + d)}{b + \sqrt{a^2 + c^2}}}}$$

command

`integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left((i a b - b c) \sqrt{-2 i a - 2 c} \operatorname{weierstrassPInverse} \left(\frac{4 (3 a^4 - 4 a^2 b^2 + 4 b^2 c^2 + 6 i a c^3 - 3 c^4 + 2 i (3 a^3 - 4 a b^2) c)}{3 (a^4 + 2 a^2 c^2 + c^4)}, -\frac{8 (-9 i a^5 b + 8 i a^3 b^3 + 27 i a b c^4 - \dots)}{\dots} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{c \cot (e x + d) + b \csc (e x + d) + a}}{\sqrt{\csc (e x + d)}}, x \right)$$

75.43 Problem number 464

$$\int \frac{\sqrt{\csc (d + e x)}}{\sqrt{a + c \cot (d + e x) + b \csc (d + e x)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos (d + e x - \arctan (c, a))}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(\sin \left(\frac{d}{2} + \frac{e x}{2} - \frac{\arctan (c, a)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}} \right) (\sqrt{\csc (e x + a)}}{\cos \left(\frac{d}{2} + \frac{e x}{2} - \frac{\arctan (c, a)}{2} \right) e \sqrt{a + c \cot (e x + d) + b \csc (e x + d)}}$$

command

`integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left((i a - c) \sqrt{-2 i a - 2 c} \operatorname{weierstrassPInverse} \left(\frac{4 (3 a^4 - 4 a^2 b^2 + 4 b^2 c^2 + 6 i a c^3 - 3 c^4 + 2 i (3 a^3 - 4 a b^2) c)}{3 (a^4 + 2 a^2 c^2 + c^4)}, -\frac{8 (-9 i a^5 b + 8 i a^3 b^3 + 27 i a b c^4 - \dots)}{\dots} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{\csc (e x + d)}}{\sqrt{c \cot (e x + d) + b \csc (e x + d) + a}}, x \right)$$

75.44 Problem number 465

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2\left(\csc^{\frac{3}{2}}(ex+d)\right)(b+c\cos(ex+d)+a\sin(ex+d))(a\cos(ex+d)-c\sin(ex+d))}{(a^2-b^2+c^2)e(a+c\cot(ex+d)+b\csc(ex+d))^{\frac{3}{2}}}$$

$$\frac{2\left(\csc^{\frac{3}{2}}(ex+d)\right)\sqrt{\frac{\cos(d+ex-\arctan(c,a))}{2}+\frac{1}{2}}\operatorname{EllipticE}\left(\sin\left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan(c,a)}{2}\right),\sqrt{2}\sqrt{\frac{\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}}\right)}{\cos\left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan(c,a)}{2}\right)(a^2-b^2+c^2)e(a+c\cot(ex+d)+b\csc(ex+d))^{\frac{3}{2}}\sqrt{\frac{b+c\cos(ex+d)}{b+\sqrt{a^2+c^2}}}}$$

command

```
integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\csc(ex+d)^{\frac{3}{2}}}{c^2\cot(ex+d)^2+b^2\csc(ex+d)^2+2ac\cot(ex+d)+a^2+2(bc\cot(ex+d)+ab)\csc(ex+d)},x\right)$$

75.45 Problem number 466

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2 \left(\csc^{\frac{5}{2}}(ex+d) \right) (b+c \cos(ex+d) + a \sin(ex+d)) (a \cos(ex+d) - c \sin(ex+d))}{3(a^2 - b^2 + c^2) e (a + c \cot(ex+d) + b \csc(ex+d))^{\frac{5}{2}}} \\
+ & \frac{8 \left(\csc^{\frac{5}{2}}(ex+d) \right) (b+c \cos(ex+d) + a \sin(ex+d))^2 (ab \cos(ex+d) - bc \sin(ex+d))}{3(a^2 - b^2 + c^2)^2 e (a + c \cot(ex+d) + b \csc(ex+d))^{\frac{5}{2}}} \\
+ & \frac{8b \left(\csc^{\frac{5}{2}}(ex+d) \right) \sqrt{\frac{\cos(d+ex - \arctan(c,a))}{2}} + \frac{1}{2} \operatorname{EllipticE} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c,a)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}} \right)}{3 \cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c,a)}{2} \right) (a^2 - b^2 + c^2)^2 e (a + c \cot(ex+d) + b \csc(ex+d))^{\frac{5}{2}} \sqrt{\frac{b+c \cos(ex+d)}{b+\sqrt{a^2+c^2}}}} \\
+ & \frac{2 \left(\csc^{\frac{5}{2}}(ex+d) \right) \sqrt{\frac{\cos(d+ex - \arctan(c,a))}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c,a)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}} \right)}{3 \cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c,a)}{2} \right) (a^2 - b^2 + c^2) e (a + c \cot(ex+d) + b \csc(ex+d))^{\frac{5}{2}}
\end{aligned}$$

command

```
integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{c \cot(ex+d) + b \csc(ex+d) + a} \csc^{\frac{5}{2}}(ex+d)}{c^3 \cot^3(ex+d) + b^3 \csc^3(ex+d) + 3ac^2 \cot^2(ex+d) + 3a^2c \cot(ex+d) + a^3 + 3(b^2c \cot(ex+d) + abc \csc(ex+d))} \right)$$

75.46 Problem number 467

$$\int (a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^{\frac{3}{2}}(d + ex) dx$$

Optimal antiderivative

$$\frac{2(a + c \cot(ex + d) + b \csc(ex + d))^{\frac{3}{2}} \left(\sin^{\frac{3}{2}}(ex + d) \right) (a \cos(ex + d) - c \sin(ex + d))}{3e(b + c \cos(ex + d) + a \sin(ex + d))}$$

$$+ \frac{8b(a + c \cot(ex + d) + b \csc(ex + d))^{\frac{3}{2}} \sqrt{\frac{\cos(d + ex - \arctan(c, a))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right)\right)}{3 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right) e(b + c \cos(ex + d) + a \sin(ex + d)) \sqrt{\frac{b + c \cos(ex + d) + a \sin(ex + d)}{b + \sqrt{a^2 + c^2}}}}$$

$$+ \frac{2(a^2 - b^2 + c^2)(a + c \cot(ex + d) + b \csc(ex + d))^{\frac{3}{2}} \sqrt{\frac{\cos(d + ex - \arctan(c, a))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right)\right)}{3 \cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right) e(b + c \cos(ex + d) + a \sin(ex + d)) \sqrt{\frac{b + c \cos(ex + d) + a \sin(ex + d)}{b + \sqrt{a^2 + c^2}}}}$$

command

```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((c \cot(ex + d) + b \csc(ex + d) + a)^{\frac{3}{2}} \sin(ex + d)^{\frac{3}{2}}, x\right)$$

75.47 Problem number 468

$$\int \sqrt{a + c \cot(d + ex) + b \csc(d + ex)} \sqrt{\sin(d + ex)} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(d + ex - \arctan(c, a))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}}\right) \sqrt{a + c \cot(ex + d)}}{\cos\left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2}\right) e \sqrt{\frac{b + c \cos(ex + d) + a \sin(ex + d)}{b + \sqrt{a^2 + c^2}}}}$$

command

```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (ab + i bc) \sqrt{ia + c} \text{weierstrassPInverse} \left(\frac{4(3a^4 - 4a^2b^2 + 4b^2c^2 + 6iac^3 - 3c^4 + 2i(3a^3 - 4ab^2)c)}{3(a^4 + 2a^2c^2 + c^4)}, -\frac{8(-9ia^5b + 8ia^3b^3 + 27iabc^4 - 8(-9ia^5b + 8ia^3b^3 + 27iabc^4 - 8(-9ia^5b + 8ia^3b^3 + 27iabc^4 - \dots))}{3(a^4 + 2a^2c^2 + c^4)} \right), x \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{c \cot(ex + d) + b \csc(ex + d) + a} \sqrt{\sin(ex + d)}, x \right)$$

75.48 Problem number 469

$$\int \frac{1}{\sqrt{a + c \cot(d + ex) + b \csc(d + ex)} \sqrt{\sin(d + ex)}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{\frac{\cos(d + ex - \arctan(c, a))}{2} + \frac{1}{2}} \text{EllipticF} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}} \right) \sqrt{\frac{b + c \cos(ex + d)}{b}}}{\cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2} \right) e \sqrt{a + c \cot(ex + d) + b \csc(ex + d)} \sqrt{\sin(ex + d)}}$$

command

`integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{2} (a + ic) \sqrt{ia + c} \text{weierstrassPInverse} \left(\frac{4(3a^4 - 4a^2b^2 + 4b^2c^2 + 6iac^3 - 3c^4 + 2i(3a^3 - 4ab^2)c)}{3(a^4 + 2a^2c^2 + c^4)}, -\frac{8(-9ia^5b + 8ia^3b^3 + 27iabc^4 - 8(-9ia^5b + 8ia^3b^3 + 27iabc^4 - \dots))}{3(a^4 + 2a^2c^2 + c^4)} \right), x \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\sqrt{c \cot(ex + d) + b \csc(ex + d) + a} \sqrt{\sin(ex + d)}}, x \right)$$

75.49 Problem number 470

$$\int \frac{1}{(a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^{3/2}(d + ex)} dx$$

Optimal antiderivative

$$\frac{2(b + c \cos(ex + d) + a \sin(ex + d)) (a \cos(ex + d) - c \sin(ex + d))}{(a^2 - b^2 + c^2) e (a + c \cot(ex + d) + b \csc(ex + d))^{3/2} \sin^3(ex + d)^{3/2}} \frac{2 \sqrt{\frac{\cos(d + ex - \arctan(c, a))}{2} + \frac{1}{2}} \text{EllipticE} \left(\sin \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2} \right), \sqrt{2} \sqrt{\frac{\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}} \right) (b + c \cos(ex + d))}{\cos \left(\frac{d}{2} + \frac{ex}{2} - \frac{\arctan(c, a)}{2} \right) (a^2 - b^2 + c^2) e (a + c \cot(ex + d) + b \csc(ex + d))^{3/2} \sin^3(ex + d)^{3/2} \sqrt{\frac{b + c \cos(ex + d)}{b + \dots}}}$$

command

```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{c \cot (ex+d)+b}}{a^2 \cos (ex+d)^2+\left(c^2 \cos (ex+d)^2-c^2\right) \cot (ex+d)^2+\left(b^2 \cos (ex+d)^2-b^2\right) \csc (ex+d)^2-a^2+2}$$

75.50 Problem number 471

$$\int \frac{1}{(a+c \cot (d+ex)+b \csc (d+ex))^{5/2} \sin ^{5/2}(d+ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(b+c \cos (ex+d)+a \sin (ex+d))(a \cos (ex+d)-c \sin (ex+d))}{3\left(a^2-b^2+c^2\right) e(a+c \cot (ex+d)+b \csc (ex+d))^{5/2} \sin (ex+d)^{5/2}} \\ & + \frac{8(b+c \cos (ex+d)+a \sin (ex+d))^2(ab \cos (ex+d)-bc \sin (ex+d))}{3\left(a^2-b^2+c^2\right)^2 e(a+c \cot (ex+d)+b \csc (ex+d))^{5/2} \sin (ex+d)^{5/2}} \\ & + \frac{8b \sqrt{\frac{\cos (d+ex-\arctan (c, a))}{2}+\frac{1}{2}} \operatorname{EllipticE}\left(\sin \left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan (c, a)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}}\right)(b+c \cos (ex+d))}{3 \cos \left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan (c, a)}{2}\right)\left(a^2-b^2+c^2\right)^2 e(a+c \cot (ex+d)+b \csc (ex+d))^{5/2} \sin (ex+d)^{5/2} \sqrt{\frac{b+c \cos (ex+d)}{b+\sqrt{a^2+c^2}}}} \\ & + \frac{2 \sqrt{\frac{\cos (d+ex-\arctan (c, a))}{2}+\frac{1}{2}} \operatorname{EllipticF}\left(\sin \left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan (c, a)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}}\right)(b+c \cos (ex+d))}{3 \cos \left(\frac{d}{2}+\frac{ex}{2}-\frac{\arctan (c, a)}{2}\right)\left(a^2-b^2+c^2\right) e(a+c \cot (ex+d)+b \csc (ex+d))^{5/2} \sin (ex+d)^{5/2}} \end{aligned}$$

command

```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(-\frac{\sqrt{c \cot (ex+d)+b}}{\left(a^3 \cos (ex+d)^2+\left(c^3 \cos (ex+d)^2-c^3\right) \cot (ex+d)^3+\left(b^3 \cos (ex+d)^2-b^3\right) \csc (ex+d)^3-a^3+2}$$

75.51 Problem number 556

$$\int (a + b \cos(x) + c \sin(x))^{5/2} (d + be \cos(x) + ce \sin(x)) dx$$

Optimal antiderivative

$$\frac{2(a + b \cos(x) + c \sin(x))^{5/2} (ce \cos(x) - be \sin(x))}{7} - \frac{2(a + b \cos(x) + c \sin(x))^{3/2} (c(5ae + 7d) \cos(x) - b(5ae + 7d) \sin(x))}{35} - \frac{2(c(56ad + 15a^2e + 25(b^2 + c^2)e) \cos(x) - b(56ad + 15a^2e + 25(b^2 + c^2)e) \sin(x)) \sqrt{a + b \cos(x) + c \sin(x)}}{105} + \frac{2(161a^2d + 63(b^2 + c^2)d + 15a^3e + 145a(b^2 + c^2)e) \sqrt{\frac{\cos(x - \arctan(b, c))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right)\right)}{105 \cos\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right) \sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}} + \frac{2(a^2 - b^2 - c^2)(56ad + 15a^2e + 25(b^2 + c^2)e) \sqrt{\frac{\cos(x - \arctan(b, c))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right)\right), \sqrt{a + b \cos(x) + c \sin(x)}}{105 \cos\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right) \sqrt{a + b \cos(x) + c \sin(x)}}$$

command

```
integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

```
integral(((b^3 - 3bc^2)e cos(x))^3 + 2ac^2e + ((b^2 - c^2)d + 2(ab^2 - ac^2)e) cos(x)^2 + (a^2 + c^2)d + (2abd + (a^2b + 3
```

75.52 Problem number 557

$$\int (a + b \cos(x) + c \sin(x))^{3/2} (d + be \cos(x) + ce \sin(x)) dx$$

Optimal antiderivative

$$\frac{2(a + b \cos(x) + c \sin(x))^{\frac{3}{2}} (ce \cos(x) - be \sin(x))}{5} - \frac{2(c(3ae + 5d) \cos(x) - b(3ae + 5d) \sin(x)) \sqrt{a + b \cos(x) + c \sin(x)}}{15}$$

$$+ \frac{2(20ad + 3a^2e + 9(b^2 + c^2)e) \sqrt{\frac{\cos(x - \arctan(b, c))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right)}{15 \cos\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right) \sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}}$$

$$- \frac{2(a^2 - b^2 - c^2)(3ae + 5d) \sqrt{\frac{\cos(x - \arctan(b, c))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right)}{15 \cos\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right) \sqrt{a + b \cos(x) + c \sin(x)}}$$

command

```
integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

```
integral(((b^2 - c^2)e*cos(x)^2 + c^2*e + ad + (abe + bd)*cos(x) + (2*b*c*e*cos(x) + ace + cd)*sin(x))*sqrt(b*cos(x) + c*sin(x)), x)
```

75.53 Problem number 558

$$\int \sqrt{a + b \cos(x) + c \sin(x)} (d + be \cos(x) + ce \sin(x)) dx$$

Optimal antiderivative

$$\frac{2(ce \cos(x) - be \sin(x)) \sqrt{a + b \cos(x) + c \sin(x)}}{3} + \frac{2(ae + 3d) \sqrt{\frac{\cos(x - \arctan(b, c))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right)}{3 \cos\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right) \sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}}$$

$$- \frac{2(a^2 - b^2 - c^2)e \sqrt{\frac{\cos(x - \arctan(b, c))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right)}{3 \cos\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right) \sqrt{a + b \cos(x) + c \sin(x)}}$$

command

```
integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((be \cos(x) + ce \sin(x) + d)\sqrt{b \cos(x) + c \sin(x) + a}, x\right)$$

75.54 Problem number 559

$$\int \frac{d + be \cos(x) + ce \sin(x)}{\sqrt{a + b \cos(x) + c \sin(x)}} dx$$

Optimal antiderivative

$$\frac{2e \sqrt{\frac{\cos(x - \arctan(b, c))}{2} + \frac{1}{2}} \text{EllipticE}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right) \sqrt{a + b \cos(x) + c \sin(x)} + \frac{\cos\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right) \sqrt{a + b \cos(x) + c \sin(x)}}{a + \sqrt{b^2 + c^2}} + \frac{2(-ae + d) \sqrt{\frac{\cos(x - \arctan(b, c))}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right) \sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}}{\cos\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right) \sqrt{a + b \cos(x) + c \sin(x)}}$$

command

```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-3i \sqrt{2} (b^2 + c^2) \sqrt{b + ic} \text{eweierstrassZeta}\left(\frac{4(4a^2b^2 - 3b^4 - 4a^2c^2 + 6ibc^3 + 3c^4 - 2i(4a^2b - 3b^3)c)}{3(b^4 + 2b^2c^2 + c^4)}, -\frac{8(8a^3b^3 - 9ab^5 + 27abc^4 - 9iac^4)}{3(b^4 + 2b^2c^2 + c^4)}\right)}{3(b^4 + 2b^2c^2 + c^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{be \cos(x) + ce \sin(x) + d}{\sqrt{b \cos(x) + c \sin(x) + a}}, x\right)$$

75.55 Problem number 560

$$\int \frac{d + be \cos(x) + ce \sin(x)}{(a + b \cos(x) + c \sin(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2c(-ae + d) \cos(x) - 2b(-ae + d) \sin(x)}{(a^2 - b^2 - c^2) \sqrt{a + b \cos(x) + c \sin(x)}}$$

$$+ \frac{2(-ae + d) \sqrt{\frac{\cos(x - \arctan(b, c))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right) \sqrt{a + b \cos(x) + c \sin(x)}}{\cos\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right) (a^2 - b^2 - c^2) \sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}}$$

$$+ \frac{2e \sqrt{\frac{\cos(x - \arctan(b, c))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right) \sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}}{\cos\left(\frac{x}{2} - \frac{\arctan(b, c)}{2}\right) \sqrt{a + b \cos(x) + c \sin(x)}}$$

command

```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(be \cos(x) + ce \sin(x) + d) \sqrt{b \cos(x) + c \sin(x) + a}}{2ab \cos(x) + (b^2 - c^2) \cos(x)^2 + a^2 + c^2 + 2(bc \cos(x) + ac) \sin(x)}, x\right)$$

75.56 Problem number 561

$$\int \frac{d + be \cos(x) + ce \sin(x)}{(a + b \cos(x) + c \sin(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2c(-ae+d)\cos(x)}{3} - \frac{2b(-ae+d)\sin(x)}{3}}{(a^2 - b^2 - c^2)(a + b\cos(x) + c\sin(x))^{\frac{3}{2}}} + \frac{\frac{2c(4ad - a^2e - 3(b^2 + c^2)e)\cos(x)}{3} - \frac{2b(4ad - a^2e - 3(b^2 + c^2)e)\sin(x)}{3}}{(a^2 - b^2 - c^2)^2 \sqrt{a + b\cos(x) + c\sin(x)}}$$

$$+ \frac{2(4ad - a^2e - 3(b^2 + c^2)e) \sqrt{\frac{\cos(x - \arctan(b,c))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b,c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right)}{3 \cos\left(\frac{x}{2} - \frac{\arctan(b,c)}{2}\right) (a^2 - b^2 - c^2)^2 \sqrt{\frac{a + b\cos(x) + c\sin(x)}{a + \sqrt{b^2 + c^2}}}}$$

$$- \frac{2(-ae + d) \sqrt{\frac{\cos(x - \arctan(b,c))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{x}{2} - \frac{\arctan(b,c)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}}\right) \sqrt{\frac{a + b\cos(x) + c\sin(x)}{a + \sqrt{b^2 + c^2}}}}{3 \cos\left(\frac{x}{2} - \frac{\arctan(b,c)}{2}\right) (a^2 - b^2 - c^2) \sqrt{a + b\cos(x) + c\sin(x)}}$$

command

```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(be\cos(x) + ce\sin(x) + d)\sqrt{b\cos(x) + c\sin(x) + a}}{(b^3 - 3bc^2)\cos(x)^3 + a^3 + 3ac^2 + 3(ab^2 - ac^2)\cos(x)^2 + 3(a^2b + bc^2)\cos(x) + (6abc\cos(x) + 3a^2c + 3b^2c^2)\sin(x) + d^2}\right)$$

75.57 Problem number 576

$$\int \frac{1}{\sqrt{a + b\cos(c + dx)\sin(c + dx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{\frac{1}{2} + \frac{\sin(2dx + 2c)}{2}} \operatorname{EllipticF}\left(\cos\left(c + \frac{\pi}{4} + dx\right), \sqrt{2} \sqrt{\frac{b}{2a + b}}\right) \sqrt{2} \sqrt{\frac{2a + b\sin(2dx + 2c)}{2a + b}}}{\sin\left(c + \frac{\pi}{4} + dx\right) d \sqrt{2a + b\sin(2dx + 2c)}}$$

command

```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(b\sqrt{-\frac{4a^2 - b^2}{b^2}} - 2ia \right) \sqrt{4ib} \sqrt{\frac{b\sqrt{-\frac{4a^2 - b^2}{b^2}} + 2ia}{b}} \operatorname{ellipticF} \left(\sqrt{\frac{b\sqrt{-\frac{4a^2 - b^2}{b^2}} + 2ia}{b}} (\cos(dx + c) + i \sin(dx + c)), x \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{b \cos(dx + c) \sin(dx + c) + a}}, x \right)$$

75.58 Problem number 938

$$\int \sin(c + dx) \left(a + \frac{b}{\sqrt{\sin(c + dx)}} + c \sin(c + dx) \right) dx$$

Optimal antiderivative

$$\frac{cx}{2} - \frac{a \cos(dx + c)}{d} - \frac{2b\sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE} \left(\cos \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right), \sqrt{2} \right)}{\sin \left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right) d} - \frac{c \cos(dx + c) \sin(dx + c)}{2d}$$

command

```
integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$c \cos(dx + c) \sin(dx + c) - 2i \sqrt{2} \sqrt{-i} b \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-c \cos(dx + c)^2 + a \sin(dx + c) + b \sqrt{\sin(dx + c)} + c, x \right)$$

75.59 Problem number 939

$$\int \sin(c + dx) \left(a + \frac{b}{\sqrt{\sin(c + dx)}} + c \sin(c + dx) \right)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & b^2 x + acx - \frac{a^2 \cos(dx + c)}{d} - \frac{c^2 \cos(dx + c)}{d} + \frac{c^2 (\cos^3(dx + c))}{3d} \\ & - \frac{4ab \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d} \\ & - \frac{4bc \sqrt{\frac{1}{2} + \frac{\sin(dx + c)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right), \sqrt{2}\right)}{3 \sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right) d} \\ & - \frac{ac \cos(dx + c) \sin(dx + c)}{d} - \frac{4bc \cos(dx + c) \left(\sqrt{\sin(dx + c)}\right)}{3d} \end{aligned}$$

command

```
integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$c^2 \cos(dx + c)^3 - 3ac \cos(dx + c) \sin(dx + c) + 2\sqrt{2} \sqrt{-i} \operatorname{bcweierstrassPInverse}(4, 0, \cos(dx + c) + i \sin(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\begin{aligned} & \operatorname{integral}\left(-2ac \cos(dx + c)^2 + b^2 + 2ac - \left(c^2 \cos(dx + c)^2 - a^2 - c^2\right) \sin(dx + c) \right. \\ & \left. + 2(bc \sin(dx + c) + ab) \sqrt{\sin(dx + c)}, x\right) \end{aligned}$$

76 Test file number 142

Test folder name:

```
test_cases/5_Inverse_trig_functions/5.1_Inverse_sine/142_5.1.2-d_x-^m-a+b_arcsin-c_x-^n
```

76.1 Problem number 203

$$\int (dx)^{5/2}(a + b\text{ArcSin}(cx)) dx$$

Optimal antiderivative

$$\frac{2(dx)^{7/2}(a + b \arcsin(cx))}{7d} - \frac{20b d^{5/2} \text{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{147c^2} + \frac{4b(dx)^{5/2} \sqrt{-c^2x^2 + 1}}{49c} + \frac{20b d^2 \sqrt{dx} \sqrt{-c^2x^2 + 1}}{147c^3}$$

command

```
integrate((d*x)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10 \sqrt{-c^2d} bd^2 \text{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right) + \left(21 bc^5 d^2 x^3 \arcsin(cx) + 21 ac^5 d^2 x^3 + 2(3bc^4 d^2 x^2 + 5bc^2 d^2) \sqrt{-c^2x^2 + 1} \right) \right)}{147c^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((bd^2x^2 \arcsin(cx) + ad^2x^2)\sqrt{dx}, x\right)$$

76.2 Problem number 204

$$\int (dx)^{3/2}(a + b\text{ArcSin}(cx)) dx$$

Optimal antiderivative

$$\frac{2(dx)^{5/2}(a + b \arcsin(cx))}{5d} - \frac{12b d^{3/2} \text{EllipticE}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{25c^2} + \frac{12b d^{3/2} \text{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{25c^2} + \frac{4b(dx)^{3/2} \sqrt{-c^2x^2 + 1}}{25c}$$

command

```
integrate((d*x)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 \sqrt{-c^2d} bd \text{weierstrassZeta}\left(\frac{4}{c^2}, 0, \text{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right)\right) - \left(5bc^3 dx^2 \arcsin(cx) + 5ac^3 dx^2 + 2 \sqrt{-c^2x^2 + 1} \right) \right)}{25c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((bdx \arcsin(cx) + adx)\sqrt{dx}, x\right)$$

76.3 Problem number 205

$$\int \sqrt{dx} (a + b \operatorname{ArcSin}(cx)) dx$$

Optimal antiderivative

$$\frac{2(dx)^{\frac{3}{2}} (a + b \arcsin(cx))}{3d} - \frac{4b \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right) \sqrt{d}}{9c^{\frac{3}{2}}} + \frac{4b \sqrt{dx} \sqrt{-c^2 x^2 + 1}}{9c}$$

command

```
integrate((d*x)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{-c^2 d} \operatorname{bweierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right) + \left(3 b c^3 x \arcsin(cx) + 3 a c^3 x + 2 \sqrt{-c^2 x^2 + 1} b c^2 \right) \sqrt{dx} \right)}{9 c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{dx} (b \arcsin(cx) + a), x\right)$$

76.4 Problem number 206

$$\int \frac{a + b \operatorname{ArcSin}(cx)}{\sqrt{dx}} dx$$

Optimal antiderivative

$$-\frac{4b \operatorname{EllipticE}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{\sqrt{c} \sqrt{d}} + \frac{4b \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{\sqrt{c} \sqrt{d}} + \frac{2(a + b \arcsin(cx)) \sqrt{dx}}{d}$$

command

```
integrate((a+b*arcsin(c*x))/(d*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2 \left(2 \sqrt{-c^2 d} \operatorname{bweierstrassZeta}\left(\frac{4}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right)\right) - (bc \arcsin(cx) + ac) \sqrt{dx} \right)}{cd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx} (b \arcsin(cx) + a)}{dx}, x\right)$$

76.5 Problem number 207

$$\int \frac{a + b \operatorname{ArcSin}(cx)}{(dx)^{3/2}} dx$$

Optimal antiderivative

$$\frac{4b \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right) \sqrt{c}}{d^{3/2}} - \frac{2(a + b \arcsin(cx))}{d\sqrt{dx}}$$

command

```
integrate((a+b*arcsin(c*x))/(d*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{-c^2d} \operatorname{bxweierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right) + (bc \arcsin(cx) + ac)\sqrt{dx}\right)}{cd^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx}(b \arcsin(cx) + a)}{d^2x^2}, x\right)$$

76.6 Problem number 208

$$\int \frac{a + b \operatorname{ArcSin}(cx)}{(dx)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(a + b \arcsin(cx))}{3d(dx)^{3/2}} - \frac{4bc^{3/2} \operatorname{EllipticE}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{3d^{5/2}} \\ & + \frac{4bc^{3/2} \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{3d^{5/2}} - \frac{4bc\sqrt{-c^2x^2 + 1}}{3d^2\sqrt{dx}} \end{aligned}$$

command

```
integrate((a+b*arcsin(c*x))/(d*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{-c^2d} \operatorname{bcx^2weierstrassZeta}\left(\frac{4}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right)\right) + \left(2\sqrt{-c^2x^2 + 1} \operatorname{bcx} + b \arcsin(cx) + a\right)\right)}{3d^3x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx}(b \arcsin(cx) + a)}{d^3x^3}, x\right)$$

77 Test file number 144

Test folder name:

test_cases/5_Inverse_trig_functions/5.1_Inverse_sine/144_5.1.5_Inverse_sine_functions

77.1 Problem number 281

$$\int (ce + dex)^{7/2} (a + b \operatorname{ArcSin}(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(e(dx + c))^{\frac{9}{2}} (a + b \arcsin(dx + c))}{9de} \\ & + \frac{28be^3 \operatorname{EllipticE}\left(\frac{\sqrt{-dx - c + 1} \sqrt{2}}{2}, \sqrt{2}\right) \sqrt{e(dx + c)}}{135d\sqrt{dx + c}} \\ & + \frac{28be^2(e(dx + c))^{\frac{3}{2}} \sqrt{1 - (dx + c)^2}}{405d} + \frac{4b(e(dx + c))^{\frac{7}{2}} \sqrt{1 - (dx + c)^2}}{81d} \end{aligned}$$

command

```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(42 \sqrt{-d^3 e} b e^3 \operatorname{weierstrassZeta}\left(\frac{4}{d^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{d^2}, 0, \frac{dx+c}{d}\right)\right) - \left(45 (bd^5 x^4 + 4bcd^4 x^3 + 6bc^2 d^3 x^2 + \dots \right) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(ad^3 e^3 x^3 + 3acd^2 e^3 x^2 + 3ac^2 de^3 x + ac^3 e^3 + (bd^3 e^3 x^3 + 3bcd^2 e^3 x^2 + 3bc^2 de^3 x + bc^3 e^3)\right) \arcsin(dx + c)\right)$$

77.2 Problem number 282

$$\int (ce + dex)^{5/2} (a + b \operatorname{ArcSin}(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(e(dx + c))^{\frac{7}{2}} (a + b \arcsin(dx + c))}{7de} - \frac{20be^{\frac{5}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}, i\right)}{147d} \\ & + \frac{4b(e(dx + c))^{\frac{5}{2}} \sqrt{1 - (dx + c)^2}}{49d} + \frac{20be^2 \sqrt{e(dx + c)} \sqrt{1 - (dx + c)^2}}{147d} \end{aligned}$$

command

```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10 \sqrt{-d^3 e} \operatorname{weierstrassPInverse}\left(\frac{4}{d^2}, 0, \frac{dx+c}{d}\right) + \left(21 (bd^5 x^3 + 3bcd^4 x^2 + 3bc^2 d^3 x + bc^3 d^2) \arcsin(dx+c) e^2 + \right. \right.}{}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(ad^2 e^2 x^2 + 2acde^2 x + ac^2 e^2 + (bd^2 e^2 x^2 + 2bcde^2 x + bc^2 e^2) \arcsin(dx+c)\right) \sqrt{dex+ce}, x\right)$$

77.3 Problem number 283

$$\int (ce + dex)^{3/2} (a + b \operatorname{ArcSin}(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(e(dx+c))^{\frac{5}{2}}(a+b \arcsin(dx+c))}{5de} + \frac{12be \operatorname{EllipticE}\left(\frac{\sqrt{-dx-c+1}\sqrt{2}}{2}, \sqrt{2}\right) \sqrt{e(dx+c)}}{25d\sqrt{dx+c}} + \frac{4b(e(dx+c))^{\frac{3}{2}} \sqrt{1-(dx+c)^2}}{25d}$$

command

```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsin(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 \sqrt{-d^3 e} \operatorname{beweierstrassZeta}\left(\frac{4}{d^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{d^2}, 0, \frac{dx+c}{d}\right)\right) - \left(5 (bd^3 x^2 + 2bcd^2 x + bc^2 d) \arcsin(dx+c) \right. \right.}{25d^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(adex+ace+(bdex+bce) \arcsin(dx+c)\right) \sqrt{dex+ce}, x\right)$$

77.4 Problem number 284

$$\int \sqrt{ce + dex} (a + b \operatorname{ArcSin}(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(e(dx + c))^{\frac{3}{2}} (a + b \arcsin(dx + c))}{3de} - \frac{4b \operatorname{EllipticF}\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}, i\right) \sqrt{e}}{9d} + \frac{4b \sqrt{e(dx + c)} \sqrt{1 - (dx + c)^2}}{9d}$$

command

`integrate((d*e*x+c*e)^(1/2)*(a+b*arcsin(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((3ad^3x + 3acd^2 + 2\sqrt{-d^2x^2 - 2cdx - c^2 + 1}bd^2 + 3(bd^3x + bcd^2) \arcsin(dx + c)) \sqrt{dx + c} e^{\frac{1}{2}} + 2\sqrt{-d^3e} \right)}{9d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{dex + ce} (b \arcsin(dx + c) + a), x\right)$$

77.5 Problem number 285

$$\int \frac{a + b \operatorname{ArcSin}(c + dx)}{\sqrt{ce + dex}} dx$$

Optimal antiderivative

$$\frac{2(a + b \arcsin(dx + c)) \sqrt{e(dx + c)}}{de} + \frac{4b \operatorname{EllipticE}\left(\frac{\sqrt{-dx - c + 1} \sqrt{2}}{2}, \sqrt{2}\right) \sqrt{e(dx + c)}}{de \sqrt{dx + c}}$$

command

`integrate((a+b*arcsin(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((bd \arcsin(dx + c) + ad) \sqrt{dx + c} e^{\frac{1}{2}} - 2\sqrt{-d^3e} b \operatorname{weierstrassZeta}\left(\frac{4}{d^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{d^2}, 0, \frac{dx+c}{d}\right)\right) \right) e^{-1}}{d^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b \arcsin(dx + c) + a}{\sqrt{dex + ce}}, x\right)$$

77.6 Problem number 286

$$\int \frac{a + b \operatorname{ArcSin}(c + dx)}{(ce + dex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{4b \operatorname{EllipticF}\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}, i\right)}{de^{\frac{3}{2}}} - \frac{2(a + b \arcsin(dx+c))}{de\sqrt{e(dx+c)}}$$

command

`integrate((a+b*arcsin(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left((bd^2 \arcsin(dx+c) + ad^2)\sqrt{dx+c}e^{\frac{1}{2}} + 2\sqrt{-d^3e}(bdx+bc)\operatorname{weierstrassPInverse}\left(\frac{4}{d^2}, 0, \frac{dx+c}{d}\right)\right)e^{(-2)}}{d^4x + cd^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dex+ce}(b \arcsin(dx+c) + a)}{d^2e^2x^2 + 2cde^2x + c^2e^2}, x\right)$$

77.7 Problem number 287

$$\int \frac{a + b \operatorname{ArcSin}(c + dx)}{(ce + dex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(a + b \arcsin(dx+c))}{3de(e(dx+c))^{\frac{3}{2}}} + \frac{4b \operatorname{EllipticE}\left(\frac{\sqrt{-dx-c+1}\sqrt{2}}{2}, \sqrt{2}\right)\sqrt{e(dx+c)}}{3de^3\sqrt{dx+c}} - \frac{4b\sqrt{1-(dx+c)^2}}{3de^2\sqrt{e(dx+c)}}$$

command

`integrate((a+b*arcsin(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((bd \arcsin(dx+c) + ad + 2(bd^2x + bcd) \sqrt{-d^2x^2 - 2cdx - c^2 + 1}) \sqrt{dx+c} e^{\frac{1}{2}} + 2(bd^2x^2 + 2bcdx + bc^2) \sqrt{-d^2x^2 - 2cdx - c^2 + 1} \right)}{3(d^4x^2 + 2cd^3x + c^2d^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{dex+ce} (b \arcsin(dx+c) + a)}{d^3e^3x^3 + 3cd^2e^3x^2 + 3c^2de^3x + c^3e^3}, x \right)$$

77.8 Problem number 288

$$\int \frac{a + b \text{ArcSin}(c + dx)}{(ce + dex)^{7/2}} dx$$

Optimal antiderivative

$$-\frac{2(a + b \arcsin(dx+c))}{5de(e(dx+c))^{\frac{5}{2}}} + \frac{4b \text{EllipticF}\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}, i\right)}{15de^{\frac{7}{2}}} - \frac{4b\sqrt{1-(dx+c)^2}}{15de^2(e(dx+c))^{\frac{3}{2}}}$$

command

`integrate((a+b*arcsin(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((3bd^2 \arcsin(dx+c) + 3ad^2 + 2(bd^3x + bcd^2) \sqrt{-d^2x^2 - 2cdx - c^2 + 1}) \sqrt{dx+c} e^{\frac{1}{2}} + 2(bd^3x^3 + 3bcd^2x^2 + 3cd^2d^2x + c^3d^3) \sqrt{-d^2x^2 - 2cdx - c^2 + 1} \right)}{15(d^6x^3 + 3cd^5x^2 + 3c^2d^4x + c^3d^3)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{dex+ce} (b \arcsin(dx+c) + a)}{d^4e^4x^4 + 4cd^3e^4x^3 + 6c^2d^2e^4x^2 + 4c^3de^4x + c^4e^4}, x \right)$$

77.9 Problem number 289

$$\int \frac{a + b \text{ArcSin}(c + dx)}{(ce + dex)^{9/2}} dx$$

Optimal antiderivative

$$-\frac{2(a + b \arcsin(dx+c))}{7de(e(dx+c))^{\frac{7}{2}}} + \frac{12b \text{EllipticE}\left(\frac{\sqrt{-dx-c+1} \sqrt{2}}{2}, \sqrt{2}\right) \sqrt{e(dx+c)}}{35de^5 \sqrt{dx+c}} - \frac{4b\sqrt{1-(dx+c)^2}}{35de^2(e(dx+c))^{\frac{5}{2}}} - \frac{12b\sqrt{1-(dx+c)^2}}{35de^4 \sqrt{e(dx+c)}}$$

command

```
integrate((a+b*arcsin(d*x+c))/(d*e*x+c*e)^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((5bd \arcsin(dx+c) + 5ad + 2(3bd^4x^3 + 9bcd^3x^2 + (9bc^2 + b)d^2x + (3bc^3 + bc)d) \sqrt{-d^2x^2 - 2cdx - c^2} + \dots \right)}{35(d^6x^4 + \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{dex+ce} (b \arcsin(dx+c) + a)}{d^5e^5x^5 + 5cd^4e^5x^4 + 10c^2d^3e^5x^3 + 10c^3d^2e^5x^2 + 5c^4de^5x + c^5e^5}, x \right)$$

77.10 Problem number 290

$$\int \frac{a + b \text{ArcSin}(c + dx)}{(ce + dex)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(a + b \arcsin(dx+c))}{9de(e(dx+c))^{\frac{9}{2}}} + \frac{20b \text{EllipticF}\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}, i\right)}{189de^{\frac{11}{2}}} \\ & -\frac{4b\sqrt{1-(dx+c)^2}}{63de^2(e(dx+c))^{\frac{7}{2}}} - \frac{20b\sqrt{1-(dx+c)^2}}{189de^4(e(dx+c))^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((a+b*arcsin(d*x+c))/(d*e*x+c*e)^(11/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((21bd^2 \arcsin(dx+c) + 21ad^2 + 2(5bd^5x^3 + 15bcd^4x^2 + 3(5bc^2 + b)d^3x + (5bc^3 + 3bc)d^2) \sqrt{-d^2x^2 - 2cdx - c^2} + \dots \right)}{189(d^8x^5 + 5cd^7x^4 + \dots)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{dex+ce} (b \arcsin(dx+c) + a)}{d^6e^6x^6 + 6cd^5e^6x^5 + 15c^2d^4e^6x^4 + 20c^3d^3e^6x^3 + 15c^4d^2e^6x^2 + 6c^5de^6x + c^6e^6}, x \right)$$

77.11 Problem number 352

$$\int x^6 (a + b \operatorname{ArcSin}(cx^2)) dx$$

Optimal antiderivative

$$\frac{x^7 (a + b \arcsin(cx^2))}{7} - \frac{10b \operatorname{EllipticF}(x\sqrt{c}, i)}{147c^{\frac{7}{2}}} + \frac{10bx\sqrt{-c^2x^4+1}}{147c^3} + \frac{2bx^5\sqrt{-c^2x^4+1}}{49c}$$

command

```
integrate(x^6*(a+b*arcsin(c*x^2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21bc^3x^7 \arcsin(cx^2) + 21ac^3x^7 + 2(3bc^2x^5 + 5bx)\sqrt{-c^2x^4+1}}{147c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(bx^6 \arcsin(cx^2) + ax^6, x)$$

77.12 Problem number 353

$$\int x^4 (a + b \operatorname{ArcSin}(cx^2)) dx$$

Optimal antiderivative

$$\frac{x^5 (a + b \arcsin(cx^2))}{5} - \frac{6b \operatorname{EllipticE}(x\sqrt{c}, i)}{25c^{\frac{5}{2}}} + \frac{6b \operatorname{EllipticF}(x\sqrt{c}, i)}{25c^{\frac{5}{2}}} + \frac{2bx^3\sqrt{-c^2x^4+1}}{25c}$$

command

```
integrate(x^4*(a+b*arcsin(c*x^2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5bc^3x^6 \arcsin(cx^2) + 5ac^3x^6 + 2(bc^2x^4 + 3b)\sqrt{-c^2x^4+1}}{25c^3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(bx^4 \arcsin(cx^2) + ax^4, x)$$

77.13 Problem number 354

$$\int x^2 (a + b \operatorname{ArcSin}(cx^2)) dx$$

Optimal antiderivative

$$\frac{x^3 (a + b \arcsin(cx^2))}{3} - \frac{2b \operatorname{EllipticF}(x\sqrt{c}, i)}{9c^{\frac{3}{2}}} + \frac{2bx\sqrt{-c^2x^4 + 1}}{9c}$$

command

```
integrate(x^2*(a+b*arcsin(c*x^2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3bcx^3 \arcsin(cx^2) + 3acx^3 + 2\sqrt{-c^2x^4 + 1}bx}{9c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(bx^2 \arcsin(cx^2) + ax^2, x)$$

77.14 Problem number 355

$$\int (a + b \operatorname{ArcSin}(cx^2)) dx$$

Optimal antiderivative

$$ax + bx \arcsin(cx^2) - \frac{2b \operatorname{EllipticE}(x\sqrt{c}, i)}{\sqrt{c}} + \frac{2b \operatorname{EllipticF}(x\sqrt{c}, i)}{\sqrt{c}}$$

command

```
integrate(a+b*arcsin(c*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bcx^2 \arcsin(cx^2) + acx^2 + 2\sqrt{-c^2x^4 + 1}b}{cx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(b \arcsin(cx^2) + a, x)$$

77.15 Problem number 356

$$\int \frac{a + b \operatorname{ArcSin}(cx^2)}{x^2} dx$$

Optimal antiderivative

$$\frac{-a - b \arcsin(cx^2)}{x} + 2b \operatorname{EllipticF}(x\sqrt{c}, i) \sqrt{c}$$

command

`integrate((a+b*arcsin(c*x^2))/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bx \arctan\left(\frac{\sqrt{-c^2x^4+1}}{cx^2}\right) + (bx-b) \arcsin(cx^2) - a}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b \arcsin(cx^2) + a}{x^2}, x\right)$$

77.16 Problem number 393

$$\int x^4(a + b \operatorname{ArcSin}(c + dx^2)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^5(a + b \arcsin(dx^2 + c))}{5} \\ & - \frac{2b(c+1)(23c^2+9) \operatorname{EllipticE}\left(\frac{x\sqrt{d}}{\sqrt{1-c}}, \sqrt{\frac{-1+c}{c+1}}\right) \sqrt{1-c} \sqrt{1-\frac{dx^2}{1-c}} \sqrt{1+\frac{dx^2}{c+1}}}{75d^{\frac{5}{2}} \sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}} \\ & + \frac{2b(c+1)(15c^2+8c+9) \operatorname{EllipticF}\left(\frac{x\sqrt{d}}{\sqrt{1-c}}, \sqrt{\frac{-1+c}{c+1}}\right) \sqrt{1-c} \sqrt{1-\frac{dx^2}{1-c}} \sqrt{1+\frac{dx^2}{c+1}}}{75d^{\frac{5}{2}} \sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}} \\ & - \frac{16bcx \sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}}{75d^2} + \frac{2bx^3 \sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}}{25d} \end{aligned}$$

command

`integrate(x^4*(a+b*arcsin(d*x^2+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{15bd^3x^6 \arcsin(dx^2 + c) + 15ad^3x^6 + 2(3bd^2x^4 - 8bcdx^2 + 23bc^2 + 9b)\sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}}{75d^3x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(bx^4 \arcsin(dx^2 + c) + ax^4, x)$$

77.17 Problem number 394

$$\int x^2(a + b\text{ArcSin}(c + dx^2)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^3(a + b \arcsin(dx^2 + c))}{3} \\ & + \frac{8bc(c+1) \text{EllipticE}\left(\frac{x\sqrt{d}}{\sqrt{1-c}}, \sqrt{\frac{-1+c}{c+1}}\right) \sqrt{1-c} \sqrt{1 - \frac{dx^2}{1-c}} \sqrt{1 + \frac{dx^2}{c+1}}}{9d^{\frac{3}{2}} \sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}} \\ & - \frac{2b(c+1)(1+3c) \text{EllipticF}\left(\frac{x\sqrt{d}}{\sqrt{1-c}}, \sqrt{\frac{-1+c}{c+1}}\right) \sqrt{1-c} \sqrt{1 - \frac{dx^2}{1-c}} \sqrt{1 + \frac{dx^2}{c+1}}}{9d^{\frac{3}{2}} \sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}} \\ & + \frac{2bx\sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}}{9d} \end{aligned}$$

command

`integrate(x^2*(a+b*arcsin(d*x^2+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3bd^2x^4 \arcsin(dx^2 + c) + 3ad^2x^4 + 2\sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}(bdx^2 - 4bc)}{9d^2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(bx^2 \arcsin(dx^2 + c) + ax^2, x)$$

77.18 Problem number 395

$$\int (a + b \operatorname{ArcSin}(c + dx^2)) dx$$

Optimal antiderivative

$$\begin{aligned} & ax + bx \operatorname{arcsin}(dx^2 + c) \\ & - \frac{2b(c+1) \operatorname{EllipticE}\left(\frac{x\sqrt{d}}{\sqrt{1-c}}, \sqrt{\frac{-1+c}{c+1}}\right) \sqrt{1-c} \sqrt{1-\frac{dx^2}{1-c}} \sqrt{1+\frac{dx^2}{c+1}}}{\sqrt{d} \sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}} \\ & + \frac{2b(c+1) \operatorname{EllipticF}\left(\frac{x\sqrt{d}}{\sqrt{1-c}}, \sqrt{\frac{-1+c}{c+1}}\right) \sqrt{1-c} \sqrt{1-\frac{dx^2}{1-c}} \sqrt{1+\frac{dx^2}{c+1}}}{\sqrt{d} \sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}} \end{aligned}$$

command

```
integrate(a+b*arcsin(d*x^2+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bdx^2 \operatorname{arcsin}(dx^2 + c) + adx^2 + 2\sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1} b}{dx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(b \operatorname{arcsin}(dx^2 + c) + a, x)$$

77.19 Problem number 396

$$\int \frac{a + b \operatorname{ArcSin}(c + dx^2)}{x^2} dx$$

Optimal antiderivative

$$\frac{-a - b \operatorname{arcsin}(dx^2 + c)}{x} + \frac{2b \operatorname{EllipticF}\left(\frac{x\sqrt{d}}{\sqrt{1-c}}, \sqrt{\frac{-1+c}{c+1}}\right) \sqrt{1-c} \sqrt{d} \sqrt{1-\frac{dx^2}{1-c}} \sqrt{1+\frac{dx^2}{c+1}}}{\sqrt{-d^2x^4 - 2cdx^2 - c^2 + 1}}$$

command

```
integrate((a+b*arcsin(d*x^2+c))/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{b \arcsin(dx^2 + c) + a}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b \arcsin(dx^2 + c) + a}{x^2}, x\right)$$

78 Test file number 145

Test folder name:

test_cases/5_Inverse_trig_functions/5.2_Inverse_cosine/145_5.2.2-d_x-^m-a+b_arccos-c_x-^n

78.1 Problem number 203

$$\int (dx)^{5/2} (a + b \text{ArcCos}(cx)) dx$$

Optimal antiderivative

$$\frac{2(dx)^{7/2} (a + b \arccos(cx))}{7d} + \frac{20b d^{5/2} \text{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{147c^{7/2}} - \frac{4b(dx)^{5/2} \sqrt{-c^2x^2 + 1}}{49c} - \frac{20b d^2 \sqrt{dx} \sqrt{-c^2x^2 + 1}}{147c^3}$$

command

`integrate((d*x)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(10 \sqrt{-c^2 d} b d^2 \text{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right) - \left(21 b c^5 d^2 x^3 \arccos(cx) + 21 a c^5 d^2 x^3 - 2 \left(3 b c^4 d^2 x^2 + 5 b c^2 d^2 \right) \sqrt{dx} \right) \right)}{147 c^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(bd^2x^2 \arccos(cx) + ad^2x^2\right)\sqrt{dx}, x\right)$$

78.2 Problem number 204

$$\int (dx)^{3/2} (a + b \operatorname{ArcCos}(cx)) dx$$

Optimal antiderivative

$$\frac{2(dx)^{\frac{5}{2}} (a + b \arccos(cx))}{5d} + \frac{12b d^{\frac{3}{2}} \operatorname{EllipticE}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{25c^{\frac{5}{2}}} - \frac{12b d^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{25c^{\frac{5}{2}}} - \frac{4b(dx)^{\frac{3}{2}} \sqrt{-c^2x^2 + 1}}{25c}$$

command

```
integrate((d*x)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(6 \sqrt{-c^2 d} \operatorname{bdweierstrassZeta}\left(\frac{4}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right)\right) + \left(5 bc^3 dx^2 \arccos(cx) + 5 ac^3 dx^2 - 2 \sqrt{-c^2 x^2 + 1} \right) \right)}{25 c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((bdx \arccos(cx) + adx)\sqrt{dx}, x\right)$$

78.3 Problem number 205

$$\int \sqrt{dx} (a + b \operatorname{ArcCos}(cx)) dx$$

Optimal antiderivative

$$\frac{2(dx)^{\frac{3}{2}} (a + b \arccos(cx))}{3d} + \frac{4b \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right) \sqrt{d}}{9c^{\frac{3}{2}}} - \frac{4b\sqrt{dx} \sqrt{-c^2x^2 + 1}}{9c}$$

command

```
integrate((a+b*arccos(c*x))*(d*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{-c^2 d} \operatorname{bweierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right) - \left(3 bc^3 x \arccos(cx) + 3 ac^3 x - 2 \sqrt{-c^2 x^2 + 1} bc^2 \right) \sqrt{dx} \right)}{9 c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{dx} (b \arccos(cx) + a), x\right)$$

78.4 Problem number 206

$$\int \frac{a + b \operatorname{ArcCos}(cx)}{\sqrt{dx}} dx$$

Optimal antiderivative

$$\frac{4b \operatorname{EllipticE}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{\sqrt{c} \sqrt{d}} - \frac{4b \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{\sqrt{c} \sqrt{d}} + \frac{2(a + b \arccos(cx)) \sqrt{dx}}{d}$$

command

```
integrate((a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{-c^2 d} b \operatorname{weierstrassZeta}\left(\frac{4}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right)\right) + (bc \arccos(cx) + ac) \sqrt{dx} \right)}{cd}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx} (b \arccos(cx) + a)}{dx}, x\right)$$

78.5 Problem number 207

$$\int \frac{a + b \operatorname{ArcCos}(cx)}{(dx)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{4b \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right) \sqrt{c}}{d^{3/2}} - \frac{2(a + b \arccos(cx))}{d \sqrt{dx}}$$

command

```
integrate((a+b*arccos(c*x))/(d*x)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{-c^2 d} b \operatorname{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right) - (bc \arccos(cx) + ac) \sqrt{dx} \right)}{cd^2 x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx} (b \arccos(cx) + a)}{d^2 x^2}, x\right)$$

78.6 Problem number 208

$$\int \frac{a + b \operatorname{ArcCos}(cx)}{(dx)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(a + b \arccos(cx))}{3d(dx)^{\frac{3}{2}}} + \frac{4b c^{\frac{3}{2}} \operatorname{EllipticE}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{3d^{\frac{5}{2}}} \\ & - \frac{4b c^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}}, i\right)}{3d^{\frac{5}{2}}} + \frac{4bc\sqrt{-c^2x^2+1}}{3d^2\sqrt{dx}} \end{aligned}$$

command

```
integrate((a+b*arccos(c*x))/(d*x)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(2\sqrt{-c^2d}bcx^2\operatorname{weierstrassZeta}\left(\frac{4}{c^2}, 0, \operatorname{weierstrassPInverse}\left(\frac{4}{c^2}, 0, x\right)\right) + \left(2\sqrt{-c^2x^2+1}bcx - b\arccos(cx) - a\right)\right)}{3d^3x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dx}(b\arccos(cx) + a)}{d^3x^3}, x\right)$$

79 Test file number 147

Test folder name:

test_cases/5_Inverse_trig_functions/5.2_Inverse_cosine/147_5.2.5_Inverse_cosine_functions

79.1 Problem number 48

$$\int x^2 \operatorname{ArcCos}(ax^2) dx$$

Optimal antiderivative

$$\frac{x^3 \arccos(ax^2)}{3} + \frac{2 \operatorname{EllipticF}(x\sqrt{a}, i)}{9a^{\frac{3}{2}}} - \frac{2x\sqrt{-a^2x^4+1}}{9a}$$

command

```
integrate(x^2*arccos(a*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3ax^3 \arccos(ax^2) - 2\sqrt{-a^2x^4 + 1}x}{9a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^2 \arccos(ax^2), x)$$

79.2 Problem number 50

$$\int \text{ArcCos}(ax^2) dx$$

Optimal antiderivative

$$x \arccos(ax^2) + \frac{2 \text{EllipticE}(x\sqrt{a}, i)}{\sqrt{a}} - \frac{2 \text{EllipticF}(x\sqrt{a}, i)}{\sqrt{a}}$$

command

```
integrate(arccos(a*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{ax^2 \arccos(ax^2) - 2\sqrt{-a^2x^4 + 1}}{ax}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(\arccos(ax^2), x)$$

79.3 Problem number 52

$$\int \frac{\text{ArcCos}(ax^2)}{x^2} dx$$

Optimal antiderivative

$$-\frac{\arccos(ax^2)}{x} - 2 \text{EllipticF}(x\sqrt{a}, i) \sqrt{a}$$

command

```
integrate(arccos(a*x^2)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\arccos(ax^2)}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\arccos(ax^2)}{x^2}, x\right)$$

80 Test file number 150

Test folder name:

test_cases/5_Inverse_trig_functions/5.3_Inverse_tangent/150_5.3.4_u-a+b_arctan-c_x-
^p

80.1 Problem number 48

$$\int \frac{a + b \operatorname{ArcTan}(cx)}{x^2(d + icdx)} dx$$

Optimal antiderivative

$$\frac{-a - b \arctan(cx)}{dx} + \frac{bc \ln(x)}{d} - \frac{bc \ln(c^2x^2 + 1)}{2d} - \frac{ic(a + b \arctan(cx)) \ln\left(2 - \frac{2}{icx+1}\right)}{d} + \frac{bc \operatorname{polylog}\left(2, -1 + \frac{2}{icx+1}\right)}{2d}$$

command

```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bcx \operatorname{Li}_2\left(\frac{cx+i}{cx-i} + 1\right) + 2(i a - b)cx \log(x) + bcx \log\left(\frac{cx+i}{c}\right) - (2i a - b)cx \log\left(\frac{cx-i}{c}\right) + i b \log\left(-\frac{cx+i}{cx-i}\right) + 2a}{2dx}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b \log\left(-\frac{cx+i}{cx-i}\right) - 2i a}{2cdx^3 - 2i dx^2}, x\right)$$

80.2 Problem number 49

$$\int \frac{a + b \operatorname{ArcTan}(cx)}{x^3(d + icdx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{bc}{2dx} - \frac{bc^2 \arctan(cx)}{2d} + \frac{-a - b \arctan(cx)}{2dx^2} + \frac{ic(a + b \arctan(cx))}{dx} - \frac{ibc^2 \ln(x)}{d} \\ & + \frac{ibc^2 \ln(c^2x^2 + 1)}{2d} - \frac{c^2(a + b \arctan(cx)) \ln\left(2 - \frac{2}{icx+1}\right)}{d} - \frac{ibc^2 \operatorname{polylog}\left(2, -1 + \frac{2}{icx+1}\right)}{2d} \end{aligned}$$

command

```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2i bc^2 x^2 \operatorname{Li}_2\left(\frac{cx+i}{cx-i} + 1\right) - 4(a + ib)c^2 x^2 \log(x) + i bc^2 x^2 \log\left(\frac{cx+i}{c}\right) + (4a + 3ib)c^2 x^2 \log\left(\frac{cx-i}{c}\right) - 2(-2ia + b)cx}{4 dx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{b \log\left(-\frac{cx+i}{cx-i}\right) - 2ia}{2cdx^4 - 2id x^3}, x\right)$$

80.3 Problem number 50

$$\int \frac{a + b \operatorname{ArcTan}(cx)}{x^4(d + icdx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{bc}{6dx^2} + \frac{ibc^2}{2dx} + \frac{ibc^3 \arctan(cx)}{2d} + \frac{-a - b \arctan(cx)}{3dx^3} + \frac{ic(a + b \arctan(cx))}{2dx^2} \\ & + \frac{c^2(a + b \arctan(cx))}{dx} - \frac{4bc^3 \ln(x)}{3d} + \frac{2bc^3 \ln(c^2x^2 + 1)}{3d} \\ & + \frac{ic^3(a + b \arctan(cx)) \ln\left(2 - \frac{2}{icx+1}\right)}{d} - \frac{bc^3 \operatorname{polylog}\left(2, -1 + \frac{2}{icx+1}\right)}{2d} \end{aligned}$$

command

```
integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6bc^3x^3\text{Li}_2\left(\frac{cx+i}{cx-i}+1\right) - 4(-3ia+4b)c^3x^3\log(x) + 5bc^3x^3\log\left(\frac{cx+i}{c}\right) + (-12ia+11b)c^3x^3\log\left(\frac{cx-i}{c}\right) + 6(2a+i)}{12dx^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{b\log\left(-\frac{cx+i}{cx-i}\right) - 2ia}{2cdx^5 - 2idx^4}, x\right)$$

80.4 Problem number 55

$$\int \frac{a + b\text{ArcTan}(cx)}{x(d + icdx)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b}{2d^2(-cx+i)} - \frac{b\arctan(cx)}{2d^2} + \frac{i(a+b\arctan(cx))}{d^2(-cx+i)} + \frac{a\ln(x)}{d^2} + \frac{(a+b\arctan(cx))\ln\left(\frac{2}{icx+1}\right)}{d^2} \\ & + \frac{ib\text{polylog}(2, -icx)}{2d^2} - \frac{ib\text{polylog}(2, icx)}{2d^2} + \frac{ib\text{polylog}\left(2, 1 - \frac{2}{icx+1}\right)}{2d^2} \end{aligned}$$

command

`integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(ibcx+b)\text{Li}_2\left(\frac{cx+i}{cx-i}+1\right) - 4(acx-ia)\log(x) - 2b\log\left(-\frac{cx+i}{cx-i}\right) - (-ibcx-b)\log\left(\frac{cx+i}{c}\right) + ((4a-ib)cx-4ib)}{4(cd^2x-id^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{-ib\log\left(-\frac{cx+i}{cx-i}\right) - 2a}{2(c^2d^2x^3 - 2icd^2x^2 - d^2x)}, x\right)$$

80.5 Problem number 56

$$\int \frac{a + b \operatorname{ArcTan}(cx)}{x^2(d + icdx)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{ibc}{2d^2(-cx+i)} + \frac{ibc \arctan(cx)}{2d^2} + \frac{-a - b \arctan(cx)}{d^2 x} + \frac{c(a + b \arctan(cx))}{d^2(-cx+i)} \\ & -\frac{2iac \ln(x)}{d^2} + \frac{bc \ln(x)}{d^2} - \frac{2ic(a + b \arctan(cx)) \ln\left(\frac{2}{icx+1}\right)}{d^2} - \frac{bc \ln(c^2 x^2 + 1)}{2d^2} \\ & + \frac{bc \operatorname{polylog}(2, -icx)}{d^2} - \frac{bc \operatorname{polylog}(2, icx)}{d^2} + \frac{bc \operatorname{polylog}\left(2, 1 - \frac{2}{icx+1}\right)}{d^2} \end{aligned}$$

command

`integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(4a - ib)cx + 4(bc^2 x^2 - ibcx) \operatorname{Li}_2\left(\frac{cx+i}{cx-i} + 1\right) + 4((2ia - b)c^2 x^2 + (2a + ib)cx) \log(x) + 2(2ibcx + b) \log\left(\frac{2}{icx+1}\right)}{4(cd^2 x^2 - id^2 x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{-ib \log\left(\frac{-cx+i}{cx-i}\right) - 2a}{2(c^2 d^2 x^4 - 2icd^2 x^3 - d^2 x^2)}, x\right)$$

80.6 Problem number 57

$$\int \frac{a + b \operatorname{ArcTan}(cx)}{x^3(d + icdx)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{bc}{2d^2 x} - \frac{bc^2}{2d^2(-cx+i)} + \frac{-a - b \arctan(cx)}{2d^2 x^2} + \frac{2ic(a + b \arctan(cx))}{d^2 x} - \frac{ic^2(a + b \arctan(cx))}{d^2(-cx+i)} \\ & -\frac{3ac^2 \ln(x)}{d^2} - \frac{2ibc^2 \ln(x)}{d^2} - \frac{3c^2(a + b \arctan(cx)) \ln\left(\frac{2}{icx+1}\right)}{d^2} + \frac{ibc^2 \ln(c^2 x^2 + 1)}{d^2} \\ & -\frac{3ibc^2 \operatorname{polylog}(2, -icx)}{2d^2} + \frac{3ibc^2 \operatorname{polylog}(2, icx)}{2d^2} - \frac{3ibc^2 \operatorname{polylog}\left(2, 1 - \frac{2}{icx+1}\right)}{2d^2} \end{aligned}$$

command

```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{12i ac^2 x^2 + 2(3a + ib)cx - 6(-ibc^3 x^3 - bc^2 x^2) \operatorname{Li}_2\left(\frac{cx+i}{cx-i} + 1\right) - 4((3a + 2ib)c^3 x^3 + (-3ia + 2b)c^2 x^2) \log(x) - 4(cd^2 x^3 - d^2 x^2)}{4(cd^2 x^3 - d^2 x^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{-ib \log\left(-\frac{cx+i}{cx-i}\right) - 2a}{2(c^2 d^2 x^5 - 2i cd^2 x^4 - d^2 x^3)}, x\right)$$

80.7 Problem number 63

$$\int \frac{a + b \operatorname{ArcTan}(cx)}{x(d + icdx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{ib}{8d^3(-cx+i)^2} + \frac{5b}{8d^3(-cx+i)} - \frac{5b \arctan(cx)}{8d^3} + \frac{-a - b \arctan(cx)}{2d^3(-cx+i)^2} \\ & + \frac{i(a + b \arctan(cx))}{d^3(-cx+i)} + \frac{a \ln(x)}{d^3} + \frac{(a + b \arctan(cx)) \ln\left(\frac{2}{icx+1}\right)}{d^3} \\ & + \frac{ib \operatorname{polylog}(2, -icx)}{2d^3} - \frac{ib \operatorname{polylog}(2, icx)}{2d^3} + \frac{ib \operatorname{polylog}\left(2, 1 - \frac{2}{icx+1}\right)}{2d^3} \end{aligned}$$

command

```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(8ia + 5b)cx + 8(ibc^2 x^2 + 2bcx - ib) \operatorname{Li}_2\left(\frac{cx+i}{cx-i} + 1\right) - 16(ac^2 x^2 - 2iacx - a) \log(x) - 4(2bcx - 3ib) \log\left(-\frac{2}{icx+1}\right) - 4(2bcx - 3ib) \log\left(1 - \frac{2}{icx+1}\right)}{16(c^2 d^3 x^4 - 6i c^2 d^3 x^3 - 6cd^3 x^2 + 2i d^3 x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{b \log\left(-\frac{cx+i}{cx-i}\right) - 2ia}{2c^3 d^3 x^4 - 6i c^2 d^3 x^3 - 6cd^3 x^2 + 2i d^3 x}, x\right)$$

80.8 Problem number 64

$$\int \frac{a + b \operatorname{ArcTan}(cx)}{x^2(d + icdx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{bc}{8d^3(-cx+i)^2} - \frac{9ibc}{8d^3(-cx+i)} + \frac{9ibc \arctan(cx)}{8d^3} + \frac{-a - b \arctan(cx)}{d^3 x} \\ & + \frac{ic(a + b \arctan(cx))}{2d^3(-cx+i)^2} + \frac{2c(a + b \arctan(cx))}{d^3(-cx+i)} - \frac{3iac \ln(x)}{d^3} \\ & + \frac{bc \ln(x)}{d^3} - \frac{3ic(a + b \arctan(cx)) \ln\left(\frac{2}{icx+1}\right)}{d^3} - \frac{bc \ln(c^2 x^2 + 1)}{2d^3} \\ & + \frac{3bc \operatorname{polylog}\left(2, -icx\right)}{2d^3} - \frac{3bc \operatorname{polylog}\left(2, icx\right)}{2d^3} + \frac{3bc \operatorname{polylog}\left(2, 1 - \frac{2}{icx+1}\right)}{2d^3} \end{aligned}$$

command

`integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6(8a - 3ib)c^2 x^2 + 4(-18ia - 5b)cx + 24(bc^3 x^3 - 2ibc^2 x^2 - bcx) \operatorname{Li}_2\left(\frac{cx+i}{cx-i} + 1\right) + 16((3ia - b)c^3 x^3 + 2(3a +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{b \log\left(-\frac{cx+i}{cx-i}\right) - 2ia}{2c^3 d^3 x^5 - 6ic^2 d^3 x^4 - 6cd^3 x^3 + 2id^3 x^2}, x\right)$$

80.9 Problem number 65

$$\int \frac{a + b \operatorname{ArcTan}(cx)}{x^3(d + icdx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{bc}{2d^3 x} - \frac{ibc^2}{8d^3(-cx+i)^2} - \frac{13bc^2}{8d^3(-cx+i)} + \frac{9bc^2 \arctan(cx)}{8d^3} + \frac{-a - b \arctan(cx)}{2d^3 x^2} \\ & + \frac{3ic(a + b \arctan(cx))}{d^3 x} + \frac{c^2(a + b \arctan(cx))}{2d^3(-cx+i)^2} - \frac{3ic^2(a + b \arctan(cx))}{d^3(-cx+i)} \\ & - \frac{6ac^2 \ln(x)}{d^3} - \frac{3ibc^2 \ln(x)}{d^3} - \frac{6c^2(a + b \arctan(cx)) \ln\left(\frac{2}{icx+1}\right)}{d^3} + \frac{3ibc^2 \ln(c^2 x^2 + 1)}{2d^3} \\ & - \frac{3ibc^2 \operatorname{polylog}\left(2, -icx\right)}{d^3} + \frac{3ibc^2 \operatorname{polylog}\left(2, icx\right)}{d^3} - \frac{3ibc^2 \operatorname{polylog}\left(2, 1 - \frac{2}{icx+1}\right)}{d^3} \end{aligned}$$

command

```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6(-16ia - 3b)c^3x^3 - 12(12a - ib)c^2x^2 + 8(4ia - b)cx + 48(-ibc^4x^4 - 2bc^3x^3 + ibc^2x^2)\text{Li}_2\left(\frac{cx+i}{cx-i} + 1\right) + 48}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(-\frac{b \log\left(-\frac{cx+i}{cx-i}\right) - 2ia}{2c^3d^3x^6 - 6ic^2d^3x^5 - 6cd^3x^4 + 2id^3x^3}, x\right)$$

80.10 Problem number 99

$$\int \frac{(a + b\text{ArcTan}(cx))^2}{x(d + icdx)} dx$$

Optimal antiderivative

$$\frac{(a + b \arctan(cx))^2 \ln\left(2 - \frac{2}{icx+1}\right)}{d} + \frac{ib(a + b \arctan(cx)) \text{polylog}\left(2, -1 + \frac{2}{icx+1}\right)}{d} + \frac{b^2 \text{polylog}\left(3, -1 + \frac{2}{icx+1}\right)}{2d}$$

command

```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{b^2 \log\left(\frac{2cx}{cx-i}\right) \log\left(-\frac{cx+i}{cx-i}\right)^2 + 2b^2 \text{Li}_2\left(-\frac{2cx}{cx-i} + 1\right) \log\left(-\frac{cx+i}{cx-i}\right) + 4iab \text{Li}_2\left(\frac{cx+i}{cx-i} + 1\right) - 4a^2 \log(x) + 4a^2 \log\left(\frac{cx}{c}\right)}{4d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{ib^2 \log\left(-\frac{cx+i}{cx-i}\right)^2 + 4ab \log\left(-\frac{cx+i}{cx-i}\right) - 4ia^2}{4cdx^2 - 4idx}, x\right)$$

80.11 Problem number 130

$$\int \frac{(a + b \operatorname{ArcTan}(cx))^3}{x(d + icdx)} dx$$

Optimal antiderivative

$$\frac{(a + b \arctan(cx))^3 \ln\left(2 - \frac{2}{icx+1}\right)}{d} + \frac{3ib(a + b \arctan(cx))^2 \operatorname{polylog}\left(2, -1 + \frac{2}{icx+1}\right)}{2d}$$

$$+ \frac{3b^2(a + b \arctan(cx)) \operatorname{polylog}\left(3, -1 + \frac{2}{icx+1}\right)}{2d} - \frac{3ib^3 \operatorname{polylog}\left(4, -1 + \frac{2}{icx+1}\right)}{4d}$$

command

```
integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-3i b^3 \operatorname{Li}_2\left(-\frac{2cx}{cx-i} + 1\right) \log\left(-\frac{cx+i}{cx-i}\right)^2 - 12ab^2 \operatorname{Li}_2\left(-\frac{2cx}{cx-i} + 1\right) \log\left(-\frac{cx+i}{cx-i}\right) - 12i a^2 b \operatorname{Li}_2\left(\frac{cx+i}{cx-i} + 1\right) + 8a^3 \log(x) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{b^3 \log\left(-\frac{cx+i}{cx-i}\right)^3 - 6iab^2 \log\left(-\frac{cx+i}{cx-i}\right)^2 - 12a^2 b \log\left(-\frac{cx+i}{cx-i}\right) + 8ia^3}{8cdx^2 - 8idx}, x\right)$$

81 Test file number 153

Test folder name:

test_cases/5_Inverse_trig_functions/5.3_Inverse_tangent/153_5.3.7_Inverse_tangent_functions

81.1 Problem number 18

$$\int x^{9/2} \operatorname{ArcTan}\left(\frac{\sqrt{-e} x}{\sqrt{d + ex^2}}\right) dx$$

Optimal antiderivative

$$\frac{2x^{\frac{11}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right)}{11} + \frac{36dx^{\frac{5}{2}}\sqrt{ex^2+d}}{847(-e)^{\frac{3}{2}}} + \frac{4x^{\frac{9}{2}}\sqrt{ex^2+d}}{121\sqrt{-e}} + \frac{60d^2\sqrt{x}\sqrt{ex^2+d}}{847(-e)^{\frac{5}{2}}}$$

$$+ \frac{30d^{\frac{11}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e} (\sqrt{d} + x\sqrt{e}) \sqrt{\left(\frac{\sqrt{d} + x\sqrt{e}}{\sqrt{d} + x\sqrt{e}}\right)}}{847 \cos\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{13}{4}} \sqrt{ex^2+d}}$$

command

```
integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{847} \left(77i x^{\frac{11}{2}} e^3 \log\left(\frac{2x^2e + 2\sqrt{x^2e+d}xe^{\frac{1}{2}} + d}{d}\right) + 60i d^3 \operatorname{weierstrassPInverse}(-4de^{(-1)}, 0, x) - 4(7ix^4e^2 - 9i) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x^{\frac{9}{2}} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right), x\right)$$

81.2 Problem number 19

$$\int x^{5/2} \operatorname{ArcTan}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$$

Optimal antiderivative

$$\frac{2x^{\frac{7}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right)}{7} + \frac{4x^{\frac{5}{2}}\sqrt{ex^2+d}}{49\sqrt{-e}} + \frac{20d\sqrt{x}\sqrt{ex^2+d}}{147(-e)^{\frac{3}{2}}}$$

$$+ \frac{10d^{\frac{7}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e} (\sqrt{d} + x\sqrt{e}) \sqrt{\left(\frac{\sqrt{d} + x\sqrt{e}}{\sqrt{d} + x\sqrt{e}}\right)}}{147 \cos\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{9}{4}} \sqrt{ex^2+d}}$$

command

```
integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{147} \left(21i x^{\frac{7}{2}} e^2 \log \left(\frac{2x^2 e + 2\sqrt{x^2 e + d} x e^{\frac{1}{2}} + d}{d} \right) - 4\sqrt{x^2 e + d} (3i x^2 e - 5i d) \sqrt{x} e^{\frac{1}{2}} - 20i d^2 \text{weierstrassPInverse} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(x^{\frac{5}{2}} \arctan \left(\frac{\sqrt{-e} x}{\sqrt{e x^2 + d}} \right), x \right)$$

81.3 Problem number 20

$$\int \sqrt{x} \text{ArcTan} \left(\frac{\sqrt{-e} x}{\sqrt{d + e x^2}} \right) dx$$

Optimal antiderivative

$$\frac{2x^{\frac{3}{2}} \arctan \left(\frac{x\sqrt{-e}}{\sqrt{e x^2 + d}} \right)}{3} + \frac{4\sqrt{x} \sqrt{e x^2 + d}}{9\sqrt{-e}}$$

$$+ \frac{2d^{\frac{3}{4}} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) \sqrt{-e} (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{\sqrt{d}}{\sqrt{d + e x^2}}}}{9 \cos \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right) e^{\frac{5}{4}} \sqrt{e x^2 + d}}$$

command

```
integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{9} \left(3i x^{\frac{3}{2}} e \log \left(\frac{2x^2 e + 2\sqrt{x^2 e + d} x e^{\frac{1}{2}} + d}{d} \right) - 4i \sqrt{x^2 e + d} \sqrt{x} e^{\frac{1}{2}} + 4i d \text{weierstrassPInverse} \left(-4 d e^{(-1)}, 0, x \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{x} \arctan \left(\frac{\sqrt{-e} x}{\sqrt{e x^2 + d}} \right), x \right)$$

81.4 Problem number 21

$$\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e} x}{\sqrt{d + ex^2}}\right)}{x^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right)}{\sqrt{x}} + \frac{2 \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e} (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{ex}{(\sqrt{d} - \dots)}}}{\cos\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) d^{\frac{1}{4}} e^{\frac{1}{4}} \sqrt{ex^2+d}}$$

command

```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4i \text{weierstrassPInverse}(-4de^{(-1)}, 0, x) - i\sqrt{x} \log\left(\frac{2x^2e+2\sqrt{x^2e+d}xe^{\frac{1}{2}+d}}{d}\right)}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e} x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}, x\right)$$

81.5 Problem number 22

$$\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e} x}{\sqrt{d + ex^2}}\right)}{x^{7/2}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right)}{5x^{\frac{5}{2}}} - \frac{4\sqrt{-e}\sqrt{ex^2+d}}{15dx^{\frac{3}{2}}}$$

$$\frac{2e^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e} (\sqrt{d} + x\sqrt{e}) \sqrt{\left(\sqrt{d} + x\sqrt{e}\right)^2 - d}}{15 \cos\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) d^{\frac{5}{4}} \sqrt{ex^2+d}}$$

command

```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-4i x^3 \operatorname{eweierstrassPInverse}(-4de^{(-1)}, 0, x) - 4i \sqrt{x^2e+d} x^{\frac{3}{2}} e^{\frac{1}{2}} - 3i d \sqrt{x} \log\left(\frac{2x^2e+2\sqrt{x^2e+d}xe^{\frac{1}{2}}+d}{d}\right)}{15 dx^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}, x\right)$$

81.6 Problem number 23

$$\int \frac{\operatorname{ArcTan}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right)}{9x^{\frac{9}{2}}} - \frac{20(-e)^{\frac{3}{2}} \sqrt{ex^2+d}}{189d^2x^{\frac{3}{2}}} - \frac{4\sqrt{-e}\sqrt{ex^2+d}}{63dx^{\frac{7}{2}}}$$

$$\frac{10e^{\frac{7}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e} (\sqrt{d} + x\sqrt{e}) \sqrt{\left(\sqrt{d} + x\sqrt{e}\right)^2 - d}}{189 \cos\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) d^{\frac{9}{4}} \sqrt{ex^2+d}}$$

command

`integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{20i x^5 e^2 \text{weierstrassPInverse}(-4 d e^{(-1)}, 0, x) - 21i d^2 \sqrt{x} \log\left(\frac{2x^2 e + 2\sqrt{x^2 e + d} x e^{\frac{1}{2}} + d}{d}\right) - 4(-5i x^3 e + 3i dx) \sqrt{x^2 e + d}}{189 d^2 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e} x}{\sqrt{e x^2 + d}}\right)}{x^{\frac{11}{2}}}, x\right)$$

81.7 Problem number 24

$$\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e} x}{\sqrt{d + e x^2}}\right)}{x^{15/2}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{x \sqrt{-e}}{\sqrt{e x^2 + d}}\right)}{13 x^{\frac{13}{2}}} - \frac{36(-e)^{\frac{3}{2}} \sqrt{e x^2 + d}}{1001 d^2 x^{\frac{7}{2}}} - \frac{60(-e)^{\frac{5}{2}} \sqrt{e x^2 + d}}{1001 d^3 x^{\frac{3}{2}}} - \frac{4 \sqrt{-e} \sqrt{e x^2 + d}}{143 d x^{\frac{11}{2}}}$$

$$\frac{30 e^{\frac{11}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2} + \frac{1}{2}} \text{EllipticF}\left(\sin\left(2 \arctan\left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e} (\sqrt{d} + x \sqrt{e}) \sqrt{\frac{e x^2 + d}{d}}}{1001 \cos\left(2 \arctan\left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}}\right)\right) d^{\frac{13}{4}} \sqrt{e x^2 + d}}$$

command

`integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-60i x^7 e^3 \text{weierstrassPInverse}(-4 d e^{(-1)}, 0, x) - 77i d^3 \sqrt{x} \log\left(\frac{2x^2 e + 2\sqrt{x^2 e + d} x e^{\frac{1}{2}} + d}{d}\right) - 4(15i x^5 e^2 - 9i dx^3 e - 4i dx) \sqrt{x^2 e + d}}{1001 d^3 x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e} x}{\sqrt{e x^2 + d}}\right)}{x^{\frac{15}{2}}}, x\right)$$

81.8 Problem number 25

$$\int x^{7/2} \operatorname{ArcTan}\left(\frac{\sqrt{-e} x}{\sqrt{d+ex^2}}\right) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^{\frac{9}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right)}{9} + \frac{28dx^{\frac{3}{2}}\sqrt{ex^2+d}}{405(-e)^{\frac{3}{2}}} + \frac{4x^{\frac{7}{2}}\sqrt{ex^2+d}}{81\sqrt{-e}} - \frac{28d^2\sqrt{-e}\sqrt{x}\sqrt{ex^2+d}}{135e^{\frac{5}{2}}(\sqrt{d}+x\sqrt{e})} \\ & + \frac{28d^{\frac{9}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)\sqrt{-e}(\sqrt{d}+x\sqrt{e})\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}}}{135\cos\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)e^{\frac{11}{4}}\sqrt{ex^2+d}} \\ & + \frac{14d^{\frac{9}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right)\sqrt{-e}(\sqrt{d}+x\sqrt{e})\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}}}{135\cos\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)e^{\frac{11}{4}}\sqrt{ex^2+d}} \end{aligned}$$

command

```
integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{405} \left(45i x^{\frac{9}{2}} e^2 \log\left(\frac{2x^2e + 2\sqrt{x^2e+d}xe^{\frac{1}{2}} + d}{d}\right) - 4(5ix^3e - 7idx)\sqrt{x^2e+d}\sqrt{x}e^{\frac{1}{2}} + 84id^2\operatorname{weierstrassZeta}\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x^{\frac{7}{2}} \arctan\left(\frac{\sqrt{-e} x}{\sqrt{ex^2+d}}\right), x\right)$$

81.9 Problem number 26

$$\int x^{3/2} \operatorname{ArcTan}\left(\frac{\sqrt{-e} x}{\sqrt{d+ex^2}}\right) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^{\frac{5}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right)}{5} + \frac{4x^{\frac{3}{2}} \sqrt{ex^2+d}}{25\sqrt{-e}} + \frac{12d\sqrt{-e} \sqrt{x} \sqrt{ex^2+d}}{25e^{\frac{3}{2}} (\sqrt{d} + x\sqrt{e})} \\ & - \frac{12d^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e} (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}}}{25 \cos\left(2 \arctan\left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{7}{4}} \sqrt{ex^2+d}} \\ & + \frac{6d^{\frac{5}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e} (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}}}{25 \cos\left(2 \arctan\left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{7}{4}} \sqrt{ex^2+d}} \end{aligned}$$

command

```
integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{25} \left(5i x^{\frac{5}{2}} e \log\left(\frac{2x^2e + 2\sqrt{x^2e+d}xe^{\frac{1}{2}} + d}{d}\right) - 4i \sqrt{x^2e+d} x^{\frac{3}{2}} e^{\frac{1}{2}} - 12i d \operatorname{weierstrassZeta}\left(-4de^{(-1)}, 0, \operatorname{weierstrassZeta}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x^{\frac{3}{2}} \arctan\left(\frac{\sqrt{-e} x}{\sqrt{ex^2+d}}\right), x\right)$$

81.10 Problem number 27

$$\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e} x}{\sqrt{d + ex^2}}\right)}{\sqrt{x}} dx$$

Optimal antiderivative

$$2\sqrt{x} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right) - \frac{4\sqrt{-e}\sqrt{x}\sqrt{ex^2+d}}{\sqrt{e}(\sqrt{d}+x\sqrt{e})}$$

$$+ \frac{4d^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{\cos\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)} \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e}(\sqrt{d}+x\sqrt{e}) \sqrt{\frac{\sqrt{d}+x\sqrt{e}}{\sqrt{d}}}}$$

$$- \frac{2d^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2}}{\cos\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)} \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e}(\sqrt{d}+x\sqrt{e}) \sqrt{\frac{\sqrt{d}+x\sqrt{e}}{\sqrt{d}}}}$$

command

```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$i\sqrt{x} \log\left(\frac{2x^2e + 2\sqrt{x^2e+d}xe^{\frac{1}{2}} + d}{d}\right)$$

$$+ 4i \text{weierstrassZeta}\left(-4de^{(-1)}, 0, \text{weierstrassPInverse}\left(-4de^{(-1)}, 0, x\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e} x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}, x\right)$$

81.11 Problem number 28

$$\int \frac{\operatorname{ArcTan}\left(\frac{\sqrt{-e} x}{\sqrt{d+ex^2}}\right)}{x^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right)}{3x^{3/2}} - \frac{4\sqrt{-e}\sqrt{ex^2+d}}{3d\sqrt{x}} + \frac{4\sqrt{-e^2}\sqrt{x}\sqrt{ex^2+d}}{3d(\sqrt{d}+x\sqrt{e})} \\ & -\frac{4e^{1/4}\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right)\sqrt{-e}(\sqrt{d}+x\sqrt{e})\sqrt{\frac{e^{1/4}\sqrt{x}}{d^{1/4}}}}{3\cos\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)d^{3/4}\sqrt{ex^2+d}} \\ & +\frac{2e^{1/4}\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right)\sqrt{-e}(\sqrt{d}+x\sqrt{e})\sqrt{\frac{e^{1/4}\sqrt{x}}{d^{1/4}}}}{3\cos\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)d^{3/4}\sqrt{ex^2+d}} \end{aligned}$$

command

```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-4i x^2 \operatorname{weierstrassZeta}(-4de^{(-1)}, 0, \operatorname{weierstrassPInverse}(-4de^{(-1)}, 0, x)) - 4i\sqrt{x^2e+d}x^{3/2}e^{1/2} - id\sqrt{x}\log\left(\frac{2x^2}{\sqrt{d}+x\sqrt{e}}\right)}{3dx^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{5/2}}, x\right)$$

81.12 Problem number 29

$$\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e} x}{\sqrt{d+ex^2}}\right)}{x^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right)}{7x^{7/2}} - \frac{4\sqrt{-e}\sqrt{ex^2+d}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{ex^2+d}}{35d^2\sqrt{x}} - \frac{12e^{3/2}\sqrt{-e}\sqrt{x}\sqrt{ex^2+d}}{35d^2(\sqrt{d}+x\sqrt{e})} \\ & + \frac{12e^{5/4}\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right)\sqrt{-e}(\sqrt{d}+x\sqrt{e})\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)}{2}}}{35\cos\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)d^{7/4}\sqrt{ex^2+d}} \\ & - \frac{6e^{5/4}\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right)\sqrt{-e}(\sqrt{d}+x\sqrt{e})\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)}{2}}}{35\cos\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)d^{7/4}\sqrt{ex^2+d}} \end{aligned}$$

command

```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{12ix^4e^2\text{weierstrassZeta}(-4de^{(-1)}, 0, \text{weierstrassPInverse}(-4de^{(-1)}, 0, x)) - 5id^2\sqrt{x}\log\left(\frac{2x^2e+2\sqrt{x^2e+d}xe^{1/2}}{d}\right)}{35d^2x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{9/2}}, x\right)$$

82 Test file number 156

Test folder name:

test_cases/5_Inverse_trig_functions/5.5_Inverse_secant/156_5.5.1_u-a+b_arcsec-c_x-
^n

82.1 Problem number 148

$$\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x(a + b \operatorname{arcsec}(cx))}{d\sqrt{ex^2 + d}} - \frac{bx \operatorname{EllipticF}\left(cx, \sqrt{-\frac{e}{c^2d}}\right) \sqrt{-c^2x^2 + 1} \sqrt{1 + \frac{ex^2}{d}}}{d\sqrt{c^2x^2} \sqrt{c^2x^2 - 1} \sqrt{ex^2 + d}}$$

command

```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(bx^2e + bd)\sqrt{-d} \operatorname{ellipticF}\left(cx, -\frac{e}{c^2d}\right) + (bcdx \operatorname{arcsec}(cx) + acdx)\sqrt{x^2e + d}}{cd^2x^2e + cd^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsec}(cx) + a)}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

82.2 Problem number 170

$$\int \frac{x^{11}(a + b \sec^{-1}(cx))}{\sqrt{1 - c^4x^4}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(-c^4x^4 + 1)^{\frac{3}{2}} (a + b \operatorname{arcsec}(cx))}{3c^{12}} - \frac{(-c^4x^4 + 1)^{\frac{5}{2}} (a + b \operatorname{arcsec}(cx))}{10c^{12}} \\
& - \frac{7b(c^2x^2 + 1)^{\frac{3}{2}} \sqrt{-c^2x^2 + 1}}{90c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} + \frac{13b(c^2x^2 + 1)^{\frac{5}{2}} \sqrt{-c^2x^2 + 1}}{150c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} \\
& - \frac{3b(c^2x^2 + 1)^{\frac{7}{2}} \sqrt{-c^2x^2 + 1}}{70c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} + \frac{b(c^2x^2 + 1)^{\frac{9}{2}} \sqrt{-c^2x^2 + 1}}{90c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} \\
& \frac{4b \operatorname{arctanh} \left(\sqrt{c^2x^2 + 1} \right) \sqrt{-c^2x^2 + 1}}{15c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} \\
& + \frac{4b \sqrt{-c^2x^2 + 1} \sqrt{c^2x^2 + 1}}{15c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} - \frac{(a + b \operatorname{arcsec}(cx)) \sqrt{-c^4x^4 + 1}}{2c^{12}}
\end{aligned}$$

command

```
integrate(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(35bc^8x^8 + 5bc^6x^6 + 78bc^4x^4 + 36bc^2x^2 + 768b) \sqrt{-c^4x^4 + 1} \sqrt{c^2x^2 - 1} - 840(bc^2x^2 - b) \arctan \left(\frac{\sqrt{-c^4x^4 + 1}}{\sqrt{c^2x^2 - 1}} \right)}{15c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

82.3 Problem number 171

$$\int \frac{x^7 (a + b \sec^{-1}(cx))}{\sqrt{1 - c^4x^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-c^4x^4 + 1)^{\frac{3}{2}} (a + b \operatorname{arcsec}(cx))}{6c^8} - \frac{b(c^2x^2 + 1)^{\frac{3}{2}} \sqrt{-c^2x^2 + 1}}{18c^9x \sqrt{1 - \frac{1}{c^2x^2}}} \\ & + \frac{b(c^2x^2 + 1)^{\frac{5}{2}} \sqrt{-c^2x^2 + 1}}{30c^9x \sqrt{1 - \frac{1}{c^2x^2}}} - \frac{b \operatorname{arctanh}\left(\sqrt{c^2x^2 + 1}\right) \sqrt{-c^2x^2 + 1}}{3c^9x \sqrt{1 - \frac{1}{c^2x^2}}} \\ & + \frac{b \sqrt{-c^2x^2 + 1} \sqrt{c^2x^2 + 1}}{3c^9x \sqrt{1 - \frac{1}{c^2x^2}}} - \frac{(a + b \operatorname{arcsec}(cx)) \sqrt{-c^4x^4 + 1}}{2c^8} \end{aligned}$$

command

```
integrate(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(3bc^4x^4 + bc^2x^2 + 28b) \sqrt{-c^4x^4 + 1} \sqrt{c^2x^2 - 1} - 30(bc^2x^2 - b) \arctan\left(\frac{\sqrt{-c^4x^4 + 1}}{\sqrt{c^2x^2 - 1}}\right) - 15(ac^6x^6 - ac^4x^4 + \dots)}{90(c^{10}x^2 - c^8)}$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

82.4 Problem number 172

$$\int \frac{x^3(a + b \sec^{-1}(cx))}{\sqrt{1 - c^4x^4}} dx$$

Optimal antiderivative

$$-\frac{bx \arctan\left(\frac{\sqrt{-c^4x^4 + 1}}{\sqrt{c^2x^2 - 1}}\right)}{2c^3 \sqrt{c^2x^2}} - \frac{(a + b \operatorname{arcsec}(cx)) \sqrt{-c^4x^4 + 1}}{2c^4} + \frac{bx \sqrt{-c^4x^4 + 1}}{2c^3 \sqrt{c^2x^2} \sqrt{c^2x^2 - 1}}$$

command

```
integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-c^4x^4 + 1} \sqrt{c^2x^2 - 1} b - (bc^2x^2 - b) \arctan\left(\frac{\sqrt{-c^4x^4 + 1}}{\sqrt{c^2x^2 - 1}}\right) - \sqrt{-c^4x^4 + 1} (ac^2x^2 + (bc^2x^2 - b) \operatorname{arcsec}(cx))}{2(c^6x^2 - c^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

83 Test file number 158

Test folder name:

test_cases/5_Inverse_trig_functions/5.6_Inverse_cosecant/158_5.6.1_u-a+b_arccsc-c_x-
^n

83.1 Problem number 154

$$\int \frac{a + b \csc^{-1}(cx)}{(d + ex^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x(a + b \operatorname{arccsc}(cx))}{d\sqrt{ex^2 + d}} + \frac{bx \operatorname{EllipticF}\left(cx, \sqrt{-\frac{e}{c^2d}}\right) \sqrt{-c^2x^2 + 1} \sqrt{1 + \frac{ex^2}{d}}}{d\sqrt{c^2x^2} \sqrt{c^2x^2 - 1} \sqrt{ex^2 + d}}$$

command

```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(bx^2e + bd)\sqrt{-d} \operatorname{ellipticF}\left(cx, -\frac{e}{c^2d}\right) - (bcdx \operatorname{arccsc}(cx) + acdx)\sqrt{x^2e + d}}{cd^2x^2e + cd^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d} (b \operatorname{arccsc}(cx) + a)}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

83.2 Problem number 174

$$\int \frac{x^{11}(a + b \csc^{-1}(cx))}{\sqrt{1 - c^4x^4}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(-c^4x^4 + 1)^{\frac{3}{2}} (a + b \operatorname{arccsc}(cx))}{3c^{12}} - \frac{(-c^4x^4 + 1)^{\frac{5}{2}} (a + b \operatorname{arccsc}(cx))}{10c^{12}} \\
& + \frac{7b(c^2x^2 + 1)^{\frac{3}{2}} \sqrt{-c^2x^2 + 1}}{90c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} - \frac{13b(c^2x^2 + 1)^{\frac{5}{2}} \sqrt{-c^2x^2 + 1}}{150c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} \\
& + \frac{3b(c^2x^2 + 1)^{\frac{7}{2}} \sqrt{-c^2x^2 + 1}}{70c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} - \frac{b(c^2x^2 + 1)^{\frac{9}{2}} \sqrt{-c^2x^2 + 1}}{90c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} \\
& + \frac{4b \operatorname{arctanh} \left(\sqrt{c^2x^2 + 1} \right) \sqrt{-c^2x^2 + 1}}{15c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} \\
& - \frac{4b \sqrt{-c^2x^2 + 1} \sqrt{c^2x^2 + 1}}{15c^{13}x \sqrt{1 - \frac{1}{c^2x^2}}} - \frac{(a + b \operatorname{arccsc}(cx)) \sqrt{-c^4x^4 + 1}}{2c^{12}}
\end{aligned}$$

command

```
integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$(35bc^8x^8 + 5bc^6x^6 + 78bc^4x^4 + 36bc^2x^2 + 768b) \sqrt{-c^4x^4 + 1} \sqrt{c^2x^2 - 1} - 840(bc^2x^2 - b) \arctan \left(\frac{\sqrt{-c^4x^4 + 1}}{\sqrt{c^2x^2 - 1}} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

83.3 Problem number 175

$$\int \frac{x^7 (a + b \operatorname{csc}^{-1}(cx))}{\sqrt{1 - c^4x^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-c^4x^4 + 1)^{\frac{3}{2}} (a + b \operatorname{arccsc}(cx))}{6c^8} + \frac{b(c^2x^2 + 1)^{\frac{3}{2}} \sqrt{-c^2x^2 + 1}}{18c^9x \sqrt{1 - \frac{1}{c^2x^2}}} \\ & - \frac{b(c^2x^2 + 1)^{\frac{5}{2}} \sqrt{-c^2x^2 + 1}}{30c^9x \sqrt{1 - \frac{1}{c^2x^2}}} + \frac{b \operatorname{arctanh}\left(\sqrt{c^2x^2 + 1}\right) \sqrt{-c^2x^2 + 1}}{3c^9x \sqrt{1 - \frac{1}{c^2x^2}}} \\ & - \frac{b \sqrt{-c^2x^2 + 1} \sqrt{c^2x^2 + 1}}{3c^9x \sqrt{1 - \frac{1}{c^2x^2}}} - \frac{(a + b \operatorname{arccsc}(cx)) \sqrt{-c^4x^4 + 1}}{2c^8} \end{aligned}$$

command

```
integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(3bc^4x^4 + bc^2x^2 + 28b) \sqrt{-c^4x^4 + 1} \sqrt{c^2x^2 - 1} - 30(bc^2x^2 - b) \arctan\left(\frac{\sqrt{-c^4x^4 + 1}}{\sqrt{c^2x^2 - 1}}\right) + 15(ac^6x^6 - ac^4x^4)}{90(c^{10}x^2 - c^8)}$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

83.4 Problem number 176

$$\int \frac{x^3(a + b \operatorname{csc}^{-1}(cx))}{\sqrt{1 - c^4x^4}} dx$$

Optimal antiderivative

$$\frac{bx \arctan\left(\frac{\sqrt{-c^4x^4 + 1}}{\sqrt{c^2x^2 - 1}}\right)}{2c^3 \sqrt{c^2x^2}} - \frac{(a + b \operatorname{arccsc}(cx)) \sqrt{-c^4x^4 + 1}}{2c^4} - \frac{bx \sqrt{-c^4x^4 + 1}}{2c^3 \sqrt{c^2x^2} \sqrt{c^2x^2 - 1}}$$

command

```
integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{-c^4x^4 + 1} \sqrt{c^2x^2 - 1} b - (bc^2x^2 - b) \arctan\left(\frac{\sqrt{-c^4x^4 + 1}}{\sqrt{c^2x^2 - 1}}\right) + \sqrt{-c^4x^4 + 1} (ac^2x^2 + (bc^2x^2 - b) \operatorname{arccsc}(cx))}{2(c^6x^2 - c^4)}$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

84 Test file number 160

Test folder name:

test_cases/6_Hyperbolic_functions/6.1_Hyperbolic_sine/160_6.1.1-c+d_x-^m-a+b_sinh-ⁿ

84.1 Problem number 248

$$\int \frac{(e + fx)^3 \operatorname{csch}^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

84.2 Problem number 401

$$\int \frac{(e + fx)^3 \cosh^3(c + dx) \sinh^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

84.3 Problem number 445

$$\int \frac{(e + fx)^2 \operatorname{csch}(c + dx) \operatorname{sech}^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((f*x+e)^2*csh(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

84.4 Problem number 473

$$\int \frac{(e + fx) \operatorname{csch}^2(c + dx) \operatorname{sech}^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{ib^2 f \operatorname{polylog}(2, ie^{dx+c})}{2a(a^2+b^2)d^2} + \frac{ib^4 f \operatorname{polylog}(2, ie^{dx+c})}{a(a^2+b^2)^2 d^2} - \frac{ib^4 f \operatorname{polylog}(2, -ie^{dx+c})}{a(a^2+b^2)^2 d^2} \\
& - \frac{ib^2 f \operatorname{polylog}(2, -ie^{dx+c})}{2a(a^2+b^2)d^2} + \frac{b^2 f \operatorname{sech}(dx+c)}{2a(a^2+b^2)d^2} + \frac{b^3(fx+e) \operatorname{sech}(dx+c)^2}{2a^2(a^2+b^2)d} \\
& - \frac{b^3 f \tanh(dx+c)}{2a^2(a^2+b^2)d^2} + \frac{b^2(fx+e) \arctan(e^{dx+c})}{a(a^2+b^2)d} + \frac{2b^4(fx+e) \arctan(e^{dx+c})}{a(a^2+b^2)^2 d} \\
& - \frac{f \operatorname{sech}(dx+c)}{2a d^2} - \frac{b(fx+e) \ln(\tanh(dx+c))}{a^2 d} - \frac{3(fx+e) \operatorname{csch}(dx+c)}{2ad} \\
& - \frac{3(fx+e) \arctan(\sinh(dx+c))}{2ad} + \frac{(fx+e) \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)^2}{2ad} \\
& + \frac{bf \tanh(dx+c)}{2a^2 d^2} - \frac{3fx \arctan(e^{dx+c})}{ad} + \frac{3fx \arctan(\sinh(dx+c))}{2ad} \\
& - \frac{f \operatorname{arctanh}(\cosh(dx+c))}{a d^2} - \frac{b^5 f \operatorname{polylog}(2, -e^{2dx+2c})}{2a^2(a^2+b^2)^2 d^2} \\
& + \frac{2bfx \operatorname{arctanh}(e^{2dx+2c})}{a^2 d} + \frac{b^5 f \operatorname{polylog}\left(2, -\frac{be^{dx+c}}{a-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^2 d^2} \\
& + \frac{b^5 f \operatorname{polylog}\left(2, -\frac{be^{dx+c}}{a+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^2 d^2} - \frac{bf \operatorname{polylog}(2, e^{2dx+2c})}{2a^2 d^2} \\
& + \frac{bf \operatorname{polylog}(2, -e^{2dx+2c})}{2a^2 d^2} + \frac{3if \operatorname{polylog}(2, -ie^{dx+c})}{2a d^2} - \frac{3if \operatorname{polylog}(2, ie^{dx+c})}{2a d^2} \\
& + \frac{b(fx+e) \operatorname{tanh}^2(dx+c)}{2a^2 d} - \frac{bfx}{2a^2 d} - \frac{b^5(fx+e) \ln(1+e^{2dx+2c})}{a^2(a^2+b^2)^2 d} \\
& + \frac{b^5(fx+e) \ln\left(1+\frac{be^{dx+c}}{a-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^2 d} + \frac{b^5(fx+e) \ln\left(1+\frac{be^{dx+c}}{a+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^2 d} \\
& + \frac{bfx \ln(\tanh(dx+c))}{a^2 d} + \frac{b^2(fx+e) \operatorname{sech}(dx+c) \tanh(dx+c)}{2a(a^2+b^2)d}
\end{aligned}$$

command

```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

84.5 Problem number 486

$$\int \frac{(e + fx)^3 \coth^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{b^2(fx+e)^4}{4a^3f} + \frac{(a^2+b^2)(fx+e)^4}{4a^3f} \\
& - \frac{(a^2+b^2)(fx+e)^3 \ln\left(1 + \frac{be^{dx+c}}{a-\sqrt{a^2+b^2}}\right)}{a^3d} \\
& - \frac{(a^2+b^2)(fx+e)^3 \ln\left(1 + \frac{be^{dx+c}}{a+\sqrt{a^2+b^2}}\right)}{a^3d} \\
& + \frac{b(fx+e)^3 \operatorname{csch}(dx+c)}{a^2d} - \frac{3f(fx+e)^2 \operatorname{coth}(dx+c)}{2ad^2} \\
& - \frac{3(a^2+b^2)f(fx+e)^2 \operatorname{polylog}\left(2, -\frac{be^{dx+c}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} \\
& + \frac{6bf(fx+e)^2 \operatorname{arctanh}(e^{dx+c})}{a^2d^2} + \frac{6bf^2(fx+e) \operatorname{polylog}(2, -e^{dx+c})}{a^2d^3} \\
& - \frac{6bf^2(fx+e) \operatorname{polylog}(2, e^{dx+c})}{a^2d^3} + \frac{3b^2f(fx+e)^2 \operatorname{polylog}(2, e^{2dx+2c})}{2a^3d^2} \\
& - \frac{3(a^2+b^2)f(fx+e)^2 \operatorname{polylog}\left(2, -\frac{be^{dx+c}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2} \\
& + \frac{6(a^2+b^2)f^2(fx+e) \operatorname{polylog}\left(3, -\frac{be^{dx+c}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3} \\
& + \frac{6(a^2+b^2)f^2(fx+e) \operatorname{polylog}\left(3, -\frac{be^{dx+c}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3} \\
& - \frac{3b^2f^2(fx+e) \operatorname{polylog}(3, e^{2dx+2c})}{2a^3d^3} - \frac{3f(fx+e)^2}{2ad^2} \\
& - \frac{(fx+e)^4}{4af} + \frac{(fx+e)^3}{2ad} + \frac{(fx+e)^3 \ln(1-e^{2dx+2c})}{ad} \\
& + \frac{3f^2(fx+e) \ln(1-e^{2dx+2c})}{ad^3} + \frac{b^2(fx+e)^3 \ln(1-e^{2dx+2c})}{a^3d} \\
& + \frac{3f^3 \operatorname{polylog}(2, e^{2dx+2c})}{2ad^4} + \frac{3f^3 \operatorname{polylog}(4, e^{2dx+2c})}{4ad^4} \\
& - \frac{(fx+e)^3 (\operatorname{coth}^2(dx+c))}{2ad} - \frac{6(a^2+b^2)f^3 \operatorname{polylog}\left(4, -\frac{be^{dx+c}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^4} \\
& - \frac{6(a^2+b^2)f^3 \operatorname{polylog}\left(4, -\frac{be^{dx+c}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^4} - \frac{6bf^3 \operatorname{polylog}(3, -e^{dx+c})}{a^2d^4} \\
& + \frac{6bf^3 \operatorname{polylog}(3, e^{dx+c})}{a^2d^4} + \frac{3b^2f^3 \operatorname{polylog}(4, e^{2dx+2c})}{4a^3d^4} \\
& + \frac{3f(fx+e)^2 \operatorname{polylog}(2, e^{2dx+2c})}{2ad^2} - \frac{3f^2(fx+e) \operatorname{polylog}(3, e^{2dx+2c})}{2ad^3}
\end{aligned}$$

command

```
integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

84.6 Problem number 491

$$\int \frac{(e + fx)^3 \operatorname{csch}^3(c + dx) \operatorname{sech}(c + dx)}{a + b \sinh(c + dx)} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

84.7 Problem number 496

$$\int \frac{(e + fx)^2 \operatorname{csch}^3(c + dx) \operatorname{sech}^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

84.8 Problem number 500

$$\int \frac{(e + fx)\operatorname{csch}^3(c + dx)\operatorname{sech}^3(c + dx)}{a + b\sinh(c + dx)} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

85 Test file number 163

Test folder name:

test_cases/6_Hyperbolic_functions/6.1_Hyperbolic_sine/163_6.1.5_Hyperbolic_sine_functions

85.1 Problem number 7

$$\int \sinh^{\frac{7}{2}}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \cosh(bx + a) \left(\sinh^{\frac{5}{2}}(bx + a) \right)}{7b} \\ & + \frac{10i \sqrt{\frac{1}{2} + \frac{\sin(ibx + ia)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right), \sqrt{2}\right) \sqrt{i \sinh(bx + a)}}{21 \sin\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right) b \sqrt{\sinh(bx + a)}} \\ & - \frac{10 \cosh(bx + a) \left(\sqrt{\sinh(bx + a)} \right)}{21b} \end{aligned}$$

command

```
integrate(sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$40 \left(\sqrt{2} \cosh (bx+a)^3 + 3 \sqrt{2} \cosh (bx+a)^2 \sinh (bx+a) + 3 \sqrt{2} \cosh (bx+a) \sinh (bx+a)^2 + \sqrt{2} \sinh (bx+a)^3 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sinh (bx+a)^{\frac{7}{2}}, x\right)$$

85.2 Problem number 8

$$\int \sinh^{\frac{5}{2}}(a+bx) dx$$

Optimal antiderivative

$$\frac{2 \cosh (bx+a) \left(\sinh^{\frac{3}{2}}(bx+a) \right)}{5b} - \frac{6i \sqrt{\frac{1}{2} + \frac{\sin (ibx+ia)}{2}} \text{EllipticE}\left(\cos\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right), \sqrt{2}\right) \left(\sqrt{\sinh (bx+a)}\right)}{5 \sin\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right) b \sqrt{i \sinh (bx+a)}}$$

command

```
integrate(sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 \left(\sqrt{2} \cosh (bx+a)^2 + 2 \sqrt{2} \cosh (bx+a) \sinh (bx+a) + \sqrt{2} \sinh (bx+a)^2 \right) \text{weierstrassZeta}(4, 0, \text{weierstrassP}(bx+a))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sinh (bx+a)^{\frac{5}{2}}, x\right)$$

85.3 Problem number 9

$$\int \sinh^{\frac{3}{2}}(a+bx) dx$$

Optimal antiderivative

$$\frac{2i\sqrt{\frac{1}{2} + \frac{\sin(ibx + ia)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right), \sqrt{2}\right) \sqrt{i \sinh(bx + a)}}{3 \sin\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right) b \sqrt{\sinh(bx + a)}} + \frac{2 \cosh(bx + a) \left(\sqrt{\sinh(bx + a)}\right)}{3b}$$

command

```
integrate(sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{2} \cosh(bx + a) + \sqrt{2} \sinh(bx + a)\right) \operatorname{weierstrassPInverse}(4, 0, \cosh(bx + a) + \sinh(bx + a)) - \left(\cosh(bx + a) + \sinh(bx + a)\right)}{3(b \cosh(bx + a) + b \sinh(bx + a))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sinh(bx + a)^{\frac{3}{2}}, x\right)$$

85.4 Problem number 10

$$\int \sqrt{\sinh(a + bx)} dx$$

Optimal antiderivative

$$\frac{2i\sqrt{\frac{1}{2} + \frac{\sin(ibx + ia)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right), \sqrt{2}\right) \left(\sqrt{\sinh(bx + a)}\right)}{\sin\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right) b \sqrt{i \sinh(bx + a)}}$$

command

```
integrate(sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{2} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cosh(bx + a) + \sinh(bx + a))) + \sqrt{\sinh(bx + a)}\right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{\sinh(bx + a)}, x\right)$$

85.5 Problem number 11

$$\int \frac{1}{\sqrt{\sinh(a+bx)}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(ibx+ia)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right), \sqrt{2}\right) \sqrt{i \sinh(bx+a)}}{\sin\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right) b \sqrt{\sinh(bx+a)}}$$

command

`integrate(1/sinh(b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2} \operatorname{weierstrassPInverse}(4, 0, \cosh(bx+a) + \sinh(bx+a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{\sinh(bx+a)}}, x\right)$$

85.6 Problem number 12

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cosh(bx+a)}{b \sqrt{\sinh(bx+a)}} \\ & + \frac{2i \sqrt{\frac{1}{2} + \frac{\sin(ibx+ia)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right), \sqrt{2}\right) \left(\sqrt{\sinh(bx+a)}\right)}{\sin\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right) b \sqrt{i \sinh(bx+a)}} \end{aligned}$$

command

`integrate(1/sinh(b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\left(\sqrt{2} \cosh(bx+a) \right)^2 + 2 \sqrt{2} \cosh(bx+a) \sinh(bx+a) + \sqrt{2} \sinh(bx+a)^2 - \sqrt{2} \right) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cosh(bx+a) + \sinh(bx+a)))}{b \cosh(bx+a)^2 + 2b \sinh(bx+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sinh(bx+a)^{\frac{3}{2}}}, x\right)$$

85.7 Problem number 13

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{-\frac{2 \cosh (bx+a)}{3b \sinh (bx+a)^{\frac{3}{2}}}-2i \sqrt{\frac{1}{2}+\frac{\sin (ibx+ia)}{2}} \operatorname{EllipticF}\left(\cos \left(\frac{1}{2}ia+\frac{1}{4}\pi+\frac{1}{2}ibx\right), \sqrt{2}\right) \sqrt{i \sinh (bx+a)}}{3 \sin \left(\frac{1}{2}ia+\frac{1}{4}\pi+\frac{1}{2}ibx\right) b \sqrt{\sinh (bx+a)}}$$

command

```
integrate(1/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\left(\sqrt{2} \cosh (bx+a)^4+4 \sqrt{2} \cosh (bx+a) \sinh (bx+a)^3+\sqrt{2} \sinh (bx+a)^4+2\left(3 \sqrt{2} \cosh (bx+a)^2-\sqrt{2}\right)\right)\right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sinh (bx+a)^{\frac{5}{2}}}, x\right)$$

85.8 Problem number 14

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{-\frac{2 \cosh (bx+a)}{5b \sinh (bx+a)^{\frac{5}{2}}}+\frac{6 \cosh (bx+a)}{5b \sqrt{\sinh (bx+a)}}-6i \sqrt{\frac{1}{2}+\frac{\sin (ibx+ia)}{2}} \operatorname{EllipticE}\left(\cos \left(\frac{1}{2}ia+\frac{1}{4}\pi+\frac{1}{2}ibx\right), \sqrt{2}\right) \left(\sqrt{\sinh (bx+a)}\right)}{5 \sin \left(\frac{1}{2}ia+\frac{1}{4}\pi+\frac{1}{2}ibx\right) b \sqrt{i \sinh (bx+a)}}$$

command

```
integrate(1/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \left(\sqrt{2} \cosh (bx + a)^6 + 6 \sqrt{2} \cosh (bx + a) \sinh (bx + a)^5 + \sqrt{2} \sinh (bx + a)^6 + 3 \left(5 \sqrt{2} \cosh (bx + a)^2 - \sqrt{2} \right) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\sinh (bx + a)^{\frac{7}{2}}}, x \right)$$

85.9 Problem number 15

$$\int (b \sinh (c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b \cosh (dx + c) (b \sinh (dx + c))^{\frac{5}{2}}}{7d} \\ & + \frac{10ib^4 \sqrt{\frac{1}{2} + \frac{\sin (idx + ic)}{2}} \text{EllipticF} \left(\cos \left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx \right), \sqrt{2} \right) \sqrt{i \sinh (dx + c)}}{21 \sin \left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx \right) d \sqrt{b \sinh (dx + c)}} \\ & - \frac{10b^3 \cosh (dx + c) \sqrt{b \sinh (dx + c)}}{21d} \end{aligned}$$

command

```
integrate((b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$40 \left(\sqrt{2} b^3 \cosh (dx + c)^3 + 3 \sqrt{2} b^3 \cosh (dx + c)^2 \sinh (dx + c) + 3 \sqrt{2} b^3 \cosh (dx + c) \sinh (dx + c)^2 + \sqrt{2} b^3 \sinh (dx + c)^3 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{b \sinh (dx + c)} b^3 \sinh (dx + c)^3, x \right)$$

85.10 Problem number 16

$$\int (b \sinh(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b \cosh(dx + c) (b \sinh(dx + c))^{3/2}}{5d} - \frac{6ib^2 \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right) \sqrt{b \sinh(dx + c)}}{5 \sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d \sqrt{i \sinh(dx + c)}}$$

command

```
integrate((b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 \left(\sqrt{2} b^2 \cosh(dx + c)^2 + 2 \sqrt{2} b^2 \cosh(dx + c) \sinh(dx + c) + \sqrt{2} b^2 \sinh(dx + c)^2 \right) \sqrt{b} \operatorname{weierstrassZeta}(4, 0, \text{we})$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sinh(dx + c)} b^2 \sinh(dx + c)^2, x\right)$$

85.11 Problem number 17

$$\int (b \sinh(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2ib^2 \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right) \sqrt{i \sinh(dx + c)}}{3 \sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d \sqrt{b \sinh(dx + c)}} + \frac{2b \cosh(dx + c) \sqrt{b \sinh(dx + c)}}{3d}$$

command

```
integrate((b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} b \cosh(dx + c) + \sqrt{2} b \sinh(dx + c) \right) \sqrt{b} \operatorname{weierstrassPInverse}(4, 0, \cosh(dx + c) + \sinh(dx + c)) - \left(b \cosh(dx + c) + b \sinh(dx + c) \right)}{3 (d \cosh(dx + c) + d \sinh(dx + c))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sinh(dx + c)} b \sinh(dx + c), x\right)$$

85.12 Problem number 18

$$\int \sqrt{b \sinh(c + dx)} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right) \sqrt{b \sinh(dx + c)}}{\sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d \sqrt{i \sinh(dx + c)}}$$

command

`integrate((b*sinh(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} \sqrt{b} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cosh(dx + c) + \sinh(dx + c))) + \sqrt{b \sinh(dx + c)} \right)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sinh(dx + c)}, x\right)$$

85.13 Problem number 19

$$\int \frac{1}{\sqrt{b \sinh(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right) \sqrt{i \sinh(dx + c)}}{\sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d \sqrt{b \sinh(dx + c)}}$$

command

```
integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2} \operatorname{weierstrassPInverse}(4, 0, \cosh(dx + c) + \sinh(dx + c))}{\sqrt{b} d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sinh(dx + c)}}{b \sinh(dx + c)}, x\right)$$

85.14 Problem number 20

$$\int \frac{1}{(b \sinh(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cosh(dx + c)}{bd \sqrt{b \sinh(dx + c)}} \\ & + \frac{2i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right) \sqrt{b \sinh(dx + c)}}{\sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) b^2 d \sqrt{i \sinh(dx + c)}} \end{aligned}$$

command

```
integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\left(\sqrt{2} \cosh(dx + c)^2 + 2\sqrt{2} \cosh(dx + c) \sinh(dx + c) + \sqrt{2} \sinh(dx + c)^2 - \sqrt{2} \right) \sqrt{b} \operatorname{weierstrassZeta}(4, 0) \right)}{b^2 d \cosh(dx + c)^2 + 2b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sinh(dx + c)}}{b^2 \sinh(dx + c)^2}, x\right)$$

85.15 Problem number 21

$$\int \frac{1}{(b \sinh(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \cosh(dx + c)}{3bd (b \sinh(dx + c))^{3/2}} - \frac{2i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right) \sqrt{i \sinh(dx + c)}}{3 \sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) b^2 d \sqrt{b \sinh(dx + c)}}$$

command

```
integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((\sqrt{2} \cosh(dx + c))^4 + 4 \sqrt{2} \cosh(dx + c) \sinh(dx + c)^3 + \sqrt{2} \sinh(dx + c)^4 + 2 \left(3 \sqrt{2} \cosh(dx + c)^2 - \sqrt{2} \right) \right)}{3 (b^3 d)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sinh(dx + c)}}{b^3 \sinh(dx + c)^3}, x\right)$$

85.16 Problem number 22

$$\int \frac{1}{(b \sinh(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2 \cosh(dx + c)}{5bd (b \sinh(dx + c))^{5/2}} + \frac{6 \cosh(dx + c)}{5b^3 d \sqrt{b \sinh(dx + c)}} - \frac{6i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right) \sqrt{b \sinh(dx + c)}}{5 \sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) b^4 d \sqrt{i \sinh(dx + c)}}$$

command

```
integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \left(\sqrt{2} \cosh(dx + c)^6 + 6 \sqrt{2} \cosh(dx + c) \sinh(dx + c)^5 + \sqrt{2} \sinh(dx + c)^6 + 3 \left(5 \sqrt{2} \cosh(dx + c)^2 - \sqrt{2} \right) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \sinh(dx + c)}}{b^4 \sinh(dx + c)^4}, x \right)$$

85.17 Problem number 23

$$\int (i \sinh(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\frac{10i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \text{EllipticF} \left(\cos \left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx \right), \sqrt{2} \right)}{21 \sin \left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx \right) d} + \frac{2i \cosh(dx + c) (i \sinh(dx + c))^{\frac{5}{2}}}{7d} + \frac{10i \cosh(dx + c) \sqrt{i \sinh(dx + c)}}{21d}$$

command

```
integrate((I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left(\sqrt{\frac{1}{2}} \left(-3i e^{(6dx+6c)} + 23i e^{(4dx+4c)} + 23i e^{(2dx+2c)} - 3i \right) \sqrt{i e^{(2dx+2c)} - i} e^{(-\frac{1}{2}dx - \frac{1}{2}c)} - 40i \sqrt{2} \sqrt{i} e^{(3dx+3c)} \right) \text{weil}$$

84 d

Fricas 1.3.7 via sagemath 9.3 output

$$\left(\sqrt{\frac{1}{2}} \left(-3i e^{(6dx+6c)} + 23i e^{(4dx+4c)} + 23i e^{(2dx+2c)} - 3i \right) \sqrt{i e^{(2dx+2c)} - i} e^{(-\frac{1}{2}dx - \frac{1}{2}c)} + 84 d e^{(3dx+3c)} \right) \text{integral} \left(- \right)$$

84 d

85.18 Problem number 24

$$\int (i \sinh(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{6i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right)}{5 \sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d} + \frac{2i \cosh(dx + c) (i \sinh(dx + c))^{\frac{3}{2}}}{5d}$$

command

```
integrate((I*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{\frac{1}{2}} (e^{(4dx+4c)} + 12e^{(2dx+2c)} - 1) \sqrt{i e^{(2dx+2c)} - i} e^{(-\frac{1}{2}dx - \frac{1}{2}c)} + 12\sqrt{2} \sqrt{i} e^{(2dx+2c)} \operatorname{weierstrassZeta}(4, 0, \dots)\right)}{10d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{\frac{1}{2}} (e^{(5dx+5c)} - 2e^{(4dx+4c)} - 12e^{(3dx+3c)} - 24e^{(2dx+2c)} - e^{(dx+c)} + 2) \sqrt{i e^{(2dx+2c)} - i} e^{(-\frac{1}{2}dx - \frac{1}{2}c)} - 10 (de^{(3dx+3c)} - 2de^{(2dx+2c)})}{10 (de^{(3dx+3c)} - 2de^{(2dx+2c)})}$$

85.19 Problem number 25

$$\int (i \sinh(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right)}{3 \sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d} + \frac{2i \cosh(dx + c) \sqrt{i \sinh(dx + c)}}{3d}$$

command

```
integrate((I*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\sqrt{\frac{1}{2}}(ie^{(2dx+2c)}+i)\sqrt{ie^{(2dx+2c)}-i}e^{(-\frac{1}{2}dx-\frac{1}{2}c)}-2i\sqrt{2}\sqrt{i}e^{(dx+c)}\text{weierstrassPInverse}(4,0,e^{(dx+c)})\right)e^{(-dx-c)}}{3d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\left(\sqrt{\frac{1}{2}}(ie^{(2dx+2c)}+i)\sqrt{ie^{(2dx+2c)}-i}e^{(-\frac{1}{2}dx-\frac{1}{2}c)}+3de^{(dx+c)}\text{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{ie^{(2dx+2c)}-i}e^{(-\frac{1}{2}dx-\frac{1}{2}c)}}{3(de^{(2dx+2c)}-d)},x\right)\right)}{3d}$$

85.20 Problem number 26

$$\int \sqrt{i \sinh(c+dx)} dx$$

Optimal antiderivative

$$\frac{2i\sqrt{\frac{1}{2}+\frac{\sin(idx+ic)}{2}}\text{EllipticE}\left(\cos\left(\frac{1}{2}ic+\frac{1}{4}\pi+\frac{1}{2}idx\right),\sqrt{2}\right)}{\sin\left(\frac{1}{2}ic+\frac{1}{4}\pi+\frac{1}{2}idx\right)d}$$

command

`integrate((I*sinh(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{\frac{1}{2}}\sqrt{ie^{(2dx+2c)}-i}e^{(-\frac{1}{2}dx-\frac{1}{2}c)}+\sqrt{2}\sqrt{i}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,e^{(dx+c)}))\right)}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{2\sqrt{\frac{1}{2}}\sqrt{ie^{(2dx+2c)}-i}(e^{(dx+c)}+2)e^{(-\frac{1}{2}dx-\frac{1}{2}c)}+(de^{(dx+c)}-2d)\text{integral}\left(\frac{2\sqrt{\frac{1}{2}}(2e^{(2dx+2c)}+3e^{(dx+c)}-2)\sqrt{ie^{(2dx+2c)}-i}}{de^{(4dx+4c)}-4de^{(3dx+3c)}+3de^{(2dx+2c)}},x\right)}{de^{(dx+c)}-2d}$$

85.21 Problem number 27

$$\int \frac{1}{\sqrt{i \sinh(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right)}{\sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d}$$

command

`integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2i \sqrt{2} \sqrt{i} \operatorname{weierstrassPInverse}(4, 0, e^{(dx+c)})}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{2i \sqrt{\frac{1}{2}} \sqrt{i e^{(2dx+2c)} - i} e^{(-\frac{1}{2}dx - \frac{1}{2}c)}}{de^{(2dx+2c)} - d}, x\right)$$

85.22 Problem number 28

$$\int \frac{1}{(i \sinh(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right)}{\sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d} + \frac{2i \cosh(dx + c)}{d \sqrt{i \sinh(dx + c)}}$$

command

`integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{\frac{1}{2}} \sqrt{i e^{(2dx+2c)} - i} e^{(\frac{3}{2}dx + \frac{3}{2}c)} + \left(\sqrt{2} \sqrt{i} e^{(2dx+2c)} - \sqrt{2} \sqrt{i} \right) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, e^{(dx+c)})) \right)}{de^{(2dx+2c)} - d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{4 \sqrt{\frac{1}{2}} \sqrt{i e^{(2 dx+2c)} - i e^{\left(\frac{3}{2} dx + \frac{3}{2} c\right)}} + (d e^{(2 dx+2c)} - d) \operatorname{integral} \left(-\frac{2 \sqrt{\frac{1}{2}} \sqrt{i e^{(2 dx+2c)} - i e^{\left(\frac{1}{2} dx + \frac{1}{2} c\right)}}}{d e^{(2 dx+2c)} - d}, x \right)}{d e^{(2 dx+2c)} - d}$$

85.23 Problem number 29

$$\int \frac{1}{(i \sinh(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{1}{2} ic + \frac{1}{4} \pi + \frac{1}{2} idx \right), \sqrt{2} \right)}{3 \sin \left(\frac{1}{2} ic + \frac{1}{4} \pi + \frac{1}{2} idx \right) d} + \frac{2i \cosh(dx + c)}{3d (i \sinh(dx + c))^{\frac{3}{2}}}$$

command

`integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{\frac{1}{2}} (i e^{(3 dx+3c)} + i e^{(dx+c)}) \sqrt{i e^{(2 dx+2c)} - i e^{\left(-\frac{1}{2} dx - \frac{1}{2} c\right)}} + (i \sqrt{2} \sqrt{i} e^{(4 dx+4c)} - 2i \sqrt{2} \sqrt{i} e^{(2 dx+2c)} + i \sqrt{2} \sqrt{i} e^{(2 dx+2c)}) \right)}{3 (d e^{(4 dx+4c)} - 2 d e^{(2 dx+2c)} + d)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\sqrt{\frac{1}{2}} (-4i e^{(3 dx+3c)} - 4i e^{(dx+c)}) \sqrt{i e^{(2 dx+2c)} - i e^{\left(-\frac{1}{2} dx - \frac{1}{2} c\right)}} + 3 (d e^{(4 dx+4c)} - 2 d e^{(2 dx+2c)} + d) \operatorname{integral} \left(-\frac{2i \sqrt{\frac{1}{2}} \sqrt{i e^{(2 dx+2c)} - i e^{\left(-\frac{1}{2} dx - \frac{1}{2} c\right)}}}{d e^{(2 dx+2c)} - d}, x \right)}{3 (d e^{(4 dx+4c)} - 2 d e^{(2 dx+2c)} + d)}$$

85.24 Problem number 30

$$\int \frac{1}{(i \sinh(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{6i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right)}{5 \sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d} + \frac{2i \cosh(dx + c)}{5d (i \sinh(dx + c))^{\frac{5}{2}}} + \frac{6i \cosh(dx + c)}{5d \sqrt{i \sinh(dx + c)}}$$

command

```
integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{\frac{1}{2}} (3e^{(6dx+6c)} - 8e^{(4dx+4c)} + e^{(2dx+2c)}) \sqrt{ie^{(2dx+2c)} - i} e^{(-\frac{1}{2}dx - \frac{1}{2}c)} + 3 \left(\sqrt{2} \sqrt{i} e^{(6dx+6c)} - 3 \sqrt{2} \sqrt{i} e^{(4dx+4c)} + 3 \sqrt{2} \sqrt{i} e^{(2dx+2c)} \right) \right)}{5 (de^{(6dx+6c)} - 3de^{(4dx+4c)} + 3de^{(2dx+2c)} - d)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{4 \sqrt{\frac{1}{2}} (3e^{(6dx+6c)} - 8e^{(4dx+4c)} + e^{(2dx+2c)}) \sqrt{ie^{(2dx+2c)} - i} e^{(-\frac{1}{2}dx - \frac{1}{2}c)} + 5 (de^{(6dx+6c)} - 3de^{(4dx+4c)} + 3de^{(2dx+2c)} - d)}{5 (de^{(6dx+6c)} - 3de^{(4dx+4c)} + 3de^{(2dx+2c)} - d)}$$

85.25 Problem number 105

$$\int (a + b \sinh(x))^{5/2} dx$$

Optimal antiderivative

$$\frac{2b \cosh(x) (a + b \sinh(x))^{\frac{3}{2}}}{5} + \frac{16ab \cosh(x) \sqrt{a + b \sinh(x)}}{15} + \frac{2i(23a^2 - 9b^2) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{a + b \sinh(x)}}{15 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}} + \frac{16ia(a^2 + b^2) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}{15 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) \sqrt{a + b \sinh(x)}}$$

command

```
integrate((a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \left(\sqrt{2} (a^3 + 33 ab^2) \cosh(x)^2 + 2 \sqrt{2} (a^3 + 33 ab^2) \cosh(x) \sinh(x) + \sqrt{2} (a^3 + 33 ab^2) \sinh(x)^2 \right) \sqrt{b} \text{weierstrassPInverse}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 \sinh(x)^2 + 2 ab \sinh(x) + a^2\right) \sqrt{b \sinh(x) + a}, x\right)$$

85.26 Problem number 106

$$\int (a + b \sinh(x))^{3/2} dx$$

Optimal antiderivative

$$\frac{2b \cosh(x) \sqrt{a + b \sinh(x)}}{3} + \frac{8ia \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \text{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{a + b \sinh(x)}}{3 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}} + \frac{2i(a^2 + b^2) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \text{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}{3 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) \sqrt{a + b \sinh(x)}}$$

command

```
integrate((a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (a^2 - 3b^2) \cosh(x) + \sqrt{2} (a^2 - 3b^2) \sinh(x) \right) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2 + 3b^2)}{3b^2}, -\frac{8(8a^3 + 9ab^2)}{27b^3}, \frac{3b \cosh(x)}{27b^3}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((b \sinh(x) + a)^{\frac{3}{2}}, x\right)$$

85.27 Problem number 107

$$\int \sqrt{a + b \sinh(x)} \, dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia+b}}\right) \sqrt{a + b \sinh(x)}}{\sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}$$

command

```
integrate((a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} a \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2+3b^2)}{3b^2}, -\frac{8(8a^3+9ab^2)}{27b^3}, \frac{3b \cosh(x)+3b \sinh(x)+2a}{3b}\right) - 3 \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}\left(\frac{4}{3}\right) \right)}{\sqrt{a + b \sinh(x)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \sinh(x) + a}, x\right)$$

85.28 Problem number 108

$$\int \frac{1}{\sqrt{a + b \sinh(x)}} \, dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia+b}}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}{\sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) \sqrt{a + b \sinh(x)}}$$

command

```
integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{2} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2+3b^2)}{3b^2}, -\frac{8(8a^3+9ab^2)}{27b^3}, \frac{3b \cosh(x)+3b \sinh(x)+2a}{3b}\right)}{\sqrt{b}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{b \sinh(x) + a}}, x\right)$$

85.29 Problem number 109

$$\int \frac{1}{(a + b \sinh(x))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b \cosh(x)}{(a^2 + b^2) \sqrt{a + b \sinh(x)}} \\ & + \frac{2i \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{a + b \sinh(x)}}{\sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) (a^2 + b^2) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}} \end{aligned}$$

command

`integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\left(\sqrt{2} ab \cosh(x)^2 + \sqrt{2} ab \sinh(x)^2 + 2 \sqrt{2} a^2 \cosh(x) - \sqrt{2} ab + 2 \left(\sqrt{2} ab \cosh(x) + \sqrt{2} a^2 \right) \sinh(x) \right) \sqrt{b}}{\dots} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \sinh(x) + a}}{b^2 \sinh(x)^2 + 2ab \sinh(x) + a^2}, x\right)$$

85.30 Problem number 110

$$\int \frac{1}{(a + b \sinh(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b \cosh(x)}{3(a^2 + b^2)(a + b \sinh(x))^{3/2}} - \frac{8ab \cosh(x)}{3(a^2 + b^2)^2 \sqrt{a + b \sinh(x)}} \\ & + \frac{8ia \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{a + b \sinh(x)}}{3 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) (a^2 + b^2)^2 \sqrt{\frac{a + b \sinh(x)}{-ib + a}}} \\ & - \frac{2i \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}{3 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) (a^2 + b^2) \sqrt{a + b \sinh(x)}} \end{aligned}$$

command

```
integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} (a^2 b^2 - 3 b^4) \cosh(x)^4 + \sqrt{2} (a^2 b^2 - 3 b^4) \sinh(x)^4 + 4 \sqrt{2} (a^3 b - 3 a b^3) \cosh(x)^3 + 4 \left(\sqrt{2} (a^2 b^2 - 3 b^4) \right) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \sinh(x) + a}}{b^3 \sinh(x)^3 + 3 a b^2 \sinh(x)^2 + 3 a^2 b \sinh(x) + a^3}, x \right)$$

85.31 Problem number 111

$$\int \frac{\sinh(x)}{\sqrt{a + b \sinh(x)}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \text{EllipticE} \left(\cos \left(\frac{\pi}{4} + \frac{ix}{2} \right), \sqrt{2} \sqrt{\frac{b}{ia + b}} \right) \sqrt{a + b \sinh(x)}}{\sin \left(\frac{\pi}{4} + \frac{ix}{2} \right) b \sqrt{\frac{a + b \sinh(x)}{-ib + a}}} - \frac{2ia \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \text{EllipticF} \left(\cos \left(\frac{\pi}{4} + \frac{ix}{2} \right), \sqrt{2} \sqrt{\frac{b}{ia + b}} \right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}{\sin \left(\frac{\pi}{4} + \frac{ix}{2} \right) b \sqrt{a + b \sinh(x)}}$$

command

```
integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2 \sqrt{2} a \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2 + 3b^2)}{3b^2}, -\frac{8(8a^3 + 9ab^2)}{27b^3}, \frac{3b \cosh(x) + 3b \sinh(x) + 2a}{3b} \right) + 3 \sqrt{2} b^{\frac{3}{2}} \text{weierstrassZeta} \left(\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sinh(x)}{\sqrt{b \sinh(x) + a}}, x \right)$$

85.32 Problem number 126

$$\int (a + b \sinh(x))^{5/2} (A + B \sinh(x)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7Ab + 5Ba) \cosh(x) (a + b \sinh(x))^{3/2}}{35} + \frac{2B \cosh(x) (a + b \sinh(x))^{5/2}}{7} \\ & + \frac{2(56Aab + 15Ba^2 - 25b^2B) \cosh(x) \sqrt{a + b \sinh(x)}}{105} \\ & + \frac{2i(161Aa^2b - 63Ab^3 + 15a^3B - 145Ba^2b^2) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{a + b \sinh(x)}}{105 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) b \sqrt{\frac{a + b \sinh(x)}{-ib + a}}} \\ & + \frac{2i(a^2 + b^2) (56Aab + 15Ba^2 - 25b^2B) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}{105 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) b \sqrt{a + b \sinh(x)}} \end{aligned}$$

command

```
integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{8 \left(\sqrt{2} (30Ba^4 + 7Aa^3b + 115Ba^2b^2 + 231Aab^3 - 75Bb^4) \cosh(x)^3 + 3\sqrt{2} (30Ba^4 + 7Aa^3b + 115Ba^2b^2 + 231Aab^3 - 75Bb^4) \cosh(x) \right)}{105 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) b \sqrt{a + b \sinh(x)}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb^2 \sinh(x)^3 + Aa^2 + (2Bab + Ab^2) \sinh(x)^2 + (Ba^2 + 2Aab) \sinh(x)\right) \sqrt{b \sinh(x) + a}, x\right)$$

85.33 Problem number 127

$$\int (a + b \sinh(x))^{3/2} (A + B \sinh(x)) dx$$

Optimal antiderivative

$$\frac{2B \cosh(x) (a + b \sinh(x))^{\frac{3}{2}}}{5} + \frac{2(5Ab + 3Ba) \cosh(x) \sqrt{a + b \sinh(x)}}{15}$$

$$+ \frac{2i(20Aab + 3B a^2 - 9b^2B) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{a + b \sinh(x)}}{15 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) b \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}$$

$$- \frac{2i(a^2 + b^2) (5Ab + 3Ba) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}{15 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) b \sqrt{a + b \sinh(x)}}$$

command

```
integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \left(\sqrt{2} (6Ba^3 - 5Aa^2b + 18Bab^2 + 15Ab^3) \cosh(x)^2 + 2\sqrt{2} (6Ba^3 - 5Aa^2b + 18Bab^2 + 15Ab^3) \cosh(x) \sinh(x) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \sinh(x)^2 + Aa + (Ba + Ab) \sinh(x)\right) \sqrt{b \sinh(x) + a}, x\right)$$

85.34 Problem number 128

$$\int \sqrt{a + b \sinh(x)} (A + B \sinh(x)) dx$$

Optimal antiderivative

$$\frac{2B \cosh(x) \sqrt{a + b \sinh(x)}}{3}$$

$$+ \frac{2i(3Ab + Ba) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{a + b \sinh(x)}}{3 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) b \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}$$

$$- \frac{2i(a^2 + b^2) B \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}{3 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) b \sqrt{a + b \sinh(x)}}$$

command

`integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} (2 B a^2 - 3 A a b + 3 B b^2) \cosh(x) + \sqrt{2} (2 B a^2 - 3 A a b + 3 B b^2) \sinh(x) \right) \sqrt{b} \operatorname{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left((B \sinh(x) + A) \sqrt{b \sinh(x) + a}, x \right)$$

85.35 Problem number 137

$$\int \frac{A + B \sinh(x)}{\sqrt{a + b \sinh(x)}} dx$$

Optimal antiderivative

$$\frac{2iB \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE} \left(\cos \left(\frac{\pi}{4} + \frac{ix}{2} \right), \sqrt{2} \sqrt{\frac{b}{ia + b}} \right) \sqrt{a + b \sinh(x)}}{\sin \left(\frac{\pi}{4} + \frac{ix}{2} \right) b \sqrt{\frac{a + b \sinh(x)}{-ib + a}}} + \frac{2i(Ab - Ba) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{\pi}{4} + \frac{ix}{2} \right), \sqrt{2} \sqrt{\frac{b}{ia + b}} \right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}{\sin \left(\frac{\pi}{4} + \frac{ix}{2} \right) b \sqrt{a + b \sinh(x)}}$$

command

`integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \sqrt{2} B b^{\frac{3}{2}} \operatorname{weierstrassZeta} \left(\frac{4(4a^2 + 3b^2)}{3b^2}, -\frac{8(8a^3 + 9ab^2)}{27b^3} \right), \operatorname{weierstrassPInverse} \left(\frac{4(4a^2 + 3b^2)}{3b^2}, -\frac{8(8a^3 + 9ab^2)}{27b^3} \right), \frac{3b \cosh(x)}{\dots} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{B \sinh(x) + A}{\sqrt{b \sinh(x) + a}}, x \right)$$

85.36 Problem number 138

$$\int \frac{A + B \sinh(x)}{(a + b \sinh(x))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(Ab - Ba) \cosh(x)}{(a^2 + b^2) \sqrt{a + b \sinh(x)}} \\ & + \frac{2i(Ab - Ba) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{a + b \sinh(x)}}{\sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) b(a^2 + b^2) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}} \\ & + \frac{2iB \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}{\sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) b \sqrt{a + b \sinh(x)}} \end{aligned}$$

command

```
integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\left(\sqrt{2} (2Ba^2b + Aab^2 + 3Bb^3) \cosh(x)^2 + \sqrt{2} (2Ba^2b + Aab^2 + 3Bb^3) \sinh(x)^2 + 2\sqrt{2} (2Ba^3 + Aa^2b + 3Bb^3) \right) \sqrt{a + b \sinh(x)} \right)}{(a + b \sinh(x))^{5/2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sinh(x) + A) \sqrt{b \sinh(x) + a}}{b^2 \sinh(x)^2 + 2ab \sinh(x) + a^2}, x\right)$$

85.37 Problem number 139

$$\int \frac{A + B \sinh(x)}{(a + b \sinh(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba) \cosh(x)}{3(a^2 + b^2)(a + b \sinh(x))^{\frac{3}{2}}} - \frac{2(4Aab - Ba^2 + 3b^2B) \cosh(x)}{3(a^2 + b^2)^2 \sqrt{a + b \sinh(x)}} + \frac{2i(4Aab - Ba^2 + 3b^2B) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{a + b \sinh(x)}}{3 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) b(a^2 + b^2)^2 \sqrt{\frac{a + b \sinh(x)}{-ib + a}}} - \frac{2i(Ab - Ba) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2} \sqrt{\frac{b}{ia + b}}\right) \sqrt{\frac{a + b \sinh(x)}{-ib + a}}}{3 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) b(a^2 + b^2) \sqrt{a + b \sinh(x)}}$$

command

```
integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \sinh(x) + A) \sqrt{b \sinh(x) + a}}{b^3 \sinh(x)^3 + 3ab^2 \sinh(x)^2 + 3a^2b \sinh(x) + a^3}, x\right)$$

85.38 Problem number 146

$$\int (a \sinh^3(x))^{5/2} dx$$

Optimal antiderivative

$$\frac{26a^2 \coth(x) \sqrt{a (\sinh^3(x))}}{77} + \frac{78a^2 \cosh(x) \sinh(x) \sqrt{a (\sinh^3(x))}}{385} - \frac{26a^2 \cosh(x) (\sinh^3(x)) \sqrt{a (\sinh^3(x))}}{165} + \frac{2a^2 \cosh(x) (\sinh^5(x)) \sqrt{a (\sinh^3(x))}}{15} + \frac{26ia^2 \operatorname{csch}(x)^2 \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2}\right) \sqrt{i \sinh(x)} \sqrt{a (\sinh^3(x))}}{77 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right)}$$

command

```
integrate((a*sinh(x)^3)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$24960 \left(\sqrt{2} a^2 \cosh(x)^7 + 7 \sqrt{2} a^2 \cosh(x)^6 \sinh(x) + 21 \sqrt{2} a^2 \cosh(x)^5 \sinh(x)^2 + 35 \sqrt{2} a^2 \cosh(x)^4 \sinh(x)^3 - \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{a \sinh(x)^3} a^2 \sinh(x)^6, x \right)$$

85.39 Problem number 147

$$\int (a \sinh^3(x))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{14a \cosh(x) \sqrt{a (\sinh^3(x))}}{45} + \frac{2a \cosh(x) (\sinh^2(x)) \sqrt{a (\sinh^3(x))}}{9} \\ & + \frac{14ia \operatorname{csch}(x) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE} \left(\cos \left(\frac{\pi}{4} + \frac{ix}{2} \right), \sqrt{2} \right) \sqrt{a (\sinh^3(x))}}{15 \sin \left(\frac{\pi}{4} + \frac{ix}{2} \right) \sqrt{i \sinh(x)}} \end{aligned}$$

command

```
integrate((a*sinh(x)^3)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$336 \left(\sqrt{2} a \cosh(x)^4 + 4 \sqrt{2} a \cosh(x)^3 \sinh(x) + 6 \sqrt{2} a \cosh(x)^2 \sinh(x)^2 + 4 \sqrt{2} a \cosh(x) \sinh(x)^3 + \sqrt{2} a \sinh(x)^4 - \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{a \sinh(x)^3} a \sinh(x)^3, x \right)$$

85.40 Problem number 148

$$\int \sqrt{a \sinh^3(x)} dx$$

Optimal antiderivative

$$\frac{2 \coth(x) \sqrt{a (\sinh^3(x))}}{3} - \frac{2i \operatorname{csch}(x)^2 \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2}\right) \sqrt{i \sinh(x)} \sqrt{a (\sinh^3(x))}}{3 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right)}$$

command

```
integrate((a*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} \cosh(x) + \sqrt{2} \sinh(x) \right) \sqrt{a} \operatorname{weierstrassPInverse}(4, 0, \cosh(x) + \sinh(x)) - \left(\cosh(x)^2 + 2 \cosh(x) \sinh(x) \right)}{3 (\cosh(x) + \sinh(x))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \sinh^3(x)}, x\right)$$

85.41 Problem number 149

$$\int \frac{1}{\sqrt{a \sinh^3(x)}} dx$$

Optimal antiderivative

$$-\frac{2 \cosh(x) \sinh(x)}{\sqrt{a (\sinh^3(x))}} + \frac{2i \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2}\right) (\sinh^2(x))}{\sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) \sqrt{i \sinh(x)} \sqrt{a (\sinh^3(x))}}$$

command

```
integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\left(\sqrt{2} \cosh(x)^2 + 2 \sqrt{2} \cosh(x) \sinh(x) + \sqrt{2} \sinh(x)^2 - \sqrt{2} \right) \sqrt{a} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(a \cosh(x)^2 + 2 a \cosh(x) \sinh(x) + a \sinh(x)^2)) \right)}{a \cosh(x)^2 + 2 a \cosh(x) \sinh(x) + a \sinh(x)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{a \sinh(x)^3}}{a \sinh(x)^3}, x \right)$$

85.42 Problem number 150

$$\int \frac{1}{(a \sinh^3(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{10 \cosh(x)}{21 a \sqrt{a (\sinh^3(x))}} - \frac{2 \coth(x) \operatorname{csch}(x)}{7 a \sqrt{a (\sinh^3(x))}} + \frac{10 i \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{\pi}{4} + \frac{i x}{2} \right), \sqrt{2} \right) \sinh(x) \sqrt{i \sinh(x)}}{21 \sin \left(\frac{\pi}{4} + \frac{i x}{2} \right) a \sqrt{a (\sinh^3(x))}}$$

command

`integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(5 \left(\sqrt{2} \cosh(x)^8 + 8 \sqrt{2} \cosh(x) \sinh(x)^7 + \sqrt{2} \sinh(x)^8 + 4 \left(7 \sqrt{2} \cosh(x)^2 - \sqrt{2} \right) \sinh(x)^6 - 4 \sqrt{2} \cosh(x) \sinh(x)^5 + \sqrt{2} \sinh(x)^4 \right) \right)}{a^2 \sinh(x)^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{a \sinh(x)^3}}{a^2 \sinh(x)^6}, x \right)$$

85.43 Problem number 151

$$\int \frac{1}{(a \sinh^3(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{154 \coth(x)}{585a^2 \sqrt{a (\sinh^3(x))}} + \frac{22 \coth(x) \operatorname{csch}(x)^2}{117a^2 \sqrt{a (\sinh^3(x))}} - \frac{2 \coth(x) \operatorname{csch}(x)^4}{13a^2 \sqrt{a (\sinh^3(x))}} \\ & + \frac{154 \cosh(x) \sinh(x)}{195a^2 \sqrt{a (\sinh^3(x))}} - \frac{154i \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2}\right) (\sinh^2(x))}{195 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) a^2 \sqrt{i \sinh(x)} \sqrt{a (\sinh^3(x))}} \end{aligned}$$

command

```
integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \sinh(x)^3}}{a^3 \sinh(x)^9}, x\right)$$

85.44 Problem number 279

$$\int \frac{\sinh^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \cosh(a + b \ln(cx^n)) (\sinh^{\frac{3}{2}}(a + b \ln(cx^n)))}{5bn} \\ & - \frac{6i \sqrt{\frac{1}{2} + \frac{\sin(ia + ib \ln(cx^n))}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right), \sqrt{2}\right) (\sqrt{\sinh}(a + b \ln(cx^n)))}{5 \sin\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right) bn \sqrt{i \sinh(a + b \ln(cx^n))}} \end{aligned}$$

command

```
integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 \left(\sqrt{2} \cosh (bn \log (x) + b \log (c) + a)^2 + 2 \sqrt{2} \cosh (bn \log (x) + b \log (c) + a) \sinh (bn \log (x) + b \log (c) + a) + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sinh (b \log (cx^n) + a)^{\frac{5}{2}}}{x}, x \right)$$

85.45 Problem number 280

$$\int \frac{\sinh^{\frac{3}{2}} (a + b \log (cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin (ia + ib \ln (cx^n))}{2}} \text{EllipticF} \left(\cos \left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln (cx^n)}{2} \right), \sqrt{2} \right) \sqrt{i \sinh (a + b \ln (cx^n))}}{3 \sin \left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln (cx^n)}{2} \right) bn \sqrt{\sinh (a + b \ln (cx^n))}} + \frac{2 \cosh (a + b \ln (cx^n)) \left(\sqrt{\sinh (a + b \ln (cx^n))} \right)}{3bn}$$

command

```
integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} \cosh (bn \log (x) + b \log (c) + a) + \sqrt{2} \sinh (bn \log (x) + b \log (c) + a) \right) \text{weierstrassPInverse}(4, 0, \cosh (bn \log (x) + b \log (c) + a))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sinh (b \log (cx^n) + a)^{\frac{3}{2}}}{x}, x \right)$$

85.46 Problem number 281

$$\int \frac{\sqrt{\sinh(a + b \log(cx^n))}}{x} dx$$

Optimal antiderivative

$$\frac{2i\sqrt{\frac{1}{2} + \frac{\sin(ia + ib \ln(cx^n))}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right), \sqrt{2}\right) \left(\sqrt{\sinh(a + b \ln(cx^n))}\right)}{\sin\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right) bn \sqrt{i \sinh(a + b \ln(cx^n))}}$$

command

`integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{2} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cosh(bn \log(x) + b \log(c) + a) + \sinh(bn \log(x) + b \log(c)))\right)}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\sinh(b \log(cx^n) + a)}}{x}, x\right)$$

85.47 Problem number 282

$$\int \frac{1}{x \sqrt{\sinh(a + b \log(cx^n))}} dx$$

Optimal antiderivative

$$\frac{2i\sqrt{\frac{1}{2} + \frac{\sin(ia + ib \ln(cx^n))}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right), \sqrt{2}\right) \sqrt{i \sinh(a + b \ln(cx^n))}}{\sin\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right) bn \sqrt{\sinh(a + b \ln(cx^n))}}$$

command

`integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2} \operatorname{weierstrassPInverse}(4, 0, \cosh(bn \log(x) + b \log(c) + a) + \sinh(bn \log(x) + b \log(c)))}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \sqrt{\sinh(b \log(cx^n) + a)}}, x\right)$$

85.48 Problem number 283

$$\int \frac{1}{x \sinh^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{2 \cosh(a + b \ln(cx^n))}{bn \sqrt{\sinh(a + b \ln(cx^n))}} + \frac{2i \sqrt{\frac{1}{2} + \frac{\sin(ia + ib \ln(cx^n))}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right), \sqrt{2}\right) \left(\sqrt{\sinh(a + b \ln(cx^n))}\right)}{\sin\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right) bn \sqrt{i \sinh(a + b \ln(cx^n))}}$$

command

```
integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} \cosh(bn \log(x) + b \log(c) + a)^2 + 2 \sqrt{2} \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \sinh(b \log(cx^n) + a)^{\frac{3}{2}}}, x\right)$$

85.49 Problem number 284

$$\int \frac{1}{x \sinh^{\frac{5}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{2 \cosh(a + b \ln(cx^n))}{3bn \sinh(a + b \ln(cx^n))^{\frac{3}{2}}} + \frac{2i \sqrt{\frac{1}{2} + \frac{\sin(ia + ib \ln(cx^n))}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right), \sqrt{2}\right) \sqrt{i \sinh(a + b \ln(cx^n))}}{3 \sin\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right) bn \sqrt{\sinh(a + b \ln(cx^n))}}$$

command

```
integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} \cosh (bn \log (x) + b \log (c) + a)^4 + 4 \sqrt{2} \cosh (bn \log (x) + b \log (c) + a) \sinh (bn \log (x) + b \log (c) + a) \right)^3 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{x \sinh (b \log (cx^n) + a)^{\frac{5}{2}}}, x \right)$$

86 Test file number 164

Test folder name:

test_cases/6_Hyperbolic_functions/6.1_Hyperbolic_sine/164_6.1.7_hyper^m-a+b_sinhⁿ-^p

86.1 Problem number 58

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^{\frac{3}{2}} (35a^2 - 56ab + 24b^2) \arctan \left(\frac{\cosh(dx+c) \sqrt{b}}{\sqrt{a-b}} \right)}{8a^4 (a-b)^{\frac{5}{2}} d} \\ & + \frac{(a+6b) \operatorname{arctanh}(\cosh(dx+c))}{2a^4 d} - \frac{(2a-3b) b \cosh(dx+c)}{4a^2 (a-b) d (a-b+b \cosh^2(dx+c))^2} \\ & - \frac{(a-4b) (4a-3b) b \cosh(dx+c)}{8a^3 (a-b)^2 d (a-b+b \cosh^2(dx+c))} - \frac{\operatorname{coth}(dx+c) \operatorname{csch}(dx+c)}{2ad (a-b+b \cosh^2(dx+c))^2} \end{aligned}$$

command

```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.2 Problem number 59

$$\int \frac{\operatorname{csch}^4(c + dx)}{(a + b \sinh^2(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{b^2(48a^2 - 80ab + 35b^2) \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tanh(dx+c)}{\sqrt{a}}\right)}{8a^{\frac{9}{2}}(a-b)^{\frac{5}{2}}d} + \frac{(8a^3 - 4a^2b - 45ab^2 + 35b^3) \operatorname{coth}(dx+c)}{8a^4(a-b)^2d} - \frac{(8a^2 - 52ab + 35b^2) (\operatorname{coth}^3(dx+c))}{24a^3(a-b)^2d} - \frac{b \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)^3}{4a(a-b)d(a-(a-b)(\tanh^2(dx+c)))^2} - \frac{(10a-7b) b \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)}{8a^2(a-b)^2d(a-(a-b)(\tanh^2(dx+c)))}$$

command

```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.3 Problem number 104

$$\int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{i \sqrt{\frac{\cos(2ifx + 2ie)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(ifx + ie), \sqrt{\frac{b}{a}}\right) \sqrt{1 + \frac{b(\sinh^2(fx + e))}{a}}}{\cos(ifx + ie) f \sqrt{a + b(\sinh^2(fx + e))}}$$

command

```
integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2b \sqrt{\frac{a^2 - ab}{b^2}} + 2a - b \right) \sqrt{\frac{2b \sqrt{\frac{a^2 - ab}{b^2}} - 2a + b}{b}} \operatorname{ellipticF} \left(\sqrt{\frac{2b \sqrt{\frac{a^2 - ab}{b^2}} - 2a + b}{b}} (\cosh(fx + \cosh(1) \dots)) \right)$$

$b^{\frac{3}{2}} f$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{b \sinh(fx + e)^2 + a}}, x \right)$$

86.4 Problem number 127

$$\int \frac{1}{\sqrt{-1 + \sinh^2(x)}} dx$$

Optimal antiderivative

$$\frac{i \sqrt{\frac{\cosh(2x)}{2} + \frac{1}{2}} \operatorname{EllipticF}(i \sinh(x), i) \sqrt{1 - (\sinh^2(x))}}{\cosh(x) \sqrt{-1 + \sinh^2(x)}}$$

command

```
integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2 \sqrt{2\sqrt{2} + 3} (2\sqrt{2} - 3) \operatorname{ellipticF} \left(\sqrt{2\sqrt{2} + 3} (\cosh(x) + \sinh(x)), -12\sqrt{2} + 17 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{\sinh(x)^2 - 1}}, x \right)$$

86.5 Problem number 129

$$\int \frac{1}{\sqrt{a + b \sinh^2(x)}} dx$$

Optimal antiderivative

$$\frac{i \sqrt{\frac{\cosh(2x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh(x), \sqrt{\frac{b}{a}}\right) \sqrt{1 + \frac{b(\sinh^2(x))}{a}}}{\cosh(x) \sqrt{a + b(\sinh^2(x))}}$$

command

`integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2b \sqrt{\frac{a^2 - ab}{b^2}} + 2a - b \right) \sqrt{\frac{2b \sqrt{\frac{a^2 - ab}{b^2}} - 2a + b}{b}} \operatorname{ellipticF}\left(\sqrt{\frac{2b \sqrt{\frac{a^2 - ab}{b^2}} - 2a + b}{b}} (\cosh(x) + \sinh(x))\right)}{b^{\frac{3}{2}}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{b \sinh(x)^2 + a}}, x\right)$$

86.6 Problem number 171

$$\int \frac{\sinh^6(c + dx)}{a + b \sinh^3(c + dx)} dx$$

Optimal antiderivative

$$\frac{-\frac{ax}{b^2} - \frac{\cosh(dx + c)}{bd} + \frac{\cosh^3(dx + c)}{3bd} - \frac{2(-1)^{\frac{2}{3}} a^{\frac{4}{3}} \operatorname{arctan}\left(\frac{(-1)^{\frac{1}{6}} \left((-1)^{\frac{5}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3b^2 d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}}}{\frac{2a^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - a^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3b^2 d \sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}} - \frac{2(-1)^{\frac{2}{3}} a^{\frac{4}{3}} \operatorname{arctan}\left(\frac{(-1)^{\frac{1}{6}} \left((-1)^{\frac{1}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3b^2 d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}}$$

command

```
integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.7 Problem number 172

$$\int \frac{\sinh^5(c + dx)}{a + b \sinh^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x}{2b} + \frac{\cosh(dx + c) \sinh(dx + c)}{2bd} + \frac{2a \arctan\left(\frac{(-1)^{\frac{1}{6}} \left((-1)^{\frac{5}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3b^{\frac{5}{3}} d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \\ & + \frac{2a \arctan\left(\frac{(-1)^{\frac{5}{6}} \left((-1)^{\frac{1}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3b^{\frac{5}{3}} d \sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}} + \frac{2a \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - a^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3b^{\frac{5}{3}} d \sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.8 Problem number 173

$$\int \frac{\sinh^4(c + dx)}{a + b \sinh^3(c + dx)} dx$$

Optimal antiderivative

$$\frac{\cosh(dx + c)}{bd} + \frac{2(-1)^{\frac{1}{3}} a^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{5}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3b^{\frac{4}{3}} d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}}$$

$$- \frac{2a^{\frac{2}{3}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - a^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3b^{\frac{4}{3}} d \sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}} - \frac{2a^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{1}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3b^{\frac{4}{3}} d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}$$

command

`integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.9 Problem number 174

$$\int \frac{\sinh^3(c + dx)}{a + b \sinh^3(c + dx)} dx$$

Optimal antiderivative

$$\frac{x}{b} + \frac{2(-1)^{\frac{2}{3}} a^{\frac{1}{3}} \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{5}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3bd \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}}$$

$$+ \frac{2a^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - a^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3bd \sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}} + \frac{2(-1)^{\frac{2}{3}} a^{\frac{1}{3}} \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{1}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3bd \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}$$

command

```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.10 Problem number 175

$$\int \frac{\sinh^2(c + dx)}{a + b \sinh^3(c + dx)} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left(\frac{(-1)^{\frac{1}{6}} \left((-1)^{\frac{5}{6}} b^{\frac{1}{3}} + i a^{\frac{1}{3}} \tanh \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \right)}{3b^{\frac{2}{3}} d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}} - \frac{2 \arctan \left(\frac{(-1)^{\frac{5}{6}} \left((-1)^{\frac{1}{6}} b^{\frac{1}{3}} + i a^{\frac{1}{3}} \tanh \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{\sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \right)}{3b^{\frac{2}{3}} d \sqrt{-(-1)^{\frac{2}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}} - \frac{2 \operatorname{arctanh} \left(\frac{b^{\frac{1}{3}} - a^{\frac{1}{3}} \tanh \left(\frac{dx}{2} + \frac{c}{2} \right)}{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}} \right)}{3b^{\frac{2}{3}} d \sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}$$

command

```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.11 Problem number 176

$$\int \frac{\sinh(c + dx)}{a + b \sinh^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-1)^{\frac{1}{3}} \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{5}{6}}b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3a^{\frac{1}{3}}b^{\frac{1}{3}}d\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \\ & + \frac{2 \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - a^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3a^{\frac{1}{3}}b^{\frac{1}{3}}d\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}} + \frac{2 \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{1}{6}}b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}} - (-1)^{\frac{2}{3}}b^{\frac{2}{3}}}}\right)}{3a^{\frac{1}{3}}b^{\frac{1}{3}}d\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}} - (-1)^{\frac{2}{3}}b^{\frac{2}{3}}}} \end{aligned}$$

command

`integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.12 Problem number 177

$$\int \frac{1}{a + b \sinh^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-1)^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{5}{6}}b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3a^{\frac{2}{3}}d\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}} - b^{\frac{2}{3}}}} - \frac{2 \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - a^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3a^{\frac{2}{3}}d\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}} \\ & - \frac{2(-1)^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{1}{6}}b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}} - (-1)^{\frac{2}{3}}b^{\frac{2}{3}}}}\right)}{3a^{\frac{2}{3}}d\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}} - (-1)^{\frac{2}{3}}b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.13 Problem number 178

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}(\cosh(dx+c))}{ad} + \frac{2b^{\frac{1}{3}} \operatorname{arctan}\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{5}{6}}b^{\frac{1}{3}}+ia^{\frac{1}{3}}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}}-b^{\frac{2}{3}}}}\right)}{3ad\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}}-b^{\frac{2}{3}}}} \\ & + \frac{2b^{\frac{1}{3}} \operatorname{arctan}\left(\frac{(-1)^{\frac{5}{6}}\left((-1)^{\frac{1}{6}}b^{\frac{1}{3}}+ia^{\frac{1}{3}}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sqrt{-(-1)^{\frac{2}{3}}a^{\frac{2}{3}}-b^{\frac{2}{3}}}}\right)}{3ad\sqrt{-(-1)^{\frac{2}{3}}a^{\frac{2}{3}}-b^{\frac{2}{3}}}} + \frac{2b^{\frac{1}{3}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}}-a^{\frac{1}{3}}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}}+b^{\frac{2}{3}}}}\right)}{3ad\sqrt{a^{\frac{2}{3}}+b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.14 Problem number 179

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{coth}(dx+c)}{ad} + \frac{2(-1)^{\frac{1}{3}} b^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{5}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3a^{\frac{4}{3}} d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \\ & - \frac{2b^{\frac{2}{3}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - a^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3a^{\frac{4}{3}} d \sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}} - \frac{2b^{\frac{2}{3}} \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{1}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3a^{\frac{4}{3}} d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.15 Problem number 180

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}(\cosh(dx+c))}{2ad} - \frac{\operatorname{coth}(dx+c) \operatorname{csch}(dx+c)}{2ad} \\ & + \frac{2(-1)^{\frac{2}{3}} b \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{5}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}}\right)}{3a^{\frac{5}{3}} d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - b^{\frac{2}{3}}}} + \frac{2b \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}} - a^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}\right)}{3a^{\frac{5}{3}} d \sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}} \\ & + \frac{2(-1)^{\frac{2}{3}} b \arctan\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{1}{6}} b^{\frac{1}{3}} + ia^{\frac{1}{3}} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}}\right)}{3a^{\frac{5}{3}} d \sqrt{(-1)^{\frac{1}{3}} a^{\frac{2}{3}} - (-1)^{\frac{2}{3}} b^{\frac{2}{3}}}} \end{aligned}$$

command

```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.16 Problem number 181

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)} dx$$

Optimal antiderivative

$$\frac{b \operatorname{arctanh}(\cosh(dx+c))}{a^2 d} + \frac{\operatorname{coth}(dx+c)}{ad} - \frac{\operatorname{coth}^3(dx+c)}{3ad} - \frac{2b^{\frac{4}{3}} \operatorname{arctan}\left(\frac{(-1)^{\frac{1}{6}}\left((-1)^{\frac{5}{6}}b^{\frac{1}{3}}+ia^{\frac{1}{3}}\tanh\left(\frac{dx+c}{2}\right)\right)}{\sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}}-b^{\frac{2}{3}}}}\right)}{3a^2 d \sqrt{(-1)^{\frac{1}{3}}a^{\frac{2}{3}}-b^{\frac{2}{3}}}} - \frac{2b^{\frac{4}{3}} \operatorname{arctan}\left(\frac{(-1)^{\frac{5}{6}}\left((-1)^{\frac{1}{6}}b^{\frac{1}{3}}+ia^{\frac{1}{3}}\tanh\left(\frac{dx+c}{2}\right)\right)}{\sqrt{-(-1)^{\frac{2}{3}}a^{\frac{2}{3}}-b^{\frac{2}{3}}}}\right)}{3a^2 d \sqrt{-(-1)^{\frac{2}{3}}a^{\frac{2}{3}}-b^{\frac{2}{3}}}} - \frac{2b^{\frac{4}{3}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{3}}-a^{\frac{1}{3}}\tanh\left(\frac{dx+c}{2}\right)}{\sqrt{a^{\frac{2}{3}}+b^{\frac{2}{3}}}}\right)}{3a^2 d \sqrt{a^{\frac{2}{3}}+b^{\frac{2}{3}}}}$$

command

```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.17 Problem number 215

$$\int \operatorname{csch}^{11}(c+dx) (a+b \sinh^4(c+dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3a(21a^2 + 80ab + 128b^2) \operatorname{arctanh}(\cosh(dx+c))}{256d} + \frac{b^3 \cosh(dx+c)}{d} \\ & - \frac{3a(21a^2 + 80ab + 128b^2) \coth(dx+c) \operatorname{csch}(dx+c)}{256d} \\ & + \frac{a^2(21a + 80b) \coth(dx+c) \operatorname{csch}(dx+c)^3}{128d} - \frac{a^2(21a + 80b) \coth(dx+c) \operatorname{csch}(dx+c)^5}{160d} \\ & + \frac{9a^3 \coth(dx+c) \operatorname{csch}(dx+c)^7}{80d} - \frac{a^3 \coth(dx+c) \operatorname{csch}(dx+c)^9}{10d} \end{aligned}$$

command

```
integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.18 Problem number 216

$$\int \operatorname{csch}^{13}(c+dx) (a+b \sinh^4(c+dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(231a^3 + 840a^2b + 1152ab^2 + 1024b^3) \operatorname{arctanh}(\cosh(dx+c))}{1024d} \\ & + \frac{3a(77a^2 + 280ab + 384b^2) \coth(dx+c) \operatorname{csch}(dx+c)}{1024d} \\ & - \frac{a(77a^2 + 280ab + 384b^2) \coth(dx+c) \operatorname{csch}(dx+c)^3}{512d} \\ & + \frac{7a^2(11a + 40b) \coth(dx+c) \operatorname{csch}(dx+c)^5}{640d} - \frac{3a^2(11a + 40b) \coth(dx+c) \operatorname{csch}(dx+c)^7}{320d} \\ & + \frac{11a^3 \coth(dx+c) \operatorname{csch}(dx+c)^9}{120d} - \frac{a^3 \coth(dx+c) \operatorname{csch}(dx+c)^{11}}{12d} \end{aligned}$$

command

`integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.19 Problem number 253

$$\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a \cosh(dx+c) (a+b-b(\cosh^2(dx+c)))}{8(a-b)b^2d(a-b+2b(\cosh^2(dx+c))-b(\cosh^4(dx+c)))^2} \\ & - \frac{\cosh(dx+c) (9a^2-11ab-10b^2-2(2a-5b)b(\cosh^2(dx+c)))}{32(a-b)^2b^2d(a-b+2b(\cosh^2(dx+c))-b(\cosh^4(dx+c)))} \\ & + \frac{\arctan\left(\frac{b^{\frac{1}{4}}\cosh(dx+c)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right) (5a+12b-14\sqrt{a}\sqrt{b})}{64b^{\frac{9}{4}}d\sqrt{a}(\sqrt{a}-\sqrt{b})^{\frac{5}{2}}} \\ & + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}\cosh(dx+c)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right) (5a+12b+14\sqrt{a}\sqrt{b})}{64b^{\frac{9}{4}}d\sqrt{a}(\sqrt{a}+\sqrt{b})^{\frac{5}{2}}} \end{aligned}$$

command

`integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.20 Problem number 254

$$\int \frac{\sinh^7(c + dx)}{(a - b \sinh^4(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a \cosh(dx + c) (2 - (\cosh^2(dx + c)))}{8(a - b)bd(a - b + 2b(\cosh^2(dx + c)) - b(\cosh^4(dx + c)))^2} \\ & + \frac{\cosh(dx + c) (5a - 17b - 3(a - 3b)(\cosh^2(dx + c)))}{32(a - b)^2bd(a - b + 2b(\cosh^2(dx + c)) - b(\cosh^4(dx + c)))} \\ & + \frac{3 \arctan\left(\frac{b^{\frac{1}{4}} \cosh(dx+c)}{\sqrt{\sqrt{a} - \sqrt{b}}}\right) (\sqrt{a} - 2\sqrt{b})}{64b^{\frac{7}{4}}d\sqrt{a} (\sqrt{a} - \sqrt{b})^{\frac{5}{2}}} - \frac{3 \operatorname{arctanh}\left(\frac{b^{\frac{1}{4}} \cosh(dx+c)}{\sqrt{\sqrt{a} + \sqrt{b}}}\right) (\sqrt{a} + 2\sqrt{b})}{64b^{\frac{7}{4}}d\sqrt{a} (\sqrt{a} + \sqrt{b})^{\frac{5}{2}}} \end{aligned}$$

command

```
integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.21 Problem number 255

$$\int \frac{\sinh^5(c + dx)}{(a - b \sinh^4(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{\cosh(dx+c)(a+b-b(\cosh^2(dx+c)))}{8(a-b)bd(a-b+2b(\cosh^2(dx+c))-b(\cosh^4(dx+c)))^2} - \frac{\cosh(dx+c)(a^2-11ab-2b^2+2b(2a+b)(\cosh^2(dx+c)))}{32a(a-b)^2bd(a-b+2b(\cosh^2(dx+c))-b(\cosh^4(dx+c)))}$$

$$- \frac{\arctan\left(\frac{b^{\frac{1}{4}}\cosh(dx+c)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)(3a+4b-10\sqrt{a}\sqrt{b})}{64a^{\frac{3}{2}}b^{\frac{5}{4}}d(\sqrt{a}-\sqrt{b})^{\frac{5}{2}}}$$

$$- \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}\cosh(dx+c)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)(3a+4b+10\sqrt{a}\sqrt{b})}{64a^{\frac{3}{2}}b^{\frac{5}{4}}d(\sqrt{a}+\sqrt{b})^{\frac{5}{2}}}$$

command

```
integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.22 Problem number 256

$$\int \frac{\sinh^3(c+dx)}{(a-b\sinh^4(c+dx))^3} dx$$

Optimal antiderivative

$$- \frac{\cosh(dx+c)(2-(\cosh^2(dx+c)))}{8(a-b)d(a-b+2b(\cosh^2(dx+c))-b(\cosh^4(dx+c)))^2} - \frac{\cosh(dx+c)(11a+b-(5a+b)(\cosh^2(dx+c)))}{32a(a-b)^2d(a-b+2b(\cosh^2(dx+c))-b(\cosh^4(dx+c)))}$$

$$- \frac{\arctan\left(\frac{b^{\frac{1}{4}}\cosh(dx+c)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)(5\sqrt{a}-2\sqrt{b})}{64a^{\frac{3}{2}}b^{\frac{3}{4}}d(\sqrt{a}-\sqrt{b})^{\frac{5}{2}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}\cosh(dx+c)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)(5\sqrt{a}+2\sqrt{b})}{64a^{\frac{3}{2}}b^{\frac{3}{4}}d(\sqrt{a}+\sqrt{b})^{\frac{5}{2}}}$$

command

```
integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.23 Problem number 257

$$\int \frac{\sinh(c+dx)}{(a-b\sinh^4(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\cosh(dx+c)(a+b-b(\cosh^2(dx+c)))}{8a(a-b)d(a-b+2b(\cosh^2(dx+c))-b(\cosh^4(dx+c)))^2} \\ & + \frac{\cosh(dx+c)((7a-3b)(a+2b)-6(2a-b)b(\cosh^2(dx+c)))}{32a^2(a-b)^2d(a-b+2b(\cosh^2(dx+c))-b(\cosh^4(dx+c)))} \\ & + \frac{3 \arctan\left(\frac{b^{\frac{1}{4}} \cosh(dx+c)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right) (7a+4b-10\sqrt{a}\sqrt{b})}{64a^{\frac{5}{2}}b^{\frac{1}{4}}d(\sqrt{a}-\sqrt{b})^{\frac{5}{2}}} \\ & + \frac{3 \operatorname{arctanh}\left(\frac{b^{\frac{1}{4}} \cosh(dx+c)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right) (7a+4b+10\sqrt{a}\sqrt{b})}{64a^{\frac{5}{2}}b^{\frac{1}{4}}d(\sqrt{a}+\sqrt{b})^{\frac{5}{2}}} \end{aligned}$$

command

```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.24 Problem number 258

$$\int \frac{\operatorname{csch}(c+dx)}{(a-b\sinh^4(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{arctanh}(\cosh(dx+c))}{a^3 d} \\ & - \frac{b \cosh(dx+c) (2 - (\cosh^2(dx+c)))}{8a(a-b)d(a-b+2b(\cosh^2(dx+c)) - b(\cosh^4(dx+c)))^2} \\ & - \frac{b \cosh(dx+c) (2 - (\cosh^2(dx+c)))}{4a^2(a-b)d(a-b+2b(\cosh^2(dx+c)) - b(\cosh^4(dx+c)))} \\ & - \frac{b \cosh(dx+c) (11a+b - (5a+b)(\cosh^2(dx+c)))}{32a^2(a-b)^2 d(a-b+2b(\cosh^2(dx+c)) - b(\cosh^4(dx+c)))} \\ & - \frac{b^{\frac{1}{4}} \operatorname{arctan}\left(\frac{b^{\frac{1}{4}} \cosh(dx+c)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right) (5\sqrt{a}-2\sqrt{b})}{64a^{\frac{5}{2}} d (\sqrt{a}-\sqrt{b})^{\frac{5}{2}}} - \frac{b^{\frac{1}{4}} \operatorname{arctan}\left(\frac{b^{\frac{1}{4}} \cosh(dx+c)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{\frac{5}{2}} d (\sqrt{a}-\sqrt{b})^{\frac{3}{2}}} \\ & + \frac{b^{\frac{1}{4}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{4}} \cosh(dx+c)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{\frac{5}{2}} d (\sqrt{a}+\sqrt{b})^{\frac{3}{2}}} + \frac{b^{\frac{1}{4}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{4}} \cosh(dx+c)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right) (5\sqrt{a}+2\sqrt{b})}{64a^{\frac{5}{2}} d (\sqrt{a}+\sqrt{b})^{\frac{5}{2}}} \\ & - \frac{b^{\frac{1}{4}} \operatorname{arctan}\left(\frac{b^{\frac{1}{4}} \cosh(dx+c)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3 d \sqrt{\sqrt{a}-\sqrt{b}}} + \frac{b^{\frac{1}{4}} \operatorname{arctanh}\left(\frac{b^{\frac{1}{4}} \cosh(dx+c)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^3 d \sqrt{\sqrt{a}+\sqrt{b}}} \end{aligned}$$

command

```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.25 Problem number 259

$$\int \frac{\sinh^8(c + dx)}{(a - b \sinh^4(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a} - \sqrt{b}} \tanh(dx+c)}{a^{\frac{1}{4}}}\right) (2\sqrt{a} - 5\sqrt{b})}{64a^{\frac{3}{4}}b^{\frac{3}{2}}d(\sqrt{a} - \sqrt{b})^{\frac{5}{2}}} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a} + \sqrt{b}} \tanh(dx+c)}{a^{\frac{1}{4}}}\right) (2\sqrt{a} + 5\sqrt{b})}{64a^{\frac{3}{4}}b^{\frac{3}{2}}d(\sqrt{a} + \sqrt{b})^{\frac{5}{2}}} - \frac{(a + 5b) \tanh(dx + c)}{32a(a - b)^2 bd} \\ & - \frac{\tanh^3(dx + c)}{32a(a - b)bd} + \frac{\tanh^9(dx + c)}{8ad(a - 2a(\tanh^2(dx + c)) + (a - b)(\tanh^4(dx + c)))^2} \\ & - \frac{\operatorname{sech}(dx + c)^2 (\tanh^5(dx + c))}{32abd(a - 2a(\tanh^2(dx + c)) + (a - b)(\tanh^4(dx + c)))} \end{aligned}$$

command

```
integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.26 Problem number 260

$$\int \frac{\sinh^6(c + dx)}{(a - b \sinh^4(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(dx+c)}{a^{\frac{1}{4}}}\right)(4a+3b-10\sqrt{a}\sqrt{b})}{64a^{\frac{5}{4}}b^{\frac{3}{2}}d(\sqrt{a}-\sqrt{b})^{\frac{5}{2}}} \\
& - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(dx+c)}{a^{\frac{1}{4}}}\right)(4a+3b+10\sqrt{a}\sqrt{b})}{64a^{\frac{5}{4}}b^{\frac{3}{2}}d(\sqrt{a}+\sqrt{b})^{\frac{5}{2}}} \\
& + \frac{\tanh(dx+c)(a(a+3b)-(a^2+6ab+b^2)(\tanh^2(dx+c)))}{8(a-b)^3d(a-2a(\tanh^2(dx+c))+(a-b)(\tanh^4(dx+c)))^2} \\
& + \frac{\tanh(dx+c)\left(\frac{2a(a^2-ab-8b^2)}{(a-b)^3}-\frac{(2a^2+15ab+3b^2)(\tanh^2(dx+c))}{(a-b)^2}\right)}{32abd(a-2a(\tanh^2(dx+c))+(a-b)(\tanh^4(dx+c)))}
\end{aligned}$$

command

```
integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.27 Problem number 261

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{3 \operatorname{arctanh} \left(\frac{\sqrt{\sqrt{a} - \sqrt{b}} \tanh(dx+c)}{a^{\frac{1}{4}}} \right) (2\sqrt{a} - \sqrt{b})}{64a^{\frac{7}{4}} d (\sqrt{a} - \sqrt{b})^{\frac{5}{2}} \sqrt{b}} \\
& - \frac{3 \operatorname{arctanh} \left(\frac{\sqrt{\sqrt{a} + \sqrt{b}} \tanh(dx+c)}{a^{\frac{1}{4}}} \right) (2\sqrt{a} + \sqrt{b})}{64a^{\frac{7}{4}} d \sqrt{b} (\sqrt{a} + \sqrt{b})^{\frac{5}{2}}} \\
& - \frac{b \tanh(dx+c) (3a+b-4(a+b)(\tanh^2(dx+c)))}{8(a-b)^3 d (a-2a(\tanh^2(dx+c))+(a-b)(\tanh^4(dx+c)))^2} \\
& - \frac{\tanh(dx+c) \left(\frac{9a^2-24ab-b^2}{(a-b)^3} - \frac{(17a+3b)(\tanh^2(dx+c))}{(a-b)^2} \right)}{32ad (a-2a(\tanh^2(dx+c))+(a-b)(\tanh^4(dx+c)))}
\end{aligned}$$

command

```
integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.28 Problem number 262

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(dx+c)}{a^{\frac{1}{4}}}\right)(12a+5b-14\sqrt{a}\sqrt{b})}{64a^{\frac{9}{4}}d(\sqrt{a}-\sqrt{b})^{\frac{5}{2}}\sqrt{b}} \\
& + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(dx+c)}{a^{\frac{1}{4}}}\right)(12a+5b+14\sqrt{a}\sqrt{b})}{64a^{\frac{9}{4}}d\sqrt{b}(\sqrt{a}+\sqrt{b})^{\frac{5}{2}}} \\
& + \frac{b\tanh(dx+c)(a(a+3b)-(a^2+6ab+b^2)(\tanh^2(dx+c)))}{8a(a-b)^3d(a-2a(\tanh^2(dx+c))+(a-b)(\tanh^4(dx+c)))^2} \\
& + \frac{\tanh(dx+c)\left(\frac{2a(5a^2-9ab-4b^2)}{(a-b)^3}-\frac{5(2a^2+3ab-b^2)(\tanh^2(dx+c))}{(a-b)^2}\right)}{32a^2d(a-2a(\tanh^2(dx+c))+(a-b)(\tanh^4(dx+c)))}
\end{aligned}$$

command

```
integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.29 Problem number 263

$$\int \frac{1}{(a-b\sinh^4(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(dx+c)}{a^{\frac{1}{4}}}\right) (32a+21b-50\sqrt{a}\sqrt{b})}{64a^{\frac{11}{4}} d (\sqrt{a}-\sqrt{b})^{\frac{5}{2}}} \\
& + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(dx+c)}{a^{\frac{1}{4}}}\right) (32a+21b+50\sqrt{a}\sqrt{b})}{64a^{\frac{11}{4}} d (\sqrt{a}+\sqrt{b})^{\frac{5}{2}}} \\
& - \frac{b^2 \tanh(dx+c) (3a+b-4(a+b) (\tanh^2(dx+c)))}{8a(a-b)^3 d (a-2a (\tanh^2(dx+c)) + (a-b) (\tanh^4(dx+c)))^2} \\
& - \frac{b \tanh(dx+c) \left(\frac{17a^2-40ab+7b^2}{(a-b)^3} - \frac{(33a-13b) (\tanh^2(dx+c))}{(a-b)^2}\right)}{32a^2 d (a-2a (\tanh^2(dx+c)) + (a-b) (\tanh^4(dx+c)))}
\end{aligned}$$

command

```
integrate(1/(a-b*sinh(d*x+c))^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.30 Problem number 264

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{\coth(dx+c)}{a^3 d} - \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(dx+c)}{a^{\frac{1}{4}}}\right) \sqrt{b} (20a+15b-34\sqrt{a}\sqrt{b})}{64a^{\frac{13}{4}} d (\sqrt{a}-\sqrt{b})^{\frac{5}{2}}} \\
 & + \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(dx+c)}{a^{\frac{1}{4}}}\right) \sqrt{b} (20a+15b+34\sqrt{a}\sqrt{b})}{64a^{\frac{13}{4}} d (\sqrt{a}+\sqrt{b})^{\frac{5}{2}}} \\
 & + \frac{b^2 \tanh(dx+c) (a(a+3b) - (a^2+6ab+b^2) (\tanh^2(dx+c)))}{8a^2 (a-b)^3 d (a-2a(\tanh^2(dx+c)) + (a-b)(\tanh^4(dx+c)))^2} \\
 & + \frac{b \tanh(dx+c) \left(\frac{2a^2(9a-17b)}{(a-b)^3} - \frac{(18a^2+15ab-13b^2)(\tanh^2(dx+c))}{(a-b)^2}\right)}{32a^3 d (a-2a(\tanh^2(dx+c)) + (a-b)(\tanh^4(dx+c)))}
 \end{aligned}$$

command

```
integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.31 Problem number 268

$$\int \frac{1}{a + b \sinh^6(x)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{\operatorname{arctanh}\left(\frac{\sqrt{a^{\frac{1}{3}}-b^{\frac{1}{3}}} \tanh(x)}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}}-b^{\frac{1}{3}}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a^{\frac{1}{3}}+(-1)^{\frac{1}{3}}b^{\frac{1}{3}}} \tanh(x)}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}}+(-1)^{\frac{1}{3}}b^{\frac{1}{3}}}} \\
 & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a^{\frac{1}{3}}-(-1)^{\frac{2}{3}}b^{\frac{1}{3}}} \tanh(x)}{a^{\frac{1}{6}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}}-(-1)^{\frac{2}{3}}b^{\frac{1}{3}}}}
 \end{aligned}$$

command

```
integrate(1/(a+b*sinh(x)^6),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.32 Problem number 269

$$\int \frac{1}{a + b \sinh^8(x)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{(-a)^{\frac{1}{4}} - b^{\frac{1}{4}}}\tanh(x)}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} - b^{\frac{1}{4}}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{(-a)^{\frac{1}{4}} - ib^{\frac{1}{4}}}\tanh(x)}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} - ib^{\frac{1}{4}}}}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{(-a)^{\frac{1}{4}} + ib^{\frac{1}{4}}}\tanh(x)}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} + ib^{\frac{1}{4}}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{(-a)^{\frac{1}{4}} + b^{\frac{1}{4}}}\tanh(x)}{(-a)^{\frac{1}{8}}}\right)}{4(-a)^{\frac{7}{8}}\sqrt{(-a)^{\frac{1}{4}} + b^{\frac{1}{4}}}}$$

command

```
integrate(1/(a+b*sinh(x)^8),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.33 Problem number 347

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\sinh^2(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a-7b)\arctan(\sinh(dx+c))}{2(a-b)^4 d} + \frac{b^{\frac{3}{2}}(35a^2-14ab+3b^2)\arctan\left(\frac{\sinh(dx+c)\sqrt{b}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}(a-b)^4 d} \\ & + \frac{b(2a+b)\sinh(dx+c)}{4a(a-b)^2 d(a+b(\sinh^2(dx+c)))^2} + \frac{(4a-b)b(a+3b)\sinh(dx+c)}{8a^2(a-b)^3 d(a+b(\sinh^2(dx+c)))} \\ & + \frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2(a-b)d(a+b(\sinh^2(dx+c)))^2} \end{aligned}$$

command

```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.34 Problem number 348

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\sinh^2(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^2(48a^2-16ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(dx+c)}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}(a-b)^{\frac{9}{2}}d} + \frac{(a-4b)\tanh(dx+c)}{(a-b)^4 d} - \frac{\tanh^3(dx+c)}{3(a-b)^3 d} \\ & + \frac{b^4\tanh(dx+c)}{4a(a-b)^4 d(a-(a-b)(\tanh^2(dx+c)))^2} - \frac{(16a-3b)b^3\tanh(dx+c)}{8a^2(a-b)^4 d(a-(a-b)(\tanh^2(dx+c)))} \end{aligned}$$

command

```
integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.35 Problem number 379

$$\int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{i \sqrt{\frac{\cos(2ifx + 2ie)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(ifx + ie), \sqrt{\frac{b}{a}}\right) \sqrt{1 + \frac{b(\sinh^2(fx + e))}{a}}}{\cos(ifx + ie) f \sqrt{a + b(\sinh^2(fx + e))}}$$

command

```
integrate(1/(a+b*sinh(f*x+e)^(1/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2b \sqrt{\frac{a^2 - ab}{b^2}} + 2a - b \right) \sqrt{\frac{2b \sqrt{\frac{a^2 - ab}{b^2}} - 2a + b}{b}} \operatorname{ellipticF}\left(\sqrt{\frac{2b \sqrt{\frac{a^2 - ab}{b^2}} - 2a + b}{b}} (\cosh(fx + e) + \cosh(e)), \sqrt{\frac{2b \sqrt{\frac{a^2 - ab}{b^2}} - 2a + b}{b}}\right)}{b^{\frac{3}{2}} f}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{b \sinh^2(fx + e) + a}}, x\right)$$

86.36 Problem number 414

$$\int \frac{\operatorname{sech}(c + dx)}{a + b \sqrt{\sinh(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{a^3 \arctan(\sinh(dx+c))}{(a^4+b^4)d} + \frac{ab^2 \ln(\cosh(dx+c))}{(a^4+b^4)d} - \frac{2ab^2 \ln\left(a+b\left(\sqrt{\sinh(dx+c)}\right)\right)}{(a^4+b^4)d} \\
& - \frac{b(a^2-b^2) \arctan\left(-1+\sqrt{2}\left(\sqrt{\sinh(dx+c)}\right)\right) \sqrt{2}}{2(a^4+b^4)d} \\
& - \frac{b(a^2-b^2) \arctan\left(1+\sqrt{2}\left(\sqrt{\sinh(dx+c)}\right)\right) \sqrt{2}}{2(a^4+b^4)d} \\
& - \frac{b(a^2+b^2) \ln\left(1+\sinh(dx+c)-\sqrt{2}\left(\sqrt{\sinh(dx+c)}\right)\right) \sqrt{2}}{4(a^4+b^4)d} \\
& + \frac{b(a^2+b^2) \ln\left(1+\sinh(dx+c)+\sqrt{2}\left(\sqrt{\sinh(dx+c)}\right)\right) \sqrt{2}}{4(a^4+b^4)d}
\end{aligned}$$

command

```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.37 Problem number 418

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{a^2(a^4 - 3b^4) \arctan(\sinh(dx + c))}{(a^4 + b^4)^2 d} + \frac{b^2(3a^4 - b^4) \ln(\cosh(dx + c))}{(a^4 + b^4)^2 d} \\
 & - \frac{2b^2(3a^4 - b^4) \ln\left(a + b\left(\sqrt{\sinh(dx + c)}\right)\right)}{(a^4 + b^4)^2 d} \\
 & - \frac{ab(a^4 + 2a^2b^2 - b^4) \ln\left(1 + \sinh(dx + c) - \sqrt{2}\left(\sqrt{\sinh(dx + c)}\right)\right) \sqrt{2}}{2(a^4 + b^4)^2 d} \\
 & + \frac{ab(a^4 + 2a^2b^2 - b^4) \ln\left(1 + \sinh(dx + c) + \sqrt{2}\left(\sqrt{\sinh(dx + c)}\right)\right) \sqrt{2}}{2(a^4 + b^4)^2 d} \\
 & - \frac{ab(a^4 - 2a^2b^2 - b^4) \arctan\left(-1 + \sqrt{2}\left(\sqrt{\sinh(dx + c)}\right)\right) \sqrt{2}}{(a^4 + b^4)^2 d} \\
 & - \frac{ab(a^4 - 2a^2b^2 - b^4) \arctan\left(1 + \sqrt{2}\left(\sqrt{\sinh(dx + c)}\right)\right) \sqrt{2}}{(a^4 + b^4)^2 d} \\
 & + \frac{2ab^2}{(a^4 + b^4)d\left(a + b\left(\sqrt{\sinh(dx + c)}\right)\right)}
 \end{aligned}$$

command

```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.38 Problem number 487

$$\int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Optimal antiderivative

$$- \frac{i \sqrt{\frac{\cos(2ifx + 2ie)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(ifx + ie), \sqrt{\frac{b}{a}}\right) \sqrt{1 + \frac{b(\sinh^2(fx + e))}{a}}}{\cos(ifx + ie) f \sqrt{a + b(\sinh^2(fx + e))}}$$

command

`integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2b \sqrt{\frac{a^2 - ab}{b^2}} + 2a - b \right) \sqrt{\frac{2b \sqrt{\frac{a^2 - ab}{b^2}} - 2a + b}{b}} \operatorname{ellipticF} \left(\sqrt{\frac{2b \sqrt{\frac{a^2 - ab}{b^2}} - 2a + b}{b}} (\cosh(fx + \cosh(1))) \right)$$

$b^{\frac{3}{2}} f$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{b \sinh^2(fx + e) + a}}, x \right)$$

86.39 Problem number 501

$$\int \frac{\tanh^5(e + fx)}{(a + b \sinh^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(8a^2 + 24ab + 3b^2) \operatorname{arctanh} \left(\frac{\sqrt{a + b (\sinh^2(fx + e))}}{\sqrt{a - b}} \right)}{8(a - b)^{\frac{9}{2}} f} \\ & + \frac{8a^2 + 24ab + 3b^2}{24(a - b)^3 f (a + b (\sinh^2(fx + e)))^{\frac{3}{2}}} + \frac{(8a - b) \operatorname{sech}(fx + e)^2}{8(a - b)^2 f (a + b (\sinh^2(fx + e)))^{\frac{3}{2}}} \\ & - \frac{\operatorname{sech}(fx + e)^4}{4(a - b) f (a + b (\sinh^2(fx + e)))^{\frac{3}{2}}} + \frac{8a^2 + 24ab + 3b^2}{8(a - b)^4 f \sqrt{a + b (\sinh^2(fx + e))}} \end{aligned}$$

command

`integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

86.40 Problem number 506

$$\int \frac{\coth^5(e + fx)}{(a + b \sinh^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(8a^2 - 40ab + 35b^2) \operatorname{arctanh}\left(\frac{\sqrt{a + b(\sinh^2(fx + e))}}{\sqrt{a}}\right)}{8a^{\frac{9}{2}}f} \\ & + \frac{8a^2 - 40ab + 35b^2}{24a^3f(a + b(\sinh^2(fx + e)))^{\frac{3}{2}}} - \frac{(8a - 7b) \operatorname{csch}(fx + e)^2}{8a^2f(a + b(\sinh^2(fx + e)))^{\frac{3}{2}}} \\ & - \frac{\operatorname{csch}(fx + e)^4}{4af(a + b(\sinh^2(fx + e)))^{\frac{3}{2}}} + \frac{8a^2 - 40ab + 35b^2}{8a^4f\sqrt{a + b(\sinh^2(fx + e))}} \end{aligned}$$

command

```
integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

87 Test file number 169

Test folder name:

test_cases/6_Hyperbolic_functions/6.2_Hyperbolic_cosine/169_6.2.5_Hyperbolic_cosine_functions

87.1 Problem number 7

$$\int \cosh^{\frac{7}{2}}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{10i \sqrt{\frac{\cosh(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{21 \cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b} \\ & + \frac{2 \left(\cosh^{\frac{5}{2}}(bx + a)\right) \sinh(bx + a)}{7b} + \frac{10 \sinh(bx + a) \left(\sqrt{\cosh(bx + a)}\right)}{21b} \end{aligned}$$

command

```
integrate(cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$40 \left(\sqrt{2} \cosh (bx + a)^3 + 3 \sqrt{2} \cosh (bx + a)^2 \sinh (bx + a) + 3 \sqrt{2} \cosh (bx + a) \sinh (bx + a)^2 + \sqrt{2} \sinh (bx + a) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\cosh (bx + a)^{\frac{7}{2}}, x \right)$$

87.2 Problem number 8

$$\int \cosh^{\frac{5}{2}}(a + bx) dx$$

Optimal antiderivative

$$-\frac{6i \sqrt{\frac{\cosh (bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(i \sinh \left(\frac{a}{2} + \frac{bx}{2} \right), \sqrt{2} \right)}{5 \cosh \left(\frac{a}{2} + \frac{bx}{2} \right) b} + \frac{2 \left(\cosh^{\frac{3}{2}}(bx + a) \right) \sinh (bx + a)}{5b}$$

command

```
integrate(cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{12 \left(\sqrt{2} \cosh (bx + a)^2 + 2 \sqrt{2} \cosh (bx + a) \sinh (bx + a) + \sqrt{2} \sinh (bx + a)^2 \right) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassZeta}(-4, 0, \dots)))}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\cosh (bx + a)^{\frac{5}{2}}, x \right)$$

87.3 Problem number 9

$$\int \cosh^{\frac{3}{2}}(a + bx) dx$$

Optimal antiderivative

$$-\frac{2i \sqrt{\frac{\cosh(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{3 \cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b} + \frac{2 \sinh(bx+a) \left(\sqrt{\cosh(bx+a)}\right)}{3b}$$

command

```
integrate(cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} \cosh(bx+a) + \sqrt{2} \sinh(bx+a) \right) \operatorname{weierstrassPInverse}(-4, 0, \cosh(bx+a) + \sinh(bx+a)) + \left(\cosh(bx+a) \right)}{3 \left(b \cosh(bx+a) + b \sinh(bx+a) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\cosh(bx+a)^{\frac{3}{2}}, x\right)$$

87.4 Problem number 10

$$\int \sqrt{\cosh(a + bx)} dx$$

Optimal antiderivative

$$-\frac{2i \sqrt{\frac{\cosh(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{\cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cosh(bx+a) + \sinh(bx+a))) + \sqrt{\cosh(bx+a)} \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{\cosh(bx+a)}, x\right)$$

87.5 Problem number 11

$$\int \frac{1}{\sqrt{\cosh(a+bx)}} dx$$

Optimal antiderivative

$$\frac{2i\sqrt{\frac{\cosh(bx+a)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(i \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{\cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cosh(bx+a) + \sinh(bx+a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{\cosh(bx+a)}}, x\right)$$

87.6 Problem number 12

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{2i\sqrt{\frac{\cosh(bx+a)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(i \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{\cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b} + \frac{2 \sinh(bx+a)}{b\sqrt{\cosh(bx+a)}}$$

command

```
integrate(1/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\left(\sqrt{2} \cosh(bx+a)\right)^2 + 2\sqrt{2} \cosh(bx+a) \sinh(bx+a) + \sqrt{2} \sinh(bx+a)^2 + \sqrt{2}\right) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cosh(bx+a) + \sinh(bx+a)))}{b \cosh(bx+a)^2 + 2b \sinh(bx+a)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\cosh(bx+a)^{\frac{3}{2}}}, x\right)$$

87.7 Problem number 13

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)} dx$$

Optimal antiderivative

$$-\frac{2i\sqrt{\frac{\cosh(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{3 \cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b} + \frac{2 \sinh(bx+a)}{3b \cosh(bx+a)^{\frac{3}{2}}}$$

command

```
integrate(1/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} \cosh(bx+a)^4 + 4 \sqrt{2} \cosh(bx+a) \sinh(bx+a)^3 + \sqrt{2} \sinh(bx+a)^4 + 2 \left(3 \sqrt{2} \cosh(bx+a)^2 + \sqrt{2} \right) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\cosh(bx+a)^{\frac{5}{2}}}, x\right)$$

87.8 Problem number 14

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{6i\sqrt{\frac{\cosh(bx+a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{5 \cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b} + \frac{2 \sinh(bx+a)}{5b \cosh(bx+a)^{\frac{5}{2}}} + \frac{6 \sinh(bx+a)}{5b \sqrt{\cosh(bx+a)}}$$

command

```
integrate(1/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \left(\sqrt{2} \cosh(bx+a)^6 + 6 \sqrt{2} \cosh(bx+a) \sinh(bx+a)^5 + \sqrt{2} \sinh(bx+a)^6 + 3 \left(5 \sqrt{2} \cosh(bx+a)^2 + \sqrt{2} \right) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\cosh(bx+a)^{\frac{7}{2}}}, x\right)$$

87.9 Problem number 15

$$\int (a \cosh(x))^{7/2} dx$$

Optimal antiderivative

$$\frac{2a(a \cosh(x))^{\frac{5}{2}} \sinh(x)}{7} - \frac{10ia^4 \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(x)}(x)\right)}{21 \cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x)}} + \frac{10a^3 \sinh(x) \sqrt{a \cosh(x)}}{21}$$

command

```
integrate((a*cosh(x))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$40 \left(\sqrt{2} a^3 \cosh(x)^3 + 3 \sqrt{2} a^3 \cosh(x)^2 \sinh(x) + 3 \sqrt{2} a^3 \cosh(x) \sinh(x)^2 + \sqrt{2} a^3 \sinh(x)^3 \right) \sqrt{a} \operatorname{weierstrassPI}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \cosh(x)} a^3 \cosh(x)^3, x\right)$$

87.10 Problem number 16

$$\int (a \cosh(x))^{5/2} dx$$

Optimal antiderivative

$$\frac{2a(a \cosh(x))^{\frac{3}{2}} \sinh(x)}{5} - \frac{6ia^2 \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a \cosh(x)}}{5 \cosh\left(\frac{x}{2}\right) \sqrt{\cosh(x)}}$$

command

```
integrate((a*cosh(x))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 \left(\sqrt{2} a^2 \cosh(x)^2 + 2 \sqrt{2} a^2 \cosh(x) \sinh(x) + \sqrt{2} a^2 \sinh(x)^2 \right) \sqrt{a} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPIInvers}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \cosh(x)} a^2 \cosh(x)^2, x\right)$$

87.11 Problem number 17

$$\int (a \cosh(x))^{3/2} dx$$

Optimal antiderivative

$$-\frac{2ia^2 \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(x)}\right)}{3 \cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x)}} + \frac{2a \sinh(x) \sqrt{a \cosh(x)}}{3}$$

command

```
integrate((a*cosh(x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} a \cosh(x) + \sqrt{2} a \sinh(x) \right) \sqrt{a} \operatorname{weierstrassPInverse}(-4, 0, \cosh(x) + \sinh(x)) + \left(a \cosh(x)^2 + 2 a \cosh(x) \right)}{3 (\cosh(x) + \sinh(x))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \cosh(x)} a \cosh(x), x\right)$$

87.12 Problem number 18

$$\int \sqrt{a \cosh(x)} dx$$

Optimal antiderivative

$$-\frac{2i \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a \cosh(x)}}{\cosh\left(\frac{x}{2}\right) \sqrt{\cosh(x)}}$$

command

```
integrate((a*cosh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2 \sqrt{2} \sqrt{a} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cosh(x) + \sinh(x))) - 2 \sqrt{a \cosh(x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \cosh(x)}, x\right)$$

87.13 Problem number 19

$$\int \frac{1}{\sqrt{a \cosh(x)}} dx$$

Optimal antiderivative

$$-\frac{2i \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(x)}\right)}{\cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x)}}$$

command

```
integrate(1/(a*cosh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cosh(x) + \sinh(x))}{\sqrt{a}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \cosh(x)}}{a \cosh(x)}, x\right)$$

87.14 Problem number 20

$$\int \frac{1}{(a \cosh(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \sinh(x)}{a \sqrt{a \cosh(x)}} + \frac{2i \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a \cosh(x)}}{\cosh\left(\frac{x}{2}\right) a^2 \sqrt{\cosh(x)}}$$

command

```
integrate(1/(a*cosh(x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\left(\sqrt{2} \cosh(x)^2 + 2 \sqrt{2} \cosh(x) \sinh(x) + \sqrt{2} \sinh(x)^2 + \sqrt{2} \right) \sqrt{a} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse} \right)}{a^2 \cosh(x)^2 + 2 a^2 \cosh(x) \sinh(x) + a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \cosh(x)}}{a^2 \cosh(x)^2}, x\right)$$

87.15 Problem number 21

$$\int \frac{1}{(a \cosh(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \sinh(x)}{3a (a \cosh(x))^{\frac{3}{2}}} - \frac{2i \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(x)}\right)}{3 \cosh\left(\frac{x}{2}\right) a^2 \sqrt{a \cosh(x)}}$$

command

```
integrate(1/(a*cosh(x))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\left(\sqrt{2} \cosh(x)^4 + 4 \sqrt{2} \cosh(x) \sinh(x)^3 + \sqrt{2} \sinh(x)^4 + 2 \left(3 \sqrt{2} \cosh(x)^2 + \sqrt{2} \right) \sinh(x)^2 + 2 \sqrt{2} \cosh(x) \right)}{3 \left(a^3 \cosh(x)^4 + 4 a^3 \cosh(x) \right)} \right)}{3 \left(a^3 \cosh(x)^4 + 4 a^3 \cosh(x) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \cosh(x)}}{a^3 \cosh(x)^3}, x\right)$$

87.16 Problem number 22

$$\int \frac{1}{(a \cosh(x))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2 \sinh(x)}{5a (a \cosh(x))^{\frac{5}{2}}} + \frac{6 \sinh(x)}{5a^3 \sqrt{a \cosh(x)}} + \frac{6i \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a \cosh(x)}}{5 \cosh\left(\frac{x}{2}\right) a^4 \sqrt{\cosh(x)}}$$

command

```
integrate(1/(a*cosh(x))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \left(\sqrt{2} \cosh(x)^6 + 6 \sqrt{2} \cosh(x) \sinh(x)^5 + \sqrt{2} \sinh(x)^6 + 3 \left(5 \sqrt{2} \cosh(x)^2 + \sqrt{2} \right) \sinh(x)^4 + 3 \sqrt{2} \cosh(x) \right)}{3 \left(a^4 \cosh(x)^4 + 4 a^4 \cosh(x) \right)} \right)}{3 \left(a^4 \cosh(x)^4 + 4 a^4 \cosh(x) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \cosh(x)}}{a^4 \cosh(x)^4}, x\right)$$

87.17 Problem number 79

$$\int (a + b \cosh(x))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(a + b \cosh(x))^{\frac{3}{2}} \sinh(x)}{5} + \frac{16ab \sinh(x) \sqrt{a + b \cosh(x)}}{15} \\ & - \frac{2i(23a^2 + 9b^2) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(x)}}{15 \cosh\left(\frac{x}{2}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}} \\ & + \frac{16ia(a^2 - b^2) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{15 \cosh\left(\frac{x}{2}\right) \sqrt{a + b \cosh(x)}} \end{aligned}$$

command

`integrate((a+b*cosh(x))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \left(\sqrt{2} (a^3 - 33ab^2) \cosh(x)^2 + 2\sqrt{2} (a^3 - 33ab^2) \cosh(x) \sinh(x) + \sqrt{2} (a^3 - 33ab^2) \sinh(x)^2 \right) \sqrt{b} \operatorname{weierstrass}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(b^2 \cosh(x)^2 + 2ab \cosh(x) + a^2\right) \sqrt{b \cosh(x) + a}, x\right)$$

87.18 Problem number 80

$$\int (a + b \cosh(x))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b \sinh(x) \sqrt{a + b \cosh(x)}}{3} \\ & - \frac{8ia \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(x)}}{3 \cosh\left(\frac{x}{2}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}} \\ & + \frac{2i(a^2 - b^2) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{3 \cosh\left(\frac{x}{2}\right) \sqrt{a + b \cosh(x)}} \end{aligned}$$

command

```
integrate((a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (a^2 + 3b^2) \cosh(x) + \sqrt{2} (a^2 + 3b^2) \sinh(x) \right) \sqrt{b} \operatorname{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cosh(x)}{3b} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left((b \cosh(x) + a)^{\frac{3}{2}}, x \right)$$

87.19 Problem number 81

$$\int \sqrt{a + b \cosh(c + dx)} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(i \sinh \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a + b \cosh(dx + c)}}{\cosh \left(\frac{dx}{2} + \frac{c}{2} \right) d \sqrt{\frac{a + b \cosh(dx + c)}{a+b}}}$$

command

```
integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} a \sqrt{b} \operatorname{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cosh(dx+c) + 3b \sinh(dx+c) + 2a}{3b} \right) - 3 \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\sqrt{b \cosh(dx + c) + a}, x \right)$$

87.20 Problem number 82

$$\int \frac{1}{\sqrt{a + b \cosh(x)}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{\cosh\left(\frac{x}{2}\right) \sqrt{a + b \cosh(x)}}$$

command

```
integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b \cosh(x)+3b \sinh(x)+2a}{3b}\right)}{\sqrt{b}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{b \cosh(x) + a}}, x\right)$$

87.21 Problem number 83

$$\int \frac{1}{(a + b \cosh(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2b \sinh(x)}{(a^2 - b^2) \sqrt{a + b \cosh(x)}} + 2i \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(x)}}{\cosh\left(\frac{x}{2}\right) (a^2 - b^2) \sqrt{\frac{a + b \cosh(x)}{a+b}}}$$

command

```
integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} ab \cosh(x)^2 + \sqrt{2} ab \sinh(x)^2 + 2 \sqrt{2} a^2 \cosh(x) + \sqrt{2} ab + 2 \left(\sqrt{2} ab \cosh(x) + \sqrt{2} a^2 \right) \sinh(x) \right) \sqrt{b} \right) \text{we}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \cosh(x) + a}}{b^2 \cosh(x)^2 + 2ab \cosh(x) + a^2}, x \right)$$

87.22 Problem number 84

$$\int \frac{1}{(a + b \cosh(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b \sinh(x)}{3(a^2 - b^2)(a + b \cosh(x))^{\frac{3}{2}}} - \frac{8ab \sinh(x)}{3(a^2 - b^2)^2 \sqrt{a + b \cosh(x)}} \\ & \frac{8ia \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \text{EllipticE} \left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a + b \cosh(x)}}{3 \cosh\left(\frac{x}{2}\right) (a^2 - b^2)^2 \sqrt{\frac{a + b \cosh(x)}{a+b}}} \\ & + \frac{2i \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \text{EllipticF} \left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{3 \cosh\left(\frac{x}{2}\right) (a^2 - b^2) \sqrt{a + b \cosh(x)}} \end{aligned}$$

command

```
integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} (a^2 b^2 + 3b^4) \cosh(x)^4 + \sqrt{2} (a^2 b^2 + 3b^4) \sinh(x)^4 + 4 \sqrt{2} (a^3 b + 3ab^3) \cosh(x)^3 + 4 \left(\sqrt{2} (a^2 b^2 + 3b^4) \right) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \cosh(x) + a}}{b^3 \cosh(x)^3 + 3ab^2 \cosh(x)^2 + 3a^2 b \cosh(x) + a^3}, x \right)$$

87.23 Problem number 85

$$\int \frac{1}{(a + b \cosh(x))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b \sinh(x)}{5(a^2 - b^2)(a + b \cosh(x))^{5/2}} - \frac{16ab \sinh(x)}{15(a^2 - b^2)^2(a + b \cosh(x))^{3/2}} \\ & - \frac{2b(23a^2 + 9b^2) \sinh(x)}{15(a^2 - b^2)^3 \sqrt{a + b \cosh(x)}} \\ & - \frac{2i(23a^2 + 9b^2) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(x)}}{15 \cosh\left(\frac{x}{2}\right) (a^2 - b^2)^3 \sqrt{\frac{a + b \cosh(x)}{a+b}}} \\ & + \frac{16ia \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{15 \cosh\left(\frac{x}{2}\right) (a^2 - b^2)^2 \sqrt{a + b \cosh(x)}} \end{aligned}$$

command

```
integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cosh(x) + a}}{b^4 \cosh(x)^4 + 4ab^3 \cosh(x)^3 + 6a^2b^2 \cosh(x)^2 + 4a^3b \cosh(x) + a^4}, x\right)$$

87.24 Problem number 86

$$\int \frac{\cosh(x)}{\sqrt{a + b \cosh(x)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2i \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(x)}}{\cosh\left(\frac{x}{2}\right) b \sqrt{\frac{a + b \cosh(x)}{a+b}}} \\ & + \frac{2ia \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{\cosh\left(\frac{x}{2}\right) b \sqrt{a + b \cosh(x)}} \end{aligned}$$

command

```
integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(2 \sqrt{2} a \sqrt{b} \operatorname{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cosh(x) + 3b \sinh(x) + 2a}{3b} \right) + 3 \sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta} \left(\frac{x}{2}, \frac{3b \cosh(x) + 3b \sinh(x) + 2a}{3b} \right) \right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\cosh(x)}{\sqrt{b \cosh(x) + a}}, x \right)$$

87.25 Problem number 107

$$\int (a + b \cosh(x))^{5/2} (A + B \cosh(x)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(7Ab + 5Ba)(a + b \cosh(x))^{\frac{3}{2}} \sinh(x)}{35} + \frac{2B(a + b \cosh(x))^{\frac{5}{2}} \sinh(x)}{7} \\ & + \frac{2(56Aab + 15Ba^2 + 25b^2B) \sinh(x) \sqrt{a + b \cosh(x)}}{105} \\ & - \frac{2i(161Aa^2b + 63Ab^3 + 15a^3B + 145Ba^2b^2) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(i \sinh \left(\frac{x}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a + b \cosh(x)}}{105 \cosh \left(\frac{x}{2} \right) b \sqrt{\frac{a + b \cosh(x)}{a+b}}} \\ & + \frac{2i(a^2 - b^2) (56Aab + 15Ba^2 + 25b^2B) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(i \sinh \left(\frac{x}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{105 \cosh \left(\frac{x}{2} \right) b \sqrt{a + b \cosh(x)}} \end{aligned}$$

command

```
integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{8 \left(\sqrt{2} (30Ba^4 + 7Aa^3b - 115Ba^2b^2 - 231Aab^3 - 75Bb^4) \cosh(x)^3 + 3 \sqrt{2} (30Ba^4 + 7Aa^3b - 115Ba^2b^2 - 231Aab^3 - 75Bb^4) \cosh(x)^2 + 3 \sqrt{2} (30Ba^4 + 7Aa^3b - 115Ba^2b^2 - 231Aab^3 - 75Bb^4) \cosh(x) + 3 \sqrt{2} (30Ba^4 + 7Aa^3b - 115Ba^2b^2 - 231Aab^3 - 75Bb^4) \right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(Bb^2 \cosh(x)^3 + Aa^2 + (2Bab + Ab^2) \cosh(x)^2 + (Ba^2 + 2Aab) \cosh(x) \right) \sqrt{b \cosh(x) + a}, x \right)$$

87.26 Problem number 108

$$\int (a + b \cosh(x))^{3/2} (A + B \cosh(x)) dx$$

Optimal antiderivative

$$\frac{2B(a + b \cosh(x))^{\frac{3}{2}} \sinh(x)}{5} + \frac{2(5Ab + 3Ba) \sinh(x) \sqrt{a + b \cosh(x)}}{15}$$

$$- \frac{2i(20Aab + 3Ba^2 + 9b^2B) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(x)}}{15 \cosh\left(\frac{x}{2}\right) b \sqrt{\frac{a + b \cosh(x)}{a+b}}}$$

$$+ \frac{2i(a^2 - b^2) (5Ab + 3Ba) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{15 \cosh\left(\frac{x}{2}\right) b \sqrt{a + b \cosh(x)}}$$

command

`integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \left(\sqrt{2} (6Ba^3 - 5Aa^2b - 18Bab^2 - 15Ab^3) \cosh(x)^2 + 2\sqrt{2} (6Ba^3 - 5Aa^2b - 18Bab^2 - 15Ab^3) \cosh(x) \sinh(x) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(Bb \cosh(x)^2 + Aa + (Ba + Ab) \cosh(x)\right) \sqrt{b \cosh(x) + a}, x\right)$$

87.27 Problem number 109

$$\int \sqrt{a + b \cosh(x)} (A + B \cosh(x)) dx$$

Optimal antiderivative

$$\frac{2B \sinh(x) \sqrt{a + b \cosh(x)}}{3}$$

$$- \frac{2i(3Ab + Ba) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(x)}}{3 \cosh\left(\frac{x}{2}\right) b \sqrt{\frac{a + b \cosh(x)}{a+b}}}$$

$$+ \frac{2i(a^2 - b^2) B \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{3 \cosh\left(\frac{x}{2}\right) b \sqrt{a + b \cosh(x)}}$$

command

```
integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} (2 B a^2 - 3 A a b - 3 B b^2) \cosh(x) + \sqrt{2} (2 B a^2 - 3 A a b - 3 B b^2) \sinh(x) \right) \sqrt{b} \operatorname{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b} \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left((B \cosh(x) + A) \sqrt{b \cosh(x) + a}, x \right)$$

87.28 Problem number 118

$$\int \frac{A + B \cosh(x)}{\sqrt{a + b \cosh(x)}} dx$$

Optimal antiderivative

$$\frac{2iB \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE} \left(i \sinh \left(\frac{x}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{a + b \cosh(x)}}{\cosh \left(\frac{x}{2} \right) b \sqrt{\frac{a + b \cosh(x)}{a+b}}} - \frac{2i(Ab - Ba) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF} \left(i \sinh \left(\frac{x}{2} \right), \sqrt{2} \sqrt{\frac{b}{a+b}} \right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{\cosh \left(\frac{x}{2} \right) b \sqrt{a + b \cosh(x)}}$$

command

```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \sqrt{2} B b^{\frac{3}{2}} \operatorname{weierstrassZeta} \left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3} \right), \operatorname{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cosh(x)}{\dots} \right) \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{B \cosh(x) + A}{\sqrt{b \cosh(x) + a}}, x \right)$$

87.29 Problem number 119

$$\int \frac{A + B \cosh(x)}{(a + b \cosh(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2(Ab - Ba) \sinh(x)}{(a^2 - b^2) \sqrt{a + b \cosh(x)}} + 2i(Ab - Ba) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(x)}}{\cosh\left(\frac{x}{2}\right) b (a^2 - b^2) \sqrt{\frac{a + b \cosh(x)}{a+b}}} + \frac{2iB \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{\cosh\left(\frac{x}{2}\right) b \sqrt{a + b \cosh(x)}}$$

command

```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} (2Ba^2b + Aab^2 - 3Bb^3) \cosh(x)^2 + \sqrt{2} (2Ba^2b + Aab^2 - 3Bb^3) \sinh(x)^2 + 2\sqrt{2} (2Ba^3 + Aa^2b - 3Bb^3) \right) \sqrt{a + b \cosh(x)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cosh(x) + A) \sqrt{b \cosh(x) + a}}{b^2 \cosh(x)^2 + 2ab \cosh(x) + a^2}, x\right)$$

87.30 Problem number 120

$$\int \frac{A + B \cosh(x)}{(a + b \cosh(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(Ab - Ba) \sinh(x)}{3(a^2 - b^2)(a + b \cosh(x))^{\frac{3}{2}}} - \frac{2(4Aab - Ba^2 - 3b^2B) \sinh(x)}{3(a^2 - b^2)^2 \sqrt{a + b \cosh(x)}}$$

$$\frac{2i(4Aab - Ba^2 - 3b^2B) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(x)}}{3 \cosh\left(\frac{x}{2}\right) b(a^2 - b^2)^2 \sqrt{\frac{a + b \cosh(x)}{a+b}}}$$

$$+ \frac{2i(Ab - Ba) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{3 \cosh\left(\frac{x}{2}\right) b(a^2 - b^2) \sqrt{a + b \cosh(x)}}$$

command

```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{(B \cosh(x) + A) \sqrt{b \cosh(x) + a}}{b^3 \cosh(x)^3 + 3ab^2 \cosh(x)^2 + 3a^2b \cosh(x) + a^3}, x\right)$$

87.31 Problem number 128

$$\int (a \cosh^3(x))^{5/2} dx$$

Optimal antiderivative

$$\frac{26ia^2 \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a (\cosh^3(x))}}{77 \cosh\left(\frac{x}{2}\right) \cosh(x)^{\frac{3}{2}}}$$

$$+ \frac{78a^2 \cosh(x) \sinh(x) \sqrt{a (\cosh^3(x))}}{385} + \frac{26a^2 (\cosh^3(x)) \sinh(x) \sqrt{a (\cosh^3(x))}}{165}$$

$$+ \frac{2a^2 (\cosh^5(x)) \sinh(x) \sqrt{a (\cosh^3(x))}}{15} + \frac{26a^2 \sqrt{a (\cosh^3(x))} \tanh(x)}{77}$$

command

```
integrate((a*cosh(x)^3)^(5/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$24960 \left(\sqrt{2} a^2 \cosh(x)^7 + 7 \sqrt{2} a^2 \cosh(x)^6 \sinh(x) + 21 \sqrt{2} a^2 \cosh(x)^5 \sinh(x)^2 + 35 \sqrt{2} a^2 \cosh(x)^4 \sinh(x)^3 - \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{a \cosh(x)^3} a^2 \cosh(x)^6, x \right)$$

87.32 Problem number 129

$$\int (a \cosh^3(x))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{14ia \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \text{EllipticE} \left(i \sinh \left(\frac{x}{2} \right), \sqrt{2} \right) \sqrt{a (\cosh^3(x))}}{15 \cosh \left(\frac{x}{2} \right) \cosh(x)^{\frac{3}{2}}} \\ & + \frac{14a \sinh(x) \sqrt{a (\cosh^3(x))}}{45} + \frac{2a (\cosh^2(x)) \sinh(x) \sqrt{a (\cosh^3(x))}}{9} \end{aligned}$$

command

```
integrate((a*cosh(x)^3)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$336 \left(\sqrt{2} a \cosh(x)^4 + 4 \sqrt{2} a \cosh(x)^3 \sinh(x) + 6 \sqrt{2} a \cosh(x)^2 \sinh(x)^2 + 4 \sqrt{2} a \cosh(x) \sinh(x)^3 + \sqrt{2} a \sinh(x)^4 - \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{a \cosh(x)^3} a \cosh(x)^3, x \right)$$

87.33 Problem number 130

$$\int \sqrt{a \cosh^3(x)} dx$$

Optimal antiderivative

$$-\frac{2i \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a (\cosh^3(x))}}{3 \cosh\left(\frac{x}{2}\right) \cosh(x)^{\frac{3}{2}}} + \frac{2 \sqrt{a (\cosh^3(x))} \tanh(x)}{3}$$

command

```
integrate((a*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} \cosh(x) + \sqrt{2} \sinh(x) \right) \sqrt{a} \operatorname{weierstrassPInverse}(-4, 0, \cosh(x) + \sinh(x)) + \sqrt{a \cosh(x)} \left(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 \right)}{3 (\cosh(x) + \sinh(x))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \cosh(x)^3}, x\right)$$

87.34 Problem number 131

$$\int \frac{1}{\sqrt{a \cosh^3(x)}} dx$$

Optimal antiderivative

$$\frac{2i \left(\cosh^{\frac{3}{2}}(x) \right) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right)}{\cosh\left(\frac{x}{2}\right) \sqrt{a (\cosh^3(x))}} + \frac{2 \cosh(x) \sinh(x)}{\sqrt{a (\cosh^3(x))}}$$

command

```
integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\left(\sqrt{2} \cosh(x)^2 + 2 \sqrt{2} \cosh(x) \sinh(x) + \sqrt{2} \sinh(x)^2 + \sqrt{2} \right) \sqrt{a} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cosh(x) + \sinh(x))) \right)}{a \cosh(x)^2 + 2 a \cosh(x) \sinh(x) + a \sinh(x)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \cosh(x)^3}}{a \cosh(x)^3}, x\right)$$

87.35 Problem number 132

$$\int \frac{1}{(a \cosh^3(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{10i \left(\cosh^{\frac{3}{2}}(x) \right) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right)}{21 \cosh\left(\frac{x}{2}\right) a \sqrt{a (\cosh^3(x))}} + \frac{10 \sinh(x)}{21a \sqrt{a (\cosh^3(x))}} + \frac{2 \operatorname{sech}(x) \tanh(x)}{7a \sqrt{a (\cosh^3(x))}}$$

command

`integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \left(\sqrt{2} \cosh(x)^8 + 8 \sqrt{2} \cosh(x) \sinh(x)^7 + \sqrt{2} \sinh(x)^8 + 4 \left(7 \sqrt{2} \cosh(x)^2 + \sqrt{2} \right) \sinh(x)^6 + 4 \sqrt{2} \cosh(x) \sinh(x)^5 + \sqrt{2} \sinh(x)^4 \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \cosh(x)^3}}{a^2 \cosh(x)^6}, x\right)$$

87.36 Problem number 133

$$\int \frac{1}{(a \cosh^3(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{154i \left(\cosh^{\frac{3}{2}}(x) \right) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right)}{195 \cosh\left(\frac{x}{2}\right) a^2 \sqrt{a (\cosh^3(x))}} + \frac{154 \cosh(x) \sinh(x)}{195a^2 \sqrt{a (\cosh^3(x))}} + \frac{154 \tanh(x)}{585a^2 \sqrt{a (\cosh^3(x))}} + \frac{22 \operatorname{sech}(x)^2 \tanh(x)}{117a^2 \sqrt{a (\cosh^3(x))}} + \frac{2 \operatorname{sech}(x)^4 \tanh(x)}{13a^2 \sqrt{a (\cosh^3(x))}}$$

command

```
integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{a \cosh(x)^3}}{a^3 \cosh(x)^9}, x \right)$$

87.37 Problem number 252

$$\int \frac{\cosh^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{6i \sqrt{\frac{\cosh(a + b \ln(cx^n))}{2}} + \frac{1}{2} \text{EllipticE} \left(i \sinh \left(\frac{a}{2} + \frac{b \ln(cx^n)}{2} \right), \sqrt{2} \right)}{5 \cosh \left(\frac{a}{2} + \frac{b \ln(cx^n)}{2} \right) bn} \\ & + \frac{2 \left(\cosh^{\frac{3}{2}}(a + b \ln(cx^n)) \right) \sinh(a + b \ln(cx^n))}{5bn} \end{aligned}$$

command

```
integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{12 \left(\sqrt{2} \cosh(bn \log(x) + b \log(c) + a)^2 + 2 \sqrt{2} \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) + \dots \right)}{\dots}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\cosh(b \log(cx^n) + a)^{\frac{5}{2}}}{x}, x \right)$$

87.38 Problem number 253

$$\int \frac{\cosh^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh(a + b \ln(cx^n))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{3 \cosh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn} + \frac{2 \sinh(a + b \ln(cx^n)) \left(\sqrt{\cosh}(a + b \ln(cx^n))\right)}{3bn}$$

command

```
integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} \cosh(bn \log(x) + b \log(c) + a) + \sqrt{2} \sinh(bn \log(x) + b \log(c) + a) \right) \operatorname{weierstrassPInverse}(-4, 0, \cosh(bn \log(x) + b \log(c) + a))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cosh(b \log(cx^n) + a)^{\frac{3}{2}}}{x}, x\right)$$

87.39 Problem number 254

$$\int \frac{\sqrt{\cosh(a + b \log(cx^n))}}{x} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh(a + b \ln(cx^n))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{\cosh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cosh(bn \log(x) + b \log(c) + a) + \sinh(bn \log(x) + b \log(c) + a)) \right)}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{\cosh(b \log(cx^n) + a)}}{x}, x \right)$$

87.40 Problem number 255

$$\int \frac{1}{x \sqrt{\cosh(a + b \log(cx^n))}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh(a + b \ln(cx^n))}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(i \sinh \left(\frac{a}{2} + \frac{b \ln(cx^n)}{2} \right), \sqrt{2} \right)}{\cosh \left(\frac{a}{2} + \frac{b \ln(cx^n)}{2} \right) bn}$$

command

`integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cosh(bn \log(x) + b \log(c) + a) + \sinh(bn \log(x) + b \log(c) + a))}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{x \sqrt{\cosh(b \log(cx^n) + a)}}, x \right)$$

87.41 Problem number 256

$$\int \frac{1}{x \cosh^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh(a + b \ln(cx^n))}{2}} + \frac{1}{2} \operatorname{EllipticE} \left(i \sinh \left(\frac{a}{2} + \frac{b \ln(cx^n)}{2} \right), \sqrt{2} \right)}{\cosh \left(\frac{a}{2} + \frac{b \ln(cx^n)}{2} \right) bn} + \frac{2 \sinh(a + b \ln(cx^n))}{bn \sqrt{\cosh(a + b \ln(cx^n))}}$$

command

```
integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} \cosh(bn \log(x) + b \log(c) + a)^2 + 2 \sqrt{2} \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) + \right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{x \cosh(b \log(cx^n) + a)^{\frac{3}{2}}}, x \right)$$

87.42 Problem number 257

$$\int \frac{1}{x \cosh^{\frac{5}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$-\frac{2i \sqrt{\frac{\cosh(a + b \ln(cx^n))}{2} + \frac{1}{2}} \text{EllipticF}\left(i \sinh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right)}{3 \cosh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn} + \frac{2 \sinh(a + b \ln(cx^n))}{3bn \cosh(a + b \ln(cx^n))^{\frac{3}{2}}}$$

command

```
integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} \cosh(bn \log(x) + b \log(c) + a)^4 + 4 \sqrt{2} \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a)^3 + \right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{x \cosh(b \log(cx^n) + a)^{\frac{5}{2}}}, x \right)$$

88 Test file number 170

Test folder name:

test_cases/6_Hyperbolic_functions/6.2_Hyperbolic_cosine/170_6.2.7_hyper^m-a+b_coshⁿ-^p

88.1 Problem number 51

$$\int \frac{1}{\sqrt{a + b \cosh^2(x)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{-(\sinh^2(x))} \operatorname{EllipticF}\left(\cosh(x), \sqrt{-\frac{b}{a}}\right) \sqrt{1 + \frac{b(\cosh^2(x))}{a}}}{\sinh(x) \sqrt{a + b(\cosh^2(x))}}$$

command

```
integrate(1/(a+b*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(2b \sqrt{\frac{a^2 + ab}{b^2}} + 2a + b \right) \sqrt{\frac{2b \sqrt{\frac{a^2 + ab}{b^2}} - 2a - b}{b}} \operatorname{ellipticF} \left(\sqrt{\frac{2b \sqrt{\frac{a^2 + ab}{b^2}} - 2a - b}{b}} (\cosh(x) + \sinh(x)) \right)$$

$$b^{\frac{3}{2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{b \cosh(x)^2 + a}}, x \right)$$

88.2 Problem number 52

$$\int \frac{1}{\sqrt{1 + \cosh^2(x)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{-(\sinh^2(x))} \operatorname{EllipticF}(\cosh(x), i)}{\sinh(x)}$$

command

```
integrate(1/(1+cosh(x)^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2 \left(2\sqrt{2} + 3 \right) \sqrt{2\sqrt{2} - 3} \operatorname{ellipticF} \left(\sqrt{2\sqrt{2} - 3} (\cosh(x) + \sinh(x)), 12\sqrt{2} + 17 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{\cosh(x)^2 + 1}}, x \right)$$

88.3 Problem number 55

$$\int \frac{1}{\sqrt{-1 - \cosh^2(x)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{-(\sinh^2(x))} \operatorname{EllipticF}(\cosh(x), i) \sqrt{1 + \cosh^2(x)}}{\sinh(x) \sqrt{-1 - (\cosh^2(x))}}$$

command

```
integrate(1/(-1-cosh(x)^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2 \sqrt{2\sqrt{2} - 3} \left(-2i\sqrt{2} - 3i \right) \operatorname{ellipticF} \left(\sqrt{2\sqrt{2} - 3} e^x, 12\sqrt{2} + 17 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(-\frac{2 \sqrt{-e^{(4x)} - 6e^{(2x)} - 1}}{e^{(4x)} + 6e^{(2x)} + 1}, x \right)$$

88.4 Problem number 56

$$\int \frac{1}{a + b \cosh^3(x)} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh} \left(\frac{\sqrt{a^{\frac{1}{3}} - b^{\frac{1}{3}}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{\frac{1}{3}} + b^{\frac{1}{3}}}} \right)}{3a^{\frac{2}{3}} \sqrt{a^{\frac{1}{3}} - b^{\frac{1}{3}}} \sqrt{a^{\frac{1}{3}} + b^{\frac{1}{3}}}} + \frac{2 \operatorname{arctanh} \left(\frac{\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{1}{3}} b^{\frac{1}{3}}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{1}{3}} b^{\frac{1}{3}}}} \right)}{3a^{\frac{2}{3}} \sqrt{a^{\frac{1}{3}} - (-1)^{\frac{1}{3}} b^{\frac{1}{3}}} \sqrt{a^{\frac{1}{3}} + (-1)^{\frac{1}{3}} b^{\frac{1}{3}}}} + \frac{2 \operatorname{arctanh} \left(\frac{\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{2}{3}} b^{\frac{1}{3}}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{2}{3}} b^{\frac{1}{3}}}} \right)}{3a^{\frac{2}{3}} \sqrt{a^{\frac{1}{3}} - (-1)^{\frac{2}{3}} b^{\frac{1}{3}}} \sqrt{a^{\frac{1}{3}} + (-1)^{\frac{2}{3}} b^{\frac{1}{3}}}}$$

command

```
integrate(1/(a+b*cosh(x)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

88.5 Problem number 57

$$\int \frac{1}{a - b \cosh^3(x)} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh} \left(\frac{\sqrt{a^{\frac{1}{3}} + b^{\frac{1}{3}}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{\frac{1}{3}} - b^{\frac{1}{3}}}} \right)}{3a^{\frac{2}{3}} \sqrt{a^{\frac{1}{3}} - b^{\frac{1}{3}}} \sqrt{a^{\frac{1}{3}} + b^{\frac{1}{3}}}} + \frac{2 \operatorname{arctanh} \left(\frac{\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{1}{3}} b^{\frac{1}{3}}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{1}{3}} b^{\frac{1}{3}}}} \right)}{3a^{\frac{2}{3}} \sqrt{a^{\frac{1}{3}} - (-1)^{\frac{1}{3}} b^{\frac{1}{3}}} \sqrt{a^{\frac{1}{3}} + (-1)^{\frac{1}{3}} b^{\frac{1}{3}}}} + \frac{2 \operatorname{arctanh} \left(\frac{\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{2}{3}} b^{\frac{1}{3}}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{2}{3}} b^{\frac{1}{3}}}} \right)}{3a^{\frac{2}{3}} \sqrt{a^{\frac{1}{3}} - (-1)^{\frac{2}{3}} b^{\frac{1}{3}}} \sqrt{a^{\frac{1}{3}} + (-1)^{\frac{2}{3}} b^{\frac{1}{3}}}}$$

command

```
integrate(1/(a-b*cosh(x)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

88.6 Problem number 65

$$\int \frac{1}{a + b \cosh^6(x)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{6}} \tanh(x)}{\sqrt{a^{\frac{1}{3}} + b^{\frac{1}{3}}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} + b^{\frac{1}{3}}}} + \frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{6}} \tanh(x)}{\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{1}{3}} b^{\frac{1}{3}}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} - (-1)^{\frac{1}{3}} b^{\frac{1}{3}}}} + \frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{6}} \tanh(x)}{\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{2}{3}} b^{\frac{1}{3}}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} + (-1)^{\frac{2}{3}} b^{\frac{1}{3}}}}$$

command

```
integrate(1/(a+b*cosh(x)^6),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

88.7 Problem number 66

$$\int \frac{1}{a + b \cosh^8(x)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}\left(\frac{(-a)^{\frac{1}{8}} \tanh(x)}{\sqrt{(-a)^{\frac{1}{4}} - b^{\frac{1}{4}}}}\right)}{4(-a)^{\frac{7}{8}} \sqrt{(-a)^{\frac{1}{4}} - b^{\frac{1}{4}}}} - \frac{\operatorname{arctanh}\left(\frac{(-a)^{\frac{1}{8}} \tanh(x)}{\sqrt{(-a)^{\frac{1}{4}} - ib^{\frac{1}{4}}}}\right)}{4(-a)^{\frac{7}{8}} \sqrt{(-a)^{\frac{1}{4}} - ib^{\frac{1}{4}}}} \\ & -\frac{\operatorname{arctanh}\left(\frac{(-a)^{\frac{1}{8}} \tanh(x)}{\sqrt{(-a)^{\frac{1}{4}} + ib^{\frac{1}{4}}}}\right)}{4(-a)^{\frac{7}{8}} \sqrt{(-a)^{\frac{1}{4}} + ib^{\frac{1}{4}}}} - \frac{\operatorname{arctanh}\left(\frac{(-a)^{\frac{1}{8}} \tanh(x)}{\sqrt{(-a)^{\frac{1}{4}} + b^{\frac{1}{4}}}}\right)}{4(-a)^{\frac{7}{8}} \sqrt{(-a)^{\frac{1}{4}} + b^{\frac{1}{4}}}} \end{aligned}$$

command

```
integrate(1/(a+b*cosh(x)^8),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

88.8 Problem number 68

$$\int \frac{1}{a - b \cosh^6(x)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{6}} \tanh(x)}{\sqrt{a^{\frac{1}{3}} - b^{\frac{1}{3}}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} - b^{\frac{1}{3}}}} + \frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{6}} \tanh(x)}{\sqrt{a^{\frac{1}{3}} + (-1)^{\frac{1}{3}} b^{\frac{1}{3}}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} + (-1)^{\frac{1}{3}} b^{\frac{1}{3}}}} + \frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{6}} \tanh(x)}{\sqrt{a^{\frac{1}{3}} - (-1)^{\frac{2}{3}} b^{\frac{1}{3}}}}\right)}{3a^{\frac{5}{6}} \sqrt{a^{\frac{1}{3}} - (-1)^{\frac{2}{3}} b^{\frac{1}{3}}}}$$

command

```
integrate(1/(a-b*cosh(x)^6),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

88.9 Problem number 69

$$\int \frac{1}{a - b \cosh^8(x)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{8}} \tanh(x)}{\sqrt{a^{\frac{1}{4}} - b^{\frac{1}{4}}}}\right)}{4a^{\frac{7}{8}} \sqrt{a^{\frac{1}{4}} - b^{\frac{1}{4}}}} + \frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{8}} \tanh(x)}{\sqrt{a^{\frac{1}{4}} - ib^{\frac{1}{4}}}}\right)}{4a^{\frac{7}{8}} \sqrt{a^{\frac{1}{4}} - ib^{\frac{1}{4}}}} + \frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{8}} \tanh(x)}{\sqrt{a^{\frac{1}{4}} + ib^{\frac{1}{4}}}}\right)}{4a^{\frac{7}{8}} \sqrt{a^{\frac{1}{4}} + ib^{\frac{1}{4}}}} + \frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{8}} \tanh(x)}{\sqrt{a^{\frac{1}{4}} + b^{\frac{1}{4}}}}\right)}{4a^{\frac{7}{8}} \sqrt{a^{\frac{1}{4}} + b^{\frac{1}{4}}}}$$

command

```
integrate(1/(a-b*cosh(x)^8),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89 Test file number 173

Test folder name:

test_cases/6_Hyperbolic_functions/6.3_Hyperbolic_tangent/173_6.3.7-d_hyper-^m-a+b-
c_tanh-ⁿ-^p

89.1 Problem number 41

$$\int \frac{\sinh^4(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(a^2 - 10ab + 5b^2)x}{8(a+b)^5} + \frac{3(5a^2 - 10ab + b^2) \arctan\left(\frac{\sqrt{b} \tanh(dx+c)}{\sqrt{a}}\right) \sqrt{b}}{8(a+b)^5 d \sqrt{a}} \\ & - \frac{(5a - 3b) \cosh(dx+c) \sinh(dx+c)}{8(a+b)^2 d (a+b (\tanh^2(dx+c)))^2} + \frac{(\cosh^3(dx+c)) \sinh(dx+c)}{4(a+b) d (a+b (\tanh^2(dx+c)))^2} \\ & + \frac{(7a - 5b) b \tanh(dx+c)}{8(a+b)^3 d (a+b (\tanh^2(dx+c)))^2} + \frac{3(a-b) b \tanh(dx+c)}{2(a+b)^4 d (a+b (\tanh^2(dx+c)))} \end{aligned}$$

command

```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.2 Problem number 42

$$\int \frac{\sinh^3(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(a - 2b) \cosh(dx + c)}{(a + b)^4 d} + \frac{\cosh^3(dx + c)}{3(a + b)^3 d} + \frac{ab \operatorname{sech}(dx + c)}{4(a + b)^3 d (a + b - b \operatorname{sech}(dx + c))^2} \\ & + \frac{(7a - 4b) b \operatorname{sech}(dx + c)}{8(a + b)^4 d (a + b - b \operatorname{sech}(dx + c))^2} + \frac{5(3a - 4b) \operatorname{arctanh}\left(\frac{\operatorname{sech}(dx + c)\sqrt{b}}{\sqrt{a + b}}\right) \sqrt{b}}{8(a + b)^{\frac{9}{2}} d} \end{aligned}$$

command

```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.3 Problem number 47

$$\int \frac{\operatorname{csch}^3(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a + 6b) \operatorname{arctanh}(\cosh(dx + c))}{2a^4 d} - \frac{\coth(dx + c) \operatorname{csch}(dx + c)}{2ad (a + b - b \operatorname{sech}(dx + c))^2} \\ & - \frac{3b \operatorname{sech}(dx + c)}{4a^2 d (a + b - b \operatorname{sech}(dx + c))^2} - \frac{b(11a + 12b) \operatorname{sech}(dx + c)}{8a^3 (a + b) d (a + b - b \operatorname{sech}(dx + c))^2} \\ & - \frac{(15a^2 + 40ab + 24b^2) \operatorname{arctanh}\left(\frac{\operatorname{sech}(dx + c)\sqrt{b}}{\sqrt{a + b}}\right) \sqrt{b}}{8a^4 (a + b)^{\frac{3}{2}} d} \end{aligned}$$

command

```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.4 Problem number 48

$$\int \frac{\operatorname{csch}^4(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a + 3b) \operatorname{coth}(dx + c)}{a^4 d} - \frac{\operatorname{coth}^3(dx + c)}{3a^3 d} + \frac{5(3a + 7b) \arctan\left(\frac{\sqrt{b} \tanh(dx + c)}{\sqrt{a}}\right) \sqrt{b}}{8a^{\frac{9}{2}} d} \\ & + \frac{b(a + b) \tanh(dx + c)}{4a^3 d (a + b (\tanh^2(dx + c)))^2} + \frac{b(7a + 11b) \tanh(dx + c)}{8a^4 d (a + b (\tanh^2(dx + c)))} \end{aligned}$$

command

```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.5 Problem number 65

$$\int \sinh^4(c + dx) (a + b \tanh^3(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3a(a^2 + 63b^2)x}{8} + \frac{3b(3a^2 + 5b^2) \ln(\cosh(dx + c))}{d} - \frac{18ab^2 \tanh(dx + c)}{d} \\ & - \frac{b(3a^2 + 10b^2) (\tanh^2(dx + c))}{2d} - \frac{3ab^2 (\tanh^3(dx + c))}{d} - \frac{3b^3 (\tanh^4(dx + c))}{2d} \\ & - \frac{3ab^2 (\tanh^5(dx + c))}{5d} - \frac{b^3 (\tanh^6(dx + c))}{2d} - \frac{b^3 (\tanh^8(dx + c))}{8d} \\ & + \frac{(\cosh^3(dx + c)) \sinh(dx + c) (a(a^2 + 3b^2) + b(3a^2 + b^2) \tanh(dx + c))}{4d} \\ & - \frac{\cosh(dx + c) \sinh(dx + c) (a(5a^2 + 51b^2) + 2b(15a^2 + 11b^2) \tanh(dx + c))}{8d} \end{aligned}$$

command

```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.6 Problem number 73

$$\int \frac{\sinh^4(c + dx)}{a + b \tanh^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3a(a-5b)\ln(1-\tanh(dx+c))}{16(a+b)^3d} + \frac{3a(a+5b)\ln(1+\tanh(dx+c))}{16(a-b)^3d} \\ & -\frac{a^{\frac{2}{3}}b^{\frac{1}{3}}\left(a^4+7a^2b^2+b^4+3a^{\frac{2}{3}}b^{\frac{4}{3}}(2a^2+b^2)\right)\ln\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\tanh(dx+c)\right)}{3(a^2-b^2)^3d} \\ & +\frac{a^{\frac{2}{3}}b^{\frac{1}{3}}\left(a^4+7a^2b^2+b^4+3a^{\frac{2}{3}}b^{\frac{4}{3}}(2a^2+b^2)\right)\ln\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}\tanh(dx+c)+b^{\frac{2}{3}}(\tanh^2(dx+c))\right)}{6(a^2-b^2)^3d} \\ & -\frac{a^2b(a^2+2b^2)\ln(a+b(\tanh^3(dx+c)))}{(a^2-b^2)^3d} \\ & -\frac{a^{\frac{2}{3}}b^{\frac{1}{3}}\left(a^2+3a^{\frac{4}{3}}b^{\frac{2}{3}}-b^2\right)\arctan\left(\frac{\left(a^{\frac{1}{3}}-2b^{\frac{1}{3}}\tanh(dx+c)\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{3\left(a^{\frac{4}{3}}+a^{\frac{2}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)^3d} \\ & +\frac{1}{16(a+b)d(1-\tanh(dx+c))^2} + \frac{-5a+b}{16(a+b)^2d(1-\tanh(dx+c))} \\ & -\frac{1}{16(a-b)d(1+\tanh(dx+c))^2} + \frac{5a+b}{16(a-b)^2d(1+\tanh(dx+c))} \end{aligned}$$

command

```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.7 Problem number 74

$$\int \frac{\sinh^3(c + dx)}{a + b \tanh^3(c + dx)} dx$$

Optimal antiderivative

$$i \text{Unintegrable} \left(-\frac{i(\sinh^3(dx + c))}{a + b(\tanh^3(dx + c))}, x \right)$$

command

```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.8 Problem number 76

$$\int \frac{\sinh(c + dx)}{a + b \tanh^3(c + dx)} dx$$

Optimal antiderivative

$$-i \text{Unintegrable} \left(\frac{i \sinh(dx + c)}{a + b(\tanh^3(dx + c))}, x \right)$$

command

```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.9 Problem number 77

$$\int \frac{\operatorname{csch}(c + dx)}{a + b \tanh^3(c + dx)} dx$$

Optimal antiderivative

$$i \operatorname{Unintegrable} \left(-\frac{i \operatorname{csch}(dx + c)}{a + b (\tanh^3(dx + c))}, x \right)$$

command

```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.10 Problem number 79

$$\int \frac{\operatorname{csch}^3(c + dx)}{a + b \tanh^3(c + dx)} dx$$

Optimal antiderivative

$$-i \operatorname{Unintegrable} \left(\frac{i \operatorname{csch}(dx + c)^3}{a + b (\tanh^3(dx + c))}, x \right)$$

command

```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.11 Problem number 200

$$\int \frac{\coth^4(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{(a+b)^3} + \frac{b^{\frac{5}{2}}(63a^2+90ab+35b^2) \arctan\left(\frac{\sqrt{b} \tanh(dx+c)}{\sqrt{a}}\right)}{8a^{\frac{9}{2}}(a+b)^3 d} \\ & - \frac{(8a^3-8a^2b-55ab^2-35b^3) \coth(dx+c)}{8a^4(a+b)^2 d} - \frac{(8a^2+55ab+35b^2) (\coth^3(dx+c))}{24a^3(a+b)^2 d} \\ & + \frac{b(\coth^3(dx+c))}{4a(a+b)d(a+b(\tanh^2(dx+c)))^2} + \frac{b(11a+7b) (\coth^3(dx+c))}{8a^2(a+b)^2 d(a+b(\tanh^2(dx+c)))} \end{aligned}$$

command

```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.12 Problem number 201

$$\int \frac{1}{(a+b \tanh^2(c+dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{(a+b)^4} + \frac{(35a^3+35a^2b+21ab^2+5b^3) \arctan\left(\frac{\sqrt{b} \tanh(dx+c)}{\sqrt{a}}\right) \sqrt{b}}{16a^{\frac{7}{2}}(a+b)^4 d} \\ & + \frac{b \tanh(dx+c)}{6a(a+b)d(a+b(\tanh^2(dx+c)))^3} + \frac{b(11a+5b) \tanh(dx+c)}{24a^2(a+b)^2 d(a+b(\tanh^2(dx+c)))^2} \\ & + \frac{b(19a^2+16ab+5b^2) \tanh(dx+c)}{16a^3(a+b)^3 d(a+b(\tanh^2(dx+c)))} \end{aligned}$$

command

```
integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.13 Problem number 220

$$\int \tanh^2(x) (a + b \tanh^2(x))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & (a + b)^{\frac{3}{2}} \operatorname{arctanh} \left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b (\tanh^2(x))}} \right) \\ & - \frac{(3a^2 + 12ab + 8b^2) \operatorname{arctanh} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b (\tanh^2(x))}} \right)}{8\sqrt{b}} \\ & - \frac{(5a + 4b) \sqrt{a + b (\tanh^2(x))} \tanh(x)}{8} - \frac{b \sqrt{a + b (\tanh^2(x))} (\tanh^3(x))}{4} \end{aligned}$$

command

`integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.14 Problem number 246

$$\int \frac{\tanh^6(x)}{(a + b \tanh^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{arctanh} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b (\tanh^2(x))}} \right)}{b^{\frac{5}{2}}} + \frac{\operatorname{arctanh} \left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b (\tanh^2(x))}} \right)}{(a + b)^{\frac{5}{2}}} \\ & + \frac{a(a + 2b) \tanh(x)}{b^2 (a + b)^2 \sqrt{a + b (\tanh^2(x))}} + \frac{a(\tanh^3(x))}{3b(a + b) (a + b (\tanh^2(x)))^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.15 Problem number 253

$$\int \frac{\coth(x)}{(a + b \tanh^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\tanh^2(x))}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\tanh^2(x))}}{\sqrt{a + b}}\right)}{(a + b)^{5/2}} \\ & + \frac{b(2a + b)}{a^2(a + b)^2 \sqrt{a + b(\tanh^2(x))}} + \frac{b}{3a(a + b)(a + b(\tanh^2(x)))^{3/2}} \end{aligned}$$

command

```
integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.16 Problem number 259

$$\int \tanh(x) (a + b \tanh^4(x))^{3/2} dx$$

Optimal antiderivative

$$\frac{(a+b)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{a+b(\tanh^2(x))}{\sqrt{a+b}\sqrt{a+b(\tanh^4(x))}}\right)}{2} - \frac{(3a+2b) \operatorname{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b(\tanh^4(x))}}\right) \sqrt{b}}{4} - \frac{\sqrt{a+b(\tanh^4(x))} (2a+2b+b(\tanh^2(x)))}{4} - \frac{(a+b(\tanh^4(x)))^{\frac{3}{2}}}{6}$$

command

```
integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

89.17 Problem number 263

$$\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{a+b(\tanh^2(x))}{\sqrt{a+b}\sqrt{a+b(\tanh^4(x))}}\right)}{2(a+b)^{\frac{5}{2}}} + \frac{-3a^2 + b(5a+2b)(\tanh^2(x))}{6a^2(a+b)^2\sqrt{a+b(\tanh^4(x))}} + \frac{-a+b(\tanh^2(x))}{6a(a+b)(a+b(\tanh^4(x)))^{\frac{3}{2}}}$$

command

```
integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

90 Test file number 175

Test folder name:

test_cases/6_Hyperbolic_functions/6.4_Hyperbolic_cotangent/175_6.4.2_Hyperbolic_cotangent_fun

90.1 Problem number 27

$$\int \frac{1}{(b \coth^2(c + dx))^{4/3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}\left(\coth^{\frac{1}{3}}(dx+c)\right)\left(\coth^{\frac{2}{3}}(dx+c)\right)}{bd\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{1}{3}}} \\ & - \frac{\left(\coth^{\frac{2}{3}}(dx+c)\right)\ln\left(1-\left(\coth^{\frac{1}{3}}(dx+c)\right)+\coth^{\frac{2}{3}}(dx+c)\right)}{4bd\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{1}{3}}} \\ & + \frac{\left(\coth^{\frac{2}{3}}(dx+c)\right)\ln\left(1+\coth^{\frac{1}{3}}(dx+c)+\coth^{\frac{2}{3}}(dx+c)\right)}{4bd\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{1}{3}}} \\ & - \frac{\operatorname{arctan}\left(\frac{\left(1-2\left(\coth^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\left(\coth^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{2bd\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{1}{3}}} \\ & + \frac{\operatorname{arctan}\left(\frac{\left(1+2\left(\coth^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\left(\coth^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{2bd\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{1}{3}}} - \frac{3\tanh(dx+c)}{5bd\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{1}{3}}} \end{aligned}$$

command

```
integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

90.2 Problem number 49

$$\int \frac{1}{(b \coth^4(c + dx))^{4/3}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3 \coth(dx + c)}{bd (b (\coth^4(dx + c)))^{\frac{1}{3}}} + \frac{\operatorname{arctanh}\left(\coth^{\frac{1}{3}}(dx + c)\right) \left(\coth^{\frac{4}{3}}(dx + c)\right)}{bd (b (\coth^4(dx + c)))^{\frac{1}{3}}} \\ & - \frac{\left(\coth^{\frac{4}{3}}(dx + c)\right) \ln\left(1 - \left(\coth^{\frac{1}{3}}(dx + c)\right) + \coth^{\frac{2}{3}}(dx + c)\right)}{4bd (b (\coth^4(dx + c)))^{\frac{1}{3}}} \\ & + \frac{\left(\coth^{\frac{4}{3}}(dx + c)\right) \ln\left(1 + \coth^{\frac{1}{3}}(dx + c) + \coth^{\frac{2}{3}}(dx + c)\right)}{4bd (b (\coth^4(dx + c)))^{\frac{1}{3}}} \\ & + \frac{\operatorname{arctan}\left(\frac{(1-2(\coth^{\frac{1}{3}}(dx+c)))\sqrt{3}}{3}\right) \left(\coth^{\frac{4}{3}}(dx + c)\right) \sqrt{3}}{2bd (b (\coth^4(dx + c)))^{\frac{1}{3}}} \\ & - \frac{\operatorname{arctan}\left(\frac{(1+2(\coth^{\frac{1}{3}}(dx+c)))\sqrt{3}}{3}\right) \left(\coth^{\frac{4}{3}}(dx + c)\right) \sqrt{3}}{2bd (b (\coth^4(dx + c)))^{\frac{1}{3}}} \\ & - \frac{3 \tanh(dx + c)}{7bd (b (\coth^4(dx + c)))^{\frac{1}{3}}} - \frac{3(\tanh^3(dx + c))}{13bd (b (\coth^4(dx + c)))^{\frac{1}{3}}} \end{aligned}$$

command

```
integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

91 Test file number 176

Test folder name:

test_cases/6_Hyperbolic_functions/6.4_Hyperbolic_cotangent/176_6.4.7-d_hyper- \hat{m} -a+b-c_coth- \hat{n} - \hat{p}

91.1 Problem number 8

$$\int \frac{1}{(a + b \coth^2(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{(a+b)^4} + \frac{b \coth(dx+c)}{6a(a+b)d(a+b(\coth^2(dx+c)))^3} \\ & + \frac{b(11a+5b)\coth(dx+c)}{24a^2(a+b)^2d(a+b(\coth^2(dx+c)))^2} + \frac{b(19a^2+16ab+5b^2)\coth(dx+c)}{16a^3(a+b)^3d(a+b(\coth^2(dx+c)))} \\ & - \frac{(35a^3+35a^2b+21ab^2+5b^3)\arctan\left(\frac{\sqrt{a}\tanh(dx+c)}{\sqrt{b}}\right)\sqrt{b}}{16a^{\frac{7}{2}}(a+b)^4d} \end{aligned}$$

command

```
integrate(1/(a+b*coth(d*x+c)^2)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

91.2 Problem number 23

$$\int \coth^2(x) (a + b \coth^2(x))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & (a+b)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\coth(x)\sqrt{a+b}}{\sqrt{a+b(\coth^2(x))}}\right) \\ & - \frac{(3a^2+12ab+8b^2)\operatorname{arctanh}\left(\frac{\coth(x)\sqrt{b}}{\sqrt{a+b(\coth^2(x))}}\right)}{8\sqrt{b}} \\ & - \frac{(5a+4b)\coth(x)\sqrt{a+b(\coth^2(x))}}{8} - \frac{b(\coth^3(x))\sqrt{a+b(\coth^2(x))}}{4} \end{aligned}$$

command

```
integrate(coth(x)^2*(a+b*coth(x)^2)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

91.3 Problem number 46

$$\int \frac{\tanh(x)}{(a + b \coth^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b(\coth^2(x))}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b(\coth^2(x))}}{\sqrt{a+b}}\right)}{(a+b)^{\frac{5}{2}}} \\ & + \frac{b}{3a(a+b)(a+b(\coth^2(x)))^{\frac{3}{2}}} + \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b(\coth^2(x))}} \end{aligned}$$

command

```
integrate(tanh(x)/(a+b*coth(x)^2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

92 Test file number 178

Test folder name:

```
test_cases/6_Hyperbolic_functions/6.5_Hyperbolic_secant/178_6.5.2-e_x~m-a+b_sech-
c+d_x~n~p
```

92.1 Problem number 84

$$\int \frac{(ex)^{-1+3n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(ex)^{3n}}{3a^2en} + \frac{b^2(ex)^{3n} x^{-n}}{a^2(a^2 - b^2)den} - \frac{2b^2(ex)^{3n} \ln\left(1 + \frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right) x^{-2n}}{a^2(a^2 - b^2)d^2en} \\ & + \frac{b^3(ex)^{3n} \ln\left(1 + \frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right) x^{-n}}{a^2(-a^2 + b^2)^{\frac{3}{2}}den} - \frac{2b^2(ex)^{3n} \ln\left(1 + \frac{ae^{c+dx^n}}{b + \sqrt{-a^2 + b^2}}\right) x^{-2n}}{a^2(a^2 - b^2)d^2en} \\ & - \frac{b^3(ex)^{3n} \ln\left(1 + \frac{ae^{c+dx^n}}{b + \sqrt{-a^2 + b^2}}\right) x^{-n}}{a^2(-a^2 + b^2)^{\frac{3}{2}}den} - \frac{2b^2(ex)^{3n} \operatorname{polylog}\left(2, -\frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right) x^{-3n}}{a^2(a^2 - b^2)d^3en} \\ & + \frac{2b^3(ex)^{3n} \operatorname{polylog}\left(2, -\frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right) x^{-2n}}{a^2(-a^2 + b^2)^{\frac{3}{2}}d^2en} - \frac{2b^2(ex)^{3n} \operatorname{polylog}\left(2, -\frac{ae^{c+dx^n}}{b + \sqrt{-a^2 + b^2}}\right) x^{-3n}}{a^2(a^2 - b^2)d^3en} \\ & - \frac{2b^3(ex)^{3n} \operatorname{polylog}\left(2, -\frac{ae^{c+dx^n}}{b + \sqrt{-a^2 + b^2}}\right) x^{-2n}}{a^2(-a^2 + b^2)^{\frac{3}{2}}d^2en} - \frac{2b^3(ex)^{3n} \operatorname{polylog}\left(3, -\frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right) x^{-3n}}{a^2(-a^2 + b^2)^{\frac{3}{2}}d^3en} \\ & + \frac{2b^3(ex)^{3n} \operatorname{polylog}\left(3, -\frac{ae^{c+dx^n}}{b + \sqrt{-a^2 + b^2}}\right) x^{-3n}}{a^2(-a^2 + b^2)^{\frac{3}{2}}d^3en} + \frac{b^2(ex)^{3n} \sinh(c + dx^n) x^{-n}}{a(a^2 - b^2)den(b + a \cosh(c + dx^n))} \\ & - \frac{2b(ex)^{3n} \ln\left(1 + \frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right) x^{-n}}{a^2den\sqrt{-a^2 + b^2}} + \frac{2b(ex)^{3n} \ln\left(1 + \frac{ae^{c+dx^n}}{b + \sqrt{-a^2 + b^2}}\right) x^{-n}}{a^2den\sqrt{-a^2 + b^2}} \\ & - \frac{4b(ex)^{3n} \operatorname{polylog}\left(2, -\frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right) x^{-2n}}{a^2d^2en\sqrt{-a^2 + b^2}} + \frac{4b(ex)^{3n} \operatorname{polylog}\left(2, -\frac{ae^{c+dx^n}}{b + \sqrt{-a^2 + b^2}}\right) x^{-2n}}{a^2d^2en\sqrt{-a^2 + b^2}} \\ & + \frac{4b(ex)^{3n} \operatorname{polylog}\left(3, -\frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right) x^{-3n}}{a^2d^3en\sqrt{-a^2 + b^2}} - \frac{4b(ex)^{3n} \operatorname{polylog}\left(3, -\frac{ae^{c+dx^n}}{b + \sqrt{-a^2 + b^2}}\right) x^{-3n}}{a^2d^3en\sqrt{-a^2 + b^2}} \end{aligned}$$

command

```
integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

93 Test file number 179

Test folder name:

test_cases/6_Hyperbolic_functions/6.5_Hyperbolic_secant/179_6.5.3_Hyperbolic_secant_functions

93.1 Problem number 9

$$\int \operatorname{sech}^{\frac{5}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{2\operatorname{sech}(bx + a)^{\frac{3}{2}} \sinh(bx + a)}{3b} - \frac{2i \sqrt{\frac{\cosh(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(bx + a)}\right) \sqrt{\operatorname{sech}(bx + a)}}{3 \cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(sech(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} \left(\cosh(bx + a)^2 + 2 \cosh(bx + a) \sinh(bx + a) + \sinh(bx + a)^2 - 1 \right) \sqrt{\frac{\cosh(bx + a) + \sinh(bx + a)}{\cosh(bx + a)^2 + 2 \cosh(bx + a) \sinh(bx + a) + \sinh(bx + a)^2}} \right) / 3 \left(b \cosh\left(\frac{a}{2} + \frac{bx}{2}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\operatorname{sech}(bx + a)^{\frac{5}{2}}, x\right)$$

93.2 Problem number 10

$$\int \operatorname{sech}^{\frac{3}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{2 \sinh(bx + a) \sqrt{\operatorname{sech}(bx + a)}}{b} + \frac{2i \sqrt{\frac{\cosh(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(bx + a)}\right) \sqrt{\operatorname{sech}(bx + a)}}{\cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(sech(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} \sqrt{\frac{\cosh (bx+a)+\sinh (bx+a)}{\cosh (bx+a)^2+2 \cosh (bx+a) \sinh (bx+a)+\sinh (bx+a)^2+1}} (\cosh (bx+a)+\sinh (bx+a))+\sqrt{2} \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\text{sech}(bx+a)^{\frac{3}{2}}, x\right)$$

93.3 Problem number 11

$$\int \sqrt{\text{sech}(a+bx)} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh (bx+a)}{2}+\frac{1}{2}} \text{EllipticF}\left(i \sinh \left(\frac{a}{2}+\frac{bx}{2}\right), \sqrt{2}\right)\left(\sqrt{\cosh (bx+a)}\right) \sqrt{\text{sech}(bx+a)}}{\cosh \left(\frac{a}{2}+\frac{bx}{2}\right) b}$$

command

```
integrate(sech(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{2} \text{weierstrassPInverse}(-4, 0, \cosh (bx+a)+\sinh (bx+a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{\text{sech}(bx+a)}, x\right)$$

93.4 Problem number 12

$$\int \frac{1}{\sqrt{\operatorname{sech}(a + bx)}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(bx + a)}\right) \sqrt{\operatorname{sech}(bx + a)}}{\cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/sech(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh(bx + a)^2 + 2 \cosh(bx + a) \sinh(bx + a) + \sinh(bx + a)^2 + 1 \right) \sqrt{\frac{\cosh(bx + a) + \sinh(bx + a)}{\cosh(bx + a)^2 + 2 \cosh(bx + a) \sinh(bx + a) + \sinh(bx + a)^2 + 1}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{\operatorname{sech}(bx + a)}}, x\right)$$

93.5 Problem number 13

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a + bx)} dx$$

Optimal antiderivative

$$\frac{\frac{2 \sinh(bx + a)}{3b \sqrt{\operatorname{sech}(bx + a)}} + 2i \sqrt{\frac{\cosh(bx + a)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(bx + a)}\right) \sqrt{\operatorname{sech}(bx + a)}}{3 \cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b}$$

command

```
integrate(1/sech(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh (bx+a)^4 + 4 \cosh (bx+a)^3 \sinh (bx+a) + 6 \cosh (bx+a)^2 \sinh (bx+a)^2 + 4 \cosh (bx+a) \sinh (bx+a)^3 + \sinh (bx+a)^4 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\text{sech}(bx+a)^{\frac{3}{2}}}, x \right)$$

93.6 Problem number 14

$$\int \frac{1}{\text{sech}^{\frac{5}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{2 \sinh (bx+a)}{5 b \text{sech}(bx+a)^{\frac{3}{2}}} - \frac{6 i \sqrt{\frac{\cosh (bx+a)}{2} + \frac{1}{2}} \text{EllipticE} \left(i \sinh \left(\frac{a}{2} + \frac{bx}{2} \right), \sqrt{2} \right) \left(\sqrt{\cosh (bx+a)} \right) \sqrt{\text{sech}(bx+a)}}{5 \cosh \left(\frac{a}{2} + \frac{bx}{2} \right) b}$$

command

```
integrate(1/sech(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh (bx+a)^6 + 6 \cosh (bx+a) \sinh (bx+a)^5 + \sinh (bx+a)^6 + \left(15 \cosh (bx+a)^2 - 11 \right) \sinh (bx+a)^4 - \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\text{sech}(bx+a)^{\frac{5}{2}}}, x \right)$$

93.7 Problem number 15

$$\int (b \operatorname{sech}(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(b \operatorname{sech}(dx + c))^{5/2} \sinh(dx + c)}{5d} \\ & + \frac{6ib^4 \sqrt{\frac{\cosh(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cosh\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cosh(dx + c)} \sqrt{b \operatorname{sech}(dx + c)}} \\ & + \frac{6b^3 \sinh(dx + c) \sqrt{b \operatorname{sech}(dx + c)}}{5d} \end{aligned}$$

command

`integrate((b*sech(d*x+c))^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{2} \left(b^3 \cosh(dx + c)^4 + 4 b^3 \cosh(dx + c) \sinh(dx + c)^3 + b^3 \sinh(dx + c)^4 + 2 b^3 \cosh(dx + c)^2 + b^3 + 2 \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \operatorname{sech}(dx + c)} b^3 \operatorname{sech}(dx + c)^3, x\right)$$

93.8 Problem number 16

$$\int (b \operatorname{sech}(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2b(b \operatorname{sech}(dx + c))^{3/2} \sinh(dx + c)}{3d} \\ & - \frac{2ib^2 \sqrt{\frac{\cosh(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(dx + c)}\right) \sqrt{b \operatorname{sech}(dx + c)}}{3 \cosh\left(\frac{dx}{2} + \frac{c}{2}\right) d} \end{aligned}$$

command

`integrate((b*sech(d*x+c))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} \left(b^2 \cosh(dx+c)^2 + 2b^2 \cosh(dx+c) \sinh(dx+c) + b^2 \sinh(dx+c)^2 + b^2 \right) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \dots) \right)$$

3

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \operatorname{sech}(dx+c)} b^2 \operatorname{sech}(dx+c)^2, x\right)$$

93.9 Problem number 17

$$\int (b \operatorname{sech}(c+dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2ib^2 \sqrt{\frac{\cosh(dx+c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cosh\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cosh(dx+c)} \sqrt{b \operatorname{sech}(dx+c)}} + \frac{2b \sinh(dx+c) \sqrt{b \operatorname{sech}(dx+c)}}{d}$$

command

`integrate((b*sech(d*x+c))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cosh(dx+c) + \sinh(dx+c))) + \sqrt{2} (b \cosh(dx+c) + \dots) \right)$$

d

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \operatorname{sech}(dx+c)} b \operatorname{sech}(dx+c), x\right)$$

93.10 Problem number 18

$$\int \sqrt{b \operatorname{sech}(c + dx)} \, dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(dx + c)}\right) \sqrt{b \operatorname{sech}(dx + c)}}{\cosh\left(\frac{dx}{2} + \frac{c}{2}\right) d}$$

command

```
integrate((b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2}\sqrt{b}\operatorname{weierstrassPInverse}(-4,0,\cosh(dx+c)+\sinh(dx+c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \operatorname{sech}(dx + c)}, x\right)$$

93.11 Problem number 19

$$\int \frac{1}{\sqrt{b \operatorname{sech}(c + dx)}} \, dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\cosh\left(\frac{dx}{2} + \frac{c}{2}\right) d \sqrt{\cosh(dx + c)} \sqrt{b \operatorname{sech}(dx + c)}}$$

command

```
integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\sqrt{2}\sqrt{b}(\cosh(dx+c)+\sinh(dx+c))\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cosh(dx+c)+\sinh(dx+c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \operatorname{sech}(dx + c)}}{b \operatorname{sech}(dx + c)}, x\right)$$

93.12 Problem number 20

$$\int \frac{1}{(b \operatorname{sech}(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2 \sinh(dx + c)}{3bd \sqrt{b \operatorname{sech}(dx + c)}} - \frac{2i \sqrt{\frac{\cosh(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(dx + c)}\right) \sqrt{b \operatorname{sech}(dx + c)}}{3 \cosh\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d}}$$

command

```
integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \sqrt{2} \left(\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 \right) \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cosh(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \operatorname{sech}(dx + c)}}{b^2 \operatorname{sech}(dx + c)^2}, x\right)$$

93.13 Problem number 21

$$\int \frac{1}{(b \operatorname{sech}(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2 \sinh(dx + c)}{5bd (b \operatorname{sech}(dx + c))^{3/2}} - \frac{6i \sqrt{\frac{\cosh(dx + c)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{5 \cosh\left(\frac{dx}{2} + \frac{c}{2}\right) b^2 d \sqrt{\cosh(dx + c)} \sqrt{b \operatorname{sech}(dx + c)}}}$$

command

```
integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$24 \sqrt{2} \left(\cosh(dx+c)^3 + 3 \cosh(dx+c)^2 \sinh(dx+c) + 3 \cosh(dx+c) \sinh(dx+c)^2 + \sinh(dx+c)^3 \right) \sqrt{b} \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \operatorname{sech}(dx+c)}}{b^3 \operatorname{sech}(dx+c)^3}, x \right)$$

93.14 Problem number 22

$$\int \frac{1}{(b \operatorname{sech}(c+dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2 \sinh(dx+c)}{7bd (b \operatorname{sech}(dx+c))^{5/2}} + \frac{10 \sinh(dx+c)}{21b^3d \sqrt{b \operatorname{sech}(dx+c)}}}{21 \cosh\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d} + \frac{10i \sqrt{\frac{\cosh(dx+c)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(i \sinh\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(dx+c)} \sqrt{b \operatorname{sech}(dx+c)}\right)}{21 \cosh\left(\frac{dx}{2} + \frac{c}{2}\right) b^4 d}$$

command

```
integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$80 \sqrt{2} \left(\cosh(dx+c)^4 + 4 \cosh(dx+c)^3 \sinh(dx+c) + 6 \cosh(dx+c)^2 \sinh(dx+c)^2 + 4 \cosh(dx+c) \sinh(dx+c)^3 + \sinh(dx+c)^4 \right) \sqrt{b} \operatorname{weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{\sqrt{b \operatorname{sech}(dx+c)}}{b^4 \operatorname{sech}(dx+c)^4}, x \right)$$

93.15 Problem number 39

$$\int (\operatorname{asech}^3(x))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{154ia^2 \left(\cosh^{\frac{3}{2}}(x)\right) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{\operatorname{asech}(x)^3}}{195 \cosh\left(\frac{x}{2}\right)} \\ & + \frac{154a^2 \cosh(x) \sinh(x) \sqrt{\operatorname{asech}(x)^3}}{195} + \frac{154a^2 \sqrt{\operatorname{asech}(x)^3} \tanh(x)}{585} \\ & + \frac{22a^2 \operatorname{sech}(x)^2 \sqrt{\operatorname{asech}(x)^3} \tanh(x)}{117} + \frac{2a^2 \operatorname{sech}(x)^4 \sqrt{\operatorname{asech}(x)^3} \tanh(x)}{13} \end{aligned}$$

command

```
integrate((a*sech(x)^3)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \operatorname{sech}(x)^3} a^2 \operatorname{sech}(x)^6, x\right)$$

93.16 Problem number 40

$$\int (\operatorname{asech}^3(x))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{10ia \left(\cosh^{\frac{3}{2}}(x)\right) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{\operatorname{asech}(x)^3}}{21 \cosh\left(\frac{x}{2}\right)} \\ & + \frac{10a \sinh(x) \sqrt{\operatorname{asech}(x)^3}}{21} + \frac{2a \operatorname{sech}(x) \sqrt{\operatorname{asech}(x)^3} \tanh(x)}{7} \end{aligned}$$

command

```
integrate((a*sech(x)^3)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \sqrt{2} \left(a \cosh(x)^6 + 6 a \cosh(x) \sinh(x)^5 + a \sinh(x)^6 + 3 a \cosh(x)^4 + 3 \left(5 a \cosh(x)^2 + a \right) \sinh(x)^4 + 4 \left(5 a \right. \right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{a \operatorname{sech}(x)^3} a \operatorname{sech}(x)^3, x\right)$$

93.17 Problem number 41

$$\int \sqrt{a \operatorname{sech}^3(x)} dx$$

Optimal antiderivative

$$\frac{2i \left(\cosh^{\frac{3}{2}}(x) \right) \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a \operatorname{sech}(x)^3}}{\cosh\left(\frac{x}{2}\right)} + 2 \cosh(x) \sinh(x) \sqrt{a \operatorname{sech}(x)^3}$$

command

`integrate((a*sech(x)^3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{2} \sqrt{\frac{a \cosh(x) + a \sinh(x)}{\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1}} (\cosh(x) + \sinh(x)) + 2 \sqrt{2} \sqrt{a} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cosh(x) + \sinh(x)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{a \operatorname{sech}(x)^3}, x\right)$$

$$672 \sqrt{2} \left(\cosh(x)^5 + 5 \cosh(x)^4 \sinh(x) + 10 \cosh(x)^3 \sinh(x)^2 + 10 \cosh(x)^2 \sinh(x)^3 + 5 \cosh(x) \sinh(x)^4 + \sinh(x)^5 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{a \operatorname{sech}(x)^3}}{a^2 \operatorname{sech}(x)^6}, x \right)$$

93.20 Problem number 44

$$\int \frac{1}{(a \operatorname{sech}^3(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{26i \sqrt{\frac{\cosh(x)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{x}{2}\right), \sqrt{2}\right)}{77 \cosh\left(\frac{x}{2}\right) a^2 \cosh(x)^{\frac{3}{2}} \sqrt{a \operatorname{sech}(x)^3}} + \frac{78 \cosh(x) \sinh(x)}{385 a^2 \sqrt{a \operatorname{sech}(x)^3}} \\ & + \frac{26 (\cosh^3(x) \sinh(x))}{165 a^2 \sqrt{a \operatorname{sech}(x)^3}} + \frac{2 (\cosh^5(x) \sinh(x))}{15 a^2 \sqrt{a \operatorname{sech}(x)^3}} + \frac{26 \tanh(x)}{77 a^2 \sqrt{a \operatorname{sech}(x)^3}} \end{aligned}$$

command

```
integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$49920 \sqrt{2} \left(\cosh(x)^8 + 8 \cosh(x)^7 \sinh(x) + 28 \cosh(x)^6 \sinh(x)^2 + 56 \cosh(x)^5 \sinh(x)^3 + 70 \cosh(x)^4 \sinh(x)^4 + 56 \cosh(x)^3 \sinh(x)^5 + 28 \cosh(x)^2 \sinh(x)^6 + 8 \cosh(x) \sinh(x)^7 + \sinh(x)^8 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{a \operatorname{sech}(x)^3}}{a^3 \operatorname{sech}(x)^9}, x \right)$$

93.21 Problem number 129

$$\int \coth^3(c + dx) \sqrt{a + b \operatorname{sech}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a}}\right) \sqrt{a}}{d} - \frac{a \operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a - b}}\right)}{d\sqrt{a - b}} \\ & + \frac{3b \operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a - b}}\right)}{4d\sqrt{a - b}} - \frac{a \operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a + b}}\right)}{d\sqrt{a + b}} \\ & - \frac{3b \operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a + b}}\right)}{4d\sqrt{a + b}} - \frac{(\coth^2(dx + c)) \sqrt{a + b \operatorname{sech}(dx + c)}}{2d} \end{aligned}$$

command

`integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

93.22 Problem number 137

$$\int \frac{\coth^3(c + dx)}{\sqrt{a + b \operatorname{sech}(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b \operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a - b}}\right)}{4(a - b)^{\frac{3}{2}}d} - \frac{b \operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a + b}}\right)}{4(a + b)^{\frac{3}{2}}d} \\ & + \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a}}\right)}{d\sqrt{a}} \\ & - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a - b}}\right)}{d\sqrt{a - b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a + b}}\right)}{d\sqrt{a + b}} \\ & - \frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{4(a + b)d(1 - \operatorname{sech}(dx + c))} - \frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{4(a - b)d(1 + \operatorname{sech}(dx + c))} \end{aligned}$$

command

```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

93.23 Problem number 145

$$\int \frac{\coth(c + dx)}{(a + b \operatorname{sech}(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a}}\right)}{a^{3/2} d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a - b}}\right)}{(a - b)^{3/2} d} \\ & - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}(dx + c)}}{\sqrt{a + b}}\right)}{(a + b)^{3/2} d} + \frac{2b^2}{a(a^2 - b^2)d\sqrt{a + b \operatorname{sech}(dx + c)}} \end{aligned}$$

command

```
integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

93.24 Problem number 146

$$\int \frac{\coth^3(c + dx)}{(a + b \operatorname{sech}(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b \operatorname{sech}(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}} d} - \frac{(2a-3b) \operatorname{arctanh}\left(\frac{\sqrt{a+b \operatorname{sech}(dx+c)}}{\sqrt{a-b}}\right)}{2(a-b)^{\frac{5}{2}} d} \\ & + \frac{b \operatorname{arctanh}\left(\frac{\sqrt{a+b \operatorname{sech}(dx+c)}}{\sqrt{a-b}}\right)}{4(a-b)^{\frac{5}{2}} d} - \frac{b \operatorname{arctanh}\left(\frac{\sqrt{a+b \operatorname{sech}(dx+c)}}{\sqrt{a+b}}\right)}{4(a+b)^{\frac{5}{2}} d} \\ & - \frac{(2a+3b) \operatorname{arctanh}\left(\frac{\sqrt{a+b \operatorname{sech}(dx+c)}}{\sqrt{a+b}}\right)}{2(a+b)^{\frac{5}{2}} d} - \frac{2b^4}{a(a^2-b^2)^2 d \sqrt{a+b \operatorname{sech}(dx+c)}} \\ & - \frac{\sqrt{a+b \operatorname{sech}(dx+c)}}{4(a+b)^2 d (1-\operatorname{sech}(dx+c))} - \frac{\sqrt{a+b \operatorname{sech}(dx+c)}}{4(a-b)^2 d (1+\operatorname{sech}(dx+c))} \end{aligned}$$

command

```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

93.25 Problem number 158

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2 \log(cx))}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^2}{21c^4 \sqrt{\operatorname{sech}(2 \ln(cx))}} + \frac{x^6}{7 \sqrt{\operatorname{sech}(2 \ln(cx))}} \\ & + \frac{\left(c^2 + \frac{1}{x^2}\right) \sqrt{\frac{\cos(4 \operatorname{arccot}(cx))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin(2 \operatorname{arccot}(cx)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}}}{21 \cos(2 \operatorname{arccot}(cx)) c^5 \left(c^4 + \frac{1}{x^4}\right) x \sqrt{\operatorname{sech}(2 \ln(cx))}} \end{aligned}$$

command

```
integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2}\sqrt{c^4}c\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}, -1\right) - \sqrt{2}(3c^8x^8 + 5c^4x^4 + 2)\sqrt{\frac{c^2x^2}{c^4x^4 + 1}}}{42c^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}, x\right)$$

93.26 Problem number 162

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}} dx$$

Optimal antiderivative

$$\frac{\frac{x^2}{3\sqrt{\operatorname{sech}(2\ln(cx))}} + \frac{(c^2 + \frac{1}{x^2})\sqrt{\frac{\cos(4\operatorname{arccot}(cx))}{2}} + \frac{1}{2}}{3\cos(2\operatorname{arccot}(cx))} \operatorname{EllipticF}\left(\sin(2\operatorname{arccot}(cx)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^4 + \frac{1}{x^4}}{(c^2 + \frac{1}{x^2})^2}}}{3\cos(2\operatorname{arccot}(cx))c\left(c^4 + \frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\ln(cx))}}$$

command

`integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2}\sqrt{c^4}c\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}, -1\right) + \sqrt{2}(c^4x^4 + 1)\sqrt{\frac{c^2x^2}{c^4x^4 + 1}}}{6c^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}, x\right)$$

93.27 Problem number 164

$$\int \frac{\sqrt{\operatorname{sech}(2 \log(cx))}}{x} dx$$

Optimal antiderivative

$$\frac{i \sqrt{\left(\frac{cx}{2} + \frac{1}{2cx}\right)^2} \operatorname{EllipticF}\left(i\left(\frac{cx}{2} - \frac{1}{2cx}\right), \sqrt{2}\right) \left(\sqrt{\cosh(2 \ln(cx))}\right) \sqrt{\operatorname{sech}(2 \ln(cx))}}{\frac{cx}{2} + \frac{1}{2cx}}$$

command

```
integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-c^4)^{\frac{3}{4}} \operatorname{ellipticF}\left((-c^4)^{\frac{1}{4}} x, -1\right)}{c^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\operatorname{sech}(2 \log(cx))}}{x}, x\right)$$

93.28 Problem number 168

$$\int \frac{\sqrt{\operatorname{sech}(2 \log(cx))}}{x^5} dx$$

Optimal antiderivative

$$\frac{(c^4 + \frac{1}{x^4}) \sqrt{\operatorname{sech}(2 \ln(cx))}}{3} + \frac{c^3(c^2 + \frac{1}{x^2}) x \sqrt{\frac{\cos(4 \operatorname{arccot}(cx))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin(2 \operatorname{arccot}(cx)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^4 + \frac{1}{x^4}}{(c^2 + \frac{1}{x^2})^2}} \sqrt{\operatorname{sech}(2 \ln(cx))}}{6 \cos(2 \operatorname{arccot}(cx))}$$

command

```
integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (-c^4)^{\frac{3}{4}} c x^4 \operatorname{ellipticF}\left((-c^4)^{\frac{1}{4}} x, -1\right) - \sqrt{2} (c^4 x^4 + 1) \sqrt{\frac{c^2 x^2}{c^4 x^4 + 1}}}{3 x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\operatorname{sech}(2 \log(cx))}}{x^5}, x\right)$$

93.29 Problem number 170

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2 \log(cx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4}{77 c^4 \left(c^4 + \frac{1}{x^4}\right) \operatorname{sech}(2 \ln(cx))^{\frac{3}{2}}} + \frac{6 x^4}{77 \left(c^4 + \frac{1}{x^4}\right) \operatorname{sech}(2 \ln(cx))^{\frac{3}{2}}} + \frac{x^8}{11 \operatorname{sech}(2 \ln(cx))^{\frac{3}{2}}} \\ & + \frac{2 \left(c^2 + \frac{1}{x^2}\right) \sqrt{\frac{\cos(4 \operatorname{arccot}(cx))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin(2 \operatorname{arccot}(cx)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}}}{77 \cos(2 \operatorname{arccot}(cx)) c^5 \left(c^4 + \frac{1}{x^4}\right)^2 x^3 \operatorname{sech}(2 \ln(cx))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \sqrt{2} \sqrt{c^4} c \left(-\frac{1}{c^4}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}, -1\right) - \sqrt{2} (7 c^{12} x^{12} + 20 c^8 x^8 + 17 c^4 x^4 + 4) \sqrt{\frac{c^2 x^2}{c^4 x^4 + 1}}}{308 c^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^7}{\operatorname{sech}(2 \log(cx))^{\frac{3}{2}}}, x\right)$$

93.30 Problem number 174

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2 \log(cx))} dx$$

Optimal antiderivative

$$\frac{2}{7(c^4 + \frac{1}{x^4}) \operatorname{sech}(2 \ln(cx))^{\frac{3}{2}}} + \frac{x^4}{7 \operatorname{sech}(2 \ln(cx))^{\frac{3}{2}}}$$

$$\frac{2(c^2 + \frac{1}{x^2}) \sqrt{\frac{\cos(4 \operatorname{arccot}(cx))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \operatorname{arccot}(cx)), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{c^4 + \frac{1}{x^4}}{(c^2 + \frac{1}{x^2})^2}}}{7 \cos(2 \operatorname{arccot}(cx)) c (c^4 + \frac{1}{x^4})^2 x^3 \operatorname{sech}(2 \ln(cx))^{\frac{3}{2}}}$$

command

```
integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \sqrt{2} \sqrt{c^4} c (-\frac{1}{c^4})^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{(-\frac{1}{c^4})^{\frac{1}{4}}}{x}, -1\right) + \sqrt{2} (c^8 x^8 + 4 c^4 x^4 + 3) \sqrt{\frac{c^2 x^2}{c^4 x^4 + 1}}}{28 c^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^3}{\operatorname{sech}(2 \log(cx))^{\frac{3}{2}}}, x\right)$$

93.31 Problem number 180

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2 \log(cx))}{x^3} dx$$

Optimal antiderivative

$$\frac{(c^4 + \frac{1}{x^4}) x^2 \operatorname{sech}(2 \ln(cx))^{\frac{3}{2}}}{2}$$

$$\frac{(c^4 + \frac{1}{x^4}) (c^2 + \frac{1}{x^2}) x^3 \sqrt{\frac{\cos(4 \operatorname{arccot}(cx))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \operatorname{arccot}(cx)), \frac{\sqrt{2}}{2}\right) \operatorname{sech}(2 \ln(cx))^{\frac{3}{2}} \sqrt{\frac{c^4 + \frac{1}{x^4}}{(c^2 + \frac{1}{x^2})^2}}}{4 \cos(2 \operatorname{arccot}(cx)) c}$$

command

```
integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{\frac{c^2 x^2}{c^4 x^4 + 1}} c^3 - \sqrt{2} (-c^4)^{\frac{3}{4}} \operatorname{ellipticF}\left((-c^4)^{\frac{1}{4}} x, -1\right)}{c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{sech}(2 \log(cx))^{\frac{3}{2}}}{x^3}, x\right)$$

93.32 Problem number 196

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{sech}(a + b \ln(cx^n))^{\frac{3}{2}} \sinh(a + b \ln(cx^n))}{3bn} - \frac{2i \sqrt{\frac{\cosh(a + b \ln(cx^n))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(i \sinh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(a + b \ln(cx^n))}\right) \sqrt{\operatorname{sech}(a + b \ln(cx^n))}}{3 \cosh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} \left(\cosh(bn \log(x) + b \log(c) + a)^2 + 2 \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) + \sinh(bn \log(x) + b \log(c) + a) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{sech}(b \log(cx^n) + a)^{\frac{5}{2}}}{x}, x\right)$$

93.33 Problem number 197

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2 \sinh(a + b \ln(cx^n)) \sqrt{\operatorname{sech}(a + b \ln(cx^n))}}{bn} + \frac{2i \sqrt{\frac{\cosh(a + b \ln(cx^n))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(a + b \ln(cx^n))}\right) \sqrt{\operatorname{sech}(a + b \ln(cx^n))}}{\cosh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} \sqrt{\frac{\cosh(bn \log(x) + b \log(c) + a) + \sinh(bn \log(x) + b \log(c) + a)}{\cosh(bn \log(x) + b \log(c) + a)^2 + 2 \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) + \sinh^2(bn \log(x) + b \log(c) + a)}} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{sech}(b \log(cx^n) + a)^{\frac{3}{2}}}{x}, x\right)$$

93.34 Problem number 198

$$\int \frac{\sqrt{\operatorname{sech}(a + b \log(cx^n))}}{x} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh(a + b \ln(cx^n))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(a + b \ln(cx^n))}\right) \sqrt{\operatorname{sech}(a + b \ln(cx^n))}}{\cosh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2} \operatorname{weierstrassPInverse}(-4, 0, \cosh(bn \log(x) + b \log(c) + a) + \sinh(bn \log(x) + b \log(c) + a))}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\operatorname{sech}(b \log(cx^n) + a)}}{x}, x\right)$$

93.35 Problem number 199

$$\int \frac{1}{x \sqrt{\operatorname{sech}(a + b \log(cx^n))}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cosh(a + b \ln(cx^n))}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(i \sinh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(a + b \ln(cx^n))}\right) \sqrt{\operatorname{sech}(a + b \ln(cx^n))}}{\cosh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

`integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh(bn \log(x) + b \log(c) + a)^2 + 2 \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) + \sinh^2(bn \log(x) + b \log(c) + a) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \sqrt{\operatorname{sech}(b \log(cx^n) + a)}}, x\right)$$

93.36 Problem number 200

$$\int \frac{1}{x \operatorname{sech}^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{2 \sinh(a + b \ln(cx^n))}{3bn \sqrt{\operatorname{sech}(a + b \ln(cx^n))}} + \frac{2i \sqrt{\frac{\cosh(a + b \ln(cx^n))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(i \sinh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right), \sqrt{2}\right) \left(\sqrt{\cosh(a + b \ln(cx^n))}\right) \sqrt{\operatorname{sech}(a + b \ln(cx^n))}}{3 \cosh\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh (bn \log (x) + b \log (c) + a)^4 + 4 \cosh (bn \log (x) + b \log (c) + a)^3 \sinh (bn \log (x) + b \log (c) + a) + 6 \cosh (bn \log (x) + b \log (c) + a)^2 \sinh^2 (bn \log (x) + b \log (c) + a) + 4 \cosh (bn \log (x) + b \log (c) + a) \sinh^3 (bn \log (x) + b \log (c) + a) + \sinh^4 (bn \log (x) + b \log (c) + a) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{x \operatorname{sech} (b \log (c x^n) + a)^{\frac{3}{2}}}, x \right)$$

93.37 Problem number 201

$$\int \frac{1}{x \operatorname{sech}^{\frac{5}{2}} (a + b \log (c x^n))} dx$$

Optimal antiderivative

$$\frac{2 \sinh (a + b \ln (c x^n))}{5 b n \operatorname{sech} (a + b \ln (c x^n))^{\frac{3}{2}}} - \frac{6 i \sqrt{\frac{\cosh (a + b \ln (c x^n))}{2}} + \frac{1}{2} \operatorname{EllipticE} \left(i \sinh \left(\frac{a}{2} + \frac{b \ln (c x^n)}{2} \right), \sqrt{2} \right) \left(\sqrt{\cosh (a + b \ln (c x^n))} \right) \sqrt{\operatorname{sech} (a + b \ln (c x^n))}}{5 \cosh \left(\frac{a}{2} + \frac{b \ln (c x^n)}{2} \right) b n}$$

command

```
integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh (bn \log (x) + b \log (c) + a)^6 + 6 \cosh (bn \log (x) + b \log (c) + a) \sinh (bn \log (x) + b \log (c) + a)^5 + \sinh (bn \log (x) + b \log (c) + a)^6 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{x \operatorname{sech} (b \log (c x^n) + a)^{\frac{5}{2}}}, x \right)$$

94 Test file number 180

Test folder name:

test_cases/6_Hyperbolic_functions/6.5_Hyperbolic_secant/180_6.5.7-d_hyper- \hat{m} -a+b-c_sech- \hat{n} - \hat{p}

94.1 Problem number 47

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a-5b)\operatorname{arctanh}(\cosh(dx+c))}{2(a+b)^4 d} + \frac{(2a-b)b\cosh(dx+c)}{4a(a+b)^2 d(b+a(\cosh^2(dx+c)))^2} \\ & - \frac{(4a^2-9ab-b^2)\cosh(dx+c)}{8a(a+b)^3 d(b+a(\cosh^2(dx+c)))} - \frac{\cosh(dx+c)(\coth^2(dx+c))}{2(a+b)d(b+a(\cosh^2(dx+c)))^2} \\ & - \frac{(15a^2-10ab-b^2)\operatorname{arctan}\left(\frac{\cosh(dx+c)\sqrt{a}}{\sqrt{b}}\right)\sqrt{b}}{8a^{\frac{3}{2}}(a+b)^4 d} \end{aligned}$$

command

```
integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

94.2 Problem number 48

$$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a-2b)\coth(dx+c)}{(a+b)^4 d} - \frac{\coth^3(dx+c)}{3(a+b)^3 d} - \frac{5(3a-4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(dx+c)}{\sqrt{a+b}}\right)\sqrt{b}}{8(a+b)^{\frac{9}{2}} d} \\ & - \frac{ab\tanh(dx+c)}{4(a+b)^3 d(a+b-b(\tanh^2(dx+c)))^2} - \frac{(7a-4b)b\tanh(dx+c)}{8(a+b)^4 d(a+b-b(\tanh^2(dx+c)))} \end{aligned}$$

command

```
integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

94.3 Problem number 168

$$\int \frac{\coth^4(c + dx)}{(a + b\operatorname{sech}^2(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{a^3} - \frac{b^{\frac{5}{2}}(63a^2 + 36ab + 8b^2) \operatorname{arctanh}\left(\frac{\sqrt{b} \tanh(dx+c)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{\frac{9}{2}}d} \\ & - \frac{(8a^3 + 32a^2b - 15ab^2 - 4b^3) \coth(dx+c)}{8a^2(a+b)^4d} - \frac{(8a^2 - 39ab - 12b^2) (\coth^3(dx+c))}{24a^2(a+b)^3d} \\ & - \frac{b(\coth^3(dx+c))}{4a(a+b)d(a+b-b(\tanh^2(dx+c)))^2} - \frac{b(11a+4b)(\coth^3(dx+c))}{8a^2(a+b)^2d(a+b-b(\tanh^2(dx+c)))} \end{aligned}$$

command

```
integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

94.4 Problem number 169

$$\int \frac{1}{(a + b \operatorname{sech}^2(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{a^4} - \frac{(35a^3 + 70a^2b + 56ab^2 + 16b^3) \operatorname{arctanh}\left(\frac{\sqrt{b} \tanh(dx+c)}{\sqrt{a+b}}\right) \sqrt{b}}{16a^4 (a+b)^{\frac{7}{2}} d} \\ & - \frac{b \tanh(dx+c)}{6a(a+b)d(a+b-b(\tanh^2(dx+c)))^3} - \frac{b(11a+6b) \tanh(dx+c)}{24a^2(a+b)^2 d(a+b-b(\tanh^2(dx+c)))^2} \\ & - \frac{b(19a^2+22ab+8b^2) \tanh(dx+c)}{16a^3(a+b)^3 d(a+b-b(\tanh^2(dx+c)))} \end{aligned}$$

command

```
integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

94.5 Problem number 186

$$\int \coth^5(x) \sqrt{a + b \operatorname{sech}^2(x)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(8a^2 + 12ab + 3b^2) \operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{\frac{3}{2}}} + \operatorname{arctanh}\left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}}\right) \sqrt{a} \\ & - \frac{(4a + 3b) (\coth^2(x)) \sqrt{a + b \operatorname{sech}^2(x)}}{8(a+b)} - \frac{(\coth^4(x)) \sqrt{a + b \operatorname{sech}^2(x)}}{4} \end{aligned}$$

command

```
integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

94.6 Problem number 193

$$\int (a + b \operatorname{sech}^2(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2} \operatorname{arctanh} \left(\frac{\sqrt{a} \tanh(dx+c)}{\sqrt{a+b-b(\tanh^2(dx+c))}} \right)}{d} \\ & + \frac{(15a^2 + 10ab + 3b^2) \operatorname{arctan} \left(\frac{\sqrt{b} \tanh(dx+c)}{\sqrt{a+b-b(\tanh^2(dx+c))}} \right) \sqrt{b}}{8d} \\ & + \frac{b(7a+3b) \sqrt{a+b-b(\tanh^2(dx+c))} \tanh(dx+c)}{8d} \\ & + \frac{b \tanh(dx+c) (a+b-b(\tanh^2(dx+c)))^{3/2}}{4d} \end{aligned}$$

command

`integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

94.7 Problem number 211

$$\int \frac{\tanh^6(x)}{(a + b \operatorname{sech}^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{arctan} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b-b(\tanh^2(x))}} \right)}{b^{5/2}} + \frac{\operatorname{arctanh} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b-b(\tanh^2(x))}} \right)}{a^{5/2}} \\ & - \frac{\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \tanh(x)}{\sqrt{a+b-b(\tanh^2(x))}} - \frac{(a+b) (\tanh^3(x))}{3ab (a+b-b(\tanh^2(x)))^{3/2}} \end{aligned}$$

command

```
integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

94.8 Problem number 218

$$\int \frac{\coth(x)}{(a + b\operatorname{sech}^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b\operatorname{sech}(x)^2}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b\operatorname{sech}(x)^2}}{\sqrt{a + b}}\right)}{(a + b)^{5/2}} - \frac{b}{3a(a + b)(a + b\operatorname{sech}(x)^2)^{3/2}} - \frac{b(2a + b)}{a^2(a + b)^2\sqrt{a + b\operatorname{sech}(x)^2}}$$

command

```
integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

94.9 Problem number 220

$$\int \frac{1}{(a + b\operatorname{sech}^2(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a} \tanh(dx+c)}{\sqrt{a+b-b(\tanh^2(dx+c))}}\right)}{a^{\frac{7}{2}}d} - \frac{b(33a^2+40ab+15b^2)\tanh(dx+c)}{15a^3(a+b)^3d\sqrt{a+b-b(\tanh^2(dx+c))}} - \frac{b\tanh(dx+c)}{5a(a+b)d(a+b-b(\tanh^2(dx+c)))^{\frac{5}{2}}} - \frac{b(9a+5b)\tanh(dx+c)}{15a^2(a+b)^2d(a+b-b(\tanh^2(dx+c)))^{\frac{3}{2}}}$$

command

```
integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

95 Test file number 182

Test folder name:

test_cases/6_Hyperbolic_functions/6.6_Hyperbolic_cosecant/182_6.6.2-e_x^-m-a+b_csch-c+d_x^n-^p

95.1 Problem number 83

$$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{csch}(c+dx^n))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(ex)^{3n}}{3a^2en} - \frac{b^2(ex)^{3n}x^{-n}}{a^2(a^2+b^2)den} + \frac{2b^2(ex)^{3n}\ln\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)x^{-2n}}{a^2(a^2+b^2)d^2en} \\
& + \frac{b^3(ex)^{3n}\ln\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)x^{-n}}{a^2(a^2+b^2)^{\frac{3}{2}}den} + \frac{2b^2(ex)^{3n}\ln\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)x^{-2n}}{a^2(a^2+b^2)d^2en} \\
& - \frac{b^3(ex)^{3n}\ln\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)x^{-n}}{a^2(a^2+b^2)^{\frac{3}{2}}den} \\
& + \frac{2b^2(ex)^{3n}\operatorname{polylog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2(a^2+b^2)d^3en} \\
& + \frac{2b^3(ex)^{3n}\operatorname{polylog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)x^{-2n}}{a^2(a^2+b^2)^{\frac{3}{2}}d^2en} \\
& + \frac{2b^2(ex)^{3n}\operatorname{polylog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2(a^2+b^2)d^3en} \\
& - \frac{2b^3(ex)^{3n}\operatorname{polylog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)x^{-2n}}{a^2(a^2+b^2)^{\frac{3}{2}}d^2en} \\
& - \frac{2b^3(ex)^{3n}\operatorname{polylog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2(a^2+b^2)^{\frac{3}{2}}d^3en} \\
& + \frac{2b^3(ex)^{3n}\operatorname{polylog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2(a^2+b^2)^{\frac{3}{2}}d^3en} \\
& - \frac{b^2(ex)^{3n}\cosh(c+dx^n)x^{-n}}{a(a^2+b^2)den(b+a\sinh(c+dx^n))} \\
& - \frac{2b(ex)^{3n}\ln\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)x^{-n}}{a^2den\sqrt{a^2+b^2}} + \frac{2b(ex)^{3n}\ln\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)x^{-n}}{a^2den\sqrt{a^2+b^2}} \\
& - \frac{4b(ex)^{3n}\operatorname{polylog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)x^{-2n}}{a^2d^2en\sqrt{a^2+b^2}} \\
& + \frac{4b(ex)^{3n}\operatorname{polylog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)x^{-2n}}{a^2d^2en\sqrt{a^2+b^2}} \\
& + \frac{4b(ex)^{3n}\operatorname{polylog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2d^3en\sqrt{a^2+b^2}} \\
& - \frac{4b(ex)^{3n}\operatorname{polylog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2d^3en\sqrt{a^2+b^2}}
\end{aligned}$$

command

```
integrate((e*x)^(-1+3*n)/(a+b*csh(c+d*x^n))^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

96 Test file number 183

Test folder name:

test_cases/6_Hyperbolic_functions/6.6_Hyperbolic_cosecant/183_6.6.3_Hyperbolic_cosecant_func

96.1 Problem number 7

$$\int \operatorname{csch}^{\frac{5}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{2 \cosh (bx + a) \operatorname{csch}(bx + a)^{\frac{3}{2}}}{3b} - \frac{2i \sqrt{\frac{1}{2} + \frac{\sin (ibx + ia)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right), \sqrt{2}\right) \sqrt{\operatorname{csch}(bx + a)} \sqrt{i \sinh (bx + a)}}{3 \sin\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right) b}$$

command

```
integrate(csch(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} \left(\cosh (bx + a)^2 + 2 \cosh (bx + a) \sinh (bx + a) + \sinh (bx + a)^2 + 1 \right) \sqrt{\frac{\cosh (bx + a) + \sinh (bx + a)}{\cosh (bx + a)^2 + 2 \cosh (bx + a) \sinh (bx + a) + \sinh (bx + a)^2 + 1}} \right)}{3(b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\operatorname{csch}(bx + a)^{\frac{5}{2}}, x\right)$$

96.2 Problem number 8

$$\int \operatorname{csch}^{\frac{3}{2}}(a + bx) dx$$

Optimal antiderivative

$$\frac{2 \cosh(bx + a) \sqrt{\operatorname{csch}(bx + a)}}{b} + \frac{2i \sqrt{\frac{1}{2} + \frac{\sin(ibx + ia)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right), \sqrt{2}\right)}{\sin\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right) b \sqrt{\operatorname{csch}(bx + a)} \sqrt{i \sinh(bx + a)}}$$

command

`integrate(csch(b*x+a)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} \sqrt{\frac{\cosh(bx + a) + \sinh(bx + a)}{\cosh(bx + a)^2 + 2 \cosh(bx + a) \sinh(bx + a) + \sinh(bx + a)^2 - 1}} (\cosh(bx + a) + \sinh(bx + a)) + \sqrt{2} \right)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\operatorname{csch}(bx + a)^{\frac{3}{2}}, x\right)$$

96.3 Problem number 9

$$\int \sqrt{\operatorname{csch}(a + bx)} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(ibx + ia)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right), \sqrt{2}\right) \sqrt{\operatorname{csch}(bx + a)} \sqrt{i \sinh(bx + a)}}{\sin\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right) b}$$

command

`integrate(csch(b*x+a)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{2} \operatorname{weierstrassPInverse}(4, 0, \cosh(bx + a) + \sinh(bx + a))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{\operatorname{csch}(bx + a)}, x\right)$$

96.4 Problem number 10

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(ibx+ia)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right), \sqrt{2}\right)}{\sin\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right) b \sqrt{\operatorname{csch}(bx+a)} \sqrt{i \sinh(bx+a)}}$$

command

```
integrate(1/csch(b*x+a)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh(bx+a)^2 + 2 \cosh(bx+a) \sinh(bx+a) + \sinh(bx+a)^2 - 1 \right) \sqrt{\frac{\cosh(bx+a) + \sinh(bx+a)}{\cosh(bx+a)^2 + 2 \cosh(bx+a) \sinh(bx+a) + \sinh(bx+a)^2}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{\operatorname{csch}(bx+a)}}, x\right)$$

96.5 Problem number 11

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)} dx$$

Optimal antiderivative

$$\frac{2 \cosh(bx+a)}{3b \sqrt{\operatorname{csch}(bx+a)}} \frac{2i \sqrt{\frac{1}{2} + \frac{\sin(ibx+ia)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right), \sqrt{2}\right) \sqrt{\operatorname{csch}(bx+a)} \sqrt{i \sinh(bx+a)}}{3 \sin\left(\frac{1}{2}ia + \frac{1}{4}\pi + \frac{1}{2}ibx\right) b}$$

command

```
integrate(1/csch(b*x+a)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh (bx + a)^4 + 4 \cosh (bx + a)^3 \sinh (bx + a) + 6 \cosh (bx + a)^2 \sinh (bx + a)^2 + 4 \cosh (bx + a) \sinh (bx + a)^3 + \sinh (bx + a)^4 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\text{csch}(bx + a)^{\frac{5}{2}}}, x \right)$$

96.6 Problem number 12

$$\int \frac{1}{\text{csch}^{\frac{5}{2}}(a + bx)} dx$$

Optimal antiderivative

$$\frac{2 \cosh (bx + a)}{5 b \text{csch}(bx + a)^{\frac{3}{2}}} - \frac{6i \sqrt{\frac{1}{2} + \frac{\sin (ibx + ia)}{2}} \text{EllipticE} \left(\cos \left(\frac{1}{2} ia + \frac{1}{4} \pi + \frac{1}{2} ibx \right), \sqrt{2} \right)}{5 \sin \left(\frac{1}{2} ia + \frac{1}{4} \pi + \frac{1}{2} ibx \right) b \sqrt{\text{csch}(bx + a)} \sqrt{i \sinh (bx + a)}}$$

command

```
integrate(1/csch(b*x+a)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh (bx + a)^6 + 6 \cosh (bx + a) \sinh (bx + a)^5 + \sinh (bx + a)^6 + \left(15 \cosh (bx + a)^2 + 11 \right) \sinh (bx + a)^4 + \sinh (bx + a)^2 \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{\text{csch}(bx + a)^{\frac{5}{2}}}, x \right)$$

96.7 Problem number 13

$$\int (b \operatorname{csch}(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b \cosh(dx + c) (b \operatorname{csch}(dx + c))^{5/2}}{5d} + \frac{6b^3 \cosh(dx + c) \sqrt{b \operatorname{csch}(dx + c)}}{5d} \\ & - \frac{6ib^4 \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right)}{5 \sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d \sqrt{b \operatorname{csch}(dx + c)} \sqrt{i \sinh(dx + c)}} \end{aligned}$$

command

```
integrate((b*csch(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 \sqrt{2} \left(b^3 \cosh(dx + c)^4 + 4b^3 \cosh(dx + c) \sinh(dx + c)^3 + b^3 \sinh(dx + c)^4 - 2b^3 \cosh(dx + c)^2 + b^3 + 2 \left(3 \sqrt{2} \right) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \operatorname{csch}(dx + c)} b^3 \operatorname{csch}(dx + c)^3, x\right)$$

96.8 Problem number 14

$$\int (b \operatorname{csch}(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b \cosh(dx + c) (b \operatorname{csch}(dx + c))^{3/2}}{3d} \\ & - \frac{2ib^2 \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right) \sqrt{b \operatorname{csch}(dx + c)} \sqrt{i \sinh(dx + c)}}{3 \sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d} \end{aligned}$$

command

```
integrate((b*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} \left(b^2 \cosh(dx+c)^2 + 2b^2 \cosh(dx+c) \sinh(dx+c) + b^2 \sinh(dx+c)^2 - b^2 \right) \sqrt{b} \operatorname{weierstrassPInverse}(4, 0, \dots) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \operatorname{csch}(dx+c)} b^2 \operatorname{csch}(dx+c)^2, x\right)$$

96.9 Problem number 15

$$\int (b \operatorname{csch}(c+dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b \cosh(dx+c) \sqrt{b \operatorname{csch}(dx+c)}}{d} \\ & + \frac{2ib^2 \sqrt{\frac{1}{2} + \frac{\sin(idx+ic)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right)}{\sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d \sqrt{b \operatorname{csch}(dx+c)} \sqrt{i \sinh(dx+c)}} \end{aligned}$$

command

```
integrate((b*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} b^{\frac{3}{2}} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cosh(dx+c) + \sinh(dx+c))) + \sqrt{2} (b \cosh(dx+c) + \dots) \right)$$

d

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \operatorname{csch}(dx+c)} b \operatorname{csch}(dx+c), x\right)$$

96.10 Problem number 16

$$\int \sqrt{b \operatorname{csch}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right) \sqrt{b \operatorname{csch}(dx + c)} \sqrt{i \sinh(dx + c)}}{\sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d}$$

command

```
integrate((b*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}(4, 0, \cosh(dx + c) + \sinh(dx + c))}{d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \operatorname{csch}(dx + c)}, x\right)$$

96.11 Problem number 17

$$\int \frac{1}{\sqrt{b \operatorname{csch}(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right)}{\sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) d \sqrt{b \operatorname{csch}(dx + c)} \sqrt{i \sinh(dx + c)}}$$

command

```
integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{2} \sqrt{b} (\cosh(dx + c) + \sinh(dx + c)) \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cosh(dx + c) + \sinh(dx + c)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \operatorname{csch}(dx + c)}}{b \operatorname{csch}(dx + c)}, x\right)$$

96.12 Problem number 18

$$\int \frac{1}{(\operatorname{bsch}(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \cosh(dx + c)}{3bd \sqrt{b \operatorname{csch}(dx + c)}} - \frac{2i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right) \sqrt{b \operatorname{csch}(dx + c)} \sqrt{i \sinh(dx + c)}}{3 \sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) b^2 d}$$

command

```
integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \sqrt{2} \left(\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 \right) \sqrt{b} \operatorname{weierstrassPInverse}(4, 0, \cosh(dx + c))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \operatorname{csch}(dx + c)}}{b^2 \operatorname{csch}(dx + c)^2}, x\right)$$

96.13 Problem number 19

$$\int \frac{1}{(\operatorname{bsch}(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \cosh(dx + c)}{5bd (b \operatorname{csch}(dx + c))^{3/2}} - \frac{6i \sqrt{\frac{1}{2} + \frac{\sin(idx + ic)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right), \sqrt{2}\right)}{5 \sin\left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx\right) b^2 d \sqrt{b \operatorname{csch}(dx + c)} \sqrt{i \sinh(dx + c)}}$$

command

```
integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$24\sqrt{2} \left(\cosh(dx+c)^3 + 3 \cosh(dx+c)^2 \sinh(dx+c) + 3 \cosh(dx+c) \sinh(dx+c)^2 + \sinh(dx+c)^3 \right) \sqrt{b} \text{ weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \operatorname{csch}(dx+c)}}{b^3 \operatorname{csch}(dx+c)^3}, x \right)$$

96.14 Problem number 20

$$\int \frac{1}{(b \operatorname{csch}(c+dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2 \cosh(dx+c)}{7bd (b \operatorname{csch}(dx+c))^{5/2}} - \frac{10 \cosh(dx+c)}{21b^3 d \sqrt{b \operatorname{csch}(dx+c)}} + \frac{10i \sqrt{\frac{1}{2} + \frac{\sin(idx+ic)}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx \right), \sqrt{2} \right) \sqrt{b \operatorname{csch}(dx+c)} \sqrt{i \sinh(dx+c)}}{21 \sin \left(\frac{1}{2}ic + \frac{1}{4}\pi + \frac{1}{2}idx \right) b^4 d}$$

command

```
integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$80\sqrt{2} \left(\cosh(dx+c)^4 + 4 \cosh(dx+c)^3 \sinh(dx+c) + 6 \cosh(dx+c)^2 \sinh(dx+c)^2 + 4 \cosh(dx+c) \sinh(dx+c)^3 + \sinh(dx+c)^4 \right) \sqrt{b} \text{ weier}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{b \operatorname{csch}(dx+c)}}{b^4 \operatorname{csch}(dx+c)^4}, x \right)$$

96.15 Problem number 36

$$\int (\operatorname{acsch}^3(x))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{154a^2 \coth(x) \sqrt{\operatorname{acsch}(x)^3}}{585} + \frac{22a^2 \coth(x) \operatorname{csch}(x)^2 \sqrt{\operatorname{acsch}(x)^3}}{117} \\ & -\frac{2a^2 \coth(x) \operatorname{csch}(x)^4 \sqrt{\operatorname{acsch}(x)^3}}{13} + \frac{154a^2 \cosh(x) \sinh(x) \sqrt{\operatorname{acsch}(x)^3}}{195} \\ & -\frac{154ia^2 \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2}\right) (\sinh^2(x)) \sqrt{\operatorname{acsch}(x)^3}}{195 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) \sqrt{i \sinh(x)}} \end{aligned}$$

command

```
integrate((a*csch(x)^3)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \operatorname{csch}(x)^3} a^2 \operatorname{csch}(x)^6, x\right)$$

96.16 Problem number 37

$$\int (\operatorname{acsch}^3(x))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10a \cosh(x) \sqrt{\operatorname{acsch}(x)^3}}{21} - \frac{2a \coth(x) \operatorname{csch}(x) \sqrt{\operatorname{acsch}(x)^3}}{7} \\ & + \frac{10ia \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2}\right) \sinh(x) \sqrt{\operatorname{acsch}(x)^3} \sqrt{i \sinh(x)}}{21 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right)} \end{aligned}$$

command

```
integrate((a*csch(x)^3)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \sqrt{2} \left(a \cosh(x)^6 + 6 a \cosh(x) \sinh(x)^5 + a \sinh(x)^6 - 3 a \cosh(x)^4 + 3 \left(5 a \cosh(x)^2 - a \right) \sinh(x)^4 + 4 \left(5 a \right. \right. \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{a \operatorname{csch}(x)^3} a \operatorname{csch}(x)^3, x\right)$$

96.17 Problem number 38

$$\int \sqrt{\operatorname{acsch}^3(x)} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2}\right) (i \sinh(x))^{\frac{3}{2}} \sqrt{\operatorname{acsch}(x)^3}}{\sin\left(\frac{\pi}{4} + \frac{ix}{2}\right)} - 2 \cosh(x) \sinh(x) \sqrt{\operatorname{acsch}(x)^3}$$

command

`integrate((a*csch(x)^3)^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-2 \sqrt{2} \sqrt{\frac{a \cosh(x) + a \sinh(x)}{\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1}} (\cosh(x) + \sinh(x)) - 2 \sqrt{2} \sqrt{a} \operatorname{weierstrassZeta}(4, 0, \operatorname{weierstrassPInverse}(4, 0, \cosh(x) + \sinh(x)))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\sqrt{a \operatorname{csch}(x)^3}, x\right)$$

96.18 Problem number 39

$$\int \frac{1}{\sqrt{\operatorname{acsch}^3(x)}} dx$$

Optimal antiderivative

$$\frac{2 \coth(x)}{3 \sqrt{\operatorname{acsch}(x)^3}} - \frac{2i \operatorname{csch}(x)^2 \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2}\right) \sqrt{i \sinh(x)}}{3 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) \sqrt{\operatorname{acsch}(x)^3}}$$

command

```
integrate(1/(a*csh(x)^3)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \sqrt{2} \left(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 \right) \sqrt{a} \operatorname{weierstrassPInverse}(4, 0, \cosh(x) + \sinh(x)) - \sqrt{2} \left(\cosh(x) \right)}{6 \left(a \cosh(x) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{a \operatorname{csch}(x)^3}}{a \operatorname{csch}(x)^3}, x\right)$$

96.19 Problem number 40

$$\int \frac{1}{(\operatorname{acsch}^3(x))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{14 \cosh(x)}{45a \sqrt{\operatorname{acsch}(x)^3}} + \frac{2 \cosh(x) (\sinh^2(x))}{9a \sqrt{\operatorname{acsch}(x)^3}} + \frac{14i \operatorname{csch}(x) \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2}\right)}{15 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) a \sqrt{\operatorname{acsch}(x)^3} \sqrt{i \sinh(x)}}$$

command

`integrate(1/(a*csh(x)^3)^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$672 \sqrt{2} \left(\cosh(x)^5 + 5 \cosh(x)^4 \sinh(x) + 10 \cosh(x)^3 \sinh(x)^2 + 10 \cosh(x)^2 \sinh(x)^3 + 5 \cosh(x) \sinh(x)^4 + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{a \operatorname{csch}(x)^3}}{a^2 \operatorname{csch}(x)^6}, x \right)$$

96.20 Problem number 41

$$\int \frac{1}{(\operatorname{acsch}^3(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{26 \operatorname{coth}(x)}{77a^2 \sqrt{\operatorname{acsch}(x)^3}} + \frac{78 \cosh(x) \sinh(x)}{385a^2 \sqrt{\operatorname{acsch}(x)^3}} - \frac{26 \cosh(x) (\sinh^3(x))}{165a^2 \sqrt{\operatorname{acsch}(x)^3}} + \frac{2 \cosh(x) (\sinh^5(x))}{15a^2 \sqrt{\operatorname{acsch}(x)^3}} \\ & + \frac{26i \operatorname{csch}(x)^2 \sqrt{\frac{1}{2} + \frac{i \sinh(x)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{ix}{2}\right), \sqrt{2}\right) \sqrt{i \sinh(x)}}{77 \sin\left(\frac{\pi}{4} + \frac{ix}{2}\right) a^2 \sqrt{\operatorname{acsch}(x)^3}} \end{aligned}$$

command

`integrate(1/(a*csh(x)^3)^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$49920 \sqrt{2} \left(\cosh(x)^8 + 8 \cosh(x)^7 \sinh(x) + 28 \cosh(x)^6 \sinh(x)^2 + 56 \cosh(x)^5 \sinh(x)^3 + 70 \cosh(x)^4 \sinh(x)^4 + \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{a \operatorname{csch}(x)^3}}{a^3 \operatorname{csch}(x)^9}, x \right)$$

96.21 Problem number 132

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2 \log(cx))}} dx$$

Optimal antiderivative

$$-\frac{2x^2}{21c^4 \sqrt{\operatorname{csch}(2 \ln(cx))}} + \frac{x^6}{7 \sqrt{\operatorname{csch}(2 \ln(cx))}} + \frac{2 \operatorname{EllipticF}\left(\frac{1}{cx}, i\right)}{21c^7 x \sqrt{1 - \frac{1}{c^4 x^4}} \sqrt{\operatorname{csch}(2 \ln(cx))}}$$

command

`integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (3c^{10}x^8 - 5c^6x^4 + 2c^2) \sqrt{\frac{c^2x^2}{c^4x^4 - 1}} + 2\sqrt{2} \sqrt{c^4} \operatorname{ellipticF}\left(\frac{1}{cx}, -1\right)}{42c^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^5}{\sqrt{\operatorname{csch}(2 \log(cx))}}, x\right)$$

96.22 Problem number 136

$$\int \frac{x}{\sqrt{\operatorname{csch}(2 \log(cx))}} dx$$

Optimal antiderivative

$$\frac{x^2}{3 \sqrt{\operatorname{csch}(2 \ln(cx))}} + \frac{2 \operatorname{EllipticF}\left(\frac{1}{cx}, i\right)}{3c^3 x \sqrt{1 - \frac{1}{c^4 x^4}} \sqrt{\operatorname{csch}(2 \ln(cx))}}$$

command

`integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (c^6x^4 - c^2) \sqrt{\frac{c^2x^2}{c^4x^4 - 1}} + 2\sqrt{2} \sqrt{c^4} \operatorname{ellipticF}\left(\frac{1}{cx}, -1\right)}{6c^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x}{\sqrt{\operatorname{csch}(2 \log(cx))}}, x\right)$$

96.23 Problem number 140

$$\int \frac{\sqrt{\operatorname{csch}(2 \log(cx))}}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -c^3 x \operatorname{EllipticE}\left(\frac{1}{cx}, i\right) \sqrt{1 - \frac{1}{c^4 x^4}} \sqrt{\operatorname{csch}(2 \ln(cx))} \\ & + c^3 x \operatorname{EllipticF}\left(\frac{1}{cx}, i\right) \sqrt{1 - \frac{1}{c^4 x^4}} \sqrt{\operatorname{csch}(2 \ln(cx))} \end{aligned}$$

command

```
integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (c^4 x^4 - 1) \sqrt{\frac{c^2 x^2}{c^4 x^4 - 1}}}{x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\operatorname{csch}(2 \log(cx))}}{x^3}, x\right)$$

96.24 Problem number 142

$$\int \frac{\sqrt{\operatorname{csch}(2 \log(cx))}}{x^5} dx$$

Optimal antiderivative

$$\frac{(c^4 - \frac{1}{x^4}) \sqrt{\operatorname{csch}(2 \ln(cx))}}{3} - \frac{c^5 x \operatorname{EllipticF}\left(\frac{1}{cx}, i\right) \sqrt{1 - \frac{1}{c^4 x^4}} \sqrt{\operatorname{csch}(2 \ln(cx))}}{3}$$

command

```
integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (c^4 x^4 - 1) \sqrt{\frac{c^2 x^2}{c^4 x^4 - 1}}}{3 x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{\operatorname{csch}(2 \log(cx))}}{x^5}, x\right)$$

96.25 Problem number 144

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2 \log(cx))} dx$$

Optimal antiderivative

$$\frac{4}{77c^4 \left(c^4 - \frac{1}{x^4}\right) \operatorname{csch}(2 \ln(cx))^{\frac{3}{2}}} - \frac{6x^4}{77 \left(c^4 - \frac{1}{x^4}\right) \operatorname{csch}(2 \ln(cx))^{\frac{3}{2}}} + \frac{x^8}{11 \operatorname{csch}(2 \ln(cx))^{\frac{3}{2}}} - \frac{4 \operatorname{EllipticF}\left(\frac{1}{cx}, i\right)}{77c^{11} \left(1 - \frac{1}{c^4x^4}\right)^{\frac{3}{2}} x^3 \operatorname{csch}(2 \ln(cx))^{\frac{3}{2}}}$$

command

`integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (7c^{14}x^{12} - 20c^{10}x^8 + 17c^6x^4 - 4c^2) \sqrt{\frac{c^2x^2}{c^4x^4 - 1}} - 4\sqrt{2} \sqrt{c^4} \operatorname{ellipticF}\left(\frac{1}{cx}, -1\right)}{308c^{10}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^7}{\operatorname{csch}(2 \log(cx))^{\frac{3}{2}}}, x\right)$$

96.26 Problem number 148

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2 \log(cx))} dx$$

Optimal antiderivative

$$-\frac{2}{7 \left(c^4 - \frac{1}{x^4}\right) \operatorname{csch}(2 \ln(cx))^{\frac{3}{2}}} + \frac{x^4}{7 \operatorname{csch}(2 \ln(cx))^{\frac{3}{2}}} - \frac{4 \operatorname{EllipticF}\left(\frac{1}{cx}, i\right)}{7c^7 \left(1 - \frac{1}{c^4x^4}\right)^{\frac{3}{2}} x^3 \operatorname{csch}(2 \ln(cx))^{\frac{3}{2}}}$$

command

`integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{2} (c^{10}x^8 - 4c^6x^4 + 3c^2) \sqrt{\frac{c^2x^2}{c^4x^4 - 1}} - 4\sqrt{2} \sqrt{c^4} \operatorname{ellipticF}\left(\frac{1}{cx}, -1\right)}{28c^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{x^3}{\operatorname{csch}(2 \log(cx))^{\frac{3}{2}}}, x\right)$$

96.27 Problem number 152

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2 \log(cx))}{x} dx$$

Optimal antiderivative

$$-\cosh(2 \ln(cx)) \sqrt{\operatorname{csch}(2 \ln(cx))} + \frac{i \sqrt{\frac{1}{2} + \frac{i \sinh(2 \ln(cx))}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{\pi}{4} + i \ln(cx)\right), \sqrt{2}\right)}{\sin\left(\frac{\pi}{4} + i \ln(cx)\right) \sqrt{\operatorname{csch}(2 \ln(cx))} \sqrt{i \sinh(2 \ln(cx))}}$$

command

`integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\sqrt{2} \sqrt{\frac{c^2 x^2}{c^4 x^4 - 1}} c^2 x^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{csch}(2 \log(cx))^{\frac{3}{2}}}{x}, x\right)$$

96.28 Problem number 154

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2 \log(cx))}{x^3} dx$$

Optimal antiderivative

$$-\frac{(c^4 - \frac{1}{x^4}) x^2 \operatorname{csch}(2 \ln(cx))^{\frac{3}{2}}}{2} + \frac{c^5 (1 - \frac{1}{c^4 x^4})^{\frac{3}{2}} x^3 \operatorname{csch}(2 \ln(cx))^{\frac{3}{2}} \operatorname{EllipticF}\left(\frac{1}{cx}, i\right)}{2}$$

command

`integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\sqrt{2} \sqrt{\frac{c^2 x^2}{c^4 x^4 - 1}} c^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{csch}(2 \log(cx))^{\frac{3}{2}}}{x^3}, x\right)$$

96.29 Problem number 170

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2 \cosh(a + b \ln(cx^n)) \operatorname{csch}(a + b \ln(cx^n))^{\frac{3}{2}}}{3bn} - \frac{2i \sqrt{\frac{1}{2} + \frac{\sin(ia + ib \ln(cx^n))}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right), \sqrt{2}\right) \sqrt{\operatorname{csch}(a + b \ln(cx^n))} \sqrt{i \sinh(a + b \ln(cx^n))}}{3 \sin\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} \left(\cosh(bn \log(x) + b \log(c) + a)^2 + 2 \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) + \sinh^2(bn \log(x) + b \log(c) + a) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{csch}(b \log(cx^n) + a)^{\frac{5}{2}}}{x}, x\right)$$

96.30 Problem number 171

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx$$

Optimal antiderivative

$$\frac{2 \cosh(a + b \ln(cx^n)) \sqrt{\operatorname{csch}(a + b \ln(cx^n))}}{bn} + \frac{2i \sqrt{\frac{1}{2} + \frac{\sin(ia + ib \ln(cx^n))}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right), \sqrt{2}\right) \sqrt{\operatorname{csch}(a + b \ln(cx^n))} \sqrt{i \sinh(a + b \ln(cx^n))}}{\sin\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right) bn \sqrt{\operatorname{csch}(a + b \ln(cx^n))} \sqrt{i \sinh(a + b \ln(cx^n))}}$$

command

`integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} \sqrt{\frac{\cosh(bn \log(x) + b \log(c) + a) + \sinh(bn \log(x) + b \log(c) + a)}{\cosh(bn \log(x) + b \log(c) + a)^2 + 2 \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) + \sinh^2(bn \log(x) + b \log(c) + a)}} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\text{csch}(b \log(cx^n) + a)^{\frac{3}{2}}}{x}, x \right)$$

96.31 Problem number 172

$$\int \frac{\sqrt{\text{csch}(a + b \log(cx^n))}}{x} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin(ia + ib \ln(cx^n))}{2}} \text{EllipticF} \left(\cos \left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2} \right), \sqrt{2} \right) \sqrt{\text{csch}(a + b \ln(cx^n))} \sqrt{i \sinh(a + b \ln(cx^n))}}{\sin \left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2} \right) bn}$$

command

`integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \sqrt{2} \text{weierstrassPInverse}(4, 0, \cosh(bn \log(x) + b \log(c) + a) + \sinh(bn \log(x) + b \log(c) + a))}{bn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{\text{csch}(b \log(cx^n) + a)}}{x}, x \right)$$

96.32 Problem number 173

$$\int \frac{1}{x \sqrt{\operatorname{csch}(a + b \log(cx^n))}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{1}{2} + \frac{\sin\left(\frac{ia + ib \ln(cx^n)}{2}\right)}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right) bn \sqrt{\operatorname{csch}(a + b \ln(cx^n))} \sqrt{i \sinh(a + b \ln(cx^n))}}$$

command

```
integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh(bn \log(x) + b \log(c) + a)^2 + 2 \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) + \sinh(bn \log(x) + b \log(c) + a) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{x \sqrt{\operatorname{csch}(b \log(cx^n) + a)}}, x\right)$$

96.33 Problem number 174

$$\int \frac{1}{x \operatorname{csch}^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Optimal antiderivative

$$\frac{2 \cosh(a + b \ln(cx^n))}{3bn \sqrt{\operatorname{csch}(a + b \ln(cx^n))}} \frac{2i \sqrt{\frac{1}{2} + \frac{\sin\left(\frac{ia + ib \ln(cx^n)}{2}\right)}{2}} \operatorname{EllipticF}\left(\cos\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right), \sqrt{2}\right) \sqrt{\operatorname{csch}(a + b \ln(cx^n))} \sqrt{i \sinh(a + b \ln(cx^n))}}{3 \sin\left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln(cx^n)}{2}\right) bn}$$

command

```
integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh (bn \log (x) + b \log (c) + a)^4 + 4 \cosh (bn \log (x) + b \log (c) + a)^3 \sinh (bn \log (x) + b \log (c) + a) + 6 \cosh (bn \log (x) + b \log (c) + a)^2 \sinh^2 (bn \log (x) + b \log (c) + a) + \sinh^3 (bn \log (x) + b \log (c) + a) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{x \operatorname{csch} (b \log (cx^n) + a)^{\frac{3}{2}}}, x \right)$$

96.34 Problem number 175

$$\int \frac{1}{x \operatorname{csch}^{\frac{5}{2}} (a + b \log (cx^n))} dx$$

Optimal antiderivative

$$\frac{2 \cosh (a + b \ln (cx^n))}{5bn \operatorname{csch} (a + b \ln (cx^n))^{\frac{3}{2}}} - \frac{6i \sqrt{\frac{1}{2} + \frac{\sin (ia + ib \ln (cx^n))}{2}} \operatorname{EllipticE} \left(\cos \left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln (cx^n)}{2} \right), \sqrt{2} \right)}{5 \sin \left(\frac{ia}{2} + \frac{\pi}{4} + \frac{ib \ln (cx^n)}{2} \right) bn \sqrt{\operatorname{csch} (a + b \ln (cx^n))} \sqrt{i \sinh (a + b \ln (cx^n))}}$$

command

```
integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{2} \left(\cosh (bn \log (x) + b \log (c) + a)^6 + 6 \cosh (bn \log (x) + b \log (c) + a)^5 \sinh (bn \log (x) + b \log (c) + a) + \sinh^2 (bn \log (x) + b \log (c) + a) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{1}{x \operatorname{csch} (b \log (cx^n) + a)^{\frac{5}{2}}}, x \right)$$

97 Test file number 184

Test folder name:

test_cases/6_Hyperbolic_functions/6.6_Hyperbolic_cosecant/184_6.6.7-d_hyper-^m-a+b-c_csch-ⁿ-^p

97.1 Problem number 8

$$\int \frac{1}{(a + b \operatorname{csch}^2(c + dx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{a^4} + \frac{b \coth(dx + c)}{6a(a-b)d(a-b+b(\coth^2(dx+c)))^3} \\ & + \frac{(11a-6b)b \coth(dx+c)}{24a^2(a-b)^2d(a-b+b(\coth^2(dx+c)))^2} + \frac{b(19a^2-22ab+8b^2) \coth(dx+c)}{16a^3(a-b)^3d(a-b+b(\coth^2(dx+c)))} \\ & - \frac{(35a^3-70a^2b+56ab^2-16b^3) \arctan\left(\frac{\sqrt{a-b} \tanh(dx+c)}{\sqrt{b}}\right) \sqrt{b}}{16a^4(a-b)^{\frac{7}{2}}d} \end{aligned}$$

command

```
integrate(1/(a+b*csch(d*x+c)^2)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

97.2 Problem number 9

$$\int (a + b \operatorname{csch}^2(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\coth(dx+c)\sqrt{a}}{\sqrt{a-b+b(\coth^2(dx+c))}}\right)}{d} - \frac{b \coth(dx+c) (a-b+b(\coth^2(dx+c)))^{\frac{3}{2}}}{4d} \\ & - \frac{(15a^2-10ab+3b^2) \operatorname{arctanh}\left(\frac{\coth(dx+c)\sqrt{b}}{\sqrt{a-b+b(\coth^2(dx+c))}}\right) \sqrt{b}}{8d} \\ & - \frac{(7a-3b)b \coth(dx+c) \sqrt{a-b+b(\coth^2(dx+c))}}{8d} \end{aligned}$$

command

```
integrate((a+b*csch(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

97.3 Problem number 15

$$\int \frac{1}{(a + b \operatorname{csch}^2(c + dx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\coth(dx+c)\sqrt{a}}{\sqrt{a-b+b(\coth^2(dx+c))}}\right)}{a^{7/2}d} + \frac{b \coth(dx+c)}{5a(a-b)d(a-b+b(\coth^2(dx+c)))^{5/2}}$$

$$+ \frac{(9a-5b)b \coth(dx+c)}{15a^2(a-b)^2 d(a-b+b(\coth^2(dx+c)))^{3/2}} + \frac{b(33a^2-40ab+15b^2) \coth(dx+c)}{15a^3(a-b)^3 d \sqrt{a-b+b(\coth^2(dx+c))}}$$

command

```
integrate(1/(a+b*csch(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

98 Test file number 185

Test folder name:

test_cases/6_Hyperbolic_functions/6.7_Miscellaneous/185_6.7.1_Hyperbolic_functions

98.1 Problem number 590

$$\int \sqrt{a \cosh(x) + b \sinh(x)} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cos(ix - \arctan(a, -ib))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2}\right), \sqrt{2}\right) \sqrt{a \cosh(x) + b \sinh(x)}}{\cos\left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2}\right) \sqrt{\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}}}}$$

command

```
integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left. -2 \sqrt{2} \sqrt{a+b} \operatorname{weierstrassZeta}\left(-\frac{4(a-b)}{a+b}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4(a-b)}{a+b}, 0, \cosh(x) + \sinh(x)\right)\right) \right) - 2 \sqrt{a \cosh(x) + b \sinh(x)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{a \cosh(x) + b \sinh(x)}, x\right)$$

98.2 Problem number 591

$$\int (a \cosh(x) + b \sinh(x))^{3/2} dx$$

Optimal antiderivative

$$\frac{2(b \cosh(x) + a \sinh(x)) \sqrt{a \cosh(x) + b \sinh(x)}}{3} + \frac{2i(a^2 - b^2) \sqrt{\frac{\cos(ix - \arctan(a, -ib))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2}\right), \sqrt{2}\right) \sqrt{\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}}}}{3 \cos\left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2}\right) \sqrt{a \cosh(x) + b \sinh(x)}}$$

command

```
integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{2} (a - b) \cosh(x) + \sqrt{2} (a - b) \sinh(x) \right) \sqrt{a + b} \operatorname{weierstrassPInverse} \left(-\frac{4(a-b)}{a+b}, 0, \cosh(x) + \sinh(x) \right) + \left((a - b) \cosh(x) + (a - b) \sinh(x) \right)}{3 (\cosh(x) + \sinh(x))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left((a \cosh(x) + b \sinh(x))^{\frac{3}{2}}, x \right)$$

98.3 Problem number 592

$$\int (a \cosh(x) + b \sinh(x))^{5/2} dx$$

Optimal antiderivative

$$\frac{2(b \cosh(x) + a \sinh(x)) (a \cosh(x) + b \sinh(x))^{\frac{3}{2}}}{5} - \frac{6i(a^2 - b^2) \sqrt{\frac{\cos(ix - \arctan(a, -ib))}{2}} + \frac{1}{2} \operatorname{EllipticE} \left(\sin \left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2} \right), \sqrt{2} \right) \sqrt{a \cosh(x) + b \sinh(x)}}{5 \cos \left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2} \right) \sqrt{\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}}}}$$

command

`integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{12 \left(\sqrt{2} (a^2 - b^2) \cosh(x)^2 + 2 \sqrt{2} (a^2 - b^2) \cosh(x) \sinh(x) + \sqrt{2} (a^2 - b^2) \sinh(x)^2 \right) \sqrt{a + b} \operatorname{weierstrassZeta} \left(\cosh(x) + \sinh(x) \right)}{3 (\cosh(x) + \sinh(x))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\left(a^2 \cosh(x)^2 + 2ab \cosh(x) \sinh(x) + b^2 \sinh(x)^2 \right) \sqrt{a \cosh(x) + b \sinh(x)}, x \right)$$

98.4 Problem number 593

$$\int \frac{1}{\sqrt{a \cosh(x) + b \sinh(x)}} dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cos(ix - \arctan(a, -ib))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2}\right), \sqrt{2}\right) \sqrt{\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}}}}{\cos\left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2}\right) \sqrt{a \cosh(x) + b \sinh(x)}}$$

command

`integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2} \operatorname{weierstrassPInverse}\left(-\frac{4(a-b)}{a+b}, 0, \cosh(x) + \sinh(x)\right)}{\sqrt{a+b}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{a \cosh(x) + b \sinh(x)}}, x\right)$$

98.5 Problem number 594

$$\int \frac{1}{(a \cosh(x) + b \sinh(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2b \cosh(x) + 2a \sinh(x)}{(a^2 - b^2) \sqrt{a \cosh(x) + b \sinh(x)}} + 2i \sqrt{\frac{\cos(ix - \arctan(a, -ib))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2}\right), \sqrt{2}\right) \sqrt{a \cosh(x) + b \sinh(x)}}{\cos\left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2}\right) (a^2 - b^2) \sqrt{\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}}}}$$

command

`integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} (a+b) \cosh(x)^2 + 2 \sqrt{2} (a+b) \cosh(x) \sinh(x) + \sqrt{2} (a+b) \sinh(x)^2 + \sqrt{2} (a-b) \right) \sqrt{a+b} \text{weierstrass} \right) \\ \frac{\quad}{a^3 - a^2b - ab^2 + b^3 + (a^3 + a^2b - ab^2)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{a \cosh(x) + b \sinh(x)}}{a^2 \cosh(x)^2 + 2ab \cosh(x) \sinh(x) + b^2 \sinh(x)^2}, x \right)$$

98.6 Problem number 595

$$\int \frac{1}{(a \cosh(x) + b \sinh(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2b \cosh(x)}{3} + \frac{2a \sinh(x)}{3}}{(a^2 - b^2) (a \cosh(x) + b \sinh(x))^{3/2}} \\ \frac{2i \sqrt{\frac{\cos(ix - \arctan(a, -ib))}{2}} + \frac{1}{2} \text{EllipticF} \left(\sin \left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2} \right), \sqrt{2} \right) \sqrt{\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}}}}{3 \cos \left(\frac{ix}{2} - \frac{\arctan(a, -ib)}{2} \right) (a^2 - b^2) \sqrt{a \cosh(x) + b \sinh(x)}}$$

command

`integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\left(\sqrt{2} (a^2 + 2ab + b^2) \cosh(x)^4 + 4 \sqrt{2} (a^2 + 2ab + b^2) \cosh(x) \sinh(x)^3 + \sqrt{2} (a^2 + 2ab + b^2) \sinh(x)^4 + 2 \sqrt{2} (a-b) \right) \sqrt{a+b} \text{weierstrass} \right) \\ \frac{\quad}{3 (a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5 + (a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5))}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{a \cosh(x) + b \sinh(x)}}{a^3 \cosh(x)^3 + 3a^2b \cosh(x)^2 \sinh(x) + 3ab^2 \cosh(x) \sinh(x)^2 + b^3 \sinh(x)^3}, x \right)$$

98.7 Problem number 745

$$\int \frac{1}{(a + b \cosh(x) + c \sinh(x))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a(2a^2 + 3b^2 - 3c^2) \operatorname{arctanh}\left(\frac{c - (a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 + c^2}}\right)}{(a^2 - b^2 + c^2)^{\frac{7}{2}}} \\ & + \frac{-c \cosh(x) - b \sinh(x)}{3(a^2 - b^2 + c^2)(a + b \cosh(x) + c \sinh(x))^3} \\ & - \frac{5(ac \cosh(x) + ab \sinh(x))}{6(a^2 - b^2 + c^2)^2(a + b \cosh(x) + c \sinh(x))^2} \\ & + \frac{-c(11a^2 + 4b^2 - 4c^2) \cosh(x) - b(11a^2 + 4b^2 - 4c^2) \sinh(x)}{6(a^2 - b^2 + c^2)^3(a + b \cosh(x) + c \sinh(x))} \end{aligned}$$

command

`integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

98.8 Problem number 761

$$\int (a + b \cosh(x) + c \sinh(x))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(c \cosh(x) + b \sinh(x))(a + b \cosh(x) + c \sinh(x))^{\frac{3}{2}}}{5} \\ & + \frac{16(ac \cosh(x) + ab \sinh(x)) \sqrt{a + b \cosh(x) + c \sinh(x)}}{15} \\ & - \frac{2i(23a^2 + 9b^2 - 9c^2) \sqrt{\frac{\cos\left(\frac{ix - \arctan(b, -ic)}{2}\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}}\right)}{15 \cos\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right) \sqrt{\frac{a + b \cosh(x) + c \sinh(x)}{a + \sqrt{b^2 - c^2}}}} \\ & + \frac{16ia(a^2 - b^2 + c^2) \sqrt{\frac{\cos\left(\frac{ix - \arctan(b, -ic)}{2}\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}}\right)}{15 \cos\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right) \sqrt{a + b \cosh(x) + c \sinh(x)}} \end{aligned}$$

command

```
integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \left(\sqrt{2} (a^3 - 33 ab^2 + 33 ac^2) \cosh(x)^2 + 2 \sqrt{2} (a^3 - 33 ab^2 + 33 ac^2) \cosh(x) \sinh(x) + \sqrt{2} (a^3 - 33 ab^2 + 33 ac^2) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(b^2 \cosh(x)^2 + c^2 \sinh(x)^2 + 2 ab \cosh(x) + a^2 + 2(bc \cosh(x) + ac) \sinh(x)\right) \sqrt{b \cosh(x) + c \sinh(x)} + \dots\right)$$

98.9 Problem number 762

$$\int (a + b \cosh(x) + c \sinh(x))^{3/2} dx$$

Optimal antiderivative

$$\frac{2(c \cosh(x) + b \sinh(x)) \sqrt{a + b \cosh(x) + c \sinh(x)}}{3} + \frac{8ia \sqrt{\frac{\cos(ix - \arctan(b, -ic))}{2}} + \frac{1}{2} \text{EllipticE}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}}\right) \sqrt{a + b \cosh(x) + c \sinh(x)}}{3 \cos\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right) \sqrt{\frac{a + b \cosh(x) + c \sinh(x)}{a + \sqrt{b^2 - c^2}}}} + \frac{2i(a^2 - b^2 + c^2) \sqrt{\frac{\cos(ix - \arctan(b, -ic))}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}}\right) \sqrt{a + b \cosh(x) + c \sinh(x)}}{3 \cos\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right) \sqrt{a + b \cosh(x) + c \sinh(x)}}$$

command

```
integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} (a^2 + 3b^2 - 3c^2) \cosh(x) + \sqrt{2} (a^2 + 3b^2 - 3c^2) \sinh(x) \right) \sqrt{b + c} \text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2 + 3c^2)}{3(b^2 + 2bc + c^2)}, -\dots\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left((b \cosh(x) + c \sinh(x) + a)^{\frac{3}{2}}, x\right)$$

98.10 Problem number 763

$$\int \sqrt{a + b \cosh(x) + c \sinh(x)} \, dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cos\left(\frac{ix - \arctan(b, -ic)}{2}\right) + \frac{1}{2}}{\cos\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right)}} \operatorname{EllipticE}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}}\right) \sqrt{a + b \cosh(x)}}{\sqrt{\frac{a + b \cosh(x) + c \sinh(x)}{a + \sqrt{b^2 - c^2}}}}$$

command

```
integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{2} a \sqrt{b + c} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2 + 3c^2)}{3(b^2 + 2bc + c^2)}, -\frac{8(8a^3 - 9ab^2 + 9ac^2)}{27(b^3 + 3b^2c + 3bc^2 + c^3)}, \frac{3(b+c)\cosh(x) + 3(b+c)\sinh(x) + 2a}{3(b+c)}\right) - 3\sqrt{2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{b \cosh(x) + c \sinh(x) + a}, x\right)$$

98.11 Problem number 764

$$\int \frac{1}{\sqrt{a + b \cosh(x) + c \sinh(x)}} \, dx$$

Optimal antiderivative

$$\frac{2i \sqrt{\frac{\cos\left(\frac{ix - \arctan(b, -ic)}{2}\right) + \frac{1}{2}}{\cos\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right)}} \operatorname{EllipticF}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}}\right) \sqrt{\frac{a + b \cosh(x)}{a + \sqrt{b^2 - c^2}}}}{\sqrt{a + b \cosh(x) + c \sinh(x)}}$$

command

```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\sqrt{2} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2+3c^2)}{3(b^2+2bc+c^2)}, -\frac{8(8a^3-9ab^2+9ac^2)}{27(b^3+3b^2c+3bc^2+c^3)}, \frac{3(b+c)\cosh(x)+3(b+c)\sinh(x)+2a}{3(b+c)}\right)}{\sqrt{b+c}}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{\sqrt{b\cosh(x)+c\sinh(x)+a}}, x\right)$$

98.12 Problem number 765

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2(c\cosh(x)+b\sinh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}}{2i\sqrt{\frac{\cos(ix-\arctan(b,-ic))}{2}}+\frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(\frac{ix}{2}-\frac{\arctan(b,-ic)}{2}\right), \sqrt{2}\sqrt{\frac{\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}}\right) \sqrt{a+b\cosh(x)}$$

$$\frac{\cos\left(\frac{ix}{2}-\frac{\arctan(b,-ic)}{2}\right)(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{\cos\left(\frac{ix}{2}-\frac{\arctan(b,-ic)}{2}\right)(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

command

```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(\left(2\sqrt{2}a^2\cosh(x)+\sqrt{2}(ab+ac)\cosh(x)^2+\sqrt{2}(ab+ac)\sinh(x)^2+2\left(\sqrt{2}a^2+\sqrt{2}(ab+ac)\cosh(x)\right)\sinh(x)\right)\sinh(x)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b\cosh(x)+c\sinh(x)+a}}{b^2\cosh(x)^2+c^2\sinh(x)^2+2ab\cosh(x)+a^2+2(bc\cosh(x)+ac)\sinh(x)}, x\right)$$

98.13 Problem number 766

$$\int \frac{1}{(a + b \cosh(x) + c \sinh(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(c \cosh(x) + b \sinh(x))}{3(a^2 - b^2 + c^2)(a + b \cosh(x) + c \sinh(x))^{3/2}} - \frac{8(ac \cosh(x) + ab \sinh(x))}{3(a^2 - b^2 + c^2)^2 \sqrt{a + b \cosh(x) + c \sinh(x)}} \\ & + \frac{8ia \sqrt{\frac{\cos(ix - \arctan(b, -ic))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}}\right) \sqrt{a + b \cosh(x)}}{3 \cos\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right) (a^2 - b^2 + c^2)^2 \sqrt{\frac{a + b \cosh(x) + c \sinh(x)}{a + \sqrt{b^2 - c^2}}}} \\ & + \frac{2i \sqrt{\frac{\cos(ix - \arctan(b, -ic))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}}\right) \sqrt{\frac{a + b \cosh(x)}{a + \sqrt{b^2 - c^2}}}}{3 \cos\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right) (a^2 - b^2 + c^2) \sqrt{a + b \cosh(x) + c \sinh(x)}} \end{aligned}$$

command

```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{b \cosh(x) + c \sinh(x) + a}}{b^3 \cosh(x)^3 + c^3 \sinh(x)^3 + 3ab^2 \cosh(x)^2 + 3a^2b \cosh(x) + a^3 + 3(bc^2 \cosh(x) + ac^2) \sinh(x)^2 + 3(b^2c \cosh(x) + abc \sinh(x))}\right)$$

98.14 Problem number 767

$$\int \frac{1}{(a + b \cosh(x) + c \sinh(x))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(c \cosh(x) + b \sinh(x))}{5(a^2 - b^2 + c^2)(a + b \cosh(x) + c \sinh(x))^{\frac{5}{2}}} - \frac{16(ac \cosh(x) + ab \sinh(x))}{15(a^2 - b^2 + c^2)^2(a + b \cosh(x) + c \sinh(x))^{\frac{3}{2}}}$$

$$- \frac{2(c(23a^2 + 9b^2 - 9c^2) \cosh(x) + b(23a^2 + 9b^2 - 9c^2) \sinh(x))}{15(a^2 - b^2 + c^2)^3 \sqrt{a + b \cosh(x) + c \sinh(x)}}$$

$$- \frac{2i(23a^2 + 9b^2 - 9c^2) \sqrt{\frac{\cos(ix - \arctan(b, -ic))}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}}\right)}{15 \cos\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right) (a^2 - b^2 + c^2)^3 \sqrt{\frac{a + b \cosh(x) + c \sinh(x)}{a + \sqrt{b^2 - c^2}}}}$$

$$+ \frac{16ia \sqrt{\frac{\cos(ix - \arctan(b, -ic))}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}}\right) \sqrt{\frac{a + b \cosh(x)}{a + \sqrt{b^2 - c^2}}}}{15 \cos\left(\frac{ix}{2} - \frac{\arctan(b, -ic)}{2}\right) (a^2 - b^2 + c^2)^2 \sqrt{a + b \cosh(x) + c \sinh(x)}}$$

command

```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{1}{b^4 \cosh(x)^4 + c^4 \sinh(x)^4 + 4ab^3 \cosh(x)^3 + 6a^2b^2 \cosh(x)^2 + 4a^3b \cosh(x) + a^4 + 4(bc^3 \cosh(x) + ac^3 \sinh(x))}\right)$$

98.15 Problem number 864

$$\int \frac{1}{\sqrt{a + b \cosh(c + dx) \sinh(c + dx)}} dx$$

Optimal antiderivative

$$\frac{i \sqrt{\frac{1}{2} + \frac{\sin(2idx + 2ic)}{2}} \operatorname{EllipticF}\left(\cos\left(ic + \frac{1}{4}\pi + idx\right), \sqrt{2} \sqrt{\frac{b}{2ia + b}}\right) \sqrt{2} \sqrt{\frac{2a + b \sinh(2dx + 2c)}{-ib + 2a}}}{\sin\left(ic + \frac{1}{4}\pi + idx\right) d \sqrt{2a + b \sinh(2dx + 2c)}}$$

command

```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{-b} b \sqrt{\frac{4a^2 + b^2}{b^2}} - 2a\sqrt{-b} \right) \sqrt{\frac{b \sqrt{\frac{4a^2 + b^2}{b^2}} + 2a}{b}} \operatorname{ellipticF} \left(\sqrt{\frac{b \sqrt{\frac{4a^2 + b^2}{b^2}} + 2a}{b}} (\cosh(dx + c) + \sinh(dx + c)), \frac{\sqrt{2}}{2} \right) \sqrt{b^2 d}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(\frac{1}{\sqrt{b \cosh(dx + c) \sinh(dx + c) + a}}, x \right)$$

99 Test file number 188

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.1_Inverse_hyperbolic_sine/188_7.1.5_Inverse_hyperbolic_sine

99.1 Problem number 228

$$\int (ce + dex)^{7/2} (a + b \sinh^{-1}(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(e(dx + c))^{\frac{9}{2}} (a + b \operatorname{arcsinh}(dx + c))}{9de} + \frac{28b e^2 (e(dx + c))^{\frac{3}{2}} \sqrt{1 + (dx + c)^2}}{405d} \\ & - \frac{4b(e(dx + c))^{\frac{7}{2}} \sqrt{1 + (dx + c)^2}}{81d} - \frac{28b e^3 \sqrt{e(dx + c)} \sqrt{1 + (dx + c)^2}}{135d(dx + c + 1)} \\ & + \frac{28b e^{\frac{7}{2}} (dx + c + 1) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{135 \cos\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right) d \sqrt{1 + (dx + c)^2}} \\ & - \frac{14b e^{\frac{7}{2}} (dx + c + 1) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{135 \cos\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right) d \sqrt{1 + (dx + c)^2}} \end{aligned}$$

command

`integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(45 \left((bd^5x^4 + 4bcd^4x^3 + 6bc^2d^3x^2 + 4bc^3d^2x + bc^4d) \cosh(1)^3 + 3(bd^5x^4 + 4bcd^4x^3 + 6bc^2d^3x^2 + 4bc^3d^2x + bc^4d) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(ad^3e^3x^3 + 3acd^2e^3x^2 + 3ac^2de^3x + ac^3e^3 + (bd^3e^3x^3 + 3bcd^2e^3x^2 + 3bc^2de^3x + bc^3e^3)\right) \text{arsinh}(dx + c)\right)$$

99.2 Problem number 229

$$\int (ce + dex)^{5/2} (a + b \sinh^{-1}(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(e(dx+c))^{\frac{7}{2}}(a+b\operatorname{arsinh}(dx+c))}{7de} - \frac{4b(e(dx+c))^{\frac{5}{2}}\sqrt{1+(dx+c)^2}}{49d} \\ & + \frac{20be^2\sqrt{e(dx+c)}\sqrt{1+(dx+c)^2}}{147d} \\ & - \frac{10be^{\frac{5}{2}}(dx+c+1)\sqrt{\frac{\cos\left(4\arctan\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)}{147\cos\left(2\arctan\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}\right)\right)d\sqrt{1+(dx+c)^2}} \end{aligned}$$

command

`integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(21 \left((bd^5x^3 + 3bcd^4x^2 + 3bc^2d^3x + bc^3d^2) \cosh(1)^2 + 2(bd^5x^3 + 3bcd^4x^2 + 3bc^2d^3x + bc^3d^2) \cosh(1) \sinh(1) \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(ad^2e^2x^2 + 2acde^2x + ac^2e^2 + (bd^2e^2x^2 + 2bcde^2x + bc^2e^2)\right) \text{arsinh}(dx + c)\sqrt{dex + ce}, x\right)$$

99.3 Problem number 230

$$\int (ce + dex)^{3/2} (a + b \sinh^{-1}(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(e(dx+c))^{\frac{5}{2}}(a+b \operatorname{arcsinh}(dx+c))}{5de} \\ & - \frac{4b(e(dx+c))^{\frac{3}{2}}\sqrt{1+(dx+c)^2}}{25d} + \frac{12be\sqrt{e(dx+c)}\sqrt{1+(dx+c)^2}}{25d(dx+c+1)} \\ & - \frac{12be^{\frac{3}{2}}(dx+c+1)\sqrt{\frac{\cos\left(4\arctan\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}\right)\right),\frac{\sqrt{2}}{2}\right)}{25\cos\left(2\arctan\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}\right)\right)d\sqrt{1+(dx+c)^2}} \\ & + \frac{6be^{\frac{3}{2}}(dx+c+1)\sqrt{\frac{\cos\left(4\arctan\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}\right)\right)}{2}} + \frac{1}{2}\operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}\right)\right),\frac{\sqrt{2}}{2}\right)}{25\cos\left(2\arctan\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}\right)\right)d\sqrt{1+(dx+c)^2}} \end{aligned}$$

command

```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\left(5\left((bd^3x^2+2bcd^2x+bc^2d)\cosh(1)+(bd^3x^2+2bcd^2x+bc^2d)\sinh(1)\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((adex+ace+(bdex+bce)\operatorname{arsinh}(dx+c))\sqrt{dex+ce},x\right)$$

99.4 Problem number 231

$$\int \sqrt{ce + dex} (a + b \sinh^{-1}(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(e(dx + c))^{\frac{3}{2}} (a + b \operatorname{arcsinh}(dx + c))}{3de} - \frac{4b\sqrt{e(dx + c)} \sqrt{1 + (dx + c)^2}}{9d}$$

$$+ \frac{2b(dx + c + 1) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{e}}{9 \cos\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right) d \sqrt{1 + (dx + c)^2}}$$

command

```
integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 (bd^3x + bcd^2) \sqrt{(dx + c) \cosh(1) + (dx + c) \sinh(1)} \log(dx + c + \sqrt{d^2x^2 + 2cdx + c^2 + 1}) + 2 \sqrt{d^3} \cosh(1) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{dex + ce} (b \operatorname{arsinh}(dx + c) + a), x\right)$$

99.5 Problem number 232

$$\int \frac{a + b \sinh^{-1}(c + dx)}{\sqrt{ce + dex}} dx$$

Optimal antiderivative

$$\frac{2(a + b \operatorname{arcsinh}(dx + c)) \sqrt{e(dx + c)}}{de} - \frac{4b\sqrt{e(dx + c)} \sqrt{1 + (dx + c)^2}}{de(dx + c + 1)}$$

$$+ \frac{4b(dx + c + 1) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{1}{d}}}{\cos\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right) d \sqrt{e} \sqrt{1 + (dx + c)^2}}$$

$$- \frac{2b(dx + c + 1) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right)}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{1}{d}}}{\cos\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right) d \sqrt{e} \sqrt{1 + (dx + c)^2}}$$

command

```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{(dx+c) \cosh(1) + (dx+c) \sinh(1)} \, bd \log \left(dx+c + \sqrt{d^2x^2 + 2cdx + c^2 + 1} \right) + \sqrt{(dx+c) \cosh(1) + (dx+c) \sinh(1)} \right)}{d^2 \cosh(1) + d^2 \sinh(1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{b \operatorname{arsinh}(dx+c) + a}{\sqrt{dex+ce}}, x \right)$$

99.6 Problem number 233

$$\int \frac{a + b \sinh^{-1}(c + dx)}{(ce + dex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(a + b \operatorname{arcsinh}(dx+c))}{de \sqrt{e(dx+c)}} + \frac{2b(dx+c+1) \sqrt{\frac{\cos \left(4 \arctan \left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}} \right) \right), \frac{\sqrt{2}}{2} \right) \sqrt{\frac{1}{(d^2x+cd^2) \cosh(1) + (d^2x+cd^2) \sinh(1)}}}{\cos \left(2 \arctan \left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}} \right) \right) d e^{\frac{3}{2}} \sqrt{1+(dx+c)^2}}$$

command

```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{(dx+c) \cosh(1) + (dx+c) \sinh(1)} \, bd^2 \log \left(dx+c + \sqrt{d^2x^2 + 2cdx + c^2 + 1} \right) + \sqrt{(dx+c) \cosh(1) + (dx+c) \sinh(1)} \right)}{(d^4x + cd^3) \cosh(1)^2 + 2(d^4x + cd^3) \cosh(1) \sinh(1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{dex+ce} (b \operatorname{arsinh}(dx+c) + a)}{d^2e^2x^2 + 2cde^2x + c^2e^2}, x \right)$$

99.7 Problem number 234

$$\int \frac{a + b \sinh^{-1}(c + dx)}{(ce + dex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(a + b \operatorname{arcsinh}(dx + c))}{3de(e(dx + c))^{\frac{3}{2}}} - \frac{4b\sqrt{1 + (dx + c)^2}}{3de^2\sqrt{e(dx + c)}} + \frac{4b\sqrt{e(dx + c)}\sqrt{1 + (dx + c)^2}}{3de^3(dx + c + 1)} \\ & - \frac{4b(dx + c + 1) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{1}{(dx + c)}}}{3 \cos\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right) de^{\frac{5}{2}} \sqrt{1 + (dx + c)^2}} \\ & + \frac{2b(dx + c + 1) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{1}{(dx + c)}}}{3 \cos\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right) de^{\frac{5}{2}} \sqrt{1 + (dx + c)^2}} \end{aligned}$$

command

```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\left(\sqrt{(dx + c) \cosh(1) + (dx + c) \sinh(1)} bd \log\left(dx + c + \sqrt{d^2x^2 + 2cdx + c^2 + 1}\right) + 2(bd^2x^2 + 2bcdx + bc^2)\right)}{3\left((d^4x^2 + 2cd^3x + c^2d^2) \cosh(1)^3 + 3(d^4x^2 + 2cd^3x + c^2d^2) \sinh(1)^3\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dex + ce}(b \operatorname{arsinh}(dx + c) + a)}{d^3e^3x^3 + 3cd^2e^3x^2 + 3c^2de^3x + c^3e^3}, x\right)$$

99.8 Problem number 235

$$\int \frac{a + b \sinh^{-1}(c + dx)}{(ce + dex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2(a + b \operatorname{arcsinh}(dx + c))}{5de (e(dx + c))^{5/2}} - \frac{4b\sqrt{1 + (dx + c)^2}}{15de^2 (e(dx + c))^{3/2}} + \frac{2b(dx + c + 1) \sqrt{\frac{\cos\left(4 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{1}{(dx + c)^2}}}{15 \cos\left(2 \arctan\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}\right)\right) de^{7/2} \sqrt{1 + (dx + c)^2}}$$

command

`integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \sqrt{(dx + c) \cosh(1) + (dx + c) \sinh(1)} bd^2 \log\left(dx + c + \sqrt{d^2x^2 + 2cdx + c^2 + 1}\right) + 2 (bd^3x^3 + 3bcd^2x^2 + \dots) \right)}{15 \left((d^6x^3 + 3cd^5x^2 + 3c^2d^4x + c^3d^3) \cosh(1)^4 + 4(d^6x^3 + 3cd^5x^2 + 3c^2d^4x + c^3d^3) \cosh(1)^3 \sinh(1) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dex + ce} (b \operatorname{arsinh}(dx + c) + a)}{d^4e^4x^4 + 4cd^3e^4x^3 + 6c^2d^2e^4x^2 + 4c^3de^4x + c^4e^4}, x\right)$$

99.9 Problem number 284

$$\int x^2 \sinh^{-1}(ax^2) dx$$

Optimal antiderivative

$$\frac{x^3 \operatorname{arcsinh}(ax^2)}{3} - \frac{2x\sqrt{a^2x^4 + 1}}{9a} + \frac{(ax^2 + 1) \sqrt{\frac{\cos\left(4 \arctan(x\sqrt{a})\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan(x\sqrt{a})\right), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^2x^4 + 1}{(ax^2 + 1)^2}}}{9 \cos\left(2 \arctan(x\sqrt{a})\right) a^{3/2} \sqrt{a^2x^4 + 1}}$$

command

`integrate(x^2*arcsinh(a*x^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3ax^3 \log(ax^2 + \sqrt{a^2x^4 + 1}) + 2a\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} \text{ellipticF}\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}, -1\right) - 2\sqrt{a^2x^4 + 1}x}{9a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^2 \text{arsinh}(ax^2), x)$$

99.10 Problem number 288

$$\int \frac{\sinh^{-1}(ax^2)}{x^2} dx$$

Optimal antiderivative

$$\frac{\frac{\text{arcsinh}(ax^2)}{x} + (ax^2 + 1) \sqrt{\frac{\cos(4 \arctan(x\sqrt{a}))}{2}} + \frac{1}{2} \text{EllipticF}\left(\sin(2 \arctan(x\sqrt{a})), \frac{\sqrt{2}}{2}\right) \sqrt{a} \sqrt{\frac{a^2x^4 + 1}{(ax^2 + 1)^2}}}{\cos(2 \arctan(x\sqrt{a})) \sqrt{a^2x^4 + 1}}$$

command

`integrate(arcsinh(a*x^2)/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2a^2x\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} \text{ellipticF}\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}, -1\right) + (x-1) \log(ax^2 + \sqrt{a^2x^4 + 1}) + x \log(ax^2 - \sqrt{a^2x^4 + 1})}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{arsinh}(ax^2)}{x^2}, x\right)$$

100 Test file number 190

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.2_Inverse_hyperbolic_cosine/190_7.2.4-f_x-^m-d+e_x²-^p-a+b_arccosh-c_x-ⁿ

100.1 Problem number 141

$$\int \frac{\cosh^{-1}(ax)}{x^2 \sqrt{1 - a^2 x^2}} dx$$

Optimal antiderivative

$$-\frac{a \ln(x) \sqrt{ax - 1}}{\sqrt{-ax + 1}} - \frac{\operatorname{arccosh}(ax) \sqrt{-a^2 x^2 + 1}}{x}$$

command

```
integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{ax \arctan\left(\frac{\sqrt{a^2 x^2 - 1} \sqrt{-a^2 x^2 + 1} (x^2 + 1)}{a^2 x^4 - (a^2 + 1)x^2 + 1}\right) - \sqrt{-a^2 x^2 + 1} \log(ax + \sqrt{a^2 x^2 - 1})}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

101 Test file number 191

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.2_Inverse_hyperbolic_cosine/191_7.2.5_Inverse_hyp

101.1 Problem number 198

$$\int (ce + dex)^{7/2} (a + b \cosh^{-1}(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(e(dx+c))^{\frac{9}{2}}(a+b \operatorname{arccosh}(dx+c))}{9de} - \frac{28be^3 \operatorname{EllipticE}\left(\frac{\sqrt{dx+c+1}\sqrt{2}}{2}, \sqrt{2}\right) \sqrt{-dx-c+1} \sqrt{e(dx+c)}}{135d\sqrt{-dx-c} \sqrt{dx+c-1}} - \frac{28be^2(e(dx+c))^{\frac{3}{2}} \sqrt{dx+c-1} \sqrt{dx+c+1}}{405d} - \frac{4b(e(dx+c))^{\frac{7}{2}} \sqrt{dx+c-1} \sqrt{dx+c+1}}{81d}$$

command

`integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(45 \left(bd^5 x^4 + 4bcd^4 x^3 + 6bc^2 d^3 x^2 + 4bc^3 d^2 x + bc^4 d \right) \cosh(1)^3 + 3 \left(bd^5 x^4 + 4bcd^4 x^3 + 6bc^2 d^3 x^2 + 4bc^3 d^2 x + bc^4 d \right) \sinh(1) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(ad^3 e^3 x^3 + 3acd^2 e^3 x^2 + 3ac^2 de^3 x + ac^3 e^3 + (bd^3 e^3 x^3 + 3bcd^2 e^3 x^2 + 3bc^2 de^3 x + bc^3 e^3) \operatorname{arccosh}(dx+c)\right)\right)$$

101.2 Problem number 199

$$\int (ce+dex)^{5/2} (a+b \cosh^{-1}(c+dx)) dx$$

Optimal antiderivative

$$\frac{2(e(dx+c))^{\frac{7}{2}}(a+b \operatorname{arccosh}(dx+c))}{7de} - \frac{20be^{\frac{5}{2}} \operatorname{EllipticF}\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}, i\right) \sqrt{-dx-c+1}}{147d\sqrt{dx+c-1}} - \frac{4b(e(dx+c))^{\frac{5}{2}} \sqrt{dx+c-1} \sqrt{dx+c+1}}{49d} - \frac{20be^2 \sqrt{dx+c-1} \sqrt{e(dx+c)} \sqrt{dx+c+1}}{147d}$$

command

`integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(21 \left(bd^5 x^3 + 3bcd^4 x^2 + 3bc^2 d^3 x + bc^3 d^2 \right) \cosh(1)^2 + 2 \left(bd^5 x^3 + 3bcd^4 x^2 + 3bc^2 d^3 x + bc^3 d^2 \right) \cosh(1) \sinh(1) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\left(ad^2 e^2 x^2 + 2acde^2 x + ac^2 e^2 + (bd^2 e^2 x^2 + 2bcde^2 x + bc^2 e^2) \operatorname{arccosh}(dx+c)\right) \sqrt{dex+ce}, x\right)$$

101.3 Problem number 200

$$\int (ce + dex)^{3/2} (a + b \cosh^{-1}(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(e(dx + c))^{5/2} (a + b \operatorname{arccosh}(dx + c))}{5de} - \frac{12be \operatorname{EllipticE}\left(\frac{\sqrt{dx + c + 1} \sqrt{2}}{2}, \sqrt{2}\right) \sqrt{-dx - c + 1} \sqrt{e(dx + c)}}{25d\sqrt{-dx - c} \sqrt{dx + c - 1}} - \frac{4b(e(dx + c))^{3/2} \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{25d}$$

command

```
integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(5 \left((bd^3x^2 + 2bcd^2x + bc^2d) \cosh(1) + (bd^3x^2 + 2bcd^2x + bc^2d) \sinh(1) \right) \sqrt{(dx + c) \cosh(1) + (dx + c) \sinh(1)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left((adex + ace + (bdex + bce) \operatorname{arccosh}(dx + c))\sqrt{dex + ce}, x\right)$$

101.4 Problem number 201

$$\int \sqrt{ce + dex} (a + b \cosh^{-1}(c + dx)) dx$$

Optimal antiderivative

$$\frac{2(e(dx + c))^{3/2} (a + b \operatorname{arccosh}(dx + c))}{3de} - \frac{4b \operatorname{EllipticF}\left(\frac{\sqrt{e(dx + c)}}{\sqrt{e}}, i\right) \sqrt{e} \sqrt{-dx - c + 1}}{9d\sqrt{dx + c - 1}} - \frac{4b\sqrt{dx + c - 1} \sqrt{e(dx + c)} \sqrt{dx + c + 1}}{9d}$$

command

```
integrate((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(3 (bd^3x + bcd^2) \sqrt{(dx + c) \cosh(1) + (dx + c) \sinh(1)} \log \left(dx + c + \sqrt{d^2x^2 + 2cdx + c^2 - 1} \right) - 2 \sqrt{d^3} \cosh(1) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\sqrt{dex + ce} (b \operatorname{arccosh}(dx + c) + a), x \right)$$

101.5 Problem number 202

$$\int \frac{a + b \cosh^{-1}(c + dx)}{\sqrt{ce + dex}} dx$$

Optimal antiderivative

$$\frac{2(a + b \operatorname{arccosh}(dx + c)) \sqrt{e(dx + c)}}{de} - \frac{4b \operatorname{EllipticE} \left(\frac{\sqrt{dx + c + 1} \sqrt{2}}{2}, \sqrt{2} \right) \sqrt{-dx - c + 1} \sqrt{e(dx + c)}}{de \sqrt{-dx - c} \sqrt{dx + c - 1}}$$

command

```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \left(\sqrt{(dx + c) \cosh(1) + (dx + c) \sinh(1)} bd \log \left(dx + c + \sqrt{d^2x^2 + 2cdx + c^2 - 1} \right) + \sqrt{(dx + c) \cosh(1) + (dx + c) \sinh(1)} \right) / (d^2 \cosh(1) + d^2 \sinh(1))$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{b \operatorname{arccosh}(dx + c) + a}{\sqrt{dex + ce}}, x \right)$$

101.6 Problem number 203

$$\int \frac{a + b \cosh^{-1}(c + dx)}{(ce + dex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{4b \operatorname{EllipticF}\left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}, i\right) \sqrt{-dx-c+1}}{de^{\frac{3}{2}} \sqrt{dx+c-1}} - \frac{2(a + b \operatorname{arccosh}(dx+c))}{de \sqrt{e(dx+c)}}$$

command

```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{(dx+c) \cosh(1) + (dx+c) \sinh(1)} b d^2 \log \left(dx+c + \sqrt{d^2 x^2 + 2cdx + c^2 - 1} \right) + \sqrt{(dx+c) \cosh(1) + (dx+c) \sinh(1)} \right)}{(d^4 x + cd^3) \cosh(1)^2 + 2(d^4 x + cd^3) \cosh(1) \sinh(1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{dex+ce} (b \operatorname{arccosh}(dx+c) + a)}{d^2 e^2 x^2 + 2cde^2 x + c^2 e^2}, x\right)$$

101.7 Problem number 204

$$\int \frac{a + b \cosh^{-1}(c + dx)}{(ce + dex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(a + b \operatorname{arccosh}(dx+c))}{3de (e(dx+c))^{\frac{3}{2}}} \\ & - \frac{4b \operatorname{EllipticE}\left(\frac{\sqrt{dx+c+1} \sqrt{2}}{2}, \sqrt{2}\right) \sqrt{-dx-c+1} \sqrt{e(dx+c)}}{3de^3 \sqrt{-dx-c} \sqrt{dx+c-1}} \\ & + \frac{4b \sqrt{dx+c-1} \sqrt{dx+c+1}}{3de^2 \sqrt{e(dx+c)}} \end{aligned}$$

command

```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\sqrt{(dx+c) \cosh(1) + (dx+c) \sinh(1)} \, bd \log \left(dx+c + \sqrt{d^2x^2 + 2cdx + c^2 - 1} \right) - 2 (bd^2x^2 + 2bcdx + bc^2) \right)}{3 \left((d^4x^2 + 2cd^3x + c^2d^2) \cosh(1)^3 + 3(d^4x^2 + 2cd^3x + c^2d^2) \sinh(1)^3 \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{dex+ce} (b \operatorname{arccosh}(dx+c) + a)}{d^3e^3x^3 + 3cd^2e^3x^2 + 3c^2de^3x + c^3e^3}, x \right)$$

101.8 Problem number 205

$$\int \frac{a + b \cosh^{-1}(c + dx)}{(ce + dex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(a + b \operatorname{arccosh}(dx+c))}{5de(e(dx+c))^{\frac{5}{2}}} + \frac{4b \operatorname{EllipticF} \left(\frac{\sqrt{e(dx+c)}}{\sqrt{e}}, i \right) \sqrt{-dx-c+1}}{15de^{\frac{7}{2}} \sqrt{dx+c-1}} \\ & + \frac{4b \sqrt{dx+c-1} \sqrt{dx+c+1}}{15de^2(e(dx+c))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(3 \sqrt{(dx+c) \cosh(1) + (dx+c) \sinh(1)} \, bd^2 \log \left(dx+c + \sqrt{d^2x^2 + 2cdx + c^2 - 1} \right) - 2 (bd^3x^3 + 3bcd^2x^2 + 3cd^2x + c^2d) \right)}{15 \left((d^6x^3 + 3cd^5x^2 + 3c^2d^4x + c^3d^3) \cosh(1)^4 + 4(d^6x^3 + 3cd^5x^2 + 3c^2d^4x + c^3d^3) \cosh(1)^3 \sinh(1) \right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\sqrt{dex+ce} (b \operatorname{arccosh}(dx+c) + a)}{d^4e^4x^4 + 4cd^3e^4x^3 + 6c^2d^2e^4x^2 + 4c^3de^4x + c^4e^4}, x \right)$$

102 Test file number 193

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.3_Inverse_hyperbolic_tangent/193_7.3.3-d+e_x^-m-a+b_arctanh-c_x^n-^p

102.1 Problem number 35

$$\int \frac{a + b \tanh^{-1}(cx^3)}{(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{-a - b \operatorname{arctanh}(cx^3)}{e(ex + d)} + \frac{bc^{\frac{1}{3}}(c^{\frac{1}{3}}d - e) \ln(1 - c^{\frac{1}{3}}x)}{2cd^3 + 2e^3} + \frac{bc^{\frac{1}{3}}(c^{\frac{1}{3}}d + e) \ln(1 + c^{\frac{1}{3}}x)}{2cd^3 - 2e^3} \\ & - \frac{3bcd^2e^2 \ln(ex + d)}{c^2d^6 - e^6} - \frac{bc^{\frac{1}{3}}(c^{\frac{1}{3}}d + e) \ln(1 - c^{\frac{1}{3}}x + c^{\frac{2}{3}}x^2)}{4(cd^3 - e^3)} \\ & - \frac{bc^{\frac{1}{3}}(c^{\frac{1}{3}}d - e) \ln(1 + c^{\frac{1}{3}}x + c^{\frac{2}{3}}x^2)}{4(cd^3 + e^3)} - \frac{bcd^2 \ln(-cx^3 + 1)}{2e(cd^3 + e^3)} + \frac{bcd^2 \ln(cx^3 + 1)}{2e(cd^3 - e^3)} \\ & - \frac{bc^{\frac{1}{3}} \operatorname{arctan}\left(\frac{(1 - 2c^{\frac{1}{3}}x)\sqrt{3}}{3}\right) \sqrt{3}}{2(c^{\frac{2}{3}}d^2 + c^{\frac{1}{3}}de + e^2)} - \frac{bc^{\frac{1}{3}}(c^{\frac{1}{3}}d + e) \operatorname{arctan}\left(\frac{(1 + 2c^{\frac{1}{3}}x)\sqrt{3}}{3}\right) \sqrt{3}}{2(cd^3 + e^3)} \end{aligned}$$

command

```
integrate((a+b*arctanh(c*x^3))/(e*x+d)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

103 Test file number 194

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.3_Inverse_hyperbolic_tangent/194_7.3.4_u-a+b_arctanh-c_x-^p

103.1 Problem number 119

$$\int \frac{\tanh^{-1}(ax)^2}{cx - acx^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(ax)^2 \ln\left(2 - \frac{2}{-ax+1}\right)}{c} + \frac{\operatorname{arctanh}(ax) \operatorname{polylog}\left(2, -1 + \frac{2}{-ax+1}\right)}{c} - \frac{\operatorname{polylog}\left(3, -1 + \frac{2}{-ax+1}\right)}{2c}$$

command

```
integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\log\left(\frac{2ax}{ax-1}\right) \log\left(-\frac{ax+1}{ax-1}\right)^2 + 2 \operatorname{Li}_2\left(-\frac{2ax}{ax-1} + 1\right) \log\left(-\frac{ax+1}{ax-1}\right) - 2 \operatorname{polylog}\left(3, -\frac{ax+1}{ax-1}\right)}{4c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\operatorname{arctanh}(ax)^2}{acx^2 - cx}, x\right)$$

103.2 Problem number 137

$$\int \frac{\tanh^{-1}(ax)^4}{x(c - acx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}(ax)^4 \ln\left(2 - \frac{2}{-ax+1}\right)}{c} + \frac{2 \operatorname{arctanh}(ax)^3 \operatorname{polylog}\left(2, -1 + \frac{2}{-ax+1}\right)}{c} \\ & - \frac{3 \operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(3, -1 + \frac{2}{-ax+1}\right)}{c} \\ & + \frac{3 \operatorname{arctanh}(ax) \operatorname{polylog}\left(4, -1 + \frac{2}{-ax+1}\right)}{c} - \frac{3 \operatorname{polylog}\left(5, -1 + \frac{2}{-ax+1}\right)}{2c} \end{aligned}$$

command

```
integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\log\left(\frac{2ax}{ax-1}\right)\log\left(-\frac{ax+1}{ax-1}\right)^4 + 4\operatorname{Li}_2\left(-\frac{2ax}{ax-1} + 1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3 - 12\log\left(-\frac{ax+1}{ax-1}\right)^2\operatorname{polylog}\left(3, -\frac{ax+1}{ax-1}\right) + 24\log\left(-\frac{ax+1}{ax-1}\right)}{16c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\operatorname{artanh}(ax)^4}{acx^2 - cx}, x\right)$$

103.3 Problem number 138

$$\int \frac{\tanh^{-1}(ax)^4}{cx - acx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}(ax)^4 \ln\left(2 - \frac{2}{-ax+1}\right)}{c} + \frac{2\operatorname{arctanh}(ax)^3 \operatorname{polylog}\left(2, -1 + \frac{2}{-ax+1}\right)}{c} \\ & - \frac{3\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(3, -1 + \frac{2}{-ax+1}\right)}{c} \\ & + \frac{3\operatorname{arctanh}(ax) \operatorname{polylog}\left(4, -1 + \frac{2}{-ax+1}\right)}{c} - \frac{3\operatorname{polylog}\left(5, -1 + \frac{2}{-ax+1}\right)}{2c} \end{aligned}$$

command

```
integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\log\left(\frac{2ax}{ax-1}\right)\log\left(-\frac{ax+1}{ax-1}\right)^4 + 4\operatorname{Li}_2\left(-\frac{2ax}{ax-1} + 1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3 - 12\log\left(-\frac{ax+1}{ax-1}\right)^2\operatorname{polylog}\left(3, -\frac{ax+1}{ax-1}\right) + 24\log\left(-\frac{ax+1}{ax-1}\right)}{16c}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(-\frac{\operatorname{artanh}(ax)^4}{acx^2 - cx}, x\right)$$

104 Test file number 197

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.3_Inverse_hyperbolic_tangent/197_7.3.7_Inverse_hy

104.1 Problem number 16

$$\int x^{9/2} \tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d + ex^2}} \right) dx$$

Optimal antiderivative

$$\frac{2x^{\frac{11}{2}} \operatorname{arctanh} \left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}} \right)}{11} + \frac{36dx^{\frac{5}{2}} \sqrt{ex^2+d}}{847e^{\frac{3}{2}}} - \frac{4x^{\frac{9}{2}} \sqrt{ex^2+d}}{121\sqrt{e}} - \frac{60d^2 \sqrt{x} \sqrt{ex^2+d}}{847e^{\frac{5}{2}}}$$

$$+ \frac{30d^{\frac{11}{4}} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{ex^2}{(\sqrt{d} + x\sqrt{e})^2}}}{847 \cos \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right) e^{\frac{11}{4}} \sqrt{ex^2+d}}$$

command

```
integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$60 d^3 \operatorname{weierstrassPInverse} \left(-\frac{4d}{\cosh(\frac{1}{2})^2 + 2 \cosh(\frac{1}{2}) \sinh(\frac{1}{2}) + \sinh(\frac{1}{2})^2}, 0, x \right) + 77 \left(x^5 \cosh \left(\frac{1}{2} \right)^6 + 6 x^5 \cosh \left(\frac{1}{2} \right)^5 \sinh \left(\frac{1}{2} \right) + \right.$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(x^{\frac{9}{2}} \operatorname{artanh} \left(\frac{\sqrt{e} x}{\sqrt{ex^2+d}} \right), x \right)$$

104.2 Problem number 17

$$\int x^{5/2} \tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d + ex^2}} \right) dx$$

Optimal antiderivative

$$\frac{2x^{7/2} \operatorname{arctanh} \left(\frac{x\sqrt{e}}{\sqrt{ex^2 + d}} \right)}{7} - \frac{4x^{5/2} \sqrt{ex^2 + d}}{49\sqrt{e}} + \frac{20d\sqrt{x} \sqrt{ex^2 + d}}{147e^{3/2}}$$

$$10d^{7/4} \sqrt{\frac{\cos \left(4 \operatorname{arctan} \left(\frac{e^{1/4} \sqrt{x}}{d^{1/4}} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(2 \operatorname{arctan} \left(\frac{e^{1/4} \sqrt{x}}{d^{1/4}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{ex^2}{(\sqrt{d} + x\sqrt{e})^2}}$$

$$147 \cos \left(2 \operatorname{arctan} \left(\frac{e^{1/4} \sqrt{x}}{d^{1/4}} \right) \right) e^{7/4} \sqrt{ex^2 + d}$$

command

```
integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$20 d^2 \operatorname{weierstrassPInverse} \left(-\frac{4d}{\cosh(\frac{1}{2})^2 + 2 \cosh(\frac{1}{2}) \sinh(\frac{1}{2}) + \sinh(\frac{1}{2})^2}, 0, x \right) - 21 \left(x^3 \cosh \left(\frac{1}{2} \right)^4 + 4x^3 \cosh \left(\frac{1}{2} \right)^3 \sinh \left(\frac{1}{2} \right) - \dots \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(x^{5/2} \operatorname{artanh} \left(\frac{\sqrt{e} x}{\sqrt{ex^2 + d}} \right), x \right)$$

104.3 Problem number 18

$$\int \sqrt{x} \tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d + ex^2}} \right) dx$$

Optimal antiderivative

$$\frac{2x^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{3} - \frac{4\sqrt{x}\sqrt{ex^2+d}}{9\sqrt{e}}$$

$$+ \frac{2d^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \operatorname{arctan}\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arctan}\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{ex^2+d}{(\sqrt{d} + x\sqrt{e})^2}}}{9 \cos\left(2 \operatorname{arctan}\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{3}{4}} \sqrt{ex^2+d}}$$

command

`integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 \left(x \cosh\left(\frac{1}{2}\right)^2 + 2x \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right) + x \sinh\left(\frac{1}{2}\right)^2 \right) \sqrt{x} \log \left(\frac{2x^2 \cosh\left(\frac{1}{2}\right)^2 + 4x^2 \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right) + 2x^2 \sinh\left(\frac{1}{2}\right)^2 + 2(x \cosh\left(\frac{1}{2}\right) + x \sinh\left(\frac{1}{2}\right)) \sqrt{d}}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\sqrt{x} \operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right), x\right)$$

104.4 Problem number 19

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}$$

$$+ \frac{2e^{\frac{1}{4}} \sqrt{\frac{\cos\left(4 \operatorname{arctan}\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arctan}\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{ex^2+d}{(\sqrt{d} + x\sqrt{e})^2}}}{\cos\left(2 \operatorname{arctan}\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) d^{\frac{1}{4}} \sqrt{ex^2+d}}$$

command

```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 x \text{weierstrassPInverse} \left(-\frac{4d}{\cosh(\frac{1}{2})^2 + 2 \cosh(\frac{1}{2}) \sinh(\frac{1}{2}) + \sinh(\frac{1}{2})^2}, 0, x \right) - \sqrt{x} \log \left(\frac{2x^2 \cosh(\frac{1}{2})^2 + 4x^2 \cosh(\frac{1}{2}) \sinh(\frac{1}{2}) + 2x^2 \sinh(\frac{1}{2})^2}{x} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\text{artanh} \left(\frac{\sqrt{e} x}{\sqrt{e x^2 + d}} \right)}{x^{\frac{3}{2}}}, x \right)$$

104.5 Problem number 20

$$\int \frac{\tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d + e x^2}} \right)}{x^{7/2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh} \left(\frac{x \sqrt{e}}{\sqrt{e x^2 + d}} \right)}{5 x^{\frac{5}{2}}} - \frac{4 \sqrt{e} \sqrt{e x^2 + d}}{15 d x^{\frac{3}{2}}} + \frac{2 e^{\frac{5}{4}} \sqrt{\frac{\cos \left(4 \operatorname{arctan} \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2}}{\operatorname{EllipticF} \left(\sin \left(2 \operatorname{arctan} \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{d} + x \sqrt{e}) \sqrt{\frac{e x^2 + d}{(\sqrt{d} + x \sqrt{e})}}} + \frac{15 \cos \left(2 \operatorname{arctan} \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right) d^{\frac{5}{4}} \sqrt{e x^2 + d}}{15 d x^{\frac{3}{2}}}$$

command

```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$3 d \sqrt{x} \log \left(\frac{2x^2 \cosh(\frac{1}{2})^2 + 4x^2 \cosh(\frac{1}{2}) \sinh(\frac{1}{2}) + 2x^2 \sinh(\frac{1}{2})^2 + 2(x \cosh(\frac{1}{2}) + x \sinh(\frac{1}{2}))}{d} \sqrt{\frac{(x^2 + d) \cosh(\frac{1}{2}) + (x^2 - d) \sinh(\frac{1}{2})}{\cosh(\frac{1}{2}) - \sinh(\frac{1}{2})}} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\operatorname{artanh} \left(\frac{\sqrt{e} x}{\sqrt{e x^2 + d}} \right)}{x^{\frac{7}{2}}}, x \right)$$

104.6 Problem number 21

$$\int \frac{\tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d + e x^2}} \right)}{x^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \operatorname{arctanh} \left(\frac{x \sqrt{e}}{\sqrt{e x^2 + d}} \right)}{9 x^{\frac{9}{2}}} + \frac{20 e^{\frac{3}{2}} \sqrt{e x^2 + d}}{189 d^2 x^{\frac{3}{2}}} - \frac{4 \sqrt{e} \sqrt{e x^2 + d}}{63 d x^{\frac{7}{2}}} \\ & + \frac{10 e^{\frac{9}{4}} \sqrt{\frac{\cos \left(4 \operatorname{arctan} \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(2 \operatorname{arctan} \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{d} + x \sqrt{e}) \sqrt{\frac{e x^2 + d}{(\sqrt{d} + x \sqrt{e})}}}{189 \cos \left(2 \operatorname{arctan} \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right) d^{\frac{9}{4}} \sqrt{e x^2 + d}} \end{aligned}$$

command

`integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$21 d^2 \sqrt{x} \log \left(\frac{2 x^2 \cosh \left(\frac{1}{2} \right)^2 + 4 x^2 \cosh \left(\frac{1}{2} \right) \sinh \left(\frac{1}{2} \right) + 2 x^2 \sinh \left(\frac{1}{2} \right)^2 + 2 (x \cosh \left(\frac{1}{2} \right) + x \sinh \left(\frac{1}{2} \right)) \sqrt{\frac{(x^2 + d) \cosh \left(\frac{1}{2} \right) + (x^2 - d) \sinh \left(\frac{1}{2} \right)}{\cosh \left(\frac{1}{2} \right) - \sinh \left(\frac{1}{2} \right)}}}{d} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{\operatorname{artanh} \left(\frac{\sqrt{e} x}{\sqrt{e x^2 + d}} \right)}{x^{\frac{11}{2}}}, x \right)$$

104.7 Problem number 22

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{13x^{\frac{13}{2}}} + \frac{36e^{\frac{3}{2}}\sqrt{ex^2+d}}{1001d^2x^{\frac{7}{2}}} - \frac{60e^{\frac{5}{2}}\sqrt{ex^2+d}}{1001d^3x^{\frac{3}{2}}} - \frac{4\sqrt{e}\sqrt{ex^2+d}}{143dx^{\frac{11}{2}}}$$

$$- \frac{30e^{\frac{13}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{ex^2}{(\sqrt{d} + x\sqrt{e})^2}}}{1001 \cos\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) d^{\frac{13}{4}} \sqrt{ex^2+d}}$$

command

`integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$77d^3\sqrt{x} \log\left(\frac{2x^2 \cosh\left(\frac{1}{2}\right)^2 + 4x^2 \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right) + 2x^2 \sinh\left(\frac{1}{2}\right)^2 + 2(x \cosh\left(\frac{1}{2}\right) + x \sinh\left(\frac{1}{2}\right)) \sqrt{\frac{(x^2+d) \cosh\left(\frac{1}{2}\right) + (x^2-d) \sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right) - \sinh\left(\frac{1}{2}\right)}}}{d}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}, x\right)$$

104.8 Problem number 23

$$\int x^{7/2} \tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d + ex^2}} \right) dx$$

Optimal antiderivative

$$\frac{2x^{\frac{9}{2}} \operatorname{arctanh} \left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}} \right)}{9} + \frac{28dx^{\frac{3}{2}} \sqrt{ex^2+d}}{405e^{\frac{3}{2}}} - \frac{4x^{\frac{7}{2}} \sqrt{ex^2+d}}{81\sqrt{e}} - \frac{28d^2 \sqrt{x} \sqrt{ex^2+d}}{135e^2 (\sqrt{d} + x\sqrt{e})}$$

$$+ \frac{28d^{\frac{9}{4}} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticE} \left(\sin \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{ex^2 - \sqrt{d}}{(\sqrt{d} + x\sqrt{e})}}}{135 \cos \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right) e^{\frac{9}{4}} \sqrt{ex^2+d}}$$

$$+ \frac{14d^{\frac{9}{4}} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{ex^2 + \sqrt{d}}{(\sqrt{d} + x\sqrt{e})}}}{135 \cos \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right) e^{\frac{9}{4}} \sqrt{ex^2+d}}$$

command

```
integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$84 d^2 \operatorname{weierstrassZeta} \left(-\frac{4d}{\cosh(\frac{1}{2})^2 + 2 \cosh(\frac{1}{2}) \sinh(\frac{1}{2}) + \sinh(\frac{1}{2})^2}, 0, \operatorname{weierstrassPInverse} \left(-\frac{4d}{\cosh(\frac{1}{2})^2 + 2 \cosh(\frac{1}{2}) \sinh(\frac{1}{2}) + \sinh(\frac{1}{2})^2} \right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(x^{\frac{7}{2}} \operatorname{artanh} \left(\frac{\sqrt{e} x}{\sqrt{ex^2+d}} \right), x \right)$$

104.9 Problem number 24

$$\int x^{3/2} \tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d + ex^2}} \right) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^{\frac{5}{2}} \operatorname{arctanh} \left(\frac{x\sqrt{e}}{\sqrt{ex^2 + d}} \right)}{5} - \frac{4x^{\frac{3}{2}} \sqrt{ex^2 + d}}{25\sqrt{e}} + \frac{12d\sqrt{x} \sqrt{ex^2 + d}}{25e(\sqrt{d} + x\sqrt{e})} \\ & - \frac{12d^{\frac{5}{4}} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticE} \left(\sin \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{ex^2 + d}{(\sqrt{d} + x\sqrt{e})^2}}}{25 \cos \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right) e^{\frac{5}{4}} \sqrt{ex^2 + d}} \\ & + \frac{6d^{\frac{5}{4}} \sqrt{\frac{\cos \left(4 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right)}{2}} + \frac{1}{2} \operatorname{EllipticF} \left(\sin \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right), \frac{\sqrt{2}}{2} \right) (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{ex^2 + d}{(\sqrt{d} + x\sqrt{e})^2}}}{25 \cos \left(2 \arctan \left(\frac{e^{\frac{1}{4}} \sqrt{x}}{d^{\frac{1}{4}}} \right) \right) e^{\frac{5}{4}} \sqrt{ex^2 + d}} \end{aligned}$$

command

```
integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5 \left(x^2 \cosh \left(\frac{1}{2} \right)^2 + 2 x^2 \cosh \left(\frac{1}{2} \right) \sinh \left(\frac{1}{2} \right) + x^2 \sinh \left(\frac{1}{2} \right)^2 \right) \sqrt{x} \log \left(\frac{2 x^2 \cosh \left(\frac{1}{2} \right)^2 + 4 x^2 \cosh \left(\frac{1}{2} \right) \sinh \left(\frac{1}{2} \right) + 2 x^2 \sinh \left(\frac{1}{2} \right)^2 + 2 (x \cosh \left(\frac{1}{2} \right) + \sqrt{d + ex^2}) \sqrt{d + ex^2}}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral} \left(x^{\frac{3}{2}} \operatorname{artanh} \left(\frac{\sqrt{e} x}{\sqrt{ex^2 + d}} \right), x \right)$$

104.10 Problem number 25

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} dx$$

Optimal antiderivative

$$2\sqrt{x} \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right) - \frac{4\sqrt{x}\sqrt{ex^2+d}}{\sqrt{d}+x\sqrt{e}}$$

$$+ \frac{4d^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{d}+x\sqrt{e}) \sqrt{\frac{ex^2+d}{(\sqrt{d}+x\sqrt{e})^2}}}{\cos\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{1}{4}}\sqrt{ex^2+d}}$$

$$- \frac{2d^{\frac{1}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{d}+x\sqrt{e}) \sqrt{\frac{ex^2+d}{(\sqrt{d}+x\sqrt{e})^2}}}{\cos\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{1}{4}}\sqrt{ex^2+d}}$$

command

```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\sqrt{x} \log\left(\frac{2x^2 \cosh\left(\frac{1}{2}\right)^2 + 4x^2 \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right) + 2x^2 \sinh\left(\frac{1}{2}\right)^2 + 2(x \cosh\left(\frac{1}{2}\right) + x \sinh\left(\frac{1}{2}\right)) \sqrt{\frac{(x^2+d) \cosh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)}}}{d}\right)$$

$$+ 4 \operatorname{weierstrassZeta}\left(-\frac{4d}{\cosh\left(\frac{1}{2}\right)^2 + 2 \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right) + \sinh\left(\frac{1}{2}\right)^2}, 0, \operatorname{weierstrassPInverse}\left(-\frac{4}{\cosh\left(\frac{1}{2}\right)^2 + 2 \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right) + \sinh\left(\frac{1}{2}\right)^2}\right)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}, x\right)$$

104.11 Problem number 26

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{3x^{3/2}} - \frac{4\sqrt{e}\sqrt{ex^2+d}}{3d\sqrt{x}} + \frac{4e\sqrt{x}\sqrt{ex^2+d}}{3d(\sqrt{d}+x\sqrt{e})} \\ & - \frac{4e^{3/4} \sqrt{\frac{\cos\left(4 \operatorname{arctan}\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2 \operatorname{arctan}\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{d}+x\sqrt{e}) \sqrt{\frac{ex^2+d}{(\sqrt{d}+x\sqrt{e})}}}{3 \cos\left(2 \operatorname{arctan}\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right) d^{3/4} \sqrt{ex^2+d}} \\ & + \frac{2e^{3/4} \sqrt{\frac{\cos\left(4 \operatorname{arctan}\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \operatorname{arctan}\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{d}+x\sqrt{e}) \sqrt{\frac{ex^2+d}{(\sqrt{d}+x\sqrt{e})}}}{3 \cos\left(2 \operatorname{arctan}\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right) d^{3/4} \sqrt{ex^2+d}} \end{aligned}$$

command

```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$d\sqrt{x} \log \left(\frac{2x^2 \cosh\left(\frac{1}{2}\right)^2 + 4x^2 \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right) + 2x^2 \sinh\left(\frac{1}{2}\right)^2 + 2(x \cosh\left(\frac{1}{2}\right) + x \sinh\left(\frac{1}{2}\right)) \sqrt{\frac{(x^2+d) \cosh\left(\frac{1}{2}\right) + (x^2-d) \sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right) - \sinh\left(\frac{1}{2}\right)}}}{d} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{5/2}}, x\right)$$

104.12 Problem number 27

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{7x^{7/2}} - \frac{4\sqrt{e}\sqrt{ex^2+d}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{ex^2+d}}{35d^2\sqrt{x}} - \frac{12e^2\sqrt{x}\sqrt{ex^2+d}}{35d^2(\sqrt{d}+x\sqrt{e})} \\ & + \frac{12e^{7/4}\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{d}+x\sqrt{e})\sqrt{\frac{ex^2+d}{(\sqrt{d}+x\sqrt{e})}}}{35\cos\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)d^{7/4}\sqrt{ex^2+d}} \\ & - \frac{6e^{7/4}\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{d}+x\sqrt{e})\sqrt{\frac{ex^2+d}{(\sqrt{d}+x\sqrt{e})}}}{35\cos\left(2\arctan\left(\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right)\right)d^{7/4}\sqrt{ex^2+d}} \end{aligned}$$

command

```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$5d^2\sqrt{x}\log\left(\frac{2x^2\cosh\left(\frac{1}{2}\right)^2+4x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2x^2\sinh\left(\frac{1}{2}\right)^2+2(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right))\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{d}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}},x\right)$$

105 Test file number 199

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.4_Inverse_hyperbolic_cotangent/199_7.4.2_Exponent

105.1 Problem number 359

$$\int e^{n \coth^{-1}(ax)} (c - acx)^{2+\frac{n}{2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(n^2 + 14n + 56) \left(1 - \frac{1}{ax}\right)^{-2-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{1+\frac{n}{2}} (-acx + c)^{2+\frac{n}{2}}}{a(4+n)(6+n)} \\ & + \frac{2(n^2 + 14n + 56) \left(1 - \frac{1}{ax}\right)^{-2-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{1+\frac{n}{2}} (-acx + c)^{2+\frac{n}{2}}}{a^2(6+n)(n^2 + 6n + 8)x} \\ & + \frac{(8+n) \left(1 - \frac{1}{ax}\right)^{-2-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{1+\frac{n}{2}} x(-acx + c)^{2+\frac{n}{2}}}{6+n} \\ & - \frac{\left(a - \frac{1}{x}\right) \left(1 - \frac{1}{ax}\right)^{-2-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{1+\frac{n}{2}} x(-acx + c)^{2+\frac{n}{2}}}{a} \end{aligned}$$

command

```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left((a^3 n^2 + 6 a^3 n + 8 a^3) x^3 - (a^2 n^2 + 14 a^2 n + 24 a^2) x^2 + n^2 - (a n^2 + 6 a n - 24 a) x + 14 n + 56 \right) (-acx + c)^{\frac{1}{2} n + 2}}{a n^3 + 12 a n^2 + (a^3 n^3 + 12 a^3 n^2 + 44 a^3 n + 48 a^3) x^2 + 44 a n - 2 (a^2 n^3 + 12 a^2 n^2 + 44 a^2 n + 48 a^2) x + 48}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left((-acx + c)^{\frac{1}{2} n + 2} \left(\frac{ax - 1}{ax + 1} \right)^{\frac{1}{2} n}, x \right)$$

105.2 Problem number 360

$$\int e^{n \coth^{-1}(ax)} (c - acx)^{1+\frac{n}{2}} dx$$

Optimal antiderivative

$$\frac{2(6+n) \left(1 - \frac{1}{ax}\right)^{-1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{1+\frac{n}{2}} (-acx+c)^{1+\frac{n}{2}}}{a(n^2+6n+8)} + \frac{2\left(1 - \frac{1}{ax}\right)^{-1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{1+\frac{n}{2}} x(-acx+c)^{1+\frac{n}{2}}}{4+n}$$

command

```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2((a^2n+2a^2)x^2-4ax-n-6)(-acx+c)^{\frac{1}{2}n+1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{an^2+6an-(a^2n^2+6a^2n+8a^2)x+8a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(-acx+c\right)^{\frac{1}{2}n+1}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

105.3 Problem number 361

$$\int e^{n \coth^{-1}(ax)} (c - acx)^{n/2} dx$$

Optimal antiderivative

$$\frac{2e^{n \operatorname{arccoth}(ax)}(ax+1)(-acx+c)^{\frac{n}{2}}}{a(2+n)}$$

command

```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(ax+1)(-acx+c)^{\frac{1}{2}n}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{an+2a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\left(-acx+c\right)^{\frac{1}{2}n}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

106 Test file number 200

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.5_Inverse_hyperbolic_secant/200_7.5.1_u-a+b_arcsech-c_x-^n

106.1 Problem number 166

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{(d + ex^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x(a + b \operatorname{arcsech}(cx))}{d\sqrt{ex^2 + d}} + \frac{b \operatorname{EllipticF}\left(cx, \sqrt{-\frac{e}{c^2d}}\right) \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1 + \frac{ex^2}{d}}}{cd\sqrt{ex^2 + d}}$$

command

```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{x^2 \cosh(1) + x^2 \sinh(1) + d} bcdx \log\left(\frac{cx \sqrt{-\frac{c^2x^2 - 1}{c^2x^2} + 1}}{cx}\right) + \sqrt{x^2 \cosh(1) + x^2 \sinh(1) + d} acdx + (bx^2 \cosh(1) + bx^2 \sinh(1) + d) cd^2}{cd^2x^2 \cosh(1) + cd^2x^2 \sinh(1) + cd^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d} (b \operatorname{ar} \operatorname{sech}(cx) + a)}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

107 Test file number 201

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.5_Inverse_hyperbolic_secant/201_7.5.2_Inverse_hyp

107.1 Problem number 46

$$\int e^{\operatorname{sech}^{-1}(ax^2)} x^6 dx$$

Optimal antiderivative

$$\frac{2x^5}{35a} + \frac{\left(\frac{1}{ax^2} + \sqrt{\frac{1}{ax^2} - 1} \sqrt{\frac{1}{ax^2} + 1}\right) x^7}{7} + \frac{2 \operatorname{EllipticF}(x\sqrt{a}, i) \sqrt{\frac{1}{ax^2} + 1} \sqrt{ax^2 + 1}}{21a^{\frac{7}{2}}} - \frac{2x \sqrt{\frac{1}{ax^2} + 1} \sqrt{ax^2 + 1} \sqrt{-a^2x^4 + 1}}{21a^3}$$

command

`integrate((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{21ax^5 + 5(3a^2x^7 - 2x^3) \sqrt{\frac{ax^2 + 1}{ax^2}} \sqrt{-\frac{ax^2 - 1}{ax^2}}}{105a^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{ax^6 \sqrt{\frac{ax^2 + 1}{ax^2}} \sqrt{-\frac{ax^2 - 1}{ax^2}} + x^4}{a}, x\right)$$

107.2 Problem number 48

$$\int e^{\operatorname{sech}^{-1}(ax^2)} x^4 dx$$

Optimal antiderivative

$$\frac{2x^3}{15a} + \frac{\left(\frac{1}{ax^2} + \sqrt{\frac{1}{ax^2} - 1} \sqrt{\frac{1}{ax^2} + 1}\right) x^5}{5} + \frac{2 \operatorname{EllipticE}(x\sqrt{a}, i) \sqrt{\frac{1}{ax^2} + 1} \sqrt{ax^2 + 1}}{5a^{\frac{5}{2}}} - \frac{2 \operatorname{EllipticF}(x\sqrt{a}, i) \sqrt{\frac{1}{ax^2} + 1} \sqrt{ax^2 + 1}}{5a^{\frac{5}{2}}}$$

command

`integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5ax^3 + 3(a^2x^5 - 2x)\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}}{15a^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{ax^4\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}} + x^2}{a}, x\right)$$

107.3 Problem number 50

$$\int e^{\text{sech}^{-1}(ax^2)} x^2 dx$$

Optimal antiderivative

$$\frac{2x}{3a} + \frac{\left(\frac{1}{ax^2} + \sqrt{\frac{1}{ax^2} - 1}\sqrt{\frac{1}{ax^2} + 1}\right)x^3}{3} + \frac{2\text{EllipticF}(x\sqrt{a}, i)\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}}{3a^{\frac{3}{2}}}$$

command

`integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{ax^3\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}} + 3x}{3a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}} + 1}{a}, x\right)$$

107.4 Problem number 54

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2} dx$$

Optimal antiderivative

$$\frac{2}{3ax^3} \frac{\frac{1}{ax^2} + \sqrt{\frac{1}{ax^2} - 1} \sqrt{\frac{1}{ax^2} + 1}}{x} - \frac{2 \operatorname{EllipticF}(x\sqrt{a}, i) \sqrt{a} \sqrt{\frac{1}{ax^2 + 1}} \sqrt{ax^2 + 1}}{3} + \frac{2 \sqrt{\frac{1}{ax^2 + 1}} \sqrt{ax^2 + 1} \sqrt{-a^2x^4 + 1}}{3ax^3}$$

command

`integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2a^{\frac{3}{2}}x^3 \operatorname{ellipticF}(\sqrt{a}x, -1) + ax^2 \sqrt{\frac{ax^2 + 1}{ax^2}} \sqrt{-\frac{ax^2 - 1}{ax^2}} + 1}{3ax^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{ax^2 \sqrt{\frac{ax^2 + 1}{ax^2}} \sqrt{-\frac{ax^2 - 1}{ax^2}} + 1}{ax^4}, x\right)$$

108 Test file number 202

Test folder name:

`test_cases/7_Inverse_hyperbolic_functions/7.6_Inverse_hyperbolic_cosecant/202_7.6.1_u-a+b_arccsch-c_x^-n`

108.1 Problem number 154

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x(a + b \operatorname{arccsch}(cx))}{d\sqrt{ex^2 + d}} - \frac{bx \sqrt{\frac{1}{c^2x^2 + 1}} \sqrt{c^2x^2 + 1} \operatorname{EllipticF}\left(\frac{cx}{\sqrt{c^2x^2 + 1}}, \sqrt{1 - \frac{e}{c^2d}}\right) \sqrt{ex^2 + d}}{d^2 \sqrt{-c^2x^2} \sqrt{-c^2x^2 - 1} \sqrt{\frac{ex^2 + d}{d(c^2x^2 + 1)}}$$

command

```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\sqrt{x^2 \cosh(1) + x^2 \sinh(1) + d} bc^2 dx \log\left(\frac{cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2} + 1}}{cx}\right) + \sqrt{x^2 \cosh(1) + x^2 \sinh(1) + d} ac^2 dx - (bx^2 \cosh(1) + bx^2 \sinh(1) + d) c^2 d^3}{c^2 d^2 x^2 \cosh(1) + c^2 d^2 x^2 \sinh(1) + c^2 d^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a)}{e^2 x^4 + 2 dex^2 + d^2}, x\right)$$

109 Test file number 203

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.6_Inverse_hyperbolic_cosecant/203_7.6.2_Inverse_h

109.1 Problem number 40

$$\int e^{\operatorname{csch}^{-1}(ax^2)} x^2 dx$$

Optimal antiderivative

$$\frac{x}{a} + \frac{x^3 \sqrt{1 + \frac{1}{a^2 x^4}}}{3} + \frac{(a + \frac{1}{x^2}) \sqrt{\frac{\cos(4 \operatorname{arccot}(x\sqrt{a}))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \operatorname{arccot}(x\sqrt{a})), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^2 + \frac{1}{x^4}}{(a + \frac{1}{x^2})^2}}}{3 \cos(2 \operatorname{arccot}(x\sqrt{a})) a^{\frac{5}{2}} \sqrt{1 + \frac{1}{a^2 x^4}}}$$

command

```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{ax^3 \sqrt{\frac{a^2 x^4 + 1}{a^2 x^4}} + 2a \left(-\frac{1}{a^2}\right)^{\frac{3}{4}} \operatorname{ellipticF}\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}, -1\right) + 3x}{3a}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{ax^2 \sqrt{\frac{a^2x^4 + 1}{a^2x^4}} + 1}{a}, x \right)$$

109.2 Problem number 44

$$\int \frac{e^{\text{csch}^{-1}(ax^2)}}{x^2} dx$$

Optimal antiderivative

$$\frac{-\frac{1}{3ax^3} - \frac{\sqrt{1 + \frac{1}{a^2x^4}}}{3x}}{(a + \frac{1}{x^2}) \sqrt{\frac{\cos(4 \operatorname{arccot}(x\sqrt{a}))}{2} + \frac{1}{2}} \operatorname{EllipticF}\left(\sin(2 \operatorname{arccot}(x\sqrt{a})), \frac{\sqrt{2}}{2}\right) \sqrt{\frac{a^2 + \frac{1}{x^4}}{(a + \frac{1}{x^2})^2}}}$$

$$3 \cos(2 \operatorname{arccot}(x\sqrt{a})) \sqrt{a} \sqrt{1 + \frac{1}{a^2x^4}}$$

command

`integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(-a^2)^{\frac{3}{4}}x^3 \operatorname{ellipticF}\left((-a^2)^{\frac{1}{4}}x, -1\right) + ax^2 \sqrt{\frac{a^2x^4 + 1}{a^2x^4}} + 1}{3ax^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral} \left(\frac{ax^2 \sqrt{\frac{a^2x^4 + 1}{ax^4}} + 1}{ax^4}, x \right)$$

110 Test file number 205

Test folder name:

test_cases/8_Special_functions/205_8.2_Fresnel_integral_functions

110.1 Problem number 1

$$\int x^7 S(bx) dx$$

Optimal antiderivative

$$-\frac{35x^3 \cos\left(\frac{b^2\pi x^2}{2}\right)}{8b^5\pi^3} + \frac{x^7 \cos\left(\frac{b^2\pi x^2}{2}\right)}{8b\pi} - \frac{105 S(bx)}{8b^8\pi^4} + \frac{x^8 S(bx)}{8} + \frac{105x \sin\left(\frac{b^2\pi x^2}{2}\right)}{8b^7\pi^4} - \frac{7x^5 \sin\left(\frac{b^2\pi x^2}{2}\right)}{8b^3\pi^2}$$

command

`integrate(x^7*fresnel_sin(b*x),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\pi^3 b^7 x^7 - 35 \pi b^3 x^3) \cos\left(\frac{1}{2} \pi b^2 x^2\right) + (\pi^4 b^8 x^8 - 105) S(bx) - 7(\pi^2 b^5 x^5 - 15 bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{8 \pi^4 b^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^7 \text{fresnels}(bx), x)$$

110.2 Problem number 2

$$\int x^6 S(bx) dx$$

Optimal antiderivative

$$-\frac{24x^2 \cos\left(\frac{b^2\pi x^2}{2}\right)}{7b^5\pi^3} + \frac{x^6 \cos\left(\frac{b^2\pi x^2}{2}\right)}{7b\pi} + \frac{x^7 S(bx)}{7} + \frac{48 \sin\left(\frac{b^2\pi x^2}{2}\right)}{7b^7\pi^4} - \frac{6x^4 \sin\left(\frac{b^2\pi x^2}{2}\right)}{7b^3\pi^2}$$

command

`integrate(x^6*fresnel_sin(b*x),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^4 b^7 x^7 S(bx) + (\pi^3 b^6 x^6 - 24 \pi b^2 x^2) \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 6(\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{7 \pi^4 b^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^6 \text{fresnels}(bx), x)$$

110.3 Problem number 3

$$\int x^5 S(bx) dx$$

Optimal antiderivative

$$-\frac{5x \cos\left(\frac{b^2 \pi x^2}{2}\right)}{2b^5 \pi^3} + \frac{x^5 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{6b\pi} + \frac{5 \operatorname{FresnelC}(bx)}{2b^6 \pi^3} + \frac{x^6 S(bx)}{6} - \frac{5x^3 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{6b^3 \pi^2}$$

command

```
integrate(x^5*fresnel_sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^3 b^7 x^6 S(bx) - 5 \pi b^4 x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right) + (\pi^2 b^6 x^5 - 15 b^2 x) \cos\left(\frac{1}{2} \pi b^2 x^2\right) + 15 \sqrt{b^2} C\left(\sqrt{b^2} x\right)}{6 \pi^3 b^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^5 \operatorname{fresnels}(bx), x)$$

110.4 Problem number 4

$$\int x^4 S(bx) dx$$

Optimal antiderivative

$$-\frac{8 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{5b^5 \pi^3} + \frac{x^4 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{5b\pi} + \frac{x^5 S(bx)}{5} - \frac{4x^2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{5b^3 \pi^2}$$

command

```
integrate(x^4*fresnel_sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^3 b^5 x^5 S(bx) - 4 \pi b^2 x^2 \sin\left(\frac{1}{2} \pi b^2 x^2\right) + (\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{5 \pi^3 b^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^4 \operatorname{fresnels}(bx), x)$$

110.5 Problem number 5

$$\int x^3 S(bx) dx$$

Optimal antiderivative

$$\frac{x^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{4b\pi} + \frac{3S(bx)}{4b^4 \pi^2} + \frac{x^4 S(bx)}{4} - \frac{3x \sin\left(\frac{b^2 \pi x^2}{2}\right)}{4b^3 \pi^2}$$

command

```
integrate(x^3*fresnel_sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^3 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 3bx \sin\left(\frac{1}{2} \pi b^2 x^2\right) + (\pi^2 b^4 x^4 + 3) S(bx)}{4 \pi^2 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(x^3fresnels(bx),x)
```

110.6 Problem number 6

$$\int x^2 S(bx) dx$$

Optimal antiderivative

$$\frac{x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{3b\pi} + \frac{x^3 S(bx)}{3} - \frac{2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{3b^3 \pi^2}$$

command

```
integrate(x^2*fresnel_sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^3 x^3 S(bx) + \pi b^2 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 2 \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{3 \pi^2 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(x^2fresnels(bx),x)
```

110.7 Problem number 7

$$\int x S(bx) dx$$

Optimal antiderivative

$$\frac{x \cos\left(\frac{b^2 \pi x^2}{2}\right)}{2b\pi} - \frac{\text{FresnelC}(bx)}{2b^2\pi} + \frac{x^2 S(bx)}{2}$$

command

```
integrate(x*fresnel_sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^3 x^2 S(bx) + b^2 x \cos\left(\frac{1}{2} \pi b^2 x^2\right) - \sqrt{b^2} C\left(\sqrt{b^2} x\right)}{2 \pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(xfresnels(bx), x)
```

110.8 Problem number 8

$$\int S(bx) dx$$

Optimal antiderivative

$$\frac{\cos\left(\frac{b^2 \pi x^2}{2}\right)}{b\pi} + x S(bx)$$

command

```
integrate(fresnel_sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b x S(bx) + \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(fresnels(bx), x)
```

110.9 Problem number 10

$$\int \frac{S(bx)}{x^2} dx$$

Optimal antiderivative

$$-\frac{S(bx)}{x} + \frac{b \operatorname{Si}\left(\frac{b^2 \pi x^2}{2}\right)}{2}$$

command

```
integrate(fresnel_sin(b*x)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bx \operatorname{Si}\left(\frac{1}{2} \pi b^2 x^2\right) - 2 S(bx)}{2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)}{x^2}, x\right)$$

110.10 Problem number 11

$$\int \frac{S(bx)}{x^3} dx$$

Optimal antiderivative

$$\frac{b^2 \pi \operatorname{FresnelC}(bx)}{2} - \frac{S(bx)}{2x^2} - \frac{b \sin\left(\frac{b^2 \pi x^2}{2}\right)}{2x}$$

command

```
integrate(fresnel_sin(b*x)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi \sqrt{b^2} bx^2 \operatorname{C}\left(\sqrt{b^2} x\right) - bx \sin\left(\frac{1}{2} \pi b^2 x^2\right) - S(bx)}{2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)}{x^3}, x\right)$$

110.11 Problem number 12

$$\int \frac{S(bx)}{x^4} dx$$

Optimal antiderivative

$$\frac{b^3 \pi \operatorname{cosineIntegral}\left(\frac{b^2 \pi x^2}{2}\right)}{12} - \frac{S(bx)}{3x^3} - \frac{b \sin\left(\frac{b^2 \pi x^2}{2}\right)}{6x^2}$$

command

```
integrate(fresnel_sin(b*x)/x^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^3 x^3 \operatorname{Ci}\left(\frac{1}{2} \pi b^2 x^2\right) + \pi b^3 x^3 \operatorname{Ci}\left(-\frac{1}{2} \pi b^2 x^2\right) - 4 b x \sin\left(\frac{1}{2} \pi b^2 x^2\right) - 8 S(bx)}{24 x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)}{x^4}, x\right)$$

110.12 Problem number 13

$$\int \frac{S(bx)}{x^5} dx$$

Optimal antiderivative

$$-\frac{b^3 \pi \cos\left(\frac{b^2 \pi x^2}{2}\right)}{12x} - \frac{b^4 \pi^2 S(bx)}{12} - \frac{S(bx)}{4x^4} - \frac{b \sin\left(\frac{b^2 \pi x^2}{2}\right)}{12x^3}$$

command

```
integrate(fresnel_sin(b*x)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\pi b^3 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) + b x \sin\left(\frac{1}{2} \pi b^2 x^2\right) + (\pi^2 b^4 x^4 + 3) S(bx)}{12 x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)}{x^5}, x\right)$$

110.13 Problem number 14

$$\int \frac{S(bx)}{x^6} dx$$

Optimal antiderivative

$$-\frac{b^3\pi \cos\left(\frac{b^2\pi x^2}{2}\right)}{40x^2} - \frac{S(bx)}{5x^5} - \frac{b^5\pi^2 \operatorname{Si}\left(\frac{b^2\pi x^2}{2}\right)}{80} - \frac{b \sin\left(\frac{b^2\pi x^2}{2}\right)}{20x^4}$$

command

```
integrate(fresnel_sin(b*x)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\pi^2 b^5 x^5 \operatorname{Si}\left(\frac{1}{2} \pi b^2 x^2\right) + 2 \pi b^3 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) + 4 b x \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 16 S(bx)}{80 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)}{x^6}, x\right)$$

110.14 Problem number 15

$$\int \frac{S(bx)}{x^7} dx$$

Optimal antiderivative

$$-\frac{b^3\pi \cos\left(\frac{b^2\pi x^2}{2}\right)}{90x^3} - \frac{b^6\pi^3 \operatorname{FresnelC}(bx)}{90} - \frac{S(bx)}{6x^6} - \frac{b \sin\left(\frac{b^2\pi x^2}{2}\right)}{30x^5} + \frac{b^5\pi^2 \sin\left(\frac{b^2\pi x^2}{2}\right)}{90x}$$

command

```
integrate(fresnel_sin(b*x)/x^7,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\pi^3 \sqrt{b^2} b^5 x^6 \operatorname{C}\left(\sqrt{b^2} x\right) + \pi b^3 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) - (\pi^2 b^5 x^5 - 3 b x) \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 15 S(bx)}{90 x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)}{x^7}, x\right)$$

110.15 Problem number 16

$$\int \frac{S(bx)}{x^8} dx$$

Optimal antiderivative

$$-\frac{b^7 \pi^3 \operatorname{cosineIntegral}\left(\frac{b^2 \pi x^2}{2}\right)}{672} - \frac{b^3 \pi \cos\left(\frac{b^2 \pi x^2}{2}\right)}{168x^4} - \frac{S(bx)}{7x^7} - \frac{b \sin\left(\frac{b^2 \pi x^2}{2}\right)}{42x^6} + \frac{b^5 \pi^2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{336x^2}$$

command

`integrate(fresnel_sin(b*x)/x^8,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\pi^3 b^7 x^7 \operatorname{Ci}\left(\frac{1}{2} \pi b^2 x^2\right) + \pi^3 b^7 x^7 \operatorname{Ci}\left(-\frac{1}{2} \pi b^2 x^2\right) + 8 \pi b^3 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 4 (\pi^2 b^5 x^5 - 8 bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 192 S(bx)}{1344 x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)}{x^8}, x\right)$$

110.16 Problem number 17

$$\int \frac{S(bx)}{x^9} dx$$

Optimal antiderivative

$$-\frac{b^3 \pi \cos\left(\frac{b^2 \pi x^2}{2}\right)}{280x^5} + \frac{b^7 \pi^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{840x} + \frac{b^8 \pi^4 S(bx)}{840} - \frac{S(bx)}{8x^8} - \frac{b \sin\left(\frac{b^2 \pi x^2}{2}\right)}{56x^7} + \frac{b^5 \pi^2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{840x^3}$$

command

`integrate(fresnel_sin(b*x)/x^9,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\pi^3 b^7 x^7 - 3 \pi b^3 x^3) \cos\left(\frac{1}{2} \pi b^2 x^2\right) + (\pi^4 b^8 x^8 - 105) S(bx) + (\pi^2 b^5 x^5 - 15 bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{840 x^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)}{x^9}, x\right)$$

110.17 Problem number 18

$$\int \frac{S(bx)}{x^{10}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b^3 \pi \cos\left(\frac{b^2 \pi x^2}{2}\right)}{432x^6} + \frac{b^7 \pi^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{3456x^2} - \frac{S(bx)}{9x^9} \\ & + \frac{b^9 \pi^4 \operatorname{Si}\left(\frac{b^2 \pi x^2}{2}\right)}{6912} - \frac{b \sin\left(\frac{b^2 \pi x^2}{2}\right)}{72x^8} + \frac{b^5 \pi^2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{1728x^4} \end{aligned}$$

command

`integrate(fresnel_sin(b*x)/x^10,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^4 b^9 x^9 \operatorname{Si}\left(\frac{1}{2} \pi b^2 x^2\right) + 2\left(\pi^3 b^7 x^7 - 8 \pi b^3 x^3\right) \cos\left(\frac{1}{2} \pi b^2 x^2\right) + 4\left(\pi^2 b^5 x^5 - 24 bx\right) \sin\left(\frac{1}{2} \pi b^2 x^2\right) - 768 S(bx)}{6912 x^9}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)}{x^{10}}, x\right)$$

110.18 Problem number 19

$$\int (c + dx)^3 S(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ad + bc)^3 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4 \pi} + \frac{3d(-ad + bc)^2 (bx + a) \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2b^4 \pi} \\ & + \frac{d^2(-ad + bc) (bx + a)^2 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4 \pi} + \frac{d^3 (bx + a)^3 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{4b^4 \pi} \\ & - \frac{3d(-ad + bc)^2 \operatorname{FresnelC}(bx + a)}{2b^4 \pi} - \frac{(-ad + bc)^4 S(bx + a)}{4b^4 d} + \frac{3d^3 S(bx + a)}{4b^4 \pi^2} \\ & + \frac{(dx + c)^4 S(bx + a)}{4d} - \frac{2d^2(-ad + bc) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4 \pi^2} - \frac{3d^3 (bx + a) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{4b^4 \pi^2} \end{aligned}$$

command

`integrate((d*x+c)^3*fresnel_sin(b*x+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$6\pi(b^2c^2d - 2abcd^2 + a^2d^3)\sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (\pi^2(4ab^3c^3 - 6a^2b^2c^2d + 4a^3bcd^2 - a^4d^3) + 3d^3)\sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}((d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3)\text{fresnels}(bx + a), x)$$

110.19 Problem number 20

$$\int (c + dx)^2 S(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ad + bc)^2 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^3\pi} + \frac{d(-ad + bc)(bx + a) \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^3\pi} \\ & + \frac{d^2(bx + a)^2 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{3b^3\pi} - \frac{d(-ad + bc) \text{FresnelC}(bx + a)}{b^3\pi} \\ & - \frac{(-ad + bc)^3 S(bx + a)}{3b^3d} + \frac{(dx + c)^3 S(bx + a)}{3d} - \frac{2d^2 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{3b^3\pi^2} \end{aligned}$$

command

`integrate((d*x+c)^2*fresnel_sin(b*x+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\pi^2(3ab^2c^2 - 3a^2bcd + a^3d^2)\sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 2bd^2 \sin\left(\frac{1}{2}\pi b^2x^2 + \pi abx + \frac{1}{2}\pi a^2\right) - 3\pi(bcd - ad^2)\sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}((d^2x^2 + 2cdx + c^2)\text{fresnels}(bx + a), x)$$

110.20 Problem number 21

$$\int (c + dx)S(a + bx) dx$$

Optimal antiderivative

$$\frac{(-ad + bc) \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^2\pi} + \frac{d(bx+a) \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2b^2\pi} - \frac{d \operatorname{FresnelC}(bx+a)}{2b^2\pi} - \frac{(-ad + bc)^2 S(bx+a)}{2b^2d} + \frac{(dx+c)^2 S(bx+a)}{2d}$$

command

```
integrate((d*x+c)*fresnel_sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi(2abc - a^2d)\sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - \sqrt{b^2} d C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + (b^2dx + 2b^2c - abd) \cos\left(\frac{1}{2}\pi b^2x^2 + \pi abx + \frac{1}{2}\pi a^2\right)}{2\pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}((dx + c)\operatorname{fresnels}(bx + a), x)$$

110.21 Problem number 22

$$\int S(a + bx) dx$$

Optimal antiderivative

$$\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b\pi} + \frac{(bx+a) S(bx+a)}{b}$$

command

```
integrate(fresnel_sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\pi bx + \pi a) S(bx + a) + \cos\left(\frac{1}{2}\pi b^2x^2 + \pi abx + \frac{1}{2}\pi a^2\right)}{\pi b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\operatorname{fresnels}(bx + a), x)$$

110.22 Problem number 25

$$\int x^3 S(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^3 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4\pi} + \frac{3a^2(bx+a) \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2b^4\pi} - \frac{a(bx+a)^2 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4\pi} \\ & + \frac{(bx+a)^3 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{4b^4\pi} - \frac{3a^2 \operatorname{FresnelC}(bx+a)}{2b^4\pi} - \frac{a^4 S(bx+a)}{4b^4} \\ & + \frac{3S(bx+a)}{4b^4\pi^2} + \frac{x^4 S(bx+a)}{4} + \frac{2a \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4\pi^2} - \frac{3(bx+a) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{4b^4\pi^2} \end{aligned}$$

command

```
integrate(x^3*fresnel_sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^5 x^4 S(bx+a) - 6\pi a^2 \sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (\pi^2 a^4 - 3)\sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + (\pi b^4 x^3 - \pi a b^3 x^2 + \pi a^2 b^2 x - \pi a^3) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{4\pi^2 b^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^3 \operatorname{fresnelS}(bx+a), x)$$

110.23 Problem number 26

$$\int x^2 S(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^3\pi} - \frac{a(bx+a) \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^3\pi} + \frac{(bx+a)^2 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{3b^3\pi} \\ & + \frac{a \operatorname{FresnelC}(bx+a)}{b^3\pi} + \frac{a^3 S(bx+a)}{3b^3} + \frac{x^3 S(bx+a)}{3} - \frac{2 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{3b^3\pi^2} \end{aligned}$$

command

```
integrate(x^2*fresnel_sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^4 x^3 S(bx+a) + \pi^2 a^3 \sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + 3\pi a \sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + (\pi b^3 x^2 - \pi a b^2 x + \pi a^2 b) \cos\left(\frac{1}{2} \pi b^2 x\right)}{3\pi^2 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^2 \text{fresnels}(bx+a), x)$$

110.24 Problem number 27

$$\int x S(a+bx) dx$$

Optimal antiderivative

$$-\frac{a \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^2 \pi} + \frac{(bx+a) \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2b^2 \pi} - \frac{\text{FresnelC}(bx+a)}{2b^2 \pi} - \frac{a^2 S(bx+a)}{2b^2} + \frac{x^2 S(bx+a)}{2}$$

command

`integrate(x*fresnel_sin(b*x+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^3 x^2 S(bx+a) - \pi a^2 \sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + (b^2 x - ab) \cos\left(\frac{1}{2} \pi b^2 x^2 + \pi abx + \frac{1}{2} \pi a^2\right) - \sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{2\pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x \text{fresnels}(bx+a), x)$$

110.25 Problem number 28

$$\int S(a+bx) dx$$

Optimal antiderivative

$$\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b\pi} + \frac{(bx+a) S(bx+a)}{b}$$

command

`integrate(fresnel_sin(b*x+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\pi bx + \pi a) S(bx+a) + \cos\left(\frac{1}{2} \pi b^2 x^2 + \pi abx + \frac{1}{2} \pi a^2\right)}{\pi b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(\text{fresnels}(bx+a), x)$$

110.26 Problem number 31

$$\int x^7 S(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} - \frac{55x^2 \cos(b^2\pi x^2)}{16b^6\pi^4} + \frac{x^6 \cos(b^2\pi x^2)}{16b^2\pi^2} - \frac{35x^3 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{4b^5\pi^3} \\ & + \frac{x^7 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{4b\pi} - \frac{105S(bx)^2}{8b^8\pi^4} + \frac{x^8 S(bx)^2}{8} + \frac{105x S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^7\pi^4} \\ & - \frac{7x^5 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^3\pi^2} + \frac{10 \sin(b^2\pi x^2)}{b^8\pi^5} - \frac{5x^4 \sin(b^2\pi x^2)}{8b^4\pi^3} \end{aligned}$$

command

```
integrate(x^7*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\pi^3 b^6 x^6 - 75\pi b^2 x^2 + 3(\pi^3 b^6 x^6 - 55\pi b^2 x^2) \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 6(\pi^4 b^7 x^7 - 35\pi^2 b^3 x^3) \cos\left(\frac{1}{2}\pi b^2 x^2\right) S(bx) - 3(105}{24\pi^5 b^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^7 \text{fresnels}(bx)^2, x\right)$$

110.27 Problem number 32

$$\int x^6 S(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} - \frac{21x \cos(b^2\pi x^2)}{8b^6\pi^4} + \frac{x^5 \cos(b^2\pi x^2)}{14b^2\pi^2} - \frac{48x^2 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{7b^5\pi^3} \\ & + \frac{2x^6 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{7b\pi} + \frac{x^7 S(bx)^2}{7} + \frac{96 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{7b^7\pi^4} \\ & - \frac{12x^4 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{7b^3\pi^2} - \frac{17x^3 \sin(b^2\pi x^2)}{28b^4\pi^3} + \frac{531 \text{FresnelC}\left(bx\sqrt{2}\right) \sqrt{2}}{112b^7\pi^4} \end{aligned}$$

command

`integrate(x^6*fresnel_sin(b*x)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$80 \pi^4 b^8 x^7 S(bx)^2 + 56 \pi^2 b^6 x^5 - 2370 b^2 x + 20 (4 \pi^2 b^6 x^5 - 147 b^2 x) \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 + 160 (\pi^3 b^7 x^6 - 24 \pi b^3 x^2) \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx)$$

560

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^6 \text{fresnels}(bx)^2, x\right)$$

110.28 Problem number 34

$$\int x^4 S(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4x^3}{15b^2\pi^2} + \frac{x^3 \cos(b^2\pi x^2)}{10b^2\pi^2} - \frac{16 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{5b^5\pi^3} + \frac{2x^4 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{5b\pi} \\ & + \frac{x^5 S(bx)^2}{5} - \frac{8x^2 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{5b^3\pi^2} - \frac{11x \sin(b^2\pi x^2)}{20b^4\pi^3} + \frac{43 S(bx\sqrt{2}) \sqrt{2}}{40b^5\pi^3} \end{aligned}$$

command

`integrate(x^4*fresnel_sin(b*x)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$24 \pi^3 b^6 x^5 S(bx)^2 + 24 \pi b^4 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 + 20 \pi b^4 x^3 + 48 (\pi^2 b^5 x^4 - 8b) \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx) + 129 \sqrt{2} \sqrt{b^2} S\left(\frac{1}{2} \pi b^2 x^2\right) S(bx)$$

$$120 \pi^3 b^6$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^4 \text{fresnels}(bx)^2, x\right)$$

110.29 Problem number 35

$$\int x^3 S(bx)^2 dx$$

Optimal antiderivative

$$\frac{3x^2}{8b^2\pi^2} + \frac{x^2 \cos(b^2\pi x^2)}{8b^2\pi^2} + \frac{x^3 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{2b\pi} + \frac{3S(bx)^2}{4b^4\pi^2}$$

$$+ \frac{x^4 S(bx)^2}{4} - \frac{3x S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^3\pi^2} - \frac{\sin(b^2\pi x^2)}{2b^4\pi^3}$$

command

```
integrate(x^3*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\pi^2 b^3 x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right) S(bx) + \pi b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + \pi b^2 x^2 + (3\pi + \pi^3 b^4 x^4) S(bx)^2 - 2(3\pi b x S(bx) + 2 \cos\left(\frac{1}{2}\pi b^2 x^2\right))}{4\pi^3 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^3 \text{fresnels}(bx)^2, x\right)$$

110.30 Problem number 36

$$\int x^2 S(bx)^2 dx$$

Optimal antiderivative

$$\frac{2x}{3b^2\pi^2} + \frac{x \cos(b^2\pi x^2)}{6b^2\pi^2} + \frac{2x^2 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{3b\pi} + \frac{x^3 S(bx)^2}{3}$$

$$- \frac{4S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{3b^3\pi^2} - \frac{5 \text{FresnelC}\left(bx\sqrt{2}\right) \sqrt{2}}{12b^3\pi^2}$$

command

```
integrate(x^2*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4\pi^2 b^4 x^3 S(bx)^2 + 8\pi b^3 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) S(bx) + 4b^2 x \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 6b^2 x - 16b S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) - 5\sqrt{2} \sqrt{b} \text{FresnelC}\left(bx\sqrt{2}\right)}{12\pi^2 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^2 \text{fresnels}(bx)^2, x\right)$$

110.31 Problem number 38

$$\int S(bx)^2 dx$$

Optimal antiderivative

$$\frac{2 \cos\left(\frac{b^2 \pi x^2}{2}\right) S(bx)}{b\pi} + xS(bx)^2 - \frac{S(bx\sqrt{2}) \sqrt{2}}{2b\pi}$$

command

`integrate(fresnel_sin(b*x)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \pi b^2 x S(bx)^2 + 4 b \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx) - \sqrt{2} \sqrt{b^2} S\left(\sqrt{2} \sqrt{b^2} x\right)}{2 \pi b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\text{fresnels}(bx)^2, x\right)$$

110.32 Problem number 43

$$\int \frac{S(bx)^2}{x^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b^2}{24x^2} + \frac{b^2 \cos(b^2 \pi x^2)}{24x^2} - \frac{b^3 \pi \cos\left(\frac{b^2 \pi x^2}{2}\right) S(bx)}{6x} - \frac{b^4 \pi^2 S(bx)^2}{12} \\ & - \frac{S(bx)^2}{4x^4} + \frac{b^4 \pi \text{sinIntegral}(b^2 \pi x^2)}{12} - \frac{b S(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right)}{6x^3} \end{aligned}$$

command

`integrate(fresnel_sin(b*x)^2/x^5,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^4 x^4 \text{Si}\left(\pi b^2 x^2\right) - 2 \pi b^3 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx) + b^2 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 - b^2 x^2 - 2 b x S(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) - \left(\pi^2 b^4 x^4 - \right)}{12 x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x^5}, x\right)$$

110.33 Problem number 47

$$\int \frac{S(bx)^2}{x^9} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2 \cos(b^2\pi x^2)}{336x^6} - \frac{b^6\pi^2 \cos(b^2\pi x^2)}{336x^2} - \frac{b^3\pi \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{140x^5} \\ & + \frac{b^7\pi^3 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{420x} + \frac{b^8\pi^4 S(bx)^2}{840} - \frac{S(bx)^2}{8x^8} - \frac{b^8\pi^3 \operatorname{sinIntegral}(b^2\pi x^2)}{280} \\ & - \frac{b S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{28x^7} + \frac{b^5\pi^2 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{420x^3} - \frac{b^4\pi \sin(b^2\pi x^2)}{420x^4} \end{aligned}$$

command

```
integrate(fresnel_sin(b*x)^2/x^9,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3\pi^3 b^8 x^8 \operatorname{Si}(\pi b^2 x^2) - 3\pi^2 b^6 x^6 + 5b^2 x^2 + 5(\pi^2 b^6 x^6 - b^2 x^2) \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 - 2(\pi^3 b^7 x^7 - 3\pi b^3 x^3) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{840 x^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)^2}{x^9}, x\right)$$

110.34 Problem number 51

$$\int S(a + bx)^2 dx$$

Optimal antiderivative

$$\frac{2 \cos\left(\frac{\pi(bx+a)^2}{2}\right) S(bx+a)}{b\pi} + \frac{(bx+a) S(bx+a)^2}{b} - \frac{S\left((bx+a)\sqrt{2}\right)\sqrt{2}}{2b\pi}$$

command

```
integrate(fresnel_sin(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4b \cos\left(\frac{1}{2}\pi b^2 x^2 + \pi abx + \frac{1}{2}\pi a^2\right) S(bx+a) + 2(\pi b^2 x + \pi ab) S(bx+a)^2 - \sqrt{2}\sqrt{b^2} S\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)}{2\pi b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\operatorname{fresnels}(bx+a)^2, x\right)$$

110.35 Problem number 54

$$\int x^2 S(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned} & \left(\frac{1}{12} - \frac{i}{12} \right) e^{-\frac{3a}{bn} + \frac{9i}{2b^2d^2n^2\pi}} x^3 \operatorname{erf} \left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{3}{n} + iab d^2 \pi + ib^2 d^2 \pi \ln(cx^n)\right)}{bd\sqrt{\pi}} \right) (cx^n)^{-\frac{3}{n}} \\ & + \left(\frac{1}{12} - \frac{i}{12} \right) e^{-\frac{3a}{bn} - \frac{9i}{2b^2d^2n^2\pi}} x^3 \operatorname{erfi} \left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{3}{n} - iab d^2 \pi - ib^2 d^2 \pi \ln(cx^n)\right)}{bd\sqrt{\pi}} \right) (cx^n)^{-\frac{3}{n}} \\ & + \frac{x^3 S(d(a + b \ln(cx^n)))}{3} \end{aligned}$$

command

```
integrate(x^2*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{3} x^3 S(bd \log(cx^n) + ad) \\ & - \frac{1}{6} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} - \frac{9i}{2 \pi b^2 d^2 n^2}\right)} C \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \\ & + \frac{1}{6} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} + \frac{9i}{2 \pi b^2 d^2 n^2}\right)} C \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \\ & - \frac{1}{6} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} - \frac{9i}{2 \pi b^2 d^2 n^2}\right)} S \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \\ & - \frac{1}{6} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} + \frac{9i}{2 \pi b^2 d^2 n^2}\right)} S \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \operatorname{fresnels}(bd \log(cx^n) + ad), x)$$

110.36 Problem number 55

$$\int x S(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned} & \left(\frac{1}{8} - \frac{i}{8}\right) e^{-\frac{2ab d^2 n \pi + 2i}{b^2 d^2 n^2 \pi}} x^2 \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} + iab d^2 \pi + ib^2 d^2 \pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right) (cx^n)^{-\frac{2}{n}} \\ & + \left(\frac{1}{8} - \frac{i}{8}\right) x^2 \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} - iab d^2 \pi - ib^2 d^2 \pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right) e^{-\frac{2(ab d^2 n \pi + i)}{b^2 d^2 n^2 \pi}} (cx^n)^{-\frac{2}{n}} \\ & + \frac{x^2 S(d(a + b \ln(cx^n)))}{2} \end{aligned}$$

command

```
integrate(x*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{1}{4} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} - \frac{2i}{\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\ & + \frac{1}{4} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} + \frac{2i}{\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\ & - \frac{1}{4} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} - \frac{2i}{\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\ & - \frac{1}{4} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} + \frac{2i}{\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\ & + \frac{1}{2} x^2 S(bd \log(cx^n) + ad) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \operatorname{fresnels}(bd \log(cx^n) + ad), x)$$

110.37 Problem number 56

$$\int S(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned} & \left(\frac{1}{4} - \frac{i}{4}\right) x \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{1}{n} + iab d^2 \pi + ib^2 d^2 \pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right) e^{-\frac{2abn - \frac{i}{2}\pi}{2b^2 d^2 n^2} (cx^n)^{-\frac{1}{n}}} \\ & + \left(\frac{1}{4} - \frac{i}{4}\right) x \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{1}{n} - iab d^2 \pi - ib^2 d^2 \pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right) e^{-\frac{2abn + \frac{i}{2}\pi}{2b^2 d^2 n^2} (cx^n)^{-\frac{1}{n}}} \\ & + x S(d(a + b \ln(cx^n))) \end{aligned}$$

command

```
integrate(fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & -\frac{1}{2} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} - \frac{i}{2\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\ & + \frac{1}{2} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} + \frac{i}{2\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\ & - \frac{1}{2} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} - \frac{i}{2\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\ & - \frac{1}{2} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} + \frac{i}{2\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\ & + x S(bd \log(cx^n) + ad) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(fresnels(bd log(cx^n) + ad), x)
```

110.38 Problem number 57

$$\int \frac{S(d(a + b \log(cx^n)))}{x} dx$$

Optimal antiderivative

$$\frac{\cos\left(\frac{d^2 \pi (a + b \ln(cx^n))^2}{2}\right)}{bdn\pi} + \frac{S(d(a + b \ln(cx^n))) (a + b \ln(cx^n))}{bn}$$

command

```
integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\pi b d n \log(x) + \pi b d \log(c) + \pi a d) S(b d \log(c x^n) + a d) + \cos\left(\frac{1}{2} \pi b^2 d^2 n^2 \log(x)^2 + \pi b^2 d^2 n \log(c) \log(x) + \frac{1}{2} \pi b^2 d^2\right)}{\pi b d n}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{fresnels}(b d \log(c x^n) + a d)}{x}, x\right)$$

110.39 Problem number 58

$$\int \frac{S(d(a + b \log(c x^n)))}{x^2} dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{2abn + \frac{i}{2}d^2\pi}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{1}{n} - iab d^2\pi - ib^2 d^2\pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{2abn - \frac{i}{2}d^2\pi}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{1}{n} + iab d^2\pi + ib^2 d^2\pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{S(d(a + b \ln(cx^n)))}{x}$$

command

```
integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{-i \pi \sqrt{b^2 d^2 n^2} x e^{\left(\frac{\log(c)}{n} + \frac{a}{bn} + \frac{i}{2\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) + i \pi \sqrt{b^2 d^2 n^2} x e^{\left(\frac{\log(c)}{n} + \frac{a}{bn}\right)}}{x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{fresnels}(b d \log(c x^n) + a d)}{x^2}, x\right)$$

110.40 Problem number 59

$$\int \frac{S(d(a + b \log(cx^n)))}{x^3} dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) e^{\frac{2ab d^2 n \pi + 2i}{b^2 d^2 n^2 \pi} (c x^n)^{\frac{2}{n}} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{2}{n} - iab d^2 \pi - ib^2 d^2 \pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right)}}{x^2} + \frac{\left(\frac{1}{8} - \frac{i}{8}\right) (c x^n)^{\frac{2}{n}} \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{2}{n} + iab d^2 \pi + ib^2 d^2 \pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right) e^{-\frac{2(-ab d^2 n \pi + i)}{b^2 d^2 n^2 \pi}}}{x^2} - \frac{S(d(a + b \ln(cx^n)))}{2x^2}$$

command

`integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i \pi \sqrt{b^2 d^2 n^2} x^2 e^{\left(\frac{2 \log(c)}{n} + \frac{2a}{bn} + \frac{2i}{\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) + i \pi \sqrt{b^2 d^2 n^2} x^2 e^{\left(\frac{2 \log(c)}{n}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bd \log(cx^n) + ad)}{x^3}, x\right)$$

110.41 Problem number 60

$$\int (ex)^m S(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{i(1+m)(2iab d^2 n \pi + m + 1)}{2b^2 d^2 n^2 \pi}} x (ex)^m \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(1+m + iab d^2 n \pi + ib^2 d^2 n \pi \ln(cx^n))}{bdn\sqrt{\pi}}\right) (c x^n)^{-\frac{1+m}{n}}}{1+m} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) x (ex)^m \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(1+m - iab d^2 n \pi - ib^2 d^2 n \pi \ln(cx^n))}{bdn\sqrt{\pi}}\right) e^{-\frac{i(1+m)(-2iab d^2 n \pi + m + 1)}{2b^2 d^2 n^2 \pi}} (c x^n)^{-\frac{1+m}{n}}}{1+m} + \frac{(ex)^{1+m} S(d(a + b \ln(cx^n)))}{e(1+m)}$$

command

```
integrate((e*x)^m*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-i\pi\sqrt{b^2d^2n^2}e^{\left(m-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{im^2}{2\pi b^2d^2n^2}-\frac{im}{\pi b^2d^2n^2}-\frac{i}{2\pi b^2d^2n^2}\right)}C\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+im+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}((ex)^m \text{fresnels}(bd \log(cx^n) + ad), x)$$

110.42 Problem number 65

$$\int S(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal antiderivative

$$\frac{S(bx)^3}{3b}$$

command

```
integrate(fresnel_sin(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{S(bx)^3}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\text{fresnels}(bx)^2 \sin\left(\frac{1}{2}\pi b^2x^2\right), x\right)$$

110.43 Problem number 66

$$\int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal antiderivative

$$\frac{S(bx)^2}{2b}$$

command

```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{S(bx)^2}{2b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\text{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right), x\right)$$

110.44 Problem number 67

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)} dx$$

Optimal antiderivative

$$\frac{\ln(S(bx))}{b}$$

command

```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\log(S(bx))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)}, x\right)$$

110.45 Problem number 68

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^2} dx$$

Optimal antiderivative

$$-\frac{1}{bS(bx)}$$

command

```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{bS(bx)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)^2}, x\right)$$

110.46 Problem number 69

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^3} dx$$

Optimal antiderivative

$$-\frac{1}{2bS(bx)^2}$$

command

```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2bS(bx)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)^3}, x\right)$$

110.47 Problem number 70

$$\int S(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal antiderivative

$$\frac{S(bx)^{1+n}}{b(1+n)}$$

command

```
integrate(fresnel_sin(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{S(bx)^n S(bx)}{bn + b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\text{fresnels}(bx)^n \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

110.48 Problem number 71

$$\int x^8 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2 \cos(b^2\pi x^2)}{4b^7\pi^4} - \frac{x^6 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{35x^3 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^6\pi^3} \\ & - \frac{x^7 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^2\pi} + \frac{105S(bx)^2}{2b^9\pi^4} - \frac{105x S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^8\pi^4} \\ & + \frac{7x^5 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2} - \frac{40 \sin(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4 \sin(b^2\pi x^2)}{2b^5\pi^3} \end{aligned}$$

command

```
integrate(x^8*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\pi^3 b^6 x^6 - 75\pi b^2 x^2 + 3(\pi^3 b^6 x^6 - 55\pi b^2 x^2) \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 + 6(\pi^4 b^7 x^7 - 35\pi^2 b^3 x^3) \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx) - 315\pi}{6\pi^5 b^9}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^8 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

110.49 Problem number 72

$$\int x^7 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} + \frac{147x \cos(b^2\pi x^2)}{16b^7\pi^4} - \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} \\ & + \frac{24x^2 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^6\pi^3} - \frac{x^6 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^2\pi} - \frac{48 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^8\pi^4} \\ & + \frac{6x^4 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2} + \frac{17x^3 \sin(b^2\pi x^2)}{8b^5\pi^3} - \frac{531 \operatorname{FresnelC}\left(bx\sqrt{2}\right) \sqrt{2}}{32b^8\pi^4} \end{aligned}$$

command

```
integrate(x^7*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{56\pi^2 b^6 x^5 - 2370b^2x + 20(4\pi^2 b^6 x^5 - 147b^2x) \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 160(\pi^3 b^7 x^6 - 24\pi b^3 x^2) \cos\left(\frac{1}{2}\pi b^2 x^2\right) S(bx) + 2}{160\pi^4 b^9}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x^7 \operatorname{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right), x\right)$$

110.50 Problem number 74

$$\int x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2x^3}{3b^3\pi^2} - \frac{x^3 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{8 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^6\pi^3} - \frac{x^4 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^2\pi} \\ & + \frac{4x^2 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2} + \frac{11x \sin(b^2\pi x^2)}{8b^5\pi^3} - \frac{43 S\left(bx\sqrt{2}\right) \sqrt{2}}{16b^6\pi^3} \end{aligned}$$

command

```
integrate(x^5*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{24 \pi b^4 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 + 20 \pi b^4 x^3 + 48 (\pi^2 b^5 x^4 - 8 b) \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx) + 129 \sqrt{2} \sqrt{b^2} S\left(\sqrt{2} \sqrt{b^2} x\right) - 12}{48 \pi^3 b^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^5 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

110.51 Problem number 75

$$\int x^4 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx$$

Optimal antiderivative

$$-\frac{3x^2}{4b^3\pi^2} - \frac{x^2 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x^3 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^2\pi} - \frac{3S(bx)^2}{2b^5\pi^2} + \frac{3x S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2} + \frac{\sin(b^2\pi x^2)}{b^5\pi^3}$$

command

```
integrate(x^4*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \pi^2 b^3 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx) + \pi b^2 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 + \pi b^2 x^2 + 3 \pi S(bx)^2 - 2 (3 \pi b x S(bx) + 2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)) \sin(bx)}{2 \pi^3 b^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^4 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

110.52 Problem number 76

$$\int x^3 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx$$

Optimal antiderivative

$$-\frac{x}{b^3\pi^2} - \frac{x \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x^2 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^2\pi} + \frac{2 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2} + \frac{5 \text{FresnelC}\left(bx\sqrt{2}\right) \sqrt{2}}{8b^4\pi^2}$$

command

```
integrate(x^3*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{8 \pi b^3 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx) + 4 b^2 x \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 + 6 b^2 x - 16 b S(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) - 5 \sqrt{2} \sqrt{b^2} C\left(\sqrt{2} \sqrt{b^2} x\right)}{8 \pi^2 b^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^3 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

110.53 Problem number 78

$$\int x S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx$$

Optimal antiderivative

$$-\frac{\cos\left(\frac{b^2 \pi x^2}{2}\right) S(bx)}{b^2 \pi} + \frac{S(bx \sqrt{2}) \sqrt{2}}{4 b^2 \pi}$$

command

```
integrate(x*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 b \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx) - \sqrt{2} \sqrt{b^2} S\left(\sqrt{2} \sqrt{b^2} x\right)}{4 \pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

110.54 Problem number 79

$$\int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal antiderivative

$$\frac{S(bx)^2}{2b}$$

command

```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{S(bx)^2}{2b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\text{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right), x\right)$$

110.55 Problem number 83

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b}{12x^2} + \frac{b \cos(b^2\pi x^2)}{12x^2} - \frac{b^2\pi \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{3x} \\ & - \frac{b^3\pi^2 S(bx)^2}{6} + \frac{b^3\pi \text{Si}(b^2\pi x^2)}{6} - \frac{S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{3x^3} \end{aligned}$$

command

```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^3 x^3 S(bx)^2 - \pi b^3 x^3 \text{Si}(\pi b^2 x^2) + 2\pi b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) S(bx) - bx \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + bx + 2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^4}, x\right)$$

110.56 Problem number 87

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b \cos(b^2\pi x^2)}{84x^6} - \frac{b^5\pi^2 \cos(b^2\pi x^2)}{84x^2} - \frac{b^2\pi \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{35x^5} \\ & + \frac{b^6\pi^3 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{105x} + \frac{b^7\pi^4 S(bx)^2}{210} - \frac{b^7\pi^3 \operatorname{sinIntegral}(b^2\pi x^2)}{70} \\ & - \frac{S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{7x^7} + \frac{b^4\pi^2 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{105x^3} - \frac{b^3\pi \sin(b^2\pi x^2)}{105x^4} \end{aligned}$$

command

```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^4 b^7 x^7 S(bx)^2 - 3\pi^3 b^7 x^7 \operatorname{Si}(\pi b^2 x^2) + 3\pi^2 b^5 x^5 - 5(\pi^2 b^5 x^5 - bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 2(\pi^3 b^6 x^6 - 3\pi b^2 x^2) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{210 x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^8}, x\right)$$

110.57 Problem number 92

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4x^3}{b^5\pi^3} - \frac{x^7}{14b\pi} + \frac{17x^3 \cos(b^2\pi x^2)}{8b^5\pi^3} - \frac{48 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^8\pi^4} \\ & + \frac{6x^4 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^4\pi^2} - \frac{24x^2 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^6\pi^3} + \frac{x^6 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} \\ & - \frac{147x \sin(b^2\pi x^2)}{16b^7\pi^4} + \frac{x^5 \sin(b^2\pi x^2)}{4b^3\pi^2} + \frac{531 S(bx\sqrt{2})\sqrt{2}}{32b^8\pi^4} \end{aligned}$$

command

```
integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{16 \pi^3 b^8 x^7 - 952 \pi b^4 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 - 420 \pi b^4 x^3 - 1344 (\pi^2 b^5 x^4 - 8 b) \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx) - 3717 \sqrt{2} \sqrt{b^2} S\left(\frac{1}{2} \pi b^2 x^2\right)}{224 \pi^4 b^9}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^7 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

110.58 Problem number 93

$$\int x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2 \cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{5x^3 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^4\pi^2} + \frac{15S(bx)^2}{2b^7\pi^3} \\ & - \frac{15x S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^6\pi^3} + \frac{x^5 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} - \frac{11 \sin(b^2\pi x^2)}{2b^7\pi^4} + \frac{x^4 \sin(b^2\pi x^2)}{4b^3\pi^2} \end{aligned}$$

command

```
integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^3 b^6 x^6 - 60 \pi^2 b^3 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx) - 42 \pi b^2 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 - 24 \pi b^2 x^2 - 90 \pi S(bx)^2 - 6 ((\pi^2 b^4 x^4 - 22) \cos\left(\frac{1}{2} \pi b^2 x^2\right) S(bx))}{12 \pi^4 b^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^6 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

110.59 Problem number 94

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$$

Optimal antiderivative

$$\frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x \cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{4x^2 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^4\pi^2} - \frac{8 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^6\pi^3}$$

$$+ \frac{x^4 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} + \frac{x^3 \sin(b^2\pi x^2)}{4b^3\pi^2} - \frac{43 \operatorname{FresnelC}\left(bx\sqrt{2}\right) \sqrt{2}}{16b^6\pi^3}$$

command

```
integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{8\pi^2 b^6 x^5 - 320\pi b^3 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) S(bx) - 220b^2 x \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 - 210b^2 x + 215\sqrt{2}\sqrt{b^2} C\left(\sqrt{2}\sqrt{b^2}x\right) - 40}{80\pi^3 b^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \operatorname{fresnels}(bx), x\right)$$

110.60 Problem number 96

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$$

Optimal antiderivative

$$-\frac{x^3}{6b\pi} + \frac{2 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{b^4\pi^2} + \frac{x^2 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} + \frac{x \sin(b^2\pi x^2)}{4b^3\pi^2} - \frac{5 S(bx\sqrt{2}) \sqrt{2}}{8b^4\pi^2}$$

command

```
integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4\pi b^4 x^3 - 48b \cos\left(\frac{1}{2}\pi b^2 x^2\right) S(bx) + 15\sqrt{2}\sqrt{b^2} S\left(\sqrt{2}\sqrt{b^2}x\right) - 12\left(2\pi b^3 x^2 S(bx) + b^2 x \cos\left(\frac{1}{2}\pi b^2 x^2\right)\right) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{24\pi^2 b^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \operatorname{fresnels}(bx), x\right)$$

110.61 Problem number 97

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$$

Optimal antiderivative

$$-\frac{x^2}{4b\pi} - \frac{S(bx)^2}{2b^3\pi} + \frac{x S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} + \frac{\sin(b^2\pi x^2)}{4b^3\pi^2}$$

command

`integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\pi b^2 x^2 + 2\pi S(bx)^2 - 2(2\pi bx S(bx) + \cos\left(\frac{1}{2}\pi b^2 x^2\right)) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

110.62 Problem number 98

$$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$$

Optimal antiderivative

$$-\frac{x}{2b\pi} + \frac{S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} + \frac{\text{FresnelC}\left(bx\sqrt{2}\right) \sqrt{2}}{4b^2\pi}$$

command

`integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2b^2x - 4bS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) - \sqrt{2} \sqrt{b^2} C\left(\sqrt{2} \sqrt{b^2} x\right)}{4\pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

110.63 Problem number 101

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^2} dx$$

Optimal antiderivative

$$-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{x} - \frac{b\pi S(bx)^2}{2} + \frac{b \operatorname{sinIntegral}(b^2\pi x^2)}{4}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\pi b x S(bx)^2 - bx \operatorname{Si}(\pi b^2 x^2) + 4 \cos\left(\frac{1}{2}\pi b^2 x^2\right) S(bx)}{4x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \operatorname{fresnels}(bx)}{x^2}, x\right)$$

110.64 Problem number 105

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^3\pi}{60x^2} - \frac{b^3\pi \cos(b^2\pi x^2)}{24x^2} - \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{5x^5} + \frac{b^4\pi^2 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{15x} \\ & + \frac{b^5\pi^3 S(bx)^2}{30} - \frac{7b^5\pi^2 \operatorname{sinIntegral}(b^2\pi x^2)}{120} + \frac{b^2\pi S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{15x^3} - \frac{b \sin(b^2\pi x^2)}{40x^4} \end{aligned}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4\pi^3 b^5 x^5 S(bx)^2 - 7\pi^2 b^5 x^5 \operatorname{Si}(\pi b^2 x^2) - 10\pi b^3 x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 7\pi b^3 x^3 + 8(\pi^2 b^4 x^4 - 3) \cos\left(\frac{1}{2}\pi b^2 x^2\right) S(bx) + b \sin(b^2\pi x^2)}{120x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \operatorname{fresnels}(bx)}{x^6}, x\right)$$

110.65 Problem number 109

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^{10}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi \cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3 \cos(b^2\pi x^2)}{2016x^2} \\ & - \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{9x^9} + \frac{b^4\pi^2 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{315x^5} - \frac{b^8\pi^4 \cos\left(\frac{b^2\pi x^2}{2}\right) S(bx)}{945x} \\ & - \frac{b^9\pi^5 S(bx)^2}{1890} + \frac{83b^9\pi^4 \sin\text{Integral}(b^2\pi x^2)}{30240} + \frac{b^2\pi S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{63x^7} \\ & - \frac{b^6\pi^3 S(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{945x^3} - \frac{b \sin(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2 \sin(b^2\pi x^2)}{30240x^4} \end{aligned}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^10,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{16\pi^5 b^9 x^9 S(bx)^2 - 83\pi^4 b^9 x^9 \text{Si}(\pi b^2 x^2) + 83\pi^3 b^7 x^7 - 150\pi b^3 x^3 - 10(15\pi^3 b^7 x^7 - 22\pi b^3 x^3) \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 3}{30240x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^{10}}, x\right)$$

110.66 Problem number 110

$$\int x^7 \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{105x \cos\left(\frac{b^2\pi x^2}{2}\right)}{8b^7\pi^4} - \frac{7x^5 \cos\left(\frac{b^2\pi x^2}{2}\right)}{8b^3\pi^2} - \frac{105 \text{FresnelC}(bx)}{8b^8\pi^4} \\ & + \frac{x^8 \text{FresnelC}(bx)}{8} + \frac{35x^3 \sin\left(\frac{b^2\pi x^2}{2}\right)}{8b^5\pi^3} - \frac{x^7 \sin\left(\frac{b^2\pi x^2}{2}\right)}{8b\pi} \end{aligned}$$

command

`integrate(x^7*fresnel_cos(b*x),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{7(\pi^2 b^5 x^5 - 15bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right) - (\pi^4 b^8 x^8 - 105) C(bx) + (\pi^3 b^7 x^7 - 35\pi b^3 x^3) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^4 b^8}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral(x^7fresnelc(bx), x)`

110.67 Problem number 111

$$\int x^6 \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$\frac{48 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{7b^7 \pi^4} - \frac{6x^4 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{7b^3 \pi^2} + \frac{x^7 \text{FresnelC}(bx)}{7} + \frac{24x^2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{7b^5 \pi^3} - \frac{x^6 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{7b\pi}$$

command

`integrate(x^6*fresnel_cos(b*x),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^4 b^7 x^7 C(bx) - 6(\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2}\pi b^2 x^2\right) - (\pi^3 b^6 x^6 - 24\pi b^2 x^2) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^4 b^7}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral(x^6fresnelc(bx), x)`

110.68 Problem number 112

$$\int x^5 \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$-\frac{5x^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{6b^3 \pi^2} + \frac{x^6 \text{FresnelC}(bx)}{6} - \frac{5S(bx)}{2b^6 \pi^3} + \frac{5x \sin\left(\frac{b^2 \pi x^2}{2}\right)}{2b^5 \pi^3} - \frac{x^5 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{6b\pi}$$

command

```
integrate(x^5*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^3 b^7 x^6 C(bx) - 5\pi b^4 x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right) - (\pi^2 b^6 x^5 - 15b^2 x) \sin\left(\frac{1}{2}\pi b^2 x^2\right) - 15\sqrt{b^2} S\left(\sqrt{b^2} x\right)}{6\pi^3 b^7}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(x^5fresnelc(bx),x)
```

110.69 Problem number 113

$$\int x^4 \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$-\frac{4x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{5b^3 \pi^2} + \frac{x^5 \text{FresnelC}(bx)}{5} + \frac{8 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{5b^5 \pi^3} - \frac{x^4 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{5b\pi}$$

command

```
integrate(x^4*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^3 b^5 x^5 C(bx) - 4\pi b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) - (\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(x^4fresnelc(bx),x)
```

110.70 Problem number 114

$$\int x^3 \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$-\frac{3x \cos\left(\frac{b^2 \pi x^2}{2}\right)}{4b^3 \pi^2} + \frac{3 \text{FresnelC}(bx)}{4b^4 \pi^2} + \frac{x^4 \text{FresnelC}(bx)}{4} - \frac{x^3 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{4b\pi}$$

command

```
integrate(x^3*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\pi b^3 x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 3 b x \cos\left(\frac{1}{2} \pi b^2 x^2\right) - (\pi^2 b^4 x^4 + 3) C(bx)}{4 \pi^2 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(x^3fresnelc(bx),x)
```

110.71 Problem number 115

$$\int x^2 \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$-\frac{2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{3 b^3 \pi^2} + \frac{x^3 \text{FresnelC}(bx)}{3} - \frac{x^2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{3 b \pi}$$

command

```
integrate(x^2*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^3 x^3 C(bx) - \pi b^2 x^2 \sin\left(\frac{1}{2} \pi b^2 x^2\right) - 2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3 \pi^2 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(x^2fresnelc(bx),x)
```

110.72 Problem number 116

$$\int x \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$\frac{x^2 \text{FresnelC}(bx)}{2} + \frac{S(bx)}{2 b^2 \pi} - \frac{x \sin\left(\frac{b^2 \pi x^2}{2}\right)}{2 b \pi}$$

command

```
integrate(x*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^3 x^2 C(bx) - b^2 x \sin\left(\frac{1}{2} \pi b^2 x^2\right) + \sqrt{b^2} S\left(\sqrt{b^2} x\right)}{2 \pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(xfresnelc(bx), x)
```

110.73 Problem number 117

$$\int \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$x \text{FresnelC}(bx) - \frac{\sin\left(\frac{b^2 \pi x^2}{2}\right)}{b\pi}$$

command

```
integrate(fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b x C(bx) - \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(fresnelc(bx), x)
```

110.74 Problem number 119

$$\int \frac{\text{FresnelC}(bx)}{x^2} dx$$

Optimal antiderivative

$$\frac{b \text{cosineIntegral}\left(\frac{b^2 \pi x^2}{2}\right)}{2} - \frac{\text{FresnelC}(bx)}{x}$$

command

```
integrate(fresnel_cos(b*x)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bx \operatorname{Ci}\left(\frac{1}{2}\pi b^2 x^2\right) + bx \operatorname{Ci}\left(-\frac{1}{2}\pi b^2 x^2\right) - 4 C(bx)}{4x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)}{x^2}, x\right)$$

110.75 Problem number 120

$$\int \frac{\operatorname{FresnelC}(bx)}{x^3} dx$$

Optimal antiderivative

$$-\frac{b \cos\left(\frac{b^2 \pi x^2}{2}\right)}{2x} - \frac{\operatorname{FresnelC}(bx)}{2x^2} - \frac{b^2 \pi S(bx)}{2}$$

command

```
integrate(fresnel_cos(b*x)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\pi \sqrt{b^2} bx^2 S\left(\sqrt{b^2} x\right) + bx \cos\left(\frac{1}{2}\pi b^2 x^2\right) + C(bx)}{2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)}{x^3}, x\right)$$

110.76 Problem number 121

$$\int \frac{\operatorname{FresnelC}(bx)}{x^4} dx$$

Optimal antiderivative

$$-\frac{b \cos\left(\frac{b^2 \pi x^2}{2}\right)}{6x^2} - \frac{\operatorname{FresnelC}(bx)}{3x^3} - \frac{b^3 \pi \operatorname{sinIntegral}\left(\frac{b^2 \pi x^2}{2}\right)}{12}$$

command

```
integrate(fresnel_cos(b*x)/x^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^3 x^3 \operatorname{Si}\left(\frac{1}{2} \pi b^2 x^2\right) + 2 b x \cos\left(\frac{1}{2} \pi b^2 x^2\right) + 4 C(bx)}{12 x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)}{x^4}, x\right)$$

110.77 Problem number 122

$$\int \frac{\operatorname{FresnelC}(bx)}{x^5} dx$$

Optimal antiderivative

$$-\frac{b \cos\left(\frac{b^2 \pi x^2}{2}\right)}{12 x^3} - \frac{b^4 \pi^2 \operatorname{FresnelC}(bx)}{12} - \frac{\operatorname{FresnelC}(bx)}{4 x^4} + \frac{b^3 \pi \sin\left(\frac{b^2 \pi x^2}{2}\right)}{12 x}$$

command

```
integrate(fresnel_cos(b*x)/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^3 x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right) - b x \cos\left(\frac{1}{2} \pi b^2 x^2\right) - (\pi^2 b^4 x^4 + 3) C(bx)}{12 x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)}{x^5}, x\right)$$

110.78 Problem number 123

$$\int \frac{\operatorname{FresnelC}(bx)}{x^6} dx$$

Optimal antiderivative

$$-\frac{b^5 \pi^2 \operatorname{cosineIntegral}\left(\frac{b^2 \pi x^2}{2}\right)}{80} - \frac{b \cos\left(\frac{b^2 \pi x^2}{2}\right)}{20 x^4} - \frac{\operatorname{FresnelC}(bx)}{5 x^5} + \frac{b^3 \pi \sin\left(\frac{b^2 \pi x^2}{2}\right)}{40 x^2}$$

command

```
integrate(fresnel_cos(b*x)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^5 x^5 \operatorname{Ci}\left(\frac{1}{2} \pi b^2 x^2\right) + \pi^2 b^5 x^5 \operatorname{Ci}\left(-\frac{1}{2} \pi b^2 x^2\right) - 4 \pi b^3 x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 8 b x \cos\left(\frac{1}{2} \pi b^2 x^2\right) + 32 C(bx)}{160 x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)}{x^6}, x\right)$$

110.79 Problem number 124

$$\int \frac{\operatorname{FresnelC}(bx)}{x^7} dx$$

Optimal antiderivative

$$-\frac{b \cos\left(\frac{b^2 \pi x^2}{2}\right)}{30 x^5} + \frac{b^5 \pi^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{90 x} - \frac{\operatorname{FresnelC}(bx)}{6 x^6} + \frac{b^6 \pi^3 \operatorname{S}(bx)}{90} + \frac{b^3 \pi \sin\left(\frac{b^2 \pi x^2}{2}\right)}{90 x^3}$$

command

```
integrate(fresnel_cos(b*x)/x^7,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^3 \sqrt{b^2} b^5 x^6 \operatorname{S}\left(\sqrt{b^2} x\right) + \pi b^3 x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right) + \left(\pi^2 b^5 x^5 - 3 b x\right) \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 15 C(bx)}{90 x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)}{x^7}, x\right)$$

110.80 Problem number 125

$$\int \frac{\operatorname{FresnelC}(bx)}{x^8} dx$$

Optimal antiderivative

$$-\frac{b \cos\left(\frac{b^2 \pi x^2}{2}\right)}{42 x^6} + \frac{b^5 \pi^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{336 x^2} - \frac{\operatorname{FresnelC}(bx)}{7 x^7} + \frac{b^7 \pi^3 \operatorname{sinIntegral}\left(\frac{b^2 \pi x^2}{2}\right)}{672} + \frac{b^3 \pi \sin\left(\frac{b^2 \pi x^2}{2}\right)}{168 x^4}$$

command

```
integrate(fresnel_cos(b*x)/x^8,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^3 b^7 x^7 \operatorname{Si}\left(\frac{1}{2} \pi b^2 x^2\right) + 4 \pi b^3 x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 2\left(\pi^2 b^5 x^5 - 8 b x\right) \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 96 C(bx)}{672 x^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)}{x^8}, x\right)$$

110.81 Problem number 126

$$\int \frac{\operatorname{FresnelC}(bx)}{x^9} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b \cos\left(\frac{b^2 \pi x^2}{2}\right)}{56 x^7} + \frac{b^5 \pi^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{840 x^3} + \frac{b^8 \pi^4 \operatorname{FresnelC}(bx)}{840} \\ & -\frac{\operatorname{FresnelC}(bx)}{8 x^8} + \frac{b^3 \pi \sin\left(\frac{b^2 \pi x^2}{2}\right)}{280 x^5} - \frac{b^7 \pi^3 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{840 x} \end{aligned}$$

command

```
integrate(fresnel_cos(b*x)/x^9,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\pi^2 b^5 x^5 - 15 b x) \cos\left(\frac{1}{2} \pi b^2 x^2\right) + (\pi^4 b^8 x^8 - 105) C(bx) - (\pi^3 b^7 x^7 - 3 \pi b^3 x^3) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{840 x^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)}{x^9}, x\right)$$

110.82 Problem number 127

$$\int \frac{\text{FresnelC}(bx)}{x^{10}} dx$$

Optimal antiderivative

$$\frac{b^9 \pi^4 \text{cosineIntegral}\left(\frac{b^2 \pi x^2}{2}\right)}{6912} - \frac{b \cos\left(\frac{b^2 \pi x^2}{2}\right)}{72x^8} + \frac{b^5 \pi^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{1728x^4}$$

$$- \frac{\text{FresnelC}(bx)}{9x^9} + \frac{b^3 \pi \sin\left(\frac{b^2 \pi x^2}{2}\right)}{432x^6} - \frac{b^7 \pi^3 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{3456x^2}$$

command

```
integrate(fresnel_cos(b*x)/x^10,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^4 b^9 x^9 \text{Ci}\left(\frac{1}{2} \pi b^2 x^2\right) + \pi^4 b^9 x^9 \text{Ci}\left(-\frac{1}{2} \pi b^2 x^2\right) + 8(\pi^2 b^5 x^5 - 24bx) \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 4(\pi^3 b^7 x^7 - 8\pi b^3 x^3) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{13824 x^9}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^{10}}, x\right)$$

110.83 Problem number 128

$$\int (c + dx)^3 \text{FresnelC}(a + bx) dx$$

Optimal antiderivative

$$-\frac{2d^2(-ad + bc) \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4 \pi^2} - \frac{3d^3(bx + a) \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{4b^4 \pi^2}$$

$$- \frac{(-ad + bc)^4 \text{FresnelC}(bx + a)}{4b^4 d} + \frac{3d^3 \text{FresnelC}(bx + a)}{4b^4 \pi^2}$$

$$+ \frac{(dx + c)^4 \text{FresnelC}(bx + a)}{4d} + \frac{3d(-ad + bc)^2 S(bx + a)}{2b^4 \pi}$$

$$- \frac{(-ad + bc)^3 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4 \pi} - \frac{3d(-ad + bc)^2 (bx + a) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2b^4 \pi}$$

$$- \frac{d^2(-ad + bc)(bx + a)^2 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4 \pi} - \frac{d^3(bx + a)^3 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{4b^4 \pi}$$

command

```
integrate((d*x+c)^3*fresnel_cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{6\pi(b^2c^2d - 2abcd^2 + a^2d^3)\sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + (\pi^2(4ab^3c^3 - 6a^2b^2c^2d + 4a^3bcd^2 - a^4d^3) + 3d^3)\sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}((d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3)\text{fresnelc}(bx + a), x)$$

110.84 Problem number 129

$$\int (c + dx)^2 \text{FresnelC}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2d^2 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{3b^3\pi^2} - \frac{(-ad + bc)^3 \text{FresnelC}(bx + a)}{3b^3d} + \frac{(dx + c)^3 \text{FresnelC}(bx + a)}{3d} \\ & + \frac{d(-ad + bc) S(bx + a)}{b^3\pi} - \frac{(-ad + bc)^2 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^3\pi} \\ & - \frac{d(-ad + bc)(bx + a) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^3\pi} - \frac{d^2(bx + a)^2 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{3b^3\pi} \end{aligned}$$

command

```
integrate((d*x+c)^2*fresnel_cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2(3ab^2c^2 - 3a^2bcd + a^3d^2)\sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 2bd^2 \cos\left(\frac{1}{2}\pi b^2x^2 + \pi abx + \frac{1}{2}\pi a^2\right) + 3\pi(bcd - ad^2)\sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{1}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}((d^2x^2 + 2cdx + c^2)\text{fresnelc}(bx + a), x)$$

110.85 Problem number 130

$$\int (c + dx) \text{FresnelC}(a + bx) dx$$

Optimal antiderivative

$$-\frac{(-ad + bc)^2 \text{FresnelC}(bx + a)}{2b^2d} + \frac{(dx + c)^2 \text{FresnelC}(bx + a)}{2d} \\ + \frac{d \text{S}(bx + a)}{2b^2\pi} - \frac{(-ad + bc) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^2\pi} - \frac{d(bx + a) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2b^2\pi}$$

command

```
integrate((d*x+c)*fresnel_cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi(2abc - a^2d)\sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + \sqrt{b^2} d \text{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + (\pi b^3 dx^2 + 2\pi b^3 cx) C(bx + a) - (b^2 dx + 2b^2 c - \dots)}{2\pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}((dx + c)\text{fresnelc}(bx + a), x)$$

110.86 Problem number 131

$$\int \text{FresnelC}(a + bx) dx$$

Optimal antiderivative

$$\frac{(bx + a) \text{FresnelC}(bx + a)}{b} - \frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b\pi}$$

command

```
integrate(fresnel_cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\pi bx + \pi a) C(bx + a) - \sin\left(\frac{1}{2} \pi b^2 x^2 + \pi abx + \frac{1}{2} \pi a^2\right)}{\pi b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(\text{fresnelc}(bx + a), x)$$

110.87 Problem number 134

$$\int x^3 \text{FresnelC}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4 \pi^2} - \frac{3(bx+a) \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{4b^4 \pi^2} - \frac{a^4 \text{FresnelC}(bx+a)}{4b^4} \\ & + \frac{3 \text{FresnelC}(bx+a)}{4b^4 \pi^2} + \frac{x^4 \text{FresnelC}(bx+a)}{4} + \frac{3a^2 \text{S}(bx+a)}{2b^4 \pi} + \frac{a^3 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4 \pi} \\ & - \frac{3a^2(bx+a) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2b^4 \pi} + \frac{a(bx+a)^2 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^4 \pi} - \frac{(bx+a)^3 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{4b^4 \pi} \end{aligned}$$

command

```
integrate(x^3*fresnel_cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^5 x^4 C(bx+a) + 6 \pi a^2 \sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (\pi^2 a^4 - 3) \sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (3b^2 x - 5ab) \cos\left(\frac{1}{2} \pi b^2 x^2 + \dots\right)}{4 \pi^2 b^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^3 \text{fresnelc}(bx+a), x)$$

110.88 Problem number 135

$$\int x^2 \text{FresnelC}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cos\left(\frac{\pi(bx+a)^2}{2}\right)}{3b^3 \pi^2} + \frac{a^3 \text{FresnelC}(bx+a)}{3b^3} + \frac{x^3 \text{FresnelC}(bx+a)}{3} - \frac{a \text{S}(bx+a)}{b^3 \pi} \\ & - \frac{a^2 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^3 \pi} + \frac{a(bx+a) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^3 \pi} - \frac{(bx+a)^2 \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{3b^3 \pi} \end{aligned}$$

command

```
integrate(x^2*fresnel_cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^4 x^3 C(bx + a) + \pi^2 a^3 \sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 3\pi a \sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 2b \cos\left(\frac{1}{2}\pi b^2 x^2 + \pi abx + \frac{1}{2}\pi a^2\right) - \dots}{3\pi^2 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^2 \text{fresnelc}(bx + a), x)$$

110.89 Problem number 136

$$\int x \text{FresnelC}(a + bx) dx$$

Optimal antiderivative

$$-\frac{a^2 \text{FresnelC}(bx + a)}{2b^2} + \frac{x^2 \text{FresnelC}(bx + a)}{2} + \frac{S(bx + a)}{2b^2\pi} + \frac{a \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b^2\pi} - \frac{(bx + a) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2b^2\pi}$$

command

`integrate(x*fresnel_cos(b*x+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^3 x^2 C(bx + a) - \pi a^2 \sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (b^2 x - ab) \sin\left(\frac{1}{2}\pi b^2 x^2 + \pi abx + \frac{1}{2}\pi a^2\right) + \sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - \dots}{2\pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x \text{fresnelc}(bx + a), x)$$

110.90 Problem number 137

$$\int \text{FresnelC}(a + bx) dx$$

Optimal antiderivative

$$\frac{(bx + a) \text{FresnelC}(bx + a)}{b} - \frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b\pi}$$

command

`integrate(fresnel_cos(b*x+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\pi b x + \pi a) C(b x + a) - \sin\left(\frac{1}{2} \pi b^2 x^2 + \pi a b x + \frac{1}{2} \pi a^2\right)}{\pi b}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral(fresnelc(bx + a), x)`

110.91 Problem number 140

$$\int x^7 \text{FresnelC}(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2 \cos(b^2\pi x^2)}{16b^6\pi^4} - \frac{x^6 \cos(b^2\pi x^2)}{16b^2\pi^2} \\ & + \frac{105x \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{4b^7\pi^4} - \frac{7x^5 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{4b^3\pi^2} \\ & - \frac{105 \text{FresnelC}(bx)^2}{8b^8\pi^4} + \frac{x^8 \text{FresnelC}(bx)^2}{8} + \frac{35x^3 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^5\pi^3} \\ & - \frac{x^7 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{4b\pi} - \frac{10 \sin(b^2\pi x^2)}{b^8\pi^5} + \frac{5x^4 \sin(b^2\pi x^2)}{8b^4\pi^3} \end{aligned}$$

command

`integrate(x^7*fresnel_cos(b*x)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5 \pi^3 b^6 x^6 - 240 \pi b^2 x^2 - 3 (\pi^3 b^6 x^6 - 55 \pi b^2 x^2) \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 - 42 (\pi^3 b^5 x^5 - 15 \pi b x) \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) - 3 (105}{24 \pi^5 b^8}$$

Fricas 1.3.7 via sagemath 9.3 output

`integral(x^7 fresnelc(bx)^2, x)`

110.92 Problem number 141

$$\int x^6 \text{FresnelC}(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} + \frac{21x \cos(b^2\pi x^2)}{8b^6\pi^4} - \frac{x^5 \cos(b^2\pi x^2)}{14b^2\pi^2} + \frac{96 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{7b^7\pi^4} \\ & - \frac{12x^4 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{7b^3\pi^2} + \frac{x^7 \text{FresnelC}(bx)^2}{7} + \frac{48x^2 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{7b^5\pi^3} \\ & - \frac{2x^6 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{7b\pi} + \frac{17x^3 \sin(b^2\pi x^2)}{28b^4\pi^3} - \frac{531 \text{FresnelC}(bx\sqrt{2})\sqrt{2}}{112b^7\pi^4} \end{aligned}$$

command

```
integrate(x^6*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{80\pi^4 b^8 x^7 C(bx)^2 + 136\pi^2 b^6 x^5 - 5310b^2 x - 20(4\pi^2 b^6 x^5 - 147b^2 x) \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 - 960(\pi^2 b^5 x^4 - 8b) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{56}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^6 \text{fresnelc}(bx)^2, x\right)$$

110.93 Problem number 143

$$\int x^4 \text{FresnelC}(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4x^3}{15b^2\pi^2} - \frac{x^3 \cos(b^2\pi x^2)}{10b^2\pi^2} - \frac{8x^2 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{5b^3\pi^2} \\ & + \frac{x^5 \text{FresnelC}(bx)^2}{5} + \frac{16 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{5b^5\pi^3} \\ & - \frac{2x^4 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{5b\pi} + \frac{11x \sin(b^2\pi x^2)}{20b^4\pi^3} - \frac{43S(bx\sqrt{2})\sqrt{2}}{40b^5\pi^3} \end{aligned}$$

command

`integrate(x^4*fresnel_cos(b*x)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{24 \pi^3 b^6 x^5 C(bx)^2 - 24 \pi b^4 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 + 44 \pi b^4 x^3 - 192 \pi b^3 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) - 129 \sqrt{2} \sqrt{b^2} S\left(\sqrt{2} \sqrt{b} x\right)}{120 \pi^3 b^6}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^4 \text{fresnelc}(bx)^2, x\right)$$

110.94 Problem number 144

$$\int x^3 \text{FresnelC}(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3x^2}{8b^2\pi^2} - \frac{x^2 \cos(b^2\pi x^2)}{8b^2\pi^2} - \frac{3x \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{2b^3\pi^2} + \frac{3 \text{FresnelC}(bx)^2}{4b^4\pi^2} \\ & + \frac{x^4 \text{FresnelC}(bx)^2}{4} - \frac{x^3 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{2b\pi} + \frac{\sin(b^2\pi x^2)}{2b^4\pi^3} \end{aligned}$$

command

`integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^2 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 - 2 \pi b^2 x^2 + 6 \pi b x \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) - (3 \pi + \pi^3 b^4 x^4) C(bx)^2 + 2 (\pi^2 b^3 x^3 C(bx) - 2 \cos(b^2 \pi x^2)) S(b \sqrt{2} x)}{4 \pi^3 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^3 \text{fresnelc}(bx)^2, x\right)$$

110.95 Problem number 145

$$\int x^2 \text{FresnelC}(bx)^2 dx$$

Optimal antiderivative

$$\frac{2x}{3b^2\pi^2} - \frac{x \cos(b^2\pi x^2)}{6b^2\pi^2} - \frac{4 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{3b^3\pi^2} + \frac{x^3 \text{FresnelC}(bx)^2}{3}$$

$$- \frac{2x^2 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{3b\pi} + \frac{5 \text{FresnelC}(bx\sqrt{2}) \sqrt{2}}{12b^3\pi^2}$$

command

```
integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4\pi^2 b^4 x^3 C(bx)^2 - 8\pi b^3 x^2 C(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) - 4b^2 x \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 10b^2 x - 16b \cos\left(\frac{1}{2}\pi b^2 x^2\right) C(bx) + 5\sqrt{2}}{12\pi^2 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^2 \text{fresnelc}(bx)^2, x\right)$$

110.96 Problem number 147

$$\int \text{FresnelC}(bx)^2 dx$$

Optimal antiderivative

$$x \text{FresnelC}(bx)^2 - \frac{2 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b\pi} + \frac{S(bx\sqrt{2}) \sqrt{2}}{2b\pi}$$

command

```
integrate(fresnel_cos(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\pi b^2 x C(bx)^2 - 4b C(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) + \sqrt{2} \sqrt{b^2} S\left(\sqrt{2} \sqrt{b^2} x\right)}{2\pi b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\text{fresnelc}(bx)^2, x\right)$$

110.97 Problem number 152

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b^2}{24x^2} - \frac{b^2 \cos(b^2\pi x^2)}{24x^2} - \frac{b \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{6x^3} - \frac{b^4\pi^2 \text{FresnelC}(bx)^2}{12} \\ & - \frac{\text{FresnelC}(bx)^2}{4x^4} - \frac{b^4\pi \sin\text{Integral}(b^2\pi x^2)}{12} + \frac{b^3\pi \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{6x} \end{aligned}$$

command

```
integrate(fresnel_cos(b*x)^2/x^5,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^4 x^4 \text{Si}(\pi b^2 x^2) - 2\pi b^3 x^3 C(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) + b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) + 2bx \cos\left(\frac{1}{2}\pi b^2 x^2\right) C(bx) + (\pi^2 b^4 x^4 + 3)}{12x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^5}, x\right)$$

110.98 Problem number 156

$$\int \frac{\text{FresnelC}(bx)^2}{x^9} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2 \cos(b^2\pi x^2)}{336x^6} + \frac{b^6\pi^2 \cos(b^2\pi x^2)}{336x^2} \\ & - \frac{b \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{28x^7} + \frac{b^5\pi^2 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{420x^3} \\ & + \frac{b^8\pi^4 \text{FresnelC}(bx)^2}{840} - \frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{b^8\pi^3 \sin\text{Integral}(b^2\pi x^2)}{280} \\ & + \frac{b^3\pi \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{140x^5} - \frac{b^7\pi^3 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{420x} + \frac{b^4\pi \sin(b^2\pi x^2)}{420x^4} \end{aligned}$$

command

`integrate(fresnel_cos(b*x)^2/x^9,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{3 \pi^3 b^8 x^8 \operatorname{Si}(\pi b^2 x^2) - 2 \pi^2 b^6 x^6 + 5 (\pi^2 b^6 x^6 - b^2 x^2) \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 + 2 (\pi^2 b^5 x^5 - 15 b x) \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) + (\pi^4 b^4 x^4 - 15 \pi^2 b^2 x^2) C(bx)^2}{840 x^8}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)^2}{x^9}, x\right)$$

110.99 Problem number 160

$$\int \operatorname{FresnelC}(a + bx)^2 dx$$

Optimal antiderivative

$$\frac{(bx + a) \operatorname{FresnelC}(bx + a)^2}{b} - \frac{2 \operatorname{FresnelC}(bx + a) \sin\left(\frac{\pi(bx+a)^2}{2}\right)}{b\pi} + \frac{S\left((bx + a) \sqrt{2}\right) \sqrt{2}}{2b\pi}$$

command

`integrate(fresnel_cos(b*x+a)^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 (\pi b^2 x + \pi ab) C(bx + a)^2 - 4 b C(bx + a) \sin\left(\frac{1}{2} \pi b^2 x^2 + \pi abx + \frac{1}{2} \pi a^2\right) + \sqrt{2} \sqrt{b^2} S\left(\frac{\sqrt{2} \sqrt{b^2} (bx+a)}{b}\right)}{2 \pi b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\operatorname{fresnelc}(bx + a)^2, x\right)$$

110.100 Problem number 163

$$\int x^2 \operatorname{FresnelC}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned} & \left(\frac{1}{12} + \frac{i}{12} \right) e^{-\frac{3a}{bn} + \frac{9i}{2b^2d^2n^2\pi}} x^3 \operatorname{erf} \left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{3}{n} + iab d^2\pi + ib^2d^2\pi \ln(cx^n)\right)}{bd\sqrt{\pi}} \right) (cx^n)^{-\frac{3}{n}} \\ & + \left(-\frac{1}{12} - \frac{i}{12} \right) e^{-\frac{3a}{bn} - \frac{9i}{2b^2d^2n^2\pi}} x^3 \operatorname{erfi} \left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{3}{n} - iab d^2\pi - ib^2d^2\pi \ln(cx^n)\right)}{bd\sqrt{\pi}} \right) (cx^n)^{-\frac{3}{n}} \\ & + \frac{x^3 \operatorname{FresnelC}(d(a + b \ln(cx^n)))}{3} \end{aligned}$$

command

```
integrate(x^2*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{3} x^3 C(bd \log(cx^n) + ad) \\ & - \frac{1}{6} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} - \frac{9i}{2\pi b^2 d^2 n^2}\right)} C \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \\ & - \frac{1}{6} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} + \frac{9i}{2\pi b^2 d^2 n^2}\right)} C \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \\ & + \frac{1}{6} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} - \frac{9i}{2\pi b^2 d^2 n^2}\right)} S \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \\ & - \frac{1}{6} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} + \frac{9i}{2\pi b^2 d^2 n^2}\right)} S \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \operatorname{fresnelc}(bd \log(cx^n) + ad), x)$$

110.101 Problem number 164

$$\int x \operatorname{FresnelC}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned} & \left(\frac{1}{8} + \frac{i}{8} \right) e^{-\frac{2ab d^2 n \pi + 2i}{b^2 d^2 n^2 \pi}} x^2 \operatorname{erf} \left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} + iab d^2\pi + ib^2d^2\pi \ln(cx^n)\right)}{bd\sqrt{\pi}} \right) (cx^n)^{-\frac{2}{n}} \\ & + \left(-\frac{1}{8} - \frac{i}{8} \right) x^2 \operatorname{erfi} \left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} - iab d^2\pi - ib^2d^2\pi \ln(cx^n)\right)}{bd\sqrt{\pi}} \right) e^{-\frac{2(ab d^2 n \pi + i)}{b^2 d^2 n^2 \pi}} (cx^n)^{-\frac{2}{n}} \\ & + \frac{x^2 \operatorname{FresnelC}(d(a + b \ln(cx^n)))}{2} \end{aligned}$$

command

```
integrate(x*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
& -\frac{1}{4} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} - \frac{2i}{\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& -\frac{1}{4} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} + \frac{2i}{\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& +\frac{1}{4} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} - \frac{2i}{\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& -\frac{1}{4} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} + \frac{2i}{\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& +\frac{1}{2} x^2 C(bd \log(cx^n) + ad)
\end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x\text{fresnelc}(bd \log(cx^n) + ad), x)$$

110.102 Problem number 165

$$\int \text{FresnelC}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned}
& \left(\frac{1}{4} + \frac{i}{4}\right) x \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} + iab d^2 \pi + ib^2 d^2 \pi \ln(cx^n)\right)}{bd \sqrt{\pi}}\right) e^{-\frac{2abn - \frac{i}{d^2} \pi}{2b^2 n^2} (cx^n)^{-\frac{1}{n}}} \\
& + \left(-\frac{1}{4} - \frac{i}{4}\right) x \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} - iab d^2 \pi - ib^2 d^2 \pi \ln(cx^n)\right)}{bd \sqrt{\pi}}\right) e^{-\frac{2abn + \frac{i}{d^2} \pi}{2b^2 n^2} (cx^n)^{-\frac{1}{n}}} \\
& + x \text{FresnelC}(d(a + b \ln(cx^n)))
\end{aligned}$$

command

```
integrate(fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
& -\frac{1}{2} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} - \frac{i}{2\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& -\frac{1}{2} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} + \frac{i}{2\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& +\frac{1}{2} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} - \frac{i}{2\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& -\frac{1}{2} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} + \frac{i}{2\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& + x C(bd \log(cx^n) + ad)
\end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(\text{fresnelc}(bd \log(cx^n) + ad), x)$$

110.103 Problem number 166

$$\int \frac{\text{FresnelC}(d(a + b \log(cx^n)))}{x} dx$$

Optimal antiderivative

$$\frac{\text{FresnelC}(d(a + b \ln(cx^n)))(a + b \ln(cx^n))}{bn} - \frac{\sin\left(\frac{d^2 \pi (a + b \ln(cx^n))^2}{2}\right)}{bdn\pi}$$

command

`integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\pi b d n \log(x) + \pi b d \log(c) + \pi a d) C(bd \log(cx^n) + ad) - \sin\left(\frac{1}{2} \pi b^2 d^2 n^2 \log(x)^2 + \pi b^2 d^2 n \log(c) \log(x) + \frac{1}{2} \pi b^2 d^2\right)}{\pi b d n}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{fresnelc}(bd \log(cx^n) + ad)}{x}, x\right)$$

110.104 Problem number 167

$$\int \frac{\text{FresnelC}(d(a + b \log(cx^n)))}{x^2} dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{\frac{2abn + \frac{i}{d^2}\pi}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{1}{n} - iab d^2\pi - ib^2 d^2\pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} + \frac{\left(-\frac{1}{4} - \frac{i}{4}\right) e^{\frac{2abn - \frac{i}{d^2}\pi}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{1}{n} + iab d^2\pi + ib^2 d^2\pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\text{FresnelC}(d(a + b \ln(cx^n)))}{x}$$

command

`integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\pi\sqrt{b^2d^2n^2} x e^{\left(\frac{\log(c)}{n} + \frac{a}{bn} + \frac{i}{2\pi b^2d^2n^2}\right)} C\left(\frac{(\pi b^2d^2n^2 \log(x) + \pi b^2d^2n \log(c) + \pi ab d^2n + i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) + \pi\sqrt{b^2d^2n^2} x e^{\left(\frac{\log(c)}{n} + \frac{a}{bn} - \frac{i}{2\pi b^2d^2n^2}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bd \log(cx^n) + ad)}{x^2}, x\right)$$

110.105 Problem number 168

$$\int \frac{\text{FresnelC}(d(a + b \log(cx^n)))}{x^3} dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) e^{\frac{2ab d^2 n \pi + 2i}{b^2 d^2 n^2 \pi}} (cx^n)^{\frac{2}{n}} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{2}{n} - iab d^2\pi - ib^2 d^2\pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} + \frac{\left(-\frac{1}{8} - \frac{i}{8}\right) (cx^n)^{\frac{2}{n}} \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{2}{n} + iab d^2\pi + ib^2 d^2\pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right) e^{-\frac{2(-ab d^2 n \pi + i)}{b^2 d^2 n^2 \pi}}}{x^2} - \frac{\text{FresnelC}(d(a + b \ln(cx^n)))}{2x^2}$$

command

```
integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\pi\sqrt{b^2d^2n^2}x^2e^{\left(\frac{2\log(c)}{n}+\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}C\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\pi\sqrt{b^2d^2n^2}x^2e^{\left(\frac{2\log(c)}{n}+\frac{2a}{bn}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{fresnelc}(bd\log(cx^n)+ad)}{x^3},x\right)$$

110.106 Problem number 169

$$\int (ex)^m \text{FresnelC}(d(a+b\log(cx^n))) dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{i(1+m)(2iab d^2 n \pi+m+1)}{2b^2 d^2 n^2 \pi}}x(ex)^m \operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m+iab d^2 n \pi+ib^2 d^2 n \pi \ln(cx^n))}{bdn\sqrt{\pi}}\right)(cx^n)^{-\frac{1+m}{n}}}{1+m} + \frac{\left(-\frac{1}{4}-\frac{i}{4}\right)x(ex)^m \operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m-iab d^2 n \pi-ib^2 d^2 n \pi \ln(cx^n))}{bdn\sqrt{\pi}}\right)e^{-\frac{i(1+m)(-2iab d^2 n \pi+m+1)}{2b^2 d^2 n^2 \pi}}(cx^n)^{-\frac{1+m}{n}}}{1+m} + \frac{(ex)^{1+m} \text{FresnelC}(d(a+b\ln(cx^n)))}{e(1+m)}$$

command

```
integrate((e*x)^m*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\pi\sqrt{b^2d^2n^2}e^{\left(m-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{im^2}{2\pi b^2d^2n^2}-\frac{im}{\pi b^2d^2n^2}-\frac{i}{2\pi b^2d^2n^2}\right)}C\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+im+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}((ex)^m \text{fresnelc}(bd\log(cx^n)+ad),x)$$

110.107 Problem number 174

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2 dx$$

Optimal antiderivative

$$\frac{\text{FresnelC}(bx)^3}{3b}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{C(bx)^3}{3b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)^2, x\right)$$

110.108 Problem number 175

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{C(bx)^2}{2b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

110.109 Problem number 176

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)} dx$$

Optimal antiderivative

$$\frac{\ln(\text{FresnelC}(bx))}{b}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\log(C(bx))}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}(bx)}, x\right)$$

110.110 Problem number 177

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2} dx$$

Optimal antiderivative

$$-\frac{1}{b \text{FresnelC}(bx)}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{b C(bx)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}(bx)^2}, x\right)$$

110.111 Problem number 178

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3} dx$$

Optimal antiderivative

$$-\frac{1}{2b \text{FresnelC}(bx)^2}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2b C(bx)^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}(bx)^3}, x\right)$$

110.112 Problem number 179

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^n dx$$

Optimal antiderivative

$$\frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^n,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{C(bx)^n C(bx)}{bn + b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\text{fresnelc}(bx)^n \cos\left(\frac{1}{2}\pi b^2 x^2\right), x\right)$$

110.113 Problem number 180

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2 \cos(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{105x \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{b^8\pi^4} \\ & + \frac{7x^5 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{b^4\pi^2} + \frac{105 \text{FresnelC}(bx)^2}{2b^9\pi^4} - \frac{35x^3 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^6\pi^3} \\ & + \frac{x^7 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} + \frac{40 \sin(b^2\pi x^2)}{b^9\pi^5} - \frac{5x^4 \sin(b^2\pi x^2)}{2b^5\pi^3} \end{aligned}$$

command

```
integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{5\pi^3 b^6 x^6 - 240\pi b^2 x^2 - 3(\pi^3 b^6 x^6 - 55\pi b^2 x^2) \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 - 42(\pi^3 b^5 x^5 - 15\pi b x) \cos\left(\frac{1}{2}\pi b^2 x^2\right) C(bx) - 315\pi}{6\pi^5 b^9}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^8 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

110.114 Problem number 181

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x \cos(b^2\pi x^2)}{16b^7\pi^4} + \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{48 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{b^8\pi^4} \\ & + \frac{6x^4 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{b^4\pi^2} - \frac{24x^2 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^6\pi^3} \\ & + \frac{x^6 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} - \frac{17x^3 \sin(b^2\pi x^2)}{8b^5\pi^3} + \frac{531 \text{FresnelC}(bx\sqrt{2})\sqrt{2}}{32b^8\pi^4} \end{aligned}$$

command

`integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{136 \pi^2 b^6 x^5 - 5310 b^2 x - 20 (4 \pi^2 b^6 x^5 - 147 b^2 x) \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 - 960 (\pi^2 b^5 x^4 - 8 b) \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) - 2655}{160 \pi^4 b^9}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^7 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

110.115 Problem number 183

$$\int x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2x^3}{3b^3\pi^2} + \frac{x^3 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{4x^2 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{b^4\pi^2} - \frac{8 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^6\pi^3} \\ & + \frac{x^4 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} - \frac{11x \sin(b^2\pi x^2)}{8b^5\pi^3} + \frac{43 S\left(bx\sqrt{2}\right) \sqrt{2}}{16b^6\pi^3} \end{aligned}$$

command

`integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{24 \pi b^4 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 - 44 \pi b^4 x^3 + 192 \pi b^3 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) + 129 \sqrt{2} \sqrt{b^2} S\left(\sqrt{2} \sqrt{b^2} x\right) - 12 (11 b^2 x \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{FresnelC}(bx) - 12 \pi b^2 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \sin\left(\frac{1}{2} \pi b^2 x^2\right))}{48 \pi^3 b^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^5 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

110.116 Problem number 184

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$-\frac{3x^2}{4b^3\pi^2} + \frac{x^2 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{3x \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{b^4\pi^2} \\ - \frac{3 \text{FresnelC}(bx)^2}{2b^5\pi^2} + \frac{x^3 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} - \frac{\sin(b^2\pi x^2)}{b^5\pi^3}$$

command

```
integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 - 2\pi b^2 x^2 + 6\pi b x \cos\left(\frac{1}{2}\pi b^2 x^2\right) C(bx) - 3\pi C(bx)^2 + 2\left(\pi^2 b^3 x^3 C(bx) - 2\cos\left(\frac{1}{2}\pi b^2 x^2\right)\right) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi^3 b^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

110.117 Problem number 185

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$-\frac{x}{b^3\pi^2} + \frac{x \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{2 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{b^4\pi^2} \\ + \frac{x^2 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} - \frac{5 \text{FresnelC}(bx\sqrt{2}) \sqrt{2}}{8b^4\pi^2}$$

command

```
integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{8\pi b^3 x^2 C(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) + 4b^2 x \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 - 10b^2 x + 16b \cos\left(\frac{1}{2}\pi b^2 x^2\right) C(bx) - 5\sqrt{2}\sqrt{b^2} C\left(\sqrt{2}\sqrt{b^2} x\right)}{8\pi^2 b^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

110.118 Problem number 187

$$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$\frac{\text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} - \frac{S(bx\sqrt{2})\sqrt{2}}{4b^2\pi}$$

command

```
integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4bC(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right) - \sqrt{2}\sqrt{b^2}S\left(\sqrt{2}\sqrt{b^2}x\right)}{4\pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x \cos\left(\frac{1}{2}\pi b^2x^2\right) \text{fresnelc}(bx), x\right)$$

110.119 Problem number 188

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$$

Optimal antiderivative

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{C(bx)^2}{2b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\cos\left(\frac{1}{2}\pi b^2x^2\right) \text{fresnelc}(bx), x\right)$$

110.120 Problem number 192

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^4} dx$$

Optimal antiderivative

$$\frac{b}{12x^2} - \frac{b \cos(b^2\pi x^2)}{12x^2} - \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{3x^3} - \frac{b^3\pi^2 \text{FresnelC}(bx)^2}{6}$$

$$- \frac{b^3\pi \sin\text{Integral}(b^2\pi x^2)}{6} + \frac{b^2\pi \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{3x}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^3 x^3 C(bx)^2 + \pi b^3 x^3 \text{Si}(\pi b^2 x^2) - 2\pi b^2 x^2 C(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) + bx \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) C(bx)}{6x^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^4}, x\right)$$

110.121 Problem number 196

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^8} dx$$

Optimal antiderivative

$$-\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b \cos(b^2\pi x^2)}{84x^6} + \frac{b^5\pi^2 \cos(b^2\pi x^2)}{84x^2} - \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{7x^7}$$

$$+ \frac{b^4\pi^2 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{105x^3} + \frac{b^7\pi^4 \text{FresnelC}(bx)^2}{210} + \frac{b^7\pi^3 \sin\text{Integral}(b^2\pi x^2)}{70}$$

$$+ \frac{b^2\pi \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{35x^5} - \frac{b^6\pi^3 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{105x} + \frac{b^3\pi \sin(b^2\pi x^2)}{105x^4}$$

command

```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^8,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^4 b^7 x^7 C(bx)^2 + 3\pi^3 b^7 x^7 \operatorname{Si}(\pi b^2 x^2) - 2\pi^2 b^5 x^5 + 5(\pi^2 b^5 x^5 - bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 2(\pi^2 b^4 x^4 - 15) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{210 x^7} C(bx)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \operatorname{fresnelc}(bx)}{x^8}, x\right)$$

110.122 Problem number 201

$$\int x^7 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3 \cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{24x^2 \cos\left(\frac{b^2\pi x^2}{2}\right) \operatorname{FresnelC}(bx)}{b^6\pi^3} \\ & - \frac{x^6 \cos\left(\frac{b^2\pi x^2}{2}\right) \operatorname{FresnelC}(bx)}{b^2\pi} - \frac{48 \operatorname{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^8\pi^4} \\ & + \frac{6x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2} - \frac{147x \sin(b^2\pi x^2)}{16b^7\pi^4} + \frac{x^5 \sin(b^2\pi x^2)}{4b^3\pi^2} + \frac{531 \operatorname{S}(bx\sqrt{2}) \sqrt{2}}{32b^8\pi^4} \end{aligned}$$

command

```
integrate(x^7*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{16\pi^3 b^8 x^7 + 952\pi b^4 x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 - 1372\pi b^4 x^3 - 224(\pi^3 b^7 x^6 - 24\pi b^3 x^2) \cos\left(\frac{1}{2}\pi b^2 x^2\right) C(bx) + 3717\sqrt{2} \sqrt{b^2}}{224\pi^4 b^9}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x^7 \operatorname{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right), x\right)$$

110.123 Problem number 202

$$\int x^6 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{15x^2}{4b^5\pi^3} + \frac{x^6}{12b\pi} + \frac{7x^2 \cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{15x \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{b^6\pi^3} \\ & - \frac{x^5 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{b^2\pi} - \frac{15 \text{FresnelC}(bx)^2}{2b^7\pi^3} \\ & + \frac{5x^3 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2} - \frac{11 \sin(b^2\pi x^2)}{2b^7\pi^4} + \frac{x^4 \sin(b^2\pi x^2)}{4b^3\pi^2} \end{aligned}$$

command

```
integrate(x^6*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^3 b^6 x^6 + 42 \pi b^2 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 - 66 \pi b^2 x^2 - 12 (\pi^3 b^5 x^5 - 15 \pi b x) \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) - 90 \pi C(bx)^2 + 6 (10 \pi^2 x^4 - 11 \pi x^2 + 3) \sin\left(\frac{1}{2} \pi b^2 x^2\right) C(bx)}{12 \pi^4 b^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^6 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

110.124 Problem number 203

$$\int x^5 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x \cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{8 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{b^6\pi^3} - \frac{x^4 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{b^2\pi} \\ & + \frac{4x^2 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2} + \frac{x^3 \sin(b^2\pi x^2)}{4b^3\pi^2} - \frac{43 \text{FresnelC}(bx\sqrt{2}) \sqrt{2}}{16b^6\pi^3} \end{aligned}$$

command

```
integrate(x^5*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{8 \pi^2 b^6 x^5 + 220 b^2 x \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 - 430 b^2 x - 80 (\pi^2 b^5 x^4 - 8 b) \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) - 215 \sqrt{2} \sqrt{b^2} C\left(\sqrt{2} \sqrt{b^2} x\right)}{80 \pi^3 b^7}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^5 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

110.125 Problem number 205

$$\int x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx$$

Optimal antiderivative

$$\frac{x^3}{6b\pi} - \frac{x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelC}(bx)}{b^2 \pi} + \frac{2 \text{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right)}{b^4 \pi^2} + \frac{x \sin(b^2 \pi x^2)}{4b^3 \pi^2} - \frac{5 S(bx\sqrt{2}) \sqrt{2}}{8b^4 \pi^2}$$

command

`integrate(x^3*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \pi b^4 x^3 - 24 \pi b^3 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) - 15 \sqrt{2} \sqrt{b^2} S\left(\sqrt{2} \sqrt{b^2} x\right) + 12 (b^2 x \cos\left(\frac{1}{2} \pi b^2 x^2\right) + 4 b C(bx)) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{24 \pi^2 b^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^3 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

110.126 Problem number 206

$$\int x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx$$

Optimal antiderivative

$$\frac{x^2}{4b\pi} - \frac{x \cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelC}(bx)}{b^2 \pi} + \frac{\text{FresnelC}(bx)^2}{2b^3 \pi} + \frac{\sin(b^2 \pi x^2)}{4b^3 \pi^2}$$

command

```
integrate(x^2*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^2 x^2 - 4 \pi b x \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) + 2 \pi C(bx)^2 + 2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{4 \pi^2 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x^2 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

110.127 Problem number 207

$$\int x \text{FresnelC}(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx$$

Optimal antiderivative

$$\frac{x}{2b\pi} - \frac{\cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelC}(bx)}{b^2 \pi} + \frac{\text{FresnelC}(bx \sqrt{2}) \sqrt{2}}{4b^2 \pi}$$

command

```
integrate(x*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 b^2 x - 4 b \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) + \sqrt{2} \sqrt{b^2} C\left(\sqrt{2} \sqrt{b^2} x\right)}{4 \pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(x \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

110.128 Problem number 210

$$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx$$

Optimal antiderivative

$$\frac{b\pi \operatorname{FresnelC}(bx)^2}{2} + \frac{b \operatorname{sinIntegral}(b^2\pi x^2)}{4} - \frac{\operatorname{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{x}$$

command

```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\pi b x C(bx)^2 + b x \operatorname{Si}(\pi b^2 x^2) - 4 C(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right)$$

110.129 Problem number 214

$$\int \frac{\operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^3\pi}{60x^2} - \frac{b^3\pi \cos(b^2\pi x^2)}{24x^2} - \frac{b^2\pi \cos\left(\frac{b^2\pi x^2}{2}\right) \operatorname{FresnelC}(bx)}{15x^3} \\ & - \frac{b^5\pi^3 \operatorname{FresnelC}(bx)^2}{30} - \frac{7b^5\pi^2 \operatorname{sinIntegral}(b^2\pi x^2)}{120} \\ & - \frac{\operatorname{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{5x^5} + \frac{b^4\pi^2 \operatorname{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{15x} - \frac{b \sin(b^2\pi x^2)}{40x^4} \end{aligned}$$

command

```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4\pi^3 b^5 x^5 C(bx)^2 + 7\pi^2 b^5 x^5 \operatorname{Si}(\pi b^2 x^2) + 10\pi b^3 x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 - 3\pi b^3 x^3 + 8\pi b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) C(bx) + 2(3b^5 \operatorname{Si}(\pi b^2 x^2) - 3b^4 \pi \operatorname{sinIntegral}(b^2 \pi x^2)) \operatorname{FresnelC}(bx) - 2b^3 \pi \cos\left(\frac{b^2 \pi x^2}{2}\right) \operatorname{FresnelC}(bx)^2 - 2b^2 \pi \cos\left(\frac{b^2 \pi x^2}{2}\right) \operatorname{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) - 2b \sin(b^2 \pi x^2)}{120x^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^6}, x\right)$$

110.130 Problem number 218

$$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi \cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3 \cos(b^2\pi x^2)}{2016x^2} \\ & - \frac{b^2\pi \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{63x^7} + \frac{b^6\pi^3 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{945x^3} + \frac{b^9\pi^5 \text{FresnelC}(bx)^2}{1890} \\ & + \frac{83b^9\pi^4 \sin\text{Integral}(b^2\pi x^2)}{30240} - \frac{\text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{9x^9} + \frac{b^4\pi^2 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{315x^5} \\ & - \frac{b^8\pi^4 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{945x} - \frac{b \sin(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2 \sin(b^2\pi x^2)}{30240x^4} \end{aligned}$$

command

```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{16\pi^5 b^9 x^9 C(bx)^2 + 83\pi^4 b^9 x^9 \text{Si}(\pi b^2 x^2) - 67\pi^3 b^7 x^7 + 70\pi b^3 x^3 + 10(15\pi^3 b^7 x^7 - 22\pi b^3 x^3) \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 32}{30240x^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^{10}}, x\right)$$

111 Test file number 206

Test folder name:

test_cases/8_Special_functions/206_8.4_Trig_integral_functions

111.1 Problem number 1

$$\int x^m \text{Si}(bx) dx$$

Optimal antiderivative

$$\frac{x^m \Gamma(1+m, -ibx) (-ibx)^{-m}}{2b(1+m)} + \frac{x^m \Gamma(1+m, ibx) (ibx)^{-m}}{2b(1+m)} + \frac{x^{1+m} \sin\text{Integral}(bx)}{1+m}$$

command

```
integrate(x^m*sin_integral(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 b x x^m \operatorname{Si}(b x) + \frac{\Gamma(m+1, i b x)}{(i b)^m} + \frac{\Gamma(m+1, -i b x)}{(-i b)^m}}{2(b m + b)}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(x^m Si(bx), x)
```

111.2 Problem number 2

$$\int x^3 \operatorname{Si}(b x) dx$$

Optimal antiderivative

$$-\frac{3x \cos(bx)}{2b^3} + \frac{x^3 \cos(bx)}{4b} + \frac{x^4 \operatorname{Si}(bx)}{4} + \frac{3 \sin(bx)}{2b^4} - \frac{3x^2 \sin(bx)}{4b^2}$$

command

```
integrate(x^3*sin_integral(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{b^4 x^4 \operatorname{Si}(b x) + (b^3 x^3 - 6 b x) \cos(b x) - 3(b^2 x^2 - 2) \sin(b x)}{4 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(x^3 Si(bx), x)
```

111.3 Problem number 3

$$\int x^2 \operatorname{Si}(b x) dx$$

Optimal antiderivative

$$-\frac{2 \cos(bx)}{3b^3} + \frac{x^2 \cos(bx)}{3b} + \frac{x^3 \operatorname{Si}(bx)}{3} - \frac{2x \sin(bx)}{3b^2}$$

command

```
integrate(x^2*sin_integral(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{b^3 x^3 \operatorname{Si}(bx) - 2bx \sin(bx) + (b^2 x^2 - 2) \cos(bx)}{3b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \operatorname{Si}(bx), x)$$

111.4 Problem number 4

$$\int x \operatorname{Si}(bx) dx$$

Optimal antiderivative

$$\frac{x \cos(bx)}{2b} + \frac{x^2 \operatorname{Si}(bx)}{2} - \frac{\sin(bx)}{2b^2}$$

command

```
integrate(x*sin_integral(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{b^2 x^2 \operatorname{Si}(bx) + bx \cos(bx) - \sin(bx)}{2b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \operatorname{Si}(bx), x)$$

111.5 Problem number 5

$$\int \operatorname{Si}(bx) dx$$

Optimal antiderivative

$$\frac{\cos(bx)}{b} + x \operatorname{Si}(bx)$$

command

```
integrate(sin_integral(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bx \operatorname{Si}(bx) + \cos(bx)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\operatorname{Si}(bx), x)$$

111.6 Problem number 7

$$\int \frac{\text{Si}(bx)}{x^2} dx$$

Optimal antiderivative

$$b \operatorname{cosineIntegral}(bx) - \frac{\operatorname{sinIntegral}(bx)}{x} - \frac{\sin(bx)}{x}$$

command

```
integrate(sin_integral(b*x)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bx \operatorname{Ci}(bx) + bx \operatorname{Ci}(-bx) - 2 \sin(bx) - 2 \operatorname{Si}(bx)}{2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{Si}(bx)}{x^2}, x\right)$$

111.7 Problem number 8

$$\int \frac{\text{Si}(bx)}{x^3} dx$$

Optimal antiderivative

$$-\frac{b \cos(bx)}{4x} - \frac{b^2 \operatorname{sinIntegral}(bx)}{4} - \frac{\operatorname{sinIntegral}(bx)}{2x^2} - \frac{\sin(bx)}{4x^2}$$

command

```
integrate(sin_integral(b*x)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{bx \cos(bx) + (b^2 x^2 + 2) \operatorname{Si}(bx) + \sin(bx)}{4x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{Si}(bx)}{x^3}, x\right)$$

111.8 Problem number 10

$$\int x^3 \text{Si}(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^2}{2b^2} + \frac{3 \cos \text{Integral}(2bx)}{2b^4} - \frac{3 \ln(x)}{2b^4} - \frac{3x \cos(bx) \sin \text{Integral}(bx)}{b^3} \\ & + \frac{x^3 \cos(bx) \sin \text{Integral}(bx)}{2b} + \frac{x^4 \sin \text{Integral}(bx)^2}{4} - \frac{x \cos(bx) \sin(bx)}{b^3} \\ & + \frac{3 \sin \text{Integral}(bx) \sin(bx)}{b^4} - \frac{3x^2 \sin \text{Integral}(bx) \sin(bx)}{2b^2} + \frac{2(\sin^2(bx))}{b^4} - \frac{x^2(\sin^2(bx))}{4b^2} \end{aligned}$$

command

```
integrate(x^3*sin_integral(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{b^4 x^4 \text{Si}(bx)^2 + b^2 x^2 + (b^2 x^2 - 8) \cos(bx)^2 + 2(b^3 x^3 - 6bx) \cos(bx) \text{Si}(bx) - 2(2bx \cos(bx) + 3(b^2 x^2 - 2) \text{Si}(bx))}{4b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^3 \text{Si}(bx)^2, x)$$

111.9 Problem number 11

$$\int x^2 \text{Si}(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5x}{6b^2} - \frac{4 \cos(bx) \sin \text{Integral}(bx)}{3b^3} + \frac{2x^2 \cos(bx) \sin \text{Integral}(bx)}{3b} + \frac{x^3 \sin \text{Integral}(bx)^2}{3} \\ & + \frac{2 \sin \text{Integral}(2bx)}{3b^3} - \frac{5 \cos(bx) \sin(bx)}{6b^3} - \frac{4x \sin \text{Integral}(bx) \sin(bx)}{3b^2} - \frac{x(\sin^2(bx))}{3b^2} \end{aligned}$$

command

```
integrate(x^2*sin_integral(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2b^3 x^3 \text{Si}(bx)^2 + 2bx \cos(bx)^2 + 4(b^2 x^2 - 2) \cos(bx) \text{Si}(bx) + 3bx - (8bx \text{Si}(bx) + 5 \cos(bx)) \sin(bx) + 4 \text{Si}(2bx)}{6b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^2 \text{Si}(bx)^2, x)$$

111.10 Problem number 12

$$\int x \operatorname{Si}(bx)^2 dx$$

Optimal antiderivative

$$-\frac{\operatorname{cosineIntegral}(2bx)}{2b^2} + \frac{\ln(x)}{2b^2} + \frac{x \cos(bx) \operatorname{sinIntegral}(bx)}{b} \\ + \frac{x^2 \operatorname{sinIntegral}(bx)^2}{2} - \frac{\operatorname{sinIntegral}(bx) \sin(bx)}{b^2} - \frac{\sin^2(bx)}{2b^2}$$

command

```
integrate(x*sin_integral(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2b^2x^2 \operatorname{Si}(bx)^2 + 4bx \cos(bx) \operatorname{Si}(bx) + 2 \cos(bx)^2 - 4 \sin(bx) \operatorname{Si}(bx) - \operatorname{Ci}(2bx) - \operatorname{Ci}(-2bx) + 2 \log(x)}{4b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x \operatorname{Si}(bx)^2, x\right)$$

111.11 Problem number 13

$$\int \operatorname{Si}(bx)^2 dx$$

Optimal antiderivative

$$\frac{2 \cos(bx) \operatorname{sinIntegral}(bx)}{b} + x \operatorname{sinIntegral}(bx)^2 - \frac{\operatorname{sinIntegral}(2bx)}{b}$$

command

```
integrate(sin_integral(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bx \operatorname{Si}(bx)^2 + 2 \cos(bx) \operatorname{Si}(bx) - \operatorname{Si}(2bx)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\operatorname{Si}(bx)^2, x\right)$$

111.12 Problem number 16

$$\int \frac{\text{Si}(bx)^2}{x^3} dx$$

Optimal antiderivative

$$\text{CannotIntegrate}\left(\frac{\text{sinIntegral}(bx)^2}{x^3}, x\right)$$

command

```
integrate(sin_integral(b*x)^2/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2b^2x^2 \text{Ci}(2bx) + 2b^2x^2 \text{Ci}(-2bx) - 2bx \cos(bx) \text{Si}(bx) - (b^2x^2 + 2) \text{Si}(bx)^2 + \cos(bx)^2 - 2(2bx \cos(bx) + \text{Si}(bx))}{4x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{Si}(bx)^2}{x^3}, x\right)$$

111.13 Problem number 18

$$\int x^3 \text{Si}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a \cos(bx + a)}{2b^4} - \frac{a^3 \cos(bx + a)}{4b^4} - \frac{3x \cos(bx + a)}{2b^3} + \frac{a^2x \cos(bx + a)}{4b^3} \\ & - \frac{ax^2 \cos(bx + a)}{4b^2} + \frac{x^3 \cos(bx + a)}{4b} - \frac{a^4 \text{sinIntegral}(bx + a)}{4b^4} + \frac{x^4 \text{sinIntegral}(bx + a)}{4} \\ & + \frac{3 \sin(bx + a)}{2b^4} - \frac{a^2 \sin(bx + a)}{4b^4} + \frac{ax \sin(bx + a)}{2b^3} - \frac{3x^2 \sin(bx + a)}{4b^2} \end{aligned}$$

command

```
integrate(x^3*sin_integral(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(b^3x^3 - ab^2x^2 - a^3 + (a^2 - 6)bx + 2a) \cos(bx + a) - (3b^2x^2 - 2abx + a^2 - 6) \sin(bx + a) + (b^4x^4 - a^4) \text{Si}(bx + a)}{4b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^3 \text{Si}(bx + a), x)$$

111.14 Problem number 19

$$\int x^2 \text{Si}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cos (bx+a)}{3b^3} + \frac{a^2 \cos (bx+a)}{3b^3} - \frac{ax \cos (bx+a)}{3b^2} + \frac{x^2 \cos (bx+a)}{3b} \\ & + \frac{a^3 \text{sinIntegral}(bx+a)}{3b^3} + \frac{x^3 \text{sinIntegral}(bx+a)}{3} + \frac{a \sin (bx+a)}{3b^3} - \frac{2x \sin (bx+a)}{3b^2} \end{aligned}$$

command

```
integrate(x^2*sin_integral(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(b^2x^2 - abx + a^2 - 2) \cos (bx+a) - (2bx - a) \sin (bx+a) + (b^3x^3 + a^3) \text{Si}(bx+a)}{3b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^2 \text{Si}(bx+a), x)$$

111.15 Problem number 20

$$\int x \text{Si}(a + bx) dx$$

Optimal antiderivative

$$-\frac{a \cos (bx+a)}{2b^2} + \frac{x \cos (bx+a)}{2b} - \frac{a^2 \text{sinIntegral}(bx+a)}{2b^2} + \frac{x^2 \text{sinIntegral}(bx+a)}{2} - \frac{\sin (bx+a)}{2b^2}$$

command

```
integrate(x*sin_integral(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(bx - a) \cos (bx+a) + (b^2x^2 - a^2) \text{Si}(bx+a) - \sin (bx+a)}{2b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x \text{Si}(bx+a), x)$$

111.16 Problem number 21

$$\int \operatorname{Si}(a + bx) dx$$

Optimal antiderivative

$$\frac{\cos(bx + a)}{b} + \frac{(bx + a) \operatorname{Si}(bx + a)}{b}$$

command

```
integrate(sin_integral(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(bx + a) \operatorname{Si}(bx + a) + \cos(bx + a)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(Si(bx + a), x)
```

111.17 Problem number 23

$$\int \frac{\operatorname{Si}(a + bx)}{x^2} dx$$

Optimal antiderivative

$$\frac{b \cos(a) \operatorname{Si}(bx)}{a} - \frac{b \operatorname{Si}(bx + a)}{a} - \frac{\operatorname{Si}(bx + a)}{x} + \frac{b \operatorname{Ci}(bx) \sin(a)}{a}$$

command

```
integrate(sin_integral(b*x+a)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2bx \cos(a) \operatorname{Si}(bx) + (bx \operatorname{Ci}(bx) + bx \operatorname{Ci}(-bx)) \sin(a) - 2(bx + a) \operatorname{Si}(bx + a)}{2ax}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(Si(bx + a)/x^2, x)
```

111.18 Problem number 24

$$\int \frac{\text{Si}(a + bx)}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^2 \text{cosineIntegral}(bx) \cos(a)}{2a} - \frac{b^2 \cos(a) \text{sinIntegral}(bx)}{2a^2} \\ & + \frac{b^2 \text{sinIntegral}(bx + a)}{2a^2} - \frac{\text{sinIntegral}(bx + a)}{2x^2} \\ & - \frac{b^2 \text{cosineIntegral}(bx) \sin(a)}{2a^2} - \frac{b^2 \text{sinIntegral}(bx) \sin(a)}{2a} - \frac{b \sin(bx + a)}{2ax} \end{aligned}$$

command

```
integrate(sin_integral(b*x+a)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2abx \sin(bx + a) - (ab^2x^2 \text{Ci}(bx) + ab^2x^2 \text{Ci}(-bx) - 2b^2x^2 \text{Si}(bx)) \cos(a) + (2ab^2x^2 \text{Si}(bx) + b^2x^2 \text{Ci}(bx) + b^2x^2 \text{Ci}(-bx)) \sin(a)}{4a^2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{Si}(bx + a)}{x^3}, x\right)$$

111.19 Problem number 26

$$\int x^2 \text{Si}(a + bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x}{3b^2} + \frac{a \text{cosineIntegral}(2bx + 2a)}{b^3} - \frac{a \cos(2bx + 2a)}{3b^3} + \frac{x \cos(2bx + 2a)}{6b^2} - \frac{a \ln(bx + a)}{b^3} \\ & - \frac{4 \cos(bx + a) \text{sinIntegral}(bx + a)}{3b^3} + \frac{2a^2 \cos(bx + a) \text{sinIntegral}(bx + a)}{3b^3} \\ & - \frac{2ax \cos(bx + a) \text{sinIntegral}(bx + a)}{3b^2} + \frac{2x^2 \cos(bx + a) \text{sinIntegral}(bx + a)}{3b} \\ & + \frac{a^2(bx + a) \text{sinIntegral}(bx + a)^2}{3b^3} - \frac{ax(bx + a) \text{sinIntegral}(bx + a)^2}{3b^2} \\ & + \frac{x^2(bx + a) \text{sinIntegral}(bx + a)^2}{3b} + \frac{2 \text{sinIntegral}(2bx + 2a)}{3b^3} - \frac{a^2 \text{sinIntegral}(2bx + 2a)}{b^3} \\ & - \frac{2 \cos(bx + a) \sin(bx + a)}{3b^3} + \frac{2a \text{sinIntegral}(bx + a) \sin(bx + a)}{3b^3} \\ & - \frac{4x \text{sinIntegral}(bx + a) \sin(bx + a)}{3b^2} - \frac{\sin(2bx + 2a)}{12b^3} \end{aligned}$$

command

```
integrate(x^2*sin_integral(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(bx - 2a) \cos(bx + a)^2 + 4(b^2x^2 - abx + a^2 - 2) \cos(bx + a) \operatorname{Si}(bx + a) + 2(b^3x^3 + a^3) \operatorname{Si}(bx + a)^2 + 3bx + 3a}{4b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x^2 \operatorname{Si}(bx + a)^2, x\right)$$

111.20 Problem number 27

$$\int x \operatorname{Si}(a + bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{cosineIntegral}(2bx + 2a)}{2b^2} + \frac{\cos(2bx + 2a)}{4b^2} + \frac{\ln(bx + a)}{2b^2} \\ & -\frac{a \cos(bx + a) \operatorname{sinIntegral}(bx + a)}{b^2} + \frac{x \cos(bx + a) \operatorname{sinIntegral}(bx + a)}{b} \\ & -\frac{a(bx + a) \operatorname{sinIntegral}(bx + a)^2}{2b^2} + \frac{x(bx + a) \operatorname{sinIntegral}(bx + a)^2}{2b} \\ & + \frac{a \operatorname{sinIntegral}(2bx + 2a)}{b^2} - \frac{\operatorname{sinIntegral}(bx + a) \sin(bx + a)}{b^2} \end{aligned}$$

command

```
integrate(x*sin_integral(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4(bx - a) \cos(bx + a) \operatorname{Si}(bx + a) + 2(b^2x^2 - a^2) \operatorname{Si}(bx + a)^2 + 2 \cos(bx + a)^2 + 4a \operatorname{Si}(2bx + 2a) - 4 \sin(bx + a)}{4b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x \operatorname{Si}(bx + a)^2, x\right)$$

111.21 Problem number 28

$$\int \operatorname{Si}(a + bx)^2 dx$$

Optimal antiderivative

$$\frac{2 \cos(bx + a) \operatorname{Si}(bx + a)}{b} + \frac{(bx + a) \operatorname{Si}(bx + a)^2}{b} - \frac{\operatorname{Si}(2bx + 2a)}{b}$$

command

```
integrate(sin_integral(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(bx + a) \operatorname{Si}(bx + a)^2 + 2 \cos(bx + a) \operatorname{Si}(bx + a) - \operatorname{Si}(2bx + 2a)}{b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\operatorname{Si}(bx + a)^2, x\right)$$

111.22 Problem number 32

$$\int x^2 \operatorname{Si}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{ix^3 \operatorname{expIntegral}\left(\frac{(-ibdn+3)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{3a}{bn}} (cx^n)^{-\frac{3}{n}}}{6} \\ & + \frac{ix^3 \operatorname{expIntegral}\left(\frac{(ibdn+3)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{3a}{bn}} (cx^n)^{-\frac{3}{n}}}{6} + \frac{x^3 \operatorname{Si}(d(a + b \ln(cx^n)))}{3} \end{aligned}$$

command

```
integrate(x^2*sin_integral(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{3} x^3 \operatorname{Si}(bd \log(cx^n) + ad) \\ & + \frac{1}{6} \left(i \operatorname{Ei}\left(\frac{iabdn + (ib^2dn + 3b) \log(c) + (ib^2dn^2 + 3bn) \log(x) + 3a}{bn}\right) - i \operatorname{Ei}\left(\frac{-iabdn + (-ib^2dn + 3b) \log(c)}{b}\right) \right) \end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \operatorname{Si}(bd \log(cx^n) + ad), x)$$

111.23 Problem number 33

$$\int x \operatorname{Si}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\frac{ix^2 \operatorname{expIntegral}\left(\frac{(-ibdn+2)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{2a}{bn}} (cx^n)^{-\frac{2}{n}}}{4} + \frac{ix^2 \operatorname{expIntegral}\left(\frac{(ibdn+2)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{2a}{bn}} (cx^n)^{-\frac{2}{n}}}{4} + \frac{x^2 \operatorname{sinIntegral}(d(a + b \ln(cx^n)))}{2}$$

command

```
integrate(x*sin_integral(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} x^2 \operatorname{Si}(bd \log(cx^n) + ad) + \frac{1}{4} \left(i \operatorname{Ei}\left(\frac{i abdn + (i b^2 dn + 2b) \log(c) + (i b^2 dn^2 + 2bn) \log(x) + 2a}{bn}\right) - i \operatorname{Ei}\left(\frac{-i abdn + (-i b^2 dn + 2b) \log(c)}{bn}\right) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \operatorname{Si}(bd \log(cx^n) + ad), x)$$

111.24 Problem number 34

$$\int \operatorname{Si}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\frac{ix \operatorname{expIntegral}\left(\frac{(-ibdn+1)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{a}{bn}} (cx^n)^{-\frac{1}{n}}}{2} + \frac{ix \operatorname{expIntegral}\left(\frac{(ibdn+1)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{a}{bn}} (cx^n)^{-\frac{1}{n}}}{2} + x \operatorname{sinIntegral}(d(a + b \ln(cx^n)))$$

command

```
integrate(sin_integral(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \left(i \operatorname{Ei} \left(\frac{i abdn + (i b^2 dn + b) \log(c) + (i b^2 dn^2 + bn) \log(x) + a}{bn} \right) - i \operatorname{Ei} \left(\frac{-i abdn + (-i b^2 dn + b) \log(c) + (-i b^2 dn^2 + bn) \log(x) + a}{bn} \right) \right) + x \operatorname{Si}(bd \log(cx^n) + ad)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\operatorname{Si}(bd \log(cx^n) + ad), x)$$

111.25 Problem number 35

$$\int \frac{\operatorname{Si}(d(a + b \log(cx^n)))}{x} dx$$

Optimal antiderivative

$$\frac{\cos(d(a + b \ln(cx^n)))}{bdn} + \frac{(a + b \ln(cx^n)) \operatorname{sinIntegral}(d(a + b \ln(cx^n)))}{bn}$$

command

`integrate(sin_integral(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(bdn \log(x) + bd \log(c) + ad) \operatorname{Si}(bd \log(cx^n) + ad) + \cos(bdn \log(x) + bd \log(c) + ad)}{bdn}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{Si}(bd \log(cx^n) + ad)}{x}, x\right)$$

111.26 Problem number 36

$$\int \frac{\operatorname{Si}(d(a + b \log(cx^n)))}{x^2} dx$$

Optimal antiderivative

$$-\frac{i e^{\frac{a}{bn}} (c x^n)^{\frac{1}{n}} \operatorname{expIntegral}\left(-\frac{(-ibdn+1)(a+b \ln(cx^n))}{bn}\right)}{2x} + \frac{i e^{\frac{a}{bn}} (c x^n)^{\frac{1}{n}} \operatorname{expIntegral}\left(-\frac{(ibdn+1)(a+b \ln(cx^n))}{bn}\right)}{2x} - \frac{\operatorname{sinIntegral}(d(a + b \ln(cx^n)))}{x}$$

command

```
integrate(sin_integral(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i x \operatorname{Ei}\left(\frac{i a b d n+(i b^2 d n-b) \log (c)+(i b^2 d n^2-b n) \log (x)-a}{b n}\right)+i x \operatorname{Ei}\left(\frac{-i a b d n+(-i b^2 d n-b) \log (c)+(-i b^2 d n^2-b n) \log (x)-a}{b n}\right)\right) e^{\left(\frac{b \log (c)+}{b n}\right)}}{2 x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{Si}(b d \log (c x^n)+a d)}{x^2}, x\right)$$

111.27 Problem number 37

$$\int \frac{\operatorname{Si}(d(a+b \log (c x^n)))}{x^3} d x$$

Optimal antiderivative

$$\begin{aligned} & -\frac{i e^{\frac{2 a}{b n}}(c x^n)^{\frac{2}{n}} \exp \operatorname{Integral}\left(-\frac{(-i b d n+2)(a+b \ln (c x^n))}{b n}\right)}{4 x^2} \\ & +\frac{i e^{\frac{2 a}{b n}}(c x^n)^{\frac{2}{n}} \exp \operatorname{Integral}\left(-\frac{(i b d n+2)(a+b \ln (c x^n))}{b n}\right)}{4 x^2}-\frac{\sin \operatorname{Integral}(d(a+b \ln (c x^n)))}{2 x^2} \end{aligned}$$

command

```
integrate(sin_integral(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(-i x^2 \operatorname{Ei}\left(\frac{i a b d n+(i b^2 d n-2 b) \log (c)+(i b^2 d n^2-2 b n) \log (x)-2 a}{b n}\right)+i x^2 \operatorname{Ei}\left(\frac{-i a b d n+(-i b^2 d n-2 b) \log (c)+(-i b^2 d n^2-2 b n) \log (x)-2 a}{b n}\right)\right)}{4 x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{Si}(b d \log (c x^n)+a d)}{x^3}, x\right)$$

111.28 Problem number 38

$$\int (ex)^m \text{Si}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{ix(ex)^m \exp\text{Integral}\left(\frac{(-ibdn+m+1)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{a(1+m)}{bn}} (cx^n)^{-\frac{1+m}{n}}}{2(1+m)} \\ & + \frac{ix(ex)^m \exp\text{Integral}\left(\frac{(ibdn+m+1)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{a(1+m)}{bn}} (cx^n)^{-\frac{1+m}{n}}}{2+2m} \\ & + \frac{(ex)^{1+m} \sin\text{Integral}(d(a + b \ln(cx^n)))}{e(1+m)} \end{aligned}$$

command

```
integrate((e*x)^m*sin_integral(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2xe^{(m \log(x)+m)} \text{Si}(bd \log(cx^n) + ad) + \left(i \text{Ei}\left(\frac{iabdn+am+(ib^2dn+bm+b) \log(c)+(ib^2dn^2+(bm+b)n) \log(x)+a}{bn}\right) - i \text{Ei}\left(\frac{-iabdn+am+(ib^2dn+bm+b) \log(c)+(ib^2dn^2+(bm+b)n) \log(x)+a}{bn}\right)\right)}{2(m+1)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}((ex)^m \text{Si}(bd \log(cx^n) + ad), x)$$

111.29 Problem number 39

$$\int \frac{\sin(bx) \text{Si}(bx)}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & b^2 \cosine\text{Integral}(2bx) - \frac{b \cos(bx) \sin\text{Integral}(bx)}{2x} - \frac{b^2 \sin\text{Integral}(bx)^2}{4} \\ & - \frac{b \cos(bx) \sin(bx)}{2x} - \frac{\sin\text{Integral}(bx) \sin(bx)}{2x^2} - \frac{\sin^2(bx)}{4x^2} - \frac{b \sin(2bx)}{4x} \end{aligned}$$

command

```
integrate(sin_integral(b*x)*sin(b*x)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{b^2x^2 \text{Si}(bx)^2 - 2b^2x^2 \text{Ci}(2bx) - 2b^2x^2 \text{Ci}(-2bx) + 2bx \cos(bx) \text{Si}(bx) - \cos(bx)^2 + 2(2bx \cos(bx) + \text{Si}(bx)) \sin(bx)}{4x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sin(bx) \text{Si}(bx)}{x^3}, x\right)$$

111.30 Problem number 41

$$\int \frac{\sin(bx)\text{Si}(bx)}{x} dx$$

Optimal antiderivative

$$\frac{\text{sinIntegral}(bx)^2}{2}$$

command

```
integrate(sin_integral(b*x)*sin(b*x)/x,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{2} \text{Si}(bx)^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\sin(bx)\text{Si}(bx)}{x}, x\right)$$

111.31 Problem number 42

$$\int \sin(bx)\text{Si}(bx) dx$$

Optimal antiderivative

$$-\frac{\cos(bx)\text{sinIntegral}(bx)}{b} + \frac{\text{sinIntegral}(2bx)}{2b}$$

command

```
integrate(sin_integral(b*x)*sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2 \cos(bx)\text{Si}(bx) - \text{Si}(2bx)}{2b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(\sin(bx)\text{Si}(bx), x)$$

111.32 Problem number 43

$$\int x \sin(bx) \operatorname{Si}(bx) dx$$

Optimal antiderivative

$$\frac{\operatorname{cosineIntegral}(2bx)}{2b^2} - \frac{\ln(x)}{2b^2} - \frac{x \cos(bx) \operatorname{sinIntegral}(bx)}{b} + \frac{\operatorname{sinIntegral}(bx) \sin(bx)}{b^2} + \frac{\sin^2(bx)}{2b^2}$$

command

```
integrate(x*sin_integral(b*x)*sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4bx \cos(bx) \operatorname{Si}(bx) + 2 \cos(bx)^2 - 4 \sin(bx) \operatorname{Si}(bx) - \operatorname{Ci}(2bx) - \operatorname{Ci}(-2bx) + 2 \log(x)}{4b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \sin(bx) \operatorname{Si}(bx), x)$$

111.33 Problem number 44

$$\int x^2 \sin(bx) \operatorname{Si}(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5x}{4b^2} + \frac{2 \cos(bx) \operatorname{sinIntegral}(bx)}{b^3} - \frac{x^2 \cos(bx) \operatorname{sinIntegral}(bx)}{b} - \frac{\operatorname{sinIntegral}(2bx)}{b^3} \\ & + \frac{5 \cos(bx) \sin(bx)}{4b^3} + \frac{2x \operatorname{sinIntegral}(bx) \sin(bx)}{b^2} + \frac{x(\sin^2(bx))}{2b^2} \end{aligned}$$

command

```
integrate(x^2*sin_integral(b*x)*sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2bx \cos(bx)^2 + 4(b^2x^2 - 2) \cos(bx) \operatorname{Si}(bx) + 3bx - (8bx \operatorname{Si}(bx) + 5 \cos(bx)) \sin(bx) + 4 \operatorname{Si}(2bx)}{4b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \sin(bx) \operatorname{Si}(bx), x)$$

111.34 Problem number 45

$$\int x^3 \sin(bx) \operatorname{Si}(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x^2}{b^2} - \frac{3 \operatorname{cosineIntegral}(2bx)}{b^4} + \frac{3 \ln(x)}{b^4} + \frac{6x \cos(bx) \operatorname{sinIntegral}(bx)}{b^3} \\ & - \frac{x^3 \cos(bx) \operatorname{sinIntegral}(bx)}{b} + \frac{2x \cos(bx) \sin(bx)}{b^3} - \frac{6 \operatorname{sinIntegral}(bx) \sin(bx)}{b^4} \\ & + \frac{3x^2 \operatorname{sinIntegral}(bx) \sin(bx)}{b^2} - \frac{4(\sin^2(bx))}{b^4} + \frac{x^2(\sin^2(bx))}{2b^2} \end{aligned}$$

command

```
integrate(x^3*sin_integral(b*x)*sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{b^2 x^2 + (b^2 x^2 - 8) \cos(bx)^2 + 2(b^3 x^3 - 6bx) \cos(bx) \operatorname{Si}(bx) - 2(2bx \cos(bx) + 3(b^2 x^2 - 2) \operatorname{Si}(bx)) \sin(bx) + 3}{2b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^3 \sin(bx) \operatorname{Si}(bx), x)$$

111.35 Problem number 47

$$\int \frac{\cos(bx) \operatorname{Si}(bx)}{x^2} dx$$

Optimal antiderivative

$$b \operatorname{cosineIntegral}(2bx) - \frac{\cos(bx) \operatorname{sinIntegral}(bx)}{x} - \frac{b \operatorname{sinIntegral}(bx)^2}{2} - \frac{\sin(2bx)}{2x}$$

command

```
integrate(cos(b*x)*sin_integral(b*x)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bx \operatorname{Si}(bx)^2 - bx \operatorname{Ci}(2bx) - bx \operatorname{Ci}(-2bx) + 2 \cos(bx) \sin(bx) + 2 \cos(bx) \operatorname{Si}(bx)}{2x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\cos(bx) \operatorname{Si}(bx)}{x^2}, x\right)$$

111.36 Problem number 49

$$\int \cos(bx) \operatorname{Si}(bx) dx$$

Optimal antiderivative

$$\frac{\operatorname{cosineIntegral}(2bx)}{2b} - \frac{\ln(x)}{2b} + \frac{\operatorname{sinIntegral}(bx) \sin(bx)}{b}$$

command

```
integrate(cos(b*x)*sin_integral(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \sin(bx) \operatorname{Si}(bx) + \operatorname{Ci}(2bx) + \operatorname{Ci}(-2bx) - 2 \log(x)}{4b}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(cos(bx) Si(bx), x)
```

111.37 Problem number 50

$$\int x \cos(bx) \operatorname{Si}(bx) dx$$

Optimal antiderivative

$$-\frac{x}{2b} + \frac{\cos(bx) \operatorname{sinIntegral}(bx)}{b^2} - \frac{\operatorname{sinIntegral}(2bx)}{2b^2} + \frac{\cos(bx) \sin(bx)}{2b^2} + \frac{x \operatorname{sinIntegral}(bx) \sin(bx)}{b}$$

command

```
integrate(x*cos(b*x)*sin_integral(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{bx - (2bx \operatorname{Si}(bx) + \cos(bx) \sin(bx)) - 2 \cos(bx) \operatorname{Si}(bx) + \operatorname{Si}(2bx)}{2b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(x cos(bx) Si(bx), x)
```

111.38 Problem number 51

$$\int x^2 \cos(bx) \operatorname{Si}(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x^2}{4b} - \frac{\operatorname{cosineIntegral}(2bx)}{b^3} + \frac{\ln(x)}{b^3} + \frac{2x \cos(bx) \operatorname{sinIntegral}(bx)}{b^2} + \frac{x \cos(bx) \sin(bx)}{2b^2} \\ & - \frac{2 \operatorname{sinIntegral}(bx) \sin(bx)}{b^3} + \frac{x^2 \operatorname{sinIntegral}(bx) \sin(bx)}{b} - \frac{5(\sin^2(bx))}{4b^3} \end{aligned}$$

command

```
integrate(x^2*cos(b*x)*sin_integral(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{b^2 x^2 - 8 b x \cos(bx) \operatorname{Si}(bx) - 5 \cos(bx)^2 - 2(bx \cos(bx) + 2(b^2 x^2 - 2) \operatorname{Si}(bx)) \sin(bx) + 2 \operatorname{Ci}(2bx) + 2 \operatorname{Ci}(-2bx)}{4 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \cos(bx) \operatorname{Si}(bx), x)$$

111.39 Problem number 52

$$\int x^3 \cos(bx) \operatorname{Si}(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4x}{b^3} - \frac{x^3}{6b} - \frac{6 \cos(bx) \operatorname{sinIntegral}(bx)}{b^4} + \frac{3x^2 \cos(bx) \operatorname{sinIntegral}(bx)}{b^2} \\ & + \frac{3 \operatorname{sinIntegral}(2bx)}{b^4} - \frac{4 \cos(bx) \sin(bx)}{b^4} + \frac{x^2 \cos(bx) \sin(bx)}{2b^2} \\ & - \frac{6x \operatorname{sinIntegral}(bx) \sin(bx)}{b^3} + \frac{x^3 \operatorname{sinIntegral}(bx) \sin(bx)}{b} - \frac{2x(\sin^2(bx))}{b^3} \end{aligned}$$

command

```
integrate(x^3*cos(b*x)*sin_integral(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{b^3 x^3 - 12 b x \cos(bx)^2 - 18(b^2 x^2 - 2) \cos(bx) \operatorname{Si}(bx) - 12 b x - 3((b^2 x^2 - 8) \cos(bx) + 2(b^3 x^3 - 6 b x) \operatorname{Si}(bx)) \sin(bx)}{6 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^3 \cos(bx) \operatorname{Si}(bx), x)$$

111.40 Problem number 53

$$\int \sin(5x)\text{Si}(2x) dx$$

Optimal antiderivative

$$-\frac{\cos(5x)\text{sinIntegral}(2x)}{5} - \frac{\text{sinIntegral}(3x)}{10} + \frac{\text{sinIntegral}(7x)}{10}$$

command

```
integrate(sin_integral(2*x)*sin(5*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{16}{5} \cos(x)^5 \text{Si}(2x) + 4 \cos(x)^3 \text{Si}(2x) - \cos(x) \text{Si}(2x) + \frac{1}{10} \text{Si}(7x) - \frac{1}{10} \text{Si}(3x)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

111.41 Problem number 54

$$\int \cos(5x)\text{Si}(2x) dx$$

Optimal antiderivative

$$-\frac{\text{cosineIntegral}(3x)}{10} + \frac{\text{cosineIntegral}(7x)}{10} + \frac{\text{sinIntegral}(2x)\sin(5x)}{5}$$

command

```
integrate(cos(5*x)*sin_integral(2*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{5} \left(16 \cos(x)^4 \text{Si}(2x) - 12 \cos(x)^2 \text{Si}(2x) + \text{Si}(2x) \right) \sin(x) + \frac{1}{20} \text{Ci}(7x) - \frac{1}{20} \text{Ci}(3x) - \frac{1}{20} \text{Ci}(-3x) + \frac{1}{20} \text{Ci}(-7x)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

111.42 Problem number 55

$$\int x^2 \sin(a + bx) \operatorname{Si}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x}{b^2} - \frac{a \operatorname{cosineIntegral}(2bx + 2a)}{b^3} + \frac{a \cos(2bx + 2a)}{4b^3} - \frac{x \cos(2bx + 2a)}{4b^2} + \frac{a \ln(bx + a)}{b^3} \\ & + \frac{2 \cos(bx + a) \operatorname{sinIntegral}(bx + a)}{b^3} - \frac{x^2 \cos(bx + a) \operatorname{sinIntegral}(bx + a)}{b} \\ & - \frac{\operatorname{sinIntegral}(2bx + 2a)}{b^3} + \frac{a^2 \operatorname{sinIntegral}(2bx + 2a)}{2b^3} + \frac{\cos(bx + a) \sin(bx + a)}{b^3} \\ & + \frac{2x \operatorname{sinIntegral}(bx + a) \sin(bx + a)}{b^2} + \frac{\sin(2bx + 2a)}{8b^3} \end{aligned}$$

command

```
integrate(x^2*sin_integral(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(bx - a) \cos(bx + a)^2 + 4(b^2x^2 - 2) \cos(bx + a) \operatorname{Si}(bx + a) + 3bx + 2a \operatorname{Ci}(2bx + 2a) + 2a \operatorname{Ci}(-2bx - 2a) - \dots}{4b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \sin(bx + a) \operatorname{Si}(bx + a), x)$$

111.43 Problem number 56

$$\int x \sin(a + bx) \operatorname{Si}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{cosineIntegral}(2bx + 2a)}{2b^2} - \frac{\cos(2bx + 2a)}{4b^2} - \frac{\ln(bx + a)}{2b^2} - \frac{x \cos(bx + a) \operatorname{sinIntegral}(bx + a)}{b} \\ & - \frac{a \operatorname{sinIntegral}(2bx + 2a)}{2b^2} + \frac{\operatorname{sinIntegral}(bx + a) \sin(bx + a)}{b^2} \end{aligned}$$

command

```
integrate(x*sin_integral(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4bx \cos(bx + a) \operatorname{Si}(bx + a) + 2 \cos(bx + a)^2 + 2a \operatorname{Si}(2bx + 2a) - 4 \sin(bx + a) \operatorname{Si}(bx + a) - \operatorname{Ci}(2bx + 2a) - \dots}{4b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \sin(bx + a) \operatorname{Si}(bx + a), x)$$

111.44 Problem number 57

$$\int \sin(a + bx) \operatorname{Si}(a + bx) dx$$

Optimal antiderivative

$$-\frac{\cos(bx + a) \operatorname{Si}(bx + a)}{b} + \frac{\operatorname{Si}(2bx + 2a)}{2b}$$

command

```
integrate(sin_integral(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{2 \cos(bx + a) \operatorname{Si}(bx + a) - \operatorname{Si}(2bx + 2a)}{2b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\sin(bx + a) \operatorname{Si}(bx + a), x)$$

111.45 Problem number 59

$$\int x^2 \cos(a + bx) \operatorname{Si}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{ax}{2b^2} - \frac{x^2}{4b} - \frac{\operatorname{cosineIntegral}(2bx + 2a)}{b^3} + \frac{a^2 \operatorname{cosineIntegral}(2bx + 2a)}{2b^3} \\ & + \frac{\cos(2bx + 2a)}{2b^3} + \frac{\ln(bx + a)}{b^3} - \frac{a^2 \ln(bx + a)}{2b^3} + \frac{2x \cos(bx + a) \operatorname{Si}(bx + a)}{b^2} \\ & + \frac{a \operatorname{Si}(2bx + 2a)}{b^3} - \frac{a \cos(bx + a) \sin(bx + a)}{2b^3} + \frac{x \cos(bx + a) \sin(bx + a)}{2b^2} \\ & - \frac{2 \operatorname{Si}(bx + a) \sin(bx + a)}{b^3} + \frac{x^2 \operatorname{Si}(bx + a) \sin(bx + a)}{b} - \frac{\sin^2(bx + a)}{4b^3} \end{aligned}$$

command

```
integrate(x^2*cos(b*x+a)*sin_integral(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{b^2 x^2 - 8bx \cos(bx + a) \operatorname{Si}(bx + a) - 2abx - 5 \cos(bx + a)^2 - (a^2 - 2) \operatorname{Ci}(2bx + 2a) - (a^2 - 2) \operatorname{Ci}(-2bx - 2a)}{4b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \cos(bx + a) \operatorname{Si}(bx + a), x)$$

111.46 Problem number 60

$$\int x \cos(a + bx) \operatorname{Si}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x}{2b} - \frac{a \operatorname{cosineIntegral}(2bx + 2a)}{2b^2} + \frac{a \ln(bx + a)}{2b^2} + \frac{\cos(bx + a) \operatorname{sinIntegral}(bx + a)}{b^2} \\ & - \frac{\operatorname{sinIntegral}(2bx + 2a)}{2b^2} + \frac{\cos(bx + a) \sin(bx + a)}{2b^2} + \frac{x \operatorname{sinIntegral}(bx + a) \sin(bx + a)}{b} \end{aligned}$$

command

```
integrate(x*cos(b*x+a)*sin_integral(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2bx + a \operatorname{Ci}(2bx + 2a) + a \operatorname{Ci}(-2bx - 2a) - 2a \log(bx + a) - 2(2bx \operatorname{Si}(bx + a) + \cos(bx + a)) \sin(bx + a) - 4}{4b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \cos(bx + a) \operatorname{Si}(bx + a), x)$$

111.47 Problem number 61

$$\int \cos(a + bx) \operatorname{Si}(a + bx) dx$$

Optimal antiderivative

$$\frac{\operatorname{cosineIntegral}(2bx + 2a)}{2b} - \frac{\ln(bx + a)}{2b} + \frac{\operatorname{sinIntegral}(bx + a) \sin(bx + a)}{b}$$

command

```
integrate(cos(b*x+a)*sin_integral(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \sin(bx + a) \operatorname{Si}(bx + a) + \operatorname{Ci}(2bx + 2a) + \operatorname{Ci}(-2bx - 2a) - 2 \log(bx + a)}{4b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\cos(bx + a) \operatorname{Si}(bx + a), x)$$

111.48 Problem number 63

$$\int x \sin(a + bx) \operatorname{Si}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b^2} \\ & + \frac{\operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b^2} + \frac{\cos(a - c + (b-d)x)}{2b(b-d)} \\ & - \frac{\cos(a + c + (b+d)x)}{2b(b+d)} + \frac{c \cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2bd} \\ & - \frac{x \cos(bx + a) \operatorname{sinIntegral}(dx + c)}{b} - \frac{c \cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2bd} \\ & + \frac{c \operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} \\ & - \frac{c \operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} \\ & + \frac{\operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} \\ & - \frac{\operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} + \frac{\operatorname{sinIntegral}(dx + c) \sin(bx + a)}{b^2} \end{aligned}$$

command

```
integrate(x*sin_integral(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4bd^2 \cos(bx + a) \cos(dx + c) + 4b^2d \sin(bx + a) \sin(dx + c) - 4(b^3d - bd^3)x \cos(bx + a) \operatorname{Si}(dx + c) + 4(b^2d - d^3) \operatorname{Si}(dx + c) \sin(bx + a)}{b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \sin(bx + a) \operatorname{Si}(dx + c), x)$$

111.49 Problem number 64

$$\int \sin(a + bx) \operatorname{Si}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2b} - \frac{\cos(bx + a) \operatorname{Si}(dx + c)}{b} \\ & + \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2b} \\ & - \frac{\operatorname{Ci}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b} \\ & + \frac{\operatorname{Ci}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b} \end{aligned}$$

command

```
integrate(sin_integral(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \left(\operatorname{Si}\left(\frac{bc+cd+(bd+d^2)x}{d}\right) + \operatorname{Si}\left(-\frac{bc-cd+(bd-d^2)x}{d}\right) \right) \cos\left(-\frac{bc-ad}{d}\right) + \left(\operatorname{Ci}\left(\frac{bc+cd+(bd+d^2)x}{d}\right) + \operatorname{Ci}\left(-\frac{bc+cd+(bd+d^2)x}{d}\right) \right) - C}{4b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\sin(bx + a) \operatorname{Si}(dx + c), x)$$

111.50 Problem number 66

$$\int x \cos(a + bx) \operatorname{Si}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{c \operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2bd} \\
& - \frac{c \operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2bd} \\
& + \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2b^2} + \frac{\cos(bx+a) \operatorname{sinIntegral}(dx+c)}{b^2} \\
& - \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2b^2} \\
& + \frac{\operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} \\
& - \frac{\operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} \\
& - \frac{c \operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} \\
& + \frac{c \operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} + \frac{x \operatorname{sinIntegral}(dx+c) \sin(bx+a)}{b} \\
& - \frac{\sin\left(a - c + (b-d)x\right)}{2b(b-d)} + \frac{\sin\left(a + c + (b+d)x\right)}{2b(b+d)}
\end{aligned}$$

command

```
integrate(x*cos(b*x+a)*sin_integral(d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4b^2d \cos(bx+a) \sin(dx+c) + 4(b^2d - d^3) \cos(bx+a) \operatorname{Si}(dx+c) - \left((b^3c - bcd^2) \operatorname{Ci}\left(\frac{bc+cd+(bd+d^2)x}{d}\right) + (b^3c - bcd^2)\right)}{b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \cos(bx+a) \operatorname{Si}(dx+c), x)$$

111.51 Problem number 67

$$\int \cos(a + bx) \operatorname{Si}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b} \\ & + \frac{\operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b} \\ & + \frac{\operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b} \\ & - \frac{\operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b} + \frac{\operatorname{sinIntegral}(dx + c) \sin(bx + a)}{b} \end{aligned}$$

command

```
integrate(cos(b*x+a)*sin_integral(d*x+c),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left(\operatorname{Ci}\left(\frac{bc+cd+(bd+d^2)x}{d}\right) + \operatorname{Ci}\left(-\frac{bc+cd+(bd+d^2)x}{d}\right) - \operatorname{Ci}\left(\frac{bc-cd+(bd-d^2)x}{d}\right) - \operatorname{Ci}\left(-\frac{bc-cd+(bd-d^2)x}{d}\right)\right) \cos\left(-\frac{bc-ad}{d}\right) - 2 \left(\operatorname{Si}\left(\frac{bc+cd+(bd+d^2)x}{d}\right) + \operatorname{Si}\left(-\frac{bc+cd+(bd+d^2)x}{d}\right) - \operatorname{Si}\left(\frac{bc-cd+(bd-d^2)x}{d}\right) - \operatorname{Si}\left(-\frac{bc-cd+(bd-d^2)x}{d}\right)\right) \sin\left(-\frac{bc-ad}{d}\right)}{4b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\cos(bx + a) \operatorname{Si}(dx + c), x)$$

111.52 Problem number 69

$$\int x^m \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$\frac{x^{1+m} \operatorname{cosineIntegral}(bx)}{1+m} + \frac{ix^m \Gamma(1+m, -ibx) (-ibx)^{-m}}{2b(1+m)} - \frac{ix^m \Gamma(1+m, ibx) (ibx)^{-m}}{2b(1+m)}$$

command

```
integrate(x^m*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\pi b x^m \operatorname{C}(bx) - i \left(\cosh\left(\frac{1}{2} m \log\left(\frac{1}{2} i \pi b^2\right)\right) - \sinh\left(\frac{1}{2} m \log\left(\frac{1}{2} i \pi b^2\right)\right)\right) \Gamma\left(\frac{1}{2} m + 1, \frac{1}{2} i \pi b^2 x^2\right) + i \left(\cosh\left(\frac{1}{2} m \log\left(-\frac{1}{2} i \pi b^2\right)\right) - \sinh\left(\frac{1}{2} m \log\left(-\frac{1}{2} i \pi b^2\right)\right)\right) \Gamma\left(\frac{1}{2} m + 1, -\frac{1}{2} i \pi b^2 x^2\right)}{2\pi(bm + b)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^m \operatorname{Ci}(bx), x)$$

111.53 Problem number 70

$$\int x^3 \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$\frac{x^4 \operatorname{cosineIntegral}(bx)}{4} + \frac{3 \cos(bx)}{2b^4} - \frac{3x^2 \cos(bx)}{4b^2} + \frac{3x \sin(bx)}{2b^3} - \frac{x^3 \sin(bx)}{4b}$$

command

```
integrate(x^3*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\pi b^3 x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 3 b x \cos\left(\frac{1}{2} \pi b^2 x^2\right) - (\pi^2 b^4 x^4 + 3) C(bx)}{4 \pi^2 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^3 \operatorname{Ci}(bx), x)$$

111.54 Problem number 71

$$\int x^2 \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$\frac{x^3 \operatorname{cosineIntegral}(bx)}{3} - \frac{2x \cos(bx)}{3b^2} + \frac{2 \sin(bx)}{3b^3} - \frac{x^2 \sin(bx)}{3b}$$

command

```
integrate(x^2*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^3 x^3 C(bx) - \pi b^2 x^2 \sin\left(\frac{1}{2} \pi b^2 x^2\right) - 2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3 \pi^2 b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \operatorname{Ci}(bx), x)$$

111.55 Problem number 72

$$\int x \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$\frac{x^2 \operatorname{cosineIntegral}(bx)}{2} - \frac{\cos(bx)}{2b^2} - \frac{x \sin(bx)}{2b}$$

command

```
integrate(x*fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^3 x^2 C(bx) - b^2 x \sin\left(\frac{1}{2} \pi b^2 x^2\right) + \sqrt{b^2} S\left(\sqrt{b^2} x\right)}{2 \pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \operatorname{Ci}(bx), x)$$

111.56 Problem number 73

$$\int \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$x \operatorname{cosineIntegral}(bx) - \frac{\sin(bx)}{b}$$

command

```
integrate(fresnel_cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b x C(bx) - \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\operatorname{Ci}(bx), x)$$

111.57 Problem number 75

$$\int \frac{\text{CosIntegral}(bx)}{x^2} dx$$

Optimal antiderivative

$$-\frac{\text{cosineIntegral}(bx)}{x} - \frac{\cos(bx)}{x} - b \text{sinIntegral}(bx)$$

command

```
integrate(fresnel_cos(b*x)/x^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{bx \text{Ci}\left(\frac{1}{2} \pi b^2 x^2\right) + bx \text{Ci}\left(-\frac{1}{2} \pi b^2 x^2\right) - 4 C(bx)}{4x}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{Ci}(bx)}{x^2}, x\right)$$

111.58 Problem number 76

$$\int \frac{\text{CosIntegral}(bx)}{x^3} dx$$

Optimal antiderivative

$$-\frac{b^2 \text{cosineIntegral}(bx)}{4} - \frac{\text{cosineIntegral}(bx)}{2x^2} - \frac{\cos(bx)}{4x^2} + \frac{b \sin(bx)}{4x}$$

command

```
integrate(fresnel_cos(b*x)/x^3,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{\pi \sqrt{b^2} bx^2 \text{S}\left(\sqrt{b^2} x\right) + bx \cos\left(\frac{1}{2} \pi b^2 x^2\right) + C(bx)}{2x^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{Ci}(bx)}{x^3}, x\right)$$

111.59 Problem number 78

$$\int x^3 \operatorname{CosIntegral}(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^2}{4b^2} + \frac{x^4 \operatorname{cosineIntegral}(bx)^2}{4} - \frac{3 \operatorname{cosineIntegral}(2bx)}{2b^4} + \frac{3 \operatorname{cosineIntegral}(bx) \cos(bx)}{b^4} \\ & - \frac{3x^2 \operatorname{cosineIntegral}(bx) \cos(bx)}{2b^2} + \frac{3(\cos^2(bx))}{8b^4} - \frac{3 \ln(x)}{2b^4} + \frac{3x \operatorname{cosineIntegral}(bx) \sin(bx)}{b^3} \\ & - \frac{x^3 \operatorname{cosineIntegral}(bx) \sin(bx)}{2b} + \frac{x \cos(bx) \sin(bx)}{b^3} - \frac{13(\sin^2(bx))}{8b^4} + \frac{x^2(\sin^2(bx))}{4b^2} \end{aligned}$$

command

```
integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^2 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 - 2 \pi b^2 x^2 + 6 \pi b x \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) - (3 \pi + \pi^3 b^4 x^4) C(bx)^2 + 2(\pi^2 b^3 x^3 C(bx) - 2 \cos(bx))}{4 \pi^3 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x^3 \operatorname{Ci}(bx)^2, x\right)$$

111.60 Problem number 79

$$\int x^2 \operatorname{CosIntegral}(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{2b^2} + \frac{x^3 \operatorname{cosineIntegral}(bx)^2}{3} - \frac{4x \operatorname{cosineIntegral}(bx) \cos(bx)}{3b^2} \\ & - \frac{2 \operatorname{sinIntegral}(2bx)}{3b^3} + \frac{4 \operatorname{cosineIntegral}(bx) \sin(bx)}{3b^3} \\ & - \frac{2x^2 \operatorname{cosineIntegral}(bx) \sin(bx)}{3b} + \frac{5 \cos(bx) \sin(bx)}{6b^3} + \frac{x(\sin^2(bx))}{3b^2} \end{aligned}$$

command

```
integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{4 \pi^2 b^4 x^3 C(bx)^2 - 8 \pi b^3 x^2 C(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) - 4 b^2 x \cos\left(\frac{1}{2} \pi b^2 x^2\right)^2 + 10 b^2 x - 16 b \cos\left(\frac{1}{2} \pi b^2 x^2\right) C(bx) + 5 \sqrt{2} \operatorname{Ci}(bx)}{12 \pi^2 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(x^2 \operatorname{Ci}(bx)^2, x\right)$$

111.61 Problem number 81

$$\int \text{CosIntegral}(bx)^2 dx$$

Optimal antiderivative

$$x \text{ cosineIntegral}(bx)^2 + \frac{\text{sinIntegral}(2bx)}{b} - \frac{2 \text{ cosineIntegral}(bx) \sin(bx)}{b}$$

command

```
integrate(fresnel_cos(b*x)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2 \pi b^2 x C(bx)^2 - 4 b C(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) + \sqrt{2} \sqrt{b^2} S\left(\sqrt{2} \sqrt{b^2} x\right)}{2 \pi b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\text{Ci}(bx)^2, x\right)$$

111.62 Problem number 86

$$\int x^3 \text{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^4 \text{ cosineIntegral}(bx + a)}{4b^4} + \frac{x^4 \text{ cosineIntegral}(bx + a)}{4} + \frac{3 \cos(bx + a)}{2b^4} \\ & -\frac{a^2 \cos(bx + a)}{4b^4} + \frac{ax \cos(bx + a)}{2b^3} - \frac{3x^2 \cos(bx + a)}{4b^2} - \frac{a \sin(bx + a)}{2b^4} + \frac{a^3 \sin(bx + a)}{4b^4} \\ & + \frac{3x \sin(bx + a)}{2b^3} - \frac{a^2 x \sin(bx + a)}{4b^3} + \frac{a x^2 \sin(bx + a)}{4b^2} - \frac{x^3 \sin(bx + a)}{4b} \end{aligned}$$

command

```
integrate(x^3*fresnel_cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^5 x^4 C(bx + a) + 6 \pi a^2 \sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (\pi^2 a^4 - 3) \sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (3b^2 x - 5ab) \cos\left(\frac{1}{2} \pi b^2 x^2 + \dots\right)}{4 \pi^2 b^5}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^3 \text{Ci}(bx + a), x)$$

111.63 Problem number 87

$$\int x^2 \operatorname{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$\frac{a^3 \operatorname{cosineIntegral}(bx + a)}{3b^3} + \frac{x^3 \operatorname{cosineIntegral}(bx + a)}{3} + \frac{a \cos(bx + a)}{3b^3} - \frac{2x \cos(bx + a)}{3b^2} + \frac{2 \sin(bx + a)}{3b^3} - \frac{a^2 \sin(bx + a)}{3b^3} + \frac{ax \sin(bx + a)}{3b^2} - \frac{x^2 \sin(bx + a)}{3b}$$

command

```
integrate(x^2*fresnel_cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi^2 b^4 x^3 C(bx + a) + \pi^2 a^3 \sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 3\pi a \sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 2b \cos\left(\frac{1}{2}\pi b^2 x^2 + \pi abx + \frac{1}{2}\pi a^2\right) - \pi^2 b^4 x^2 \sin(bx + a) + \pi^2 a^2 \sqrt{b^2} \sin\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 2\pi a \sqrt{b^2} \cos\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + 2b \sin\left(\frac{1}{2}\pi b^2 x^2 + \pi abx + \frac{1}{2}\pi a^2\right) + \pi^2 b^4 x \cos(bx + a) - \pi^2 a \sqrt{b^2} \cos\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + \pi^2 a^2 \sqrt{b^2} \sin\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - \pi^2 b^4 x \sin(bx + a) + \pi^2 a \sqrt{b^2} \sin\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - \pi^2 a^2 \sqrt{b^2} \cos\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{3\pi^2 b^4}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \operatorname{Ci}(bx + a), x)$$

111.64 Problem number 88

$$\int x \operatorname{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$-\frac{a^2 \operatorname{cosineIntegral}(bx + a)}{2b^2} + \frac{x^2 \operatorname{cosineIntegral}(bx + a)}{2} - \frac{\cos(bx + a)}{2b^2} + \frac{a \sin(bx + a)}{2b^2} - \frac{x \sin(bx + a)}{2b}$$

command

```
integrate(x*fresnel_cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\pi b^3 x^2 C(bx + a) - \pi a^2 \sqrt{b^2} C\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (b^2 x - ab) \sin\left(\frac{1}{2}\pi b^2 x^2 + \pi abx + \frac{1}{2}\pi a^2\right) + \sqrt{b^2} S\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - \pi b^3 x \cos(bx + a) + \pi a \sqrt{b^2} \cos\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - \pi a^2 \sqrt{b^2} \sin\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + \pi b^3 x \sin(bx + a) - \pi a \sqrt{b^2} \sin\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + \pi a^2 \sqrt{b^2} \cos\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{2\pi b^3}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \operatorname{Ci}(bx + a), x)$$

111.65 Problem number 89

$$\int \text{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$\frac{(bx + a) \text{cosineIntegral}(bx + a)}{b} - \frac{\sin(bx + a)}{b}$$

command

```
integrate(fresnel_cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\pi bx + \pi a) C(bx + a) - \sin\left(\frac{1}{2} \pi b^2 x^2 + \pi abx + \frac{1}{2} \pi a^2\right)}{\pi b}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(Ci(bx + a), x)
```

111.66 Problem number 96

$$\int \text{CosIntegral}(a + bx)^2 dx$$

Optimal antiderivative

$$\frac{(bx + a) \text{cosineIntegral}(bx + a)^2}{b} + \frac{\text{sinIntegral}(2bx + 2a)}{b} - \frac{2 \text{cosineIntegral}(bx + a) \sin(bx + a)}{b}$$

command

```
integrate(fresnel_cos(b*x+a)^2,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2(\pi b^2 x + \pi ab) C(bx + a)^2 - 4b C(bx + a) \sin\left(\frac{1}{2} \pi b^2 x^2 + \pi abx + \frac{1}{2} \pi a^2\right) + \sqrt{2} \sqrt{b^2} S\left(\frac{\sqrt{2} \sqrt{b^2} (bx+a)}{b}\right)}{2 \pi b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

```
integral(Ci(bx + a)^2, x)
```

111.67 Problem number 100

$$\int x^2 \operatorname{CosIntegral}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\frac{x^3 \operatorname{cosineIntegral}(d(a + b \ln(cx^n)))}{3} - \frac{x^3 \operatorname{expIntegral}\left(\frac{(-ibdn+3)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{3a}{bn}} (cx^n)^{-\frac{3}{n}}}{6}$$

$$- \frac{x^3 \operatorname{expIntegral}\left(\frac{(ibdn+3)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{3a}{bn}} (cx^n)^{-\frac{3}{n}}}{6}$$

command

```
integrate(x^2*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{1}{3} x^3 C(bd \log(cx^n) + ad)$$

$$- \frac{1}{6} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} - \frac{9i}{2\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right)$$

$$- \frac{1}{6} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} + \frac{9i}{2\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right)$$

$$+ \frac{1}{6} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} - \frac{9i}{2\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right)$$

$$- \frac{1}{6} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{3 \log(c)}{n} - \frac{3a}{bn} + \frac{9i}{2\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 3i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \operatorname{Ci}(bd \log(cx^n) + ad), x)$$

111.68 Problem number 101

$$\int x \operatorname{CosIntegral}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\frac{x^2 \operatorname{cosineIntegral}(d(a + b \ln(cx^n)))}{2} - \frac{x^2 \operatorname{expIntegral}\left(\frac{(-ibdn+2)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{2a}{bn}} (cx^n)^{-\frac{2}{n}}}{4}$$

$$- \frac{x^2 \operatorname{expIntegral}\left(\frac{(ibdn+2)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{2a}{bn}} (cx^n)^{-\frac{2}{n}}}{4}$$

command

```
integrate(x*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
& -\frac{1}{4} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} - \frac{2i}{\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& -\frac{1}{4} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} + \frac{2i}{\pi b^2 d^2 n^2}\right)} C\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& +\frac{1}{4} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} - \frac{2i}{\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& -\frac{1}{4} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} + \frac{2i}{\pi b^2 d^2 n^2}\right)} S\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) \\
& +\frac{1}{2} x^2 C(bd \log(cx^n) + ad)
\end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x \text{ Ci}(bd \log(cx^n) + ad), x)$$

111.69 Problem number 102

$$\int \text{CosIntegral}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned}
& x \text{ cosineIntegral}(d(a + b \ln(cx^n))) - \frac{x \text{ expIntegral}\left(\frac{(-ibdn+1)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{a}{bn}} (cx^n)^{-\frac{1}{n}}}{2} \\
& - \frac{x \text{ expIntegral}\left(\frac{(ibdn+1)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{a}{bn}} (cx^n)^{-\frac{1}{n}}}{2}
\end{aligned}$$

command

```
integrate(fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
& -\frac{1}{2} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} - \frac{i}{2\pi b^2 d^2 n^2}\right)} \operatorname{C} \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \\
& -\frac{1}{2} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} + \frac{i}{2\pi b^2 d^2 n^2}\right)} \operatorname{C} \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \\
& +\frac{1}{2} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} - \frac{i}{2\pi b^2 d^2 n^2}\right)} \operatorname{S} \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \\
& -\frac{1}{2} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} + \frac{i}{2\pi b^2 d^2 n^2}\right)} \operatorname{S} \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) \\
& + x \operatorname{C}(bd \log(cx^n) + ad)
\end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\operatorname{Ci}(bd \log(cx^n) + ad), x)$$

111.70 Problem number 103

$$\int \frac{\operatorname{CosIntegral}(d(a + b \log(cx^n)))}{x} dx$$

Optimal antiderivative

$$\frac{\operatorname{cosineIntegral}(d(a + b \ln(cx^n)))(a + b \ln(cx^n))}{bn} - \frac{\sin(d(a + b \ln(cx^n)))}{bdn}$$

command

`integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{(\pi b d n \log(x) + \pi b d \log(c) + \pi a d) \operatorname{C}(bd \log(cx^n) + ad) - \sin\left(\frac{1}{2} \pi b^2 d^2 n^2 \log(x)^2 + \pi b^2 d^2 n \log(c) \log(x) + \frac{1}{2} \pi b^2 d^2\right)}{\pi b d n}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{Ci}(bd \log(cx^n) + ad)}{x}, x\right)$$

111.71 Problem number 104

$$\int \frac{\text{CosIntegral}(d(a + b \log(cx^n)))}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\text{cosineIntegral}(d(a + b \ln(cx^n)))}{x} + \frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}} \text{expIntegral}\left(-\frac{(-ibdn+1)(a+b \ln(cx^n))}{bn}\right)}{2x} \\ & + \frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}} \text{expIntegral}\left(-\frac{(ibdn+1)(a+b \ln(cx^n))}{bn}\right)}{2x} \end{aligned}$$

command

`integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\pi \sqrt{b^2 d^2 n^2} x e^{\left(\frac{\log(c)}{n} + \frac{a}{bn} + \frac{i}{2\pi b^2 d^2 n^2}\right)} \text{C}\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) + \pi \sqrt{b^2 d^2 n^2} x e^{\left(\frac{\log(c)}{n} + \frac{a}{bn} - \frac{i}{2\pi b^2 d^2 n^2}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}\left(\frac{\text{Ci}(bd \log(cx^n) + ad)}{x^2}, x\right)$$

111.72 Problem number 105

$$\int \frac{\text{CosIntegral}(d(a + b \log(cx^n)))}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\text{cosineIntegral}(d(a + b \ln(cx^n)))}{2x^2} + \frac{e^{\frac{2a}{bn}}(cx^n)^{\frac{2}{n}} \text{expIntegral}\left(-\frac{(-ibdn+2)(a+b \ln(cx^n))}{bn}\right)}{4x^2} \\ & + \frac{e^{\frac{2a}{bn}}(cx^n)^{\frac{2}{n}} \text{expIntegral}\left(-\frac{(ibdn+2)(a+b \ln(cx^n))}{bn}\right)}{4x^2} \end{aligned}$$

command

`integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\pi \sqrt{b^2 d^2 n^2} x^2 e^{\left(\frac{2 \log(c)}{n} + \frac{2a}{bn} + \frac{2i}{\pi b^2 d^2 n^2}\right)} \operatorname{C} \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 2i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right) + \pi \sqrt{b^2 d^2 n^2} x^2 e^{\left(\frac{2 \log(c)}{n} + \frac{2a}{bn}\right)}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{Ci}(bd \log(cx^n) + ad)}{x^3}, x\right)$$

111.73 Problem number 106

$$\int (ex)^m \operatorname{CosIntegral}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(ex)^{1+m} \operatorname{cosineIntegral}(d(a + b \ln(cx^n)))}{e(1+m)} \\ & - \frac{x(ex)^m \operatorname{expIntegral}\left(\frac{(-ibdn+m+1)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{a(1+m)}{bn}} (cx^n)^{-\frac{1+m}{n}}}{2(1+m)} \\ & - \frac{x(ex)^m \operatorname{expIntegral}\left(\frac{(ibdn+m+1)(a+b \ln(cx^n))}{bn}\right) e^{-\frac{a(1+m)}{bn}} (cx^n)^{-\frac{1+m}{n}}}{2(1+m)} \end{aligned}$$

command

```
integrate((e*x)^m*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\pi \sqrt{b^2 d^2 n^2} e^{\left(m - \frac{m \log(c)}{n} - \frac{am}{bn} - \frac{\log(c)}{n} - \frac{a}{bn} - \frac{im^2}{2\pi b^2 d^2 n^2} - \frac{im}{\pi b^2 d^2 n^2} - \frac{i}{2\pi b^2 d^2 n^2}\right)} \operatorname{C} \left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + i m + i) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}((ex)^m \operatorname{Ci}(bd \log(cx^n) + ad), x)$$

111.74 Problem number 110

$$\int \text{CosIntegral}(bx) \sin(bx) dx$$

Optimal antiderivative

$$\frac{\text{cosineIntegral}(2bx)}{2b} - \frac{\text{cosineIntegral}(bx) \cos(bx)}{b} + \frac{\ln(x)}{2b}$$

command

```
integrate(fresnel_cos(b*x)*sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2b \cos(bx) C(bx) - \sqrt{b^2} \cos\left(\frac{1}{2\pi}\right) C\left(\frac{(\pi bx+1)\sqrt{b^2}}{\pi b}\right) - \sqrt{b^2} \cos\left(\frac{1}{2\pi}\right) C\left(\frac{(\pi bx-1)\sqrt{b^2}}{\pi b}\right) - \sqrt{b^2} S\left(\frac{(\pi bx+1)\sqrt{b^2}}{\pi b}\right)}{2b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(\text{Ci}(bx) \sin(bx), x)$$

111.75 Problem number 111

$$\int x \text{CosIntegral}(bx) \sin(bx) dx$$

Optimal antiderivative

$$\frac{x}{2b} - \frac{x \text{cosineIntegral}(bx) \cos(bx)}{b} - \frac{\text{sinIntegral}(2bx)}{2b^2} + \frac{\text{cosineIntegral}(bx) \sin(bx)}{b^2} + \frac{\cos(bx) \sin(bx)}{2b^2}$$

command

```
integrate(x*fresnel_cos(b*x)*sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\pi b^2 x \cos(bx) C(bx) - 2\pi b C(bx) \sin(bx) - 2b \cos(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) - \sqrt{b^2} \left(\pi \sin\left(\frac{1}{2\pi}\right) - \cos\left(\frac{1}{2\pi}\right)\right) C\left(\frac{(\pi bx+1)}{\pi b}\right)}{2b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x \text{Ci}(bx) \sin(bx), x)$$

111.76 Problem number 112

$$\int x^2 \operatorname{CosIntegral}(bx) \sin(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^2}{4b} - \frac{\operatorname{cosineIntegral}(2bx)}{b^3} + \frac{2 \operatorname{cosineIntegral}(bx) \cos(bx)}{b^3} - \frac{x^2 \operatorname{cosineIntegral}(bx) \cos(bx)}{b} \\ & + \frac{\cos^2(bx)}{4b^3} - \frac{\ln(x)}{b^3} + \frac{2x \operatorname{cosineIntegral}(bx) \sin(bx)}{b^2} + \frac{x \cos(bx) \sin(bx)}{2b^2} - \frac{\sin^2(bx)}{b^3} \end{aligned}$$

command

```
integrate(x^2*fresnel_cos(b*x)*sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(\pi^2 b^3 x^2 - 2\pi^2 b) \cos(bx) C(bx) + \sqrt{b^2} \left((2\pi^2 - 1) \cos\left(\frac{1}{2\pi}\right) + \pi \sin\left(\frac{1}{2\pi}\right) \right) C\left(\frac{(\pi bx + 1)\sqrt{b^2}}{\pi b}\right) + \sqrt{b^2} \left((2\pi^2 - 1) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \operatorname{Ci}(bx) \sin(bx), x)$$

111.77 Problem number 113

$$\int x^3 \operatorname{CosIntegral}(bx) \sin(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5x}{2b^3} + \frac{x^3}{6b} + \frac{6x \operatorname{cosineIntegral}(bx) \cos(bx)}{b^3} - \frac{x^3 \operatorname{cosineIntegral}(bx) \cos(bx)}{b} \\ & + \frac{x(\cos^2(bx))}{2b^3} + \frac{3 \operatorname{sinIntegral}(2bx)}{b^4} - \frac{6 \operatorname{cosineIntegral}(bx) \sin(bx)}{b^4} \\ & + \frac{3x^2 \operatorname{cosineIntegral}(bx) \sin(bx)}{b^2} - \frac{4 \cos(bx) \sin(bx)}{b^4} + \frac{x^2 \cos(bx) \sin(bx)}{2b^2} - \frac{3x(\sin^2(bx))}{2b^3} \end{aligned}$$

command

```
integrate(x^3*fresnel_cos(b*x)*sin(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\pi b \cos\left(\frac{1}{2}\pi b^2 x^2\right) \cos(bx) + 2(\pi^3 b^4 x^3 - 6\pi^3 b^2 x) \cos(bx) C(bx) + (6\pi^3 \sin\left(\frac{1}{2\pi}\right) - (3\pi^2 - 1) \cos\left(\frac{1}{2\pi}\right)) \sqrt{b^2} C\left(\frac{(\pi bx + 1)\sqrt{b^2}}{\pi b}\right) + \sqrt{b^2} \left((2\pi^2 - 1) \right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^3 \operatorname{Ci}(bx) \sin(bx), x)$$

111.78 Problem number 117

$$\int \cos(bx) \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$-\frac{\operatorname{sinIntegral}(2bx)}{2b} + \frac{\operatorname{cosineIntegral}(bx) \sin(bx)}{b}$$

command

```
integrate(fresnel_cos(b*x)*cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2b C(bx) \sin(bx) - \sqrt{b^2} \cos\left(\frac{1}{2\pi}\right) S\left(\frac{(\pi bx+1)\sqrt{b^2}}{\pi b}\right) + \sqrt{b^2} \cos\left(\frac{1}{2\pi}\right) S\left(\frac{(\pi bx-1)\sqrt{b^2}}{\pi b}\right) + \sqrt{b^2} C\left(\frac{(\pi bx+1)\sqrt{b^2}}{\pi b}\right) \sin\left(\frac{(\pi bx+1)\sqrt{b^2}}{\pi b}\right) + \sqrt{b^2} C\left(\frac{(\pi bx-1)\sqrt{b^2}}{\pi b}\right) \sin\left(\frac{(\pi bx-1)\sqrt{b^2}}{\pi b}\right)}{2b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\cos(bx) \operatorname{Ci}(bx), x)$$

111.79 Problem number 118

$$\int x \cos(bx) \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$-\frac{\operatorname{cosineIntegral}(2bx)}{2b^2} + \frac{\operatorname{cosineIntegral}(bx) \cos(bx)}{b^2} - \frac{\ln(x)}{2b^2} + \frac{x \operatorname{cosineIntegral}(bx) \sin(bx)}{b} - \frac{\sin^2(bx)}{2b^2}$$

command

```
integrate(x*fresnel_cos(b*x)*cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2\pi b^2 x C(bx) \sin(bx) + 2\pi b \cos(bx) C(bx) - 2b \sin\left(\frac{1}{2}\pi b^2 x^2\right) \sin(bx) - \sqrt{b^2} \left(\pi \cos\left(\frac{1}{2\pi}\right) + \sin\left(\frac{1}{2\pi}\right)\right) C\left(\frac{(\pi bx+1)\sqrt{b^2}}{\pi b}\right) \sin\left(\frac{(\pi bx+1)\sqrt{b^2}}{\pi b}\right) + \sqrt{b^2} \left(\pi \cos\left(\frac{1}{2\pi}\right) + \sin\left(\frac{1}{2\pi}\right)\right) C\left(\frac{(\pi bx-1)\sqrt{b^2}}{\pi b}\right) \sin\left(\frac{(\pi bx-1)\sqrt{b^2}}{\pi b}\right)}{2b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \cos(bx) \operatorname{Ci}(bx), x)$$

111.80 Problem number 119

$$\int x^2 \cos(bx) \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$-\frac{3x}{4b^2} + \frac{2x \operatorname{cosineIntegral}(bx) \cos(bx)}{b^2} + \frac{\operatorname{sinIntegral}(2bx)}{b^3} - \frac{2 \operatorname{cosineIntegral}(bx) \sin(bx)}{b^3} \\ + \frac{x^2 \operatorname{cosineIntegral}(bx) \sin(bx)}{b} - \frac{5 \cos(bx) \sin(bx)}{4b^3} - \frac{x(\sin^2(bx))}{2b^2}$$

command

```
integrate(x^2*fresnel_cos(b*x)*cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4\pi^2 b^2 x \cos(bx) C(bx) - 2b \cos\left(\frac{1}{2}\pi b^2 x^2\right) \cos(bx) + 2(\pi^2 b^3 x^2 - 2\pi^2 b) C(bx) \sin(bx) + \sqrt{b^2} \left(\pi \cos\left(\frac{1}{2\pi}\right) - (2\pi^2 - \dots)\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \cos(bx) \operatorname{Ci}(bx), x)$$

111.81 Problem number 120

$$\int x^3 \cos(bx) \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$-\frac{x^2}{2b^2} + \frac{3 \operatorname{cosineIntegral}(2bx)}{b^4} - \frac{6 \operatorname{cosineIntegral}(bx) \cos(bx)}{b^4} \\ + \frac{3x^2 \operatorname{cosineIntegral}(bx) \cos(bx)}{b^2} - \frac{3(\cos^2(bx))}{4b^4} + \frac{3 \ln(x)}{b^4} - \frac{6x \operatorname{cosineIntegral}(bx) \sin(bx)}{b^3} \\ + \frac{x^3 \operatorname{cosineIntegral}(bx) \sin(bx)}{b} - \frac{2x \cos(bx) \sin(bx)}{b^3} + \frac{13(\sin^2(bx))}{4b^4} - \frac{x^2(\sin^2(bx))}{2b^2}$$

command

```
integrate(x^3*fresnel_cos(b*x)*cos(b*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\pi b^2 x \cos\left(\frac{1}{2}\pi b^2 x^2\right) \cos(bx) - 6(\pi^3 b^3 x^2 - 2\pi^3 b) \cos(bx) C(bx) - (6\pi^3 \cos\left(\frac{1}{2\pi}\right) + (3\pi^2 - 1) \sin\left(\frac{1}{2\pi}\right)) \sqrt{b^2} C\left(\dots\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^3 \cos(bx) \operatorname{Ci}(bx), x)$$

111.82 Problem number 121

$$\int \text{CosIntegral}(2x) \sin(5x) dx$$

Optimal antiderivative

$$\frac{\text{cosineIntegral}(3x)}{10} + \frac{\text{cosineIntegral}(7x)}{10} - \frac{\text{cosineIntegral}(2x) \cos(5x)}{5}$$

command

```
integrate(fresnel_cos(2*x)*sin(5*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{5} \cos(5x) C(2x) + \frac{1}{10} \cos\left(\frac{25}{8\pi}\right) C\left(\frac{4\pi x + 5}{2\pi}\right) + \frac{1}{10} \cos\left(\frac{25}{8\pi}\right) C\left(\frac{4\pi x - 5}{2\pi}\right) \\ + \frac{1}{10} \left(S\left(\frac{4\pi x + 5}{2\pi}\right) + S\left(\frac{4\pi x - 5}{2\pi}\right) \right) \sin\left(\frac{25}{8\pi}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

111.83 Problem number 122

$$\int \cos(5x) \text{CosIntegral}(2x) dx$$

Optimal antiderivative

$$-\frac{\text{sinIntegral}(3x)}{10} - \frac{\text{sinIntegral}(7x)}{10} + \frac{\text{cosineIntegral}(2x) \sin(5x)}{5}$$

command

```
integrate(fresnel_cos(2*x)*cos(5*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{10} \cos\left(\frac{25}{8\pi}\right) S\left(\frac{4\pi x + 5}{2\pi}\right) + \frac{1}{10} \cos\left(\frac{25}{8\pi}\right) S\left(\frac{4\pi x - 5}{2\pi}\right) \\ + \frac{1}{5} C(2x) \sin(5x) + \frac{1}{10} \left(C\left(\frac{4\pi x + 5}{2\pi}\right) - C\left(\frac{4\pi x - 5}{2\pi}\right) \right) \sin\left(\frac{25}{8\pi}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: TypeError

111.84 Problem number 123

$$\int x^2 \operatorname{CosIntegral}(a + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{ax}{2b^2} + \frac{x^2}{4b} - \frac{\operatorname{cosineIntegral}(2bx + 2a)}{b^3} + \frac{a^2 \operatorname{cosineIntegral}(2bx + 2a)}{2b^3} \\ & + \frac{2 \operatorname{cosineIntegral}(bx + a) \cos(bx + a)}{b^3} - \frac{x^2 \operatorname{cosineIntegral}(bx + a) \cos(bx + a)}{b} \\ & + \frac{\cos^2(bx + a)}{4b^3} + \frac{\cos(2bx + 2a)}{2b^3} - \frac{\ln(bx + a)}{b^3} + \frac{a^2 \ln(bx + a)}{2b^3} \\ & + \frac{a \operatorname{sinIntegral}(2bx + 2a)}{b^3} + \frac{2x \operatorname{cosineIntegral}(bx + a) \sin(bx + a)}{b^2} \\ & - \frac{a \cos(bx + a) \sin(bx + a)}{2b^3} + \frac{x \cos(bx + a) \sin(bx + a)}{2b^2} \end{aligned}$$

command

```
integrate(x^2*fresnel_cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2(\pi^2 b^3 x^2 - 2\pi^2 b) \cos(bx + a) C(bx + a) - \sqrt{b^2} \left((\pi^2 (a^2 - 2) + 2\pi a + 1) \cos\left(\frac{1}{2\pi}\right) - (\pi + 2\pi^2 a) \sin\left(\frac{1}{2\pi}\right) \right) C\left(\frac{1}{2\pi}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x^2 \operatorname{Ci}(bx + a) \sin(bx + a), x)$$

111.85 Problem number 124

$$\int x \operatorname{CosIntegral}(a + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{2b} - \frac{a \operatorname{cosineIntegral}(2bx + 2a)}{2b^2} - \frac{x \operatorname{cosineIntegral}(bx + a) \cos(bx + a)}{b} - \frac{a \ln(bx + a)}{2b^2} \\ & - \frac{\operatorname{sinIntegral}(2bx + 2a)}{2b^2} + \frac{\operatorname{cosineIntegral}(bx + a) \sin(bx + a)}{b^2} + \frac{\cos(bx + a) \sin(bx + a)}{2b^2} \end{aligned}$$

command

```
integrate(x*fresnel_cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\pi b^2 x \cos(bx + a) C(bx + a) - 2\pi b C(bx + a) \sin(bx + a) - 2b \cos(bx + a) \sin\left(\frac{1}{2}\pi b^2 x^2 + \pi abx + \frac{1}{2}\pi a^2\right) + \sqrt{b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x \text{Ci}(bx + a) \sin(bx + a), x)$$

111.86 Problem number 125

$$\int \text{CosIntegral}(a + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$\frac{\text{cosineIntegral}(2bx + 2a)}{2b} - \frac{\text{cosineIntegral}(bx + a) \cos(bx + a)}{b} + \frac{\ln(bx + a)}{2b}$$

command

`integrate(fresnel_cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2b \cos(bx + a) C(bx + a) - \sqrt{b^2} \cos\left(\frac{1}{2\pi}\right) C\left(\frac{(\pi bx + \pi a + 1)\sqrt{b^2}}{\pi b}\right) - \sqrt{b^2} \cos\left(\frac{1}{2\pi}\right) C\left(\frac{(\pi bx + \pi a - 1)\sqrt{b^2}}{\pi b}\right) - \sqrt{b^2} S$$

$$2b^2$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(\text{Ci}(bx + a) \sin(bx + a), x)$$

111.87 Problem number 127

$$\int x^2 \cos(a + bx) \text{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x}{b^2} + \frac{a \text{cosineIntegral}(2bx + 2a)}{b^3} + \frac{2x \text{cosineIntegral}(bx + a) \cos(bx + a)}{b^2} \\ & - \frac{a \cos(2bx + 2a)}{4b^3} + \frac{x \cos(2bx + 2a)}{4b^2} + \frac{a \ln(bx + a)}{b^3} + \frac{\text{sinIntegral}(2bx + 2a)}{b^3} \\ & - \frac{a^2 \text{sinIntegral}(2bx + 2a)}{2b^3} - \frac{2 \text{cosineIntegral}(bx + a) \sin(bx + a)}{b^3} \\ & + \frac{x^2 \text{cosineIntegral}(bx + a) \sin(bx + a)}{b} - \frac{\cos(bx + a) \sin(bx + a)}{b^3} - \frac{\sin(2bx + 2a)}{8b^3} \end{aligned}$$

command

```
integrate(x^2*fresnel_cos(b*x+a)*cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \pi^2 b^2 x \cos (bx + a) C (bx + a) - 2 b \cos \left(\frac{1}{2} \pi b^2 x^2 + \pi abx + \frac{1}{2} \pi a^2 \right) \cos (bx + a) + 2 \left(\pi^2 b^3 x^2 - 2 \pi^2 b \right) C (bx + a) \sin (bx + a)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x^2 \cos (bx + a) \text{Ci}(bx + a), x)$$

111.88 Problem number 128

$$\int x \cos (a + bx) \text{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\text{cosineIntegral}(2bx + 2a)}{2b^2} + \frac{\text{cosineIntegral}(bx + a) \cos (bx + a)}{b^2} + \frac{\cos (2bx + 2a)}{4b^2} \\ & -\frac{\ln (bx + a)}{2b^2} + \frac{a \text{sinIntegral}(2bx + 2a)}{2b^2} + \frac{x \text{cosineIntegral}(bx + a) \sin (bx + a)}{b} \end{aligned}$$

command

```
integrate(x*fresnel_cos(b*x+a)*cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \pi b^2 x C (bx + a) \sin (bx + a) + 2 \pi b \cos (bx + a) C (bx + a) - 2 b \sin \left(\frac{1}{2} \pi b^2 x^2 + \pi abx + \frac{1}{2} \pi a^2 \right) \sin (bx + a) - \sqrt{b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(x \cos (bx + a) \text{Ci}(bx + a), x)$$

111.89 Problem number 129

$$\int \cos(a + bx) \operatorname{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$-\frac{\operatorname{sinIntegral}(2bx + 2a)}{2b} + \frac{\operatorname{cosineIntegral}(bx + a) \sin(bx + a)}{b}$$

command

```
integrate(fresnel_cos(b*x+a)*cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2b C(bx + a) \sin(bx + a) - \sqrt{b^2} \cos\left(\frac{1}{2\pi}\right) S\left(\frac{(\pi bx + \pi a + 1)\sqrt{b^2}}{\pi b}\right) + \sqrt{b^2} \cos\left(\frac{1}{2\pi}\right) S\left(\frac{(\pi bx + \pi a - 1)\sqrt{b^2}}{\pi b}\right) + \sqrt{b^2} C\left(\frac{1}{2\pi}\right)}{2b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\cos(bx + a) \operatorname{Ci}(bx + a), x)$$

111.90 Problem number 131

$$\int x \operatorname{CosIntegral}(c + dx) \sin(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c \operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2bd} \\ & - \frac{c \operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2bd} \\ & - \frac{x \operatorname{cosineIntegral}(dx + c) \cos(bx + a)}{b} - \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2b^2} \\ & - \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2b^2} \\ & - \frac{\operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} \\ & - \frac{\operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} \\ & + \frac{c \operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} \\ & + \frac{c \operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} + \frac{\operatorname{cosineIntegral}(dx + c) \sin(bx + a)}{b^2} \\ & + \frac{\sin\left(a - c + (b-d)x\right)}{2b(b-d)} + \frac{\sin\left(a + c + (b+d)x\right)}{2b(b+d)} \end{aligned}$$

command

```
integrate(x*fresnel_cos(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2\pi bd^3 x \cos(bx + a) C(dx + c) - 2\pi d^3 C(dx + c) \sin(bx + a) - 2bd^2 \cos(bx + a) \sin\left(\frac{1}{2}\pi d^2 x^2 + \pi cdx + \frac{1}{2}\pi c^2\right) +$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \operatorname{Ci}(dx + c) \sin(bx + a), x)$$

111.91 Problem number 132

$$\int \text{CosIntegral}(c + dx) \sin(a + bx) dx$$

Optimal antiderivative

$$\frac{\text{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b} + \frac{\text{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b} - \frac{\text{cosineIntegral}(dx + c) \cos(bx + a)}{b} - \frac{\text{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b} - \frac{\text{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b}$$

command

```
integrate(fresnel_cos(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{2d \cos(bx + a) C(dx + c) - \sqrt{d^2} \cos\left(a - \frac{bc}{d} - \frac{b^2}{2\pi d^2}\right) C\left(\frac{(\pi d^2 x + \pi c d + b)\sqrt{d^2}}{\pi d^2}\right) - \sqrt{d^2} \cos\left(a - \frac{bc}{d} + \frac{b^2}{2\pi d^2}\right) C\left(\frac{(\pi d^2 x + \pi c d - b)\sqrt{d^2}}{\pi d^2}\right)}{2b}$$

Fricas 1.3.7 via sagemath 9.3 output

$$\text{integral}(\text{Ci}(dx + c) \sin(bx + a), x)$$

111.92 Problem number 134

$$\int x \cos(a + bx) \text{CosIntegral}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{\operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b^2} \\
& - \frac{\operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b^2} \\
& + \frac{\operatorname{cosineIntegral}(dx+c) \cos(bx+a)}{b^2} + \frac{\cos(a-c+(b-d)x)}{2b(b-d)} \\
& + \frac{\cos(a+c+(b+d)x)}{2b(b+d)} + \frac{c \cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2bd} \\
& + \frac{c \cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2bd} \\
& + \frac{c \operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} \\
& + \frac{c \operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} \\
& + \frac{\operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} \\
& + \frac{\operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} + \frac{x \operatorname{cosineIntegral}(dx+c) \sin(bx+a)}{b}
\end{aligned}$$

command

```
integrate(x*fresnel_cos(d*x+c)*cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \pi b d^3 x C(dx+c) \sin(bx+a) + 2 \pi d^3 \cos(bx+a) C(dx+c) - 2 b d^2 \sin\left(\frac{1}{2} \pi d^2 x^2 + \pi c d x + \frac{1}{2} \pi c^2\right) \sin(bx+a) -$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(x \cos(bx+a) \operatorname{Ci}(dx+c), x)$$

111.93 Problem number 135

$$\int \cos(a + bx) \operatorname{CosIntegral}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2b} \\ & - \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2b} \\ & - \frac{\operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b} \\ & - \frac{\operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b} + \frac{\operatorname{cosineIntegral}(dx + c) \sin(bx + a)}{b} \end{aligned}$$

command

```
integrate(fresnel_cos(d*x+c)*cos(b*x+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 d C(dx + c) \sin(bx + a) - \sqrt{d^2} \cos\left(a - \frac{bc}{d} - \frac{b^2}{2\pi d^2}\right) S\left(\frac{(\pi d^2 x + \pi c d + b) \sqrt{d^2}}{\pi d^2}\right) + \sqrt{d^2} \cos\left(a - \frac{bc}{d} + \frac{b^2}{2\pi d^2}\right) S\left(\frac{(\pi d^2 x + \pi c d - b) \sqrt{d^2}}{\pi d^2}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}(\cos(bx + a) \operatorname{Ci}(dx + c), x)$$

112 Test file number 208

Test folder name:

test_cases/8_Special_functions/208_8.8_Polylogarithm_function

112.1 Problem number 145

$$\int \frac{\operatorname{PolyLog}(2, x)}{-1 + x} dx$$

Optimal antiderivative

$$\ln(1-x)^2 \ln(x) + 2 \ln(1-x) \operatorname{polylog}(2, 1-x) + \ln(1-x) \operatorname{polylog}(2, x) - 2 \operatorname{polylog}(3, 1-x)$$

command

```
integrate(polylog(2,x)/(-1+x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\log(x) \log(-x+1)^2 + (\operatorname{Li}_2(x) + 2\operatorname{Li}_2(-x+1)) \log(-x+1) - 2 \operatorname{polylog}(3, -x+1)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{Li}_2(x)}{x-1}, x\right)$$

112.2 Problem number 146

$$\int -\frac{\operatorname{PolyLog}(2, x)}{1-x} dx$$

Optimal antiderivative

$$\ln(1-x)^2 \ln(x) + 2 \ln(1-x) \operatorname{polylog}(2, 1-x) + \ln(1-x) \operatorname{polylog}(2, x) - 2 \operatorname{polylog}(3, 1-x)$$

command

```
integrate(-polylog(2,x)/(1-x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\log(x) \log(-x+1)^2 + (\operatorname{Li}_2(x) + 2\operatorname{Li}_2(-x+1)) \log(-x+1) - 2 \operatorname{polylog}(3, -x+1)$$

Fricas 1.3.7 via sagemath 9.3 output

$$\operatorname{integral}\left(\frac{\operatorname{Li}_2(x)}{x-1}, x\right)$$

113 Test file number 209

Test folder name:

test_cases/209_Blake_problems

113.1 Problem number 204

$$\int \frac{1}{(-1+x^2)\sqrt[3]{-x+x^3}} dx$$

Optimal antiderivative

$$-\frac{3(x^3-x)^{\frac{2}{3}}}{2x^2-2}$$

command

```
integrate(1/(x^2-1)/(x^3-x)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{3(x^3-x)^{\frac{2}{3}}}{2(x^2-1)}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.2 Problem number 587

$$\int \frac{-1+x^{16}}{\sqrt{-1+x^4}(1-x^8+x^{16})} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((x^16-1)/(x^4-1)^(1/2)/(x^16-x^8+1),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.3 Problem number 946

$$\int \frac{\sqrt[3]{1+2x+x^2}}{4+x+x^2+x^3} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((x^2+2*x+1)^(1/3)/(x^3+x^2+x+4),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.4 Problem number 1062

$$\int \frac{1}{(b+a^3x^3)\sqrt[3]{-bx^2+a^3x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(1/(a^3*x^3+b)/(a^3*x^3-b*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.5 Problem number 1063

$$\int \frac{1}{(b + a^3x^3) \sqrt[3]{-bx^2 + a^3x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(1/(a^3*x^3+b)/(a^3*x^3-b*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.6 Problem number 1074

$$\int \frac{1}{(-b + a^3x^3) \sqrt[3]{-bx^2 + a^3x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(1/(a^3*x^3-b)/(a^3*x^3-b*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.7 Problem number 1075

$$\int \frac{1}{(-b + a^3x^3) \sqrt[3]{-bx^2 + a^3x^3}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate(1/(a^3*x^3-b)/(a^3*x^3-b*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.8 Problem number 1087

$$\int \frac{1}{\sqrt[3]{-bx^2 + ax^3} (b + ax^4)} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate(1/(a*x^3-b*x^2)^(1/3)/(a*x^4+b),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.9 Problem number 1088

$$\int \frac{1}{\sqrt[3]{-bx^2 + ax^3} (b + ax^4)} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(1/(a*x^3-b*x^2)^(1/3)/(a*x^4+b),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.10 Problem number 1100

$$\int \frac{\sqrt[4]{-bx^3 + ax^4}}{x (b + ax^3)} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((a*x^4-b*x^3)^(1/4)/x/(a*x^3+b),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.11 Problem number 1101

$$\int \frac{\sqrt[4]{-bx^3 + ax^4}}{x(b + ax^3)} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((a*x^4-b*x^3)^(1/4)/x/(a*x^3+b),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.12 Problem number 1115

$$\int \frac{1}{\sqrt[4]{b + ax^4} (2b + ax^4)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{a^{\frac{1}{4}}x2^{\frac{3}{4}}}{2(ax^4+b)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{4a^{\frac{1}{4}}b} + \frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{4}}x2^{\frac{3}{4}}}{2(ax^4+b)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{4a^{\frac{1}{4}}b}$$

command

```
integrate(1/(a*x^4+b)^(1/4)/(a*x^4+2*b),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$-\frac{1}{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} \left(\frac{1}{ab^4}\right)^{\frac{1}{4}} \arctan \left(\frac{4 \left(4 \left(\frac{1}{8}\right)^{\frac{3}{4}} (ax^4 + b)^{\frac{3}{4}} ab^3 x \left(\frac{1}{ab^4}\right)^{\frac{3}{4}} + \left(\frac{1}{8}\right)^{\frac{1}{4}} (ax^4 + b)^{\frac{1}{4}} abx^3 \left(\frac{1}{ab^4}\right)^{\frac{1}{4}} - 2 \sqrt{\frac{1}{2}} \left(\left(\frac{1}{8}\right)^{\frac{1}{4}} \sqrt{ax^4 + b}\right)}{ax^4 + 2b} \right)$$

$$+\frac{1}{8} \left(\frac{1}{8}\right)^{\frac{1}{4}} \left(\frac{1}{ab^4}\right)^{\frac{1}{4}} \log \left(\frac{8 \left(\frac{1}{8}\right)^{\frac{3}{4}} \sqrt{ax^4 + b} ab^3 x^2 \left(\frac{1}{ab^4}\right)^{\frac{3}{4}} + 2 \sqrt{\frac{1}{2}} (ax^4 + b)^{\frac{1}{4}} ab^2 x^3 \sqrt{\frac{1}{ab^4}} + 2 (ax^4 + b)^{\frac{3}{4}} x + \left(\frac{1}{8}\right)^{\frac{1}{4}}}{2(ax^4 + 2b)} \right)$$

$$-\frac{1}{8} \left(\frac{1}{8}\right)^{\frac{1}{4}} \left(\frac{1}{ab^4}\right)^{\frac{1}{4}} \log \left(\frac{8 \left(\frac{1}{8}\right)^{\frac{3}{4}} \sqrt{ax^4 + b} ab^3 x^2 \left(\frac{1}{ab^4}\right)^{\frac{3}{4}} - 2 \sqrt{\frac{1}{2}} (ax^4 + b)^{\frac{1}{4}} ab^2 x^3 \sqrt{\frac{1}{ab^4}} - 2 (ax^4 + b)^{\frac{3}{4}} x + \left(\frac{1}{8}\right)^{\frac{1}{4}}}{2(ax^4 + 2b)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.13 Problem number 1197

$$\int \frac{1}{x + \sqrt{x + \sqrt{1 + x^2}}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate(1/(x+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.14 Problem number 1198

$$\int \frac{1}{x + \sqrt{x + \sqrt{1 + x^2}}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate(1/(x+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.15 Problem number 1344

$$\int \frac{1+x}{(-1-x+x^3)\sqrt[3]{-x^2+x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((1+x)/(x^3-x-1)/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.16 Problem number 1345

$$\int \frac{1+x}{(-1-x+x^3)\sqrt[3]{-x^2+x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((1+x)/(x^3-x-1)/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.17 Problem number 1366

$$\int \frac{(4 + x^5)(1 - x^4 - 2x^5 + x^8 + x^9 + x^{10})}{x^2(-1 + x^5)^{3/4}(1 + x^4 - 2x^5 - x^8 - x^9 + x^{10})} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((x^5+4)*(x^10+x^9+x^8-2*x^5-x^4+1)/x^2/(x^5-1)^(3/4)/(x^10-x^9-x^8-2*x^5+x^4+1),x,
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.18 Problem number 1367

$$\int \frac{(4 + x^5)(1 - x^4 - 2x^5 + x^8 + x^9 + x^{10})}{x^2(-1 + x^5)^{3/4}(1 + x^4 - 2x^5 - x^8 - x^9 + x^{10})} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((x^5+4)*(x^10+x^9+x^8-2*x^5-x^4+1)/x^2/(x^5-1)^(3/4)/(x^10-x^9-x^8-2*x^5+x^4+1),x,
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.19 Problem number 1456

$$\int \frac{x}{x + \sqrt{x + \sqrt{1 + x^2}}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(x/(x+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.20 Problem number 1457

$$\int \frac{x}{x + \sqrt{x + \sqrt{1 + x^2}}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(x/(x+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.21 Problem number 1541

$$\int \frac{(1+x^6)(-1+2x^6)(-1+x^4+2x^6)^{5/4}}{x^{10}(-1-x^4+2x^6)} dx$$

Optimal antiderivative

$$\frac{(2x^6+x^4-1)^{\frac{1}{4}}(20x^{12}+38x^{10}+104x^8-20x^6-19x^4+5)}{45x^9} + 2^{\frac{1}{4}} \arctan\left(\frac{2^{\frac{1}{4}}x}{(2x^6+x^4-1)^{\frac{1}{4}}}\right) - 2^{\frac{1}{4}} \operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x}{(2x^6+x^4-1)^{\frac{1}{4}}}\right)$$

command

```
integrate((x^6+1)*(2*x^6-1)*(2*x^6+x^4-1)^(5/4)/x^10/(2*x^6-x^4-1),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$180 \cdot 8^{\frac{3}{4}} x^9 \arctan\left(-\frac{16 \cdot 8^{\frac{1}{4}} (2x^6+x^4-1)^{\frac{1}{4}} x^3 + 4 \cdot 8^{\frac{3}{4}} (2x^6+x^4-1)^{\frac{3}{4}} x - 2^{\frac{3}{4}} \left(8 \cdot 8^{\frac{1}{4}} \sqrt{2x^6+x^4-1} x^2 + 8^{\frac{3}{4}} (2x^6+3x^4-1)\right)}{8(2x^6-x^4-1)}\right) + 45 \cdot 8$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.22 Problem number 1601

$$\int \frac{(2+x^6)(1-2x^6+x^8+x^{12})}{x^8 \sqrt[4]{-1+x^6} (-1+x^4+x^6)} dx$$

Optimal antiderivative

$$\frac{2(x^6-1)^{\frac{3}{4}}(3x^6-7x^4-3)}{21x^7} - \sqrt{2} \arctan\left(\frac{\sqrt{2} x (x^6-1)^{\frac{1}{4}}}{-x^2 + \sqrt{x^6-1}}\right) - \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} x (x^6-1)^{\frac{1}{4}}}{x^2 + \sqrt{x^6-1}}\right)$$

command

```
integrate((x^6+2)*(x^12+x^8-2*x^6+1)/x^8/(x^6-1)^(1/4)/(x^6+x^4-1),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$84\sqrt{2}x^7 \arctan \left(\frac{x^{12} + 2x^{10} + x^8 - 2x^6 - 2x^4 + 2\sqrt{2}(x^7 - 3x^5 - x)(x^6 - 1)^{\frac{3}{4}} + 2\sqrt{2}(3x^9 - x^7 - 3x^3)(x^6 - 1)^{\frac{1}{4}} + 4(x^8 + x^6 - x^2)\sqrt{x^6 - 1}}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.23 Problem number 1868

$$\int \frac{\sqrt{ax - \sqrt{b + a^2x^2}}}{a^2x^2 + \sqrt{b + a^2x^2}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((a*x-(a^2*x^2+b)^(1/2))^(1/2)/(a^2*x^2+(a^2*x^2+b)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.24 Problem number 1872

$$\int \frac{1}{x^3 \sqrt[3]{-bx^2 + ax^3} (d + cx^3)} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate(1/x^3/(a*x^3-b*x^2)^(1/3)/(c*x^3+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.25 Problem number 1886

$$\int \frac{1 + ax^2}{(-1 + ax^2) \sqrt{x + \sqrt{1 + x^2}}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((a*x^2+1)/(a*x^2-1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\begin{aligned}
& -\frac{2}{3} \left(x^2 - \sqrt{x^2+1} x - 1 \right) \sqrt{x + \sqrt{x^2+1}} \\
& + 4 \left(-\frac{2a^3 \sqrt{\frac{a+1}{a^6}} - a - 2}{a^3} \right)^{\frac{1}{4}} \arctan \left(\left(a^4 \sqrt{\frac{a+1}{a^6}} + a \right) \sqrt{\left(2a^3 \sqrt{\frac{a+1}{a^6}} + a + 2 \right) \sqrt{-\frac{2a^3 \sqrt{\frac{a+1}{a^6}} - a - 2}{a^3}}} \right. \\
& \left. - \left(a^4 \sqrt{\frac{a+1}{a^6}} + a \right) \sqrt{x + \sqrt{x^2+1}} \left(-\frac{2a^3 \sqrt{\frac{a+1}{a^6}} - a - 2}{a^3} \right)^{\frac{3}{4}} \right) \\
& - 4 \left(\frac{2a^3 \sqrt{\frac{a+1}{a^6}} + a + 2}{a^3} \right)^{\frac{1}{4}} \arctan \left(\frac{1}{8} \left(\left(a^4 \sqrt{\frac{a+1}{a^6}} - a \right) \sqrt{-64 \left(2a^3 \sqrt{\frac{a+1}{a^6}} - a - 2 \right) \sqrt{\frac{2a^3 \sqrt{\frac{a+1}{a^6}} + a + 2}{a^3}}} \right. \right. \\
& \left. \left. + \left(\frac{2a^3 \sqrt{\frac{a+1}{a^6}} + a + 2}{a^3} \right)^{\frac{1}{4}} \log \left(8 \left(a^3 \sqrt{\frac{a+1}{a^6}} - 1 \right) \left(\frac{2a^3 \sqrt{\frac{a+1}{a^6}} + a + 2}{a^3} \right)^{\frac{1}{4}} \right. \right. \right. \\
& \left. \left. + 8 \sqrt{x + \sqrt{x^2+1}} \right) \right) \\
& - \left(\frac{2a^3 \sqrt{\frac{a+1}{a^6}} + a + 2}{a^3} \right)^{\frac{1}{4}} \log \left(-8 \left(a^3 \sqrt{\frac{a+1}{a^6}} - 1 \right) \left(\frac{2a^3 \sqrt{\frac{a+1}{a^6}} + a + 2}{a^3} \right)^{\frac{1}{4}} \right. \\
& \left. + 8 \sqrt{x + \sqrt{x^2+1}} \right) \\
& - \left(-\frac{2a^3 \sqrt{\frac{a+1}{a^6}} - a - 2}{a^3} \right)^{\frac{1}{4}} \log \left(8 \left(a^3 \sqrt{\frac{a+1}{a^6}} + 1 \right) \left(-\frac{2a^3 \sqrt{\frac{a+1}{a^6}} - a - 2}{a^3} \right)^{\frac{1}{4}} \right. \\
& \left. + 8 \sqrt{x + \sqrt{x^2+1}} \right) \\
& + \left(-\frac{2a^3 \sqrt{\frac{a+1}{a^6}} - a - 2}{a^3} \right)^{\frac{1}{4}} \log \left(-8 \left(a^3 \sqrt{\frac{a+1}{a^6}} + 1 \right) \left(-\frac{2a^3 \sqrt{\frac{a+1}{a^6}} - a - 2}{a^3} \right)^{\frac{1}{4}} \right.
\end{aligned}$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

113.26 Problem number 1907

$$\int \frac{1}{x^3 (-b + ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate(1/x^3/(a*x^3-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.27 Problem number 1908

$$\int \frac{1}{x^3 (-b + ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate(1/x^3/(a*x^3-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.28 Problem number 1926

$$\int \frac{1}{x^3 (b + ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(1/x^3/(a*x^3+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.29 Problem number 1927

$$\int \frac{1}{x^3 (b + ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(1/x^3/(a*x^3+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.30 Problem number 1979

$$\int \frac{(1+x^5)^{2/3}(-3+2x^5)(2+x^3+2x^5)}{x^6(2-x^3+2x^5)} dx$$

Optimal antiderivative

$$\frac{3(x^5+1)^{\frac{2}{3}}(2x^5+5x^3+2)}{10x^5} - \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}x}{x+2\sqrt[3]{x^5+1}}\right) 2^{\frac{1}{3}}}{2} + \frac{\ln\left(-x+2\sqrt[3]{x^5+1}\right) 2^{\frac{1}{3}}}{2} - \frac{\ln\left(x^2+2\sqrt[3]{x^5+1}+2\sqrt[3]{x^5+1}\right) 2^{\frac{1}{3}}}{4}$$

command

`integrate((x^5+1)^(2/3)*(2*x^5-3)*(2*x^5+x^3+2)/x^6/(2*x^5-x^3+2),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$20 \cdot 4^{\frac{1}{6}} \sqrt{3} x^5 \arctan\left(\frac{4^{\frac{1}{6}} \sqrt{3} \left(12 \cdot 4^{\frac{2}{3}} (2x^{11} + x^9 - x^7 + 4x^6 + x^4 + 2x)(x^5 + 1)^{\frac{2}{3}} + 4^{\frac{1}{3}} (8x^{15} + 60x^{13} + 24x^{11} + 24x^{10} - x^9 + 120x^8 + 24x^6 + 24x^5 - 12x^4 - 12x^3 - 48x^2 - 48x - 48)\right)}{6(8x^{15} - 12x^{13} - 48x^{11} + 24x^{10} - x^9 - 24x^8 - 48x^6 + 24x^5 - 12x^4 - 12x^3 - 48x^2 - 48x - 48)}}\right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.31 Problem number 2026

$$\int \frac{\sqrt[3]{1-x^7}(-2+x^3+2x^7)(3+4x^7)}{x^2(-1+x^7)(-4+x^3+4x^7)} dx$$

Optimal antiderivative

$$\frac{3(-x^7+1)^{\frac{1}{3}}}{2x} - \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}x}{x+2\sqrt[3]{-x^7+1}}\right) 2^{\frac{1}{3}}}{4} - \frac{\ln\left(-x+2\sqrt[3]{-x^7+1}\right) 2^{\frac{1}{3}}}{4} + \frac{\ln\left(x^2+2\sqrt[3]{-x^7+1}+2\sqrt[3]{-x^7+1}\right) 2^{\frac{1}{3}}}{8}$$

command

`integrate((-x^7+1)^(1/3)*(2*x^7+x^3-2)*(4*x^7+3)/x^2/(x^7-1)/(4*x^7+x^3-4),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$4 \cdot 4^{\frac{1}{6}} \sqrt{3} (-1)^{\frac{1}{3}} x \arctan \left(-\frac{4^{\frac{1}{6}} \sqrt{3} \left(6 \cdot 4^{\frac{2}{3}} (-1)^{\frac{2}{3}} (16x^{16} - 28x^{12} - 32x^9 + x^8 + 28x^5 + 16x^2) (-x^7 + 1) \right)^{\frac{1}{3}} - 48 (-1)^{\frac{1}{3}} (8x^{15} - 2x^{11} - 16x^8 - x^7)}{6(64x^{21} + 48x^{17} - 192x^{14} - 96x^{13} - 96x^{10} - 48x^7 - 16x^4 - 16x)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.32 Problem number 2067

$$\int \frac{\sqrt[3]{-1 + 2x^3 + x^8} (3 + 5x^8)}{x^2 (-1 + x^8)} dx$$

Optimal antiderivative

$$\frac{3(x^8 + 2x^3 - 1)^{\frac{1}{3}}}{x} + 2^{\frac{1}{3}} \sqrt{3} \arctan \left(\frac{\sqrt{3} x}{x + 2^{\frac{2}{3}} (x^8 + 2x^3 - 1)^{\frac{1}{3}}} \right) + 2^{\frac{1}{3}} \ln \left(-2x + 2^{\frac{2}{3}} (x^8 + 2x^3 - 1)^{\frac{1}{3}} \right) - \frac{\ln \left(2x^2 + 2^{\frac{2}{3}} x (x^8 + 2x^3 - 1)^{\frac{1}{3}} + 2^{\frac{1}{3}} (x^8 + 2x^3 - 1)^{\frac{2}{3}} \right) 2^{\frac{1}{3}}}{2}$$

command

`integrate((x^8+2*x^3-1)^(1/3)*(5*x^8+3)/x^2/(x^8-1),x, algorithm="fricas")`

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$2 \sqrt{3} 2^{\frac{1}{3}} x \arctan \left(\frac{6 \sqrt{3} 2^{\frac{2}{3}} (x^{18} + 18x^{13} - 2x^{10} + 36x^8 - 18x^5 + x^2) (x^8 + 2x^3 - 1)^{\frac{1}{3}} + 6 \sqrt{3} 2^{\frac{1}{3}} (x^{17} + 6x^{12} - 2x^9 - 6x^4 + x) (x^8 + 2x^3 - 1)^{\frac{2}{3}} + \sqrt{3}}{3(x^{24} - 3x^{16} - 108x^{14} - 216x^9 + 3x^8 + 108x^6 - 1)} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.33 Problem number 2068

$$\int \frac{\sqrt{b^2 + ax^2}}{\sqrt{b + \sqrt{b^2 + ax^2}}} dx$$

Optimal antiderivative

$$\frac{2bx}{3\sqrt{b+\sqrt{ax^2+b^2}}} + \frac{2x\sqrt{ax^2+b^2}}{3\sqrt{b+\sqrt{ax^2+b^2}}} + \frac{2\sqrt{2} b^{\frac{3}{2}} \arctan\left(\frac{\sqrt{a} x \sqrt{2}}{2\sqrt{b} \sqrt{b+\sqrt{ax^2+b^2}}}\right) - \frac{\sqrt{b+\sqrt{ax^2+b^2}} \sqrt{2}}{2\sqrt{b}}}{\sqrt{a}}$$

command

```
integrate((a*x^2+b^2)^(1/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{3\sqrt{2} abx \sqrt{-\frac{b}{a}} \log\left(\frac{ax^3+4b^2x-4\sqrt{ax^2+b^2}bx-2\left(2\sqrt{2}\sqrt{ax^2+b^2}b\sqrt{-\frac{b}{a}}-\sqrt{2}(ax^2+2b^2)\sqrt{-\frac{b}{a}}\right)\sqrt{b+\sqrt{ax^2+b^2}}}{x^3}}{6ax}\right)}{3ax} \right]$$

$$\left[\frac{3\sqrt{2} abx \sqrt{\frac{b}{a}} \arctan\left(\frac{\sqrt{2}\sqrt{b+\sqrt{ax^2+b^2}}\sqrt{\frac{b}{a}}}{x}\right) - 2(ax^2+2b^2-2\sqrt{ax^2+b^2}b)\sqrt{b+\sqrt{ax^2+b^2}}}{3ax} \right]$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.34 Problem number 2141

$$\int \frac{x^3}{\sqrt[3]{-bx^2 + ax^3} (-b^2 + a^2x^6)} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(x^3/(a*x^3-b*x^2)^(1/3)/(a^2*x^6-b^2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.35 Problem number 2144

$$\int \frac{-1 + x + x^3}{(1 - x + x^3) \sqrt[3]{-x^2 + x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((x^3+x-1)/(x^3-x+1)/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.36 Problem number 2145

$$\int \frac{-1 + x + x^3}{(1 - x + x^3) \sqrt[3]{-x^2 + x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((x^3+x-1)/(x^3-x+1)/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.37 Problem number 2279

$$\int \frac{(1 + 2x^8) \sqrt[4]{-1 - 2x^4 + 2x^8} (1 - 3x^8 + 4x^{16})}{x^{10} (-1 + 2x^8)} dx$$

Optimal antiderivative

$$\frac{(2x^8 - 2x^4 - 1)^{\frac{1}{4}} (20x^{16} - 4x^{12} + 9x^8 + 2x^4 + 5)}{45x^9} - \frac{\arctan\left(\frac{2^{\frac{3}{4}}x(2x^8-2x^4-1)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{2x^8-2x^4-1}}\right) 2^{\frac{3}{4}}}{4} - \frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x(2x^8-2x^4-1)^{\frac{1}{4}}}{2x^2+\sqrt{2}\sqrt{2x^8-2x^4-1}}\right) 2^{\frac{3}{4}}}{4}$$

command

```
integrate((2*x^8+1)*(2*x^8-2*x^4-1)^(1/4)*(4*x^16-3*x^8+1)/x^10/(2*x^8-1),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$180 \cdot 8^{\frac{3}{4}} \sqrt{2} x^9 \arctan \left(\frac{32x^{16} - 32x^8 + 4 \cdot 8^{\frac{3}{4}} \sqrt{2} (2x^9 - 8x^5 - x)(2x^8 - 2x^4 - 1)^{\frac{3}{4}} + 16 \cdot 8^{\frac{1}{4}} \sqrt{2} (6x^{11} - 8x^7 - 3x^3)(2x^8 - 2x^4 - 1)^{\frac{1}{4}} + 32 \sqrt{2} x^9}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.38 Problem number 2333

$$\int \frac{-d + cx}{x^7 \sqrt[3]{-b + ax^3}} dx$$

Optimal antiderivative

$$\frac{(ax^3 - b)^{\frac{2}{3}} (27acx^4 - 20adx^3 + 18bcx - 15bd)}{90b^2x^6} - \frac{2a^2d \arctan\left(-\frac{\sqrt{3}}{3} + \frac{2(ax^3 - b)^{\frac{1}{3}}\sqrt{3}}{3b^{\frac{1}{3}}}\right) \sqrt{3}}{27b^{\frac{7}{3}}} + \frac{2a^2d \ln\left(b^{\frac{1}{3}} + (ax^3 - b)^{\frac{1}{3}}\right)}{27b^{\frac{7}{3}}} - \frac{a^2d \ln\left(b^{\frac{2}{3}} - b^{\frac{1}{3}}(ax^3 - b)^{\frac{1}{3}} + (ax^3 - b)^{\frac{2}{3}}\right)}{27b^{\frac{7}{3}}}$$

command

```
integrate((c*x-d)/x^7/(a*x^3-b)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\frac{\left[30 \sqrt{\frac{1}{3}} a^2 b d x^6 \sqrt{-\frac{1}{b^{\frac{2}{3}}}} \log\left(\frac{2ax^3 - 3\sqrt{\frac{1}{3}}\left(2(ax^3 - b)^{\frac{2}{3}}b^{\frac{2}{3}} + (ax^3 - b)^{\frac{1}{3}}b - b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}} - 3(ax^3 - b)^{\frac{1}{3}}b^{\frac{2}{3}} - 3b}{x^3}\right) + 20a^2b^{\frac{2}{3}}dx^6 \log\left(\frac{(ax^3 - b)^{\frac{1}{3}} + b^{\frac{1}{3}}}{x}\right) \right]}{270b^3x^6} - \frac{60\sqrt{\frac{1}{3}}a^2b^{\frac{2}{3}}dx^6 \arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(ax^3 - b)^{\frac{1}{3}} - b^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\right) - 20a^2b^{\frac{2}{3}}dx^6 \log\left(\frac{(ax^3 - b)^{\frac{1}{3}} + b^{\frac{1}{3}}}{x}\right) + 10a^2b^{\frac{2}{3}}dx^6 \log\left(\frac{(ax^3 - b)^{\frac{2}{3}} - (ax^3 - b)^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}}{x}\right)}{270b^3x^6}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.39 Problem number 2357

$$\int \frac{1}{\sqrt[4]{-1+3x-3x^2+x^3} (-1-2x+x^2+3x^3)^4} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(1/(x^3-3*x^2+3*x-1)^(1/4)/(3*x^3+x^2-2*x-1)^4,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.40 Problem number 2377

$$\int \frac{1}{x \sqrt{ax + \sqrt{-b + a^2x^2}} \sqrt{c + \sqrt{ax + \sqrt{-b + a^2x^2}}}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(1/x/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.41 Problem number 2426

$$\int \frac{(d + cx^2) \sqrt{ax + \sqrt{b^2 + a^2x^2}}}{-d + cx^2} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((c*x^2+d)*(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)/(c*x^2-d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12a \left(\frac{b^2cd^2 + 2a^2d^3 + 2c^3 \sqrt{\frac{a^2b^2cd^5 + a^4d^6}{c^6}}}{c^3} \right)^{\frac{1}{4}} \arctan \left(\frac{\sqrt{a^3b^4d^8x + \sqrt{a^2x^2 + b^2}} a^2b^4d^8 + a^2b^4cd^7 \sqrt{\frac{b^2cd^2 + 2a^2d^3 + 2c^3 \sqrt{\frac{a^2b^2cd^5 + a^4d^6}{c^6}}}{c^3}}}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

113.42 Problem number 2427

$$\int \frac{(d + cx^2) \sqrt{ax + \sqrt{b^2 + a^2x^2}}}{-d + cx^2} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((c*x^2+d)*(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)/(c*x^2-d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 a \left(\frac{b^2 c d^2 + 2 a^2 d^3 + 2 c^3 \sqrt{\frac{a^2 b^2 c d^5 + a^4 d^6}{c^6}}}{c^3} \right)^{\frac{1}{4}} \arctan \left(\frac{\sqrt{a^3 b^4 d^8 x + \sqrt{a^2 x^2 + b^2} a^2 b^4 d^8 + a^2 b^4 c d^7} \sqrt{\frac{b^2 c d^2 + 2 a^2 d^3 + 2 c^3 \sqrt{\frac{a^2 b^2 c d^5 + a^4 d^6}{c^6}}}{c^3}}}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

113.43 Problem number 2435

$$\int \frac{(-d + cx^2) \sqrt{ax + \sqrt{b^2 + a^2 x^2}}}{d + cx^2} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((c*x^2-d)*(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)/(c*x^2+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 a \left(\frac{b^2 c d^2 - 2 a^2 d^3 + 2 c^3 \sqrt{\frac{a^2 b^2 c d^5 - a^4 d^6}{c^6}}}{c^3} \right)^{\frac{1}{4}} \arctan \left(\frac{\sqrt{a^3 b^4 d^8 x + \sqrt{a^2 x^2 + b^2} a^2 b^4 d^8 - a^2 b^4 c d^7} \sqrt{\frac{b^2 c d^2 - 2 a^2 d^3 + 2 c^3 \sqrt{\frac{a^2 b^2 c d^5 - a^4 d^6}{c^6}}}{c^3}}}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

113.44 Problem number 2436

$$\int \frac{(-d + cx^2) \sqrt{ax + \sqrt{b^2 + a^2x^2}}}{d + cx^2} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((c*x^2-d)*(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)/(c*x^2+d),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 a \left(\frac{b^2 c d^2 - 2 a^2 d^3 + 2 c^3 \sqrt{-\frac{a^2 b^2 c d^5 - a^4 d^6}{c^6}}}{c^3} \right)^{\frac{1}{4}} \arctan \left(\frac{\sqrt{a^3 b^4 d^8 x + \sqrt{a^2 x^2 + b^2} a^2 b^4 d^8 - a^2 b^4 c d^7} \sqrt{\frac{b^2 c d^2 - 2 a^2 d^3 + 2 c^3 \sqrt{-\frac{a^2 b^2 c d^5 - a^4 d^6}{c^6}}}{c^3}}}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

113.45 Problem number 2443

$$\int \frac{-b + cx^4 + 2ax^8}{\sqrt[4]{-b + ax^4} (b - cx^4 + ax^8)} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((2*a*x^8+c*x^4-b)/(a*x^4-b)^(1/4)/(a*x^8-c*x^4+b),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.46 Problem number 2444

$$\int \frac{-b + cx^4 + 2ax^8}{\sqrt[4]{-b + ax^4} (b - cx^4 + ax^8)} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((2*a*x^8+c*x^4-b)/(a*x^4-b)^(1/4)/(a*x^8-c*x^4+b),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.47 Problem number 2446

$$\int \frac{\sqrt{b + \sqrt{b^2 + ax^2}}}{x^6} dx$$

Optimal antiderivative

$$\frac{\sqrt{b + \sqrt{ax^2 + b^2}} \left(105a^{\frac{5}{2}}x^4 + \sqrt{a} \left(-432b^4 + 48b^3 \sqrt{ax^2 + b^2} \right) + a^{\frac{3}{2}} \left(14b^2x^2 - 70bx^2 \sqrt{ax^2 + b^2} \right) \right)}{1920\sqrt{a} b^4x^5} + \frac{7a^{\frac{5}{2}} \arctan \left(\frac{\sqrt{a} x \sqrt{2}}{2\sqrt{b} \sqrt{b + \sqrt{ax^2 + b^2}}} - \frac{\sqrt{b + \sqrt{ax^2 + b^2}} \sqrt{2}}{2\sqrt{b}} \right) \sqrt{2}}{128b^{\frac{9}{2}}}$$

command

```
integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^6,x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$\left[\frac{105 \sqrt{\frac{1}{2}} a^2 x^5 \sqrt{-\frac{a}{b}} \log \left(\frac{a^2 x^3 + 4 ab^2 x - 4 \sqrt{ax^2 + b^2} abx - 4 \left(2 \sqrt{\frac{1}{2}} \sqrt{ax^2 + b^2} b^2 \sqrt{-\frac{a}{b}} - \sqrt{\frac{1}{2}} (abx^2 + 2b^3) \sqrt{-\frac{a}{b}} \right) \sqrt{ax^2 + b^2}}{x^3} \right)}{3840 b^4} \right.$$

$$\left. \frac{105 \sqrt{\frac{1}{2}} a^2 x^5 \sqrt{\frac{a}{b}} \arctan \left(\frac{2 \sqrt{\frac{1}{2}} \sqrt{b + \sqrt{ax^2 + b^2}} b \sqrt{\frac{a}{b}}}{ax} \right) - (105 a^2 x^4 + 14 ab^2 x^2 - 432 b^4 - 2 (35 abx^2 - 24 ab^2)) \sqrt{ax^2 + b^2}}{1920 b^4 x^5} \right.$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.48 Problem number 2447

$$\int \frac{\sqrt{b + \sqrt{b^2 + ax^2}}}{x^6 \sqrt{b^2 + ax^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{b + \sqrt{ax^2 + b^2}} \left(-315a^{\frac{5}{2}}x^4 + \sqrt{a} \left(16b^4 - 144b^3 \sqrt{ax^2 + b^2} \right) + a^{\frac{3}{2}} \left(-42b^2x^2 + 210bx^2 \sqrt{ax^2 + b^2} \right) \right)}{640\sqrt{a} b^5 x^5}$$

$$\frac{63a^{\frac{5}{2}} \arctan \left(\frac{\sqrt{a} x \sqrt{2}}{2\sqrt{b} \sqrt{b + \sqrt{ax^2 + b^2}}} - \frac{\sqrt{b + \sqrt{ax^2 + b^2}} \sqrt{2}}{2\sqrt{b}} \right) \sqrt{2}}{128b^{\frac{11}{2}}}$$

command

```
integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^6/(a*x^2+b^2)^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$315 \sqrt{\frac{1}{2}} a^2 x^5 \sqrt{-\frac{a}{b}} \log \left(\frac{a^2 x^3 + 4 a b^2 x - 4 \sqrt{a x^2 + b^2} a b x + 4 \left(2 \sqrt{\frac{1}{2}} \sqrt{a x^2 + b^2} b^2 \sqrt{-\frac{a}{b}} - \sqrt{\frac{1}{2}} (a b x^2 + 2 b^3) \sqrt{-\frac{a}{b}} \right) \sqrt{a x^2 + b^2}}{x^3} \right) \sqrt{a x^2 + b^2}$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.49 Problem number 2452

$$\int \frac{1}{x^6 (-b + a x^3) \sqrt[3]{b^2 x^2 + a^3 x^3}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate(1/x^6/(a*x^3-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.50 Problem number 2453

$$\int \frac{1}{x^6 (-b + a x^3) \sqrt[3]{b^2 x^2 + a^3 x^3}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate(1/x^6/(a*x^3-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.51 Problem number 2458

$$\int \frac{d + cx^2}{(-d + cx^2) \sqrt{ax + \sqrt{b^2 + a^2x^2}}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((c*x^2+d)/(c*x^2-d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 ab^2 \left(\frac{2b^4c^3 \sqrt{\frac{a^2b^2cd^5 + a^4d^6}{b^8c^6}} + b^2cd^2 + 2a^2d^3}{b^4c^3} \right)^{\frac{1}{4}} \arctan \left(\frac{\sqrt{a^3d^6x + \sqrt{a^2x^2 + b^2}} a^2d^6 - \left(2a^2b^4c^3d^3 \sqrt{\frac{a^2b^2cd^5}{b^8}} \right)}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

113.52 Problem number 2459

$$\int \frac{d + cx^2}{(-d + cx^2) \sqrt{ax + \sqrt{b^2 + a^2x^2}}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((c*x^2+d)/(c*x^2-d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 ab^2 \left(\frac{2b^4c^3 \sqrt{\frac{a^2b^2cd^5 + a^4d^6}{b^8c^6}} + b^2cd^2 + 2a^2d^3}{b^4c^3} \right)^{\frac{1}{4}} \arctan \left(\frac{\sqrt{a^3d^6x + \sqrt{a^2x^2 + b^2} a^2d^6} - \left(2a^2b^4c^3d^3 \sqrt{\frac{a^2b^2cd^5}{b^8}} \right)}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

113.53 Problem number 2460

$$\int \frac{1}{x^6 (b + ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate(1/x^6/(a*x^3+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.54 Problem number 2461

$$\int \frac{1}{x^6 (b + ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate(1/x^6/(a*x^3+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.55 Problem number 2464

$$\int \frac{-d + cx^2}{(d + cx^2) \sqrt{ax + \sqrt{b^2 + a^2x^2}}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((c*x^2-d)/(c*x^2+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 ab^2 \left(-\frac{2b^4c^3 \sqrt{-\frac{a^2b^2cd^5 - a^4d^6}{b^8c^6}} - b^2cd^2 + 2a^2d^3}{b^4c^3} \right)^{\frac{1}{4}} \arctan \left(\frac{\sqrt{a^3d^6x + \sqrt{a^2x^2 + b^2}} a^2d^6 - \left(2a^2b^4c^3d^3 \sqrt{-\frac{a^2b^2}{b^8c^6}} \right)}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

113.56 Problem number 2465

$$\int \frac{-d + cx^2}{(d + cx^2) \sqrt{ax + \sqrt{b^2 + a^2x^2}}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((c*x^2-d)/(c*x^2+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$12 ab^2 \left(-\frac{2b^4c^3 \sqrt{-\frac{a^2b^2cd^5 - a^4d^6}{b^8c^6}} - b^2cd^2 + 2a^2d^3}{b^4c^3} \right)^{\frac{1}{4}} \arctan \left(\frac{\sqrt{a^3d^6x + \sqrt{a^2x^2 + b^2} a^2d^6} - \left(2a^2b^4c^3d^3 \sqrt{-\frac{a^2b^2}{b^8c^6}} \right)}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

113.57 Problem number 2486

$$\int \frac{\sqrt{x + \sqrt{1+x}}}{x^2 - \sqrt{1+x}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((x+(1+x)^(1/2))^(1/2)/(x^2-(1+x)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.58 Problem number 2487

$$\int \frac{\sqrt{x + \sqrt{1+x}}}{x^2 - \sqrt{1+x}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((x+(1+x)^(1/2))^(1/2)/(x^2-(1+x)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.59 Problem number 2520

$$\int \frac{b - 2cx^4 + 2ax^8}{\sqrt[4]{-b + ax^4} (-2b - cx^4 + 2ax^8)} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((2*a*x^8-2*c*x^4+b)/(a*x^4-b)^(1/4)/(2*a*x^8-c*x^4-2*b),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.60 Problem number 2521

$$\int \frac{b - 2cx^4 + 2ax^8}{\sqrt[4]{-b + ax^4} (-2b - cx^4 + 2ax^8)} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((2*a*x^8-2*c*x^4+b)/(a*x^4-b)^(1/4)/(2*a*x^8-c*x^4-2*b),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.61 Problem number 2603

$$\int \frac{-b + ax^2}{(-b + 2ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((a*x^2-b)/(2*a*x^3-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.62 Problem number 2604

$$\int \frac{-b + ax^2}{(-b + 2ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((a*x^2-b)/(2*a*x^3-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.63 Problem number 2646

$$\int \frac{\sqrt{ax + \sqrt{b + a^2x^2}} \sqrt{c + \sqrt{ax + \sqrt{b + a^2x^2}}}}{(b + a^2x^2)^{3/2}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2)/(a^2*x^2+b)^(3/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.64 Problem number 2647

$$\int \frac{\sqrt{ax + \sqrt{b + a^2x^2}} \sqrt{c + \sqrt{ax + \sqrt{b + a^2x^2}}}}{(b + a^2x^2)^{3/2}} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2)/(a^2*x^2+b)^(3/2), x)
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.65 Problem number 2720

$$\int \frac{\sqrt[4]{\frac{1 + 4x^2 + ax^2 + 6x^4 + 4ax^4 + 4x^6 + 6ax^6 + x^8 + 4ax^8 + ax^{10}}{x^2}}}{x} dx$$

Optimal antiderivative

$$\left(\frac{ax^{10} + 4ax^8 + x^8 + 6ax^6 + 4x^6 + 4ax^4 + 6x^4 + ax^2 + 4x^2 + 1}{x^2} \right)^{\frac{1}{4}} \left(-2(ax^2 + 1)^{\frac{1}{4}} + \frac{x^2(ax^2 + 1)^{\frac{1}{4}}}{2} + \frac{\sqrt{x} \arctan\left(\frac{(ax^2 + 1)^{\frac{1}{4}}}{a^{\frac{1}{4}}\sqrt{x}}\right)}{4a^{\frac{3}{4}}} + a^{\frac{1}{4}}\sqrt{x} \right) \frac{1}{(x^2 + 1)(ax^2 + 1)^{\frac{1}{4}}}$$

command

```
integrate(((a*x^10+4*a*x^8+x^8+6*a*x^6+4*x^6+4*a*x^4+6*x^4+a*x^2+4*x^2+1)/x^2)^(1/4)/x,x, alg
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.66 Problem number 2799

$$\int \frac{(1-x^4) \sqrt[4]{1-x-4x^2+4x^3+6x^4-6x^5-4x^6+4x^7+x^8-x^9}}{1+x^4} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((-x^4+1)*(-x^9+x^8+4*x^7-4*x^6-6*x^5+6*x^4+4*x^3-4*x^2-x+1)^(1/4)/(x^4+1),x, algo=
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.67 Problem number 2830

$$\int \frac{\sqrt{c + \sqrt{ax + \sqrt{b + a^2x^2}}}}{(b + a^2x^2)^{3/2} \sqrt{ax + \sqrt{b + a^2x^2}}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2)/(a^2*x^2+b)^(3/2)/(a*x+(a^2*x^2+b)^(1/2))^(
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.68 Problem number 2831

$$\int \frac{\sqrt{c + \sqrt{ax + \sqrt{b + a^2x^2}}}}{(b + a^2x^2)^{3/2} \sqrt{ax + \sqrt{b + a^2x^2}}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2)/(a^2*x^2+b)^(3/2)/(a*x+(a^2*x^2+b)^(1/2))^(1/2),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.69 Problem number 2876

$$\int \frac{-bx + ax^3}{(-b + 2ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((a*x^3-b*x)/(2*a*x^3-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.70 Problem number 2877

$$\int \frac{-bx + ax^3}{(-b + 2ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((a*x^3-b*x)/(2*a*x^3-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.71 Problem number 2878

$$\int \frac{bx + ax^3}{(b + 2ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((a*x^3+b*x)/(2*a*x^3+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.72 Problem number 2891

$$\int \frac{bx + ax^3}{(b + 2ax^3) \sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((a*x^3+b*x)/(2*a*x^3+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.73 Problem number 2892

$$\int \frac{1}{\sqrt{-1+2x} (4+3x) + (1+x)\sqrt{-3+4x}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(1/((-1+2*x)^(1/2)*(4+3*x)+(1+x)*(-3+4*x)^(1/2)),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.74 Problem number 2901

$$\int \frac{(1+x^2)^2 \sqrt{x^2 + \sqrt{1+x^4}}}{\sqrt{1+x^4} (-1+x^2+x^4)} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((x^2+1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2)/(x^4+x^2-1),x, algorithm="fricas"
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.75 Problem number 2902

$$\int \frac{(1+x^2)^2 \sqrt{x^2 + \sqrt{1+x^4}}}{\sqrt{1+x^4} (-1+x^2+x^4)} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((x^2+1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2)/(x^4+x^2-1),x, algorithm="fricas"
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.76 Problem number 2903

$$\int \frac{x^2 \sqrt[4]{bx^3 + ax^4}}{-b + ax^4} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(x^2*(a*x^4+b*x^3)^(1/4)/(a*x^4-b),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.77 Problem number 2904

$$\int \frac{x^2 \sqrt[4]{bx^3 + ax^4}}{-b + ax^4} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate(x^2*(a*x^4+b*x^3)^(1/4)/(a*x^4-b),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.78 Problem number 2939

$$\int \frac{(d + cx^2) \sqrt{ax + \sqrt{b^2 + a^2x^2}}}{f + ex^2} dx$$

Optimal antiderivative

Unintegrable

command

```
integrate((c*x^2+d)*(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)/(e*x^2+f),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Exception raised: NotImplementedError

113.79 Problem number 2946

$$\int \frac{x^3}{\sqrt{a + bx + cx^2 + bx^3 + ax^4} (1 - x^6)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{-2a - 2b - c} \arctan\left(\frac{\sqrt{-2a - 2b - c} x}{\sqrt{a - 2x\sqrt{a} + x^2\sqrt{a} - \sqrt{a}x^4 + bx^3 + cx^2 + bx + a}}\right)}{12a + 12b + 6c} \\ & + \frac{\arctan\left(\frac{\sqrt{a - b - c} x}{\sqrt{a - x\sqrt{a} + x^2\sqrt{a} - \sqrt{a}x^4 + bx^3 + cx^2 + bx + a}}\right)}{3\sqrt{a - b - c}} \\ & - \frac{\arctan\left(\frac{\sqrt{a + b - c} x}{\sqrt{a + x\sqrt{a} + x^2\sqrt{a} - \sqrt{a}x^4 + bx^3 + cx^2 + bx + a}}\right)}{3\sqrt{a + b - c}} \\ & - \frac{\sqrt{-2a + 2b - c} \arctan\left(\frac{\sqrt{-2a + 2b - c} x}{\sqrt{a + 2x\sqrt{a} + x^2\sqrt{a} - \sqrt{a}x^4 + bx^3 + cx^2 + bx + a}}\right)}{12a - 12b + 6c} \end{aligned}$$

command

```
integrate(x^3/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)/(-x^6+1),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.80 Problem number 2951

$$\int \frac{x + \sqrt{1 + 4x + 7x^2 + 8x^3 + 5x^4 + 2x^5}}{1 - \sqrt{1 + 4x + 7x^2 + 8x^3 + 5x^4 + 2x^5}} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((x+(2*x^5+5*x^4+8*x^3+7*x^2+4*x+1)^(1/2))/(1-(2*x^5+5*x^4+8*x^3+7*x^2+4*x+1)^(1/2)))
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.81 Problem number 3037

$$\int \frac{x^2}{(1+x^4)\sqrt[4]{-x^2+x^6}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{\sqrt{1+\sqrt{2}} \arctan\left(\frac{\sqrt{2-\sqrt{2}} x}{-x\sqrt{2+\sqrt{2}} + 2^{\frac{3}{4}}(x^6-x^2)^{\frac{1}{4}}}\right)}{8} \\
& - \frac{\sqrt{1+\sqrt{2}} \arctan\left(\frac{\sqrt{2-\sqrt{2}} x}{x\sqrt{2+\sqrt{2}} + 2^{\frac{3}{4}}(x^6-x^2)^{\frac{1}{4}}}\right)}{8} \\
& + \frac{\sqrt{\sqrt{2}-1} \arctan\left(\frac{2^{\frac{3}{4}}\sqrt{2+\sqrt{2}} x(x^6-x^2)^{\frac{1}{4}}}{-2x^2+\sqrt{2}\sqrt{x^6-x^2}}\right)}{8} \\
& - \frac{\sqrt{1+\sqrt{2}} \operatorname{arctanh}\left(\frac{\frac{2^{\frac{1}{4}}x^2}{\sqrt{2-\sqrt{2}}} + \frac{\sqrt{x^6-x^2} 2^{\frac{3}{4}}}{2\sqrt{2-\sqrt{2}}}}{x(x^6-x^2)^{\frac{1}{4}}}\right)}{8} \\
& - \frac{\sqrt{\sqrt{2}-1} \ln\left(-2x^2 + 2^{\frac{3}{4}}\sqrt{2+\sqrt{2}} x(x^6-x^2)^{\frac{1}{4}} - \sqrt{2}\sqrt{x^6-x^2}\right)}{16} \\
& + \frac{\sqrt{\sqrt{2}-1} \ln\left(2\sqrt{2-\sqrt{2}} x^2 + 2 \cdot 2^{\frac{1}{4}} x(x^6-x^2)^{\frac{1}{4}} + \sqrt{4-2\sqrt{2}}\sqrt{x^6-x^2}\right)}{16}
\end{aligned}$$

command

```
integrate(x^2/(x^4+1)/(x^6-x^2)^(1/4),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.82 Problem number 3038

$$\int \frac{x^2}{(1+x^4)\sqrt[4]{-x^2+x^6}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{1+\sqrt{2}} \arctan\left(\frac{\sqrt{2-\sqrt{2}} x}{-x\sqrt{2+\sqrt{2}}+2^{\frac{3}{4}}(x^6-x^2)^{\frac{1}{4}}}\right)}{8} \\ & - \frac{\sqrt{1+\sqrt{2}} \arctan\left(\frac{\sqrt{2-\sqrt{2}} x}{x\sqrt{2+\sqrt{2}}+2^{\frac{3}{4}}(x^6-x^2)^{\frac{1}{4}}}\right)}{8} \\ & + \frac{\sqrt{\sqrt{2}-1} \arctan\left(\frac{2^{\frac{3}{4}}\sqrt{2+\sqrt{2}} x(x^6-x^2)^{\frac{1}{4}}}{-2x^2+\sqrt{2}\sqrt{x^6-x^2}}\right)}{8} \\ & - \frac{\sqrt{1+\sqrt{2}} \operatorname{arctanh}\left(\frac{\frac{2^{\frac{1}{4}}x^2}{\sqrt{2-\sqrt{2}}} + \frac{\sqrt{x^6-x^2} 2^{\frac{3}{4}}}{2\sqrt{2-\sqrt{2}}}}{x(x^6-x^2)^{\frac{1}{4}}}\right)}{8} \\ & - \frac{\sqrt{\sqrt{2}-1} \ln\left(-2x^2+2^{\frac{3}{4}}\sqrt{2+\sqrt{2}} x(x^6-x^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{x^6-x^2}\right)}{16} \\ & + \frac{\sqrt{\sqrt{2}-1} \ln\left(2\sqrt{2-\sqrt{2}} x^2+2\cdot 2^{\frac{1}{4}}x(x^6-x^2)^{\frac{1}{4}}+\sqrt{4-2\sqrt{2}}\sqrt{x^6-x^2}\right)}{16} \end{aligned}$$

command

```
integrate(x^2/(x^4+1)/(x^6-x^2)^(1/4),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.83 Problem number 3085

$$\int \frac{(1+x^2+x^4)^2 \sqrt{x^2 + \sqrt{1+x^4}}}{\sqrt{1+x^4} (-1+x^2+x^4)^2} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((x^4+x^2+1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2)/(x^4+x^2-1)^2,x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.84 Problem number 3086

$$\int \frac{(1+x^2+x^4)^2 \sqrt{x^2 + \sqrt{1+x^4}}}{\sqrt{1+x^4} (-1+x^2+x^4)^2} dx$$

Optimal antiderivative*Unintegrable*command

```
integrate((x^4+x^2+1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2)/(x^4+x^2-1)^2,x, algorithm="f
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.85 Problem number 3099

$$\int \frac{\sqrt{x^2 + \sqrt{1+x^4}}}{1+ax} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{a} + \frac{(1 + a^4 + \sqrt{a^4 + 1}) \arctan\left(\frac{a\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{-1 - \sqrt{a^4 + 1}}}\right)}{a^2 \sqrt{a^4 + 1} \sqrt{-1 - \sqrt{a^4 + 1}}} \\ & + \frac{(-1 - a^4 + \sqrt{a^4 + 1}) \arctan\left(\frac{a\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{-1 + \sqrt{a^4 + 1}}}\right)}{a^2 \sqrt{a^4 + 1} \sqrt{-1 + \sqrt{a^4 + 1}}} \\ & + \frac{\left(\sqrt{2} \sqrt{-a^2 - \sqrt{a^4 + 1}} - \sqrt{2} a^2 \sqrt{-a^2 - \sqrt{a^4 + 1}} + \sqrt{2} \sqrt{a^4 + 1} \sqrt{-a^2 - \sqrt{a^4 + 1}}\right) \arctan\left(\frac{\sqrt{2} \sqrt{-a^2 - \sqrt{a^4 + 1}}}{\sqrt{-a^2 - \sqrt{a^4 + 1}}}\right)}{2a^2} \\ & + \frac{\left(-\sqrt{2} \sqrt{-a^2 + \sqrt{a^4 + 1}} + \sqrt{2} a^2 \sqrt{-a^2 + \sqrt{a^4 + 1}} + \sqrt{2} \sqrt{a^4 + 1} \sqrt{-a^2 + \sqrt{a^4 + 1}}\right) \arctan\left(\frac{\sqrt{2} \sqrt{-a^2 + \sqrt{a^4 + 1}}}{\sqrt{-a^2 + \sqrt{a^4 + 1}}}\right)}{2a^2} \\ & - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} x \sqrt{x^2 + \sqrt{x^4 + 1}}}{1+x^2+\sqrt{x^4 + 1}}\right)}{a^2} \end{aligned}$$

command

```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(a*x+1),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

$$a^2 \sqrt{\frac{a^4 \sqrt{\frac{a^4 + 1}{a^8}} + 1}{a^4}} \log \left(\frac{4 \left(\left(a^5 - (a^4 + 1)x^3 + \sqrt{x^4 + 1} \left((a^4 + 1)x + (a^7 - a^4)x \sqrt{\frac{a^4 + 1}{a^8}} \right) - (a^7 x^2 + a^6 x - a^4 x^3 + a^5) \sqrt{\frac{a^4 + 1}{a^8}} \right)}{\dots} \right)}{\dots} \right)$$

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.86 Problem number 3100

$$\int \frac{1}{\sqrt{c_4 + \sqrt{\frac{c_0 + xc_1}{c_2 + xc_3}} c_5} (c_6 + xc_7)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \operatorname{arctanh} \left(\frac{-C_3^{\frac{1}{4}} \sqrt{-C_4 + \sqrt{\frac{-C_1 x + -C_0}{-C_3 x + -C_2}} - C_5}}{\sqrt{\sqrt{-C_3} - C_4 - \sqrt{-C_1} - C_5}} \right) - C_3^{\frac{1}{4}}}{\sqrt{\sqrt{-C_3} - C_4 - \sqrt{-C_1} - C_5} - C_7} \\ & + \frac{2 \operatorname{arctanh} \left(\frac{-C_3^{\frac{1}{4}} \sqrt{-C_4 + \sqrt{\frac{-C_1 x + -C_0}{-C_3 x + -C_2}} - C_5}}{\sqrt{\sqrt{-C_3} - C_4 + \sqrt{-C_1} - C_5}} \right) - C_3^{\frac{1}{4}}}{\sqrt{\sqrt{-C_3} - C_4 + \sqrt{-C_1} - C_5} - C_7} \\ & + \frac{2 \operatorname{arctan} \left(\frac{\sqrt{-C_4 + \sqrt{\frac{-C_1 x + -C_0}{-C_3 x + -C_2}} - C_5} \sqrt{-C_2 - C_7 + -C_3 - C_6}}{\sqrt{-C_3 - C_4 - C_6 + -C_2 - C_4 - C_7 - C_5 \sqrt{-C_0 - C_7 + -C_1 - C_6}} \sqrt{-C_2 - C_7 + -C_3 - C_6}} \right) \sqrt{-C_2 - C_7 + -C_3 - C_6}}{-C_7 \sqrt{-C_3 - C_4 - C_6 + -C_2 - C_4 - C_7 - C_5 \sqrt{-C_0 - C_7 + -C_1 - C_6}} \sqrt{-C_2 - C_7 + -C_3 - C_6}} \\ & + \frac{2 \operatorname{arctan} \left(\frac{\sqrt{-C_4 + \sqrt{\frac{-C_1 x + -C_0}{-C_3 x + -C_2}} - C_5} \sqrt{-C_2 - C_7 + -C_3 - C_6}}{\sqrt{-C_3 - C_4 - C_6 + -C_2 - C_4 - C_7 + -C_5 \sqrt{-C_0 - C_7 + -C_1 - C_6}} \sqrt{-C_2 - C_7 + -C_3 - C_6}} \right) \sqrt{-C_2 - C_7 + -C_3 - C_6}}{-C_7 \sqrt{-C_3 - C_4 - C_6 + -C_2 - C_4 - C_7 + -C_5 \sqrt{-C_0 - C_7 + -C_1 - C_6}} \sqrt{-C_2 - C_7 + -C_3 - C_6}} \end{aligned}$$

command

```
integrate(1/(_C4+((C1*x+_C0)/(C3*x+_C2))^(1/2)*_C5)^(1/2)/(_C7*x+_C6),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.87 Problem number 3137

$$\int \frac{1}{\sqrt{c_4 + \sqrt{\frac{c_0 + xc_1}{c_2 + xc_3}}} c_5 (c_6 + xc_7)^2} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate(1/(_C4+((C1*x+_C0)/(C3*x+_C2))^(1/2)*_C5)^(1/2)/(_C7*x+_C6)^2,x, algorithm="fricas")
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output
```

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out

113.88 Problem number 3153

$$\int \frac{x^2 - cx^2 \left(\frac{b+ax}{d+cx} \right)^{3/2}}{a - b \sqrt{\frac{b+ax}{d+cx}}} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate((x^2-c*x^2*((a*x+b)/(c*x+d))^(3/2))/(a-b*((a*x+b)/(c*x+d))^(1/2)),x, algorithm="fricas")
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output
```

output too large to display

Fricas 1.3.7 via sagemath 9.3 output

Timed out