Computer algebra independent integration tests

8-Special-functions/8.2-Fresnel-integral-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [218]. This is test number [205].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (218)	% 0. (0)
Mathematica	% 87.16 (190)	% 12.84 (28)
Maple	% 70.64 (154)	% 29.36 (64)
Maxima	% 27.52 (60)	% 72.48 (158)
Fricas	% 27.52 (60)	% 72.48 (158)
Sympy	% 48.62 (106)	% 51.38 (112)
Giac	% 27.52 (60)	% 72.48 (158)

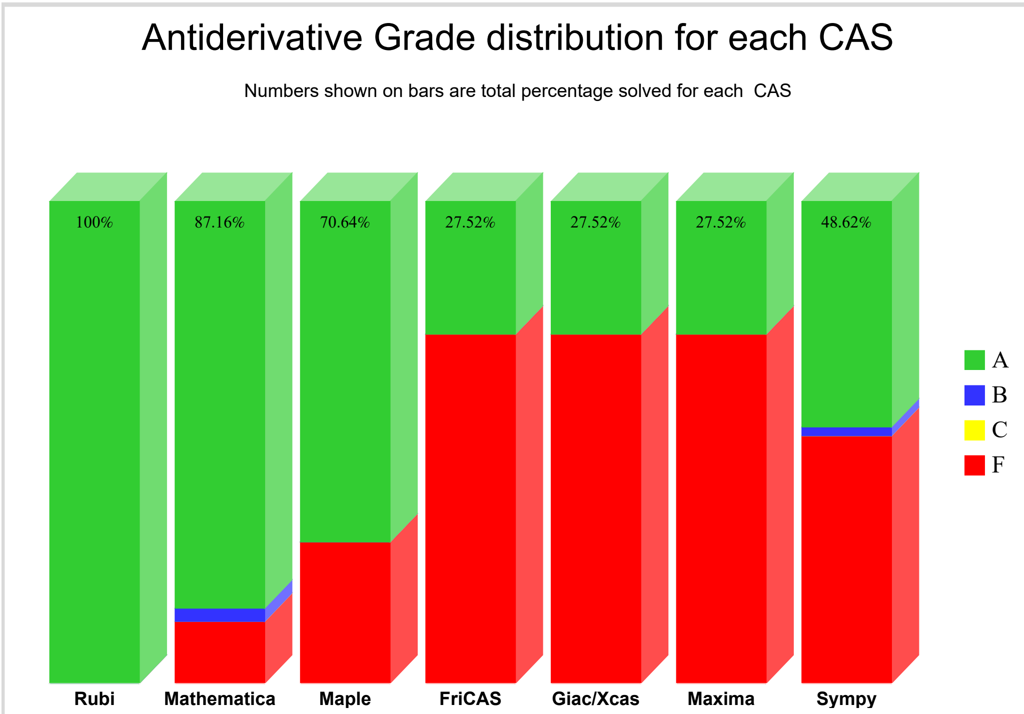
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

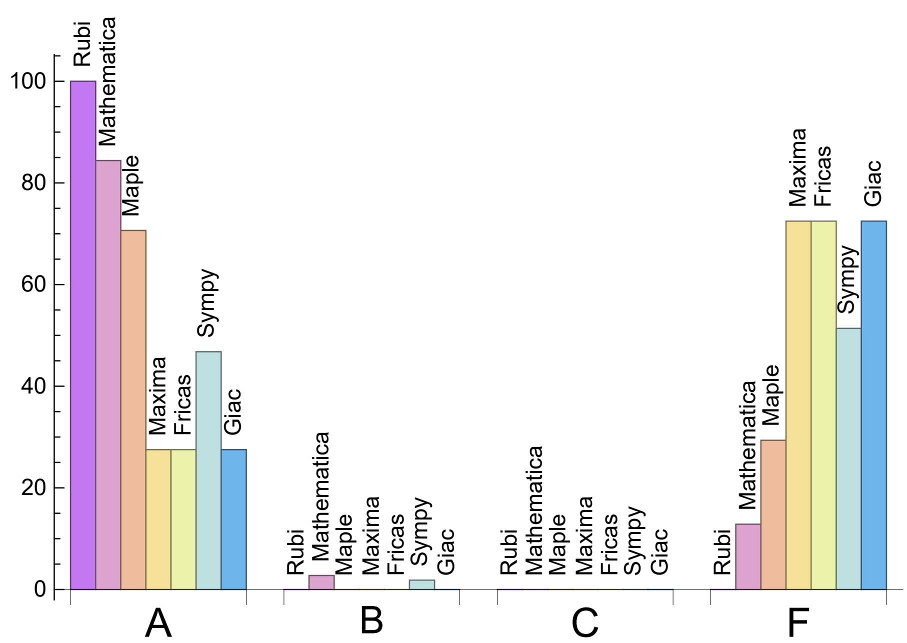
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	84.4	2.75	0.	12.84
Maple	70.64	0.	0.	29.36
Maxima	27.52	0.	0.	72.48
Fricas	27.52	0.	0.	72.48
Sympy	46.79	1.83	0.	51.38
Giac	27.52	0.	0.	72.48

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.15	96.61	0.72	71.5	1.
Mathematica	0.47	81.86	0.69	59.	0.76
Maple	0.08	62.54	0.61	28.	0.9
Maxima	0.	0.	0.	0.	0.
Fricas	0.	0.	0.	0.	0.
Sympy	3.68	39.53	0.59	9.	0.4
Giac	0.	0.	0.	0.	0.

1.4 list of integrals that has no closed form antiderivative

{23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {54, 55, 56, 163, 164, 165}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

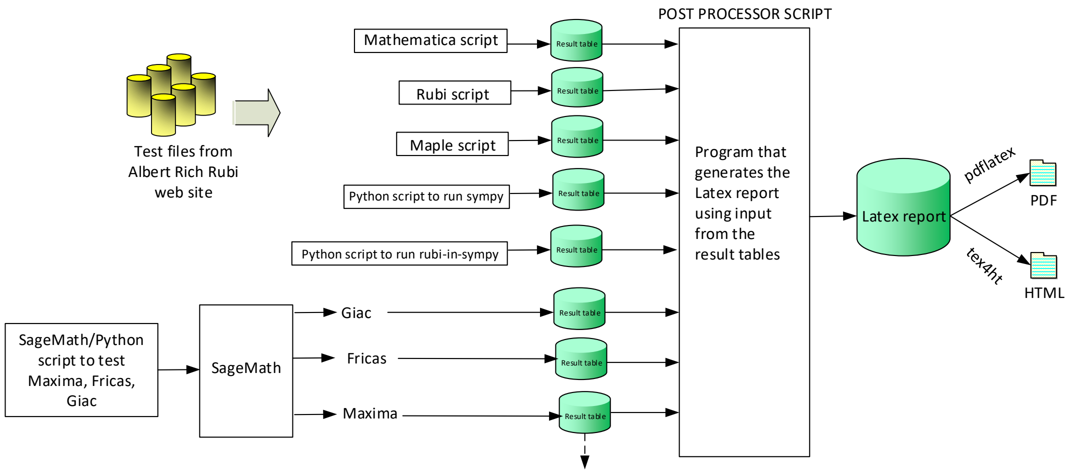
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 30, 31, 32, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 138, 139, 140, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 167, 168, 169, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 201, 202, 203, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218 }

B grade: { 22, 28, 57, 131, 137, 166 }

C grade: { }

F grade: { 9, 33, 37, 49, 50, 61, 62, 63, 64, 73, 77, 91, 95, 99, 118, 142, 146, 158, 159, 170, 171, 172, 173, 182, 186, 200, 204, 208}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 36, 38, 39, 40, 41, 42, 44, 45, 46, 48, 51, 52, 53, 57, 65, 66, 67, 68, 69, 70, 72, 74, 76, 78, 79, 80, 81, 82, 84, 85, 86, 88, 89, 90, 92, 94, 96, 98, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 143, 145, 147, 148, 149, 150, 151, 153, 154, 155, 157, 160, 161, 162, 166, 174, 175, 176, 177, 178, 179, 181, 183, 185, 187, 188, 189, 190, 191, 193, 194, 195, 197, 198, 199, 201, 203, 205, 207, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 31, 33, 35, 37, 43, 47, 49, 50, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 71, 73, 75, 77, 83, 87, 91, 93, 95, 97, 99, 101, 105, 109, 140, 142, 144, 146, 152, 156, 158, 159, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 180, 182, 184, 186, 192, 196, 200, 202, 204, 206, 208, 210, 214, 218 }

2.1.4 Maxima

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.1.5 FriCAS

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 65, 66, 67, 68, 69, 70, 75, 79, 80, 81, 82, 84, 85, 86, 90, 93, 97, 100, 102, 103, 104, 106, 110, 111, 112, 113, 114, 115, 116, 120, 121, 122, 123, 124, 125, 126, 127, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 174, 175, 176, 177, 178, 179, 184, 188, 189, 190, 191, 193, 194, 195, 199, 202, 206, 209, 211, 212, 213, 215 }

B grade: { 8, 10, 117, 119 }

C grade: { }

F grade: { 9, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 71, 72, 73, 74, 76, 77, 78, 83, 87, 88, 89, 91, 92, 94, 95, 96, 98, 99, 101, 105, 107, 108, 109, 118, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 180, 181, 182, 183, 185, 186, 187, 192, 196, 197, 198, 200, 201, 203, 204, 205, 207, 208, 210, 214, 216, 217, 218 }

2.1.7 Giac

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	88	123	0	0	184	0
normalized size	1	1.	0.71	0.99	0.	0.	1.48	0.
time (sec)	N/A	0.088	0.068	0.049	0.	0.	2.421	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	83	107	0	0	156	0
normalized size	1	1.	0.76	0.98	0.	0.	1.43	0.
time (sec)	N/A	0.111	0.052	0.049	0.	0.	1.766	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	79	96	0	0	53	0
normalized size	1	1.	0.8	0.97	0.	0.	0.54	0.
time (sec)	N/A	0.064	0.069	0.049	0.	0.	1.047	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	71	80	0	0	121	0
normalized size	1	1.	0.85	0.95	0.	0.	1.44	0.
time (sec)	N/A	0.08	0.041	0.052	0.	0.	1.629	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	70	0	0	112	0
normalized size	1	1.	1.	0.95	0.	0.	1.51	0.
time (sec)	N/A	0.047	0.016	0.046	0.	0.	0.934	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	0	0	80	0
normalized size	1	1.	1.	0.92	0.	0.	1.36	0.
time (sec)	N/A	0.053	0.012	0.049	0.	0.	0.828	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	0	0	53	0
normalized size	1	1.	1.	0.9	0.	0.	1.08	0.
time (sec)	N/A	0.024	0.011	0.049	0.	0.	0.524	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	0	0	48	0
normalized size	1	1.	1.	1.04	0.	0.	1.85	0.
time (sec)	N/A	0.005	0.002	0.046	0.	0.	0.833	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	0	29	0	0	0	0
normalized size	1	1.	0.	0.4	0.	0.	0.	0.
time (sec)	N/A	0.047	0.014	0.08	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	0	0	42	0
normalized size	1	1.	1.	1.04	0.	0.	1.56	0.
time (sec)	N/A	0.022	0.011	0.049	0.	0.	0.545	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	43	0	0	51	0
normalized size	1	1.	1.	0.98	0.	0.	1.16	0.
time (sec)	N/A	0.029	0.011	0.048	0.	0.	0.624	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	49	0	0	56	0
normalized size	1	1.	1.	0.94	0.	0.	1.08	0.
time (sec)	N/A	0.064	0.014	0.052	0.	0.	1.387	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	65	0	0	110	0
normalized size	1	1.	1.	0.94	0.	0.	1.59	0.
time (sec)	N/A	0.043	0.015	0.05	0.	0.	1.117	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	71	0	0	46	0
normalized size	1	1.	1.	0.92	0.	0.	0.6	0.
time (sec)	N/A	0.089	0.018	0.049	0.	0.	1.211	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	76	86	0	0	56	0
normalized size	1	1.	0.81	0.91	0.	0.	0.6	0.
time (sec)	N/A	0.059	0.057	0.05	0.	0.	1.529	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	85	93	0	0	68	0
normalized size	1	1.	0.83	0.91	0.	0.	0.67	0.
time (sec)	N/A	0.122	0.062	0.047	0.	0.	2.944	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	84	109	0	0	185	0
normalized size	1	1.	0.71	0.92	0.	0.	1.55	0.
time (sec)	N/A	0.081	0.055	0.047	0.	0.	3.71	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	96	115	0	0	48	0
normalized size	1	1.	0.76	0.91	0.	0.	0.38	0.
time (sec)	N/A	0.153	0.166	0.047	0.	0.	5.553	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	424	400	0	0	0	0
normalized size	1	1.	1.43	1.35	0.	0.	0.	0.
time (sec)	N/A	0.401	0.814	0.054	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	236	251	0	0	0	0
normalized size	1	1.	1.22	1.3	0.	0.	0.	0.
time (sec)	N/A	0.226	0.457	0.055	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	61	108	0	0	0	0
normalized size	1	1.	0.5	0.89	0.	0.	0.	0.
time (sec)	N/A	0.114	0.209	0.051	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	0	0	0	0
normalized size	1	1.	2.47	0.92	0.	0.	0.	0.
time (sec)	N/A	0.006	0.029	0.046	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.025	0.369	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	4.864	0.413	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	166	189	0	0	0	0
normalized size	1	1.	0.72	0.83	0.	0.	0.	0.
time (sec)	N/A	0.184	0.324	0.052	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	115	121	0	0	0	0
normalized size	1	1.	0.78	0.82	0.	0.	0.	0.
time (sec)	N/A	0.129	0.238	0.05	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	51	80	0	0	0	0
normalized size	1	1.	0.53	0.83	0.	0.	0.	0.
time (sec)	N/A	0.067	0.166	0.051	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	0	0	0	0
normalized size	1	1.	2.47	0.92	0.	0.	0.	0.
time (sec)	N/A	0.007	0.03	0.046	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.026	0.054	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	3.246	0.1	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	253	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.428	0.021	0.053	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	171	324	0	0	0	0
normalized size	1	1.	0.72	1.36	0.	0.	0.	0.
time (sec)	N/A	0.318	0.289	0.077	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	265	265	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	0.207	0.053	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	137	208	0	0	0	0
normalized size	1	1.	0.77	1.18	0.	0.	0.	0.
time (sec)	N/A	0.188	0.133	0.075	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	140	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	0.006	0.051	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	100	122	0	0	0	0
normalized size	1	1.	0.81	0.98	0.	0.	0.	0.
time (sec)	N/A	0.111	0.108	0.072	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.172	0.053	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	0	0	0	0
normalized size	1	1.	1.	0.89	0.	0.	0.	0.
time (sec)	N/A	0.038	0.009	0.056	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.017	0.05	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.027	0.052	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.02	0.053	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	119	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.026	0.055	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	0.006	0.062	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	170	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.027	0.056	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	165	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.222	0.02	0.05	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	258	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.245	0.027	0.057	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	242	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.393	0.011	0.062	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	285	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.358	0.027	0.056	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	497	497	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.41	0.664	0.244	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	0.585	0.056	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	67	60	0	0	0	0
normalized size	1	1.	0.96	0.86	0.	0.	0.	0.
time (sec)	N/A	0.168	0.01	0.056	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.036	0.36	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.082	0.355	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	319	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.571	6.888	0.701	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	319	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.435	6.65	0.535	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	316	0	0	0	0	0
normalized size	1	1.	1.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.326	6.659	0.541	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	164	80	0	0	0	0
normalized size	1	1.	2.52	1.23	0.	0.	0.	0.
time (sec)	N/A	0.041	0.093	0.216	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	195	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.511	3.968	0.517	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	200	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.523	3.989	0.522	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	244	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.702	5.504	0.306	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.03	0.114	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.032	0.069	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.049	0.085	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.044	0.073	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.015	0.004	0.046	0.	0.	1.404	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.011	0.003	0.045	0.	0.	0.434	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	8	0
normalized size	1	1.	1.	1.11	0.	0.	0.89	0.
time (sec)	N/A	0.015	0.01	0.118	0.	0.	0.235	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	0	10	0
normalized size	1	1.	1.	1.09	0.	0.	0.91	0.
time (sec)	N/A	0.015	0.003	0.043	0.	0.	0.78	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	14	0
normalized size	1	1.	1.	0.92	0.	0.	1.08	0.
time (sec)	N/A	0.015	0.004	0.044	0.	0.	1.934	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	0	31	0
normalized size	1	1.	1.	1.06	0.	0.	1.82	0.
time (sec)	N/A	0.018	0.006	0.046	0.	0.	4.624	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	232	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.384	0.013	0.077	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	153	318	0	0	0	0
normalized size	1	1.	0.71	1.47	0.	0.	0.	0.
time (sec)	N/A	0.265	0.276	0.076	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	248	248	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.255	0.469	0.089	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	120	202	0	0	0	0
normalized size	1	1.	0.76	1.28	0.	0.	0.	0.
time (sec)	N/A	0.156	0.173	0.078	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	151	0
normalized size	1	1.	1.	0.	0.	0.	1.26	0.
time (sec)	N/A	0.118	0.006	0.079	0.	0.	26.593	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	83	115	0	0	0	0
normalized size	1	1.	0.79	1.1	0.	0.	0.	0.
time (sec)	N/A	0.081	0.102	0.083	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.216	0.095	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	46	0	0	0	0
normalized size	1	1.	0.9	0.94	0.	0.	0.	0.
time (sec)	N/A	0.022	0.024	0.063	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.011	0.002	0.046	0.	0.	0.451	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.03	0.079	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.03	0.054	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.03	0.067	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.008	0.062	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	152	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	0.03	0.062	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	147	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	0.031	0.062	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	240	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	0.03	0.056	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	224	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.353	0.013	0.055	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	267	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.306	0.032	0.062	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	262	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.477	0.032	0.053	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.073	0.062	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	307	307	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.442	0.046	0.088	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	163	321	0	0	0	0
normalized size	1	1.	0.75	1.48	0.	0.	0.	0.
time (sec)	N/A	0.271	0.176	0.084	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	184	0	0	0	264	0
normalized size	1	1.	1.	0.	0.	0.	1.43	0.
time (sec)	N/A	0.253	0.009	0.088	0.	0.	124.888	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	126	212	0	0	0	0
normalized size	1	1.	0.76	1.28	0.	0.	0.	0.
time (sec)	N/A	0.169	0.173	0.079	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.038	0.084	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	90	119	0	0	0	0
normalized size	1	1.	0.83	1.1	0.	0.	0.	0.
time (sec)	N/A	0.09	0.081	0.075	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	114	0
normalized size	1	1.	1.	0.	0.	0.	1.56	0.
time (sec)	N/A	0.055	0.005	0.086	0.	0.	4.716	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	52	0	0	0	0
normalized size	1	1.	0.81	0.88	0.	0.	0.	0.
time (sec)	N/A	0.032	0.028	0.066	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.014	0.058	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.03	0.086	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.005	0.063	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.031	0.063	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.034	0.061	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	155	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.032	0.063	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	0.01	0.062	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	230	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.205	0.033	0.065	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	201	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.319	0.033	0.063	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	270	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.31	0.032	0.064	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	278	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.516	0.016	0.064	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	89	123	0	0	184	0
normalized size	1	1.	0.72	0.99	0.	0.	1.48	0.
time (sec)	N/A	0.082	0.071	0.049	0.	0.	2.6	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	83	107	0	0	153	0
normalized size	1	1.	0.76	0.98	0.	0.	1.4	0.
time (sec)	N/A	0.114	0.056	0.049	0.	0.	2.361	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	80	97	0	0	49	0
normalized size	1	1.	0.81	0.98	0.	0.	0.49	0.
time (sec)	N/A	0.062	0.063	0.048	0.	0.	1.129	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	71	81	0	0	116	0
normalized size	1	1.	0.85	0.96	0.	0.	1.38	0.
time (sec)	N/A	0.077	0.043	0.046	0.	0.	1.223	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	70	0	0	112	0
normalized size	1	1.	1.	0.95	0.	0.	1.51	0.
time (sec)	N/A	0.043	0.016	0.048	0.	0.	1.238	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	0	0	80	0
normalized size	1	1.	1.	0.92	0.	0.	1.36	0.
time (sec)	N/A	0.054	0.014	0.05	0.	0.	1.03	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	0	0	49	0
normalized size	1	1.	1.	0.9	0.	0.	1.	0.
time (sec)	N/A	0.026	0.012	0.048	0.	0.	0.526	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	0	0	44	0
normalized size	1	1.	1.	1.04	0.	0.	1.63	0.
time (sec)	N/A	0.005	0.002	0.051	0.	0.	0.633	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	0	23	0	0	0	0
normalized size	1	1.	0.	0.33	0.	0.	0.	0.
time (sec)	N/A	0.044	0.014	0.073	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	0	0	53	0
normalized size	1	1.	1.	1.04	0.	0.	1.96	0.
time (sec)	N/A	0.021	0.011	0.049	0.	0.	1.134	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	43	0	0	51	0
normalized size	1	1.	1.	0.98	0.	0.	1.16	0.
time (sec)	N/A	0.029	0.011	0.048	0.	0.	0.633	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	49	0	0	42	0
normalized size	1	1.	1.	0.94	0.	0.	0.81	0.
time (sec)	N/A	0.064	0.015	0.047	0.	0.	0.735	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	64	0	0	110	0
normalized size	1	1.	1.	0.93	0.	0.	1.59	0.
time (sec)	N/A	0.041	0.016	0.051	0.	0.	1.125	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	71	0	0	65	0
normalized size	1	1.	1.	0.92	0.	0.	0.84	0.
time (sec)	N/A	0.089	0.019	0.052	0.	0.	1.992	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	74	87	0	0	56	0
normalized size	1	1.	0.79	0.93	0.	0.	0.6	0.
time (sec)	N/A	0.059	0.115	0.047	0.	0.	1.528	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	84	93	0	0	44	0
normalized size	1	1.	0.82	0.91	0.	0.	0.43	0.
time (sec)	N/A	0.118	0.127	0.05	0.	0.	2.123	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	85	108	0	0	185	0
normalized size	1	1.	0.71	0.91	0.	0.	1.55	0.
time (sec)	N/A	0.078	0.058	0.048	0.	0.	3.706	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	96	115	0	0	76	0
normalized size	1	1.	0.76	0.91	0.	0.	0.6	0.
time (sec)	N/A	0.144	0.081	0.047	0.	0.	6.865	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	424	397	0	0	0	0
normalized size	1	1.	1.42	1.33	0.	0.	0.	0.
time (sec)	N/A	0.373	0.817	0.056	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	237	249	0	0	0	0
normalized size	1	1.	1.22	1.28	0.	0.	0.	0.
time (sec)	N/A	0.205	0.466	0.054	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	74	107	0	0	0	0
normalized size	1	1.	0.61	0.88	0.	0.	0.	0.
time (sec)	N/A	0.113	0.236	0.053	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	0	0	0	0
normalized size	1	1.	2.43	0.92	0.	0.	0.	0.
time (sec)	N/A	0.006	0.029	0.049	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.026	0.365	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	2.287	0.373	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	166	187	0	0	0	0
normalized size	1	1.	0.73	0.82	0.	0.	0.	0.
time (sec)	N/A	0.187	0.294	0.05	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	116	122	0	0	0	0
normalized size	1	1.	0.78	0.82	0.	0.	0.	0.
time (sec)	N/A	0.123	0.297	0.052	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	59	79	0	0	0	0
normalized size	1	1.	0.62	0.83	0.	0.	0.	0.
time (sec)	N/A	0.071	0.148	0.052	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	0	0	0	0
normalized size	1	1.	2.43	0.92	0.	0.	0.	0.
time (sec)	N/A	0.007	0.029	0.049	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.018	0.056	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	1.464	0.085	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	253	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.422	0.017	0.052	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	170	324	0	0	0	0
normalized size	1	1.	0.71	1.36	0.	0.	0.	0.
time (sec)	N/A	0.298	0.24	0.077	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	265	265	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.288	0.198	0.052	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	137	209	0	0	0	0
normalized size	1	1.	0.77	1.18	0.	0.	0.	0.
time (sec)	N/A	0.188	0.134	0.076	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	140	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	0.009	0.055	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	100	122	0	0	0	0
normalized size	1	1.	0.81	0.98	0.	0.	0.	0.
time (sec)	N/A	0.105	0.088	0.075	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.174	0.051	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	0	0	0	0
normalized size	1	1.	1.	0.91	0.	0.	0.	0.
time (sec)	N/A	0.036	0.01	0.053	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.019	0.046	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.026	0.056	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.02	0.056	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	119	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.026	0.053	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.007	0.053	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	170	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.027	0.053	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	165	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.22	0.02	0.053	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	258	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.24	0.027	0.053	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	242	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.388	0.013	0.053	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	285	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.342	0.027	0.052	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	495	495	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.396	0.649	0.227	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.548	0.054	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	60	0	0	0	0
normalized size	1	1.	0.96	0.87	0.	0.	0.	0.
time (sec)	N/A	0.151	0.01	0.054	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.035	0.357	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.08	0.347	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	318	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.522	6.775	0.523	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	318	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.408	6.65	0.539	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	315	0	0	0	0	0
normalized size	1	1.	1.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.263	6.632	0.523	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	165	81	0	0	0	0
normalized size	1	1.	2.5	1.23	0.	0.	0.	0.
time (sec)	N/A	0.04	0.095	0.069	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	194	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.493	3.93	0.523	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	199	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.493	4.006	0.527	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	244	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.607	5.352	0.205	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.029	0.056	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.03	0.056	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.044	0.056	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.039	0.061	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.015	0.009	0.044	0.	0.	1.41	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.012	0.006	0.044	0.	0.	0.441	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	10	0
normalized size	1	1.	1.	1.11	0.	0.	1.11	0.
time (sec)	N/A	0.016	0.011	0.122	0.	0.	0.243	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	0	12	0
normalized size	1	1.	1.	1.09	0.	0.	1.09	0.
time (sec)	N/A	0.015	0.006	0.044	0.	0.	0.798	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	15	0
normalized size	1	1.	1.	0.92	0.	0.	1.15	0.
time (sec)	N/A	0.016	0.006	0.046	0.	0.	1.972	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	0	34	0
normalized size	1	1.	1.	1.06	0.	0.	2.	0.
time (sec)	N/A	0.019	0.01	0.047	0.	0.	4.591	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	231	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.383	0.012	0.083	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	154	317	0	0	0	0
normalized size	1	1.	0.72	1.47	0.	0.	0.	0.
time (sec)	N/A	0.259	0.234	0.085	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	247	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.253	0.381	0.082	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	120	202	0	0	0	0
normalized size	1	1.	0.76	1.29	0.	0.	0.	0.
time (sec)	N/A	0.151	0.132	0.079	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	151	0
normalized size	1	1.	1.	0.	0.	0.	1.26	0.
time (sec)	N/A	0.118	0.007	0.079	0.	0.	26.668	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	83	114	0	0	0	0
normalized size	1	1.	0.8	1.1	0.	0.	0.	0.
time (sec)	N/A	0.078	0.085	0.076	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.209	0.079	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	44	45	0	0	0	0
normalized size	1	1.	0.92	0.94	0.	0.	0.	0.
time (sec)	N/A	0.021	0.027	0.054	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.011	0.002	0.046	0.	0.	0.459	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.029	0.082	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.031	0.059	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.03	0.058	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.007	0.057	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	152	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.03	0.065	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	147	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.03	0.057	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	240	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	0.029	0.059	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	224	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.352	0.014	0.057	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	267	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.304	0.031	0.061	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	262	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.495	0.033	0.06	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.069	0.059	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.431	0.045	0.078	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	163	322	0	0	0	0
normalized size	1	1.	0.75	1.48	0.	0.	0.	0.
time (sec)	N/A	0.271	0.174	0.083	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	185	0	0	0	264	0
normalized size	1	1.	1.	0.	0.	0.	1.43	0.
time (sec)	N/A	0.254	0.009	0.08	0.	0.	123.898	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	126	212	0	0	0	0
normalized size	1	1.	0.75	1.27	0.	0.	0.	0.
time (sec)	N/A	0.174	0.152	0.079	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.037	0.078	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	90	120	0	0	0	0
normalized size	1	1.	0.83	1.1	0.	0.	0.	0.
time (sec)	N/A	0.089	0.084	0.08	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	114	0
normalized size	1	1.	1.	0.	0.	0.	1.54	0.
time (sec)	N/A	0.057	0.008	0.086	0.	0.	4.371	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	48	52	0	0	0	0
normalized size	1	1.	0.8	0.87	0.	0.	0.	0.
time (sec)	N/A	0.032	0.026	0.061	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.015	0.059	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.032	0.084	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.005	0.059	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.033	0.059	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.032	0.06	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	155	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.032	0.059	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.218	0.011	0.058	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	230	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.2	0.031	0.063	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	201	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.314	0.033	0.069	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	270	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.306	0.033	0.063	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	278	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.52	0.017	0.056	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [140] had the largest ratio of [1.1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.	8	0.5
2	A	6	4	1.	8	0.5
3	A	5	4	1.	8	0.5
4	A	5	4	1.	8	0.5
5	A	4	4	1.	8	0.5
6	A	4	4	1.	8	0.5
7	A	3	3	1.	6	0.5
8	A	1	1	1.	4	0.25
9	A	3	3	1.	8	0.375
10	A	2	2	1.	8	0.25
11	A	3	3	1.	8	0.375
12	A	4	4	1.	8	0.5
13	A	4	4	1.	8	0.5
14	A	5	4	1.	8	0.5
15	A	5	4	1.	8	0.5
16	A	6	4	1.	8	0.5
17	A	6	4	1.	8	0.5
18	A	7	4	1.	8	0.5
19	A	14	10	1.	14	0.714
20	A	11	9	1.	14	0.643
21	A	8	7	1.	12	0.583
22	A	1	1	1.	6	0.167
23	A	0	0	0.	0	0.
24	A	0	0	0.	0	0.
25	A	14	10	1.	10	1.
26	A	11	9	1.	10	0.9
27	A	8	7	1.	8	0.875
28	A	1	1	1.	6	0.167
29	A	0	0	0.	0	0.
30	A	0	0	0.	0	0.
31	A	23	10	1.	10	1.
32	A	19	10	1.	10	1.
33	A	16	9	1.	10	0.9
34	A	12	9	1.	10	0.9
35	A	10	9	1.	10	0.9
36	A	8	6	1.	10	0.6
37	A	5	5	1.	8	0.625
38	A	4	4	1.	6	0.667
39	A	0	0	0.	0	0.
40	A	0	0	0.	0	0.
41	A	0	0	0.	0	0.
42	A	0	0	0.	0	0.
43	A	9	9	1.	10	0.9

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
44	A	0	0	0.	0	0.
45	A	0	0	0.	0	0.
46	A	0	0	0.	0	0.
47	A	20	10	1.	10	1.
48	A	0	0	0.	0	0.
49	A	18	13	1.	16	0.812
50	A	10	9	1.	14	0.643
51	A	4	3	1.	8	0.375
52	A	0	0	0.	0	0.
53	A	0	0	0.	0	0.
54	A	14	9	1.	17	0.529
55	A	14	9	1.	15	0.6
56	A	14	9	1.	13	0.692
57	A	3	1	1.	17	0.059
58	A	14	9	1.	17	0.529
59	A	14	9	1.	17	0.529
60	A	16	10	1.	19	0.526
61	A	4	4	1.	22	0.182
62	A	4	4	1.	22	0.182
63	A	4	4	1.	19	0.21
64	A	4	4	1.	19	0.21
65	A	2	2	1.	19	0.105
66	A	2	2	1.	17	0.118
67	A	2	2	1.	19	0.105
68	A	2	2	1.	19	0.105
69	A	2	2	1.	19	0.105
70	A	2	2	1.	19	0.105
71	A	22	9	1.	20	0.45
72	A	18	9	1.	20	0.45
73	A	15	8	1.	20	0.4
74	A	11	8	1.	20	0.4
75	A	9	8	1.	20	0.4
76	A	7	5	1.	20	0.25
77	A	4	4	1.	20	0.2
78	A	2	2	1.	18	0.111
79	A	2	2	1.	17	0.118
80	A	0	0	0.	0	0.
81	A	0	0	0.	0	0.
82	A	0	0	0.	0	0.
83	A	8	8	1.	20	0.4
84	A	0	0	0.	0	0.
85	A	0	0	0.	0	0.
86	A	0	0	0.	0	0.
87	A	19	9	1.	20	0.45

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	0	0	0.	0	0.
89	A	0	0	0.	0	0.
90	A	0	0	0.	0	0.
91	A	23	8	1.	20	0.4
92	A	18	8	1.	20	0.4
93	A	16	9	1.	20	0.45
94	A	13	9	1.	20	0.45
95	A	10	8	1.	20	0.4
96	A	7	6	1.	20	0.3
97	A	5	5	1.	20	0.25
98	A	4	3	1.	18	0.167
99	A	1	1	1.	17	0.059
100	A	0	0	0.	0	0.
101	A	4	4	1.	20	0.2
102	A	0	0	0.	0	0.
103	A	0	0	0.	0	0.
104	A	0	0	0.	0	0.
105	A	13	9	1.	20	0.45
106	A	0	0	0.	0	0.
107	A	0	0	0.	0	0.
108	A	0	0	0.	0	0.
109	A	26	9	1.	20	0.45
110	A	6	4	1.	8	0.5
111	A	6	4	1.	8	0.5
112	A	5	4	1.	8	0.5
113	A	5	4	1.	8	0.5
114	A	4	4	1.	8	0.5
115	A	4	4	1.	8	0.5
116	A	3	3	1.	6	0.5
117	A	1	1	1.	4	0.25
118	A	3	3	1.	8	0.375
119	A	2	2	1.	8	0.25
120	A	3	3	1.	8	0.375
121	A	4	4	1.	8	0.5
122	A	4	4	1.	8	0.5
123	A	5	4	1.	8	0.5
124	A	5	4	1.	8	0.5
125	A	6	4	1.	8	0.5
126	A	6	4	1.	8	0.5
127	A	7	4	1.	8	0.5
128	A	14	10	1.	14	0.714
129	A	11	9	1.	14	0.643
130	A	8	7	1.	12	0.583
131	A	1	1	1.	6	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
132	A	0	0	0.	0	0.
133	A	0	0	0.	0	0.
134	A	14	10	1.	10	1.
135	A	11	9	1.	10	0.9
136	A	8	7	1.	8	0.875
137	A	1	1	1.	6	0.167
138	A	0	0	0.	0	0.
139	A	0	0	0.	0	0.
140	A	23	11	1.	10	1.1
141	A	19	10	1.	10	1.
142	A	16	10	1.	10	1.
143	A	12	9	1.	10	0.9
144	A	10	10	1.	10	1.
145	A	8	6	1.	10	0.6
146	A	5	5	1.	8	0.625
147	A	4	4	1.	6	0.667
148	A	0	0	0.	0	0.
149	A	0	0	0.	0	0.
150	A	0	0	0.	0	0.
151	A	0	0	0.	0	0.
152	A	9	9	1.	10	0.9
153	A	0	0	0.	0	0.
154	A	0	0	0.	0	0.
155	A	0	0	0.	0	0.
156	A	20	10	1.	10	1.
157	A	0	0	0.	0	0.
158	A	18	13	1.	16	0.812
159	A	10	9	1.	14	0.643
160	A	4	3	1.	8	0.375
161	A	0	0	0.	0	0.
162	A	0	0	0.	0	0.
163	A	14	9	1.	17	0.529
164	A	14	9	1.	15	0.6
165	A	14	9	1.	13	0.692
166	A	3	1	1.	17	0.059
167	A	14	9	1.	17	0.529
168	A	14	9	1.	17	0.529
169	A	16	10	1.	19	0.526
170	A	4	4	1.	22	0.182
171	A	4	4	1.	22	0.182
172	A	4	4	1.	19	0.21
173	A	4	4	1.	19	0.21
174	A	2	2	1.	19	0.105
175	A	2	2	1.	17	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	2	2	1.	19	0.105
177	A	2	2	1.	19	0.105
178	A	2	2	1.	19	0.105
179	A	2	2	1.	19	0.105
180	A	22	10	1.	20	0.5
181	A	18	9	1.	20	0.45
182	A	15	9	1.	20	0.45
183	A	11	8	1.	20	0.4
184	A	9	9	1.	20	0.45
185	A	7	5	1.	20	0.25
186	A	4	4	1.	20	0.2
187	A	2	2	1.	18	0.111
188	A	2	2	1.	17	0.118
189	A	0	0	0.	0	0.
190	A	0	0	0.	0	0.
191	A	0	0	0.	0	0.
192	A	8	8	1.	20	0.4
193	A	0	0	0.	0	0.
194	A	0	0	0.	0	0.
195	A	0	0	0.	0	0.
196	A	19	9	1.	20	0.45
197	A	0	0	0.	0	0.
198	A	0	0	0.	0	0.
199	A	0	0	0.	0	0.
200	A	23	9	1.	20	0.45
201	A	18	8	1.	20	0.4
202	A	16	10	1.	20	0.5
203	A	13	9	1.	20	0.45
204	A	10	9	1.	20	0.45
205	A	7	6	1.	20	0.3
206	A	5	5	1.	20	0.25
207	A	4	3	1.	18	0.167
208	A	1	1	1.	17	0.059
209	A	0	0	0.	0	0.
210	A	4	4	1.	20	0.2
211	A	0	0	0.	0	0.
212	A	0	0	0.	0	0.
213	A	0	0	0.	0	0.
214	A	13	9	1.	20	0.45
215	A	0	0	0.	0	0.
216	A	0	0	0.	0	0.
217	A	0	0	0.	0	0.
218	A	26	9	1.	20	0.45

Chapter 3

Listing of integrals

3.1 $\int x^7 S(bx)\,dx$

Optimal. Leaf size=124

$$-\frac{105S(bx)}{8\pi^4b^8}-\frac{7x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3}+\frac{105x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7}+\frac{x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b}-\frac{35x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5}+\frac{1}{8}x^8S(bx)$$

```
[Out] (-35*x^3*Cos[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2])/(8*b
*Pi) - (105*FresnelS[b*x])/(8*b^8*Pi^4) + (x^8*FresnelS[b*x])/8 + (105*x*Si
n[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*Sin[(b^2*Pi*x^2)/2])/(8*b^3*Pi^2)
```

Rubi [A] time = 0.0883233, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3385, 3386, 3351}

$$-\frac{105S(bx)}{8\pi^4b^8}-\frac{7x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3}+\frac{105x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7}+\frac{x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b}-\frac{35x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5}+\frac{1}{8}x^8S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelS[b*x],x]
```

```
[Out] (-35*x^3*Cos[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2])/(8*b
*Pi) - (105*FresnelS[b*x])/(8*b^8*Pi^4) + (x^8*FresnelS[b*x])/8 + (105*x*Si
n[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*Sin[(b^2*Pi*x^2)/2])/(8*b^3*Pi^2)
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int x^7 S(bx)\,dx &= \frac{1}{8}x^8 S(bx) - \frac{1}{8}b\int x^8 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) - \frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{8b\pi} \\
&= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{35\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{8b^3\pi^2} \\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{105\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{8b^5\pi^3} \\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) + \frac{105x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} - \frac{105\int s
\\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105S(bx)}{8b^8\pi^4} + \frac{1}{8}x^8 S(bx) + \frac{105x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x
\end{aligned}$$

Mathematica [A] time = 0.0682553, size = 88, normalized size = 0.71

$$\frac{\left(\pi^4b^8x^8-105\right)S(bx)-7bx\left(\pi^2b^4x^4-15\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)+\pi b^3x^3\left(\pi^2b^4x^4-35\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelS[b*x],x]
```

```
[Out] (b^3*Pi*x^3*(-35 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8
)*FresnelS[b*x] - 7*b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*Pi
^4)
```

Maple [A] time = 0.049, size = 123, normalized size = 1.

$$\frac{1}{b^8}\left(\frac{\mathrm{FresnelS}\,(bx)\,b^8x^8}{8}+\frac{b^7x^7}{8\,\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right)-\frac{7}{8\,\pi}\left(\frac{b^5x^5}{\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right)-5\,\frac{1}{\pi}\left(-\frac{x^3b^3\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi}+3\,\frac{1}{\pi}\left(\frac{\sin\left(1/2\,b^2
}{\pi}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x),x)
```

```
[Out] 1/b^8*(1/8*FresnelS(b*x)*b^8*x^8+1/8/Pi*b^7*x^7*cos(1/2*b^2*Pi*x^2)-7/8/Pi*
(1/Pi*b^5*x^5*sin(1/2*b^2*Pi*x^2)-5/Pi*(-1/Pi*b^3*x^3*cos(1/2*b^2*Pi*x^2)+3
```

```
/Pi*(1/Pi*b*x*sin(1/2*b^2*Pi*x^2)-1/Pi*FresnelS(b*x)))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnels}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^7 \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnels(b*x), x)
```

Sympy [A] time = 2.42098, size = 184, normalized size = 1.48

$$\frac{231x^8 S(bx)\Gamma\left(\frac{3}{4}\right)}{512\Gamma\left(\frac{15}{4}\right)} + \frac{231x^7\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi b\Gamma\left(\frac{15}{4}\right)} - \frac{1617x^5\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^2b^3\Gamma\left(\frac{15}{4}\right)} - \frac{8085x^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^3b^5\Gamma\left(\frac{15}{4}\right)} + \frac{24255x\sin\left(\frac{\pi b^2x}{2}\right.}{512\pi^4b^7\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnels(b*x),x)
```

```
[Out] 231*x**8*fresnels(b*x)*gamma(3/4)/(512*gamma(15/4)) + 231*x**7*cos(pi*b**2*
x**2/2)*gamma(3/4)/(512*pi*b*gamma(15/4)) - 1617*x**5*sin(pi*b**2*x**2/2)*g
amma(3/4)/(512*pi**2*b**3*gamma(15/4)) - 8085*x**3*cos(pi*b**2*x**2/2)*gamm
a(3/4)/(512*pi**3*b**5*gamma(15/4)) + 24255*x*sin(pi*b**2*x**2/2)*gamma(3/4
)/(512*pi**4*b**7*gamma(15/4)) - 24255*fresnels(b*x)*gamma(3/4)/(512*pi**4*
b**8*gamma(15/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnels}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnels(b*x), x)
```

3.2 $\int x^6 S(bx)\,dx$

Optimal. Leaf size=109

$$-\frac{6x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{48\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}-\frac{24x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}+\frac{1}{7}x^7S(bx)$$

```
[Out] (-24*x^2*Cos[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^6*Cos[(b^2*Pi*x^2)/2])/(7*b
*Pi) + (x^7*FresnelS[b*x])/7 + (48*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x
^4*Sin[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2)
```

Rubi [A] time = 0.110693, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3379, 3296, 2637}

$$-\frac{6x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{48\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}-\frac{24x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}+\frac{1}{7}x^7S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelS[b*x],x]
```

```
[Out] (-24*x^2*Cos[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^6*Cos[(b^2*Pi*x^2)/2])/(7*b
*Pi) + (x^7*FresnelS[b*x])/7 + (48*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x
^4*Sin[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2)
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^6 S(bx)\,dx &= \frac{1}{7}x^7 S(bx) - \frac{1}{7}b\int x^7 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= \frac{1}{7}x^7 S(bx) - \frac{1}{14}b\,\text{Subst}\left(\int x^3 \sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)\\
&= \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) - \frac{3\,\text{Subst}\left(\int x^2\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{7b\pi}\\
&= \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{12\,\text{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{7b^3\pi^2}\\
&= -\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{24\,\text{Subst}\left(\int \cos\left(\frac{1}{2}b^2\pi x\right)\right.}{7b^5\pi^3}\\
&= -\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) + \frac{48\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}\end{aligned}$$

Mathematica [A] time = 0.0516547, size = 83, normalized size = 0.76

$$-\frac{6\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7} + \frac{x^2\left(\pi^2b^4x^4-24\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5} + \frac{1}{7}x^7 S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelS[b*x],x]
```

```
[Out] (x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^7*FresnelS
[b*x])/7 - (6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4)
```

Maple [A] time = 0.049, size = 107, normalized size = 1.

$$\frac{1}{b^7}\left(\frac{b^7x^7\text{FresnelS}\,(bx)}{7} + \frac{b^6x^6}{7\,\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{6}{7\,\pi}\left(\frac{x^4b^4}{\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right) - 4\,\frac{1}{\pi}\left(-\frac{b^2x^2\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi} + 2\,\frac{\sin\left(1/2\,b^2\pi\right.}{\pi^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x),x)
```

```
[Out] 1/b^7*(1/7*b^7*x^7*FresnelS(b*x)+1/7/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)-6/7/Pi*
(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2
/Pi^2*sin(1/2*b^2*Pi*x^2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\text{fresnels}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^6\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^6*fresnels(b*x), x)
```

Sympy [A] time = 1.76553, size = 156, normalized size = 1.43

$$\frac{3x^7 S\,(bx)\,\Gamma\left(\frac{3}{4}\right)}{28\Gamma\left(\frac{7}{4}\right)}+\frac{3x^6\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{28\pi b\Gamma\left(\frac{7}{4}\right)}-\frac{9x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{14\pi^2b^3\Gamma\left(\frac{7}{4}\right)}-\frac{18x^2\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{7\pi^3b^5\Gamma\left(\frac{7}{4}\right)}+\frac{36\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{7\pi^4b^7\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnels(b*x),x)
```

```
[Out] 3*x**7*fresnels(b*x)*gamma(3/4)/(28*gamma(7/4)) + 3*x**6*cos(pi*b**2*x**2/2
)*gamma(3/4)/(28*pi*b*gamma(7/4)) - 9*x**4*sin(pi*b**2*x**2/2)*gamma(3/4)/(
14*pi**2*b**3*gamma(7/4)) - 18*x**2*cos(pi*b**2*x**2/2)*gamma(3/4)/(7*pi**3
*b**5*gamma(7/4)) + 36*sin(pi*b**2*x**2/2)*gamma(3/4)/(7*pi**4*b**7*gamma(7
/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnels(b*x), x)
```

3.3 $\int x^5 S(bx)\,dx$

Optimal. Leaf size=99

$$\frac{5\text{FresnelC}(bx)}{2\pi^3 b^6}-\frac{5x^3\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6\pi^2 b^3}+\frac{x^5\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6\pi b}-\frac{5x\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi^3 b^5}+\frac{1}{6}x^6 S(bx)$$

```
[Out] (-5*x*Cos[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) + (x^5*Cos[(b^2*Pi*x^2)/2])/(6*b*Pi
) + (5*FresnelC[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x])/6 - (5*x^3*Sin[(b^
2*Pi*x^2)/2])/(6*b^3*Pi^2)
```

Rubi [A] time = 0.0643839, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3385, 3386, 3352}

$$\frac{5\text{FresnelC}(bx)}{2\pi^3 b^6}-\frac{5x^3\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6\pi^2 b^3}+\frac{x^5\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6\pi b}-\frac{5x\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi^3 b^5}+\frac{1}{6}x^6 S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelS[b*x],x]
```

```
[Out] (-5*x*Cos[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) + (x^5*Cos[(b^2*Pi*x^2)/2])/(6*b*Pi
) + (5*FresnelC[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x])/6 - (5*x^3*Sin[(b^
2*Pi*x^2)/2])/(6*b^3*Pi^2)
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^5 S(bx)\,dx &= \frac{1}{6}x^6 S(bx) - \frac{1}{6}b\int x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6 S(bx) - \frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{6b\pi}\\
&= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6 S(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{5\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b^3\pi^2}\\
&= -\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6 S(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{5\int\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b^5\pi^3}\\
&= -\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5C(bx)}{2b^6\pi^3} + \frac{1}{6}x^6 S(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2}
\end{aligned}
$$

Mathematica [A] time = 0.0685961, size = 79, normalized size = 0.8

$$
\frac{\pi^3 b^6 x^6 S(bx) - 5\pi b^3 x^3\sin\left(\frac{1}{2}\pi b^2 x^2\right) + bx\left(\pi^2 b^4 x^4 - 15\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right) + 15\text{FresnelC}(bx)}{6\pi^3 b^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelS[b*x],x]
```

```
[Out] (b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + 15*FresnelC[b*x] + b^6*Pi^3
*x^6*FresnelS[b*x] - 5*b^3*Pi*x^3*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```

Maple [A] time = 0.049, size = 96, normalized size = 1.

$$
\frac{1}{b^6}\left(\frac{b^6x^6\text{FresnelS}(bx)}{6} + \frac{b^5x^5}{6\pi}\cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{5}{6\pi}\left(\frac{x^3b^3}{\pi}\sin\left(\frac{b^2\pi x^2}{2}\right) - 3\frac{1}{\pi}\left(-\frac{bx\cos\left(1/2\,b^2\pi x^2\right)}{\pi} + \frac{\text{FresnelC}(bx)}{\pi}\right)\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x),x)
```

```
[Out] 1/b^6*(1/6*b^6*x^6*FresnelS(b*x)+1/6/Pi*b^5*x^5*cos(1/2*b^2*Pi*x^2)-5/6/Pi*
(1/Pi*b^3*x^3*sin(1/2*b^2*Pi*x^2)-3/Pi*(-1/Pi*b*x*cos(1/2*b^2*Pi*x^2)+1/Pi*
FresnelC(b*x))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int x^5\text{fresnels}(bx)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^5\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnels(b*x), x)
```

Sympy [A] time = 1.04664, size = 53, normalized size = 0.54

$$\frac{\pi b^3x^9\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{9}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{3}{2},\frac{7}{4},\frac{13}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnels(b*x),x)
```

```
[Out] pi*b**3*x**9*gamma(3/4)*gamma(9/4)*hyper((3/4, 9/4), (3/2, 7/4, 13/4), -pi*
*2*b**4*x**4/16)/(32*gamma(7/4)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnels(b*x), x)
```

3.4 $\int x^4 S(bx)\,dx$

Optimal. Leaf size=84

$$-\frac{4x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}-\frac{8\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}+\frac{1}{5}x^5S(bx)$$

```
[Out] (-8*Cos[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^4*Cos[(b^2*Pi*x^2)/2])/(5*b*Pi)
+ (x^5*FresnelS[b*x])/5 - (4*x^2*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
```

Rubi [A] time = 0.0801435, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3379, 3296, 2638}

$$-\frac{4x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}-\frac{8\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}+\frac{1}{5}x^5S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelS[b*x],x]
```

```
[Out] (-8*Cos[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^4*Cos[(b^2*Pi*x^2)/2])/(5*b*Pi)
+ (x^5*FresnelS[b*x])/5 - (4*x^2*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^4 S(bx)\,dx &= \frac{1}{5}x^5 S(bx) - \frac{1}{5}b\int x^5 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= \frac{1}{5}x^5 S(bx) - \frac{1}{10}b\,\text{Subst}\left(\int x^2 \sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right) \\
&= \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5 S(bx) - \frac{2\,\text{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{5b\pi} \\
&= \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5 S(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{4\,\text{Subst}\left(\int \sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{5b^3\pi^2} \\
&= -\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} + \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5 S(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}
\end{aligned}
$$

Mathematica [A] time = 0.0407404, size = 71, normalized size = 0.85

$$
-\frac{4x^2\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3} + \frac{\left(\pi^2 b^4 x^4 - 8\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5} + \frac{1}{5}x^5 S(bx)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelS[b*x],x]
```

```
[Out] ((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^5*FresnelS[b*x]
)/5 - (4*x^2*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
```

Maple [A] time = 0.052, size = 80, normalized size = 1.

$$
\frac{1}{b^5}\left(\frac{b^5x^5\text{FresnelS}\,(bx)}{5} + \frac{x^4b^4}{5\,\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{4}{5\,\pi}\left(\frac{b^2x^2}{\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + 2\,\frac{\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x),x)
```

```
[Out] 1/b^5*(1/5*b^5*x^5*FresnelS(b*x)+1/5/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)-4/5/Pi*
(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int x^4\text{fresnels}\,(bx)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnels(b*x), x)
```

Sympy [A] time = 1.62895, size = 121, normalized size = 1.44

$$\frac{3x^5S\,(bx)\,\Gamma\left(\frac{3}{4}\right)}{20\Gamma\left(\frac{7}{4}\right)}+\frac{3x^4\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{20\pi b\Gamma\left(\frac{7}{4}\right)}-\frac{3x^2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{5\pi^2b^3\Gamma\left(\frac{7}{4}\right)}-\frac{6\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{5\pi^3b^5\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnels(b*x),x)
```

```
[Out] 3*x**5*fresnels(b*x)*gamma(3/4)/(20*gamma(7/4)) + 3*x**4*cos(pi*b**2*x**2/2
)*gamma(3/4)/(20*pi*b*gamma(7/4)) - 3*x**2*sin(pi*b**2*x**2/2)*gamma(3/4)/(
5*pi**2*b**3*gamma(7/4)) - 6*cos(pi*b**2*x**2/2)*gamma(3/4)/(5*pi**3*b**5*g
amma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnels(b*x), x)
```

3.5 $\int x^3 S(bx)\, dx$

Optimal. Leaf size=74

$$\frac{3S(bx)}{4\pi^2 b^4} - \frac{3x\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{x^3\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} + \frac{1}{4}x^4 S(bx)$$

```
[Out] (x^3*Cos[(b^2*Pi*x^2)/2])/(4*b*Pi) + (3*FresnelS[b*x])/(4*b^4*Pi^2) + (x^4*
FresnelS[b*x])/4 - (3*x*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.0474889, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3385, 3386, 3351}

$$\frac{3S(bx)}{4\pi^2 b^4} - \frac{3x\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{x^3\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} + \frac{1}{4}x^4 S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[b*x],x]
```

```
[Out] (x^3*Cos[(b^2*Pi*x^2)/2])/(4*b*Pi) + (3*FresnelS[b*x])/(4*b^4*Pi^2) + (x^4*
FresnelS[b*x])/4 - (3*x*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2)
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int x^3 S(bx)\,dx &= \frac{1}{4}x^4 S(bx) - \frac{1}{4}b\int x^4 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\ &= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{1}{4}x^4 S(bx) - \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{4b\pi}\\ &= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{1}{4}x^4 S(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{4b^3\pi^2}\\ &= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3S(bx)}{4b^4\pi^2} + \frac{1}{4}x^4 S(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}\end{aligned}$$

Mathematica [A] time = 0.0159426, size = 74, normalized size = 1.

$$\frac{3S(bx)}{4\pi^2 b^4} - \frac{3x\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{x^3\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} + \frac{1}{4}x^4 S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[b*x],x]
```

```
[Out] (x^3*Cos[(b^2*Pi*x^2)/2])/(4*b*Pi) + (3*FresnelS[b*x])/(4*b^4*Pi^2) + (x^4*
FresnelS[b*x])/4 - (3*x*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2)
```

Maple [A] time = 0.046, size = 70, normalized size = 1.

$$\frac{1}{b^4}\left(\frac{b^4x^4\mathrm{FresnelS}\,(bx)}{4} + \frac{x^3b^3}{4\,\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{3}{4\,\pi}\left(\frac{bx}{\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{\mathrm{FresnelS}\,(bx)}{\pi}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x),x)
```

```
[Out] 1/b^4*(1/4*b^4*x^4*FresnelS(b*x)+1/4/Pi*b^3*x^3*cos(1/2*b^2*Pi*x^2)-3/4/Pi*
(1/Pi*b*x*sin(1/2*b^2*Pi*x^2)-1/Pi*FresnelS(b*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \mathrm{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^3\mathrm{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnels(b*x), x)
```

Sympy [A] time = 0.93364, size = 112, normalized size = 1.51

$$\frac{21x^4S(bx)\Gamma\left(\frac{3}{4}\right)}{64\Gamma\left(\frac{11}{4}\right)}+\frac{21x^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{64\pi b\Gamma\left(\frac{11}{4}\right)}-\frac{63x\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{64\pi^2b^3\Gamma\left(\frac{11}{4}\right)}+\frac{63S(bx)\Gamma\left(\frac{3}{4}\right)}{64\pi^2b^4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x),x)
```

```
[Out] 21*x**4*fresnels(b*x)*gamma(3/4)/(64*gamma(11/4)) + 21*x**3*cos(pi*b**2*x**
2/2)*gamma(3/4)/(64*pi*b*gamma(11/4)) - 63*x*sin(pi*b**2*x**2/2)*gamma(3/4)
/(64*pi**2*b**3*gamma(11/4)) + 63*fresnels(b*x)*gamma(3/4)/(64*pi**2*b**4*g
amma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnels(b*x), x)
```

3.6 $\int x^2 S(bx)\,dx$

Optimal. Leaf size=59

$$-\frac{2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+\frac{1}{3}x^3S(bx)$$

```
[Out] (x^2*Cos[(b^2*Pi*x^2)/2])/(3*b*Pi) + (x^3*FresnelS[b*x])/3 - (2*Sin[(b^2*Pi
*x^2)/2])/(3*b^3*Pi^2)
```

Rubi [A] time = 0.052703, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3379, 3296, 2637}

$$-\frac{2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+\frac{1}{3}x^3S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[b*x],x]
```

```
[Out] (x^2*Cos[(b^2*Pi*x^2)/2])/(3*b*Pi) + (x^3*FresnelS[b*x])/3 - (2*Sin[(b^2*Pi
*x^2)/2])/(3*b^3*Pi^2)
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2 S(bx)\,dx &= \frac{1}{3}x^3 S(bx) - \frac{1}{3}b\int x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\ &= \frac{1}{3}x^3 S(bx) - \frac{1}{6}b\,\text{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)\\ &= \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3 S(bx) - \frac{\text{Subst}\left(\int\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{3b\pi}\\ &= \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3 S(bx) - \frac{2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}\end{aligned}$$

Mathematica [A] time = 0.0115254, size = 59, normalized size = 1.

$$-\frac{2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3} + \frac{x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b} + \frac{1}{3}x^3 S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelS[b*x],x]
```

```
[Out] (x^2*Cos[(b^2*Pi*x^2)/2])/(3*b*Pi) + (x^3*FresnelS[b*x])/3 - (2*Sin[(b^2*Pi
*x^2)/2])/(3*b^3*Pi^2)
```

Maple [A] time = 0.049, size = 54, normalized size = 0.9

$$\frac{1}{b^3}\left(\frac{b^3x^3\text{FresnelS}(bx)}{3} + \frac{b^2x^2}{3\pi}\cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{2}{3\pi^2}\sin\left(\frac{b^2\pi x^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelS(b*x)+1/3/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)-2/3/Pi^
2*sin(1/2*b^2*Pi*x^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\text{fresnels}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnels(b*x), x)
```

Sympy [A] time = 0.828279, size = 80, normalized size = 1.36

$$\frac{x^3 S(bx)\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x^2\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi b\Gamma\left(\frac{7}{4}\right)}-\frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{2\pi^2b^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x),x)
```

```
[Out] x**3*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + x**2*cos(pi*b**2*x**2/2)*gam
ma(3/4)/(4*pi*b*gamma(7/4)) - sin(pi*b**2*x**2/2)*gamma(3/4)/(2*pi**2*b**3*
gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\text{fresnels}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnels(b*x), x)
```

3.7 $\int xS(bx)\,dx$

Optimal. Leaf size=49

$$-\frac{\text{FresnelC}(bx)}{2\pi b^2}+\frac{x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{1}{2}x^2S(bx)$$

```
[Out] (x*Cos[(b^2*Pi*x^2)/2])/(2*b*Pi) - FresnelC[b*x]/(2*b^2*Pi) + (x^2*FresnelS
[b*x])/2
```

Rubi [A] time = 0.0240392, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3385, 3352}

$$-\frac{\text{FresnelC}(bx)}{2\pi b^2}+\frac{x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{1}{2}x^2S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[b*x],x]
```

```
[Out] (x*Cos[(b^2*Pi*x^2)/2])/(2*b*Pi) - FresnelC[b*x]/(2*b^2*Pi) + (x^2*FresnelS
[b*x])/2
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int xS(bx)\,dx&=\frac{1}{2}x^2S(bx)-\frac{1}{2}b\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\&=\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}+\frac{1}{2}x^2S(bx)-\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b\pi}\\&=\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}-\frac{C(bx)}{2b^2\pi}+\frac{1}{2}x^2S(bx)\end{aligned}$$

Mathematica [A] time = 0.0110994, size = 49, normalized size = 1.

$$-\frac{\text{FresnelC}(bx)}{2\pi b^2}+\frac{x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{1}{2}x^2S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelS[b*x],x]
```

```
[Out] (x*Cos[(b^2*Pi*x^2)/2])/(2*b*Pi) - FresnelC[b*x]/(2*b^2*Pi) + (x^2*FresnelS
[b*x])/2
```

Maple [A] time = 0.049, size = 44, normalized size = 0.9

$$\frac{1}{b^2}\left(\frac{b^2x^2\text{FresnelS}\,(bx)}{2}+\frac{bx}{2\,\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right)-\frac{\text{FresnelC}\,(bx)}{2\,\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x),x)
```

```
[Out] 1/b^2*(1/2*b^2*x^2*FresnelS(b*x)+1/2/Pi*b*x*cos(1/2*b^2*Pi*x^2)-1/2/Pi*Fres
nelC(b*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,(x\text{fresnels}\,(bx)\,,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x*fresnels(b*x), x)
```

Sympy [A] time = 0.524181, size = 53, normalized size = 1.08

$$\frac{\pi b^3x^5\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x),x)
```

```
[Out] pi*b**3*x**5*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -pi**
2*b**4*x**4/16)/(32*gamma(7/4)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*fresnels(b*x), x)
```

3.8 $\int S(bx)\,dx$

Optimal. Leaf size=26

$$\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)$$

```
[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]
```

Rubi [A] time = 0.0048741, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6418}

$$\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x],x]
```

```
[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]
```

Rule 6418

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelS[a +
 b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int S(bx)\,dx=\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}+xS(bx)$$

Mathematica [A] time = 0.0020637, size = 26, normalized size = 1.

$$\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x],x]
```

```
[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]
```

Maple [A] time = 0.046, size = 27, normalized size = 1.

$$\frac{1}{b}\left(bx\mathrm{FresnelS}\left(bx\right)+\frac{1}{\pi}\cos\left(\frac{b^2\pi x^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x),x)
```

```
[Out] 1/b*(b*x*FresnelS(b*x)+1/Pi*cos(1/2*b^2*Pi*x^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,(\text{fresnels}\,(bx)\,,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x), x)
```

Sympy [B] time = 0.83294, size = 48, normalized size = 1.85

$$\frac{3xS\,(bx)\,\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi b\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x),x)
```

```
[Out] 3*x*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + 3*cos(pi*b**2*x**2/2)*gamma(3
/4)/(4*pi*b*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x), x)
```

3.9 $\int \frac{S(bx)}{x}\,dx$

Optimal. Leaf size=73

$$\frac{1}{2} ibx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-\frac{1}{2}i\pi b^2x^2\right)-\frac{1}{2}ibx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},\frac{1}{2}i\pi b^2x^2\right)$$

```
[Out] (I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-I/2)*b^2*Pi*x^2] - (I
/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2]
```

Rubi [A] time = 0.0466434, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6424, 6358, 6360}

$$\frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x,x]
```

```
[Out] (I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-I/2)*b^2*Pi*x^2] - (I
/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2]
```

Rule 6424

```
Int[FresnelS[(b_.)*(x_)]/(x_), x_Symbol] :> Dist[(1 + I)/4, Int[Erf[(Sqrt[P
i]*(1 + I)*b*x)/2]/x, x], x] + Dist[(1 - I)/4, Int[Erf[(Sqrt[Pi]*(1 - I)*b*
x)/2]/x, x], x] /; FreeQ[b, x]
```

Rule 6358

```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(2*b*x*HypergeometricPFQ[{1/2,
1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[b, x]
```

Rule 6360

```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(2*b*x*HypergeometricPFQ[{1/2,
 1/2}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[b, x]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)}{x}\,dx &= \left(-\frac{1}{4}-\frac{i}{4}\right)\int \frac{\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}\,dx+\left(\frac{1}{4}+\frac{i}{4}\right)\int \frac{\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}\,dx\\ &= \frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)\end{aligned}$$

Mathematica [F] time = 0.0136888, size = 0, normalized size = 0.

$$\int \frac{S(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]/x,x]
```

```
[Out] Integrate[FresnelS[b*x]/x, x]
```

Maple [A] time = 0.08, size = 29, normalized size = 0.4

$$\frac{\pi x^3 b^3}{18} {}_2F_3\left(\frac{3}{4}, \frac{3}{4}; \frac{3}{2}, \frac{7}{4}, \frac{7}{4}; -\frac{x^4 \pi^2 b^4}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x,x)
```

```
[Out] 1/18*Pi*x^3*b^3*hypergeom([3/4,3/4],[3/2,7/4,7/4],-1/16*x^4*Pi^2*b^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x, x)
```

3.10 $\int \frac{S(bx)}{x^2} dx$

Optimal. Leaf size=27

$$\frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{S(bx)}{x}$$

```
[Out] -(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2
```

Rubi [A] time = 0.0215755, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6426, 3375}

$$\frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{S(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^2,x]
```

```
[Out] -(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*Sin[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)}{x^2} dx &= -\frac{S(bx)}{x} + b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx\\ &= -\frac{S(bx)}{x} + \frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)\end{aligned}$$

Mathematica [A] time = 0.010818, size = 27, normalized size = 1.

$$\frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{S(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^2,x]
```

```
[Out] -(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2
```

Maple [A] time = 0.049, size = 28, normalized size = 1.

$$b\left(-\frac{\text{FresnelS}\,(bx)}{bx}+\frac{1}{2}\text{Si}\left(\frac{b^2\pi\,x^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^2,x)
```

```
[Out] b*(-FresnelS(b*x)/b/x+1/2*Si(1/2*b^2*Pi*x^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^2, x)
```

Sympy [B] time = 0.544513, size = 42, normalized size = 1.56

$$\frac{\pi b^3x^2\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**2,x)
```

```
[Out] pi*b**3*x**2*gamma(3/4)*hyper((1/2, 3/4), (3/2, 3/2, 7/4), -pi**2*b**4*x**4
/16)/(16*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^2, x)
```

3.11 $\int \frac{S(bx)}{x^3}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}\pi b^2\text{FresnelC}(bx)-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x}-\frac{S(bx)}{2x^2}$$

```
[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/
(2*x)
```

Rubi [A] time = 0.0287987, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6426, 3387, 3352}

$$\frac{1}{2}\pi b^2\text{FresnelC}(bx)-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x}-\frac{S(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/
(2*x)
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^3}\,dx &= -\frac{S(bx)}{2x^2}+\frac{1}{2}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{S(bx)}{2x^2}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}+\frac{1}{2}\left(b^3\pi\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= \frac{1}{2}b^2\pi C(bx)-\frac{S(bx)}{2x^2}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}
\end{aligned}$$

Mathematica [A] time = 0.0112385, size = 44, normalized size = 1.

$$\frac{1}{2}\pi b^2\text{FresnelC}(bx) - \frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x} - \frac{S(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/
(2*x)
```

Maple [A] time = 0.048, size = 43, normalized size = 1.

$$b^2\left(-\frac{\text{FresnelS}\,(bx)}{2\,b^2x^2} - \frac{1}{2\,bx}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\pi\,\text{FresnelC}\,(bx)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^3,x)
```

```
[Out] b^2*(-1/2*FresnelS(b*x)/b^2/x^2-1/2*sin(1/2*b^2*Pi*x^2)/b/x+1/2*Pi*FresnelC
(b*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^3, x)
```

Sympy [A] time = 0.623655, size = 51, normalized size = 1.16

$$\frac{\pi b^3x\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{5}{4},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**3,x)
```

```
[Out] pi*b**3*x*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (5/4, 3/2, 7/4), -pi**2*b
**4*x**4/16)/(32*gamma(5/4)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^3, x)
```

3.12 $\int \frac{S(bx)}{x^4} \, dx$

Optimal. Leaf size=52

$$\frac{1}{12}\pi b^3 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^2} - \frac{S(bx)}{3x^3}$$

```
[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b
^2*Pi*x^2)/2])/(6*x^2)
```

Rubi [A] time = 0.0637342, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3379, 3297, 3302}

$$\frac{1}{12}\pi b^3 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^2} - \frac{S(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b
^2*Pi*x^2)/2])/(6*x^2)
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^4}\,dx &= -\frac{S(bx)}{3x^3} + \frac{1}{3}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx \\
&= -\frac{S(bx)}{3x^3} + \frac{1}{6}b\,\mathrm{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&= -\frac{S(bx)}{3x^3} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2} + \frac{1}{12}\left(b^3\pi\right)\mathrm{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&= \frac{1}{12}b^3\pi\,\mathrm{Ci}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{S(bx)}{3x^3} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}
\end{aligned}$$

Mathematica [A] time = 0.0143616, size = 52, normalized size = 1.

$$\frac{1}{12}\pi b^3\,\mathrm{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right) - \frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2} - \frac{S(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b
^2*Pi*x^2)/2])/(6*x^2)
```

Maple [A] time = 0.052, size = 49, normalized size = 0.9

$$b^3\left(-\frac{\mathrm{FresnelS}\,(bx)}{3\,x^3b^3} - \frac{1}{6\,b^2x^2}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\pi}{12}\mathrm{Ci}\left(\frac{b^2\pi\,x^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^4,x)
```

```
[Out] b^3*(-1/3*FresnelS(b*x)/b^3/x^3-1/6*sin(1/2*b^2*Pi*x^2)/b^2/x^2+1/12*Pi*Ci(
1/2*b^2*Pi*x^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^4, x)
```

Sympy [A] time = 1.38716, size = 56, normalized size = 1.08

$$-\frac{\pi^3 b^7 x^4 \Gamma\left(\frac{7}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{7}{4} \\ 2, 2, \frac{5}{2}, \frac{11}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{768\Gamma\left(\frac{11}{4}\right)} + \frac{\pi b^3 \log\left(b^4 x^4\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**4,x)
```

```
[Out] -pi**3*b**7*x**4*gamma(7/4)*hyper((1, 1, 7/4), (2, 2, 5/2, 11/4), -pi**2*b*
*4*x**4/16)/(768*gamma(11/4)) + pi*b**3*log(b**4*x**4)/24
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^4, x)
```

3.13 $\int \frac{S(bx)}{x^5} \, dx$

Optimal. Leaf size=69

$$-\frac{1}{12}\pi^2 b^4 S(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x} - \frac{S(bx)}{4x^4}$$

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(12*x) - (b^4*Pi^2*FresnelS[b*x])/12 - Fresne
lS[b*x]/(4*x^4) - (b*Sin[(b^2*Pi*x^2)/2])/(12*x^3)
```

Rubi [A] time = 0.0433676, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3387, 3388, 3351}

$$-\frac{1}{12}\pi^2 b^4 S(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x} - \frac{S(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^5,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(12*x) - (b^4*Pi^2*FresnelS[b*x])/12 - Fresne
lS[b*x]/(4*x^4) - (b*Sin[(b^2*Pi*x^2)/2])/(12*x^3)
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{S(bx)}{x^5}\,dx &= -\frac{S(bx)}{4x^4} + \frac{1}{4} b \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx \\
&= -\frac{S(bx)}{4x^4} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} + \frac{1}{12}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{S(bx)}{4x^4} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{1}{12}\left(b^5\pi^2\right)\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{1}{12}b^4\pi^2 S(bx) - \frac{S(bx)}{4x^4} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}
\end{aligned}
$$

Mathematica [A] time = 0.0145281, size = 69, normalized size = 1.

$$
-\frac{1}{12}\pi^2 b^4 S(bx) - \frac{b\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3} - \frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x} - \frac{S(bx)}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^5,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(12*x) - (b^4*Pi^2*FresnelS[b*x])/12 - Fresne
lS[b*x]/(4*x^4) - (b*Sin[(b^2*Pi*x^2)/2])/(12*x^3)
```

Maple [A] time = 0.05, size = 65, normalized size = 0.9

$$
b^4\left(-\frac{\mathrm{FresnelS}\,(bx)}{4\,x^4 b^4} - \frac{1}{12\,x^3 b^3}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\pi}{12}\left(-\frac{1}{bx}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \pi\,\mathrm{FresnelS}\,(bx)\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^5,x)
```

```
[Out] b^4*(-1/4*FresnelS(b*x)/b^4/x^4-1/12*sin(1/2*b^2*Pi*x^2)/b^3/x^3+1/12*Pi*(-
1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*FresnelS(b*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\mathrm{fresnels}\,(bx)}{x^5}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^5, x)
```

Sympy [A] time = 1.11749, size = 110, normalized size = 1.59

$$\frac{\pi^2 b^4 S\,(bx)\,\Gamma\left(-\frac{1}{4}\right)}{64\Gamma\left(\frac{7}{4}\right)}+\frac{\pi b^3\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{1}{4}\right)}{64x\Gamma\left(\frac{7}{4}\right)}+\frac{b\sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{1}{4}\right)}{64x^3\Gamma\left(\frac{7}{4}\right)}+\frac{3S\,(bx)\,\Gamma\left(-\frac{1}{4}\right)}{64x^4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**5,x)
```

```
[Out] pi**2*b**4*fresnels(b*x)*gamma(-1/4)/(64*gamma(7/4)) + pi*b**3*cos(pi*b**2*
x**2/2)*gamma(-1/4)/(64*x*gamma(7/4)) + b*sin(pi*b**2*x**2/2)*gamma(-1/4)/(
64*x**3*gamma(7/4)) + 3*fresnels(b*x)*gamma(-1/4)/(64*x**4*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{fresnels}\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^5, x)
```

3.14 $\int \frac{S(bx)}{x^6} dx$

Optimal. Leaf size=77

$$-\frac{1}{80}\pi^2 b^5 \mathrm{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}$$

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(40*x^2) - FresnelS[b*x]/(5*x^5) - (b*Sin[(b^
2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*SinIntegral[(b^2*Pi*x^2)/2])/80
```

Rubi [A] time = 0.0892369, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3379, 3297, 3299}

$$-\frac{1}{80}\pi^2 b^5 \mathrm{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^6,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(40*x^2) - FresnelS[b*x]/(5*x^5) - (b*Sin[(b^
2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*SinIntegral[(b^2*Pi*x^2)/2])/80
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^6}\,dx &= -\frac{S(bx)}{5x^5} + \frac{1}{5}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx \\
&= -\frac{S(bx)}{5x^5} + \frac{1}{10}b\,\mathrm{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx, x, x^2\right) \\
&= -\frac{S(bx)}{5x^5} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} + \frac{1}{40}\left(b^3\pi\right)\mathrm{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2} - \frac{S(bx)}{5x^5} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}\left(b^5\pi^2\right)\mathrm{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2} - \frac{S(bx)}{5x^5} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2\mathrm{Si}\left(\frac{1}{2}b^2\pi x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0184752, size = 77, normalized size = 1.

$$-\frac{1}{80}\pi^2 b^5\mathrm{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4} - \frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2} - \frac{S(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^6,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(40*x^2) - FresnelS[b*x]/(5*x^5) - (b*Sin[(b^
2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*SinIntegral[(b^2*Pi*x^2)/2])/80
```

Maple [A] time = 0.049, size = 71, normalized size = 0.9

$$b^5\left(-\frac{\mathrm{FresnelS}\,(bx)}{5\,b^5x^5} - \frac{1}{20\,x^4b^4}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\pi}{20}\left(-\frac{1}{2\,b^2x^2}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{\pi}{4}\mathrm{Si}\left(\frac{b^2\pi\,x^2}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^6,x)
```

```
[Out] b^5*(-1/5*FresnelS(b*x)/b^5/x^5-1/20*sin(1/2*b^2*Pi*x^2)/b^4/x^4+1/20*Pi*(-
1/2/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/4*Pi*Si(1/2*b^2*Pi*x^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\,(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^6, x)
```

Sympy [A] time = 1.21138, size = 46, normalized size = 0.6

$$-\frac{\pi b^3\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16x^2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**6,x)
```

```
[Out] -pi*b**3*gamma(3/4)*hyper((-1/2, 3/4), (1/2, 3/2, 7/4), -pi**2*b**4*x**4/16
)/(16*x**2*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^6, x)
```

3.15 $\int \frac{S(bx)}{x^7}\,dx$

Optimal. Leaf size=94

$$-\frac{1}{90}\pi^3 b^6 \text{FresnelC}(bx)+\frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{90x}-\frac{b\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{30x^5}-\frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{90x^3}-\frac{S(bx)}{6x^6}$$

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(90*x^3) - (b^6*Pi^3*FresnelC[b*x])/90 - Fres
nelS[b*x]/(6*x^6) - (b*Sin[(b^2*Pi*x^2)/2])/(30*x^5) + (b^5*Pi^2*Sin[(b^2*P
i*x^2)/2])/(90*x)
```

Rubi [A] time = 0.0593807, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3387, 3388, 3352}

$$-\frac{1}{90}\pi^3 b^6 \text{FresnelC}(bx)+\frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{90x}-\frac{b\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{30x^5}-\frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{90x^3}-\frac{S(bx)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^7,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(90*x^3) - (b^6*Pi^3*FresnelC[b*x])/90 - Fres
nelS[b*x]/(6*x^6) - (b*Sin[(b^2*Pi*x^2)/2])/(30*x^5) + (b^5*Pi^2*Sin[(b^2*P
i*x^2)/2])/(90*x)
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)}{x^7}\,dx &= -\frac{S(bx)}{6x^6} + \frac{1}{6}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx \\ &= -\frac{S(bx)}{6x^6} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{1}{30}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx \\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3} - \frac{S(bx)}{6x^6} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} - \frac{1}{90}\left(b^5\pi^2\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3} - \frac{S(bx)}{6x^6} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x} - \frac{1}{90}\left(b^7\pi^3\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)dx \\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3} - \frac{1}{90}b^6\pi^3 C(bx) - \frac{S(bx)}{6x^6} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x}\end{aligned}$$

Mathematica [A] time = 0.0574427, size = 76, normalized size = 0.81

$$\frac{1}{90}\left(-\pi^3 b^6 \text{FresnelC}(bx) + \frac{b\left(\pi^2 b^4 x^4 - 3\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^5} - \frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^3} - \frac{15S(bx)}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^7,x]
```

```
[Out] (-((b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^3) - b^6*Pi^3*FresnelC[b*x] - (15*Fresnel
S[b*x])/x^6 + (b*(-3 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^5)/90
```

Maple [A] time = 0.05, size = 86, normalized size = 0.9

$$b^6\left(-\frac{\text{FresnelS}(bx)}{6b^6x^6} - \frac{1}{30b^5x^5}\sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{30}\left(-\frac{1}{3x^3b^3}\cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{3}\left(-\frac{1}{bx}\sin\left(\frac{b^2\pi x^2}{2}\right) + \pi\,\text{FresnelC}(bx)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^7,x)
```

```
[Out] b^6*(-1/6*FresnelS(b*x)/b^6/x^6-1/30*sin(1/2*b^2*Pi*x^2)/b^5/x^5+1/30*Pi*(-
1/3/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/3*Pi*(-sin(1/2*b^2*Pi*x^2)/b/x+Pi*Fresnel
C(b*x))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^7, x)
```

Sympy [A] time = 1.52855, size = 56, normalized size = 0.6

$$\frac{\pi b^3\Gamma\left(-\frac{3}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{1}{4},\frac{3}{2},\frac{7}{4}\end{matrix}\middle| -\frac{\pi^2b^4x^4}{16}\right)}{32x^3\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**7,x)
```

```
[Out] pi*b**3*gamma(-3/4)*gamma(3/4)*hyper((-3/4, 3/4), (1/4, 3/2, 7/4), -pi**2*b
**4*x**4/16)/(32*x**3*gamma(1/4)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^7, x)
```

3.16 $\int \frac{S(bx)}{x^8} \, dx$

Optimal. Leaf size=102

$$-\frac{1}{672}\pi^3 b^7 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right)+\frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{336 x^2}-\frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{42 x^6}-\frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{168 x^4}-\frac{S(bx)}{7 x^7}$$

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(168*x^4) - (b^7*Pi^3*CosIntegral[(b^2*Pi*x^2
)/2])/672 - FresnelS[b*x]/(7*x^7) - (b*Sin[(b^2*Pi*x^2)/2])/(42*x^6) + (b^5
*Pi^2*Sin[(b^2*Pi*x^2)/2])/(336*x^2)
```

Rubi [A] time = 0.121533, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3379, 3297, 3302}

$$-\frac{1}{672}\pi^3 b^7 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right)+\frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{336 x^2}-\frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{42 x^6}-\frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{168 x^4}-\frac{S(bx)}{7 x^7}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^8,x]
```


```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(168*x^4) - (b^7*Pi^3*CosIntegral[(b^2*Pi*x^2
)/2])/672 - FresnelS[b*x]/(7*x^7) - (b*Sin[(b^2*Pi*x^2)/2])/(42*x^6) + (b^5
*Pi^2*Sin[(b^2*Pi*x^2)/2])/(336*x^2)
```


Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{S(bx)}{x^8}\,dx &= -\frac{S(bx)}{7x^7} + \frac{1}{7}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx \\
&= -\frac{S(bx)}{7x^7} + \frac{1}{14}b\,\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx, x, x^2\right) \\
&= -\frac{S(bx)}{7x^7} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx, x, x^2\right) \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{S(bx)}{7x^7} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} - \frac{1}{336}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{S(bx)}{7x^7} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{1}{672}\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\right. \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{1}{672}b^7\pi^3\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{S(bx)}{7x^7} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}
\end{aligned}
$$

Mathematica [A] time = 0.0618826, size = 85, normalized size = 0.83

$$
\frac{1}{672}\left(-\pi^3 b^7\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right) + \frac{2b\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^6} - \frac{4\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^4} - \frac{96S(bx)}{x^7}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^8,x]
```

```
[Out] ((-4*b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^4 - b^7*Pi^3*CosIntegral[(b^2*Pi*x^2)/2]
 - (96*FresnelS[b*x])/x^7 + (2*b*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x
^6)/672
```

Maple [A] time = 0.047, size = 93, normalized size = 0.9

$$
b^7\left(-\frac{\text{FresnelS}\,(bx)}{7\,b^7x^7} - \frac{1}{42\,b^6x^6}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\pi}{42}\left(-\frac{1}{4\,x^4b^4}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{\pi}{4}\left(-\frac{1}{2\,b^2x^2}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\pi}{4}\text{Ci}\left(\frac{b^2\pi\,x^2}{2}\right)\right)\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^8,x)
```

```
[Out] b^7*(-1/7*FresnelS(b*x)/b^7/x^7-1/42*sin(1/2*b^2*Pi*x^2)/b^6/x^6+1/42*Pi*(-
1/4/b^4/x^4*cos(1/2*b^2*Pi*x^2)-1/4*Pi*(-1/2*sin(1/2*b^2*Pi*x^2)/b^2/x^2+1/
4*Pi*Ci(1/2*b^2*Pi*x^2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\text{fresnels}\,(bx)}{x^8}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^8, x)
```

Sympy [A] time = 2.94366, size = 68, normalized size = 0.67

$$\frac{\pi^5 b^{11} x^4 \Gamma\left(\frac{11}{4}\right) {}_3F_4\left(\begin{matrix} 1,1,\frac{11}{4} \\ 2,3,\frac{7}{2},\frac{15}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{61440\Gamma\left(\frac{15}{4}\right)} - \frac{\pi^3 b^7 \log\left(b^4 x^4\right)}{1344} - \frac{\pi b^3}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**8,x)
```

```
[Out] pi**5*b**11*x**4*gamma(11/4)*hyper((1, 1, 11/4), (2, 3, 7/2, 15/4), -pi**2*
b**4*x**4/16)/(61440*gamma(15/4)) - pi**3*b**7*log(b**4*x**4)/1344 - pi*b**
3/(24*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^8, x)
```

3.17 $\int \frac{S(bx)}{x^9}\,dx$

Optimal. Leaf size=119

$$\frac{1}{840}\pi^4b^8S(bx)+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{840x^3}-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{56x^7}+\frac{\pi^3b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{280x^5}-\frac{S(bx)}{8x^8}$$

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(280*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2])/(8
40*x) + (b^8*Pi^4*FresnelS[b*x])/840 - FresnelS[b*x]/(8*x^8) - (b*Sin[(b^2*
Pi*x^2)/2])/(56*x^7) + (b^5*Pi^2*Sin[(b^2*Pi*x^2)/2])/(840*x^3)
```

Rubi [A] time = 0.0810103, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3387, 3388, 3351}

$$\frac{1}{840}\pi^4b^8S(bx)+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{840x^3}-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{56x^7}+\frac{\pi^3b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{280x^5}-\frac{S(bx)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^9,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(280*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2])/(8
40*x) + (b^8*Pi^4*FresnelS[b*x])/840 - FresnelS[b*x]/(8*x^8) - (b*Sin[(b^2*
Pi*x^2)/2])/(56*x^7) + (b^5*Pi^2*Sin[(b^2*Pi*x^2)/2])/(840*x^3)
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{S(bx)}{x^9}\,dx &= -\frac{S(bx)}{8x^8} + \frac{1}{8}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx \\
&= -\frac{S(bx)}{8x^8} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{1}{56}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{S(bx)}{8x^8} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} - \frac{1}{280}\left(b^5\pi^2\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{S(bx)}{8x^8} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} - \frac{1}{840}\left(b^7\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x} - \frac{S(bx)}{8x^8} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} + \frac{1}{840}\left(b^9\pi\right. \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x} + \frac{1}{840}b^8\pi^4S(bx) - \frac{S(bx)}{8x^8} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\right.}{840x^3}
\end{aligned}
$$

Mathematica [A] time = 0.0552173, size = 84, normalized size = 0.71

$$\frac{\left(\pi^4b^8x^8-105\right)S(bx)+bx\left(\pi^2b^4x^4-15\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)+\pi b^3x^3\left(\pi^2b^4x^4-3\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^9,x]
```

```
[Out] (b^3*Pi*x^3*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)
*FresnelS[b*x] + b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)
```

Maple [A] time = 0.047, size = 109, normalized size = 0.9

$$b^8\left(-\frac{\mathrm{FresnelS}\,(bx)}{8\,b^8x^8}-\frac{1}{56\,b^7x^7}\sin\left(\frac{b^2\pi\,x^2}{2}\right)+\frac{\pi}{56}\left(-\frac{1}{5\,b^5x^5}\cos\left(\frac{b^2\pi\,x^2}{2}\right)-\frac{\pi}{5}\left(-\frac{1}{3\,x^3b^3}\sin\left(\frac{b^2\pi\,x^2}{2}\right)+\frac{\pi}{3}\left(-\frac{1}{bx}\cos\left(\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^9,x)
```

```
[Out] b^8*(-1/8*FresnelS(b*x)/b^8/x^8-1/56*sin(1/2*b^2*Pi*x^2)/b^7/x^7+1/56*Pi*(-
1/5/b^5/x^5*cos(1/2*b^2*Pi*x^2)-1/5*Pi*(-1/3*sin(1/2*b^2*Pi*x^2)/b^3/x^3+1/
3*Pi*(-1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*FresnelS(b*x)))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^9, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^9,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^9, x)
```

Sympy [A] time = 3.70966, size = 185, normalized size = 1.55

$$\frac{\pi^4 b^8 S\,(bx)\,\Gamma\left(-\frac{5}{4}\right)}{3584\Gamma\left(\frac{7}{4}\right)}+\frac{\pi^3 b^7\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x\Gamma\left(\frac{7}{4}\right)}+\frac{\pi^2 b^5\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^3\Gamma\left(\frac{7}{4}\right)}-\frac{3\pi b^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^5\Gamma\left(\frac{7}{4}\right)}-\frac{15b\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^7\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**9,x)
```

```
[Out] pi**4*b**8*fresnels(b*x)*gamma(-5/4)/(3584*gamma(7/4)) + pi**3*b**7*cos(pi*
b**2*x**2/2)*gamma(-5/4)/(3584*x*gamma(7/4)) + pi**2*b**5*sin(pi*b**2*x**2/
2)*gamma(-5/4)/(3584*x**3*gamma(7/4)) - 3*pi*b**3*cos(pi*b**2*x**2/2)*gamma
(-5/4)/(3584*x**5*gamma(7/4)) - 15*b*sin(pi*b**2*x**2/2)*gamma(-5/4)/(3584*
x**7*gamma(7/4)) - 15*fresnels(b*x)*gamma(-5/4)/(512*x**8*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^9, x)
```

3.18 $\int \frac{S(bx)}{x^{10}}\,dx$

Optimal. Leaf size=127

$$\frac{\pi^4 b^9 \mathrm{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}+\frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2x^2\right)}{1728x^4}-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{72x^8}+\frac{\pi^3 b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3456x^2}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{432x^6}-\frac{S(bx)}{9x^9}$$

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(432*x^6) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2])/(3
456*x^2) - FresnelS[b*x]/(9*x^9) - (b*Sin[(b^2*Pi*x^2)/2])/(72*x^8) + (b^5*
Pi^2*Sin[(b^2*Pi*x^2)/2])/(1728*x^4) + (b^9*Pi^4*SinIntegral[(b^2*Pi*x^2)/2
])/6912
```

Rubi [A] time = 0.152954, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6426, 3379, 3297, 3299}

$$\frac{\pi^4 b^9 \mathrm{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}+\frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2x^2\right)}{1728x^4}-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{72x^8}+\frac{\pi^3 b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3456x^2}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{432x^6}-\frac{S(bx)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^10,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(432*x^6) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2])/(3
456*x^2) - FresnelS[b*x]/(9*x^9) - (b*Sin[(b^2*Pi*x^2)/2])/(72*x^8) + (b^5*
Pi^2*Sin[(b^2*Pi*x^2)/2])/(1728*x^4) + (b^9*Pi^4*SinIntegral[(b^2*Pi*x^2)/2
])/6912
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{S(bx)}{x^{10}}\,dx = -\frac{S(bx)}{9x^9} + \frac{1}{9}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$
$$= -\frac{S(bx)}{9x^9} + \frac{1}{18}b\,\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{S(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{S(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} - \frac{1}{864}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{S(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} - \frac{\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right)}{3456}$$
$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2} - \frac{S(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{\left(b^9\pi^4\right)\text{Subs}}{}$$
$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2} - \frac{S(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{b^9\pi^4\text{Si}\left(\frac{1}{2}b^2\right.}{6912}$$

Mathematica [A] time = 0.165594, size = 96, normalized size = 0.76

$$\frac{\pi^4 b^9\,\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) + \frac{4b\left(\pi^2b^4x^4-24\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^8} + \frac{2\pi b^3\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^6} - \frac{768S(bx)}{x^9}}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^10,x]
```

```
[Out] ((2*b^3*Pi*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^6 - (768*FresnelS[b*x
])/x^9 + (4*b*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^8 + b^9*Pi^4*SinI
ntegral[(b^2*Pi*x^2)/2])/6912
```

Maple [A] time = 0.047, size = 115, normalized size = 0.9

$$b^9\left(-\frac{\text{FresnelS}\,(bx)}{9\,b^9x^9} - \frac{1}{72\,b^8x^8}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\pi}{72}\left(-\frac{1}{6\,b^6x^6}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{\pi}{6}\left(-\frac{1}{4\,x^4b^4}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\pi}{4}\left(-\frac{1}{2\,b^2x^2}\cos\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^10,x)
```

```
[Out] b^9*(-1/9*FresnelS(b*x)/b^9/x^9-1/72*sin(1/2*b^2*Pi*x^2)/b^8/x^8+1/72*Pi*(-
1/6/b^6/x^6*cos(1/2*b^2*Pi*x^2)-1/6*Pi*(-1/4*sin(1/2*b^2*Pi*x^2)/b^4/x^4+1/
4*Pi*(-1/2/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/4*Pi*Si(1/2*b^2*Pi*x^2)))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^10, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^10, x)
```

Sympy [A] time = 5.55253, size = 48, normalized size = 0.38

$$-\frac{\pi b^3\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\ -\frac{1}{2},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{48x^6\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**10,x)
```

```
[Out] -pi*b**3*gamma(3/4)*hyper((-3/2, 3/4), (-1/2, 3/2, 7/4), -pi**2*b**4*x**4/1
6)/(48*x**6*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^10, x)
```

3.19 $\int (c + dx)^3 S(a + bx)\,dx$

Optimal. Leaf size=296

$$-\frac{2d^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2b^4}+\frac{d^2(a+bx)^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{3d(bc-ad)^2\text{FresnelC}(a+bx)}{2\pi b^4}-\frac{(bc-ad)^4}{4b}$$

```
[Out] ((b*c - a*d)^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + (3*d*(b*c - a*d)^2*(a +
b*x)*Cos[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) + (d^2*(b*c - a*d)*(a + b*x)^2*Cos
[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + (d^3*(a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/
(4*b^4*Pi) - (3*d*(b*c - a*d)^2*FresnelC[a + b*x])/(2*b^4*Pi) - ((b*c - a*d
)^4*FresnelS[a + b*x])/(4*b^4*d) + (3*d^3*FresnelS[a + b*x])/(4*b^4*Pi^2) +
 ((c + d*x)^4*FresnelS[a + b*x])/(4*d) - (2*d^2*(b*c - a*d)*Sin[(Pi*(a + b*
x)^2)/2])/(b^4*Pi^2) - (3*d^3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^
2)
```

Rubi [A] time = 0.400987, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637, 3386}

$$-\frac{2d^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2b^4}+\frac{d^2(a+bx)^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{3d(bc-ad)^2\text{FresnelC}(a+bx)}{2\pi b^4}-\frac{(bc-ad)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*FresnelS[a + b*x],x]
```

```
[Out] ((b*c - a*d)^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + (3*d*(b*c - a*d)^2*(a +
b*x)*Cos[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) + (d^2*(b*c - a*d)*(a + b*x)^2*Cos
[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + (d^3*(a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/
(4*b^4*Pi) - (3*d*(b*c - a*d)^2*FresnelC[a + b*x])/(2*b^4*Pi) - ((b*c - a*d
)^4*FresnelS[a + b*x])/(4*b^4*d) + (3*d^3*FresnelS[a + b*x])/(4*b^4*Pi^2) +
 ((c + d*x)^4*FresnelS[a + b*x])/(4*d) - (2*d^2*(b*c - a*d)*Sin[(Pi*(a + b*
x)^2)/2])/(b^4*Pi^2) - (3*d^3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^
2)
```

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\int (c+dx)^3 S(a+bx)\,dx = \frac{(c+dx)^4 S(a+bx)}{4d} - \frac{b\int (c+dx)^4 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{4d}$$
$$= \frac{(c+dx)^4 S(a+bx)}{4d} - \frac{\text{Subst}\left(\int\left(b^4c^4\left(1+\frac{ad\left(-4b^3c^3+6ab^2c^2d-4a^2bcd^2+a^3d^3\right)}{b^4c^4}\right)\sin\left(\frac{\pi x^2}{2}\right)+4b^3c^3d\left(1\right.\right.\right.}{}$$
$$= \frac{(c+dx)^4 S(a+bx)}{4d} - \frac{d^3\,\text{Subst}\left(\int x^4\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{4b^4} - \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int x^3\sin\right.}{b^4}$$
$$= \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^3(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{(bc-ad)^4 S(a+bx}{4b^4d}$$
$$= \frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^2(bc-ad)(a+bx)^2}{b^4}$$
$$= \frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^2(bc-ad)(a+bx)^2}{b^4}$$

Mathematica [A] time = 0.813954, size = 424, normalized size = 1.43

$$\frac{S(a+bx)\left(6\pi^2b^2c^2d\left(b^2x^2-a^2\right)+4\pi^2bcd^2\left(a^3+b^3x^3\right)+d^3\left(-\pi^2a^4+\pi^2b^4x^4+3\right)+4\pi^2b^3c^3(a+bx)\right)+4\pi a^2bcd^2\cos\left(\frac{1}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*FresnelS[a + b*x],x]
```

```
[Out] (4*b^3*c^3*Pi*Cos[(Pi*(a + b*x)^2)/2] - 6*a*b^2*c^2*d*Pi*Cos[(Pi*(a + b*x)^
2)/2] + 4*a^2*b*c*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a^3*d^3*Pi*Cos[(Pi*(a +
b*x)^2)/2] + 6*b^3*c^2*d*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - 4*a*b^2*c*d^2*Pi*x*
Cos[(Pi*(a + b*x)^2)/2] + a^2*b*d^3*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + 4*b^3*c*
d^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] - a*b^2*d^3*Pi*x^2*Cos[(Pi*(a + b*x)^2)/
2] + b^3*d^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2] - 6*d*(b*c - a*d)^2*Pi*FresnelC
[a + b*x] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*Pi^2*(-a^2 + b^2*x^2) +
 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2 + b^4*Pi^2*x^4))*Fresne
lS[a + b*x] - 8*b*c*d^2*Sin[(Pi*(a + b*x)^2)/2] + 5*a*d^3*Sin[(Pi*(a + b*x)
^2)/2] - 3*b*d^3*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Maple [A] time = 0.054, size = 400, normalized size = 1.4

$$\frac{1}{b}\left(\frac{\text{FresnelS}\,(bx+a)\,(d\,(bx+a)-ad+bc)^4}{4\,db^3} - \frac{1}{4\,db^3}\left(-\frac{d^4\,(bx+a)^3}{\pi}\cos\left(\frac{\pi\,(bx+a)^2}{2}\right)+3\,\frac{d^4}{\pi}\left(\frac{(bx+a)\sin\left(1/2\,\pi\,(bx+\right.}{\pi}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*FresnelS(b*x+a),x)
```

```
[Out] 1/b*(1/4*FresnelS(b*x+a)*(d*(b*x+a)-a*d+b*c)^4/b^3/d-1/4/b^3/d*(-d^4/Pi*(b*
x+a)^3*cos(1/2*Pi*(b*x+a)^2)+3*d^4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-1
/Pi*FresnelS(b*x+a))-(-4*a*d^4+4*b*c*d^3)/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2
)+2*(-4*a*d^4+4*b*c*d^3)/Pi^2*sin(1/2*Pi*(b*x+a)^2)-(6*a^2*d^4-12*a*b*c*d^3
+6*b^2*c^2*d^2)/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+(6*a^2*d^4-12*a*b*c*d^3+6*
b^2*c^2*d^2)/Pi*FresnelC(b*x+a)-(-4*a^3*d^4+12*a^2*b*c*d^3-12*a*b^2*c^2*d^2
```

```
+4*b^3*c^3*d)/Pi*cos(1/2*Pi*(b*x+a)^2)+a^4*d^4*FresnelS(b*x+a)-4*a^3*b*c*d^
3*FresnelS(b*x+a)+6*a^2*b^2*c^2*d^2*FresnelS(b*x+a)-4*a*b^3*c^3*d*FresnelS(
b*x+a)+b^4*c^4*FresnelS(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \text{fresnels}\,(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^3*fresnels(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\text{fresnels}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*fresnels(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3\,S\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*fresnels(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*fresnels(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \text{fresnels}\,(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*fresnels(b*x + a), x)
```

3.20 $\int (c+dx)^2 S(a+bx)\,dx$

Optimal. Leaf size=193

$$-\frac{d(bc-ad)\text{FresnelC}(a+bx)}{\pi b^3}-\frac{(bc-ad)^3 S(a+bx)}{3b^3 d}+\frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{d(a+bx)(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx\right.}{\pi b^3}$$

```
[Out] ((b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d*(b*c - a*d)*(a + b*x)
*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/
2])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x])/(b^3*Pi) - ((b*c - a*d)^
3*FresnelS[a + b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelS[a + b*x])/(3*d) - (2
*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rubi [A] time = 0.225743, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637}

$$-\frac{d(bc-ad)\text{FresnelC}(a+bx)}{\pi b^3}-\frac{(bc-ad)^3 S(a+bx)}{3b^3 d}+\frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{d(a+bx)(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx\right.}{\pi b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelS[a + b*x],x]
```


```
[Out] ((b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d*(b*c - a*d)*(a + b*x)
*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/
2])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x])/(b^3*Pi) - ((b*c - a*d)^
3*FresnelS[a + b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelS[a + b*x])/(3*d) - (2
*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```


Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2 S(a+bx)\,dx = \frac{(c+dx)^3 S(a+bx)}{3d} - \frac{b\int (c+dx)^3 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{3d}$$

$$= \frac{(c+dx)^3 S(a+bx)}{3d} - \frac{\text{Subst}\left(\int\left(b^3c^3\left(1-\frac{ad\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3c^3}\right)\sin\left(\frac{\pi x^2}{2}\right)+3b^2c^2d\left(1+\frac{ad(-2\cdots}{b^2\cdots}\right.\right.\right.}{\cdots}$$

$$= \frac{(c+dx)^3 S(a+bx)}{3d} - \frac{d^2\,\text{Subst}\left(\int x^3\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{3b^3} - \frac{(d(bc-ad))\,\text{Subst}\left(\int x^2\sin\cdots\right.}{b^3}$$

$$= \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{(bc-ad)^3 S(a+bx)}{3b^3d} + \frac{(c+dx)^3 S(a+bx)}{3d} - \frac{d^2\,\text{Sub}\cdots}{\cdots}$$

$$= \frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d^2(a+bx)^2\cos\left(\frac{1}{2}\cdots\right.}{3b^3\pi}$$

$$= \frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d^2(a+bx)^2\cos\left(\frac{1}{2}\cdots\right.}{3b^3\pi}$$

Mathematica [A] time = 0.457134, size = 236, normalized size = 1.22

$$\frac{\pi^2 S(a+bx)\left(-3a^2bcd+a^3d^2+3ab^2c^2+b^3x\left(3c^2+3cdx+d^2x^2\right)\right)+\pi a^2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+3\pi b^2c^2\cos\left(\frac{1}{2}\pi(a+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*FresnelS[a + b*x],x]
```

```
[Out] (3*b^2*c^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - 3*a*b*c*d*Pi*Cos[(Pi*(a + b*x)^2)/2
] + a^2*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] + 3*b^2*c*d*Pi*x*Cos[(Pi*(a + b*x)^2
)/2] - a*b*d^2*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*d^2*Pi*x^2*Cos[(Pi*(a + b
*x)^2)/2] + 3*d*(-(b*c) + a*d)*Pi*FresnelC[a + b*x] + Pi^2*(3*a*b^2*c^2 - 3
*a^2*b*c*d + a^3*d^2 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelS[a + b*x]
 - 2*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Maple [A] time = 0.055, size = 251, normalized size = 1.3

$$\frac{1}{b}\left(\frac{\mathrm{FresnelS}\,(bx+a)\,(d\,(bx+a)-ad+bc)^3}{3\,db^2}-\frac{1}{3\,db^2}\left(-\frac{d^3\,(bx+a)^2}{\pi}\cos\left(\frac{\pi\,(bx+a)^2}{2}\right)+2\,\frac{d^3\sin\left(1/2\,\pi\,(bx+a)^2\right)}{\pi^2}-\frac{(-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelS(b*x+a),x)
```

```
[Out] 1/b*(1/3*FresnelS(b*x+a)*(d*(b*x+a)-a*d+b*c)^3/b^2/d-1/3/b^2/d*(-d^3/Pi*(b*
x+a)^2*cos(1/2*Pi*(b*x+a)^2)+2*d^3/Pi^2*sin(1/2*Pi*(b*x+a)^2)-(-3*a*d^3+3*b
*c*d^2)/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+(-3*a*d^3+3*b*c*d^2)/Pi*FresnelC(b
*x+a)-(3*a^2*d^3-6*a*b*c*d^2+3*b^2*c^2*d)/Pi*cos(1/2*Pi*(b*x+a)^2)-a^3*d^3*
FresnelS(b*x+a)+3*a^2*b*c*d^2*FresnelS(b*x+a)-3*a*b^2*c^2*d*FresnelS(b*x+a)
+b^3*c^3*FresnelS(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \mathrm{fresnels}\,(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*fresnels(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d^2x^2+2\,cdx+c^2\right)\mathrm{fresnels}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnels(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2\,S\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnels(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*fresnels(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \text{fresnels}\,(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*fresnels(b*x + a), x)
```

3.21 $\int (c+dx)S(a+bx)\,dx$

Optimal. Leaf size=121

$$-\frac{(bc-ad)^2S(a+bx)}{2b^2d}+\frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{d\text{FresnelC}(a+bx)}{2\pi b^2}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{(c+dx)^2S(a}{2d}$$

```
[Out] ((b*c - a*d)*Cos[(Pi*(a + b*x)^2)/2])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a +
b*x)^2)/2])/(2*b^2*Pi) - (d*FresnelC[a + b*x])/(2*b^2*Pi) - ((b*c - a*d)^2*
FresnelS[a + b*x])/(2*b^2*d) + ((c + d*x)^2*FresnelS[a + b*x])/(2*d)
```

Rubi [A] time = 0.1145, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352}

$$-\frac{(bc-ad)^2S(a+bx)}{2b^2d}+\frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{d\text{FresnelC}(a+bx)}{2\pi b^2}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{(c+dx)^2S(a}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelS[a + b*x],x]
```


```
[Out] ((b*c - a*d)*Cos[(Pi*(a + b*x)^2)/2])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a +
b*x)^2)/2])/(2*b^2*Pi) - (d*FresnelC[a + b*x])/(2*b^2*Pi) - ((b*c - a*d)^2*
FresnelS[a + b*x])/(2*b^2*d) + ((c + d*x)^2*FresnelS[a + b*x])/(2*d)
```


Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)S(a+bx)\,dx = \frac{(c+dx)^2S(a+bx)}{2d} - \frac{b\int (c+dx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{2d}$$
$$= \frac{(c+dx)^2S(a+bx)}{2d} - \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\sin\left(\frac{\pi x^2}{2}\right)+2bcd\left(1-\frac{ad}{bc}\right)x\sin\left(\frac{\pi x^2}{2}\right)\right.}{2b^2d}$$
$$= \frac{(c+dx)^2S(a+bx)}{2d} - \frac{d\,\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{2b^2} - \frac{(bc-ad)\,\text{Subst}\left(\int x\sin\left(\frac{\pi x^2}{2}\right.\right.}{b^2}$$
$$= \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{(bc-ad)^2S(a+bx)}{2b^2d} + \frac{(c+dx)^2S(a+bx)}{2d} - \frac{(bc-ad)\,\text{Subst}(}{}$$
$$= \frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{dC(a+bx)}{2b^2\pi} - \frac{(bc-ad)^2S(a+}{2b^2d}$$

Mathematica [A] time = 0.208895, size = 61, normalized size = 0.5

$$\frac{(-ad+2bc+bdx)\left(\pi(a+bx)S(a+bx)+\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\right)-d\text{FresnelC}(a+bx)}{2\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*FresnelS[a + b*x],x]
```

```
[Out] (-(d*FresnelC[a + b*x]) + (2*b*c - a*d + b*d*x)*(Cos[(Pi*(a + b*x)^2)/2] +
Pi*(a + b*x)*FresnelS[a + b*x]))/(2*b^2*Pi)
```

Maple [A] time = 0.051, size = 108, normalized size = 0.9

$$\frac{1}{b}\left(\frac{\text{FresnelS}(bx+a)}{b}\left(\frac{d(bx+a)^2}{2}-ad(bx+a)+bc(bx+a)\right)-\frac{1}{2b}\left(-\frac{d(bx+a)}{\pi}\cos\left(\frac{\pi(bx+a)^2}{2}\right)+\frac{d\text{FresnelC}(bx}{\pi}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelS(b*x+a),x)
```

```
[Out] 1/b*(FresnelS(b*x+a)/b*(1/2*d*(b*x+a)^2-a*d*(b*x+a)+b*c*(b*x+a))-1/2/b*(-d/
Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+d/Pi*FresnelC(b*x+a)-(-2*a*d+2*b*c)/Pi*cos
(1/2*Pi*(b*x+a)^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\text{fresnels}\,(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*fresnels(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,((dx+c)\text{fresnels}\,(bx+a)\,,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*fresnels(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\,S\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a),x)
```

```
[Out] Integral((c + d*x)*fresnels(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\text{fresnels}\,(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnels(b*x + a), x)
```

3.22 $\int S(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx)S(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

Rubi [A] time = 0.0063385, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6418}

$$\frac{(a+bx)S(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[a + b*x],x]
```

```
[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

Rule 6418

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelS[a +
 b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int S(a+bx)\,dx=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)S(a+bx)}{b}$$

Mathematica [B] time = 0.0286256, size = 89, normalized size = 2.47

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+\frac{\cos\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(a+bx)+\frac{aS(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[a + b*x],x]
```

```
[Out] (Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a + b
*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

Maple [A] time = 0.046, size = 33, normalized size = 0.9

$$\frac{1}{b}\left((bx+a)\,\text{FresnelS}\,(bx+a)+\frac{1}{\pi}\cos\left(\frac{\pi\,(bx+a)^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*FresnelS(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\,(bx + a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,(\text{fresnels}\,(bx + a)\,, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int S\,(a + bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x)
```

```
[Out] Integral(fresnels(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\,(bx + a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a), x)
```

3.23 $\int \frac{S(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{S(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[FresnelS[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.0146781, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{S(a+bx)}{c+dx} \, dx = \int \frac{S(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 0.0248727, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[FresnelS[a + b*x]/(c + d*x), x]
```

Maple [A] time = 0.369, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx+a)}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/(d*x+c),x)
```

```
[Out] int(FresnelS(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S\,(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(fresnels(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)/(d*x + c), x)
```

3.24 $\int \frac{S(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{S(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[FresnelS[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.0142623, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{S(a+bx)}{(c+dx)^2} dx = \int \frac{S(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 4.8644, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[FresnelS[a + b*x]/(c + d*x)^2, x]
```

Maple [A] time = 0.413, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(FresnelS(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a)/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S\,(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(fresnels(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)/(d*x + c)^2, x)
```

3.25 $\int x^3 S(a+bx)\,dx$

Optimal. Leaf size=229

$$-\frac{3a^2\text{FresnelC}(a+bx)}{2\pi b^4}-\frac{a^4S(a+bx)}{4b^4}-\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}+\frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3S(a+bx)}{4\pi^2b^4}+\frac{2a\sin}{}$$

```
[Out] -((a^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi)) + (3*a^2*(a + b*x)*Cos[(Pi*(a + b
*x)^2)/2])/(2*b^4*Pi) - (a*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) +
((a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi) - (3*a^2*FresnelC[a + b*x]
)/(2*b^4*Pi) - (a^4*FresnelS[a + b*x])/(4*b^4) + (3*FresnelS[a + b*x])/(4*b
^4*Pi^2) + (x^4*FresnelS[a + b*x])/4 + (2*a*Sin[(Pi*(a + b*x)^2)/2])/(b^4*P
i^2) - (3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Rubi [A] time = 0.184451, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637, 3386}

$$-\frac{3a^2\text{FresnelC}(a+bx)}{2\pi b^4}-\frac{a^4S(a+bx)}{4b^4}-\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}+\frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3S(a+bx)}{4\pi^2b^4}+\frac{2a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[a + b*x],x]
```


```
[Out] -((a^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi)) + (3*a^2*(a + b*x)*Cos[(Pi*(a + b
*x)^2)/2])/(2*b^4*Pi) - (a*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) +
((a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi) - (3*a^2*FresnelC[a + b*x]
)/(2*b^4*Pi) - (a^4*FresnelS[a + b*x])/(4*b^4) + (3*FresnelS[a + b*x])/(4*b
^4*Pi^2) + (x^4*FresnelS[a + b*x])/4 + (2*a*Sin[(Pi*(a + b*x)^2)/2])/(b^4*P
i^2) - (3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```


Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^3 S(a+bx)\,dx &= \frac{1}{4}x^4 S(a+bx) - \frac{1}{4}b\int x^4 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx \\
&= \frac{1}{4}x^4 S(a+bx) - \frac{\text{Subst}\left(\int\left(a^4\sin\left(\frac{\pi x^2}{2}\right) - 4a^3x\sin\left(\frac{\pi x^2}{2}\right) + 6a^2x^2\sin\left(\frac{\pi x^2}{2}\right) - 4ax^3\sin\left(\frac{\pi x^2}{2}\right) + x^4\sin\right.}{4b^4} \\
&= \frac{1}{4}x^4 S(a+bx) - \frac{\text{Subst}\left(\int x^4\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{4b^4} + \frac{a\,\text{Subst}\left(\int x^3\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{b^4} - \frac{(3a^2}{} \\
&= \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{a^4S(a+bx)}{4b^4} + \frac{1}{4}x^4S(a+bx) + \frac{a\,\text{Su}}{} \\
&= -\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{(a+bx)^3}{} \\
&= -\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{(a+bx)^3}{}
\end{aligned}$$

Mathematica [A] time = 0.323819, size = 166, normalized size = 0.72

$$\frac{\left(-\pi^2a^4+\pi^2b^4x^4+3\right)S(a+bx)-6\pi a^2\text{FresnelC}(a+bx)-\pi a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi a^2bx\cos\left(\frac{1}{2}\pi(a+bx)^2\right)-\pi ab^2}{4\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[a + b*x],x]
```

```
[Out] (-(a^3*Pi*Cos[(Pi*(a + b*x)^2)/2]) + a^2*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - a
*b^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] + b^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2] -
6*a^2*Pi*FresnelC[a + b*x] + (3 - a^4*Pi^2 + b^4*Pi^2*x^4)*FresnelS[a + b*x
] + 5*a*Sin[(Pi*(a + b*x)^2)/2] - 3*b*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^
2)
```

Maple [A] time = 0.052, size = 189, normalized size = 0.8

$$\frac{1}{b^4}\left(\frac{\text{FresnelS}\,(bx+a)\,b^4x^4}{4}+\frac{(bx+a)^3}{4\,\pi}\cos\left(\frac{\pi\,(bx+a)^2}{2}\right)-\frac{3}{4\,\pi}\left(\frac{bx+a}{\pi}\sin\left(\frac{\pi\,(bx+a)^2}{2}\right)-\frac{\text{FresnelS}\,(bx+a)}{\pi}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*FresnelS(b*x+a)*b^4*x^4+1/4/Pi*(b*x+a)^3*cos(1/2*Pi*(b*x+a)^2)-3
/4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-1/Pi*FresnelS(b*x+a))-a/Pi*(b*x+a
)^2*cos(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*sin(1/2*Pi*(b*x+a)^2)+3/2*a^2/Pi*(b*x+a)
*cos(1/2*Pi*(b*x+a)^2)-3/2*a^2/Pi*FresnelC(b*x+a)-a^3/Pi*cos(1/2*Pi*(b*x+a)
^2)-1/4*a^4*FresnelS(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\text{fresnels}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnels(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3\text{fresnels}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnels(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 S(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x+a),x)
```

```
[Out] Integral(x**3*fresnels(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnels}(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnels(b*x + a), x)
```

3.26 $\int x^2 S(a+bx)\,dx$

Optimal. Leaf size=147

$$\frac{a^3 S(a+bx)}{3b^3}+\frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{a\text{FresnelC}(a+bx)}{\pi b^3}-\frac{2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2 b^3}-\frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{(a$$

```
[Out] (a^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (a*(a + b*x)*Cos[(Pi*(a + b*x)^2)/
2])/(b^3*Pi) + ((a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi) + (a*Fresne
lC[a + b*x])/(b^3*Pi) + (a^3*FresnelS[a + b*x])/(3*b^3) + (x^3*FresnelS[a +
 b*x])/3 - (2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rubi [A] time = 0.128683, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637}

$$\frac{a^3 S(a+bx)}{3b^3}+\frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{a\text{FresnelC}(a+bx)}{\pi b^3}-\frac{2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2 b^3}-\frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{(a$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[a + b*x],x]
```


```
[Out] (a^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (a*(a + b*x)*Cos[(Pi*(a + b*x)^2)/
2])/(b^3*Pi) + ((a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi) + (a*Fresne
lC[a + b*x])/(b^3*Pi) + (a^3*FresnelS[a + b*x])/(3*b^3) + (x^3*FresnelS[a +
 b*x])/3 - (2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```


Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2 S(a+bx)\,dx &= \frac{1}{3}x^3 S(a+bx) - \frac{1}{3}b\int x^3 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx\\
&= \frac{1}{3}x^3 S(a+bx) - \frac{\text{Subst}\left(\int\left(-a^3\sin\left(\frac{\pi x^2}{2}\right)+3a^2x\sin\left(\frac{\pi x^2}{2}\right)-3ax^2\sin\left(\frac{\pi x^2}{2}\right)+x^3\sin\left(\frac{\pi x^2}{2}\right)\right)dx,x,a+\right.}{3b^3}\\
&= \frac{1}{3}x^3 S(a+bx) - \frac{\text{Subst}\left(\int x^3\sin\left(\frac{\pi x^2}{2}\right)dx,x,a+bx\right)}{3b^3} + \frac{a\,\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)dx,x,a+bx\right)}{b^3} - \frac{a^2\,\text{S}}{}\\
&= -\frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{a^3S(a+bx)}{3b^3} + \frac{1}{3}x^3S(a+bx) - \frac{\text{Subst}\left(\int x\sin\left(\frac{\pi x}{2}\right)dx,x,(a+bx)^2\right)}{6b^3}\\
&= \frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} + \frac{aC(a+bx)}{b^3\pi} +\\
&= \frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} + \frac{aC(a+bx)}{b^3\pi} +\end{aligned}$$

Mathematica [A] time = 0.23802, size = 115, normalized size = 0.78

$$\frac{\pi^2\left(a^3+b^3x^3\right)S(a+bx)+\pi a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi b^2x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+3\pi a\text{FresnelC}(a+bx)-2\sin\left(\frac{1}{2}\pi(a+bx\right.}{3\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelS[a + b*x],x]
```

```
[Out] (a^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*Pi
*x^2*Cos[(Pi*(a + b*x)^2)/2] + 3*a*Pi*FresnelC[a + b*x] + Pi^2*(a^3 + b^3*x
^3)*FresnelS[a + b*x] - 2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Maple [A] time = 0.05, size = 121, normalized size = 0.8

$$\frac{1}{b^3}\left(\frac{b^3x^3\text{FresnelS}(bx+a)}{3}+\frac{(bx+a)^2}{3\pi}\cos\left(\frac{\pi\,(bx+a)^2}{2}\right)-\frac{2}{3\pi^2}\sin\left(\frac{\pi\,(bx+a)^2}{2}\right)-\frac{a\,(bx+a)}{\pi}\cos\left(\frac{\pi\,(bx+a)^2}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelS(b*x+a)+1/3/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)-2
/3/Pi^2*sin(1/2*Pi*(b*x+a)^2)-a/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+a/Pi*Fresn
elC(b*x+a)+a^2/Pi*cos(1/2*Pi*(b*x+a)^2)+1/3*a^3*FresnelS(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\text{fresnels}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnels(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\text{fresnels}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnels(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2S\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x+a),x)
```

```
[Out] Integral(x**2*fresnels(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}\,(bx + a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnels(b*x + a), x)
```

3.27 $\int xS(a+bx)\,dx$

Optimal. Leaf size=96

$$-\frac{a^2S(a+bx)}{2b^2}-\frac{\text{FresnelC}(a+bx)}{2\pi b^2}-\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2S(a+bx)$$

```
[Out] -((a*Cos[(Pi*(a + b*x)^2)/2])/(b^2*Pi)) + ((a + b*x)*Cos[(Pi*(a + b*x)^2)/2
])/(2*b^2*Pi) - FresnelC[a + b*x]/(2*b^2*Pi) - (a^2*FresnelS[a + b*x])/(2*b
^2) + (x^2*FresnelS[a + b*x])/2
```

Rubi [A] time = 0.0672627, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352}

$$-\frac{a^2S(a+bx)}{2b^2}-\frac{\text{FresnelC}(a+bx)}{2\pi b^2}-\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2S(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[a + b*x],x]
```

```
[Out] -((a*Cos[(Pi*(a + b*x)^2)/2])/(b^2*Pi)) + ((a + b*x)*Cos[(Pi*(a + b*x)^2)/2
])/(2*b^2*Pi) - FresnelC[a + b*x]/(2*b^2*Pi) - (a^2*FresnelS[a + b*x])/(2*b
^2) + (x^2*FresnelS[a + b*x])/2
```

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int xS(a+bx)\,dx &= \frac{1}{2}x^2S(a+bx) - \frac{1}{2}b\int x^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx \\
&= \frac{1}{2}x^2S(a+bx) - \frac{\text{Subst}\left(\int\left(a^2\sin\left(\frac{\pi x^2}{2}\right) - 2ax\sin\left(\frac{\pi x^2}{2}\right) + x^2\sin\left(\frac{\pi x^2}{2}\right)\right)dx, x, a+bx\right)}{2b^2} \\
&= \frac{1}{2}x^2S(a+bx) - \frac{\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{2b^2} + \frac{a\,\text{Subst}\left(\int x\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{b^2} - \frac{a^2\,\text{Sul}}{} \\
&= \frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{a^2S(a+bx)}{2b^2} + \frac{1}{2}x^2S(a+bx) + \frac{a\,\text{Subst}\left(\int\sin\left(\frac{\pi x}{2}\right)dx, x, (a+bx)^2\right)}{2b^2} - \\
&= -\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} + \frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{C(a+bx)}{2b^2\pi} - \frac{a^2S(a+bx)}{2b^2} + \frac{1}{2}x^2S(a+bx)
\end{aligned}$$

Mathematica [A] time = 0.166339, size = 51, normalized size = 0.53

$$-\frac{\text{FresnelC}(a+bx) + (a-bx)\left(\pi(a+bx)S(a+bx) + \cos\left(\frac{1}{2}\pi(a+bx)^2\right)\right)}{2\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelS[a + b*x],x]
```

```
[Out] -(FresnelC[a + b*x] + (a - b*x)*(Cos[(Pi*(a + b*x)^2)/2] + Pi*(a + b*x)*Fre
snelS[a + b*x]))/(2*b^2*Pi)
```

Maple [A] time = 0.051, size = 80, normalized size = 0.8

$$\frac{1}{b^2}\left(\text{FresnelS}(bx+a)\left(\frac{(bx+a)^2}{2} - a(bx+a)\right) + \frac{bx+a}{2\pi}\cos\left(\frac{\pi(bx+a)^2}{2}\right) - \frac{\text{FresnelC}(bx+a)}{2\pi} - \frac{a}{\pi}\cos\left(\frac{\pi(bx+a)^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x+a),x)
```

```
[Out] 1/b^2*(FresnelS(b*x+a)*(1/2*(b*x+a)^2-a*(b*x+a))+1/2/Pi*(b*x+a)*cos(1/2*Pi*
(b*x+a)^2)-1/2/Pi*FresnelC(b*x+a)-a/Pi*cos(1/2*Pi*(b*x+a)^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnels}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnels(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,(x\text{fresnels}\,(bx+a)\,,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*fresnels(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xS\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x+a),x)
```

```
[Out] Integral(x*fresnels(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnels}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*fresnels(b*x + a), x)
```

3.28 $\int S(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx)S(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

Rubi [A] time = 0.0065408, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6418}

$$\frac{(a+bx)S(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[a + b*x],x]
```

```
[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

Rule 6418

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelS[a +
 b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int S(a+bx)\,dx=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)S(a+bx)}{b}$$

Mathematica [B] time = 0.0300116, size = 89, normalized size = 2.47

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+\frac{\cos\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(a+bx)+\frac{aS(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[a + b*x],x]
```

```
[Out] (Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a + b
*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

Maple [A] time = 0.046, size = 33, normalized size = 0.9

$$\frac{1}{b}\left((bx+a)\,\mathrm{FresnelS}\,(bx+a)+\frac{1}{\pi}\cos\left(\frac{\pi\,(bx+a)^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*FresnelS(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,(\text{fresnels}\,(bx+a)\,,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int S\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x)
```

```
[Out] Integral(fresnels(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a), x)
```

3.29 $\int \frac{S(a+bx)}{x} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{S(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[FresnelS[a + b*x]/x, x]
```

Rubi [A] time = 0.0105536, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/x,x]

[Out] Defer[Int][FresnelS[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{S(a+bx)}{x} \, dx = \int \frac{S(a+bx)}{x} \, dx$$

Mathematica [A] time = 0.0263527, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/x,x]

[Out] Integrate[FresnelS[a + b*x]/x, x]
```

Maple [A] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx+a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/x,x)

[Out] int(FresnelS(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S\,(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x,x)
```

```
[Out] Integral(fresnels(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)/x, x)
```

3.30 $\int \frac{S(a+bx)}{x^2}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{S(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable[FresnelS[a + b*x]/x^2, x]
```

Rubi [A] time = 0.0112494, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{S(a+bx)}{x^2}\,dx = \int \frac{S(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 3.24649, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/x^2,x]
```

```
[Out] Integrate[FresnelS[a + b*x]/x^2, x]
```

Maple [A] time = 0.1, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/x^2,x)
```

```
[Out] int(FresnelS(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx+a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S\,(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x**2,x)
```

```
[Out] Integral(fresnels(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)/x^2, x)
```

3.31 $\int x^7 S(bx)^2\,dx$

Optimal. Leaf size=253

$$-\frac{7x^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2b^3}+\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^4b^7}+\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}-\frac{35x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^3b^5}-\frac{105S(bx)^2}{8\pi^4b^8}+$$

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) - (35*x^3*Cos[(b^2*Pi*
x^2)/2]*FresnelS[b*x])/(4*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x
])/(4*b*Pi) - (105*FresnelS[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelS[b*x]^2)/8
+ (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^7*Pi^4) - (7*x^5*FresnelS[
b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) -
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Rubi [A] time = 0.428415, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6430, 6454, 6462, 3379, 3309, 30, 3296, 2637, 2634, 6440}

$$-\frac{7x^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2b^3}+\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^4b^7}+\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}-\frac{35x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^3b^5}-\frac{105S(bx)^2}{8\pi^4b^8}+$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelS[b*x]^2,x]
```


```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) - (35*x^3*Cos[(b^2*Pi*
x^2)/2]*FresnelS[b*x])/(4*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x
])/(4*b*Pi) - (105*FresnelS[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelS[b*x]^2)/8
+ (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^7*Pi^4) - (7*x^5*FresnelS[
b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) -
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```


Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int x^7 S(bx)^2\,dx = \frac{1}{8}x^8 S(bx)^2 - \frac{1}{4}b\int x^8 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 - \frac{\int x^7\sin\left(b^2\pi x^2\right)dx}{8\pi} - \frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{4b\pi}$$
$$= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 - \frac{7x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35\int x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{4b^3\pi^2} + \frac{7\int x^5}{}$$
$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 - \frac{7x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{4b^3\pi^2}$$
$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 + \frac{105xS(bx)\sin\left(\frac{1}{2}b^2\right.}{4b^7\pi^4}$$
$$= \frac{7x^6}{48b^2\pi^2} - \frac{41x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}$$
$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right.}{4b\pi}$$
$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right.}{4b\pi}$$

<u>**Mathematica [A]**</u> time = 0.0209369, size = 253, normalized size = 1.

$$-\frac{7x^5 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^4 b^7} + \frac{x^7 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} - \frac{35x^3 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^3 b^5} - \frac{105S(bx)^2}{8\pi^4 b^8} +$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelS[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) - (35*x^3*Cos[(b^2*Pi*
x^2)/2]*FresnelS[b*x])/(4*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x
])/(4*b*Pi) - (105*FresnelS[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelS[b*x]^2)/8
+ (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^7*Pi^4) - (7*x^5*FresnelS[
b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) -
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

<u>**Maple [F]**</u> time = 0.053, size = 0, normalized size = 0.

$$\int x^7\left(\mathrm{FresnelS}\left(bx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)^2,x)
```

```
[Out] int(x^7*FresnelS(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnels}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnels(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^7 \text{fresnels}(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnels(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 S^2(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**7*fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnels}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnels(b*x)^2, x)
```

3.32 $\int x^6 S(bx)^2 \, dx$

Optimal. Leaf size=239

$$\frac{531\text{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}\pi^4b^7}-\frac{12x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{96S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{2x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}-\frac{48x^2S(bx)\cos\left(\frac{1}{2}\pi\right.}{7\pi^3b^5}$$

```
[Out] (-48*x)/(7*b^6*Pi^4) + (6*x^5)/(35*b^2*Pi^2) - (21*x*Cos[b^2*Pi*x^2])/(8*b^
6*Pi^4) + (x^5*Cos[b^2*Pi*x^2])/(14*b^2*Pi^2) + (531*FresnelC[Sqrt[2]*b*x])
/(56*Sqrt[2]*b^7*Pi^4) - (48*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*b^5*
Pi^3) + (2*x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*b*Pi) + (x^7*FresnelS[
b*x]^2)/7 + (96*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (12*x^4*F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2) - (17*x^3*Sin[b^2*Pi*x^2])/(
28*b^4*Pi^3)
```

Rubi [A] time = 0.317892, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6430, 6454, 6462, 3391, 30, 3386, 3385, 3352, 6460, 3357}

$$\frac{531\text{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}\pi^4b^7}-\frac{12x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{96S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{2x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}-\frac{48x^2S(bx)\cos\left(\frac{1}{2}\pi\right.}{7\pi^3b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelS[b*x]^2,x]
```


```
[Out] (-48*x)/(7*b^6*Pi^4) + (6*x^5)/(35*b^2*Pi^2) - (21*x*Cos[b^2*Pi*x^2])/(8*b^
6*Pi^4) + (x^5*Cos[b^2*Pi*x^2])/(14*b^2*Pi^2) + (531*FresnelC[Sqrt[2]*b*x])
/(56*Sqrt[2]*b^7*Pi^4) - (48*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*b^5*
Pi^3) + (2*x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*b*Pi) + (x^7*FresnelS[
b*x]^2)/7 + (96*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (12*x^4*F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2) - (17*x^3*Sin[b^2*Pi*x^2])/(
28*b^4*Pi^3)
```


Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\int x^6 S(bx)^2\,dx = \frac{1}{7}x^7S(bx)^2 - \frac{1}{7}(2b)\int x^7S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b\pi} + \frac{1}{7}x^7S(bx)^2 - \frac{\int x^6\sin\left(b^2\pi x^2\right)dx}{7\pi} - \frac{12\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{7b\pi}$$
$$= \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b\pi} + \frac{1}{7}x^7S(bx)^2 - \frac{12x^4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{48\int x^3S(bx)\sin\left(\right.}{7b^3\pi^2}$$
$$= \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b^5\pi^3} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b\pi} + \frac{1}{7}x^7S(bx)^2 - \frac{12x^4S(bx)\sin\left(\frac{1}{2}b\right.}{7b^3\pi^2}$$
$$= \frac{6x^5}{35b^2\pi^2} - \frac{111x\cos\left(b^2\pi x^2\right)}{56b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b^5\pi^3} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b\pi} +$$
$$= \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{15C\left(\sqrt{2}bx\right)}{56\sqrt{2}b^7\pi^4} + \frac{6\sqrt{2}C\left(\sqrt{2}bx\right)}{7b^7\pi^4} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{7b^5\pi^3}$$
$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{51C\left(\sqrt{2}bx\right)}{56\sqrt{2}b^7\pi^4} + \frac{6\sqrt{2}C\left(\sqrt{2}bx\right)}{7b^7\pi^4} - \frac{48x^2\cos\left(\frac{1}{2}\right.}{7b}$$
$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{51C\left(\sqrt{2}bx\right)}{56\sqrt{2}b^7\pi^4} + \frac{30\sqrt{2}C\left(\sqrt{2}bx\right)}{7b^7\pi^4} - \frac{48x^2\cos\left(\right.}{7}$$

Mathematica [A] time = 0.289179, size = 171, normalized size = 0.72

$$\frac{80\pi^4b^7x^7S(bx)^2 + 160S(bx)\left(\pi b^2x^2\left(\pi^2b^4x^4-24\right)\cos\left(\frac{1}{2}\pi b^2x^2\right) - 6\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\right) + 2bx\left(5\left(4\pi^2b^4x^4-147\right.\right.}{560\pi^4b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelS[b*x]^2,x]
```

```
[Out] (2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*b^7*Pi^4*x^7*FresnelS[b*x]^2 + 160
*FresnelS[b*x]*(b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8
 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(5*(-147 + 4*b^4*Pi^2*x^4)*Co
s[b^2*Pi*x^2] - 2*(960 - 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))
/(560*b^7*Pi^4)
```

Maple [A] time = 0.077, size = 324, normalized size = 1.4

$$\frac{1}{b^7}\left(\frac{b^7x^7\left(\mathrm{FresnelS}\left(bx\right)\right)^2}{7} - 2\,\mathrm{FresnelS}\left(bx\right)\left(-1/7\,\frac{b^6x^6\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi} + 6/7\,\frac{1}{\pi}\left(\frac{x^4b^4\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi} - 4\,\frac{1}{\pi}\left(-\frac{b^2x^2\cos}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)^2,x)
```

```
[Out] 1/b^7*(1/7*b^7*x^7*FresnelS(b*x)^2-2*FresnelS(b*x)*(-1/7/Pi*b^6*x^6*cos(1/2
*b^2*Pi*x^2)+6/7/Pi*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*c
os(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*Pi^2*b^5*x^5
-8*b*x)-6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^
2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1
```

```
/2)))-1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(
b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x
*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnels(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^6 \text{fresnels}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^6*fresnels(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 S^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**6*fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnels(b*x)^2, x)
```

3.33 $\int x^5 S(bx)^2\, dx$

Optimal. Leaf size=265

$$-\frac{5ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi^3b^4}+\frac{5ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi^3b^4}+\frac{5\text{Fresn}}{}$$

```
[Out] (5*x^4)/(24*b^2*Pi^2) - (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) + (x^4*Cos[b^2*Pi
*x^2])/(12*b^2*Pi^2) - (5*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^5*Pi^3) +
 (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*b*Pi) + (5*FresnelC[b*x]*Fresne
lS[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x]^2)/6 - (((5*I)/8)*x^2*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^4*Pi^3) + (((5*I)/8)*x^2*H
ypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^4*Pi^3) - (5*x^3*F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) - (7*x^2*Sin[b^2*Pi*x^2])/(1
2*b^4*Pi^3)
```

Rubi [A] time = 0.300067, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {6430, 6454, 6462, 3379, 3309, 30, 3296, 2638, 6446}

$$-\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5\text{FresnelC}(bx)S(bx)}{2\pi^3b^6}-\frac{5x^3S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{x^5S(b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelS[b*x]^2,x]
```


```
[Out] (5*x^4)/(24*b^2*Pi^2) - (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) + (x^4*Cos[b^2*Pi
*x^2])/(12*b^2*Pi^2) - (5*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^5*Pi^3) +
 (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*b*Pi) + (5*FresnelC[b*x]*Fresne
lS[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x]^2)/6 - (((5*I)/8)*x^2*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^4*Pi^3) + (((5*I)/8)*x^2*H
ypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^4*Pi^3) - (5*x^3*F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) - (7*x^2*Sin[b^2*Pi*x^2])/(1
2*b^4*Pi^3)
```


Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
```

```
1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rubi steps

$$\int x^5 S(bx)^2\,dx = \frac{1}{6}x^6 S(bx)^2 - \frac{1}{3}b\int x^6 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{6}x^6 S(bx)^2 - \frac{\int x^5\sin\left(b^2\pi x^2\right)dx}{6\pi} - \frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{3b\pi}$$
$$= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{6}x^6 S(bx)^2 - \frac{5x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{5\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^3\pi^2} + \frac{5\int x^3 \ldots}{\ldots}$$
$$= \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{6}x^6 S(bx)^2 - \frac{5x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} \ldots$$
$$= \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{5C(bx)S(bx)}{2b^6\pi^3} + \frac{1}{6}x^6 S(bx)^2 - \frac{5ix^2 \ldots}{\ldots}$$
$$= \frac{5x^4}{24b^2\pi^2} - \frac{17\cos\left(b^2\pi x^2\right)}{12b^6\pi^4} + \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{5C(b \ldots}{2b \ldots}$$
$$= \frac{5x^4}{24b^2\pi^2} - \frac{11\cos\left(b^2\pi x^2\right)}{6b^6\pi^4} + \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{5C(b \ldots}{2b \ldots}$$

Mathematica [F] time = 0.207061, size = 0, normalized size = 0.

$$\int x^5 S(bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5*FresnelS[b*x]^2,x]
```

```
[Out] Integrate[x^5*FresnelS[b*x]^2, x]
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^5\left(\mathrm{FresnelS}\left(bx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)^2,x)
```

```
[Out] int(x^5*FresnelS(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\,\mathrm{fresnels}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnels(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^5\text{fresnels}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnels(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 S^2\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**5*fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\text{fresnels}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnels(b*x)^2, x)
```

3.34 $\int x^4 S(bx)^2\,dx$

Optimal. Leaf size=177

$$-\frac{8x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{2x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}-\frac{16S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}+\frac{43S\left(\sqrt{2}bx\right)}{20\sqrt{2}\pi^3b^5}+\frac{4x^3}{15\pi^2b^2}-\frac{11x\sin\left(\pi b^2x^2\right)}{20\pi^3b^4}$$

```
[Out] (4*x^3)/(15*b^2*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (16*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(5*b^5*Pi^3) + (2*x^4*Cos[(b^2*Pi*x^2)/2]*Fresnel
S[b*x])/(5*b*Pi) + (x^5*FresnelS[b*x]^2)/5 + (43*FresnelS[Sqrt[2]*b*x])/(20
*Sqrt[2]*b^5*Pi^3) - (8*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
 - (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)
```

Rubi [A] time = 0.187754, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {6430, 6454, 6462, 3391, 30, 3386, 3351, 6452, 3385}

$$-\frac{8x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{2x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}-\frac{16S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}+\frac{43S\left(\sqrt{2}bx\right)}{20\sqrt{2}\pi^3b^5}+\frac{4x^3}{15\pi^2b^2}-\frac{11x\sin\left(\pi b^2x^2\right)}{20\pi^3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelS[b*x]^2,x]
```

```
[Out] (4*x^3)/(15*b^2*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (16*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(5*b^5*Pi^3) + (2*x^4*Cos[(b^2*Pi*x^2)/2]*Fresnel
S[b*x])/(5*b*Pi) + (x^5*FresnelS[b*x]^2)/5 + (43*FresnelS[Sqrt[2]*b*x])/(20
*Sqrt[2]*b^5*Pi^3) - (8*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
 - (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
```

```
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$
\begin{aligned}
\int x^4 S(bx)^2\,dx &= \frac{1}{5}x^5 S(bx)^2 - \frac{1}{5}(2b)\int x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 - \frac{\int x^4\sin\left(b^2\pi x^2\right)dx}{5\pi} - \frac{8\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{5b\pi}\\
&= \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 - \frac{8x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{16\int xS(bx)\sin\left(\cdots\right.}{5b^3\pi\cdots}\\
&= \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 - \frac{8x^2 S(bx)\sin\left(\frac{1}{2}b\cdots\right.}{5b^3\pi^2}\\
&= \frac{4x^3}{15b^2\pi^2} + \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 + \frac{3S\left(\sqrt{\cdots}\right.}{20\sqrt{2}\cdots}\\
&= \frac{4x^3}{15b^2\pi^2} + \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 + \frac{3S\left(\sqrt{\cdots}\right.}{20\sqrt{2}\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.132727, size = 137, normalized size = 0.77

$$
\frac{24\pi^3 b^5 x^5 S(bx)^2 + 48S(bx)\left(\left(\pi^2 b^4 x^4 - 8\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right) - 4\pi b^2 x^2\sin\left(\frac{1}{2}\pi b^2 x^2\right)\right) + 32\pi b^3 x^3 - 66bx\sin\left(\pi b^2 x^2\right) + 12\pi\cdots}{120\pi^3 b^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelS[b*x]^2,x]
```

```
[Out] (32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelS[b
*x]^2 + 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*Pi^
2*x^4)*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin
[b^2*Pi*x^2])/(120*b^5*Pi^3)
```

Maple [A] time = 0.075, size = 208, normalized size = 1.2

$$\frac{1}{b^5}\left(\frac{b^5x^5\,(\mathrm{FresnelS}\,(bx))^2}{5}-2\,\mathrm{FresnelS}\,(bx)\left(-1/5\,\frac{x^4b^4\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi}+4/5\,\frac{1}{\pi}\left(\frac{b^2x^2\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi}+2\,\frac{\cos\left(1/2\,b^2\pi\right.}{\pi^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)^2,x)
```

```
[Out] 1/b^5*(1/5*b^5*x^5*FresnelS(b*x)^2-2*FresnelS(b*x)*(-1/5/Pi*b^4*x^4*cos(1/2
*b^2*Pi*x^2)+4/5/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi
*x^2)))+4/15/Pi^2*b^3*x^3-4/5/Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/
2)*FresnelS(b*x*2^(1/2)))-1/5/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)
*FresnelS(b*x*2^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\mathrm{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnels(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^4\mathrm{fresnels}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnels(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4S^2\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**4*fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnels(b*x)^2, x)
```

3.35 $\int x^3 S(bx)^2 \, dx$

Optimal. Leaf size=140

$$-\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3}+\frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{3S(bx)^2}{4\pi^2b^4}+\frac{3x^2}{8\pi^2b^2}-\frac{\sin\left(\pi b^2x^2\right)}{2\pi^3b^4}+\frac{x^2\cos\left(\pi b^2x^2\right)}{8\pi^2b^2}+\frac{1}{4}x^4S(bx)^2$$

```
[Out] (3*x^2)/(8*b^2*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) + (x^3*Cos[(b^2*P
i*x^2)/2]*FresnelS[b*x])/(2*b*Pi) + (3*FresnelS[b*x]^2)/(4*b^4*Pi^2) + (x^4
*FresnelS[b*x]^2)/4 - (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b^3*Pi^2)
- Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Rubi [A] time = 0.150878, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {6430, 6454, 6462, 3379, 2634, 6440, 30, 3296, 2637}

$$-\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3}+\frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{3S(bx)^2}{4\pi^2b^4}+\frac{3x^2}{8\pi^2b^2}-\frac{\sin\left(\pi b^2x^2\right)}{2\pi^3b^4}+\frac{x^2\cos\left(\pi b^2x^2\right)}{8\pi^2b^2}+\frac{1}{4}x^4S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[b*x]^2,x]
```


```
[Out] (3*x^2)/(8*b^2*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) + (x^3*Cos[(b^2*P
i*x^2)/2]*FresnelS[b*x])/(2*b*Pi) + (3*FresnelS[b*x]^2)/(4*b^4*Pi^2) + (x^4
*FresnelS[b*x]^2)/4 - (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b^3*Pi^2)
- Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```


Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 S(bx)^2\,dx &= \frac{1}{4}x^4 S(bx)^2 - \frac{1}{2}b\int x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{1}{4}x^4 S(bx)^2 - \frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{4\pi} - \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{2b\pi} \\
&= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{1}{4}x^4 S(bx)^2 - \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2} + \frac{3\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b^3\pi^2} + \frac{3\int x\,\mathrm{si}}{} \\
&= \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} + \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{1}{4}x^4 S(bx)^2 - \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2} + \frac{3\,\mathrm{Subst}(\int x\,dx, x,}{2b^4\pi^2} \\
&= \frac{3x^2}{8b^2\pi^2} + \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} + \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{3S(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4 S(bx)^2 - \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2}
\end{aligned}$$

Mathematica [A] time = 0.0055243, size = 140, normalized size = 1.

$$-\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3} + \frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} + \frac{3S(bx)^2}{4\pi^2b^4} + \frac{3x^2}{8\pi^2b^2} - \frac{\sin\left(\pi b^2x^2\right)}{2\pi^3b^4} + \frac{x^2\cos\left(\pi b^2x^2\right)}{8\pi^2b^2} + \frac{1}{4}x^4S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) + (x^3*Cos[(b^2*P
i*x^2)/2]*FresnelS[b*x])/(2*b*Pi) + (3*FresnelS[b*x]^2)/(4*b^4*Pi^2) + (x^4
```

```
*FresnelS[b*x]^2)/4 - (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b^3*Pi^2)
- Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int x^3 (\text{FresnelS}(bx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)^2,x)
```

```
[Out] int(x^3*FresnelS(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnels}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnels(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \text{fresnels}(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnels(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 S^2(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**3*fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnels}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnels(b*x)^2, x)
```

3.36 $\int x^2 S(bx)^2 \, dx$

Optimal. Leaf size=124

$$-\frac{5\text{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}\pi^2b^3}-\frac{4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{2x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+\frac{x\cos\left(\pi b^2x^2\right)}{6\pi^2b^2}+\frac{2x}{3\pi^2b^2}+\frac{1}{3}x^3S(bx)^2$$

```
[Out] (2*x)/(3*b^2*Pi^2) + (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (5*FresnelC[Sqrt[2]
*b*x])/(6*Sqrt[2]*b^3*Pi^2) + (2*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*
b*Pi) + (x^3*FresnelS[b*x]^2)/3 - (4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*
b^3*Pi^2)
```

Rubi [A] time = 0.110876, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6430, 6454, 6460, 3357, 3352, 3385}

$$-\frac{5\text{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}\pi^2b^3}-\frac{4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{2x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+\frac{x\cos\left(\pi b^2x^2\right)}{6\pi^2b^2}+\frac{2x}{3\pi^2b^2}+\frac{1}{3}x^3S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[b*x]^2,x]
```

```
[Out] (2*x)/(3*b^2*Pi^2) + (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (5*FresnelC[Sqrt[2]
*b*x])/(6*Sqrt[2]*b^3*Pi^2) + (2*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*
b*Pi) + (x^3*FresnelS[b*x]^2)/3 - (4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*
b^3*Pi^2)
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\int x^2 S(bx)^2\,dx = \frac{1}{3}x^3 S(bx)^2 - \frac{1}{3}(2b)\int x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$= \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3S(bx)^2 - \frac{\int x^2\sin\left(b^2\pi x^2\right)dx}{3\pi} - \frac{4\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{3b\pi}$$

$$= \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3S(bx)^2 - \frac{4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} - \frac{\int\cos\left(b^2\pi x^2\right)dx}{6b^2\pi^2} +$$

$$= \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{C\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3S(bx)^2 - \frac{4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{4\int\left(\frac{1}{2}\right.}{}$$

$$= \frac{2x}{3b^2\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{C\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3S(bx)^2 - \frac{4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}$$

$$= \frac{2x}{3b^2\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{C\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2} - \frac{\sqrt{2}C\left(\sqrt{2}bx\right)}{3b^3\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3S(bx)^2 - \frac{4S(b}{}$$

Mathematica [A] time = 0.107693, size = 100, normalized size = 0.81

$$\frac{4\pi^2b^3x^3S(bx)^2 + 8S(bx)\left(\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right) - 2\sin\left(\frac{1}{2}\pi b^2x^2\right)\right) + 2bx\left(\cos\left(\pi b^2x^2\right)+4\right) - 5\sqrt{2}\text{FresnelC}\left(\sqrt{2}bx\right)}{12\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelS[b*x]^2,x]
```

```
[Out] (2*b*x*(4 + Cos[b^2*Pi*x^2]) - 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*b^3*Pi^2
*x^3*FresnelS[b*x]^2 + 8*FresnelS[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 2*
Sin[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)
```

Maple [A] time = 0.072, size = 122, normalized size = 1.

$$\frac{1}{b^3}\left(\frac{b^3x^3\left(\text{FresnelS}\,(bx)\right)^2}{3} - 2\,\text{FresnelS}\,(bx)\left(-1/3\,\frac{b^2x^2\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi} + 2/3\,\frac{\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right) + \frac{2\,bx}{3\,\pi^2} - \frac{\sqrt{2}\text{Fresne}}{3\,\pi}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)^2,x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelS(b*x)^2-2*FresnelS(b*x)*(-1/3/Pi*b^2*x^2*cos(1/2
*b^2*Pi*x^2)+2/3/Pi^2*sin(1/2*b^2*Pi*x^2))+2/3/Pi^2*b*x-1/3/Pi^2*2^(1/2)*Fr
```

```
esnelC(b*x*2^(1/2))-1/3/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fres
nelC(b*x*2^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnels(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\text{fresnels}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnels(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2S^2\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**2*fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnels(b*x)^2, x)
```

3.37 $\int xS(bx)^2\,dx$

Optimal. Leaf size=143

$$\frac{ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi}-\frac{ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi}-\frac{\text{Fresnel}}{2}$$

```
[Out] Cos[b^2*Pi*x^2]/(4*b^2*Pi^2) + (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi)
 - (FresnelC[b*x]*FresnelS[b*x])/(2*b^2*Pi) + (x^2*FresnelS[b*x]^2)/2 + ((I
/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/Pi - ((I/8)
*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/Pi
```

Rubi [A] time = 0.0863939, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6430, 6454, 6446, 3379, 2638}

$$\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^2}+\frac{xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+\frac{\cos\left(\pi b^2x\right.}{4\pi^2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[b*x]^2,x]
```

```
[Out] Cos[b^2*Pi*x^2]/(4*b^2*Pi^2) + (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi)
 - (FresnelC[b*x]*FresnelS[b*x])/(2*b^2*Pi) + (x^2*FresnelS[b*x]^2)/2 + ((I
/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/Pi - ((I/8)
*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/Pi
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int xS(bx)^2\,dx &= \frac{1}{2}x^2S(bx)^2 - b\int x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} + \frac{1}{2}x^2S(bx)^2 - \frac{\int x\sin\left(b^2\pi x^2\right)dx}{2\pi} - \frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b\pi} \\
&= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} - \frac{C(bx)S(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)^2 + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib\right.}{8\pi} \\
&= \frac{\cos\left(b^2\pi x^2\right)}{4b^2\pi^2} + \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} - \frac{C(bx)S(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)^2 + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F}{}
\end{aligned}
$$

Mathematica [F] time = 0.171656, size = 0, normalized size = 0.

$$\int xS(bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*FresnelS[b*x]^2,x]
```

```
[Out] Integrate[x*FresnelS[b*x]^2, x]
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x\left(\mathrm{FresnelS}\left(bx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)^2,x)
```

```
[Out] int(x*FresnelS(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{fresnels}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x*fresnels(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\text{fresnels}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x*fresnels(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xS^2\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)**2,x)
```

```
[Out] Integral(x*fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnels}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*fresnels(b*x)^2, x)
```

3.38 $\int S(bx)^2\,dx$

Optimal. Leaf size=55

$$\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)^2-\frac{S\left(\sqrt{2}bx\right)}{\sqrt{2}\pi b}$$

```
[Out] (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - FresnelS
[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)
```

Rubi [A] time = 0.0383974, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6420, 12, 6452, 3351}

$$\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)^2-\frac{S\left(\sqrt{2}bx\right)}{\sqrt{2}\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^2,x]
```

```
[Out] (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - FresnelS
[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)
```

Rule 6420

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelS[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Sin[(Pi*(a + b*x)^2)/2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int S(bx)^2\,dx &= xS(bx)^2 - 2\int bxS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\ &= xS(bx)^2 - (2b)\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\ &= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} + xS(bx)^2 - \frac{\int \sin\left(b^2\pi x^2\right)dx}{\pi}\\ &= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} + xS(bx)^2 - \frac{S\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi}\end{aligned}$$

Mathematica [A] time = 0.0093133, size = 55, normalized size = 1.

$$\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b} + xS(bx)^2 - \frac{S\left(\sqrt{2}bx\right)}{\sqrt{2}\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^2,x]
```

```
[Out] (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - FresnelS
[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)
```

Maple [A] time = 0.056, size = 49, normalized size = 0.9

$$\frac{1}{b}\left(bx\left(\mathrm{FresnelS}\left(bx\right)\right)^2 + 2\,\frac{\cos\left(1/2\,b^2\pi\,x^2\right)\mathrm{FresnelS}\left(bx\right)}{\pi} - \frac{\sqrt{2}\mathrm{FresnelS}\left(bx\sqrt{2}\right)}{2\,\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2,x)
```

```
[Out] 1/b*(b*x*FresnelS(b*x)^2+2*FresnelS(b*x)/Pi*cos(1/2*b^2*Pi*x^2)-1/2/Pi*2^(1
/2)*FresnelS(b*x*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{fresnels}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{fresnels}\left(bx\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int S^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2,x)
```

```
[Out] Integral(fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2, x)
```

3.39 $\int \frac{S(bx)^2}{x} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{S(bx)^2}{x}, x\right)$$

```
[Out] Unintegrable[FresnelS[b*x]^2/x, x]
```

Rubi [A] time = 0.0166764, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)^2}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x,x]
```

```
[Out] Defer[Int][FresnelS[b*x]^2/x, x]
```

Rubi steps

$$\int \frac{S(bx)^2}{x} \, dx = \int \frac{S(bx)^2}{x} \, dx$$

Mathematica [A] time = 0.0170186, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x, x]
```

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x,x)
```

```
[Out] int(FresnelS(b*x)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x,x)
```

```
[Out] Integral(fresnels(b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x, x)
```

3.40 $\int \frac{S(bx)^2}{x^2} dx$

Optimal. Leaf size=37

$$2b\text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{S(bx)^2}{x}$$

```
[Out] -(FresnelS[b*x]^2/x) + 2*b*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])
/x, x]
```

Rubi [A] time = 0.0372656, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)^2}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^2,x]
```

```
[Out] -(FresnelS[b*x]^2/x) + 2*b*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x
, x]
```

Rubi steps

$$\int \frac{S(bx)^2}{x^2} dx = -\frac{S(bx)^2}{x} + (2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx$$

Mathematica [A] time = 0.0266247, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^2,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^2, x]
```

Maple [A] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^2,x)
```

```
[Out] int(FresnelS(b*x)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**2,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^2, x)
```

3.41 $\int \frac{S(bx)^2}{x^3} \, dx$

Optimal. Leaf size=38

$$b\text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)-\frac{S(bx)^2}{2x^2}$$

```
[Out] -FresnelS[b*x]^2/(2*x^2) + b*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2
])/x^2, x]
```

Rubi [A] time = 0.0383717, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)^2}{x^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^3,x]
```

```
[Out] -FresnelS[b*x]^2/(2*x^2) + b*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])
/x^2, x]
```

Rubi steps

$$\int \frac{S(bx)^2}{x^3} \, dx = -\frac{S(bx)^2}{2x^2} + b\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} \, dx$$

Mathematica [A] time = 0.0198748, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^3,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^3, x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^3,x)
```

```
[Out] int(FresnelS(b*x)^2/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**3,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^3, x)
```

3.42 $\int \frac{S(bx)^2}{x^4} dx$

Optimal. Leaf size=119

$$\frac{1}{3}\pi b^3 \text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^2}+\frac{\pi b^3 S\left(\sqrt{2}bx\right)}{3\sqrt{2}}+\frac{b^2\cos\left(\pi b^2x^2\right)}{6x}-\frac{b^2}{6x}-\frac{S(bx)^2}{3x^3}$$

```
[Out] -b^2/(6*x) + (b^2*Cos[b^2*Pi*x^2])/(6*x) - FresnelS[b*x]^2/(3*x^3) + (b^3*P
i*FresnelS[Sqrt[2]*b*x])/(3*Sqrt[2]) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(3*x^2) + (b^3*Pi*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])
/3
```

Rubi [A] time = 0.0873615, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^4,x]
```

```
[Out] -b^2/(6*x) + (b^2*Cos[b^2*Pi*x^2])/(6*x) - FresnelS[b*x]^2/(3*x^3) + (b^3*P
i*FresnelS[Sqrt[2]*b*x])/(3*Sqrt[2]) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(3*x^2) + (b^3*Pi*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^4}\,dx &= -\frac{S(bx)^2}{3x^3}+\frac{1}{3}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx\\
&= -\frac{b^2}{6x}-\frac{S(bx)^2}{3x^3}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2}-\frac{1}{6}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx+\frac{1}{3}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx\\
&= -\frac{b^2}{6x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{S(bx)^2}{3x^3}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2}+\frac{1}{3}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx+\frac{1}{3}\left(b^4\pi\right.\\
&= -\frac{b^2}{6x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{S(bx)^2}{3x^3}+\frac{b^3\pi S\left(\sqrt{2}bx\right)}{3\sqrt{2}}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2}+\frac{1}{3}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{x}
\end{aligned}$$

Mathematica [A] time = 0.0259727, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^4,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^4, x]
```

Maple [A] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{\left(\mathrm{FresnelS}\left(bx\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^4,x)
```

```
[Out] int(FresnelS(b*x)^2/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\left(bx\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\left(bx\right)^{2}}{x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^{2}\left(bx\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**4,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\left(bx\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^4, x)
```

3.43 $\int \frac{S(bx)^2}{x^5}\,dx$

Optimal. Leaf size=127

$$-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3}-\frac{\pi b^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x}-\frac{1}{12}\pi^2b^4S(bx)^2+\frac{1}{12}\pi b^4\text{Si}\left(b^2\pi x^2\right)-\frac{b^2}{24x^2}+\frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2}-\frac{S(bx)^2}{4x^4}$$

```
[Out] -b^2/(24*x^2) + (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(6*x) - (b^4*Pi^2*FresnelS[b*x]^2)/12 - FresnelS[b*x]^2/(4
*x^4) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^3) + (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Rubi [A] time = 0.143154, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {6430, 6456, 6464, 6440, 30, 3375, 3380, 3297, 3299}

$$-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3}-\frac{\pi b^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x}-\frac{1}{12}\pi^2b^4S(bx)^2+\frac{1}{12}\pi b^4\text{Si}\left(b^2\pi x^2\right)-\frac{b^2}{24x^2}+\frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2}-\frac{S(bx)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^2/x^5,x]
```


```
[Out] -b^2/(24*x^2) + (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(6*x) - (b^4*Pi^2*FresnelS[b*x]^2)/12 - FresnelS[b*x]^2/(4
*x^4) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^3) + (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```


Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)^2}{x^5}\,dx &= -\frac{S(bx)^2}{4x^4}+\frac{1}{2}b\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b^2}{24x^2}-\frac{S(bx)^2}{4x^4}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3}-\frac{1}{12}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx+\frac{1}{6}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^2}\,d\\
&= -\frac{b^2}{24x^2}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x}-\frac{S(bx)^2}{4x^4}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3}-\frac{1}{24}b^2\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx,x\right.\\
&= -\frac{b^2}{24x^2}+\frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x}-\frac{S(bx)^2}{4x^4}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3}+\frac{1}{24}b^4\pi\mathrm{Si}\left(b^2\pi\right.\\
&= -\frac{b^2}{24x^2}+\frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x}-\frac{1}{12}b^4\pi^2S(bx)^2-\frac{S(bx)^2}{4x^4}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3}\end{aligned}$$

Mathematica [A] time = 0.0060233, size = 127, normalized size = 1.

$$-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3}-\frac{\pi b^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x}-\frac{1}{12}\pi^2b^4S(bx)^2+\frac{1}{12}\pi b^4\mathrm{Si}\left(b^2\pi x^2\right)-\frac{b^2}{24x^2}+\frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2}-\frac{S(b}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^2/x^5,x]
```

```
[Out] -b^2/(24*x^2) + (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(6*x) - (b^4*Pi^2*FresnelS[b*x]^2)/12 - FresnelS[b*x]^2/(4
*x^4) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^3) + (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}\,(bx))^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^5,x)
```

```
[Out] int(FresnelS(b*x)^2/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)^2}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**5,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^5, x)
```

3.44 $\int \frac{S(bx)^2}{x^6}\,dx$

Optimal. Leaf size=170

$$-\frac{1}{20}\pi^2 b^5 \text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{7\pi^2b^5\text{FresnelC}\left(\sqrt{2}bx\right)}{60\sqrt{2}}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{10x^4}-\frac{\pi b^3S(bx)\cos\left(\frac{1}{2}\pi b\right.}{20x^2}$$

```
[Out] -b^2/(60*x^3) + (b^2*Cos[b^2*Pi*x^2])/(60*x^3) + (7*b^5*Pi^2*FresnelC[Sqrt[
2]*b*x])/(60*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(20*x^2)
 - FresnelS[b*x]^2/(5*x^5) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(10*x^4)
 - (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Unintegrable[(FresnelS[b*
x]*Sin[(b^2*Pi*x^2)/2])/x, x])/20
```

Rubi [A] time = 0.153846, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^6,x]
```

```
[Out] -b^2/(60*x^3) + (b^2*Cos[b^2*Pi*x^2])/(60*x^3) + (7*b^5*Pi^2*FresnelC[Sqrt[
2]*b*x])/(60*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(20*x^2)
 - FresnelS[b*x]^2/(5*x^5) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(10*x^4)
 - (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Defer[Int][(FresnelS[b*x]
*Sin[(b^2*Pi*x^2)/2])/x, x])/20
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^6}\,dx &= -\frac{S(bx)^2}{5x^5}+\frac{1}{5}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx\\
&= -\frac{b^2}{60x^3}-\frac{S(bx)^2}{5x^5}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4}-\frac{1}{20}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx+\frac{1}{10}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx\\
&= -\frac{b^2}{60x^3}+\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{20x^2}-\frac{S(bx)^2}{5x^5}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4}+\frac{1}{40}\left(b^4\pi\right)\int \frac{\sin\left(\right.}{}\\
&= -\frac{b^2}{60x^3}+\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{20x^2}-\frac{S(bx)^2}{5x^5}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4}-\frac{7b^4\pi\sin\left(b^2\pi x^2\right)}{120x}\\
&= -\frac{b^2}{60x^3}+\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}+\frac{7b^5\pi^2C\left(\sqrt{2}bx\right)}{60\sqrt{2}}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{20x^2}-\frac{S(bx)^2}{5x^5}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4}-
\end{aligned}$$

Mathematica [A] time = 0.0267984, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^6,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^6, x]
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^6,x)
```

```
[Out] int(FresnelS(b*x)^2/x^6,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^6, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^6, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**6,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**6, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^6, x)
```

3.45 $\int \frac{S(bx)^2}{x^7}\,dx$

Optimal. Leaf size=165

$$-\frac{1}{45}\pi^2 b^5 \text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2},x\right)+\frac{1}{72}\pi^2 b^6 \text{CosIntegral}\left(\pi b^2 x^2\right)-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^5}-\frac{\pi b^3 S(bx)\,c}{4}$$

```
[Out] -b^2/(120*x^4) + (b^2*Cos[b^2*Pi*x^2])/(120*x^4) + (b^6*Pi^2*CosIntegral[b^
2*Pi*x^2])/72 - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(45*x^3) - Fresn
elS[b*x]^2/(6*x^6) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x^5) - (b^4*
Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Unintegrable[(FresnelS[b*x]*Sin[(b
^2*Pi*x^2)/2])/x^2, x])/45
```

Rubi [A] time = 0.222055, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)^2}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^7,x]
```

```
[Out] -b^2/(120*x^4) + (b^2*Cos[b^2*Pi*x^2])/(120*x^4) + (b^6*Pi^2*CosIntegral[b^
2*Pi*x^2])/72 - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(45*x^3) - Fresn
elS[b*x]^2/(6*x^6) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x^5) - (b^4*
Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/x^2, x])/45
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^7}\,dx &= -\frac{S(bx)^2}{6x^6}+\frac{1}{3}b\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\
&= -\frac{b^2}{120x^4}-\frac{S(bx)^2}{6x^6}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5}-\frac{1}{30}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx+\frac{1}{15}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{x^4}\\
&= -\frac{b^2}{120x^4}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3}-\frac{S(bx)^2}{6x^6}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5}-\frac{1}{60}b^2\,\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^3}\,dx,\right.\\
&= -\frac{b^2}{120x^4}+\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3}-\frac{S(bx)^2}{6x^6}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5}+\frac{1}{180}\left(b^4\pi\right)\text{Su}\\
&= -\frac{b^2}{120x^4}+\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3}-\frac{S(bx)^2}{6x^6}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5}-\frac{b^4\pi\sin\left(b^2\pi x\right.}{72x^2}\\
&= -\frac{b^2}{120x^4}+\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}+\frac{1}{72}b^6\pi^2\text{Ci}\left(b^2\pi x^2\right)-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3}-\frac{S(bx)^2}{6x^6}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\right.}{15x^5}
\end{aligned}$$

Mathematica [A] time = 0.0202359, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^7,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^7, x]
```

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{FresnelS}\,(bx))^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^7,x)
```

```
[Out] int(FresnelS(b*x)^2/x^7,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\,(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^7, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\,(bx)^2}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^7,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^7, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**7,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**7, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^7, x)
```

3.46 $\int \frac{S(bx)^2}{x^8} dx$

Optimal. Leaf size=258

$$-\frac{1}{168}\pi^3 b^7 \text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{\pi^2b^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{168x^2}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{21x^6}-\frac{\pi b^3S(bx)\cos\left(\frac{1}{2}\pi\right.}{84x^4}$$

```
[Out] -b^2/(210*x^5) + (b^6*Pi^2)/(336*x) + (b^2*Cos[b^2*Pi*x^2])/(210*x^5) - (67
*b^6*Pi^2*Cos[b^2*Pi*x^2])/(5040*x) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/(84*x^4) - FresnelS[b*x]^2/(7*x^7) - (b^7*Pi^3*FresnelS[Sqrt[2]*b*x])
/(72*Sqrt[2]) - (2*Sqrt[2]*b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/315 - (b*Fresnel
S[b*x]*Sin[(b^2*Pi*x^2)/2])/(21*x^6) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*
x^2)/2])/(168*x^2) - (13*b^4*Pi*Sin[b^2*Pi*x^2])/(2520*x^3) - (b^7*Pi^3*Uni
ntegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/168
```

Rubi [A] time = 0.244556, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)^2}{x^8} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^8,x]
```

```
[Out] -b^2/(210*x^5) + (b^6*Pi^2)/(336*x) + (b^2*Cos[b^2*Pi*x^2])/(210*x^5) - (67
*b^6*Pi^2*Cos[b^2*Pi*x^2])/(5040*x) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/(84*x^4) - FresnelS[b*x]^2/(7*x^7) - (b^7*Pi^3*FresnelS[Sqrt[2]*b*x])
/(72*Sqrt[2]) - (2*Sqrt[2]*b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/315 - (b*Fresnel
S[b*x]*Sin[(b^2*Pi*x^2)/2])/(21*x^6) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*
x^2)/2])/(168*x^2) - (13*b^4*Pi*Sin[b^2*Pi*x^2])/(2520*x^3) - (b^7*Pi^3*Def
er[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/168
```

Rubi steps

$$\int \frac{S(bx)^2}{x^8} dx = -\frac{S(bx)^2}{7x^7}+\frac{1}{7}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx$$

$$= -\frac{b^2}{210x^5}-\frac{S(bx)^2}{7x^7}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6}-\frac{1}{42}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6} dx+\frac{1}{21}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5} dx$$

$$= -\frac{b^2}{210x^5}+\frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4}-\frac{S(bx)^2}{7x^7}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6}+\frac{1}{168}\left(b^4\pi\right)\int \frac{\sin\ldots}{\ldots}$$

$$= -\frac{b^2}{210x^5}+\frac{b^6\pi^2}{336x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4}-\frac{S(bx)^2}{7x^7}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6}+\frac{b^5\pi^2S(b\ldots}{\ldots}$$

$$= -\frac{b^2}{210x^5}+\frac{b^6\pi^2}{336x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5}-\frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4}-\frac{S(bx)^2}{7x^7}-\frac{bS(bx)\sin\ldots}{2\ldots}$$

$$= -\frac{b^2}{210x^5}+\frac{b^6\pi^2}{336x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5}-\frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4}-\frac{S(bx)^2}{7x^7}-\frac{b^7\pi^3S\left(\sqrt{\ldots}\right.}{72\sqrt{\ldots}}$$

Mathematica [A] time = 0.0272621, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^8,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^8, x]
```

Maple [A] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{FresnelS}\,(bx))^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^8,x)
```

```
[Out] int(FresnelS(b*x)^2/x^8,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\,(bx)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^8, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\,(bx)^2}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^8, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**8,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**8, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^8, x)
```

3.47 $\int \frac{S(bx)^2}{x^9}\,dx$

Optimal. Leaf size=242

$$\frac{\pi^2 b^5 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{420x^3}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{28x^7}+\frac{\pi^3 b^7 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{420x}-\frac{\pi b^3 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{140x^5}+\frac{1}{840}\pi^4 b^8 S(bx$$

```
[Out] -b^2/(336*x^6) + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) -
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS
[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x) + (
b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b*x]*
Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Rubi [A] time = 0.393047, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6430, 6456, 6464, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi^2 b^5 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{420x^3}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{28x^7}+\frac{\pi^3 b^7 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{420x}-\frac{\pi b^3 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{140x^5}+\frac{1}{840}\pi^4 b^8 S(bx$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^2/x^9,x]
```


```
[Out] -b^2/(336*x^6) + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) -
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS
[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x) + (
b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b*x]*
Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```


Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{S(bx)^2}{x^9}\,dx = -\frac{S(bx)^2}{8x^8} + \frac{1}{4}b\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

$$= -\frac{b^2}{336x^6} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} - \frac{1}{56}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx + \frac{1}{28}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{x^6}$$

$$= -\frac{b^2}{336x^6} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} - \frac{1}{112}b^2\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^4}\,dx\right.$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} + \frac{b^5}{}$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{420x} - \frac{S(bx)^2}{8x^8}$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi\right.}{420x}$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi\right.}{420x}$$

Mathematica [A] time = 0.0113531, size = 242, normalized size = 1.

$$\frac{\pi^2 b^5 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{420x^3} - \frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{28x^7} + \frac{\pi^3 b^7 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{420x} - \frac{\pi b^3 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{140x^5} + \frac{1}{840}\pi^4 b^8 S(b$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^2/x^9,x]
```

```
[Out] -b^2/(336*x^6) + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) -
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS
[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x) + (
b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b*x]*
Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\left(\mathrm{FresnelS}\left(bx\right)\right)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^9,x)
```

```
[Out] int(FresnelS(b*x)^2/x^9,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^9,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^9, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)^2}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^9,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^9, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**9,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**9, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^9, x)
```

3.48 $\int \frac{S(bx)^2}{x^{10}}\, dx$

Optimal. Leaf size=285

$$\frac{\pi^4 b^9 \text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)}{1728}-\frac{853\pi^4b^9\text{FresnelC}\left(\sqrt{2}bx\right)}{181440\sqrt{2}}+\frac{\pi^2b^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{864x^4}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{36x^8}$$

```
[Out] -b^2/(504*x^7) + (b^6*Pi^2)/(5184*x^3) + (b^2*Cos[b^2*Pi*x^2])/(504*x^7) -
(187*b^6*Pi^2*Cos[b^2*Pi*x^2])/(181440*x^3) - (853*b^9*Pi^4*FresnelC[Sqrt[2
]*b*x])/(181440*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(216*
x^6) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(1728*x^2) - FresnelS[b
*x]^2/(9*x^9) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(36*x^8) + (b^5*Pi^2*
FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(864*x^4) - (19*b^4*Pi*Sin[b^2*Pi*x^2])/
(15120*x^5) + (853*b^8*Pi^3*Sin[b^2*Pi*x^2])/(362880*x) + (b^9*Pi^4*Uninteg
rable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/1728
```

Rubi [A] time = 0.358317, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)^2}{x^{10}}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^10,x]
```

```
[Out] -b^2/(504*x^7) + (b^6*Pi^2)/(5184*x^3) + (b^2*Cos[b^2*Pi*x^2])/(504*x^7) -
(187*b^6*Pi^2*Cos[b^2*Pi*x^2])/(181440*x^3) - (853*b^9*Pi^4*FresnelC[Sqrt[2
]*b*x])/(181440*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(216*
x^6) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(1728*x^2) - FresnelS[b
*x]^2/(9*x^9) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(36*x^8) + (b^5*Pi^2*
FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(864*x^4) - (19*b^4*Pi*Sin[b^2*Pi*x^2])/
(15120*x^5) + (853*b^8*Pi^3*Sin[b^2*Pi*x^2])/(362880*x) + (b^9*Pi^4*Defer[I
nt][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/1728
```

Rubi steps

$$\int \frac{S(bx)^2}{x^{10}}\,dx = -\frac{S(bx)^2}{9x^9} + \frac{1}{9}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$
$$= -\frac{b^2}{504x^7} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} - \frac{1}{72}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx + \frac{1}{36}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx$$
$$= -\frac{b^2}{504x^7} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} + \frac{1}{432}\left(b^4\pi\right)\int \frac{\sin}{}$$
$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} + \frac{b^5\pi^2 S}{}$$
$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi\right.}{1728x}$$
$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi\right.}{1728x}$$
$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{67b^9\pi^4C\left(\sqrt{2}bx\right)}{25920\sqrt{2}} - \frac{1}{945}\sqrt{2}b^9\pi^4C\left(\sqrt{2}bx\right)$$

Mathematica [A] time = 0.0271738, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^10,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^10, x]
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{FresnelS}\,(bx))^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^10,x)
```

```
[Out] int(FresnelS(b*x)^2/x^10,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\,(bx)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^10, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)^2}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^10, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2\,(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**10,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**10, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^10, x)
```

3.49 $\int (c+dx)^2 S(a+bx)^2\,dx$

Optimal. Leaf size=497

$$\frac{id(a+bx)^2(bc-ad)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{id(a+bx)^2(bc-ad)\text{HypergeometricPFQ}}{4\pi b^3}$$

```
[Out] (2*d^2*x)/(3*b^2*Pi^2) + (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) +
 (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (5*d^2*FresnelC[Sqrt[2]
*(a + b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (2*(b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2
]*FresnelS[a + b*x])/(b^3*Pi) + (2*d*(b*c - a*d)*(a + b*x)*Cos[(Pi*(a + b*x
)^2)/2]*FresnelS[a + b*x])/(b^3*Pi) + (2*d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^
2)/2]*FresnelS[a + b*x])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x]*Fres
nelS[a + b*x])/(b^3*Pi) + ((b*c - a*d)^2*(a + b*x)*FresnelS[a + b*x]^2)/b^3
 + (d*(b*c - a*d)*(a + b*x)^2*FresnelS[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*F
resnelS[a + b*x]^2)/(3*b^3) - ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(
Sqrt[2]*b^3*Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a + b*
x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi) -
(4*d^2*FresnelS[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rubi [A] time = 0.410257, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {6432, 6420, 6452, 3351, 6430, 6454, 6446, 3379, 2638, 6460, 3357, 3352, 3385}

$$\frac{id(a+bx)^2(bc-ad)\,_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{id(a+bx)^2(bc-ad)\,_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{d(bc-ad)\text{Fresn}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelS[a + b*x]^2,x]
```



```
[Out] (2*d^2*x)/(3*b^2*Pi^2) + (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) +
 (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (5*d^2*FresnelC[Sqrt[2]
*(a + b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (2*(b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2
]*FresnelS[a + b*x])/(b^3*Pi) + (2*d*(b*c - a*d)*(a + b*x)*Cos[(Pi*(a + b*x
)^2)/2]*FresnelS[a + b*x])/(b^3*Pi) + (2*d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^
2)/2]*FresnelS[a + b*x])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x]*Fres
nelS[a + b*x])/(b^3*Pi) + ((b*c - a*d)^2*(a + b*x)*FresnelS[a + b*x]^2)/b^3
 + (d*(b*c - a*d)*(a + b*x)^2*FresnelS[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*F
resnelS[a + b*x]^2)/(3*b^3) - ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(
Sqrt[2]*b^3*Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a + b*
x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi) -
(4*d^2*FresnelS[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```


Rule 6432

```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6420

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelS[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Sin[(Pi*(a + b*x)^2)/2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\int (c+dx)^2 S(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)S(x)^2+2bcd\left(1-\frac{ad}{bc}\right)xS(x)^2+d^2x^2S(x)^2\right)dx,x,a+bx\right)}{b^3}$$

$$= \frac{d^2\,\text{Subst}\left(\int x^2S(x)^2\,dx,x,a+bx\right)}{b^3}+\frac{(2d(bc-ad))\,\text{Subst}\left(\int xS(x)^2\,dx,x,a+bx\right)}{b^3}+\frac{(bc-ad)}{}$$

$$= \frac{(bc-ad)^2(a+bx)S(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2S(a+bx)^2}{b^3}+\frac{d^2(a+bx)^3S(a+bx)^2}{3b^3}-\frac{(2d}{}$$

$$= \frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^3\pi}+\frac{2d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^3\pi}+$$

$$= \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}+\frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^3\pi}+\frac{2d(bc-ad)(a+bx)}{}$$

$$= \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{d^2C\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2}+\frac{2(bc-ad)^2}{}$$

$$= \frac{2d^2x}{3b^2\pi^2}+\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{d^2C\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2}+\frac{2(}{}$$

$$= \frac{2d^2x}{3b^2\pi^2}+\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{d^2C\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2}-\frac{\sqrt{}}{}$$

Mathematica [F] time = 0.664232, size = 0, normalized size = 0.

$$\int (c+dx)^2 S(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^2*FresnelS[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^2*FresnelS[a + b*x]^2, x]
```

Maple [F] time = 0.244, size = 0, normalized size = 0.

$$\int (dx+c)^2\left(\text{FresnelS}\,(bx+a)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelS(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^2*FresnelS(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \text{fresnels}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*fresnels(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^2+2\,cdx+c^2\right)\text{fresnels}(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnels(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 S^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnels(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*fresnels(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \text{fresnels}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*fresnels(b*x + a)^2, x)
```

3.50 $\int (c + dx)S(a + bx)^2\,dx$

Optimal. Leaf size=279

$$\frac{id(a+bx)^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}-\frac{id(a+bx)^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi\right.}{8\pi b^2}$$

```
[Out] (d*Cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + (2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2)/
2]*FresnelS[a + b*x])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*Fresn
elS[a + b*x])/(b^2*Pi) - (d*FresnelC[a + b*x]*FresnelS[a + b*x])/(2*b^2*Pi)
 + ((b*c - a*d)*(a + b*x)*FresnelS[a + b*x]^2)/b^2 + (d*(a + b*x)^2*Fresnel
S[a + b*x]^2)/(2*b^2) - ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*
b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*P
i*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {
3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi)
```

Rubi [A] time = 0.196207, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6432, 6420, 6452, 3351, 6430, 6454, 6446, 3379, 2638}

$$\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}+\frac{(a+bx)(bc-ad)S(a+bx)^2}{b^2}-\frac{(bc\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelS[a + b*x]^2,x]
```

```
[Out] (d*Cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + (2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2)/
2]*FresnelS[a + b*x])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*Fresn
elS[a + b*x])/(b^2*Pi) - (d*FresnelC[a + b*x]*FresnelS[a + b*x])/(2*b^2*Pi)
 + ((b*c - a*d)*(a + b*x)*FresnelS[a + b*x]^2)/b^2 + (d*(a + b*x)^2*Fresnel
S[a + b*x]^2)/(2*b^2) - ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*
b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*P
i*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {
3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi)
```

Rule 6432

```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6420

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelS[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Sin[(Pi*(a + b*x)^2)/2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)S(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int \left(bc\left(1-\frac{ad}{bc}\right)S(x)^2+dxS(x)^2\right)dx,x,a+bx\right)}{b^2}\\
&= \frac{d\,\text{Subst}\left(\int xS(x)^2\,dx,x,a+bx\right)}{b^2}+\frac{(bc-ad)\,\text{Subst}\left(\int S(x)^2\,dx,x,a+bx\right)}{b^2}\\
&= \frac{(bc-ad)(a+bx)S(a+bx)^2}{b^2}+\frac{d(a+bx)^2S(a+bx)^2}{2b^2}-\frac{d\,\text{Subst}\left(\int x^2S(x)\sin\left(\frac{\pi x^2}{2}\right)dx,x,a\right.}{b^2}\\
&= \frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{(bc-ad}{}\\
&= \frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}-\frac{dC(a+}{}\\
&= \frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2}+\frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right.}{b^2\pi}
\end{aligned}
$$

Mathematica [F] time = 0.585015, size = 0, normalized size = 0.

$$\int (c + dx)S(a + bx)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)*FresnelS[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)*FresnelS[a + b*x]^2, x]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int (dx + c)\left(\mathrm{FresnelS}\left(bx + a\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelS(b*x+a)^2,x)
```

```
[Out] int((d*x+c)*FresnelS(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\mathrm{fresnels}\left(bx + a\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*fresnels(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx + c)\mathrm{fresnels}\left(bx + a\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*fresnels(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)\, S^2\left(a + bx\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*fresnels(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\text{fresnels}\,(bx + a)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnels(b*x + a)^2, x)
```

3.51 $\int S(a+bx)^2\,dx$

Optimal. Leaf size=70

$$\frac{(a+bx)S(a+bx)^2}{b}-\frac{S\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b}+\frac{2S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[Out] (2*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x])/(b*Pi) + ((a + b*x)*FresnelS[
a + b*x]^2)/b - FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*Pi)
```

Rubi [A] time = 0.167726, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6420, 6452, 3351}

$$\frac{(a+bx)S(a+bx)^2}{b}-\frac{S\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b}+\frac{2S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[a + b*x]^2,x]
```

```
[Out] (2*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x])/(b*Pi) + ((a + b*x)*FresnelS[
a + b*x]^2)/b - FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*Pi)
```

Rule 6420

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelS[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Sin[(Pi*(a + b*x)^2)/2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int S(a+bx)^2\,dx &= \frac{(a+bx)S(a+bx)^2}{b}-2\int (a+bx)S(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx\\
&= \frac{(a+bx)S(a+bx)^2}{b}-\frac{2\,\text{Subst}\left(\int xS(x)\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b}\\
&= \frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b\pi}+\frac{(a+bx)S(a+bx)^2}{b}-\frac{\text{Subst}\left(\int \sin\left(\pi x^2\right)\,dx,x,a+bx\right)}{b\pi}\\
&= \frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b\pi}+\frac{(a+bx)S(a+bx)^2}{b}-\frac{S\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b\pi}
\end{aligned}$$

Mathematica [A] time = 0.0104769, size = 67, normalized size = 0.96

$$\frac{2\pi(a+bx)S(a+bx)^2-\sqrt{2}S\left(\sqrt{2}(a+bx)\right)+4S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[a + b*x]^2,x]
```

```
[Out] (4*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x] + 2*Pi*(a + b*x)*FresnelS[a +
b*x]^2 - Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)])/(2*b*Pi)
```

Maple [A] time = 0.056, size = 60, normalized size = 0.9

$$\frac{1}{b}\left((bx+a)\left(\mathrm{FresnelS}\left(bx+a\right)\right)^2+2\,\frac{\mathrm{FresnelS}\left(bx+a\right)\cos\left(1/2\,\pi\,(bx+a)^2\right)}{\pi}-\frac{\sqrt{2}\mathrm{FresnelS}\left((bx+a)\sqrt{2}\right)}{2\,\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)^2,x)
```

```
[Out] 1/b*((b*x+a)*FresnelS(b*x+a)^2+2*FresnelS(b*x+a)/Pi*cos(1/2*Pi*(b*x+a)^2)-1
/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{fresnels}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{fresnels}\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int S^2\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)**2,x)
```

```
[Out] Integral(fresnels(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\,(bx+a)^2 \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)^2, x)
```

3.52 $\int \frac{S(a+bx)^2}{c+dx} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{S(a+bx)^2}{c+dx},x\right)$$

```
[Out] Unintegrable[FresnelS[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0232799, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(a+bx)^2}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{S(a+bx)^2}{c+dx} \, dx = \int \frac{S(a+bx)^2}{c+dx} \, dx$$

Mathematica [A] time = 0.0357935, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)^2}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[FresnelS[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.36, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx+a))^2}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(FresnelS(b*x+a)^2/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a)^2/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2\,(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(fresnels(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)^2/(d*x + c), x)
```

3.53 $\int \frac{S(a+bx)^2}{(c+dx)^2} dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{S(a+bx)^2}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[FresnelS[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [A] time = 0.0228937, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{S(a+bx)^2}{(c+dx)^2} dx = \int \frac{S(a+bx)^2}{(c+dx)^2} dx$$

Mathematica [A] time = 0.0817917, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[FresnelS[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A] time = 0.355, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a)^2/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2\,(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(fresnels(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)^2/(d*x + c)^2, x)
```

3.54 $\int x^2 S(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=231

$$\left(\frac{1}{12}-\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)+\left(\frac{1}{12}-\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2}}$$

```
[Out] ((1/12 - I/12)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2
+ I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*
x^n)^(3/n) + ((1/12 - I/12)*E^((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(c*x^n)^(3/n) + (x^3*FresnelS[d*(a + b*Log[c*x^n])])/3
```

Rubi [A] time = 0.570596, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{12}-\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)+\left(\frac{1}{12}-\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/12 - I/12)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2
+ I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*
x^n)^(3/n) + ((1/12 - I/12)*E^((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(c*x^n)^(3/n) + (x^3*FresnelS[d*(a + b*Log[c*x^n])])/3
```

Rule 6471

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelS[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Sin[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 4617

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^2 S\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx = \frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{1}{3}(bdn)\int x^2\sin\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)dx$$

$$= \frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{1}{6}(ibdn)\int e^{-\frac{1}{2}id^2\pi\left(a+b\log\left(cx^n\right)\right)^2}x^2\,dx + \frac{1}{6}(ibdn)\int e^{\frac{1}{2}id^2\pi(a+b\log$$

$$= \frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{1}{6}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log\left(cx^n\right) - \frac{1}{2}ib^2d^2\pi\log$$

$$= \frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{1}{6}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^2\left(cx^n\right)^{-ia}$$

$$= \frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{1}{6}\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log$$

$$= \frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{1}{6}\left(ibdx^3\left(cx^n\right)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi + \right.\right.$$

$$= \frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{1}{6}\left(ibde^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3\left(cx^n\right)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(\frac{i}{-}\right.\right.$$

$$= \left(\frac{1}{12}-\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3\left(cx^n\right)^{-3/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right) + \left(\frac{1}{12}\right.$$

Mathematica [A] time = 6.88797, size = 319, normalized size = 1.38

$$\frac{1}{12}x^3\left(4S(d(a+b\log(cx^n)))+\sqrt[4]{-1}\sqrt{2}(cx^n)^{-3/n}\left(e^{\frac{9i}{\pi b^2d^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(\pi abd^2n+\pi b^2d^2n\log(cx^n)-3i)}{\sqrt{\pi}bdn}\right)+i\text{Erfi}\left(\frac{(-\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*(4*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^(((-6*a)/(b*
n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - L
og[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(E^((9*I)/(b^2*d
^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n
]))/(b*d*n*Sqrt[Pi])] + I*Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(3/n)))/12
```

Maple [F] time = 0.701, size = 0, normalized size = 0.

$$\int x^2\text{FresnelS}(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\text{fresnels}((b\log(cx^n)+a)d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnels((b*log(c*x^n) + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\text{fresnels}(bd\log(cx^n)+ad),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnels(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 S\left(ad + bd\log\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*fresnels(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}\left((b\log\left(cx^n\right) + a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnels((b*log(c*x^n) + a)*d), x)
```

3.55 $\int xS(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=227

$$\left(\frac{1}{8}-\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)+\left(\frac{1}{8}-\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{-\frac{2\left(\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\right.$$

```
[Out] ((1/8 - I/8)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[((1/2 + I/
2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)
^(2/n) + ((1/8 - I/8)*x^2*Erfi[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*(c*x^n)^(2/n)) + (x^2*FresnelS[d*(a + b*Log[c*x^n])])/2
```

Rubi [A] time = 0.435235, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{8}-\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)+\left(\frac{1}{8}-\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{-\frac{2\left(\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[d*(a + b*Log[c*x^n])],x]
```



```
[Out] ((1/8 - I/8)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[((1/2 + I/
2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)
^(2/n) + ((1/8 - I/8)*x^2*Erfi[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*(c*x^n)^(2/n)) + (x^2*FresnelS[d*(a + b*Log[c*x^n])])/2
```


Rule 6471

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelS[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Sin[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 4617

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx &= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int x\sin\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)\,dx\\
&= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(ibdn)\int e^{-\frac{1}{2}id^2\pi\left(a+b\log\left(cx^n\right)\right)^2}x\,dx+\frac{1}{4}(ibdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(c
\\
&= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}ib^2d^2\pi\log^2
\\
&= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x\left(cx^n\right)^{-iabd^2
\\
&= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log
\\
&= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(ibdx^2\left(cx^n\right)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(2
\\
&= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(ibde^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(\frac{i\left(2-
\\
&= \left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-2/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)+\left(\frac{1}{8}-\frac{i}{8}\right)e
\end{aligned}
$$

Mathematica [A] time = 6.64993, size = 319, normalized size = 1.41

$$\frac{1}{8}x^2\left(4S\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\sqrt[4]{-1}\sqrt{2}\left(cx^n\right)^{-2/n}\left(e^{\frac{4i}{\pi b^2d^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)-2i\right)}{\sqrt{\pi}bdn}\right)+i\text{Erfi}\left(\frac{(-1\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*(4*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*n
) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] - Lo
g[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(E^((4*I)/(b^2*d
^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n
]))/(b*d*n*Sqrt[Pi])] + I*Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(2/n)))/8
```

Maple [F] time = 0.535, size = 0, normalized size = 0.

$$\int x\text{FresnelS}\left(d\left(a+b\ln\left(cx^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*FresnelS(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnels}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnels((b*log(c*x^n) + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\text{fresnels}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x*fresnels(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xS\left(ad + bd\log\left(cx^{n}\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*fresnels(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{fresnels}\left((b\log\left(cx^{n}\right) + a)d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x*fresnels((b*log(c*x^n) + a)*d), x)
```

3.56 $\int S(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=214

$$\left(\frac{1}{4}-\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)+\left(\frac{1}{4}-\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}-\right.}{}\right.$$

```
[Out] ((1/4 - I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c
*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)) + ((1/4 - I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(
c*x^n)^n^(-1)) + x*FresnelS[d*(a + b*Log[c*x^n])]
```

Rubi [A] time = 0.326252, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {6468, 4615, 2277, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{4}-\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)+\left(\frac{1}{4}-\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}-\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 - I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c
*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)) + ((1/4 - I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(
c*x^n)^n^(-1)) + x*FresnelS[d*(a + b*Log[c*x^n])]
```

Rule 6468

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Sim
p[x*FresnelS[d*(a + b*Log[c*x^n])], x] - Dist[b*d*n, Int[Sin[(Pi*(d*(a + b*
Log[c*x^n]))^2)/2], x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 4615

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)], x_Symbol] :> Dist[I
/2, Int[E^(-(I*d*(a + b*Log[c*x^n])^2)), x], x] - Dist[I/2, Int[E^(I*d*(a +
 b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 2277

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)), x_Symbol] :> Int[
F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b,
c, d, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int S(d(a+b\log(cx^n)))\,dx = xS(d(a+b\log(cx^n))) - (bdn)\int \sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)dx$$
$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}(ibdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}\,dx + \frac{1}{2}(ibdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}\,d$$
$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \frac{1}{2}ib^2d^2\pi\log^2(cx$$
$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(cx^n)^{-iabd^2\pi}\,dx$$
$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(c$$
$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}\left(ibdx(cx^n)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(1-ia}{}\right.\right.$$
$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}\left(ibde^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(\frac{i\left(\frac{1-iabd^2n\pi}{n}\right.}{2b}\right.\right.$$
$$= \left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-1/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right) + \left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2}{}}$$

Mathematica [A] time = 6.65872, size = 316, normalized size = 1.48

$$xS(d(a+b\log(cx^n))) + \frac{\sqrt[4]{-1}x(cx^n)^{-1/n}\left(e^{\frac{i}{\pi b^2d^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(\pi abd^2n+\pi b^2d^2n\log(cx^n)-i)}{\sqrt{\pi}bdn}\right) + i\,\text{Erfi}\left(\frac{(-1)^{3/4}(\pi abd^2n+\pi b^2d^2n\log(cx^n)+i)}{\sqrt{2\pi}bdn}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((-2*a)/(b*n) - I/(b^2*d^
2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2
*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1
/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]
+ I*Erfi[((-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*S
qrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*Sin[(d^
2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-1))
```

Maple [F] time = 0.541, size = 0, normalized size = 0.

$$\int \text{FresnelS}\left(d\left(a+b\ln\left(cx^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(FresnelS(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(fresnels((b*log(c*x^n) + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}\left(bd\log\left(cx^{n}\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int S\left(d\left(a+b\log\left(cx^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(fresnels(d*(a + b*log(c*x**n))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(fresnels((b*log(c*x^n) + a)*d), x)
```

3.57 $\int \frac{S(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=65

$$\frac{\cos\left(\frac{1}{2}\pi d^2(a+b\log(cx^n))^2\right)}{\pi bdn}+\frac{(a+b\log(cx^n))S(d(a+b\log(cx^n)))}{bn}$$

```
[Out] Cos[(d^2*Pi*(a + b*Log[c*x^n])^2)/2]/(b*d*n*Pi) + (FresnelS[d*(a + b*Log[c*
x^n])]*(a + b*Log[c*x^n]))/(b*n)
```

Rubi [A] time = 0.0411381, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6418}

$$\frac{\cos\left(\frac{1}{2}\pi d^2(a+b\log(cx^n))^2\right)}{\pi bdn}+\frac{(a+b\log(cx^n))S(d(a+b\log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] Cos[(d^2*Pi*(a + b*Log[c*x^n])^2)/2]/(b*d*n*Pi) + (FresnelS[d*(a + b*Log[c*
x^n])]*(a + b*Log[c*x^n]))/(b*n)
```

Rule 6418

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelS[a +
 b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{S(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\text{Subst}\left(\int S(d(a+bx))\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{\text{Subst}\left(\int S(x)\,dx,x,ad+bd\log(cx^n)\right)}{bdn}\\ &= \frac{\cos\left(\frac{1}{2}\pi(ad+bd\log(cx^n))^2\right)}{bdn\pi}+\frac{S(ad+bd\log(cx^n))(a+b\log(cx^n))}{bn}\end{aligned}$$

Mathematica [B] time = 0.0931173, size = 164, normalized size = 2.52

$$-\frac{\sin\left(\frac{1}{2}\pi a^2d^2\right)\sin\left(\pi abd^2\log(cx^n)+\frac{1}{2}\pi b^2d^2\log^2(cx^n)\right)}{\pi bdn}+\frac{\cos\left(\frac{1}{2}\pi a^2d^2\right)\cos\left(\pi abd^2\log(cx^n)+\frac{1}{2}\pi b^2d^2\log^2(cx^n)\right)}{\pi bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (Cos[(a^2*d^2*Pi)/2]*Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/
2])/(b*d*n*Pi) + (a*FresnelS[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelS[d*(a
+ b*Log[c*x^n])]*Log[c*x^n])/n - (Sin[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log[c*
```

```
x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)
```

Maple [A] time = 0.216, size = 80, normalized size = 1.2

$$\frac{\ln(cx^n)\,\mathrm{FresnelS}(ad+bd\ln(cx^n))}{n}+\frac{\mathrm{FresnelS}(ad+bd\ln(cx^n))\,a}{bn}+\frac{1}{bdn\pi}\cos\left(\frac{\pi\,(ad+bd\ln(cx^n))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] 1/n*ln(c*x^n)*FresnelS(a*d+b*d*ln(c*x^n))+1/n/b*FresnelS(a*d+b*d*ln(c*x^n))
*a+1/n/b/d/Pi*cos(1/2*Pi*(a*d+b*d*ln(c*x^n))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\left((b\log(cx^n)+a)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}(bd\log(cx^n)+ad)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*d*log(c*x^n) + a*d)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{S(ad+bd\log(cx^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\left((b\log\left(cx^n\right) + a)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x, x)
```

3.58 $\int \frac{S(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=217

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x}$$

```
[Out] ((1/4 - I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Erf[((1/
2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])]
)/x + ((1/4 - I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Er
fi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])])/x - FresnelS[d*(a + b*Log[c*x^n])]/x
```

Rubi [A] time = 0.511188, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] ((1/4 - I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Erf[((1/
2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])]
)/x + ((1/4 - I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Er
fi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])])/x - FresnelS[d*(a + b*Log[c*x^n])]/x
```

Rule 6471

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelS[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Sin[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 4617

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{S(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{S(d(a+b\log(cx^n)))}{x} + (bdn)\int \frac{\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{x^2}\,dx$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(ibdn)\int \frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}\,dx - \frac{1}{2}(ibdn)\int \frac{e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \frac{1}{2}ib^2d^2\pi\log^2(cx\right.}{x^2}$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(cx^n)^{-iabd^2\pi}}{x^2}\,dx$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{1}{2}\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(\right.$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{\left(ibd(cx^n)^{-iabd^2\pi - \frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(-1-iabd^2n}{n}\right.\right.}{2x}$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{\left(ibde^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{-iabd^2\pi - \frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(\frac{i\left(\frac{-1-iabd^2n\pi}{n} - ib^2\right.}{2b^2d^2\pi}\right.\right.}{2x}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} + \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\operatorname{erfi}\left(\right.}{x}$$

Mathematica [A] time = 3.96764, size = 195, normalized size = 0.9

$$-\frac{4S(d(a+b\log(cx^n))) + \sqrt[4]{-1}\sqrt{2}(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\left(e^{\frac{i}{\pi b^2d^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log(cx^n)+i\right)}{\sqrt{\pi}bdn}\right) + i\text{Erfi}\left(\frac{(-1)^{3/4}\left(\pi abd^2n+\pi b^2d^2\right.}{\sqrt{2\pi}bdn}\right.\right.}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*
(I*Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*S
qrt[2*Pi])] + E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(I + a*b*d^2*n*Pi +
b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]) + 4*FresnelS[d*(a + b*Log[c*x^
n])])/(4*x)
```

Maple [F] time = 0.517, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(d(a+b\ln(cx^n)))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\left((b\log\left(cx^{n}\right)+a)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\left(bd\log\left(cx^{n}\right)+ad\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{S\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\left((b\log\left(cx^{n}\right)+a)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x^2, x)
```

3.59 $\int \frac{S(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=228

$$\frac{\left(\frac{1}{8}-\frac{i}{8}\right)(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{x^2}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)(cx^n)^{2/n}e^{-\frac{2(-\pi abd^2n+i)}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n\right.}{\sqrt{\pi}bd}\right.}{x^2}$$

```
[Out] ((1/8 - I/8)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(2/n)*Erf[
((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])
])/x^2 + ((1/8 - I/8)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi +
 I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I - a*b*d^2*n*Pi))/(b^2
*d^2*n^2*Pi))*x^2) - FresnelS[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rubi [A] time = 0.523086, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{8}-\frac{i}{8}\right)(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{x^2}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)(cx^n)^{2/n}e^{-\frac{2(-\pi abd^2n+i)}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n\right.}{\sqrt{\pi}bd}\right.}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] ((1/8 - I/8)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(2/n)*Erf[
((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])
])/x^2 + ((1/8 - I/8)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi +
 I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I - a*b*d^2*n*Pi))/(b^2
*d^2*n^2*Pi))*x^2) - FresnelS[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rule 6471

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelS[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Sin[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 4617

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{S(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{2}(bdn)\int \frac{\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{x^3}\,dx$$

$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{4}(ibdn)\int \frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}\,dx - \frac{1}{4}(ibdn)\int \frac{e^{\frac{1}{2}id^2\pi(a+b\log(c\ldots}}{x^3}$$

$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{4}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \frac{1}{2}ib^2d^2\pi\log^2\ldots\right.}{x^3}$$

$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{4}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(cx^n)^{-iabd^2\pi}}{x^3}$$

$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{4}\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log\ldots\right.$$

$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(ibd(cx^n)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\mathrm{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(-2-iab\ldots}{n}\right.\right.}{4x^2}$$

$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(ibde^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}}(cx^n)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\mathrm{Subst}\left(\int\exp\left(\frac{i\left(\frac{-2-iabd^2\ldots}{n}\right.}{2\ldots}\right.\right.}{4x^2}$$

$$= \frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}(cx^n)^{2/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} + \frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}}(cx^n)\ldots}{\ldots}$$

Mathematica [A] time = 3.98948, size = 200, normalized size = 0.88

$$-\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}-\frac{\sqrt[4]{-1}\left(e^{\frac{4i}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\frac{\sqrt[4]{-1}\left(\pi abd^2n+\pi b^2d^2n\log(cx^n)+2i\right)}{\sqrt{2\pi}bdn}\right)+i\operatorname{Erfi}\left(\frac{(-1)^{3/4}\left(\pi abd^2n+\pi b^2d^2n\log(cx^n)-2i\right)}{\sqrt{2\pi}bdn}\right)\right)\exp\left(\cdots\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*x^n])
))/n^2)*(I*Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n])
)/(b*d*n*Sqrt[2*Pi])] + E^((4*I)/(b^2*d^2*n^2*Pi))*Erfi[((-1)^(1/4)*(2*I +
a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])]))/(4*Sqrt[2])
- FresnelS[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Maple [F] time = 0.522, size = 0, normalized size = 0.

$$\int \frac{\operatorname{FresnelS}\left(d\left(a+b\ln\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{fresnels}\left((b\log\left(cx^n\right)+a)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}\left(bd\log\left(cx^n\right)+ad\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{S\left(ad + bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{fresnels}\left((b\log\left(cx^{n}\right) + a)d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x^3, x)
```

3.60 $\int (ex)^m S\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=280

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)\left(2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2n+i\pi b^2d^2n\log\left(cx^n\right)+m+1\right)}{\sqrt{\pi}bdn}\right)}{m+1}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\text{ex}}{}$$

```
[Out] ((1/4 - I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*x*(e*x)^m*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])])/((1 + m)*(c*x^n)^((1 + m)/n)) + ((1/4 - I/4)*x
*(e*x)^m*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])])/(E^(((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))/(
b^2*d^2*n^2*Pi))*(1 + m)*(c*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*FresnelS[d*(
a + b*Log[c*x^n])])/(e*(1 + m))
```

Rubi [A] time = 0.701812, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {6471, 4617, 2278, 2274, 15, 20, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)\left(2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2n+i\pi b^2d^2n\log\left(cx^n\right)+m+1\right)}{\sqrt{\pi}bdn}\right)}{m+1}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\text{ex}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*FresnelS[d*(a + b*Log[c*x^n])],x]
```


```
[Out] ((1/4 - I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*x*(e*x)^m*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])])/((1 + m)*(c*x^n)^((1 + m)/n)) + ((1/4 - I/4)*x
*(e*x)^m*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])])/(E^(((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))/(
b^2*d^2*n^2*Pi))*(1 + m)*(c*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*FresnelS[d*(
a + b*Log[c*x^n])])/(e*(1 + m))
```


Rule 6471

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelS[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Sin[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 4617

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ex)^m S(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(bdn)\int (ex)^m \sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)dx}{1+m}$$

$$= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(ibdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^m\,dx}{2(1+m)} + \frac{(ibdn)\int e^{\frac{1}{2}id^2\pi(}}{2(}$$

$$= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \frac{1}{2}ib^2d^2\pi}{2(1+m)}$$

$$= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(ex)^m (}{2(1+m)}$$

$$= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2}{2(1+m)}$$

$$= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(ibdnx^{-m+iabd^2n\pi}(ex)^m(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi}{2(1+m)}$$

$$= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(ibdx(ex)^m(cx^n)^{-iabd^2\pi-\frac{1+m-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}\right.\right.}{2(1}$$

$$= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(ibd\exp\left(-\frac{i(1+m)(1+m-2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m(cx^n)^{-iabd^2\pi-\frac{1+}{}}\right.}{}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\exp\left(\frac{i(1+m)(1+m+2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m(cx^n)^{-\frac{1+m}{n}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m+iabd^2n\pi+ib^2d^2n\pi\log(cx}{bdn\sqrt{\pi}}\right.}{1+m}$$

Mathematica [A] time = 5.50402, size = 244, normalized size = 0.87

$$\frac{(ex)^m\left(4xS(d(a+b\log(cx^n))) - \sqrt[4]{-1}\sqrt{2}x^{-m}\exp\left(-\frac{(m+1)(2\pi abd^2n+2\pi b^2d^2n(\log(cx^n)-n\log(x))+im+i)}{2\pi b^2d^2n^2}\right)\left(\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(\pi abd^2n+\pi b^2d^2n\,\text{lo}}{\sqrt{\pi}bdn}\right.\right.\right.}{4(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*(-(((-1)^(1/4)*Sqrt[2]*(Erf[((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi +
 b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] + E^((I*(1 + m)^2)/(b^2*d^2*n^
2*Pi))*Erfi[((-1)^(3/4)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]
))/(b*d*n*Sqrt[2*Pi])]))/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2*d^2
*n*Pi*(-(n*Log[x]) + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m)) + 4*x*FresnelS
[d*(a + b*Log[c*x^n])]))/(4*(1 + m))
```

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int (ex)^m \,\text{FresnelS}(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{fresnels}\left((b\log\left(cx^n\right) + a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*fresnels((b*log(c*x^n) + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((ex)^m \operatorname{fresnels}\left(bd\log\left(cx^n\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*fresnels(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m S\left(ad + bd\log\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*fresnels(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*fresnels(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{fresnels}\left((b\log\left(cx^n\right) + a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*fresnels((b*log(c*x^n) + a)*d), x)
```

3.61 $\int e^{c+\frac{1}{2}ib^2\pi x^2}S(bx)\,dx$

Optimal. Leaf size=64

$$\frac{1}{4}ibe^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)-\frac{e^c\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

```
[Out] -(E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/(8*b) + (I/4)*b*E^c*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]
```

Rubi [A] time = 0.0692265, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6436, 6376, 6375, 30}

$$\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{e^c\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```



```
[Out] -(E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/(8*b) + (I/4)*b*E^c*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]
```


Rule 6436

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[(1 + I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 + I)*b*x)/2], x], x] + Dist[(1 - I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 - I)*b*x)/2], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, -((Pi^2*b^4)/4)]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}S(bx)\,dx = \left(-\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx+\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)d$$
$$=\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{e^c\,\text{Subst}\left(\int x\,dx,x,\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\right)}{4b}$$
$$=-\frac{e^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}+\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.0304085, size = 0, normalized size = 0.

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}S(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

```
[Out] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int e^{c+\frac{i}{2}b^2\pi x^2}\text{FresnelS}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)}\text{fresnels}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)}\text{fresnels}(bx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{c}\int e^{\frac{i\pi b^{2}x^{2}}{2}}S\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\frac{1}{2}i\,\pi b^{2}x^{2}+c\right)}\mathrm{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

3.62 $\int e^{c-\frac{1}{2}ib^2\pi x^2} S(bx)\,dx$

Optimal. Leaf size=64

$$\frac{e^c\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}-\frac{1}{4}ibe^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)$$

```
[Out] (E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/(8*b) - (I/4)*b*E^c*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]
```

Rubi [A] time = 0.0681901, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6436, 6373, 30, 6378}

$$\frac{e^c\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}-\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

```
[Out] (E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/(8*b) - (I/4)*b*E^c*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]
```

Rule 6436

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[(1 + I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 + I)*b*x)/2], x], x] + Dist[(1 - I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 - I)*b*x)/2], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, -((Pi^2*b^4)/4)]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rubi steps

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}S(bx)\,dx = \left(-\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx+\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$
$$= -\frac{1}{4}ibe^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{e^c\,\mathrm{Subst}\left(\int x\,dx,x,\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\right)}{4b}$$
$$= \frac{e^c\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}-\frac{1}{4}ibe^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.0321957, size = 0, normalized size = 0.

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}S(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

```
[Out] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int e^{c-\frac{i}{2}b^2\pi x^2}\mathrm{FresnelS}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)}\mathrm{fresnels}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)}\mathrm{fresnels}(bx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{c}\int e^{-\frac{i\pi b^{2}x^{2}}{2}}S\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(-\frac{1}{2}i\pi b^{2}x^{2}+c\right)}\operatorname{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

3.63 $\int S(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=101

$$-\frac{1}{8}ibx^2\sin(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)+\frac{1}{8}ibx^2\sin(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}$$

```
[Out] (Cos[c]*FresnelS[b*x]^2)/(2*b) + (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
 - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]*Sin[c
] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin[c
]
```

Rubi [A] time = 0.0566188, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6442, 6446, 6440, 30}

$$-\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\sin(c)\text{FresnelC}(bx)S(bx)}{2b}+\frac{\cos(c)S(bx}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```


```
[Out] (Cos[c]*FresnelS[b*x]^2)/(2*b) + (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
 - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]*Sin[c
] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin[c
]
```


Rule 6442

```
Int[FresnelS[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[Sin[c]
, Int[Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[Cos[c], Int[Sin[d*x^2]*Fresne
lS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int S(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)dx &= \cos(c)\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx+\sin(c)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx\\ &= \frac{C(bx)S(bx)\sin(c)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\right.\\ &= \frac{\cos(c)S(bx)^2}{2b}+\frac{C(bx)S(bx)\sin(c)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c)+\frac{1}{8}ibx^2\,{}_2\end{aligned}$$

Mathematica [F] time = 0.0490403, size = 0, normalized size = 0.

$$\int S(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

```
[Out] Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \mathrm{FresnelS}\left(bx\right)\sin\left(c+\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

```
[Out] int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{\pi b^2 x^2}{2} + c\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(c+1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2 + c)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

3.64 $\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=101

$$-\frac{1}{8}ibx^2\cos(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)+\frac{1}{8}ibx^2\cos(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\}\right.$$

```
[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*Cos[c]*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] + (I/8)*b*x^2*Cos[c]*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] - (FresnelS[b*x]^2*Sin[c])/(2*b
)
```

Rubi [A] time = 0.049668, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6448, 6446, 6440, 30}

$$-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\cos(c)\text{FresnelC}(bx)S(bx)}{2b}-\frac{\sin(c)S}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*Cos[c]*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] + (I/8)*b*x^2*Cos[c]*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] - (FresnelS[b*x]^2*Sin[c])/(2*b
)
```

Rule 6448

```
Int[Cos[(c_) + (d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[Cos[c]
, Int[Cos[d*x^2]*FresnelS[b*x], x], x] - Dist[Sin[c], Int[Sin[d*x^2]*Fresne
lS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx = \cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx - \sin(c)\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= \frac{\cos(c)C(bx)S(bx)}{2b} - \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\right.$$
$$= \frac{\cos(c)C(bx)S(bx)}{2b} - \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\right.$$

Mathematica [F] time = 0.043668, size = 0, normalized size = 0.

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \cos\left(c+\frac{b^2\pi x^2}{2}\right)\mathrm{FresnelS}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2 x^2 + c\right)\mathrm{fresnels}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos\left(\frac{1}{2}\pi b^2 x^2 + c\right)\mathrm{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{\pi b^2 x^2}{2} + c\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2 + c)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2 x^2 + c\right) \operatorname{fresnels}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

3.65 $\int S(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=13

$$\frac{S(bx)^3}{3b}$$

[Out] FresnelS[b*x]^3/(3*b)

Rubi [A] time = 0.0147598, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6440, 30}

$$\frac{S(bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^3/(3*b)

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int S(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, S(bx)\right)}{b}\\ &= \frac{S(bx)^3}{3b}\end{aligned}$$

Mathematica [A] time = 0.0037452, size = 13, normalized size = 1.

$$\frac{S(bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^3/(3*b)

Maple [A] time = 0.046, size = 12, normalized size = 0.9

$$\frac{(\mathrm{FresnelS}\,(bx))^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] 1/3*FresnelS(b*x)^3/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{fresnels}\,(bx)^2 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{fresnels}\,(bx)^2 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 1.4036, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^3(bx)}{3b} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((fresnels(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{fresnels}\,(bx)^2 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2*sin(1/2*pi*b^2*x^2), x)
```

3.66 $\int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=13

$$\frac{S(bx)^2}{2b}$$

[Out] FresnelS[b*x]^2/(2*b)

Rubi [A] time = 0.0113005, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6440, 30}

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^2/(2*b)

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= \frac{\text{Subst}\left(\int x\,dx, x, S(bx)\right)}{b}\\ &= \frac{S(bx)^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.0026855, size = 13, normalized size = 1.

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^2/(2*b)

Maple [A] time = 0.045, size = 12, normalized size = 0.9

$$\frac{(\text{FresnelS}(bx))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] 1/2*FresnelS(b*x)^2/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 0.434449, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.67 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(S(bx))}{b}$$

```
[Out] Log[FresnelS[b*x]]/b
```

Rubi [A] time = 0.0147908, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6440, 29}

$$\frac{\log(S(bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x],x]
```

```
[Out] Log[FresnelS[b*x]]/b
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, S(bx)\right)}{b}\\ &= \frac{\log(S(bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.010098, size = 9, normalized size = 1.

$$\frac{\log(S(bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x],x]
```

```
[Out] Log[FresnelS[b*x]]/b
```

Maple [A] time = 0.118, size = 10, normalized size = 1.1

$$\frac{\ln\left(\text{FresnelS}\left(bx\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x),x)
```

```
[Out] ln(FresnelS(b*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnels}\left(bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnels}\left(bx\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(sin(1/2*pi*b^2*x^2)/fresnels(b*x), x)
```

Sympy [A] time = 0.235245, size = 8, normalized size = 0.89

$$\begin{cases} \frac{\log\left(S\left(bx\right)\right)}{b} & \text{for}\ b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x),x)
```

```
[Out] Piecewise((log(fresnels(b*x))/b, Ne(b, 0)), (nan, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnels}\left(bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x), x)
```

3.68 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^2}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{bS(bx)}$$

```
[Out] -(1/(b*FresnelS[b*x]))
```

Rubi [A] time = 0.0148235, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6440, 30}

$$-\frac{1}{bS(bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]

[Out] -(1/(b*FresnelS[b*x]))
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, S(bx)\right)}{b}\\ &= -\frac{1}{bS(bx)}\end{aligned}$$

Mathematica [A] time = 0.0029559, size = 11, normalized size = 1.

$$-\frac{1}{bS(bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]

[Out] -(1/(b*FresnelS[b*x]))
```

Maple [A] time = 0.043, size = 12, normalized size = 1.1

$$-\frac{1}{b\text{FresnelS}\left(bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^2,x)
```

```
[Out] -1/b/FresnelS(b*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnels}\left(bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnels}\left(bx\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^2, x)
```

Sympy [A] time = 0.779841, size = 10, normalized size = 0.91

$$\begin{cases} -\frac{1}{bS(bx)} & \text{for}\ b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**2,x)
```

```
[Out] Piecewise((-1/(b*fresnels(b*x)), Ne(b, 0)), (nan, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\text{fresnels}\,(bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^2, x)
```

3.69 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2bS(bx)^2}$$

```
[Out] -1/(2*b*FresnelS[b*x]^2)
```

Rubi [A] time = 0.0149547, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6440, 30}

$$-\frac{1}{2bS(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]
```

```
[Out] -1/(2*b*FresnelS[b*x]^2)
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, S(bx)\right)}{b}\\ &= -\frac{1}{2bS(bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0040066, size = 13, normalized size = 1.

$$-\frac{1}{2bS(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]
```

```
[Out] -1/(2*b*FresnelS[b*x]^2)
```

Maple [A] time = 0.044, size = 12, normalized size = 0.9

$$-\frac{1}{2\,b\,(\mathrm{FresnelS}\,(bx))^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^3,x)
```

```
[Out] -1/2/b/FresnelS(b*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\mathrm{fresnels}\,(bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\mathrm{fresnels}\,(bx)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^3, x)
```

Sympy [A] time = 1.93431, size = 14, normalized size = 1.08

$$\begin{cases} -\frac{1}{2bS^{2}(bx)} & \text{for}\ b \neq 0 \\ \mathrm{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**3,x)
```

```
[Out] Piecewise((-1/(2*b*fresnels(b*x)**2), Ne(b, 0)), (nan, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}\,(bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^3, x)
```

3.70 $\int S(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=17

$$\frac{S(bx)^{n+1}}{b(n+1)}$$

```
[Out] FresnelS[b*x]^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.0176465, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6440, 30}

$$\frac{S(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^(1 + n)/(b*(1 + n))
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int S(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= \frac{\text{Subst}\left(\int x^n\, dx, x, S(bx)\right)}{b}\\ &= \frac{S(bx)^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0057795, size = 17, normalized size = 1.

$$\frac{S(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^(1 + n)/(b*(1 + n))
```

Maple [A] time = 0.046, size = 18, normalized size = 1.1

$$\frac{(\text{FresnelS}(bx))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] FresnelS(b*x)^(1+n)/b/(1+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)^n \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}(bx)^n \sin\left(\frac{1}{2}\pi b^2x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 4.62388, size = 31, normalized size = 1.82

$$\begin{cases} 0 & \text{for } b = 0 \wedge (b = 0 \vee n = -1) \\ \frac{\log(S(bx))}{b} & \text{for } n = -1 \\ \frac{S(bx)S^n(bx)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**n*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(n, -1))), (log(fresnels(b*x))/b, Eq
(n, -1)), (fresnels(b*x)*fresnels(b*x)**n/(b*n + b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)^n \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

3.71 $\int x^8 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=232

$$\frac{7x^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{35x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{105S(bx)^2}{2\pi^4b^9}-\frac{}{12}$$

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)
/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^
2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Rubi [A] time = 0.384108, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6454, 6462, 3379, 3309, 30, 3296, 2637, 2634, 6440}

$$\frac{7x^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{35x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{105S(bx)^2}{2\pi^4b^9}-\frac{}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```



```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)
/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^
2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```


Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int x^8 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x^7\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$

$$= -\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{7x^5S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{35\int x^4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^4\pi^2}-\ldots$$

$$= -\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{7x^5S(bx)\sin\left(\ldots\right)}{b^4\pi^2}\ldots$$

$$= -\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}-\frac{105xS(bx)\sin\ldots}{b^8\pi^4}\ldots$$

$$= -\frac{7x^6}{12b^3\pi^2}+\frac{41x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4}-\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^7\cos\left(\ldots\right)}{\ldots}\ldots$$

$$= \frac{105x^2}{4b^7\pi^4}-\frac{7x^6}{12b^3\pi^2}+\frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4}-\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\ldots$$

$$= \frac{105x^2}{4b^7\pi^4}-\frac{7x^6}{12b^3\pi^2}+\frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4}-\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\ldots$$

Mathematica [A] time = 0.0130934, size = 232, normalized size = 1.

$$\frac{7x^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{35x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{105S(bx)^2}{2\pi^4b^9}-\frac{}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)
/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^
2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int x^8\text{FresnelS}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^8*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^8\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^8*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^8*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.72 $\int x^7 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=216

$$-\frac{531\text{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}\pi^4b^8}+\frac{6x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{48S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{24x^2S(bx)\cos\left(\frac{1}{2}\pi\right.}{\pi^3b^6}$$

```
[Out] (24*x)/(b^7*Pi^4) - (3*x^5)/(5*b^3*Pi^2) + (147*x*Cos[b^2*Pi*x^2])/(16*b^7*
Pi^4) - (x^5*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (531*FresnelC[Sqrt[2]*b*x])/(1
6*Sqrt[2]*b^8*Pi^4) + (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3)
 - (x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (48*FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b
^4*Pi^2) + (17*x^3*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)
```

Rubi [A] time = 0.264901, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6454, 6462, 3391, 30, 3386, 3385, 3352, 6460, 3357}

$$-\frac{531\text{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}\pi^4b^8}+\frac{6x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{48S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{24x^2S(bx)\cos\left(\frac{1}{2}\pi\right.}{\pi^3b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (24*x)/(b^7*Pi^4) - (3*x^5)/(5*b^3*Pi^2) + (147*x*Cos[b^2*Pi*x^2])/(16*b^7*
Pi^4) - (x^5*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (531*FresnelC[Sqrt[2]*b*x])/(1
6*Sqrt[2]*b^8*Pi^4) + (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3)
 - (x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (48*FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b
^4*Pi^2) + (17*x^3*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\int x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x^6\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$

$$= -\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{6x^4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{24\int x^3S(bx)\sin\cdots}{b^4\pi\cdots}$$

$$= -\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{6x^4S(bx)\sin\cdots}{b^4\pi^2}\cdots$$

$$= -\frac{3x^5}{5b^3\pi^2}+\frac{111x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4}-\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^6\cos\left(\frac{1}{2}\cdots\right.}{\cdots}$$

$$= -\frac{3x^5}{5b^3\pi^2}+\frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4}-\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{15C\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4}-\frac{3\sqrt{2}C\left(\sqrt{2}bx\right)}{b^8\pi^4}+\frac{24x^2\cdots}{\cdots}$$

$$= \frac{24x}{b^7\pi^4}-\frac{3x^5}{5b^3\pi^2}+\frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4}-\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{51C\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4}-\frac{3\sqrt{2}C\left(\sqrt{2}bx\right)}{b^8\pi^4}\cdots$$

$$= \frac{24x}{b^7\pi^4}-\frac{3x^5}{5b^3\pi^2}+\frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4}-\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{51C\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4}-\frac{15\sqrt{2}C\left(\sqrt{2}bx\right)}{b^8\pi^4}\cdots$$

Mathematica [A] time = 0.276432, size = 153, normalized size = 0.71

$$\frac{-160S(bx)\left(\pi b^2x^2\left(\pi^2b^4x^4-24\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)-6\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)+2bx\left(2\left(-24\pi^2b^4x^4+85\pi b^2x^2\sin\left(\pi b^2\right.\right.\right.}{160\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 160*FresnelS[b*x]*(b^2*Pi*x^2*(-24 +
 b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)
/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] + 2*(960 - 24*b^4*Pi^
2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/(160*b^8*Pi^4)
```

Maple [A] time = 0.076, size = 318, normalized size = 1.5

$$\frac{1}{b}\left(\frac{\mathrm{FresnelS}\left(bx\right)}{b^7}\left(-\frac{b^6x^6}{\pi}\cos\left(\frac{b^2\pi x^2}{2}\right)+6\frac{1}{\pi}\left(\frac{x^4b^4\sin\left(1/2\,b^2\pi x^2\right)}{\pi}-4\frac{1}{\pi}\left(-\frac{b^2x^2\cos\left(1/2\,b^2\pi x^2\right)}{\pi}+2\frac{\sin\left(1/2\,b^2\pi x^2\right)}{\pi^2}\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelS(b*x)/b^7*(-1/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/Pi*(1/Pi*b^4*x^4*si
n(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^
2*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*Pi^2*b^5*x^5-8*b*x)-3/Pi^4*(1/2*Pi*b^3*x^3*
sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC
(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))-1/2/Pi^3*(-1/2*Pi*b^5*x^5*c
os(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*c
os(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*
x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^7\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnels}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.73 $\int x^6 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=248

$$\frac{15ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi^3b^5}-\frac{15ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi^3b^5}-15\text{Fres}$$

```
[Out] (-5*x^4)/(8*b^3*Pi^2) + (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) - (x^4*Cos[b^2*Pi
*x^2])/(4*b^3*Pi^2) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) -
 (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (15*FresnelC[b*x]*Fresn
elS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, (-I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) + (5*x^3*FresnelS[b*x]*Sin[(b^2*P
i*x^2)/2])/(b^4*Pi^2) + (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```

Rubi [A] time = 0.254643, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6454, 6462, 3379, 3309, 30, 3296, 2638, 6446}

$$\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15\text{FresnelC}(bx)S(bx)}{2\pi^3b^7}+\frac{5x^3S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^5S(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```


```
[Out] (-5*x^4)/(8*b^3*Pi^2) + (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) - (x^4*Cos[b^2*Pi
*x^2])/(4*b^3*Pi^2) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) -
 (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (15*FresnelC[b*x]*Fresn
elS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, (-I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) + (5*x^3*FresnelS[b*x]*Sin[(b^2*P
i*x^2)/2])/(b^4*Pi^2) + (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```


Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}
\int x^6 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x^5\sin\left(b^2\pi x^2\right)dx}{2b\pi}\\
&= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{5x^3S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{15\int x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^4\pi^2}-\ldots\\
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{5x^3S(bx)\sin\left(\ldots\right)}{b^4\pi^2}\ldots\\
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}-\frac{15C(bx)S(bx)}{2b^7\pi^3}\ldots\\
&= -\frac{5x^4}{8b^3\pi^2}+\frac{17\cos\left(b^2\pi x^2\right)}{4b^7\pi^4}-\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\ldots\right)}{b^2\pi}\ldots\\
&= -\frac{5x^4}{8b^3\pi^2}+\frac{11\cos\left(b^2\pi x^2\right)}{2b^7\pi^4}-\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\ldots\right)}{b^2\pi}\ldots
\end{aligned}$$

Mathematica [F] time = 0.468768, size = 0, normalized size = 0.

$$\int x^6 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x^{6}\mathrm{FresnelS}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{6}\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{6}\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^6*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.74 $\int x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=158

$$\frac{4x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{8S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{43S\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3b^6}-\frac{2x^3}{3\pi^2b^3}+\frac{11x\sin\left(\pi b^2x^2\right)}{8\pi^3b^5}-\frac{x^3}{}$$

```
[Out] (-2*x^3)/(3*b^3*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (8*Cos[(b^2*Pi
*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]
)/(b^2*Pi) - (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*Fresn
elS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi
^3)
```

Rubi [A] time = 0.156292, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6454, 6462, 3391, 30, 3386, 3351, 6452, 3385}

$$\frac{4x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{8S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{43S\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3b^6}-\frac{2x^3}{3\pi^2b^3}+\frac{11x\sin\left(\pi b^2x^2\right)}{8\pi^3b^5}-\frac{x^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2*x^3)/(3*b^3*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (8*Cos[(b^2*Pi
*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]
)/(b^2*Pi) - (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*Fresn
elS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi
^3)
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$
\begin{aligned}
\int x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x^4\sin\left(b^2\pi x^2\right)dx}{2b\pi}\\
&= -\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{4x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{8\int xS(bx)\sin\left(\right.}{b^4\pi^2}\\
&= -\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{4x^2S(bx)\sin\left(\frac{1}{2}b^2\right.}{b^4\pi^2}\\
&= -\frac{2x^3}{3b^3\pi^2}-\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}-\frac{3S\left(\sqrt{2}b\right.}{8\sqrt{2}b^6\pi}\\
&= -\frac{2x^3}{3b^3\pi^2}-\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}-\frac{11S\left(\sqrt{2}\right.}{8\sqrt{2}b^6}
\end{aligned}
$$

Mathematica [A] time = 0.172595, size = 120, normalized size = 0.76

$$
-\frac{48S(bx)\left(\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)-4\pi b^2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)+32\pi b^3x^3-66bx\sin\left(\pi b^2x^2\right)+12\pi b^3x^3\cos\left(\pi b^2x^2\right)}{48\pi^3b^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -(32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*Sqrt[2]*FresnelS[Sqrt
[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 4*b^
```

```
2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(48*b^6*Pi^3)
```

Maple [A] time = 0.078, size = 202, normalized size = 1.3

$$\frac{1}{b}\left(\frac{\mathrm{FresnelS}\,(bx)}{b^5}\left(-\frac{x^4b^4}{\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right)+4\,\frac{1}{\pi}\left(\frac{b^2x^2\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi}+2\,\frac{\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right)\right)-\frac{1}{b^5}\left(\frac{2\,x^3b^3}{3\,\pi^2}-2\,\frac{1}{\pi^2}\left(1/2\,\frac{bx\,\mathrm{s}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelS(b*x)/b^5*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))-1/b^5*(2/3/Pi^2*b^3*x^3-2/Pi
^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi
^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/
Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^5\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\sin\left(\frac{\pi b^2x^2}{2}\right)S\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**5*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnels}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.75 $\int x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=120

$$\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{3S(bx)^2}{2\pi^2b^5}-\frac{3x^2}{4\pi^2b^3}+\frac{\sin\left(\pi b^2x^2\right)}{\pi^3b^5}-\frac{x^2\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

```
[Out] (-3*x^2)/(4*b^3*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (x^3*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (3*FresnelS[b*x]^2)/(2*b^5*Pi^2) + (3*
x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Rubi [A] time = 0.118406, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6454, 6462, 3379, 2634, 6440, 30, 3296, 2637}

$$\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{3S(bx)^2}{2\pi^2b^5}-\frac{3x^2}{4\pi^2b^3}+\frac{\sin\left(\pi b^2x^2\right)}{\pi^3b^5}-\frac{x^2\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (x^3*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (3*FresnelS[b*x]^2)/(2*b^5*Pi^2) + (3*
x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi} + \frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$

$$= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^4\pi^2} - \frac{3\int x}{}$$

$$= -\frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3\,\mathrm{Subst}(\int x\,dx, x}{b^5\pi^2}$$

$$= -\frac{3x^2}{4b^3\pi^2} - \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} - \frac{3S(bx)^2}{2b^5\pi^2} + \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$

Mathematica [A] time = 0.0058869, size = 120, normalized size = 1.

$$\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{3S(bx)^2}{2\pi^2b^5} - \frac{3x^2}{4\pi^2b^3} + \frac{\sin\left(\pi b^2x^2\right)}{\pi^3b^5} - \frac{x^2\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (x^3*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (3*FresnelS[b*x]^2)/(2*b^5*Pi^2) + (3*
x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x^4 \text{FresnelS}\,(bx) \sin\left(\frac{b^2\pi\,x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnels}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \text{fresnels}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 26.5934, size = 151, normalized size = 1.26

$$\begin{cases} -\frac{x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{\pi b^2} - \frac{x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} - \frac{x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^2 b^3} + \frac{3x \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{\pi^2 b^4} + \frac{2 \sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^3 b^5} - \frac{3 S^2(bx)}{2\pi^2 b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((-x**3*cos(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) - x**2*sin(pi*
b**2*x**2/2)**2/(2*pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(pi**2*b**3) +
 3*x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*sin(pi*b**2*x**2/2)
*cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnels(b*x)**2/(2*pi**2*b**5), Ne(b
, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnels}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.76 $\int x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=105

$$\frac{5\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2b^4}+\frac{2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x}{\pi^2b^3}$$

```
[Out] -(x/(b^3*Pi^2)) - (x*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (5*FresnelC[Sqrt[2]*b*
x])/(4*Sqrt[2]*b^4*Pi^2) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)
 + (2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2)
```

Rubi [A] time = 0.081234, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6454, 6460, 3357, 3352, 3385}

$$\frac{5\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2b^4}+\frac{2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x}{\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -(x/(b^3*Pi^2)) - (x*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (5*FresnelC[Sqrt[2]*b*
x])/(4*Sqrt[2]*b^4*Pi^2) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)
 + (2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2)
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
```

```
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\int x^3S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi} + \frac{\int x^2\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$

$$= -\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{\int\cos\left(b^2\pi x^2\right)dx}{4b^3\pi^2}$$

$$= -\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{C\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} - \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - 2\int\left(\frac{1}{2}\right.$$

$$= -\frac{x}{b^3\pi^2} - \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{C\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} - \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$

$$= -\frac{x}{b^3\pi^2} - \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5C\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} - \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$

Mathematica [A] time = 0.102146, size = 83, normalized size = 0.79

$$\frac{-8S(bx)\left(\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)-2\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)-2bx\left(\cos\left(\pi b^2x^2\right)+4\right)+5\sqrt{2}\text{FresnelC}\left(\sqrt{2}bx\right)}{8\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2*b*x*(4 + Cos[b^2*Pi*x^2]) + 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 8*Fresnel
S[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]))/(8*b^4*Pi^
2)
```

Maple [A] time = 0.083, size = 115, normalized size = 1.1

$$\frac{1}{b}\left(\frac{\text{FresnelS}\,(bx)}{b^3}\left(-\frac{b^2x^2}{\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right)+2\,\frac{\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right)-\frac{1}{b^3}\left(\frac{bx}{\pi^2}-\frac{\sqrt{2}\text{FresnelC}\left(bx\sqrt{2}\right)}{2\,\pi^2}-\frac{1}{2\,\pi}\left(-\frac{bx\cos\left(b^2\pi\,x^2\right)}{2\,\pi}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelS(b*x)/b^3*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2))-1/b^3*(1/Pi^2*b*x-1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))-1/2/Pi*(-1/
2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3\text{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sin\left(\frac{\pi b^2x^2}{2}\right)S\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**3*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\text{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.77 $\int x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=137

$$-\frac{ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi b}+\frac{ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi b}+\text{Fresn}$$

```
[Out] -Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*
Pi) + (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - ((I/8)*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b*Pi) + ((I/8)*x^2*Hypergeometric
PFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi)
```

Rubi [A] time = 0.0606421, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6454, 6446, 3379, 2638}

$$-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^3}-\frac{xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2\right.}{4\pi^2 b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*
Pi) + (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - ((I/8)*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b*Pi) + ((I/8)*x^2*Hypergeometric
PFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi)
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$
$$= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{C(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\right.}{8b\pi}$$
$$= -\frac{\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{C(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{ix^2}{}$$

Mathematica [F] time = 0.215816, size = 0, normalized size = 0.

$$\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x^2 \mathrm{FresnelS}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)

[Out] int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(x^2*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sin\left(\frac{\pi b^2x^2}{2}\right)S\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**2*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.78 $\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=49

$$\frac{S\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + FresnelS[Sqrt[2]*b*x]/(2*
Sqrt[2]*b^2*Pi)
```

Rubi [A] time = 0.0216188, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6452, 3351}

$$\frac{S\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + FresnelS[Sqrt[2]*b*x]/(2*
Sqrt[2]*b^2*Pi)
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx&=-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{\int\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\&=-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{S\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi}\end{aligned}$$

Mathematica [A] time = 0.0244937, size = 44, normalized size = 0.9

$$\frac{\sqrt{2}S\left(\sqrt{2}bx\right)-4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x] + Sqrt[2]*FresnelS[Sqrt[2]*b*x])/(4*b
^2*Pi)
```

Maple [A] time = 0.063, size = 46, normalized size = 0.9

$$\frac{1}{b}\left(-\frac{\mathrm{FresnelS}(bx)}{b\pi}\cos\left(\frac{b^2\pi x^2}{2}\right)+\frac{\sqrt{2}\mathrm{FresnelS}\left(bx\sqrt{2}\right)}{4b\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (-cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^(
1/2))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\mathrm{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.79 $\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=13

$$\frac{S(bx)^2}{2b}$$

```
[Out] FresnelS[b*x]^2/(2*b)
```

Rubi [A] time = 0.0114849, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6440, 30}

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^2/(2*b)
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, S(bx)\right)}{b}\\ &= \frac{S(bx)^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.0019935, size = 13, normalized size = 1.

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^2/(2*b)
```

Maple [A] time = 0.046, size = 12, normalized size = 0.9

$$\frac{(\text{FresnelS}(bx))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] 1/2*FresnelS(b*x)^2/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 0.451291, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.80 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)$$

```
[Out] Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Rubi [A] time = 0.0183464, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

```
[Out] Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx=\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [A] time = 0.0302, size = 0, normalized size = 0.

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Maple [A] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x}\sin\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

3.81 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)$$

```
[Out] Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]
```

Rubi [A] time = 0.0184603, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

```
[Out] Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx=\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Mathematica [A] time = 0.0304836, size = 0, normalized size = 0.

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]
```

Maple [A] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}\,(bx)}{x^2}\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S\left(bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

3.82 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$

Optimal. Leaf size=101

$$\frac{1}{2}\pi b^2\text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{\pi b^2S\left(\sqrt{2}bx\right)}{2\sqrt{2}}+\frac{b\cos\left(\pi b^2x^2\right)}{4x}-\frac{b}{4x}$$

```
[Out] -b/(4*x) + (b*Cos[b^2*Pi*x^2])/(4*x) + (b^2*Pi*FresnelS[Sqrt[2]*b*x])/(2*Sq
rt[2]) - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*x^2) + (b^2*Pi*Unintegrable
[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/2
```

Rubi [A] time = 0.0584933, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] -b/(4*x) + (b*Cos[b^2*Pi*x^2])/(4*x) + (b^2*Pi*FresnelS[Sqrt[2]*b*x])/(2*Sq
rt[2]) - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*x^2) + (b^2*Pi*Defer[Int][(
Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx &= -\frac{b}{4x}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}-\frac{1}{4}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx\\ &= -\frac{b}{4x}+\frac{b\cos\left(b^2\pi x^2\right)}{4x}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx+\frac{1}{2}\left(b^3\pi\right)\\ &= -\frac{b}{4x}+\frac{b\cos\left(b^2\pi x^2\right)}{4x}+\frac{b^2\pi S\left(\sqrt{2}bx\right)}{2\sqrt{2}}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{x}\end{aligned}$$

Mathematica [A] time = 0.0303193, size = 0, normalized size = 0.

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]
```

Maple [A] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\left(bx\right)}{x^{3}}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)S\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

3.83 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$

Optimal. Leaf size=109

$$-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{\pi b^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{1}{6}\pi^2b^3S(bx)^2+\frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right)+\frac{b\cos\left(\pi b^2x^2\right)}{12x^2}-\frac{b}{12x^2}$$

```
[Out] -b/(12*x^2) + (b*Cos[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*Fr
esnelS[b*x])/(3*x) - (b^3*Pi^2*FresnelS[b*x]^2)/6 - (FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Rubi [A] time = 0.116202, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6456, 6464, 6440, 30, 3375, 3380, 3297, 3299}

$$-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{\pi b^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{1}{6}\pi^2b^3S(bx)^2+\frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right)+\frac{b\cos\left(\pi b^2x^2\right)}{12x^2}-\frac{b}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] -b/(12*x^2) + (b*Cos[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*Fr
esnelS[b*x])/(3*x) - (b^3*Pi^2*FresnelS[b*x]^2)/6 - (FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx &= -\frac{b}{12x^2}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}-\frac{1}{6}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx+\frac{1}{3}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^2}\,dx\\ &= -\frac{b}{12x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}-\frac{1}{12}b\,\mathrm{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^2}\,dx,x,x\right.\\ &= -\frac{b}{12x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{12x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}+\frac{1}{12}b^3\pi\mathrm{Si}\left(b^2\pi x^2\right)\\ &= -\frac{b}{12x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{12x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x}-\frac{1}{6}b^3\pi^2S(bx)^2-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}+\frac{1}{6}\end{aligned}$$

Mathematica [A] time = 0.007643, size = 109, normalized size = 1.

$$-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{\pi b^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{1}{6}\pi^2b^3S(bx)^2+\frac{1}{6}\pi b^3\mathrm{Si}\left(b^2\pi x^2\right)+\frac{b\cos\left(\pi b^2x^2\right)}{12x^2}-\frac{b}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] -b/(12*x^2) + (b*Cos[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*Fr
esnelS[b*x])/(3*x) - (b^3*Pi^2*FresnelS[b*x]^2)/6 - (FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\,(bx)}{x^4}\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

3.84 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

Optimal. Leaf size=152

$$-\frac{1}{8}\pi^2 b^4 \text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{7\pi^2b^4\text{FresnelC}\left(\sqrt{2}bx\right)}{24\sqrt{2}}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{\pi b^2S(bx)\cos\left(\frac{1}{2}\pi\right.}{8x^2}$$

```
[Out] -b/(24*x^3) + (b*Cos[b^2*Pi*x^2])/(24*x^3) + (7*b^4*Pi^2*FresnelC[Sqrt[2]*b
*x])/(24*Sqrt[2]) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(8*x^2) - (F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*x^4) - (7*b^3*Pi*Sin[b^2*Pi*x^2])/(48*
x) - (b^4*Pi^2*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/8
```

Rubi [A] time = 0.121331, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

```
[Out] -b/(24*x^3) + (b*Cos[b^2*Pi*x^2])/(24*x^3) + (7*b^4*Pi^2*FresnelC[Sqrt[2]*b
*x])/(24*Sqrt[2]) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(8*x^2) - (F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*x^4) - (7*b^3*Pi*Sin[b^2*Pi*x^2])/(48*
x) - (b^4*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx &= -\frac{b}{24x^3}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx+\frac{1}{4}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,d\\
&= -\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^2}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}+\frac{1}{16}\left(b^3\pi\right)\int\frac{\mathrm{s}}{}\\
&= -\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^2}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{7b^3\pi\sin\left(b^2\pi\right.}{48x}\\
&= -\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}+\frac{7b^4\pi^2C\left(\sqrt{2}bx\right)}{24\sqrt{2}}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^2}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x\right.}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.0303067, size = 0, normalized size = 0.

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]
```

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}(bx)}{x^5}\sin\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)
```

<u>**Giac**</u> **[A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

3.85 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

Optimal. Leaf size=147

$$-\frac{1}{15}\pi^2 b^4 \text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2},x\right)+\frac{1}{24}\pi^2 b^5 \text{CosIntegral}\left(\pi b^2 x^2\right)-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5}-\frac{\pi b^2 S(bx)\cos\left(\right.}{15x^3}$$

```
[Out] -b/(40*x^4) + (b*Cos[b^2*Pi*x^2])/(40*x^4) + (b^5*Pi^2*CosIntegral[b^2*Pi*x
^2])/24 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15*x^3) - (FresnelS[b
*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) - (b^3*Pi*Sin[b^2*Pi*x^2])/(24*x^2) - (b^4
*Pi^2*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/15
```

Rubi [A] time = 0.192541, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -b/(40*x^4) + (b*Cos[b^2*Pi*x^2])/(40*x^4) + (b^5*Pi^2*CosIntegral[b^2*Pi*x
^2])/24 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15*x^3) - (FresnelS[b
*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) - (b^3*Pi*Sin[b^2*Pi*x^2])/(24*x^2) - (b^4
*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/15
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx &= -\frac{b}{40x^4}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}-\frac{1}{10}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx+\frac{1}{5}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx\\
&= -\frac{b}{40x^4}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}-\frac{1}{20}b\,\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^3}\,dx,x,x\right.\\
&= -\frac{b}{40x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}+\frac{1}{60}\left(b^3\pi\right)\text{Subst}\left(\right.\\
&= -\frac{b}{40x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}-\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{24x^2}\\
&= -\frac{b}{40x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}+\frac{1}{24}b^5\pi^2\text{Ci}\left(b^2\pi x^2\right)-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}
\end{aligned}$$

Mathematica [A] time = 0.0314466, size = 0, normalized size = 0.

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6, x]
```

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\left(bx\right)}{x^{6}}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)S\left(bx\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

3.86 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

Optimal. Leaf size=240

$$-\frac{1}{48}\pi^3b^6\text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{\pi^2b^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{48x^2}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{\pi b^2S(bx)\cos\left(\frac{1}{2}\right.}{24x^4}$$

```
[Out] -b/(60*x^5) + (b^5*Pi^2)/(96*x) + (b*Cos[b^2*Pi*x^2])/(60*x^5) - (67*b^5*Pi
^2*Cos[b^2*Pi*x^2])/(1440*x) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
24*x^4) - (7*b^6*Pi^3*FresnelS[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*P
i^3*FresnelS[Sqrt[2]*b*x])/45 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^6)
 + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^2) - (13*b^3*Pi*Sin[b
^2*Pi*x^2])/(720*x^3) - (b^6*Pi^3*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*Fresnel
S[b*x])/x, x])/48
```

Rubi [A] time = 0.206718, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

```
[Out] -b/(60*x^5) + (b^5*Pi^2)/(96*x) + (b*Cos[b^2*Pi*x^2])/(60*x^5) - (67*b^5*Pi
^2*Cos[b^2*Pi*x^2])/(1440*x) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
24*x^4) - (7*b^6*Pi^3*FresnelS[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*P
i^3*FresnelS[Sqrt[2]*b*x])/45 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^6)
 + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^2) - (13*b^3*Pi*Sin[b
^2*Pi*x^2])/(720*x^3) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/x, x])/48
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx=-\frac{b}{60x^5}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}-\frac{1}{12}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx+\frac{1}{6}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}$$

$$=-\frac{b}{60x^5}+\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}+\frac{1}{48}\left(b^3\pi\right)\int$$

$$=-\frac{b}{60x^5}+\frac{b^5\pi^2}{96x}+\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}+$$

$$=-\frac{b}{60x^5}+\frac{b^5\pi^2}{96x}+\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}-\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4}-$$

$$=-\frac{b}{60x^5}+\frac{b^5\pi^2}{96x}+\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}-\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4}-$$

Mathematica [A] time = 0.0298719, size = 0, normalized size = 0.

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\,(bx)}{x^7}\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)S\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

3.87 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

Optimal. Leaf size=224

$$\frac{\pi^2 b^4 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{105x^3}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{\pi^3 b^6 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{105x}-\frac{\pi b^2 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}+\frac{1}{210}\pi^4 b^7 S(bx)^2 -$$

```
[Out] -b/(84*x^6) + (b^5*Pi^2)/(420*x^2) + (b*Cos[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
35*x^5) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x) + (b^7*Pi^4*
FresnelS[b*x]^2)/210 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*P
i^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) - (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Rubi [A] time = 0.353489, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6456, 6464, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi^2 b^4 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{105x^3}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{\pi^3 b^6 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{105x}-\frac{\pi b^2 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}+\frac{1}{210}\pi^4 b^7 S(bx)^2 -$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```


```
[Out] -b/(84*x^6) + (b^5*Pi^2)/(420*x^2) + (b*Cos[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
35*x^5) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x) + (b^7*Pi^4*
FresnelS[b*x]^2)/210 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*P
i^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) - (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```


Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx &= -\frac{b}{84x^6} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} - \frac{1}{14}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx + \frac{1}{7}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^6}\\
&= -\frac{b}{84x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} - \frac{1}{28}b\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^4}\,dx,\right.\\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x} - \\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5} + \\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5} +
\end{aligned}$$

Mathematica [A] time = 0.012694, size = 224, normalized size = 1.

$$\frac{\pi^2 b^4 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{105x^3}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{\pi^3 b^6 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{105x}-\frac{\pi b^2 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}+\frac{1}{210}\pi^4 b^7 S(bx)^2-$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] -b/(84*x^6) + (b^5*Pi^2)/(420*x^2) + (b*Cos[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
35*x^5) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x) + (b^7*Pi^4*
FresnelS[b*x]^2)/210 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*P
i^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) - (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\left(bx\right)}{x^8}\sin\left(\frac{b^2\pi\, x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

3.88 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

Optimal. Leaf size=267

$$\frac{1}{384}\pi^4 b^8 \text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)-\frac{853\pi^4 b^8 \text{FresnelC}\left(\sqrt{2}bx\right)}{40320\sqrt{2}}+\frac{\pi^2 b^4 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{192x^4}-\frac{S(bx)\sin\left(\frac{1}{2}\pi\right.}{8x^8}$$

```
[Out] -b/(112*x^7) + (b^5*Pi^2)/(1152*x^3) + (b*Cos[b^2*Pi*x^2])/(112*x^7) - (187
*b^5*Pi^2*Cos[b^2*Pi*x^2])/(40320*x^3) - (853*b^8*Pi^4*FresnelC[Sqrt[2]*b*x
])/(40320*Sqrt[2]) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(48*x^6) +
(b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(384*x^2) - (FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(8*x^8) + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(1
92*x^4) - (19*b^3*Pi*Sin[b^2*Pi*x^2])/(3360*x^5) + (853*b^7*Pi^3*Sin[b^2*Pi
*x^2])/(80640*x) + (b^8*Pi^4*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2
])/x, x])/384
```

Rubi [A] time = 0.306247, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

```
[Out] -b/(112*x^7) + (b^5*Pi^2)/(1152*x^3) + (b*Cos[b^2*Pi*x^2])/(112*x^7) - (187
*b^5*Pi^2*Cos[b^2*Pi*x^2])/(40320*x^3) - (853*b^8*Pi^4*FresnelC[Sqrt[2]*b*x
])/(40320*Sqrt[2]) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(48*x^6) +
(b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(384*x^2) - (FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(8*x^8) + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(1
92*x^4) - (19*b^3*Pi*Sin[b^2*Pi*x^2])/(3360*x^5) + (853*b^7*Pi^3*Sin[b^2*Pi
*x^2])/(80640*x) + (b^8*Pi^4*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])
/x, x])/384
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = -\frac{b}{112x^7} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} - \frac{1}{16}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx + \frac{1}{8}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}$$

$$= -\frac{b}{112x^7} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^6} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{1}{96}\left(b^3\pi\right)\int$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^6} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{b^4}{}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^6} + $$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^6} + $$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{853b^8\pi^4C\left(\sqrt{2}bx\right)}{40320\sqrt{2}} - \frac{b^2\pi\,\mathrm{cc}}{}$$

Mathematica [A] time = 0.0316223, size = 0, normalized size = 0.

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]
```

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\,(bx)}{x^9}\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

3.89 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

Optimal. Leaf size=262

$$\frac{1}{945}\pi^4 b^8 \text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2},x\right)-\frac{5\pi^4 b^9 \text{CosIntegral}\left(\pi b^2 x^2\right)}{2016}+\frac{\pi^2 b^4 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{315x^5}-\frac{S(bx)\sin}{9}$$

```
[Out] -b/(144*x^8) + (b^5*Pi^2)/(2520*x^4) + (b*Cos[b^2*Pi*x^2])/(144*x^8) - (67*
b^5*Pi^2*Cos[b^2*Pi*x^2])/(30240*x^4) - (5*b^9*Pi^4*CosIntegral[b^2*Pi*x^2]
)/2016 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(63*x^7) + (b^6*Pi^3*Co
s[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x^3) - (FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(9*x^9) + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5) - (1
1*b^3*Pi*Sin[b^2*Pi*x^2])/(3024*x^6) + (5*b^7*Pi^3*Sin[b^2*Pi*x^2])/(2016*x
^2) + (b^8*Pi^4*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/9
45
```

Rubi [A] time = 0.476823, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

```
[Out] -b/(144*x^8) + (b^5*Pi^2)/(2520*x^4) + (b*Cos[b^2*Pi*x^2])/(144*x^8) - (67*
b^5*Pi^2*Cos[b^2*Pi*x^2])/(30240*x^4) - (5*b^9*Pi^4*CosIntegral[b^2*Pi*x^2]
)/2016 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(63*x^7) + (b^6*Pi^3*Co
s[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x^3) - (FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(9*x^9) + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5) - (1
1*b^3*Pi*Sin[b^2*Pi*x^2])/(3024*x^6) + (5*b^7*Pi^3*Sin[b^2*Pi*x^2])/(2016*x
^2) + (b^8*Pi^4*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/945
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}dx = -\frac{b}{144x^8} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} - \frac{1}{18}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^9}dx + \frac{1}{9}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}dx$$

$$= -\frac{b}{144x^8} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} - \frac{1}{36}b\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^5}dx, x,\right.$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{b^4\pi^2}{}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{945x^3} -$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7} + \frac{b^6\pi^3}{}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7} + \frac{b^6\pi^3}{}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{5b^9\pi^4\mathrm{Ci}\left(b^2\pi x^2\right)}{2016} - \frac{b^2\pi\cos\left(\frac{1}{2}\right.}{63}$$

Mathematica [A] time = 0.0322328, size = 0, normalized size = 0.

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10, x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\left(bx\right)}{x^{10}}\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

3.90 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\cos\left(\frac{1}{2}\pi b^2x^2\right)S(bx)^n,x\right)$$

[Out] Unintegrable[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]

Rubi [A] time = 0.0146736, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)^n\,dx$$

Verification is Not applicable to the result.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n,x]

[Out] Defer[Int][Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]

Rubi steps

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)^n\,dx = \int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)^n\,dx$$

Mathematica [A] time = 0.0730368, size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)^n\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n,x]

[Out] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int \cos\left(\frac{b^2\pi x^2}{2}\right)(\text{FresnelS}(bx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\,(bx)^n \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}\,(bx)^n \cos\left(\frac{1}{2}\,\pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)^n,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{\pi b^2 x^2}{2}\right) S^n\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)**n,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)**n, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}\,(bx)^n \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)^n,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

3.91 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=307

$$-\frac{105ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi^4b^7}+\frac{105ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi^4b^7}+\frac{10}{}$$

```
[Out] (35*x^4)/(8*b^5*Pi^3) - x^8/(16*b*Pi) - (40*Cos[b^2*Pi*x^2])/(b^9*Pi^5) + (
5*x^4*Cos[b^2*Pi*x^2])/(2*b^5*Pi^3) - (105*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b
*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (1
05*FresnelC[b*x]*FresnelS[b*x])/(2*b^9*Pi^4) - (((105*I)/8)*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^7*Pi^4) + (((105*I)/8)*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (35*x^3
*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelS[b*x]*Sin[(b^
2*Pi*x^2)/2])/(b^2*Pi) - (55*x^2*Sin[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Sin[b
^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.441621, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6462, 3379, 3309, 30, 3296, 2638, 6454, 6446}

$$-\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105\text{FresnelC}(bx)S(bx)}{2\pi^4b^9}+\frac{x^7S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```


```
[Out] (35*x^4)/(8*b^5*Pi^3) - x^8/(16*b*Pi) - (40*Cos[b^2*Pi*x^2])/(b^9*Pi^5) + (
5*x^4*Cos[b^2*Pi*x^2])/(2*b^5*Pi^3) - (105*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b
*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (1
05*FresnelC[b*x]*FresnelS[b*x])/(2*b^9*Pi^4) - (((105*I)/8)*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^7*Pi^4) + (((105*I)/8)*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (35*x^3
*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelS[b*x]*Sin[(b^
2*Pi*x^2)/2])/(b^2*Pi) - (55*x^2*Sin[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Sin[b
^2*Pi*x^2])/(4*b^3*Pi^2)
```


Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rubi steps

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx = \frac{x^7S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7\int x^6S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^7\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$

$$= \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^7S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^4\pi^2} - \frac{7}{}$$

$$= \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{35x^3S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{105\int x}{}$$

$$= -\frac{x^8}{16b\pi} + \frac{7x^4\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{35}{}$$

$$= -\frac{x^8}{16b\pi} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{10}{}$$

$$= \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{119\cos\left(b^2\pi x^2\right)}{4b^9\pi^5} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4} + \frac{7x}{}$$

$$= \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{40\cos\left(b^2\pi x^2\right)}{b^9\pi^5} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4} + \frac{7x^5}{}$$

Mathematica [F] time = 0.0459479, size = 0, normalized size = 0.

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int x^8 \cos\left(\frac{b^2\pi\, x^2}{2}\right) \mathrm{FresnelS}\,(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{fresnels}\,(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^8 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{fresnels}\,(bx)\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^8*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \cos\left(\frac{1}{2}\pi b^2x^2\right) \text{fresnels}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.92 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\, dx$

Optimal. Leaf size=217

$$\frac{x^6 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{24x^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{6x^4 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{48S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}+\frac{531S\left(\sqrt{2}bx\right)}{16\sqrt{2}\pi^4b^8}+$$

```
[Out] (4*x^3)/(b^5*Pi^3) - x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
- (48*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^
2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*b
^8*Pi^4) - (24*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*Fre
snelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7*
Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.271429, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6462, 3391, 30, 3386, 3385, 3351, 6454, 6452}

$$\frac{x^6 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{24x^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{6x^4 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{48S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}+\frac{531S\left(\sqrt{2}bx\right)}{16\sqrt{2}\pi^4b^8}+$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (4*x^3)/(b^5*Pi^3) - x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
- (48*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^
2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*b
^8*Pi^4) - (24*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*Fre
snelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7*
Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
```

```
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}
\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx &= \frac{x^6S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{6\int x^5S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi}-\frac{\int x^6\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}\\
&= \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}+\frac{x^6S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{24\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^4\pi^2}-\frac{3}{\cdots}\\
&= -\frac{x^7}{14b\pi}+\frac{3x^3\cos\left(b^2\pi x^2\right)}{2b^5\pi^3}+\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}-\frac{24x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}+\frac{x^6S\cdots}{\cdots}\\
&= -\frac{x^7}{14b\pi}+\frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3}-\frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4}+\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}-\frac{24x\cdots}{\cdots}\\
&= \frac{4x^3}{b^5\pi^3}-\frac{x^7}{14b\pi}+\frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3}-\frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4}+\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}\cdots\\
&= \frac{4x^3}{b^5\pi^3}-\frac{x^7}{14b\pi}+\frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3}-\frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4}+\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}\cdots
\end{aligned}$$

Mathematica [A] time = 0.175812, size = 163, normalized size = 0.75

$$\frac{224S(bx)\left(\pi b^2x^2\left(\pi^2b^4x^4-24\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)+6\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)\right)-16\pi^3b^7x^7+896\pi b^3x^3+56\pi^2b^5x^5\sin\cdots}{224\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (896*b^3*Pi*x^3 - 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717*S
qrt[2]*FresnelS[Sqrt[2]*b*x] + 224*FresnelS[b*x]*(6*(-8 + b^4*Pi^2*x^4)*Cos
[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 2
058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi^4)
```

Maple [A] time = 0.084, size = 321, normalized size = 1.5

$$\frac{1}{b}\left(\frac{\mathrm{FresnelS}\,(bx)}{b^7}\left(\frac{b^6x^6}{\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right)-6\,\frac{1}{\pi}\left(-\frac{x^4b^4\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi}+4\,\frac{1}{\pi}\left(\frac{b^2x^2\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi}+2\,\frac{\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] (FresnelS(b*x)/b^7*(1/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/Pi*(-1/Pi*b^4*x^4*co
s(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2
*Pi*x^2))))-1/b^7*(1/2/Pi^3*(1/7*Pi^2*b^7*x^7-8*x^3*b^3)+3/Pi^4*(-1/2*Pi*b^
3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*Fre
snelS(b*x*2^(1/2))))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^5*x
^5*sin(b^2*Pi*x^2)-5/2*Pi*(-1/2/Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2/Pi*(1/2/Pi*b
*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))-12/Pi*b*x*sin(b^2
*Pi*x^2)+6/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^7\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^7*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}\,(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.93 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\, dx$

Optimal. Leaf size=184

$$\frac{x^5 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{15xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{5x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{15S(bx)^2}{2\pi^3b^7}+\frac{15x^2}{4\pi^3b^5}+\frac{x^4\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{11\sin}{2}$$

```
[Out] (15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3
) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x
]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (
x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b
^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.253349, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6462, 3379, 3309, 30, 3296, 2637, 6454, 2634, 6440}

$$\frac{x^5 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{15xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{5x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{15S(bx)^2}{2\pi^3b^7}+\frac{15x^2}{4\pi^3b^5}+\frac{x^4\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{11\sin}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```


```
[Out] (15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3
) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x
]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (
x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b
^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```


Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx = \frac{x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{5\int x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^5\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$

$$= \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{15\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^4\pi^2} - \frac{5}{\quad}$$

$$= \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{15xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{15\int S(b}{\quad}$$

$$= -\frac{x^6}{12b\pi} + \frac{5x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{15xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5S(}{\quad}$$

$$= \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{15S(bx)^2}{2b^7\pi^3} - \frac{15xS(bx)\,\mathrm{s}}{b^6}$$

$$= \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{15S(bx)^2}{2b^7\pi^3} - \frac{15xS(bx)\,\mathrm{s}}{b^6}$$

Mathematica [A] time = 0.008825, size = 184, normalized size = 1.

$$\frac{x^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{15xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} + \frac{5x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{15S(bx)^2}{2\pi^3b^7} + \frac{15x^2}{4\pi^3b^5} + \frac{x^4\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} - 1\!1$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3
) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x
]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (
x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b
^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int x^{6} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right) \mathrm{FresnelS}\left(bx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{6} \cos\left(\frac{1}{2} \pi b^{2} x^{2}\right) \mathrm{fresnels}\left(bx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{6} \cos\left(\frac{1}{2} \pi b^{2} x^{2}\right) \mathrm{fresnels}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^6*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Sympy [A] time = 124.888, size = 264, normalized size = 1.43

$$\begin{cases} -\frac{x^{6} \sin^{2}\left(\frac{\pi b^{2} x^{2}}{2}\right)}{12 \pi b} - \frac{x^{6} \cos^{2}\left(\frac{\pi b^{2} x^{2}}{2}\right)}{12 \pi b} + \frac{x^{5} \sin\left(\frac{\pi b^{2} x^{2}}{2}\right) S(bx)}{\pi b^{2}} + \frac{x^{4} \sin\left(\frac{\pi b^{2} x^{2}}{2}\right) \cos\left(\frac{\pi b^{2} x^{2}}{2}\right)}{2 \pi^{2} b^{3}} + \frac{5 x^{3} \cos\left(\frac{\pi b^{2} x^{2}}{2}\right) S(bx)}{\pi^{2} b^{4}} + \frac{2 x^{2} \sin^{2}\left(\frac{\pi b^{2} x^{2}}{2}\right)}{\pi^{3} b^{5}} + \frac{11 x^{2} \cos^{2}\left(\frac{\pi b^{2} x^{2}}{2}\right)}{2 \pi^{3} b^{5}} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Piecewise((-x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) - x**6*cos(pi*b**2*x**2/2
)**2/(12*pi*b) + x**5*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + x**4*si
n(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*cos(pi*b**2*x
**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*x**2*sin(pi*b**2*x**2/2)**2/(pi**3*b*
*5) + 11*x**2*cos(pi*b**2*x**2/2)**2/(2*pi**3*b**5) - 15*x*sin(pi*b**2*x**2
/2)*fresnels(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)
/(pi**4*b**7) + 15*fresnels(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.94 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\, dx$

Optimal. Leaf size=166

$$-\frac{43\text{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3 b^6}+\frac{x^4 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}-\frac{8S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6}+\frac{4x^2 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4}+\frac{x^3\sin\left(\pi b^2 x^2\right)}{4\pi^2 b^3}+\frac{11x}{}$$

```
[Out] (4*x)/(b^5*Pi^3) - x^5/(10*b*Pi) + (11*x*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) - (4
3*FresnelC[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*Cos[(b^2*Pi*x^2)/2]*
FresnelS[b*x])/(b^4*Pi^2) - (8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3
) + (x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (x^3*Sin[b^2*Pi*x^2]
)/(4*b^3*Pi^2)
```

Rubi [A] time = 0.168881, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6462, 3391, 30, 3386, 3385, 3352, 6454, 6460, 3357}

$$-\frac{43\text{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3 b^6}+\frac{x^4 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}-\frac{8S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6}+\frac{4x^2 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4}+\frac{x^3\sin\left(\pi b^2 x^2\right)}{4\pi^2 b^3}+\frac{11x}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```


```
[Out] (4*x)/(b^5*Pi^3) - x^5/(10*b*Pi) + (11*x*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) - (4
3*FresnelC[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*Cos[(b^2*Pi*x^2)/2]*
FresnelS[b*x])/(b^4*Pi^2) - (8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3
) + (x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (x^3*Sin[b^2*Pi*x^2]
)/(4*b^3*Pi^2)
```


Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
```

```
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx = \frac{x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{4\int x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^4\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$

$$= \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{8\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^4\pi^2} - \frac{2\int}{}$$

$$= -\frac{x^5}{10b\pi} + \frac{x\cos\left(b^2\pi x^2\right)}{b^5\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{8S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4 S(bx)\,\mathrm{s}}{}$$

$$= -\frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{C\left(\sqrt{2}bx\right)}{\sqrt{2}b^6\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{8S(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{b^6\pi^3}$$

$$= \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{11C\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{8S(bx)\,\mathrm{si}}{b}$$

$$= \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{11C\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} - \frac{2\sqrt{2}C\left(\sqrt{2}bx\right)}{b^6\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$

Mathematica [A] time = 0.173342, size = 126, normalized size = 0.76

$$\frac{80S(bx)\left(\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)+4\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)\right)+2bx\left(-4\pi^2b^4x^4+10\pi b^2x^2\sin\left(\pi b^2x^2\right)+55\cos\left(\pi b^2x^2\right)\right.}{80\pi^3b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (-215*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*FresnelS[b*x]*(4*b^2*Pi*x^2*Cos[(b
^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(160 - 4*b
^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*b^6*
Pi^3)
```

Maple [A] time = 0.079, size = 212, normalized size = 1.3

$$\frac{1}{b}\left(\frac{\text{FresnelS}\,(bx)}{b^5}\left(\frac{x^4b^4}{\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right)-4\,\frac{1}{\pi}\left(-\frac{b^2x^2\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi}+2\,\frac{\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right)\right)-\frac{1}{b^5}\left(\frac{1}{2\,\pi^3}\left(\frac{\pi^2b^5x^5}{5}-8\,bx\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] (FresnelS(b*x)/b^5*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*co
s(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))-1/b^5*(1/2/Pi^3*(1/5*Pi^2*b^
5*x^5-8*b*x)+2/Pi^2*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*
x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*co
s(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*
2^(1/2)))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^5\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^5*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**5*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \operatorname{fresnels}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.95 $\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=195

$$\frac{3ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi^2b^3}-\frac{3ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi^2b^3}-\frac{3\text{Fresne}}{2}$$

```
[Out] -x^4/(8*b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) + (3*x*Cos[(b^2*Pi*x^2)/2]*Fresn
elS[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) + (((3*
I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^3*Pi^2
) - (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(
b^3*Pi^2) + (x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (x^2*Sin[b^2
*Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.157324, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6462, 3379, 3309, 30, 3296, 2638, 6454, 6446}

$$\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3\text{FresnelC}(bx)S(bx)}{2\pi^2b^5}+\frac{x^3S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3xS(bx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -x^4/(8*b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) + (3*x*Cos[(b^2*Pi*x^2)/2]*Fresn
elS[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) + (((3*
I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^3*Pi^2
) - (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(
b^3*Pi^2) + (x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (x^2*Sin[b^2
*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}
\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx &= \frac{x^3S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{3\int x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi}-\frac{\int x^3\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}\\
&= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}+\frac{x^3S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^4\pi^2}-\frac{3\int x\,\mathrm{s}
\end{aligned}$$

$$\begin{aligned}
&= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}-\frac{3C(bx)S(bx)}{2b^5\pi^2}+\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2}-\frac{3ix^2\,{}_2F_2\left(1,1\right.}{8}\\
&= -\frac{x^4}{8b\pi}+\frac{3\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}+\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}-\frac{3C(bx)S(bx)}{2b^5\pi^2}+\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\right.}{8b^3\pi^2}\\
&= -\frac{x^4}{8b\pi}+\frac{\cos\left(b^2\pi x^2\right)}{b^5\pi^3}+\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}-\frac{3C(bx)S(bx)}{2b^5\pi^2}+\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\right.}{8b^3\pi^2}
\end{aligned}$$

Mathematica [F] time = 0.038016, size = 0, normalized size = 0.

$$\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{b^2\pi\, x^2}{2}\right) \mathrm{FresnelS}\,(bx)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{fresnels}\,(bx)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^4 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^4*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{\pi b^2 x^2}{2}\right) S\,(bx)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**4*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.96 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\, dx$

Optimal. Leaf size=108

$$\frac{x^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{5S\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2b^4}+\frac{x\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^3}{6\pi b}$$

```
[Out] -x^3/(6*b*Pi) + (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) - (5*Fresn
elS[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2) + (x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(b^2*Pi) + (x*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.0898809, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6462, 3391, 30, 3386, 3351, 6452}

$$\frac{x^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{5S\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2b^4}+\frac{x\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^3}{6\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```


```
[Out] -x^3/(6*b*Pi) + (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) - (5*Fresn
elS[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2) + (x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(b^2*Pi) + (x*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```


Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx = \frac{x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{2\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$
$$= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int\sin\left(b^2\pi x^2\right)dx}{b^3\pi^2} - \frac{\int x^2\,dx}{2b\pi} + \frac{\int x^2\,\mathrm{c}}{}$$
$$= -\frac{x^3}{6b\pi} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{S\left(\sqrt{2}bx\right)}{\sqrt{2}b^4\pi^2} + \frac{x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} -$$
$$= -\frac{x^3}{6b\pi} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{5S\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Mathematica [A] time = 0.081416, size = 90, normalized size = 0.83

$$\frac{24S(bx)\left(\pi b^2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)+2\cos\left(\frac{1}{2}\pi b^2x^2\right)\right)-4\pi b^3x^3+6bx\sin\left(\pi b^2x^2\right)-15\sqrt{2}S\left(\sqrt{2}bx\right)}{24\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (-4*b^3*Pi*x^3 - 15*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 24*FresnelS[b*x]*(2*Cos
[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) + 6*b*x*Sin[b^2*Pi*x^2])
/(24*b^4*Pi^2)
```

Maple [A] time = 0.075, size = 119, normalized size = 1.1

$$\frac{1}{b}\left(\frac{\mathrm{FresnelS}\left(bx\right)}{b^3}\left(\frac{b^2x^2}{\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right)+2\,\frac{\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right)-\frac{1}{b^3}\left(\frac{\sqrt{2}\mathrm{FresnelS}\left(bx\sqrt{2}\right)}{2\,\pi^2}+\frac{x^3b^3}{6\,\pi}-\frac{1}{2\,\pi}\left(\frac{bx\sin\left(b^2\pi\,x^2\right)}{2\,\pi}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] (FresnelS(b*x)/b^3*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*
x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))+1/6/Pi*b^3*x^3-1/2/Pi*(
1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}\,(bx)\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) S\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**3*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.97 $\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=73

$$\frac{xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{S(bx)^2}{2\pi b^3}+\frac{\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^2}{4\pi b}$$

```
[Out] -x^2/(4*b*Pi) - FresnelS[b*x]^2/(2*b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(b^2*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Rubi [A] time = 0.0547892, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6462, 3379, 2634, 6440, 30}

$$\frac{xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{S(bx)^2}{2\pi b^3}+\frac{\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```


```
[Out] -x^2/(4*b*Pi) - FresnelS[b*x]^2/(2*b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(b^2*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```


Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx = \frac{xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$
$$= \frac{xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\text{Subst}(\int x\,dx, x, S(bx))}{b^3\pi} - \frac{\text{Subst}\left(\int \sin^2\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{2b\pi}$$
$$= -\frac{x^2}{4b\pi} - \frac{S(bx)^2}{2b^3\pi} + \frac{xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Mathematica [A] time = 0.0045326, size = 73, normalized size = 1.

$$\frac{xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{S(bx)^2}{2\pi b^3} + \frac{\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} - \frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -x^2/(4*b*Pi) - FresnelS[b*x]^2/(2*b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(b^2*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x^2\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelS}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Sympy [A] time = 4.71556, size = 114, normalized size = 1.56

$$\begin{cases} -\frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} - \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} + \frac{x\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2} + \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} - \frac{S^2(bx)}{2\pi b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Piecewise((-x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) - x**2*cos(pi*b**2*x**2/2)
**2/(4*pi*b) + x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + sin(pi*b**2*
x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) - fresnels(b*x)**2/(2*pi*b**3),
Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.98 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\, dx$

Optimal. Leaf size=59

$$\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}+\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{x}{2\pi b}$$

```
[Out] -x/(2*b*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi) + (FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rubi [A] time = 0.0321775, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6460, 3357, 3352}

$$\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}+\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{x}{2\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -x/(2*b*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi) + (FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx &= \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi} \\ &= \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{b\pi} \\ &= -\frac{x}{2b\pi} + \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\int \cos\left(b^2\pi x^2\right)dx}{2b\pi} \\ &= -\frac{x}{2b\pi} + \frac{C\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi} + \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}\end{aligned}$$

Mathematica [A] time = 0.0279266, size = 48, normalized size = 0.81

$$\frac{4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)+\sqrt{2}\text{FresnelC}\left(\sqrt{2}bx\right)-2bx}{4\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (-2*b*x + Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/
2])/(4*b^2*Pi)
```

Maple [A] time = 0.066, size = 52, normalized size = 0.9

$$\frac{1}{b}\left(\frac{\text{FresnelS}\,(bx)}{b\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right)-\frac{1}{b\pi}\left(\frac{bx}{2}-\frac{\sqrt{2}\text{FresnelC}\left(bx\sqrt{2}\right)}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] (FresnelS(b*x)/b/Pi*sin(1/2*b^2*Pi*x^2)-1/b/Pi*(1/2*b*x-1/4*2^(1/2)*Fresnel
C(b*x*2^(1/2))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos\left(\frac{\pi b^2x^2}{2}\right)S\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.99 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=80

$$-\frac{1}{8}ibx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)+\frac{1}{8}ibx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)+$$

```
[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-I/2)*b^2*Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2,
 2}, (I/2)*b^2*Pi*x^2]
```

Rubi [A] time = 0.0167872, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6446}

$$-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)S(bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-I/2)*b^2*Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2,
 2}, (I/2)*b^2*Pi*x^2]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rubi steps

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx=\frac{C(bx)S(bx)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.0139307, size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \cos\left(\frac{b^2\pi x^2}{2}\right) \mathrm{FresnelS}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{fresnels}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{fresnels}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.100 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)$$

```
[Out] Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]
```

Rubi [A] time = 0.0178919, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x,x]
```

```
[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$$

Mathematica [A] time = 0.0304774, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]
```

Maple [A] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}\,(bx)}{x}\cos\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnels}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnels}\,(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)S\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnels}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x, x)
```

3.101 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\operatorname{Si}\left(b^2\pi x^2\right)-\frac{1}{2}\pi bS(bx)^2$$

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*Si
nIntegral[b^2*Pi*x^2])/4
```

Rubi [A] time = 0.0413899, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6464, 6440, 30, 3375}

$$-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\operatorname{Si}\left(b^2\pi x^2\right)-\frac{1}{2}\pi bS(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]
```

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*Si
nIntegral[b^2*Pi*x^2])/4
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^2}dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} + \frac{1}{2}b\int \frac{\sin\left(b^2\pi x^2\right)}{x}dx - \left(b^2\pi\right)\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} + \frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right) - (b\pi)\,\mathrm{Subst}(\int x\,dx, x, S(bx))$$
$$= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} - \frac{1}{2}b\pi S(bx)^2 + \frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)$$

Mathematica [A] time = 0.0045022, size = 48, normalized size = 1.

$$-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x} + \frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right) - \frac{1}{2}\pi bS(bx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]
```

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*Si
nIntegral[b^2*Pi*x^2])/4
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}(bx)}{x^2}\cos\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnels}(bx)}{x^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnels}(bx)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**2,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \operatorname{fresnels}(bx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^2, x)
```

3.102 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx$

Optimal. Leaf size=93

$$-\frac{1}{2}\pi b^2\text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{\pi b^2\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{b\sin\left(\pi b^2x^2\right)}{4x}$$

```
[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) - (b^2*Pi*Unintegrable[(FresnelS[
b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2
```

Rubi [A] time = 0.0588469, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) - (b^2*Pi*Defer[Int][(FresnelS[b*
x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2}+\frac{1}{4}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{2}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\
&= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}-\frac{1}{2}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx+\frac{1}{2}\left(b^3\pi\right)\int\\
&= \frac{b^2\pi C\left(\sqrt{2}bx\right)}{2\sqrt{2}}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}-\frac{1}{2}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 0.0308939, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3, x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\left(bx\right)}{x^{3}}\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{fresnels}\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{fresnels}\left(bx\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^{2}x^{2}}{2}\right)S\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**3,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{fresnels}\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^3, x)
```

3.103 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx$

Optimal. Leaf size=88

$$-\frac{1}{3}\pi b^2\text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)+\frac{1}{12}\pi b^3\text{CosIntegral}\left(\pi b^2x^2\right)-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b\sin\left(\pi b^2x^2\right)}{12x^2}$$

```
[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Unintegrable[(FresnelS[b*x]
*Sin[(b^2*Pi*x^2)/2])/x^2, x])/3
```

Rubi [A] time = 0.0878529, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Defer[Int][(FresnelS[b*x]*S
in[(b^2*Pi*x^2)/2])/x^2, x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}+\frac{1}{6}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}+\frac{1}{12}b\,\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^2}\,dx,x,x^2\right)-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\\
&= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx+\frac{1}{12}\left(b^3\pi\right)\text{Subs}\\
&= \frac{1}{12}b^3\pi\text{Ci}\left(b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,d
\end{aligned}$$

Mathematica [A] time = 0.0336194, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4, x]
```

Maple [A] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}\,(bx)}{x^4}\cos\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnels}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnels}\,(bx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)S\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**4,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}\left(bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^4, x)
```

3.104 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx$

Optimal. Leaf size=155

$$-\frac{1}{8}\pi^2b^4\text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{\pi b^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^2}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{7\pi^2b^4S\left(\sqrt{2}bx\right)}{24\sqrt{2}}-$$

```
[Out] (b^3*Pi)/(16*x) - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (Cos[(b^2*Pi*x^2)/2]*
FresnelS[b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqrt[2]) +
(b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) - (b*Sin[b^2*Pi*x^2])/(2
4*x^3) - (b^4*Pi^2*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/
8
```

Rubi [A] time = 0.126207, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]
```

```
[Out] (b^3*Pi)/(16*x) - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (Cos[(b^2*Pi*x^2)/2]*
FresnelS[b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqrt[2]) +
(b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) - (b*Sin[b^2*Pi*x^2])/(2
4*x^3) - (b^4*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/8
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}+\frac{1}{8}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx-\frac{1}{4}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx\\ &= \frac{b^3\pi}{16x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}+\frac{1}{16}\left(b^3\pi\right)\int \frac{\cos}{}\\ &= \frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}\\ &= \frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}-\frac{7b^4\pi^2S\left(\sqrt{2}bx\right)}{24\sqrt{2}}+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{8x^2}\end{aligned}$$

Mathematica [A] time = 0.0318803, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5, x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^5}\cos\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^5, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}(bx)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^5, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**5,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnels}\left(bx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^5, x)
```

3.105 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^6}\,dx$

Optimal. Leaf size=163

$$\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3}+\frac{\pi^2 b^4 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{30}\pi^3b^5S(bx)^2-\frac{7}{120}\pi^2b^5\text{Si}\left(b^2\pi x^2\right)+\frac{\pi b^3}{60x^2}$$

```
[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15
*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2
*Pi*x^2])/120
```

Rubi [A] time = 0.213637, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6464, 6456, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3}+\frac{\pi^2 b^4 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{30}\pi^3b^5S(bx)^2-\frac{7}{120}\pi^2b^5\text{Si}\left(b^2\pi x^2\right)+\frac{\pi b^3}{60x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]
```


```
[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15
*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2
*Pi*x^2])/120
```


Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^6}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{1}{10}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx - \frac{1}{5}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

$$= \frac{b^3\pi}{60x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} + \frac{1}{20}b\,\mathrm{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x,\right.$$

$$= \frac{b^3\pi}{60x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} - \frac{b\sin}{}$$

$$= \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x} + \frac{b^2\pi S(bx)\sin}{15}$$

$$= \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x} + \frac{1}{30}b^5\pi^3 S(b$$

Mathematica [A] time = 0.0100253, size = 163, normalized size = 1.

$$\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3} + \frac{\pi^2 b^4 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x} - \frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5} + \frac{1}{30}\pi^3 b^5 S(bx)^2 - \frac{7}{120}\pi^2 b^5\,\mathrm{Si}\left(b^2\pi x^2\right) + \frac{\pi}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]
```

```
[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15
*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2
*Pi*x^2])/120
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\left(bx\right)}{x^{6}}\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^{2} x^{2}\right)\mathrm{fresnels}\left(bx\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^{2} x^{2}\right)\mathrm{fresnels}\left(bx\right)}{x^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^{2} x^{2}}{2}\right)S\left(bx\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**6,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}\left(bx\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^6, x)
```

3.106 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx$

Optimal. Leaf size=230

$$\frac{1}{48}\pi^3 b^6 \text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)-\frac{1}{45}\sqrt{2}\pi^3 b^6 \text{FresnelC}\left(\sqrt{2}bx\right)-\frac{7\pi^3 b^6 \text{FresnelC}\left(\sqrt{2}bx\right)}{144\sqrt{2}}+\frac{\pi b^2 S(bx)\sin}{24}$$

```
[Out] (b^3*Pi)/(144*x^3) - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (7*b^6*Pi^3*Fr
esnelC[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x
])/45 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(6*x^6) + (b^4*Pi^2*Cos[(b^2*Pi
*x^2)/2]*FresnelS[b*x])/(48*x^2) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2
])/(24*x^4) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/
(1440*x) + (b^6*Pi^3*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
)/48
```

Rubi [A] time = 0.204738, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]
```

```
[Out] (b^3*Pi)/(144*x^3) - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (7*b^6*Pi^3*Fr
esnelC[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x
])/45 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(6*x^6) + (b^4*Pi^2*Cos[(b^2*Pi
*x^2)/2]*FresnelS[b*x])/(48*x^2) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2
])/(24*x^4) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/
(1440*x) + (b^6*Pi^3*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/
48
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6}+\frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx-\frac{1}{6}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx\\
&= \frac{b^3\pi}{144x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6}+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4}-\frac{b\sin\left(b^2\pi x^2\right)}{60x^5}+\frac{1}{48}\left(b^3\pi\right)\int \frac{\cos(}{}\\
&= \frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^2}+\frac{b^2\pi S(bx)\sin}{2}\\
&= \frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^2}+\frac{b^2\pi S(bx)\sin}{2}\\
&= \frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3}-\frac{7b^6\pi^3 C\left(\sqrt{2}bx\right)}{144\sqrt{2}}-\frac{1}{45}\sqrt{2}b^6\pi^3 C\left(\sqrt{2}bx\right)-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{6x^6}
\end{aligned}$$

Mathematica [A] time = 0.032583, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7, x]
```

Maple [A] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\,(bx)}{x^7}\cos\left(\frac{b^2\pi\, x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnels}\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^7, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnels}\,(bx)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^7, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)S\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**7,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnels}\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^7, x)
```

3.107 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx$

Optimal. Leaf size=201

$$\frac{1}{105}\pi^3 b^6 \text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2},x\right)-\frac{1}{84}\pi^3 b^7 \text{CosIntegral}\left(\pi b^2 x^2\right)+\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{35x^5}+\frac{\pi^2 b^4 S(b\ldots}{}$$

```
[Out] (b^3*Pi)/(280*x^4) - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosInte
gral[b^2*Pi*x^2])/84 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*x^7) + (b^4*P
i^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x^3) + (b^2*Pi*FresnelS[b*x]*Si
n[(b^2*Pi*x^2)/2])/(35*x^5) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin[
b^2*Pi*x^2])/(84*x^2) + (b^6*Pi^3*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/x^2, x])/105
```

Rubi [A] time = 0.319427, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]
```

```
[Out] (b^3*Pi)/(280*x^4) - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosInte
gral[b^2*Pi*x^2])/84 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*x^7) + (b^4*P
i^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x^3) + (b^2*Pi*FresnelS[b*x]*Si
n[(b^2*Pi*x^2)/2])/(35*x^5) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin[
b^2*Pi*x^2])/(84*x^2) + (b^6*Pi^3*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/x^2, x])/105
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7}+\frac{1}{14}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}\,dx-\frac{1}{7}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\ &= \frac{b^3\pi}{280x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7}+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}+\frac{1}{28}b\,\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^4}\,dx,\ldots\right.\\ &= \frac{b^3\pi}{280x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x^3}+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}-\frac{b\,\mathrm{s}\ldots}{}\\ &= \frac{b^3\pi}{280x^4}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x^3}+\frac{b^2\pi S(bx)\ldots}{}\\ &= \frac{b^3\pi}{280x^4}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x^3}+\frac{b^2\pi S(bx)\ldots}{}\\ &= \frac{b^3\pi}{280x^4}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4}-\frac{1}{84}b^7\pi^3\text{Ci}\left(b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi\ldots\right.}{105x^3}\end{aligned}$$

Mathematica [A] time = 0.0334512, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8, x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\,(bx)}{x^8}\cos\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnels}\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^8, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnels}\,(bx)}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^8, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnels}\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^8, x)
```

3.108 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}\,dx$

Optimal. Leaf size=270

$$\frac{1}{384}\pi^4 b^8 \text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{\pi^3b^6S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{384x^2}+\frac{\pi b^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{48x^6}+\frac{\pi^2b^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{192x^4}$$

```
[Out] (b^3*Pi)/(480*x^5) - (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(3360
*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (Cos[(b^2*Pi*x^2)/2]*Fre
snelS[b*x])/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(192*x^4
) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelS
[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(384*x^2) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*P
i*x^2])/(40320*x^3) + (b^8*Pi^4*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/x, x])/384
```

Rubi [A] time = 0.309526, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9,x]
```

```
[Out] (b^3*Pi)/(480*x^5) - (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(3360
*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (Cos[(b^2*Pi*x^2)/2]*Fre
snelS[b*x])/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(192*x^4
) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelS
[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(384*x^2) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*P
i*x^2])/(40320*x^3) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*
x])/x, x])/384
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{1}{16}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx - \frac{1}{8}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx$$
$$= \frac{b^3\pi}{480x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{1}{96}\left(b^3\pi\right)\int \frac{\mathrm{c}}{}$$
$$= \frac{b^3\pi}{480x^5} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{192x^4} + \frac{b^2\pi S(b}{}$$
$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{192x^4} +$$
$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{b^4}{}$$
$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{b^4}{}$$

Mathematica [A] time = 0.031928, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9, x]
```

Maple [A] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\,(bx)}{x^9}\cos\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnels}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^9, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\,(bx)}{x^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^9,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^9, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**9,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^9,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^9, x)
```

3.109 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^{10}}\,dx$

Optimal. Leaf size=278

$$-\frac{\pi^3 b^6 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{945x^3}+\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7}-\frac{\pi^4 b^8 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{945x}+\frac{\pi^2 b^4 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{315x^5}-\frac{S(bx)\,\mathrm{cc}}{}$$

```
[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3
024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x
^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*Fres
nelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (
b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])
/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIn
tegral[b^2*Pi*x^2])/30240
```

Rubi [A] time = 0.516173, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6464, 6456, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$-\frac{\pi^3 b^6 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{945x^3}+\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7}-\frac{\pi^4 b^8 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{945x}+\frac{\pi^2 b^4 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{315x^5}-\frac{S(bx)\,\mathrm{cc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]
```


```
[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3
024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x
^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*Fres
nelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (
b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])
/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIn
tegral[b^2*Pi*x^2])/30240
```


Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^{10}}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{1}{18}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^9}\,dx - \frac{1}{9}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

$$= \frac{b^3\pi}{756x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} + \frac{1}{36}b\,\mathrm{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^5}\,dx,\right.$$

$$= \frac{b^3\pi}{756x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{315x^5} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b\,\mathrm{s}}{}$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{315x^5}$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{315x^5}$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi}{}$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi}{}$$

Mathematica [A] time = 0.01589, size = 278, normalized size = 1.

$$-\frac{\pi^3b^6S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{945x^3} + \frac{\pi b^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7} - \frac{\pi^4b^8S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{945x} + \frac{\pi^2b^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{315x^5} - \frac{S(bx)\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]
```

```
[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3
024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x
^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*Fres
nelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (
b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])
/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIn
tegral[b^2*Pi*x^2])/30240
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelS}\,(bx)}{x^{10}}\cos\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^10, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^10, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**10,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^10,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^10, x)
```

3.110 $\int x^7 \mathbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=124

$$-\frac{105\text{FresnelC}(bx)}{8\pi^4b^8}-\frac{x^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b}+\frac{35x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5}-\frac{7x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3}+\frac{105x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7}+\frac{1}{8}x^8\text{Fresnel}$$

```
[Out] (105*x*Cos[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2])/(8*b
^3*Pi^2) - (105*FresnelC[b*x])/(8*b^8*Pi^4) + (x^8*FresnelC[b*x])/8 + (35*x
^3*Sin[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) - (x^7*Sin[(b^2*Pi*x^2)/2])/(8*b*Pi)
```

Rubi [A] time = 0.0821382, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3386, 3385, 3352}

$$-\frac{105\text{FresnelC}(bx)}{8\pi^4b^8}-\frac{x^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b}+\frac{35x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5}-\frac{7x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3}+\frac{105x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7}+\frac{1}{8}x^8\text{Fresnel}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelC[b*x],x]
```

```
[Out] (105*x*Cos[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2])/(8*b
^3*Pi^2) - (105*FresnelC[b*x])/(8*b^8*Pi^4) + (x^8*FresnelC[b*x])/8 + (35*x
^3*Sin[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) - (x^7*Sin[(b^2*Pi*x^2)/2])/(8*b*Pi)
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x^7C(bx)\,dx = \frac{1}{8}x^8C(bx) - \frac{1}{8}b\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= \frac{1}{8}x^8C(bx) - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{7\int x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{8b\pi}$$
$$= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8C(bx) - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{8b^3\pi^2}$$
$$= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8C(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{8b^5\pi^3}$$
$$= \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8C(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105\int c}{}$$
$$= \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} - \frac{105C(bx)}{8b^8\pi^4} + \frac{1}{8}x^8C(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi}$$

Mathematica [A] time = 0.0712275, size = 89, normalized size = 0.72

$$\frac{\left(\pi^4b^8x^8-105\right)\text{FresnelC}(bx)+\pi b^3x^3\left(35-\pi^2b^4x^4\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)-7bx\left(\pi^2b^4x^4-15\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelC[b*x],x]
```

```
[Out] (-7*b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*Fr
esnelC[b*x] + b^3*Pi*x^3*(35 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*Pi
^4)
```

Maple [A] time = 0.049, size = 123, normalized size = 1.

$$\frac{1}{b^8}\left(\frac{\text{FresnelC}\,(bx)\,b^8x^8}{8} - \frac{b^7x^7}{8\,\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{7}{8\,\pi}\left(-\frac{b^5x^5}{\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right) + 5\,\frac{1}{\pi}\left(\frac{x^3b^3\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi} - 3\,\frac{1}{\pi}\left(-\frac{\cos\left(1/2\,b\right.}{\pi}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x),x)
```

```
[Out] 1/b^8*(1/8*FresnelC(b*x)*b^8*x^8-1/8/Pi*b^7*x^7*sin(1/2*b^2*Pi*x^2)+7/8/Pi*
(-1/Pi*b^5*x^5*cos(1/2*b^2*Pi*x^2)+5/Pi*(1/Pi*b^3*x^3*sin(1/2*b^2*Pi*x^2)-3
/Pi*(-1/Pi*b*x*cos(1/2*b^2*Pi*x^2)+1/Pi*FresnelC(b*x)))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^7\text{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnelc(b*x), x)
```

Sympy [A] time = 2.60003, size = 184, normalized size = 1.48

$$\frac{45x^8C\left(bx\right)\Gamma\left(\frac{1}{4}\right)}{512\Gamma\left(\frac{13}{4}\right)}-\frac{45x^7\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi b\Gamma\left(\frac{13}{4}\right)}-\frac{315x^5\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^2b^3\Gamma\left(\frac{13}{4}\right)}+\frac{1575x^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^3b^5\Gamma\left(\frac{13}{4}\right)}+\frac{4725x\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma}{512\pi^4b^7\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnelc(b*x),x)
```

```
[Out] 45*x**8*fresnelc(b*x)*gamma(1/4)/(512*gamma(13/4)) - 45*x**7*sin(pi*b**2*x*
*2/2)*gamma(1/4)/(512*pi*b*gamma(13/4)) - 315*x**5*cos(pi*b**2*x**2/2)*gamm
a(1/4)/(512*pi**2*b**3*gamma(13/4)) + 1575*x**3*sin(pi*b**2*x**2/2)*gamma(1
/4)/(512*pi**3*b**5*gamma(13/4)) + 4725*x*cos(pi*b**2*x**2/2)*gamma(1/4)/(5
12*pi**4*b**7*gamma(13/4)) - 4725*fresnelc(b*x)*gamma(1/4)/(512*pi**4*b**8*
gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnelc(b*x), x)
```

3.111 $\int x^6 \mathbf{FresnelC}(bx)\, dx$

Optimal. Leaf size=109

$$-\frac{x^6\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{24x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}-\frac{6x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{48\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{1}{7}x^7\text{FresnelC}(bx)$$

```
[Out] (48*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x^4*Cos[(b^2*Pi*x^2)/2])/(7*b^3*
Pi^2) + (x^7*FresnelC[b*x])/7 + (24*x^2*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) -
 (x^6*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi)
```

Rubi [A] time = 0.114325, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3380, 3296, 2638}

$$-\frac{x^6\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{24x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}-\frac{6x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{48\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{1}{7}x^7\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelC[b*x],x]
```

```
[Out] (48*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x^4*Cos[(b^2*Pi*x^2)/2])/(7*b^3*
Pi^2) + (x^7*FresnelC[b*x])/7 + (24*x^2*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) -
 (x^6*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi)
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^6 C(bx)\,dx &= \frac{1}{7}x^7 C(bx) - \frac{1}{7}b\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= \frac{1}{7}x^7 C(bx) - \frac{1}{14}b\,\text{Subst}\left(\int x^3\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right) \\
&= \frac{1}{7}x^7 C(bx) - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{3\,\text{Subst}\left(\int x^2\sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{7b\pi} \\
&= -\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{1}{7}x^7 C(bx) - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{12\,\text{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{7b^3\pi^2} \\
&= -\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{1}{7}x^7 C(bx) + \frac{24x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} - \frac{24\,\text{Subst}\left(\int \sin\left(\frac{1}{2}b^2\pi x\right)\right.}{7b^5\pi^3} \\
&= \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{1}{7}x^7 C(bx) + \frac{24x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi}
\end{aligned}
$$

Mathematica [A] time = 0.0559025, size = 83, normalized size = 0.76

$$
-\frac{x^2\left(\pi^2b^4x^4-24\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5} - \frac{6\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7} + \frac{1}{7}x^7\text{FresnelC}(bx)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelC[b*x],x]
```

```
[Out] (-6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) + (x^7*FresnelC[b
*x])/7 - (x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3)
```

Maple [A] time = 0.049, size = 107, normalized size = 1.

$$
\frac{1}{b^7}\left(\frac{b^7x^7\text{FresnelC}(bx)}{7} - \frac{b^6x^6}{7\pi}\sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{6}{7\pi}\left(-\frac{x^4b^4}{\pi}\cos\left(\frac{b^2\pi x^2}{2}\right) + 4\frac{1}{\pi}\left(\frac{b^2x^2\sin\left(1/2\,b^2\pi x^2\right)}{\pi} + 2\frac{\cos\left(1/2\,b^2\right.}{\pi^2}\right.\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x),x)
```

```
[Out] 1/b^7*(1/7*b^7*x^7*FresnelC(b*x)-1/7/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)+6/7/Pi*
(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2
/Pi^2*cos(1/2*b^2*Pi*x^2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int x^6\text{fresnelc}(bx)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^6\text{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^6*fresnelc(b*x), x)
```

Sympy [A] time = 2.36106, size = 153, normalized size = 1.4

$$\frac{x^7C\left(bx\right)\Gamma\left(\frac{1}{4}\right)}{28\Gamma\left(\frac{5}{4}\right)}-\frac{x^6\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{28\pi b\Gamma\left(\frac{5}{4}\right)}-\frac{3x^4\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{14\pi^2b^3\Gamma\left(\frac{5}{4}\right)}+\frac{6x^2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{7\pi^3b^5\Gamma\left(\frac{5}{4}\right)}+\frac{12\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{7\pi^4b^7\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnelc(b*x),x)
```

```
[Out] x**7*fresnelc(b*x)*gamma(1/4)/(28*gamma(5/4)) - x**6*sin(pi*b**2*x**2/2)*ga
mma(1/4)/(28*pi*b*gamma(5/4)) - 3*x**4*cos(pi*b**2*x**2/2)*gamma(1/4)/(14*p
i**2*b**3*gamma(5/4)) + 6*x**2*sin(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**3*b**5
*gamma(5/4)) + 12*cos(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**4*b**7*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnelc(b*x), x)
```

3.112 $\int x^5 \mathbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=99

$$-\frac{5S(bx)}{2\pi^3b^6}-\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b}+\frac{5x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5}-\frac{5x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2b^3}+\frac{1}{6}x^6\text{FresnelC}(bx)$$

```
[Out] (-5*x^3*Cos[(b^2*Pi*x^2)/2])/(6*b^3*Pi^2) + (x^6*FresnelC[b*x])/6 - (5*Fres
nelS[b*x])/(2*b^6*Pi^3) + (5*x*Sin[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) - (x^5*Sin
[(b^2*Pi*x^2)/2])/(6*b*Pi)
```

Rubi [A] time = 0.0624975, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3386, 3385, 3351}

$$-\frac{5S(bx)}{2\pi^3b^6}-\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b}+\frac{5x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5}-\frac{5x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2b^3}+\frac{1}{6}x^6\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelC[b*x],x]
```

```
[Out] (-5*x^3*Cos[(b^2*Pi*x^2)/2])/(6*b^3*Pi^2) + (x^6*FresnelC[b*x])/6 - (5*Fres
nelS[b*x])/(2*b^6*Pi^3) + (5*x*Sin[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) - (x^5*Sin
[(b^2*Pi*x^2)/2])/(6*b*Pi)
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^5 C(bx)\,dx &= \frac{1}{6}x^6 C(bx) - \frac{1}{6}b\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= \frac{1}{6}x^6 C(bx) - \frac{x^5 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5\int x^4 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{6b\pi} \\
&= -\frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6 C(bx) - \frac{x^5 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b^3\pi^2} \\
&= -\frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6 C(bx) + \frac{5x \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} - \frac{x^5 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} - \frac{5\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b^5\pi^3} \\
&= -\frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6 C(bx) - \frac{5S(bx)}{2b^6\pi^3} + \frac{5x \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} - \frac{x^5 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi}
\end{aligned}
$$

Mathematica [A] time = 0.0634354, size = 80, normalized size = 0.81

$$
\frac{\pi^3 b^6 x^6 \text{FresnelC}(bx) + bx\left(15 - \pi^2 b^4 x^4\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right) - 5\pi b^3 x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right) - 15S(bx)}{6\pi^3 b^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelC[b*x],x]
```

```
[Out] (-5*b^3*Pi*x^3*Cos[(b^2*Pi*x^2)/2] + b^6*Pi^3*x^6*FresnelC[b*x] - 15*Fresne
lS[b*x] + b*x*(15 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```

Maple [A] time = 0.048, size = 97, normalized size = 1.

$$
\frac{1}{b^6}\left(\frac{b^6x^6 \text{FresnelC}\,(bx)}{6} - \frac{b^5x^5}{6\,\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{5}{6\,\pi}\left(-\frac{x^3b^3}{\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right) + 3\,\frac{1}{\pi}\left(\frac{bx\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi} - \frac{\text{FresnelS}\,(bx)}{\pi}\right)\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x),x)
```

```
[Out] 1/b^6*(1/6*b^6*x^6*FresnelC(b*x)-1/6/Pi*b^5*x^5*sin(1/2*b^2*Pi*x^2)+5/6/Pi*
(-1/Pi*b^3*x^3*cos(1/2*b^2*Pi*x^2)+3/Pi*(1/Pi*b*x*sin(1/2*b^2*Pi*x^2)-1/Pi*
FresnelS(b*x))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int x^5 \text{fresnelc}\,(bx)\;dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^5\text{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnelc(b*x), x)
```

Sympy [A] time = 1.12886, size = 49, normalized size = 0.49

$$\frac{bx^7\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{7}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{1}{2},\frac{5}{4},\frac{11}{4}\end{matrix}\middle| -\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnelc(b*x),x)
```

```
[Out] b*x**7*gamma(1/4)*gamma(7/4)*hyper((1/4, 7/4), (1/2, 5/4, 11/4), -pi**2*b**
4*x**4/16)/(16*gamma(5/4)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnelc(b*x), x)
```

3.113 $\int x^4 \mathbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=84

$$-\frac{x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}+\frac{8\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}-\frac{4x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{1}{5}x^5\text{FresnelC}(bx)$$

```
[Out] (-4*x^2*Cos[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x])/5 + (8*Sin[
(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) - (x^4*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi)
```

Rubi [A] time = 0.0766497, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3380, 3296, 2637}

$$-\frac{x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}+\frac{8\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}-\frac{4x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{1}{5}x^5\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelC[b*x],x]
```

```
[Out] (-4*x^2*Cos[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x])/5 + (8*Sin[
(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) - (x^4*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi)
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^4C(bx)\,dx &= \frac{1}{5}x^5C(bx) - \frac{1}{5}b\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)dx \\ &= \frac{1}{5}x^5C(bx) - \frac{1}{10}b\,\mathrm{Subst}\left(\int x^2\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right) \\ &= \frac{1}{5}x^5C(bx) - \frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{2\,\mathrm{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{5b\pi} \\ &= -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{1}{5}x^5C(bx) - \frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{4\,\mathrm{Subst}\left(\int\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{5b^3\pi^2} \\ &= -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{1}{5}x^5C(bx) + \frac{8\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}\end{aligned}$$

Mathematica [A] time = 0.0434825, size = 71, normalized size = 0.85

$$-\frac{\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5} - \frac{4x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3} + \frac{1}{5}x^5\mathrm{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelC[b*x],x]
```

```
[Out] (-4*x^2*Cos[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x])/5 - ((-8 +
b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3)
```

Maple [A] time = 0.046, size = 81, normalized size = 1.

$$\frac{1}{b^5}\left(\frac{b^5x^5\mathrm{FresnelC}(bx)}{5} - \frac{x^4b^4}{5\pi}\sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{4}{5\pi}\left(-\frac{b^2x^2}{\pi}\cos\left(\frac{b^2\pi x^2}{2}\right) + 2\frac{\sin\left(1/2\,b^2\pi x^2\right)}{\pi^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x),x)
```

```
[Out] 1/b^5*(1/5*b^5*x^5*FresnelC(b*x)-1/5/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)+4/5/Pi*
(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\mathrm{fresnelc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4\text{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnelc(b*x), x)
```

Sympy [A] time = 1.223, size = 116, normalized size = 1.38

$$\frac{x^5C\left(bx\right)\Gamma\left(\frac{1}{4}\right)}{20\Gamma\left(\frac{5}{4}\right)}-\frac{x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{20\pi b\Gamma\left(\frac{5}{4}\right)}-\frac{x^2\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{5\pi^2b^3\Gamma\left(\frac{5}{4}\right)}+\frac{2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{5\pi^3b^5\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnelc(b*x),x)
```

```
[Out] x**5*fresnelc(b*x)*gamma(1/4)/(20*gamma(5/4)) - x**4*sin(pi*b**2*x**2/2)*ga
mma(1/4)/(20*pi*b*gamma(5/4)) - x**2*cos(pi*b**2*x**2/2)*gamma(1/4)/(5*pi**
2*b**3*gamma(5/4)) + 2*sin(pi*b**2*x**2/2)*gamma(1/4)/(5*pi**3*b**5*gamma(5
/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnelc(b*x), x)
```

3.114 $\int x^3 \mathbf{FresnelC}(bx)\, dx$

Optimal. Leaf size=74

$$\frac{3\text{FresnelC}(bx)}{4\pi^2 b^4} - \frac{x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} - \frac{3x \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{1}{4}x^4 \text{FresnelC}(bx)$$

```
[Out] (-3*x*Cos[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*FresnelC[b*x])/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x])/4 - (x^3*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi)
```

Rubi [A] time = 0.0429744, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3386, 3385, 3352}

$$\frac{3\text{FresnelC}(bx)}{4\pi^2 b^4} - \frac{x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} - \frac{3x \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{1}{4}x^4 \text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[b*x],x]
```

```
[Out] (-3*x*Cos[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*FresnelC[b*x])/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x])/4 - (x^3*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi)
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^3C(bx)\,dx &= \frac{1}{4}x^4C(bx) - \frac{1}{4}b\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= \frac{1}{4}x^4C(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{4b\pi} \\
&= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{1}{4}x^4C(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{4b^3\pi^2} \\
&= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3C(bx)}{4b^4\pi^2} + \frac{1}{4}x^4C(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi}
\end{aligned}
$$

Mathematica [A] time = 0.016423, size = 74, normalized size = 1.

$$
\frac{3\text{FresnelC}(bx)}{4\pi^2b^4} - \frac{x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b} - \frac{3x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2b^3} + \frac{1}{4}x^4\text{FresnelC}(bx)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[b*x],x]
```

```
[Out] (-3*x*Cos[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*FresnelC[b*x])/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x])/4 - (x^3*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi)
```

Maple [A] time = 0.048, size = 70, normalized size = 1.

$$
\frac{1}{b^4}\left(\frac{b^4x^4\text{FresnelC}\,(bx)}{4} - \frac{x^3b^3}{4\,\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{3}{4\,\pi}\left(-\frac{bx}{\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\text{FresnelC}\,(bx)}{\pi}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x),x)
```

```
[Out] 1/b^4*(1/4*b^4*x^4*FresnelC(b*x)-1/4/Pi*b^3*x^3*sin(1/2*b^2*Pi*x^2)+3/4/Pi*
(-1/Pi*b*x*cos(1/2*b^2*Pi*x^2)+1/Pi*FresnelC(b*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int x^3\text{fresnelc}\,(bx)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$
\text{integral}\left(x^3\text{fresnelc}\,(bx)\,,x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnelc(b*x), x)
```

Sympy [A] time = 1.23752, size = 112, normalized size = 1.51

$$\frac{5x^4C(bx)\,\Gamma\left(\frac{1}{4}\right)}{64\Gamma\left(\frac{9}{4}\right)} - \frac{5x^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{64\pi b\Gamma\left(\frac{9}{4}\right)} - \frac{15x\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{64\pi^2b^3\Gamma\left(\frac{9}{4}\right)} + \frac{15C(bx)\,\Gamma\left(\frac{1}{4}\right)}{64\pi^2b^4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x),x)
```

```
[Out] 5*x**4*fresnelc(b*x)*gamma(1/4)/(64*gamma(9/4)) - 5*x**3*sin(pi*b**2*x**2/2
)*gamma(1/4)/(64*pi*b*gamma(9/4)) - 15*x*cos(pi*b**2*x**2/2)*gamma(1/4)/(64
*pi**2*b**3*gamma(9/4)) + 15*fresnelc(b*x)*gamma(1/4)/(64*pi**2*b**4*gamma(
9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\text{fresnelc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnelc(b*x), x)
```

3.115 $\int x^2 \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=59

$$-\frac{x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{3}x^3\text{FresnelC}(bx)$$

```
[Out] (-2*Cos[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x])/3 - (x^2*Sin[(b
^2*Pi*x^2)/2])/(3*b*Pi)
```

Rubi [A] time = 0.0543763, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3380, 3296, 2638}

$$-\frac{x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{3}x^3\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[b*x],x]
```

```
[Out] (-2*Cos[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x])/3 - (x^2*Sin[(b
^2*Pi*x^2)/2])/(3*b*Pi)
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2C(bx)\,dx &= \frac{1}{3}x^3C(bx) - \frac{1}{3}b\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)dx \\ &= \frac{1}{3}x^3C(bx) - \frac{1}{6}b\,\text{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right) \\ &= \frac{1}{3}x^3C(bx) - \frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\text{Subst}\left(\int\sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{3b\pi} \\ &= -\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{1}{3}x^3C(bx) - \frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}\end{aligned}$$

Mathematica [A] time = 0.0136692, size = 59, normalized size = 1.

$$-\frac{x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b} - \frac{2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3} + \frac{1}{3}x^3\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[b*x],x]
```

```
[Out] (-2*Cos[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x])/3 - (x^2*Sin[(b
^2*Pi*x^2)/2])/(3*b*Pi)
```

Maple [A] time = 0.05, size = 54, normalized size = 0.9

$$\frac{1}{b^3}\left(\frac{b^3x^3\text{FresnelC}\,(bx)}{3} - \frac{b^2x^2}{3\,\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{2}{3\,\pi^2}\cos\left(\frac{b^2\pi\,x^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelC(b*x)-1/3/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)-2/3/Pi^
2*cos(1/2*b^2*Pi*x^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\text{fresnelc}\,(bx)\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnelc(b*x), x)
```

Sympy [A] time = 1.03035, size = 80, normalized size = 1.36

$$\frac{x^3C(bx)\Gamma\left(\frac{1}{4}\right)}{12\Gamma\left(\frac{5}{4}\right)}-\frac{x^2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi b\Gamma\left(\frac{5}{4}\right)}-\frac{\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{6\pi^2b^3\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x),x)
```

```
[Out] x**3*fresnelc(b*x)*gamma(1/4)/(12*gamma(5/4)) - x**2*sin(pi*b**2*x**2/2)*ga
mma(1/4)/(12*pi*b*gamma(5/4)) - cos(pi*b**2*x**2/2)*gamma(1/4)/(6*pi**2*b**
3*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\text{fresnelc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc(b*x), x)
```

3.116 $\int x\textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=49

$$\frac{S(bx)}{2\pi b^2}-\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{1}{2}x^2\text{FresnelC}(bx)$$

```
[Out] (x^2*FresnelC[b*x])/2 + FresnelS[b*x]/(2*b^2*Pi) - (x*Sin[(b^2*Pi*x^2)/2])/
(2*b*Pi)
```

Rubi [A] time = 0.0258736, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3386, 3351}

$$\frac{S(bx)}{2\pi b^2}-\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{1}{2}x^2\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[b*x],x]
```


```
[Out] (x^2*FresnelC[b*x])/2 + FresnelS[b*x]/(2*b^2*Pi) - (x*Sin[(b^2*Pi*x^2)/2])/
(2*b*Pi)
```


Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int xC(bx)\,dx&=\frac{1}{2}x^2C(bx)-\frac{1}{2}b\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)dx\\&=\frac{1}{2}x^2C(bx)-\frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}+\frac{\int\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b\pi}\\&=\frac{1}{2}x^2C(bx)+\frac{S(bx)}{2b^2\pi}-\frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}\end{aligned}$$

Mathematica [A] time = 0.0117981, size = 49, normalized size = 1.

$$\frac{S(bx)}{2\pi b^2}-\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{1}{2}x^2\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelC[b*x],x]
```

```
[Out] (x^2*FresnelC[b*x])/2 + FresnelS[b*x]/(2*b^2*Pi) - (x*Sin[(b^2*Pi*x^2)/2])/
(2*b*Pi)
```

Maple [A] time = 0.048, size = 44, normalized size = 0.9

$$\frac{1}{b^2}\left(\frac{b^2x^2\text{FresnelC}\,(bx)}{2}-\frac{bx}{2\,\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right)+\frac{\text{FresnelS}\,(bx)}{2\,\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x),x)
```

```
[Out] 1/b^2*(1/2*b^2*x^2*FresnelC(b*x)-1/2/Pi*b*x*sin(1/2*b^2*Pi*x^2)+1/2/Pi*Fres
nelS(b*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnelc}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,(x\text{fresnelc}\,(bx)\,,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x*fresnelc(b*x), x)
```

Sympy [A] time = 0.525939, size = 49, normalized size = 1.

$$\frac{bx^3\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x),x)
```

```
[Out] b*x**3*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -pi**2*b**4
*x**4/16)/(16*gamma(5/4)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnelc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*fresnelc(b*x), x)
```

3.117 $\int \text{FresnelC}(bx)\,dx$

Optimal. Leaf size=27

$$x\text{FresnelC}(bx)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}$$

```
[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)
```

Rubi [A] time = 0.0047574, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6419}

$$x\text{FresnelC}(bx)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x],x]
```

```
[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)
```

Rule 6419

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelC[a +
 b*x])/b, x] - Simp[Sin[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int C(bx)\,dx=xC(bx)-\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

Mathematica [A] time = 0.0021411, size = 27, normalized size = 1.

$$x\text{FresnelC}(bx)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x],x]
```

```
[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)
```

Maple [A] time = 0.051, size = 28, normalized size = 1.

$$\frac{1}{b}\left(bx\text{FresnelC}\,(bx)-\frac{1}{\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x),x)
```

```
[Out] 1/b*(b*x*FresnelC(b*x)-1/Pi*sin(1/2*b^2*Pi*x^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,(\text{fresnelc}\,(bx)\,, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x), x)
```

Sympy [B] time = 0.633405, size = 44, normalized size = 1.63

$$\frac{xC\,(bx)\,\Gamma\left(\frac{1}{4}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{4\pi b\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x),x)
```

```
[Out] x*fresnelc(b*x)*gamma(1/4)/(4*gamma(5/4)) - sin(pi*b**2*x**2/2)*gamma(1/4)/
(4*pi*b*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x), x)
```

3.118 $\int \frac{\textbf{FresnelC}(bx)}{x}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}bx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-\frac{1}{2}i\pi b^2x^2\right)+\frac{1}{2}bx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},\frac{1}{2}i\pi b^2x^2\right)$$

```
[Out] (b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-I/2)*b^2*Pi*x^2])/2 + (b*x
*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2])/2
```

Rubi [A] time = 0.0443547, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6425, 6358, 6360}

$$\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x,x]
```

```
[Out] (b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-I/2)*b^2*Pi*x^2])/2 + (b*x
*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2])/2
```

Rule 6425

```
Int[FresnelC[(b_.)*(x_)]/(x_), x_Symbol] :> Dist[(1 - I)/4, Int[Erf[(Sqrt[P
i]*(1 + I)*b*x)/2]/x, x], x] + Dist[(1 + I)/4, Int[Erf[(Sqrt[Pi]*(1 - I)*b*
x)/2]/x, x], x] /; FreeQ[b, x]
```

Rule 6358

```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(2*b*x*HypergeometricPFQ[{1/2,
1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[b, x]
```

Rule 6360

```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(2*b*x*HypergeometricPFQ[{1/2,
 1/2}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[b, x]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x}\,dx &= \left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}\,dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}\,dx\\ &= \frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)\end{aligned}$$

Mathematica [F] time = 0.0140928, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]/x,x]

[Out] Integrate[FresnelC[b*x]/x, x]
```

Maple [A] time = 0.073, size = 23, normalized size = 0.3

$$bx_2F_3\left(\frac{1}{4},\frac{1}{4};\frac{1}{2},\frac{5}{4},\frac{5}{4};-\frac{x^4\pi^2b^4}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x,x)

[Out] b*x*hypergeom([1/4,1/4],[1/2,5/4,5/4],-1/16*x^4*Pi^2*b^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)/x, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x,x)

[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x, x)
```

3.119 $\int \frac{\text{FresnelC}(bx)}{x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}b\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)-\frac{\text{FresnelC}(bx)}{x}$$

```
[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x
```

Rubi [A] time = 0.0212742, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6427, 3376}

$$\frac{1}{2}b\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)-\frac{\text{FresnelC}(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^2,x]
```

```
[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x^2}\,dx &= -\frac{C(bx)}{x}+b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\ &= \frac{1}{2}b\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{C(bx)}{x}\end{aligned}$$

Mathematica [A] time = 0.0108917, size = 27, normalized size = 1.

$$\frac{1}{2}b\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)-\frac{\text{FresnelC}(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^2,x]
```

```
[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x
```

Maple [A] time = 0.049, size = 28, normalized size = 1.

$$b\left(-\frac{\mathrm{FresnelC}\left(bx\right)}{bx}+\frac{1}{2}\mathrm{Ci}\left(\frac{b^{2}\pi\,x^{2}}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^2,x)
```

```
[Out] b*(-FresnelC(b*x)/b/x+1/2*Ci(1/2*b^2*Pi*x^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}\left(bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\left(bx\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^2, x)
```

Sympy [B] time = 1.13432, size = 53, normalized size = 1.96

$$-\frac{\pi^{2}b^{5}x^{4}\Gamma\left(\frac{5}{4}\right){}_{3}F_{4}\left(\begin{matrix}1,1,\frac{5}{4}\\ \frac{3}{2},2,2,\frac{9}{4}\end{matrix}\middle|-\frac{\pi^{2}b^{4}x^{4}}{16}\right)}{128\Gamma\left(\frac{9}{4}\right)}+\frac{b\log\left(b^{4}x^{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**2,x)
```

```
[Out] -pi**2*b**5*x**4*gamma(5/4)*hyper((1, 1, 5/4), (3/2, 2, 2, 9/4), -pi**2*b**
4*x**4/16)/(128*gamma(9/4)) + b*log(b**4*x**4)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}\left(bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^2, x)
```

3.120 $\int \frac{\text{FresnelC}(bx)}{x^3}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\pi b^2 S(bx)-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x}-\frac{\text{FresnelC}(bx)}{2x^2}$$

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(2*x) - FresnelC[b*x]/(2*x^2) - (b^2*Pi*FresnelS[b
*x])/2
```

Rubi [A] time = 0.0285715, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6427, 3388, 3351}

$$-\frac{1}{2}\pi b^2 S(bx)-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x}-\frac{\text{FresnelC}(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^3,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(2*x) - FresnelC[b*x]/(2*x^2) - (b^2*Pi*FresnelS[b
*x])/2
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)}{x^3}\,dx &= -\frac{C(bx)}{2x^2}+\frac{1}{2}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x}-\frac{C(bx)}{2x^2}-\frac{1}{2}\left(b^3\pi\right)\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x}-\frac{C(bx)}{2x^2}-\frac{1}{2}b^2\pi S(bx)
\end{aligned}$$

Mathematica [A] time = 0.0109718, size = 44, normalized size = 1.

$$-\frac{1}{2}\pi b^2 S(bx) - \frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^3,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(2*x) - FresnelC[b*x]/(2*x^2) - (b^2*Pi*FresnelS[b
*x])/2
```

Maple [A] time = 0.048, size = 43, normalized size = 1.

$$b^2\left(-\frac{\text{FresnelC}\,(bx)}{2\,b^2x^2} - \frac{1}{2\,bx}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{\pi\,\text{FresnelS}\,(bx)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^3,x)
```

```
[Out] b^2*(-1/2*FresnelC(b*x)/b^2/x^2-1/2/b/x*cos(1/2*b^2*Pi*x^2)-1/2*Pi*FresnelS
(b*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^3, x)
```

Sympy [A] time = 0.633295, size = 51, normalized size = 1.16

$$\frac{b\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_2F_3\left(\begin{matrix}-\frac{1}{4}, \frac{1}{4}\\ \frac{1}{2}, \frac{3}{4}, \frac{5}{4}\end{matrix}\middle| -\frac{\pi^2b^4x^4}{16}\right)}{16x\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**3,x)
```

```
[Out] b*gamma(-1/4)*gamma(1/4)*hyper((-1/4, 1/4), (1/2, 3/4, 5/4), -pi**2*b**4*x*
*4/16)/(16*x*gamma(3/4)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^3, x)
```

3.121 $\int \frac{\text{FresnelC}(bx)}{x^4}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{12}\pi b^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2}-\frac{\text{FresnelC}(bx)}{3x^3}$$

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(6*x^2) - FresnelC[b*x]/(3*x^3) - (b^3*Pi*SinInteg
ral[(b^2*Pi*x^2)/2])/12
```

Rubi [A] time = 0.0636539, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3380, 3297, 3299}

$$-\frac{1}{12}\pi b^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2}-\frac{\text{FresnelC}(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^4,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(6*x^2) - FresnelC[b*x]/(3*x^3) - (b^3*Pi*SinInteg
ral[(b^2*Pi*x^2)/2])/12
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{C(bx)}{x^4}\,dx = -\frac{C(bx)}{3x^3} + \frac{1}{3}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$
$$= -\frac{C(bx)}{3x^3} + \frac{1}{6}b\,\mathrm{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2} - \frac{C(bx)}{3x^3} - \frac{1}{12}\left(b^3\pi\right)\mathrm{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx, x, x^2\right)$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2} - \frac{C(bx)}{3x^3} - \frac{1}{12}b^3\pi\,\mathrm{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

Mathematica [A] time = 0.0152894, size = 52, normalized size = 1.

$$-\frac{1}{12}\pi b^3\mathrm{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2} - \frac{\mathrm{FresnelC}(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^4,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(6*x^2) - FresnelC[b*x]/(3*x^3) - (b^3*Pi*SinInteg
ral[(b^2*Pi*x^2)/2])/12
```

Maple [A] time = 0.047, size = 49, normalized size = 0.9

$$b^3\left(-\frac{\mathrm{FresnelC}\,(bx)}{3\,x^3b^3} - \frac{1}{6\,b^2x^2}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{\pi}{12}\mathrm{Si}\left(\frac{b^2\pi\,x^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^4,x)
```

```
[Out] b^3*(-1/3*FresnelC(b*x)/b^3/x^3-1/6/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/12*Pi*Si(
1/2*b^2*Pi*x^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^4, x)
```

Sympy [A] time = 0.734969, size = 42, normalized size = 0.81

$$-\frac{b\Gamma\left(\frac{1}{4}\right){}_2F_3\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2},\frac{1}{2},\frac{5}{4}\end{matrix}\middle| -\frac{\pi^2b^4x^4}{16}\right)}{8x^2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**4,x)
```

```
[Out] -b*gamma(1/4)*hyper((-1/2, 1/4), (1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(8*x
**2*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^4, x)
```

3.122 $\int \frac{\text{FresnelC}(bx)}{x^5}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{12}\pi^2 b^4 \text{FresnelC}(bx)+\frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x}-\frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3}-\frac{\text{FresnelC}(bx)}{4x^4}$$

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(12*x^3) - (b^4*Pi^2*FresnelC[b*x])/12 - FresnelC[
b*x]/(4*x^4) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(12*x)
```

Rubi [A] time = 0.0411592, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3388, 3387, 3352}

$$-\frac{1}{12}\pi^2 b^4 \text{FresnelC}(bx)+\frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x}-\frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3}-\frac{\text{FresnelC}(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^5,x]
```


```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(12*x^3) - (b^4*Pi^2*FresnelC[b*x])/12 - FresnelC[
b*x]/(4*x^4) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(12*x)
```


Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{C(bx)}{x^5}\,dx &= -\frac{C(bx)}{4x^4}+\frac{1}{4}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{C(bx)}{4x^4}-\frac{1}{12}\left(b^3\pi\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{C(bx)}{4x^4}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}-\frac{1}{12}\left(b^5\pi^2\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{1}{12}b^4\pi^2C(bx)-\frac{C(bx)}{4x^4}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}
\end{aligned}
$$

Mathematica [A] time = 0.0158539, size = 69, normalized size = 1.

$$
-\frac{1}{12}\pi^2b^4\text{FresnelC}(bx)+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{12x}-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{12x^3}-\frac{\text{FresnelC}(bx)}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^5,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(12*x^3) - (b^4*Pi^2*FresnelC[b*x])/12 - FresnelC[
b*x]/(4*x^4) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(12*x)
```

Maple [A] time = 0.051, size = 64, normalized size = 0.9

$$
b^4\left(-\frac{\text{FresnelC}\,(bx)}{4\,x^4b^4}-\frac{1}{12\,x^3b^3}\cos\left(\frac{b^2\pi\,x^2}{2}\right)-\frac{\pi}{12}\left(-\frac{1}{bx}\sin\left(\frac{b^2\pi\,x^2}{2}\right)+\pi\,\text{FresnelC}\,(bx)\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^5,x)
```

```
[Out] b^4*(-1/4*FresnelC(b*x)/b^4/x^4-1/12/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/12*Pi*(-
sin(1/2*b^2*Pi*x^2)/b/x+Pi*FresnelC(b*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\text{fresnelc}\,(bx)}{x^5}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^5, x)
```

Sympy [A] time = 1.12457, size = 110, normalized size = 1.59

$$\frac{\pi^2 b^4 C(bx)\Gamma\left(-\frac{3}{4}\right)}{64\Gamma\left(\frac{5}{4}\right)} - \frac{\pi b^3 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{3}{4}\right)}{64x\Gamma\left(\frac{5}{4}\right)} + \frac{b\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{3}{4}\right)}{64x^3\Gamma\left(\frac{5}{4}\right)} + \frac{3C(bx)\Gamma\left(-\frac{3}{4}\right)}{64x^4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**5,x)
```

```
[Out] pi**2*b**4*fresnelc(b*x)*gamma(-3/4)/(64*gamma(5/4)) - pi*b**3*sin(pi*b**2*
x**2/2)*gamma(-3/4)/(64*x*gamma(5/4)) + b*cos(pi*b**2*x**2/2)*gamma(-3/4)/(
64*x**3*gamma(5/4)) + 3*fresnelc(b*x)*gamma(-3/4)/(64*x**4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^5, x)
```

3.123 $\int \frac{\text{FresnelC}(bx)}{x^6}\,dx$

Optimal. Leaf size=77

$$-\frac{1}{80}\pi^2 b^5 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right)+\frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{40x^2}-\frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{20x^4}-\frac{\text{FresnelC}(bx)}{5x^5}$$

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*CosIntegral[(b^2*Pi*x^2)/2])/
80 - FresnelC[b*x]/(5*x^5) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(40*x^2)
```

Rubi [A] time = 0.0893477, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3380, 3297, 3302}

$$-\frac{1}{80}\pi^2 b^5 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right)+\frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{40x^2}-\frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{20x^4}-\frac{\text{FresnelC}(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^6,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*CosIntegral[(b^2*Pi*x^2)/2])/
80 - FresnelC[b*x]/(5*x^5) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(40*x^2)
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x^6}\,dx &= -\frac{C(bx)}{5x^5} + \frac{1}{5}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx\\
&= -\frac{C(bx)}{5x^5} + \frac{1}{10}b\,\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx, x, x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{C(bx)}{5x^5} - \frac{1}{40}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{C(bx)}{5x^5} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2} - \frac{1}{80}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx, x, x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{C(bx)}{5x^5} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}\end{aligned}$$

Mathematica [A] time = 0.0190792, size = 77, normalized size = 1.

$$-\frac{1}{80}\pi^2 b^5\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right) + \frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2} - \frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4} - \frac{\text{FresnelC}(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^6,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*CosIntegral[(b^2*Pi*x^2)/2])/
80 - FresnelC[b*x]/(5*x^5) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(40*x^2)
```

Maple [A] time = 0.052, size = 71, normalized size = 0.9

$$b^5\left(-\frac{\text{FresnelC}\,(bx)}{5\,b^5x^5} - \frac{1}{20\,x^4b^4}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{\pi}{20}\left(-\frac{1}{2\,b^2x^2}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\pi}{4}\text{Ci}\left(\frac{b^2\pi\,x^2}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^6,x)
```

```
[Out] b^5*(-1/5*FresnelC(b*x)/b^5/x^5-1/20/b^4/x^4*cos(1/2*b^2*Pi*x^2)-1/20*Pi*(-
1/2*sin(1/2*b^2*Pi*x^2)/b^2/x^2+1/4*Pi*Ci(1/2*b^2*Pi*x^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^6, x)
```

Sympy [A] time = 1.99193, size = 65, normalized size = 0.84

$$\frac{\pi^4 b^9 x^4 \Gamma\left(\frac{9}{4}\right) {}_3F_4\left(\begin{matrix} 1,1,\frac{9}{4} \\ 2,\frac{5}{2},3,\frac{13}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{6144\Gamma\left(\frac{13}{4}\right)} - \frac{\pi^2 b^5 \log\left(b^4 x^4\right)}{160} - \frac{b}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**6,x)
```

```
[Out] pi**4*b**9*x**4*gamma(9/4)*hyper((1, 1, 9/4), (2, 5/2, 3, 13/4), -pi**2*b**
4*x**4/16)/(6144*gamma(13/4)) - pi**2*b**5*log(b**4*x**4)/160 - b/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^6, x)
```

3.124 $\int \frac{\textbf{FresnelC}(bx)}{x^7}\,dx$

Optimal. Leaf size=94

$$\frac{1}{90}\pi^3 b^6 S(bx) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{90x^3} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{90x} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{30x^5} - \frac{\text{FresnelC}(bx)}{6x^6}$$

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(30*x^5) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(90*x) -
 FresnelC[b*x]/(6*x^6) + (b^6*Pi^3*FresnelS[b*x])/90 + (b^3*Pi*Sin[(b^2*Pi*
x^2)/2])/(90*x^3)
```

Rubi [A] time = 0.0587288, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3388, 3387, 3351}

$$\frac{1}{90}\pi^3 b^6 S(bx) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{90x^3} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{90x} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{30x^5} - \frac{\text{FresnelC}(bx)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^7,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(30*x^5) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(90*x) -
 FresnelC[b*x]/(6*x^6) + (b^6*Pi^3*FresnelS[b*x])/90 + (b^3*Pi*Sin[(b^2*Pi*
x^2)/2])/(90*x^3)
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{C(bx)}{x^7} dx = -\frac{C(bx)}{6x^6} + \frac{1}{6} b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} - \frac{C(bx)}{6x^6} - \frac{1}{30}\left(b^3\pi\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} - \frac{C(bx)}{6x^6} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3} - \frac{1}{90}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x} - \frac{C(bx)}{6x^6} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3} + \frac{1}{90}\left(b^7\pi^3\right)\int \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x} - \frac{C(bx)}{6x^6} + \frac{1}{90}b^6\pi^3S(bx) + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}$$

Mathematica [A] time = 0.115146, size = 74, normalized size = 0.79

$$\frac{1}{90}\left(\pi^3b^6S(bx) + \frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^3} + \frac{b\left(\pi^2b^4x^4-3\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^5} - \frac{15\text{FresnelC}(bx)}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^7,x]
```

```
[Out] ((b*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^5 - (15*FresnelC[b*x])/x^6 +
 b^6*Pi^3*FresnelS[b*x] + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/x^3)/90
```

Maple [A] time = 0.047, size = 87, normalized size = 0.9

$$b^6\left(-\frac{\text{FresnelC}(bx)}{6b^6x^6} - \frac{1}{30b^5x^5}\cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{30}\left(-\frac{1}{3x^3b^3}\sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{3}\left(-\frac{1}{bx}\cos\left(\frac{b^2\pi x^2}{2}\right) - \pi\,\text{FresnelS}(bx)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^7,x)
```

```
[Out] b^6*(-1/6*FresnelC(b*x)/b^6/x^6-1/30/b^5/x^5*cos(1/2*b^2*Pi*x^2)-1/30*Pi*(-
1/3*sin(1/2*b^2*Pi*x^2)/b^3/x^3+1/3*Pi*(-1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*Fresn
elS(b*x))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^7, x)
```

Sympy [A] time = 1.52775, size = 56, normalized size = 0.6

$$\frac{b\Gamma\left(-\frac{5}{4}\right)\Gamma\left(\frac{1}{4}\right){}_2F_3\left(\begin{matrix} -\frac{5}{4},\frac{1}{4} \\ -\frac{1}{4},\frac{1}{2},\frac{5}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16x^5\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**7,x)
```

```
[Out] b*gamma(-5/4)*gamma(1/4)*hyper((-5/4, 1/4), (-1/4, 1/2, 5/4), -pi**2*b**4*x
**4/16)/(16*x**5*gamma(-1/4)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^7, x)
```

3.125 $\int \frac{\text{FresnelC}(bx)}{x^8}\,dx$

Optimal. Leaf size=102

$$\frac{1}{672}\pi^3 b^7 \text{Si}\left(\frac{1}{2}b^2\pi x^2\right)+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{168x^4}+\frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{336x^2}-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{42x^6}-\frac{\text{FresnelC}(bx)}{7x^7}$$

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(42*x^6) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(336*x^2
) - FresnelC[b*x]/(7*x^7) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(168*x^4) + (b^7*P
i^3*SinIntegral[(b^2*Pi*x^2)/2])/672
```

Rubi [A] time = 0.118397, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3380, 3297, 3299}

$$\frac{1}{672}\pi^3 b^7 \text{Si}\left(\frac{1}{2}b^2\pi x^2\right)+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{168x^4}+\frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{336x^2}-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{42x^6}-\frac{\text{FresnelC}(bx)}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^8,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(42*x^6) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(336*x^2
) - FresnelC[b*x]/(7*x^7) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(168*x^4) + (b^7*P
i^3*SinIntegral[(b^2*Pi*x^2)/2])/672
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{C(bx)}{x^8}\,dx = -\frac{C(bx)}{7x^7} + \frac{1}{7}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$
$$= -\frac{C(bx)}{7x^7} + \frac{1}{14}b\,\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} - \frac{C(bx)}{7x^7} - \frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} - \frac{C(bx)}{7x^7} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{1}{336}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{C(bx)}{7x^7} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} + \frac{1}{672}\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x}\right.$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{C(bx)}{7x^7} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} + \frac{1}{672}b^7\pi^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

Mathematica [A] time = 0.126535, size = 84, normalized size = 0.82

$$\frac{1}{672}\left(\pi^3b^7\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) + \frac{4\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^4} + \frac{2b\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^6} - \frac{96\text{FresnelC}(bx)}{x^7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^8,x]
```

```
[Out] ((2*b*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^6 - (96*FresnelC[b*x])/x^7
 + (4*b^3*Pi*Sin[(b^2*Pi*x^2)/2])/x^4 + b^7*Pi^3*SinIntegral[(b^2*Pi*x^2)/2
])/672
```

Maple [A] time = 0.05, size = 93, normalized size = 0.9

$$b^7\left(-\frac{\text{FresnelC}\,(bx)}{7\,b^7x^7} - \frac{1}{42\,b^6x^6}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{\pi}{42}\left(-\frac{1}{4\,x^4b^4}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\pi}{4}\left(-\frac{1}{2\,b^2x^2}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{\pi}{4}\text{Si}\left(\frac{b^2\pi\,x^2}{2}\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^8,x)
```

```
[Out] b^7*(-1/7*FresnelC(b*x)/b^7/x^7-1/42/b^6/x^6*cos(1/2*b^2*Pi*x^2)-1/42*Pi*(-
1/4*sin(1/2*b^2*Pi*x^2)/b^4/x^4+1/4*Pi*(-1/2/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/
4*Pi*Si(1/2*b^2*Pi*x^2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^8, x)
```

Sympy [A] time = 2.12267, size = 44, normalized size = 0.43

$$-\frac{b\Gamma\left(\frac{1}{4}\right){}_{2}F_{3}\left(\begin{matrix} -\frac{3}{2},\frac{1}{4} \\ -\frac{1}{2},\frac{1}{2},\frac{5}{4} \end{matrix}\middle| -\frac{\pi^2b^4x^4}{16}\right)}{24x^6\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**8,x)
```

```
[Out] -b*gamma(1/4)*hyper((-3/2, 1/4), (-1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(24
*x**6*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^8, x)
```

3.126 $\int \frac{\textbf{FresnelC}(bx)}{x^9}\,dx$

Optimal. Leaf size=119

$$\frac{1}{840}\pi^4 b^8\text{FresnelC}(bx)-\frac{\pi^3 b^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{840x}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{280x^5}+\frac{\pi^2 b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x^3}-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{56x^7}-\frac{\text{FresnelC}(b}{8x^8}$$

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(56*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(840*x^3
) + (b^8*Pi^4*FresnelC[b*x])/840 - FresnelC[b*x]/(8*x^8) + (b^3*Pi*Sin[(b^2
*Pi*x^2)/2])/(280*x^5) - (b^7*Pi^3*Sin[(b^2*Pi*x^2)/2])/(840*x)
```

Rubi [A] time = 0.0784687, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3388, 3387, 3352}

$$\frac{1}{840}\pi^4 b^8\text{FresnelC}(bx)-\frac{\pi^3 b^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{840x}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{280x^5}+\frac{\pi^2 b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x^3}-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{56x^7}-\frac{\text{FresnelC}(b}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^9,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(56*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(840*x^3
) + (b^8*Pi^4*FresnelC[b*x])/840 - FresnelC[b*x]/(8*x^8) + (b^3*Pi*Sin[(b^2
*Pi*x^2)/2])/(280*x^5) - (b^7*Pi^3*Sin[(b^2*Pi*x^2)/2])/(840*x)
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x^9}\,dx &= -\frac{C(bx)}{8x^8}+\frac{1}{8}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}-\frac{C(bx)}{8x^8}-\frac{1}{56}\left(b^3\pi\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}-\frac{C(bx)}{8x^8}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}-\frac{1}{280}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}-\frac{C(bx)}{8x^8}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}+\frac{1}{840}\left(b^7\pi^3\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}-\frac{C(bx)}{8x^8}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}-\frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x}+\frac{1}{840}\left(b^9\pi\right.\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}+\frac{1}{840}b^8\pi^4C(bx)-\frac{C(bx)}{8x^8}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}-\frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\right.}{840x}\end{aligned}$$

Mathematica [A] time = 0.0577638, size = 85, normalized size = 0.71

$$\frac{\left(\pi^4b^8x^8-105\right)\text{FresnelC}(bx)+\pi b^3x^3\left(3-\pi^2b^4x^4\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)+bx\left(\pi^2b^4x^4-15\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^9,x]
```

```
[Out] (b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*Fresn
elC[b*x] + b^3*Pi*x^3*(3 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)
```

Maple [A] time = 0.048, size = 108, normalized size = 0.9

$$b^8\left(-\frac{\text{FresnelC}\,(bx)}{8\,b^8x^8}-\frac{1}{56\,b^7x^7}\cos\left(\frac{b^2\pi\,x^2}{2}\right)-\frac{\pi}{56}\left(-\frac{1}{5\,b^5x^5}\sin\left(\frac{b^2\pi\,x^2}{2}\right)+\frac{\pi}{5}\left(-\frac{1}{3\,x^3b^3}\cos\left(\frac{b^2\pi\,x^2}{2}\right)-\frac{\pi}{3}\left(-\frac{1}{bx}\sin\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^9,x)
```

```
[Out] b^8*(-1/8*FresnelC(b*x)/b^8/x^8-1/56/b^7/x^7*cos(1/2*b^2*Pi*x^2)-1/56*Pi*(-
1/5*sin(1/2*b^2*Pi*x^2)/b^5/x^5+1/5*Pi*(-1/3/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/
3*Pi*(-sin(1/2*b^2*Pi*x^2)/b/x+Pi*FresnelC(b*x)))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^9,x, algorithm="maxima")
```

[Out] integrate(fresnelc(b*x)/x^9, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^9,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)/x^9, x)

Sympy [A] time = 3.70601, size = 185, normalized size = 1.55

$$\frac{\pi^4 b^8 C\,(bx)\,\Gamma\left(-\frac{7}{4}\right)}{2560\Gamma\left(\frac{5}{4}\right)} - \frac{\pi^3 b^7 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x\Gamma\left(\frac{5}{4}\right)} + \frac{\pi^2 b^5 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x^3\Gamma\left(\frac{5}{4}\right)} + \frac{3\pi b^3 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x^5\Gamma\left(\frac{5}{4}\right)} - \frac{3b\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{512x^7\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x**9,x)

[Out] pi**4*b**8*fresnelc(b*x)*gamma(-7/4)/(2560*gamma(5/4)) - pi**3*b**7*sin(pi*
b**2*x**2/2)*gamma(-7/4)/(2560*x*gamma(5/4)) + pi**2*b**5*cos(pi*b**2*x**2/
2)*gamma(-7/4)/(2560*x**3*gamma(5/4)) + 3*pi*b**3*sin(pi*b**2*x**2/2)*gamma
(-7/4)/(2560*x**5*gamma(5/4)) - 3*b*cos(pi*b**2*x**2/2)*gamma(-7/4)/(512*x*
*7*gamma(5/4)) - 21*fresnelc(b*x)*gamma(-7/4)/(512*x**8*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^9,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)/x^9, x)

3.127 $\int \frac{\textbf{FresnelC}(bx)}{x^{10}}\,dx$

Optimal. Leaf size=127

$$\frac{\pi^4 b^9 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right)}{6912} - \frac{\pi^3 b^7 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3456x^2} + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{432x^6} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{1728x^4} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{72x^8} - \text{Fre}$$

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(72*x^8) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(1728*x^
4) + (b^9*Pi^4*CosIntegral[(b^2*Pi*x^2)/2])/6912 - FresnelC[b*x]/(9*x^9) +
(b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(432*x^6) - (b^7*Pi^3*Sin[(b^2*Pi*x^2)/2])/(34
56*x^2)
```

Rubi [A] time = 0.143552, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6427, 3380, 3297, 3302}

$$\frac{\pi^4 b^9 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right)}{6912} - \frac{\pi^3 b^7 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3456x^2} + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{432x^6} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{1728x^4} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{72x^8} - \text{Fre}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^10,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(72*x^8) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(1728*x^
4) + (b^9*Pi^4*CosIntegral[(b^2*Pi*x^2)/2])/6912 - FresnelC[b*x]/(9*x^9) +
(b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(432*x^6) - (b^7*Pi^3*Sin[(b^2*Pi*x^2)/2])/(34
56*x^2)
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{C(bx)}{x^{10}}\,dx = -\frac{C(bx)}{9x^9} + \frac{1}{9}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$
$$= -\frac{C(bx)}{9x^9} + \frac{1}{18}b\,\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} - \frac{C(bx)}{9x^9} - \frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} - \frac{C(bx)}{9x^9} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{1}{864}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} - \frac{C(bx)}{9x^9} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} + \frac{\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x\right.}{3456}$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} - \frac{C(bx)}{9x^9} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2} + \frac{\left(b^9\pi^4\right)\text{Subs}}{}$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{b^9\pi^4\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)}{6912} - \frac{C(bx)}{9x^9} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\right.}{3456x^2}$$

Mathematica [A] time = 0.0805378, size = 96, normalized size = 0.76

$$\frac{\pi^4b^9\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right) - \frac{2\pi b^3\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^6} + \frac{4b\left(\pi^2b^4x^4-24\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^8} - \frac{768\text{FresnelC}(bx)}{x^9}}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^10,x]
```

```
[Out] ((4*b*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^8 + b^9*Pi^4*CosIntegral[
(b^2*Pi*x^2)/2] - (768*FresnelC[b*x])/x^9 - (2*b^3*Pi*(-8 + b^4*Pi^2*x^4)*S
in[(b^2*Pi*x^2)/2])/x^6)/6912
```

Maple [A] time = 0.047, size = 115, normalized size = 0.9

$$b^9\left(-\frac{\text{FresnelC}\,(bx)}{9\,b^9x^9} - \frac{1}{72\,b^8x^8}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{\pi}{72}\left(-\frac{1}{6\,b^6x^6}\sin\left(\frac{b^2\pi\,x^2}{2}\right) + \frac{\pi}{6}\left(-\frac{1}{4\,x^4b^4}\cos\left(\frac{b^2\pi\,x^2}{2}\right) - \frac{\pi}{4}\left(-\frac{1}{2\,b^2x^2}\sin\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^10,x)
```

```
[Out] b^9*(-1/9*FresnelC(b*x)/b^9/x^9-1/72/b^8/x^8*cos(1/2*b^2*Pi*x^2)-1/72*Pi*(-
1/6*sin(1/2*b^2*Pi*x^2)/b^6/x^6+1/6*Pi*(-1/4/b^4/x^4*cos(1/2*b^2*Pi*x^2)-1/
4*Pi*(-1/2*sin(1/2*b^2*Pi*x^2)/b^2/x^2+1/4*Pi*Ci(1/2*b^2*Pi*x^2)))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^10, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^10, x)
```

Sympy [A] time = 6.86518, size = 76, normalized size = 0.6

$$-\frac{\pi^6 b^{13} x^4 \Gamma\left(\frac{13}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{13}{4} \\ 2, \frac{7}{2}, 4, \frac{17}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{737280\Gamma\left(\frac{17}{4}\right)} + \frac{\pi^4 b^9 \log\left(b^4 x^4\right)}{13824} + \frac{\pi^2 b^5}{160 x^4} - \frac{b}{8 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**10,x)
```

```
[Out] -pi**6*b**13*x**4*gamma(13/4)*hyper((1, 1, 13/4), (2, 7/2, 4, 17/4), -pi**2
*b**4*x**4/16)/(737280*gamma(17/4)) + pi**4*b**9*log(b**4*x**4)/13824 + pi*
*2*b**5/(160*x**4) - b/(8*x**8)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^10, x)
```

3.128 $\int (c+dx)^3 \text{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=298

$$-\frac{d^2(a+bx)^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2b^4}-\frac{(bc-ad)^4\text{FresnelC}(a+bx)}{4b^4d}+\frac{3d(bc-ad)^2}{2\pi b}$$

```
[Out] (-2*d^2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*d^3*(a + b*x)*
Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2) - ((b*c - a*d)^4*FresnelC[a + b*x])/(
4*b^4*d) + (3*d^3*FresnelC[a + b*x])/(4*b^4*Pi^2) + ((c + d*x)^4*FresnelC[a
 + b*x])/(4*d) + (3*d*(b*c - a*d)^2*FresnelS[a + b*x])/(2*b^4*Pi) - ((b*c -
 a*d)^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*d*(b*c - a*d)^2*(a + b*x)*Si
n[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) - (d^2*(b*c - a*d)*(a + b*x)^2*Sin[(Pi*(a
 + b*x)^2)/2])/(b^4*Pi) - (d^3*(a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*
Pi)
```

Rubi [A] time = 0.37306, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638, 3385}

$$-\frac{d^2(a+bx)^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2b^4}-\frac{(bc-ad)^4\text{FresnelC}(a+bx)}{4b^4d}+\frac{3d(bc-ad)^2}{2\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*FresnelC[a + b*x],x]
```


```
[Out] (-2*d^2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*d^3*(a + b*x)*
Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2) - ((b*c - a*d)^4*FresnelC[a + b*x])/(
4*b^4*d) + (3*d^3*FresnelC[a + b*x])/(4*b^4*Pi^2) + ((c + d*x)^4*FresnelC[a
 + b*x])/(4*d) + (3*d*(b*c - a*d)^2*FresnelS[a + b*x])/(2*b^4*Pi) - ((b*c -
 a*d)^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*d*(b*c - a*d)^2*(a + b*x)*Si
n[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) - (d^2*(b*c - a*d)*(a + b*x)^2*Sin[(Pi*(a
 + b*x)^2)/2])/(b^4*Pi) - (d^3*(a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*
Pi)
```


Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\int (c+dx)^3 C(a+bx)\,dx = \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{b\int (c+dx)^4 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{4d}$$

$$= \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{\text{Subst}\left(\int \left(b^4c^4\left(1+\frac{ad(-4b^3c^3+6ab^2c^2d-4a^2bcd^2+a^3d^3)}{b^4c^4}\right)\cos\left(\frac{\pi x^2}{2}\right)+4b^3c^3d\left(1\right.\right.\right.}{}$$

$$= \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{d^3\,\text{Subst}\left(\int x^4\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{4b^4} - \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int x^3\cos\right.}{b^4}$$

$$= -\frac{(bc-ad)^4 C(a+bx)}{4b^4 d} + \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{3d(bc-ad)^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{d^3(a}{}$$

$$= -\frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{(bc-ad)^4 C(a+bx)}{4b^4 d} + \frac{(c+dx)^4 C(a+bx)}{4d} + \frac{3d(bc-ad)^2 S}{2b^4\pi}$$

$$= -\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{(bc-ad)^4 C(a+bx)}{4b^4 d} + \frac{3a}{}$$

Mathematica [A] time = 0.816556, size = 424, normalized size = 1.42

$$\frac{\text{FresnelC}(a+bx)\left(6\pi^2b^2c^2d\left(b^2x^2-a^2\right)+4\pi^2bcd^2\left(a^3+b^3x^3\right)+d^3\left(-\pi^2a^4+\pi^2b^4x^4+3\right)+4\pi^2b^3c^3(a+bx)\right)-4\pi a^2bc}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*FresnelC[a + b*x],x]
```

```
[Out] (-8*b*c*d^2*Cos[(Pi*(a + b*x)^2)/2] + 5*a*d^3*Cos[(Pi*(a + b*x)^2)/2] - 3*b
*d^3*x*Cos[(Pi*(a + b*x)^2)/2] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*Pi
^2*(-a^2 + b^2*x^2) + 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2 +
b^4*Pi^2*x^4))*FresnelC[a + b*x] + 6*d*(b*c - a*d)^2*Pi*FresnelS[a + b*x] -
 4*b^3*c^3*Pi*Sin[(Pi*(a + b*x)^2)/2] + 6*a*b^2*c^2*d*Pi*Sin[(Pi*(a + b*x)^
2)/2] - 4*a^2*b*c*d^2*Pi*Sin[(Pi*(a + b*x)^2)/2] + a^3*d^3*Pi*Sin[(Pi*(a +
b*x)^2)/2] - 6*b^3*c^2*d*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + 4*a*b^2*c*d^2*Pi*x*
Sin[(Pi*(a + b*x)^2)/2] - a^2*b*d^3*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - 4*b^3*c*
d^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2] + a*b^2*d^3*Pi*x^2*Sin[(Pi*(a + b*x)^2)/
2] - b^3*d^3*Pi*x^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Maple [A] time = 0.056, size = 397, normalized size = 1.3

$$\frac{1}{b}\left(\frac{\text{FresnelC}\,(bx+a)\,(d\,(bx+a)-ad+bc)^4}{4\,db^3} - \frac{1}{4\,db^3}\left(\frac{d^4\,(bx+a)^3}{\pi}\sin\left(\frac{\pi\,(bx+a)^2}{2}\right) - 3\,\frac{d^4}{\pi}\left(-\frac{(bx+a)\cos\left(1/2\,\pi\,(bx+\right.}{\pi}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*FresnelC(b*x+a),x)
```

```
[Out] 1/b*(1/4*FresnelC(b*x+a)*(d*(b*x+a)-a*d+b*c)^4/b^3/d-1/4/b^3/d*(d^4/Pi*(b*x
+a)^3*sin(1/2*Pi*(b*x+a)^2)-3*d^4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+1
/Pi*FresnelC(b*x+a))+(-4*a*d^4+4*b*c*d^3)/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2
)+2*(-4*a*d^4+4*b*c*d^3)/Pi^2*cos(1/2*Pi*(b*x+a)^2)+(6*a^2*d^4-12*a*b*c*d^3
+6*b^2*c^2*d^2)/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-(6*a^2*d^4-12*a*b*c*d^3+6*
b^2*c^2*d^2)/Pi*FresnelS(b*x+a)+(-4*a^3*d^4+12*a^2*b*c*d^3-12*a*b^2*c^2*d^2
```

```
+4*b^3*c^3*d)/Pi*sin(1/2*Pi*(b*x+a)^2)+a^4*d^4*FresnelC(b*x+a)-4*a^3*b*c*d^
3*FresnelC(b*x+a)+6*a^2*b^2*c^2*d^2*FresnelC(b*x+a)-4*a*b^3*c^3*d*FresnelC(
b*x+a)+b^4*c^4*FresnelC(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \text{fresnelc}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^3*fresnelc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\text{fresnelc}(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*fresnelc(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3\,C(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*fresnelc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*fresnelc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \text{fresnelc}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*fresnelc(b*x + a), x)
```

3.129 $\int (c+dx)^2 \mathbf{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=194

$$-\frac{(bc-ad)^3\text{FresnelC}(a+bx)}{3b^3d}+\frac{d(bc-ad)S(a+bx)}{\pi b^3}-\frac{(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{d(a+bx)(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx\right.}{\pi b^3}$$

```
[Out] (-2*d^2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) - ((b*c - a*d)^3*FresnelC[a +
 b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelC[a + b*x])/(3*d) + (d*(b*c - a*d)*F
resnelS[a + b*x])/(b^3*Pi) - ((b*c - a*d)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^3*P
i) - (d*(b*c - a*d)*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (d^2*(a +
 b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rubi [A] time = 0.204614, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638}

$$-\frac{(bc-ad)^3\text{FresnelC}(a+bx)}{3b^3d}+\frac{d(bc-ad)S(a+bx)}{\pi b^3}-\frac{(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{d(a+bx)(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx\right.}{\pi b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelC[a + b*x],x]
```


```
[Out] (-2*d^2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) - ((b*c - a*d)^3*FresnelC[a +
 b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelC[a + b*x])/(3*d) + (d*(b*c - a*d)*F
resnelS[a + b*x])/(b^3*Pi) - ((b*c - a*d)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^3*P
i) - (d*(b*c - a*d)*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (d^2*(a +
 b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```


Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2 C(a+bx)\,dx = \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{b\int (c+dx)^3 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{3d}$$

$$= \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{\text{Subst}\left(\int \left(b^3c^3\left(1-\frac{ad\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3c^3}\right)\cos\left(\frac{\pi x^2}{2}\right) + 3b^2c^2d\left(1+\frac{ad(-\cdots}{b\cdots}\right.\right.\right.}{\cdots}$$

$$= \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{d^2\,\text{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{3b^3} - \frac{(d(bc-ad))\,\text{Subst}\left(\int x^2 \cdots\right.}{b^3}$$

$$= -\frac{(bc-ad)^3 C(a+bx)}{3b^3d} + \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{d(bc-ad)(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{d^2\,\text{S}\cdots}{\cdots}$$

$$= -\frac{(bc-ad)^3 C(a+bx)}{3b^3d} + \frac{(c+dx)^3 C(a+bx)}{3d} + \frac{d(bc-ad)S(a+bx)}{b^3\pi} - \frac{(bc-ad)^2\sin\left(\frac{1}{2}\pi(a\cdots\right.}{b^3\pi}$$

$$= -\frac{2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2} - \frac{(bc-ad)^3 C(a+bx)}{3b^3d} + \frac{(c+dx)^3 C(a+bx)}{3d} + \frac{d(bc-ad)S(a+bx\cdots}{b^3\pi}$$

Mathematica [A] time = 0.46643, size = 237, normalized size = 1.22

$$\frac{\pi^2\text{FresnelC}(a+bx)\left(-3a^2bcd+a^3d^2+3ab^2c^2+b^3x\left(3c^2+3cdx+d^2x^2\right)\right)-\pi a^2d^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)-3\pi b^2c^2\sin\left(\frac{1}{2}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*FresnelC[a + b*x],x]
```

```
[Out] (-2*d^2*Cos[(Pi*(a + b*x)^2)/2] + Pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2
 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelC[a + b*x] + 3*d*(b*c - a*d)*P
i*FresnelS[a + b*x] - 3*b^2*c^2*Pi*Sin[(Pi*(a + b*x)^2)/2] + 3*a*b*c*d*Pi*S
in[(Pi*(a + b*x)^2)/2] - a^2*d^2*Pi*Sin[(Pi*(a + b*x)^2)/2] - 3*b^2*c*d*Pi*
x*Sin[(Pi*(a + b*x)^2)/2] + a*b*d^2*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - b^2*d^2*
Pi*x^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Maple [A] time = 0.054, size = 249, normalized size = 1.3

$$\frac{1}{b}\left(\frac{\mathrm{FresnelC}\,(bx+a)\,(d\,(bx+a)-ad+bc)^3}{3\,db^2}-\frac{1}{3\,db^2}\left(\frac{d^3\,(bx+a)^2}{\pi}\sin\left(\frac{\pi\,(bx+a)^2}{2}\right)+2\,\frac{d^3\cos\left(1/2\,\pi\,(bx+a)^2\right)}{\pi^2}+\frac{(-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelC(b*x+a),x)
```

```
[Out] 1/b*(1/3*FresnelC(b*x+a)*(d*(b*x+a)-a*d+b*c)^3/b^2/d-1/3/b^2/d*(d^3/Pi*(b*x
+a)^2*sin(1/2*Pi*(b*x+a)^2)+2*d^3/Pi^2*cos(1/2*Pi*(b*x+a)^2)+(-3*a*d^3+3*b*
c*d^2)/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-(-3*a*d^3+3*b*c*d^2)/Pi*FresnelS(b*
x+a)+(3*a^2*d^3-6*a*b*c*d^2+3*b^2*c^2*d)/Pi*sin(1/2*Pi*(b*x+a)^2)-a^3*d^3*F
resnelC(b*x+a)+3*a^2*b*c*d^2*FresnelC(b*x+a)-3*a*b^2*c^2*d*FresnelC(b*x+a)+
b^3*c^3*FresnelC(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \mathrm{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*fresnelc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d^2x^2+2\,cdx+c^2\right)\mathrm{fresnelc}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnelc(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2\, C\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnelc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*fresnelc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \operatorname{fresnelc}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*fresnelc(b*x + a), x)
```

3.130 $\int (c+dx)\,\textbf{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=122

$$-\frac{(bc-ad)^2\text{FresnelC}(a+bx)}{2b^2d}-\frac{(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{dS(a+bx)}{2\pi b^2}-\frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{(c+dx)^2\text{Fres}}{2}$$

```
[Out] -((b*c - a*d)^2*FresnelC[a + b*x])/(2*b^2*d) + ((c + d*x)^2*FresnelC[a + b*
x])/(2*d) + (d*FresnelS[a + b*x])/(2*b^2*Pi) - ((b*c - a*d)*Sin[(Pi*(a + b*
x)^2)/2])/(b^2*Pi) - (d*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Rubi [A] time = 0.112805, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351}

$$-\frac{(bc-ad)^2\text{FresnelC}(a+bx)}{2b^2d}-\frac{(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{dS(a+bx)}{2\pi b^2}-\frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{(c+dx)^2\text{Fres}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelC[a + b*x],x]
```

```
[Out] -((b*c - a*d)^2*FresnelC[a + b*x])/(2*b^2*d) + ((c + d*x)^2*FresnelC[a + b*
x])/(2*d) + (d*FresnelS[a + b*x])/(2*b^2*Pi) - ((b*c - a*d)*Sin[(Pi*(a + b*
x)^2)/2])/(b^2*Pi) - (d*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)C(a+bx)\,dx &= \frac{(c+dx)^2C(a+bx)}{2d}-\frac{b\int (c+dx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{2d}\\ &= \frac{(c+dx)^2C(a+bx)}{2d}-\frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\cos\left(\frac{\pi x^2}{2}\right)+2bcd\left(1-\frac{ad}{bc}\right)x\cos\left(\frac{\pi x^2}{2}\right)\right.}{2b^2d}\\ &= \frac{(c+dx)^2C(a+bx)}{2d}-\frac{d\,\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)dx,x,a+bx\right)}{2b^2}-\frac{(bc-ad)\,\text{Subst}\left(\int x\cos\left(\frac{\pi x}{2}\right.\right.}{b^2}\\ &= -\frac{(bc-ad)^2C(a+bx)}{2b^2d}+\frac{(c+dx)^2C(a+bx)}{2d}-\frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}-\frac{(bc-ad)\,\text{Subs}}{}\\ &= -\frac{(bc-ad)^2C(a+bx)}{2b^2d}+\frac{(c+dx)^2C(a+bx)}{2d}+\frac{dS(a+bx)}{2b^2\pi}-\frac{(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}-\end{aligned}$$

Mathematica [A] time = 0.235525, size = 74, normalized size = 0.61

$$\frac{-\pi(a+bx)\text{FresnelC}(a+bx)(ad-b(2c+dx))+\sin\left(\frac{1}{2}\pi(a+bx)^2\right)(ad-2bc-bdx)+dS(a+bx)}{2\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*FresnelC[a + b*x],x]
```

```
[Out] (-(Pi*(a + b*x)*(a*d - b*(2*c + d*x))*FresnelC[a + b*x]) + d*FresnelS[a + b
*x] + (-2*b*c + a*d - b*d*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Maple [A] time = 0.053, size = 107, normalized size = 0.9

$$\frac{1}{b}\left(\frac{\text{FresnelC}\,(bx+a)}{b}\left(\frac{d\,(bx+a)^2}{2}-ad\,(bx+a)+bc\,(bx+a)\right)-\frac{1}{2\,b}\left(\frac{d\,(bx+a)}{\pi}\sin\left(\frac{\pi\,(bx+a)^2}{2}\right)-\frac{d\text{FresnelS}\,(bx+}{\pi}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelC(b*x+a),x)
```

```
[Out] 1/b*(FresnelC(b*x+a)/b*(1/2*d*(b*x+a)^2-a*d*(b*x+a)+b*c*(b*x+a))-1/2/b*(d/P
i*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-d/Pi*FresnelS(b*x+a)+(-2*a*d+2*b*c)/Pi*sin(
1/2*Pi*(b*x+a)^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\text{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*fresnelc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,((dx+c)\text{fresnelc}\,(bx+a)\,,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*fresnelc(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\,C\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*fresnelc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\text{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnelc(b*x + a), x)
```

3.131 $\int \text{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{(a+bx)\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[Out] ((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

Rubi [A] time = 0.0062556, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6419}

$$\frac{(a+bx)\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[a + b*x],x]
```

```
[Out] ((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

Rule 6419

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelC[a +
 b*x])/b, x] - Simp[Sin[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int C(a+bx)\,dx=\frac{(a+bx)C(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

Mathematica [B] time = 0.0292309, size = 90, normalized size = 2.43

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}-\frac{\cos\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+x\text{FresnelC}(a+bx)+\frac{a\text{FresnelC}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[a + b*x],x]
```

```
[Out] (a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^2
)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

Maple [A] time = 0.049, size = 34, normalized size = 0.9

$$\frac{1}{b}\left(\text{FresnelC}\left(bx+a\right)\left(bx+a\right)-\frac{1}{\pi}\sin\left(\frac{\pi\left(bx+a\right)^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a),x)
```

```
[Out] 1/b*(FresnelC(b*x+a)*(b*x+a)-1/Pi*sin(1/2*Pi*(b*x+a)^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,(\text{fresnelc}\,(bx+a)\,,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int C\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x)
```

```
[Out] Integral(fresnelc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a), x)
```

3.132 $\int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[FresnelC[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.014454, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{C(a+bx)}{c+dx} \, dx = \int \frac{C(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 0.025616, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[FresnelC[a + b*x]/(c + d*x), x]
```

Maple [A] time = 0.365, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx+a)}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/(d*x+c),x)
```

```
[Out] int(FresnelC(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\,(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(fresnelc(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a)/(d*x + c), x)
```

3.133 $\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(a+bx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable[FresnelC[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.0142711, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{C(a+bx)}{(c+dx)^2} dx = \int \frac{C(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 2.28707, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[FresnelC[a + b*x]/(c + d*x)^2, x]
```

Maple [A] time = 0.373, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(FresnelC(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a)/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx+a)}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a)/(d*x + c)^2, x)
```

3.134 $\int x^3 \textbf{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=227

$$-\frac{a^4\text{FresnelC}(a+bx)}{4b^4}+\frac{3a^2S(a+bx)}{2\pi b^4}+\frac{a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3\text{FresnelC}(a+bx)}{4\pi^2b^4}$$

```
[Out] (2*a*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*(a + b*x)*Cos[(Pi*(a + b*x)^2
)/2])/(4*b^4*Pi^2) - (a^4*FresnelC[a + b*x])/(4*b^4) + (3*FresnelC[a + b*x]
)/(4*b^4*Pi^2) + (x^4*FresnelC[a + b*x])/4 + (3*a^2*FresnelS[a + b*x])/(2*b
^4*Pi) + (a^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*a^2*(a + b*x)*Sin[(Pi*
(a + b*x)^2)/2])/(2*b^4*Pi) + (a*(a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^4*
Pi) - ((a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi)
```

Rubi [A] time = 0.18712, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638, 3385}

$$-\frac{a^4\text{FresnelC}(a+bx)}{4b^4}+\frac{3a^2S(a+bx)}{2\pi b^4}+\frac{a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3\text{FresnelC}(a+bx)}{4\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[a + b*x],x]
```

```
[Out] (2*a*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*(a + b*x)*Cos[(Pi*(a + b*x)^2
)/2])/(4*b^4*Pi^2) - (a^4*FresnelC[a + b*x])/(4*b^4) + (3*FresnelC[a + b*x]
)/(4*b^4*Pi^2) + (x^4*FresnelC[a + b*x])/4 + (3*a^2*FresnelS[a + b*x])/(2*b
^4*Pi) + (a^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*a^2*(a + b*x)*Sin[(Pi*
(a + b*x)^2)/2])/(2*b^4*Pi) + (a*(a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^4*
Pi) - ((a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi)
```

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\int x^3 C(a+bx)\,dx = \frac{1}{4}x^4C(a+bx) - \frac{1}{4}b\int x^4\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$

$$= \frac{1}{4}x^4C(a+bx) - \frac{\text{Subst}\left(\int\left(a^4\cos\left(\frac{\pi x^2}{2}\right) - 4a^3x\cos\left(\frac{\pi x^2}{2}\right) + 6a^2x^2\cos\left(\frac{\pi x^2}{2}\right) - 4ax^3\cos\left(\frac{\pi x^2}{2}\right) + x^4\,\text{co}\right.}{4b^4}$$

$$= \frac{1}{4}x^4C(a+bx) - \frac{\text{Subst}\left(\int x^4\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{4b^4} + \frac{a\,\text{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{b^4} - \frac{(3a}{}$$

$$= -\frac{a^4C(a+bx)}{4b^4} + \frac{1}{4}x^4C(a+bx) - \frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} + \frac{a\,\text{S}}{}$$

$$= -\frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{a^4C(a+bx)}{4b^4} + \frac{1}{4}x^4C(a+bx) + \frac{3a^2S(a+bx)}{2b^4\pi} + \frac{a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right.}{b^4\pi}$$

$$= \frac{2a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{a^4C(a+bx)}{4b^4} + \frac{3C(a+bx)}{4b^4\pi^2} + \frac{1}{4}x^4C(a+bx) +$$

Mathematica [A] time = 0.294206, size = 166, normalized size = 0.73

$$\frac{\left(-\pi^2a^4+\pi^2b^4x^4+3\right)\text{FresnelC}(a+bx)+6\pi a^2S(a+bx)+\pi a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)-\pi a^2bx\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi ab^2}{4\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[a + b*x],x]
```

```
[Out] (5*a*Cos[(Pi*(a + b*x)^2)/2] - 3*b*x*Cos[(Pi*(a + b*x)^2)/2] + (3 - a^4*Pi^
2 + b^4*Pi^2*x^4)*FresnelC[a + b*x] + 6*a^2*Pi*FresnelS[a + b*x] + a^3*Pi*S
in[(Pi*(a + b*x)^2)/2] - a^2*b*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + a*b^2*Pi*x^2*
Sin[(Pi*(a + b*x)^2)/2] - b^3*Pi*x^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Maple [A] time = 0.05, size = 187, normalized size = 0.8

$$\frac{1}{b^4}\left(\frac{\text{FresnelC}\,(bx+a)\,b^4x^4}{4}-\frac{(bx+a)^3}{4\,\pi}\sin\left(\frac{\pi\,(bx+a)^2}{2}\right)+\frac{3}{4\,\pi}\left(-\frac{bx+a}{\pi}\cos\left(\frac{\pi\,(bx+a)^2}{2}\right)+\frac{\text{FresnelC}\,(bx+a)}{\pi}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*FresnelC(b*x+a)*b^4*x^4-1/4/Pi*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)+3
/4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+1/Pi*FresnelC(b*x+a))+a/Pi*(b*x+
a)^2*sin(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*cos(1/2*Pi*(b*x+a)^2)-3/2*a^2/Pi*(b*x+a
)*sin(1/2*Pi*(b*x+a)^2)+3/2*a^2/Pi*FresnelS(b*x+a)+a^3/Pi*sin(1/2*Pi*(b*x+a
)^2)-1/4*a^4*FresnelC(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\text{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnelc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3\text{fresnelc}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnelc(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 C(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x+a),x)
```

```
[Out] Integral(x**3*fresnelc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \mathrm{fresnelc}\,(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnelc(b*x + a), x)
```

3.135 $\int x^2\mathbf{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=148

$$\frac{a^3\text{FresnelC}(a+bx)}{3b^3}-\frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{aS(a+bx)}{\pi b^3}+\frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3}$$

```
[Out] (-2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) + (a^3*FresnelC[a + b*x])/(3*b^3)
 + (x^3*FresnelC[a + b*x])/3 - (a*FresnelS[a + b*x])/(b^3*Pi) - (a^2*Sin[(P
i*(a + b*x)^2)/2])/(b^3*Pi) + (a*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi
) - ((a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rubi [A] time = 0.122512, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638}

$$\frac{a^3\text{FresnelC}(a+bx)}{3b^3}-\frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{aS(a+bx)}{\pi b^3}+\frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[a + b*x],x]
```



```
[Out] (-2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) + (a^3*FresnelC[a + b*x])/(3*b^3)
 + (x^3*FresnelC[a + b*x])/3 - (a*FresnelS[a + b*x])/(b^3*Pi) - (a^2*Sin[(P
i*(a + b*x)^2)/2])/(b^3*Pi) + (a*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi
) - ((a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```


Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2 C(a+bx)\,dx &= \frac{1}{3}x^3 C(a+bx) - \frac{1}{3}b\int x^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx \\
&= \frac{1}{3}x^3C(a+bx) - \frac{\text{Subst}\left(\int\left(-a^3\cos\left(\frac{\pi x^2}{2}\right)+3a^2x\cos\left(\frac{\pi x^2}{2}\right)-3ax^2\cos\left(\frac{\pi x^2}{2}\right)+x^3\cos\left(\frac{\pi x^2}{2}\right)\right)dx,x,a\right.}{3b^3} \\
&= \frac{1}{3}x^3C(a+bx) - \frac{\text{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)dx,x,a+bx\right)}{3b^3} + \frac{a\,\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)dx,x,a+bx\right)}{b^3} - \frac{a^2}{} \\
&= \frac{a^3C(a+bx)}{3b^3} + \frac{1}{3}x^3C(a+bx) + \frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{\text{Subst}\left(\int x\cos\left(\frac{\pi x}{2}\right)dx,x,(a+bx)^2\right)}{6b^3} \\
&= \frac{a^3C(a+bx)}{3b^3} + \frac{1}{3}x^3C(a+bx) - \frac{aS(a+bx)}{b^3\pi} - \frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \\
&= -\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2} + \frac{a^3C(a+bx)}{3b^3} + \frac{1}{3}x^3C(a+bx) - \frac{aS(a+bx)}{b^3\pi} - \frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{a(a+}{}
\end{aligned}
$$

Mathematica [A] time = 0.29704, size = 116, normalized size = 0.78

$$
-\frac{-\pi^2\left(a^3+b^3x^3\right)\text{FresnelC}(a+bx)+\pi a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi b^2x^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+3\pi aS(a+bx)-\pi abx\sin\left(\frac{1}{2}\pi(\right.}{3\pi^2b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[a + b*x],x]
```

```
[Out] -(2*Cos[(Pi*(a + b*x)^2)/2] - Pi^2*(a^3 + b^3*x^3)*FresnelC[a + b*x] + 3*a*
Pi*FresnelS[a + b*x] + a^2*Pi*Sin[(Pi*(a + b*x)^2)/2] - a*b*Pi*x*Sin[(Pi*(a
 + b*x)^2)/2] + b^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Maple [A] time = 0.052, size = 122, normalized size = 0.8

$$\frac{1}{b^3}\left(\frac{b^3x^3\mathrm{FresnelC}\,(bx+a)}{3}-\frac{(bx+a)^2}{3\,\pi}\sin\left(\frac{\pi\,(bx+a)^2}{2}\right)-\frac{2}{3\,\pi^2}\cos\left(\frac{\pi\,(bx+a)^2}{2}\right)+\frac{a\,(bx+a)}{\pi}\sin\left(\frac{\pi\,(bx+a)^2}{2}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelC(b*x+a)-1/3/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)-2
/3/Pi^2*cos(1/2*Pi*(b*x+a)^2)+a/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-a/Pi*Fresn
elS(b*x+a)-a^2/Pi*sin(1/2*Pi*(b*x+a)^2)+1/3*a^3*FresnelC(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\mathrm{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnelc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2\mathrm{fresnelc}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnelc(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2C\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x+a),x)
```

```
[Out] Integral(x**2*fresnelc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc(b*x + a), x)
```

3.136 $\int x\textbf{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{a^2\text{FresnelC}(a+bx)}{2b^2}+\frac{S(a+bx)}{2\pi b^2}+\frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2\text{FresnelC}(a+bx)$$

```
[Out] -(a^2*FresnelC[a + b*x])/(2*b^2) + (x^2*FresnelC[a + b*x])/2 + FresnelS[a +
 b*x]/(2*b^2*Pi) + (a*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi) - ((a + b*x)*Sin[(P
i*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Rubi [A] time = 0.0707863, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351}

$$-\frac{a^2\text{FresnelC}(a+bx)}{2b^2}+\frac{S(a+bx)}{2\pi b^2}+\frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2\text{FresnelC}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[a + b*x],x]
```

```
[Out] -(a^2*FresnelC[a + b*x])/(2*b^2) + (x^2*FresnelC[a + b*x])/2 + FresnelS[a +
 b*x]/(2*b^2*Pi) + (a*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi) - ((a + b*x)*Sin[(P
i*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int xC(a+bx)\,dx &= \frac{1}{2}x^2C(a+bx) - \frac{1}{2}b\int x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx \\
&= \frac{1}{2}x^2C(a+bx) - \frac{\operatorname{Subst}\left(\int\left(a^2\cos\left(\frac{\pi x^2}{2}\right) - 2ax\cos\left(\frac{\pi x^2}{2}\right) + x^2\cos\left(\frac{\pi x^2}{2}\right)\right)dx, x, a+bx\right)}{2b^2} \\
&= \frac{1}{2}x^2C(a+bx) - \frac{\operatorname{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{2b^2} + \frac{a\operatorname{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{b^2} - \frac{a^2\operatorname{Su}\ldots}{\ldots} \\
&= -\frac{a^2C(a+bx)}{2b^2} + \frac{1}{2}x^2C(a+bx) - \frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} + \frac{a\operatorname{Subst}\left(\int\cos\left(\frac{\pi x}{2}\right)dx, x, (a+bx)^2\right)}{2b^2} \ldots \\
&= -\frac{a^2C(a+bx)}{2b^2} + \frac{1}{2}x^2C(a+bx) + \frac{S(a+bx)}{2b^2\pi} + \frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} - \frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}
\end{aligned}$$

Mathematica [A] time = 0.148444, size = 59, normalized size = 0.62

$$\frac{\left(\pi b^2x^2 - \pi a^2\right)\operatorname{FresnelC}(a+bx) + S(a+bx) + (a-bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelC[a + b*x],x]
```

```
[Out] ((-(a^2*Pi) + b^2*Pi*x^2)*FresnelC[a + b*x] + FresnelS[a + b*x] + (a - b*x)
*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Maple [A] time = 0.052, size = 79, normalized size = 0.8

$$\frac{1}{b^2}\left(\operatorname{FresnelC}(bx+a)\left(\frac{(bx+a)^2}{2} - a(bx+a)\right) - \frac{bx+a}{2\pi}\sin\left(\frac{\pi(bx+a)^2}{2}\right) + \frac{\operatorname{FresnelS}(bx+a)}{2\pi} + \frac{a}{\pi}\sin\left(\frac{\pi(bx+a)^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x+a),x)
```

```
[Out] 1/b^2*(FresnelC(b*x+a)*(1/2*(b*x+a)^2-a*(b*x+a))-1/2/Pi*(b*x+a)*sin(1/2*Pi*
(b*x+a)^2)+1/2/Pi*FresnelS(b*x+a)+a/Pi*sin(1/2*Pi*(b*x+a)^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnelc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,(x\text{fresnelc}\,(bx+a)\,,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*fresnelc(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xC\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x+a),x)
```

```
[Out] Integral(x*fresnelc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*fresnelc(b*x + a), x)
```

3.137 $\int \text{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{(a+bx)\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[Out] ((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

Rubi [A] time = 0.0067759, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6419}

$$\frac{(a+bx)\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[a + b*x],x]
```

```
[Out] ((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

Rule 6419

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelC[a +
 b*x])/b, x] - Simp[Sin[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int C(a+bx)\,dx=\frac{(a+bx)C(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

Mathematica [B] time = 0.0290167, size = 90, normalized size = 2.43

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}-\frac{\cos\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+x\text{FresnelC}(a+bx)+\frac{a\text{FresnelC}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[a + b*x],x]
```

```
[Out] (a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^2
)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

Maple [A] time = 0.049, size = 34, normalized size = 0.9

$$\frac{1}{b}\left(\text{FresnelC}\,(bx+a)\,(bx+a)-\frac{1}{\pi}\sin\left(\frac{\pi\,(bx+a)^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a),x)
```

```
[Out] 1/b*(FresnelC(b*x+a)*(b*x+a)-1/Pi*sin(1/2*Pi*(b*x+a)^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\,(\text{fresnelc}\,(bx+a)\,,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int C\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x)
```

```
[Out] Integral(fresnelc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a), x)
```

3.138 $\int \frac{\text{FresnelC}(a+bx)}{x} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[FresnelC[a + b*x]/x, x]
```

Rubi [A] time = 0.0103739, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/x,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{C(a+bx)}{x} \, dx = \int \frac{C(a+bx)}{x} \, dx$$

Mathematica [A] time = 0.0175946, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/x,x]
```

```
[Out] Integrate[FresnelC[a + b*x]/x, x]
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}\,(bx+a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/x,x)
```

```
[Out] int(FresnelC(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x,x)
```

```
[Out] Integral(fresnelc(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a)/x, x)
```

3.139 $\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable[FresnelC[a + b*x]/x^2, x]
```

Rubi [A] time = 0.0109038, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{C(a+bx)}{x^2}\,dx = \int \frac{C(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 1.46409, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/x^2,x]
```

```
[Out] Integrate[FresnelC[a + b*x]/x^2, x]
```

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/x^2,x)
```

```
[Out] int(FresnelC(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C(a + bx)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a)/x^2, x)
```

3.140 $\int x^7 \text{FresnelC}(bx)^2\, dx$

Optimal. Leaf size=253

$$-\frac{x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b}+\frac{35x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^3 b^5}-\frac{7x^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3}+\frac{105x\text{FresnelC}(}{4\pi}$$

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) + (105*x*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(4*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*
x])/(4*b^3*Pi^2) - (105*FresnelC[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelC[b*x]^
2)/8 + (35*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^5*Pi^3) - (x^7*Fresn
elC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi) - (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) +
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Rubi [A] time = 0.422278, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.1, Rules used = {6431, 6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637, 3309}

$$-\frac{x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b}+\frac{35x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^3 b^5}-\frac{7x^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3}+\frac{105x\text{FresnelC}(}{4\pi}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelC[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) + (105*x*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(4*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*
x])/(4*b^3*Pi^2) - (105*FresnelC[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelC[b*x]^
2)/8 + (35*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^5*Pi^3) - (x^7*Fresn
elC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi) - (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) +
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int x^7 C(bx)^2\,dx = \frac{1}{8}x^8 C(bx)^2 - \frac{1}{4}b\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$

$$= \frac{1}{8}x^8C(bx)^2 - \frac{x^7C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{\int x^7\sin\left(b^2\pi x^2\right)dx}{8\pi} + \frac{7\int x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{4b\pi}$$

$$= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^3\pi^2} + \frac{1}{8}x^8C(bx)^2 - \frac{x^7C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{4b^3\pi^2} + \frac{7}{}$$

$$= -\frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^3\pi^2} + \frac{1}{8}x^8C(bx)^2 + \frac{35x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^7C(bx)\sin\left(\frac{1}{2}b\right.}{4b\pi}$$

$$= -\frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^3\pi^2} + \frac{1}{8}x^8C(bx)^2 + \frac{35x^3C(bx)\sin}{4b^5\pi^3}$$

$$= \frac{7x^6}{48b^2\pi^2} + \frac{41x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^3\pi^2} +$$

$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi\right.}{4b^3\pi}$$

$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi\right.}{4b^3\pi}$$

Mathematica [A] time = 0.0172749, size = 253, normalized size = 1.

$$-\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b} + \frac{35x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^3b^5} - \frac{7x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2b^3} + \frac{105x\text{FresnelC}(}{4\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelC[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) + (105*x*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(4*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*
x])/(4*b^3*Pi^2) - (105*FresnelC[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelC[b*x]^
2)/8 + (35*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^5*Pi^3) - (x^7*Fresn
elC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi) - (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) +
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int x^7\left(\text{FresnelC}\left(bx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)^2,x)
```

```
[Out] int(x^7*FresnelC(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnelc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnelc(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^7 \text{fresnelc}(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnelc(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 C^2(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**7*fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnelc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnelc(b*x)^2, x)
```

3.141 $\int x^6 \mathbf{FresnelC}(bx)^2\,dx$

Optimal. Leaf size=239

$$-\frac{2x^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{48x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}-\frac{12x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{96\text{FresnelC}(}{7}$$

```
[Out] (-48*x)/(7*b^6*Pi^4) + (6*x^5)/(35*b^2*Pi^2) + (21*x*Cos[b^2*Pi*x^2])/(8*b^
6*Pi^4) - (x^5*Cos[b^2*Pi*x^2])/(14*b^2*Pi^2) + (96*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(7*b^7*Pi^4) - (12*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*b^
3*Pi^2) + (x^7*FresnelC[b*x]^2)/7 - (531*FresnelC[Sqrt[2]*b*x])/(56*Sqrt[2]
*b^7*Pi^4) + (48*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) - (2*x
^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi) + (17*x^3*Sin[b^2*Pi*x^2])/(
28*b^4*Pi^3)
```

Rubi [A] time = 0.298269, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6431, 6455, 6463, 6461, 3358, 3352, 3385, 3392, 30, 3386}

$$-\frac{2x^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{48x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}-\frac{12x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{96\text{FresnelC}(}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelC[b*x]^2,x]
```


```
[Out] (-48*x)/(7*b^6*Pi^4) + (6*x^5)/(35*b^2*Pi^2) + (21*x*Cos[b^2*Pi*x^2])/(8*b^
6*Pi^4) - (x^5*Cos[b^2*Pi*x^2])/(14*b^2*Pi^2) + (96*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(7*b^7*Pi^4) - (12*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*b^
3*Pi^2) + (x^7*FresnelC[b*x]^2)/7 - (531*FresnelC[Sqrt[2]*b*x])/(56*Sqrt[2]
*b^7*Pi^4) + (48*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) - (2*x
^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi) + (17*x^3*Sin[b^2*Pi*x^2])/(
28*b^4*Pi^3)
```


Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\int x^6C(bx)^2\,dx = \frac{1}{7}x^7C(bx)^2 - \frac{1}{7}(2b)\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$
$$= \frac{1}{7}x^7C(bx)^2 - \frac{2x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{\int x^6\sin\left(b^2\pi x^2\right)dx}{7\pi} + \frac{12\int x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{7b\pi}$$
$$= -\frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^3\pi^2} + \frac{1}{7}x^7C(bx)^2 - \frac{2x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{48\int x^3\cos\left(\frac{1}{2}b^{\cdots}\right.}{7b^3\cdots}$$
$$= -\frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^3\pi^2} + \frac{1}{7}x^7C(bx)^2 + \frac{48x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{2x^6C(bx)\sin\left(\cdots\right.}{7b\pi}$$
$$= \frac{6x^5}{35b^2\pi^2} + \frac{111x\cos\left(b^2\pi x^2\right)}{56b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^3\pi^2} + \cdots$$
$$= \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^3\pi^2} + \frac{1}{7}\cdots$$
$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\cdots}{7b^3\pi^2}$$
$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\cdots}{7b^3\pi^2}$$

Mathematica [A] time = 0.240461, size = 170, normalized size = 0.71

$$\frac{80\pi^4b^7x^7\text{FresnelC}(bx)^2 - 160\text{FresnelC}(bx)\left(\pi b^2x^2\left(\pi^2b^4x^4-24\right)\sin\left(\frac{1}{2}\pi b^2x^2\right) + 6\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)\right) + 2bx\left(\cdots\right.}{560\pi^4b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelC[b*x]^2,x]
```

```
[Out] (80*b^7*Pi^4*x^7*FresnelC[b*x]^2 - 2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 160
*FresnelC[b*x]*(6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24
 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*Cos[
b^2*Pi*x^2] + 2*(-960 + 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/
(560*b^7*Pi^4)
```

Maple [A] time = 0.077, size = 324, normalized size = 1.4

$$\frac{1}{b^7}\left(\frac{b^7x^7\left(\text{FresnelC}\left(bx\right)\right)^2}{7} - 2\,\text{FresnelC}\left(bx\right)\left(1/7\,\frac{b^6x^6\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi} - 6/7\,\frac{1}{\pi}\left(-\frac{x^4b^4\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi} + 4\,\frac{1}{\pi}\left(\frac{b^2x^2\sin\cdots}{\cdots}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)^2,x)
```

```
[Out] 1/b^7*(1/7*b^7*x^7*FresnelC(b*x)^2-2*FresnelC(b*x)*(1/7/Pi*b^6*x^6*sin(1/2*
b^2*Pi*x^2)-6/7/Pi*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*Pi^2*b^5*x^5-
8*b*x)+6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2
*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))-4*2^(1/2)*FresnelC(b*x*2^(1/
```

```
2))+1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b
^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*
2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnelc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnelc(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^6 \text{fresnelc}(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^6*fresnelc(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 C^2(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**6*fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnelc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnelc(b*x)^2, x)
```

3.142 $\int x^5 \mathbf{FresnelC}(bx)^2\,dx$

Optimal. Leaf size=265

$$-\frac{5ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi^3b^4}+\frac{5ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi^3b^4}-\frac{5\text{Fresn}}{}$$

```
[Out] (5*x^4)/(24*b^2*Pi^2) + (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) - (x^4*Cos[b^2*Pi
*x^2])/(12*b^2*Pi^2) - (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*b^3*Pi^
2) + (x^6*FresnelC[b*x]^2)/6 - (5*FresnelC[b*x]*FresnelS[b*x])/(2*b^6*Pi^3)
 - (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(
b^4*Pi^3) + (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi
*x^2])/(b^4*Pi^3) + (5*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^5*Pi^3) - (x
^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b*Pi) + (7*x^2*Sin[b^2*Pi*x^2])/(1
2*b^4*Pi^3)
```

Rubi [A] time = 0.288327, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6431, 6455, 6463, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$-\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}-\frac{5\text{FresnelC}(bx)S(bx)}{2\pi^3b^6}-\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelC[b*x]^2,x]
```


```
[Out] (5*x^4)/(24*b^2*Pi^2) + (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) - (x^4*Cos[b^2*Pi
*x^2])/(12*b^2*Pi^2) - (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*b^3*Pi^
2) + (x^6*FresnelC[b*x]^2)/6 - (5*FresnelC[b*x]*FresnelS[b*x])/(2*b^6*Pi^3)
 - (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(
b^4*Pi^3) + (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi
*x^2])/(b^4*Pi^3) + (5*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^5*Pi^3) - (x
^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b*Pi) + (7*x^2*Sin[b^2*Pi*x^2])/(1
2*b^4*Pi^3)
```


Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
```

```
]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^5C(bx)^2\,dx = \frac{1}{6}x^6C(bx)^2 - \frac{1}{3}b\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$
$$= \frac{1}{6}x^6C(bx)^2 - \frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\int x^5\sin\left(b^2\pi x^2\right)dx}{6\pi} + \frac{5\int x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{3b\pi}$$
$$= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6C(bx)^2 - \frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{5\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^3\pi^2} + \frac{5\int}{}$$
$$= -\frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6C(bx)^2 + \frac{5xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^5\pi^3} - \frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{3b\pi}$$
$$= -\frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6C(bx)^2 - \frac{5C(bx)S(bx)}{2b^6\pi^3} - \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x\right.}{8b^4\pi^3}$$
$$= \frac{5x^4}{24b^2\pi^2} + \frac{17\cos\left(b^2\pi x^2\right)}{12b^6\pi^4} - \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6C(bx)^2 - \frac{5C(bx)S(bx)}{2b^6\pi^3} - $$
$$= \frac{5x^4}{24b^2\pi^2} + \frac{11\cos\left(b^2\pi x^2\right)}{6b^6\pi^4} - \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6C(bx)^2 - \frac{5C(bx)S(bx)}{2b^6\pi^3} - $$

Mathematica [F] time = 0.197811, size = 0, normalized size = 0.

$$\int x^5\text{FresnelC}(bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5*FresnelC[b*x]^2,x]
```

```
[Out] Integrate[x^5*FresnelC[b*x]^2, x]
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int x^5\left(\text{FresnelC}\left(bx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x)^2,x)
```

```
[Out] int(x^5*FresnelC(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\text{fresnelc}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnelc(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^5\text{fresnelc}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnelc(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5C^2\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**5*fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\text{fresnelc}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnelc(b*x)^2, x)
```

3.143 $\int x^4 \textbf{FresnelC}(bx)^2\,dx$

Optimal. Leaf size=177

$$-\frac{2x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}+\frac{16\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}-\frac{8x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}-\frac{43S\left(\sqrt{2}bx\right)}{20\sqrt{2}\pi^3b^5}+\frac{1}{15}$$

```
[Out] (4*x^3)/(15*b^2*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (8*x^2*Cos[(b
^2*Pi*x^2)/2]*FresnelC[b*x])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x]^2)/5 - (43*F
resnelS[Sqrt[2]*b*x])/(20*Sqrt[2]*b^5*Pi^3) + (16*FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(5*b^5*Pi^3) - (2*x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi)
 + (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)
```

Rubi [A] time = 0.187988, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {6431, 6455, 6463, 6453, 3351, 3392, 30, 3386, 3385}

$$-\frac{2x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}+\frac{16\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}-\frac{8x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}-\frac{43S\left(\sqrt{2}bx\right)}{20\sqrt{2}\pi^3b^5}+\frac{1}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelC[b*x]^2,x]
```

```
[Out] (4*x^3)/(15*b^2*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (8*x^2*Cos[(b
^2*Pi*x^2)/2]*FresnelC[b*x])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x]^2)/5 - (43*F
resnelS[Sqrt[2]*b*x])/(20*Sqrt[2]*b^5*Pi^3) + (16*FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(5*b^5*Pi^3) - (2*x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi)
 + (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
```

```
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$
\begin{aligned}
\int x^4 C(bx)^2\,dx &= \frac{1}{5}x^5 C(bx)^2 - \frac{1}{5}(2b)\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx \\
&= \frac{1}{5}x^5 C(bx)^2 - \frac{2x^4 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{\int x^4 \sin\left(b^2\pi x^2\right)\,dx}{5\pi} + \frac{8\int x^3 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{5b\pi} \\
&= -\frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5 C(bx)^2 - \frac{2x^4 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{16\int x\cos\left(\frac{1}{2}\right.}{5} \\
&= -\frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5 C(bx)^2 + \frac{16C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{2x^4 C(bx)\sin\left(\right.}{5b\pi} \\
&= \frac{4x^3}{15b^2\pi^2} - \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5 C(bx)^2 - \frac{3S\left(\sqrt{2}bx\right)}{20\sqrt{2}b^5\pi^3} - \frac{4\sqrt{2}S\left(\sqrt{2}bx\right)}{5b^5\pi^3} + \\
&= \frac{4x^3}{15b^2\pi^2} - \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5 C(bx)^2 - \frac{3S\left(\sqrt{2}bx\right)}{20\sqrt{2}b^5\pi^3} - \frac{\sqrt{2}S\left(\sqrt{2}bx\right)}{b^5\pi^3} +
\end{aligned}
$$

Mathematica [A] time = 0.133759, size = 137, normalized size = 0.77

$$
\frac{24\pi^3 b^5 x^5 \text{FresnelC}(bx)^2 - 48\text{FresnelC}(bx)\left(\left(\pi^2 b^4 x^4 - 8\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right) + 4\pi b^2 x^2\cos\left(\frac{1}{2}\pi b^2 x^2\right)\right) + 32\pi b^3 x^3 + 66bx\,\mathrm{s}}{120\pi^3 b^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelC[b*x]^2,x]
```

```
[Out] (32*b^3*Pi*x^3 - 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelC[b
*x]^2 - 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 48*FresnelC[b*x]*(4*b^2*Pi*x^2*
Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 66*b*x*Sin
[b^2*Pi*x^2])/(120*b^5*Pi^3)
```

Maple [A] time = 0.076, size = 209, normalized size = 1.2

$$\frac{1}{b^5}\left(\frac{b^5x^5\left(\text{FresnelC}\left(bx\right)\right)^2}{5}-2\,\text{FresnelC}\left(bx\right)\left(1/5\,\frac{x^4b^4\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi}-4/5\,\frac{1}{\pi}\left(-\frac{b^2x^2\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi}+2\,\frac{\sin\left(1/2\,b^2\pi\right.}{\pi^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)^2,x)
```

```
[Out] 1/b^5*(1/5*b^5*x^5*FresnelC(b*x)^2-2*FresnelC(b*x)*(1/5/Pi*b^4*x^4*sin(1/2*
b^2*Pi*x^2)-4/5/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2)))+4/15/Pi^2*b^3*x^3+4/5/Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/
2)*FresnelS(b*x*2^(1/2)))+1/5/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)
*FresnelS(b*x*2^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\text{fresnelc}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnelc(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4\text{fresnelc}\left(bx\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnelc(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4C^2\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**4*fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnelc}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnelc(b*x)^2, x)
```

3.144 $\int x^3 \textbf{FresnelC}(bx)^2\,dx$

Optimal. Leaf size=140

$$-\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}-\frac{3x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3}+\frac{3\text{FresnelC}(bx)^2}{4\pi^2b^4}+\frac{3x^2}{8\pi^2b^2}+\frac{\sin\left(\pi b^2x^2\right)}{2\pi^3b^4}-\frac{x^2\cos\left(\pi\right.}{8\pi^2}$$

```
[Out] (3*x^2)/(8*b^2*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) - (3*x*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(2*b^3*Pi^2) + (3*FresnelC[b*x]^2)/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x]^2)/4 - (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b*Pi)
+ Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Rubi [A] time = 0.155816, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6431, 6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637}

$$-\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}-\frac{3x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3}+\frac{3\text{FresnelC}(bx)^2}{4\pi^2b^4}+\frac{3x^2}{8\pi^2b^2}+\frac{\sin\left(\pi b^2x^2\right)}{2\pi^3b^4}-\frac{x^2\cos\left(\pi\right.}{8\pi^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) - (3*x*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(2*b^3*Pi^2) + (3*FresnelC[b*x]^2)/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x]^2)/4 - (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b*Pi)
+ Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^3 C(bx)^2\,dx &= \frac{1}{4}x^4 C(bx)^2 - \frac{1}{2}b\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\
&= \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{\int x^3\sin\left(b^2\pi x^2\right)\,dx}{4\pi} + \frac{3\int x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b\pi}\\
&= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2b^3\pi^2} + \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{2b^3\pi^2} + \frac{3\int}{}\\
&= -\frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} - \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2b^3\pi^2} + \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{3\,\mathrm{Subst}(\int x\,dx,}{2b^4\pi^2}\\
&= \frac{3x^2}{8b^2\pi^2} - \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} - \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2b^3\pi^2} + \frac{3C(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}\end{aligned}$$

Mathematica [A] time = 0.0087466, size = 140, normalized size = 1.

$$-\frac{x^3\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} - \frac{3x\mathrm{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3} + \frac{3\mathrm{FresnelC}(bx)^2}{4\pi^2b^4} + \frac{3x^2}{8\pi^2b^2} + \frac{\sin\left(\pi b^2x^2\right)}{2\pi^3b^4} - \frac{x^2\cos}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) - (3*x*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(2*b^3*Pi^2) + (3*FresnelC[b*x]^2)/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x]^2)/4 - (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b*Pi)
+ Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int x^3 (\mathrm{FresnelC}(bx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)^2,x)
```

```
[Out] int(x^3*FresnelC(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \mathrm{fresnelc}(bx)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnelc(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^3 \mathrm{fresnelc}(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnelc(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 C^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**3*fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnelc}(bx)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnelc(b*x)^2, x)
```

3.145 $\int x^2 \mathbf{FresnelC}(bx)^2\,dx$

Optimal. Leaf size=124

$$-\frac{2x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{5\text{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}\pi^2b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{6\pi^2b^2}+\frac{2x}{3\pi^2b^2}+\frac{1}{3}x^3$$

```
[Out] (2*x)/(3*b^2*Pi^2) - (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (4*Cos[(b^2*Pi*x^2)
/2]*FresnelC[b*x])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x]^2)/3 + (5*FresnelC[Sqr
t[2]*b*x])/(6*Sqrt[2]*b^3*Pi^2) - (2*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])
/(3*b*Pi)
```

Rubi [A] time = 0.105312, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6431, 6455, 6461, 3358, 3352, 3385}

$$-\frac{2x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{5\text{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}\pi^2b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{6\pi^2b^2}+\frac{2x}{3\pi^2b^2}+\frac{1}{3}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[b*x]^2,x]
```

```
[Out] (2*x)/(3*b^2*Pi^2) - (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (4*Cos[(b^2*Pi*x^2)
/2]*FresnelC[b*x])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x]^2)/3 + (5*FresnelC[Sqr
t[2]*b*x])/(6*Sqrt[2]*b^3*Pi^2) - (2*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])
/(3*b*Pi)
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^2C(bx)^2\,dx &= \frac{1}{3}x^3C(bx)^2-\frac{1}{3}(2b)\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\
&= \frac{1}{3}x^3C(bx)^2-\frac{2x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx}{3\pi}+\frac{4\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{3b\pi}\\
&= -\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2}-\frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2}+\frac{1}{3}x^3C(bx)^2-\frac{2x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}+\frac{\int\cos\left(b^2\pi x^2\right)dx}{6b^2\pi^2}\\
&= -\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2}-\frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2}+\frac{1}{3}x^3C(bx)^2+\frac{C\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2}-\frac{2x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}+\frac{4\int}{}\\
&= \frac{2x}{3b^2\pi^2}-\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2}-\frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2}+\frac{1}{3}x^3C(bx)^2+\frac{C\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2}-\frac{2x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}\\
&= \frac{2x}{3b^2\pi^2}-\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2}-\frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2}+\frac{1}{3}x^3C(bx)^2+\frac{C\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2}+\frac{\sqrt{2}C\left(\sqrt{2}bx\right)}{3b^3\pi^2}-\frac{2x^2C(}{}
\end{aligned}$$

Mathematica [A] time = 0.0882343, size = 100, normalized size = 0.81

$$\frac{4\pi^2b^3x^3\text{FresnelC}(bx)^2-8\text{FresnelC}(bx)\left(\pi b^2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)+2\cos\left(\frac{1}{2}\pi b^2x^2\right)\right)-2bx\left(\cos\left(\pi b^2x^2\right)-4\right)+5\sqrt{2}\text{Fres}}{12\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[b*x]^2,x]
```

```
[Out] (-2*b*x*(-4 + Cos[b^2*Pi*x^2]) + 4*b^3*Pi^2*x^3*FresnelC[b*x]^2 + 5*Sqrt[2]
*FresnelC[Sqrt[2]*b*x] - 8*FresnelC[b*x]*(2*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^
2*Sin[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)
```

Maple [A] time = 0.075, size = 122, normalized size = 1.

$$\frac{1}{b^3}\left(\frac{b^3x^3\left(\text{FresnelC}\left(bx\right)\right)^2}{3}-2\,\text{FresnelC}\left(bx\right)\left(1/3\,\frac{b^2x^2\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi}+2/3\,\frac{\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right)+\frac{2\,bx}{3\,\pi^2}+\frac{\sqrt{2}\text{Fresnel}}{3\,\pi}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)^2,x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelC(b*x)^2-2*FresnelC(b*x)*(1/3/Pi*b^2*x^2*sin(1/2*
b^2*Pi*x^2)+2/3/Pi^2*cos(1/2*b^2*Pi*x^2))+2/3/Pi^2*b*x+1/3/Pi^2*2^(1/2)*Fre
```

```
snelC(b*x*2^(1/2))+1/3/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fresn
elC(b*x*2^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnelc(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \text{fresnelc}(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnelc(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 C^2(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**2*fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc(b*x)^2, x)
```

3.146 $\int x\text{FresnelC}(bx)^2\,dx$

Optimal. Leaf size=144

$$\frac{ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi}-\frac{ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi}+\frac{\text{Fresnel}}{2}$$

```
[Out] -Cos[b^2*Pi*x^2]/(4*b^2*Pi^2) + (x^2*FresnelC[b*x]^2)/2 + (FresnelC[b*x]*Fr
esnelS[b*x])/(2*b^2*Pi) + ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-
I/2)*b^2*Pi*x^2])/Pi - ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)
*b^2*Pi*x^2])/Pi - (x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Rubi [A] time = 0.0839067, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6431, 6455, 6447, 3379, 2638}

$$\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}+\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^2}-\frac{x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}-\frac{\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[b*x]^2,x]
```

```
[Out] -Cos[b^2*Pi*x^2]/(4*b^2*Pi^2) + (x^2*FresnelC[b*x]^2)/2 + (FresnelC[b*x]*Fr
esnelS[b*x])/(2*b^2*Pi) + ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-
I/2)*b^2*Pi*x^2])/Pi - ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)
*b^2*Pi*x^2])/Pi - (x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int xC(bx)^2\,dx &= \frac{1}{2}x^2C(bx)^2 - b\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\ &= \frac{1}{2}x^2C(bx)^2 - \frac{xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + \frac{\int x\sin\left(b^2\pi x^2\right)dx}{2\pi} + \frac{\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}\\ &= \frac{1}{2}x^2C(bx)^2 + \frac{C(bx)S(bx)}{2b^2\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{xC(bx)\sin\left(\frac{1}{2}b\right.}{b\pi}\\ &= -\frac{\cos\left(b^2\pi x^2\right)}{4b^2\pi^2} + \frac{1}{2}x^2C(bx)^2 + \frac{C(bx)S(bx)}{2b^2\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}\end{aligned}$$

Mathematica [F] time = 0.174331, size = 0, normalized size = 0.

$$\int x\text{FresnelC}(bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*FresnelC[b*x]^2,x]

[Out] Integrate[x*FresnelC[b*x]^2, x]
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int x\left(\text{FresnelC}\left(bx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)^2,x)

[Out] int(x*FresnelC(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnelc}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)^2,x, algorithm="maxima")

[Out] integrate(x*fresnelc(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\text{fresnelc}\left(bx\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x*fresnelc(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xC^{2}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x*fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnelc}\left(bx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*fresnelc(b*x)^2, x)
```

3.147 $\int \text{FresnelC}(bx)^2\,dx$

Optimal. Leaf size=54

$$-\frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+x\text{FresnelC}(bx)^2+\frac{S\left(\sqrt{2}bx\right)}{\sqrt{2}\pi b}$$

```
[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Rubi [A] time = 0.0360543, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6421, 12, 6453, 3351}

$$-\frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+x\text{FresnelC}(bx)^2+\frac{S\left(\sqrt{2}bx\right)}{\sqrt{2}\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]^2,x]
```

```
[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Rule 6421

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelC[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int C(bx)^2\,dx &= xC(bx)^2 - 2\int bx\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\ &= xC(bx)^2 - (2b)\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\ &= xC(bx)^2 - \frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + \frac{\int\sin\left(b^2\pi x^2\right)\,dx}{\pi}\\ &= xC(bx)^2 + \frac{S\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi} - \frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}\end{aligned}$$

Mathematica [A] time = 0.0100313, size = 54, normalized size = 1.

$$-\frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b} + x\text{FresnelC}(bx)^2 + \frac{S\left(\sqrt{2}bx\right)}{\sqrt{2}\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]^2,x]
```

```
[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Maple [A] time = 0.053, size = 49, normalized size = 0.9

$$\frac{1}{b}\left(bx\left(\text{FresnelC}\left(bx\right)\right)^2 - 2\frac{\text{FresnelC}\left(bx\right)\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi} + \frac{\sqrt{2}\text{FresnelS}\left(bx\sqrt{2}\right)}{2\,\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2,x)
```

```
[Out] 1/b*(b*x*FresnelC(b*x)^2-2*FresnelC(b*x)/Pi*sin(1/2*b^2*Pi*x^2)+1/2/Pi*2^(1
/2)*FresnelS(b*x*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnelc}\left(bx\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int C^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2,x)
```

```
[Out] Integral(fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2, x)
```

3.148 $\int \frac{\text{FresnelC}(bx)^2}{x}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)^2}{x},x\right)$$

```
[Out] Unintegrable[FresnelC[b*x]^2/x, x]
```

Rubi [A] time = 0.0161198, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x,x]
```

```
[Out] Defer[Int][FresnelC[b*x]^2/x, x]
```

Rubi steps

$$\int \frac{C(bx)^2}{x}\,dx = \int \frac{C(bx)^2}{x}\,dx$$

Mathematica [A] time = 0.0188291, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x, x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x,x)
```

```
[Out] int(FresnelC(b*x)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x, x)
```

3.149 $\int \frac{\text{FresnelC}(bx)^2}{x^2} dx$

Optimal. Leaf size=37

$$2b\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{\text{FresnelC}(bx)^2}{x}$$

```
[Out] -(FresnelC[b*x]^2/x) + 2*b*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/x, x]
```

Rubi [A] time = 0.0373831, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^2,x]
```

```
[Out] -(FresnelC[b*x]^2/x) + 2*b*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x
, x]
```

Rubi steps

$$\int \frac{C(bx)^2}{x^2} dx = -\frac{C(bx)^2}{x} + (2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x} dx$$

Mathematica [A] time = 0.0259263, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^2,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^2, x]
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^2,x)
```

```
[Out] int(FresnelC(b*x)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**2,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^2, x)
```

3.150 $\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx$

Optimal. Leaf size=38

$$b\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)-\frac{\text{FresnelC}(bx)^2}{2x^2}$$

```
[Out] -FresnelC[b*x]^2/(2*x^2) + b*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x
])/x^2, x]
```

Rubi [A] time = 0.0371539, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^3,x]
```

```
[Out] -FresnelC[b*x]^2/(2*x^2) + b*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/x^2, x]
```

Rubi steps

$$\int \frac{C(bx)^2}{x^3}\,dx=-\frac{C(bx)^2}{2x^2}+b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx$$

Mathematica [A] time = 0.0198106, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^3,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^3, x]
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^3,x)
```

```
[Out] int(FresnelC(b*x)^2/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**3,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^3, x)
```

3.151 $\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{3}\pi b^3\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{b\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^2}-\frac{\pi b^3S\left(\sqrt{2}bx\right)}{3\sqrt{2}}-\frac{b^2\cos\left(\pi b^2\right.}{6x}$$

```
[Out] -b^2/(6*x) - (b^2*Cos[b^2*Pi*x^2])/(6*x) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^2) - FresnelC[b*x]^2/(3*x^3) - (b^3*Pi*FresnelS[Sqrt[2]*b*x])/(3
*Sqrt[2]) - (b^3*Pi*Unintegrable[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])
/3
```

Rubi [A] time = 0.0828138, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^4,x]
```

```
[Out] -b^2/(6*x) - (b^2*Cos[b^2*Pi*x^2])/(6*x) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^2) - FresnelC[b*x]^2/(3*x^3) - (b^3*Pi*FresnelS[Sqrt[2]*b*x])/(3
*Sqrt[2]) - (b^3*Pi*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^4}\,dx &= -\frac{C(bx)^2}{3x^3}+\frac{1}{3}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^3}\,dx\\
&= -\frac{b^2}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^2}-\frac{C(bx)^2}{3x^3}+\frac{1}{6}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{3}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\
&= -\frac{b^2}{6x}-\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^2}-\frac{C(bx)^2}{3x^3}-\frac{1}{3}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx-\frac{1}{3}\left(b^4\right.\\
&= -\frac{b^2}{6x}-\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^2}-\frac{C(bx)^2}{3x^3}-\frac{b^3\pi S\left(\sqrt{2}bx\right)}{3\sqrt{2}}-\frac{1}{3}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\right.}{x}
\end{aligned}$$

Mathematica [A] time = 0.0262054, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^4,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^4, x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^4,x)
```

```
[Out] int(FresnelC(b*x)^2/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**4,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^4, x)
```

3.152 $\int \frac{\text{FresnelC}(bx)^2}{x^5} \, dx$

Optimal. Leaf size=127

$$\frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x} - \frac{b \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^3} - \frac{1}{12}\pi^2 b^4 \text{FresnelC}(bx)^2 - \frac{1}{12}\pi b^4 \text{Si}\left(b^2 \pi x^2\right) - \frac{b^2}{24x^2} - \cdots$$

```
[Out] -b^2/(24*x^2) - (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(6*x^3) - (b^4*Pi^2*FresnelC[b*x]^2)/12 - FresnelC[b*x]^2/(4*x^
4) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x) - (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Rubi [A] time = 0.137878, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {6431, 6457, 6465, 6441, 30, 3375, 3380, 3297, 3299}

$$\frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x} - \frac{b \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^3} - \frac{1}{12}\pi^2 b^4 \text{FresnelC}(bx)^2 - \frac{1}{12}\pi b^4 \text{Si}\left(b^2 \pi x^2\right) - \frac{b^2}{24x^2} - \cdots$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]^2/x^5,x]
```

```
[Out] -b^2/(24*x^2) - (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(6*x^3) - (b^4*Pi^2*FresnelC[b*x]^2)/12 - FresnelC[b*x]^2/(4*x^
4) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x) - (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^5}\,dx &= -\frac{C(bx)^2}{4x^4} + \frac{1}{2}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^4}\,dx \\
&= -\frac{b^2}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{1}{12}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx - \frac{1}{6}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} \\
&= -\frac{b^2}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} + \frac{1}{24}b^2\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx\right. \\
&= -\frac{b^2}{24x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} - \frac{1}{24}b^4\pi\mathrm{Si}\left(b^2\pi\right. \\
&= -\frac{b^2}{24x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{1}{12}b^4\pi^2C(bx)^2 - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x}
\end{aligned}$$

Mathematica [A] time = 0.0069876, size = 127, normalized size = 1.

$$\frac{\pi b^3\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x} - \frac{b\mathrm{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3} - \frac{1}{12}\pi^2b^4\mathrm{FresnelC}(bx)^2 - \frac{1}{12}\pi b^4\mathrm{Si}\left(b^2\pi x^2\right) - \frac{b^2}{24x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]^2/x^5,x]
```

```
[Out] -b^2/(24*x^2) - (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(6*x^3) - (b^4*Pi^2*FresnelC[b*x]^2)/12 - FresnelC[b*x]^2/(4*x^
4) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x) - (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{FresnelC}\,(bx))^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^5,x)
```

```
[Out] int(FresnelC(b*x)^2/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}\,(bx)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)^2}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**5,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^5, x)
```

3.153 $\int \frac{\text{FresnelC}(bx)^2}{x^6} dx$

Optimal. Leaf size=170

$$-\frac{1}{20}\pi^2 b^5 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) + \frac{\pi b^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{20x^2} - \frac{b\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2\right.}{10x^4}$$

```
[Out] -b^2/(60*x^3) - (b^2*Cos[b^2*Pi*x^2])/(60*x^3) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(10*x^4) - FresnelC[b*x]^2/(5*x^5) - (7*b^5*Pi^2*FresnelC[Sqrt[
2]*b*x])/(60*Sqrt[2]) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(20*x^2)
 + (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Unintegrable[(Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/x, x])/20
```

Rubi [A] time = 0.145268, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^6} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^6,x]
```

```
[Out] -b^2/(60*x^3) - (b^2*Cos[b^2*Pi*x^2])/(60*x^3) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(10*x^4) - FresnelC[b*x]^2/(5*x^5) - (7*b^5*Pi^2*FresnelC[Sqrt[
2]*b*x])/(60*Sqrt[2]) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(20*x^2)
 + (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Defer[Int][(Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/x, x])/20
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^6} dx &= -\frac{C(bx)^2}{5x^5} + \frac{1}{5}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^5} dx \\
&= -\frac{b^2}{60x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} + \frac{1}{20}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4} dx - \frac{1}{10}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} d \\
&= -\frac{b^2}{60x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2} - \frac{1}{40}\left(b^4\pi\right)\int \frac{\sin}{} \\
&= -\frac{b^2}{60x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2} + \frac{7b^4\pi\sin\left(b^2\pi x^2\right.}{120x} \\
&= -\frac{b^2}{60x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} - \frac{7b^5\pi^2 C\left(\sqrt{2}bx\right)}{60\sqrt{2}} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2}
\end{aligned}$$

Mathematica [A] time = 0.0265742, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^6} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^6,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^6, x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{FresnelC}(bx))^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^6,x)
```

```
[Out] int(FresnelC(b*x)^2/x^6,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}(bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^6, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}(bx)^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^6, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**6,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**6, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^6, x)
```

3.154 $\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$

Optimal. Leaf size=165

$$-\frac{1}{45}\pi^2 b^5 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)-\frac{1}{72}\pi^2 b^6 \text{CosIntegral}\left(\pi b^2x^2\right)+\frac{\pi b^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi\right.}{45x^3}$$

```
[Out] -b^2/(120*x^4) - (b^2*Cos[b^2*Pi*x^2])/(120*x^4) - (b^6*Pi^2*CosIntegral[b^
2*Pi*x^2])/72 - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(15*x^5) - FresnelC[b
*x]^2/(6*x^6) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(45*x^3) + (b^4*
Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*
FresnelC[b*x])/x^2, x])/45
```

Rubi [A] time = 0.220258, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^7,x]
```

```
[Out] -b^2/(120*x^4) - (b^2*Cos[b^2*Pi*x^2])/(120*x^4) - (b^6*Pi^2*CosIntegral[b^
2*Pi*x^2])/72 - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(15*x^5) - FresnelC[b
*x]^2/(6*x^6) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(45*x^3) + (b^4*
Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*Fr
esnelC[b*x])/x^2, x])/45
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^7}\,dx &= -\frac{C(bx)^2}{6x^6}+\frac{1}{3}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^6}\,dx\\
&= -\frac{b^2}{120x^4}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5}-\frac{C(bx)^2}{6x^6}+\frac{1}{30}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx-\frac{1}{15}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x\right.}{x^4}\\
&= -\frac{b^2}{120x^4}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5}-\frac{C(bx)^2}{6x^6}+\frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3}+\frac{1}{60}b^2\,\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^3}\,d\right.\\
&= -\frac{b^2}{120x^4}-\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5}-\frac{C(bx)^2}{6x^6}+\frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3}-\frac{1}{180}\left(b^4\pi\right)\text{S}\\
&= -\frac{b^2}{120x^4}-\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5}-\frac{C(bx)^2}{6x^6}+\frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3}+\frac{b^4\pi\sin\left(b^2\right.}{72x^2}\\
&= -\frac{b^2}{120x^4}-\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}-\frac{1}{72}b^6\pi^2\text{Ci}\left(b^2\pi x^2\right)-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5}-\frac{C(bx)^2}{6x^6}+\frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b\right.}{45x^3}
\end{aligned}$$

Mathematica [A] time = 0.0201974, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^7,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^7, x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}\,(bx))^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^7,x)
```

```
[Out] int(FresnelC(b*x)^2/x^7,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^7, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)^2}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^7,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^7, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**7,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**7, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^7, x)
```

3.155 $\int \frac{\textbf{FresnelC}(bx)^2}{x^8} dx$

Optimal. Leaf size=258

$$\frac{1}{168}\pi^3 b^7 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}, x\right) + \frac{\pi b^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{84x^4} + \frac{\pi^2 b^5 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi\right.}{168x^2}$$

```
[Out] -b^2/(210*x^5) + (b^6*Pi^2)/(336*x) - (b^2*Cos[b^2*Pi*x^2])/(210*x^5) + (67
*b^6*Pi^2*Cos[b^2*Pi*x^2])/(5040*x) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/(21*x^6) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(168*x^2) - Fresne
lC[b*x]^2/(7*x^7) + (b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/(72*Sqrt[2]) + (2*Sqrt
[2]*b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/315 + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(84*x^4) + (13*b^4*Pi*Sin[b^2*Pi*x^2])/(2520*x^3) + (b^7*Pi^3*Uni
ntegrable[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/168
```

Rubi [A] time = 0.239743, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^8} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^8,x]
```

```
[Out] -b^2/(210*x^5) + (b^6*Pi^2)/(336*x) - (b^2*Cos[b^2*Pi*x^2])/(210*x^5) + (67
*b^6*Pi^2*Cos[b^2*Pi*x^2])/(5040*x) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/(21*x^6) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(168*x^2) - Fresne
lC[b*x]^2/(7*x^7) + (b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/(72*Sqrt[2]) + (2*Sqrt
[2]*b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/315 + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(84*x^4) + (13*b^4*Pi*Sin[b^2*Pi*x^2])/(2520*x^3) + (b^7*Pi^3*Def
er[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/168
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^8} dx &= -\frac{C(bx)^2}{7x^7} + \frac{1}{7}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^7} dx \\
&= -\frac{b^2}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} - \frac{C(bx)^2}{7x^7} + \frac{1}{42}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6} dx - \frac{1}{21}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} \\
&= -\frac{b^2}{210x^5} - \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} - \frac{C(bx)^2}{7x^7} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{84x^4} - \frac{1}{168}\left(b^4\pi\right)\int \frac{\sin}{} \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{168x^2} - \frac{C(bx)^2}{7x^7} + \frac{b^3\pi C}{} \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} + \frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C}{168x^2} \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} + \frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C}{168x^2}
\end{aligned}$$

Mathematica [A] time = 0.0270006, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^8,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^8, x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}\,(bx))^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^8,x)
```

```
[Out] int(FresnelC(b*x)^2/x^8,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^8, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)^2}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^8, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**8,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**8, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^8, x)
```

3.156 $\int \frac{\textbf{FresnelC}(bx)^2}{x^9}\,dx$

Optimal. Leaf size=242

$$-\frac{\pi^3 b^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{420x}+\frac{\pi b^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{140x^5}+\frac{\pi^2 b^5 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{420x^3}-\frac{b\text{Fresnel}}{}$$

```
[Out] -b^2/(336*x^6) + (b^6*Pi^2)/(1680*x^2) - (b^2*Cos[b^2*Pi*x^2])/(336*x^6) +
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(28*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(420*x^3) + (b^8*
Pi^4*FresnelC[b*x]^2)/840 - FresnelC[b*x]^2/(8*x^8) + (b^3*Pi*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(140*x^5) - (b^7*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(420*x) + (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) + (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Rubi [A] time = 0.388084, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6431, 6457, 6465, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$-\frac{\pi^3 b^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{420x}+\frac{\pi b^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{140x^5}+\frac{\pi^2 b^5 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{420x^3}-\frac{b\text{Fresnel}}{}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]^2/x^9,x]
```


```
[Out] -b^2/(336*x^6) + (b^6*Pi^2)/(1680*x^2) - (b^2*Cos[b^2*Pi*x^2])/(336*x^6) +
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(28*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(420*x^3) + (b^8*
Pi^4*FresnelC[b*x]^2)/840 - FresnelC[b*x]^2/(8*x^8) + (b^3*Pi*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(140*x^5) - (b^7*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(420*x) + (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) + (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```


Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{C(bx)^2}{x^9}\,dx = -\frac{C(bx)^2}{8x^8} + \frac{1}{4}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8}\,dx$$

$$= -\frac{b^2}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} - \frac{C(bx)^2}{8x^8} + \frac{1}{56}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx - \frac{1}{28}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x\right.}{x^6}$$

$$= -\frac{b^2}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} - \frac{C(bx)^2}{8x^8} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5} + \frac{1}{112}b^2\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^4}\right.$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{420x^3} - \frac{C(bx)^2}{8x^8} +$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{420x^3} - \frac{C(bx)^2}{8x^8} +$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} + \frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right.}{420x^3}$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} + \frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right.}{420x^3}$$

Mathematica [A] time = 0.0126167, size = 242, normalized size = 1.

$$-\frac{\pi^3 b^7\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{420x} + \frac{\pi b^3\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{140x^5} + \frac{\pi^2 b^5\mathrm{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{420x^3} - \frac{b\mathrm{Fresnel}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]^2/x^9,x]
```

```
[Out] -b^2/(336*x^6) + (b^6*Pi^2)/(1680*x^2) - (b^2*Cos[b^2*Pi*x^2])/(336*x^6) +
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(28*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(420*x^3) + (b^8*
Pi^4*FresnelC[b*x]^2)/840 - FresnelC[b*x]^2/(8*x^8) + (b^3*Pi*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(140*x^5) - (b^7*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(420*x) + (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) + (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{\left(\mathrm{FresnelC}\left(bx\right)\right)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^9,x)
```

```
[Out] int(FresnelC(b*x)^2/x^9,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^9,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^9, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^9,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^9, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**9,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**9, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^9, x)
```

3.157 $\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$

Optimal. Leaf size=285

$$\frac{\pi^4 b^9 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)}{1728}-\frac{\pi^3 b^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{1728x^2}+\frac{\pi b^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{216x^6}$$

```
[Out] -b^2/(504*x^7) + (b^6*Pi^2)/(5184*x^3) - (b^2*Cos[b^2*Pi*x^2])/(504*x^7) +
(187*b^6*Pi^2*Cos[b^2*Pi*x^2])/(181440*x^3) - (b*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(36*x^8) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(864*x^4)
- FresnelC[b*x]^2/(9*x^9) + (853*b^9*Pi^4*FresnelC[Sqrt[2]*b*x])/(181440*Sq
rt[2]) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(216*x^6) - (b^7*Pi^3*F
resnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(1728*x^2) + (19*b^4*Pi*Sin[b^2*Pi*x^2])/
(15120*x^5) - (853*b^8*Pi^3*Sin[b^2*Pi*x^2])/(362880*x) + (b^9*Pi^4*Uninteg
rable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/1728
```

Rubi [A] time = 0.342115, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^10,x]
```

```
[Out] -b^2/(504*x^7) + (b^6*Pi^2)/(5184*x^3) - (b^2*Cos[b^2*Pi*x^2])/(504*x^7) +
(187*b^6*Pi^2*Cos[b^2*Pi*x^2])/(181440*x^3) - (b*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(36*x^8) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(864*x^4)
- FresnelC[b*x]^2/(9*x^9) + (853*b^9*Pi^4*FresnelC[Sqrt[2]*b*x])/(181440*Sq
rt[2]) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(216*x^6) - (b^7*Pi^3*F
resnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(1728*x^2) + (19*b^4*Pi*Sin[b^2*Pi*x^2])/
(15120*x^5) - (853*b^8*Pi^3*Sin[b^2*Pi*x^2])/(362880*x) + (b^9*Pi^4*Defer[I
nt][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/1728
```

Rubi steps

$$\int \frac{C(bx)^2}{x^{10}}\,dx = -\frac{C(bx)^2}{9x^9} + \frac{1}{9}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^9}\,dx$$

$$= -\frac{b^2}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} - \frac{C(bx)^2}{9x^9} + \frac{1}{72}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx - \frac{1}{36}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}$$

$$= -\frac{b^2}{504x^7} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} - \frac{C(bx)^2}{9x^9} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{216x^6} - \frac{1}{432}\left(b^4\pi\right)\int$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{864x^4} - \frac{C(bx)^2}{9x^9} +$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} + \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{864x^4}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} + \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{864x^4}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} + \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{864x^4}$$

Mathematica [A] time = 0.0265919, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^10,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^10, x]
```

Maple [A] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}\,(bx))^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^10,x)
```

```
[Out] int(FresnelC(b*x)^2/x^10,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^10, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)^2}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^10, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2\,(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**10,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**10, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^10, x)
```

3.158 $\int (c+dx)^2 \textbf{FresnelC}(a+bx)^2\,dx$

Optimal. Leaf size=495

$$\frac{id(a+bx)^2(bc-ad)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{id(a+bx)^2(bc-ad)\text{HypergeometricPFQ}}{4\pi b^3}$$

```
[Out] (2*d^2*x)/(3*b^2*Pi^2) - (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) -
 (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (4*d^2*Cos[(Pi*(a + b*x
)^2)/2]*FresnelC[a + b*x])/(3*b^3*Pi^2) + ((b*c - a*d)^2*(a + b*x)*FresnelC
[a + b*x]^2)/b^3 + (d*(b*c - a*d)*(a + b*x)^2*FresnelC[a + b*x]^2)/b^3 + (d
^2*(a + b*x)^3*FresnelC[a + b*x]^2)/(3*b^3) + (5*d^2*FresnelC[Sqrt[2]*(a +
b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (d*(b*c - a*d)*FresnelC[a + b*x]*FresnelS[a +
 b*x])/(b^3*Pi) + ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*b^3*
Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2},
(-I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a + b*x)^2*Hyperge
ometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - (2*(b*c - a*
d)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (2*d*(b*c - a*d)
*(a + b*x)*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (2*d^2*(a
+ b*x)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rubi [A] time = 0.396293, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {6433, 6421, 6453, 3351, 6431, 6455, 6447, 3379, 2638, 6461, 3358, 3352, 3385}

$$\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}+\frac{d(bc-ad)\text{Fresn}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelC[a + b*x]^2,x]
```


```
[Out] (2*d^2*x)/(3*b^2*Pi^2) - (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) -
 (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (4*d^2*Cos[(Pi*(a + b*x
)^2)/2]*FresnelC[a + b*x])/(3*b^3*Pi^2) + ((b*c - a*d)^2*(a + b*x)*FresnelC
[a + b*x]^2)/b^3 + (d*(b*c - a*d)*(a + b*x)^2*FresnelC[a + b*x]^2)/b^3 + (d
^2*(a + b*x)^3*FresnelC[a + b*x]^2)/(3*b^3) + (5*d^2*FresnelC[Sqrt[2]*(a +
b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (d*(b*c - a*d)*FresnelC[a + b*x]*FresnelS[a +
 b*x])/(b^3*Pi) + ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*b^3*
Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2},
(-I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a + b*x)^2*Hyperge
ometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - (2*(b*c - a*
d)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (2*d*(b*c - a*d)
*(a + b*x)*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (2*d^2*(a
+ b*x)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```


Rule 6433

```
Int[FresnelC[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelC[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6421

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelC[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\int (c+dx)^2 C(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)C(x)^2+2bcd\left(1-\frac{ad}{bc}\right)xC(x)^2+d^2x^2C(x)^2\right)dx, x, a+bx\right)}{b^3}$$
$$= \frac{d^2\,\text{Subst}\left(\int x^2C(x)^2\,dx, x, a+bx\right)}{b^3}+\frac{(2d(bc-ad))\,\text{Subst}\left(\int xC(x)^2\,dx, x, a+bx\right)}{b^3}+\frac{(bc-ad}{}$$
$$= \frac{(bc-ad)^2(a+bx)C(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2C(a+bx)^2}{b^3}+\frac{d^2(a+bx)^3C(a+bx)^2}{3b^3}-\frac{(2}{}$$
$$= \frac{(bc-ad)^2(a+bx)C(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2C(a+bx)^2}{b^3}+\frac{d^2(a+bx)^3C(a+bx)^2}{3b^3}-\frac{2(}{}$$
$$= -\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)C(a+bx)}{3b^3\pi^2}+\frac{(bc-ad)^2(a+bx)C(a+b}{b^3}$$
$$= -\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}-\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)C(a+b}{3b^3\pi^2}$$
$$= \frac{2d^2x}{3b^2\pi^2}-\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}-\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}$$
$$= \frac{2d^2x}{3b^2\pi^2}-\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}-\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}$$

Mathematica [F] time = 0.6492, size = 0, normalized size = 0.

$$\int (c+dx)^2\text{FresnelC}(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^2*FresnelC[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^2*FresnelC[a + b*x]^2, x]
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int (dx+c)^2\left(\text{FresnelC}\,(bx+a)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelC(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^2*FresnelC(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \text{fresnelc}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*fresnelc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^2+2\,cdx+c^2\right)\text{fresnelc}(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnelc(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 C^2(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnelc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*fresnelc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \text{fresnelc}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*fresnelc(b*x + a)^2, x)
```

3.159 $\int (c+dx)\textbf{FresnelC}(a+bx)^2\,dx$

Optimal. Leaf size=279

$$\frac{id(a+bx)^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}-\frac{id(a+bx)^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi\right.}{8\pi b^2}$$

```
[Out] -(d*Cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + ((b*c - a*d)*(a + b*x)*FresnelC[a +
 b*x]^2)/b^2 + (d*(a + b*x)^2*FresnelC[a + b*x]^2)/(2*b^2) + (d*FresnelC[a
+ b*x]*FresnelS[a + b*x])/(2*b^2*Pi) + ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b
*x)])/(Sqrt[2]*b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - (2*(b*c - a*d)*Fr
esnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi) - (d*(a + b*x)*FresnelC[a
 + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi)
```

Rubi [A] time = 0.185127, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6433, 6421, 6453, 3351, 6431, 6455, 6447, 3379, 2638}

$$\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}+\frac{(a+bx)(bc-ad)\text{FresnelC}(a+bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelC[a + b*x]^2,x]
```

```
[Out] -(d*Cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + ((b*c - a*d)*(a + b*x)*FresnelC[a +
 b*x]^2)/b^2 + (d*(a + b*x)^2*FresnelC[a + b*x]^2)/(2*b^2) + (d*FresnelC[a
+ b*x]*FresnelS[a + b*x])/(2*b^2*Pi) + ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b
*x)])/(Sqrt[2]*b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - (2*(b*c - a*d)*Fr
esnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi) - (d*(a + b*x)*FresnelC[a
 + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi)
```

Rule 6433

```
Int[FresnelC[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelC[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6421

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelC[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)C(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(bc\left(1-\frac{ad}{bc}\right)C(x)^2+dxC(x)^2\right)dx,x,a+bx\right)}{b^2}$$

$$= \frac{d\,\text{Subst}\left(\int xC(x)^2\,dx,x,a+bx\right)}{b^2}+\frac{(bc-ad)\,\text{Subst}\left(\int C(x)^2\,dx,x,a+bx\right)}{b^2}$$

$$= \frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}-\frac{d\,\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)C(x)\,dx,x,\right.}{b^2}$$

$$= \frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}-\frac{2(bc-ad)C(a+bx)\sin\left(\frac{1}{2}\pi(a+bx\right.}{b^2\pi}$$

$$= \frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}+\frac{dC(a+bx)S(a+bx)}{2b^2\pi}+\frac{(bc-ad)S\left(\right.}{\sqrt{2}}$$

$$= -\frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2}+\frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}+\frac{dC(a+bx)S}{2b^2\pi}$$

Mathematica [F] time = 0.548492, size = 0, normalized size = 0.

$$\int (c+dx)\text{FresnelC}(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)*FresnelC[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)*FresnelC[a + b*x]^2, x]
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int (dx+c)\left(\text{FresnelC}\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelC(b*x+a)^2,x)
```

```
[Out] int((d*x+c)*FresnelC(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\text{fresnelc}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*fresnelc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)\text{fresnelc}\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*fresnelc(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\,C^2\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*fresnelc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\text{fresnelc}\,(bx + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnelc(b*x + a)^2, x)
```

3.160 $\int \text{FresnelC}(a+bx)^2\,dx$

Optimal. Leaf size=69

$$\frac{(a+bx)\text{FresnelC}(a+bx)^2}{b}-\frac{2\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}+\frac{S\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b}$$

```
[Out] ((a + b*x)*FresnelC[a + b*x]^2)/b + FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*
Pi) - (2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b*Pi)
```

Rubi [A] time = 0.150686, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6421, 6453, 3351}

$$\frac{(a+bx)\text{FresnelC}(a+bx)^2}{b}-\frac{2\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}+\frac{S\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*FresnelC[a + b*x]^2)/b + FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*
Pi) - (2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b*Pi)
```

Rule 6421

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelC[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int C(a+bx)^2\,dx &= \frac{(a+bx)C(a+bx)^2}{b}-2\int (a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)C(a+bx)\,dx\\
&= \frac{(a+bx)C(a+bx)^2}{b}-\frac{2\,\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)C(x)\,dx,x,a+bx\right)}{b}\\
&= \frac{(a+bx)C(a+bx)^2}{b}-\frac{2C(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{\text{Subst}\left(\int \sin\left(\pi x^2\right)dx,x,a+bx\right)}{b\pi}\\
&= \frac{(a+bx)C(a+bx)^2}{b}+\frac{S\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b\pi}-\frac{2C(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}
\end{aligned}$$

Mathematica [A] time = 0.0099422, size = 66, normalized size = 0.96

$$\frac{2\pi(a+bx)\text{FresnelC}(a+bx)^2-4\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+\sqrt{2}S\left(\sqrt{2}(a+bx)\right)}{2\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[a + b*x]^2,x]
```

```
[Out] (2*Pi*(a + b*x)*FresnelC[a + b*x]^2 + Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)] -
 4*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(2*b*Pi)
```

Maple [A] time = 0.054, size = 60, normalized size = 0.9

$$\frac{1}{b}\left((bx+a)\left(\text{FresnelC}\left(bx+a\right)\right)^2-2\,\frac{\text{FresnelC}\left(bx+a\right)\sin\left(1/2\,\pi\,\left(bx+a\right)^2\right)}{\pi}+\frac{\sqrt{2}\text{FresnelS}\left(\left(bx+a\right)\sqrt{2}\right)}{2\,\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)^2,x)
```

```
[Out] 1/b*((b*x+a)*FresnelC(b*x+a)^2-2*FresnelC(b*x+a)/Pi*sin(1/2*Pi*(b*x+a)^2)+1
/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnelc}\left(bx+a\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int C^2\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx+a)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a)^2, x)
```

3.161 $\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(a+bx)^2}{c+dx},x\right)$$

```
[Out] Unintegrable[FresnelC[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.023169, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{C(a+bx)^2}{c+dx}\,dx=\int \frac{C(a+bx)^2}{c+dx}\,dx$$

Mathematica [A] time = 0.0351198, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[FresnelC[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.357, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}\,(bx+a))^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(FresnelC(b*x+a)^2/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a)^2/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2\,(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(fresnelc(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a)^2/(d*x + c), x)
```

3.162 $\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable[FresnelC[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [A] time = 0.0228539, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{C(a+bx)^2}{(c+dx)^2} dx = \int \frac{C(a+bx)^2}{(c+dx)^2} dx$$

Mathematica [A] time = 0.0795137, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[FresnelC[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A] time = 0.347, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a)^2/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a)^2/(d*x + c)^2, x)
```

3.163 $\int x^2 \textbf{FresnelC}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=231

$$\left(\frac{1}{12}+\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)-\left(\frac{1}{12}+\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2}}$$

```
[Out] ((1/12 + I/12)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2
+ I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*
x^n)^(3/n) - ((1/12 + I/12)*E^((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(c*x^n)^(3/n) + (x^3*FresnelC[d*(a + b*Log[c*x^n])])/3
```

Rubi [A] time = 0.522269, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{12}+\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)-\left(\frac{1}{12}+\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[d*(a + b*Log[c*x^n])],x]
```


```
[Out] ((1/12 + I/12)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2
+ I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*
x^n)^(3/n) - ((1/12 + I/12)*E^((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(c*x^n)^(3/n) + (x^3*FresnelC[d*(a + b*Log[c*x^n])])/3
```


Rule 6472

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 4618

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^2 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = \frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{3}(bdn)\int x^2\cos\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)\,dx$$

$$= \frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x^2\,dx-\frac{1}{6}(bdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(}$$

$$= \frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}(bdn)\int \exp\Bigl(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}ib^2d^2\pi\log$$

$$= \frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}(bdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^2\left(cx^n\right)^{-iab}$$

$$= \frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\Bigl(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\,\mathrm{lo}$$

$$= \frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}\left(bdx^3\left(cx^n\right)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\mathrm{Subst}\Bigl(\int \exp\Bigl(-\frac{1}{2}ia^2d^2\pi+$$

$$= \frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}\left(bde^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3\left(cx^n\right)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\mathrm{Subst}\Bigl(\int \exp\Bigl(\frac{i}{}$$

$$= \left(\frac{1}{12}+\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3\left(cx^n\right)^{-3/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)-\Bigl(\frac{1}{12}$$

Mathematica [A] time = 6.7753, size = 318, normalized size = 1.38

$$\frac{1}{12}x^3\left(4\text{FresnelC}\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\sqrt[4]{-1}\sqrt{2}\left(cx^n\right)^{-3/n}\left(ie^{\frac{9i}{\pi b^2d^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)-3i\right)}{\sqrt{\pi}bdn}\right)+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*(4*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^(((-6*a)/(b*
n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - L
og[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(I*E^((9*I)/(b^2
*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(3/n)))/12
```

Maple [F] time = 0.523, size = 0, normalized size = 0.

$$\int x^2\text{FresnelC}\left(d\left(a+b\ln\left(cx^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\text{fresnelc}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnelc((b*log(c*x^n) + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\text{fresnelc}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnelc(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 C\left(ad + bd\log\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*fresnelc(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}\left((b\log\left(cx^n\right) + a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc((b*log(c*x^n) + a)*d), x)
```

3.164 $\int x\text{FresnelC}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=227

$$\left(\frac{1}{8}+\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{-\frac{2\left(\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\text{Erfi}\Bigg($$

```
[Out] ((1/8 + I/8)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[((1/2 + I/
2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)
^(2/n) - ((1/8 + I/8)*x^2*Erfi[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*(c*x^n)^(2/n)) + (x^2*FresnelC[d*(a + b*Log[c*x^n])])/2
```

Rubi [A] time = 0.407877, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{8}+\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{-\frac{2\left(\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\text{Erfi}\Bigg($$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/8 + I/8)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[((1/2 + I/
2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)
^(2/n) - ((1/8 + I/8)*x^2*Erfi[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*(c*x^n)^(2/n)) + (x^2*FresnelC[d*(a + b*Log[c*x^n])])/2
```

Rule 6472

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 4618

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx &= \frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int x\cos\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)\,dx\\
&= \frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x\,dx-\frac{1}{4}(bdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(cx\cdots}\\
&= \frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}ib^2d^2\pi\log^2\cdots\right.\\
&= \frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x\left(cx^n\right)^{-iabd^2\cdots}\\
&= \frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log\cdots\right.\\
&= \frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(bdx^2\left(cx^n\right)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(2\cdots}{\cdots}\right.\right.\\
&= \frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(bde^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{2\cdots}{\cdots}\right.}{\cdots}\right.\right.\\
&= \left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-2/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)e\cdots
\end{aligned}
$$

Mathematica [A] time = 6.64992, size = 318, normalized size = 1.4

$$\frac{1}{8}x^2\left(4\text{FresnelC}\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\sqrt[4]{-1}\sqrt{2}\left(cx^n\right)^{-2/n}\left(ie^{\frac{4i}{\pi b^2d^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)-2i\right)}{\sqrt{\pi}bdn}\right)+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*(4*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*n
) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] - Lo
g[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(I*E^((4*I)/(b^2
*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(2/n)))/8
```

Maple [F] time = 0.539, size = 0, normalized size = 0.

$$\int x\text{FresnelC}\left(d\left(a+b\ln\left(cx^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*FresnelC(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnelc}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnelc((b*log(c*x^n) + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\text{fresnelc}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x*fresnelc(b*d*log(c*x^n) + a*d), x)
```

<u>**Sympy**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int xC\left(ad + bd\log\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*fresnelc(a*d + b*d*log(c*x**n)), x)
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{fresnelc}\left(\left(b\log\left(cx^n\right) + a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x*fresnelc((b*log(c*x^n) + a)*d), x)
```

3.165 $\int \text{FresnelC}\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx$

Optimal. Leaf size=214

$$\left(\frac{1}{4}+\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)-\left(\frac{1}{4}+\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erfi}\left(\left(\frac{1}{2}\right.\right.$$

```
[Out] ((1/4 + I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c
*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)) - ((1/4 + I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(
c*x^n)^n^(-1)) + x*FresnelC[d*(a + b*Log[c*x^n])]
```

Rubi [A] time = 0.263422, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {6469, 4616, 2277, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{4}+\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)-\left(\frac{1}{4}+\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erfi}\left(\left(\frac{1}{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])],x]
```


```
[Out] ((1/4 + I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c
*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)) - ((1/4 + I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(
c*x^n)^n^(-1)) + x*FresnelC[d*(a + b*Log[c*x^n])]
```


Rule 6469

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Sim
p[x*FresnelC[d*(a + b*Log[c*x^n])], x] - Dist[b*d*n, Int[Cos[(Pi*(d*(a + b*
Log[c*x^n]))^2)/2], x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 4616

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)], x_Symbol] :> Dist[1
/2, Int[E^(-(I*d*(a + b*Log[c*x^n])^2)), x], x] + Dist[1/2, Int[E^(I*d*(a +
 b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 2277

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)), x_Symbol] :> Int[
F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b,
c, d, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int C\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx = xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-(bdn)\int\cos\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)dx$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}dx-\frac{1}{2}(bdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}dx$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^{\cdots}\right.\right.$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)\left(cx^n\right)^{-iabd^2\pi}dx\ \cdots$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(c\cdots\right.\right.$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}\left(bdx\left(cx^n\right)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{\left(1-ia\cdots\right.}{\cdots}\right.\right.$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}\left(bde^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{1-iabd^2n\cdots}{n}\right.}{2b\cdots}\right.\right.$$

$$= \left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-1/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)-\left(\frac{1}{4}+\frac{i}{4}\right)e^{\cdots}$$

Mathematica [A] time = 6.63245, size = 315, normalized size = 1.47

$$x\text{FresnelC}\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\frac{\sqrt[4]{-1}x\left(cx^n\right)^{-1/n}\left(ie^{\frac{i}{\pi b^2d^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)-i\right)}{\sqrt{\pi}bdn}\right)+\text{Erfi}\left(\frac{(-1)^{3/4}\left(\pi abd^2n+\pi b^2d^2n\,\mathrm{l}\cdots\right)}{\sqrt{2\pi}bdn}\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((-2*a)/(b*n) - I/(b^2*d^
2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2
*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(I*E^(I/(b^2*d^2*n^2*Pi))*Erfi[(
(1/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])
] + Erfi[((-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*S
qrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*Sin[(d^
2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-1))
```

Maple [F] time = 0.523, size = 0, normalized size = 0.

$$\int \mathrm{FresnelC}\left(d\left(a+b\ln\left(cx^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(FresnelC(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{fresnelc}\left((b\log\left(cx^{n}\right)+a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{fresnelc}\left(bd\log\left(cx^{n}\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int C\left(d\left(a+b\log\left(cx^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(fresnelc(d*(a + b*log(c*x**n))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d), x)
```

3.166 $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=66

$$\frac{(a+b\log(cx^n))\,\text{FresnelC}(d(a+b\log(cx^n)))}{bn}-\frac{\sin\left(\frac{1}{2}\pi d^2(a+b\log(cx^n))^2\right)}{\pi bdn}$$

```
[Out] (FresnelC[d*(a + b*Log[c*x^n])]*(a + b*Log[c*x^n]))/(b*n) - Sin[(d^2*Pi*(a
+ b*Log[c*x^n])^2)/2]/(b*d*n*Pi)
```

Rubi [A] time = 0.0399113, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6419}

$$\frac{(a+b\log(cx^n))\,\text{FresnelC}(d(a+b\log(cx^n)))}{bn}-\frac{\sin\left(\frac{1}{2}\pi d^2(a+b\log(cx^n))^2\right)}{\pi bdn}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (FresnelC[d*(a + b*Log[c*x^n])]*(a + b*Log[c*x^n]))/(b*n) - Sin[(d^2*Pi*(a
+ b*Log[c*x^n])^2)/2]/(b*d*n*Pi)
```

Rule 6419

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelC[a +
 b*x])/b, x] - Simp[Sin[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\text{Subst}\left(\int C(d(a+bx))\,dx, x, \log(cx^n)\right)}{n}\\
&= \frac{\text{Subst}\left(\int C(x)\,dx, x, ad+bd\log(cx^n)\right)}{bdn}\\
&= \frac{C(ad+bd\log(cx^n))\,(a+b\log(cx^n))}{bn}-\frac{\sin\left(\frac{1}{2}\pi(ad+bd\log(cx^n))^2\right)}{bdn\pi}
\end{aligned}$$

Mathematica [B] time = 0.0946098, size = 165, normalized size = 2.5

$$-\frac{\sin\left(\frac{1}{2}\pi a^2d^2\right)\cos\left(\pi abd^2\log(cx^n)+\frac{1}{2}\pi b^2d^2\log^2(cx^n)\right)}{\pi bdn}-\frac{\cos\left(\frac{1}{2}\pi a^2d^2\right)\sin\left(\pi abd^2\log(cx^n)+\frac{1}{2}\pi b^2d^2\log^2(cx^n)\right)}{\pi bdn}\cdot$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (a*FresnelC[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelC[d*(a + b*Log[c*x^n])]*
Log[c*x^n])/n - (Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2]*S
in[(a^2*d^2*Pi)/2])/(b*d*n*Pi) - (Cos[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log[c*
```

```
x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)
```

Maple [A] time = 0.069, size = 81, normalized size = 1.2

$$\frac{\ln(cx^n)\,\mathrm{FresnelC}\,(ad+bd\ln(cx^n))}{n}+\frac{\mathrm{FresnelC}\,(ad+bd\ln(cx^n))\,a}{bn}-\frac{1}{bdn\pi}\sin\left(\frac{\pi\,(ad+bd\ln(cx^n))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] 1/n*ln(c*x^n)*FresnelC(a*d+b*d*ln(c*x^n))+1/n/b*FresnelC(a*d+b*d*ln(c*x^n))
*a-1/n/b/d/Pi*sin(1/2*Pi*(a*d+b*d*ln(c*x^n))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}\,((b\log(cx^n)+a)d)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bd\log(cx^n)+ad)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*d*log(c*x^n) + a*d)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\,(ad+bd\log(cx^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\left((b\log\left(cx^{n}\right)+a)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x, x)
```

3.167 $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=217

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x}$$

```
[Out] ((1/4 + I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Erf[((1/
2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])]
)/x - ((1/4 + I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Er
fi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])])/x - FresnelC[d*(a + b*Log[c*x^n])]/x
```

Rubi [A] time = 0.49256, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] ((1/4 + I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Erf[((1/
2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])]
)/x - ((1/4 + I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Er
fi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])])/x - FresnelC[d*(a + b*Log[c*x^n])]/x
```

Rule 6472

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 4618

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{C(d(a+b\log(cx^n)))}{x^2}dx = -\frac{C(d(a+b\log(cx^n)))}{x} + (bdn)\int \frac{\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{x^2}dx$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(bdn)\int \frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}dx + \frac{1}{2}(bdn)\int \frac{e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \frac{1}{2}ib^2d^2\pi\log^2(cx}{x^2}$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(cx^n)^{-iabd^2\pi}}{x^2}dx$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{1}{2}\left(bdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(\right.$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{\left(bd(cx^n)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(-1-iabd^2n}{n}\right.\right.}{2x}$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{\left(bde^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{-1-iabd^2n\pi}{n}-ib^2\right.}{2b^2d^2\pi}\right.\right.}{2x}$$
$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\text{erfi}\left(\right.}{}$$

Mathematica [A] time = 3.92977, size = 194, normalized size = 0.89

$$-\frac{4\text{FresnelC}(d(a+b\log(cx^n))) + \sqrt[4]{-1}\sqrt{2}(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\left(ie^{\frac{i}{\pi b^2d^2n^2}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(\pi abd^2n+\pi b^2d^2n\log(cx^n)+i)}{\sqrt{\pi}bdn}\right) + \text{Erfi}\left(\frac{(-1)^{3/4}(\pi ab}{}\right.\right.}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*
(Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqr
t[2*Pi])] + I*E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(I + a*b*d^2*n*Pi +
b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]) + 4*FresnelC[d*(a + b*Log[c*x^
n])])/(4*x)
```

Maple [F] time = 0.523, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(d(a+b\ln(cx^n)))}{x^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\left((b\log\left(cx^n\right)+a)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\left(bd\log\left(cx^n\right)+ad\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\left(ad+bd\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\left((b\log\left(cx^n\right)+a)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^2, x)
```

3.168 $\int \frac{\mathbf{FresnelC}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=228

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{x^2}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(cx^n)^{2/n}e^{-\frac{2(-\pi abd^2n+i)}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n\right.}{\sqrt{\pi}bd}\right.}{x^2}$$

```
[Out] ((1/8 + I/8)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(2/n)*Erf[
((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])
])/x^2 - ((1/8 + I/8)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi +
 I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I - a*b*d^2*n*Pi))/(b^2
*d^2*n^2*Pi))*x^2) - FresnelC[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rubi [A] time = 0.493126, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{x^2}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(cx^n)^{2/n}e^{-\frac{2(-\pi abd^2n+i)}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n\right.}{\sqrt{\pi}bd}\right.}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] ((1/8 + I/8)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(2/n)*Erf[
((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])
])/x^2 - ((1/8 + I/8)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi +
 I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I - a*b*d^2*n*Pi))/(b^2
*d^2*n^2*Pi))*x^2) - FresnelC[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rule 6472

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 4618

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx = -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{2}(bdn)\int \frac{\cos\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)}{x^3}\,dx$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{4}(bdn)\int \frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}\,dx+\frac{1}{4}(bdn)\int \frac{e^{\frac{1}{2}id^2\pi(a+b\log(cx}}{x^3}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{4}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}ib^2d^2\pi\log^2}{x^3}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{4}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)\left(cx^n\right)^{-iabd^2\pi}}{x^3}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{4}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log\right.$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{\left(bd\left(cx^n\right)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(-2-iab}{n}\right.\right.}{4x^2}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{\left(bde^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}\left(cx^n\right)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{-2-iabd^2}{n}\right.}{2b}\right.\right.}{4x^2}$$

$$= \frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}\left(cx^n\right)^{2/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}\left(cx^n\right.}{}$$

Mathematica [A] time = 4.00641, size = 199, normalized size = 0.87

$$-\frac{\text{FresnelC}(d(a+b\log(cx^n)))}{2x^2}-\frac{\sqrt[4]{-1}\left(ie^{\frac{4i}{\pi b^2d^2n^2}}\text{Erfi}\left(\frac{\sqrt[4]{-1}(\pi abd^2n+\pi b^2d^2n\log(cx^n)+2i)}{\sqrt{2\pi}bdn}\right)+\text{Erfi}\left(\frac{(-1)^{3/4}(\pi abd^2n+\pi b^2d^2n\log(cx^n)-2i)}{\sqrt{2\pi}bdn}\right)\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*x^n])
))/n^2)*(Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/
(b*d*n*Sqrt[2*Pi])] + I*E^((4*I)/(b^2*d^2*n^2*Pi))*Erfi[((-1)^(1/4)*(2*I +
a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])]))/(4*Sqrt[2])
- FresnelC[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Maple [F] time = 0.527, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(d(a+b\ln(cx^n)))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}((b\log(cx^n)+a)d)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bd\log(cx^n)+ad)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\left(ad + bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{fresnelc}\left(\left(b\log\left(cx^{n}\right) + a\right)d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^3, x)
```

3.169 $\int (ex)^m \text{FresnelC}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=280

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m(cx^n)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)(2i\pi abd^2n+m+1)}{2\pi b^2d^2n^2}\right)\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i\pi abd^2n+i\pi b^2d^2n\log(cx^n)+m+1)}{\sqrt{\pi}bdn}\right)}{m+1}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m(cx^n)^{-\frac{m+1}{n}}\text{ex}}{}$$

```
[Out] ((1/4 + I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*x*(e*x)^m*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])])/((1 + m)*(c*x^n)^((1 + m)/n)) - ((1/4 + I/4)*x
*(e*x)^m*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])])/(E^(((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))/(
b^2*d^2*n^2*Pi))*(1 + m)*(c*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*FresnelC[d*(
a + b*Log[c*x^n])])/(e*(1 + m))
```

Rubi [A] time = 0.607006, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {6472, 4618, 2278, 2274, 15, 20, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m(cx^n)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)(2i\pi abd^2n+m+1)}{2\pi b^2d^2n^2}\right)\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i\pi abd^2n+i\pi b^2d^2n\log(cx^n)+m+1)}{\sqrt{\pi}bdn}\right)}{m+1}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m(cx^n)^{-\frac{m+1}{n}}\text{ex}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*FresnelC[d*(a + b*Log[c*x^n])],x]
```


```
[Out] ((1/4 + I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*x*(e*x)^m*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])])/((1 + m)*(c*x^n)^((1 + m)/n)) - ((1/4 + I/4)*x
*(e*x)^m*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])])/(E^(((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))/(
b^2*d^2*n^2*Pi))*(1 + m)*(c*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*FresnelC[d*(
a + b*Log[c*x^n])])/(e*(1 + m))
```


Rule 6472

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 4618

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ex)^m C(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(bdn)\int (ex)^m \cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)dx}{1+m}$$

$$= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^m\,dx}{2(1+m)} - \frac{(bdn)\int e^{\frac{1}{2}id^2\pi(a}}{2(}$$

$$= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(bdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \frac{1}{2}ib^2d^2\pi}{2(1+m)}$$

$$= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(bdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(ex)^m(}{2(1+m)}$$

$$= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(bdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2}{2(1+m)}$$

$$= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(bdnx^{-m+iabd^2n\pi}(ex)^m(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi}{2(1+m)}$$

$$= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(bdx(ex)^m(cx^n)^{-iabd^2\pi-\frac{1+m-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}\right.\right.}{2(1}$$

$$= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(bd\exp\left(-\frac{i(1+m)(1+m-2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m(cx^n)^{-iabd^2\pi-\frac{1+}{}}\right.}{}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\exp\left(\frac{i(1+m)(1+m+2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m+iabd^2n\pi+ib^2d^2n\pi\log(c}{bdn\sqrt{\pi}}\right.}{1+m}$$

Mathematica [A] time = 5.35204, size = 244, normalized size = 0.87

$$\frac{(ex)^m\left(4x\text{FresnelC}(d(a+b\log(cx^n))) + (-1)^{3/4}\sqrt{2}x^{-m}\exp\left(-\frac{(m+1)(2\pi abd^2n+2\pi b^2d^2n(\log(cx^n)-n\log(x))+im+i)}{2\pi b^2d^2n^2}\right)\left(\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(\pi ab}{}\right.\right.\right.}{4(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*(((-1)^(3/4)*Sqrt[2]*(Erf[((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi + b
^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] - E^((I*(1 + m)^2)/(b^2*d^2*n^2*
Pi))*Erfi[((-1)^(3/4)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]))
/(b*d*n*Sqrt[2*Pi])]))/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2*d^2*n
*Pi*(-(n*Log[x]) + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m) + 4*x*FresnelC[d*
(a + b*Log[c*x^n])]))/(4*(1 + m))
```

Maple [F] time = 0.205, size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{FresnelC}(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{fresnelc}\left((b\log\left(cx^n\right) + a)d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*fresnelc((b*log(c*x^n) + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((ex)^m \operatorname{fresnelc}\left(bd\log\left(cx^n\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*fresnelc(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m C\left(ad + bd\log\left(cx^n\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*fresnelc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*fresnelc(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{fresnelc}\left((b\log\left(cx^n\right) + a)d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*fresnelc((b*log(c*x^n) + a)*d), x)
```

3.170 $\int e^{c+\frac{1}{2}ib^2\pi x^2}\textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=64

$$\frac{1}{4}be^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)-\frac{ie^c\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

```
[Out] ((-I/8)*E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*Hypergeometric
PFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/4
```

Rubi [A] time = 0.0693593, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6437, 6376, 6375, 30}

$$\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

```
[Out] ((-I/8)*E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*Hypergeometric
PFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/4
```

Rule 6437

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[(1 - I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 + I)*b*x)/2], x], x] + Dist[(1 + I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 - I)*b*x)/2], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, -((Pi^2*b^4)/4)]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c+\frac{1}{2}ib^2\pi x^2}C(bx)\,dx &= \left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx\\ &= \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{(ie^c)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\right)}{4b}\\ &= -\frac{ie^c\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}+\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)\end{aligned}$$

Mathematica [F] time = 0.0291736, size = 0, normalized size = 0.

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\operatorname{FresnelC}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

```
[Out] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int e^{c+\frac{i}{2}b^2\pi x^2}\operatorname{FresnelC}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)}\operatorname{fresnelc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)}\operatorname{fresnelc}(bx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{c}\int e^{\frac{i\pi b^{2}x^{2}}{2}}C\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\frac{1}{2}i\,\pi b^{2}x^{2}+c\right)}\mathrm{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

3.171 $\int e^{c-\frac{1}{2}ib^2\pi x^2}$ **FresnelC**$(bx)\,dx$

Optimal. Leaf size=64

$$\frac{1}{4}be^c x^2 \text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)-\frac{ie^c\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

```
[Out] ((-I/8)*E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/4
```

Rubi [A] time = 0.0662754, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6437, 6373, 30, 6378}

$$\frac{1}{4}be^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

```
[Out] ((-I/8)*E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/4
```

Rule 6437

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[(1 - I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 + I)*b*x)/2], x], x] + Dist[(1 + I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 - I)*b*x)/2], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, -((Pi^2*b^4)/4)]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rubi steps

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}C(bx)\,dx = \left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$
$$= \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{(ie^c)\,\text{Subst}\left(\int x\,dx,x,\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\right)}{4b}$$
$$= -\frac{ie^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}+\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.0304312, size = 0, normalized size = 0.

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{FresnelC}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

```
[Out] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int e^{c-\frac{i}{2}b^2\pi x^2}\text{FresnelC}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)}\text{fresnelc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)}\text{fresnelc}(bx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{c}\int e^{-\frac{i\pi b^{2}x^{2}}{2}}C\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(-\frac{1}{2}i\pi b^{2}x^{2}+c\right)}\mathrm{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

3.172 $\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=101

$$\frac{1}{8}ibx^2\cos(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)-\frac{1}{8}ibx^2\cos(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi$$

```
[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*Cos[c]*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] - (I/8)*b*x^2*Cos[c]*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] + (FresnelC[b*x]^2*Sin[c])/(2*b
)
```

Rubi [A] time = 0.050629, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6449, 6441, 30, 6447}

$$\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\cos(c)\text{FresnelC}(bx)S(bx)}{2b}+\frac{\sin(c)\text{Fresn}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```


```
[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*Cos[c]*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] - (I/8)*b*x^2*Cos[c]*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] + (FresnelC[b*x]^2*Sin[c])/(2*b
)
```


Rule 6449

```
Int[FresnelC[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[Sin[c]
, Int[Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[Cos[c], Int[Sin[d*x^2]*Fresne
lC[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int C(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)dx = \cos(c)\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx + \sin(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$
$$= \frac{\cos(c)C(bx)S(bx)}{2b} + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2}\right.$$
$$= \frac{\cos(c)C(bx)S(bx)}{2b} + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2}\right.$$

Mathematica [F] time = 0.0440036, size = 0, normalized size = 0.

$$\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

```
[Out] Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \text{FresnelC}\,(bx)\sin\left(c+\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

```
[Out] int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{\pi b^2 x^2}{2} + c\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(c+1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

3.173 $\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=101

$$-\frac{1}{8}ibx^2\sin(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)+\frac{1}{8}ibx^2\sin(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\}\right.$$

```
[Out] (Cos[c]*FresnelC[b*x]^2)/(2*b) - (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
 - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]*Sin[c
] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin[c
]
```

Rubi [A] time = 0.0478725, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6443, 6441, 30, 6447}

$$-\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{\sin(c)\text{FresnelC}(bx)S(bx)}{2b}+\frac{\cos(c)\text{F}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (Cos[c]*FresnelC[b*x]^2)/(2*b) - (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
 - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]*Sin[c
] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin[c
]
```

Rule 6443

```
Int[Cos[(c_) + (d_.)*(x_)^2]*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[Cos[c]
, Int[Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[Sin[c], Int[Sin[d*x^2]*Fresne
lC[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx = \cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx - \sin(c)\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= -\frac{C(bx)S(bx)\sin(c)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c) + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\right.$$
$$= \frac{\cos(c)C(bx)^2}{2b} - \frac{C(bx)S(bx)\sin(c)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c) + \frac{1}{8}ibx^2\,{}_2F_2\left(\right.$$

Mathematica [F] time = 0.0389271, size = 0, normalized size = 0.

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \cos\left(c+\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\,\pi b^2x^2+c\right)\text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos\left(\frac{1}{2}\,\pi b^2x^2+c\right)\text{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos \left(\frac{\pi b^2 x^2}{2} + c \right) C\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos \left(\frac{1}{2}\,\pi b^2 x^2 + c \right) \mathrm{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

3.174 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)^2 \, dx$

Optimal. Leaf size=13

$$\frac{\text{FresnelC}(bx)^3}{3b}$$

[Out] FresnelC[b*x]^3/(3*b)

Rubi [A] time = 0.0148672, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6441, 30}

$$\frac{\text{FresnelC}(bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]

[Out] FresnelC[b*x]^3/(3*b)

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)^2 \, dx &= \frac{\text{Subst}\left(\int x^2 \, dx, x, C(bx)\right)}{b}\\ &= \frac{C(bx)^3}{3b}\end{aligned}$$

Mathematica [A] time = 0.0087997, size = 13, normalized size = 1.

$$\frac{\text{FresnelC}(bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]

[Out] FresnelC[b*x]^3/(3*b)

Maple [A] time = 0.044, size = 12, normalized size = 0.9

$$\frac{(\text{FresnelC}(bx))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^2,x)
```

```
[Out] 1/3*FresnelC(b*x)^3/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2x^2\right) \text{fresnelc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos\left(\frac{1}{2}\pi b^2x^2\right) \text{fresnelc}(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)^2, x)
```

Sympy [A] time = 1.41009, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^3(bx)}{3b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**2,x)
```

```
[Out] Piecewise((fresnelc(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2x^2\right) \text{fresnelc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)^2, x)
```

3.175 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=13

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

```
[Out] FresnelC[b*x]^2/(2*b)
```

Rubi [A] time = 0.0116175, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6441, 30}

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] FresnelC[b*x]^2/(2*b)
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, C(bx)\right)}{b}\\ &= \frac{C(bx)^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.0058943, size = 13, normalized size = 1.

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] FresnelC[b*x]^2/(2*b)
```

Maple [A] time = 0.044, size = 12, normalized size = 0.9

$$\frac{(\mathrm{FresnelC}(bx))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] 1/2*FresnelC(b*x)^2/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{fresnelc}\,(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [A] time = 0.44121, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.176 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\textbf{FresnelC}(bx)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(\text{FresnelC}(bx))}{b}$$

```
[Out] Log[FresnelC[b*x]]/b
```

Rubi [A] time = 0.0155068, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6441, 29}

$$\frac{\log(\text{FresnelC}(bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x],x]
```

```
[Out] Log[FresnelC[b*x]]/b
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, C(bx)\right)}{b}\\ &= \frac{\log(C(bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.0109443, size = 9, normalized size = 1.

$$\frac{\log(\text{FresnelC}(bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x],x]
```

```
[Out] Log[FresnelC[b*x]]/b
```

Maple [A] time = 0.122, size = 10, normalized size = 1.1

$$\frac{\ln(\mathrm{FresnelC}(bx))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x),x)
```

```
[Out] ln(FresnelC(b*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\mathrm{fresnelc}(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\mathrm{fresnelc}(bx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)/fresnelc(b*x), x)
```

Sympy [A] time = 0.242884, size = 10, normalized size = 1.11

$$\begin{cases} \frac{\log(C(bx))}{b} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x),x)
```

```
[Out] Piecewise((log(fresnelc(b*x))/b, Ne(b, 0)), (zoo*x, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\mathrm{fresnelc}(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x), x)
```

3.177 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\textbf{FresnelC}(bx)^2}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{b\text{FresnelC}(bx)}$$

[Out] -(1/(b*FresnelC[b*x]))

Rubi [A] time = 0.0149059, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6441, 30}

$$-\frac{1}{b\text{FresnelC}(bx)}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]

[Out] -(1/(b*FresnelC[b*x]))

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, C(bx)\right)}{b}\\ &= -\frac{1}{bC(bx)}\end{aligned}$$

Mathematica [A] time = 0.0055501, size = 11, normalized size = 1.

$$-\frac{1}{b\text{FresnelC}(bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]

[Out] -(1/(b*FresnelC[b*x]))

Maple [A] time = 0.044, size = 12, normalized size = 1.1

$$-\frac{1}{b\text{FresnelC}\left(bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^2,x)
```

```
[Out] -1/b/FresnelC(b*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnelc}\left(bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnelc}\left(bx\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^2, x)
```

Sympy [A] time = 0.797724, size = 12, normalized size = 1.09

$$\begin{cases} -\frac{1}{bC(bx)} & \text{for}\: b \neq 0 \\ \tilde{\infty} x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**2,x)
```

```
[Out] Piecewise((-1/(b*fresnelc(b*x)), Ne(b, 0)), (zoo*x, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}\,(bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^2, x)
```

3.178 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\textbf{FresnelC}(bx)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2b\text{FresnelC}(bx)^2}$$

```
[Out] -1/(2*b*FresnelC[b*x]^2)
```

Rubi [A] time = 0.0156124, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6441, 30}

$$-\frac{1}{2b\text{FresnelC}(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]
```

```
[Out] -1/(2*b*FresnelC[b*x]^2)
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, C(bx)\right)}{b}\\ &= -\frac{1}{2bC(bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0059558, size = 13, normalized size = 1.

$$-\frac{1}{2b\text{FresnelC}(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]
```

```
[Out] -1/(2*b*FresnelC[b*x]^2)
```

Maple [A] time = 0.046, size = 12, normalized size = 0.9

$$-\frac{1}{2\,b\,(\mathrm{FresnelC}\,(bx))^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^3,x)
```

```
[Out] -1/2/b/FresnelC(b*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{\mathrm{fresnelc}\,(bx)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{\mathrm{fresnelc}\,(bx)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^3,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^3, x)
```

Sympy [A] time = 1.97152, size = 15, normalized size = 1.15

$$\begin{cases} -\frac{1}{2bC^{2}(bx)} & \text{for}\ b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**3,x)
```

```
[Out] Piecewise((-1/(2*b*fresnelc(b*x)**2), Ne(b, 0)), (zoo*x, True))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}\,(bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^3,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^3, x)
```

3.179 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^n \, dx$

Optimal. Leaf size=17

$$\frac{\text{FresnelC}(bx)^{n+1}}{b(n+1)}$$

```
[Out] FresnelC[b*x]^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.0187874, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6441, 30}

$$\frac{\text{FresnelC}(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]
```

```
[Out] FresnelC[b*x]^(1 + n)/(b*(1 + n))
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)^n \, dx &= \frac{\text{Subst}\left(\int x^n \, dx, x, C(bx)\right)}{b} \\ &= \frac{C(bx)^{1+n}}{b(1+n)} \end{aligned}$$

Mathematica [A] time = 0.0102103, size = 17, normalized size = 1.

$$\frac{\text{FresnelC}(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]
```

```
[Out] FresnelC[b*x]^(1 + n)/(b*(1 + n))
```

Maple [A] time = 0.047, size = 18, normalized size = 1.1

$$\frac{(\text{FresnelC}\,(bx))^{1+n}}{b\,(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^n,x)
```

```
[Out] FresnelC(b*x)^(1+n)/b/(1+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx)^n \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnelc}\,(bx)^n \cos\left(\frac{1}{2}\,\pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^n,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 4.59086, size = 34, normalized size = 2.

$$\begin{cases} \tilde{\infty} x & \text{for}\ b = 0 \wedge n = -1 \\ 0^n x & \text{for}\ b = 0 \\ \frac{\log(C(bx))}{b} & \text{for}\ n = -1 \\ \frac{C(bx) C^n(bx)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**n,x)
```

```
[Out] Piecewise((zoo*x, Eq(b, 0) & Eq(n, -1)), (0**n*x, Eq(b, 0)), (log(fresnelc(
b*x))/b, Eq(n, -1)), (fresnelc(b*x)*fresnelc(b*x)**n/(b*n + b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx)^n \cos\left(\frac{1}{2}\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^n,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

3.180 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=231

$$\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{35x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{7x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{105x\text{FresnelC}(b}{\pi^4 b}$$

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2)/
2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b
^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(
b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*P
i) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Rubi [A] time = 0.382908, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637, 3309}

$$\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{35x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{7x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{105x\text{FresnelC}(b}{\pi^4 b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```


```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2)/
2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b
^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(
b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*P
i) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```


Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx = \frac{x^7 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7\int x^6 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^7\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^7 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^4\pi^2} - \frac{7\int}{}$$

$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{35x^3 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7 C(bx)\sin\left(\frac{1}{2}b^2\right.}{b^2\pi}$$

$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{35x^3 C(bx)\sin\left(\right.}{b^6\pi^3}$$

$$= -\frac{7x^6}{12b^3\pi^2} - \frac{41x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}\right.}{b}$$

$$= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{7x^5}{}$$

$$= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{7x^5}{}$$

Mathematica [A] time = 0.0117987, size = 231, normalized size = 1.

$$\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{35x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3 b^6} + \frac{7x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2 b^4} - \frac{105x\text{FresnelC}(b}{\pi^4 b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2)/
2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b
^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(
b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*P
i) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int x^8\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^8\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^8*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.181 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=215

$$\frac{x^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{24x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{6x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{48\text{FresnelC}(bx)}{\pi^4b^8}$$

```
[Out] (24*x)/(b^7*Pi^4) - (3*x^5)/(5*b^3*Pi^2) - (147*x*Cos[b^2*Pi*x^2])/(16*b^7*
Pi^4) + (x^5*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (48*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2)
+ (531*FresnelC[Sqrt[2]*b*x])/(16*Sqrt[2]*b^8*Pi^4) - (24*x^2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/
(b^2*Pi) - (17*x^3*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)
```

Rubi [A] time = 0.259335, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6455, 6463, 6461, 3358, 3352, 3385, 3392, 30, 3386}

$$\frac{x^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{24x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{6x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{48\text{FresnelC}(bx)}{\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```


```
[Out] (24*x)/(b^7*Pi^4) - (3*x^5)/(5*b^3*Pi^2) - (147*x*Cos[b^2*Pi*x^2])/(16*b^7*
Pi^4) + (x^5*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (48*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2)
+ (531*FresnelC[Sqrt[2]*b*x])/(16*Sqrt[2]*b^8*Pi^4) - (24*x^2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/
(b^2*Pi) - (17*x^3*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)
```


Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx &= \frac{x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{6\int x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^6\sin\left(b^2\pi x^2\right)dx}{2b\pi} \\
&= \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{24\int x^3\cos\left(\frac{1}{2}b^2\right.}{b^4\pi} \\
&= \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{24x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^6C(bx)\sin\left(\right.}{b^2\pi} \\
&= -\frac{3x^5}{5b^3\pi^2} - \frac{111x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{6x^4\cos\left(\frac{1}{2}b\right.}{b^4} \\
&= -\frac{3x^5}{5b^3\pi^2} - \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{6x^4\cos\left(\frac{1}{2}\right.}{b} \\
&= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{6x^4}{} \\
&= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{6x^4}{}
\end{aligned}$$

Mathematica [A] time = 0.234203, size = 154, normalized size = 0.72

$$\frac{160\text{FresnelC}(bx)\left(\pi b^2x^2\left(\pi^2b^4x^4-24\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)+6\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)\right)+2bx\left(5\left(4\pi^2b^4x^4-147\right)\cos\left(\pi b^2\right.\right.}{160\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 160*FresnelC[b*x]*(6*(-8 + b^4*Pi^2*x
^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/
2]) + 2*b*x*(5*(-147 + 4*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] - 2*(-960 + 24*b^4*P
i^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/(160*b^8*Pi^4)
```

Maple [A] time = 0.085, size = 317, normalized size = 1.5

$$\frac{1}{b}\left(\frac{\text{FresnelC}\left(bx\right)}{b^7}\left(\frac{b^6x^6}{\pi}\sin\left(\frac{b^2\pi x^2}{2}\right)-6\frac{1}{\pi}\left(-\frac{x^4b^4\cos\left(1/2\,b^2\pi x^2\right)}{\pi}+4\frac{1}{\pi}\left(\frac{b^2x^2\sin\left(1/2\,b^2\pi x^2\right)}{\pi}+2\frac{\cos\left(1/2\,b^2\pi x^2\right)}{\pi^2}\right)\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] (FresnelC(b*x)/b^7*(1/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/Pi*(-1/Pi*b^4*x^4*co
s(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2
*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*Pi^2*b^5*x^5-8*b*x)+3/Pi^4*(1/2*Pi*b^3*x^3*s
in(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(
b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))+1/2/Pi^3*(-1/2*Pi*b^5*x^5*co
s(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*co
s(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x
^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^7\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^7*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.182 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=247

$$\frac{15ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi^3b^5}-\frac{15ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi^3b^5}+\frac{15\text{Fres}}{}$$

```
[Out] (-5*x^4)/(8*b^3*Pi^2) - (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*Cos[b^2*Pi
*x^2])/(4*b^3*Pi^2) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2)
+ (15*FresnelC[b*x]*FresnelS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*Hypergeom
etricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (15*x*
FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^5*FresnelC[b*x]*Sin[(b^2
*Pi*x^2)/2])/(b^2*Pi) - (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```

Rubi [A] time = 0.25316, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6455, 6463, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}+\frac{15\text{FresnelC}(bx)S(bx)}{2\pi^3b^7}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```


```
[Out] (-5*x^4)/(8*b^3*Pi^2) - (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*Cos[b^2*Pi
*x^2])/(4*b^3*Pi^2) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2)
+ (15*FresnelC[b*x]*FresnelS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*Hypergeom
etricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (15*x*
FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^5*FresnelC[b*x]*Sin[(b^2
*Pi*x^2)/2])/(b^2*Pi) - (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```


Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
```

```
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx = \frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{5\int x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^5\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$
$$= \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{15\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^4\pi^2} - \frac{5\int$$
$$= \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{15xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{b^2\pi}$$
$$= \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{15C(bx)S(bx)}{2b^7\pi^3} + \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi\right.}{8b^5\pi^3}$$
$$= -\frac{5x^4}{8b^3\pi^2} - \frac{17\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{15C(bx)S(bx)}{2b^7\pi^3} +$$
$$= -\frac{5x^4}{8b^3\pi^2} - \frac{11\cos\left(b^2\pi x^2\right)}{2b^7\pi^4} + \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{15C(bx)S(bx)}{2b^7\pi^3} +$$

Mathematica [F] time = 0.381465, size = 0, normalized size = 0.

$$\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int x^6\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^6*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.183 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=157

$$\frac{x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{8\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{4x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{43S\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3b^6}-\frac{2x^3}{3\pi^2b^3}$$

```
[Out] (-2*x^3)/(3*b^3*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (4*x^2*Cos[(b^
2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt
[2]*b^6*Pi^3) - (8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^4*Fre
snelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi
^3)
```

Rubi [A] time = 0.151098, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6455, 6463, 6453, 3351, 3392, 30, 3386, 3385}

$$\frac{x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{8\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{4x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{43S\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3b^6}-\frac{2x^3}{3\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```


```
[Out] (-2*x^3)/(3*b^3*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (4*x^2*Cos[(b^
2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt
[2]*b^6*Pi^3) - (8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^4*Fre
snelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi
^3)
```


Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx &= \frac{x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{4\int x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi}-\frac{\int x^4\sin\left(b^2\pi x^2\right)dx}{2b\pi}\\
&=\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}+\frac{x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{8\int x\cos\left(\frac{1}{2}b^2\pi\right.}{b^4\pi^2}\\
&=\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}-\frac{8C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}+\frac{x^4C(bx)\sin\left(\frac{1}{2}b^2\right.}{b^2\pi}\\
&=-\frac{2x^3}{3b^3\pi^2}+\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}+\frac{3S\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3}+\frac{2\sqrt{2}S\left(\sqrt{2}bx\right)}{b^6\pi^3}-\\
&=-\frac{2x^3}{3b^3\pi^2}+\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}+\frac{11S\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3}+\frac{2\sqrt{2}S\left(\sqrt{2}bx\right)}{b^6\pi^3}-\end{aligned}$$

Mathematica [A] time = 0.131994, size = 120, normalized size = 0.76

$$\frac{48\text{FresnelC}(bx)\left(\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)+4\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)\right)-32\pi b^3x^3-66bx\sin\left(\pi b^2x^2\right)+12\pi b^3x^3\cos\left(\right.}{48\pi^3b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*Sqrt[2]*FresnelS[Sqrt
[2]*b*x] + 48*FresnelC[b*x]*(4*b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*P
```

```
i^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(48*b^6*Pi^3)
```

Maple [A] time = 0.079, size = 202, normalized size = 1.3

$$\frac{1}{b}\left(\frac{\mathrm{FresnelC}\,(bx)}{b^5}\left(\frac{x^4b^4}{\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right)-4\,\frac{1}{\pi}\left(-\frac{b^2x^2\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi}+2\,\frac{\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right)\right)-\frac{1}{b^5}\left(\frac{2\,x^3b^3}{3\,\pi^2}+2\,\frac{1}{\pi^2}\left(1/2\,\frac{bx\,s}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] (FresnelC(b*x)/b^5*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*co
s(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))-1/b^5*(2/3/Pi^2*b^3*x^3+2/Pi
^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))+1/2/Pi
^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/
Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^5\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^5*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\cos\left(\frac{\pi b^2x^2}{2}\right)C\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.184 $\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=120

$$\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{3\text{FresnelC}(bx)^2}{2\pi^2b^5}-\frac{3x^2}{4\pi^2b^3}-\frac{\sin\left(\pi b^2x^2\right)}{\pi^3b^5}+\frac{x^2\cos\left(\pi b\right.}{4\pi^2b^3}$$

```
[Out] (-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (
x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Rubi [A] time = 0.118108, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637}

$$\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{3\text{FresnelC}(bx)^2}{2\pi^2b^5}-\frac{3x^2}{4\pi^2b^3}-\frac{\sin\left(\pi b^2x^2\right)}{\pi^3b^5}+\frac{x^2\cos\left(\pi b\right.}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (
x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx &= \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{3\int x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi}-\frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{2b\pi}\\ &= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}+\frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^4\pi^2}-\frac{3\int x\\ &= \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}+\frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{3\,\text{Subst}(\int x\,dx,x}{b^5\pi^2}\\ &= -\frac{3x^2}{4b^3\pi^2}+\frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}-\frac{3C(bx)^2}{2b^5\pi^2}+\frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x\right.}{b^2\pi}\end{aligned}$$

Mathematica [A] time = 0.006659, size = 120, normalized size = 1.

$$\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{3\text{FresnelC}(bx)^2}{2\pi^2b^5}-\frac{3x^2}{4\pi^2b^3}-\frac{\sin\left(\pi b^2x^2\right)}{\pi^3b^5}+\frac{x^2\cos\left(\right.}{4\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (
```

```
x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{b^2 \pi x^2}{2}\right) \mathrm{FresnelC}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{fresnelc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^4*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [A] time = 26.6684, size = 151, normalized size = 1.26

$$\begin{cases} \frac{x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} - \frac{x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^2 b^3} - \frac{x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{3x \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi^2 b^4} - \frac{2 \sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^3 b^5} - \frac{3C^2(bx)}{2\pi^2 b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Piecewise((x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) - x**2*sin(pi*b
**2*x**2/2)**2/(pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(2*pi**2*b**3) +
3*x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi**2*b**4) - 2*sin(pi*b**2*x**2/2)*
```

```
cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnelc(b*x)**2/(2*pi**2*b**5), Ne(b,
 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.185 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=104

$$\frac{x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{5\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2b^4}+\frac{x\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x}{\pi^2b^3}$$

```
[Out] -(x/(b^3*Pi^2)) + (x*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (2*Cos[(b^2*Pi*x^2)/2]
*FresnelC[b*x])/(b^4*Pi^2) - (5*FresnelC[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2)
 + (x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rubi [A] time = 0.0782079, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6455, 6461, 3358, 3352, 3385}

$$\frac{x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{5\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2b^4}+\frac{x\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x}{\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```


```
[Out] -(x/(b^3*Pi^2)) + (x*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (2*Cos[(b^2*Pi*x^2)/2]
*FresnelC[b*x])/(b^4*Pi^2) - (5*FresnelC[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2)
 + (x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```


Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
```

```
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx = \frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{2\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^2\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$

$$= \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int\cos\left(b^2\pi x^2\right)dx}{4b^3\pi^2} -$$

$$= \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{C\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{2\int\left(\frac{1}{2}\right.}{}$$

$$= -\frac{x}{b^3\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{C\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

$$= -\frac{x}{b^3\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{5C\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right.}{b^2\pi}$$

Mathematica [A] time = 0.084731, size = 83, normalized size = 0.8

$$\frac{8\text{FresnelC}(bx)\left(\pi b^2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)+2\cos\left(\frac{1}{2}\pi b^2x^2\right)\right)+2bx\left(\cos\left(\pi b^2x^2\right)-4\right)-5\sqrt{2}\text{FresnelC}\left(\sqrt{2}bx\right)}{8\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (2*b*x*(-4 + Cos[b^2*Pi*x^2]) - 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 8*Fresnel
C[b*x]*(2*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]))/(8*b^4*Pi^
2)
```

Maple [A] time = 0.076, size = 114, normalized size = 1.1

$$\frac{1}{b}\left(\frac{\text{FresnelC}\,(bx)}{b^3}\left(\frac{b^2x^2}{\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right)+2\,\frac{\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right)-\frac{1}{b^3}\left(\frac{bx}{\pi^2}+\frac{\sqrt{2}\text{FresnelC}\left(bx\sqrt{2}\right)}{2\,\pi^2}+\frac{1}{2\,\pi}\left(-\frac{bx\cos\left(b^2\pi\,x^{\cdot}\right.}{2\,\pi}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] (FresnelC(b*x)/b^3*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*
x^2))-1/b^3*(1/Pi^2*b*x+1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))+1/2/Pi*(-1/2
/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\cos\left(\frac{\pi b^2x^2}{2}\right)C\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**3*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.186 $\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=136

$$-\frac{ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi b}+\frac{ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi b}-\text{Fresn}$$

```
[Out] Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - (
(I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b*Pi) +
((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi) +
(x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rubi [A] time = 0.0595254, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6455, 6447, 3379, 2638}

$$-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}-\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^3}+\frac{x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - (
(I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b*Pi) +
((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi) +
(x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx = \frac{xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$
$$= -\frac{C(bx)S(bx)}{2b^3\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{xC(bx)\sin\left(\right.}{b^2\pi}$$
$$= \frac{\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{C(bx)S(bx)}{2b^3\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}$$

Mathematica [F] time = 0.208928, size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**2*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{fresnelc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.187 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=48

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{S\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}$$

```
[Out] -FresnelS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi) + (FresnelC[b*x]*Sin[(b^2*Pi*x^2)
/2])/(b^2*Pi)
```

Rubi [A] time = 0.0208259, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6453, 3351}

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{S\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] -FresnelS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi) + (FresnelC[b*x]*Sin[(b^2*Pi*x^2)
/2])/(b^2*Pi)
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx&=\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{\int\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\&=-\frac{S\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi}+\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}\end{aligned}$$

Mathematica [A] time = 0.0273675, size = 44, normalized size = 0.92

$$-\frac{\sqrt{2}S\left(\sqrt{2}bx\right)-4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] -(Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b
^2*Pi)
```

Maple [A] time = 0.054, size = 45, normalized size = 0.9

$$\frac{1}{b}\left(\frac{\text{FresnelC}\,(bx)}{b\pi}\sin\left(\frac{b^2\pi\,x^2}{2}\right)-\frac{\sqrt{2}\text{FresnelS}\,\left(bx\sqrt{2}\right)}{4\,b\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] (FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi-1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^(1
/2))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos\left(\frac{\pi b^2x^2}{2}\right)C\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.188 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx)\, dx$

Optimal. Leaf size=13

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

[Out] FresnelC[b*x]^2/(2*b)

Rubi [A] time = 0.0114821, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6441, 30}

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] FresnelC[b*x]^2/(2*b)

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\, dx &= \frac{\text{Subst}\left(\int x\, dx, x, C(bx)\right)}{b} \\ &= \frac{C(bx)^2}{2b} \end{aligned}$$

Mathematica [A] time = 0.0024072, size = 13, normalized size = 1.

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] FresnelC[b*x]^2/(2*b)

Maple [A] time = 0.046, size = 12, normalized size = 0.9

$$\frac{(\text{FresnelC}\,(bx))^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] 1/2*FresnelC(b*x)^2/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [A] time = 0.458888, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^2(bx)}{2b} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.189 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)$$

```
[Out] Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]
```

Rubi [A] time = 0.0168393, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x,x]
```

```
[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx=\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx$$

Mathematica [A] time = 0.029494, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]
```

Maple [A] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}\,(bx)}{x}\cos\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\operatorname{fresnelc}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\operatorname{fresnelc}(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\operatorname{fresnelc}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x, x)
```

3.190 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)$$

```
[Out] Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]
```

Rubi [A] time = 0.0187246, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2,x]
```

```
[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx$$

Mathematica [A] time = 0.030572, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]
```

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^2}\cos\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^2,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^2,x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C\left(bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**2,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^2,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^2, x)

3.191 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\textbf{FresnelC}(bx)}{x^3}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{2}\pi b^2\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{\pi b^2S\left(\sqrt{2}bx\right)}{2\sqrt{2}}-\frac{b\cos\left(\pi b^2x^2\right)}{4x}-$$

```
[Out] -b/(4*x) - (b*Cos[b^2*Pi*x^2])/(4*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/
(2*x^2) - (b^2*Pi*FresnelS[Sqrt[2]*b*x])/(2*Sqrt[2]) - (b^2*Pi*Unintegrable
[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2
```

Rubi [A] time = 0.0564858, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3,x]
```

```
[Out] -b/(4*x) - (b*Cos[b^2*Pi*x^2])/(4*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/
(2*x^2) - (b^2*Pi*FresnelS[Sqrt[2]*b*x])/(2*Sqrt[2]) - (b^2*Pi*Defer[Int][(
FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^3}\,dx &= -\frac{b}{4x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2x^2}+\frac{1}{4}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{2}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\ &= -\frac{b}{4x}-\frac{b\cos\left(b^2\pi x^2\right)}{4x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2x^2}-\frac{1}{2}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx-\frac{1}{2}\left(b^3\pi\right)\\ &= -\frac{b}{4x}-\frac{b\cos\left(b^2\pi x^2\right)}{4x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2x^2}-\frac{b^2\pi S\left(\sqrt{2}bx\right)}{2\sqrt{2}}-\frac{1}{2}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x\right.}{x}\end{aligned}$$

Mathematica [A] time = 0.0296671, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3, x]
```

Maple [A] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelC}\left(bx\right)}{x^{3}}\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{fresnelc}\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{fresnelc}\left(bx\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^{2}x^{2}}{2}\right)C\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**3,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{fresnelc}\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^3, x)
```

3.192 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\textbf{FresnelC}(bx)}{x^4}\,dx$

Optimal. Leaf size=109

$$\frac{\pi b^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{6}\pi^2b^3\text{FresnelC}(bx)^2-\frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right)-\frac{b\cos\left(\pi b\right.}{12x^2}$$

```
[Out] -b/(12*x^2) - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Rubi [A] time = 0.115158, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6457, 6465, 6441, 30, 3375, 3380, 3297, 3299}

$$\frac{\pi b^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{6}\pi^2b^3\text{FresnelC}(bx)^2-\frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right)-\frac{b\cos\left(\pi b\right.}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]
```

```
[Out] -b/(12*x^2) - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^4}\,dx &= -\frac{b}{12x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3}+\frac{1}{6}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx-\frac{1}{3}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\ &= -\frac{b}{12x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x}+\frac{1}{12}b\,\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx,x,\right.\\ &= -\frac{b}{12x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{12x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x}-\frac{1}{12}b^3\pi\text{Si}\left(b^2\pi x^2\right.\\ &= -\frac{b}{12x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{12x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3}-\frac{1}{6}b^3\pi^2C(bx)^2+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x}-\end{aligned}$$

Mathematica [A] time = 0.0069185, size = 109, normalized size = 1.

$$\frac{\pi b^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{6}\pi^2b^3\text{FresnelC}(bx)^2-\frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right)-\frac{b\cos\left(\pi b^2x^2\right.}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]
```

```
[Out] -b/(12*x^2) - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}\,(bx)}{x^4}\cos\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\operatorname{fresnelc}(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\operatorname{fresnelc}(bx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**4,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\operatorname{fresnelc}(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^4,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^4, x)
```

3.193 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx$

Optimal. Leaf size=152

$$-\frac{1}{8}\pi^2 b^4\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{\pi b^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^2}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b\right.}{4x^4}$$

```
[Out] -b/(24*x^3) - (b*Cos[b^2*Pi*x^2])/(24*x^3) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelC[Sqrt[2]*b*x])/(24*Sqrt[2]) + (b^2*Pi*F
resnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) + (7*b^3*Pi*Sin[b^2*Pi*x^2])/(48*
x) - (b^4*Pi^2*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/8
```

Rubi [A] time = 0.115349, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5,x]
```

```
[Out] -b/(24*x^3) - (b*Cos[b^2*Pi*x^2])/(24*x^3) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelC[Sqrt[2]*b*x])/(24*Sqrt[2]) + (b^2*Pi*F
resnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) + (7*b^3*Pi*Sin[b^2*Pi*x^2])/(48*
x) - (b^4*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^5}\,dx &= -\frac{b}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}+\frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx-\frac{1}{4}\left(b^2\pi\right)\int\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\\
&= -\frac{b}{24x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{1}{16}\left(b^3\pi\right)\int\\
&= -\frac{b}{24x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}+\frac{7b^3\pi\sin\left(b^2\right.}{48x}\\
&= -\frac{b}{24x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}-\frac{7b^4\pi^2C\left(\sqrt{2}bx\right)}{24\sqrt{2}}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{8x^2}
\end{aligned}$$

Mathematica [A] time = 0.0300349, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5, x]
```

Maple [A] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelC}(bx)}{x^5}\cos\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnelc}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^5, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnelc}(bx)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^5, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**5,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \operatorname{fresnelc}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^5, x)
```

3.194 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^6}\,dx$

Optimal. Leaf size=147

$$-\frac{1}{15}\pi^2 b^4 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2},x\right)-\frac{1}{24}\pi^2 b^5 \text{CosIntegral}\left(\pi b^2 x^2\right)+\frac{\pi b^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3}$$

```
[Out] -b/(40*x^4) - (b*Cos[b^2*Pi*x^2])/(40*x^4) - (b^5*Pi^2*CosIntegral[b^2*Pi*x
^2])/24 - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(5*x^5) + (b^2*Pi*FresnelC[b*
x]*Sin[(b^2*Pi*x^2)/2])/(15*x^3) + (b^3*Pi*Sin[b^2*Pi*x^2])/(24*x^2) - (b^4
*Pi^2*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/15
```

Rubi [A] time = 0.192316, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6,x]
```

```
[Out] -b/(40*x^4) - (b*Cos[b^2*Pi*x^2])/(40*x^4) - (b^5*Pi^2*CosIntegral[b^2*Pi*x
^2])/24 - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(5*x^5) + (b^2*Pi*FresnelC[b*
x]*Sin[(b^2*Pi*x^2)/2])/(15*x^3) + (b^3*Pi*Sin[b^2*Pi*x^2])/(24*x^2) - (b^4
*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/15
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^6}\,dx &= -\frac{b}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5}+\frac{1}{10}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx-\frac{1}{5}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}+\frac{1}{20}b\,\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^3}\,dx,x,\right.\\
&= -\frac{b}{40x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}-\frac{1}{60}\left(b^3\pi\right)\text{Subst}\\
&= -\frac{b}{40x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}+\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{24x^2}\\
&= -\frac{b}{40x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{1}{24}b^5\pi^2\text{Ci}\left(b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}\end{aligned}$$

Mathematica [A] time = 0.0296004, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6, x]
```

Maple [A] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelC}\left(bx\right)}{x^{6}}\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{fresnelc}\left(bx\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^6, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{fresnelc}\left(bx\right)}{x^{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^6, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^{2}x^{2}}{2}\right)C\left(bx\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**6,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\left(bx\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^6, x)
```

3.195 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx$

Optimal. Leaf size=240

$$\frac{1}{48}\pi^3 b^6\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{\pi b^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{24x^4}+\frac{\pi^2b^4\text{FresnelC}(bx)\cos\left(\right.}{48x^2}$$

```
[Out] -b/(60*x^5) + (b^5*Pi^2)/(96*x) - (b*Cos[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi
^2*Cos[b^2*Pi*x^2])/(1440*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(6*x^6)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(48*x^2) + (7*b^6*Pi^3*Fresn
elS[Sqrt[2]*b*x])/(144*Sqrt[2]) + (Sqrt[2]*b^6*Pi^3*FresnelS[Sqrt[2]*b*x])/
45 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(24*x^4) + (13*b^3*Pi*Sin[b
^2*Pi*x^2])/(720*x^3) + (b^6*Pi^3*Unintegrable[(FresnelC[b*x]*Sin[(b^2*Pi*x
^2)/2])/x, x])/48
```

Rubi [A] time = 0.20372, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7,x]
```

```
[Out] -b/(60*x^5) + (b^5*Pi^2)/(96*x) - (b*Cos[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi
^2*Cos[b^2*Pi*x^2])/(1440*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(6*x^6)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(48*x^2) + (7*b^6*Pi^3*Fresn
elS[Sqrt[2]*b*x])/(144*Sqrt[2]) + (Sqrt[2]*b^6*Pi^3*FresnelS[Sqrt[2]*b*x])/
45 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(24*x^4) + (13*b^3*Pi*Sin[b
^2*Pi*x^2])/(720*x^3) + (b^6*Pi^3*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2
)/2])/x, x])/48
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^7}\,dx &= -\frac{b}{60x^5}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6}+\frac{1}{12}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx-\frac{1}{6}\left(b^2\pi\right)\int\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} \\
&= -\frac{b}{60x^5}-\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4}-\frac{1}{48}\left(b^3\pi\right)\int \\
&= -\frac{b}{60x^5}+\frac{b^5\pi^2}{96x}-\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^2}+ \\
&= -\frac{b}{60x^5}+\frac{b^5\pi^2}{96x}-\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}+\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6}+ \\
&= -\frac{b}{60x^5}+\frac{b^5\pi^2}{96x}-\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}+\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6}+
\end{aligned}$$

Mathematica [A] time = 0.0290908, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7, x]
```

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}\,(bx)}{x^7}\cos\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^7, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\,(bx)}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^7, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**7,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^7, x)
```

3.196 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^8}\,dx$

Optimal. Leaf size=224

$$-\frac{\pi^3 b^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{105x}+\frac{\pi b^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}+\frac{\pi^2b^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{105x^3}-\frac{\text{FresnelC}(bx}{7}$$

```
[Out] -b/(84*x^6) + (b^5*Pi^2)/(420*x^2) - (b*Cos[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*x^7)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(105*x^3) + (b^7*Pi^4*Fresne
lC[b*x]^2)/210 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6
*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) + (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Rubi [A] time = 0.351595, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6457, 6465, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$-\frac{\pi^3 b^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{105x}+\frac{\pi b^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}+\frac{\pi^2b^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{105x^3}-\frac{\text{FresnelC}(bx}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^8,x]
```


```
[Out] -b/(84*x^6) + (b^5*Pi^2)/(420*x^2) - (b*Cos[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*x^7)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(105*x^3) + (b^7*Pi^4*Fresne
lC[b*x]^2)/210 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6
*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) + (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```


Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8}\,dx = -\frac{b}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{1}{14}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx - \frac{1}{7}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}$$

$$= -\frac{b}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} + \frac{1}{28}b\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^4}\,dx\right.$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{105x^3} + -$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{105x^3} + -$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} + \frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi^2\cos}{}$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} + \frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi^2\cos}{}$$

Mathematica [A] time = 0.0135264, size = 224, normalized size = 1.

$$-\frac{\pi^3 b^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{105x}+\frac{\pi b^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}+\frac{\pi^2 b^4 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{105x^3}-\frac{\text{FresnelC}(b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^8,x]
```

```
[Out] -b/(84*x^6) + (b^5*Pi^2)/(420*x^2) - (b*Cos[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*x^7)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(105*x^3) + (b^7*Pi^4*Fresne
lC[b*x]^2)/210 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6
*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) + (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}\,(bx)}{x^8}\cos\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\,(bx)}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^8, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}\left(bx\right)}{x^8} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^8, x)
```

3.197 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^9}\,dx$

Optimal. Leaf size=267

$$\frac{1}{384}\pi^4 b^8 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)-\frac{\pi^3 b^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{384x^2}+\frac{\pi b^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi\right.}{48x^6}$$

```
[Out] -b/(112*x^7) + (b^5*Pi^2)/(1152*x^3) - (b*Cos[b^2*Pi*x^2])/(112*x^7) + (187
*b^5*Pi^2*Cos[b^2*Pi*x^2])/(40320*x^3) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(192*x^4) + (853*b
^8*Pi^4*FresnelC[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelC[b*x]*Sin[
(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3
84*x^2) + (19*b^3*Pi*Sin[b^2*Pi*x^2])/(3360*x^5) - (853*b^7*Pi^3*Sin[b^2*Pi
*x^2])/(80640*x) + (b^8*Pi^4*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x
])/x, x])/384
```

Rubi [A] time = 0.303801, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9,x]
```

```
[Out] -b/(112*x^7) + (b^5*Pi^2)/(1152*x^3) - (b*Cos[b^2*Pi*x^2])/(112*x^7) + (187
*b^5*Pi^2*Cos[b^2*Pi*x^2])/(40320*x^3) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(192*x^4) + (853*b
^8*Pi^4*FresnelC[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelC[b*x]*Sin[
(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3
84*x^2) + (19*b^3*Pi*Sin[b^2*Pi*x^2])/(3360*x^5) - (853*b^7*Pi^3*Sin[b^2*Pi
*x^2])/(80640*x) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/x, x])/384
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^9}dx = -\frac{b}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{1}{16}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}dx - \frac{1}{8}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right.}{x^7}$$

$$= -\frac{b}{112x^7} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{1}{96}\left(b^3\pi\right)\int$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{192x^4} +$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^4\pi}{}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^4\pi}{}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^4\pi}{}$$

Mathematica [A] time = 0.0311179, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9, x]
```

Maple [A] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}\left(bx\right)}{x^9}\cos\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right)}{x^9}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^9, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\left(bx\right)}{x^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^9,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^9, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**9,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\left(bx\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^9,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^9, x)
```

3.198 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\textbf{FresnelC}(bx)}{x^{10}}\,dx$

Optimal. Leaf size=262

$$\frac{1}{945}\pi^4 b^8 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right) + \frac{5\pi^4 b^9 \text{CosIntegral}\left(\pi b^2 x^2\right)}{2016} - \frac{\pi^3 b^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi\right.}{945x^3}$$

```
[Out] -b/(144*x^8) + (b^5*Pi^2)/(2520*x^4) - (b*Cos[b^2*Pi*x^2])/(144*x^8) + (67*
b^5*Pi^2*Cos[b^2*Pi*x^2])/(30240*x^4) + (5*b^9*Pi^4*CosIntegral[b^2*Pi*x^2]
)/2016 - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(315*x^5) + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)
/2])/(63*x^7) - (b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) + (1
1*b^3*Pi*Sin[b^2*Pi*x^2])/(3024*x^6) - (5*b^7*Pi^3*Sin[b^2*Pi*x^2])/(2016*x
^2) + (b^8*Pi^4*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/9
45
```

Rubi [A] time = 0.495136, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10,x]
```

```
[Out] -b/(144*x^8) + (b^5*Pi^2)/(2520*x^4) - (b*Cos[b^2*Pi*x^2])/(144*x^8) + (67*
b^5*Pi^2*Cos[b^2*Pi*x^2])/(30240*x^4) + (5*b^9*Pi^4*CosIntegral[b^2*Pi*x^2]
)/2016 - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(315*x^5) + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)
/2])/(63*x^7) - (b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) + (1
1*b^3*Pi*Sin[b^2*Pi*x^2])/(3024*x^6) - (5*b^7*Pi^3*Sin[b^2*Pi*x^2])/(2016*x
^2) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/945
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^{10}}dx = -\frac{b}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{1}{18}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^9}dx - \frac{1}{9}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} d$$

$$= -\frac{b}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} + \frac{1}{36}b\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^5}dx, x\right.$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{315x^5} + \frac{b^2}{}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{315x^5} + \frac{b^2}{}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{b^4\pi^2\cos}{}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{b^4\pi^2\cos}{}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} + \frac{5b^9\pi^4\mathrm{Ci}\left(b^2\pi x^2\right)}{2016} - \frac{\cos\left(\frac{1}{2}b^2\pi\right.}{9x}$$

Mathematica [A] time = 0.0328045, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^{10}}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10, x]
```

Maple [A] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelC}(bx)}{x^{10}}\cos\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnelc}(bx)}{x^{10}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^10, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnelc}(bx)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^10, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**10,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnelc}(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^10,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^10, x)
```

3.199 $\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\sin\left(\frac{1}{2}\pi b^2x^2\right)\text{FresnelC}(bx)^n, x\right)$$

```
[Out] Unintegrable[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]
```

Rubi [A] time = 0.0129616, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] Defer[Int][FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]
```

Rubi steps

$$\int C(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int C(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Mathematica [A] time = 0.0690703, size = 0, normalized size = 0.

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int (\text{FresnelC}(bx))^n \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx)^n \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnelc}\,(bx)^n \sin\left(\frac{1}{2}\,\pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{\pi b^2 x^2}{2}\right) C^n\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**n*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)**n, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}\,(bx)^n \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

3.200 $\int x^8 \textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=308

$$\frac{105ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi^4b^7}-\frac{105ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi^4b^7}+\frac{105}{}$$

```
[Out] (-35*x^4)/(8*b^5*Pi^3) + x^8/(16*b*Pi) - (40*Cos[b^2*Pi*x^2])/(b^9*Pi^5) +
(5*x^4*Cos[b^2*Pi*x^2])/(2*b^5*Pi^3) + (35*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC
[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (105
*FresnelC[b*x]*FresnelS[b*x])/(2*b^9*Pi^4) + (((105*I)/8)*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (((105*I)/8)*x^2*Hy
pergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (105*x*Fr
esnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelC[b*x]*Sin[(b^2
*Pi*x^2)/2])/(b^4*Pi^2) - (55*x^2*Sin[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Sin[
b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.43063, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6463, 6455, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}-\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105\text{FresnelC}(bx)S(bx)}{2\pi^4b^9}+\frac{7x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi\right.}{\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```


```
[Out] (-35*x^4)/(8*b^5*Pi^3) + x^8/(16*b*Pi) - (40*Cos[b^2*Pi*x^2])/(b^9*Pi^5) +
(5*x^4*Cos[b^2*Pi*x^2])/(2*b^5*Pi^3) + (35*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC
[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (105
*FresnelC[b*x]*FresnelS[b*x])/(2*b^9*Pi^4) + (((105*I)/8)*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (((105*I)/8)*x^2*Hy
pergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (105*x*Fr
esnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelC[b*x]*Sin[(b^2
*Pi*x^2)/2])/(b^4*Pi^2) - (55*x^2*Sin[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Sin[
b^2*Pi*x^2])/(4*b^3*Pi^2)
```


Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^8 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{7\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx}{b^2\pi} + \frac{\int x^7 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi}$$
$$= -\frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{7x^5 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{35\int x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} - 7\int$$
$$= \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{7x^5 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{105\int x^2}{}$$
$$= \frac{x^8}{16b\pi} + \frac{7x^4 \cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} - \frac{105xC(}{}$$
$$= \frac{x^8}{16b\pi} + \frac{5x^4 \cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{105C(b}{2b^9}$$
$$= -\frac{35x^4}{8b^5\pi^3} + \frac{x^8}{16b\pi} - \frac{119 \cos\left(b^2\pi x^2\right)}{4b^9\pi^5} + \frac{5x^4 \cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^7 c}{}$$
$$= -\frac{35x^4}{8b^5\pi^3} + \frac{x^8}{16b\pi} - \frac{40 \cos\left(b^2\pi x^2\right)}{b^9\pi^5} + \frac{5x^4 \cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^7 c}{}$$

Mathematica [F] time = 0.0454054, size = 0, normalized size = 0.

$$\int x^8 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int x^8 \text{FresnelC}\,(bx) \sin\left(\frac{b^2\pi\, x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \text{fresnelc}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^8*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^8\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^8*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^8*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.201 $\int x^7 \textbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=218

$$\frac{6x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{48 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^8} - \frac{x^6 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{24x^2 \text{FresnelC}(bx)}{\pi^3 b^6}$$

```
[Out] (-4*x^3)/(b^5*Pi^3) + x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
 + (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^6*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*
b^8*Pi^4) - (48*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*Fres
nelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7
*Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.27088, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6463, 6455, 6453, 3351, 3392, 30, 3386, 3385}

$$\frac{6x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{48 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^8} - \frac{x^6 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{24x^2 \text{FresnelC}(bx)}{\pi^3 b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*x^3)/(b^5*Pi^3) + x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
 + (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^6*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*
b^8*Pi^4) - (48*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*Fres
nelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7
*Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\int x^7 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi} + \frac{\int x^6\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$

$$= -\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{6x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{24\int x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^4\pi^2} -$$

$$= \frac{x^7}{14b\pi} + \frac{3x^3\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{6x^4C}{}$$

$$= \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} - \frac{48C}{}$$

$$= -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{b^2\pi}$$

$$= -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{b^2\pi}$$

Mathematica [A] time = 0.173901, size = 163, normalized size = 0.75

$$\frac{-224\text{FresnelC}(bx)\left(\pi b^2x^2\left(\pi^2b^4x^4-24\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)-6\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)+16\pi^3b^7x^7-896\pi b^3x^3+56}{224\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-896*b^3*Pi*x^3 + 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717*
Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 224*FresnelC[b*x]*(b^2*Pi*x^2*(-24 + b^4*Pi
^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) -
2058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi^4)
```

Maple [A] time = 0.083, size = 322, normalized size = 1.5

$$\frac{1}{b}\left(\frac{\text{FresnelC}\,(bx)}{b^7}\left(-\frac{b^6x^6}{\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right)+6\,\frac{1}{\pi}\left(\frac{x^4b^4\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi}-4\,\frac{1}{\pi}\left(-\frac{b^2x^2\cos\left(1/2\,b^2\pi\,x^2\right)}{\pi}+2\,\frac{\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelC(b*x)/b^7*(-1/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/Pi*(1/Pi*b^4*x^4*si
n(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^
2*Pi*x^2))))-1/b^7*(-1/2/Pi^3*(1/7*Pi^2*b^7*x^7-8*x^3*b^3)+3/Pi^4*(-1/2*Pi*
b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*F
resnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^5
*x^5*sin(b^2*Pi*x^2)-5/2*Pi*(-1/2/Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2/Pi*(1/2/Pi
*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))-12/Pi*b*x*sin(b
^2*Pi*x^2)+6/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^7\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.202 $\int x^6 \textbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=185

$$\frac{5x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{15x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} - \frac{15\text{FresnelC}(bx)^2}{2\pi^3b^7}$$

```
[Out] (-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^
3) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^5*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (15*FresnelC[b*x]^2)/(2*b^7*Pi^3) + (5*x
^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*Sin[b^2*Pi*x^2])/(2*
b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.254223, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6463, 6455, 6441, 30, 3380, 2634, 3379, 3296, 2637, 3309}

$$\frac{5x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{15x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} - \frac{15\text{FresnelC}(bx)^2}{2\pi^3b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```


```
[Out] (-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^
3) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^5*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (15*FresnelC[b*x]^2)/(2*b^7*Pi^3) + (5*x
^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*Sin[b^2*Pi*x^2])/(2*
b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```


Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int x^6 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi} + \frac{\int x^5\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$

$$= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{5x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{15\int x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^4\pi^2} - 5\int$$

$$= \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{5x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - 15\int\cos($$

$$= \frac{x^6}{12b\pi} + \frac{5x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + 5x^3C(bx$$

$$= -\frac{15x^2}{4b^5\pi^3} + \frac{x^6}{12b\pi} + \frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} -$$

$$= -\frac{15x^2}{4b^5\pi^3} + \frac{x^6}{12b\pi} + \frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} -$$

Mathematica [A] time = 0.0092096, size = 185, normalized size = 1.

$$\frac{5x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{15x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} - \frac{15\text{FresnelC}(bx)^2}{2\pi^3b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^
3) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^5*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (15*FresnelC[b*x]^2)/(2*b^7*Pi^3) + (5*x
^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*Sin[b^2*Pi*x^2])/(2*
b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int x^6\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^6\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^6*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 123.898, size = 264, normalized size = 1.43

$$\begin{cases} \frac{x^6\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{12\pi b} + \frac{x^6\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{12\pi b} - \frac{x^5\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi b^2} + \frac{x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} + \frac{5x^3\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi^2b^4} - \frac{11x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^3b^5} - \frac{2x^2\cos^2\left(\right.}{\pi^3 b} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) + x**6*cos(pi*b**2*x**2/2)
**2/(12*pi*b) - x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + x**4*sin
(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*sin(pi*b**2*x*
*2/2)*fresnelc(b*x)/(pi**2*b**4) - 11*x**2*sin(pi*b**2*x**2/2)**2/(2*pi**3*
b**5) - 2*x**2*cos(pi*b**2*x**2/2)**2/(pi**3*b**5) + 15*x*cos(pi*b**2*x**2/
2)*fresnelc(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/
(pi**4*b**7) - 15*fresnelc(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.203 $\int x^5 \textbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=167

$$\frac{4x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{8\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{43\text{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3b^6}$$

```
[Out] (-4*x)/(b^5*Pi^3) + x^5/(10*b*Pi) + (11*x*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) + (
8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*
FresnelC[b*x])/(b^2*Pi) - (43*FresnelC[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) +
 (4*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^3*Sin[b^2*Pi*x^2
])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.174375, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6463, 6455, 6461, 3358, 3352, 3385, 3392, 30, 3386}

$$\frac{4x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{8\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{43\text{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*x)/(b^5*Pi^3) + x^5/(10*b*Pi) + (11*x*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) + (
8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*
FresnelC[b*x])/(b^2*Pi) - (43*FresnelC[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) +
 (4*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^3*Sin[b^2*Pi*x^2
])/(4*b^3*Pi^2)
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^5 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi} + \frac{\int x^4\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi} \\
&= -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{4x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{8\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^4\pi^2} - \frac{2}{} \\
&= \frac{x^5}{10b\pi} + \frac{x\cos\left(b^2\pi x^2\right)}{b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{4x^2C(bx)\mathrm{s}}{b} \\
&= \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} - \frac{C\left(\sqrt{2}bx\right.}{\sqrt{2}b^6\pi^3} \\
&= -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} - \\
&= -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} -
\end{aligned}$$

Mathematica [A] time = 0.152275, size = 126, normalized size = 0.75

$$\frac{-80\text{FresnelC}(bx)\left(\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)-4\pi b^2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)+2bx\left(4\pi^2b^4x^4+10\pi b^2x^2\sin\left(\pi b^2x^2\right)+55\cos\left(\right.\right.}{80\pi^3b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-215*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 80*FresnelC[b*x]*((-8 + b^4*Pi^2*x^4)
*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(-160 + 4*
b^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*b^6
*Pi^3)
```

Maple [A] time = 0.079, size = 212, normalized size = 1.3

$$\frac{1}{b}\left(\frac{\text{FresnelC}(bx)}{b^5}\left(-\frac{x^4b^4}{\pi}\cos\left(\frac{b^2\pi x^2}{2}\right)+4\frac{1}{\pi}\left(\frac{b^2x^2\sin\left(1/2\,b^2\pi x^2\right)}{\pi}+2\frac{\cos\left(1/2\,b^2\pi x^2\right)}{\pi^2}\right)\right)-\frac{1}{b^5}\left(-\frac{1}{2\pi^3}\left(\frac{\pi^2b^5x^5}{5}-8\,bx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelC(b*x)/b^5*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))-1/b^5*(-1/2/Pi^3*(1/5*Pi^2*b
^5*x^5-8*b*x)+2/Pi^2*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b
*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*c
os(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x
*2^(1/2)))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^5\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**5*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \mathrm{fresnelc}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.204 $\int x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=196

$$-\frac{3ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi^2b^3}+\frac{3ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi^2b^3}-\frac{3\text{Fresn}}{}$$

```
[Out] x^4/(8*b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) - (x^3*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(b^2*Pi) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) - (((3*I)/
8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^3*Pi^2) +
 (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^3
*Pi^2) + (3*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^2*Sin[b^2*
Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.15681, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6463, 6455, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}+\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3\text{FresnelC}(bx)S(bx)}{2\pi^2b^5}+\frac{3x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^4/(8*b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) - (x^3*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(b^2*Pi) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) - (((3*I)/
8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^3*Pi^2) +
 (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^3
*Pi^2) + (3*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^2*Sin[b^2*
Pi*x^2])/(4*b^3*Pi^2)
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^4 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi} + \frac{\int x^3\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$

$$= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{3xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^4\pi^2} - \frac{3\int$$

$$= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} - \frac{3C(bx)S(bx)}{2b^5\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{3ix^2\,{}_2F_2\left(1\right.}{}$$

$$= \frac{x^4}{8b\pi} + \frac{3\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} - \frac{3C(bx)S(bx)}{2b^5\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}\right.}{8b^3\pi^2}$$

$$= \frac{x^4}{8b\pi} + \frac{\cos\left(b^2\pi x^2\right)}{b^5\pi^3} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} - \frac{3C(bx)S(bx)}{2b^5\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\right.}{8b^3\pi^2}$$

Mathematica [F] time = 0.0371357, size = 0, normalized size = 0.

$$\int x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int x^4 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**4*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.205 $\int x^3\textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=109

$$\frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5S\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2b^4}+\frac{x\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^3}{6\pi b}$$

```
[Out] x^3/(6*b*Pi) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (5*Fresne
lS[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2) + (2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2
])/(b^4*Pi^2) + (x*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.0893288, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6463, 6453, 3351, 3392, 30, 3386}

$$\frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5S\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2b^4}+\frac{x\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^3}{6\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^3/(6*b*Pi) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (5*Fresne
lS[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2) + (2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2
])/(b^4*Pi^2) + (x*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\int x^3 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi}+\frac{\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$

$$= -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx}{b^3\pi^2}+\frac{\int x^2\,dx}{2b\pi}+\frac{\int x^2}{}$$

$$= \frac{x^3}{6b\pi}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}-\frac{S\left(\sqrt{2}bx\right)}{\sqrt{2}b^4\pi^2}+\frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}-$$

$$= \frac{x^3}{6b\pi}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}-\frac{5S\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2}+\frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Mathematica [A] time = 0.0841006, size = 90, normalized size = 0.83

$$\frac{-24\text{FresnelC}(bx)\left(\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)-2\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)+4\pi b^3x^3+6bx\sin\left(\pi b^2x^2\right)-15\sqrt{2}S\left(\sqrt{2}bx\right)}{24\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (4*b^3*Pi*x^3 - 15*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 24*FresnelC[b*x]*(b^2*Pi
*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]) + 6*b*x*Sin[b^2*Pi*x^2])/
(24*b^4*Pi^2)
```

Maple [A] time = 0.08, size = 120, normalized size = 1.1

$$\frac{1}{b}\left(\frac{\text{FresnelC}\left(bx\right)}{b^3}\left(-\frac{b^2x^2}{\pi}\cos\left(\frac{b^2\pi\,x^2}{2}\right)+2\,\frac{\sin\left(1/2\,b^2\pi\,x^2\right)}{\pi^2}\right)-\frac{1}{b^3}\left(\frac{\sqrt{2}\text{FresnelS}\left(bx\sqrt{2}\right)}{2\,\pi^2}-\frac{x^3b^3}{6\,\pi}-\frac{1}{2\,\pi}\left(\frac{bx\sin\left(b^2\pi\,x\right.}{2\,\pi}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelC(b*x)/b^3*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))-1/6/Pi*b^3*x^3-1/2/Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sin\left(\frac{\pi b^2x^2}{2}\right)C\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.206 $\int x^2 \textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=74

$$-\frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\text{FresnelC}(bx)^2}{2\pi b^3}+\frac{\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^2}{4\pi b}$$

```
[Out] x^2/(4*b*Pi) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[b*
x]^2/(2*b^3*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Rubi [A] time = 0.0570508, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6463, 6441, 30, 3380, 2634}

$$-\frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\text{FresnelC}(bx)^2}{2\pi b^3}+\frac{\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```


```
[Out] x^2/(4*b*Pi) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[b*
x]^2/(2*b^3*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```


Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi}+\frac{\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}\\
&= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{\operatorname{Subst}(\int x\,dx, x, C(bx))}{b^3\pi}+\frac{\operatorname{Subst}\left(\int \cos^2\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{2b\pi}\\
&= \frac{x^2}{4b\pi}-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{C(bx)^2}{2b^3\pi}+\frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}
\end{aligned}
$$

Mathematica [A] time = 0.007526, size = 74, normalized size = 1.

$$
-\frac{x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\text{FresnelC}(bx)^2}{2\pi b^3}+\frac{\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^2}{4\pi b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^2/(4*b*Pi) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[b*
x]^2/(2*b^3*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$
\int x^2\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int x^2\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$
\text{integral}\left(x^2\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right), x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 4.37083, size = 114, normalized size = 1.54

$$\begin{cases} \frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} + \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} - \frac{x\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi b^2} + \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} + \frac{C^2(bx)}{2\pi b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) + x**2*cos(pi*b**2*x**2/2)*
*2/(4*pi*b) - x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + sin(pi*b**2*x
**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + fresnelc(b*x)**2/(2*pi*b**3), N
e(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.207 $\int x\textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=60

$$-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}+\frac{x}{2\pi b}$$

```
[Out] x/(2*b*Pi) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[Sqrt[2
]*b*x]/(2*Sqrt[2]*b^2*Pi)
```

Rubi [A] time = 0.0317032, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6461, 3358, 3352}

$$-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}+\frac{x}{2\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x/(2*b*Pi) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[Sqrt[2
]*b*x]/(2*Sqrt[2]*b^2*Pi)
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{\int\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}\\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{\int\left(\frac{1}{2}+\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{b\pi}\\ &= \frac{x}{2b\pi} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{\int\cos\left(b^2\pi x^2\right)dx}{2b\pi}\\ &= \frac{x}{2b\pi} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{C\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi}\end{aligned}$$

Mathematica [A] time = 0.0263026, size = 48, normalized size = 0.8

$$\frac{-4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)+\sqrt{2}\text{FresnelC}\left(\sqrt{2}bx\right)+2bx}{4\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (2*b*x - 4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x] + Sqrt[2]*FresnelC[Sqrt[2]*b*x
])/(4*b^2*Pi)
```

Maple [A] time = 0.061, size = 52, normalized size = 0.9

$$\frac{1}{b}\left(-\frac{\text{FresnelC}(bx)}{b\pi}\cos\left(\frac{b^2\pi x^2}{2}\right)+\frac{1}{b\pi}\left(\frac{bx}{2}+\frac{\sqrt{2}\text{FresnelC}\left(bx\sqrt{2}\right)}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (-FresnelC(b*x)/b/Pi*cos(1/2*b^2*Pi*x^2)+1/b/Pi*(1/2*b*x+1/4*2^(1/2)*Fresne
lC(b*x*2^(1/2))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sin\left(\frac{\pi b^2x^2}{2}\right)C\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.208 $\int \textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=80

$$\frac{1}{8}ibx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)-\frac{1}{8}ibx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)+$$

```
[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-I/2)*b^2*Pi*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2,
 2}, (I/2)*b^2*Pi*x^2]
```

Rubi [A] time = 0.0157251, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6447}

$$\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)S(bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-I/2)*b^2*Pi*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2,
 2}, (I/2)*b^2*Pi*x^2]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)]])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{C(bx)S(bx)}{2b}+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.0153241, size = 0, normalized size = 0.

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \mathrm{FresnelC}(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{fresnelc}(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{fresnelc}(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{fresnelc}(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.209 $\int \frac{\textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)$$

[Out] Unintegrable[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

Rubi [A] time = 0.0172122, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]

[Out] Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [A] time = 0.0320878, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]

[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

Maple [A] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}\left(bx\right)}{x}\sin\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

3.210 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)+\frac{1}{2}\pi b\text{FresnelC}(bx)^2$$

```
[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIn
tegral[b^2*Pi*x^2])/4
```

Rubi [A] time = 0.0427923, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6465, 6441, 30, 3375}

$$-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)+\frac{1}{2}\pi b\text{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

```
[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIn
tegral[b^2*Pi*x^2])/4
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} + \frac{1}{2}b\int \frac{\sin\left(b^2\pi x^2\right)}{x}\,dx + \left(b^2\pi\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$
$$= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} + \frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right) + (b\pi)\,\mathrm{Subst}\left(\int x\,dx, x, C(bx)\right)$$
$$= \frac{1}{2}b\pi C(bx)^2 - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} + \frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)$$

Mathematica [A] time = 0.0050088, size = 48, normalized size = 1.

$$-\frac{\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x} + \frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right) + \frac{1}{2}\pi b\mathrm{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

```
[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIn
tegral[b^2*Pi*x^2])/4
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelC}(bx)}{x^2}\sin\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

3.211 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$

Optimal. Leaf size=93

$$\frac{1}{2}\pi b^2\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{\pi b^2\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}}-\frac{b\sin}{}$$

```
[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2
)/2])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) + (b^2*Pi*Unintegrable[(Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/x, x])/2
```

Rubi [A] time = 0.0563796, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2
)/2])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) + (b^2*Pi*Defer[Int][(Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/x, x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}+\frac{1}{4}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx\\ &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx+\frac{1}{2}\left(b^3\pi\right)\int\\ &= \frac{b^2\pi C\left(\sqrt{2}bx\right)}{2\sqrt{2}}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,d\end{aligned}$$

Mathematica [A] time = 0.0330993, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]
```

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelC}\,(bx)}{x^3}\sin\left(\frac{b^2\pi\, x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)C\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

3.212 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$

Optimal. Leaf size=88

$$\frac{1}{3}\pi b^2\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)+\frac{1}{12}\pi b^3\text{CosIntegral}\left(\pi b^2x^2\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b\,\text{s}}{}$$

```
[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(
3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) + (b^2*Pi*Unintegrable[(Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/x^2, x])/3
```

Rubi [A] time = 0.0893682, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(
3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) + (b^2*Pi*Defer[Int][(Cos[(b^2*Pi*x^2
)/2]*FresnelC[b*x])/x^2, x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}+\frac{1}{6}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx\\
&= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}+\frac{1}{12}b\,\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^2}\,dx,x,x^2\right)+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(b}{x^2}\\
&= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx+\frac{1}{12}\left(b^3\pi\right)\text{Sub}\\
&= \frac{1}{12}b^3\pi\text{Ci}\left(b^2\pi x^2\right)-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}
\end{aligned}$$

Mathematica [A] time = 0.0318848, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4, x]
```

Maple [A] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^4}\sin\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)
```

<u>Giac</u> [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

3.213 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

Optimal. Leaf size=155

$$-\frac{1}{8}\pi^2 b^4 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{\pi b^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2\right.}{8x^2}$$

```
[Out] -(b^3*Pi)/(16*x) - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (b^2*Pi*Cos[(b^2*Pi*
x^2)/2]*FresnelC[b*x])/(8*x^2) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqr
t[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*x^4) - (b*Sin[b^2*Pi*x^2])/(
24*x^3) - (b^4*Pi^2*Unintegrable[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])
/8
```

Rubi [A] time = 0.119052, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

```
[Out] -(b^3*Pi)/(16*x) - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (b^2*Pi*Cos[(b^2*Pi*
x^2)/2]*FresnelC[b*x])/(8*x^2) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqr
t[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*x^4) - (b*Sin[b^2*Pi*x^2])/(
24*x^3) - (b^4*Pi^2*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}+\frac{1}{8}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx+\frac{1}{4}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^3}\,dx\\
&= -\frac{b^3\pi}{16x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^2}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}+\frac{1}{16}\left(b^3\pi\right)\int \frac{\mathrm{c}}{}\\
&= -\frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^2}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{b\sin\left(b^2\pi\right.}{24x^3}\\
&= -\frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^2}-\frac{7b^4\pi^2S\left(\sqrt{2}bx\right)}{24\sqrt{2}}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\right.}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.0324145, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

```
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]
```

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelC}(bx)}{x^5}\sin\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^5}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^5}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

3.214 $\int \frac{\textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

Optimal. Leaf size=163

$$\frac{\pi^2 b^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{\pi b^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3}-\frac{1}{30}\pi^3b^5\text{FresnelC}(b$$

```
[Out] -(b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(15*x) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^
2*Pi*x^2])/120
```

Rubi [A] time = 0.218457, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6465, 6457, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi^2 b^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{\pi b^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3}-\frac{1}{30}\pi^3b^5\text{FresnelC}(b$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```


```
[Out] -(b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(15*x) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^
2*Pi*x^2])/120
```


Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{10}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx + \frac{1}{5}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^4}\,dx$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{20}b\,\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^3}\,dx\right.$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x} - $$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + $$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{1}{30}b^5\pi^3C(bx)^2 - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi \right.}{5x^5}$$

Mathematica [A] time = 0.0105432, size = 163, normalized size = 1.

$$\frac{\pi^2b^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5} - \frac{\pi b^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3} - \frac{1}{30}\pi^3b^5\text{Fresnel}$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -(b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(15*x) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^
2*Pi*x^2])/120
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelC}\,(bx)}{x^6}\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)C\,(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

3.215 $\int \frac{\textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

Optimal. Leaf size=230

$$-\frac{1}{48}\pi^3 b^6 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{\pi^2b^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{48x^2}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 \right.}{6x^6}$$

```
[Out] -(b^3*Pi)/(144*x^3) - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (b^2*Pi*Cos[(
b^2*Pi*x^2)/2]*FresnelC[b*x])/(24*x^4) - (7*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])
/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])/45 - (FresnelC[b*
x]*Sin[(b^2*Pi*x^2)/2])/(6*x^6) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(48*x^2) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])
/(1440*x) - (b^6*Pi^3*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x
])/48
```

Rubi [A] time = 0.199967, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

```
[Out] -(b^3*Pi)/(144*x^3) - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (b^2*Pi*Cos[(
b^2*Pi*x^2)/2]*FresnelC[b*x])/(24*x^4) - (7*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])
/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])/45 - (FresnelC[b*
x]*Sin[(b^2*Pi*x^2)/2])/(6*x^6) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(48*x^2) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])
/(1440*x) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])
/48
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}+\frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx+\frac{1}{6}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^5}\,dx\\
&= -\frac{b^3\pi}{144x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}-\frac{b\sin\left(b^2\pi x^2\right)}{60x^5}+\frac{1}{48}\left(b^3\pi\right)\int \frac{\text{co}}{}\\
&= -\frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}+\frac{b^4\pi^2C(bx}{}\\
&= -\frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}+\frac{b^4\pi^2C(bx}{}\\
&= -\frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4}-\frac{7b^6\pi^3C\left(\sqrt{2}bx\right)}{144\sqrt{2}}-\frac{1}{45}\sqrt{2}b^6\pi^3C\left(\right.
\end{aligned}$$

Mathematica [A] time = 0.0313279, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

```
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^7}\sin\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

3.216 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

Optimal. Leaf size=201

$$-\frac{1}{105}\pi^3 b^6 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2},x\right)-\frac{1}{84}\pi^3 b^7 \text{CosIntegral}\left(\pi b^2 x^2\right)+\frac{\pi^2 b^4 \text{FresnelC}(bx)\sin\left(\right.}{105x^3}$$

```
[Out] -(b^3*Pi)/(280*x^4) - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosInt
egral[b^2*Pi*x^2])/84 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(35*x^5)
 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*Pi^2*FresnelC[b*x]*Si
n[(b^2*Pi*x^2)/2])/(105*x^3) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin
[b^2*Pi*x^2])/(84*x^2) - (b^6*Pi^3*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/x^2, x])/105
```

Rubi [A] time = 0.314121, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] -(b^3*Pi)/(280*x^4) - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosInt
egral[b^2*Pi*x^2])/84 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(35*x^5)
 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*Pi^2*FresnelC[b*x]*Si
n[(b^2*Pi*x^2)/2])/(105*x^3) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin
[b^2*Pi*x^2])/(84*x^2) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC
[b*x])/x^2, x])/105
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}+\frac{1}{14}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}\,dx+\frac{1}{7}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^6}\,dx\\
&= -\frac{b^3\pi}{280x^4}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}+\frac{1}{28}b\,\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^4}\,d\right.\\
&= -\frac{b^3\pi}{280x^4}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}+\frac{b^4\pi^2 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3}-\\
&= -\frac{b^3\pi}{280x^4}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}+\frac{b^4\pi^2 C(b}{}\\
&= -\frac{b^3\pi}{280x^4}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}+\frac{b^4\pi^2 C(b}{}\\
&= -\frac{b^3\pi}{280x^4}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4}-\frac{1}{84}b^7\pi^3\text{Ci}\left(b^2\pi x^2\right)-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5}-\frac{C(bx)\sin}{7x}
\end{aligned}$$

Mathematica [A] time = 0.0328026, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8, x]
```

Maple [A] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}\,(bx)}{x^8}\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

3.217 $\int \frac{\textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

Optimal. Leaf size=270

$$\frac{1}{384}\pi^4 b^8 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{\pi^2 b^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{192x^4}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x\right.}{8x^8}$$

```
[Out] -(b^3*Pi)/(480*x^5) + (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(336
0*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (b^2*Pi*Cos[(b^2*Pi*x^2
)/2]*FresnelC[b*x])/(48*x^6) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/(384*x^2) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^8) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(192*x^4) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*
Pi*x^2])/(40320*x^3) + (b^8*Pi^4*Unintegrable[(FresnelC[b*x]*Sin[(b^2*Pi*x^
2)/2])/x, x])/384
```

Rubi [A] time = 0.306279, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

```
[Out] -(b^3*Pi)/(480*x^5) + (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(336
0*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (b^2*Pi*Cos[(b^2*Pi*x^2
)/2]*FresnelC[b*x])/(48*x^6) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/(384*x^2) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^8) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(192*x^4) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*
Pi*x^2])/(40320*x^3) + (b^8*Pi^4*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)
/2])/x, x])/384
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{1}{16}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}\,dx + \frac{1}{8}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^7}\,dx$$

$$= -\frac{b^3\pi}{480x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^6} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{1}{96}\left(b^3\pi\right)\int$$

$$= -\frac{b^3\pi}{480x^5} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^6} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{b^4\pi^2 C}{}$$

$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^6} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C}{384x^2}$$

$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^6}$$

$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^6}$$

Mathematica [A] time = 0.0332526, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

```
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^9}\sin\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

3.218 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

Optimal. Leaf size=278

$$-\frac{\pi^4 b^8 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{945x}+\frac{\pi^2 b^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{315x^5}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9}+\frac{\pi^3 b^6 \text{FresnelC}}{}$$

```
[Out] -(b^3*Pi)/(756*x^6) + (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(
3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (b^2*Pi*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(63*x^7) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x
])/(945*x^3) + (b^9*Pi^5*FresnelC[b*x]^2)/1890 - (FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(9*x^9) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5)
- (b^8*Pi^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x) - (b*Sin[b^2*Pi*x^2]
)/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinI
ntegral[b^2*Pi*x^2])/30240
```

Rubi [A] time = 0.520338, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6465, 6457, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$-\frac{\pi^4 b^8 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{945x}+\frac{\pi^2 b^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{315x^5}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9}+\frac{\pi^3 b^6 \text{FresnelC}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```


```
[Out] -(b^3*Pi)/(756*x^6) + (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(
3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (b^2*Pi*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(63*x^7) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x
])/(945*x^3) + (b^9*Pi^5*FresnelC[b*x]^2)/1890 - (FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(9*x^9) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5)
- (b^8*Pi^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x) - (b*Sin[b^2*Pi*x^2]
)/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinI
ntegral[b^2*Pi*x^2])/30240
```


Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{1}{18}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^9}\,dx + \frac{1}{9}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8}\,dx$$

$$= -\frac{b^3\pi}{756x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{1}{36}b\,\mathrm{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^5}\,d\right.$$

$$= -\frac{b^3\pi}{756x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{b^4\pi^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5} -$$

$$= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3}$$

$$= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3}$$

$$= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7}$$

$$= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7}$$

Mathematica [A] time = 0.0174078, size = 278, normalized size = 1.

$$-\frac{\pi^4b^8\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{945x} + \frac{\pi^2b^4\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{315x^5} - \frac{\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9} + \frac{\pi^3b^6\mathrm{FresnelC}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

```
[Out] -(b^3*Pi)/(756*x^6) + (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(
3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (b^2*Pi*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(63*x^7) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x
])/(945*x^3) + (b^9*Pi^5*FresnelC[b*x]^2)/1890 - (FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(9*x^9) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5)
- (b^8*Pi^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x) - (b*Sin[b^2*Pi*x^2]
)/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinI
ntegral[b^2*Pi*x^2])/30240
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{\mathrm{FresnelC}\,(bx)}{x^{10}}\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^{10}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```