

Computer algebra independent integration tests

8-Special-functions/8.2-Fresnel-integral-functions

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3.136	$\int x \text{FresnelC}(a + bx) dx$	622
3.137	$\int \text{FresnelC}(a + bx) dx$	627
3.138	$\int \frac{\text{FresnelC}(a+bx)}{x} dx$	630
3.139	$\int \frac{\text{FresnelC}(a+bx)}{x^2} dx$	633
3.140	$\int x^7 \text{FresnelC}(bx)^2 dx$	636
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3.142	$\int x^5 \text{FresnelC}(bx)^2 dx$	646
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3.152	$\int \frac{\text{FresnelC}(bx)^2}{x^5} dx$	686
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3.159	$\int (c + dx) \text{FresnelC}(a + bx)^2 dx$	718
3.160	$\int \text{FresnelC}(a + bx)^2 dx$	723
3.161	$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} dx$	727
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3.163	$\int x^2 \text{FresnelC}(d(a + b \log(cx^n))) dx$	733
3.164	$\int x \text{FresnelC}(d(a + b \log(cx^n))) dx$	738
3.165	$\int \text{FresnelC}(d(a + b \log(cx^n))) dx$	743
3.166	$\int \frac{\text{FresnelC}(d(a+b \log(cx^n)))}{x} dx$	748
3.167	$\int \frac{\text{FresnelC}(d(a+b \log(cx^n)))}{x^2} dx$	751
3.168	$\int \frac{\text{FresnelC}(d(a+b \log(cx^n)))}{x^3} dx$	756
3.169	$\int (ex)^m \text{FresnelC}(d(a + b \log(cx^n))) dx$	761
3.170	$\int e^{c+\frac{1}{2}ib^2\pi x^2} \text{FresnelC}(bx) dx$	766
3.171	$\int e^{c-\frac{1}{2}ib^2\pi x^2} \text{FresnelC}(bx) dx$	770
3.172	$\int \text{FresnelC}(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) dx$	774
3.173	$\int \cos\left(c + \frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$	778
3.174	$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2 dx$	782
3.175	$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$	785
3.176	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)} dx$	788
3.177	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2} dx$	791
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3.179	$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^n dx$	797
3.180	$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$	800
3.181	$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$	805
3.182	$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$	810
3.183	$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$	815
3.184	$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$	820
3.185	$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$	825
3.186	$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$	829
3.187	$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$	833
3.188	$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$	836
3.189	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x} dx$	839
3.190	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^2} dx$	842

3.191	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3} dx$	845
3.192	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4} dx$	848
3.193	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5} dx$	853
3.194	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6} dx$	857
3.195	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7} dx$	861
3.196	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8} dx$	865
3.197	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9} dx$	870
3.198	$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}} dx$	874
3.199	$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$	878
3.200	$\int x^8 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$	881
3.201	$\int x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$	886
3.202	$\int x^6 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$	891
3.203	$\int x^5 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$	896
3.204	$\int x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$	901
3.205	$\int x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$	906
3.206	$\int x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$	910
3.207	$\int x \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$	914
3.208	$\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$	918
3.209	$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx$	921
3.210	$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx$	924
3.211	$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx$	928
3.212	$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx$	931
3.213	$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx$	935
3.214	$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx$	939

3.215	$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx$	944
3.216	$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx$	948
3.217	$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx$	952
3.218	$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}} dx$	956

4 Listing of Grading functions 961

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [218]. This is test number [205].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (218)	% 0. (0)
Mathematica	% 87.16 (190)	% 12.84 (28)
Maple	% 70.64 (154)	% 29.36 (64)
Maxima	% 27.52 (60)	% 72.48 (158)
Fricas	% 27.52 (60)	% 72.48 (158)
Sympy	% 48.62 (106)	% 51.38 (112)
Giac	% 27.52 (60)	% 72.48 (158)

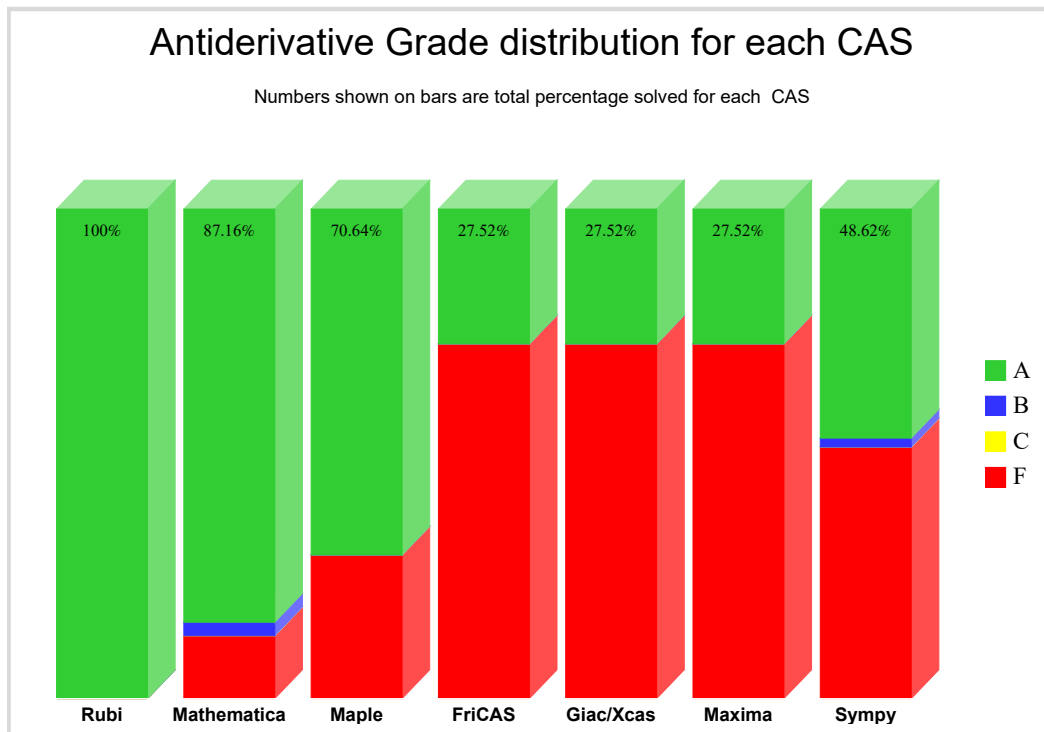
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

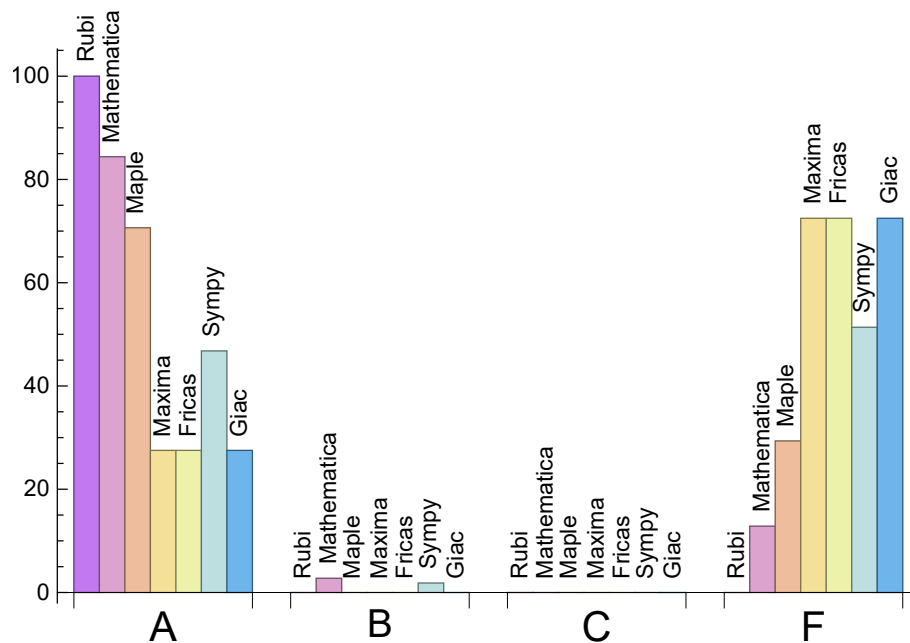
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	84.4	2.75	0.	12.84
Maple	70.64	0.	0.	29.36
Maxima	27.52	0.	0.	72.48
Fricas	27.52	0.	0.	72.48
Sympy	46.79	1.83	0.	51.38
Giac	27.52	0.	0.	72.48

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.15	96.61	0.72	71.5	1.
Mathematica	0.47	81.86	0.69	59.	0.76
Maple	0.08	62.54	0.61	28.	0.9
Maxima	0.	0.	0.	0.	0.
Fricas	0.	0.	0.	0.	0.
Sympy	3.68	39.53	0.59	9.	0.4
Giac	0.	0.	0.	0.	0.

1.4 list of integrals that has no closed form antiderivative

{23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {54, 55, 56, 163, 164, 165}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

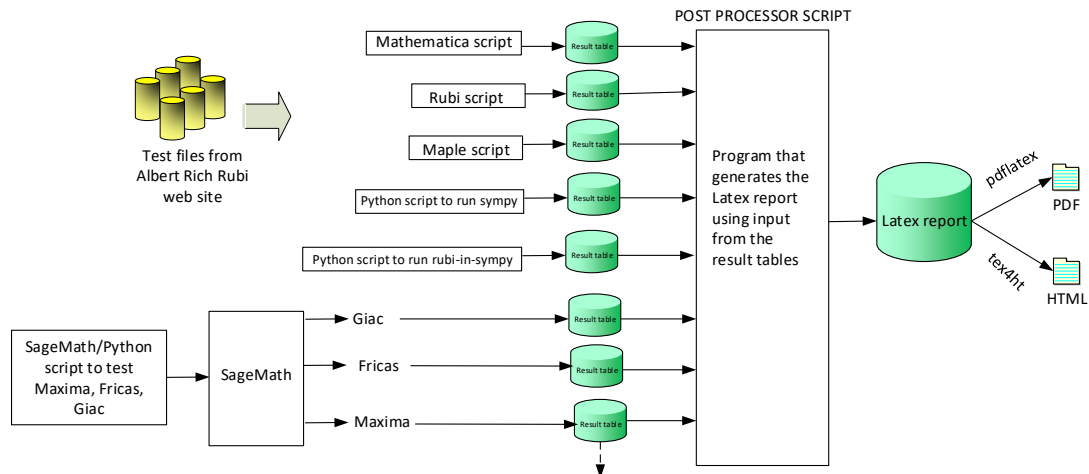
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 30, 31, 32, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 138, 139, 140, 141, 143, 144, 145, 147,

148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 167, 168, 169, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 201, 202, 203, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218 }

B grade: { 22, 28, 57, 131, 137, 166 }

C grade: { }

F grade: { 9, 33, 37, 49, 50, 61, 62, 63, 64, 73, 77, 91, 95, 99, 118, 142, 146, 158, 159, 170, 171, 172, 173, 182, 186, 200, 204, 208 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 36, 38, 39, 40, 41, 42, 44, 45, 46, 48, 51, 52, 53, 57, 65, 66, 67, 68, 69, 70, 72, 74, 76, 78, 79, 80, 81, 82, 84, 85, 86, 88, 89, 90, 92, 94, 96, 98, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 143, 145, 147, 148, 149, 150, 151, 153, 154, 155, 157, 160, 161, 162, 166, 174, 175, 176, 177, 178, 179, 181, 183, 185, 187, 188, 189, 190, 191, 193, 194, 195, 197, 198, 199, 201, 203, 205, 207, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 31, 33, 35, 37, 43, 47, 49, 50, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 71, 73, 75, 77, 83, 87, 91, 93, 95, 97, 99, 101, 105, 109, 140, 142, 144, 146, 152, 156, 158, 159, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 180, 182, 184, 186, 192, 196, 200, 202, 204, 206, 208, 210, 214, 218 }

2.1.4 Maxima

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.1.5 FriCAS

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 65, 66, 67, 68, 69, 70, 75, 79, 80, 81, 82, 84, 85, 86, 90, 93, 97, 100, 102, 103, 104, 106, 110, 111, 112, 113, 114, 115, 116, 120, 121, 122, 123, 124, 125, 126, 127, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 174, 175, 176, 177, 178, 179, 184, 188, 189, 190, 191, 193, 194, 195, 199, 202, 206, 209, 211, 212, 213, 215 }

B grade: { 8, 10, 117, 119 }

C grade: { }

F grade: { 9, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 71, 72, 73, 74, 76, 77, 78, 83, 87, 88, 89, 91, 92, 94, 95, 96, 98, 99, 101, 105, 107, 108, 109, 118, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 180, 181, 182, 183, 185, 186, 187, 192, 196, 197, 198, 200, 201, 203, 204, 205, 207, 208, 210, 214, 216, 217, 218 }

2.1.7 Giac

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69,

70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	88	123	0	0	184	0
normalized size	1	1.	0.71	0.99	0.	0.	1.48	0.
time (sec)	N/A	0.088	0.068	0.049	0.	0.	2.421	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	83	107	0	0	156	0
normalized size	1	1.	0.76	0.98	0.	0.	1.43	0.
time (sec)	N/A	0.111	0.052	0.049	0.	0.	1.766	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	79	96	0	0	53	0
normalized size	1	1.	0.8	0.97	0.	0.	0.54	0.
time (sec)	N/A	0.064	0.069	0.049	0.	0.	1.047	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	71	80	0	0	121	0
normalized size	1	1.	0.85	0.95	0.	0.	1.44	0.
time (sec)	N/A	0.08	0.041	0.052	0.	0.	1.629	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	70	0	0	112	0
normalized size	1	1.	1.	0.95	0.	0.	1.51	0.
time (sec)	N/A	0.047	0.016	0.046	0.	0.	0.934	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	0	0	80	0
normalized size	1	1.	1.	0.92	0.	0.	1.36	0.
time (sec)	N/A	0.053	0.012	0.049	0.	0.	0.828	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	0	0	53	0
normalized size	1	1.	1.	0.9	0.	0.	1.08	0.
time (sec)	N/A	0.024	0.011	0.049	0.	0.	0.524	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	0	0	48	0
normalized size	1	1.	1.	1.04	0.	0.	1.85	0.
time (sec)	N/A	0.005	0.002	0.046	0.	0.	0.833	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	0	29	0	0	0	0
normalized size	1	1.	0.	0.4	0.	0.	0.	0.
time (sec)	N/A	0.047	0.014	0.08	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	0	0	42	0
normalized size	1	1.	1.	1.04	0.	0.	1.56	0.
time (sec)	N/A	0.022	0.011	0.049	0.	0.	0.545	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	43	0	0	51	0
normalized size	1	1.	1.	0.98	0.	0.	1.16	0.
time (sec)	N/A	0.029	0.011	0.048	0.	0.	0.624	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	49	0	0	56	0
normalized size	1	1.	1.	0.94	0.	0.	1.08	0.
time (sec)	N/A	0.064	0.014	0.052	0.	0.	1.387	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	65	0	0	110	0
normalized size	1	1.	1.	0.94	0.	0.	1.59	0.
time (sec)	N/A	0.043	0.015	0.05	0.	0.	1.117	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	71	0	0	46	0
normalized size	1	1.	1.	0.92	0.	0.	0.6	0.
time (sec)	N/A	0.089	0.018	0.049	0.	0.	1.211	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	76	86	0	0	56	0
normalized size	1	1.	0.81	0.91	0.	0.	0.6	0.
time (sec)	N/A	0.059	0.057	0.05	0.	0.	1.529	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	85	93	0	0	68	0
normalized size	1	1.	0.83	0.91	0.	0.	0.67	0.
time (sec)	N/A	0.122	0.062	0.047	0.	0.	2.944	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	84	109	0	0	185	0
normalized size	1	1.	0.71	0.92	0.	0.	1.55	0.
time (sec)	N/A	0.081	0.055	0.047	0.	0.	3.71	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	96	115	0	0	48	0
normalized size	1	1.	0.76	0.91	0.	0.	0.38	0.
time (sec)	N/A	0.153	0.166	0.047	0.	0.	5.553	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	424	400	0	0	0	0
normalized size	1	1.	1.43	1.35	0.	0.	0.	0.
time (sec)	N/A	0.401	0.814	0.054	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	236	251	0	0	0	0
normalized size	1	1.	1.22	1.3	0.	0.	0.	0.
time (sec)	N/A	0.226	0.457	0.055	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	61	108	0	0	0	0
normalized size	1	1.	0.5	0.89	0.	0.	0.	0.
time (sec)	N/A	0.114	0.209	0.051	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	0	0	0	0
normalized size	1	1.	2.47	0.92	0.	0.	0.	0.
time (sec)	N/A	0.006	0.029	0.046	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.025	0.369	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	4.864	0.413	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	166	189	0	0	0	0
normalized size	1	1.	0.72	0.83	0.	0.	0.	0.
time (sec)	N/A	0.184	0.324	0.052	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	115	121	0	0	0	0
normalized size	1	1.	0.78	0.82	0.	0.	0.	0.
time (sec)	N/A	0.129	0.238	0.05	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	51	80	0	0	0	0
normalized size	1	1.	0.53	0.83	0.	0.	0.	0.
time (sec)	N/A	0.067	0.166	0.051	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	0	0	0	0
normalized size	1	1.	2.47	0.92	0.	0.	0.	0.
time (sec)	N/A	0.007	0.03	0.046	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.026	0.054	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	3.246	0.1	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	253	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.428	0.021	0.053	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	171	324	0	0	0	0
normalized size	1	1.	0.72	1.36	0.	0.	0.	0.
time (sec)	N/A	0.318	0.289	0.077	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	265	265	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	0.207	0.053	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	137	208	0	0	0	0
normalized size	1	1.	0.77	1.18	0.	0.	0.	0.
time (sec)	N/A	0.188	0.133	0.075	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	140	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	0.006	0.051	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	100	122	0	0	0	0
normalized size	1	1.	0.81	0.98	0.	0.	0.	0.
time (sec)	N/A	0.111	0.108	0.072	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.172	0.053	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	0	0	0	0
normalized size	1	1.	1.	0.89	0.	0.	0.	0.
time (sec)	N/A	0.038	0.009	0.056	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.017	0.05	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.027	0.052	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.02	0.053	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	119	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.026	0.055	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	0.006	0.062	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	170	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.027	0.056	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	165	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.222	0.02	0.05	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	258	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.245	0.027	0.057	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	242	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.393	0.011	0.062	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	285	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.358	0.027	0.056	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	497	497	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.41	0.664	0.244	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	0.585	0.056	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	67	60	0	0	0	0
normalized size	1	1.	0.96	0.86	0.	0.	0.	0.
time (sec)	N/A	0.168	0.01	0.056	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.036	0.36	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.082	0.355	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	319	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.571	6.888	0.701	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	319	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.435	6.65	0.535	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	316	0	0	0	0	0
normalized size	1	1.	1.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.326	6.659	0.541	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	164	80	0	0	0	0
normalized size	1	1.	2.52	1.23	0.	0.	0.	0.
time (sec)	N/A	0.041	0.093	0.216	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	195	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.511	3.968	0.517	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	200	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.523	3.989	0.522	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	244	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.702	5.504	0.306	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.03	0.114	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.032	0.069	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.049	0.085	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.044	0.073	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.015	0.004	0.046	0.	0.	1.404	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.011	0.003	0.045	0.	0.	0.434	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	8	0
normalized size	1	1.	1.	1.11	0.	0.	0.89	0.
time (sec)	N/A	0.015	0.01	0.118	0.	0.	0.235	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	0	10	0
normalized size	1	1.	1.	1.09	0.	0.	0.91	0.
time (sec)	N/A	0.015	0.003	0.043	0.	0.	0.78	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	14	0
normalized size	1	1.	1.	0.92	0.	0.	1.08	0.
time (sec)	N/A	0.015	0.004	0.044	0.	0.	1.934	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	0	31	0
normalized size	1	1.	1.	1.06	0.	0.	1.82	0.
time (sec)	N/A	0.018	0.006	0.046	0.	0.	4.624	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	232	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.384	0.013	0.077	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	153	318	0	0	0	0
normalized size	1	1.	0.71	1.47	0.	0.	0.	0.
time (sec)	N/A	0.265	0.276	0.076	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	248	248	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.255	0.469	0.089	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	120	202	0	0	0	0
normalized size	1	1.	0.76	1.28	0.	0.	0.	0.
time (sec)	N/A	0.156	0.173	0.078	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	151	0
normalized size	1	1.	1.	0.	0.	0.	1.26	0.
time (sec)	N/A	0.118	0.006	0.079	0.	0.	26.593	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	83	115	0	0	0	0
normalized size	1	1.	0.79	1.1	0.	0.	0.	0.
time (sec)	N/A	0.081	0.102	0.083	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.216	0.095	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	46	0	0	0	0
normalized size	1	1.	0.9	0.94	0.	0.	0.	0.
time (sec)	N/A	0.022	0.024	0.063	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.011	0.002	0.046	0.	0.	0.451	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.03	0.079	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.03	0.054	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.03	0.067	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.008	0.062	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	152	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	0.03	0.062	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	147	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	0.031	0.062	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	240	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	0.03	0.056	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	224	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.353	0.013	0.055	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	267	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.306	0.032	0.062	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	262	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.477	0.032	0.053	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.073	0.062	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	307	307	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.442	0.046	0.088	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	163	321	0	0	0	0
normalized size	1	1.	0.75	1.48	0.	0.	0.	0.
time (sec)	N/A	0.271	0.176	0.084	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	184	0	0	0	264	0
normalized size	1	1.	1.	0.	0.	0.	1.43	0.
time (sec)	N/A	0.253	0.009	0.088	0.	0.	124.888	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	126	212	0	0	0	0
normalized size	1	1.	0.76	1.28	0.	0.	0.	0.
time (sec)	N/A	0.169	0.173	0.079	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.038	0.084	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	90	119	0	0	0	0
normalized size	1	1.	0.83	1.1	0.	0.	0.	0.
time (sec)	N/A	0.09	0.081	0.075	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	114	0
normalized size	1	1.	1.	0.	0.	0.	1.56	0.
time (sec)	N/A	0.055	0.005	0.086	0.	0.	4.716	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	52	0	0	0	0
normalized size	1	1.	0.81	0.88	0.	0.	0.	0.
time (sec)	N/A	0.032	0.028	0.066	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.014	0.058	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.03	0.086	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.005	0.063	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.031	0.063	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.034	0.061	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	155	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.032	0.063	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	0.01	0.062	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	230	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.205	0.033	0.065	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	201	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.319	0.033	0.063	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	270	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.31	0.032	0.064	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	278	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.516	0.016	0.064	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	89	123	0	0	184	0
normalized size	1	1.	0.72	0.99	0.	0.	1.48	0.
time (sec)	N/A	0.082	0.071	0.049	0.	0.	2.6	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	83	107	0	0	153	0
normalized size	1	1.	0.76	0.98	0.	0.	1.4	0.
time (sec)	N/A	0.114	0.056	0.049	0.	0.	2.361	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	80	97	0	0	49	0
normalized size	1	1.	0.81	0.98	0.	0.	0.49	0.
time (sec)	N/A	0.062	0.063	0.048	0.	0.	1.129	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	71	81	0	0	116	0
normalized size	1	1.	0.85	0.96	0.	0.	1.38	0.
time (sec)	N/A	0.077	0.043	0.046	0.	0.	1.223	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	70	0	0	112	0
normalized size	1	1.	1.	0.95	0.	0.	1.51	0.
time (sec)	N/A	0.043	0.016	0.048	0.	0.	1.238	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	0	0	80	0
normalized size	1	1.	1.	0.92	0.	0.	1.36	0.
time (sec)	N/A	0.054	0.014	0.05	0.	0.	1.03	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	0	0	49	0
normalized size	1	1.	1.	0.9	0.	0.	1.	0.
time (sec)	N/A	0.026	0.012	0.048	0.	0.	0.526	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	0	0	44	0
normalized size	1	1.	1.	1.04	0.	0.	1.63	0.
time (sec)	N/A	0.005	0.002	0.051	0.	0.	0.633	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	0	23	0	0	0	0
normalized size	1	1.	0.	0.33	0.	0.	0.	0.
time (sec)	N/A	0.044	0.014	0.073	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	0	0	53	0
normalized size	1	1.	1.	1.04	0.	0.	1.96	0.
time (sec)	N/A	0.021	0.011	0.049	0.	0.	1.134	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	43	0	0	51	0
normalized size	1	1.	1.	0.98	0.	0.	1.16	0.
time (sec)	N/A	0.029	0.011	0.048	0.	0.	0.633	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	49	0	0	42	0
normalized size	1	1.	1.	0.94	0.	0.	0.81	0.
time (sec)	N/A	0.064	0.015	0.047	0.	0.	0.735	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	64	0	0	110	0
normalized size	1	1.	1.	0.93	0.	0.	1.59	0.
time (sec)	N/A	0.041	0.016	0.051	0.	0.	1.125	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	71	0	0	65	0
normalized size	1	1.	1.	0.92	0.	0.	0.84	0.
time (sec)	N/A	0.089	0.019	0.052	0.	0.	1.992	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	74	87	0	0	56	0
normalized size	1	1.	0.79	0.93	0.	0.	0.6	0.
time (sec)	N/A	0.059	0.115	0.047	0.	0.	1.528	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	84	93	0	0	44	0
normalized size	1	1.	0.82	0.91	0.	0.	0.43	0.
time (sec)	N/A	0.118	0.127	0.05	0.	0.	2.123	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	85	108	0	0	185	0
normalized size	1	1.	0.71	0.91	0.	0.	1.55	0.
time (sec)	N/A	0.078	0.058	0.048	0.	0.	3.706	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	96	115	0	0	76	0
normalized size	1	1.	0.76	0.91	0.	0.	0.6	0.
time (sec)	N/A	0.144	0.081	0.047	0.	0.	6.865	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	424	397	0	0	0	0
normalized size	1	1.	1.42	1.33	0.	0.	0.	0.
time (sec)	N/A	0.373	0.817	0.056	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	237	249	0	0	0	0
normalized size	1	1.	1.22	1.28	0.	0.	0.	0.
time (sec)	N/A	0.205	0.466	0.054	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	74	107	0	0	0	0
normalized size	1	1.	0.61	0.88	0.	0.	0.	0.
time (sec)	N/A	0.113	0.236	0.053	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	0	0	0	0
normalized size	1	1.	2.43	0.92	0.	0.	0.	0.
time (sec)	N/A	0.006	0.029	0.049	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.026	0.365	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	2.287	0.373	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	166	187	0	0	0	0
normalized size	1	1.	0.73	0.82	0.	0.	0.	0.
time (sec)	N/A	0.187	0.294	0.05	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	116	122	0	0	0	0
normalized size	1	1.	0.78	0.82	0.	0.	0.	0.
time (sec)	N/A	0.123	0.297	0.052	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	59	79	0	0	0	0
normalized size	1	1.	0.62	0.83	0.	0.	0.	0.
time (sec)	N/A	0.071	0.148	0.052	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	0	0	0	0
normalized size	1	1.	2.43	0.92	0.	0.	0.	0.
time (sec)	N/A	0.007	0.029	0.049	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.018	0.056	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	1.464	0.085	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	253	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.422	0.017	0.052	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	170	324	0	0	0	0
normalized size	1	1.	0.71	1.36	0.	0.	0.	0.
time (sec)	N/A	0.298	0.24	0.077	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	265	265	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.288	0.198	0.052	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	137	209	0	0	0	0
normalized size	1	1.	0.77	1.18	0.	0.	0.	0.
time (sec)	N/A	0.188	0.134	0.076	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	140	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	0.009	0.055	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	100	122	0	0	0	0
normalized size	1	1.	0.81	0.98	0.	0.	0.	0.
time (sec)	N/A	0.105	0.088	0.075	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.174	0.051	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	0	0	0	0
normalized size	1	1.	1.	0.91	0.	0.	0.	0.
time (sec)	N/A	0.036	0.01	0.053	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.019	0.046	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.026	0.056	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.02	0.056	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	119	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.026	0.053	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.007	0.053	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	170	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.027	0.053	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	165	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.22	0.02	0.053	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	258	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.24	0.027	0.053	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	242	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.388	0.013	0.053	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	285	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.342	0.027	0.052	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	495	495	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.396	0.649	0.227	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.548	0.054	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	60	0	0	0	0
normalized size	1	1.	0.96	0.87	0.	0.	0.	0.
time (sec)	N/A	0.151	0.01	0.054	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.035	0.357	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.08	0.347	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	318	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.522	6.775	0.523	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	318	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.408	6.65	0.539	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	315	0	0	0	0	0
normalized size	1	1.	1.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.263	6.632	0.523	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	165	81	0	0	0	0
normalized size	1	1.	2.5	1.23	0.	0.	0.	0.
time (sec)	N/A	0.04	0.095	0.069	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	194	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.493	3.93	0.523	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	199	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.493	4.006	0.527	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	244	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.607	5.352	0.205	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.029	0.056	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.03	0.056	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.044	0.056	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.039	0.061	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.015	0.009	0.044	0.	0.	1.41	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.012	0.006	0.044	0.	0.	0.441	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	10	0
normalized size	1	1.	1.	1.11	0.	0.	1.11	0.
time (sec)	N/A	0.016	0.011	0.122	0.	0.	0.243	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	0	12	0
normalized size	1	1.	1.	1.09	0.	0.	1.09	0.
time (sec)	N/A	0.015	0.006	0.044	0.	0.	0.798	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	15	0
normalized size	1	1.	1.	0.92	0.	0.	1.15	0.
time (sec)	N/A	0.016	0.006	0.046	0.	0.	1.972	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	0	34	0
normalized size	1	1.	1.	1.06	0.	0.	2.	0.
time (sec)	N/A	0.019	0.01	0.047	0.	0.	4.591	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	231	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.383	0.012	0.083	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	154	317	0	0	0	0
normalized size	1	1.	0.72	1.47	0.	0.	0.	0.
time (sec)	N/A	0.259	0.234	0.085	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	247	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.253	0.381	0.082	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	120	202	0	0	0	0
normalized size	1	1.	0.76	1.29	0.	0.	0.	0.
time (sec)	N/A	0.151	0.132	0.079	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	151	0
normalized size	1	1.	1.	0.	0.	0.	1.26	0.
time (sec)	N/A	0.118	0.007	0.079	0.	0.	26.668	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	83	114	0	0	0	0
normalized size	1	1.	0.8	1.1	0.	0.	0.	0.
time (sec)	N/A	0.078	0.085	0.076	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.209	0.079	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	44	45	0	0	0	0
normalized size	1	1.	0.92	0.94	0.	0.	0.	0.
time (sec)	N/A	0.021	0.027	0.054	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0
normalized size	1	1.	1.	0.92	0.	0.	0.77	0.
time (sec)	N/A	0.011	0.002	0.046	0.	0.	0.459	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.029	0.082	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.031	0.059	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.03	0.058	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.007	0.057	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	152	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.03	0.065	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	147	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.03	0.057	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	240	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	0.029	0.059	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	224	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.352	0.014	0.057	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	267	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.304	0.031	0.061	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	262	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.495	0.033	0.06	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.069	0.059	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.431	0.045	0.078	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	163	322	0	0	0	0
normalized size	1	1.	0.75	1.48	0.	0.	0.	0.
time (sec)	N/A	0.271	0.174	0.083	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	185	0	0	0	264	0
normalized size	1	1.	1.	0.	0.	0.	1.43	0.
time (sec)	N/A	0.254	0.009	0.08	0.	0.	123.898	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	126	212	0	0	0	0
normalized size	1	1.	0.75	1.27	0.	0.	0.	0.
time (sec)	N/A	0.174	0.152	0.079	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.037	0.078	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	90	120	0	0	0	0
normalized size	1	1.	0.83	1.1	0.	0.	0.	0.
time (sec)	N/A	0.089	0.084	0.08	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	114	0
normalized size	1	1.	1.	0.	0.	0.	1.54	0.
time (sec)	N/A	0.057	0.008	0.086	0.	0.	4.371	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	48	52	0	0	0	0
normalized size	1	1.	0.8	0.87	0.	0.	0.	0.
time (sec)	N/A	0.032	0.026	0.061	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.015	0.059	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.032	0.084	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.005	0.059	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.033	0.059	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.032	0.06	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	155	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.032	0.059	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.218	0.011	0.058	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	230	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.2	0.031	0.063	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	201	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.314	0.033	0.069	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	270	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.306	0.033	0.063	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	278	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.52	0.017	0.056	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [140] had the largest ratio of [1.1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.	8	0.5
2	A	6	4	1.	8	0.5
3	A	5	4	1.	8	0.5
4	A	5	4	1.	8	0.5
5	A	4	4	1.	8	0.5
6	A	4	4	1.	8	0.5
7	A	3	3	1.	6	0.5
8	A	1	1	1.	4	0.25
9	A	3	3	1.	8	0.375
10	A	2	2	1.	8	0.25
11	A	3	3	1.	8	0.375
12	A	4	4	1.	8	0.5
13	A	4	4	1.	8	0.5
14	A	5	4	1.	8	0.5
15	A	5	4	1.	8	0.5
16	A	6	4	1.	8	0.5
17	A	6	4	1.	8	0.5
18	A	7	4	1.	8	0.5
19	A	14	10	1.	14	0.714
20	A	11	9	1.	14	0.643
21	A	8	7	1.	12	0.583
22	A	1	1	1.	6	0.167
23	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	0	0	0.	0	0.
25	A	14	10	1.	10	1.
26	A	11	9	1.	10	0.9
27	A	8	7	1.	8	0.875
28	A	1	1	1.	6	0.167
29	A	0	0	0.	0	0.
30	A	0	0	0.	0	0.
31	A	23	10	1.	10	1.
32	A	19	10	1.	10	1.
33	A	16	9	1.	10	0.9
34	A	12	9	1.	10	0.9
35	A	10	9	1.	10	0.9
36	A	8	6	1.	10	0.6
37	A	5	5	1.	8	0.625
38	A	4	4	1.	6	0.667
39	A	0	0	0.	0	0.
40	A	0	0	0.	0	0.
41	A	0	0	0.	0	0.
42	A	0	0	0.	0	0.
43	A	9	9	1.	10	0.9
44	A	0	0	0.	0	0.
45	A	0	0	0.	0	0.
46	A	0	0	0.	0	0.
47	A	20	10	1.	10	1.
48	A	0	0	0.	0	0.
49	A	18	13	1.	16	0.812
50	A	10	9	1.	14	0.643
51	A	4	3	1.	8	0.375
52	A	0	0	0.	0	0.
53	A	0	0	0.	0	0.
54	A	14	9	1.	17	0.529
55	A	14	9	1.	15	0.6

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	14	9	1.	13	0.692
57	A	3	1	1.	17	0.059
58	A	14	9	1.	17	0.529
59	A	14	9	1.	17	0.529
60	A	16	10	1.	19	0.526
61	A	4	4	1.	22	0.182
62	A	4	4	1.	22	0.182
63	A	4	4	1.	19	0.21
64	A	4	4	1.	19	0.21
65	A	2	2	1.	19	0.105
66	A	2	2	1.	17	0.118
67	A	2	2	1.	19	0.105
68	A	2	2	1.	19	0.105
69	A	2	2	1.	19	0.105
70	A	2	2	1.	19	0.105
71	A	22	9	1.	20	0.45
72	A	18	9	1.	20	0.45
73	A	15	8	1.	20	0.4
74	A	11	8	1.	20	0.4
75	A	9	8	1.	20	0.4
76	A	7	5	1.	20	0.25
77	A	4	4	1.	20	0.2
78	A	2	2	1.	18	0.111
79	A	2	2	1.	17	0.118
80	A	0	0	0.	0	0.
81	A	0	0	0.	0	0.
82	A	0	0	0.	0	0.
83	A	8	8	1.	20	0.4
84	A	0	0	0.	0	0.
85	A	0	0	0.	0	0.
86	A	0	0	0.	0	0.
87	A	19	9	1.	20	0.45

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	0	0	0.	0	0.
89	A	0	0	0.	0	0.
90	A	0	0	0.	0	0.
91	A	23	8	1.	20	0.4
92	A	18	8	1.	20	0.4
93	A	16	9	1.	20	0.45
94	A	13	9	1.	20	0.45
95	A	10	8	1.	20	0.4
96	A	7	6	1.	20	0.3
97	A	5	5	1.	20	0.25
98	A	4	3	1.	18	0.167
99	A	1	1	1.	17	0.059
100	A	0	0	0.	0	0.
101	A	4	4	1.	20	0.2
102	A	0	0	0.	0	0.
103	A	0	0	0.	0	0.
104	A	0	0	0.	0	0.
105	A	13	9	1.	20	0.45
106	A	0	0	0.	0	0.
107	A	0	0	0.	0	0.
108	A	0	0	0.	0	0.
109	A	26	9	1.	20	0.45
110	A	6	4	1.	8	0.5
111	A	6	4	1.	8	0.5
112	A	5	4	1.	8	0.5
113	A	5	4	1.	8	0.5
114	A	4	4	1.	8	0.5
115	A	4	4	1.	8	0.5
116	A	3	3	1.	6	0.5
117	A	1	1	1.	4	0.25
118	A	3	3	1.	8	0.375
119	A	2	2	1.	8	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	3	3	1.	8	0.375
121	A	4	4	1.	8	0.5
122	A	4	4	1.	8	0.5
123	A	5	4	1.	8	0.5
124	A	5	4	1.	8	0.5
125	A	6	4	1.	8	0.5
126	A	6	4	1.	8	0.5
127	A	7	4	1.	8	0.5
128	A	14	10	1.	14	0.714
129	A	11	9	1.	14	0.643
130	A	8	7	1.	12	0.583
131	A	1	1	1.	6	0.167
132	A	0	0	0.	0	0.
133	A	0	0	0.	0	0.
134	A	14	10	1.	10	1.
135	A	11	9	1.	10	0.9
136	A	8	7	1.	8	0.875
137	A	1	1	1.	6	0.167
138	A	0	0	0.	0	0.
139	A	0	0	0.	0	0.
140	A	23	11	1.	10	1.1
141	A	19	10	1.	10	1.
142	A	16	10	1.	10	1.
143	A	12	9	1.	10	0.9
144	A	10	10	1.	10	1.
145	A	8	6	1.	10	0.6
146	A	5	5	1.	8	0.625
147	A	4	4	1.	6	0.667
148	A	0	0	0.	0	0.
149	A	0	0	0.	0	0.
150	A	0	0	0.	0	0.
151	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	9	9	1.	10	0.9
153	A	0	0	0.	0	0.
154	A	0	0	0.	0	0.
155	A	0	0	0.	0	0.
156	A	20	10	1.	10	1.
157	A	0	0	0.	0	0.
158	A	18	13	1.	16	0.812
159	A	10	9	1.	14	0.643
160	A	4	3	1.	8	0.375
161	A	0	0	0.	0	0.
162	A	0	0	0.	0	0.
163	A	14	9	1.	17	0.529
164	A	14	9	1.	15	0.6
165	A	14	9	1.	13	0.692
166	A	3	1	1.	17	0.059
167	A	14	9	1.	17	0.529
168	A	14	9	1.	17	0.529
169	A	16	10	1.	19	0.526
170	A	4	4	1.	22	0.182
171	A	4	4	1.	22	0.182
172	A	4	4	1.	19	0.21
173	A	4	4	1.	19	0.21
174	A	2	2	1.	19	0.105
175	A	2	2	1.	17	0.118
176	A	2	2	1.	19	0.105
177	A	2	2	1.	19	0.105
178	A	2	2	1.	19	0.105
179	A	2	2	1.	19	0.105
180	A	22	10	1.	20	0.5
181	A	18	9	1.	20	0.45
182	A	15	9	1.	20	0.45
183	A	11	8	1.	20	0.4

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	9	9	1.	20	0.45
185	A	7	5	1.	20	0.25
186	A	4	4	1.	20	0.2
187	A	2	2	1.	18	0.111
188	A	2	2	1.	17	0.118
189	A	0	0	0.	0	0.
190	A	0	0	0.	0	0.
191	A	0	0	0.	0	0.
192	A	8	8	1.	20	0.4
193	A	0	0	0.	0	0.
194	A	0	0	0.	0	0.
195	A	0	0	0.	0	0.
196	A	19	9	1.	20	0.45
197	A	0	0	0.	0	0.
198	A	0	0	0.	0	0.
199	A	0	0	0.	0	0.
200	A	23	9	1.	20	0.45
201	A	18	8	1.	20	0.4
202	A	16	10	1.	20	0.5
203	A	13	9	1.	20	0.45
204	A	10	9	1.	20	0.45
205	A	7	6	1.	20	0.3
206	A	5	5	1.	20	0.25
207	A	4	3	1.	18	0.167
208	A	1	1	1.	17	0.059
209	A	0	0	0.	0	0.
210	A	4	4	1.	20	0.2
211	A	0	0	0.	0	0.
212	A	0	0	0.	0	0.
213	A	0	0	0.	0	0.
214	A	13	9	1.	20	0.45
215	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	0	0	0.	0	0.
217	A	0	0	0.	0	0.
218	A	26	9	1.	20	0.45

Chapter 3

Listing of integrals

3.1 $\int x^7 S(bx) dx$

Optimal. Leaf size=124

$$-\frac{105S(bx)}{8\pi^4b^8} - \frac{7x^5 \sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3} + \frac{105x \sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7} + \frac{x^7 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b} - \frac{35x^3 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5} + \frac{1}{8}x^8S(bx)$$

[Out] $(-35*x^3*\text{Cos}[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) + (x^7*\text{Cos}[(b^2*Pi*x^2)/2])/(8*b*Pi) - (105*\text{FresnelS}[b*x])/(8*b^8*Pi^4) + (x^8*\text{FresnelS}[b*x])/8 + (105*x*\text{Sin}[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*\text{Sin}[(b^2*Pi*x^2)/2])/(8*b^3*Pi^2)$

Rubi [A] time = 0.0883233, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3385, 3386, 3351}

$$-\frac{105S(bx)}{8\pi^4b^8} - \frac{7x^5 \sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3} + \frac{105x \sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7} + \frac{x^7 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b} - \frac{35x^3 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5} + \frac{1}{8}x^8S(bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^7*\text{FresnelS}[b*x], x]$

[Out] $(-35*x^3*\text{Cos}[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) + (x^7*\text{Cos}[(b^2*Pi*x^2)/2])/(8*b*Pi) - (105*\text{FresnelS}[b*x])/(8*b^8*Pi^4) + (x^8*\text{FresnelS}[b*x])/8 + (105*x*\text{Sin}[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*\text{Sin}[(b^2*Pi*x^2)/2])/(8*b^3*Pi^2)$

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*Sin[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_.)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_.)]*((e_.)*(x_))^(m_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int x^7 S(bx) dx &= \frac{1}{8} x^8 S(bx) - \frac{1}{8} b \int x^8 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
&= \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b\pi} + \frac{1}{8} x^8 S(bx) - \frac{7 \int x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{8b\pi} \\
&= \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b\pi} + \frac{1}{8} x^8 S(bx) - \frac{7x^5 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^3 \pi^2} + \frac{35 \int x^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{8b^3 \pi^2} \\
&= -\frac{35x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^5 \pi^3} + \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b\pi} + \frac{1}{8} x^8 S(bx) - \frac{7x^5 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^3 \pi^2} + \frac{105 \int x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{8b^5 \pi^3} \\
&= -\frac{35x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^5 \pi^3} + \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b\pi} + \frac{1}{8} x^8 S(bx) + \frac{105x \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^7 \pi^4} - \frac{7x^5 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^3 \pi^2} - \frac{105 \int \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{8b^5 \pi^3} \\
&= -\frac{35x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^5 \pi^3} + \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b\pi} - \frac{105S(bx)}{8b^8 \pi^4} + \frac{1}{8} x^8 S(bx) + \frac{105x \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^7 \pi^4} - \frac{7x^5 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^3 \pi^2}
\end{aligned}$$

Mathematica [A] time = 0.0682553, size = 88, normalized size = 0.71

$$\frac{(\pi^4 b^8 x^8 - 105) S(bx) - 7bx (\pi^2 b^4 x^4 - 15) \sin\left(\frac{1}{2}\pi b^2 x^2\right) + \pi b^3 x^3 (\pi^2 b^4 x^4 - 35) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^4 b^8}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*FresnelS[b*x], x]

[Out] (b^3*Pi*x^3*(-35 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*FresnelS[b*x] - 7*b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*Pi^4)

Maple [A] time = 0.049, size = 123, normalized size = 1.

$$\frac{1}{b^8} \left(\frac{\text{FresnelS}(bx) b^8 x^8}{8} + \frac{b^7 x^7}{8\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) - \frac{7}{8\pi} \left(\frac{b^5 x^5}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) - 5 \frac{1}{\pi} \left(-\frac{x^3 b^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 3 \frac{1}{\pi} \left(\frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*FresnelS(b*x), x)

[Out] 1/b^8*(1/8*FresnelS(b*x)*b^8*x^8+1/8/Pi*b^7*x^7*cos(1/2*b^2*Pi*x^2)-7/8/Pi*(1/Pi*b^5*x^5*sin(1/2*b^2*Pi*x^2)-5/Pi*(-1/Pi*b^3*x^3*cos(1/2*b^2*Pi*x^2)+3/Pi*(1/Pi*b*x*sin(1/2*b^2*Pi*x^2)-1/Pi*FresnelS(b*x))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnels(b*x), x, algorithm="maxima")

[Out] integrate(x^7*fresnels(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^7 \text{fresnels}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnels(b*x),x, algorithm="fricas")

[Out] integral(x^7*fresnels(b*x), x)

Sympy [A] time = 2.42098, size = 184, normalized size = 1.48

$$\frac{231x^8 S(bx) \Gamma\left(\frac{3}{4}\right)}{512 \Gamma\left(\frac{15}{4}\right)} + \frac{231x^7 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{512 \pi b \Gamma\left(\frac{15}{4}\right)} - \frac{1617x^5 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{512 \pi^2 b^3 \Gamma\left(\frac{15}{4}\right)} - \frac{8085x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{512 \pi^3 b^5 \Gamma\left(\frac{15}{4}\right)} + \frac{24255x \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{512 \pi^4 b^7 \Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*fresnels(b*x),x)

[Out] 231*x**8*fresnels(b*x)*gamma(3/4)/(512*gamma(15/4)) + 231*x**7*cos(pi*b**2*x**2/2)*gamma(3/4)/(512*pi*b*gamma(15/4)) - 1617*x**5*sin(pi*b**2*x**2/2)*gamma(3/4)/(512*pi**2*b**3*gamma(15/4)) - 8085*x**3*cos(pi*b**2*x**2/2)*gamma(3/4)/(512*pi**3*b**5*gamma(15/4)) + 24255*x*sin(pi*b**2*x**2/2)*gamma(3/4)/(512*pi**4*b**7*gamma(15/4)) - 24255*fresnels(b*x)*gamma(3/4)/(512*pi**4*b**8*gamma(15/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnels(b*x),x, algorithm="giac")

[Out] integrate(x^7*fresnels(b*x), x)

3.2 $\int x^6 S(bx) dx$

Optimal. Leaf size=109

$$-\frac{6x^4 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^2 b^3} + \frac{48 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^4 b^7} + \frac{x^6 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi b} - \frac{24x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^3 b^5} + \frac{1}{7}x^7 S(bx)$$

[Out] $(-24*x^2*\text{Cos}[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^6*\text{Cos}[(b^2*Pi*x^2)/2])/(7*b*Pi) + (x^7*\text{FresnelS}[b*x])/7 + (48*\text{Sin}[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x^4*\text{Sin}[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2)$

Rubi [A] time = 0.110693, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3379, 3296, 2637}

$$-\frac{6x^4 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^2 b^3} + \frac{48 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^4 b^7} + \frac{x^6 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi b} - \frac{24x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^3 b^5} + \frac{1}{7}x^7 S(bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^6*\text{FresnelS}[b*x], x]$

[Out] $(-24*x^2*\text{Cos}[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^6*\text{Cos}[(b^2*Pi*x^2)/2])/(7*b*Pi) + (x^7*\text{FresnelS}[b*x])/7 + (48*\text{Sin}[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x^4*\text{Sin}[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2)$

Rule 6426

$\text{Int}[\text{FresnelS}[(b_.)*(x_.)]*((d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[\text{((d*x)}^{(m+1)}*\text{FresnelS}[b*x])/(\text{d}*(m+1)), x] - \text{Dist}[b/(\text{d}*(m+1)), \text{Int}[\text{((d*x)}^{(m+1)}*\text{Sin}[(\text{Pi}*b^2*x^2)/2]), x], x] /; \text{FreeQ}\{b, d, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 3379

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*\text{Sin}[(c_.) + (d_.)*(x_.)^{(n_.)])^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Sin}[c + d*x])^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{EqQ}[m, n-1] \ || \ (\text{IntegerQ}[p] \ \&\& \ \text{GtQ}[\text{Simplify}[(m+1)/n], 0]))$

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^6 S(bx) dx &= \frac{1}{7} x^7 S(bx) - \frac{1}{7} b \int x^7 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
&= \frac{1}{7} x^7 S(bx) - \frac{1}{14} b \operatorname{Subst}\left(\int x^3 \sin\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right) \\
&= \frac{x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} + \frac{1}{7} x^7 S(bx) - \frac{3 \operatorname{Subst}\left(\int x^2 \cos\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{7b\pi} \\
&= \frac{x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} + \frac{1}{7} x^7 S(bx) - \frac{6x^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^3 \pi^2} + \frac{12 \operatorname{Subst}\left(\int x \sin\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{7b^3 \pi^2} \\
&= -\frac{24x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^5 \pi^3} + \frac{x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} + \frac{1}{7} x^7 S(bx) - \frac{6x^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^3 \pi^2} + \frac{24 \operatorname{Subst}\left(\int \cos\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{7b^5 \pi^3} \\
&= -\frac{24x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^5 \pi^3} + \frac{x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} + \frac{1}{7} x^7 S(bx) + \frac{48 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^7 \pi^4} - \frac{6x^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^3 \pi^2}
\end{aligned}$$

Mathematica [A] time = 0.0516547, size = 83, normalized size = 0.76

$$-\frac{6(\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{7\pi^4 b^7} + \frac{x^2(\pi^2 b^4 x^4 - 24) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{7\pi^3 b^5} + \frac{1}{7} x^7 S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelS[b*x], x]
```

```
[Out] (x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^7*FresnelS
[b*x])/7 - (6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4)
```

Maple [A] time = 0.049, size = 107, normalized size = 1.

$$\frac{1}{b^7} \left(\frac{b^7 x^7 \text{FresnelS}(bx)}{7} + \frac{b^6 x^6}{7\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) - \frac{6}{7\pi} \left(\frac{x^4 b^4}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) - 4 \frac{1}{\pi} \left(-\frac{b^2 x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 2 \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*FresnelS(b*x),x)

[Out] 1/b^7*(1/7*b^7*x^7*FresnelS(b*x)+1/7/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)-6/7/Pi*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*fresnels(b*x),x, algorithm="maxima")

[Out] integrate(x^6*fresnels(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^6 \text{fresnels}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*fresnels(b*x),x, algorithm="fricas")

[Out] integral(x^6*fresnels(b*x), x)

Sympy [A] time = 1.76553, size = 156, normalized size = 1.43

$$\frac{3x^7 S(bx) \Gamma\left(\frac{3}{4}\right)}{28\Gamma\left(\frac{7}{4}\right)} + \frac{3x^6 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{28\pi b \Gamma\left(\frac{7}{4}\right)} - \frac{9x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{14\pi^2 b^3 \Gamma\left(\frac{7}{4}\right)} - \frac{18x^2 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{7\pi^3 b^5 \Gamma\left(\frac{7}{4}\right)} + \frac{36 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{7\pi^4 b^7 \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnels(b*x),x)
```

```
[Out] 3*x**7*fresnels(b*x)*gamma(3/4)/(28*gamma(7/4)) + 3*x**6*cos(pi*b**2*x**2/2)
*gamma(3/4)/(28*pi*b*gamma(7/4)) - 9*x**4*sin(pi*b**2*x**2/2)*gamma(3/4)/(
14*pi**2*b**3*gamma(7/4)) - 18*x**2*cos(pi*b**2*x**2/2)*gamma(3/4)/(7*pi**3
*b**5*gamma(7/4)) + 36*sin(pi*b**2*x**2/2)*gamma(3/4)/(7*pi**4*b**7*gamma(7
/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnels(b*x), x)
```

3.3 $\int x^5 S(bx) dx$

Optimal. Leaf size=99

$$\frac{5\text{FresnelC}(bx)}{2\pi^3b^6} - \frac{5x^3 \sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2b^3} + \frac{x^5 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b} - \frac{5x \cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5} + \frac{1}{6}x^6S(bx)$$

[Out] $(-5*x*\text{Cos}[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) + (x^5*\text{Cos}[(b^2*Pi*x^2)/2])/(6*b*Pi) + (5*\text{FresnelC}[b*x])/(2*b^6*Pi^3) + (x^6*\text{FresnelS}[b*x])/6 - (5*x^3*\text{Sin}[(b^2*Pi*x^2)/2])/(6*b^3*Pi^2)$

Rubi [A] time = 0.0643839, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3385, 3386, 3352}

$$\frac{5\text{FresnelC}(bx)}{2\pi^3b^6} - \frac{5x^3 \sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2b^3} + \frac{x^5 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b} - \frac{5x \cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5} + \frac{1}{6}x^6S(bx)$$

Antiderivative was successfully verified.

[In] Int[x^5*FresnelS[b*x],x]

[Out] $(-5*x*\text{Cos}[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) + (x^5*\text{Cos}[(b^2*Pi*x^2)/2])/(6*b*Pi) + (5*\text{FresnelC}[b*x])/(2*b^6*Pi^3) + (x^6*\text{FresnelS}[b*x])/6 - (5*x^3*\text{Sin}[(b^2*Pi*x^2)/2])/(6*b^3*Pi^2)$

Rule 6426

Int[FresnelS[(b_.)*(x_.)]*((d_.)*(x_.))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*Sin[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3385

Int[((e_.)*(x_.))^(m_.)*Sin[(c_.) + (d_.)*(x_.)^(n_.)], x_Symbol] :> -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] := Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int x^5 S(bx) dx &= \frac{1}{6} x^6 S(bx) - \frac{1}{6} b \int x^6 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
&= \frac{x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{6b\pi} + \frac{1}{6} x^6 S(bx) - \frac{5 \int x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{6b\pi} \\
&= \frac{x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{6b\pi} + \frac{1}{6} x^6 S(bx) - \frac{5x^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{6b^3 \pi^2} + \frac{5 \int x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{2b^3 \pi^2} \\
&= -\frac{5x \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{2b^5 \pi^3} + \frac{x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{6b\pi} + \frac{1}{6} x^6 S(bx) - \frac{5x^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{6b^3 \pi^2} + \frac{5 \int \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{2b^5 \pi^3} \\
&= -\frac{5x \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{2b^5 \pi^3} + \frac{x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{6b\pi} + \frac{5C(bx)}{2b^6 \pi^3} + \frac{1}{6} x^6 S(bx) - \frac{5x^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{6b^3 \pi^2}
\end{aligned}$$

Mathematica [A] time = 0.0685961, size = 79, normalized size = 0.8

$$\frac{\pi^3 b^6 x^6 S(bx) - 5\pi b^3 x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right) + bx (\pi^2 b^4 x^4 - 15) \cos\left(\frac{1}{2} \pi b^2 x^2\right) + 15 \text{FresnelC}(bx)}{6\pi^3 b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelS[b*x], x]
```

```
[Out] (b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + 15*FresnelC[b*x] + b^6*Pi^3
*x^6*FresnelS[b*x] - 5*b^3*Pi*x^3*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```

Maple [A] time = 0.049, size = 96, normalized size = 1.

$$\frac{1}{b^6} \left(\frac{b^6 x^6 \text{FresnelS}(bx)}{6} + \frac{b^5 x^5}{6\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) - \frac{5}{6\pi} \left(\frac{x^3 b^3}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) - 3 \frac{1}{\pi} \left(-\frac{bx \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + \frac{\text{FresnelC}(bx)}{\pi} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*FresnelS(b*x),x)

[Out] 1/b^6*(1/6*b^6*x^6*FresnelS(b*x)+1/6/Pi*b^5*x^5*cos(1/2*b^2*Pi*x^2)-5/6/Pi*(1/Pi*b^3*x^3*sin(1/2*b^2*Pi*x^2)-3/Pi*(-1/Pi*b*x*cos(1/2*b^2*Pi*x^2)+1/Pi*FresnelC(b*x))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*fresnels(b*x),x, algorithm="maxima")

[Out] integrate(x^5*fresnels(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^5 \text{fresnels}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*fresnels(b*x),x, algorithm="fricas")

[Out] integral(x^5*fresnels(b*x), x)

Sympy [A] time = 1.04664, size = 53, normalized size = 0.54

$$\frac{\pi b^3 x^9 \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{9}{4}\right) {}_2F_3\left(\frac{3}{4}, \frac{9}{4} \middle| \frac{3}{2}, \frac{7}{4}, \frac{13}{4}; -\frac{\pi^2 b^4 x^4}{16}\right)}{32 \Gamma\left(\frac{7}{4}\right) \Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*fresnels(b*x), x)

[Out] pi*b**3*x**9*gamma(3/4)*gamma(9/4)*hyper((3/4, 9/4), (3/2, 7/4, 13/4), -pi*
*2*b**4*x**4/16)/(32*gamma(7/4)*gamma(13/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*fresnels(b*x), x, algorithm="giac")

[Out] integrate(x^5*fresnels(b*x), x)

3.4 $\int x^4 S(bx) dx$

Optimal. Leaf size=84

$$-\frac{4x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3} + \frac{x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi b} - \frac{8 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5} + \frac{1}{5}x^5 S(bx)$$

[Out] $(-8*\text{Cos}[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^4*\text{Cos}[(b^2*Pi*x^2)/2])/(5*b*Pi) + (x^5*\text{FresnelS}[b*x])/5 - (4*x^2*\text{Sin}[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)$

Rubi [A] time = 0.0801435, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3379, 3296, 2638}

$$-\frac{4x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3} + \frac{x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi b} - \frac{8 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5} + \frac{1}{5}x^5 S(bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4*\text{FresnelS}[b*x], x]$

[Out] $(-8*\text{Cos}[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^4*\text{Cos}[(b^2*Pi*x^2)/2])/(5*b*Pi) + (x^5*\text{FresnelS}[b*x])/5 - (4*x^2*\text{Sin}[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)$

Rule 6426

$\text{Int}[\text{FresnelS}[(b_)*(x_)]*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[((d*x)^{(m+1)}*\text{FresnelS}[b*x])/(d*(m+1)), x] - \text{Dist}[b/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*\text{Sin}[(Pi*b^2*x^2)/2], x], x] /; \text{FreeQ}[\{b, d, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 3379

$\text{Int}[(x_)^{(m_)}*((a_)+(b_)*\text{Sin}[(c_)+(d_)*(x_)]^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n]-1)*(a+b*\text{Sin}[c+d*x])^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{EqQ}[m, n-1] \ || \ (\text{IntegerQ}[p] \ \&\& \ \text{GtQ}[\text{Simplify}[(m+1)/n], 0]))$

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^4 S(bx) dx &= \frac{1}{5} x^5 S(bx) - \frac{1}{5} b \int x^5 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
&= \frac{1}{5} x^5 S(bx) - \frac{1}{10} b \operatorname{Subst}\left(\int x^2 \sin\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right) \\
&= \frac{x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{5b\pi} + \frac{1}{5} x^5 S(bx) - \frac{2 \operatorname{Subst}\left(\int x \cos\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{5b\pi} \\
&= \frac{x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{5b\pi} + \frac{1}{5} x^5 S(bx) - \frac{4x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b^3 \pi^2} + \frac{4 \operatorname{Subst}\left(\int \sin\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{5b^3 \pi^2} \\
&= -\frac{8 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{5b^5 \pi^3} + \frac{x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{5b\pi} + \frac{1}{5} x^5 S(bx) - \frac{4x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b^3 \pi^2}
\end{aligned}$$

Mathematica [A] time = 0.0407404, size = 71, normalized size = 0.85

$$-\frac{4x^2 \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{5\pi^2 b^3} + \frac{(\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{5\pi^3 b^5} + \frac{1}{5} x^5 S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelS[b*x], x]
```

```
[Out] ((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^5*FresnelS[b*x]
)/5 - (4*x^2*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
```

Maple [A] time = 0.052, size = 80, normalized size = 1.

$$\frac{1}{b^5} \left(\frac{b^5 x^5 \text{FresnelS}(bx)}{5} + \frac{x^4 b^4}{5\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) - \frac{4}{5\pi} \left(\frac{b^2 x^2}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) + 2 \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*FresnelS(b*x), x)

[Out] 1/b^5*(1/5*b^5*x^5*FresnelS(b*x)+1/5/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)-4/5/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*fresnels(b*x), x, algorithm="maxima")

[Out] integrate(x^4*fresnels(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^4 \text{fresnels}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*fresnels(b*x), x, algorithm="fricas")

[Out] integral(x^4*fresnels(b*x), x)

Sympy [A] time = 1.62895, size = 121, normalized size = 1.44

$$\frac{3x^5 S(bx) \Gamma\left(\frac{3}{4}\right)}{20\Gamma\left(\frac{7}{4}\right)} + \frac{3x^4 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{20\pi b \Gamma\left(\frac{7}{4}\right)} - \frac{3x^2 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{5\pi^2 b^3 \Gamma\left(\frac{7}{4}\right)} - \frac{6 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{5\pi^3 b^5 \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnels(b*x),x)
```

```
[Out] 3*x**5*fresnels(b*x)*gamma(3/4)/(20*gamma(7/4)) + 3*x**4*cos(pi*b**2*x**2/2)
)*gamma(3/4)/(20*pi*b*gamma(7/4)) - 3*x**2*sin(pi*b**2*x**2/2)*gamma(3/4)/(
5*pi**2*b**3*gamma(7/4)) - 6*cos(pi*b**2*x**2/2)*gamma(3/4)/(5*pi**3*b**5*g
amma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnels(b*x), x)
```

3.5 $\int x^3 S(bx) dx$

Optimal. Leaf size=74

$$\frac{3S(bx)}{4\pi^2 b^4} - \frac{3x \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} + \frac{1}{4}x^4 S(bx)$$

[Out] $(x^3 \cos[(b^2 \pi x^2)/2])/(4b\pi) + (3 \text{FresnelS}[bx])/(4b^4 \pi^2) + (x^4 \text{FresnelS}[bx])/4 - (3x \sin[(b^2 \pi x^2)/2])/(4b^3 \pi^2)$

Rubi [A] time = 0.0474889, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3385, 3386, 3351}

$$\frac{3S(bx)}{4\pi^2 b^4} - \frac{3x \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} + \frac{1}{4}x^4 S(bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3 \text{FresnelS}[bx], x]$

[Out] $(x^3 \cos[(b^2 \pi x^2)/2])/(4b\pi) + (3 \text{FresnelS}[bx])/(4b^4 \pi^2) + (x^4 \text{FresnelS}[bx])/4 - (3x \sin[(b^2 \pi x^2)/2])/(4b^3 \pi^2)$

Rule 6426

$\text{Int}[\text{FresnelS}[(b_)(x_)]*((d_)(x_))^{(m_)}, x_Symbol] :> \text{Simp}[\frac{(d*x)^{(m+1)} \text{FresnelS}[bx]}{d*(m+1)}, x] - \text{Dist}[b/(d*(m+1)), \text{Int}[(d*x)^{(m+1)} \text{Sin}[(\pi*b^2*x^2)/2], x], x] /; \text{FreeQ}\{b, d, m\}, x \&\& \text{NeQ}[m, -1]$

Rule 3385

$\text{Int}[\frac{(e_)(x_)^{(m_)} \text{Sin}[(c_)+(d_)(x_)^{(n_)}], x_Symbol] :> -\text{Simp}[\frac{e^{(n-1)}(e*x)^{(m-n+1)} \text{Cos}[c+d*x^n]}{d*n}, x] + \text{Dist}[\frac{e^n*(m-n+1)}{d*n}, \text{Int}[(e*x)^{(m-n)} \text{Cos}[c+d*x^n], x], x] /; \text{FreeQ}\{c, d, e\}, x \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[n, m+1]$

Rule 3386

$\text{Int}[\text{Cos}[(c_)+(d_)(x_)^{(n_)]*((e_)(x_))^{(m_)}, x_Symbol] :> \text{Simp}[\frac{e^{(n-1)}(e*x)^{(m-n+1)} \text{Sin}[c+d*x^n]}{d*n}, x] - \text{Dist}[\frac{e^n*(m-n+1)}{d*n}, \text{Int}[(e*x)^{(m-n)} \text{Cos}[c+d*x^n], x], x]$

/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned} \int x^3 S(bx) dx &= \frac{1}{4}x^4 S(bx) - \frac{1}{4}b \int x^4 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx \\ &= \frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{1}{4}x^4 S(bx) - \frac{3 \int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) dx}{4b\pi} \\ &= \frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{1}{4}x^4 S(bx) - \frac{3x \sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3 \int \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{4b^3\pi^2} \\ &= \frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3S(bx)}{4b^4\pi^2} + \frac{1}{4}x^4 S(bx) - \frac{3x \sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} \end{aligned}$$

Mathematica [A] time = 0.0159426, size = 74, normalized size = 1.

$$\frac{3S(bx)}{4\pi^2 b^4} - \frac{3x \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} + \frac{1}{4}x^4 S(bx)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*FresnelS[b*x], x]

[Out] (x^3*Cos[(b^2*Pi*x^2)/2])/(4*b*Pi) + (3*FresnelS[b*x])/(4*b^4*Pi^2) + (x^4*FresnelS[b*x])/4 - (3*x*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2)

Maple [A] time = 0.046, size = 70, normalized size = 1.

$$\frac{1}{b^4} \left(\frac{b^4 x^4 \text{FresnelS}(bx)}{4} + \frac{x^3 b^3}{4\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) - \frac{3}{4\pi} \left(\frac{bx}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) - \frac{\text{FresnelS}(bx)}{\pi} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*FresnelS(b*x),x)`

[Out] $\frac{1}{b^4} \left(\frac{1}{4} b^4 x^4 \text{FresnelS}(bx) + \frac{1}{4} \pi b^3 x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{3}{4} \pi \left(\frac{1}{\pi} b x \sin\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{1}{\pi} \text{FresnelS}(bx) \right) \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*fresnels(b*x),x, algorithm="maxima")`

[Out] `integrate(x^3*fresnels(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(x^3*fresnels(b*x), x)`

Sympy [A] time = 0.93364, size = 112, normalized size = 1.51

$$\frac{21x^4 S(bx) \Gamma\left(\frac{3}{4}\right)}{64 \Gamma\left(\frac{11}{4}\right)} + \frac{21x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{64 \pi b \Gamma\left(\frac{11}{4}\right)} - \frac{63x \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{64 \pi^2 b^3 \Gamma\left(\frac{11}{4}\right)} + \frac{63 S(bx) \Gamma\left(\frac{3}{4}\right)}{64 \pi^2 b^4 \Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*fresnels(b*x),x)`

```
[Out] 21*x**4*fresnels(b*x)*gamma(3/4)/(64*gamma(11/4)) + 21*x**3*cos(pi*b**2*x**
2/2)*gamma(3/4)/(64*pi*b*gamma(11/4)) - 63*x*sin(pi*b**2*x**2/2)*gamma(3/4)
/(64*pi**2*b**3*gamma(11/4)) + 63*fresnels(b*x)*gamma(3/4)/(64*pi**2*b**4*g
amma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnels(b*x), x)
```


3.6 $\int x^2 S(bx) dx$

Optimal. Leaf size=59

$$-\frac{2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi^2 b^3} + \frac{x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi b} + \frac{1}{3}x^3 S(bx)$$

[Out] $(x^2 \cos[(b^2 \pi x^2)/2]) / (3 b \pi) + (x^3 \text{FresnelS}[b x]) / 3 - (2 \sin[(b^2 \pi x^2)/2]) / (3 b^3 \pi^2)$

Rubi [A] time = 0.052703, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3379, 3296, 2637}

$$-\frac{2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi^2 b^3} + \frac{x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi b} + \frac{1}{3}x^3 S(bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2 \text{FresnelS}[b x], x]$

[Out] $(x^2 \cos[(b^2 \pi x^2)/2]) / (3 b \pi) + (x^3 \text{FresnelS}[b x]) / 3 - (2 \sin[(b^2 \pi x^2)/2]) / (3 b^3 \pi^2)$

Rule 6426

$\text{Int}[\text{FresnelS}[(b \cdot)(x)] \cdot ((d \cdot)(x))^m, x_Symbol] \rightarrow \text{Simp}[\frac{(d \cdot x)^{m+1} \text{FresnelS}[b \cdot x]}{d \cdot (m+1)}, x] - \text{Dist}[b / (d \cdot (m+1)), \text{Int}[(d \cdot x)^{m+1} \text{Sin}[(\pi \cdot b^2 \cdot x^2) / 2], x], x] /;$ FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3379

$\text{Int}[(x \cdot)^m \cdot ((a \cdot) + (b \cdot) \cdot \text{Sin}[(c \cdot) + (d \cdot)(x \cdot)^n])^p, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1) \cdot (a + b \cdot \text{Sin}[c + d \cdot x])^p}, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]] && (EqQ[p, 1] || EqQ[m, n-1] || (IntegerQ[p] && GtQ[Simplify[(m+1)/n], 0]))

Rule 3296

```
Int[((c_.) + (d_.)*(x_)^(m_.))*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 S(bx) dx &= \frac{1}{3} x^3 S(bx) - \frac{1}{3} b \int x^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
&= \frac{1}{3} x^3 S(bx) - \frac{1}{6} b \operatorname{Subst}\left(\int x \sin\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right) \\
&= \frac{x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi} + \frac{1}{3} x^3 S(bx) - \frac{\operatorname{Subst}\left(\int \cos\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{3b\pi} \\
&= \frac{x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi} + \frac{1}{3} x^3 S(bx) - \frac{2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b^3 \pi^2}
\end{aligned}$$

Mathematica [A] time = 0.0115254, size = 59, normalized size = 1.

$$-\frac{2 \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{3 \pi^2 b^3} + \frac{x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3 \pi b} + \frac{1}{3} x^3 S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelS[b*x], x]
```

```
[Out] (x^2*Cos[(b^2*Pi*x^2)/2])/(3*b*Pi) + (x^3*FresnelS[b*x])/3 - (2*Sin[(b^2*Pi
*x^2)/2])/(3*b^3*Pi^2)
```

Maple [A] time = 0.049, size = 54, normalized size = 0.9

$$\frac{1}{b^3} \left(\frac{b^3 x^3 \operatorname{FresnelS}(bx)}{3} + \frac{b^2 x^2}{3\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) - \frac{2}{3\pi^2} \sin\left(\frac{b^2 \pi x^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*FresnelS(b*x),x)`

[Out] $1/b^3*(1/3*b^3*x^3*FresnelS(b*x)+1/3/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)-2/3/Pi^2*sin(1/2*b^2*Pi*x^2))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnels(b*x),x, algorithm="maxima")`

[Out] `integrate(x^2*fresnels(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \text{fresnels}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(x^2*fresnels(b*x), x)`

Sympy [A] time = 0.828279, size = 80, normalized size = 1.36

$$\frac{x^3 S(bx) \Gamma\left(\frac{3}{4}\right)}{4 \Gamma\left(\frac{7}{4}\right)} + \frac{x^2 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{4 \pi b \Gamma\left(\frac{7}{4}\right)} - \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{3}{4}\right)}{2 \pi^2 b^3 \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*fresnels(b*x),x)`

```
[Out] x**3*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + x**2*cos(pi*b**2*x**2/2)*gamma(3/4)/(4*pi*b*gamma(7/4)) - sin(pi*b**2*x**2/2)*gamma(3/4)/(2*pi**2*b**3*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnels(b*x), x)
```

3.7 $\int xS(bx) dx$

Optimal. Leaf size=49

$$-\frac{\text{FresnelC}(bx)}{2\pi b^2} + \frac{x \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b} + \frac{1}{2}x^2 S(bx)$$

[Out] $(x \cos[(b^2 \pi x^2)/2]) / (2 b \pi) - \text{FresnelC}[b x] / (2 b^2 \pi) + (x^2 \text{FresnelS}[b x]) / 2$

Rubi [A] time = 0.0240392, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3385, 3352}

$$-\frac{\text{FresnelC}(bx)}{2\pi b^2} + \frac{x \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b} + \frac{1}{2}x^2 S(bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x \text{FresnelS}[b x], x]$

[Out] $(x \cos[(b^2 \pi x^2)/2]) / (2 b \pi) - \text{FresnelC}[b x] / (2 b^2 \pi) + (x^2 \text{FresnelS}[b x]) / 2$

Rule 6426

$\text{Int}[\text{FresnelS}[(b \cdot)(x)] \cdot ((d \cdot)(x))^m, x_Symbol] \rightarrow \text{Simp}[(d x)^{m+1} \text{FresnelS}[b x] / (d(m+1)), x] - \text{Dist}[b / (d(m+1)), \text{Int}[(d x)^{m+1} S(\pi b^2 x^2 / 2), x], x] /;$ $\text{FreeQ}\{b, d, m, x\} \ \&\& \ \text{NeQ}[m, -1]$

Rule 3385

$\text{Int}[(e \cdot)(x))^m \cdot \text{Sin}[c \cdot + (d \cdot)(x)^n], x_Symbol] \rightarrow -\text{Simp}[(e^{(n-1)} (e x)^{m-n+1} \text{Cos}[c + d x^n]) / (d n), x] + \text{Dist}[(e^n (m-n+1)) / (d n), \text{Int}[(e x)^{m-n} \text{Cos}[c + d x^n], x], x] /;$ $\text{FreeQ}\{c, d, e, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[n, m+1]$

Rule 3352

$\text{Int}[\text{Cos}[(d \cdot) \cdot ((e \cdot) + (f \cdot)(x))^2], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\pi/2] \text{FresnelC}[\text{Sqrt}[2/\pi] \text{Rt}[d, 2] (e + f x)]) / (f \text{Rt}[d, 2]), x] /;$ $\text{FreeQ}\{d, e, f, x\}$

Rubi steps

$$\begin{aligned}
\int xS(bx) dx &= \frac{1}{2}x^2S(bx) - \frac{1}{2}b \int x^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx \\
&= \frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{1}{2}x^2S(bx) - \frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right) dx}{2b\pi} \\
&= \frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} - \frac{C(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)
\end{aligned}$$

Mathematica [A] time = 0.0110994, size = 49, normalized size = 1.

$$-\frac{\text{FresnelC}(bx)}{2\pi b^2} + \frac{x \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b} + \frac{1}{2}x^2S(bx)$$

Antiderivative was successfully verified.

[In] Integrate[x*FresnelS[b*x],x]

[Out] (x*Cos[(b^2*Pi*x^2)/2])/(2*b*Pi) - FresnelC[b*x]/(2*b^2*Pi) + (x^2*FresnelS[b*x])/2

Maple [A] time = 0.049, size = 44, normalized size = 0.9

$$\frac{1}{b^2} \left(\frac{b^2 x^2 \text{FresnelS}(bx)}{2} + \frac{bx}{2\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) - \frac{\text{FresnelC}(bx)}{2\pi} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*FresnelS(b*x),x)

[Out] 1/b^2*(1/2*b^2*x^2*FresnelS(b*x)+1/2/Pi*b*x*cos(1/2*b^2*Pi*x^2)-1/2/Pi*FresnelC(b*x))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{xfresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnels(b*x),x, algorithm="maxima")

[Out] integrate(x*fresnels(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

integral(xfresnels(bx),x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnels(b*x),x, algorithm="fricas")

[Out] integral(x*fresnels(b*x), x)

Sympy [A] time = 0.524181, size = 53, normalized size = 1.08

$$\frac{\pi b^3 x^5 \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{5}{4}\right) {}_2F_3\left(\frac{3}{4}, \frac{5}{4} \middle| \frac{3}{2}, \frac{7}{4}, \frac{9}{4}; -\frac{\pi^2 b^4 x^4}{16}\right)}{32 \Gamma\left(\frac{7}{4}\right) \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnels(b*x),x)

[Out] pi*b**3*x**5*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -pi**2*b**4*x**4/16)/(32*gamma(7/4)*gamma(9/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{xfresnels}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*fresnels(b*x), x)
```


3.8 $\int S(bx) dx$

Optimal. Leaf size=26

$$\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b} + xS(bx)$$

[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]

Rubi [A] time = 0.0048741, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6418}

$$\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b} + xS(bx)$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x], x]

[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]

Rule 6418

Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] := Simp[((a + b*x)*FresnelS[a + b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int S(bx) dx = \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + xS(bx)$$

Mathematica [A] time = 0.0020637, size = 26, normalized size = 1.

$$\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b} + xS(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x],x]
```

```
[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]
```

Maple [A] time = 0.046, size = 27, normalized size = 1.

$$\frac{1}{b} \left(bx \operatorname{FresnelS}(bx) + \frac{1}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x),x)
```

```
[Out] 1/b*(b*x*FresnelS(b*x)+1/Pi*cos(1/2*b^2*Pi*x^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(\operatorname{fresnels}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x), x)
```

Sympy [B] time = 0.83294, size = 48, normalized size = 1.85

$$\frac{3xS(bx)\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi b\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x),x)

[Out] 3*x*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + 3*cos(pi*b**2*x**2/2)*gamma(3/4)/(4*pi*b*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x),x, algorithm="giac")

[Out] integrate(fresnels(b*x), x)

3.9 $\int \frac{S(bx)}{x} dx$

Optimal. Leaf size=73

$$\frac{1}{2}ibx \text{HypergeometricPFQ}\left(\left\{\frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{1}{2}i\pi b^2 x^2\right) - \frac{1}{2}ibx \text{HypergeometricPFQ}\left(\left\{\frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, \frac{1}{2}i\pi b^2 x^2\right)$$

[Out] (I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-I/2)*b^2*Pi*x^2] - (I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2]

Rubi [A] time = 0.0466434, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6424, 6358, 6360}

$$\frac{1}{2}ibx {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{2}ibx {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{1}{2}ib^2\pi x^2\right)$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]/x, x]

[Out] (I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-I/2)*b^2*Pi*x^2] - (I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2]

Rule 6424

Int[FresnelS[(b_.)*(x_)]/(x_), x_Symbol] := Dist[(1 + I)/4, Int[Erf[(Sqrt[Pi]*(1 + I)*b*x)/2]/x, x], x] + Dist[(1 - I)/4, Int[Erf[(Sqrt[Pi]*(1 - I)*b*x)/2]/x, x], x] /; FreeQ[b, x]

Rule 6358

Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] := Simp[(2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[b, x]

Rule 6360

Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] := Simp[(2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[b, x]

Rubi steps

$$\int \frac{S(bx)}{x} dx = \left(-\frac{1}{4} - \frac{i}{4}\right) \int \frac{\operatorname{erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi x}\right)}{x} dx + \left(\frac{1}{4} + \frac{i}{4}\right) \int \frac{\operatorname{erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi x}\right)}{x} dx$$

$$= \frac{1}{2} ibx {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{1}{2} ib^2 \pi x^2\right) - \frac{1}{2} ibx {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{1}{2} ib^2 \pi x^2\right)$$

Mathematica [F] time = 0.0136888, size = 0, normalized size = 0.

$$\int \frac{S(bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[b*x]/x,x]

[Out] Integrate[FresnelS[b*x]/x, x]

Maple [A] time = 0.08, size = 29, normalized size = 0.4

$$\frac{\pi x^3 b^3}{18} {}_2F_3\left(\frac{3}{4}, \frac{3}{4}; \frac{3}{2}, \frac{7}{4}, \frac{7}{4}; -\frac{x^4 \pi^2 b^4}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)/x,x)

[Out] 1/18*Pi*x^3*b^3*hypergeom([3/4,3/4],[3/2,7/4,7/4],-1/16*x^4*Pi^2*b^4)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{fresnels}(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x,x, algorithm="fricas")

[Out] integral(fresnels(b*x)/x, x)

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x,x)

[Out] Exception raised: AttributeError

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x,x, algorithm="giac")

[Out] integrate(fresnels(b*x)/x, x)

$$3.10 \quad \int \frac{S(bx)}{x^2} dx$$

Optimal. Leaf size=27

$$\frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{S(bx)}{x}$$

[Out] $-(\text{FresnelS}[b*x]/x) + (b*\text{SinIntegral}[(b^2*\text{Pi}*x^2)/2])/2$

Rubi [A] time = 0.0215755, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6426, 3375}

$$\frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{S(bx)}{x}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{FresnelS}[b*x]/x^2, x]$

[Out] $-(\text{FresnelS}[b*x]/x) + (b*\text{SinIntegral}[(b^2*\text{Pi}*x^2)/2])/2$

Rule 6426

$\text{Int}[\text{FresnelS}[(b_)*(x_)]*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[\frac{((d*x)^{(m+1)}*\text{FresnelS}[b*x])}{(d*(m+1)), x} - \text{Dist}[b/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*\text{Sin}[(\text{Pi}*b^2*x^2)/2], x], x] /; \text{FreeQ}\{b, d, m\}, x] \&\& \text{NeQ}[m, -1]$

Rule 3375

$\text{Int}[\text{Sin}[(d_)*(x_)^{(n_)}]/(x_), x_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[d*x^n]/n, x] /; \text{FreeQ}\{d, n\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{S(bx)}{x^2} dx &= -\frac{S(bx)}{x} + b \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx \\ &= -\frac{S(bx)}{x} + \frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) \end{aligned}$$

Mathematica [A] time = 0.010818, size = 27, normalized size = 1.

$$\frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{S(bx)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]/x^2,x]

[Out] -(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2

Maple [A] time = 0.049, size = 28, normalized size = 1.

$$b\left(-\frac{\text{FresnelS}(bx)}{bx} + \frac{1}{2}\text{Si}\left(\frac{b^2\pi x^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)/x^2,x)

[Out] b*(-FresnelS(b*x)/b/x+1/2*Si(1/2*b^2*Pi*x^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^2,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)/x^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^2, x)
```

Sympy [B] time = 0.544513, size = 42, normalized size = 1.56

$$\frac{\pi b^3 x^2 \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{3}{2}, \frac{3}{2}, \frac{7}{4} \right) - \frac{\pi^2 b^4 x^4}{16}}{16 \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**2,x)
```

```
[Out] pi*b**3*x**2*gamma(3/4)*hyper((1/2, 3/4), (3/2, 3/2, 7/4), -pi**2*b**4*x**4/16)/(16*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^2, x)
```

3.11 $\int \frac{S(bx)}{x^3} dx$

Optimal. Leaf size=44

$$\frac{1}{2}\pi b^2 \text{FresnelC}(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{S(bx)}{2x^2}$$

[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/(2*x)

Rubi [A] time = 0.0287987, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6426, 3387, 3352}

$$\frac{1}{2}\pi b^2 \text{FresnelC}(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{S(bx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]/x^3,x]

[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/(2*x)

Rule 6426

Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*Sin[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3387

Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^3} dx &= -\frac{S(bx)}{2x^2} + \frac{1}{2}b \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
&= -\frac{S(bx)}{2x^2} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x} + \frac{1}{2}(b^3\pi) \int \cos\left(\frac{1}{2}b^2\pi x^2\right) dx \\
&= \frac{1}{2}b^2\pi C(bx) - \frac{S(bx)}{2x^2} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}
\end{aligned}$$

Mathematica [A] time = 0.0112385, size = 44, normalized size = 1.

$$\frac{1}{2}\pi b^2 \text{FresnelC}(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{S(bx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]/x^3,x]

[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/ (2*x)

Maple [A] time = 0.048, size = 43, normalized size = 1.

$$b^2 \left(-\frac{\text{FresnelS}(bx)}{2b^2x^2} - \frac{1}{2bx} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi \text{FresnelC}(bx)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)/x^3,x)

[Out] b^2*(-1/2*FresnelS(b*x)/b^2/x^2-1/2*sin(1/2*b^2*Pi*x^2)/b/x+1/2*Pi*FresnelC(b*x))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^3,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)/x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^3,x, algorithm="fricas")

[Out] integral(fresnels(b*x)/x^3, x)

Sympy [A] time = 0.623655, size = 51, normalized size = 1.16

$$\frac{\pi b^3 x \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\frac{1}{4}, \frac{3}{4} \middle| \frac{5}{4}, \frac{3}{2}, \frac{7}{4} \right) - \frac{\pi^2 b^4 x^4}{16}}{32 \Gamma\left(\frac{5}{4}\right) \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x**3,x)

[Out] pi*b**3*x*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (5/4, 3/2, 7/4), -pi**2*b**4*x**4/16)/(32*gamma(5/4)*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^3, x)
```

3.12 $\int \frac{S(bx)}{x^4} dx$

Optimal. Leaf size=52

$$\frac{1}{12}\pi b^3 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^2} - \frac{S(bx)}{3x^3}$$

[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b^2*Pi*x^2)/2])/(6*x^2)

Rubi [A] time = 0.0637342, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3379, 3297, 3302}

$$\frac{1}{12}\pi b^3 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^2} - \frac{S(bx)}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]/x^4,x]

[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b^2*Pi*x^2)/2])/(6*x^2)

Rule 6426

Int[FresnelS[(b_.)*(x_.)]*((d_.)*(x_.))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*Sin[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{S(bx)}{x^4} dx &= -\frac{S(bx)}{3x^3} + \frac{1}{3}b \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx \\
 &= -\frac{S(bx)}{3x^3} + \frac{1}{6}b \operatorname{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2} dx, x, x^2\right) \\
 &= -\frac{S(bx)}{3x^3} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2} + \frac{1}{12}(b^3\pi) \operatorname{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x} dx, x, x^2\right) \\
 &= \frac{1}{12}b^3\pi \operatorname{Ci}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{S(bx)}{3x^3} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}
 \end{aligned}$$

Mathematica [A] time = 0.0143616, size = 52, normalized size = 1.

$$\frac{1}{12}\pi b^3 \operatorname{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^2} - \frac{S(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^4, x]
```

```
[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b^2*Pi*x^2)/2])/(6*x^2)
```

Maple [A] time = 0.052, size = 49, normalized size = 0.9

$$b^3 \left(-\frac{\text{FresnelS}(bx)}{3x^3b^3} - \frac{1}{6b^2x^2} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{12} \text{Ci}\left(\frac{b^2\pi x^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)/x^4,x)

[Out] b^3*(-1/3*FresnelS(b*x)/b^3/x^3-1/6*sin(1/2*b^2*Pi*x^2)/b^2/x^2+1/12*Pi*Ci(1/2*b^2*Pi*x^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^4,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)/x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^4,x, algorithm="fricas")

[Out] integral(fresnels(b*x)/x^4, x)

Sympy [A] time = 1.38716, size = 56, normalized size = 1.08

$$-\frac{\pi^3 b^7 x^4 \Gamma\left(\frac{7}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{7}{4} \\ 2, 2, \frac{5}{2}, \frac{11}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{768 \Gamma\left(\frac{11}{4}\right)} + \frac{\pi b^3 \log(b^4 x^4)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x**4, x)

[Out] $-\pi^{**3}b^{**7}x^{**4}\text{gamma}(7/4)*\text{hyper}((1, 1, 7/4), (2, 2, 5/2, 11/4), -\pi^{**2}b^{**4}x^{**4}/16)/(768*\text{gamma}(11/4)) + \pi*b^{**3}*\log(b^{**4}x^{**4})/24$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^4, x, algorithm="giac")

[Out] integrate(fresnels(b*x)/x^4, x)

3.13 $\int \frac{S(bx)}{x^5} dx$

Optimal. Leaf size=69

$$-\frac{1}{12}\pi^2 b^4 S(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x} - \frac{S(bx)}{4x^4}$$

[Out] $-(b^3 \pi \cos[(b^2 \pi x^2)/2])/(12x) - (b^4 \pi^2 \text{FresnelS}[b*x])/12 - \text{FresnelS}[b*x]/(4*x^4) - (b \sin[(b^2 \pi x^2)/2])/(12*x^3)$

Rubi [A] time = 0.0433676, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3387, 3388, 3351}

$$-\frac{1}{12}\pi^2 b^4 S(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x} - \frac{S(bx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]/x^5,x]

[Out] $-(b^3 \pi \cos[(b^2 \pi x^2)/2])/(12x) - (b^4 \pi^2 \text{FresnelS}[b*x])/12 - \text{FresnelS}[b*x]/(4*x^4) - (b \sin[(b^2 \pi x^2)/2])/(12*x^3)$

Rule 6426

Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*FresnelS[b*x])/(d*(m+1)), x] - Dist[b/(d*(m+1)), Int[(d*x)^(m+1)*Sin[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3387

Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Simp[((e*x)^(m+1)*Sin[c + d*x^n])/(e*(m+1)), x] - Dist[(d*n)/(e^n*(m+1)), Int[(e*x)^(m+n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3388

Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] := Simp[((e*x)^(m+1)*Cos[c + d*x^n])/(e*(m+1)), x] + Dist[(d*n)/(e^n*(m+1)), Int[(e*x)^(m+n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

$e^x)^{(m+n)} \sin[c + d x^n], x, x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[m, -1]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.) * ((e_.) + (f_.) * (x_.)^2)], x_Symbol] :> \text{Simp}[(\text{Sqrt}[\text{Pi}/2] * \text{FresnelS}[\text{Sqrt}[2/\text{Pi}] * \text{Rt}[d, 2] * (e + f * x)]) / (f * \text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{S(bx)}{x^5} dx &= -\frac{S(bx)}{4x^4} + \frac{1}{4}b \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx \\ &= -\frac{S(bx)}{4x^4} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} + \frac{1}{12}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\ &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{S(bx)}{4x^4} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{1}{12}(b^5\pi^2) \int \sin\left(\frac{1}{2}b^2\pi x^2\right) dx \\ &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{1}{12}b^4\pi^2 S(bx) - \frac{S(bx)}{4x^4} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} \end{aligned}$$

Mathematica [A] time = 0.0145281, size = 69, normalized size = 1.

$$-\frac{1}{12}\pi^2 b^4 S(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x} - \frac{S(bx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]/x^5,x]

[Out] $-(b^3\pi \cos[(b^2\pi x^2)/2])/(12x) - (b^4\pi^2 \text{FresnelS}[b*x])/12 - \text{FresnelS}[b*x]/(4x^4) - (b \sin[(b^2\pi x^2)/2])/(12x^3)$

Maple [A] time = 0.05, size = 65, normalized size = 0.9

$$b^4 \left(-\frac{\text{FresnelS}(bx)}{4x^4 b^4} - \frac{1}{12x^3 b^3} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{12} \left(-\frac{1}{bx} \cos\left(\frac{b^2\pi x^2}{2}\right) - \pi \text{FresnelS}(bx) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(FresnelS(b*x)/x^5,x)`

[Out] $b^4*(-1/4*\text{FresnelS}(b*x)/b^4/x^4-1/12*\sin(1/2*b^2*\text{Pi}*x^2)/b^3/x^3+1/12*\text{Pi}*(-1/b/x*\cos(1/2*b^2*\text{Pi}*x^2)-\text{Pi}*\text{FresnelS}(b*x)))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)/x^5,x, algorithm="maxima")`

[Out] `integrate(fresnels(b*x)/x^5, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)/x^5,x, algorithm="fricas")`

[Out] `integral(fresnels(b*x)/x^5, x)`

Sympy [A] time = 1.11749, size = 110, normalized size = 1.59

$$\frac{\pi^2 b^4 S(bx) \Gamma\left(-\frac{1}{4}\right)}{64 \Gamma\left(\frac{7}{4}\right)} + \frac{\pi b^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{1}{4}\right)}{64 x \Gamma\left(\frac{7}{4}\right)} + \frac{b \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{1}{4}\right)}{64 x^3 \Gamma\left(\frac{7}{4}\right)} + \frac{3 S(bx) \Gamma\left(-\frac{1}{4}\right)}{64 x^4 \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**5,x)
```

```
[Out] pi**2*b**4*fresnels(b*x)*gamma(-1/4)/(64*gamma(7/4)) + pi*b**3*cos(pi*b**2*
x**2/2)*gamma(-1/4)/(64*x*gamma(7/4)) + b*sin(pi*b**2*x**2/2)*gamma(-1/4)/(
64*x**3*gamma(7/4)) + 3*fresnels(b*x)*gamma(-1/4)/(64*x**4*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^5, x)
```

3.14 $\int \frac{S(bx)}{x^6} dx$

Optimal. Leaf size=77

$$-\frac{1}{80}\pi^2 b^5 \operatorname{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{20x^4} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{40x^2} - \frac{S(bx)}{5x^5}$$

[Out] $-(b^3\pi\cos[(b^2\pi x^2)/2])/(40x^2) - \operatorname{FresnelS}[b*x]/(5x^5) - (b\sin[(b^2\pi x^2)/2])/(20x^4) - (b^5\pi^2\operatorname{SinIntegral}[(b^2\pi x^2)/2])/80$

Rubi [A] time = 0.0892369, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3379, 3297, 3299}

$$-\frac{1}{80}\pi^2 b^5 \operatorname{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{20x^4} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{40x^2} - \frac{S(bx)}{5x^5}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{FresnelS}[b*x]/x^6, x]$

[Out] $-(b^3\pi\cos[(b^2\pi x^2)/2])/(40x^2) - \operatorname{FresnelS}[b*x]/(5x^5) - (b\sin[(b^2\pi x^2)/2])/(20x^4) - (b^5\pi^2\operatorname{SinIntegral}[(b^2\pi x^2)/2])/80$

Rule 6426

$\operatorname{Int}[\operatorname{FresnelS}[(b_)*(x_)]*((d_)*(x_))^{\wedge}(m_), x_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{\wedge}(m+1)*\operatorname{FresnelS}[b*x]/(d*(m+1)), x] - \operatorname{Dist}[b/(d*(m+1)), \operatorname{Int}[(d*x)^{\wedge}(m+1)*\operatorname{Sin}[(\pi*b^2*x^2)/2], x], x] /;$ $\operatorname{FreeQ}\{b, d, m\}, x \ \&\& \ \operatorname{NeQ}[m, -1]$

Rule 3379

$\operatorname{Int}[(x_)^{\wedge}(m_)*((a_)+(b_)*\operatorname{Sin}[(c_)+(d_)*(x_)^{\wedge}(n_)])^{\wedge}(p_), x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{\wedge}(\operatorname{Simplify}[(m+1)/n]-1)*(a+b*\operatorname{Sin}[c+d*x])^{\wedge}p, x], x, x^{\wedge}n], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x \ \&\& \ \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]] \ \&\& \ (\operatorname{EqQ}[p, 1] \ || \ \operatorname{EqQ}[m, n-1] \ || \ (\operatorname{IntegerQ}[p] \ \&\& \ \operatorname{GtQ}[\operatorname{Simplify}[(m+1)/n], 0]))$

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{S(bx)}{x^6} dx &= -\frac{S(bx)}{5x^5} + \frac{1}{5}b \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx \\
 &= -\frac{S(bx)}{5x^5} + \frac{1}{10}b \operatorname{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3} dx, x, x^2\right) \\
 &= -\frac{S(bx)}{5x^5} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} + \frac{1}{40}(b^3\pi) \operatorname{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2} dx, x, x^2\right) \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2} - \frac{S(bx)}{5x^5} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}(b^5\pi^2) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x} dx, x, x^2\right) \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2} - \frac{S(bx)}{5x^5} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2 \operatorname{Si}\left(\frac{1}{2}b^2\pi x^2\right)
 \end{aligned}$$

Mathematica [A] time = 0.0184752, size = 77, normalized size = 1.

$$-\frac{1}{80}\pi^2 b^5 \operatorname{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{20x^4} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{40x^2} - \frac{S(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^6, x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(40*x^2) - FresnelS[b*x]/(5*x^5) - (b*Sin[(b^2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*SinIntegral[(b^2*Pi*x^2)/2])/80
```

Maple [A] time = 0.049, size = 71, normalized size = 0.9

$$b^5 \left(-\frac{\text{FresnelS}(bx)}{5b^5x^5} - \frac{1}{20x^4b^4} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{20} \left(-\frac{1}{2b^2x^2} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{4} \text{Si}\left(\frac{b^2\pi x^2}{2}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)/x^6,x)

[Out] b^5*(-1/5*FresnelS(b*x)/b^5/x^5-1/20*sin(1/2*b^2*Pi*x^2)/b^4/x^4+1/20*Pi*(-1/2/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/4*Pi*Si(1/2*b^2*Pi*x^2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^6,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)/x^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^6,x, algorithm="fricas")

[Out] integral(fresnels(b*x)/x^6, x)

Sympy [A] time = 1.21138, size = 46, normalized size = 0.6

$$-\frac{\pi b^3 \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2}, \frac{3}{2}, \frac{7}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16} \right)}{16 x^2 \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x**6,x)

[Out] -pi*b**3*gamma(3/4)*hyper((-1/2, 3/4), (1/2, 3/2, 7/4), -pi**2*b**4*x**4/16)/(16*x**2*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^6,x, algorithm="giac")

[Out] integrate(fresnels(b*x)/x^6, x)

3.15 $\int \frac{S(bx)}{x^7} dx$

Optimal. Leaf size=94

$$-\frac{1}{90}\pi^3 b^6 \text{FresnelC}(bx) + \frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{90x} - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{30x^5} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{90x^3} - \frac{S(bx)}{6x^6}$$

[Out] $-(b^3 \pi \cos[(b^2 \pi x^2)/2])/(90 x^3) - (b^6 \pi^3 \text{FresnelC}[b x])/90 - \text{FresnelS}[b x]/(6 x^6) - (b \sin[(b^2 \pi x^2)/2])/(30 x^5) + (b^5 \pi^2 \sin[(b^2 \pi x^2)/2])/(90 x)$

Rubi [A] time = 0.0593807, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3387, 3388, 3352}

$$-\frac{1}{90}\pi^3 b^6 \text{FresnelC}(bx) + \frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{90x} - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{30x^5} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{90x^3} - \frac{S(bx)}{6x^6}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]/x^7,x]

[Out] $-(b^3 \pi \cos[(b^2 \pi x^2)/2])/(90 x^3) - (b^6 \pi^3 \text{FresnelC}[b x])/90 - \text{FresnelS}[b x]/(6 x^6) - (b \sin[(b^2 \pi x^2)/2])/(30 x^5) + (b^5 \pi^2 \sin[(b^2 \pi x^2)/2])/(90 x)$

Rule 6426

Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*FresnelS[b*x])/(d*(m+1)), x] - Dist[b/(d*(m+1)), Int[(d*x)^(m+1)*Sin[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3387

Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Simp[((e*x)^(m+1)*Sin[c + d*x^n])/(e*(m+1)), x] - Dist[(d*n)/(e^n*(m+1)), Int[(e*x)^(m+n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_), x_Symbol] := Simp[((e*x)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{S(bx)}{x^7} dx &= -\frac{S(bx)}{6x^6} + \frac{1}{6}b \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx \\
 &= -\frac{S(bx)}{6x^6} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{1}{30}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3} - \frac{S(bx)}{6x^6} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} - \frac{1}{90}(b^5\pi^2) \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3} - \frac{S(bx)}{6x^6} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{b^5\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x} - \frac{1}{90}(b^7\pi^3) \int \cos\left(\frac{1}{2}b^2\pi x^2\right) dx \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3} - \frac{1}{90}b^6\pi^3 C(bx) - \frac{S(bx)}{6x^6} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{b^5\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x}
 \end{aligned}$$

Mathematica [A] time = 0.0574427, size = 76, normalized size = 0.81

$$\frac{1}{90} \left(-\pi^3 b^6 \text{FresnelC}(bx) + \frac{b(\pi^2 b^4 x^4 - 3) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^5} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^3} - \frac{15S(bx)}{x^6} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^7,x]
```

```
[Out] (-((b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^3) - b^6*Pi^3*FresnelC[b*x] - (15*FresnelS[b*x])/x^6 + (b*(-3 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^5)/90
```

Maple [A] time = 0.05, size = 86, normalized size = 0.9

$$b^6 \left(-\frac{\text{FresnelS}(bx)}{6b^6x^6} - \frac{1}{30b^5x^5} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{30} \left(-\frac{1}{3x^3b^3} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{3} \left(-\frac{1}{bx} \sin\left(\frac{b^2\pi x^2}{2}\right) + \pi \text{FresnelC}(bx) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)/x^7,x)

[Out] b^6*(-1/6*FresnelS(b*x)/b^6/x^6-1/30*sin(1/2*b^2*Pi*x^2)/b^5/x^5+1/30*Pi*(-1/3/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/3*Pi*(-sin(1/2*b^2*Pi*x^2)/b/x+Pi*FresnelC(b*x))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^7,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)/x^7, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^7,x, algorithm="fricas")

[Out] integral(fresnels(b*x)/x^7, x)

Sympy [A] time = 1.52855, size = 56, normalized size = 0.6

$$\frac{\pi b^3 \Gamma\left(-\frac{3}{4}\right) \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{3}{4}, \frac{3}{4} \\ \frac{1}{4}, \frac{3}{2}, \frac{7}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{32 x^3 \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x**7, x)

[Out] pi*b**3*gamma(-3/4)*gamma(3/4)*hyper((-3/4, 3/4), (1/4, 3/2, 7/4), -pi**2*b**4*x**4/16)/(32*x**3*gamma(1/4)*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^7,x, algorithm="giac")

[Out] integrate(fresnels(b*x)/x^7, x)

3.16 $\int \frac{S(bx)}{x^8} dx$

Optimal. Leaf size=102

$$-\frac{1}{672}\pi^3 b^7 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) + \frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{336x^2} - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{42x^6} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{168x^4} - \frac{S(bx)}{7x^7}$$

[Out] $-(b^3 \pi \text{Cos}[(b^2 \pi x^2)/2])/(168 x^4) - (b^7 \pi^3 \text{CosIntegral}[(b^2 \pi x^2)/2])/672 - \text{FresnelS}[b x]/(7 x^7) - (b \text{Sin}[(b^2 \pi x^2)/2])/(42 x^6) + (b^5 \pi^2 \text{Sin}[(b^2 \pi x^2)/2])/(336 x^2)$

Rubi [A] time = 0.121533, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3379, 3297, 3302}

$$-\frac{1}{672}\pi^3 b^7 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) + \frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{336x^2} - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{42x^6} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{168x^4} - \frac{S(bx)}{7x^7}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{FresnelS}[b x]/x^8, x]$

[Out] $-(b^3 \pi \text{Cos}[(b^2 \pi x^2)/2])/(168 x^4) - (b^7 \pi^3 \text{CosIntegral}[(b^2 \pi x^2)/2])/672 - \text{FresnelS}[b x]/(7 x^7) - (b \text{Sin}[(b^2 \pi x^2)/2])/(42 x^6) + (b^5 \pi^2 \text{Sin}[(b^2 \pi x^2)/2])/(336 x^2)$

Rule 6426

$\text{Int}[\text{FresnelS}[(b \cdot)(x)] \cdot ((d \cdot)(x))^m, x_Symbol] \rightarrow \text{Simp}[(d \cdot x)^{m+1} \text{FresnelS}[b x]/(d(m+1)), x] - \text{Dist}[b/(d(m+1)), \text{Int}[(d \cdot x)^{m+1} \text{Sin}[(\pi b^2 x^2)/2], x], x] /; \text{FreeQ}\{b, d, m, x\} \ \&\& \ \text{NeQ}[m, -1]$

Rule 3379

$\text{Int}[(x \cdot)^m \cdot ((a \cdot) + (b \cdot) \text{Sin}[(c \cdot) + (d \cdot)(x \cdot)^n])^p, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{\text{Simplify}[(m+1)/n] - 1} \cdot (a + b \text{Sin}[c + d x])^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, x\} \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{EqQ}[m, n-1] \ || \ (\text{IntegerQ}[p] \ \&\& \ \text{GtQ}[\text{Simplify}[(m+1)/n], 0]))$

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^8} dx &= -\frac{S(bx)}{7x^7} + \frac{1}{7}b \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx \\
&= -\frac{S(bx)}{7x^7} + \frac{1}{14}b \operatorname{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^4} dx, x, x^2\right) \\
&= -\frac{S(bx)}{7x^7} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{1}{84}(b^3\pi) \operatorname{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3} dx, x, x^2\right) \\
&= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{S(bx)}{7x^7} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} - \frac{1}{336}(b^5\pi^2) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2} dx, x, x^2\right) \\
&= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{S(bx)}{7x^7} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{1}{672}(b^7\pi^3) \operatorname{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x} dx, x, x^2\right) \\
&= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{1}{672}b^7\pi^3 \operatorname{Ci}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{S(bx)}{7x^7} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}
\end{aligned}$$

Mathematica [A] time = 0.0618826, size = 85, normalized size = 0.83

$$\frac{1}{672} \left(-\pi^3 b^7 \operatorname{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) + \frac{2b(\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^6} - \frac{4\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^4} - \frac{96S(bx)}{x^7} \right)$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]/x^8,x]

[Out] $\left(\frac{-4b^3\pi\cos\left(\frac{b^2\pi x^2}{2}\right)}{x^4} - b^7\pi^3\text{CosIntegral}\left[\frac{b^2\pi x^2}{2}\right] - \frac{96\text{FresnelS}[b*x]}{x^7} + \frac{(2b*(-8 + b^4\pi^2x^4))\text{Sin}\left[\frac{b^2\pi x^2}{2}\right]}{x^6}\right)/672$

Maple [A] time = 0.047, size = 93, normalized size = 0.9

$$b^7 \left(-\frac{\text{FresnelS}(bx)}{7b^7x^7} - \frac{1}{42b^6x^6} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{42} \left(-\frac{1}{4x^4b^4} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{4} \left(-\frac{1}{2b^2x^2} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{4} \text{Ci}\left(\frac{b^2\pi x^2}{2}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)/x^8,x)

[Out] $b^7 \left(-\frac{1}{7} \text{FresnelS}(bx) / b^7 / x^7 - \frac{1}{42} \sin\left(\frac{1}{2} b^2 \pi x^2\right) / b^6 / x^6 + \frac{1}{42} \pi \left(-\frac{1}{4} / b^4 / x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{1}{4} \pi \left(-\frac{1}{2} \sin\left(\frac{1}{2} b^2 \pi x^2\right) / b^2 / x^2 + \frac{1}{4} \pi \text{Ci}\left(\frac{1}{2} b^2 \pi x^2\right) \right) \right) \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^8,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)/x^8, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^8,x, algorithm="fricas")

[Out] integral(fresnels(b*x)/x^8, x)

Sympy [A] time = 2.94366, size = 68, normalized size = 0.67

$$\frac{\pi^5 b^{11} x^4 \Gamma\left(\frac{11}{4}\right) {}_3F_4\left(1, 1, \frac{11}{4} \middle| -\frac{\pi^2 b^4 x^4}{16} \right)}{61440 \Gamma\left(\frac{15}{4}\right)} - \frac{\pi^3 b^7 \log(b^4 x^4)}{1344} - \frac{\pi b^3}{24 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x**8,x)

[Out] pi**5*b**11*x**4*gamma(11/4)*hyper((1, 1, 11/4), (2, 3, 7/2, 15/4), -pi**2*b**4*x**4/16)/(61440*gamma(15/4)) - pi**3*b**7*log(b**4*x**4)/1344 - pi*b**3/(24*x**4)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^8,x, algorithm="giac")

[Out] integrate(fresnels(b*x)/x^8, x)

3.17 $\int \frac{S(bx)}{x^9} dx$

Optimal. Leaf size=119

$$\frac{1}{840}\pi^4 b^8 S(bx) + \frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{840x^3} - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{56x^7} + \frac{\pi^3 b^7 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{840x} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{280x^5} - \frac{S(bx)}{8x^8}$$

[Out] $-(b^3 \pi \cos[(b^2 \pi x^2)/2])/(280 x^5) + (b^7 \pi^3 \cos[(b^2 \pi x^2)/2])/(840 x) + (b^8 \pi^4 \text{FresnelS}[b x])/840 - \text{FresnelS}[b x]/(8 x^8) - (b \sin[(b^2 \pi x^2)/2])/(56 x^7) + (b^5 \pi^2 \sin[(b^2 \pi x^2)/2])/(840 x^3)$

Rubi [A] time = 0.0810103, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3387, 3388, 3351}

$$\frac{1}{840}\pi^4 b^8 S(bx) + \frac{\pi^2 b^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{840x^3} - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{56x^7} + \frac{\pi^3 b^7 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{840x} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{280x^5} - \frac{S(bx)}{8x^8}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]/x^9,x]

[Out] $-(b^3 \pi \cos[(b^2 \pi x^2)/2])/(280 x^5) + (b^7 \pi^3 \cos[(b^2 \pi x^2)/2])/(840 x) + (b^8 \pi^4 \text{FresnelS}[b x])/840 - \text{FresnelS}[b x]/(8 x^8) - (b \sin[(b^2 \pi x^2)/2])/(56 x^7) + (b^5 \pi^2 \sin[(b^2 \pi x^2)/2])/(840 x^3)$

Rule 6426

Int[FresnelS[(b_.)*(x_.)]*((d_.)*(x_.))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*FresnelS[b*x])/(d*(m+1)), x] - Dist[b/(d*(m+1)), Int[(d*x)^(m+1)*Sin[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3387

Int[((e_.)*(x_.))^(m_.)*Sin[(c_.) + (d_.)*(x_.)^(n_.)], x_Symbol] := Simp[((e*x)^(m+1)*Sin[c + d*x^n])/(e*(m+1)), x] - Dist[(d*n)/(e^n*(m+1)), Int[(e*x)^(m+n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_), x_Symbol] := Simp[((e*x)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{S(bx)}{x^9} dx &= -\frac{S(bx)}{8x^8} + \frac{1}{8}b \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx \\
 &= -\frac{S(bx)}{8x^8} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{1}{56}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{S(bx)}{8x^8} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} - \frac{1}{280}(b^5\pi^2) \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{S(bx)}{8x^8} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} - \frac{1}{840}(b^7\pi^3) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} + \frac{b^7\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x} - \frac{S(bx)}{8x^8} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} + \frac{1}{840}(b^9\pi^4) \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} + \frac{b^7\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x} + \frac{1}{840}b^8\pi^4 S(bx) - \frac{S(bx)}{8x^8} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}
 \end{aligned}$$

Mathematica [A] time = 0.0552173, size = 84, normalized size = 0.71

$$\frac{(\pi^4 b^8 x^8 - 105) S(bx) + bx (\pi^2 b^4 x^4 - 15) \sin\left(\frac{1}{2}\pi b^2 x^2\right) + \pi b^3 x^3 (\pi^2 b^4 x^4 - 3) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{840x^8}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]/x^9, x]

[Out] (b^3*Pi*x^3*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*FresnelS[b*x] + b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)

Maple [A] time = 0.047, size = 109, normalized size = 0.9

$$b^8 \left(-\frac{\text{FresnelS}(bx)}{8b^8x^8} - \frac{1}{56b^7x^7} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{56} \left(-\frac{1}{5b^5x^5} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{5} \left(-\frac{1}{3x^3b^3} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{3} \left(-\frac{1}{bx} \cos\left(\frac{b^2\pi x^2}{2}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)/x^9,x)

[Out] b^8*(-1/8*FresnelS(b*x)/b^8/x^8-1/56*sin(1/2*b^2*Pi*x^2)/b^7/x^7+1/56*Pi*(-1/5/b^5/x^5*cos(1/2*b^2*Pi*x^2)-1/5*Pi*(-1/3*sin(1/2*b^2*Pi*x^2)/b^3/x^3+1/3*Pi*(-1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*FresnelS(b*x))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^9,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)/x^9, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^9,x, algorithm="fricas")

[Out] integral(fresnels(b*x)/x^9, x)

Sympy [A] time = 3.70966, size = 185, normalized size = 1.55

$$\frac{\pi^4 b^8 S(bx) \Gamma\left(-\frac{5}{4}\right)}{3584 \Gamma\left(\frac{7}{4}\right)} + \frac{\pi^3 b^7 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{5}{4}\right)}{3584 x \Gamma\left(\frac{7}{4}\right)} + \frac{\pi^2 b^5 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{5}{4}\right)}{3584 x^3 \Gamma\left(\frac{7}{4}\right)} - \frac{3 \pi b^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{5}{4}\right)}{3584 x^5 \Gamma\left(\frac{7}{4}\right)} - \frac{15 b \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{5}{4}\right)}{3584 x^7 \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x**9, x)

[Out] pi**4*b**8*fresnels(b*x)*gamma(-5/4)/(3584*gamma(7/4)) + pi**3*b**7*cos(pi*b**2*x**2/2)*gamma(-5/4)/(3584*x*gamma(7/4)) + pi**2*b**5*sin(pi*b**2*x**2/2)*gamma(-5/4)/(3584*x**3*gamma(7/4)) - 3*pi*b**3*cos(pi*b**2*x**2/2)*gamma(-5/4)/(3584*x**5*gamma(7/4)) - 15*b*sin(pi*b**2*x**2/2)*gamma(-5/4)/(3584*x**7*gamma(7/4)) - 15*fresnels(b*x)*gamma(-5/4)/(512*x**8*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^9, x, algorithm="giac")

[Out] integrate(fresnels(b*x)/x^9, x)

3.18 $\int \frac{S(bx)}{x^{10}} dx$

Optimal. Leaf size=127

$$\frac{\pi^4 b^9 \operatorname{Si}\left(\frac{1}{2} b^2 \pi x^2\right)}{6912} + \frac{\pi^2 b^5 \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{1728 x^4} - \frac{b \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{72 x^8} + \frac{\pi^3 b^7 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3456 x^2} - \frac{\pi b^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{432 x^6} - \frac{S(bx)}{9 x^9}$$

[Out] $-(b^3 \pi \operatorname{Cos}[(b^2 \pi x^2)/2])/(432 x^6) + (b^7 \pi^3 \operatorname{Cos}[(b^2 \pi x^2)/2])/(3456 x^2) - \operatorname{FresnelS}[b x]/(9 x^9) - (b \operatorname{Sin}[(b^2 \pi x^2)/2])/(72 x^8) + (b^5 \pi^2 \operatorname{Sin}[(b^2 \pi x^2)/2])/(1728 x^4) + (b^9 \pi^4 \operatorname{SinIntegral}[(b^2 \pi x^2)/2])/6912$

Rubi [A] time = 0.152954, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6426, 3379, 3297, 3299}

$$\frac{\pi^4 b^9 \operatorname{Si}\left(\frac{1}{2} b^2 \pi x^2\right)}{6912} + \frac{\pi^2 b^5 \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{1728 x^4} - \frac{b \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{72 x^8} + \frac{\pi^3 b^7 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3456 x^2} - \frac{\pi b^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{432 x^6} - \frac{S(bx)}{9 x^9}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{FresnelS}[b x]/x^{10}, x]$

[Out] $-(b^3 \pi \operatorname{Cos}[(b^2 \pi x^2)/2])/(432 x^6) + (b^7 \pi^3 \operatorname{Cos}[(b^2 \pi x^2)/2])/(3456 x^2) - \operatorname{FresnelS}[b x]/(9 x^9) - (b \operatorname{Sin}[(b^2 \pi x^2)/2])/(72 x^8) + (b^5 \pi^2 \operatorname{Sin}[(b^2 \pi x^2)/2])/(1728 x^4) + (b^9 \pi^4 \operatorname{SinIntegral}[(b^2 \pi x^2)/2])/6912$

Rule 6426

$\operatorname{Int}[\operatorname{FresnelS}[(b_.)(x_)]*((d_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d x)^{(m+1)} \operatorname{FresnelS}[b x]/(d(m+1)), x] - \operatorname{Dist}[b/(d(m+1)), \operatorname{Int}[(d x)^{(m+1)} \operatorname{Sin}[(\pi b^2 x^2)/2], x], x] /;$ $\operatorname{FreeQ}\{b, d, m\}, x \} \ \&\& \ \operatorname{NeQ}[m, -1]$

Rule 3379

$\operatorname{Int}[(x_)^{(m_.)}*((a_.) + (b_.)\operatorname{Sin}[(c_.) + (d_.)(x_)^{(n_.)}])^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n] - 1)}(a + b \operatorname{Sin}[c + d x])^p, x], x, x^n], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x \} \ \&\& \ \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]] \ \&\& \ (\operatorname{EqQ}[p, 1] \ || \ \operatorname{EqQ}[m, n-1] \ || \ (\operatorname{IntegerQ}[p] \ \&\& \ \operatorname{GtQ}[\operatorname{Simplify}[($

$m + 1)/n], 0])$

Rule 3297

$\text{Int}[\text{((c_.) + (d_.)*(x_.))}^{(m_*)} \sin[(e_.) + (f_.)*(x_.)], x_Symbol] \text{ :> Simp}[\text{((c + d*x)}^{(m + 1)} \text{Sin}[e + f*x]) / (d*(m + 1)), x] - \text{Dist}[f / (d*(m + 1)), \text{Int}[(c + d*x)}^{(m + 1)} \text{Cos}[e + f*x], x], x] \text{ /; FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{LtQ}[m, -1]$

Rule 3299

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)] / ((c_.) + (d_.)*(x_.)), x_Symbol] \text{ :> Simp}[\text{SinIntegral}[e + f*x] / d, x] \text{ /; FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

Rubi steps

$$\begin{aligned}
 \int \frac{S(bx)}{x^{10}} dx &= -\frac{S(bx)}{9x^9} + \frac{1}{9}b \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx \\
 &= -\frac{S(bx)}{9x^9} + \frac{1}{18}b \text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^5} dx, x, x^2\right) \\
 &= -\frac{S(bx)}{9x^9} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{1}{144}(b^3\pi) \text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^4} dx, x, x^2\right) \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{S(bx)}{9x^9} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} - \frac{1}{864}(b^5\pi^2) \text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3} dx, x, x^2\right) \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{S(bx)}{9x^9} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} - \frac{(b^7\pi^3) \text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2} dx, x, x^2\right)}{3456} \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} + \frac{b^7\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2} - \frac{S(bx)}{9x^9} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{(b^9\pi^4) \text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x} dx, x, x^2\right)}{6912} \\
 &= -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} + \frac{b^7\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2} - \frac{S(bx)}{9x^9} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{b^9\pi^4 \text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}
 \end{aligned}$$

Mathematica [A] time = 0.165594, size = 96, normalized size = 0.76

$$\frac{\pi^4 b^9 \operatorname{Si}\left(\frac{1}{2} b^2 \pi x^2\right) + \frac{4b(\pi^2 b^4 x^4 - 24) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^8} + \frac{2\pi b^3(\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{x^6} - \frac{768S(bx)}{x^9}}{6912}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]/x^10,x]

[Out] $\left(\frac{2b^3\pi(-8 + b^4\pi^2x^4)\operatorname{Cos}\left[\frac{b^2\pi x^2}{2}\right]}{x^6} - \frac{768\operatorname{FresnelS}[bx]}{x^9} + \frac{4b(-24 + b^4\pi^2x^4)\operatorname{Sin}\left[\frac{b^2\pi x^2}{2}\right]}{x^8} + b^9\pi^4\operatorname{SinIntegral}\left[\frac{b^2\pi x^2}{2}\right]\right)/6912$

Maple [A] time = 0.047, size = 115, normalized size = 0.9

$$b^9 \left(-\frac{\operatorname{FresnelS}(bx)}{9b^9x^9} - \frac{1}{72b^8x^8} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{72} \left(-\frac{1}{6b^6x^6} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{6} \left(-\frac{1}{4x^4b^4} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{4} \left(-\frac{1}{2b^2x^2} \cos\left(\frac{b^2\pi x^2}{2}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)/x^10,x)

[Out] $b^9 \left(-\frac{1}{9} \operatorname{FresnelS}(bx) / b^9 / x^9 - \frac{1}{72} \sin(1/2 * b^2 * \pi * x^2) / b^8 / x^8 + \frac{1}{72} \pi \left(-\frac{1}{6} / b^6 / x^6 * \cos(1/2 * b^2 * \pi * x^2) - \frac{1}{6} \pi \left(-\frac{1}{4} \sin(1/2 * b^2 * \pi * x^2) / b^4 / x^4 + \frac{1}{4} \pi \left(-\frac{1}{2} / b^2 / x^2 * \cos(1/2 * b^2 * \pi * x^2) - \frac{1}{4} \pi \operatorname{Si}(1/2 * b^2 * \pi * x^2) \right) \right) \right) \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{fresnels}(bx)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^10,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)/x^10, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^10,x, algorithm="fricas")

[Out] integral(fresnels(b*x)/x^10, x)

Sympy [A] time = 5.55253, size = 48, normalized size = 0.38

$$-\frac{\pi b^3 \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ \frac{1}{2}, \frac{3}{2}, \frac{7}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{48 x^6 \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x**10,x)

[Out] $-\pi * b^{**3} * \text{gamma}(3/4) * \text{hyper}((-3/2, 3/4), (-1/2, 3/2, 7/4), -\pi^{**2} * b^{**4} * x^{**4} / 16) / (48 * x^{**6} * \text{gamma}(7/4))$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)/x^10,x, algorithm="giac")

[Out] integrate(fresnels(b*x)/x^10, x)

3.19 $\int (c + dx)^3 S(a + bx) dx$

Optimal. Leaf size=296

$$-\frac{2d^2(bc - ad) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi^2 b^4} + \frac{d^2(a + bx)^2(bc - ad) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^4} - \frac{3d(bc - ad)^2 \text{FresnelC}(a + bx)}{2\pi b^4} - \frac{(bc - ad)^4}{4b}$$

[Out] $((b*c - a*d)^3 \text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}) + (3*d*(b*c - a*d)^2*(a + b*x)*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(2*b^4*\text{Pi}) + (d^2*(b*c - a*d)*(a + b*x)^2*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}) + (d^3*(a + b*x)^3*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(4*b^4*\text{Pi}) - (3*d*(b*c - a*d)^2*\text{FresnelC}[a + b*x])/(2*b^4*\text{Pi}) - ((b*c - a*d)^4*\text{FresnelS}[a + b*x])/(4*b^4*d) + (3*d^3*\text{FresnelS}[a + b*x])/(4*b^4*\text{Pi}^2) + ((c + d*x)^4*\text{FresnelS}[a + b*x])/(4*d) - (2*d^2*(b*c - a*d)*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}^2) - (3*d^3*(a + b*x)*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(4*b^4*\text{Pi}^2)$

Rubi [A] time = 0.400987, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637, 3386}

$$-\frac{2d^2(bc - ad) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi^2 b^4} + \frac{d^2(a + bx)^2(bc - ad) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^4} - \frac{3d(bc - ad)^2 \text{FresnelC}(a + bx)}{2\pi b^4} - \frac{(bc - ad)^4}{4b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{FresnelS}[a + b*x], x]$

[Out] $((b*c - a*d)^3 \text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}) + (3*d*(b*c - a*d)^2*(a + b*x)*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(2*b^4*\text{Pi}) + (d^2*(b*c - a*d)*(a + b*x)^2*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}) + (d^3*(a + b*x)^3*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(4*b^4*\text{Pi}) - (3*d*(b*c - a*d)^2*\text{FresnelC}[a + b*x])/(2*b^4*\text{Pi}) - ((b*c - a*d)^4*\text{FresnelS}[a + b*x])/(4*b^4*d) + (3*d^3*\text{FresnelS}[a + b*x])/(4*b^4*\text{Pi}^2) + ((c + d*x)^4*\text{FresnelS}[a + b*x])/(4*d) - (2*d^2*(b*c - a*d)*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}^2) - (3*d^3*(a + b*x)*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(4*b^4*\text{Pi}^2)$

Rule 6428

$\text{Int}[\text{FresnelS}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] :> \text{Simp}[(c + d*x)^{(m + 1)}*\text{FresnelS}[a + b*x]/(d*(m + 1)), x] - \text{Dist}[b/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2], x], x] /; \text{FreeQ}\{a, b,$

c, d}, x] && IGtQ[m, 0]

Rule 3433

Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_Symbol] := Module[{k = If[FractionQ[n], Denominator[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3385

Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[

$e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[\sin[c + d*x]/d, x] /;$
 $\text{FreeQ}\{c, d\}, x]$

Rule 3386

$\text{Int}[\text{Cos}[(c_.) + (d_.)*(x_.)^{(n_.)}]*((e_.)*(x_.))^{(m_.)}, x_Symbol] \text{ :> } \text{Simp}[(e^{(n-1)}*(e*x)^{(m-n+1)}*\sin[c + d*x^n])/(d*n), x] - \text{Dist}[(e^n*(m-n+1))/(d*n), \text{Int}[(e*x)^{(m-n)}*\sin[c + d*x^n], x], x] /;$
 $\text{FreeQ}\{c, d, e\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[n, m + 1]$

Rubi steps

$$\begin{aligned} \int (c + dx)^3 S(a + bx) dx &= \frac{(c + dx)^4 S(a + bx)}{4d} - \frac{b \int (c + dx)^4 \sin\left(\frac{1}{2}\pi(a + bx)^2\right) dx}{4d} \\ &= \frac{(c + dx)^4 S(a + bx)}{4d} - \frac{\text{Subst}\left(\int \left(b^4 c^4 \left(1 + \frac{ad(-4b^3 c^3 + 6ab^2 c^2 d - 4a^2 bcd^2 + a^3 d^3)}{b^4 c^4}\right) \sin\left(\frac{\pi x^2}{2}\right) + 4b^3 c^3 d\right) dx}{b^4 c^4}, x, a + bx\right)}{b^4 c^4} \\ &= \frac{(c + dx)^4 S(a + bx)}{4d} - \frac{d^3 \text{Subst}\left(\int x^4 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{4b^4} - \frac{(d^2(bc - ad)) \text{Subst}\left(\int x^3 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^4} \\ &= \frac{3d(bc - ad)^2(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{2b^4\pi} + \frac{d^3(a + bx)^3 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{4b^4\pi} - \frac{(bc - ad)^4 S(a + bx)}{4b^4 d} \\ &= \frac{(bc - ad)^3 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^4\pi} + \frac{3d(bc - ad)^2(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{2b^4\pi} + \frac{d^2(bc - ad)(a + bx)^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^4} \\ &= \frac{(bc - ad)^3 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^4\pi} + \frac{3d(bc - ad)^2(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{2b^4\pi} + \frac{d^2(bc - ad)(a + bx)^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^4} \end{aligned}$$

Mathematica [A] time = 0.813954, size = 424, normalized size = 1.43

$$\frac{S(a + bx) \left(6\pi^2 b^2 c^2 d (b^2 x^2 - a^2) + 4\pi^2 bcd^2 (a^3 + b^3 x^3) + d^3 (-\pi^2 a^4 + \pi^2 b^4 x^4 + 3) + 4\pi^2 b^3 c^3 (a + bx) + 4\pi a^2 bcd^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)\right)}{b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*FresnelS[a + b*x],x]

```
[Out] (4*b^3*c^3*Pi*Cos[(Pi*(a + b*x)^2)/2] - 6*a*b^2*c^2*d*Pi*Cos[(Pi*(a + b*x)^2)/2] + 4*a^2*b*c*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a^3*d^3*Pi*Cos[(Pi*(a + b*x)^2)/2] + 6*b^3*c^2*d*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - 4*a*b^2*c*d^2*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + a^2*b*d^3*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + 4*b^3*c*d^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] - a*b^2*d^3*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] + b^3*d^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2] - 6*d*(b*c - a*d)^2*Pi*FresnelC[a + b*x] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*Pi^2*(-a^2 + b^2*x^2) + 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2 + b^4*Pi^2*x^4))*FresnelS[a + b*x] - 8*b*c*d^2*Sin[(Pi*(a + b*x)^2)/2] + 5*a*d^3*Sin[(Pi*(a + b*x)^2)/2] - 3*b*d^3*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Maple [A] time = 0.054, size = 400, normalized size = 1.4

$$\frac{1}{b} \left(\frac{\text{FresnelS}(bx + a) (d(bx + a) - ad + bc)^4}{4db^3} - \frac{1}{4db^3} \left(-\frac{d^4(bx + a)^3}{\pi} \cos\left(\frac{\pi(bx + a)^2}{2}\right) + 3 \frac{d^4}{\pi} \left(\frac{(bx + a) \sin\left(\frac{1}{2}\pi(bx + a)^2\right)}{\pi} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*FresnelS(b*x+a), x)
```

```
[Out] 1/b*(1/4*FresnelS(b*x+a)*(d*(b*x+a)-a*d+b*c)^4/b^3/d-1/4/b^3/d*(-d^4/Pi*(b*x+a)^3*cos(1/2*Pi*(b*x+a)^2)+3*d^4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-1/Pi*FresnelS(b*x+a))-(-4*a*d^4+4*b*c*d^3)/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)+2*(-4*a*d^4+4*b*c*d^3)/Pi^2*sin(1/2*Pi*(b*x+a)^2)-(6*a^2*d^4-12*a*b*c*d^3+6*b^2*c^2*d^2)/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+(6*a^2*d^4-12*a*b*c*d^3+6*b^2*c^2*d^2)/Pi*FresnelC(b*x+a)-(-4*a^3*d^4+12*a^2*b*c*d^3-12*a*b^2*c^2*d^2+4*b^3*c^3*d)/Pi*cos(1/2*Pi*(b*x+a)^2)+a^4*d^4*FresnelS(b*x+a)-4*a^3*b*c*d^3*FresnelS(b*x+a)+6*a^2*b^2*c^2*d^2*FresnelS(b*x+a)-4*a*b^3*c^3*d*FresnelS(b*x+a)+b^4*c^4*FresnelS(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \text{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnels(b*x+a), x, algorithm="maxima")
```

[Out] integrate((d*x + c)^3*fresnels(b*x + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3\right)\text{fresnels}(bx + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*fresnels(b*x+a),x, algorithm="fricas")

[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*fresnels(b*x + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 S(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*fresnels(b*x+a),x)

[Out] Integral((c + d*x)**3*fresnels(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \text{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*fresnels(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^3*fresnels(b*x + a), x)

3.20 $\int (c + dx)^2 S(a + bx) dx$

Optimal. Leaf size=193

$$\frac{d(bc - ad)\text{FresnelC}(a + bx)}{\pi b^3} - \frac{(bc - ad)^3 S(a + bx)}{3b^3 d} + \frac{(bc - ad)^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} + \frac{d(a + bx)(bc - ad) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3}$$

```
[Out] ((b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d*(b*c - a*d)*(a + b*x)
*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/
2])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x])/(b^3*Pi) - ((b*c - a*d)^
3*FresnelS[a + b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelS[a + b*x])/(3*d) - (2
*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rubi [A] time = 0.225743, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637}

$$\frac{d(bc - ad)\text{FresnelC}(a + bx)}{\pi b^3} - \frac{(bc - ad)^3 S(a + bx)}{3b^3 d} + \frac{(bc - ad)^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} + \frac{d(a + bx)(bc - ad) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelS[a + b*x], x]
```

```
[Out] ((b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d*(b*c - a*d)*(a + b*x)
*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/
2])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x])/(b^3*Pi) - ((b*c - a*d)^
3*FresnelS[a + b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelS[a + b*x])/(3*d) - (2
*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)]^(p_.), x_Symbol] := Module[{k = If[FractionQ[n], Denominat
```

or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3385

Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 S(a + bx) dx &= \frac{(c + dx)^3 S(a + bx)}{3d} - \frac{b \int (c + dx)^3 \sin\left(\frac{1}{2}\pi(a + bx)^2\right) dx}{3d} \\
&= \frac{(c + dx)^3 S(a + bx)}{3d} - \frac{\text{Subst}\left(\int\left(b^3 c^3\left(1 - \frac{ad(3b^2 c^2 - 3abcd + a^2 d^2)}{b^3 c^3}\right)\sin\left(\frac{\pi x^2}{2}\right) + 3b^2 c^2 d\left(1 + \frac{ad(-2bc)}{b^2 c^2}\right)\right) dx}{3d} \\
&= \frac{(c + dx)^3 S(a + bx)}{3d} - \frac{d^2 \text{Subst}\left(\int x^3 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{3b^3} - \frac{(d(bc - ad)) \text{Subst}\left(\int x^2 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^3} \\
&= \frac{d(bc - ad)(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^3 \pi} - \frac{(bc - ad)^3 S(a + bx)}{3b^3 d} + \frac{(c + dx)^3 S(a + bx)}{3d} - \frac{d^2 \text{Subst}\left(\int x^2 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^3} \\
&= \frac{(bc - ad)^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^3 \pi} + \frac{d(bc - ad)(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^3 \pi} + \frac{d^2(a + bx)^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{3b^3 \pi} \\
&= \frac{(bc - ad)^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^3 \pi} + \frac{d(bc - ad)(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^3 \pi} + \frac{d^2(a + bx)^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{3b^3 \pi}
\end{aligned}$$

Mathematica [A] time = 0.457134, size = 236, normalized size = 1.22

$$\pi^2 S(a + bx) \left(-3a^2 bcd + a^3 d^2 + 3ab^2 c^2 + b^3 x(3c^2 + 3cdx + d^2 x^2)\right) + \pi a^2 d^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right) + 3\pi b^2 c^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*FresnelS[a + b*x],x]

[Out] (3*b^2*c^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - 3*a*b*c*d*Pi*Cos[(Pi*(a + b*x)^2)/2] + a^2*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] + 3*b^2*c*d*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - a*b*d^2*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*d^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] + 3*d*(-(b*c) + a*d)*Pi*FresnelC[a + b*x] + Pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelS[a + b*x] - 2*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)

Maple [A] time = 0.055, size = 251, normalized size = 1.3

$$\frac{1}{b} \left(\frac{\text{FresnelS}(bx + a) (d(bx + a) - ad + bc)^3}{3db^2} - \frac{1}{3db^2} \left(-\frac{d^3(bx + a)^2}{\pi} \cos\left(\frac{\pi(bx + a)^2}{2}\right) + 2 \frac{d^3 \sin\left(\frac{1}{2}\pi(bx + a)^2\right)}{\pi^2} - \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^2*FresnelS(b*x+a),x)`

[Out] $\frac{1}{b} \left(\frac{1}{3} \text{FresnelS}(b*x+a) * (d*(b*x+a) - a*d + b*c)^3 / b^2 / d - \frac{1}{3} / b^2 / d * (-d^3 / \text{Pi} * (b*x+a)^2 * \cos(1/2 * \text{Pi} * (b*x+a)^2) + 2*d^3 / \text{Pi}^2 * \sin(1/2 * \text{Pi} * (b*x+a)^2) - (-3*a*d^3 + 3*b*c*d^2) / \text{Pi} * (b*x+a) * \cos(1/2 * \text{Pi} * (b*x+a)^2) + (-3*a*d^3 + 3*b*c*d^2) / \text{Pi} * \text{FresnelC}(b*x+a) - (3*a^2*d^3 - 6*a*b*c*d^2 + 3*b^2*c^2*d) / \text{Pi} * \cos(1/2 * \text{Pi} * (b*x+a)^2) - a^3*d^3 * \text{FresnelS}(b*x+a) + 3*a^2*b*c*d^2 * \text{FresnelS}(b*x+a) - 3*a*b^2*c^2*d * \text{FresnelS}(b*x+a) + b^3*c^3 * \text{FresnelS}(b*x+a) \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \text{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*fresnels(b*x+a),x, algorithm="maxima")`

[Out] `integrate((d*x + c)^2*fresnels(b*x + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((d^2 x^2 + 2 c d x + c^2) \text{fresnels}(bx + a), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*fresnels(b*x+a),x, algorithm="fricas")`

[Out] `integral((d^2*x^2 + 2*c*d*x + c^2)*fresnels(b*x + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 S(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnels(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*fresnels(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \text{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*fresnels(b*x + a), x)
```

3.21 $\int (c + dx)S(a + bx) dx$

Optimal. Leaf size=121

$$-\frac{(bc - ad)^2 S(a + bx)}{2b^2 d} + \frac{(bc - ad) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^2} - \frac{d \operatorname{FresnelC}(a + bx)}{2\pi b^2} + \frac{d(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^2} + \frac{(c + dx)^2 S(a + bx)}{2d}$$

[Out] $((b*c - a*d)*\operatorname{Cos}[(\operatorname{Pi}*(a + b*x)^2)/2])/(b^2*\operatorname{Pi}) + (d*(a + b*x)*\operatorname{Cos}[(\operatorname{Pi}*(a + b*x)^2)/2])/(2*b^2*\operatorname{Pi}) - (d*\operatorname{FresnelC}[a + b*x])/(2*b^2*\operatorname{Pi}) - ((b*c - a*d)^2*\operatorname{FresnelS}[a + b*x])/(2*b^2*d) + ((c + d*x)^2*\operatorname{FresnelS}[a + b*x])/(2*d)$

Rubi [A] time = 0.1145, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352}

$$-\frac{(bc - ad)^2 S(a + bx)}{2b^2 d} + \frac{(bc - ad) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^2} - \frac{d \operatorname{FresnelC}(a + bx)}{2\pi b^2} + \frac{d(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^2} + \frac{(c + dx)^2 S(a + bx)}{2d}$$

Antiderivative was successfully verified.

[In] `Int[(c + d*x)*FresnelS[a + b*x],x]`

[Out] $((b*c - a*d)*\operatorname{Cos}[(\operatorname{Pi}*(a + b*x)^2)/2])/(b^2*\operatorname{Pi}) + (d*(a + b*x)*\operatorname{Cos}[(\operatorname{Pi}*(a + b*x)^2)/2])/(2*b^2*\operatorname{Pi}) - (d*\operatorname{FresnelC}[a + b*x])/(2*b^2*\operatorname{Pi}) - ((b*c - a*d)^2*\operatorname{FresnelS}[a + b*x])/(2*b^2*d) + ((c + d*x)^2*\operatorname{FresnelS}[a + b*x])/(2*d)$

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] :> S
imp[(((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_.))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_.))^(n_.)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, xⁿ], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3385

Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*xⁿ]/(d*n), x] + Dist[(eⁿ*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*xⁿ], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)S(a + bx) dx &= \frac{(c + dx)^2 S(a + bx)}{2d} - \frac{b \int (c + dx)^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right) dx}{2d} \\
&= \frac{(c + dx)^2 S(a + bx)}{2d} - \frac{\text{Subst}\left(\int\left(b^2 c^2\left(1 + \frac{ad(-2bc+ad)}{b^2 c^2}\right)\sin\left(\frac{\pi x^2}{2}\right) + 2bcd\left(1 - \frac{ad}{bc}\right)x \sin\left(\frac{\pi x^2}{2}\right) + d^2 x^2 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{2b^2 d} \\
&= \frac{(c + dx)^2 S(a + bx)}{2d} - \frac{d \text{Subst}\left(\int x^2 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{2b^2} - \frac{(bc - ad) \text{Subst}\left(\int x \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^2} \\
&= \frac{d(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{2b^2 \pi} - \frac{(bc - ad)^2 S(a + bx)}{2b^2 d} + \frac{(c + dx)^2 S(a + bx)}{2d} - \frac{(bc - ad) \text{Subst}\left(\int x \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^2} \\
&= \frac{(bc - ad) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^2 \pi} + \frac{d(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{2b^2 \pi} - \frac{dC(a + bx)}{2b^2 \pi} - \frac{(bc - ad)^2 S(a + bx)}{2b^2 d}
\end{aligned}$$

Mathematica [A] time = 0.208895, size = 61, normalized size = 0.5

$$\frac{(-ad + 2bc + bdx) \left(\pi(a + bx)S(a + bx) + \cos\left(\frac{1}{2}\pi(a + bx)^2\right) \right) - d\text{FresnelC}(a + bx)}{2\pi b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*FresnelS[a + b*x], x]

[Out] $(-(d*\text{FresnelC}[a + b*x]) + (2*b*c - a*d + b*d*x)*(Cos[(Pi*(a + b*x)^2]/2) + Pi*(a + b*x)*\text{FresnelS}[a + b*x]))/(2*b^2*Pi)$

Maple [A] time = 0.051, size = 108, normalized size = 0.9

$$\frac{1}{b} \left(\frac{\text{FresnelS}(bx + a)}{b} \left(\frac{d(bx + a)^2}{2} - ad(bx + a) + bc(bx + a) \right) - \frac{1}{2b} \left(-\frac{d(bx + a)}{\pi} \cos\left(\frac{\pi(bx + a)^2}{2}\right) + \frac{d\text{FresnelC}(bx + a)}{\pi} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*FresnelS(b*x+a), x)

[Out] $1/b*(\text{FresnelS}(b*x+a)/b*(1/2*d*(b*x+a)^2-a*d*(b*x+a)+b*c*(b*x+a))-1/2/b*(-d/Pi*(b*x+a)*\cos(1/2*Pi*(b*x+a)^2)+d/Pi*\text{FresnelC}(b*x+a)-(-2*a*d+2*b*c)/Pi*\cos$

(1/2*Pi*(b*x+a)^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\text{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*fresnels(b*x+a),x, algorithm="maxima")

[Out] integrate((d*x + c)*fresnels(b*x + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)\text{fresnels}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*fresnels(b*x+a),x, algorithm="fricas")

[Out] integral((d*x + c)*fresnels(b*x + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) S(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*fresnels(b*x+a),x)

[Out] Integral((c + d*x)*fresnels(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\text{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnels(b*x + a), x)
```


3.22 $\int S(a + bx) dx$

Optimal. Leaf size=36

$$\frac{(a + bx)S(a + bx)}{b} + \frac{\cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b}$$

[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b

Rubi [A] time = 0.0063385, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6418}

$$\frac{(a + bx)S(a + bx)}{b} + \frac{\cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[a + b*x], x]

[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b

Rule 6418

Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] := Simp[((a + b*x)*FresnelS[a + b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int S(a + bx) dx = \frac{\cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b\pi} + \frac{(a + bx)S(a + bx)}{b}$$

Mathematica [B] time = 0.0286256, size = 89, normalized size = 2.47

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx + \frac{1}{2}\pi b^2 x^2\right)}{\pi b} + \frac{\cos\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx + \frac{1}{2}\pi b^2 x^2\right)}{\pi b} + xS(a + bx) + \frac{aS(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[a + b*x],x]

[Out] (Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a + b*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi)

Maple [A] time = 0.046, size = 33, normalized size = 0.9

$$\frac{1}{b} \left((bx + a) \operatorname{FresnelS}(bx + a) + \frac{1}{\pi} \cos \left(\frac{\pi (bx + a)^2}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x+a),x)

[Out] 1/b*((b*x+a)*FresnelS(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a),x, algorithm="maxima")

[Out] integrate(fresnels(b*x + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(\operatorname{fresnels}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a),x, algorithm="fricas")

```
[Out] integral(fresnels(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int S(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x)
```

```
[Out] Integral(fresnels(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a), x)
```

$$3.23 \quad \int \frac{S(a+bx)}{c+dx} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{S(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable[FresnelS[a + b*x]/(c + d*x), x]

Rubi [A] time = 0.0146781, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[a + b*x]/(c + d*x), x]

[Out] Defer[Int][FresnelS[a + b*x]/(c + d*x), x]

Rubi steps

$$\int \frac{S(a+bx)}{c+dx} dx = \int \frac{S(a+bx)}{c+dx} dx$$

Mathematica [A] time = 0.0248727, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[a + b*x]/(c + d*x), x]

[Out] Integrate[FresnelS[a + b*x]/(c + d*x), x]

Maple [A] time = 0.369, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x+a)/(d*x+c),x)

[Out] int(FresnelS(b*x+a)/(d*x+c),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] integrate(fresnels(b*x + a)/(d*x + c), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral(fresnels(b*x + a)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(fresnels(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)/(d*x + c), x)
```

$$3.24 \quad \int \frac{S(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{S(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[FresnelS[a + b*x]/(c + d*x)^2, x]

Rubi [A] time = 0.0142623, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[a + b*x]/(c + d*x)^2,x]

[Out] Defer[Int][FresnelS[a + b*x]/(c + d*x)^2, x]

Rubi steps

$$\int \frac{S(a+bx)}{(c+dx)^2} dx = \int \frac{S(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 4.8644, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[a + b*x]/(c + d*x)^2,x]

[Out] Integrate[FresnelS[a + b*x]/(c + d*x)^2, x]

Maple [A] time = 0.413, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x+a)/(d*x+c)^2,x)

[Out] int(FresnelS(b*x+a)/(d*x+c)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] integrate(fresnels(b*x + a)/(d*x + c)^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx + a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(fresnels(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x+a)/(d*x+c)**2,x)`

[Out] `Integral(fresnels(a + b*x)/(c + d*x)**2, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x+a)/(d*x+c)^2,x, algorithm="giac")`

[Out] `integrate(fresnels(b*x + a)/(d*x + c)^2, x)`

3.25 $\int x^3 S(a + bx) dx$

Optimal. Leaf size=229

$$-\frac{3a^2 \text{FresnelC}(a + bx)}{2\pi b^4} - \frac{a^4 S(a + bx)}{4b^4} - \frac{a^3 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^4} + \frac{3a^2(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^4} + \frac{3S(a + bx)}{4\pi^2 b^4} + \frac{2a \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{4\pi^2 b^4}$$

```
[Out] -((a^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi)) + (3*a^2*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) - (a*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + ((a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi) - (3*a^2*FresnelC[a + b*x])/(2*b^4*Pi) - (a^4*FresnelS[a + b*x])/(4*b^4) + (3*FresnelS[a + b*x])/(4*b^4*Pi^2) + (x^4*FresnelS[a + b*x])/4 + (2*a*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Rubi [A] time = 0.184451, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637, 3386}

$$-\frac{3a^2 \text{FresnelC}(a + bx)}{2\pi b^4} - \frac{a^4 S(a + bx)}{4b^4} - \frac{a^3 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^4} + \frac{3a^2(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^4} + \frac{3S(a + bx)}{4\pi^2 b^4} + \frac{2a \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{4\pi^2 b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[a + b*x], x]
```

```
[Out] -((a^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi)) + (3*a^2*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) - (a*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + ((a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi) - (3*a^2*FresnelC[a + b*x])/(2*b^4*Pi) - (a^4*FresnelS[a + b*x])/(4*b^4) + (3*FresnelS[a + b*x])/(4*b^4*Pi^2) + (x^4*FresnelS[a + b*x])/4 + (2*a*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_Symbol] := Module[{k = If[FractionQ[n], Denominator[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] := Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^3 S(a + bx) dx &= \frac{1}{4} x^4 S(a + bx) - \frac{1}{4} b \int x^4 \sin\left(\frac{1}{2} \pi (a + bx)^2\right) dx \\
&= \frac{1}{4} x^4 S(a + bx) - \frac{\text{Subst}\left(\int \left(a^4 \sin\left(\frac{\pi x^2}{2}\right) - 4a^3 x \sin\left(\frac{\pi x^2}{2}\right) + 6a^2 x^2 \sin\left(\frac{\pi x^2}{2}\right) - 4ax^3 \sin\left(\frac{\pi x^2}{2}\right) + x^4 \sin\left(\frac{\pi x^2}{2}\right)\right) dx, x, a + bx}{4b^4}}{4b^4} \\
&= \frac{1}{4} x^4 S(a + bx) - \frac{\text{Subst}\left(\int x^4 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{4b^4} + \frac{a \text{Subst}\left(\int x^3 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^4} - \frac{(3a^2)}{b^4} \\
&= \frac{3a^2(a + bx) \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{2b^4 \pi} + \frac{(a + bx)^3 \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{4b^4 \pi} - \frac{a^4 S(a + bx)}{4b^4} + \frac{1}{4} x^4 S(a + bx) + \frac{a \text{Subst}\left(\int x^3 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^4} \\
&= -\frac{a^3 \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{b^4 \pi} + \frac{3a^2(a + bx) \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{2b^4 \pi} - \frac{a(a + bx)^2 \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{b^4 \pi} + \frac{(a + bx)^3 \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{b^4 \pi} \\
&= -\frac{a^3 \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{b^4 \pi} + \frac{3a^2(a + bx) \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{2b^4 \pi} - \frac{a(a + bx)^2 \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{b^4 \pi} + \frac{(a + bx)^3 \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{b^4 \pi}
\end{aligned}$$

Mathematica [A] time = 0.323819, size = 166, normalized size = 0.72

$$\frac{(-\pi^2 a^4 + \pi^2 b^4 x^4 + 3) S(a + bx) - 6\pi a^2 \text{FresnelC}(a + bx) - \pi a^3 \cos\left(\frac{1}{2} \pi (a + bx)^2\right) + \pi a^2 b x \cos\left(\frac{1}{2} \pi (a + bx)^2\right) - \pi a b^2 x^2 \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{4\pi^2 b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[a + b*x], x]
```

```
[Out] (-a^3*Pi*Cos[(Pi*(a + b*x)^2)/2]) + a^2*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - a
*b^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] + b^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2] -
6*a^2*Pi*FresnelC[a + b*x] + (3 - a^4*Pi^2 + b^4*Pi^2*x^4)*FresnelS[a + b*x]
```

] + 5*a*Sin[(Pi*(a + b*x)^2)/2] - 3*b*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)

Maple [A] time = 0.052, size = 189, normalized size = 0.8

$$\frac{1}{b^4} \left(\frac{\text{FresnelS}(bx+a)b^4x^4}{4} + \frac{(bx+a)^3}{4\pi} \cos\left(\frac{\pi(bx+a)^2}{2}\right) - \frac{3}{4\pi} \left(\frac{bx+a}{\pi} \sin\left(\frac{\pi(bx+a)^2}{2}\right) - \frac{\text{FresnelS}(bx+a)}{\pi} \right) - \frac{a}{\pi} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*FresnelS(b*x+a),x)

[Out] 1/b^4*(1/4*FresnelS(b*x+a)*b^4*x^4+1/4/Pi*(b*x+a)^3*cos(1/2*Pi*(b*x+a)^2)-3/4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-1/Pi*FresnelS(b*x+a))-a/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*sin(1/2*Pi*(b*x+a)^2)+3/2*a^2/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)-3/2*a^2/Pi*FresnelC(b*x+a)-a^3/Pi*cos(1/2*Pi*(b*x+a)^2)-1/4*a^4*FresnelS(b*x+a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnels}(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*fresnels(b*x+a),x, algorithm="maxima")

[Out] integrate(x^3*fresnels(b*x + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \text{fresnels}(bx+a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*fresnels(b*x+a),x, algorithm="fricas")

[Out] `integral(x^3*fresnels(b*x + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 S(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*fresnels(b*x+a),x)`

[Out] `Integral(x**3*fresnels(a + b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*fresnels(b*x+a),x, algorithm="giac")`

[Out] `integrate(x^3*fresnels(b*x + a), x)`

3.26 $\int x^2 S(a + bx) dx$

Optimal. Leaf size=147

$$\frac{a^3 S(a + bx)}{3b^3} + \frac{a^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} + \frac{a \operatorname{FresnelC}(a + bx)}{\pi b^3} - \frac{2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{3\pi^2 b^3} - \frac{a(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} + \frac{(a + bx)^3 \operatorname{FresnelS}(a + bx)}{3} - \frac{(2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right))}{(3b^3 \pi^2)}$$

[Out] $(a^2 \operatorname{Cos}[(\operatorname{Pi}(a + b*x)^2)/2])/(b^3 \operatorname{Pi}) - (a*(a + b*x)*\operatorname{Cos}[(\operatorname{Pi}(a + b*x)^2)/2])/(b^3 \operatorname{Pi}) + ((a + b*x)^2 * \operatorname{Cos}[(\operatorname{Pi}(a + b*x)^2)/2])/(3*b^3 \operatorname{Pi}) + (a*\operatorname{FresnelC}[a + b*x])/(b^3 \operatorname{Pi}) + (a^3*\operatorname{FresnelS}[a + b*x])/(3*b^3) + (x^3*\operatorname{FresnelS}[a + b*x])/3 - (2*\operatorname{Sin}[(\operatorname{Pi}(a + b*x)^2)/2])/(3*b^3*\operatorname{Pi}^2)$

Rubi [A] time = 0.128683, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637}

$$\frac{a^3 S(a + bx)}{3b^3} + \frac{a^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} + \frac{a \operatorname{FresnelC}(a + bx)}{\pi b^3} - \frac{2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{3\pi^2 b^3} - \frac{a(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} + \frac{(a + bx)^3 \operatorname{FresnelS}(a + bx)}{3} - \frac{(2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right))}{(3b^3 \pi^2)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2*\operatorname{FresnelS}[a + b*x], x]$

[Out] $(a^2 \operatorname{Cos}[(\operatorname{Pi}(a + b*x)^2)/2])/(b^3 \operatorname{Pi}) - (a*(a + b*x)*\operatorname{Cos}[(\operatorname{Pi}(a + b*x)^2)/2])/(b^3 \operatorname{Pi}) + ((a + b*x)^2 * \operatorname{Cos}[(\operatorname{Pi}(a + b*x)^2)/2])/(3*b^3 \operatorname{Pi}) + (a*\operatorname{FresnelC}[a + b*x])/(b^3 \operatorname{Pi}) + (a^3*\operatorname{FresnelS}[a + b*x])/(3*b^3) + (x^3*\operatorname{FresnelS}[a + b*x])/3 - (2*\operatorname{Sin}[(\operatorname{Pi}(a + b*x)^2)/2])/(3*b^3*\operatorname{Pi}^2)$

Rule 6428

$\operatorname{Int}[\operatorname{FresnelS}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] := \operatorname{Simp}[((c + d*x)^{(m + 1)}*\operatorname{FresnelS}[a + b*x])/(d*(m + 1)), x] - \operatorname{Dist}[b/(d*(m + 1)), \operatorname{Int}[(c + d*x)^{(m + 1)}*\operatorname{Sin}[(\operatorname{Pi}(a + b*x)^2)/2], x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{IGtQ}[m, 0]$

Rule 3433

$\operatorname{Int}(((g_.) + (h_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*\operatorname{Sin}[(c_.) + (d_.)*((e_.) + (f_.)*(x_.))^{(n_.)}])^{(p_.)}, x_Symbol] := \operatorname{Module}\{k = \operatorname{If}[\operatorname{FractionQ}[n], \operatorname{Denominator}[n], 1]\}, \operatorname{Dist}[k/f^{(m + 1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*\operatorname{Sin}[c + d*x^{(k*n)}])^p, x^{(k - 1)}*(f*g - e*h + h*x^k)^m, x], x], x], (e + f*x)^{(1/k)], x]$

] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, xⁿ], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3385

Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*xⁿ])/(d*n), x] + Dist[(eⁿ*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*xⁿ], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x^2 S(a+bx) dx &= \frac{1}{3} x^3 S(a+bx) - \frac{1}{3} b \int x^3 \sin\left(\frac{1}{2} \pi(a+bx)^2\right) dx \\
&= \frac{1}{3} x^3 S(a+bx) - \frac{\text{Subst}\left(\int\left(-a^3 \sin\left(\frac{\pi x^2}{2}\right) + 3a^2 x \sin\left(\frac{\pi x^2}{2}\right) - 3ax^2 \sin\left(\frac{\pi x^2}{2}\right) + x^3 \sin\left(\frac{\pi x^2}{2}\right)\right) dx, x, a\right)}{3b^3} \\
&= \frac{1}{3} x^3 S(a+bx) - \frac{\text{Subst}\left(\int x^3 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{3b^3} + \frac{a \text{Subst}\left(\int x^2 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{b^3} - \frac{a^2}{3b^3} \\
&= -\frac{a(a+bx) \cos\left(\frac{1}{2} \pi(a+bx)^2\right)}{b^3 \pi} + \frac{a^3 S(a+bx)}{3b^3} + \frac{1}{3} x^3 S(a+bx) - \frac{\text{Subst}\left(\int x \sin\left(\frac{\pi x}{2}\right) dx, x, (a+bx)\right)}{6b^3} \\
&= \frac{a^2 \cos\left(\frac{1}{2} \pi(a+bx)^2\right)}{b^3 \pi} - \frac{a(a+bx) \cos\left(\frac{1}{2} \pi(a+bx)^2\right)}{b^3 \pi} + \frac{(a+bx)^2 \cos\left(\frac{1}{2} \pi(a+bx)^2\right)}{3b^3 \pi} + \frac{aC(a+bx)}{b^3 \pi} \\
&= \frac{a^2 \cos\left(\frac{1}{2} \pi(a+bx)^2\right)}{b^3 \pi} - \frac{a(a+bx) \cos\left(\frac{1}{2} \pi(a+bx)^2\right)}{b^3 \pi} + \frac{(a+bx)^2 \cos\left(\frac{1}{2} \pi(a+bx)^2\right)}{3b^3 \pi} + \frac{aC(a+bx)}{b^3 \pi}
\end{aligned}$$

Mathematica [A] time = 0.23802, size = 115, normalized size = 0.78

$$\frac{\pi^2 (a^3 + b^3 x^3) S(a+bx) + \pi a^2 \cos\left(\frac{1}{2} \pi(a+bx)^2\right) + \pi b^2 x^2 \cos\left(\frac{1}{2} \pi(a+bx)^2\right) + 3\pi a \text{FresnelC}(a+bx) - 2 \sin\left(\frac{1}{2} \pi(a+bx)^2\right)}{3\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*FresnelS[a + b*x],x]

[Out] (a^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] + 3*a*Pi*FresnelC[a + b*x] + Pi^2*(a^3 + b^3*x^3)*FresnelS[a + b*x] - 2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)

Maple [A] time = 0.05, size = 121, normalized size = 0.8

$$\frac{1}{b^3} \left(\frac{b^3 x^3 \text{FresnelS}(bx+a)}{3} + \frac{(bx+a)^2}{3\pi} \cos\left(\frac{\pi (bx+a)^2}{2}\right) - \frac{2}{3\pi^2} \sin\left(\frac{\pi (bx+a)^2}{2}\right) - \frac{a(bx+a)}{\pi} \cos\left(\frac{\pi (bx+a)^2}{2}\right) + \frac{a^2}{3\pi} \cos\left(\frac{\pi (bx+a)^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*FresnelS(b*x+a),x)

[Out] $1/b^3*(1/3*b^3*x^3*\text{FresnelS}(b*x+a)+1/3/\text{Pi}*(b*x+a)^2*\cos(1/2*\text{Pi}*(b*x+a)^2)-2/3/\text{Pi}^2*\sin(1/2*\text{Pi}*(b*x+a)^2)-a/\text{Pi}*(b*x+a)*\cos(1/2*\text{Pi}*(b*x+a)^2)+a/\text{Pi}*\text{FresnelC}(b*x+a)+a^2/\text{Pi}*\cos(1/2*\text{Pi}*(b*x+a)^2)+1/3*a^3*\text{FresnelS}(b*x+a)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnels(b*x+a),x, algorithm="maxima")`

[Out] `integrate(x^2*fresnels(b*x + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \text{fresnels}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnels(b*x+a),x, algorithm="fricas")`

[Out] `integral(x^2*fresnels(b*x + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 S(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*fresnels(b*x+a),x)`

[Out] `Integral(x**2*fresnels(a + b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnels(b*x + a), x)
```

3.27 $\int xS(a + bx) dx$

Optimal. Leaf size=96

$$-\frac{a^2S(a + bx)}{2b^2} - \frac{\text{FresnelC}(a + bx)}{2\pi b^2} - \frac{a \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^2} + \frac{(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^2} + \frac{1}{2}x^2S(a + bx)$$

[Out] $-\left(\frac{a \cos\left[\frac{\pi}{2}(a + b^2x^2)\right]}{\pi b^2}\right) + \left(\frac{(a + b^2x^2) \cos\left[\frac{\pi}{2}(a + b^2x^2)\right]}{2\pi b^2}\right) - \frac{\text{FresnelC}[a + b^2x^2]}{2\pi b^2} - \frac{a^2 \text{FresnelS}[a + b^2x^2]}{2b^2} + \frac{x^2 \text{FresnelS}[a + b^2x^2]}{2}$

Rubi [A] time = 0.0672627, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352}

$$-\frac{a^2S(a + bx)}{2b^2} - \frac{\text{FresnelC}(a + bx)}{2\pi b^2} - \frac{a \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^2} + \frac{(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^2} + \frac{1}{2}x^2S(a + bx)$$

Antiderivative was successfully verified.

[In] Int[x*FresnelS[a + b*x],x]

[Out] $-\left(\frac{a \cos\left[\frac{\pi}{2}(a + b^2x^2)\right]}{\pi b^2}\right) + \left(\frac{(a + b^2x^2) \cos\left[\frac{\pi}{2}(a + b^2x^2)\right]}{2\pi b^2}\right) - \frac{\text{FresnelC}[a + b^2x^2]}{2\pi b^2} - \frac{a^2 \text{FresnelS}[a + b^2x^2]}{2b^2} + \frac{x^2 \text{FresnelS}[a + b^2x^2]}{2}$

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] :> Simp[(((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2]/2], x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_.))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_.))^(n_.)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominator[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x], (e + f*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, xⁿ], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3385

Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*xⁿ]/(d*n), x] + Dist[(eⁿ*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*xⁿ], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int xS(a+bx) dx &= \frac{1}{2}x^2S(a+bx) - \frac{1}{2}b \int x^2 \sin\left(\frac{1}{2}\pi(a+bx)^2\right) dx \\
&= \frac{1}{2}x^2S(a+bx) - \frac{\text{Subst}\left(\int\left(a^2 \sin\left(\frac{\pi x^2}{2}\right) - 2ax \sin\left(\frac{\pi x^2}{2}\right) + x^2 \sin\left(\frac{\pi x^2}{2}\right)\right) dx, x, a+bx\right)}{2b^2} \\
&= \frac{1}{2}x^2S(a+bx) - \frac{\text{Subst}\left(\int x^2 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{2b^2} + \frac{a \text{Subst}\left(\int x \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{b^2} - \frac{a^2 \text{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{2b^2} \\
&= \frac{(a+bx) \cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{a^2S(a+bx)}{2b^2} + \frac{1}{2}x^2S(a+bx) + \frac{a \text{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right) dx, x, (a+bx)^2\right)}{2b^2} - \frac{a^2 \text{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right) dx, x, (a+bx)^2\right)}{2b^2} \\
&= -\frac{a \cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} + \frac{(a+bx) \cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{C(a+bx)}{2b^2\pi} - \frac{a^2S(a+bx)}{2b^2} + \frac{1}{2}x^2S(a+bx)
\end{aligned}$$

Mathematica [A] time = 0.166339, size = 51, normalized size = 0.53

$$\frac{\text{FresnelC}(a+bx) + (a-bx)\left(\pi(a+bx)S(a+bx) + \cos\left(\frac{1}{2}\pi(a+bx)^2\right)\right)}{2\pi b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*FresnelS[a + b*x], x]

[Out] -(FresnelC[a + b*x] + (a - b*x)*(Cos[(Pi*(a + b*x)^2]/2] + Pi*(a + b*x)*FresnelS[a + b*x]))/(2*b^2*Pi)

Maple [A] time = 0.051, size = 80, normalized size = 0.8

$$\frac{1}{b^2} \left(\text{FresnelS}(bx+a) \left(\frac{(bx+a)^2}{2} - a(bx+a) \right) + \frac{bx+a}{2\pi} \cos\left(\frac{\pi(bx+a)^2}{2}\right) - \frac{\text{FresnelC}(bx+a)}{2\pi} - \frac{a}{\pi} \cos\left(\frac{\pi(bx+a)^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*FresnelS(b*x+a), x)

[Out] 1/b^2*(FresnelS(b*x+a)*(1/2*(b*x+a)^2-a*(b*x+a))+1/2/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)-1/2/Pi*FresnelC(b*x+a)-a/Pi*cos(1/2*Pi*(b*x+a)^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*fresnels(b*x+a),x, algorithm="maxima")`

[Out] `integrate(x*fresnels(b*x + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x \operatorname{fresnels}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*fresnels(b*x+a),x, algorithm="fricas")`

[Out] `integral(x*fresnels(b*x + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x S(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*fresnels(b*x+a),x)`

[Out] `Integral(x*fresnels(a + b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*fresnels(b*x + a), x)
```


3.28 $\int S(a + bx) dx$

Optimal. Leaf size=36

$$\frac{(a + bx)S(a + bx)}{b} + \frac{\cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b}$$

[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b

Rubi [A] time = 0.0065408, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6418}

$$\frac{(a + bx)S(a + bx)}{b} + \frac{\cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[a + b*x], x]

[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b

Rule 6418

Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] := Simp[((a + b*x)*FresnelS[a + b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int S(a + bx) dx = \frac{\cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b\pi} + \frac{(a + bx)S(a + bx)}{b}$$

Mathematica [B] time = 0.0300116, size = 89, normalized size = 2.47

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx + \frac{1}{2}\pi b^2 x^2\right)}{\pi b} + \frac{\cos\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx + \frac{1}{2}\pi b^2 x^2\right)}{\pi b} + xS(a + bx) + \frac{aS(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[a + b*x],x]

[Out] (Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a + b*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi)

Maple [A] time = 0.046, size = 33, normalized size = 0.9

$$\frac{1}{b} \left((bx + a) \operatorname{FresnelS}(bx + a) + \frac{1}{\pi} \cos \left(\frac{\pi (bx + a)^2}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x+a),x)

[Out] 1/b*((b*x+a)*FresnelS(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a),x, algorithm="maxima")

[Out] integrate(fresnels(b*x + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(\operatorname{fresnels}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a),x, algorithm="fricas")

[Out] `integral(fresnels(b*x + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int S(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x+a),x)`

[Out] `Integral(fresnels(a + b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x+a),x, algorithm="giac")`

[Out] `integrate(fresnels(b*x + a), x)`

$$3.29 \quad \int \frac{S(a+bx)}{x} dx$$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{S(a+bx)}{x}, x\right)$$

[Out] Unintegrable[FresnelS[a + b*x]/x, x]

Rubi [A] time = 0.0105536, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[a + b*x]/x,x]

[Out] Defer[Int][FresnelS[a + b*x]/x, x]

Rubi steps

$$\int \frac{S(a+bx)}{x} dx = \int \frac{S(a+bx)}{x} dx$$

Mathematica [A] time = 0.0263527, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[a + b*x]/x,x]

[Out] Integrate[FresnelS[a + b*x]/x, x]

Maple [A] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x+a)/x,x)

[Out] int(FresnelS(b*x+a)/x,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)/x,x, algorithm="maxima")

[Out] integrate(fresnels(b*x + a)/x, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)/x,x, algorithm="fricas")

[Out] integral(fresnels(b*x + a)/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x,x)
```

```
[Out] Integral(fresnels(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)/x, x)
```

$$3.30 \quad \int \frac{S(a+bx)}{x^2} dx$$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{S(a+bx)}{x^2}, x\right)$$

[Out] Unintegrable[FresnelS[a + b*x]/x^2, x]

Rubi [A] time = 0.0112494, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[a + b*x]/x^2, x]

[Out] Defer[Int][FresnelS[a + b*x]/x^2, x]

Rubi steps

$$\int \frac{S(a+bx)}{x^2} dx = \int \frac{S(a+bx)}{x^2} dx$$

Mathematica [A] time = 3.24649, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[a + b*x]/x^2, x]

[Out] Integrate[FresnelS[a + b*x]/x^2, x]

Maple [A] time = 0.1, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x+a)/x^2,x)

[Out] int(FresnelS(b*x+a)/x^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)/x^2,x, algorithm="maxima")

[Out] integrate(fresnels(b*x + a)/x^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)/x^2,x, algorithm="fricas")

[Out] integral(fresnels(b*x + a)/x^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S(a + bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x**2,x)
```

```
[Out] Integral(fresnels(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)/x^2, x)
```

3.31 $\int x^7 S(bx)^2 dx$

Optimal. Leaf size=253

$$-\frac{7x^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{105xS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^4 b^7} + \frac{x^7 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} - \frac{35x^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^3 b^5} - \frac{105S(bx)^2}{8\pi^4 b^8} + \dots$$

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) - (35*x^3*Cos[(b^2*Pi*
x^2)/2]*FresnelS[b*x])/(4*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x
])/ (4*b*Pi) - (105*FresnelS[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelS[b*x]^2)/8
+ (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^7*Pi^4) - (7*x^5*FresnelS[
b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) -
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Rubi [A] time = 0.428415, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6430, 6454, 6462, 3379, 3309, 30, 3296, 2637, 2634, 6440}

$$-\frac{7x^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{105xS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^4 b^7} + \frac{x^7 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} - \frac{35x^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^3 b^5} - \frac{105S(bx)^2}{8\pi^4 b^8} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelS[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) - (35*x^3*Cos[(b^2*Pi*
x^2)/2]*FresnelS[b*x])/(4*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x
])/ (4*b*Pi) - (105*FresnelS[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelS[b*x]^2)/8
+ (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^7*Pi^4) - (7*x^5*FresnelS[
b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) -
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] := Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :=>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :=> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :=> -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :=> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 2634

$\text{Int}[\sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] \rightarrow \text{Simp}[x/2, x] - \text{Simp}[\text{Sin}[2*c + d*x]/(2*d), x] /; \text{FreeQ}\{c, d\}, x]$

Rule 6440

$\text{Int}[\text{FresnelS}[(b_.)*(x_)]^{(n_.)*\text{Sin}[(d_.)*(x_)^2]}, x_Symbol] \rightarrow \text{Dist}[(\text{Pi}*b)/(2*d), \text{Subst}[\text{Int}[x^n, x], x, \text{FresnelS}[b*x]], x] /; \text{FreeQ}\{b, d, n\}, x] \&\& \text{E} \text{qQ}[d^2, (\text{Pi}^2*b^4)/4]$

Rubi steps

$$\begin{aligned} \int x^7 S(bx)^2 dx &= \frac{1}{8} x^8 S(bx)^2 - \frac{1}{4} b \int x^8 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx \\ &= \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b\pi} + \frac{1}{8} x^8 S(bx)^2 - \frac{\int x^7 \sin(b^2 \pi x^2) dx}{8\pi} - \frac{7 \int x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx) dx}{4b\pi} \\ &= \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b\pi} + \frac{1}{8} x^8 S(bx)^2 - \frac{7x^5 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{4b^3 \pi^2} + \frac{35 \int x^4 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{4b^3 \pi^2} + \frac{7 \int x^5 S(bx) \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{4b^3 \pi^2} \\ &= \frac{x^6 \cos(b^2 \pi x^2)}{16b^2 \pi^2} - \frac{35x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b^5 \pi^3} + \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b\pi} + \frac{1}{8} x^8 S(bx)^2 - \frac{7x^5 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{4b^3 \pi^2} \\ &= \frac{x^6 \cos(b^2 \pi x^2)}{16b^2 \pi^2} - \frac{35x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b^5 \pi^3} + \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b\pi} + \frac{1}{8} x^8 S(bx)^2 + \frac{105x S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{4b^7 \pi^4} \\ &= \frac{7x^6}{48b^2 \pi^2} - \frac{41x^2 \cos(b^2 \pi x^2)}{16b^6 \pi^4} + \frac{x^6 \cos(b^2 \pi x^2)}{16b^2 \pi^2} - \frac{35x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b^5 \pi^3} + \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b\pi} + \frac{1}{8} x^8 S(bx)^2 \\ &= -\frac{105x^2}{16b^6 \pi^4} + \frac{7x^6}{48b^2 \pi^2} - \frac{55x^2 \cos(b^2 \pi x^2)}{16b^6 \pi^4} + \frac{x^6 \cos(b^2 \pi x^2)}{16b^2 \pi^2} - \frac{35x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b^5 \pi^3} + \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b\pi} \\ &= -\frac{105x^2}{16b^6 \pi^4} + \frac{7x^6}{48b^2 \pi^2} - \frac{55x^2 \cos(b^2 \pi x^2)}{16b^6 \pi^4} + \frac{x^6 \cos(b^2 \pi x^2)}{16b^2 \pi^2} - \frac{35x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b^5 \pi^3} + \frac{x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{4b\pi} \end{aligned}$$

Mathematica [A] time = 0.0209369, size = 253, normalized size = 1.

$$-\frac{7x^5 S(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{105x S(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{4\pi^4 b^7} + \frac{x^7 S(bx) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{4\pi b} - \frac{35x^3 S(bx) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{4\pi^3 b^5} - \frac{105S(bx)^2}{8\pi^4 b^8} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[x^7*FresnelS[b*x]^2,x]

[Out]
$$\begin{aligned} & (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) - (55*x^2*\text{Cos}[b^2*Pi*x^2]) \\ & / (16*b^6*Pi^4) + (x^6*\text{Cos}[b^2*Pi*x^2]) / (16*b^2*Pi^2) - (35*x^3*\text{Cos}[(b^2*Pi* \\ & x^2)/2]*\text{FresnelS}[b*x]) / (4*b^5*Pi^3) + (x^7*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x \\ &]) / (4*b*Pi) - (105*\text{FresnelS}[b*x]^2) / (8*b^8*Pi^4) + (x^8*\text{FresnelS}[b*x]^2) / 8 \\ & + (105*x*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2]) / (4*b^7*Pi^4) - (7*x^5*\text{FresnelS}[\\ & b*x]*\text{Sin}[(b^2*Pi*x^2)/2]) / (4*b^3*Pi^2) + (10*\text{Sin}[b^2*Pi*x^2]) / (b^8*Pi^5) - \\ & (5*x^4*\text{Sin}[b^2*Pi*x^2]) / (8*b^4*Pi^3) \end{aligned}$$

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^7 (\text{FresnelS}(bx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*FresnelS(b*x)^2,x)

[Out] int(x^7*FresnelS(b*x)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnels(b*x)^2,x, algorithm="maxima")

[Out] integrate(x^7*fresnels(b*x)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^7 \text{fresnels}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnels(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 S^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**7*fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnels(b*x)^2, x)
```

3.32 $\int x^6 S(bx)^2 dx$

Optimal. Leaf size=239

$$\frac{531\text{FresnelC}(\sqrt{2}bx)}{56\sqrt{2}\pi^4b^7} - \frac{12x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3} + \frac{96S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7} + \frac{2x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b} - \frac{48x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}$$

[Out] $(-48*x)/(7*b^6*\text{Pi}^4) + (6*x^5)/(35*b^2*\text{Pi}^2) - (21*x*\text{Cos}[b^2*\text{Pi}*x^2])/(8*b^6*\text{Pi}^4) + (x^5*\text{Cos}[b^2*\text{Pi}*x^2])/(14*b^2*\text{Pi}^2) + (531*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(56*\text{Sqrt}[2]*b^7*\text{Pi}^4) - (48*x^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(7*b^5*\text{Pi}^3) + (2*x^6*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(7*b*\text{Pi}) + (x^7*\text{FresnelS}[b*x]^2)/7 + (96*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(7*b^7*\text{Pi}^4) - (12*x^4*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(7*b^3*\text{Pi}^2) - (17*x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(28*b^4*\text{Pi}^3)$

Rubi [A] time = 0.317892, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6430, 6454, 6462, 3391, 30, 3386, 3385, 3352, 6460, 3357}

$$\frac{531\text{FresnelC}(\sqrt{2}bx)}{56\sqrt{2}\pi^4b^7} - \frac{12x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3} + \frac{96S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7} + \frac{2x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b} - \frac{48x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^6*\text{FresnelS}[b*x]^2, x]$

[Out] $(-48*x)/(7*b^6*\text{Pi}^4) + (6*x^5)/(35*b^2*\text{Pi}^2) - (21*x*\text{Cos}[b^2*\text{Pi}*x^2])/(8*b^6*\text{Pi}^4) + (x^5*\text{Cos}[b^2*\text{Pi}*x^2])/(14*b^2*\text{Pi}^2) + (531*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(56*\text{Sqrt}[2]*b^7*\text{Pi}^4) - (48*x^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(7*b^5*\text{Pi}^3) + (2*x^6*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(7*b*\text{Pi}) + (x^7*\text{FresnelS}[b*x]^2)/7 + (96*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(7*b^7*\text{Pi}^4) - (12*x^4*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(7*b^3*\text{Pi}^2) - (17*x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(28*b^4*\text{Pi}^3)$

Rule 6430

$\text{Int}[\text{FresnelS}[(b_.)*(x_.)]^2*(x_)^(m_.), x_Symbol] \text{ :> } \text{Simp}[(x^(m+1)*\text{FresnelS}[b*x]^2)/(m+1), x] - \text{Dist}[(2*b)/(m+1), \text{Int}[x^(m+1)*\text{Sin}[(\text{Pi}*b^2*x^2)/2]*\text{FresnelS}[b*x], x], x] /; \text{FreeQ}[b, x] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{NeQ}[m, -1]$

Rule 6454

Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 6462

Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 3391

Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] := Dist[1/2, Int[x^m, x], x] - Dist[1/2, Int[x^m*cos[2*a + b*x^n], x], x] /; FreeQ[{a, b, m, n}, x]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3386

Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3385

Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 6460

Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)*(x_), x_Symbol] :> Simp[(Sin[d*x^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 3357

Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Symbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]

Rubi steps

$$\begin{aligned}
 \int x^6 S(bx)^2 dx &= \frac{1}{7} x^7 S(bx)^2 - \frac{1}{7} (2b) \int x^7 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
 &= \frac{2x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{7b\pi} + \frac{1}{7} x^7 S(bx)^2 - \frac{\int x^6 \sin(b^2 \pi x^2) dx}{7\pi} - \frac{12 \int x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx) dx}{7b\pi} \\
 &= \frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} + \frac{2x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{7b\pi} + \frac{1}{7} x^7 S(bx)^2 - \frac{12x^4 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^3 \pi^2} + \frac{48 \int x^3 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{7b^3 \pi^2} \\
 &= \frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} - \frac{48x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{7b^5 \pi^3} + \frac{2x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{7b\pi} + \frac{1}{7} x^7 S(bx)^2 - \frac{12x^4 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^3 \pi^2} \\
 &= \frac{6x^5}{35b^2 \pi^2} - \frac{111x \cos(b^2 \pi x^2)}{56b^6 \pi^4} + \frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} - \frac{48x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{7b^5 \pi^3} + \frac{2x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{7b\pi} \\
 &= \frac{6x^5}{35b^2 \pi^2} - \frac{21x \cos(b^2 \pi x^2)}{8b^6 \pi^4} + \frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} + \frac{15C(\sqrt{2}bx)}{56\sqrt{2}b^7 \pi^4} + \frac{6\sqrt{2}C(\sqrt{2}bx)}{7b^7 \pi^4} - \frac{48x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{7b^5 \pi^3} \\
 &= -\frac{48x}{7b^6 \pi^4} + \frac{6x^5}{35b^2 \pi^2} - \frac{21x \cos(b^2 \pi x^2)}{8b^6 \pi^4} + \frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} + \frac{51C(\sqrt{2}bx)}{56\sqrt{2}b^7 \pi^4} + \frac{6\sqrt{2}C(\sqrt{2}bx)}{7b^7 \pi^4} - \frac{48x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{7b^5 \pi^3} \\
 &= -\frac{48x}{7b^6 \pi^4} + \frac{6x^5}{35b^2 \pi^2} - \frac{21x \cos(b^2 \pi x^2)}{8b^6 \pi^4} + \frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} + \frac{51C(\sqrt{2}bx)}{56\sqrt{2}b^7 \pi^4} + \frac{30\sqrt{2}C(\sqrt{2}bx)}{7b^7 \pi^4} - \frac{48x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{7b^5 \pi^3}
 \end{aligned}$$

Mathematica [A] time = 0.289179, size = 171, normalized size = 0.72

$$\frac{80\pi^4 b^7 x^7 S(bx)^2 + 160S(bx) \left(\pi b^2 x^2 (\pi^2 b^4 x^4 - 24) \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 6(\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2} \pi b^2 x^2\right) \right) + 2bx (5(4\pi^2 b^4 x^4 - 14) \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 6 \sin\left(\frac{1}{2} \pi b^2 x^2\right))}{560\pi^4 b^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^6*FresnelS[b*x]^2,x]

[Out] (2655*sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*b^7*Pi^4*x^7*FresnelS[b*x]^2 + 160*FresnelS[b*x]*(b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(5*(-147 + 4*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] - 2*(960 - 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Ssin[b^2*Pi*x^2]))) / (560*b^7*Pi^4)

Maple [A] time = 0.077, size = 324, normalized size = 1.4

$$\frac{1}{b^7} \left(\frac{b^7 x^7 (\text{FresnelS}(bx))^2}{7} - 2 \text{FresnelS}(bx) \left(-\frac{1}{7} \frac{b^6 x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 6/7 \frac{1}{\pi} \left(\frac{x^4 b^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} - 4 \frac{1}{\pi} \left(-\frac{b^2 x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*FresnelS(b*x)^2,x)

[Out] 1/b^7*(1/7*b^7*x^7*FresnelS(b*x)^2-2*FresnelS(b*x)*(-1/7/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/7/Pi*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*Pi^2*b^5*x^5-8*b*x)-6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))-1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*fresnels(b*x)^2,x, algorithm="maxima")

[Out] integrate(x^6*fresnels(b*x)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^6 \text{fresnels}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*fresnels(b*x)^2,x, algorithm="fricas")

[Out] integral(x^6*fresnels(b*x)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 S^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*fresnels(b*x)**2,x)

[Out] Integral(x**6*fresnels(b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*fresnels(b*x)^2,x, algorithm="giac")

[Out] integrate(x^6*fresnels(b*x)^2, x)

3.33 $\int x^5 S(bx)^2 dx$

Optimal. Leaf size=265

$$\frac{5ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right)}{8\pi^3 b^4} + \frac{5ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)}{8\pi^3 b^4} + \frac{5 \text{FresnelS}[bx]}{b^4}$$

[Out] (5*x^4)/(24*b^2*Pi^2) - (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) + (x^4*Cos[b^2*Pi*x^2])/(12*b^2*Pi^2) - (5*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^5*Pi^3) + (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*b*Pi) + (5*FresnelC[b*x]*FresnelS[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x]^2)/6 - (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^4*Pi^3) + (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^4*Pi^3) - (5*x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) - (7*x^2*Sin[b^2*Pi*x^2])/(12*b^4*Pi^3)

Rubi [A] time = 0.300067, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$, Rules used = {6430, 6454, 6462, 3379, 3309, 30, 3296, 2638, 6446}

$$\frac{5ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3 b^4} + \frac{5ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi^3 b^4} + \frac{5 \text{FresnelC}(bx)S(bx)}{2\pi^3 b^6} - \frac{5x^3 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi^2 b^3} + \frac{x^5 S(bx)^2}{b^4}$$

Antiderivative was successfully verified.

[In] Int[x^5*FresnelS[b*x]^2,x]

[Out] (5*x^4)/(24*b^2*Pi^2) - (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) + (x^4*Cos[b^2*Pi*x^2])/(12*b^2*Pi^2) - (5*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^5*Pi^3) + (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*b*Pi) + (5*FresnelC[b*x]*FresnelS[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x]^2)/6 - (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^4*Pi^3) + (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^4*Pi^3) - (5*x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) - (7*x^2*Sin[b^2*Pi*x^2])/(12*b^4*Pi^3)

Rule 6430

Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*FresnelS[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/2], x]]

2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6454

Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 6462

Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 3309

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :> Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*cos[2*e + f*x], x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 6446

Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_.)], x_Symbol] := Simp[(FresnelC[b*x]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(I*b^2*Pi*x^2)/2])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rubi steps

$$\begin{aligned}
 \int x^5 S(bx)^2 dx &= \frac{1}{6} x^6 S(bx)^2 - \frac{1}{3} b \int x^6 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
 &= \frac{x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{3b\pi} + \frac{1}{6} x^6 S(bx)^2 - \frac{\int x^5 \sin(b^2 \pi x^2) dx}{6\pi} - \frac{5 \int x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx) dx}{3b\pi} \\
 &= \frac{x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{3b\pi} + \frac{1}{6} x^6 S(bx)^2 - \frac{5x^3 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b^3 \pi^2} + \frac{5 \int x^2 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{b^3 \pi^2} + \frac{5 \int x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx) dx}{b^3 \pi^2} \\
 &= \frac{x^4 \cos(b^2 \pi x^2)}{12b^2 \pi^2} - \frac{5x \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{b^5 \pi^3} + \frac{x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{3b\pi} + \frac{1}{6} x^6 S(bx)^2 - \frac{5x^3 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b^3 \pi^2} \\
 &= \frac{x^4 \cos(b^2 \pi x^2)}{12b^2 \pi^2} - \frac{5x \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{b^5 \pi^3} + \frac{x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{3b\pi} + \frac{5C(bx)S(bx)}{2b^6 \pi^3} + \frac{1}{6} x^6 S(bx)^2 - \frac{5ix^2 \cos(b^2 \pi x^2)}{2b^3 \pi^2} \\
 &= \frac{5x^4}{24b^2 \pi^2} - \frac{17 \cos(b^2 \pi x^2)}{12b^6 \pi^4} + \frac{x^4 \cos(b^2 \pi x^2)}{12b^2 \pi^2} - \frac{5x \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{b^5 \pi^3} + \frac{x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{3b\pi} + \frac{5C(bx)S(bx)}{2b^6 \pi^3} \\
 &= \frac{5x^4}{24b^2 \pi^2} - \frac{11 \cos(b^2 \pi x^2)}{6b^6 \pi^4} + \frac{x^4 \cos(b^2 \pi x^2)}{12b^2 \pi^2} - \frac{5x \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{b^5 \pi^3} + \frac{x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{3b\pi} + \frac{5C(bx)S(bx)}{2b^6 \pi^3}
 \end{aligned}$$

Mathematica [F] time = 0.207061, size = 0, normalized size = 0.

$$\int x^5 S(bx)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x^5*FresnelS[b*x]^2,x]

[Out] Integrate[x^5*FresnelS[b*x]^2, x]

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^5 (\text{FresnelS}(bx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*FresnelS(b*x)^2,x)

[Out] int(x^5*FresnelS(b*x)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*fresnels(b*x)^2,x, algorithm="maxima")

[Out] integrate(x^5*fresnels(b*x)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^5 \text{fresnels}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*fresnels(b*x)^2,x, algorithm="fricas")

[Out] integral(x^5*fresnels(b*x)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 S^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**5*fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnels(b*x)^2, x)
```


3.34 $\int x^4 S(bx)^2 dx$

Optimal. Leaf size=177

$$-\frac{8x^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3} + \frac{2x^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi b} - \frac{16S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5} + \frac{43S(\sqrt{2}bx)}{20\sqrt{2}\pi^3 b^5} + \frac{4x^3}{15\pi^2 b^2} - \frac{11x \sin(\pi b^2 x^2)}{20\pi^3 b^4}$$

[Out] $(4x^3)/(15b^2\pi^2) + (x^3\cos[b^2\pi x^2])/(10b^2\pi^2) - (16\cos[(b^2\pi x^2)/2]*\text{FresnelS}[bx])/(5b^5\pi^3) + (2x^4\cos[(b^2\pi x^2)/2]*\text{FresnelS}[bx])/(5b\pi) + (x^5\text{FresnelS}[bx]^2)/5 + (43\text{FresnelS}[\text{Sqrt}[2]*bx])/(20*\text{Sqrt}[2]*b^5\pi^3) - (8x^2*\text{FresnelS}[bx]*\text{Sin}[(b^2\pi x^2)/2])/(5b^3\pi^2) - (11x*\text{Sin}[b^2\pi x^2])/(20b^4\pi^3)$

Rubi [A] time = 0.187754, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$, Rules used = {6430, 6454, 6462, 3391, 30, 3386, 3351, 6452, 3385}

$$-\frac{8x^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3} + \frac{2x^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi b} - \frac{16S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5} + \frac{43S(\sqrt{2}bx)}{20\sqrt{2}\pi^3 b^5} + \frac{4x^3}{15\pi^2 b^2} - \frac{11x \sin(\pi b^2 x^2)}{20\pi^3 b^4}$$

Antiderivative was successfully verified.

[In] Int[x^4*FresnelS[b*x]^2,x]

[Out] $(4x^3)/(15b^2\pi^2) + (x^3\cos[b^2\pi x^2])/(10b^2\pi^2) - (16\cos[(b^2\pi x^2)/2]*\text{FresnelS}[bx])/(5b^5\pi^3) + (2x^4\cos[(b^2\pi x^2)/2]*\text{FresnelS}[bx])/(5b\pi) + (x^5\text{FresnelS}[bx]^2)/5 + (43\text{FresnelS}[\text{Sqrt}[2]*bx])/(20*\text{Sqrt}[2]*b^5\pi^3) - (8x^2*\text{FresnelS}[bx]*\text{Sin}[(b^2\pi x^2)/2])/(5b^3\pi^2) - (11x*\text{Sin}[b^2\pi x^2])/(20b^4\pi^3)$

Rule 6430

Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*FresnelS[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6454

Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(

$m - 2) \cdot \cos[d \cdot x^2] \cdot \text{FresnelS}[b \cdot x], x], x] + \text{Dist}[1/(2 \cdot b \cdot \text{Pi}), \text{Int}[x^{(m - 1)} \cdot \text{Sin}[2 \cdot d \cdot x^2], x], x)] /;$ FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 6462

$\text{Int}[\text{Cos}[(d \cdot x)^2] \cdot \text{FresnelS}[(b \cdot x) \cdot (x)] \cdot (x)^{(m)}, x_Symbol] :> \text{Simp}[(x^{(m - 1)} \cdot \text{Sin}[d \cdot x^2] \cdot \text{FresnelS}[b \cdot x]) / (2 \cdot d), x] + (-\text{Dist}[1/(\text{Pi} \cdot b), \text{Int}[x^{(m - 1)} \cdot \text{Sin}[d \cdot x^2]^2, x], x] - \text{Dist}[(m - 1) / (2 \cdot d), \text{Int}[x^{(m - 2)} \cdot \text{Sin}[d \cdot x^2] \cdot \text{FresnelS}[b \cdot x], x], x)] /;$ FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 3391

$\text{Int}[(x)^{(m)} \cdot \text{Sin}[(a) + ((b) \cdot (x)^{(n)}) / 2]^2, x_Symbol] :> \text{Dist}[1/2, \text{Int}[x^m, x], x] - \text{Dist}[1/2, \text{Int}[x^m \cdot \text{Cos}[2 \cdot a + b \cdot x^n], x], x] /;$ FreeQ[{a, b, m, n}, x]

Rule 30

$\text{Int}[(x)^{(m)}, x_Symbol] :> \text{Simp}[x^{(m + 1)} / (m + 1), x] /;$ FreeQ[m, x] && NeQ[m, -1]

Rule 3386

$\text{Int}[\text{Cos}[(c) + (d) \cdot (x)^{(n)}] \cdot ((e) \cdot (x))^{(m)}, x_Symbol] :> \text{Simp}[(e^{(n - 1)} \cdot (e \cdot x)^{(m - n + 1)} \cdot \text{Sin}[c + d \cdot x^n]) / (d \cdot n), x] - \text{Dist}[(e^n \cdot (m - n + 1)) / (d \cdot n), \text{Int}[(e \cdot x)^{(m - n)} \cdot \text{Sin}[c + d \cdot x^n], x], x] /;$ FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3351

$\text{Int}[\text{Sin}[(d) \cdot ((e) + (f) \cdot (x))^2], x_Symbol] :> \text{Simp}[(\text{Sqrt}[\text{Pi}/2] \cdot \text{FresnelS}[\text{Sqrt}[2/\text{Pi}] \cdot \text{Rt}[d, 2] \cdot (e + f \cdot x)]) / (f \cdot \text{Rt}[d, 2]), x] /;$ FreeQ[{d, e, f}, x]

Rule 6452

$\text{Int}[\text{FresnelS}[(b) \cdot (x)] \cdot (x) \cdot \text{Sin}[(d) \cdot (x)^2], x_Symbol] :> -\text{Simp}[(\text{Cos}[d \cdot x^2] \cdot \text{FresnelS}[b \cdot x]) / (2 \cdot d), x] + \text{Dist}[1/(2 \cdot b \cdot \text{Pi}), \text{Int}[\text{Sin}[2 \cdot d \cdot x^2], x], x] /;$ FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 3385

$\text{Int}[(e) \cdot (x)^{(m)} \cdot \text{Sin}[(c) + (d) \cdot (x)^{(n)}], x_Symbol] :> -\text{Simp}[(e^{(n - 1)} \cdot (e \cdot x)^{(m - n + 1)} \cdot \text{Cos}[c + d \cdot x^n]) / (d \cdot n), x] + \text{Dist}[(e^n \cdot (m - n + 1)) / (d \cdot n), \text{Int}[(e \cdot x)^{(m - n)} \cdot \text{Sin}[c + d \cdot x^n], x], x] /;$

)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]

Rubi steps

$$\begin{aligned}
 \int x^4 S(bx)^2 dx &= \frac{1}{5} x^5 S(bx)^2 - \frac{1}{5} (2b) \int x^5 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
 &= \frac{2x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{5b\pi} + \frac{1}{5} x^5 S(bx)^2 - \frac{\int x^4 \sin(b^2 \pi x^2) dx}{5\pi} - \frac{8 \int x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx) dx}{5b\pi} \\
 &= \frac{x^3 \cos(b^2 \pi x^2)}{10b^2 \pi^2} + \frac{2x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{5b\pi} + \frac{1}{5} x^5 S(bx)^2 - \frac{8x^2 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b^3 \pi^2} + \frac{16 \int x S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{5b^3 \pi^2} \\
 &= \frac{x^3 \cos(b^2 \pi x^2)}{10b^2 \pi^2} - \frac{16 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{5b^5 \pi^3} + \frac{2x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{5b\pi} + \frac{1}{5} x^5 S(bx)^2 - \frac{8x^2 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b^3 \pi^2} \\
 &= \frac{4x^3}{15b^2 \pi^2} + \frac{x^3 \cos(b^2 \pi x^2)}{10b^2 \pi^2} - \frac{16 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{5b^5 \pi^3} + \frac{2x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{5b\pi} + \frac{1}{5} x^5 S(bx)^2 + \frac{3S(\sqrt{2}bx)}{20\sqrt{2}b^5} \\
 &= \frac{4x^3}{15b^2 \pi^2} + \frac{x^3 \cos(b^2 \pi x^2)}{10b^2 \pi^2} - \frac{16 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{5b^5 \pi^3} + \frac{2x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{5b\pi} + \frac{1}{5} x^5 S(bx)^2 + \frac{3S(\sqrt{2}bx)}{20\sqrt{2}b^5}
 \end{aligned}$$

Mathematica [A] time = 0.132727, size = 137, normalized size = 0.77

$$\frac{24\pi^3 b^5 x^5 S(bx)^2 + 48S(bx) \left((\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 4\pi b^2 x^2 \sin\left(\frac{1}{2} \pi b^2 x^2\right) \right) + 32\pi b^3 x^3 - 66bx \sin(\pi b^2 x^2) + 12\pi b^5}{120\pi^3 b^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*FresnelS[b*x]^2,x]

[Out] (32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelS[b*x]^2 + 129*sqrt[2]*FresnelS[sqrt[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(120*b^5*Pi^3)

Maple [A] time = 0.075, size = 208, normalized size = 1.2

$$\frac{1}{b^5} \left(\frac{b^5 x^5 (\text{FresnelS}(bx))^2}{5} - 2 \text{FresnelS}(bx) \left(-1/5 \frac{x^4 b^4 \cos(1/2 b^2 \pi x^2)}{\pi} + 4/5 \frac{1}{\pi} \left(\frac{b^2 x^2 \sin(1/2 b^2 \pi x^2)}{\pi} + 2 \frac{\cos(1/2 b^2 \pi x^2)}{\pi^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*fresnels(b*x)^2,x)`

[Out] $\frac{1}{b^5} \left(\frac{1}{5} b^5 x^5 \operatorname{FresnelS}(bx)^2 - 2 \operatorname{FresnelS}(bx) \left(-\frac{1}{5} \pi b^4 x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) + \frac{4}{5} \pi \left(\frac{1}{\pi} b^2 x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right) + \frac{2}{\pi^2} \cos\left(\frac{1}{2} b^2 \pi x^2\right) \right) \right) + \frac{4}{15} \pi^2 b^3 x^3 - \frac{4}{5} \pi^2 \left(\frac{1}{2} \pi b x \sin(b^2 \pi x^2) - \frac{1}{4} \pi^2 \left(\frac{1}{2} \right) \operatorname{FresnelS}(bx^2)^{\frac{1}{2}} \right) - \frac{1}{5} \pi^3 \left(-\frac{1}{2} \pi b^3 x^3 \cos(b^2 \pi x^2) + \frac{3}{2} \pi \left(\frac{1}{2} \pi b x \sin(b^2 \pi x^2) - \frac{1}{4} \pi^2 \left(\frac{1}{2} \right) \operatorname{FresnelS}(bx^2)^{\frac{1}{2}} \right) \right) - 4x^2 \left(\frac{1}{2} \right) \operatorname{FresnelS}(bx^2)^{\frac{1}{2}} \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*fresnels(b*x)^2,x, algorithm="maxima")`

[Out] `integrate(x^4*fresnels(b*x)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^4 \operatorname{fresnels}(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*fresnels(b*x)^2,x, algorithm="fricas")`

[Out] `integral(x^4*fresnels(b*x)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 S^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**4*fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnels(b*x)^2, x)
```

3.35 $\int x^3 S(bx)^2 dx$

Optimal. Leaf size=140

$$-\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3} + \frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} + \frac{3S(bx)^2}{4\pi^2b^4} + \frac{3x^2}{8\pi^2b^2} - \frac{\sin(\pi b^2x^2)}{2\pi^3b^4} + \frac{x^2\cos(\pi b^2x^2)}{8\pi^2b^2} + \frac{1}{4}x^4S(bx)^2$$

[Out] (3*x^2)/(8*b^2*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) + (x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(2*b*Pi) + (3*FresnelS[b*x]^2)/(4*b^4*Pi^2) + (x^4*FresnelS[b*x]^2)/4 - (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b^3*Pi^2) - Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)

Rubi [A] time = 0.150878, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$, Rules used = {6430, 6454, 6462, 3379, 2634, 6440, 30, 3296, 2637}

$$-\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3} + \frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} + \frac{3S(bx)^2}{4\pi^2b^4} + \frac{3x^2}{8\pi^2b^2} - \frac{\sin(\pi b^2x^2)}{2\pi^3b^4} + \frac{x^2\cos(\pi b^2x^2)}{8\pi^2b^2} + \frac{1}{4}x^4S(bx)^2$$

Antiderivative was successfully verified.

[In] Int[x^3*FresnelS[b*x]^2,x]

[Out] (3*x^2)/(8*b^2*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) + (x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(2*b*Pi) + (3*FresnelS[b*x]^2)/(4*b^4*Pi^2) + (x^4*FresnelS[b*x]^2)/4 - (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b^3*Pi^2) - Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)

Rule 6430

Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*FresnelS[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6454

Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m

, 1]

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] := Simp[x/2, x] - Simp[Sin[2*c + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 S(bx)^2 dx &= \frac{1}{4} x^4 S(bx)^2 - \frac{1}{2} b \int x^4 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
&= \frac{x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{2b\pi} + \frac{1}{4} x^4 S(bx)^2 - \frac{\int x^3 \sin(b^2 \pi x^2) dx}{4\pi} - \frac{3 \int x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx) dx}{2b\pi} \\
&= \frac{x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{2b\pi} + \frac{1}{4} x^4 S(bx)^2 - \frac{3xS(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{2b^3 \pi^2} + \frac{3 \int S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{2b^3 \pi^2} + \frac{3 \int x \sin^2}{2} \\
&= \frac{x^2 \cos(b^2 \pi x^2)}{8b^2 \pi^2} + \frac{x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{2b\pi} + \frac{1}{4} x^4 S(bx)^2 - \frac{3xS(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{2b^3 \pi^2} + \frac{3 \text{Subst}(\int x dx, x, S(bx))}{2b^4 \pi^2} \\
&= \frac{3x^2}{8b^2 \pi^2} + \frac{x^2 \cos(b^2 \pi x^2)}{8b^2 \pi^2} + \frac{x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{2b\pi} + \frac{3S(bx)^2}{4b^4 \pi^2} + \frac{1}{4} x^4 S(bx)^2 - \frac{3xS(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{2b^3 \pi^2} - \frac{3 \int x \sin^2}{2}
\end{aligned}$$

Mathematica [A] time = 0.0055243, size = 140, normalized size = 1.

$$-\frac{3xS(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{2\pi^2 b^3} + \frac{x^3 S(bx) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{2\pi b} + \frac{3S(bx)^2}{4\pi^2 b^4} + \frac{3x^2}{8\pi^2 b^2} - \frac{\sin(\pi b^2 x^2)}{2\pi^3 b^4} + \frac{x^2 \cos(\pi b^2 x^2)}{8\pi^2 b^2} + \frac{1}{4} x^4 S(bx)^2$$

Antiderivative was successfully verified.

[In] Integrate[x^3*FresnelS[b*x]^2,x]

[Out] (3*x^2)/(8*b^2*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) + (x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(2*b*Pi) + (3*FresnelS[b*x]^2)/(4*b^4*Pi^2) + (x^4*FresnelS[b*x]^2)/4 - (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b^3*Pi^2) - Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int x^3 (\text{FresnelS}(bx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*FresnelS(b*x)^2,x)

[Out] `int(x^3*FresnelS(b*x)^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*fresnels(b*x)^2,x, algorithm="maxima")`

[Out] `integrate(x^3*fresnels(b*x)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \text{fresnels}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*fresnels(b*x)^2,x, algorithm="fricas")`

[Out] `integral(x^3*fresnels(b*x)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 S^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*fresnels(b*x)**2,x)`

[Out] `Integral(x**3*fresnels(b*x)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnels(b*x)^2, x)
```

3.36 $\int x^2 S(bx)^2 dx$

Optimal. Leaf size=124

$$-\frac{5\text{FresnelC}(\sqrt{2}bx)}{6\sqrt{2}\pi^2b^3} - \frac{4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3} + \frac{2x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b} + \frac{x\cos(\pi b^2x^2)}{6\pi^2b^2} + \frac{2x}{3\pi^2b^2} + \frac{1}{3}x^3S(bx)^2$$

[Out] (2*x)/(3*b^2*Pi^2) + (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (5*FresnelC[Sqrt[2]*b*x])/(6*Sqrt[2]*b^3*Pi^2) + (2*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*b*Pi) + (x^3*FresnelS[b*x]^2)/3 - (4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2)

Rubi [A] time = 0.110876, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6430, 6454, 6460, 3357, 3352, 3385}

$$-\frac{5\text{FresnelC}(\sqrt{2}bx)}{6\sqrt{2}\pi^2b^3} - \frac{4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3} + \frac{2x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b} + \frac{x\cos(\pi b^2x^2)}{6\pi^2b^2} + \frac{2x}{3\pi^2b^2} + \frac{1}{3}x^3S(bx)^2$$

Antiderivative was successfully verified.

[In] Int[x^2*FresnelS[b*x]^2,x]

[Out] (2*x)/(3*b^2*Pi^2) + (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (5*FresnelC[Sqrt[2]*b*x])/(6*Sqrt[2]*b^3*Pi^2) + (2*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*b*Pi) + (x^3*FresnelS[b*x]^2)/3 - (4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2)

Rule 6430

Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*FresnelS[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6454

Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m

, 1]

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)](x_), x_Symbol] := Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_)^(n_))]^(p_), x_Sy
mbol] := Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_)^2)], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_)^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)]), x_Symbol] := -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^2 S(bx)^2 dx &= \frac{1}{3} x^3 S(bx)^2 - \frac{1}{3} (2b) \int x^3 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
&= \frac{2x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{3b\pi} + \frac{1}{3} x^3 S(bx)^2 - \frac{\int x^2 \sin(b^2 \pi x^2) dx}{3\pi} - \frac{4 \int x \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx) dx}{3b\pi} \\
&= \frac{x \cos(b^2 \pi x^2)}{6b^2 \pi^2} + \frac{2x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{3b\pi} + \frac{1}{3} x^3 S(bx)^2 - \frac{4S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b^3 \pi^2} - \frac{\int \cos(b^2 \pi x^2) dx}{6b^2 \pi^2} + \dots \\
&= \frac{x \cos(b^2 \pi x^2)}{6b^2 \pi^2} - \frac{C(\sqrt{2}bx)}{6\sqrt{2}b^3 \pi^2} + \frac{2x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{3b\pi} + \frac{1}{3} x^3 S(bx)^2 - \frac{4S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b^3 \pi^2} + \frac{4 \int \left(\frac{1}{2} - \dots\right)}{\dots} \\
&= \frac{2x}{3b^2 \pi^2} + \frac{x \cos(b^2 \pi x^2)}{6b^2 \pi^2} - \frac{C(\sqrt{2}bx)}{6\sqrt{2}b^3 \pi^2} + \frac{2x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{3b\pi} + \frac{1}{3} x^3 S(bx)^2 - \frac{4S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b^3 \pi^2} - \dots \\
&= \frac{2x}{3b^2 \pi^2} + \frac{x \cos(b^2 \pi x^2)}{6b^2 \pi^2} - \frac{C(\sqrt{2}bx)}{6\sqrt{2}b^3 \pi^2} - \frac{\sqrt{2}C(\sqrt{2}bx)}{3b^3 \pi^2} + \frac{2x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)}{3b\pi} + \frac{1}{3} x^3 S(bx)^2 - \frac{4S(bx)}{\dots}
\end{aligned}$$

Mathematica [A] time = 0.107693, size = 100, normalized size = 0.81

$$\frac{4\pi^2 b^3 x^3 S(bx)^2 + 8S(bx) \left(\pi b^2 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 2 \sin\left(\frac{1}{2} \pi b^2 x^2\right) \right) + 2bx \left(\cos(\pi b^2 x^2) + 4 \right) - 5\sqrt{2} \text{FresnelC}(\sqrt{2}bx)}{12\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*FresnelS[b*x]^2,x]

[Out] (2*b*x*(4 + Cos[b^2*Pi*x^2]) - 5*sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*b^3*Pi^2*x^3*FresnelS[b*x]^2 + 8*FresnelS[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)

Maple [A] time = 0.072, size = 122, normalized size = 1.

$$\frac{1}{b^3} \left(\frac{b^3 x^3 (\text{FresnelS}(bx))^2}{3} - 2 \text{FresnelS}(bx) \left(-1/3 \frac{b^2 x^2 \cos(1/2 b^2 \pi x^2)}{\pi} + 2/3 \frac{\sin(1/2 b^2 \pi x^2)}{\pi^2} \right) + \frac{2bx}{3\pi^2} - \frac{\sqrt{2} \text{FresnelC}}{3\pi^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*FresnelS(b*x)^2,x)

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelS(b*x)^2-2*FresnelS(b*x)*(-1/3/Pi*b^2*x^2*cos(1/2
*b^2*Pi*x^2)+2/3/Pi^2*sin(1/2*b^2*Pi*x^2))+2/3/Pi^2*b*x-1/3/Pi^2*2^(1/2)*Fr
esnelC(b*x*2^(1/2))-1/3/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fres
nelC(b*x*2^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnels(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \text{fresnels}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnels(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 S^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**2*fresnels(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnels(b*x)^2, x)
```

3.37 $\int xS(bx)^2 dx$

Optimal. Leaf size=143

$$\frac{ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right)}{8\pi} - \frac{ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)}{8\pi} - \frac{\text{FresnelC}(bx)}{2\pi b}$$

[Out] Cos[b^2*Pi*x^2]/(4*b^2*Pi^2) + (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) - (FresnelC[b*x]*FresnelS[b*x])/(2*b^2*Pi) + (x^2*FresnelS[b*x]^2)/2 + ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/Pi - ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/Pi

Rubi [A] time = 0.0863939, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6430, 6454, 6446, 3379, 2638}

$$\frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{\text{FresnelC}(bx)S(bx)}{2\pi b^2} + \frac{xS(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b} + \frac{\cos(\pi b^2 x^2)}{4\pi^2 b^2}$$

Antiderivative was successfully verified.

[In] Int[x*FresnelS[b*x]^2, x]

[Out] Cos[b^2*Pi*x^2]/(4*b^2*Pi^2) + (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) - (FresnelC[b*x]*FresnelS[b*x])/(2*b^2*Pi) + (x^2*FresnelS[b*x]^2)/2 + ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/Pi - ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/Pi

Rule 6430

Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*FresnelS[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6454

Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m

, 1]

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] := Simp[(FresnelC[b*x]
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2)]/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int xS(bx)^2 dx &= \frac{1}{2}x^2S(bx)^2 - b \int x^2S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx \\
&= \frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b\pi} + \frac{1}{2}x^2S(bx)^2 - \frac{\int x \sin(b^2\pi x^2) dx}{2\pi} - \frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b\pi} \\
&= \frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b\pi} - \frac{C(bx)S(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)^2 + \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi} \\
&= \frac{\cos(b^2\pi x^2)}{4b^2\pi^2} + \frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b\pi} - \frac{C(bx)S(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)^2 + \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi}
\end{aligned}$$

Mathematica [F] time = 0.171656, size = 0, normalized size = 0.

$$\int xS(bx)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x*FresnelS[b*x]^2,x]

[Out] Integrate[x*FresnelS[b*x]^2, x]

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x (\text{FresnelS}(bx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*FresnelS(b*x)^2,x)

[Out] int(x*FresnelS(b*x)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int xfresnels(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnels(b*x)^2,x, algorithm="maxima")

[Out] integrate(x*fresnels(b*x)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(xfresnels(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnels(b*x)^2,x, algorithm="fricas")

[Out] integral(x*fresnels(b*x)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x S^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnels(b*x)**2,x)

[Out] Integral(x*fresnels(b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnels(b*x)^2,x, algorithm="giac")

[Out] integrate(x*fresnels(b*x)^2, x)

3.38 $\int S(bx)^2 dx$

Optimal. Leaf size=55

$$\frac{2S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b} + xS(bx)^2 - \frac{S(\sqrt{2}bx)}{\sqrt{2}\pi b}$$

[Out] (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)

Rubi [A] time = 0.0383974, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6420, 12, 6452, 3351}

$$\frac{2S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b} + xS(bx)^2 - \frac{S(\sqrt{2}bx)}{\sqrt{2}\pi b}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]^2, x]

[Out] (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)

Rule 6420

Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] := Simp[((a + b*x)*FresnelS[a + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Sin[(Pi*(a + b*x)^2]/2]*FresnelS[a + b*x], x], x] /; FreeQ[{a, b}, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 6452

Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 3351

`Int[Sin[(d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]`

Rubi steps

$$\begin{aligned}
 \int S(bx)^2 dx &= xS(bx)^2 - 2 \int bxS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx \\
 &= xS(bx)^2 - (2b) \int xS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx \\
 &= \frac{2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b\pi} + xS(bx)^2 - \frac{\int \sin(b^2\pi x^2) dx}{\pi} \\
 &= \frac{2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b\pi} + xS(bx)^2 - \frac{S(\sqrt{2}bx)}{\sqrt{2}b\pi}
 \end{aligned}$$

Mathematica [A] time = 0.0093133, size = 55, normalized size = 1.

$$\frac{2S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b} + xS(bx)^2 - \frac{S(\sqrt{2}bx)}{\sqrt{2}\pi b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]², x]

[Out] (2*Cos[(b²*Pi*x²)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]² - FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)

Maple [A] time = 0.056, size = 49, normalized size = 0.9

$$\frac{1}{b} \left(bx (\text{FresnelS}(bx))^2 + 2 \frac{\cos(1/2 b^2 \pi x^2) \text{FresnelS}(bx)}{\pi} - \frac{\sqrt{2} \text{FresnelS}(bx\sqrt{2})}{2\pi} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)², x)

[Out] $1/b*(b*x*\text{FresnelS}(b*x)^2+2*\text{FresnelS}(b*x)/\text{Pi}*\cos(1/2*b^2*\text{Pi}*x^2)-1/2/\text{Pi}*2^(1/2)*\text{FresnelS}(b*x*2^(1/2)))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)^2,x, algorithm="maxima")`

[Out] `integrate(fresnels(b*x)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{fresnels}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)^2,x, algorithm="fricas")`

[Out] `integral(fresnels(b*x)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int S^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)**2,x)`

[Out] `Integral(fresnels(b*x)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2, x)
```

$$3.39 \quad \int \frac{S(bx)^2}{x} dx$$

Optimal. Leaf size=12

Unintegrable $\left(\frac{S(bx)^2}{x}, x\right)$

[Out] Unintegrable[FresnelS[b*x]^2/x, x]

Rubi [A] time = 0.0166764, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx)^2}{x} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[b*x]^2/x, x]

[Out] Defer[Int][FresnelS[b*x]^2/x, x]

Rubi steps

$$\int \frac{S(bx)^2}{x} dx = \int \frac{S(bx)^2}{x} dx$$

Mathematica [A] time = 0.0170186, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[b*x]^2/x, x]

[Out] Integrate[FresnelS[b*x]^2/x, x]

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)^2/x,x)

[Out] int(FresnelS(b*x)^2/x,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)^2/x, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x,x, algorithm="fricas")

[Out] integral(fresnels(b*x)^2/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x,x)
```

```
[Out] Integral(fresnels(b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x, x)
```

$$3.40 \quad \int \frac{S(bx)^2}{x^2} dx$$

Optimal. Leaf size=37

$$2b\text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}, x\right) - \frac{S(bx)^2}{x}$$

[Out] -(FresnelS[b*x]^2/x) + 2*b*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

Rubi [A] time = 0.0372656, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx)^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[b*x]^2/x^2, x]

[Out] -(FresnelS[b*x]^2/x) + 2*b*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

Rubi steps

$$\int \frac{S(bx)^2}{x^2} dx = -\frac{S(bx)^2}{x} + (2b) \int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx$$

Mathematica [A] time = 0.0266247, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[b*x]^2/x^2, x]

[Out] Integrate[FresnelS[b*x]^2/x^2, x]

Maple [A] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)^2/x^2,x)

[Out] int(FresnelS(b*x)^2/x^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^2,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)^2/x^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^2,x, algorithm="fricas")

[Out] integral(fresnels(b*x)^2/x^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)**2/x**2,x)

[Out] Integral(fresnels(b*x)**2/x**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^2,x, algorithm="giac")

[Out] integrate(fresnels(b*x)^2/x^2, x)

3.41 $\int \frac{S(bx)^2}{x^3} dx$

Optimal. Leaf size=38

$$b\text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}, x\right) - \frac{S(bx)^2}{2x^2}$$

[Out] -FresnelS[b*x]^2/(2*x^2) + b*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]

Rubi [A] time = 0.0383717, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx)^2}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[b*x]^2/x^3, x]

[Out] -FresnelS[b*x]^2/(2*x^2) + b*Defer[Int] [(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]

Rubi steps

$$\int \frac{S(bx)^2}{x^3} dx = -\frac{S(bx)^2}{2x^2} + b \int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx$$

Mathematica [A] time = 0.0198748, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[b*x]^2/x^3, x]

[Out] Integrate[FresnelS[b*x]^2/x^3, x]

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)^2/x^3,x)

[Out] int(FresnelS(b*x)^2/x^3,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^3,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)^2/x^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^3,x, algorithm="fricas")

[Out] integral(fresnels(b*x)^2/x^3, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)**2/x**3,x)

[Out] Integral(fresnels(b*x)**2/x**3, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^3,x, algorithm="giac")

[Out] integrate(fresnels(b*x)^2/x^3, x)

$$3.42 \quad \int \frac{S(bx)^2}{x^4} dx$$

Optimal. Leaf size=119

$$\frac{1}{3}\pi b^3 \text{Unintegrable} \left(\frac{S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x \right) - \frac{bS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^2} + \frac{\pi b^3 S(\sqrt{2}bx)}{3\sqrt{2}} + \frac{b^2 \cos(\pi b^2 x^2)}{6x} - \frac{b^2}{6x} - \frac{S(bx)^2}{3x^3}$$

[Out] $-b^2/(6*x) + (b^2*\text{Cos}[b^2*Pi*x^2])/(6*x) - \text{FresnelS}[b*x]^2/(3*x^3) + (b^3*Pi*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(3*\text{Sqrt}[2]) - (b*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(3*x^2) + (b^3*Pi*\text{Unintegrable}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/x, x])/3$

Rubi [A] time = 0.0873615, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx)^2}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[b*x]^2/x^4,x]

[Out] $-b^2/(6*x) + (b^2*\text{Cos}[b^2*Pi*x^2])/(6*x) - \text{FresnelS}[b*x]^2/(3*x^3) + (b^3*Pi*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(3*\text{Sqrt}[2]) - (b*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(3*x^2) + (b^3*Pi*\text{Defer}[\text{Int}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/x, x])/3$

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^4} dx &= -\frac{S(bx)^2}{3x^3} + \frac{1}{3}(2b) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx \\
&= -\frac{b^2}{6x} - \frac{S(bx)^2}{3x^3} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2} - \frac{1}{6}b^2 \int \frac{\cos(b^2\pi x^2)}{x^2} dx + \frac{1}{3}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x} dx \\
&= -\frac{b^2}{6x} + \frac{b^2 \cos(b^2\pi x^2)}{6x} - \frac{S(bx)^2}{3x^3} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2} + \frac{1}{3}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x} dx + \frac{1}{3}(b^4\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x} dx \\
&= -\frac{b^2}{6x} + \frac{b^2 \cos(b^2\pi x^2)}{6x} - \frac{S(bx)^2}{3x^3} + \frac{b^3\pi S(\sqrt{2}bx)}{3\sqrt{2}} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2} + \frac{1}{3}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x} dx
\end{aligned}$$

Mathematica [A] time = 0.0259727, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[b*x]^2/x^4, x]

[Out] Integrate[FresnelS[b*x]^2/x^4, x]

Maple [A] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)^2/x^4, x)

[Out] int(FresnelS(b*x)^2/x^4, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^4,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)^2/x^4, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^4,x, algorithm="fricas")

[Out] integral(fresnels(b*x)^2/x^4, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)**2/x**4,x)

[Out] Integral(fresnels(b*x)**2/x**4, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^4,x, algorithm="giac")

[Out] integrate(fresnels(b*x)^2/x^4, x)

3.43 $\int \frac{S(bx)^2}{x^5} dx$

Optimal. Leaf size=127

$$-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3} - \frac{\pi b^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x} - \frac{1}{12}\pi^2b^4S(bx)^2 + \frac{1}{12}\pi b^4\text{Si}\left(b^2\pi x^2\right) - \frac{b^2}{24x^2} + \frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2} - \frac{S(bx)^2}{4x^4}$$

[Out] $-b^2/(24*x^2) + (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(24*x^2) - (b^3*\text{Pi}*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(6*x) - (b^4*\text{Pi}^2*\text{FresnelS}[b*x]^2)/12 - \text{FresnelS}[b*x]^2/(4*x^4) - (b*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(6*x^3) + (b^4*\text{Pi}*\text{SinIntegral}[b^2*\text{Pi}*x^2])/12$

Rubi [A] time = 0.143154, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$, Rules used = {6430, 6456, 6464, 6440, 30, 3375, 3380, 3297, 3299}

$$-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3} - \frac{\pi b^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x} - \frac{1}{12}\pi^2b^4S(bx)^2 + \frac{1}{12}\pi b^4\text{Si}\left(b^2\pi x^2\right) - \frac{b^2}{24x^2} + \frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2} - \frac{S(bx)^2}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]^2/x^5,x]

[Out] $-b^2/(24*x^2) + (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(24*x^2) - (b^3*\text{Pi}*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(6*x) - (b^4*\text{Pi}^2*\text{FresnelS}[b*x]^2)/12 - \text{FresnelS}[b*x]^2/(4*x^4) - (b*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(6*x^3) + (b^4*\text{Pi}*\text{SinIntegral}[b^2*\text{Pi}*x^2])/12$

Rule 6430

Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*FresnelS[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6456

Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(x^(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])

/; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]

Rule 6464

Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)*(x_)^(m_)], x_Symbol] := Simp[(x^(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -1]

Rule 6440

Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3375

Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^5} dx &= -\frac{S(bx)^2}{4x^4} + \frac{1}{2}b \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx \\
&= -\frac{b^2}{24x^2} - \frac{S(bx)^2}{4x^4} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} - \frac{1}{12}b^2 \int \frac{\cos(b^2\pi x^2)}{x^3} dx + \frac{1}{6}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^2} dx \\
&= -\frac{b^2}{24x^2} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x} - \frac{S(bx)^2}{4x^4} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} - \frac{1}{24}b^2 \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^2} dx, x, x^2\right) \\
&= -\frac{b^2}{24x^2} + \frac{b^2 \cos(b^2\pi x^2)}{24x^2} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x} - \frac{S(bx)^2}{4x^4} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} + \frac{1}{24}b^4\pi \text{Si}(b^2\pi x^2) \\
&= -\frac{b^2}{24x^2} + \frac{b^2 \cos(b^2\pi x^2)}{24x^2} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x} - \frac{1}{12}b^4\pi^2 S(bx)^2 - \frac{S(bx)^2}{4x^4} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} + \frac{1}{24}b^4\pi \text{Si}(b^2\pi x^2)
\end{aligned}$$

Mathematica [A] time = 0.0060233, size = 127, normalized size = 1.

$$-\frac{bS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^3} - \frac{\pi b^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x} - \frac{1}{12}\pi^2 b^4 S(bx)^2 + \frac{1}{12}\pi b^4 \text{Si}(b^2\pi x^2) - \frac{b^2}{24x^2} + \frac{b^2 \cos(\pi b^2 x^2)}{24x^2} - \frac{S(bx)^2}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]^2/x^5, x]

[Out] $-\frac{b^2}{24x^2} + \frac{(b^2 \cos(b^2 \pi x^2))}{(24x^2)} - \frac{(b^3 \pi \cos((b^2 \pi x^2)/2)) * \text{FresnelS}[b*x]}{(6x)} - \frac{(b^4 \pi^2 \text{FresnelS}[b*x]^2)}{12} - \frac{\text{FresnelS}[b*x]^2}{(4x^4)} - \frac{(b \text{FresnelS}[b*x] * \text{Sin}[(b^2 \pi x^2)/2])}{(6x^3)} + \frac{(b^4 \pi \text{SinIntegral}[b^2 \pi x^2])}{12}$

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^5,x)
```

```
[Out] int(FresnelS(b*x)^2/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**5,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^5, x)
```


$$3.44 \quad \int \frac{S(bx)^2}{x^6} dx$$

Optimal. Leaf size=170

$$-\frac{1}{20}\pi^2b^5\text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}, x\right) + \frac{7\pi^2b^5\text{FresnelC}(\sqrt{2}bx)}{60\sqrt{2}} - \frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{10x^4} - \frac{\pi b^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{20x^2}$$

```
[Out] -b^2/(60*x^3) + (b^2*Cos[b^2*Pi*x^2])/(60*x^3) + (7*b^5*Pi^2*FresnelC[Sqrt[2]*b*x])/(60*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(20*x^2)
- FresnelS[b*x]^2/(5*x^5) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(10*x^4)
- (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/20
```

Rubi [A] time = 0.153846, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx)^2}{x^6} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^6,x]
```

```
[Out] -b^2/(60*x^3) + (b^2*Cos[b^2*Pi*x^2])/(60*x^3) + (7*b^5*Pi^2*FresnelC[Sqrt[2]*b*x])/(60*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(20*x^2)
- FresnelS[b*x]^2/(5*x^5) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(10*x^4)
- (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/20
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^6} dx &= -\frac{S(bx)^2}{5x^5} + \frac{1}{5}(2b) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx \\
&= -\frac{b^2}{60x^3} - \frac{S(bx)^2}{5x^5} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4} - \frac{1}{20}b^2 \int \frac{\cos(b^2\pi x^2)}{x^4} dx + \frac{1}{10}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^3} dx \\
&= -\frac{b^2}{60x^3} + \frac{b^2 \cos(b^2\pi x^2)}{60x^3} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{20x^2} - \frac{S(bx)^2}{5x^5} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4} + \frac{1}{40}(b^4\pi) \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
&= -\frac{b^2}{60x^3} + \frac{b^2 \cos(b^2\pi x^2)}{60x^3} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{20x^2} - \frac{S(bx)^2}{5x^5} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4} - \frac{7b^4\pi \sin(b^2\pi x^2)}{120x} \\
&= -\frac{b^2}{60x^3} + \frac{b^2 \cos(b^2\pi x^2)}{60x^3} + \frac{7b^5\pi^2 C(\sqrt{2}bx)}{60\sqrt{2}} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{20x^2} - \frac{S(bx)^2}{5x^5} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4}
\end{aligned}$$

Mathematica [A] time = 0.0267984, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^6} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[b*x]^2/x^6, x]

[Out] Integrate[FresnelS[b*x]^2/x^6, x]

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)^2/x^6, x)

[Out] int(FresnelS(b*x)^2/x^6, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^6,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)^2/x^6, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^6,x, algorithm="fricas")

[Out] integral(fresnels(b*x)^2/x^6, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)**2/x**6,x)

[Out] Integral(fresnels(b*x)**2/x**6, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^6, x)
```

3.45 $\int \frac{S(bx)^2}{x^7} dx$

Optimal. Leaf size=165

$$-\frac{1}{45}\pi^2 b^5 \text{Unintegrable}\left(\frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right) + \frac{1}{72}\pi^2 b^6 \text{CosIntegral}(\pi b^2 x^2) - \frac{bS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^5} - \frac{\pi b^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{45x^3}$$

```
[Out] -b^2/(120*x^4) + (b^2*Cos[b^2*Pi*x^2])/(120*x^4) + (b^6*Pi^2*CosIntegral[b^2*Pi*x^2])/72 - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(45*x^3) - FresnelS[b*x]^2/(6*x^6) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x^5) - (b^4*Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/45
```

Rubi [A] time = 0.222055, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx)^2}{x^7} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^7,x]
```

```
[Out] -b^2/(120*x^4) + (b^2*Cos[b^2*Pi*x^2])/(120*x^4) + (b^6*Pi^2*CosIntegral[b^2*Pi*x^2])/72 - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(45*x^3) - FresnelS[b*x]^2/(6*x^6) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x^5) - (b^4*Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/45
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^7} dx &= -\frac{S(bx)^2}{6x^6} + \frac{1}{3}b \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx \\
&= -\frac{b^2}{120x^4} - \frac{S(bx)^2}{6x^6} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} - \frac{1}{30}b^2 \int \frac{\cos(b^2\pi x^2)}{x^5} dx + \frac{1}{15}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^4} dx \\
&= -\frac{b^2}{120x^4} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{45x^3} - \frac{S(bx)^2}{6x^6} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} - \frac{1}{60}b^2 \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^3} dx, x, x^2\right) \\
&= -\frac{b^2}{120x^4} + \frac{b^2 \cos(b^2\pi x^2)}{120x^4} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{45x^3} - \frac{S(bx)^2}{6x^6} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} + \frac{1}{180}(b^4\pi) \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^3} dx, x, x^2\right) \\
&= -\frac{b^2}{120x^4} + \frac{b^2 \cos(b^2\pi x^2)}{120x^4} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{45x^3} - \frac{S(bx)^2}{6x^6} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} - \frac{b^4\pi \sin(b^2\pi x^2)}{72x^2} \\
&= -\frac{b^2}{120x^4} + \frac{b^2 \cos(b^2\pi x^2)}{120x^4} + \frac{1}{72}b^6\pi^2 \text{Ci}(b^2\pi x^2) - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{45x^3} - \frac{S(bx)^2}{6x^6} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5}
\end{aligned}$$

Mathematica [A] time = 0.0202359, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^7} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[b*x]^2/x^7, x]

[Out] Integrate[FresnelS[b*x]^2/x^7, x]

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)^2/x^7, x)

[Out] `int(Fresnels(b*x)^2/x^7,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)^2/x^7,x, algorithm="maxima")`

[Out] `integrate(fresnels(b*x)^2/x^7, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)^2/x^7,x, algorithm="fricas")`

[Out] `integral(fresnels(b*x)^2/x^7, x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)**2/x**7,x)`

[Out] `Integral(fresnels(b*x)**2/x**7, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^7, x)
```


3.46 $\int \frac{S(bx)^2}{x^8} dx$

Optimal. Leaf size=258

$$-\frac{1}{168}\pi^3b^7\text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{\pi^2b^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{168x^2}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{21x^6}-\frac{\pi b^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{84x^4}$$

[Out] $-b^2/(210*x^5) + (b^6*Pi^2)/(336*x) + (b^2*\text{Cos}[b^2*Pi*x^2])/(210*x^5) - (67*b^6*Pi^2*\text{Cos}[b^2*Pi*x^2])/(5040*x) - (b^3*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(84*x^4) - \text{FresnelS}[b*x]^2/(7*x^7) - (b^7*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(72*\text{Sqrt}[2]) - (2*\text{Sqrt}[2]*b^7*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/315 - (b*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(21*x^6) + (b^5*Pi^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(168*x^2) - (13*b^4*Pi*\text{Sin}[b^2*Pi*x^2])/(2520*x^3) - (b^7*Pi^3*\text{Unintegrable}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/x, x])/168$

Rubi [A] time = 0.244556, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx)^2}{x^8} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[b*x]^2/x^8,x]

[Out] $-b^2/(210*x^5) + (b^6*Pi^2)/(336*x) + (b^2*\text{Cos}[b^2*Pi*x^2])/(210*x^5) - (67*b^6*Pi^2*\text{Cos}[b^2*Pi*x^2])/(5040*x) - (b^3*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(84*x^4) - \text{FresnelS}[b*x]^2/(7*x^7) - (b^7*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(72*\text{Sqrt}[2]) - (2*\text{Sqrt}[2]*b^7*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/315 - (b*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(21*x^6) + (b^5*Pi^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(168*x^2) - (13*b^4*Pi*\text{Sin}[b^2*Pi*x^2])/(2520*x^3) - (b^7*Pi^3*\text{Defier[Int]}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/x, x])/168$

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^8} dx &= -\frac{S(bx)^2}{7x^7} + \frac{1}{7}(2b) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx \\
&= -\frac{b^2}{210x^5} - \frac{S(bx)^2}{7x^7} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} - \frac{1}{42}b^2 \int \frac{\cos(b^2\pi x^2)}{x^6} dx + \frac{1}{21}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^5} dx \\
&= -\frac{b^2}{210x^5} + \frac{b^2 \cos(b^2\pi x^2)}{210x^5} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} + \frac{1}{168}(b^4\pi) \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^4} dx \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2 \cos(b^2\pi x^2)}{210x^5} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} + \frac{b^5\pi^2 S(bx)}{72\sqrt{x}} \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2 \cos(b^2\pi x^2)}{210x^5} - \frac{67b^6\pi^2 \cos(b^2\pi x^2)}{5040x} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} + \frac{b^5\pi^2 S(bx)}{72\sqrt{x}} \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2 \cos(b^2\pi x^2)}{210x^5} - \frac{67b^6\pi^2 \cos(b^2\pi x^2)}{5040x} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7} - \frac{b^7\pi^3 S(bx)}{72\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0272621, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^8} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[b*x]^2/x^8, x]

[Out] Integrate[FresnelS[b*x]^2/x^8, x]

Maple [A] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)^2/x^8, x)

[Out] int(FresnelS(b*x)^2/x^8, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^8,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)^2/x^8, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^8,x, algorithm="fricas")

[Out] integral(fresnels(b*x)^2/x^8, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)**2/x**8,x)

[Out] Integral(fresnels(b*x)**2/x**8, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^8, x)
```

3.47 $\int \frac{S(bx)^2}{x^9} dx$

Optimal. Leaf size=242

$$\frac{\pi^2 b^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{420x^3} - \frac{bS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{28x^7} + \frac{\pi^3 b^7 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{420x} - \frac{\pi b^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{140x^5} + \frac{1}{840}\pi^4 b^8 S(bx)$$

[Out] $-b^2/(336*x^6) + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) - (b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x) + (b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinIntegral[b^2*Pi*x^2])/280$

Rubi [A] time = 0.393047, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6430, 6456, 6464, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi^2 b^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{420x^3} - \frac{bS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{28x^7} + \frac{\pi^3 b^7 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{420x} - \frac{\pi b^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{140x^5} + \frac{1}{840}\pi^4 b^8 S(bx)$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]^2/x^9, x]

[Out] $-b^2/(336*x^6) + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) - (b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x) + (b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinIntegral[b^2*Pi*x^2])/280$

Rule 6430

Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*FresnelS[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(x^(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] := Simp[(x^(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_)^(m_))*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned}
 \int \frac{S(bx)^2}{x^9} dx &= -\frac{S(bx)^2}{8x^8} + \frac{1}{4}b \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx \\
 &= -\frac{b^2}{336x^6} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} - \frac{1}{56}b^2 \int \frac{\cos(b^2\pi x^2)}{x^7} dx + \frac{1}{28}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^6} dx \\
 &= -\frac{b^2}{336x^6} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{140x^5} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} - \frac{1}{112}b^2 \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^4} dx, x, x^2\right) \\
 &= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2 \cos(b^2\pi x^2)}{336x^6} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{140x^5} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{420x} \\
 &= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2 \cos(b^2\pi x^2)}{336x^6} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{140x^5} + \frac{b^7\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{420x} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} \\
 &= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2 \cos(b^2\pi x^2)}{336x^6} - \frac{b^6\pi^2 \cos(b^2\pi x^2)}{336x^2} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{140x^5} + \frac{b^7\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{420x} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} \\
 &= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2 \cos(b^2\pi x^2)}{336x^6} - \frac{b^6\pi^2 \cos(b^2\pi x^2)}{336x^2} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{140x^5} + \frac{b^7\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{420x} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7}
 \end{aligned}$$

Mathematica [A] time = 0.0113531, size = 242, normalized size = 1.

$$\frac{\pi^2 b^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{420x^3} - \frac{bS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{28x^7} + \frac{\pi^3 b^7 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{420x} - \frac{\pi b^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{140x^5} + \frac{1}{840}\pi^4 b^8 S(bx)^2$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]^2/x^9,x]

[Out] $-b^2/(336*x^6) + (b^6*\pi^2)/(1680*x^2) + (b^2*\cos[b^2*\pi*x^2])/(336*x^6) - (b^6*\pi^2*\cos[b^2*\pi*x^2])/(336*x^2) - (b^3*\pi*\cos[(b^2*\pi*x^2)/2]*\text{FresnelS}[b*x])/(140*x^5) + (b^7*\pi^3*\cos[(b^2*\pi*x^2)/2]*\text{FresnelS}[b*x])/(420*x) + (b^8*\pi^4*\text{FresnelS}[b*x]^2)/840 - \text{FresnelS}[b*x]^2/(8*x^8) - (b*\text{FresnelS}[b*x]*\sin[(b^2*\pi*x^2)/2])/(28*x^7) + (b^5*\pi^2*\text{FresnelS}[b*x]*\sin[(b^2*\pi*x^2)/2])/(420*x^3) - (b^4*\pi*\sin[b^2*\pi*x^2])/(420*x^4) - (b^8*\pi^3*\text{SinIntegral}[b^2*\pi*x^2])/280$

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)^2/x^9,x)

[Out] int(FresnelS(b*x)^2/x^9,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^9,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)^2/x^9, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^9,x, algorithm="fricas")

[Out] integral(fresnels(b*x)^2/x^9, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)**2/x**9,x)

[Out] Integral(fresnels(b*x)**2/x**9, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^9,x, algorithm="giac")

[Out] integrate(fresnels(b*x)^2/x^9, x)

3.48 $\int \frac{S(bx)^2}{x^{10}} dx$

Optimal. Leaf size=285

$$\frac{\pi^4 b^9 \text{Unintegrable}\left(\frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right)}{1728} - \frac{853\pi^4 b^9 \text{FresnelC}\left(\sqrt{2}bx\right)}{181440\sqrt{2}} + \frac{\pi^2 b^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{864x^4} - \frac{bS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{36x^8}$$

[Out] $-b^2/(504*x^7) + (b^6*\text{Pi}^2)/(5184*x^3) + (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(504*x^7) - (187*b^6*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(181440*x^3) - (853*b^9*\text{Pi}^4*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(181440*\text{Sqrt}[2]) - (b^3*\text{Pi}*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(216*x^6) + (b^7*\text{Pi}^3*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(1728*x^2) - \text{FresnelS}[b*x]^2/(9*x^9) - (b*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(36*x^8) + (b^5*\text{Pi}^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(864*x^4) - (19*b^4*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(15120*x^5) + (853*b^8*\text{Pi}^3*\text{Sin}[b^2*\text{Pi}*x^2])/(362880*x) + (b^9*\text{Pi}^4*\text{Unintegrable}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/x, x])/1728$

Rubi [A] time = 0.358317, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx)^2}{x^{10}} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[b*x]^2/x^10, x]

[Out] $-b^2/(504*x^7) + (b^6*\text{Pi}^2)/(5184*x^3) + (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(504*x^7) - (187*b^6*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(181440*x^3) - (853*b^9*\text{Pi}^4*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(181440*\text{Sqrt}[2]) - (b^3*\text{Pi}*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(216*x^6) + (b^7*\text{Pi}^3*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(1728*x^2) - \text{FresnelS}[b*x]^2/(9*x^9) - (b*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(36*x^8) + (b^5*\text{Pi}^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(864*x^4) - (19*b^4*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(15120*x^5) + (853*b^8*\text{Pi}^3*\text{Sin}[b^2*\text{Pi}*x^2])/(362880*x) + (b^9*\text{Pi}^4*\text{Defer[Int]}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/x, x])/1728$

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^{10}} dx &= -\frac{S(bx)^2}{9x^9} + \frac{1}{9}(2b) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx \\
&= -\frac{b^2}{504x^7} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} - \frac{1}{72}b^2 \int \frac{\cos(b^2\pi x^2)}{x^8} dx + \frac{1}{36}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^7} dx \\
&= -\frac{b^2}{504x^7} + \frac{b^2 \cos(b^2\pi x^2)}{504x^7} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{216x^6} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} + \frac{1}{432}(b^4\pi) \int \frac{S(bx)^2}{x^6} dx \\
&= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2 \cos(b^2\pi x^2)}{504x^7} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{216x^6} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} + \frac{b^5\pi^2}{1728x^3} \\
&= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2 \cos(b^2\pi x^2)}{504x^7} - \frac{187b^6\pi^2 \cos(b^2\pi x^2)}{181440x^3} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{216x^6} + \frac{b^7\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^3} \\
&= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2 \cos(b^2\pi x^2)}{504x^7} - \frac{187b^6\pi^2 \cos(b^2\pi x^2)}{181440x^3} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{216x^6} + \frac{b^7\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^3} \\
&= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2 \cos(b^2\pi x^2)}{504x^7} - \frac{187b^6\pi^2 \cos(b^2\pi x^2)}{181440x^3} - \frac{67b^9\pi^4 C(\sqrt{2}bx)}{25920\sqrt{2}} - \frac{1}{945}\sqrt{2}b^9\pi^4 C(\sqrt{2}bx)
\end{aligned}$$

Mathematica [A] time = 0.0271738, size = 0, normalized size = 0.

$$\int \frac{S(bx)^2}{x^{10}} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[b*x]^2/x^10,x]

[Out] Integrate[FresnelS[b*x]^2/x^10, x]

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx))^2}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^10,x)
```

```
[Out] int(FresnelS(b*x)^2/x^10,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^10, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx)^2}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^10, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(bx)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**10,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**10, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx)^2}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2/x^10,x, algorithm="giac")

[Out] integrate(fresnels(b*x)^2/x^10, x)

3.49 $\int (c + dx)^2 S(a + bx)^2 dx$

Optimal. Leaf size=497

$$\frac{id(a + bx)^2(bc - ad)\text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi(a + bx)^2\right)}{4\pi b^3} - \frac{id(a + bx)^2(bc - ad)\text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi(a + bx)^2\right)}{4\pi b^3}$$

```
[Out] (2*d^2*x)/(3*b^2*Pi^2) + (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) +
(d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (5*d^2*FresnelC[Sqrt[2]
*(a + b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (2*(b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2]
*FresnelS[a + b*x])/(b^3*Pi) + (2*d*(b*c - a*d)*(a + b*x)*Cos[(Pi*(a + b*x)
^2)/2]*FresnelS[a + b*x])/(b^3*Pi) + (2*d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)
^2)/2]*FresnelS[a + b*x])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x]*Fres
nelS[a + b*x])/(b^3*Pi) + ((b*c - a*d)^2*(a + b*x)*FresnelS[a + b*x]^2)/b^3
+ (d*(b*c - a*d)*(a + b*x)^2*FresnelS[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*F
resnelS[a + b*x]^2)/(3*b^3) - ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(
Sqrt[2]*b^3*Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a + b
*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi) -
(4*d^2*FresnelS[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rubi [A] time = 0.410257, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {6432, 6420, 6452, 3351, 6430, 6454, 6446, 3379, 2638, 6460, 3357, 3352, 3385}

$$\frac{id(a + bx)^2(bc - ad) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}i\pi(a + bx)^2\right)}{4\pi b^3} - \frac{id(a + bx)^2(bc - ad) {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}i\pi(a + bx)^2\right)}{4\pi b^3} - \frac{d(bc - ad)\text{FresnelS}[a + bx]}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelS[a + b*x]^2,x]
```

```
[Out] (2*d^2*x)/(3*b^2*Pi^2) + (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) +
(d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (5*d^2*FresnelC[Sqrt[2]
*(a + b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (2*(b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2]
*FresnelS[a + b*x])/(b^3*Pi) + (2*d*(b*c - a*d)*(a + b*x)*Cos[(Pi*(a + b*x)
^2)/2]*FresnelS[a + b*x])/(b^3*Pi) + (2*d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)
^2)/2]*FresnelS[a + b*x])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x]*Fres
nelS[a + b*x])/(b^3*Pi) + ((b*c - a*d)^2*(a + b*x)*FresnelS[a + b*x]^2)/b^3
```

$$+ (d*(b*c - a*d)*(a + b*x)^2*\text{FresnelS}[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*\text{FresnelS}[a + b*x]^2)/(3*b^3) - ((b*c - a*d)^2*\text{FresnelS}[\text{Sqrt}[2]*(a + b*x)])/(\text{Sqrt}[2]*b^3*\text{Pi}) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*\text{Pi}*(a + b*x)^2])/(b^3*\text{Pi}) - ((I/4)*d*(b*c - a*d)*(a + b*x)^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*\text{Pi}*(a + b*x)^2])/(b^3*\text{Pi}) - (4*d^2*\text{FresnelS}[a + b*x]*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(3*b^3*\text{Pi}^2)$$

Rule 6432

```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_), x_Symbol] :=
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*x)
]^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6420

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] := Simp[((a + b*x)*FresnelS[a
+ b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Sin[(Pi*(a + b*x)^2)/2]*FresnelS[a
+ b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(m_)], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_), x_Symbol] := Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] := Simp[(FresnelC[b*x]
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -(I*b^2*Pi*x^2)/2])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)*(x_)], x_Symbol] := Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_)^(n_)])^(p_.), x_Sy
mbol] := Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_)^2)], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_)^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```


Rubi steps

$$\begin{aligned}
\int (c + dx)^2 S(a + bx)^2 dx &= \frac{\text{Subst} \left(\int \left(b^2 c^2 \left(1 + \frac{ad(-2bc+ad)}{b^2 c^2} \right) S(x)^2 + 2bcd \left(1 - \frac{ad}{bc} \right) x S(x)^2 + d^2 x^2 S(x)^2 \right) dx, x, a + bx \right)}{b^3} \\
&= \frac{d^2 \text{Subst} \left(\int x^2 S(x)^2 dx, x, a + bx \right)}{b^3} + \frac{(2d(bc - ad)) \text{Subst} \left(\int x S(x)^2 dx, x, a + bx \right)}{b^3} + \frac{(bc - ad) \text{Subst} \left(\int S(x)^2 dx, x, a + bx \right)}{b^3} \\
&= \frac{(bc - ad)^2 (a + bx) S(a + bx)^2}{b^3} + \frac{d(bc - ad)(a + bx)^2 S(a + bx)^2}{b^3} + \frac{d^2 (a + bx)^3 S(a + bx)^2}{3b^3} - \frac{2d(bc - ad)(a + bx) \cos \left(\frac{1}{2} \pi (a + bx)^2 \right) S(a + bx)}{b^3 \pi} \\
&= \frac{2(bc - ad)^2 \cos \left(\frac{1}{2} \pi (a + bx)^2 \right) S(a + bx)}{b^3 \pi} + \frac{2d(bc - ad)(a + bx) \cos \left(\frac{1}{2} \pi (a + bx)^2 \right) S(a + bx)}{b^3 \pi} \\
&= \frac{d^2 (a + bx) \cos \left(\pi (a + bx)^2 \right)}{6b^3 \pi^2} + \frac{2(bc - ad)^2 \cos \left(\frac{1}{2} \pi (a + bx)^2 \right) S(a + bx)}{b^3 \pi} + \frac{2d(bc - ad)(a + bx) \cos \left(\frac{1}{2} \pi (a + bx)^2 \right) S(a + bx)}{b^3 \pi} \\
&= \frac{d(bc - ad) \cos \left(\pi (a + bx)^2 \right)}{2b^3 \pi^2} + \frac{d^2 (a + bx) \cos \left(\pi (a + bx)^2 \right)}{6b^3 \pi^2} - \frac{d^2 C \left(\sqrt{2}(a + bx) \right)}{6\sqrt{2}b^3 \pi^2} + \frac{2(bc - ad)(a + bx) \cos \left(\frac{1}{2} \pi (a + bx)^2 \right) S(a + bx)}{b^3 \pi} \\
&= \frac{2d^2 x}{3b^2 \pi^2} + \frac{d(bc - ad) \cos \left(\pi (a + bx)^2 \right)}{2b^3 \pi^2} + \frac{d^2 (a + bx) \cos \left(\pi (a + bx)^2 \right)}{6b^3 \pi^2} - \frac{d^2 C \left(\sqrt{2}(a + bx) \right)}{6\sqrt{2}b^3 \pi^2} + \frac{2(bc - ad)(a + bx) \cos \left(\frac{1}{2} \pi (a + bx)^2 \right) S(a + bx)}{b^3 \pi} \\
&= \frac{2d^2 x}{3b^2 \pi^2} + \frac{d(bc - ad) \cos \left(\pi (a + bx)^2 \right)}{2b^3 \pi^2} + \frac{d^2 (a + bx) \cos \left(\pi (a + bx)^2 \right)}{6b^3 \pi^2} - \frac{d^2 C \left(\sqrt{2}(a + bx) \right)}{6\sqrt{2}b^3 \pi^2} + \frac{2(bc - ad)(a + bx) \cos \left(\frac{1}{2} \pi (a + bx)^2 \right) S(a + bx)}{b^3 \pi}
\end{aligned}$$

Mathematica [F] time = 0.664232, size = 0, normalized size = 0.

$$\int (c + dx)^2 S(a + bx)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^2*FresnelS[a + b*x]^2,x]

[Out] Integrate[(c + d*x)^2*FresnelS[a + b*x]^2, x]

Maple [F] time = 0.244, size = 0, normalized size = 0.

$$\int (dx + c)^2 (\text{FresnelS}(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelS(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^2*FresnelS(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \text{fresnels}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*fresnels(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^2 + 2cdx + c^2\right)\text{fresnels}(bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnels(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 S^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnels(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*fresnels(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \text{fresnels}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*fresnels(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*fresnels(b*x + a)^2, x)

3.50 $\int (c + dx)S(a + bx)^2 dx$

Optimal. Leaf size=279

$$\frac{id(a + bx)^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi(a + bx)^2\right)}{8\pi b^2} - \frac{id(a + bx)^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi(a + bx)^2\right)}{8\pi b^2}$$

```
[Out] (d*Cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + (2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2]/2]*FresnelS[a + b*x])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a + b*x)^2]/2]*FresnelS[a + b*x])/(b^2*Pi) - (d*FresnelC[a + b*x]*FresnelS[a + b*x])/(2*b^2*Pi) + ((b*c - a*d)*(a + b*x)*FresnelS[a + b*x]^2)/b^2 + (d*(a + b*x)^2*FresnelS[a + b*x]^2)/(2*b^2) - ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi)
```

Rubi [A] time = 0.196207, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6432, 6420, 6452, 3351, 6430, 6454, 6446, 3379, 2638}

$$\frac{id(a + bx)^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}i\pi(a + bx)^2\right)}{8\pi b^2} - \frac{id(a + bx)^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}i\pi(a + bx)^2\right)}{8\pi b^2} + \frac{(a + bx)(bc - ad)S(a + bx)^2}{b^2} - \frac{(bc - ad)S(a + bx)^2}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelS[a + b*x]^2, x]
```

```
[Out] (d*Cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + (2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2]/2]*FresnelS[a + b*x])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a + b*x)^2]/2]*FresnelS[a + b*x])/(b^2*Pi) - (d*FresnelC[a + b*x]*FresnelS[a + b*x])/(2*b^2*Pi) + ((b*c - a*d)*(a + b*x)*FresnelS[a + b*x]^2)/b^2 + (d*(a + b*x)^2*FresnelS[a + b*x]^2)/(2*b^2) - ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi)
```

Rule 6432

```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*x
```

$^m, x], x], x, a + b*x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 6420

$\text{Int}[\text{FresnelS}[(a_.) + (b_.)*(x_.)]^2, x_Symbol] \rightarrow \text{Simp}[(a + b*x)*\text{FresnelS}[a + b*x]^2/b, x] - \text{Dist}[2, \text{Int}[(a + b*x)*\text{Sin}[(\text{Pi}*(a + b*x)^2]/2)*\text{FresnelS}[a + b*x], x], x] /; \text{FreeQ}\{a, b\}, x]$

Rule 6452

$\text{Int}[\text{FresnelS}[(b_.)*(x_.)]*(x_.)*\text{Sin}[(d_.)*(x_.)^2], x_Symbol] \rightarrow -\text{Simp}[(\text{Cos}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + \text{Dist}[1/(2*b*\text{Pi}), \text{Int}[\text{Sin}[2*d*x^2], x], x] /; \text{FreeQ}\{b, d\}, x] \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

Rule 6430

$\text{Int}[\text{FresnelS}[(b_.)*(x_.)]^2*(x_.)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m+1)}*\text{FresnelS}[b*x]^2)/(m+1), x] - \text{Dist}[(2*b)/(m+1), \text{Int}[x^{(m+1)}*\text{Sin}[(\text{Pi}*b^2*x^2)/2]*\text{FresnelS}[b*x], x], x] /; \text{FreeQ}[b, x] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{NeQ}[m, -1]$

Rule 6454

$\text{Int}[\text{FresnelS}[(b_.)*(x_.)]*(x_.)^{(m_.)*\text{Sin}[(d_.)*(x_.)^2], x_Symbol] \rightarrow -\text{Simp}[(x^{(m-1)}*\text{Cos}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + (\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\text{Cos}[d*x^2]*\text{FresnelS}[b*x], x], x] + \text{Dist}[1/(2*b*\text{Pi}), \text{Int}[x^{(m-1)}*\text{Sin}[2*d*x^2], x], x]) /; \text{FreeQ}\{b, d\}, x] \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

Rule 6446

$\text{Int}[\text{Cos}[(d_.)*(x_.)^2]*\text{FresnelS}[(b_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[(\text{FresnelC}[b*x]*\text{FresnelS}[b*x])/(2*b), x] + (-\text{Simp}[(1*I*b*x^2*\text{HypergeometricPFQ}\{1, 1\}, \{3/2, 2\}, -((I*b^2*\text{Pi}*x^2)/2))]/8, x] + \text{Simp}[(1*I*b*x^2*\text{HypergeometricPFQ}\{1, 1\}, \{3/2, 2\}, (1*I*b^2*\text{Pi}*x^2)/2)]/8, x]) /; \text{FreeQ}\{b, d\}, x] \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4]$

Rule 3379

$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*\text{Sin}[(c_.) + (d_.)*(x_.)^{(n_.)])}^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Sin}[c + d*x])^p}], x]$

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int (c + dx)S(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \left(bc\left(1 - \frac{ad}{bc}\right)S(x)^2 + dxS(x)^2\right) dx, x, a + bx\right)}{b^2} \\
 &= \frac{d \text{Subst}\left(\int xS(x)^2 dx, x, a + bx\right)}{b^2} + \frac{(bc - ad) \text{Subst}\left(\int S(x)^2 dx, x, a + bx\right)}{b^2} \\
 &= \frac{(bc - ad)(a + bx)S(a + bx)^2}{b^2} + \frac{d(a + bx)^2S(a + bx)^2}{2b^2} - \frac{d \text{Subst}\left(\int x^2S(x) \sin\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^2} \\
 &= \frac{2(bc - ad) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)S(a + bx)}{b^2\pi} + \frac{d(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)S(a + bx)}{b^2\pi} + \frac{(bc - ad)(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)S(a + bx)}{b^2\pi} \\
 &= \frac{2(bc - ad) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)S(a + bx)}{b^2\pi} + \frac{d(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)S(a + bx)}{b^2\pi} - \frac{dC(a + bx)}{2b^2} \\
 &= \frac{d \cos\left(\pi(a + bx)^2\right)}{4b^2\pi^2} + \frac{2(bc - ad) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)S(a + bx)}{b^2\pi} + \frac{d(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)S(a + bx)}{b^2\pi}
 \end{aligned}$$

Mathematica [F] time = 0.585015, size = 0, normalized size = 0.

$$\int (c + dx)S(a + bx)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)*FresnelS[a + b*x]^2, x]
```

```
[Out] Integrate[(c + d*x)*FresnelS[a + b*x]^2, x]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int (dx + c) (\text{FresnelS}(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*FresnelS(b*x+a)^2,x)

[Out] int((d*x+c)*FresnelS(b*x+a)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \text{fresnels}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*fresnels(b*x+a)^2,x, algorithm="maxima")

[Out] integrate((d*x + c)*fresnels(b*x + a)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c) \text{fresnels}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*fresnels(b*x+a)^2,x, algorithm="fricas")

[Out] integral((d*x + c)*fresnels(b*x + a)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) S^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*fresnels(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\text{fresnels}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnels(b*x + a)^2, x)
```


3.51 $\int S(a + bx)^2 dx$

Optimal. Leaf size=70

$$\frac{(a + bx)S(a + bx)^2}{b} - \frac{S(\sqrt{2}(a + bx))}{\sqrt{2}\pi b} + \frac{2S(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b}$$

[Out] (2*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x])/(b*Pi) + ((a + b*x)*FresnelS[a + b*x]^2)/b - FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*Pi)

Rubi [A] time = 0.167726, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6420, 6452, 3351}

$$\frac{(a + bx)S(a + bx)^2}{b} - \frac{S(\sqrt{2}(a + bx))}{\sqrt{2}\pi b} + \frac{2S(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[a + b*x]^2, x]

[Out] (2*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x])/(b*Pi) + ((a + b*x)*FresnelS[a + b*x]^2)/b - FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*Pi)

Rule 6420

Int[FresnelS[(a_.) + (b_.)*(x_.)]^2, x_Symbol] := Simp[((a + b*x)*FresnelS[a + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Sin[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x], x], x] /; FreeQ[{a, b}, x]

Rule 6452

Int[FresnelS[(b_.)*(x_.)]*(x_.)*Sin[(d_.)*(x_.)^2], x_Symbol] := -Simp[(Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int S(a+bx)^2 dx &= \frac{(a+bx)S(a+bx)^2}{b} - 2 \int (a+bx)S(a+bx) \sin\left(\frac{1}{2}\pi(a+bx)^2\right) dx \\
&= \frac{(a+bx)S(a+bx)^2}{b} - \frac{2 \operatorname{Subst}\left(\int xS(x) \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{b} \\
&= \frac{2 \cos\left(\frac{1}{2}\pi(a+bx)^2\right) S(a+bx)}{b\pi} + \frac{(a+bx)S(a+bx)^2}{b} - \frac{\operatorname{Subst}\left(\int \sin(\pi x^2) dx, x, a+bx\right)}{b\pi} \\
&= \frac{2 \cos\left(\frac{1}{2}\pi(a+bx)^2\right) S(a+bx)}{b\pi} + \frac{(a+bx)S(a+bx)^2}{b} - \frac{S\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b\pi}
\end{aligned}$$

Mathematica [A] time = 0.0104769, size = 67, normalized size = 0.96

$$\frac{2\pi(a+bx)S(a+bx)^2 - \sqrt{2}S\left(\sqrt{2}(a+bx)\right) + 4S(a+bx) \cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[a + b*x]^2, x]

[Out] (4*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x] + 2*Pi*(a + b*x)*FresnelS[a + b*x]^2 - Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)])/(2*b*Pi)

Maple [A] time = 0.056, size = 60, normalized size = 0.9

$$\frac{1}{b} \left((bx+a) (\operatorname{FresnelS}(bx+a))^2 + 2 \frac{\operatorname{FresnelS}(bx+a) \cos\left(\frac{1}{2}\pi(bx+a)^2\right)}{\pi} - \frac{\sqrt{2}\operatorname{FresnelS}\left((bx+a)\sqrt{2}\right)}{2\pi} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x+a)^2, x)

[Out] 1/b*((b*x+a)*FresnelS(b*x+a)^2+2*FresnelS(b*x+a)/Pi*cos(1/2*Pi*(b*x+a)^2)-1/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)^2,x, algorithm="maxima")

[Out] integrate(fresnels(b*x + a)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}(bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)^2,x, algorithm="fricas")

[Out] integral(fresnels(b*x + a)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int S^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)**2,x)

[Out] Integral(fresnels(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)^2, x)
```

$$3.52 \quad \int \frac{S(a+bx)^2}{c+dx} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable} \left(\frac{S(a+bx)^2}{c+dx}, x \right)$$

[Out] Unintegrable[FresnelS[a + b*x]^2/(c + d*x), x]

Rubi [A] time = 0.0232799, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(a+bx)^2}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[a + b*x]^2/(c + d*x), x]

[Out] Defer[Int][FresnelS[a + b*x]^2/(c + d*x), x]

Rubi steps

$$\int \frac{S(a+bx)^2}{c+dx} dx = \int \frac{S(a+bx)^2}{c+dx} dx$$

Mathematica [A] time = 0.0357935, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)^2}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[a + b*x]^2/(c + d*x), x]

[Out] Integrate[FresnelS[a + b*x]^2/(c + d*x), x]

Maple [A] time = 0.36, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx + a))^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x+a)^2/(d*x+c), x)

[Out] int(FresnelS(b*x+a)^2/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)^2/(d*x+c), x, algorithm="maxima")

[Out] integrate(fresnels(b*x + a)^2/(d*x + c), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx + a)^2}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)^2/(d*x+c), x, algorithm="fricas")

[Out] integral(fresnels(b*x + a)^2/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(fresnels(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)^2/(d*x + c), x)
```

$$3.53 \quad \int \frac{S(a+bx)^2}{(c+dx)^2} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{S(a+bx)^2}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[FresnelS[a + b*x]^2/(c + d*x)^2, x]

Rubi [A] time = 0.0228937, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelS[a + b*x]^2/(c + d*x)^2,x]

[Out] Defer[Int][FresnelS[a + b*x]^2/(c + d*x)^2, x]

Rubi steps

$$\int \frac{S(a+bx)^2}{(c+dx)^2} dx = \int \frac{S(a+bx)^2}{(c+dx)^2} dx$$

Mathematica [A] time = 0.0817917, size = 0, normalized size = 0.

$$\int \frac{S(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelS[a + b*x]^2/(c + d*x)^2,x]

[Out] Integrate[FresnelS[a + b*x]^2/(c + d*x)^2, x]

Maple [A] time = 0.355, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelS}(bx + a))^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x+a)^2/(d*x+c)^2,x)

[Out] int(FresnelS(b*x+a)^2/(d*x+c)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] integrate(fresnels(b*x + a)^2/(d*x + c)^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx + a)^2}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(fresnels(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{S^2(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(fresnels(a + b*x)**2/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx + a)^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(fresnels(b*x + a)^2/(d*x + c)^2, x)

3.54 $\int x^2 S(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=231

$$\left(\frac{1}{12} - \frac{i}{12}\right) x^3 (cx^n)^{-3/n} e^{-\frac{3a}{bn} + \frac{9i}{2\pi b^2 d^2 n^2}} \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi a b d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{3}{n}\right)}{\sqrt{\pi} b d}\right) + \left(\frac{1}{12} - \frac{i}{12}\right) x^3 (cx^n)^{-3/n} e^{-\frac{3a}{bn} - \frac{9i}{2\pi b^2 d^2 n^2}}$$

[Out] $((1/12 - I/12)*E^{((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2 + I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)^{(3/n) + ((1/12 - I/12)*E^{((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)^{(3/n) + (x^3*FresnelS[d*(a + b*Log[c*x^n]))]/3}$

Rubi [A] time = 0.570596, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{12} - \frac{i}{12}\right) x^3 (cx^n)^{-3/n} e^{-\frac{3a}{bn} + \frac{9i}{2\pi b^2 d^2 n^2}} \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi a b d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{3}{n}\right)}{\sqrt{\pi} b d}\right) + \left(\frac{1}{12} - \frac{i}{12}\right) x^3 (cx^n)^{-3/n} e^{-\frac{3a}{bn} - \frac{9i}{2\pi b^2 d^2 n^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2 \operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])], x]$

[Out] $((1/12 - I/12)*E^{((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2 + I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)^{(3/n) + ((1/12 - I/12)*E^{((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)^{(3/n) + (x^3*FresnelS[d*(a + b*Log[c*x^n]))]/3}$

Rule 6471

$\operatorname{Int}[\operatorname{FresnelS}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*(d_.)]*((e_.)*(x_.))^{(m_.)}, x_Symbol] :> \operatorname{Simp}[(e*x)^{(m+1)}*\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])]/(e*(m+1)), x] - \operatorname{Dist}[(b*d*n)/(m+1), \operatorname{Int}[(e*x)^m*\operatorname{Sin}[(\operatorname{Pi}*(d*(a + b*\operatorname{Log}[c*x^n]))^2)/2], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m, n\}, x\} \&\& \operatorname{NeQ}[m, -1]$

Rule 4617

Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
 x_Symbol] := Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - Dist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]

Rule 2278

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.),
 x_Symbol] := Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2274

Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] := Int[u*F^(a*v)*z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2276

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.),
 x_Symbol] := Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2234

Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int x^2 S(d(a + b \log(cx^n))) dx &= \frac{1}{3} x^3 S(d(a + b \log(cx^n))) - \frac{1}{3} (bdn) \int x^2 \sin\left(\frac{1}{2} d^2 \pi (a + b \log(cx^n))^2\right) dx \\
 &= \frac{1}{3} x^3 S(d(a + b \log(cx^n))) - \frac{1}{6} (ibdn) \int e^{-\frac{1}{2} i d^2 \pi (a + b \log(cx^n))^2} x^2 dx + \frac{1}{6} (ibdn) \int e^{\frac{1}{2} i d^2 \pi (a + b \log(cx^n))^2} x^2 dx \\
 &= \frac{1}{3} x^3 S(d(a + b \log(cx^n))) - \frac{1}{6} (ibdn) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - i a b d^2 \pi \log(cx^n) - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) x^2 dx \\
 &= \frac{1}{3} x^3 S(d(a + b \log(cx^n))) - \frac{1}{6} (ibdn) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) x^2 (cx^n)^{-n} dx \\
 &= \frac{1}{3} x^3 S(d(a + b \log(cx^n))) - \frac{1}{6} \left(i b d n x^{i a b d^2 n \pi} (cx^n)^{-i a b d^2 \pi} \right) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) x^2 dx \\
 &= \frac{1}{3} x^3 S(d(a + b \log(cx^n))) - \frac{1}{6} \left(i b d x^3 (cx^n)^{-i a b d^2 \pi - \frac{3 - i a b d^2 n \pi}{n}} \right) \text{Subst}\left(\int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) dx\right) \\
 &= \frac{1}{3} x^3 S(d(a + b \log(cx^n))) - \frac{1}{6} \left(i b d e^{-\frac{3a}{bn} - \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (cx^n)^{-i a b d^2 \pi - \frac{3 - i a b d^2 n \pi}{n}} \right) \text{Subst}\left(\int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) dx\right) \\
 &= \left(\frac{1}{12} - \frac{i}{12}\right) e^{-\frac{3a}{bn} + \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (cx^n)^{-3/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{3}{n} + i a b d^2 \pi + i b^2 d^2 \pi \log(cx^n)\right)}{b d \sqrt{\pi}}\right) + \left(\frac{1}{12} + \frac{i}{12}\right) e^{-\frac{3a}{bn} - \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (cx^n)^{-3/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{3}{n} + i a b d^2 \pi + i b^2 d^2 \pi \log(cx^n)\right)}{b d \sqrt{\pi}}\right)
 \end{aligned}$$

Mathematica [A] time = 6.88797, size = 319, normalized size = 1.38

$$\frac{1}{12} x^3 \left(4S(d(a + b \log(cx^n))) + \sqrt[4]{-1} \sqrt{2} (cx^n)^{-3/n} \left(\frac{e^{\frac{9i}{\pi b^2 d^2 n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) - 3i)}{\sqrt{\pi} b d n}\right)} \right) + i \operatorname{Erfi}\left(\frac{(-1)^{1/4} \sqrt{2} (cx^n)^{-3/n} \left(\frac{1}{2} + \frac{i}{2}\right) (\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) - 3i)}{\sqrt{\pi} b d n}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*FresnelS[d*(a + b*Log[c*x^n])], x]

[Out] (x^3*(4*FresnelS[d*(a + b*Log[c*x^n])]) + ((-1)^(1/4)*Sqrt[2]*E^(((6*a)/(b*n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(E^(((9*I)/(b^2*d^2*n^2*Pi)))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]) + I*Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n])]/(b*d*n*Sqrt[Pi])))

$$\frac{\text{Pi} \cdot \text{Log}[c \cdot x^n]}{(b \cdot d \cdot n \cdot \text{Sqrt}[2 \cdot \text{Pi}])} \cdot (\text{Cos}[(d^2 \cdot \text{Pi} \cdot (a - b \cdot n \cdot \text{Log}[x] + b \cdot \text{Log}[c \cdot x^n])^2) / 2] + \text{I} \cdot \text{Sin}[(d^2 \cdot \text{Pi} \cdot (a - b \cdot n \cdot \text{Log}[x] + b \cdot \text{Log}[c \cdot x^n])^2) / 2]) / (c \cdot x^n)^{(3/n)}) / 12$$

Maple [F] time = 0.701, size = 0, normalized size = 0.

$$\int x^2 \text{FresnelS}(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)

[Out] int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")

[Out] integrate(x^2*fresnels((b*log(c*x^n) + a)*d), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \text{fresnels}(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] integral(x^2*fresnels(b*d*log(c*x^n) + a*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 S(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*fresnels(d*(a+b*ln(c*x**n))),x)

[Out] Integral(x**2*fresnels(a*d + b*d*log(c*x**n)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")

[Out] integrate(x^2*fresnels((b*log(c*x^n) + a)*d), x)

3.55 $\int x S(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=227

$$\left(\frac{1}{8} - \frac{i}{8}\right) x^2 (cx^n)^{-2/n} e^{-\frac{2\pi ab d^2 n + 2i}{\pi b^2 d^2 n^2}} \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n}\right)}{\sqrt{\pi} b d}\right) + \left(\frac{1}{8} - \frac{i}{8}\right) x^2 (cx^n)^{-2/n} e^{-\frac{2(\pi ab d^2 n + i)}{\pi b^2 d^2 n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n}\right)}{\sqrt{\pi} b d}\right)$$

[Out] $((1/8 - I/8)*E^{((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*\operatorname{Erf}(((1/2 + I/2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*\operatorname{Log}[c*x^n]))/(b*d*\operatorname{Sqrt}[Pi])))/(c*x^n)^{(2/n)} + ((1/8 - I/8)*x^2*\operatorname{Erfi}(((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*\operatorname{Log}[c*x^n]))/(b*d*\operatorname{Sqrt}[Pi])))/(E^{((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*Pi))}*(c*x^n)^{(2/n)} + (x^2*\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])]))/2$

Rubi [A] time = 0.435235, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{8} - \frac{i}{8}\right) x^2 (cx^n)^{-2/n} e^{-\frac{2\pi ab d^2 n + 2i}{\pi b^2 d^2 n^2}} \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n}\right)}{\sqrt{\pi} b d}\right) + \left(\frac{1}{8} - \frac{i}{8}\right) x^2 (cx^n)^{-2/n} e^{-\frac{2(\pi ab d^2 n + i)}{\pi b^2 d^2 n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n}\right)}{\sqrt{\pi} b d}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])], x]$

[Out] $((1/8 - I/8)*E^{((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*\operatorname{Erf}(((1/2 + I/2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*\operatorname{Log}[c*x^n]))/(b*d*\operatorname{Sqrt}[Pi])))/(c*x^n)^{(2/n)} + ((1/8 - I/8)*x^2*\operatorname{Erfi}(((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*\operatorname{Log}[c*x^n]))/(b*d*\operatorname{Sqrt}[Pi])))/(E^{((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*Pi))}*(c*x^n)^{(2/n)} + (x^2*\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])]))/2$

Rule 6471

$\operatorname{Int}[\operatorname{FresnelS}(((a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.))*(d_.))*((e_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}(((e*x)^{(m+1)}*\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])])/(e*(m+1)), x) - \operatorname{Dist}[(b*d*n)/(m+1), \operatorname{Int}[(e*x)^m*\operatorname{Sin}[(\operatorname{Pi}*(d*(a + b*\operatorname{Log}[c*x^n]))^2)/2], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 4617

Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)], x_Symbol] := Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - Dist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]

Rule 2278

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.), x_Symbol] := Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2274

Int[(u_.)*(F_)^(((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] := Int[u*F^(a*v)*z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2276

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.), x_Symbol] := Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2234

Int[(F_)^(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]

Rule 2204

Int[(F_)^(((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[(F^a*sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F_)^(((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[(F^a*sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int x S(d(a + b \log(cx^n))) dx &= \frac{1}{2} x^2 S(d(a + b \log(cx^n))) - \frac{1}{2} (bdn) \int x \sin\left(\frac{1}{2} d^2 \pi (a + b \log(cx^n))^2\right) dx \\
 &= \frac{1}{2} x^2 S(d(a + b \log(cx^n))) - \frac{1}{4} (ibdn) \int e^{-\frac{1}{2} i d^2 \pi (a + b \log(cx^n))^2} x dx + \frac{1}{4} (ibdn) \int e^{\frac{1}{2} i d^2 \pi (a + b \log(cx^n))^2} x dx \\
 &= \frac{1}{2} x^2 S(d(a + b \log(cx^n))) - \frac{1}{4} (ibdn) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - i a b d^2 \pi \log(cx^n) - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) x dx \\
 &= \frac{1}{2} x^2 S(d(a + b \log(cx^n))) - \frac{1}{4} (ibdn) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) x (cx^n)^{-i a b d^2 \pi} dx \\
 &= \frac{1}{2} x^2 S(d(a + b \log(cx^n))) - \frac{1}{4} (ibdn x^{i a b d^2 \pi} (cx^n)^{-i a b d^2 \pi}) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) dx \\
 &= \frac{1}{2} x^2 S(d(a + b \log(cx^n))) - \frac{1}{4} \left(i b d x^2 (cx^n)^{-i a b d^2 \pi - \frac{2 - i a b d^2 \pi}{n}} \right) \text{Subst} \left(\int \exp\left(-\frac{1}{2} i a^2 d^2 \pi + \frac{1}{2} i b^2 d^2 \pi \log^2\left(\frac{x}{cx^n}\right)\right) dx \right) \\
 &= \frac{1}{2} x^2 S(d(a + b \log(cx^n))) - \frac{1}{4} \left(i b d e^{-\frac{2(i + a b d^2 \pi)}{b^2 d^2 n^2 \pi}} x^2 (cx^n)^{-i a b d^2 \pi - \frac{2 - i a b d^2 \pi}{n}} \right) \text{Subst} \left(\int \exp\left(\frac{i}{2} \left(\frac{2}{n} + i a b d^2 \pi + i b^2 d^2 \pi \log\left(\frac{x}{cx^n}\right)\right)^2\right) dx \right) \\
 &= \left(\frac{1}{8} - \frac{i}{8}\right) e^{\frac{2i - 2 a b d^2 \pi}{b^2 d^2 n^2 \pi}} x^2 (cx^n)^{-2/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} + i a b d^2 \pi + i b^2 d^2 \pi \log(cx^n)\right)}{b d \sqrt{\pi}}\right) + \left(\frac{1}{8} - \frac{i}{8}\right) e^{-\frac{2i - 2 a b d^2 \pi}{b^2 d^2 n^2 \pi}} x^2 (cx^n)^{-2/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} - \frac{i}{2}\right) \left(\frac{2}{n} + i a b d^2 \pi + i b^2 d^2 \pi \log(cx^n)\right)}{b d \sqrt{\pi}}\right)
 \end{aligned}$$

Mathematica [A] time = 6.64993, size = 319, normalized size = 1.41

$$\frac{1}{8} x^2 \left(4 S(d(a + b \log(cx^n))) + \sqrt[4]{-1} \sqrt{2} (cx^n)^{-2/n} \left(\frac{4i}{e^{\frac{2i - 2 a b d^2 \pi}{b^2 d^2 n^2 \pi}}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) - 2i)}{\sqrt{\pi} b d n}\right) \right) + i \operatorname{Erfi}\left(\frac{(-1)^{3/4}}{\sqrt{\pi} b d n}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x*FresnelS[d*(a + b*Log[c*x^n])],x]

[Out] (x^2*(4*FresnelS[d*(a + b*Log[c*x^n])]) + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*n)) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(E^((4*I)/(b^2*d^2*n^2*Pi)))*Erfi[(((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] + I*Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])])

$$\frac{\text{Pi} \cdot \text{Log}[c \cdot x^n]}{(b \cdot d \cdot n \cdot \text{Sqrt}[2 \cdot \text{Pi}])} \cdot (\text{Cos}[(d^2 \cdot \text{Pi} \cdot (a - b \cdot n \cdot \text{Log}[x] + b \cdot \text{Log}[c \cdot x^n])^2) / 2] + \text{I} \cdot \text{Sin}[(d^2 \cdot \text{Pi} \cdot (a - b \cdot n \cdot \text{Log}[x] + b \cdot \text{Log}[c \cdot x^n])^2) / 2]) / (c \cdot x^n)^{(2/n)}) / 8$$

Maple [F] time = 0.535, size = 0, normalized size = 0.

$$\int x \text{FresnelS}(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*FresnelS(d*(a+b*ln(c*x^n))),x)

[Out] int(x*FresnelS(d*(a+b*ln(c*x^n))),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \text{fresnels}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")

[Out] integrate(x*fresnels((b*log(c*x^n) + a)*d), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \text{fresnels}(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] integral(x*fresnels(b*d*log(c*x^n) + a*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x S(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnels(d*(a+b*ln(c*x**n))),x)

[Out] Integral(x*fresnels(a*d + b*d*log(c*x**n)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \text{fresnels}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")

[Out] integrate(x*fresnels((b*log(c*x^n) + a)*d), x)

3.56 $\int S(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=214

$$\left(\frac{1}{4} - \frac{i}{4}\right) x (cx^n)^{-1/n} e^{-\frac{2abn - \frac{i}{\pi d^2}}{2b^2 n^2}} \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n}\right)}{\sqrt{\pi} b d}\right) + \left(\frac{1}{4} - \frac{i}{4}\right) x (cx^n)^{-1/n} e^{-\frac{2abn + \frac{i}{\pi d^2}}{2b^2 n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n}\right)}{\sqrt{\pi} b d}\right)$$

[Out] $((1/4 - I/4)*x*\operatorname{Erf}(((1/2 + I/2)*(n^{-1}) + I*a*b*d^2*\Pi + I*b^2*d^2*\Pi*\operatorname{Log}[c*x^n]))/(b*d*\operatorname{Sqrt}[\Pi]))/(E^{((2*a*b*n - I/(d^2*\Pi))/(2*b^2*n^2))*(c*x^n)^{n^{-1}}}) + ((1/4 - I/4)*x*\operatorname{Erfi}(((1/2 + I/2)*(n^{-1}) - I*a*b*d^2*\Pi - I*b^2*d^2*\Pi*\operatorname{Log}[c*x^n]))/(b*d*\operatorname{Sqrt}[\Pi]))/(E^{((2*a*b*n + I/(d^2*\Pi))/(2*b^2*n^2))*(c*x^n)^{n^{-1}}}) + x*\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])]$

Rubi [A] time = 0.326252, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {6468, 4615, 2277, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{4} - \frac{i}{4}\right) x (cx^n)^{-1/n} e^{-\frac{2abn - \frac{i}{\pi d^2}}{2b^2 n^2}} \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n}\right)}{\sqrt{\pi} b d}\right) + \left(\frac{1}{4} - \frac{i}{4}\right) x (cx^n)^{-1/n} e^{-\frac{2abn + \frac{i}{\pi d^2}}{2b^2 n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n}\right)}{\sqrt{\pi} b d}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])], x]$

[Out] $((1/4 - I/4)*x*\operatorname{Erf}(((1/2 + I/2)*(n^{-1}) + I*a*b*d^2*\Pi + I*b^2*d^2*\Pi*\operatorname{Log}[c*x^n]))/(b*d*\operatorname{Sqrt}[\Pi]))/(E^{((2*a*b*n - I/(d^2*\Pi))/(2*b^2*n^2))*(c*x^n)^{n^{-1}}}) + ((1/4 - I/4)*x*\operatorname{Erfi}(((1/2 + I/2)*(n^{-1}) - I*a*b*d^2*\Pi - I*b^2*d^2*\Pi*\operatorname{Log}[c*x^n]))/(b*d*\operatorname{Sqrt}[\Pi]))/(E^{((2*a*b*n + I/(d^2*\Pi))/(2*b^2*n^2))*(c*x^n)^{n^{-1}}}) + x*\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])]$

Rule 6468

$\operatorname{Int}[\operatorname{FresnelS}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*(d_.)], x_Symbol] \rightarrow \operatorname{Simp}[x*\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])], x] - \operatorname{Dist}[b*d*n, \operatorname{Int}[\operatorname{Sin}[(\Pi*(d*(a + b*\operatorname{Log}[c*x^n]))^2)/2], x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x]$

Rule 4615

$\operatorname{Int}[\operatorname{Sin}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^2*(d_.)], x_Symbol] \rightarrow \operatorname{Dist}[I/2, \operatorname{Int}[E^{-(I*d*(a + b*\operatorname{Log}[c*x^n]))^2}], x], x] - \operatorname{Dist}[I/2, \operatorname{Int}[E^{(I*d*(a + b*\operatorname{Log}[c*x^n]))^2}], x], x]$

$b \cdot \log[c \cdot x^n]^2, x, x] /; \text{FreeQ}\{a, b, c, d, n\}, x]$

Rule 2277

$\text{Int}[(F_)^{\left(\left(a_{.}\right) + \log\left[\left(c_{.}\right) \cdot \left(x_{.}\right)^{\left(n_{.}\right)}\right] \cdot \left(b_{.}\right)\right)^2 \cdot \left(d_{.}\right)}, x_Symbol] :> \text{Int}\left[F^{\left(a^2 \cdot d + 2 \cdot a \cdot b \cdot d \cdot \log\left[c \cdot x^n\right] + b^2 \cdot d \cdot \log\left[c \cdot x^n\right]^2\right)}, x\right] /; \text{FreeQ}\{F, a, b, c, d, n\}, x]$

Rule 2274

$\text{Int}\left[\left(u_{.}\right) \cdot \left(F_{.}\right)^{\left(\left(a_{.}\right) \cdot \left(\log\left[z_{.}\right] \cdot \left(b_{.}\right) + \left(v_{.}\right)\right)\right)}, x_Symbol] :> \text{Int}\left[u \cdot F^{\left(a \cdot v\right)} \cdot z^{\left(a \cdot b \cdot \log[F]\right)}, x\right] /; \text{FreeQ}\{F, a, b\}, x]$

Rule 15

$\text{Int}\left[\left(u_{.}\right) \cdot \left(\left(a_{.}\right) \cdot \left(x_{.}\right)^{\left(n_{.}\right)}\right)^{\left(m_{.}\right)}, x_Symbol] :> \text{Dist}\left[\left(a^{\text{IntPart}[m]} \cdot \left(a \cdot x^n\right)^{\text{FracPart}[m]}\right) / x^{\left(n \cdot \text{FracPart}[m]\right)}, \text{Int}\left[u \cdot x^{\left(m \cdot n\right)}, x\right], x\right] /; \text{FreeQ}\{a, m, n\}, x] \&\amp; \text{IntegerQ}[m]$

Rule 2276

$\text{Int}\left[\left(F_{.}\right)^{\left(\left(a_{.}\right) + \log\left[\left(c_{.}\right) \cdot \left(x_{.}\right)^{\left(n_{.}\right)}\right]^2 \cdot \left(b_{.}\right)\right) \cdot \left(d_{.}\right)} \cdot \left(\left(e_{.}\right) \cdot \left(x_{.}\right)\right)^{\left(m_{.}\right)}, x_Symbol] :> \text{Dist}\left[\left(e \cdot x\right)^{\left(m + 1\right)} / \left(e \cdot n \cdot \left(c \cdot x^n\right)^{\left(m + 1\right) / n}\right), \text{Subst}\left[\text{Int}\left[E^{\left(a \cdot d \cdot \log[F] + \left(m + 1\right) \cdot x / n + b \cdot d \cdot \log[F] \cdot x^2\right)}, x\right], x, \log\left[c \cdot x^n\right]\right], x\right] /; \text{FreeQ}\{F, a, b, c, d, e, m, n\}, x]$

Rule 2234

$\text{Int}\left[\left(F_{.}\right)^{\left(\left(a_{.}\right) + \left(b_{.}\right) \cdot \left(x_{.}\right) + \left(c_{.}\right) \cdot \left(x_{.}\right)^2\right)}, x_Symbol] :> \text{Dist}\left[F^{\left(a - b^2 / \left(4 \cdot c\right)\right)}, \text{Int}\left[F^{\left(\left(b + 2 \cdot c \cdot x\right)^2 / \left(4 \cdot c\right)\right)}, x\right], x\right] /; \text{FreeQ}\{F, a, b, c\}, x]$

Rule 2204

$\text{Int}\left[\left(F_{.}\right)^{\left(\left(a_{.}\right) + \left(b_{.}\right) \cdot \left(\left(c_{.}\right) + \left(d_{.}\right) \cdot \left(x_{.}\right)\right)^2\right)}, x_Symbol] :> \text{Simp}\left[\left(F^a \cdot \text{Sqrt}\left[\text{Pi}\right] \cdot \text{Erfi}\left[\left(c + d \cdot x\right) \cdot \text{Rt}\left[b \cdot \log[F], 2\right]\right]\right) / \left(2 \cdot d \cdot \text{Rt}\left[b \cdot \log[F], 2\right]\right), x\right] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\amp; \text{PosQ}[b]$

Rule 2205

$\text{Int}\left[\left(F_{.}\right)^{\left(\left(a_{.}\right) + \left(b_{.}\right) \cdot \left(\left(c_{.}\right) + \left(d_{.}\right) \cdot \left(x_{.}\right)\right)^2\right)}, x_Symbol] :> \text{Simp}\left[\left(F^a \cdot \text{Sqrt}\left[\text{Pi}\right] \cdot \text{Erf}\left[\left(c + d \cdot x\right) \cdot \text{Rt}\left[-\left(b \cdot \log[F]\right), 2\right]\right]\right) / \left(2 \cdot d \cdot \text{Rt}\left[-\left(b \cdot \log[F]\right), 2\right]\right), x\right] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\amp; \text{NegQ}[b]$

Rubi steps

$$\begin{aligned}
\int S(d(a + b \log(cx^n))) dx &= xS(d(a + b \log(cx^n))) - (bdn) \int \sin\left(\frac{1}{2}d^2\pi(a + b \log(cx^n))^2\right) dx \\
&= xS(d(a + b \log(cx^n))) - \frac{1}{2}(ibdn) \int e^{-\frac{1}{2}id^2\pi(a+b \log(cx^n))^2} dx + \frac{1}{2}(ibdn) \int e^{\frac{1}{2}id^2\pi(a+b \log(cx^n))^2} dx \\
&= xS(d(a + b \log(cx^n))) - \frac{1}{2}(ibdn) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi \log(cx^n) - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) dx \\
&= xS(d(a + b \log(cx^n))) - \frac{1}{2}(ibdn) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) (cx^n)^{-iabd^2\pi} dx \\
&= xS(d(a + b \log(cx^n))) - \frac{1}{2}\left(ibdnx^{iabd^2n\pi} (cx^n)^{-iabd^2\pi}\right) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2\right) dx \\
&= xS(d(a + b \log(cx^n))) - \frac{1}{2}\left(ibdx (cx^n)^{-iabd^2\pi - \frac{1-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(1-iabd^2n\pi)\log^2}{n}\right) dx\right) \\
&= xS(d(a + b \log(cx^n))) - \frac{1}{2}\left(ibde^{-\frac{2abn + \frac{i}{d^2\pi}}{2b^2n^2}} x (cx^n)^{-iabd^2\pi - \frac{1-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(\frac{i\left(\frac{1-iabd^2n\pi}{n}\right)\log^2}{n}\right) dx\right) \\
&= \left(\frac{1}{4} - \frac{i}{4}\right) e^{-\frac{2abn - \frac{i}{d^2\pi}}{2b^2n^2}} x (cx^n)^{-1/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{1}{n} + iabd^2\pi + ib^2d^2\pi \log(cx^n)\right)}{bd\sqrt{\pi}}\right) + \left(\frac{1}{4} - \frac{i}{4}\right) e^{-\frac{2abn + \frac{i}{d^2\pi}}{2b^2n^2}} x (cx^n)^{-1/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} - \frac{i}{2}\right)\left(\frac{1}{n} + iabd^2\pi + ib^2d^2\pi \log(cx^n)\right)}{bd\sqrt{\pi}}\right)
\end{aligned}$$

Mathematica [A] time = 6.65872, size = 316, normalized size = 1.48

$$xS(d(a + b \log(cx^n))) + \frac{\sqrt[4]{-1}x (cx^n)^{-1/n} \left(e^{\frac{i}{\pi b^2 d^2 n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) - i)}{\sqrt{\pi} b d n}\right) + i \operatorname{Erfi}\left(\frac{(-1)^{3/4}(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) + i)}{\sqrt{2\pi} b d n}\right) \right)}{\sqrt{2\pi} b d n}$$

Warning: Unable to verify antiderivative.

[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])], x]

[Out] x*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((-2*a)/(b*n) - I/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(E^(I/(b^2*d^2*n^2*Pi))*Erfi[(((1/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi]))] + I*Erfi[(((-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi]))])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-1))

Maple [F] time = 0.541, size = 0, normalized size = 0.

$$\int \text{FresnelS}(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(d*(a+b*ln(c*x^n))),x)

[Out] int(FresnelS(d*(a+b*ln(c*x^n))),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")

[Out] integrate(fresnels((b*log(c*x^n) + a)*d), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{fresnels}(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] integral(fresnels(b*d*log(c*x^n) + a*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int S(d(a + b \log(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(fresnels(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(fresnels(d*(a + b*log(c*x**n))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(fresnels((b*log(c*x^n) + a)*d), x)
```

$$3.57 \quad \int \frac{S(d(a+b \log(cx^n)))}{x} dx$$

Optimal. Leaf size=65

$$\frac{\cos\left(\frac{1}{2}\pi d^2 (a + b \log(cx^n))^2\right)}{\pi b d n} + \frac{(a + b \log(cx^n)) S(d(a + b \log(cx^n)))}{b n}$$

[Out] Cos[(d^2*Pi*(a + b*Log[c*x^n])^2)/2]/(b*d*n*Pi) + (FresnelS[d*(a + b*Log[c*x^n]))*(a + b*Log[c*x^n])/(b*n)

Rubi [A] time = 0.0411381, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6418}

$$\frac{\cos\left(\frac{1}{2}\pi d^2 (a + b \log(cx^n))^2\right)}{\pi b d n} + \frac{(a + b \log(cx^n)) S(d(a + b \log(cx^n)))}{b n}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x,x]

[Out] Cos[(d^2*Pi*(a + b*Log[c*x^n])^2)/2]/(b*d*n*Pi) + (FresnelS[d*(a + b*Log[c*x^n]))*(a + b*Log[c*x^n])/(b*n)

Rule 6418

Int[FresnelS[(a_.) + (b_.)*(x_.)], x_Symbol] :> Simp[((a + b*x)*FresnelS[a + b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{S(d(a+b \log(cx^n)))}{x} dx &= \frac{\text{Subst}\left(\int S(d(a+bx)) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\text{Subst}\left(\int S(x) dx, x, ad + bd \log(cx^n)\right)}{bdn} \\ &= \frac{\cos\left(\frac{1}{2}\pi (ad + bd \log(cx^n))^2\right)}{bdn\pi} + \frac{S(ad + bd \log(cx^n)) (a + b \log(cx^n))}{bn} \end{aligned}$$

Mathematica [B] time = 0.0931173, size = 164, normalized size = 2.52

$$\frac{\sin\left(\frac{1}{2}\pi a^2 d^2\right) \sin\left(\pi a b d^2 \log(cx^n) + \frac{1}{2}\pi b^2 d^2 \log^2(cx^n)\right)}{\pi b d n} + \frac{\cos\left(\frac{1}{2}\pi a^2 d^2\right) \cos\left(\pi a b d^2 \log(cx^n) + \frac{1}{2}\pi b^2 d^2 \log^2(cx^n)\right)}{\pi b d n} +$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x,x]

[Out] (Cos[(a^2*d^2*Pi)/2]*Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi) + (a*FresnelS[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelS[d*(a + b*Log[c*x^n])]*Log[c*x^n])/n - (Sin[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)

Maple [A] time = 0.216, size = 80, normalized size = 1.2

$$\frac{\ln(cx^n) \operatorname{FresnelS}(ad + bd \ln(cx^n))}{n} + \frac{\operatorname{FresnelS}(ad + bd \ln(cx^n)) a}{bn} + \frac{1}{bdn\pi} \cos\left(\frac{\pi (ad + bd \ln(cx^n))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x,x)

[Out] 1/n*ln(c*x^n)*FresnelS(a*d+b*d*ln(c*x^n))+1/n/b*FresnelS(a*d+b*d*ln(c*x^n))*a+1/n/b/d/Pi*cos(1/2*Pi*(a*d+b*d*ln(c*x^n))^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{fresnels}((b \log(cx^n) + a)d)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")

[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bd \log(cx^n) + ad)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")

[Out] integral(fresnels(b*d*log(c*x^n) + a*d)/x, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{S(ad + bd \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x,x)

[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}((b \log(cx^n) + a)d)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")

[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x, x)

$$3.58 \quad \int \frac{S(d(a+b \log(cx^n)))}{x^2} dx$$

Optimal. Leaf size=217

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (cx^n)^{\frac{1}{n}} e^{\frac{2abn + \frac{i}{\pi d^2}}{2b^2 n^2}} \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (-i\pi abd^2 - i\pi b^2 d^2 \log(cx^n) + \frac{1}{n})}{\sqrt{\pi} bd}\right)}{x} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (cx^n)^{\frac{1}{n}} e^{\frac{2abn - \frac{i}{\pi d^2}}{2b^2 n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (i\pi abd^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n})}{\sqrt{\pi} bd}\right)}{x}$$

[Out] $((1/4 - I/4)*E^{((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^{(-1)}*Erf[((1/2 + I/2)*(n^{(-1)} - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/x + ((1/4 - I/4)*E^{((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^{(-1)}*Erfi[((1/2 + I/2)*(n^{(-1)} + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/x - FresnelS[d*(a + b*Log[c*x^n])]/x$

Rubi [A] time = 0.511188, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (cx^n)^{\frac{1}{n}} e^{\frac{2abn + \frac{i}{\pi d^2}}{2b^2 n^2}} \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (-i\pi abd^2 - i\pi b^2 d^2 \log(cx^n) + \frac{1}{n})}{\sqrt{\pi} bd}\right)}{x} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (cx^n)^{\frac{1}{n}} e^{\frac{2abn - \frac{i}{\pi d^2}}{2b^2 n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (i\pi abd^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n})}{\sqrt{\pi} bd}\right)}{x}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])]/x^2, x]$

[Out] $((1/4 - I/4)*E^{((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^{(-1)}*Erf[((1/2 + I/2)*(n^{(-1)} - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/x + ((1/4 - I/4)*E^{((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^{(-1)}*Erfi[((1/2 + I/2)*(n^{(-1)} + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/x - FresnelS[d*(a + b*Log[c*x^n])]/x$

Rule 6471

$\operatorname{Int}[\operatorname{FresnelS}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*(d_.)]*((e_.)*(x_.))^{(m_.)}, x_Symbol] :> \operatorname{Simp}[(e*x)^{(m+1)}*\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])]/(e*(m+1)), x] - \operatorname{Dist}[(b*d*n)/(m+1), \operatorname{Int}[(e*x)^m*\operatorname{Sin}[(\operatorname{Pi}*(d*(a + b*\operatorname{Log}[c*x^n]))^2)/2], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\amp; \operatorname{NeQ}[m, -1]$

Rule 4617

Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
 x_Symbol] := Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - Dist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]

Rule 2278

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.),
 x_Symbol] := Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2274

Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] := Int[u*F^(a*v)*z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2276

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.),
 x_Symbol] := Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2234

Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int \frac{S(d(a+b \log(cx^n)))}{x^2} dx &= -\frac{S(d(a+b \log(cx^n)))}{x} + (bdn) \int \frac{\sin\left(\frac{1}{2}d^2\pi(a+b \log(cx^n))^2\right)}{x^2} dx \\
 &= -\frac{S(d(a+b \log(cx^n)))}{x} + \frac{1}{2}(ibdn) \int \frac{e^{-\frac{1}{2}id^2\pi(a+b \log(cx^n))^2}}{x^2} dx - \frac{1}{2}(ibdn) \int \frac{e^{\frac{1}{2}id^2\pi(a+b \log(cx^n))^2}}{x^2} dx \\
 &= -\frac{S(d(a+b \log(cx^n)))}{x} + \frac{1}{2}(ibdn) \int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi \log(cx^n) - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right)}{x^2} dx \\
 &= -\frac{S(d(a+b \log(cx^n)))}{x} + \frac{1}{2}(ibdn) \int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right)(cx^n)^{-iabd^2\pi}}{x^2} dx \\
 &= -\frac{S(d(a+b \log(cx^n)))}{x} + \frac{1}{2}\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) dx \\
 &= -\frac{S(d(a+b \log(cx^n)))}{x} + \frac{\left(ibd(cx^n)^{-iabd^2\pi - \frac{-1-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(-1-iabd^2n\pi)}{n} \log^2(u)\right) du\right)}{2x} \\
 &= -\frac{S(d(a+b \log(cx^n)))}{x} + \frac{\left(ibde^{\frac{2abn-i}{2b^2n^2}}(cx^n)^{-iabd^2\pi - \frac{-1-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(\frac{i\left(\frac{-1-iabd^2n\pi}{n} - ib^2d^2\pi \log^2(u)\right)}{2b^2d^2\pi}\right) du\right)}{2x} \\
 &= \frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{2abn-i}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \text{erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{1}{n} - iabd^2\pi - ib^2d^2\pi \log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{2abn-i}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \text{erfi}\left(\frac{\left(\frac{1}{2} - \frac{i}{2}\right)\left(\frac{1}{n} - iabd^2\pi - ib^2d^2\pi \log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x}
 \end{aligned}$$

Mathematica [A] time = 3.96764, size = 195, normalized size = 0.9

$$\frac{4S(d(a+b \log(cx^n))) + \sqrt[4]{-1}\sqrt{2}(cx^n)^{\frac{1}{n}} e^{\frac{2abn-i}{2b^2n^2}} \left(e^{\frac{i}{\pi b^2 d^2 n^2}} \text{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) + i)}{\sqrt{\pi} b d n}\right) + i \text{Erfi}\left(\frac{(-1)^{3/4}(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) + i)}{\sqrt{2} \pi b d}\right) \right)}{4x}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^2,x]

[Out] -((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*(I*Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))]/(b*d*n*S

$\sqrt[3]{2\pi}] + E^{(I/(b^2d^2n^2\pi))} \operatorname{Erfi}\left(\frac{(1/2 + I/2)(I + a b d^2 n \pi + b^2 d^2 n \pi \operatorname{Log}[c x^n])}{b d n \sqrt{\pi}}\right) + 4 \operatorname{FresnelS}\left[\frac{d(a + b \operatorname{Log}[c x^n])}{4x}\right]$

Maple [F] time = 0.517, size = 0, normalized size = 0.

$$\int \frac{\operatorname{FresnelS}(d(a + b \ln(cx^n)))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)`

[Out] `int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{fresnels}((b \log(cx^n) + a)d)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")`

[Out] `integrate(fresnels((b*log(c*x^n) + a)*d)/x^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bd \log(cx^n) + ad)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")`

[Out] `integral(fresnels(b*d*log(c*x^n) + a*d)/x^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{S(ad + bd \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x**2,x)

[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}((b \log(cx^n) + a)d)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")

[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x^2, x)

$$3.59 \quad \int \frac{S(d(a+b \log(cx^n)))}{x^3} dx$$

Optimal. Leaf size=228

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) (cx^n)^{2/n} e^{\frac{2\pi ab d^2 n + 2i}{\pi b^2 d^2 n^2} \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (-i\pi ab d^2 - i\pi b^2 d^2 \log(cx^n) + \frac{2}{n})}{\sqrt{\pi} b d}\right)}}{x^2} + \frac{\left(\frac{1}{8} - \frac{i}{8}\right) (cx^n)^{2/n} e^{-\frac{2(-\pi ab d^2 n + i)}{\pi b^2 d^2 n^2} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n})}{\sqrt{\pi} b d}\right)}}{x^2}$$

[Out] $((1/8 - I/8)*E^{\left(\frac{2*I + 2*a*b*d^2*n*Pi}{b^2*d^2*n^2*Pi}\right)}*(c*x^n)^{(2/n)}*\operatorname{Erf}\left[\frac{\left(\frac{1}{2} + I/2\right)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*\operatorname{Log}[c*x^n])}{b*d*\operatorname{Sqrt}[Pi]}\right])/x^2 + ((1/8 - I/8)*(c*x^n)^{(2/n)}*\operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + I/2\right)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*\operatorname{Log}[c*x^n])}{b*d*\operatorname{Sqrt}[Pi]}\right])/(E^{\left(\frac{2*(I - a*b*d^2*n*Pi)}{b^2*d^2*n^2*Pi}\right)}*x^2) - \operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])]/(2*x^2)$

Rubi [A] time = 0.523086, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) (cx^n)^{2/n} e^{\frac{2\pi ab d^2 n + 2i}{\pi b^2 d^2 n^2} \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (-i\pi ab d^2 - i\pi b^2 d^2 \log(cx^n) + \frac{2}{n})}{\sqrt{\pi} b d}\right)}}{x^2} + \frac{\left(\frac{1}{8} - \frac{i}{8}\right) (cx^n)^{2/n} e^{-\frac{2(-\pi ab d^2 n + i)}{\pi b^2 d^2 n^2} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n})}{\sqrt{\pi} b d}\right)}}{x^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])]/x^3, x]$

[Out] $((1/8 - I/8)*E^{\left(\frac{2*I + 2*a*b*d^2*n*Pi}{b^2*d^2*n^2*Pi}\right)}*(c*x^n)^{(2/n)}*\operatorname{Erf}\left[\frac{\left(\frac{1}{2} + I/2\right)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*\operatorname{Log}[c*x^n])}{b*d*\operatorname{Sqrt}[Pi]}\right])/x^2 + ((1/8 - I/8)*(c*x^n)^{(2/n)}*\operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + I/2\right)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*\operatorname{Log}[c*x^n])}{b*d*\operatorname{Sqrt}[Pi]}\right])/(E^{\left(\frac{2*(I - a*b*d^2*n*Pi)}{b^2*d^2*n^2*Pi}\right)}*x^2) - \operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])]/(2*x^2)$

Rule 6471

$\operatorname{Int}[\operatorname{FresnelS}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*(d_.)]*(e_.)*(x_.)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(e*x)^{(m+1)}*\operatorname{FresnelS}[d*(a + b*\operatorname{Log}[c*x^n])]/(e*(m+1)), x] - \operatorname{Dist}[(b*d*n)/(m+1), \operatorname{Int}[(e*x)^m*\operatorname{Sin}[(\pi*(d*(a + b*\operatorname{Log}[c*x^n]))^2)/2], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 4617

Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
 x_Symbol] := Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - Dist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]

Rule 2278

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.),
 x_Symbol] := Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2274

Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] := Int[u*F^(a*v)*z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2276

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.),
 x_Symbol] := Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2234

Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int \frac{S(d(a+b \log(cx^n)))}{x^3} dx &= -\frac{S(d(a+b \log(cx^n)))}{2x^2} + \frac{1}{2}(bdn) \int \frac{\sin\left(\frac{1}{2}d^2\pi(a+b \log(cx^n))^2\right)}{x^3} dx \\
 &= -\frac{S(d(a+b \log(cx^n)))}{2x^2} + \frac{1}{4}(ibdn) \int \frac{e^{-\frac{1}{2}id^2\pi(a+b \log(cx^n))^2}}{x^3} dx - \frac{1}{4}(ibdn) \int \frac{e^{\frac{1}{2}id^2\pi(a+b \log(cx^n))^2}}{x^3} dx \\
 &= -\frac{S(d(a+b \log(cx^n)))}{2x^2} + \frac{1}{4}(ibdn) \int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi \log(cx^n) - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right)}{x^3} dx \\
 &= -\frac{S(d(a+b \log(cx^n)))}{2x^2} + \frac{1}{4}(ibdn) \int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right)(cx^n)^{-iabd^2\pi}}{x^3} dx \\
 &= -\frac{S(d(a+b \log(cx^n)))}{2x^2} + \frac{1}{4}\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2\left(\frac{cx^n}{x}\right)\right) dx \\
 &= -\frac{S(d(a+b \log(cx^n)))}{2x^2} + \frac{\left(ibd(cx^n)^{-iabd^2\pi - \frac{-2-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(-2-iabd^2n\pi)}{n} \log\left(\frac{cx^n}{x}\right)\right) dx}{4x^2}}{4x^2} \\
 &= -\frac{S(d(a+b \log(cx^n)))}{2x^2} + \frac{\left(ibde^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}}(cx^n)^{-iabd^2\pi - \frac{-2-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(i\left(\frac{-2-iabd^2n\pi}{n} \log\left(\frac{cx^n}{x}\right) - \frac{1}{2}ib^2d^2\pi \log^2\left(\frac{cx^n}{x}\right)\right)\right) dx}{4x^2}}{4x^2} \\
 &= \frac{\left(\frac{1}{8} - \frac{i}{8}\right) e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}} (cx^n)^{2/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{2}{n} - iabd^2\pi - ib^2d^2\pi \log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} + \frac{\left(\frac{1}{8} - \frac{i}{8}\right) e^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}} (cx^n)^{2/n}}{x^2}
 \end{aligned}$$

Mathematica [A] time = 3.98948, size = 200, normalized size = 0.88

$$\frac{S(d(a+b \log(cx^n)))}{2x^2} - \frac{\sqrt[4]{-1} \left(e^{\frac{4i}{\pi b^2 d^2 n^2}} \operatorname{Erfi}\left(\frac{\sqrt[4]{-1}(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) + 2i)}{\sqrt{2\pi} b d n}\right) + i \operatorname{Erfi}\left(\frac{(-1)^{3/4}(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) - 2i)}{\sqrt{2\pi} b d n}\right) \right) \exp\left(\frac{2i}{n}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^3,x]

[Out] -((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*x^n])))/n^2)*(I*Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n])])

)/(b*d*n*Sqrt[2*Pi]]) + E^((4*I)/(b^2*d^2*n^2*Pi))*Erfi[((-1)^(1/4)*(2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi]))]/(4*Sqrt[2]) - FresnelS[d*(a + b*Log[c*x^n])]/(2*x^2)

Maple [F] time = 0.522, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(d(a + b \ln(cx^n)))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)

[Out] int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}((b \log(cx^n) + a)d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")

[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bd \log(cx^n) + ad)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")

[Out] integral(fresnels(b*d*log(c*x^n) + a*d)/x^3, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{S(ad + bd \log(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x**3,x)

[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}((b \log(cx^n) + a)d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")

[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x^3, x)

3.60 $\int (ex)^m S(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=280

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) x(ex)^m (cx^n)^{-\frac{m+1}{n}} \exp\left(\frac{i(m+1)(2i\pi abd^2n+m+1)}{2\pi b^2 d^2 n^2}\right) \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(i\pi abd^2n+i\pi b^2 d^2 n \log(cx^n)+m+1)}{\sqrt{\pi} b d n}\right)}{m+1} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) x(ex)^m (cx^n)^{-\frac{m+1}{n}} e}{m+1}$$

[Out] $((1/4 - I/4)*E^{((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))}/(b^2*d^2*n^2*Pi)) * x * (e*x)^m * \operatorname{Erf} [((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log [c*x^n])) / (b*d*n*sqrt [Pi])] / ((1 + m)*(c*x^n)^{((1 + m)/n)}) + ((1/4 - I/4)*x * (e*x)^m * \operatorname{Erfi} [((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log [c*x^n])) / (b*d*n*sqrt [Pi])] / (E^{((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))} / (b^2*d^2*n^2*Pi)) * (1 + m)*(c*x^n)^{((1 + m)/n)}) + ((e*x)^{(1 + m)} * \operatorname{FresnelS} [d*(a + b*Log [c*x^n])]) / (e*(1 + m))$

Rubi [A] time = 0.701812, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {6471, 4617, 2278, 2274, 15, 20, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) x(ex)^m (cx^n)^{-\frac{m+1}{n}} \exp\left(\frac{i(m+1)(2i\pi abd^2n+m+1)}{2\pi b^2 d^2 n^2}\right) \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(i\pi abd^2n+i\pi b^2 d^2 n \log(cx^n)+m+1)}{\sqrt{\pi} b d n}\right)}{m+1} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) x(ex)^m (cx^n)^{-\frac{m+1}{n}} e}{m+1}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(e*x)^m * \operatorname{FresnelS}[d*(a + b*Log [c*x^n])], x]$

[Out] $((1/4 - I/4)*E^{((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))}/(b^2*d^2*n^2*Pi)) * x * (e*x)^m * \operatorname{Erf} [((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log [c*x^n])) / (b*d*n*sqrt [Pi])] / ((1 + m)*(c*x^n)^{((1 + m)/n)}) + ((1/4 - I/4)*x * (e*x)^m * \operatorname{Erfi} [((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log [c*x^n])) / (b*d*n*sqrt [Pi])] / (E^{((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))} / (b^2*d^2*n^2*Pi)) * (1 + m)*(c*x^n)^{((1 + m)/n)}) + ((e*x)^{(1 + m)} * \operatorname{FresnelS} [d*(a + b*Log [c*x^n])]) / (e*(1 + m))$

Rule 6471

$\operatorname{Int}[\operatorname{FresnelS}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}] * (b_.) * (d_.)] * ((e_.)*(x_.))^{(m_.)}, x_Symbol] :> \operatorname{Simp}[(e*x)^{(m+1)} * \operatorname{FresnelS}[d*(a + b*Log [c*x^n])]) / (e*(m+1)), x] - \operatorname{Dist}[(b*d*n)/(m+1), \operatorname{Int}[(e*x)^m * \operatorname{Sin}[(Pi*(d*(a + b*Log [c*x^n])])]]$

$\wedge 2)/2], x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x\} \&\& \text{NeQ}[m, -1]$

Rule 4617

$\text{Int}[(e \cdot x)^m \cdot \sin((a + \log(cx^n) \cdot b)^{2d}), x_Symbol] \rightarrow \text{Dist}[I/2, \text{Int}[(e \cdot x)^m / E^{(I \cdot d \cdot (a + b \cdot \log(cx^n))^2)}, x], x] - \text{Dist}[I/2, \text{Int}[(e \cdot x)^m \cdot E^{(I \cdot d \cdot (a + b \cdot \log(cx^n))^2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x]$

Rule 2278

$\text{Int}(F^{((a + \log(cx^n) \cdot b)^{2d})} \cdot (e \cdot x)^m, x_Symbol) \rightarrow \text{Int}[(e \cdot x)^m \cdot F^{(a^2 \cdot d + 2 \cdot a \cdot b \cdot d \cdot \log(cx^n) + b^2 \cdot d \cdot \log(cx^n)^2)}, x] /; \text{FreeQ}\{F, a, b, c, d, e, m, n\}, x]$

Rule 2274

$\text{Int}(u \cdot F^{(a \cdot (\log[z] \cdot b) + v)}, x_Symbol) \rightarrow \text{Int}[u \cdot F^{(a \cdot v)} \cdot z^{(a \cdot b \cdot \log[F])}, x] /; \text{FreeQ}\{F, a, b\}, x]$

Rule 15

$\text{Int}(u \cdot (a \cdot x^n)^m, x_Symbol) \rightarrow \text{Dist}[(a^{\text{IntPart}[m]} \cdot (a \cdot x^n)^{\text{FracPart}[m]}) / x^{(n \cdot \text{FracPart}[m])}, \text{Int}[u \cdot x^{(m \cdot n)}, x], x] /; \text{FreeQ}\{a, m, n\}, x \&\& \text{IntegerQ}[m]$

Rule 20

$\text{Int}(u \cdot (a \cdot v)^m \cdot (b \cdot v)^n, x_Symbol) \rightarrow \text{Dist}[(b^{\text{IntPart}[n]} \cdot (b \cdot v)^{\text{FracPart}[n]}) / (a^{\text{IntPart}[n]} \cdot (a \cdot v)^{\text{FracPart}[n]}), \text{Int}[u \cdot (a \cdot v)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, m, n\}, x \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n] \&\& \text{IntegerQ}[m+n]$

Rule 2276

$\text{Int}(F^{((a + \log(cx^n) \cdot b)^{2d})} \cdot (e \cdot x)^m, x_Symbol) \rightarrow \text{Dist}[(e \cdot x)^{(m+1)} / (e \cdot n \cdot (c \cdot x^n)^{(m+1)/n}), \text{Subst}[\text{Int}[E^{(a \cdot d \cdot \log[F] + ((m+1) \cdot x)/n + b \cdot d \cdot \log[F] \cdot x^2)}, x], x, \log[cx^n]], x] /; \text{FreeQ}\{F, a, b, c, d, e, m, n\}, x]$

Rule 2234

$\text{Int}(F^{(a + (b \cdot x) + (c \cdot x)^2)}, x_Symbol) \rightarrow \text{Dist}[F^{(a - b^2/(4 \cdot c))}, \text{Int}[F^{((b + 2 \cdot c \cdot x)^2/(4 \cdot c))}, x], x] /; \text{FreeQ}\{F, a, b, c\}, x]$

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
\int (ex)^m S(d(a + b \log(cx^n))) dx &= \frac{(ex)^{1+m} S(d(a + b \log(cx^n)))}{e(1+m)} - \frac{(bdn) \int (ex)^m \sin\left(\frac{1}{2}d^2\pi(a + b \log(cx^n))^2\right) dx}{1+m} \\
&= \frac{(ex)^{1+m} S(d(a + b \log(cx^n)))}{e(1+m)} - \frac{(ibdn) \int e^{-\frac{1}{2}id^2\pi(a+b \log(cx^n))^2} (ex)^m dx}{2(1+m)} + \frac{(ibdn) \int e^{\frac{1}{2}id^2\pi(a+b \log(cx^n))^2} (ex)^m dx}{2(1+m)} \\
&= \frac{(ex)^{1+m} S(d(a + b \log(cx^n)))}{e(1+m)} - \frac{(ibdn) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi \log(cx^n) - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) (ex)^m dx}{2(1+m)} \\
&= \frac{(ex)^{1+m} S(d(a + b \log(cx^n)))}{e(1+m)} - \frac{(ibdn) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) (ex)^m dx}{2(1+m)} \\
&= \frac{(ex)^{1+m} S(d(a + b \log(cx^n)))}{e(1+m)} - \frac{\left(ibdnx^{iabd^2n\pi} (cx^n)^{-iabd^2\pi}\right) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) (ex)^m dx}{2(1+m)} \\
&= \frac{(ex)^{1+m} S(d(a + b \log(cx^n)))}{e(1+m)} - \frac{\left(ibdnx^{-m+iabd^2n\pi} (ex)^m (cx^n)^{-iabd^2\pi}\right) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) dx}{2(1+m)} \\
&= \frac{(ex)^{1+m} S(d(a + b \log(cx^n)))}{e(1+m)} - \frac{\left(ibdx(ex)^m (cx^n)^{-iabd^2\pi - \frac{1+m-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2\left(\frac{x}{cx^n}\right)\right) dx, x, cx^n\right)}{2(1+m)} \\
&= \frac{(ex)^{1+m} S(d(a + b \log(cx^n)))}{e(1+m)} - \frac{\left(ibd \exp\left(-\frac{i(1+m)(1+m-2iabd^2n\pi)}{2b^2d^2n^2\pi}\right) x(ex)^m (cx^n)^{-iabd^2\pi - \frac{1+m-iabd^2n\pi}{n}}\right)}{2(1+m)} \\
&= \frac{\left(\frac{1}{4} - \frac{i}{4}\right) \exp\left(\frac{i(1+m)(1+m+2iabd^2n\pi)}{2b^2d^2n^2\pi}\right) x(ex)^m (cx^n)^{-\frac{1+m}{n}} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(1+m+iabd^2n\pi+ib^2d^2n\pi \log^2(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m}
\end{aligned}$$

Mathematica [A] time = 5.50402, size = 244, normalized size = 0.87

$$\frac{(ex)^m \left(4xS(d(a + b \log(cx^n))) - \sqrt[4]{-1} \sqrt{2} x^{-m} \exp\left(-\frac{(m+1)(2\pi ab d^2 n + 2\pi b^2 d^2 n (\log(cx^n) - n \log(x)) + im + i)}{2\pi b^2 d^2 n^2}\right) \right) \left(\operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(\pi ab d^2 n + \pi b^2 d^2 n \log(cx^n))}{\sqrt{\pi} b d n}\right) \right)}{4(m+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(e*x)^m*FresnelS[d*(a + b*Log[c*x^n])],x]

[Out] ((e*x)^m*(-(((1/4)*Sqrt[2]*(Erf[((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n])]/(b*d*n*Sqrt[Pi]))] + E^((I*(1 + m)^2)/(b^2*d^2*n^2*Pi))*Erfi[(((1/4)*(-1)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]))]/(b*d*n*Sqrt[2*Pi]))]/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2*d^2*n*Pi*(-n*Log[x]) + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m) + 4*x*FresnelS[d*(a + b*Log[c*x^n])]))/(4*(1 + m))

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{FresnelS}(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)

[Out] int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{fresnels}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")

[Out] integrate((e*x)^m*fresnels((b*log(c*x^n) + a)*d), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \text{fresnels}(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] integral((e*x)^m*fresnels(b*d*log(c*x^n) + a*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m S(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*fresnels(d*(a+b*ln(c*x**n))),x)

[Out] Integral((e*x)**m*fresnels(a*d + b*d*log(c*x**n)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \text{fresnels}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")

[Out] integrate((e*x)^m*fresnels((b*log(c*x^n) + a)*d), x)

3.61 $\int e^{c+\frac{1}{2}ib^2\pi x^2} S(bx) dx$

Optimal. Leaf size=64

$$\frac{1}{4}ibe^c x^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right) - \frac{e^c \text{Erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

[Out] $-(E^c \text{Erfi}[(1/2 + I/2)*b*\text{Sqrt}[\text{Pi}]*x]^2)/(8*b) + (I/4)*b*E^c*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2*\text{Pi}*x^2]$

Rubi [A] time = 0.0692265, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6436, 6376, 6375, 30}

$$\frac{1}{4}ibe^c x^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) - \frac{e^c \text{Erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(c + (I/2)*b^2*\text{Pi}*x^2)}*\text{FresnelS}[b*x], x]$

[Out] $-(E^c \text{Erfi}[(1/2 + I/2)*b*\text{Sqrt}[\text{Pi}]*x]^2)/(8*b) + (I/4)*b*E^c*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2*\text{Pi}*x^2]$

Rule 6436

$\text{Int}[E^{((c_.) + (d_.)*(x_)^2)}*\text{FresnelS}[(b_.)*(x_)], x_Symbol] \rightarrow \text{Dist}[(1 + I)/4, \text{Int}[E^{(c + d*x^2)}*\text{Erf}[(\text{Sqrt}[\text{Pi}]*(1 + I)*b*x)/2], x], x] + \text{Dist}[(1 - I)/4, \text{Int}[E^{(c + d*x^2)}*\text{Erf}[(\text{Sqrt}[\text{Pi}]*(1 - I)*b*x)/2], x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{EqQ}[d^2, -((\text{Pi}^2*b^4)/4)]$

Rule 6376

$\text{Int}[E^{((c_.) + (d_.)*(x_)^2)}*\text{Erf}[(b_.)*(x_)], x_Symbol] \rightarrow \text{Simp}[(b*E^c*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, b^2*x^2])/\text{Sqrt}[\text{Pi}], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{EqQ}[d, b^2]$

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)^(n_.), x_Symbol] := Dist[(E^c
*sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int e^{c+\frac{1}{2}ib^2\pi x^2} S(bx) dx &= \left(-\frac{1}{4} - \frac{i}{4}\right) \int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right) dx + \left(\frac{1}{4} + \frac{i}{4}\right) \int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right) dx \\ &= \frac{1}{4}ibe^c x^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) - \frac{e^c \operatorname{Subst}\left(\int x dx, x, \operatorname{erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right)\right)}{4b} \\ &= -\frac{e^c \operatorname{erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b} + \frac{1}{4}ibe^c x^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) \end{aligned}$$

Mathematica [F] time = 0.0304085, size = 0, normalized size = 0.

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} S(bx) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

```
[Out] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int e^{c+\frac{i}{2}b^2\pi x^2} \operatorname{FresnelS}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x), x)
```

[Out] `int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\frac{1}{2}i\pi b^2 x^2 + c\right)} \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")`

[Out] `integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{\left(\frac{1}{2}i\pi b^2 x^2 + c\right)} \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^c \int e^{\frac{i\pi b^2 x^2}{2}} S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnels(b*x),x)`

[Out] `exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnels(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\frac{1}{2}i\pi b^2 x^2 + c\right)} \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnels(b*x), x, algorithm="giac")
```

```
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

3.62 $\int e^{c-\frac{1}{2}ib^2\pi x^2} S(bx) dx$

Optimal. Leaf size=64

$$\frac{e^c \operatorname{Erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{\pi} bx\right)^2}{8b} - \frac{1}{4} i b e^c x^2 \operatorname{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2} i \pi b^2 x^2\right)$$

[Out] (E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/(8*b) - (I/4)*b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]

Rubi [A] time = 0.0681901, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6436, 6373, 30, 6378}

$$\frac{e^c \operatorname{Erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{\pi} bx\right)^2}{8b} - \frac{1}{4} i b e^c x^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2} i b^2 \pi x^2\right)$$

Antiderivative was successfully verified.

[In] Int[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]

[Out] (E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/(8*b) - (I/4)*b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]

Rule 6436

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelS[(b_.)*(x_)], x_Symbol] := Dist[(1 + I)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 + I)*b*x)/2], x], x] + Dist[(1 - I)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 - I)*b*x)/2], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -(Pi^2*b^4)/4]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] := Dist[(E^c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}, x] && EqQ[d, -b^2]
```

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 6378

Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] := Simp[(b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)]/Sqrt[Pi], x] /; FreeQ[{b, c, d}, x] && EqQ[d, -b^2]

Rubi steps

$$\begin{aligned} \int e^{c-\frac{1}{2}ib^2\pi x^2} S(bx) dx &= \left(-\frac{1}{4} - \frac{i}{4}\right) \int e^{c-\frac{1}{2}ib^2\pi x^2} \operatorname{erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right) dx + \left(\frac{1}{4} + \frac{i}{4}\right) \int e^{c-\frac{1}{2}ib^2\pi x^2} \operatorname{erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right) dx \\ &= -\frac{1}{4}ibe^c x^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) + \frac{e^c \operatorname{Subst}\left(\int x dx, x, \operatorname{erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right)\right)}{4b} \\ &= \frac{e^c \operatorname{erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b} - \frac{1}{4}ibe^c x^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) \end{aligned}$$

Mathematica [F] time = 0.0321957, size = 0, normalized size = 0.

$$\int e^{c-\frac{1}{2}ib^2\pi x^2} S(bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]

[Out] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int e^{c-\frac{i}{2}b^2\pi x^2} \operatorname{FresnelS}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x), x)

[Out] `int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)} \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")`

[Out] `integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)} \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^c \int e^{-\frac{i\pi b^2x^2}{2}} S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnels(b*x),x)`

[Out] `exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnels(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(-\frac{1}{2}i\pi b^2 x^2 + c\right)} \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnels(b*x), x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

3.63 $\int S(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=101

$$-\frac{1}{8}ibx^2 \sin(c) \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right) + \frac{1}{8}ibx^2 \sin(c) \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}\right)$$

```
[Out] (Cos[c]*FresnelS[b*x]^2)/(2*b) + (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
- (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]*Sin[c]
+ (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin[c]
```

Rubi [A] time = 0.0566188, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {6442, 6446, 6440, 30}

$$-\frac{1}{8}ibx^2 \sin(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2 \sin(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) + \frac{\sin(c)\text{FresnelC}(bx)S(bx)}{2b} + \frac{\cos(c)S(bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

```
[Out] (Cos[c]*FresnelS[b*x]^2)/(2*b) + (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
- (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]*Sin[c]
+ (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin[c]
```

Rule 6442

```
Int[FresnelS[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[Sin[c], Int[Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[Cos[c], Int[Sin[d*x^2]*FresnelS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int S(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) dx &= \cos(c) \int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx + \sin(c) \int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx \\ &= \frac{C(bx)S(bx) \sin(c)}{2b} - \frac{1}{8}ibx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) \sin(c) + \frac{1}{8}ibx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) \sin(c) \\ &= \frac{\cos(c)S(bx)^2}{2b} + \frac{C(bx)S(bx) \sin(c)}{2b} - \frac{1}{8}ibx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) \sin(c) + \frac{1}{8}ibx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) \sin(c) \end{aligned}$$

Mathematica [F] time = 0.0490403, size = 0, normalized size = 0.

$$\int S(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

```
[Out] Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \text{FresnelS}(bx) \sin\left(c + \frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2), x)
```

[Out] `int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")`

[Out] `integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2 + c), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2 + c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2 + c), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{\pi b^2 x^2}{2} + c\right) S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)*sin(c+1/2*b**2*pi*x**2),x)`

[Out] `Integral(sin(pi*b**2*x**2/2 + c)*fresnels(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

3.64 $\int \cos\left(c + \frac{1}{2}b^2\pi x^2\right) S(bx) dx$

Optimal. Leaf size=101

$$-\frac{1}{8}ibx^2 \cos(c) \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right) + \frac{1}{8}ibx^2 \cos(c) \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)$$

[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*Cos[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] + (I/8)*b*x^2*Cos[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] - (FresnelS[b*x]^2*Sin[c])/(2*b)

Rubi [A] time = 0.049668, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {6448, 6446, 6440, 30}

$$-\frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) + \frac{\cos(c)\text{FresnelC}(bx)S(bx)}{2b} - \frac{\sin(c)S(bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x], x]

[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*Cos[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] + (I/8)*b*x^2*Cos[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] - (FresnelS[b*x]^2*Sin[c])/(2*b)

Rule 6448

Int[Cos[(c_) + (d_)*(x_)^2]*FresnelS[(b_)*(x_)], x_Symbol] :> Dist[Cos[c], Int[Cos[d*x^2]*FresnelS[b*x], x], x] - Dist[Sin[c], Int[Sin[d*x^2]*FresnelS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 6446

Int[Cos[(d_)*(x_)^2]*FresnelS[(b_)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \cos\left(c + \frac{1}{2}b^2\pi x^2\right) S(bx) dx &= \cos(c) \int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx - \sin(c) \int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx \\ &= \frac{\cos(c)C(bx)S(bx)}{2b} - \frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) \\ &= \frac{\cos(c)C(bx)S(bx)}{2b} - \frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) \end{aligned}$$

Mathematica [F] time = 0.043668, size = 0, normalized size = 0.

$$\int \cos\left(c + \frac{1}{2}b^2\pi x^2\right) S(bx) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

```
[Out] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \cos\left(c + \frac{b^2\pi x^2}{2}\right) \text{FresnelS}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x), x)
```

[Out] `int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2} \pi b^2 x^2 + c\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(c+1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")`

[Out] `integrate(cos(1/2*pi*b^2*x^2 + c)*fresnels(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos\left(\frac{1}{2} \pi b^2 x^2 + c\right) \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(c+1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2 + c)*fresnels(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{\pi b^2 x^2}{2} + c\right) S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(c+1/2*b**2*pi*x**2)*fresnels(b*x),x)`

[Out] `Integral(cos(pi*b**2*x**2/2 + c)*fresnels(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2 x^2 + c\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

$$3.65 \quad \int S(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal. Leaf size=13

$$\frac{S(bx)^3}{3b}$$

[Out] FresnelS[b*x]^3/(3*b)

Rubi [A] time = 0.0147598, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6440, 30}

$$\frac{S(bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^3/(3*b)

Rule 6440

Int[FresnelS[(b_.)*(x_.)]^(n_.)*Sin[(d_.)*(x_.)^2], x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int S(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= \frac{\text{Subst}\left(\int x^2 dx, x, S(bx)\right)}{b} \\ &= \frac{S(bx)^3}{3b} \end{aligned}$$

Mathematica [A] time = 0.0037452, size = 13, normalized size = 1.

$$\frac{S(bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^3/(3*b)

Maple [A] time = 0.046, size = 12, normalized size = 0.9

$$\frac{(\text{FresnelS}(bx))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)^2*sin(1/2*b^2*Pi*x^2),x)

[Out] 1/3*FresnelS(b*x)^3/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)^2 \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(fresnels(b*x)^2*sin(1/2*pi*b^2*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}(bx)^2 \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 1.4036, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^3(bx)}{3b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((fresnels(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)^2 \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2*sin(1/2*pi*b^2*x^2), x)
```

$$3.66 \quad \int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal. Leaf size=13

$$\frac{S(bx)^2}{2b}$$

[Out] FresnelS[b*x]^2/(2*b)

Rubi [A] time = 0.0113005, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6440, 30}

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]

[Out] FresnelS[b*x]^2/(2*b)

Rule 6440

Int[FresnelS[(b_.)*(x_.)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= \frac{\text{Subst}\left(\int x dx, x, S(bx)\right)}{b} \\ &= \frac{S(bx)^2}{2b} \end{aligned}$$

Mathematica [A] time = 0.0026855, size = 13, normalized size = 1.

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^2/(2*b)

Maple [A] time = 0.045, size = 12, normalized size = 0.9

$$\frac{(\text{FresnelS}(bx))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)

[Out] 1/2*FresnelS(b*x)^2/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 0.434449, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

$$3.67 \quad \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)} dx$$

Optimal. Leaf size=9

$$\frac{\log(S(bx))}{b}$$

[Out] Log[FresnelS[b*x]]/b

Rubi [A] time = 0.0147908, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6440, 29}

$$\frac{\log(S(bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x],x]

[Out] Log[FresnelS[b*x]]/b

Rule 6440

Int[FresnelS[(b_.)*(x_)^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rubi steps

$$\begin{aligned} \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, S(bx)\right)}{b} \\ &= \frac{\log(S(bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.010098, size = 9, normalized size = 1.

$$\frac{\log(S(bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x], x]

[Out] Log[FresnelS[b*x]]/b

Maple [A] time = 0.118, size = 10, normalized size = 1.1

$$\frac{\ln(\text{FresnelS}(bx))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x), x)

[Out] ln(FresnelS(b*x))/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x), x, algorithm="maxima")

[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(sin(1/2*pi*b^2*x^2)/fresnels(b*x), x)
```

Sympy [A] time = 0.235245, size = 8, normalized size = 0.89

$$\begin{cases} \frac{\log(S(bx))}{b} & \text{for } b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x),x)
```

```
[Out] Piecewise((log(fresnels(b*x))/b, Ne(b, 0)), (nan, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x), x)
```

$$3.68 \quad \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^2} dx$$

Optimal. Leaf size=11

$$-\frac{1}{bS(bx)}$$

[Out] -(1/(b*FresnelS[b*x]))

Rubi [A] time = 0.0148235, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6440, 30}

$$-\frac{1}{bS(bx)}$$

Antiderivative was successfully verified.

[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]

[Out] -(1/(b*FresnelS[b*x]))

Rule 6440

Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2} dx, x, S(bx)\right)}{b} \\ &= -\frac{1}{bS(bx)} \end{aligned}$$

Mathematica [A] time = 0.0029559, size = 11, normalized size = 1.

$$-\frac{1}{bS(bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]

[Out] -(1/(b*FresnelS[b*x]))

Maple [A] time = 0.043, size = 12, normalized size = 1.1

$$-\frac{1}{b\text{FresnelS}(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^2,x)

[Out] -1/b/FresnelS(b*x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^2,x, algorithm="maxima")

[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^2, x)
```

Sympy [A] time = 0.779841, size = 10, normalized size = 0.91

$$\begin{cases} -\frac{1}{bS(bx)} & \text{for } b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**2,x)
```

```
[Out] Piecewise((-1/(b*fresnels(b*x)), Ne(b, 0)), (nan, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^2, x)
```

$$3.69 \quad \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^3} dx$$

Optimal. Leaf size=13

$$-\frac{1}{2bS(bx)^2}$$

[Out] -1/(2*b*FresnelS[b*x]^2)

Rubi [A] time = 0.0149547, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6440, 30}

$$-\frac{1}{2bS(bx)^2}$$

Antiderivative was successfully verified.

[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]

[Out] -1/(2*b*FresnelS[b*x]^2)

Rule 6440

Int[FresnelS[(b_.)*(x_.)]^(n_.)*Sin[(d_.)*(x_.)^2], x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3} dx, x, S(bx)\right)}{b} \\ &= -\frac{1}{2bS(bx)^2} \end{aligned}$$

Mathematica [A] time = 0.0040066, size = 13, normalized size = 1.

$$-\frac{1}{2bS(bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]

[Out] -1/(2*b*FresnelS[b*x]^2)

Maple [A] time = 0.044, size = 12, normalized size = 0.9

$$-\frac{1}{2b(\text{FresnelS}(bx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^3,x)

[Out] -1/2/b/FresnelS(b*x)^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^3,x, algorithm="maxima")

[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^3, x)
```

Sympy [A] time = 1.93431, size = 14, normalized size = 1.08

$$\begin{cases} -\frac{1}{2bs^2(bx)} & \text{for } b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**3,x)
```

```
[Out] Piecewise((-1/(2*b*fresnels(b*x)**2), Ne(b, 0)), (nan, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}(bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^3, x)
```

$$3.70 \quad \int S(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal. Leaf size=17

$$\frac{S(bx)^{n+1}}{b(n+1)}$$

[Out] FresnelS[b*x]^(1 + n)/(b*(1 + n))

Rubi [A] time = 0.0176465, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6440, 30}

$$\frac{S(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]

[Out] FresnelS[b*x]^(1 + n)/(b*(1 + n))

Rule 6440

Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int S(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= \frac{\text{Subst}\left(\int x^n dx, x, S(bx)\right)}{b} \\ &= \frac{S(bx)^{1+n}}{b(1+n)} \end{aligned}$$

Mathematica [A] time = 0.0057795, size = 17, normalized size = 1.

$$\frac{S(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^(1+n)/(b*(1+n))

Maple [A] time = 0.046, size = 18, normalized size = 1.1

$$\frac{(\text{FresnelS}(bx))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)^n*sin(1/2*b^2*Pi*x^2),x)

[Out] FresnelS(b*x)^(1+n)/b/(1+n)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)^n \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(fresnels(b*x)^n*sin(1/2*pi*b^2*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}(bx)^n \sin\left(\frac{1}{2}\pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")

[Out] integral(fresnels(b*x)^n*sin(1/2*pi*b^2*x^2), x)

Sympy [A] time = 4.62388, size = 31, normalized size = 1.82

$$\begin{cases} 0 & \text{for } b = 0 \wedge (b = 0 \vee n = -1) \\ \frac{\log(S(bx))}{b} & \text{for } n = -1 \\ \frac{S(bx)S^n(bx)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)**n*sin(1/2*b**2*pi*x**2),x)

[Out] Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(n, -1))), (log(fresnels(b*x))/b, Eq(n, -1)), (fresnels(b*x)*fresnels(b*x)**n/(b*n + b), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)^n \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")

[Out] integrate(fresnels(b*x)^n*sin(1/2*pi*b^2*x^2), x)

3.71 $\int x^8 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=232

$$\frac{7x^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{105x S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^8} - \frac{x^7 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{35x^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{105S(bx)^2}{2\pi^4 b^9} - \frac{1}{12}$$

[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)

Rubi [A] time = 0.384108, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6454, 6462, 3379, 3309, 30, 3296, 2637, 2634, 6440}

$$\frac{7x^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{105x S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^8} - \frac{x^7 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{35x^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{105S(bx)^2}{2\pi^4 b^9} - \frac{1}{12}$$

Antiderivative was successfully verified.

[In] Int[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)

Rule 6454

Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x^(m-1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m-1)/(2*d), Int[x^(m-2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m-1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)*(x_)^(m_)], x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] := Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] := Simp[x/2, x] - Simp[Sin[2*c + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/(
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}
\int x^8 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{7 \int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^2\pi} + \frac{\int x^7 \sin\left(b^2\pi x^2\right) dx}{2b\pi} \\
&= -\frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{7x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{35 \int x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} - \frac{7 \int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^4\pi^2} \\
&= -\frac{x^6 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{7x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&= -\frac{x^6 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} - \frac{105x S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} \\
&= -\frac{7x^6}{12b^3\pi^2} + \frac{41x^2 \cos\left(b^2\pi x^2\right)}{4b^7\pi^4} - \frac{x^6 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} \\
&= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2 \cos\left(b^2\pi x^2\right)}{4b^7\pi^4} - \frac{x^6 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} \\
&= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2 \cos\left(b^2\pi x^2\right)}{4b^7\pi^4} - \frac{x^6 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi}
\end{aligned}$$

Mathematica [A] time = 0.0130934, size = 232, normalized size = 1.

$$\frac{7x^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{105x S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^8} - \frac{x^7 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{35x^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{105S(bx)^2}{2\pi^4 b^9} - \frac{7x^6}{12b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)
/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^
4)
```


$$2) - (40*\text{Sin}[b^2*\text{Pi}*x^2])/(b^9*\text{Pi}^5) + (5*x^4*\text{Sin}[b^2*\text{Pi}*x^2])/(2*b^5*\text{Pi}^3)$$

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int x^8 \text{FresnelS}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)`

[Out] `int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")`

[Out] `integrate(x^8*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^8 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(x^8*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \operatorname{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")`

[Out] `integrate(x^8*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

3.72 $\int x^7 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=216

$$-\frac{531\text{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}\pi^4b^8} + \frac{6x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{48S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8} - \frac{x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{24x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}$$

[Out] $(24*x)/(b^7*\text{Pi}^4) - (3*x^5)/(5*b^3*\text{Pi}^2) + (147*x*\text{Cos}[b^2*\text{Pi}*x^2])/(16*b^7*\text{Pi}^4) - (x^5*\text{Cos}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2) - (531*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(16*\text{Sqrt}[2]*b^8*\text{Pi}^4) + (24*x^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(b^6*\text{Pi}^3) - (x^6*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(b^2*\text{Pi}) - (48*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^8*\text{Pi}^4) + (6*x^4*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^4*\text{Pi}^2) + (17*x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(8*b^5*\text{Pi}^3)$

Rubi [A] time = 0.264901, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6454, 6462, 3391, 30, 3386, 3385, 3352, 6460, 3357}

$$-\frac{531\text{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}\pi^4b^8} + \frac{6x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{48S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8} - \frac{x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{24x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^7*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2], x]$

[Out] $(24*x)/(b^7*\text{Pi}^4) - (3*x^5)/(5*b^3*\text{Pi}^2) + (147*x*\text{Cos}[b^2*\text{Pi}*x^2])/(16*b^7*\text{Pi}^4) - (x^5*\text{Cos}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2) - (531*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(16*\text{Sqrt}[2]*b^8*\text{Pi}^4) + (24*x^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(b^6*\text{Pi}^3) - (x^6*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(b^2*\text{Pi}) - (48*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^8*\text{Pi}^4) + (6*x^4*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^4*\text{Pi}^2) + (17*x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(8*b^5*\text{Pi}^3)$

Rule 6454

$\text{Int}[\text{FresnelS}[(b_.)*(x_.)]*(x_.)^{(m_.)}*\text{Sin}[(d_.)*(x_.)^2], x_Symbol] \text{ :> } -\text{Simp}[(x^{(m-1)}*\text{Cos}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + (\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\text{Cos}[d*x^2]*\text{FresnelS}[b*x], x], x] + \text{Dist}[1/(2*b*\text{Pi}), \text{Int}[x^{(m-1)}*\text{Sin}[2*d*x^2], x], x]) /;$ FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)](x_)^(m_), x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] := Dist[1/2, Int[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b, m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)](x_), x_Symbol] := Simp[(Sin[d*x^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
 \int x^7 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{6 \int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^2\pi} + \frac{\int x^6 \sin(b^2\pi x^2) dx}{2b\pi} \\
 &= -\frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{6x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{24 \int x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} \\
 &= -\frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{6x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
 &= -\frac{3x^5}{5b^3\pi^2} + \frac{111x \cos(b^2\pi x^2)}{16b^7\pi^4} - \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} \\
 &= -\frac{3x^5}{5b^3\pi^2} + \frac{147x \cos(b^2\pi x^2)}{16b^7\pi^4} - \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{15C(\sqrt{2}bx)}{16\sqrt{2}b^8\pi^4} - \frac{3\sqrt{2}C(\sqrt{2}bx)}{b^8\pi^4} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} \\
 &= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} + \frac{147x \cos(b^2\pi x^2)}{16b^7\pi^4} - \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{51C(\sqrt{2}bx)}{16\sqrt{2}b^8\pi^4} - \frac{3\sqrt{2}C(\sqrt{2}bx)}{b^8\pi^4} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} \\
 &= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} + \frac{147x \cos(b^2\pi x^2)}{16b^7\pi^4} - \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{51C(\sqrt{2}bx)}{16\sqrt{2}b^8\pi^4} - \frac{15\sqrt{2}C(\sqrt{2}bx)}{b^8\pi^4} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3}
 \end{aligned}$$

Mathematica [A] time = 0.276432, size = 153, normalized size = 0.71

$$\frac{-160S(bx) \left(\pi b^2 x^2 (\pi^2 b^4 x^4 - 24) \cos\left(\frac{1}{2}\pi b^2 x^2\right) - 6(\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2}\pi b^2 x^2\right) \right) + 2bx \left(2(-24\pi^2 b^4 x^4 + 85\pi b^2 x^2 \sin(\pi b^2 x^2)) \right)}{160\pi^4 b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

```
[Out] (-2655*sqrt[2]*FresnelC[sqrt[2]*b*x] - 160*FresnelS[b*x]*(b^2*Pi*x^2*(-24 +
b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)
/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] + 2*(960 - 24*b^4*Pi^
2*x^4 + 85*b^2*Pi*x^2*SIN[b^2*Pi*x^2]))) / (160*b^8*Pi^4)
```

Maple [A] time = 0.076, size = 318, normalized size = 1.5

$$\frac{1}{b} \left(\frac{\text{FresnelS}(bx)}{b^7} \left(-\frac{b^6 x^6}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + 6 \frac{1}{\pi} \left(\frac{x^4 b^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} - 4 \frac{1}{\pi} \left(-\frac{b^2 x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 2 \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)

[Out] (FresnelS(b*x)/b^7*(-1/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/Pi*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*Pi^2*b^5*x^5-8*b*x)-3/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))-1/2/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(x^7*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^7 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")

[Out] `integral(x^7*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")`

[Out] `integrate(x^7*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

3.73 $\int x^6 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=248

$$\frac{15ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right)}{8\pi^3 b^5} - \frac{15ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)}{8\pi^3 b^5} - \frac{15 \text{FresnelS}[bx]}{\pi^2 b^4} - \frac{x^5 S(bx)}{\pi^2 b^4}$$

[Out] $(-5x^4)/(8b^3\pi^2) + (11\cos[b^2\pi x^2])/(2b^7\pi^4) - (x^4\cos[b^2\pi x^2])/(4b^3\pi^2) + (15x\cos[(b^2\pi x^2)/2]*\text{FresnelS}[bx])/(b^6\pi^3) - (x^5\cos[(b^2\pi x^2)/2]*\text{FresnelS}[bx])/(b^2\pi) - (15*\text{FresnelC}[bx]*\text{FresnelS}[bx])/(2b^7\pi^3) + (((15I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2\pi x^2])/(b^5\pi^3) - (((15I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2\pi x^2])/(b^5\pi^3) + (5x^3*\text{FresnelS}[bx]*\sin[(b^2\pi x^2)/2])/(b^4\pi^2) + (7x^2*\sin[b^2\pi x^2])/(4b^5\pi^3)$

Rubi [A] time = 0.254643, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6454, 6462, 3379, 3309, 30, 3296, 2638, 6446}

$$\frac{15ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3 b^5} - \frac{15ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi^3 b^5} - \frac{15\text{FresnelC}(bx)S(bx)}{2\pi^3 b^7} + \frac{5x^3 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^5 S(bx)}{\pi^2 b^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^6*\text{FresnelS}[bx]*\sin[(b^2\pi x^2)/2], x]$

[Out] $(-5x^4)/(8b^3\pi^2) + (11\cos[b^2\pi x^2])/(2b^7\pi^4) - (x^4\cos[b^2\pi x^2])/(4b^3\pi^2) + (15x\cos[(b^2\pi x^2)/2]*\text{FresnelS}[bx])/(b^6\pi^3) - (x^5\cos[(b^2\pi x^2)/2]*\text{FresnelS}[bx])/(b^2\pi) - (15*\text{FresnelC}[bx]*\text{FresnelS}[bx])/(2b^7\pi^3) + (((15I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2\pi x^2])/(b^5\pi^3) - (((15I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2\pi x^2])/(b^5\pi^3) + (5x^3*\text{FresnelS}[bx]*\sin[(b^2\pi x^2)/2])/(b^4\pi^2) + (7x^2*\sin[b^2\pi x^2])/(4b^5\pi^3)$

Rule 6454

$\text{Int}[\text{FresnelS}[(b \cdot x)]*(x)^{(m)}*\sin[(d \cdot x)^2], x_Symbol] \rightarrow -\text{Simp}[(x^{(m-1)}*\cos[d*x^2]*\text{FresnelS}[bx])/(2*d), x] + (\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\cos[d*x^2]*\text{FresnelS}[bx], x], x] + \text{Dist}[1/(2*b*\pi), \text{Int}[x^{(m-1)}*\sin[2*d*x^2], x], x]) /; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (\pi^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 2]$

, 1]

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)*(x_)^(m_)], x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] := Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] := Simp[(FresnelC[b*x]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(I*b^2*Pi*x^2)/2])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
```

1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2))/8, x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rubi steps

$$\begin{aligned}
 \int x^6 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{5 \int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^2\pi} + \frac{\int x^5 \sin(b^2\pi x^2) dx}{2b\pi} \\
 &= -\frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{5x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{15 \int x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} - \frac{5 \int x \cos(b^2\pi x^2) S(bx) dx}{b^4\pi^2} \\
 &= -\frac{x^4 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{5x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
 &= -\frac{x^4 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} - \frac{15C(bx)S(bx)}{2b^7\pi^3} + \frac{15 \int x \cos(b^2\pi x^2) S(bx) dx}{b^6\pi^3} \\
 &= -\frac{5x^4}{8b^3\pi^2} + \frac{17 \cos(b^2\pi x^2)}{4b^7\pi^4} - \frac{x^4 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
 &= -\frac{5x^4}{8b^3\pi^2} + \frac{11 \cos(b^2\pi x^2)}{2b^7\pi^4} - \frac{x^4 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}
 \end{aligned}$$

Mathematica [F] time = 0.468768, size = 0, normalized size = 0.

$$\int x^6 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]

[Out] Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x^6 \text{FresnelS}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)`

[Out] `int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")`

[Out] `integrate(x^6*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^6 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(x^6*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")

[Out] integrate(x^6*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)

3.74 $\int x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=158

$$\frac{4x^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{8S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} - \frac{43S(\sqrt{2}bx)}{8\sqrt{2}\pi^3 b^6} - \frac{2x^3}{3\pi^2 b^3} + \frac{11x \sin(\pi b^2 x^2)}{8\pi^3 b^5} - \frac{x^5 \sin(\pi b^2 x^2)}{5\pi^3 b^5}$$

[Out] $(-2*x^3)/(3*b^3*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)$

Rubi [A] time = 0.156292, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6454, 6462, 3391, 30, 3386, 3351, 6452, 3385}

$$\frac{4x^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{8S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} - \frac{43S(\sqrt{2}bx)}{8\sqrt{2}\pi^3 b^6} - \frac{2x^3}{3\pi^2 b^3} + \frac{11x \sin(\pi b^2 x^2)}{8\pi^3 b^5} - \frac{x^5 \sin(\pi b^2 x^2)}{5\pi^3 b^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^5 * \text{FresnelS}[b*x] * \text{Sin}[(b^2*Pi*x^2)/2], x]$

[Out] $(-2*x^3)/(3*b^3*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)$

Rule 6454

$\text{Int}[\text{FresnelS}[(b_)*(x_)]*(x_)^{(m_)}*\text{Sin}[(d_)*(x_)^2], x_Symbol] \rightarrow -\text{Simp}[(x^{(m-1)}*\text{Cos}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + (\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\text{Cos}[d*x^2]*\text{FresnelS}[b*x], x], x] + \text{Dist}[1/(2*b*Pi), \text{Int}[x^{(m-1)}*\text{Sin}[2*d*x^2], x], x]) /; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (Pi^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] := Dist[1/2, Int[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b, m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.)), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_)^2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)^(m_)]*(x_)^2*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3385

```
Int[((e_.)*(x_)^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)]), x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{4 \int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^2\pi} + \frac{\int x^4 \sin(b^2\pi x^2) dx}{2b\pi} \\
&= -\frac{x^3 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{4x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{8 \int x S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} \\
&= -\frac{x^3 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{4x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&= -\frac{2x^3}{3b^3\pi^2} - \frac{x^3 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} - \frac{3S(\sqrt{2}bx)}{8\sqrt{2}b^6\pi^3} \\
&= -\frac{2x^3}{3b^3\pi^2} - \frac{x^3 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^6\pi^3} - \frac{x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} - \frac{11S(\sqrt{2}bx)}{8\sqrt{2}b^6\pi^3}
\end{aligned}$$

Mathematica [A] time = 0.172595, size = 120, normalized size = 0.76

$$\frac{48S(bx) \left((\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2}\pi b^2 x^2\right) - 4\pi b^2 x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right) \right) + 32\pi b^3 x^3 - 66bx \sin(\pi b^2 x^2) + 12\pi b^3 x^3 \cos(\pi b^2 x^2) + \dots}{48\pi^3 b^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]

[Out] $-(32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*sqrt[2]*FresnelS[Sqrt[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(48*b^6*Pi^3)$

Maple [A] time = 0.078, size = 202, normalized size = 1.3

$$\frac{1}{b} \left(\frac{FresnelS(bx)}{b^5} \left(-\frac{x^4 b^4}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + 4 \frac{1}{\pi} \left(\frac{b^2 x^2 \sin(1/2 b^2 \pi x^2)}{\pi} + 2 \frac{\cos(1/2 b^2 \pi x^2)}{\pi^2} \right) \right) - \frac{1}{b^5} \left(\frac{2x^3 b^3}{3\pi^2} - 2 \frac{1}{\pi^2} \left(\frac{1}{2} b^2 \pi x^2 \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2), x)

```
[Out] (FresnelS(b*x)/b^5*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))-1/b^5*(2/3/Pi^2*b^3*x^3-2/Pi
^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi
^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/
Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^5 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```


[Out] Integral(x**5*sin(pi*b**2*x**2/2)*fresnels(b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")

[Out] integrate(x^5*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)

3.75 $\int x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=120

$$\frac{3xS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{3S(bx)^2}{2\pi^2 b^5} - \frac{3x^2}{4\pi^2 b^3} + \frac{\sin(\pi b^2 x^2)}{\pi^3 b^5} - \frac{x^2 \cos(\pi b^2 x^2)}{4\pi^2 b^3}$$

[Out] $(-3x^2)/(4b^3\pi^2) - (x^2\cos[b^2\pi x^2])/(4b^3\pi^2) - (x^3\cos[(b^2\pi x^2)/2]*\text{FresnelS}[b*x])/(b^2\pi) - (3*\text{FresnelS}[b*x]^2)/(2*b^5*\pi^2) + (3*x*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(b^4*\pi^2) + \text{Sin}[b^2*\pi*x^2]/(b^5*\pi^3)$

Rubi [A] time = 0.118406, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6454, 6462, 3379, 2634, 6440, 30, 3296, 2637}

$$\frac{3xS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{3S(bx)^2}{2\pi^2 b^5} - \frac{3x^2}{4\pi^2 b^3} + \frac{\sin(\pi b^2 x^2)}{\pi^3 b^5} - \frac{x^2 \cos(\pi b^2 x^2)}{4\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2], x]$

[Out] $(-3x^2)/(4b^3\pi^2) - (x^2\cos[b^2\pi x^2])/(4b^3\pi^2) - (x^3\cos[(b^2\pi x^2)/2]*\text{FresnelS}[b*x])/(b^2\pi) - (3*\text{FresnelS}[b*x]^2)/(2*b^5*\pi^2) + (3*x*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(b^4*\pi^2) + \text{Sin}[b^2*\pi*x^2]/(b^5*\pi^3)$

Rule 6454

$\text{Int}[\text{FresnelS}[(b_)*(x_)]*(x_)^{(m_)}*\text{Sin}[(d_)*(x_)^2], x_Symbol] \rightarrow -\text{Simp}[(x^{(m-1)}*\cos[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + (\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\cos[d*x^2]*\text{FresnelS}[b*x], x], x] + \text{Dist}[1/(2*b*\pi), \text{Int}[x^{(m-1)}*\text{Sin}[2*d*x^2], x], x]) /; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (\pi^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

Rule 6462

$\text{Int}[\cos[(d_)*(x_)^2]*\text{FresnelS}[(b_)*(x_)]*(x_)^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m-1)}*\sin[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + (-\text{Dist}[1/(\pi*b), \text{Int}[x^{(m-1)}*\sin[d*x^2]^2, x], x] - \text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\sin[d*x^2]*\text{FresnelS}[b*x], x], x]) /; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (\pi^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

```
e1S[b*x], x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{3 \int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^2\pi} + \frac{\int x^3 \sin(b^2\pi x^2) dx}{2b\pi} \\
&= -\frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{3xS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3 \int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} - \frac{3 \int x \sin(b^2\pi x^2) dx}{b^5\pi^2} \\
&= -\frac{x^2 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{3xS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3 \text{Subst}\left(\int x dx, x, S(bx)\right)}{b^5\pi^2} \\
&= -\frac{3x^2}{4b^3\pi^2} - \frac{x^2 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} - \frac{3S(bx)^2}{2b^5\pi^2} + \frac{3xS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \dots
\end{aligned}$$

Mathematica [A] time = 0.0058869, size = 120, normalized size = 1.

$$\frac{3xS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{3S(bx)^2}{2\pi^2 b^5} - \frac{3x^2}{4\pi^2 b^3} + \frac{\sin(\pi b^2 x^2)}{\pi^3 b^5} - \frac{x^2 \cos(\pi b^2 x^2)}{4\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] (-3*x^2)/(4*b^3*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (3*FresnelS[b*x]^2)/(2*b^5*Pi^2) + (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + Sin[b^2*Pi*x^2]/(b^5*Pi^3)

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x^4 \text{FresnelS}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)

[Out] int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(x^4*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")

[Out] integral(x^4*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)

Sympy [A] time = 26.5934, size = 151, normalized size = 1.26

$$\begin{cases} \frac{x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{\pi b^2} - \frac{x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} - \frac{x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^2 b^3} + \frac{3x \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{\pi^2 b^4} + \frac{2 \sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^3 b^5} - \frac{3S^2(bx)}{2\pi^2 b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)

[Out] Piecewise((-x**3*cos(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) - x**2*sin(pi*b**2*x**2/2)**2/(2*pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(pi**2*b**3) + 3*x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnels(b*x)**2/(2*pi**2*b**5), Ne(b, 0)), (0, True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.76 $\int x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=105

$$\frac{5\text{FresnelC}(\sqrt{2}bx)}{4\sqrt{2}\pi^2b^4} + \frac{2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{x\cos(\pi b^2x^2)}{4\pi^2b^3} - \frac{x}{\pi^2b^3}$$

[Out] $-(x/(b^3\pi^2)) - (x\cos[b^2\pi x^2])/(4b^3\pi^2) + (5\text{FresnelC}[\text{Sqrt}[2]*b*x])/(4\text{Sqrt}[2]*b^4\pi^2) - (x^2\cos[(b^2\pi x^2)/2]*\text{FresnelS}[b*x])/(b^2\pi) + (2*\text{FresnelS}[b*x]*\text{Sin}[(b^2\pi x^2)/2])/(b^4\pi^2)$

Rubi [A] time = 0.081234, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6454, 6460, 3357, 3352, 3385}

$$\frac{5\text{FresnelC}(\sqrt{2}bx)}{4\sqrt{2}\pi^2b^4} + \frac{2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{x\cos(\pi b^2x^2)}{4\pi^2b^3} - \frac{x}{\pi^2b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2], x]$

[Out] $-(x/(b^3\pi^2)) - (x\cos[b^2\pi x^2])/(4b^3\pi^2) + (5\text{FresnelC}[\text{Sqrt}[2]*b*x])/(4\text{Sqrt}[2]*b^4\pi^2) - (x^2\cos[(b^2\pi x^2)/2]*\text{FresnelS}[b*x])/(b^2\pi) + (2*\text{FresnelS}[b*x]*\text{Sin}[(b^2\pi x^2)/2])/(b^4\pi^2)$

Rule 6454

$\text{Int}[\text{FresnelS}[(b_.)*(x_)]*(x_)^(m_)*\text{Sin}[(d_.)*(x_)^2], x_Symbol] \rightarrow -\text{Simp}[(x^(m-1)*\text{Cos}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + (\text{Dist}[(m-1)/(2*d), \text{Int}[x^(m-2)*\text{Cos}[d*x^2]*\text{FresnelS}[b*x], x], x] + \text{Dist}[1/(2*b*\pi), \text{Int}[x^(m-1)*\text{Sin}[2*d*x^2], x], x]) / ; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (\pi^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

Rule 6460

$\text{Int}[\text{Cos}[(d_.)*(x_)^2]*\text{FresnelS}[(b_.)*(x_)]*(x_), x_Symbol] \rightarrow \text{Simp}[(\text{Sin}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] - \text{Dist}[1/(\pi*b), \text{Int}[\text{Sin}[d*x^2]^2, x], x] / ; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (\pi^2*b^4)/4]$

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Symbol]
:> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol]
:> Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol]
:> -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{2 \int x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^2\pi} + \frac{\int x^2 \sin(b^2\pi x^2) dx}{2b\pi} \\
&= -\frac{x \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{2S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{\int \cos(b^2\pi x^2) dx}{4b^3\pi^2} - \frac{2 \int \left(\frac{1}{2} - \dots\right)}{4b^3\pi^2} \\
&= -\frac{x \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{C(\sqrt{2}bx)}{4\sqrt{2}b^4\pi^2} - \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{2S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{2 \int \left(\frac{1}{2} - \dots\right)}{4b^3\pi^2} \\
&= -\frac{x}{b^3\pi^2} - \frac{x \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{C(\sqrt{2}bx)}{4\sqrt{2}b^4\pi^2} - \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{2S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{\int \dots}{4b^3\pi^2} \\
&= -\frac{x}{b^3\pi^2} - \frac{x \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{5C(\sqrt{2}bx)}{4\sqrt{2}b^4\pi^2} - \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{2S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}
\end{aligned}$$

Mathematica [A] time = 0.102146, size = 83, normalized size = 0.79

$$\frac{-8S(bx) \left(\pi b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) - 2 \sin\left(\frac{1}{2}\pi b^2 x^2\right) \right) - 2bx \left(\cos(\pi b^2 x^2) + 4 \right) + 5\sqrt{2}\text{FresnelC}(\sqrt{2}bx)}{8\pi^2 b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] $(-2*b*x*(4 + \cos[b^2*Pi*x^2]) + 5*\sqrt{2}*FresnelC[\sqrt{2}*b*x] - 8*FresnelS[b*x]*(b^2*Pi*x^2*\cos[(b^2*Pi*x^2)/2] - 2*\sin[(b^2*Pi*x^2)/2]))/(8*b^4*Pi^2)$

Maple [A] time = 0.083, size = 115, normalized size = 1.1

$$\frac{1}{b} \left(\frac{FresnelS(bx)}{b^3} \left(-\frac{b^2 x^2}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + 2 \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) - \frac{1}{b^3} \left(\frac{bx}{\pi^2} - \frac{\sqrt{2} FresnelC(bx\sqrt{2})}{2\pi^2} - \frac{1}{2\pi} \left(-\frac{bx \cos(b^2 \pi x^2)}{2\pi} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)

[Out] $(FresnelS(b*x)/b^3*(-1/Pi*b^2*x^2*\cos(1/2*b^2*Pi*x^2)+2/Pi^2*\sin(1/2*b^2*Pi*x^2))-1/b^3*(1/Pi^2*b*x-1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))-1/2/Pi*(-1/2/Pi*b*x*\cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))/b$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(x^3*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**3*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.77 $\int x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=137

$$\frac{ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right)}{8\pi b} + \frac{ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)}{8\pi b} + \frac{\text{FresnelC}[bx] \text{FresnelS}[bx]}{2\pi b^3}$$

[Out] $-\text{Cos}[b^2\pi x^2]/(4b^3\pi^2) - (x\text{Cos}[(b^2\pi x^2)/2]*\text{FresnelS}[bx])/(b^2\pi) + (\text{FresnelC}[bx]*\text{FresnelS}[bx])/(2b^3\pi) - ((I/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2\pi x^2])/(b\pi) + ((I/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2\pi x^2])/(b\pi)$

Rubi [A] time = 0.0606421, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {6454, 6446, 3379, 2638}

$$\frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi b} + \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi b} + \frac{\text{FresnelC}(bx)S(bx)}{2\pi b^3} - \frac{xS(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{\cos(\pi b^2 x^2)}{4\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*\text{FresnelS}[bx]*\text{Sin}[(b^2\pi x^2)/2], x]$

[Out] $-\text{Cos}[b^2\pi x^2]/(4b^3\pi^2) - (x\text{Cos}[(b^2\pi x^2)/2]*\text{FresnelS}[bx])/(b^2\pi) + (\text{FresnelC}[bx]*\text{FresnelS}[bx])/(2b^3\pi) - ((I/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2\pi x^2])/(b\pi) + ((I/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2\pi x^2])/(b\pi)$

Rule 6454

$\text{Int}[\text{FresnelS}[(b_*)(x_)]*(x_)^{(m_*)}\text{Sin}[(d_*)(x_)^2], x_Symbol] \rightarrow -\text{Simp}[(x)^{(m-1)}*\text{Cos}[d*x^2]*\text{FresnelS}[bx])/(2*d), x] + (\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\text{Cos}[d*x^2]*\text{FresnelS}[bx], x], x] + \text{Dist}[1/(2*b*\pi), \text{Int}[x^{(m-1)}*\text{Sin}[2*d*x^2], x], x]) /; \text{FreeQ}[\{b, d\}, x] \&\& \text{EqQ}[d^2, (\pi^2*b^4)/4] \&\& \text{IGtQ}[m, 1]$

Rule 6446

$\text{Int}[\text{Cos}[(d_*)(x_)^2]*\text{FresnelS}[(b_*)(x_)], x_Symbol] \rightarrow \text{Simp}[(\text{FresnelC}[bx]*\text{FresnelS}[bx])/(2*b), x] + (-\text{Simp}[(1*I*b*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3$

/2, 2}, -((I*b^2*Pi*x^2)/2))/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^2\pi} + \frac{\int x \sin(b^2\pi x^2) dx}{2b\pi} \\ &= -\frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{C(bx)S(bx)}{2b^3\pi} - \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8b\pi} \\ &= -\frac{\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^2\pi} + \frac{C(bx)S(bx)}{2b^3\pi} - \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8b\pi} \end{aligned}$$

Mathematica [F] time = 0.215816, size = 0, normalized size = 0.

$$\int x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]

[Out] Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x^2 \text{FresnelS}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)`

[Out] `int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")`

[Out] `integrate(x^2*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(x^2*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)`

[Out] `Integral(x**2*sin(pi*b**2*x**2/2)*fresnels(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")`

[Out] `integrate(x^2*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

3.78 $\int xS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=49

$$\frac{S(\sqrt{2}bx)}{2\sqrt{2}\pi b^2} - \frac{S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

[Out] $-\left(\frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}[b*x]}{b^2\pi}\right) + \frac{\text{FresnelS}[\sqrt{2}*b*x]}{2*\sqrt{2}*b^2\pi}$

Rubi [A] time = 0.0216188, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6452, 3351}

$$\frac{S(\sqrt{2}bx)}{2\sqrt{2}\pi b^2} - \frac{S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{FresnelS}[b*x]*\text{Sin}\left[\frac{b^2\pi x^2}{2}\right], x]$

[Out] $-\left(\frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}[b*x]}{b^2\pi}\right) + \frac{\text{FresnelS}[\sqrt{2}*b*x]}{2*\sqrt{2}*b^2\pi}$

Rule 6452

$\text{Int}[\text{FresnelS}[(b_.)*(x_.)]*(x_.)*\text{Sin}[(d_.)*(x_.)^2], x_Symbol] \text{ :> } -\text{Simp}[(\text{Cos}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + \text{Dist}[1/(2*b*\pi), \text{Int}[\text{Sin}[2*d*x^2], x], x] / ; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (\pi^2*b^4)/4]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \text{ :> } \text{Simp}[(\text{Sqrt}[\pi/2]*\text{FresnelS}[\text{Sqrt}[2/\pi]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] / ; \text{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\int xS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{\int \sin(b^2\pi x^2) dx}{2b\pi}$$

$$= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{S(\sqrt{2}bx)}{2\sqrt{2}b^2\pi}$$

Mathematica [A] time = 0.0244937, size = 44, normalized size = 0.9

$$\frac{\sqrt{2}S(\sqrt{2}bx) - 4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] (-4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x] + Sqrt[2]*FresnelS[Sqrt[2]*b*x])/(4*b^2*Pi)

Maple [A] time = 0.063, size = 46, normalized size = 0.9

$$\frac{1}{b} \left(-\frac{\text{FresnelS}(bx)}{b\pi} \cos\left(\frac{b^2\pi x^2}{2}\right) + \frac{\sqrt{2}\text{FresnelS}(bx\sqrt{2})}{4b\pi} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)

[Out] (-cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^(1/2))/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int xfresnels(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")`

[Out] `integrate(x*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\text{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(x*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)`

[Out] `Integral(x*sin(pi*b**2*x**2/2)*fresnels(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")`

[Out] `integrate(x*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)`

$$3.79 \quad \int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Optimal. Leaf size=13

$$\frac{S(bx)^2}{2b}$$

[Out] FresnelS[b*x]^2/(2*b)

Rubi [A] time = 0.0114849, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6440, 30}

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^2/(2*b)

Rule 6440

Int[FresnelS[(b_.)*(x_.)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= \frac{\text{Subst}\left(\int x dx, x, S(bx)\right)}{b} \\ &= \frac{S(bx)^2}{2b} \end{aligned}$$

Mathematica [A] time = 0.0019935, size = 13, normalized size = 1.

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^2/(2*b)

Maple [A] time = 0.046, size = 12, normalized size = 0.9

$$\frac{(\text{FresnelS}(bx))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)

[Out] 1/2*FresnelS(b*x)^2/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A] time = 0.451291, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

$$3.80 \quad \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right)$$

[Out] Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

Rubi [A] time = 0.0183464, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

[Out] Defer[Int] [(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

Rubi steps

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx = \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx$$

Mathematica [A] time = 0.0302, size = 0, normalized size = 0.

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

Maple [A] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)

[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")

[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x,x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x, x)

$$3.81 \quad \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right)$$

[Out] Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]

Rubi [A] time = 0.0184603, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]

[Out] Defer[Int] [(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]

Rubi steps

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx = \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx$$

Mathematica [A] time = 0.0304836, size = 0, normalized size = 0.

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]

[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]

Maple [A] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^2} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)

[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")

[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)

$$3.82 \quad \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx$$

Optimal. Leaf size=101

$$\frac{1}{2}\pi b^2 \text{Unintegrable}\left(\frac{S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) - \frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2x^2} + \frac{\pi b^2 S(\sqrt{2}bx)}{2\sqrt{2}} + \frac{b \cos(\pi b^2 x^2)}{4x} - \frac{b}{4x}$$

[Out] $-b/(4*x) + (b*\text{Cos}[b^2*\text{Pi}*x^2])/(4*x) + (b^2*\text{Pi}*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(2*\text{Sqrt}[2]) - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(2*x^2) + (b^2*\text{Pi}*\text{Unintegrable}[(\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/x, x])/2$

Rubi [A] time = 0.0584933, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/x^3, x]$

[Out] $-b/(4*x) + (b*\text{Cos}[b^2*\text{Pi}*x^2])/(4*x) + (b^2*\text{Pi}*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(2*\text{Sqrt}[2]) - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(2*x^2) + (b^2*\text{Pi}*\text{Defer}[\text{Int}[(\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/x, x])/2$

Rubi steps

$$\begin{aligned} \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx &= -\frac{b}{4x} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} - \frac{1}{4}b \int \frac{\cos(b^2\pi x^2)}{x^2} dx + \frac{1}{2}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x} dx \\ &= -\frac{b}{4x} + \frac{b \cos(b^2\pi x^2)}{4x} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} + \frac{1}{2}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x} dx + \frac{1}{2}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x} dx \\ &= -\frac{b}{4x} + \frac{b \cos(b^2\pi x^2)}{4x} + \frac{b^2\pi S(\sqrt{2}bx)}{2\sqrt{2}} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} + \frac{1}{2}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x} dx \end{aligned}$$

Mathematica [A] time = 0.0303193, size = 0, normalized size = 0.

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]

[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]

Maple [A] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^3} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)

[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

$$3.83 \quad \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx$$

Optimal. Leaf size=109

$$-\frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^3} - \frac{\pi b^2 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3x} - \frac{1}{6}\pi^2 b^3 S(bx)^2 + \frac{1}{6}\pi b^3 \text{Si}\left(b^2\pi x^2\right) + \frac{b \cos\left(\pi b^2 x^2\right)}{12x^2} - \frac{b}{12x^2}$$

[Out] $-b/(12*x^2) + (b*\text{Cos}[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(3*x) - (b^3*Pi^2*\text{FresnelS}[b*x]^2)/6 - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*\text{SinIntegral}[b^2*Pi*x^2])/6$

Rubi [A] time = 0.116202, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6456, 6464, 6440, 30, 3375, 3380, 3297, 3299}

$$-\frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^3} - \frac{\pi b^2 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3x} - \frac{1}{6}\pi^2 b^3 S(bx)^2 + \frac{1}{6}\pi b^3 \text{Si}\left(b^2\pi x^2\right) + \frac{b \cos\left(\pi b^2 x^2\right)}{12x^2} - \frac{b}{12x^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x^4, x]$

[Out] $-b/(12*x^2) + (b*\text{Cos}[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(3*x) - (b^3*Pi^2*\text{FresnelS}[b*x]^2)/6 - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*\text{SinIntegral}[b^2*Pi*x^2])/6$

Rule 6456

$\text{Int}[\text{FresnelS}[(b_)*(x_)]*(x_)^{(m_)}*\text{Sin}[(d_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(x^{(m+1)}*\text{Sin}[d*x^2]*\text{FresnelS}[b*x])/(m+1), x] + (-\text{Dist}[(2*d)/(m+1), \text{Int}[x^{(m+2)}*\text{Cos}[d*x^2]*\text{FresnelS}[b*x], x], x] + \text{Dist}[d/(Pi*b*(m+1)), \text{Int}[x^{(m+1)}*\text{Cos}[2*d*x^2], x], x] - \text{Simp}[(d*x^{(m+2)})/(Pi*b*(m+1)*(m+2)), x]) /; \text{FreeQ}\{b, d\}, x] \&\& \text{EqQ}[d^2, (Pi^2*b^4)/4] \&\& \text{ILtQ}[m, -2]$

Rule 6464

$\text{Int}[\text{Cos}[(d_)*(x_)^2]*\text{FresnelS}[(b_)*(x_)]*(x_)^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m+1)}*\text{Cos}[d*x^2]*\text{FresnelS}[b*x])/(m+1), x] + (\text{Dist}[(2*d)/(m+1), \text{Int}[x^{(m+2)}*\text{Sin}[d*x^2]*\text{FresnelS}[b*x], x], x] - \text{Dist}[d/(Pi*b*(m+1)), \text{Int}[x^{(m+1)}*\text{Sin}[d*x^2]*\text{FresnelS}[b*x], x], x])$

+ 1)*Sin[2*d*x^2], x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -1]

Rule 6440

Int[FresnelS[(b_.)*(x_)^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3375

Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n/n, x] /; FreeQ[{d, n}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx &= -\frac{b}{12x^2} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} - \frac{1}{6}b \int \frac{\cos(b^2\pi x^2)}{x^3} dx + \frac{1}{3}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^2} dx \\
&= -\frac{b}{12x^2} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{3x} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} - \frac{1}{12}b \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^2} dx, x, x\right) \\
&= -\frac{b}{12x^2} + \frac{b \cos(b^2\pi x^2)}{12x^2} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{3x} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{12}b^3\pi \text{Si}(b^2\pi x^2) \\
&= -\frac{b}{12x^2} + \frac{b \cos(b^2\pi x^2)}{12x^2} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{3x} - \frac{1}{6}b^3\pi^2 S(bx)^2 - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{6}
\end{aligned}$$

Mathematica [A] time = 0.007643, size = 109, normalized size = 1.

$$-\frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^3} - \frac{\pi b^2 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3x} - \frac{1}{6}\pi^2 b^3 S(bx)^2 + \frac{1}{6}\pi b^3 \text{Si}(b^2\pi x^2) + \frac{b \cos(\pi b^2 x^2)}{12x^2} - \frac{b}{12x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]

[Out] -b/(12*x^2) + (b*Cos[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*x) - (b^3*Pi^2*FresnelS[b*x]^2)/6 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^4} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)

[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")

[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

$$3.84 \quad \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx$$

Optimal. Leaf size=152

$$-\frac{1}{8}\pi^2 b^4 \text{Unintegrable}\left(\frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) + \frac{7\pi^2 b^4 \text{FresnelC}(\sqrt{2}bx)}{24\sqrt{2}} - \frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4x^4} - \frac{\pi b^2 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^2}$$

[Out] $-b/(24*x^3) + (b*\text{Cos}[b^2*Pi*x^2])/(24*x^3) + (7*b^4*Pi^2*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(24*\text{Sqrt}[2]) - (b^2*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(8*x^2) - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(4*x^4) - (7*b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(48*x) - (b^4*Pi^2*\text{Unintegrable}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x, x])/8$

Rubi [A] time = 0.121331, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x^5, x]$

[Out] $-b/(24*x^3) + (b*\text{Cos}[b^2*Pi*x^2])/(24*x^3) + (7*b^4*Pi^2*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(24*\text{Sqrt}[2]) - (b^2*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(8*x^2) - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(4*x^4) - (7*b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(48*x) - (b^4*Pi^2*\text{Defer}[\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x, x])/8$

Rubi steps

$$\begin{aligned}
\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx &= -\frac{b}{24x^3} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} - \frac{1}{8}b \int \frac{\cos(b^2\pi x^2)}{x^4} dx + \frac{1}{4}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^3} dx \\
&= -\frac{b}{24x^3} + \frac{b \cos(b^2\pi x^2)}{24x^3} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{8x^2} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} + \frac{1}{16}(b^3\pi) \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
&= -\frac{b}{24x^3} + \frac{b \cos(b^2\pi x^2)}{24x^3} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{8x^2} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} - \frac{7b^3\pi \sin(b^2\pi x^2)}{48x} \\
&= -\frac{b}{24x^3} + \frac{b \cos(b^2\pi x^2)}{24x^3} + \frac{7b^4\pi^2 C(\sqrt{2}bx)}{24\sqrt{2}} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{8x^2} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.0303067, size = 0, normalized size = 0.

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]

[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^5} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)

[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")`

[Out] `integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")`

[Out] `integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)`

[Out] `Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

$$3.85 \quad \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx$$

Optimal. Leaf size=147

$$-\frac{1}{15}\pi^2 b^4 \text{Unintegrable}\left(\frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right) + \frac{1}{24}\pi^2 b^5 \text{CosIntegral}(\pi b^2 x^2) - \frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5} - \frac{\pi b^2 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3}$$

[Out] $-b/(40*x^4) + (b*\text{Cos}[b^2*Pi*x^2])/(40*x^4) + (b^5*Pi^2*\text{CosIntegral}[b^2*Pi*x^2])/24 - (b^2*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(15*x^3) - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(5*x^5) - (b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(24*x^2) - (b^4*Pi^2*\text{Unintegrable}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x^2, x])/15$

Rubi [A] time = 0.192541, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x^6, x]$

[Out] $-b/(40*x^4) + (b*\text{Cos}[b^2*Pi*x^2])/(40*x^4) + (b^5*Pi^2*\text{CosIntegral}[b^2*Pi*x^2])/24 - (b^2*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(15*x^3) - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(5*x^5) - (b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(24*x^2) - (b^4*Pi^2*\text{Defer}[\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x^2, x])/15$

Rubi steps

$$\begin{aligned}
\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx &= -\frac{b}{40x^4} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} - \frac{1}{10}b \int \frac{\cos(b^2\pi x^2)}{x^5} dx + \frac{1}{5}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^4} dx \\
&= -\frac{b}{40x^4} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{15x^3} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} - \frac{1}{20}b \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^3} dx, x, x\right) \\
&= -\frac{b}{40x^4} + \frac{b \cos(b^2\pi x^2)}{40x^4} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{15x^3} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{60}(b^3\pi) \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^3} dx, x, x\right) \\
&= -\frac{b}{40x^4} + \frac{b \cos(b^2\pi x^2)}{40x^4} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{15x^3} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} - \frac{b^3\pi \sin(b^2\pi x^2)}{24x^2} \\
&= -\frac{b}{40x^4} + \frac{b \cos(b^2\pi x^2)}{40x^4} + \frac{1}{24}b^5\pi^2 \text{Ci}(b^2\pi x^2) - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{15x^3} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}
\end{aligned}$$

Mathematica [A] time = 0.0314466, size = 0, normalized size = 0.

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]

[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6, x]

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^6} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)

[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")

[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

$$3.86 \quad \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx$$

Optimal. Leaf size=240

$$-\frac{1}{48}\pi^3b^6\text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}, x\right) + \frac{\pi^2b^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{48x^2} - \frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6} - \frac{\pi b^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{24x^4}$$

[Out] $-b/(60*x^5) + (b^5*Pi^2)/(96*x) + (b*\text{Cos}[b^2*Pi*x^2])/(60*x^5) - (67*b^5*Pi^2*\text{Cos}[b^2*Pi*x^2])/(1440*x) - (b^2*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(24*x^4) - (7*b^6*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(144*\text{Sqrt}[2]) - (\text{Sqrt}[2]*b^6*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/45 - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(6*x^6) + (b^4*Pi^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(48*x^2) - (13*b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(720*x^3) - (b^6*Pi^3*\text{Unintegrable}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/x, x])/48$

Rubi [A] time = 0.206718, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x^7, x]$

[Out] $-b/(60*x^5) + (b^5*Pi^2)/(96*x) + (b*\text{Cos}[b^2*Pi*x^2])/(60*x^5) - (67*b^5*Pi^2*\text{Cos}[b^2*Pi*x^2])/(1440*x) - (b^2*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(24*x^4) - (7*b^6*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(144*\text{Sqrt}[2]) - (\text{Sqrt}[2]*b^6*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/45 - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(6*x^6) + (b^4*Pi^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(48*x^2) - (13*b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(720*x^3) - (b^6*Pi^3*\text{Defer}[\text{Int}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/x, x])/48$

Rubi steps

$$\begin{aligned}
\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx &= -\frac{b}{60x^5} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} - \frac{1}{12}b \int \frac{\cos(b^2\pi x^2)}{x^6} dx + \frac{1}{6}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^5} dx \\
&= -\frac{b}{60x^5} + \frac{b \cos(b^2\pi x^2)}{60x^5} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{24x^4} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{1}{48}(b^3\pi) \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^4} dx \\
&= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b \cos(b^2\pi x^2)}{60x^5} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{24x^4} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{b^4\pi^2 S(bx)}{1440x} \\
&= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b \cos(b^2\pi x^2)}{60x^5} - \frac{67b^5\pi^2 \cos(b^2\pi x^2)}{1440x} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{24x^4} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} \\
&= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b \cos(b^2\pi x^2)}{60x^5} - \frac{67b^5\pi^2 \cos(b^2\pi x^2)}{1440x} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{24x^4} - \frac{7b^6\pi^3 S(bx)}{1440x^6}
\end{aligned}$$

Mathematica [A] time = 0.0298719, size = 0, normalized size = 0.

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]

[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^7} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)

[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")

[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

$$3.87 \quad \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx$$

Optimal. Leaf size=224

$$\frac{\pi^2 b^4 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{105x^3} - \frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7x^7} + \frac{\pi^3 b^6 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{105x} - \frac{\pi b^2 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{35x^5} + \frac{1}{210} \pi^4 b^7 S(bx)^2$$

[Out] $-b/(84*x^6) + (b^5*Pi^2)/(420*x^2) + (b*\text{Cos}[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi^2*\text{Cos}[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(35*x^5) + (b^6*Pi^3*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(105*x) + (b^7*Pi^4*\text{FresnelS}[b*x]^2)/210 - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*Pi^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*\text{SinIntegral}[b^2*Pi*x^2])/70$

Rubi [A] time = 0.353489, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6456, 6464, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi^2 b^4 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{105x^3} - \frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7x^7} + \frac{\pi^3 b^6 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{105x} - \frac{\pi b^2 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{35x^5} + \frac{1}{210} \pi^4 b^7 S(bx)^2$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x^8, x]$

[Out] $-b/(84*x^6) + (b^5*Pi^2)/(420*x^2) + (b*\text{Cos}[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi^2*\text{Cos}[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(35*x^5) + (b^6*Pi^3*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(105*x) + (b^7*Pi^4*\text{FresnelS}[b*x]^2)/210 - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*Pi^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*\text{SinIntegral}[b^2*Pi*x^2])/70$

Rule 6456

$\text{Int}[\text{FresnelS}[(b_.)*(x_.)]*(x_.)^{(m_.)}*\text{Sin}[(d_.)*(x_.)^2], x_Symbol] \rightarrow \text{Simp}[(x^{(m+1)}*\text{Sin}[d*x^2]*\text{FresnelS}[b*x])/(m+1), x] + (-\text{Dist}[(2*d)/(m+1), \text{Int}[x^{(m+2)}*\text{Cos}[d*x^2]*\text{FresnelS}[b*x], x], x] + \text{Dist}[d/(Pi*b*(m+1)), \text{Int}[x^{(m+1)}*\text{Cos}[2*d*x^2], x], x] - \text{Simp}[(d*x^{(m+2)})/(Pi*b*(m+1)*(m+2)), x]) /; \text{FreeQ}\{b, d\}, x \&\& \text{EqQ}[d^2, (Pi^2*b^4)/4] \&\& \text{ILtQ}[m, -2]$

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_)^(m_))*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379


```
Int[(x_)^(m_)*((a_) + (b_)*Sin[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx &= -\frac{b}{84x^6} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} - \frac{1}{14}b \int \frac{\cos(b^2\pi x^2)}{x^7} dx + \frac{1}{7}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^6} dx \\
&= -\frac{b}{84x^6} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{35x^5} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} - \frac{1}{28}b \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^4} dx, x\right) \\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b \cos(b^2\pi x^2)}{84x^6} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{35x^5} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2 S(bx)}{210x} \\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b \cos(b^2\pi x^2)}{84x^6} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{35x^5} + \frac{b^6\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{105x} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} \\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b \cos(b^2\pi x^2)}{84x^6} - \frac{b^5\pi^2 \cos(b^2\pi x^2)}{84x^2} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{35x^5} + \frac{b^6\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{105x} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} \\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b \cos(b^2\pi x^2)}{84x^6} - \frac{b^5\pi^2 \cos(b^2\pi x^2)}{84x^2} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{35x^5} + \frac{b^6\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{105x} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}
\end{aligned}$$

Mathematica [A] time = 0.012694, size = 224, normalized size = 1.

$$\frac{\pi^2 b^4 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{105x^3} - \frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7x^7} + \frac{\pi^3 b^6 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{105x} - \frac{\pi b^2 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{35x^5} + \frac{1}{210}\pi^4 b^7 S(bx)^2$$

Antiderivative was successfully verified.

[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]

[Out] -b/(84*x^6) + (b^5*Pi^2)/(420*x^2) + (b*Cos[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi^2*Cos[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(35*x^5) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x) + (b^7*Pi^4*FresnelS[b*x]^2)/210 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*Sin[b^2*Pi*x^2])

$/(105*x^4) - (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70$

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^8} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)

[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")

[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)

$$3.88 \quad \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx$$

Optimal. Leaf size=267

$$\frac{1}{384}\pi^4 b^8 \text{Unintegrable}\left(\frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) - \frac{853\pi^4 b^8 \text{FresnelC}(\sqrt{2}bx)}{40320\sqrt{2}} + \frac{\pi^2 b^4 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{192x^4} - \frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8}$$

[Out] $-b/(112*x^7) + (b^5*\pi^2)/(1152*x^3) + (b*\text{Cos}[b^2*\pi*x^2])/(112*x^7) - (187*b^5*\pi^2*\text{Cos}[b^2*\pi*x^2])/(40320*x^3) - (853*b^8*\pi^4*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(40320*\text{Sqrt}[2]) - (b^2*\pi*\text{Cos}[(b^2*\pi*x^2)/2]*\text{FresnelS}[b*x])/(48*x^6) + (b^6*\pi^3*\text{Cos}[(b^2*\pi*x^2)/2]*\text{FresnelS}[b*x])/(384*x^2) - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(8*x^8) + (b^4*\pi^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(192*x^4) - (19*b^3*\pi*\text{Sin}[b^2*\pi*x^2])/(3360*x^5) + (853*b^7*\pi^3*\text{Sin}[b^2*\pi*x^2])/(80640*x) + (b^8*\pi^4*\text{Unintegrable}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/x, x])/384$

Rubi [A] time = 0.306247, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/x^9, x]$

[Out] $-b/(112*x^7) + (b^5*\pi^2)/(1152*x^3) + (b*\text{Cos}[b^2*\pi*x^2])/(112*x^7) - (187*b^5*\pi^2*\text{Cos}[b^2*\pi*x^2])/(40320*x^3) - (853*b^8*\pi^4*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(40320*\text{Sqrt}[2]) - (b^2*\pi*\text{Cos}[(b^2*\pi*x^2)/2]*\text{FresnelS}[b*x])/(48*x^6) + (b^6*\pi^3*\text{Cos}[(b^2*\pi*x^2)/2]*\text{FresnelS}[b*x])/(384*x^2) - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(8*x^8) + (b^4*\pi^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(192*x^4) - (19*b^3*\pi*\text{Sin}[b^2*\pi*x^2])/(3360*x^5) + (853*b^7*\pi^3*\text{Sin}[b^2*\pi*x^2])/(80640*x) + (b^8*\pi^4*\text{Defer}[\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/x, x])/384$

Rubi steps

$$\begin{aligned}
\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx &= -\frac{b}{112x^7} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} - \frac{1}{16}b \int \frac{\cos(b^2\pi x^2)}{x^8} dx + \frac{1}{8}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^7} dx \\
&= -\frac{b}{112x^7} + \frac{b \cos(b^2\pi x^2)}{112x^7} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{48x^6} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{1}{96}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^6} dx \\
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b \cos(b^2\pi x^2)}{112x^7} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{48x^6} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{b^4\pi^2}{96x^5} \\
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b \cos(b^2\pi x^2)}{112x^7} - \frac{187b^5\pi^2 \cos(b^2\pi x^2)}{40320x^3} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{48x^6} + \frac{b^6\pi^3}{96x^4} \\
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b \cos(b^2\pi x^2)}{112x^7} - \frac{187b^5\pi^2 \cos(b^2\pi x^2)}{40320x^3} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{48x^6} + \frac{b^6\pi^3}{96x^4} \\
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b \cos(b^2\pi x^2)}{112x^7} - \frac{187b^5\pi^2 \cos(b^2\pi x^2)}{40320x^3} - \frac{853b^8\pi^4 C(\sqrt{2}bx)}{40320\sqrt{2}} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{48x^6}
\end{aligned}$$

Mathematica [A] time = 0.0316223, size = 0, normalized size = 0.

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]

[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^9} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)

[Out] `int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")`

[Out] `integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")`

[Out] `integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

$$3.89 \quad \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}} dx$$

Optimal. Leaf size=262

$$\frac{1}{945}\pi^4 b^8 \text{Unintegrable}\left(\frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right) - \frac{5\pi^4 b^9 \text{CosIntegral}(\pi b^2 x^2)}{2016} + \frac{\pi^2 b^4 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{315x^5} - \frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9}$$

[Out] $-b/(144*x^8) + (b^5*\text{Pi}^2)/(2520*x^4) + (b*\text{Cos}[b^2*\text{Pi}*x^2])/(144*x^8) - (67*b^5*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(30240*x^4) - (5*b^9*\text{Pi}^4*\text{CosIntegral}[b^2*\text{Pi}*x^2])/2016 - (b^2*\text{Pi}*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(63*x^7) + (b^6*\text{Pi}^3*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(945*x^3) - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(9*x^9) + (b^4*\text{Pi}^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(315*x^5) - (11*b^3*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(3024*x^6) + (5*b^7*\text{Pi}^3*\text{Sin}[b^2*\text{Pi}*x^2])/(2016*x^2) + (b^8*\text{Pi}^4*\text{Unintegrable}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/x^2, x])/945$

Rubi [A] time = 0.476823, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/x^{10}, x]$

[Out] $-b/(144*x^8) + (b^5*\text{Pi}^2)/(2520*x^4) + (b*\text{Cos}[b^2*\text{Pi}*x^2])/(144*x^8) - (67*b^5*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(30240*x^4) - (5*b^9*\text{Pi}^4*\text{CosIntegral}[b^2*\text{Pi}*x^2])/2016 - (b^2*\text{Pi}*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(63*x^7) + (b^6*\text{Pi}^3*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(945*x^3) - (\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(9*x^9) + (b^4*\text{Pi}^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(315*x^5) - (11*b^3*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(3024*x^6) + (5*b^7*\text{Pi}^3*\text{Sin}[b^2*\text{Pi}*x^2])/(2016*x^2) + (b^8*\text{Pi}^4*\text{Defer}[\text{Int}[(\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/x^2, x])/945$

Rubi steps

$$\begin{aligned}
\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}} dx &= -\frac{b}{144x^8} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} - \frac{1}{18}b \int \frac{\cos(b^2\pi x^2)}{x^9} dx + \frac{1}{9}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^8} dx \\
&= -\frac{b}{144x^8} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{63x^7} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} - \frac{1}{36}b \operatorname{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^5} dx, x, x^2\right) \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b \cos(b^2\pi x^2)}{144x^8} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{63x^7} - \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{b^4\pi^3}{945x^3} \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b \cos(b^2\pi x^2)}{144x^8} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{63x^7} + \frac{b^6\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{945x^3} \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b \cos(b^2\pi x^2)}{144x^8} - \frac{67b^5\pi^2 \cos(b^2\pi x^2)}{30240x^4} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{63x^7} + \frac{b^6\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{945x^3} \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b \cos(b^2\pi x^2)}{144x^8} - \frac{67b^5\pi^2 \cos(b^2\pi x^2)}{30240x^4} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{63x^7} + \frac{b^6\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{945x^3} \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b \cos(b^2\pi x^2)}{144x^8} - \frac{67b^5\pi^2 \cos(b^2\pi x^2)}{30240x^4} - \frac{5b^9\pi^4 \operatorname{Ci}(b^2\pi x^2)}{2016} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{63x^7}
\end{aligned}$$

Mathematica [A] time = 0.0322328, size = 0, normalized size = 0.

$$\int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10, x]

[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10, x]

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{\operatorname{FresnelS}(bx)}{x^{10}} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)
```

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")

[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)

$$3.90 \quad \int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n dx$$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\cos\left(\frac{1}{2}\pi b^2 x^2\right) S(bx)^n, x\right)$$

[Out] Unintegrable[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]

Rubi [A] time = 0.0146736, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n dx$$

Verification is Not applicable to the result.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]

[Out] Defer[Int][Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]

Rubi steps

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n dx = \int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n dx$$

Mathematica [A] time = 0.0730368, size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n dx$$

Verification is Not applicable to the result.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]

[Out] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int \cos\left(\frac{b^2\pi x^2}{2}\right) (\text{FresnelS}(bx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)`

[Out] `int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)^n \cos\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)^n,x, algorithm="maxima")`

[Out] `integrate(fresnels(b*x)^n*cos(1/2*pi*b^2*x^2), x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnels}(bx)^n \cos\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)^n,x, algorithm="fricas")`

[Out] `integral(fresnels(b*x)^n*cos(1/2*pi*b^2*x^2), x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{\pi b^2 x^2}{2}\right) S^n(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)**n,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)**n, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnels}(bx)^n \cos\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)^n,x, algorithm="giac")

[Out] integrate(fresnels(b*x)^n*cos(1/2*pi*b^2*x^2), x)

3.91 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$

Optimal. Leaf size=307

$$\frac{105ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)}{8\pi^4b^7} + \frac{105ix^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)}{8\pi^4b^7} + \dots$$

[Out] $(35*x^4)/(8*b^5*Pi^3) - x^8/(16*b*Pi) - (40*\text{Cos}[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*\text{Cos}[b^2*Pi*x^2])/(2*b^5*Pi^3) - (105*x*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(b^8*Pi^4) + (7*x^5*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(b^4*Pi^2) + (105*\text{FresnelC}[b*x]*\text{FresnelS}[b*x])/(2*b^9*Pi^4) - (((105*I)/8)*x^2*\text{HypergeometricPFQ}[\{1,1\},\{3/2,2\},(-I/2)*b^2*Pi*x^2])/(b^7*Pi^4) + (((105*I)/8)*x^2*\text{HypergeometricPFQ}[\{1,1\},\{3/2,2\},(I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (35*x^3*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(b^2*Pi) - (55*x^2*\text{Sin}[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*\text{Sin}[b^2*Pi*x^2])/(4*b^3*Pi^2)$

Rubi [A] time = 0.441621, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6462, 3379, 3309, 30, 3296, 2638, 6454, 6446}

$$\frac{105ix^2 {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7} + \frac{105ix^2 {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7} + \frac{105\text{FresnelC}(bx)S(bx)}{2\pi^4b^9} + \frac{x^7S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^8*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x],x]$

[Out] $(35*x^4)/(8*b^5*Pi^3) - x^8/(16*b*Pi) - (40*\text{Cos}[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*\text{Cos}[b^2*Pi*x^2])/(2*b^5*Pi^3) - (105*x*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(b^8*Pi^4) + (7*x^5*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x])/(b^4*Pi^2) + (105*\text{FresnelC}[b*x]*\text{FresnelS}[b*x])/(2*b^9*Pi^4) - (((105*I)/8)*x^2*\text{HypergeometricPFQ}[\{1,1\},\{3/2,2\},(-I/2)*b^2*Pi*x^2])/(b^7*Pi^4) + (((105*I)/8)*x^2*\text{HypergeometricPFQ}[\{1,1\},\{3/2,2\},(I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (35*x^3*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*\text{FresnelS}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(b^2*Pi) - (55*x^2*\text{Sin}[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*\text{Sin}[b^2*Pi*x^2])/(4*b^3*Pi^2)$

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] := Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*cos[2*e + f*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)^(m_) * Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```


Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] := Simp[(FresnelC[b*x]
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])]/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2)])]/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}
\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx &= \frac{x^7 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7 \int x^6 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^7 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{x^7 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{35 \int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^4\pi^2} - \frac{7 \int x^5 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{35x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{105 \int x^2 S(bx) \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} \\
&= -\frac{x^8}{16b\pi} + \frac{7x^4 \cos(b^2\pi x^2)}{4b^5\pi^3} - \frac{105x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^8\pi^4} + \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{35x^5 \sin^2\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&= -\frac{x^8}{16b\pi} + \frac{5x^4 \cos(b^2\pi x^2)}{2b^5\pi^3} - \frac{105x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^8\pi^4} + \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{105 \int x^2 S(bx) \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} \\
&= \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{119 \cos(b^2\pi x^2)}{4b^9\pi^5} + \frac{5x^4 \cos(b^2\pi x^2)}{2b^5\pi^3} - \frac{105x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^8\pi^4} + \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} \\
&= \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{40 \cos(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4 \cos(b^2\pi x^2)}{2b^5\pi^3} - \frac{105x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^8\pi^4} + \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2}
\end{aligned}$$

Mathematica [F] time = 0.0459479, size = 0, normalized size = 0.

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]

[Out] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int x^8 \cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelS}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)`

[Out] `int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")`

[Out] `integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^8 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(x^8*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")`

[Out] `integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

3.92 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$

Optimal. Leaf size=217

$$\frac{x^6 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{24x^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{6x^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{48S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^8} + \frac{531S(\sqrt{2}bx)}{16\sqrt{2}\pi^4 b^8} + \frac{4}{\pi}$$

[Out] $(4*x^3)/(b^5*Pi^3) - x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) - (48*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*b^8*Pi^4) - (24*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7*Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)$

Rubi [A] time = 0.271429, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6462, 3391, 30, 3386, 3385, 3351, 6454, 6452}

$$\frac{x^6 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{24x^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{6x^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{48S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^8} + \frac{531S(\sqrt{2}bx)}{16\sqrt{2}\pi^4 b^8} + \frac{4}{\pi}$$

Antiderivative was successfully verified.

[In] Int[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]

[Out] $(4*x^3)/(b^5*Pi^3) - x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) - (48*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*b^8*Pi^4) - (24*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7*Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)$

Rule 6462

Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^(m-1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m-1)*Sin[d*x^2]^2, x], x] - Dist[(m-1)/(2*d), Int[x^(m-2)*Sin[d*x^2]*FresnelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 3391

Int[(x_)^(m_)*Sin[(a_) + ((b_)*(x_)^(n_))/2]^2, x_Symbol] := Dist[1/2, Int[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b, m, n}, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3386

Int[Cos[(c_) + (d_)*(x_)^(n_)]*((e_)*(x_)^(m_)), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3385

Int[((e_)*(x_))^(m_)*Sin[(c_) + (d_)*(x_)^(n_)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3351

Int[Sin[(d_)*((e_) + (f_)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 6454

Int[FresnelS[(b_)*(x_)]*(x_)^(m_)*Sin[(d_)*(x_)^2], x_Symbol] := -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 6452

Int[FresnelS[(b_)*(x_)]*(x_)*Sin[(d_)*(x_)^2], x_Symbol] := -Simp[(Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rubi steps

$$\begin{aligned}
\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx &= \frac{x^6 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{6 \int x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^6 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{x^6 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{24 \int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^4\pi^2} - \frac{3 \int x^6 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= -\frac{x^7}{14b\pi} + \frac{3x^3 \cos(b^2\pi x^2)}{2b^5\pi^3} + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{24x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^6 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&= -\frac{x^7}{14b\pi} + \frac{17x^3 \cos(b^2\pi x^2)}{8b^5\pi^3} - \frac{48 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^8\pi^4} + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{24x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&= \frac{4x^3}{b^5\pi^3} - \frac{x^7}{14b\pi} + \frac{17x^3 \cos(b^2\pi x^2)}{8b^5\pi^3} - \frac{48 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^8\pi^4} + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{3 \int x^6 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= \frac{4x^3}{b^5\pi^3} - \frac{x^7}{14b\pi} + \frac{17x^3 \cos(b^2\pi x^2)}{8b^5\pi^3} - \frac{48 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^8\pi^4} + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{3 \int x^6 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi}
\end{aligned}$$

Mathematica [A] time = 0.175812, size = 163, normalized size = 0.75

$$\frac{224S(bx) \left(\pi b^2 x^2 (\pi^2 b^4 x^4 - 24) \sin\left(\frac{1}{2}\pi b^2 x^2\right) + 6 (\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2}\pi b^2 x^2\right) \right) - 16\pi^3 b^7 x^7 + 896\pi b^3 x^3 + 56\pi^2 b^5 x^5 \sin(\pi b^2 x^2)}{224\pi^4 b^8}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]

[Out] (896*b^3*Pi*x^3 - 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 224*FresnelS[b*x]*(6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 2058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi^4)

Maple [A] time = 0.084, size = 321, normalized size = 1.5

$$\frac{1}{b} \left(\frac{\text{FresnelS}(bx)}{b^7} \left(\frac{b^6 x^6}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) - 6 \frac{1}{\pi} \left(-\frac{x^4 b^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 4 \frac{1}{\pi} \left(\frac{b^2 x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 2 \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)`

[Out]
$$\frac{\text{FresnelS}(bx)/b^7 \cdot \left(\frac{1}{\text{Pi}} b^6 x^6 \sin\left(\frac{1}{2} b^2 \text{Pi} x^2\right) - \frac{6}{\text{Pi}} \left(-\frac{1}{\text{Pi}} b^4 x^4 \cos\left(\frac{1}{2} b^2 \text{Pi} x^2\right) + \frac{4}{\text{Pi}} \left(\frac{1}{\text{Pi}} b^2 x^2 \sin\left(\frac{1}{2} b^2 \text{Pi} x^2\right) + \frac{2}{\text{Pi}^2} \cos\left(\frac{1}{2} b^2 \text{Pi} x^2\right) \right) \right) - \frac{1}{b^7} \left(\frac{1}{2} \text{Pi}^3 \left(\frac{1}{7} \text{Pi}^2 b^7 x^7 - 8 x^3 b^3 \right) + \frac{3}{\text{Pi}^4} \left(-\frac{1}{2} \text{Pi} b^3 x^3 \cos(b^2 \text{Pi} x^2) + \frac{3}{2} \text{Pi} \left(\frac{1}{2} \text{Pi} b x \sin(b^2 \text{Pi} x^2) - \frac{1}{4} \text{Pi}^2 \left(\frac{1}{2} \right) \text{FresnelS}(b x^2 \left(\frac{1}{2} \right)) - 4 \cdot 2 \left(\frac{1}{2} \right) \text{FresnelS}(b x^2 \left(\frac{1}{2} \right)) - \frac{1}{2} \text{Pi}^3 \left(\frac{1}{2} \text{Pi} b^5 x^5 \sin(b^2 \text{Pi} x^2) - \frac{5}{2} \text{Pi} \left(-\frac{1}{2} \text{Pi} b^3 x^3 \cos(b^2 \text{Pi} x^2) + \frac{3}{2} \text{Pi} \left(\frac{1}{2} \text{Pi} b x \sin(b^2 \text{Pi} x^2) - \frac{1}{4} \text{Pi}^2 \left(\frac{1}{2} \right) \text{FresnelS}(b x^2 \left(\frac{1}{2} \right)) \right) \right) - \frac{12}{\text{Pi}} b x \sin(b^2 \text{Pi} x^2) + \frac{6}{\text{Pi}^2} \left(\frac{1}{2} \right) \text{FresnelS}(b x^2 \left(\frac{1}{2} \right)) \right) \right) \right) / b$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")`

[Out] `integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^7 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(x^7*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")

[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)

3.93 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$

Optimal. Leaf size=184

$$\frac{x^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{15x S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{5x^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} + \frac{15S(bx)^2}{2\pi^3 b^7} + \frac{15x^2}{4\pi^3 b^5} + \frac{x^4 \sin(\pi b^2 x^2)}{4\pi^2 b^3} - \frac{11S(bx)}{4\pi^2 b^3}$$

[Out] $(15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)$

Rubi [A] time = 0.253349, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6462, 3379, 3309, 30, 3296, 2637, 6454, 2634, 6440}

$$\frac{x^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{15x S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{5x^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} + \frac{15S(bx)^2}{2\pi^3 b^7} + \frac{15x^2}{4\pi^3 b^5} + \frac{x^4 \sin(\pi b^2 x^2)}{4\pi^2 b^3} - \frac{11S(bx)}{4\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^6*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelS}[b*x], x]$

[Out] $(15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)$

Rule 6462

$\text{Int}[\text{Cos}[(d_*)*(x_)^2]*\text{FresnelS}[(b_*)*(x_)]*(x_)^{(m_)}, x_Symbol] \text{ :> } \text{Simp}[(x^{(m-1)}*\text{Sin}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + (-\text{Dist}[1/(Pi*b), \text{Int}[x^{(m-1)}*\text{Sin}[d*x^2]^2, x], x] - \text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\text{Sin}[d*x^2]*\text{FresnelS}[b*x], x], x]) /; \text{FreeQ}[\{b, d\}, x] \ \&\& \ \text{EqQ}[d^2, (Pi^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*SIN[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :=
  Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*cos[2*e + f*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
  ((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[SIN[c + d*x]/d, x] /;
  FreeQ[{c, d}, x]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] := Simp[x/2, x] - Simp[SIN[2*c + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
```

qQ[d^2, (Pi^2*b^4)/4]

Rubi steps

$$\begin{aligned}
 \int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx &= \frac{x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{5 \int x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^5 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
 &= \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{15 \int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^4\pi^2} - \frac{5 \int x^5 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
 &= \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{15x S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{15 \int S(bx) dx}{b^6\pi^3} \\
 &= -\frac{x^6}{12b\pi} + \frac{5x^2 \cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{15x S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
 &= \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2 \cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{15S(bx)^2}{2b^7\pi^3} - \frac{15x S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
 &= \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2 \cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{15S(bx)^2}{2b^7\pi^3} - \frac{15x S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}
 \end{aligned}$$

Mathematica [A] time = 0.008825, size = 184, normalized size = 1.

$$\frac{x^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{15x S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{5x^3 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} + \frac{15S(bx)^2}{2\pi^3 b^7} + \frac{15x^2}{4\pi^3 b^5} + \frac{x^4 \sin(\pi b^2 x^2)}{4\pi^2 b^3} - \frac{11x^5 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{b^6\pi^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]

[Out] (15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int x^6 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)`

[Out] `int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")`

[Out] `integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^6 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(x^6*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Sympy [A] time = 124.888, size = 264, normalized size = 1.43

$$\left\{ \begin{array}{l} -\frac{x^6 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} - \frac{x^6 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} + \frac{x^5 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{\pi b^2} + \frac{x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{5x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{\pi^2 b^4} + \frac{2x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^3 b^5} + \frac{11x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^3 b^5} \\ 0 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)`

```
[Out] Piecewise((-x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) - x**6*cos(pi*b**2*x**2/2)
)**2/(12*pi*b) + x**5*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + x**4*si
n(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*cos(pi*b**2*x
**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*x**2*sin(pi*b**2*x**2/2)**2/(pi**3*b
**5) + 11*x**2*cos(pi*b**2*x**2/2)**2/(2*pi**3*b**5) - 15*x*sin(pi*b**2*x**2
/2)*fresnels(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)
/(pi**4*b**7) + 15*fresnels(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.94 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$

Optimal. Leaf size=166

$$-\frac{43\text{FresnelC}(\sqrt{2}bx)}{8\sqrt{2}\pi^3b^6} + \frac{x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{8S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} + \frac{4x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^3\sin(\pi b^2x^2)}{4\pi^2b^3} + \frac{11x^5}{10\pi b^5}$$

[Out] $(4*x)/(b^5*\text{Pi}^3) - x^5/(10*b*\text{Pi}) + (11*x*\text{Cos}[b^2*\text{Pi}*x^2])/(8*b^5*\text{Pi}^3) - (4*3*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(8*\text{Sqrt}[2]*b^6*\text{Pi}^3) + (4*x^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(b^4*\text{Pi}^2) - (8*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^6*\text{Pi}^3) + (x^4*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi}) + (x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2)$

Rubi [A] time = 0.168881, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6462, 3391, 30, 3386, 3385, 3352, 6454, 6460, 3357}

$$-\frac{43\text{FresnelC}(\sqrt{2}bx)}{8\sqrt{2}\pi^3b^6} + \frac{x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{8S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} + \frac{4x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^3\sin(\pi b^2x^2)}{4\pi^2b^3} + \frac{11x^5}{10\pi b^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^5*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x], x]$

[Out] $(4*x)/(b^5*\text{Pi}^3) - x^5/(10*b*\text{Pi}) + (11*x*\text{Cos}[b^2*\text{Pi}*x^2])/(8*b^5*\text{Pi}^3) - (4*3*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(8*\text{Sqrt}[2]*b^6*\text{Pi}^3) + (4*x^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(b^4*\text{Pi}^2) - (8*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^6*\text{Pi}^3) + (x^4*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi}) + (x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2)$

Rule 6462

$\text{Int}[\text{Cos}[(d_.)*(x_)^2]*\text{FresnelS}[(b_.)*(x_)]*(x_)^(m_), x_Symbol] \text{ :> } \text{Simp}[(x^(m-1)*\text{Sin}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + (-\text{Dist}[1/(\text{Pi}*b), \text{Int}[x^(m-1)*\text{Sin}[d*x^2]^2, x], x] - \text{Dist}[(m-1)/(2*d), \text{Int}[x^(m-2)*\text{Sin}[d*x^2]*\text{FresnelS}[b*x], x], x]) /; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

Rule 3391

Int[(x_)^(m_)*Sin[(a_) + ((b_)*(x_)^(n_))/2]^2, x_Symbol] := Dist[1/2, Int[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b, m, n}, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3386

Int[Cos[(c_) + (d_)*(x_)^(n_)]*((e_)*(x_))^(m_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3385

Int[((e_)*(x_))^(m_)*Sin[(c_) + (d_)*(x_)^(n_)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3352

Int[Cos[(d_)*((e_) + (f_)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 6454

Int[FresnelS[(b_)*(x_)]*(x_)^(m_)*Sin[(d_)*(x_)^2], x_Symbol] := -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 6460

Int[Cos[(d_)*(x_)^2]*FresnelS[(b_)*(x_)]*(x_), x_Symbol] := Simp[(Sin[d*x^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_)^n_)])^p_, x_Sy
mbol] := Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
 \int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx &= \frac{x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{4 \int x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^4 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
 &= \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{8 \int x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^4\pi^2} - \frac{2 \int x^2 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
 &= -\frac{x^5}{10b\pi} + \frac{x \cos(b^2\pi x^2)}{b^5\pi^3} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{8S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
 &= -\frac{x^5}{10b\pi} + \frac{11x \cos(b^2\pi x^2)}{8b^5\pi^3} - \frac{C(\sqrt{2}bx)}{\sqrt{2}b^6\pi^3} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{8S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
 &= \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x \cos(b^2\pi x^2)}{8b^5\pi^3} - \frac{11C(\sqrt{2}bx)}{8\sqrt{2}b^6\pi^3} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{8S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
 &= \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x \cos(b^2\pi x^2)}{8b^5\pi^3} - \frac{11C(\sqrt{2}bx)}{8\sqrt{2}b^6\pi^3} - \frac{2\sqrt{2}C(\sqrt{2}bx)}{b^6\pi^3} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2}
 \end{aligned}$$

Mathematica [A] time = 0.173342, size = 126, normalized size = 0.76

$$\frac{80S(bx) \left((\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2}\pi b^2 x^2\right) + 4\pi b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \right) + 2bx \left(-4\pi^2 b^4 x^4 + 10\pi b^2 x^2 \sin(\pi b^2 x^2) + 55 \cos(\pi b^2 x^2) \right)}{80\pi^3 b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

```
[Out] (-215*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*FresnelS[b*x]*(4*b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(160 - 4*b^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*b^6*Pi^3)
```


Maple [A] time = 0.079, size = 212, normalized size = 1.3

$$\frac{1}{b} \left(\frac{\text{FresnelS}(bx)}{b^5} \left(\frac{x^4 b^4}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) - 4 \frac{1}{\pi} \left(-\frac{b^2 x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 2 \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right) - \frac{1}{b^5} \left(\frac{1}{2 \pi^3} \left(\frac{\pi^2 b^5 x^5}{5} - 8 b x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x), x)

[Out] (FresnelS(b*x)/b^5*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))-1/b^5*(1/2/Pi^3*(1/5*Pi^2*b^5*x^5-8*b*x)+2/Pi^2*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2))))/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnels(b*x), x, algorithm="maxima")

[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^5 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnels(b*x), x, algorithm="fricas")

[Out] integral(x^5*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnels(b*x), x)

[Out] Integral(x**5*cos(pi*b**2*x**2/2)*fresnels(b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnels(b*x), x, algorithm="giac")

[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)

3.95 $\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$

Optimal. Leaf size=195

$$\frac{3ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right)}{8\pi^2 b^3} - \frac{3ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)}{8\pi^2 b^3} - \frac{3 \text{FresnelS}(bx)}{3 \text{FresnelS}(bx)}$$

[Out] $-x^4/(8*b*\text{Pi}) + \text{Cos}[b^2*\text{Pi}*x^2]/(b^5*\text{Pi}^3) + (3*x*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(b^4*\text{Pi}^2) - (3*\text{FresnelC}[b*x]*\text{FresnelS}[b*x])/(2*b^5*\text{Pi}^2) + (((3*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2*\text{Pi}*x^2])/(b^3*\text{Pi}^2) - (((3*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2*\text{Pi}*x^2])/(b^3*\text{Pi}^2) + (x^3*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi}) + (x^2*\text{Sin}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2)$

Rubi [A] time = 0.157324, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6462, 3379, 3309, 30, 3296, 2638, 6454, 6446}

$$\frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2 b^3} - \frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi^2 b^3} - \frac{3 \text{FresnelC}(bx)S(bx)}{2\pi^2 b^5} + \frac{x^3 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{3xS(bx)}{3xS(bx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x], x]$

[Out] $-x^4/(8*b*\text{Pi}) + \text{Cos}[b^2*\text{Pi}*x^2]/(b^5*\text{Pi}^3) + (3*x*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(b^4*\text{Pi}^2) - (3*\text{FresnelC}[b*x]*\text{FresnelS}[b*x])/(2*b^5*\text{Pi}^2) + (((3*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2*\text{Pi}*x^2])/(b^3*\text{Pi}^2) - (((3*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2*\text{Pi}*x^2])/(b^3*\text{Pi}^2) + (x^3*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi}) + (x^2*\text{Sin}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2)$

Rule 6462

$\text{Int}[\text{Cos}[(d \cdot x)^2] * \text{FresnelS}[b \cdot x] * (x)^m, x_Symbol] \rightarrow \text{Simp}[(x^{m-1} * \text{Sin}[d \cdot x^2] * \text{FresnelS}[b \cdot x]) / (2 \cdot d), x] + (-\text{Dist}[1 / (\text{Pi} \cdot b), \text{Int}[x^{m-1} * \text{Sin}[d \cdot x^2]^2, x], x] - \text{Dist}[(m-1) / (2 \cdot d), \text{Int}[x^{m-2} * \text{Sin}[d \cdot x^2] * \text{FresnelS}[b \cdot x], x], x]) / ; \text{FreeQ}\{b, d, x\} \&\& \text{EqQ}[d^2, (\text{Pi}^2 * b^4) / 4] \&\& \text{IGtQ}[m, 1]$

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p,
    x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
  && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :=
  Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x],
    x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
  ((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x],
    x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_.)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d),
  x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Sin[2*d*x^2],
  x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] := Simp[(FresnelC[b*x]*FresnelS[b*x])/(2*b),
  x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(I*b^2*Pi*x^2)/2])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2},
  (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
```

(Pi^2*b^4)/4]

Rubi steps

$$\begin{aligned}
\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx &= \frac{x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3 \int x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^3 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= \frac{3x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3 \int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx}{b^4\pi^2} - \frac{3 \int x \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{2b^3\pi} \\
&= \frac{3x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{3C(bx)S(bx)}{2b^5\pi^2} + \frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} - \frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} \\
&= -\frac{x^4}{8b\pi} + \frac{3 \cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{3x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{3C(bx)S(bx)}{2b^5\pi^2} + \frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} \\
&= -\frac{x^4}{8b\pi} + \frac{\cos(b^2\pi x^2)}{b^5\pi^3} + \frac{3x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{3C(bx)S(bx)}{2b^5\pi^2} + \frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2}
\end{aligned}$$

Mathematica [F] time = 0.038016, size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]

[Out] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x), x)

[Out] `int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")`

[Out] `integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(x^4*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)`

[Out] `Integral(x**4*cos(pi*b**2*x**2/2)*fresnels(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.96 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$

Optimal. Leaf size=108

$$\frac{x^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{2S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{5S(\sqrt{2}bx)}{4\sqrt{2}\pi^2 b^4} + \frac{x \sin(\pi b^2 x^2)}{4\pi^2 b^3} - \frac{x^3}{6\pi b}$$

[Out] $-x^3/(6*b*\text{Pi}) + (2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(b^4*\text{Pi}^2) - (5*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(4*\text{Sqrt}[2]*b^4*\text{Pi}^2) + (x^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi}) + (x*\text{Sin}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2)$

Rubi [A] time = 0.0898809, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {6462, 3391, 30, 3386, 3351, 6452}

$$\frac{x^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{2S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{5S(\sqrt{2}bx)}{4\sqrt{2}\pi^2 b^4} + \frac{x \sin(\pi b^2 x^2)}{4\pi^2 b^3} - \frac{x^3}{6\pi b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x], x]$

[Out] $-x^3/(6*b*\text{Pi}) + (2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x])/(b^4*\text{Pi}^2) - (5*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(4*\text{Sqrt}[2]*b^4*\text{Pi}^2) + (x^2*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi}) + (x*\text{Sin}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2)$

Rule 6462

$\text{Int}[\text{Cos}[(d_.)*(x_)^2]*\text{FresnelS}[(b_.)*(x_)]*(x_)^(m_), x_Symbol] \rightarrow \text{Simp}[(x^(m-1)*\text{Sin}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + (-\text{Dist}[1/(\text{Pi}*b), \text{Int}[x^(m-1)*\text{Sin}[d*x^2]^2, x], x] - \text{Dist}[(m-1)/(2*d), \text{Int}[x^(m-2)*\text{Sin}[d*x^2]*\text{FresnelS}[b*x], x], x]) /; \text{FreeQ}\{b, d\}, x \&\& \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4] \&\& \text{IGtQ}[m, 1]$

Rule 3391

$\text{Int}[(x_)^(m_.)*\text{Sin}[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[x^m, x], x] - \text{Dist}[1/2, \text{Int}[x^m*\text{Cos}[2*a + b*x^n], x], x] /; \text{FreeQ}\{a, b, m, n\}, x]$

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3386

Int[Cos[(c_) + (d_)*(x_)^(n_)]*((e_)*(x_))^(m_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n]/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3351

Int[Sin[(d_)*((e_) + (f_)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 6452

Int[FresnelS[(b_)*(x_)]*(x_)*Sin[(d_)*(x_)^2], x_Symbol] := -Simp[(Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rubi steps

$$\begin{aligned}
 \int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx &= \frac{x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{2 \int x S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^2 \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
 &= \frac{2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \sin(b^2\pi x^2) dx}{b^3\pi^2} - \frac{\int x^2 dx}{2b\pi} + \frac{\int x^2 \cos(b^2\pi x^2) dx}{4b^3\pi^2} \\
 &= -\frac{x^3}{6b\pi} + \frac{2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{S(\sqrt{2}bx)}{\sqrt{2}b^4\pi^2} + \frac{x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x \sin(b^2\pi x^2)}{4b^3\pi^2} - \frac{\int x^2 \cos(b^2\pi x^2) dx}{4b^3\pi^2} \\
 &= -\frac{x^3}{6b\pi} + \frac{2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{5S(\sqrt{2}bx)}{4\sqrt{2}b^4\pi^2} + \frac{x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x \sin(b^2\pi x^2)}{4b^3\pi^2}
 \end{aligned}$$

Mathematica [A] time = 0.081416, size = 90, normalized size = 0.83

$$\frac{24S(bx) \left(\pi b^2 x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right) + 2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \right) - 4\pi b^3 x^3 + 6bx \sin(\pi b^2 x^2) - 15\sqrt{2}S(\sqrt{2}bx)}{24\pi^2 b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]

[Out] $(-4*b^3*Pi*x^3 - 15*sqrt{2}*FresnelS[sqrt{2}*b*x] + 24*FresnelS[b*x]*(2*\cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*\sin[(b^2*Pi*x^2)/2]) + 6*b*x*\sin[b^2*Pi*x^2]) / (24*b^4*Pi^2)$

Maple [A] time = 0.075, size = 119, normalized size = 1.1

$$\frac{1}{b} \left(\frac{FresnelS(bx)}{b^3} \left(\frac{b^2 x^2}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) + 2 \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) - \frac{1}{b^3} \left(\frac{\sqrt{2} FresnelS(bx\sqrt{2})}{2\pi^2} + \frac{x^3 b^3}{6\pi} - \frac{1}{2\pi} \left(\frac{bx \sin(b^2 \pi x^2)}{2\pi} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)

[Out] $(FresnelS(b*x)/b^3*(1/Pi*b^2*x^2*\sin(1/2*b^2*Pi*x^2)+2/Pi^2*\cos(1/2*b^2*Pi*x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))+1/6/Pi*b^3*x^3-1/2/Pi*(1/2/Pi*b*x*\sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))/b$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")

[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(x^3*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)`

[Out] `Integral(x**3*cos(pi*b**2*x**2/2)*fresnels(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")`

[Out] `integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

3.97 $\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$

Optimal. Leaf size=73

$$\frac{xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{S(bx)^2}{2\pi b^3} + \frac{\sin(\pi b^2x^2)}{4\pi^2 b^3} - \frac{x^2}{4\pi b}$$

[Out] $-x^2/(4*b*\text{Pi}) - \text{FresnelS}[b*x]^2/(2*b^3*\text{Pi}) + (x*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi}) + \text{Sin}[b^2*\text{Pi}*x^2]/(4*b^3*\text{Pi}^2)$

Rubi [A] time = 0.0547892, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6462, 3379, 2634, 6440, 30}

$$\frac{xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{S(bx)^2}{2\pi b^3} + \frac{\sin(\pi b^2x^2)}{4\pi^2 b^3} - \frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x], x]$

[Out] $-x^2/(4*b*\text{Pi}) - \text{FresnelS}[b*x]^2/(2*b^3*\text{Pi}) + (x*\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi}) + \text{Sin}[b^2*\text{Pi}*x^2]/(4*b^3*\text{Pi}^2)$

Rule 6462

$\text{Int}[\text{Cos}[(d_.)*(x_)^2]*\text{FresnelS}[(b_.)*(x_)]*(x_)^(m_), x_Symbol] \rightarrow \text{Simp}[(x^(m-1)*\text{Sin}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] + (-\text{Dist}[1/(\text{Pi}*b), \text{Int}[x^(m-1)*\text{Sin}[d*x^2]^2, x], x] - \text{Dist}[(m-1)/(2*d), \text{Int}[x^(m-2)*\text{Sin}[d*x^2]*\text{FresnelS}[b*x], x], x]) /; \text{FreeQ}\{b, d\}, x \&\& \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4] \&\& \text{IGtQ}[m, 1]$

Rule 3379

$\text{Int}[(x_)^(m_.)*((a_.) + (b_.)*\text{Sin}[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Sin}[c + d*x])^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \&\& (\text{EqQ}[p, 1] \parallel \text{EqQ}[m, n-1] \parallel (\text{IntegerQ}[p] \&\& \text{GtQ}[\text{Simplify}[(m+1)/n], 0]))$

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] := Simp[x/2, x] - Simp[Sin[2*c
+ d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx &= \frac{xS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\ &= \frac{xS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\text{Subst}\left(\int x dx, x, S(bx)\right)}{b^3\pi} - \frac{\text{Subst}\left(\int \sin^2\left(\frac{1}{2}b^2\pi x\right) dx, x, x^2\right)}{2b\pi} \\ &= -\frac{x^2}{4b\pi} - \frac{S(bx)^2}{2b^3\pi} + \frac{xS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} \end{aligned}$$

Mathematica [A] time = 0.0045326, size = 73, normalized size = 1.

$$\frac{xS(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{S(bx)^2}{2\pi b^3} + \frac{\sin\left(\pi b^2 x^2\right)}{4\pi^2 b^3} - \frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

```
[Out] -x^2/(4*b*Pi) - FresnelS[b*x]^2/(2*b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(b^2*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelS}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)`

[Out] `int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")`

[Out] `integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(x^2*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Sympy [A] time = 4.71556, size = 114, normalized size = 1.56

$$\begin{cases} -\frac{x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{4\pi b} - \frac{x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{4\pi b} + \frac{x \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{\pi b^2} + \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} - \frac{S^2(bx)}{2\pi b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Piecewise((-x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) - x**2*cos(pi*b**2*x**2/2)
**2/(4*pi*b) + x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + sin(pi*b**2*
x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) - fresnels(b*x)**2/(2*pi*b**3),
Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.98 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$

Optimal. Leaf size=59

$$\frac{\text{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}\pi b^2} + \frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{x}{2\pi b}$$

[Out] $-x/(2*b*\text{Pi}) + \text{FresnelC}[\text{Sqrt}[2]*b*x]/(2*\text{Sqrt}[2]*b^2*\text{Pi}) + (\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi})$

Rubi [A] time = 0.0321775, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6460, 3357, 3352}

$$\frac{\text{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}\pi b^2} + \frac{S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{x}{2\pi b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelS}[b*x], x]$

[Out] $-x/(2*b*\text{Pi}) + \text{FresnelC}[\text{Sqrt}[2]*b*x]/(2*\text{Sqrt}[2]*b^2*\text{Pi}) + (\text{FresnelS}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi})$

Rule 6460

$\text{Int}[\text{Cos}[(d_.)*(x_)^2]*\text{FresnelS}[(b_.)*(x_)]*(x_), x_Symbol] \rightarrow \text{Simp}[(\text{Sin}[d*x^2]*\text{FresnelS}[b*x])/(2*d), x] - \text{Dist}[1/(\text{Pi}*b), \text{Int}[\text{Sin}[d*x^2]^2, x], x] /; \text{FreeQ}\{b, d, x\} \&\& \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4]$

Rule 3357

$\text{Int}[(a_. + (b_.)*\text{Sin}[c_. + (d_.)*((e_.) + (f_.)*(x_))^{(n_)}])^{(p_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(a + b*\text{Sin}[c + d*(e + f*x)^n])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{IGtQ}[p, 1] \&\& \text{IGtQ}[n, 1]$

Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_))^{(n_)}], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f, x\}$

Rubi steps

$$\begin{aligned}
\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx &= \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int\left(\frac{1}{2} - \frac{1}{2}\cos(b^2\pi x^2)\right) dx}{b\pi} \\
&= -\frac{x}{2b\pi} + \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\int \cos(b^2\pi x^2) dx}{2b\pi} \\
&= -\frac{x}{2b\pi} + \frac{C(\sqrt{2}bx)}{2\sqrt{2}b^2\pi} + \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}
\end{aligned}$$

Mathematica [A] time = 0.0279266, size = 48, normalized size = 0.81

$$\frac{4S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) + \sqrt{2}\text{FresnelC}(\sqrt{2}bx) - 2bx}{4\pi b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]

[Out] (-2*b*x + Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^2*Pi)

Maple [A] time = 0.066, size = 52, normalized size = 0.9

$$\frac{1}{b} \left(\frac{\text{FresnelS}(bx)}{b\pi} \sin\left(\frac{b^2\pi x^2}{2}\right) - \frac{1}{b\pi} \left(\frac{bx}{2} - \frac{\sqrt{2}\text{FresnelC}(bx\sqrt{2})}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x), x)

[Out] (FresnelS(b*x)/b/Pi*sin(1/2*b^2*Pi*x^2)-1/b/Pi*(1/2*b*x-1/4*2^(1/2)*FresnelC(b*x*2^(1/2))))/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")

[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")

[Out] integral(x*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)

[Out] Integral(x*cos(pi*b**2*x**2/2)*fresnels(b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

3.99 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$

Optimal. Leaf size=80

$$-\frac{1}{8}ibx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)+\frac{1}{8}ibx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)+\frac{\text{FresnelC}(bx)S(bx)}{2b}$$

[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]

Rubi [A] time = 0.0167872, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6446}

$$-\frac{1}{8}ibx^2 {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2 {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)S(bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]

[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]

Rule 6446

Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(I*b^2*Pi*x^2)/2])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rubi steps

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx = \frac{C(bx)S(bx)}{2b} - \frac{1}{8}ibx^2 {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2 {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.0139307, size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]

[Out] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x), x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x), x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)`

[Out] `Integral(cos(pi*b**2*x**2/2)*fresnels(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)`

$$3.100 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}, x\right)$$

[Out] Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]

Rubi [A] time = 0.0178919, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]

[Out] Defer[Int] [(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} dx$$

Mathematica [A] time = 0.0304774, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]

Maple [A] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x,x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x, x)

$$3.101 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^2} dx$$

Optimal. Leaf size=48

$$-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x} + \frac{1}{4}b\text{Si}(b^2\pi x^2) - \frac{1}{2}\pi bS(bx)^2$$

[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*Si
nIntegral[b^2*Pi*x^2])/4

Rubi [A] time = 0.0413899, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {6464, 6440, 30, 3375}

$$-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x} + \frac{1}{4}b\text{Si}(b^2\pi x^2) - \frac{1}{2}\pi bS(bx)^2$$

Antiderivative was successfully verified.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]

[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*Si
nIntegral[b^2*Pi*x^2])/4

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3375

Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^2} dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x} + \frac{1}{2}b \int \frac{\sin\left(b^2\pi x^2\right)}{x} dx - (b^2\pi) \int S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx \\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x} + \frac{1}{4}b \operatorname{Si}\left(b^2\pi x^2\right) - (b\pi) \operatorname{Subst}\left(\int x dx, x, S(bx)\right) \\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x} - \frac{1}{2}b\pi S(bx)^2 + \frac{1}{4}b \operatorname{Si}\left(b^2\pi x^2\right) \end{aligned}$$

Mathematica [A] time = 0.0045022, size = 48, normalized size = 1.

$$-\frac{S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x} + \frac{1}{4}b \operatorname{Si}\left(b^2\pi x^2\right) - \frac{1}{2}\pi b S(bx)^2$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]

[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*SinIntegral[b^2*Pi*x^2])/4

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\operatorname{FresnelS}(bx)}{x^2} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)`

[Out] `int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^2,x, algorithm="maxima")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^2,x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**2,x)`

[Out] `Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^2,x, algorithm="giac")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^2, x)`

$$3.102 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3} dx$$

Optimal. Leaf size=93

$$-\frac{1}{2}\pi b^2 \text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) + \frac{\pi b^2 \text{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}} - \frac{S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2x^2} - \frac{b\sin(\pi b^2 x^2)}{4x}$$

[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) - (b^2*Pi*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2

Rubi [A] time = 0.0588469, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3,x]

[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) - (b^2*Pi*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3} dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2} + \frac{1}{4}b \int \frac{\sin(b^2\pi x^2)}{x^2} dx - \frac{1}{2}(b^2\pi) \int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx \\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2} - \frac{b\sin(b^2\pi x^2)}{4x} - \frac{1}{2}(b^2\pi) \int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx + \frac{1}{2}(b^3\pi) \int \cos \\ &= \frac{b^2\pi C(\sqrt{2}bx)}{2\sqrt{2}} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2} - \frac{b\sin(b^2\pi x^2)}{4x} - \frac{1}{2}(b^2\pi) \int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx \end{aligned}$$

Mathematica [A] time = 0.0308939, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3, x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3, x]

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^3} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3, x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^3, x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^3,x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^3, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**3,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^3,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^3, x)

$$3.103 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4} dx$$

Optimal. Leaf size=88

$$-\frac{1}{3}\pi b^2 \text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right) + \frac{1}{12}\pi b^3 \text{CosIntegral}(\pi b^2 x^2) - \frac{S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^3} - \frac{b\sin(\pi b^2 x^2)}{12x^2}$$

[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/3

Rubi [A] time = 0.0878529, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4, x]

[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Defer[Int] [(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/3

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^4} dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{3x^3} + \frac{1}{6}b \int \frac{\sin(b^2\pi x^2)}{x^3} dx - \frac{1}{3}(b^2\pi) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
&= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{3x^3} + \frac{1}{12}b \operatorname{Subst}\left(\int \frac{\sin(b^2\pi x)}{x^2} dx, x, x^2\right) - \frac{1}{3}(b^2\pi) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
&= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{3x^3} - \frac{b \sin(b^2\pi x^2)}{12x^2} - \frac{1}{3}(b^2\pi) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx + \frac{1}{12}(b^3\pi) \operatorname{Subst}\left(\int \frac{\sin(b^2\pi x)}{x^2} dx, x, x^2\right) \\
&= \frac{1}{12}b^3\pi \operatorname{Ci}(b^2\pi x^2) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{3x^3} - \frac{b \sin(b^2\pi x^2)}{12x^2} - \frac{1}{3}(b^2\pi) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx
\end{aligned}$$

Mathematica [A] time = 0.0336194, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4,x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4, x]

Maple [A] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{\operatorname{FresnelS}(bx)}{x^4} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^4,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^4, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^4,x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^4, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**4,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^4, x)
```

$$3.104 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5} dx$$

Optimal. Leaf size=155

$$-\frac{1}{8}\pi^2b^4\text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}, x\right) + \frac{\pi b^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^2} - \frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} - \frac{7\pi^2b^4S(\sqrt{2}bx)}{24\sqrt{2}} - \frac{b}{8}$$

[Out] (b^3*Pi)/(16*x) - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqrt[2]) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) - (b*Sin[b^2*Pi*x^2])/(24*x^3) - (b^4*Pi^2*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/8

Rubi [A] time = 0.126207, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]

[Out] (b^3*Pi)/(16*x) - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqrt[2]) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) - (b*Sin[b^2*Pi*x^2])/(24*x^3) - (b^4*Pi^2*Defer[Int] [(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/8

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^5} dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{4x^4} + \frac{1}{8}b \int \frac{\sin(b^2\pi x^2)}{x^4} dx - \frac{1}{4}(b^2\pi) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx \\
&= \frac{b^3\pi}{16x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{4x^4} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2} - \frac{b \sin(b^2\pi x^2)}{24x^3} + \frac{1}{16}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
&= \frac{b^3\pi}{16x} - \frac{7b^3\pi \cos(b^2\pi x^2)}{48x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{4x^4} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2} - \frac{b \sin(b^2\pi x^2)}{24x^3} \\
&= \frac{b^3\pi}{16x} - \frac{7b^3\pi \cos(b^2\pi x^2)}{48x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{4x^4} - \frac{7b^4\pi^2 S(\sqrt{2}bx)}{24\sqrt{2}} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}
\end{aligned}$$

Mathematica [A] time = 0.0318803, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^5} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5, x]

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^5} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^5,x, algorithm="maxima")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^5, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnels}(bx)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^5,x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^5, x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)S(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**5,x)`

[Out] `Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnels}(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^5, x)
```


$$3.105 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^6} dx$$

Optimal. Leaf size=163

$$\frac{\pi b^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3} + \frac{\pi^2 b^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{15x} - \frac{S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5} + \frac{1}{30}\pi^3 b^5 S(bx)^2 - \frac{7}{120}\pi^2 b^5 \text{Si}(b^2 \pi x^2) + \frac{\pi b^5}{60x}$$

[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2*Pi*x^2])/120

Rubi [A] time = 0.213637, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6464, 6456, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi b^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3} + \frac{\pi^2 b^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{15x} - \frac{S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5} + \frac{1}{30}\pi^3 b^5 S(bx)^2 - \frac{7}{120}\pi^2 b^5 \text{Si}(b^2 \pi x^2) + \frac{\pi b^5}{60x}$$

Antiderivative was successfully verified.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]

[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2*Pi*x^2])/120

Rule 6464

Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)^(m_)], x_Symbol] :> Simp[(x^(m+1)*Cos[d*x^2]*FresnelS[b*x])/(m+1), x] + (Dist[(2*d)/(m+1), Int[x^(m+2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m+1)), Int[x^(m+1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -1]

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(x^(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_)*Sin[(d_.)*(x_)^2], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^6} dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{5x^5} + \frac{1}{10}b \int \frac{\sin(b^2\pi x^2)}{x^5} dx - \frac{1}{5}(b^2\pi) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx \\
&= \frac{b^3\pi}{60x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{5x^5} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} + \frac{1}{20}b \operatorname{Subst}\left(\int \frac{\sin(b^2\pi x)}{x^3} dx, x, x^2\right) \\
&= \frac{b^3\pi}{60x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{5x^5} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{15x} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} - \frac{b \sin(b^2\pi x)}{4x} \\
&= \frac{b^3\pi}{60x^2} - \frac{b^3\pi \cos(b^2\pi x^2)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{5x^5} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{15x} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} \\
&= \frac{b^3\pi}{60x^2} - \frac{b^3\pi \cos(b^2\pi x^2)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{5x^5} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{15x} + \frac{1}{30}b^5\pi^3 S(bx)
\end{aligned}$$

Mathematica [A] time = 0.0100253, size = 163, normalized size = 1.

$$\frac{\pi b^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3} + \frac{\pi^2 b^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{15x} - \frac{S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5} + \frac{1}{30}\pi^3 b^5 S(bx)^2 - \frac{7}{120}\pi^2 b^5 \operatorname{Si}(b^2\pi x^2) + \frac{\pi b \sin(b^2\pi x)}{60x}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]

[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2*Pi*x^2])/120

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^6} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^6,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^6,x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^6, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**6,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^6,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^6, x)

$$3.106 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7} dx$$

Optimal. Leaf size=230

$$\frac{1}{48}\pi^3b^6\text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}, x\right) - \frac{1}{45}\sqrt{2}\pi^3b^6\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{7\pi^3b^6\text{FresnelC}\left(\sqrt{2}bx\right)}{144\sqrt{2}} + \frac{\pi b^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{24x^4}$$

[Out] (b^3*Pi)/(144*x^3) - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (7*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])/45 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(6*x^6) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(48*x^2) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(24*x^4) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(1440*x) + (b^6*Pi^3*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/48

Rubi [A] time = 0.204738, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7, x]

[Out] (b^3*Pi)/(144*x^3) - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (7*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])/45 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(6*x^6) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(48*x^2) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(24*x^4) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(1440*x) + (b^6*Pi^3*Defer[Int] [(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/48

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^7} dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x^6} + \frac{1}{12}b \int \frac{\sin\left(b^2\pi x^2\right)}{x^6} dx - \frac{1}{6}(b^2\pi) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx \\
&= \frac{b^3\pi}{144x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x^6} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} - \frac{b \sin\left(b^2\pi x^2\right)}{60x^5} + \frac{1}{48}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^4} dx \\
&= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi \cos\left(b^2\pi x^2\right)}{720x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x^6} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{48x^2} + \frac{b^2\pi S(bx)}{6x^6} \\
&= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi \cos\left(b^2\pi x^2\right)}{720x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x^6} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{48x^2} + \frac{b^2\pi S(bx)}{6x^6} \\
&= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi \cos\left(b^2\pi x^2\right)}{720x^3} - \frac{7b^6\pi^3 C\left(\sqrt{2}bx\right)}{144\sqrt{2}} - \frac{1}{45}\sqrt{2}b^6\pi^3 C\left(\sqrt{2}bx\right) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x^6}
\end{aligned}$$

Mathematica [A] time = 0.032583, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^7} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]

Maple [A] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^7} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^7,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^7, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^7,x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^7, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**7,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^7, x)
```

$$3.107 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8} dx$$

Optimal. Leaf size=201

$$\frac{1}{105}\pi^3b^6\text{Unintegrable}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}, x\right) - \frac{1}{84}\pi^3b^7\text{CosIntegral}(\pi b^2x^2) + \frac{\pi b^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5} + \frac{\pi^2b^4S(bx)}{10}$$

[Out] (b^3*Pi)/(280*x^4) - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosIntegral[b^2*Pi*x^2])/84 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*x^7) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x^3) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin[b^2*Pi*x^2])/(84*x^2) + (b^6*Pi^3*Unintegrable[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/105

Rubi [A] time = 0.319427, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]

[Out] (b^3*Pi)/(280*x^4) - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosIntegral[b^2*Pi*x^2])/84 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*x^7) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x^3) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin[b^2*Pi*x^2])/(84*x^2) + (b^6*Pi^3*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/105

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^8} dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{7x^7} + \frac{1}{14}b \int \frac{\sin\left(b^2\pi x^2\right)}{x^7} dx - \frac{1}{7}(b^2\pi) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx \\
&= \frac{b^3\pi}{280x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{7x^7} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} + \frac{1}{28}b \text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^4} dx, x, bx\right) \\
&= \frac{b^3\pi}{280x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{7x^7} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{105x^3} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} - \frac{b \sin\left(b^2\pi x\right)}{35x^4} \\
&= \frac{b^3\pi}{280x^4} - \frac{b^3\pi \cos\left(b^2\pi x^2\right)}{105x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{7x^7} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{105x^3} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} \\
&= \frac{b^3\pi}{280x^4} - \frac{b^3\pi \cos\left(b^2\pi x^2\right)}{105x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{7x^7} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{105x^3} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} \\
&= \frac{b^3\pi}{280x^4} - \frac{b^3\pi \cos\left(b^2\pi x^2\right)}{105x^4} - \frac{1}{84}b^7\pi^3 \text{Ci}\left(b^2\pi x^2\right) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{7x^7} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{105x^3}
\end{aligned}$$

Mathematica [A] time = 0.0334512, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^8} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8, x]

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^8} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)

[Out] `int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^8,x, algorithm="maxima")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^8, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^8,x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^8, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**8,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^8, x)
```

$$3.108 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9} dx$$

Optimal. Leaf size=270

$$\frac{1}{384}\pi^4 b^8 \text{Unintegrable}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) - \frac{\pi^3 b^6 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{384x^2} + \frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{48x^6} + \frac{\pi^2 b^4 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{192x^4}$$

[Out] (b^3*Pi)/(480*x^5) - (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(3360*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(192*x^4) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(384*x^2) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*Pi*x^2])/(40320*x^3) + (b^8*Pi^4*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/384

Rubi [A] time = 0.309526, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9, x]

[Out] (b^3*Pi)/(480*x^5) - (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(3360*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(192*x^4) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(384*x^2) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*Pi*x^2])/(40320*x^3) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/384

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^9} dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{8x^8} + \frac{1}{16}b \int \frac{\sin\left(b^2\pi x^2\right)}{x^8} dx - \frac{1}{8}(b^2\pi) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx \\
&= \frac{b^3\pi}{480x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{8x^8} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{b \sin\left(b^2\pi x^2\right)}{112x^7} + \frac{1}{96}(b^3\pi) \int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx) dx \\
&= \frac{b^3\pi}{480x^5} - \frac{19b^3\pi \cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{8x^8} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{192x^4} + \frac{b^4\pi^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{112x^7} \\
&= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi \cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{8x^8} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{192x^4} + \frac{b^4\pi^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{112x^7} \\
&= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi \cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3 \cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{8x^8} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{192x^4} + \frac{b^4\pi^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{112x^7} \\
&= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi \cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3 \cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{8x^8} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{192x^4} + \frac{b^4\pi^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{112x^7}
\end{aligned}$$

Mathematica [A] time = 0.031928, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^9} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9,x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9, x]

Maple [A] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^9} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9,x)

[Out] `int(cos(1/2*b^2*Pi*x^2)*Fresnels(b*x)/x^9,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^9,x, algorithm="maxima")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^9, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^9,x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^9, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**9,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnels}(bx)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^9,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^9, x)
```

$$3.109 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^{10}} dx$$

Optimal. Leaf size=278

$$-\frac{\pi^3 b^6 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{945x^3} + \frac{\pi b^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{63x^7} - \frac{\pi^4 b^8 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{945x} + \frac{\pi^2 b^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{315x^5} - \frac{S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9}$$

[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*FresnelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIntegral[b^2*Pi*x^2])/30240

Rubi [A] time = 0.516173, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6464, 6456, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$-\frac{\pi^3 b^6 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{945x^3} + \frac{\pi b^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{63x^7} - \frac{\pi^4 b^8 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{945x} + \frac{\pi^2 b^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{315x^5} - \frac{S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9}$$

Antiderivative was successfully verified.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]

[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*FresnelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIntegral[b^2*Pi*x^2])/30240

Rule 6464

Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)^m], x_Symbol] :> Simp[(x^(m+1)*Cos[d*x^2]*FresnelS[b*x])/(m+1), x] + (Dist[(2*d)/(m+1), Int[x^

$(m + 2) \cdot \sin[d \cdot x^2] \cdot \text{FresnelS}[b \cdot x], x], x] - \text{Dist}[d / (\text{Pi} \cdot b \cdot (m + 1)), \text{Int}[x^{(m + 1)} \cdot \sin[2 \cdot d \cdot x^2], x], x)] /; \text{FreeQ}[\{b, d\}, x] \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2 \cdot b^4) / 4] \ \&\& \ \text{ILtQ}[m, -1]$

Rule 6456

$\text{Int}[\text{FresnelS}[(b \cdot x)] \cdot (x)^{(m)} \cdot \sin[(d \cdot x)^2], x_Symbol] \rightarrow \text{Simp}[(x^{(m + 1)} \cdot \sin[d \cdot x^2] \cdot \text{FresnelS}[b \cdot x]) / (m + 1), x] + (-\text{Dist}[(2 \cdot d) / (m + 1), \text{Int}[x^{(m + 2)} \cdot \cos[d \cdot x^2] \cdot \text{FresnelS}[b \cdot x], x], x] + \text{Dist}[d / (\text{Pi} \cdot b \cdot (m + 1)), \text{Int}[x^{(m + 1)} \cdot \cos[2 \cdot d \cdot x^2], x], x] - \text{Simp}[(d \cdot x^{(m + 2)}) / (\text{Pi} \cdot b \cdot (m + 1) \cdot (m + 2)), x]) /; \text{FreeQ}[\{b, d\}, x] \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2 \cdot b^4) / 4] \ \&\& \ \text{ILtQ}[m, -2]$

Rule 6440

$\text{Int}[\text{FresnelS}[(b \cdot x)]^{(n)} \cdot \sin[(d \cdot x)^2], x_Symbol] \rightarrow \text{Dist}[(\text{Pi} \cdot b) / (2 \cdot d), \text{Subst}[\text{Int}[x^n, x], x, \text{FresnelS}[b \cdot x]], x] /; \text{FreeQ}[\{b, d, n\}, x] \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2 \cdot b^4) / 4]$

Rule 30

$\text{Int}[(x)^{(m)}, x_Symbol] \rightarrow \text{Simp}[x^{(m + 1)} / (m + 1), x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 3375

$\text{Int}[\sin[(d \cdot x)^n] / (x), x_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[d \cdot x^n] / n, x] /; \text{FreeQ}[\{d, n\}, x]$

Rule 3380

$\text{Int}[(a + \cos[c + d \cdot x] \cdot (x)^n] \cdot (b \cdot x)^p \cdot (x)^m, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot \cos[c + d \cdot x])^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{EqQ}[m, n - 1] \ || \ (\text{IntegerQ}[p] \ \&\& \ \text{GtQ}[\text{Simplify}[(m + 1)/n], 0]))$

Rule 3297

$\text{Int}[(c + d \cdot x)^m \cdot \sin[e + f \cdot x], x_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^{(m + 1)} \cdot \sin[e + f \cdot x] / (d \cdot (m + 1)), x] - \text{Dist}[f / (d \cdot (m + 1)), \text{Int}[(c + d \cdot x)^{(m + 1)} \cdot \cos[e + f \cdot x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{LtQ}[m, -1]$

Rule 3299

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[SinInte
 gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
 , x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
 m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
 m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{x^{10}} dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{9x^9} + \frac{1}{18}b \int \frac{\sin(b^2\pi x^2)}{x^9} dx - \frac{1}{9}(b^2\pi) \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx \\ &= \frac{b^3\pi}{756x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{9x^9} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} + \frac{1}{36}b \text{Subst}\left(\int \frac{\sin(b^2\pi x)}{x^5} dx, x, x^2\right) \\ &= \frac{b^3\pi}{756x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{9x^9} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{315x^5} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b \sin\left(\frac{1}{2}b^2\pi x^2\right)}{14x^4} \\ &= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi \cos(b^2\pi x^2)}{3024x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{9x^9} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{315x^5} + \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} \\ &= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi \cos(b^2\pi x^2)}{3024x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{9x^9} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{315x^5} - \frac{b^2\pi S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} \\ &= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi \cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3 \cos(b^2\pi x^2)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{9x^9} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{315x^5} \\ &= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi \cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3 \cos(b^2\pi x^2)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{9x^9} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{315x^5} \end{aligned}$$

Mathematica [A] time = 0.01589, size = 278, normalized size = 1.

$$-\frac{\pi^3 b^6 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{945x^3} + \frac{\pi b^2 S(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{63x^7} - \frac{\pi^4 b^8 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{945x} + \frac{\pi^2 b^4 S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{315x^5} - \frac{S(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]

[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*FresnelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIntegral[b^2*Pi*x^2])/30240

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelS}(bx)}{x^{10}} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^10,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^10, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^10, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**10,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}(bx)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^10,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^10, x)
```

3.110 $\int x^7 \text{FresnelC}(bx) dx$

Optimal. Leaf size=124

$$-\frac{105 \text{FresnelC}(bx)}{8\pi^4 b^8} - \frac{x^7 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi b} + \frac{35x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^3 b^5} - \frac{7x^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^2 b^3} + \frac{105x \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^4 b^7} + \frac{1}{8}x^8 \text{FresnelC}(bx)$$

[Out] $(105*x*\text{Cos}[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*\text{Cos}[(b^2*Pi*x^2)/2])/(8*b^3*Pi^2) - (105*\text{FresnelC}[b*x])/(8*b^8*Pi^4) + (x^8*\text{FresnelC}[b*x])/8 + (35*x^3*\text{Sin}[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) - (x^7*\text{Sin}[(b^2*Pi*x^2)/2])/(8*b*Pi)$

Rubi [A] time = 0.0821382, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3386, 3385, 3352}

$$-\frac{105 \text{FresnelC}(bx)}{8\pi^4 b^8} - \frac{x^7 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi b} + \frac{35x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^3 b^5} - \frac{7x^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^2 b^3} + \frac{105x \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^4 b^7} + \frac{1}{8}x^8 \text{FresnelC}(bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^7*\text{FresnelC}[b*x], x]$

[Out] $(105*x*\text{Cos}[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*\text{Cos}[(b^2*Pi*x^2)/2])/(8*b^3*Pi^2) - (105*\text{FresnelC}[b*x])/(8*b^8*Pi^4) + (x^8*\text{FresnelC}[b*x])/8 + (35*x^3*\text{Sin}[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) - (x^7*\text{Sin}[(b^2*Pi*x^2)/2])/(8*b*Pi)$

Rule 6427

$\text{Int}[\text{FresnelC}[(b_)*(x_)]*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[\frac{(d*x)^{(m+1)}*\text{FresnelC}[b*x]}{(d*(m+1)), x] - \text{Dist}[b/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*\text{Cos}[(Pi*b^2*x^2)/2], x], x] /; \text{FreeQ}\{b, d, m, x\} \ \&\& \ \text{NeQ}[m, -1]$

Rule 3386

$\text{Int}[\text{Cos}[(c_)+(d_)*(x_)]^{(n_)}*((e_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[\frac{(e*x)^{(m-n+1)}*\text{Sin}[c+d*x^n]}{(d*n), x] - \text{Dist}[\frac{(e^n*(m-n+1))}{(d*n)}, \text{Int}[(e*x)^{(m-n)}*\text{Sin}[c+d*x^n], x], x] /; \text{FreeQ}\{c, d, e, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[n, m+1]$

Rule 3385

```
Int[((e_.)*(x_)^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_.)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_)^2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
 \int x^7 C(bx) dx &= \frac{1}{8} x^8 C(bx) - \frac{1}{8} b \int x^8 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
 &= \frac{1}{8} x^8 C(bx) - \frac{x^7 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b\pi} + \frac{7 \int x^6 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{8b\pi} \\
 &= -\frac{7x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^3 \pi^2} + \frac{1}{8} x^8 C(bx) - \frac{x^7 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b\pi} + \frac{35 \int x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{8b^3 \pi^2} \\
 &= -\frac{7x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^3 \pi^2} + \frac{1}{8} x^8 C(bx) + \frac{35x^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^5 \pi^3} - \frac{x^7 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b\pi} - \frac{105 \int x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{8b^5 \pi^3} \\
 &= \frac{105x \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^7 \pi^4} - \frac{7x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^3 \pi^2} + \frac{1}{8} x^8 C(bx) + \frac{35x^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^5 \pi^3} - \frac{x^7 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b\pi} - \frac{105 \int \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{8b^5 \pi^3} \\
 &= \frac{105x \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^7 \pi^4} - \frac{7x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^3 \pi^2} - \frac{105C(bx)}{8b^8 \pi^4} + \frac{1}{8} x^8 C(bx) + \frac{35x^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b^5 \pi^3} - \frac{x^7 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{8b\pi}
 \end{aligned}$$

Mathematica [A] time = 0.0712275, size = 89, normalized size = 0.72

$$\frac{(\pi^4 b^8 x^8 - 105) \text{FresnelC}(bx) + \pi b^3 x^3 (35 - \pi^2 b^4 x^4) \sin\left(\frac{1}{2} \pi b^2 x^2\right) - 7bx (\pi^2 b^4 x^4 - 15) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{8\pi^4 b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelC[b*x],x]
```

```
[Out] (-7*b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*FresnelC[b*x] + b^3*Pi*x^3*(35 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*Pi^4)
```

Maple [A] time = 0.049, size = 123, normalized size = 1.

$$\frac{1}{b^8} \left(\frac{\text{FresnelC}(bx) b^8 x^8}{8} - \frac{b^7 x^7}{8\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) + \frac{7}{8\pi} \left(-\frac{b^5 x^5}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + 5 \frac{1}{\pi} \left(\frac{x^3 b^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} - 3 \frac{1}{\pi} \left(-\frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*FresnelC(b*x), x)

[Out] 1/b^8*(1/8*FresnelC(b*x)*b^8*x^8-1/8/Pi*b^7*x^7*sin(1/2*b^2*Pi*x^2)+7/8/Pi*(-1/Pi*b^5*x^5*cos(1/2*b^2*Pi*x^2)+5/Pi*(1/Pi*b^3*x^3*sin(1/2*b^2*Pi*x^2)-3/Pi*(-1/Pi*b*x*cos(1/2*b^2*Pi*x^2)+1/Pi*FresnelC(b*x))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnelc(b*x), x, algorithm="maxima")

[Out] integrate(x^7*fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^7 \text{fresnelc}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnelc(b*x), x, algorithm="fricas")

[Out] integral(x^7*fresnelc(b*x), x)

Sympy [A] time = 2.60003, size = 184, normalized size = 1.48

$$\frac{45x^8 C(bx) \Gamma\left(\frac{1}{4}\right)}{512\Gamma\left(\frac{13}{4}\right)} - \frac{45x^7 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{512\pi b \Gamma\left(\frac{13}{4}\right)} - \frac{315x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{512\pi^2 b^3 \Gamma\left(\frac{13}{4}\right)} + \frac{1575x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{512\pi^3 b^5 \Gamma\left(\frac{13}{4}\right)} + \frac{4725x \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{512\pi^4 b^7 \Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*fresnelc(b*x),x)

[Out] 45*x**8*fresnelc(b*x)*gamma(1/4)/(512*gamma(13/4)) - 45*x**7*sin(pi*b**2*x**2/2)*gamma(1/4)/(512*pi*b*gamma(13/4)) - 315*x**5*cos(pi*b**2*x**2/2)*gamma(1/4)/(512*pi**2*b**3*gamma(13/4)) + 1575*x**3*sin(pi*b**2*x**2/2)*gamma(1/4)/(512*pi**3*b**5*gamma(13/4)) + 4725*x*cos(pi*b**2*x**2/2)*gamma(1/4)/(512*pi**4*b**7*gamma(13/4)) - 4725*fresnelc(b*x)*gamma(1/4)/(512*pi**4*b**8*gamma(13/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnelc(b*x),x, algorithm="giac")

[Out] integrate(x^7*fresnelc(b*x), x)

3.111 $\int x^6 \text{FresnelC}(bx) dx$

Optimal. Leaf size=109

$$-\frac{x^6 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi b} + \frac{24x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^3 b^5} - \frac{6x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^2 b^3} + \frac{48 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^4 b^7} + \frac{1}{7}x^7 \text{FresnelC}(bx)$$

[Out] (48*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x^4*Cos[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2) + (x^7*FresnelC[b*x])/7 + (24*x^2*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) - (x^6*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi)

Rubi [A] time = 0.114325, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3380, 3296, 2638}

$$-\frac{x^6 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi b} + \frac{24x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^3 b^5} - \frac{6x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^2 b^3} + \frac{48 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^4 b^7} + \frac{1}{7}x^7 \text{FresnelC}(bx)$$

Antiderivative was successfully verified.

[In] Int[x^6*FresnelC[b*x], x]

[Out] (48*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x^4*Cos[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2) + (x^7*FresnelC[b*x])/7 + (24*x^2*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) - (x^6*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi)

Rule 6427

Int[FresnelC[(b_.)*(x_.)]*((d_.)*(x_.))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^6 C(bx) dx &= \frac{1}{7} x^7 C(bx) - \frac{1}{7} b \int x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
&= \frac{1}{7} x^7 C(bx) - \frac{1}{14} b \operatorname{Subst}\left(\int x^3 \cos\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right) \\
&= \frac{1}{7} x^7 C(bx) - \frac{x^6 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} + \frac{3 \operatorname{Subst}\left(\int x^2 \sin\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{7b\pi} \\
&= -\frac{6x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^3 \pi^2} + \frac{1}{7} x^7 C(bx) - \frac{x^6 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} + \frac{12 \operatorname{Subst}\left(\int x \cos\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{7b^3 \pi^2} \\
&= -\frac{6x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^3 \pi^2} + \frac{1}{7} x^7 C(bx) + \frac{24x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^5 \pi^3} - \frac{x^6 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} - \frac{24 \operatorname{Subst}\left(\int \sin\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{7b^5 \pi^3} \\
&= \frac{48 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^7 \pi^4} - \frac{6x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^3 \pi^2} + \frac{1}{7} x^7 C(bx) + \frac{24x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^5 \pi^3} - \frac{x^6 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi}
\end{aligned}$$

Mathematica [A] time = 0.0559025, size = 83, normalized size = 0.76

$$-\frac{x^2 (\pi^2 b^4 x^4 - 24) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{7\pi^3 b^5} - \frac{6 (\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{7\pi^4 b^7} + \frac{1}{7} x^7 \operatorname{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelC[b*x], x]
```

```
[Out] (-6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) + (x^7*FresnelC[b
*x])/7 - (x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3)
```

Maple [A] time = 0.049, size = 107, normalized size = 1.

$$\frac{1}{b^7} \left(\frac{b^7 x^7 \text{FresnelC}(bx)}{7} - \frac{b^6 x^6}{7\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) + \frac{6}{7\pi} \left(-\frac{x^4 b^4}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + 4 \frac{1}{\pi} \left(\frac{b^2 x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 2 \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*FresnelC(b*x), x)

[Out] 1/b^7*(1/7*b^7*x^7*FresnelC(b*x)-1/7/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)+6/7/Pi*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*fresnelc(b*x), x, algorithm="maxima")

[Out] integrate(x^6*fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^6 \text{fresnelc}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*fresnelc(b*x), x, algorithm="fricas")

[Out] integral(x^6*fresnelc(b*x), x)

Sympy [A] time = 2.36106, size = 153, normalized size = 1.4

$$\frac{x^7 C(bx) \Gamma\left(\frac{1}{4}\right)}{28 \Gamma\left(\frac{5}{4}\right)} - \frac{x^6 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{28 \pi b \Gamma\left(\frac{5}{4}\right)} - \frac{3 x^4 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{14 \pi^2 b^3 \Gamma\left(\frac{5}{4}\right)} + \frac{6 x^2 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{7 \pi^3 b^5 \Gamma\left(\frac{5}{4}\right)} + \frac{12 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{7 \pi^4 b^7 \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnelc(b*x),x)
```

```
[Out] x**7*fresnelc(b*x)*gamma(1/4)/(28*gamma(5/4)) - x**6*sin(pi*b**2*x**2/2)*gamma(1/4)/(28*pi*b*gamma(5/4)) - 3*x**4*cos(pi*b**2*x**2/2)*gamma(1/4)/(14*pi**2*b**3*gamma(5/4)) + 6*x**2*sin(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**3*b**5*gamma(5/4)) + 12*cos(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**4*b**7*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnelc(b*x), x)
```

3.112 $\int x^5 \text{FresnelC}(bx) dx$

Optimal. Leaf size=99

$$-\frac{5S(bx)}{2\pi^3b^6} - \frac{x^5 \sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b} + \frac{5x \sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5} - \frac{5x^3 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2b^3} + \frac{1}{6}x^6 \text{FresnelC}(bx)$$

[Out] $(-5*x^3*\text{Cos}[(b^2*Pi*x^2)/2])/(6*b^3*Pi^2) + (x^6*\text{FresnelC}[b*x])/6 - (5*\text{FresnelS}[b*x])/(2*b^6*Pi^3) + (5*x*\text{Sin}[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) - (x^5*\text{Sin}[(b^2*Pi*x^2)/2])/(6*b*Pi)$

Rubi [A] time = 0.0624975, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3386, 3385, 3351}

$$-\frac{5S(bx)}{2\pi^3b^6} - \frac{x^5 \sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b} + \frac{5x \sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5} - \frac{5x^3 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2b^3} + \frac{1}{6}x^6 \text{FresnelC}(bx)$$

Antiderivative was successfully verified.

[In] Int[x^5*FresnelC[b*x],x]

[Out] $(-5*x^3*\text{Cos}[(b^2*Pi*x^2)/2])/(6*b^3*Pi^2) + (x^6*\text{FresnelC}[b*x])/6 - (5*\text{FresnelS}[b*x])/(2*b^6*Pi^3) + (5*x*\text{Sin}[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) - (x^5*\text{Sin}[(b^2*Pi*x^2)/2])/(6*b*Pi)$

Rule 6427

Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*FresnelC[b*x])/(d*(m+1)), x] - Dist[b/(d*(m+1)), Int[(d*x)^(m+1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3386

Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*Sin[c+d*x^n])/(d*n), x] - Dist[(e^n*(m-n+1))/(d*n), Int[(e*x)^(m-n)*Sin[c+d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m+1]

Rule 3385

```
Int[((e_.)*(x_)^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_.)], x_Symbol] :> -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_)^2)], x_Symbol] :> Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
 \int x^5 C(bx) dx &= \frac{1}{6} x^6 C(bx) - \frac{1}{6} b \int x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
 &= \frac{1}{6} x^6 C(bx) - \frac{x^5 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{6b\pi} + \frac{5 \int x^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{6b\pi} \\
 &= -\frac{5x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{6b^3 \pi^2} + \frac{1}{6} x^6 C(bx) - \frac{x^5 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{6b\pi} + \frac{5 \int x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{2b^3 \pi^2} \\
 &= -\frac{5x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{6b^3 \pi^2} + \frac{1}{6} x^6 C(bx) + \frac{5x \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{2b^5 \pi^3} - \frac{x^5 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{6b\pi} - \frac{5 \int \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{2b^5 \pi^3} \\
 &= -\frac{5x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{6b^3 \pi^2} + \frac{1}{6} x^6 C(bx) - \frac{5S(bx)}{2b^6 \pi^3} + \frac{5x \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{2b^5 \pi^3} - \frac{x^5 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{6b\pi}
 \end{aligned}$$

Mathematica [A] time = 0.0634354, size = 80, normalized size = 0.81

$$\frac{\pi^3 b^6 x^6 \text{FresnelC}(bx) + bx \left(15 - \pi^2 b^4 x^4\right) \sin\left(\frac{1}{2} \pi b^2 x^2\right) - 5\pi b^3 x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) - 15S(bx)}{6\pi^3 b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelC[b*x], x]
```

```
[Out] (-5*b^3*Pi*x^3*Cos[(b^2*Pi*x^2)/2] + b^6*Pi^3*x^6*FresnelC[b*x] - 15*FresnelS[b*x] + b*x*(15 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```


Maple [A] time = 0.048, size = 97, normalized size = 1.

$$\frac{1}{b^6} \left(\frac{b^6 x^6 \text{FresnelC}(bx)}{6} - \frac{b^5 x^5}{6\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) + \frac{5}{6\pi} \left(-\frac{x^3 b^3}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + 3 \frac{1}{\pi} \left(\frac{bx \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} - \frac{\text{FresnelS}(bx)}{\pi} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*FresnelC(b*x),x)

[Out] 1/b^6*(1/6*b^6*x^6*FresnelC(b*x)-1/6/Pi*b^5*x^5*sin(1/2*b^2*Pi*x^2)+5/6/Pi*(-1/Pi*b^3*x^3*cos(1/2*b^2*Pi*x^2)+3/Pi*(1/Pi*b*x*sin(1/2*b^2*Pi*x^2)-1/Pi*FresnelS(b*x))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*fresnelc(b*x),x, algorithm="maxima")

[Out] integrate(x^5*fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^5 \text{fresnelc}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*fresnelc(b*x),x, algorithm="fricas")

[Out] integral(x^5*fresnelc(b*x), x)

Sympy [A] time = 1.12886, size = 49, normalized size = 0.49

$$\frac{bx^7 \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{7}{4}\right) {}_2F_3\left(\begin{matrix} \frac{1}{4}, \frac{7}{4} \\ \frac{1}{2}, \frac{5}{4}, \frac{11}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16 \Gamma\left(\frac{5}{4}\right) \Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*fresnelc(b*x),x)

[Out] b*x**7*gamma(1/4)*gamma(7/4)*hyper((1/4, 7/4), (1/2, 5/4, 11/4), -pi**2*b**4*x**4/16)/(16*gamma(5/4)*gamma(11/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*fresnelc(b*x),x, algorithm="giac")

[Out] integrate(x^5*fresnelc(b*x), x)

3.113 $\int x^4 \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=84

$$-\frac{x^4 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi b} + \frac{8 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5} - \frac{4x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3} + \frac{1}{5}x^5 \mathbf{FresnelC}(bx)$$

[Out] $(-4*x^2*\text{Cos}[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*\text{FresnelC}[b*x])/5 + (8*\text{Sin}[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) - (x^4*\text{Sin}[(b^2*Pi*x^2)/2])/(5*b*Pi)$

Rubi [A] time = 0.0766497, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3380, 3296, 2637}

$$-\frac{x^4 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi b} + \frac{8 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5} - \frac{4x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3} + \frac{1}{5}x^5 \mathbf{FresnelC}(bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4*\text{FresnelC}[b*x], x]$

[Out] $(-4*x^2*\text{Cos}[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*\text{FresnelC}[b*x])/5 + (8*\text{Sin}[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) - (x^4*\text{Sin}[(b^2*Pi*x^2)/2])/(5*b*Pi)$

Rule 6427

$\text{Int}[\text{FresnelC}[(b_)*(x_)]*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[\frac{(d*x)^{(m+1)}*\text{FresnelC}[b*x]}{d*(m+1)}, x] - \text{Dist}[b/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*\text{Cos}[(Pi*b^2*x^2)/2], x], x] /; \text{FreeQ}\{b, d, m\}, x \} \ \&\& \ \text{NeQ}[m, -1]$

Rule 3380

$\text{Int}[(a_)+\text{Cos}[(c_)+(d_)*(x_)]^{(n_)}*(b_)]^{(p_)}*(x_)^{(m_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n]-1)*(a+b*\text{Cos}[c+dx])^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x \} \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{EqQ}[m, n-1] \ || \ (\text{IntegerQ}[p] \ \&\& \ \text{GtQ}[\text{Simplify}[(m+1)/n], 0]))$

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^4 C(bx) dx &= \frac{1}{5} x^5 C(bx) - \frac{1}{5} b \int x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
&= \frac{1}{5} x^5 C(bx) - \frac{1}{10} b \operatorname{Subst}\left(\int x^2 \cos\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right) \\
&= \frac{1}{5} x^5 C(bx) - \frac{x^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b\pi} + \frac{2 \operatorname{Subst}\left(\int x \sin\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{5b\pi} \\
&= -\frac{4x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{5b^3 \pi^2} + \frac{1}{5} x^5 C(bx) - \frac{x^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b\pi} + \frac{4 \operatorname{Subst}\left(\int \cos\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{5b^3 \pi^2} \\
&= -\frac{4x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{5b^3 \pi^2} + \frac{1}{5} x^5 C(bx) + \frac{8 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b^5 \pi^3} - \frac{x^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b\pi}
\end{aligned}$$

Mathematica [A] time = 0.0434825, size = 71, normalized size = 0.85

$$-\frac{(\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{5\pi^3 b^5} - \frac{4x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{5\pi^2 b^3} + \frac{1}{5} x^5 \operatorname{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelC[b*x], x]
```

```
[Out] (-4*x^2*Cos[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x])/5 - ((-8 +
b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3)
```

Maple [A] time = 0.046, size = 81, normalized size = 1.

$$\frac{1}{b^5} \left(\frac{b^5 x^5 \text{FresnelC}(bx)}{5} - \frac{x^4 b^4}{5\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) + \frac{4}{5\pi} \left(-\frac{b^2 x^2}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + 2 \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*FresnelC(b*x),x)

[Out] 1/b^5*(1/5*b^5*x^5*FresnelC(b*x)-1/5/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)+4/5/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*fresnelc(b*x),x, algorithm="maxima")

[Out] integrate(x^4*fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^4 \text{fresnelc}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*fresnelc(b*x),x, algorithm="fricas")

[Out] integral(x^4*fresnelc(b*x), x)

Sympy [A] time = 1.223, size = 116, normalized size = 1.38

$$\frac{x^5 C(bx) \Gamma\left(\frac{1}{4}\right)}{20\Gamma\left(\frac{5}{4}\right)} - \frac{x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{20\pi b \Gamma\left(\frac{5}{4}\right)} - \frac{x^2 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{5\pi^2 b^3 \Gamma\left(\frac{5}{4}\right)} + \frac{2 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{5\pi^3 b^5 \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnelc(b*x),x)
```

```
[Out] x**5*fresnelc(b*x)*gamma(1/4)/(20*gamma(5/4)) - x**4*sin(pi*b**2*x**2/2)*gamma(1/4)/(20*pi*b*gamma(5/4)) - x**2*cos(pi*b**2*x**2/2)*gamma(1/4)/(5*pi**2*b**3*gamma(5/4)) + 2*sin(pi*b**2*x**2/2)*gamma(1/4)/(5*pi**3*b**5*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnelc(b*x), x)
```

3.114 $\int x^3 \text{FresnelC}(bx) dx$

Optimal. Leaf size=74

$$\frac{3\text{FresnelC}(bx)}{4\pi^2 b^4} - \frac{x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} - \frac{3x \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{1}{4}x^4 \text{FresnelC}(bx)$$

[Out] $(-3*x*\text{Cos}[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*\text{FresnelC}[b*x])/(4*b^4*Pi^2) + (x^4*\text{FresnelC}[b*x])/4 - (x^3*\text{Sin}[(b^2*Pi*x^2)/2])/(4*b*Pi)$

Rubi [A] time = 0.0429744, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3386, 3385, 3352}

$$\frac{3\text{FresnelC}(bx)}{4\pi^2 b^4} - \frac{x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} - \frac{3x \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{1}{4}x^4 \text{FresnelC}(bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*\text{FresnelC}[b*x], x]$

[Out] $(-3*x*\text{Cos}[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*\text{FresnelC}[b*x])/(4*b^4*Pi^2) + (x^4*\text{FresnelC}[b*x])/4 - (x^3*\text{Sin}[(b^2*Pi*x^2)/2])/(4*b*Pi)$

Rule 6427

$\text{Int}[\text{FresnelC}[(b_)*(x_)]*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[\frac{(d*x)^{(m+1)}*\text{FresnelC}[b*x]}{d*(m+1)}, x] - \text{Dist}[b/(d*(m+1)), \text{Int}[\frac{(d*x)^{(m+1)}*\text{Cos}[(Pi*b^2*x^2)/2]}{d}, x], x] /; \text{FreeQ}\{b, d, m, x\} \&\& \text{NeQ}[m, -1]$

Rule 3386

$\text{Int}[\text{Cos}[(c_)+(d_)*(x_)]^{(n_)}*((e_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[\frac{e^{(n-1)}*(e*x)^{(m-n+1)}*\text{Sin}[c+d*x^n]}{d*n}, x] - \text{Dist}[\frac{e^n*(m-n+1)}{d*n}, \text{Int}[\frac{e*x^{(m-n)}*\text{Sin}[c+d*x^n]}{d}, x], x] /; \text{FreeQ}\{c, d, e, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[n, m+1]$

Rule 3385

$\text{Int}[\frac{(e_)*(x_)]^{(m_)}*\text{Sin}[(c_)+(d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[\frac{e^{(n-1)}*(e*x)^{(m-n+1)}*\text{Cos}[c+d*x^n]}{d*n}, x] + \text{Dist}[\frac{e^n*(m-n+1)}{d*n}, \text{Int}[\frac{e*x^{(m-n)}*\text{Sin}[c+d*x^n]}{d}, x], x]$

)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}\int x^3 C(bx) dx &= \frac{1}{4} x^4 C(bx) - \frac{1}{4} b \int x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx \\ &= \frac{1}{4} x^4 C(bx) - \frac{x^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{4b\pi} + \frac{3 \int x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{4b\pi} \\ &= -\frac{3x \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{4b^3 \pi^2} + \frac{1}{4} x^4 C(bx) - \frac{x^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{4b\pi} + \frac{3 \int \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{4b^3 \pi^2} \\ &= -\frac{3x \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{4b^3 \pi^2} + \frac{3C(bx)}{4b^4 \pi^2} + \frac{1}{4} x^4 C(bx) - \frac{x^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{4b\pi}\end{aligned}$$

Mathematica [A] time = 0.016423, size = 74, normalized size = 1.

$$\frac{3\text{FresnelC}(bx)}{4\pi^2 b^4} - \frac{x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{4\pi b} - \frac{3x \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{1}{4} x^4 \text{FresnelC}(bx)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*FresnelC[b*x], x]

[Out] (-3*x*Cos[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*FresnelC[b*x])/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x])/4 - (x^3*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi)

Maple [A] time = 0.048, size = 70, normalized size = 1.

$$\frac{1}{b^4} \left(\frac{b^4 x^4 \text{FresnelC}(bx)}{4} - \frac{x^3 b^3}{4\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) + \frac{3}{4\pi} \left(-\frac{bx}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + \frac{\text{FresnelC}(bx)}{\pi} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*FresnelC(b*x),x)`

[Out] $\frac{1}{b^4} \left(\frac{1}{4} b^4 x^4 \text{FresnelC}(bx) - \frac{1}{4} \pi b^3 x^3 \sin\left(\frac{1}{2} b^2 \pi x^2\right) + \frac{3}{4} \pi \left(-\frac{1}{\pi} b x \cos\left(\frac{1}{2} b^2 \pi x^2\right) + \frac{1}{\pi} \text{FresnelC}(bx) \right) \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*fresnelc(b*x),x, algorithm="maxima")`

[Out] `integrate(x^3*fresnelc(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*fresnelc(b*x),x, algorithm="fricas")`

[Out] `integral(x^3*fresnelc(b*x), x)`

Sympy [A] time = 1.23752, size = 112, normalized size = 1.51

$$\frac{5x^4 C(bx) \Gamma\left(\frac{1}{4}\right)}{64 \Gamma\left(\frac{9}{4}\right)} - \frac{5x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{64 \pi b \Gamma\left(\frac{9}{4}\right)} - \frac{15x \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{64 \pi^2 b^3 \Gamma\left(\frac{9}{4}\right)} + \frac{15 C(bx) \Gamma\left(\frac{1}{4}\right)}{64 \pi^2 b^4 \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*fresnelc(b*x),x)`

```
[Out] 5*x**4*fresnelc(b*x)*gamma(1/4)/(64*gamma(9/4)) - 5*x**3*sin(pi*b**2*x**2/2)
*gamma(1/4)/(64*pi*b*gamma(9/4)) - 15*x*cos(pi*b**2*x**2/2)*gamma(1/4)/(64
*pi**2*b**3*gamma(9/4)) + 15*fresnelc(b*x)*gamma(1/4)/(64*pi**2*b**4*gamma(
9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnelc(b*x), x)
```

3.115 $\int x^2 \text{FresnelC}(bx) dx$

Optimal. Leaf size=59

$$-\frac{x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi b} - \frac{2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi^2 b^3} + \frac{1}{3}x^3 \text{FresnelC}(bx)$$

[Out] $(-2*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(3*b^3*\text{Pi}^2) + (x^3*\text{FresnelC}[b*x])/3 - (x^2*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(3*b*\text{Pi})$

Rubi [A] time = 0.0543763, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3380, 3296, 2638}

$$-\frac{x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi b} - \frac{2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi^2 b^3} + \frac{1}{3}x^3 \text{FresnelC}(bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*\text{FresnelC}[b*x], x]$

[Out] $(-2*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(3*b^3*\text{Pi}^2) + (x^3*\text{FresnelC}[b*x])/3 - (x^2*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(3*b*\text{Pi})$

Rule 6427

$\text{Int}[\text{FresnelC}[(b_)*(x_)]*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[\frac{(d*x)^{(m+1)}*\text{FresnelC}[b*x]}{d*(m+1)}, x] - \text{Dist}[b/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*\text{Cos}[(\text{Pi}*b^2*x^2)/2], x], x] /; \text{FreeQ}\{b, d, m\}, x \} \ \&\& \ \text{NeQ}[m, -1]$

Rule 3380

$\text{Int}[(a_ + \text{Cos}[c_ + (d_)*(x_)^{(n_)]*(b_)]^{(p_)}*(x_)^{(m_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Cos}[c + d*x])^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x \} \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{EqQ}[m, n-1] \ || \ (\text{IntegerQ}[p] \ \&\& \ \text{GtQ}[\text{Simplify}[(m+1)/n], 0]))$

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 C(bx) dx &= \frac{1}{3} x^3 C(bx) - \frac{1}{3} b \int x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx \\
&= \frac{1}{3} x^3 C(bx) - \frac{1}{6} b \operatorname{Subst}\left(\int x \cos\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right) \\
&= \frac{1}{3} x^3 C(bx) - \frac{x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi} + \frac{\operatorname{Subst}\left(\int \sin\left(\frac{1}{2} b^2 \pi x\right) dx, x, x^2\right)}{3b\pi} \\
&= -\frac{2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{3b^3 \pi^2} + \frac{1}{3} x^3 C(bx) - \frac{x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi}
\end{aligned}$$

Mathematica [A] time = 0.0136692, size = 59, normalized size = 1.

$$-\frac{x^2 \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{3\pi b} - \frac{2 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3\pi^2 b^3} + \frac{1}{3} x^3 \operatorname{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[b*x], x]
```

```
[Out] (-2*Cos[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x])/3 - (x^2*Sin[(b
^2*Pi*x^2)/2])/(3*b*Pi)
```

Maple [A] time = 0.05, size = 54, normalized size = 0.9

$$\frac{1}{b^3} \left(\frac{b^3 x^3 \operatorname{FresnelC}(bx)}{3} - \frac{b^2 x^2}{3\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) - \frac{2}{3\pi^2} \cos\left(\frac{b^2 \pi x^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*FresnelC(b*x),x)`

[Out] $1/b^3*(1/3*b^3*x^3*FresnelC(b*x)-1/3/Pi*b^2*x^2*\sin(1/2*b^2*Pi*x^2)-2/3/Pi^2*\cos(1/2*b^2*Pi*x^2))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnelc(b*x),x, algorithm="maxima")`

[Out] `integrate(x^2*fresnelc(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \text{fresnelc}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnelc(b*x),x, algorithm="fricas")`

[Out] `integral(x^2*fresnelc(b*x), x)`

Sympy [A] time = 1.03035, size = 80, normalized size = 1.36

$$\frac{x^3 C(bx) \Gamma\left(\frac{1}{4}\right)}{12 \Gamma\left(\frac{5}{4}\right)} - \frac{x^2 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{12 \pi b \Gamma\left(\frac{5}{4}\right)} - \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{6 \pi^2 b^3 \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*fresnelc(b*x),x)`

```
[Out] x**3*fresnelc(b*x)*gamma(1/4)/(12*gamma(5/4)) - x**2*sin(pi*b**2*x**2/2)*gamma(1/4)/(12*pi*b*gamma(5/4)) - cos(pi*b**2*x**2/2)*gamma(1/4)/(6*pi**2*b**3*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc(b*x), x)
```

3.116 $\int x \text{FresnelC}(bx) dx$

Optimal. Leaf size=49

$$\frac{S(bx)}{2\pi b^2} - \frac{x \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b} + \frac{1}{2}x^2 \text{FresnelC}(bx)$$

[Out] $(x^2 \text{FresnelC}[b*x])/2 + \text{FresnelS}[b*x]/(2*b^2*\text{Pi}) - (x*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/ (2*b*\text{Pi})$

Rubi [A] time = 0.0258736, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3386, 3351}

$$\frac{S(bx)}{2\pi b^2} - \frac{x \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b} + \frac{1}{2}x^2 \text{FresnelC}(bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{FresnelC}[b*x], x]$

[Out] $(x^2*\text{FresnelC}[b*x])/2 + \text{FresnelS}[b*x]/(2*b^2*\text{Pi}) - (x*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/ (2*b*\text{Pi})$

Rule 6427

$\text{Int}[\text{FresnelC}[(b_.)*(x_.)]*((d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[\frac{(d*x)^{(m+1)}*\text{FresnelC}[b*x]}{d*(m+1)}, x] - \text{Dist}[b/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*\text{Cos}[(\text{Pi}*b^2*x^2)/2], x], x] /; \text{FreeQ}\{b, d, m\}, x \ \&\& \ \text{NeQ}[m, -1]$

Rule 3386

$\text{Int}[\text{Cos}[(c_.) + (d_.)*(x_.)^{(n_.)]*((e_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(e^{(n-1)}*(e*x)^{(m-n+1)}*\text{Sin}[c + d*x^n])/d^n, x] - \text{Dist}[(e^n*(m-n+1))/d^n, \text{Int}[(e*x)^{(m-n)}*\text{Sin}[c + d*x^n], x], x] /; \text{FreeQ}\{c, d, e\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[n, m+1]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned}
\int xC(bx) dx &= \frac{1}{2}x^2C(bx) - \frac{1}{2}b \int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) dx \\
&= \frac{1}{2}x^2C(bx) - \frac{x \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{\int \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{2b\pi} \\
&= \frac{1}{2}x^2C(bx) + \frac{S(bx)}{2b^2\pi} - \frac{x \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}
\end{aligned}$$

Mathematica [A] time = 0.0117981, size = 49, normalized size = 1.

$$\frac{S(bx)}{2\pi b^2} - \frac{x \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b} + \frac{1}{2}x^2 \text{FresnelC}(bx)$$

Antiderivative was successfully verified.

[In] Integrate[x*FresnelC[b*x],x]

[Out] (x^2*FresnelC[b*x])/2 + FresnelS[b*x]/(2*b^2*Pi) - (x*Sin[(b^2*Pi*x^2)/2])/(2*b*Pi)

Maple [A] time = 0.048, size = 44, normalized size = 0.9

$$\frac{1}{b^2} \left(\frac{b^2 x^2 \text{FresnelC}(bx)}{2} - \frac{bx}{2\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) + \frac{\text{FresnelS}(bx)}{2\pi} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*FresnelC(b*x),x)

[Out] 1/b^2*(1/2*b^2*x^2*FresnelC(b*x)-1/2/Pi*b*x*sin(1/2*b^2*Pi*x^2)+1/2/Pi*FresnelS(b*x))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{xfresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(b*x),x, algorithm="maxima")

[Out] integrate(x*fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

integral(xfresnelc(bx),x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(b*x),x, algorithm="fricas")

[Out] integral(x*fresnelc(b*x), x)

Sympy [A] time = 0.525939, size = 49, normalized size = 1.

$$\frac{bx^3\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{5}{4}, \frac{7}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(b*x),x)

[Out] b*x**3*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -pi**2*b**4*x**4/16)/(16*gamma(5/4)*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{xfresnelc}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*fresnelc(b*x), x)
```

3.117 $\int \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=27

$$x\mathbf{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b}$$

[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)

Rubi [A] time = 0.0047574, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6419}

$$x\mathbf{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x], x]

[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)

Rule 6419

Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] := Simp[((a + b*x)*FresnelC[a + b*x])/b, x] - Simp[Sin[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int C(bx) dx = xC(bx) - \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

Mathematica [A] time = 0.0021411, size = 27, normalized size = 1.

$$x\mathbf{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x],x]

[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)

Maple [A] time = 0.051, size = 28, normalized size = 1.

$$\frac{1}{b} \left(bx \operatorname{FresnelC}(bx) - \frac{1}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x),x)

[Out] 1/b*(b*x*FresnelC(b*x)-1/Pi*sin(1/2*b^2*Pi*x^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x),x, algorithm="maxima")

[Out] integrate(fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(\operatorname{fresnelc}(bx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x),x, algorithm="fricas")

[Out] integral(fresnelc(b*x), x)

Sympy [B] time = 0.633405, size = 44, normalized size = 1.63

$$\frac{x C(bx) \Gamma\left(\frac{1}{4}\right)}{4 \Gamma\left(\frac{5}{4}\right)} - \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{4 \pi b \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x), x)

[Out] x*fresnelc(b*x)*gamma(1/4)/(4*gamma(5/4)) - sin(pi*b**2*x**2/2)*gamma(1/4)/(4*pi*b*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x), x, algorithm="giac")

[Out] integrate(fresnelc(b*x), x)

$$3.118 \quad \int \frac{\text{FresnelC}(bx)}{x} dx$$

Optimal. Leaf size=69

$$\frac{1}{2}bx \text{HypergeometricPFQ}\left(\left\{\frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{1}{2}i\pi b^2 x^2\right) + \frac{1}{2}bx \text{HypergeometricPFQ}\left(\left\{\frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, \frac{1}{2}i\pi b^2 x^2\right)$$

[Out] (b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-I/2)*b^2*Pi*x^2])/2 + (b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2])/2

Rubi [A] time = 0.0443547, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6425, 6358, 6360}

$$\frac{1}{2}bx {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{2}bx {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{1}{2}ib^2\pi x^2\right)$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]/x, x]

[Out] (b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-I/2)*b^2*Pi*x^2])/2 + (b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2])/2

Rule 6425

Int[FresnelC[(b_.)*(x_)]/(x_), x_Symbol] := Dist[(1 - I)/4, Int[Erf[(Sqrt[Pi]*(1 + I)*b*x)/2]/x, x], x] + Dist[(1 + I)/4, Int[Erf[(Sqrt[Pi]*(1 - I)*b*x)/2]/x, x], x] /; FreeQ[b, x]

Rule 6358

Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] := Simp[(2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)]/Sqrt[Pi], x] /; FreeQ[b, x]

Rule 6360

Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] := Simp[(2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2)]/Sqrt[Pi], x] /; FreeQ[b, x]

Rubi steps

$$\int \frac{C(bx)}{x} dx = \left(\frac{1}{4} - \frac{i}{4}\right) \int \frac{\operatorname{erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi x}\right)}{x} dx + \left(\frac{1}{4} - \frac{i}{4}\right) \int \frac{\operatorname{erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi x}\right)}{x} dx$$

$$= \frac{1}{2}bx {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{2}bx {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.0140928, size = 0, normalized size = 0.

$$\int \frac{\operatorname{FresnelC}(bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]/x,x]

[Out] Integrate[FresnelC[b*x]/x, x]

Maple [A] time = 0.073, size = 23, normalized size = 0.3

$$bx {}_2F_3\left(\frac{1}{4}, \frac{1}{4}; \frac{1}{2}, \frac{5}{4}, \frac{5}{4}; -\frac{x^4\pi^2b^4}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x,x)

[Out] b*x*hypergeom([1/4,1/4],[1/2,5/4,5/4],-1/16*x^4*Pi^2*b^4)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{fresnelc}(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)/x, x)

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x,x)

[Out] Exception raised: AttributeError

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)/x, x)

$$3.119 \quad \int \frac{\mathbf{FresnelC}(bx)}{x^2} dx$$

Optimal. Leaf size=27

$$\frac{1}{2}b\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right) - \frac{\text{FresnelC}(bx)}{x}$$

[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x

Rubi [A] time = 0.0212742, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6427, 3376}

$$\frac{1}{2}b\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right) - \frac{\text{FresnelC}(bx)}{x}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]/x^2,x]

[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x

Rule 6427

Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3376

Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[CosIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{C(bx)}{x^2} dx &= -\frac{C(bx)}{x} + b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx \\ &= \frac{1}{2}b\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{C(bx)}{x} \end{aligned}$$

Mathematica [A] time = 0.0108917, size = 27, normalized size = 1.

$$\frac{1}{2}b\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right) - \frac{\text{FresnelC}(bx)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x]/x^2,x]

[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x

Maple [A] time = 0.049, size = 28, normalized size = 1.

$$b\left(-\frac{\text{FresnelC}(bx)}{bx} + \frac{1}{2}\text{Ci}\left(\frac{b^2\pi x^2}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x^2,x)

[Out] b*(-FresnelC(b*x)/b/x+1/2*Ci(1/2*b^2*Pi*x^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^2,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)/x^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^2,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)/x^2, x)

Sympy [B] time = 1.13432, size = 53, normalized size = 1.96

$$-\frac{\pi^2 b^5 x^4 \Gamma\left(\frac{5}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{5}{4} \\ \frac{3}{2}, 2, 2, \frac{9}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{128 \Gamma\left(\frac{9}{4}\right)} + \frac{b \log(b^4 x^4)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x**2,x)

[Out] -pi**2*b**5*x**4*gamma(5/4)*hyper((1, 1, 5/4), (3/2, 2, 2, 9/4), -pi**2*b**4*x**4/16)/(128*gamma(9/4)) + b*log(b**4*x**4)/4

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^2,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)/x^2, x)

$$3.120 \quad \int \frac{\text{FresnelC}(bx)}{x^3} dx$$

Optimal. Leaf size=44

$$-\frac{1}{2}\pi b^2 S(bx) - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2}$$

[Out] $-(b*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(2*x) - \text{FresnelC}[b*x]/(2*x^2) - (b^2*\text{Pi}*\text{FresnelS}[b*x])/2$

Rubi [A] time = 0.0285715, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6427, 3388, 3351}

$$-\frac{1}{2}\pi b^2 S(bx) - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]/x^3,x]

[Out] $-(b*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(2*x) - \text{FresnelC}[b*x]/(2*x^2) - (b^2*\text{Pi}*\text{FresnelS}[b*x])/2$

Rule 6427

Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3388

Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] := Simp[((e*x)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)}{x^3} dx &= -\frac{C(bx)}{2x^2} + \frac{1}{2}b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
&= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{C(bx)}{2x^2} - \frac{1}{2}(b^3\pi) \int \sin\left(\frac{1}{2}b^2\pi x^2\right) dx \\
&= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{C(bx)}{2x^2} - \frac{1}{2}b^2\pi S(bx)
\end{aligned}$$

Mathematica [A] time = 0.0109718, size = 44, normalized size = 1.

$$-\frac{1}{2}\pi b^2 S(bx) - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x]/x^3,x]

[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(2*x) - FresnelC[b*x]/(2*x^2) - (b^2*Pi*FresnelS[b*x])/2

Maple [A] time = 0.048, size = 43, normalized size = 1.

$$b^2 \left(-\frac{\text{FresnelC}(bx)}{2b^2x^2} - \frac{1}{2bx} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi \text{FresnelS}(bx)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x^3,x)

[Out] b^2*(-1/2*FresnelC(b*x)/b^2/x^2-1/2/b/x*cos(1/2*b^2*Pi*x^2)-1/2*Pi*FresnelS(b*x))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^3,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)/x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^3,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)/x^3, x)

Sympy [A] time = 0.633295, size = 51, normalized size = 1.16

$$\frac{b\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ \frac{1}{2}, \frac{3}{4}, \frac{5}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16x\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x**3,x)

[Out] b*gamma(-1/4)*gamma(1/4)*hyper((-1/4, 1/4), (1/2, 3/4, 5/4), -pi**2*b**4*x**4/16)/(16*x*gamma(3/4)*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^3, x)
```

$$3.121 \quad \int \frac{\text{FresnelC}(bx)}{x^4} dx$$

Optimal. Leaf size=52

$$-\frac{1}{12}\pi b^3 \text{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^2} - \frac{\text{FresnelC}(bx)}{3x^3}$$

[Out] $-(b*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(6*x^2) - \text{FresnelC}[b*x]/(3*x^3) - (b^3*\text{Pi}*\text{SinIntegral}[(b^2*\text{Pi}*x^2)/2])/12$

Rubi [A] time = 0.0636539, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3380, 3297, 3299}

$$-\frac{1}{12}\pi b^3 \text{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^2} - \frac{\text{FresnelC}(bx)}{3x^3}$$

Antiderivative was successfully verified.

[In] `Int[FresnelC[b*x]/x^4,x]`

[Out] $-(b*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(6*x^2) - \text{FresnelC}[b*x]/(3*x^3) - (b^3*\text{Pi}*\text{SinIntegral}[(b^2*\text{Pi}*x^2)/2])/12$

Rule 6427

`Int[FresnelC[(b_.)*(x_.)]*((d_.)*(x_.))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]`

Rule 3380

`Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))`

Rule 3297


```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{C(bx)}{x^4} dx &= -\frac{C(bx)}{3x^3} + \frac{1}{3}b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx \\
 &= -\frac{C(bx)}{3x^3} + \frac{1}{6}b \operatorname{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2} dx, x, x^2\right) \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2} - \frac{C(bx)}{3x^3} - \frac{1}{12}(b^3\pi) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x} dx, x, x^2\right) \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2} - \frac{C(bx)}{3x^3} - \frac{1}{12}b^3\pi \operatorname{Si}\left(\frac{1}{2}b^2\pi x^2\right)
 \end{aligned}$$

Mathematica [A] time = 0.0152894, size = 52, normalized size = 1.

$$-\frac{1}{12}\pi b^3 \operatorname{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^2} - \frac{\operatorname{FresnelC}(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^4, x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(6*x^2) - FresnelC[b*x]/(3*x^3) - (b^3*Pi*SinIntegral[(b^2*Pi*x^2)/2])/12
```

Maple [A] time = 0.047, size = 49, normalized size = 0.9

$$b^3 \left(-\frac{\text{FresnelC}(bx)}{3x^3b^3} - \frac{1}{6b^2x^2} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{12} \text{Si}\left(\frac{b^2\pi x^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x^4,x)

[Out] b^3*(-1/3*FresnelC(b*x)/b^3/x^3-1/6/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/12*Pi*Si(1/2*b^2*Pi*x^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^4,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)/x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^4,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)/x^4, x)

Sympy [A] time = 0.734969, size = 42, normalized size = 0.81

$$\frac{b\Gamma\left(\frac{1}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2}, \frac{1}{2}, \frac{5}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16} \right)}{8x^2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x**4, x)

[Out] -b*gamma(1/4)*hyper((-1/2, 1/4), (1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(8*x**2*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^4,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)/x^4, x)

3.122 $\int \frac{\text{FresnelC}(bx)}{x^5} dx$

Optimal. Leaf size=69

$$-\frac{1}{12}\pi^2 b^4 \text{FresnelC}(bx) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3} - \frac{\text{FresnelC}(bx)}{4x^4}$$

[Out] $-(b \cos[(b^2 \pi x^2)/2])/(12 x^3) - (b^4 \pi^2 \text{FresnelC}[b x])/12 - \text{FresnelC}[b x]/(4 x^4) + (b^3 \pi \sin[(b^2 \pi x^2)/2])/(12 x)$

Rubi [A] time = 0.0411592, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3388, 3387, 3352}

$$-\frac{1}{12}\pi^2 b^4 \text{FresnelC}(bx) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3} - \frac{\text{FresnelC}(bx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]/x^5, x]

[Out] $-(b \cos[(b^2 \pi x^2)/2])/(12 x^3) - (b^4 \pi^2 \text{FresnelC}[b x])/12 - \text{FresnelC}[b x]/(4 x^4) + (b^3 \pi \sin[(b^2 \pi x^2)/2])/(12 x)$

Rule 6427

Int[FresnelC[(b_.)*(x_.)]*((d_.)*(x_.))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*FresnelC[b*x])/(d*(m+1)), x] - Dist[b/(d*(m+1)), Int[(d*x)^(m+1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3388

Int[Cos[(c_.) + (d_.)*(x_)^(n_.)]*((e_.)*(x_.))^(m_.), x_Symbol] := Simp[((e*x)^(m+1)*Cos[c + d*x^n])/(e*(m+1)), x] + Dist[(d*n)/(e^n*(m+1)), Int[(e*x)^(m+n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3387

Int[((e_.)*(x_.))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_.)], x_Symbol] := Simp[((e*x)^(m+1)*Sin[c + d*x^n])/(e*(m+1)), x] - Dist[(d*n)/(e^n*(m+1)), Int[(

$e^x)^{(m+n)} \cdot \text{Cos}[c + d \cdot x^n], x], x] /; \text{FreeQ}\{c, d, e\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[m, -1]$

Rule 3352

$\text{Int}[\text{Cos}[(d \cdot _) * ((e \cdot _) + (f \cdot _) * (x \cdot _))^{2}], x_Symbol] :> \text{Simp}[(\text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[2/\text{Pi}] * \text{Rt}[d, 2] * (e + f * x)]) / (f * \text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{C(bx)}{x^5} dx &= -\frac{C(bx)}{4x^4} + \frac{1}{4}b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx \\ &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{C(bx)}{4x^4} - \frac{1}{12}(b^3\pi) \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\ &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{C(bx)}{4x^4} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{1}{12}(b^5\pi^2) \int \cos\left(\frac{1}{2}b^2\pi x^2\right) dx \\ &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{1}{12}b^4\pi^2 C(bx) - \frac{C(bx)}{4x^4} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x} \end{aligned}$$

Mathematica [A] time = 0.0158539, size = 69, normalized size = 1.

$$-\frac{1}{12}\pi^2 b^4 \text{FresnelC}(bx) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3} - \frac{\text{FresnelC}(bx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x]/x^5,x]

[Out] $-(b \cdot \text{Cos}[(b^2 \cdot \text{Pi} \cdot x^2)/2]) / (12 \cdot x^3) - (b^4 \cdot \text{Pi}^2 \cdot \text{FresnelC}[b \cdot x]) / 12 - \text{FresnelC}[b \cdot x] / (4 \cdot x^4) + (b^3 \cdot \text{Pi} \cdot \text{Sin}[(b^2 \cdot \text{Pi} \cdot x^2)/2]) / (12 \cdot x)$

Maple [A] time = 0.051, size = 64, normalized size = 0.9

$$b^4 \left(-\frac{\text{FresnelC}(bx)}{4x^4 b^4} - \frac{1}{12x^3 b^3} \cos\left(\frac{b^2 \pi x^2}{2}\right) - \frac{\pi}{12} \left(-\frac{1}{bx} \sin\left(\frac{b^2 \pi x^2}{2}\right) + \pi \text{FresnelC}(bx) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(FresnelC(b*x)/x^5,x)`

[Out] $b^4 * (-1/4 * \text{FresnelC}(b*x) / b^4 / x^4 - 1/12 / b^3 / x^3 * \cos(1/2 * b^2 * \pi * x^2) - 1/12 * \pi * (-\sin(1/2 * b^2 * \pi * x^2) / b / x + \pi * \text{FresnelC}(b*x)))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)/x^5,x, algorithm="maxima")`

[Out] `integrate(fresnelc(b*x)/x^5, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)/x^5,x, algorithm="fricas")`

[Out] `integral(fresnelc(b*x)/x^5, x)`

Sympy [A] time = 1.12457, size = 110, normalized size = 1.59

$$\frac{\pi^2 b^4 C(bx) \Gamma\left(-\frac{3}{4}\right)}{64 \Gamma\left(\frac{5}{4}\right)} - \frac{\pi b^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{3}{4}\right)}{64 x \Gamma\left(\frac{5}{4}\right)} + \frac{b \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{3}{4}\right)}{64 x^3 \Gamma\left(\frac{5}{4}\right)} + \frac{3 C(bx) \Gamma\left(-\frac{3}{4}\right)}{64 x^4 \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**5,x)
```

```
[Out] pi**2*b**4*fresnelc(b*x)*gamma(-3/4)/(64*gamma(5/4)) - pi*b**3*sin(pi*b**2*
x**2/2)*gamma(-3/4)/(64*x*gamma(5/4)) + b*cos(pi*b**2*x**2/2)*gamma(-3/4)/(
64*x**3*gamma(5/4)) + 3*fresnelc(b*x)*gamma(-3/4)/(64*x**4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^5, x)
```

3.123 $\int \frac{\text{FresnelC}(bx)}{x^6} dx$

Optimal. Leaf size=77

$$-\frac{1}{80}\pi^2 b^5 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{40x^2} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{20x^4} - \frac{\text{FresnelC}(bx)}{5x^5}$$

[Out] $-(b \cdot \text{Cos}[(b^2 \cdot \text{Pi} \cdot x^2)/2])/(20 \cdot x^4) - (b^5 \cdot \text{Pi}^2 \cdot \text{CosIntegral}[(b^2 \cdot \text{Pi} \cdot x^2)/2])/80 - \text{FresnelC}[b \cdot x]/(5 \cdot x^5) + (b^3 \cdot \text{Pi} \cdot \text{Sin}[(b^2 \cdot \text{Pi} \cdot x^2)/2])/(40 \cdot x^2)$

Rubi [A] time = 0.0893477, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3380, 3297, 3302}

$$-\frac{1}{80}\pi^2 b^5 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{40x^2} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{20x^4} - \frac{\text{FresnelC}(bx)}{5x^5}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]/x^6,x]

[Out] $-(b \cdot \text{Cos}[(b^2 \cdot \text{Pi} \cdot x^2)/2])/(20 \cdot x^4) - (b^5 \cdot \text{Pi}^2 \cdot \text{CosIntegral}[(b^2 \cdot \text{Pi} \cdot x^2)/2])/80 - \text{FresnelC}[b \cdot x]/(5 \cdot x^5) + (b^3 \cdot \text{Pi} \cdot \text{Sin}[(b^2 \cdot \text{Pi} \cdot x^2)/2])/(40 \cdot x^2)$

Rule 6427

Int[FresnelC[(b_.)*(x_.)]*((d_.)*(x_.))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*FresnelC[b*x])/(d*(m+1)), x] - Dist[b/(d*(m+1)), Int[(d*x)^(m+1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]] && (EqQ[p, 1] || EqQ[m, n-1] || (IntegerQ[p] && GtQ[Simplify[(m+1)/n], 0]))

Rule 3297


```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{C(bx)}{x^6} dx &= -\frac{C(bx)}{5x^5} + \frac{1}{5}b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx \\
 &= -\frac{C(bx)}{5x^5} + \frac{1}{10}b \operatorname{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3} dx, x, x^2\right) \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{C(bx)}{5x^5} - \frac{1}{40}(b^3\pi) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2} dx, x, x^2\right) \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{C(bx)}{5x^5} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2} - \frac{1}{80}(b^5\pi^2) \operatorname{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x} dx, x, x^2\right) \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2 \operatorname{Ci}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{C(bx)}{5x^5} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}
 \end{aligned}$$

Mathematica [A] time = 0.0190792, size = 77, normalized size = 1.

$$-\frac{1}{80}\pi^2 b^5 \operatorname{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{40x^2} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{20x^4} - \frac{\operatorname{FresnelC}(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^6, x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*CosIntegral[(b^2*Pi*x^2)/2])/80 - FresnelC[b*x]/(5*x^5) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(40*x^2)
```

Maple [A] time = 0.052, size = 71, normalized size = 0.9

$$b^5 \left(-\frac{\text{FresnelC}(bx)}{5b^5x^5} - \frac{1}{20x^4b^4} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{20} \left(-\frac{1}{2b^2x^2} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{4} \text{Ci}\left(\frac{b^2\pi x^2}{2}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x^6,x)

[Out] b^5*(-1/5*FresnelC(b*x)/b^5/x^5-1/20/b^4/x^4*cos(1/2*b^2*Pi*x^2)-1/20*Pi*(-1/2*sin(1/2*b^2*Pi*x^2)/b^2/x^2+1/4*Pi*Ci(1/2*b^2*Pi*x^2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^6,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)/x^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^6,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)/x^6, x)

Sympy [A] time = 1.99193, size = 65, normalized size = 0.84

$$\frac{\pi^4 b^9 x^4 \Gamma\left(\frac{9}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{9}{4} \\ 2, \frac{5}{2}, 3, \frac{13}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{6144 \Gamma\left(\frac{13}{4}\right)} - \frac{\pi^2 b^5 \log(b^4 x^4)}{160} - \frac{b}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x**6, x)

[Out] pi**4*b**9*x**4*gamma(9/4)*hyper((1, 1, 9/4), (2, 5/2, 3, 13/4), -pi**2*b**4*x**4/16)/(6144*gamma(13/4)) - pi**2*b**5*log(b**4*x**4)/160 - b/(4*x**4)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^6, x, algorithm="giac")

[Out] integrate(fresnelc(b*x)/x^6, x)

3.124 $\int \frac{\text{FresnelC}(bx)}{x^7} dx$

Optimal. Leaf size=94

$$\frac{1}{90}\pi^3 b^6 S(bx) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{90x^3} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{90x} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{30x^5} - \frac{\text{FresnelC}(bx)}{6x^6}$$

[Out] $-(b*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(30*x^5) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(90*x) - \text{FresnelC}[b*x]/(6*x^6) + (b^6*\text{Pi}^3*\text{FresnelS}[b*x])/90 + (b^3*\text{Pi}*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(90*x^3)$

Rubi [A] time = 0.0587288, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3388, 3387, 3351}

$$\frac{1}{90}\pi^3 b^6 S(bx) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{90x^3} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{90x} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{30x^5} - \frac{\text{FresnelC}(bx)}{6x^6}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]/x^7,x]

[Out] $-(b*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(30*x^5) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(90*x) - \text{FresnelC}[b*x]/(6*x^6) + (b^6*\text{Pi}^3*\text{FresnelS}[b*x])/90 + (b^3*\text{Pi}*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(90*x^3)$

Rule 6427

Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*FresnelC[b*x])/(d*(m+1)), x] - Dist[b/(d*(m+1)), Int[(d*x)^(m+1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3388

Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] := Simp[((e*x)^(m+1)*Cos[c + d*x^n])/(e*(m+1)), x] + Dist[(d*n)/(e^n*(m+1)), Int[(e*x)^(m+n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Simp[((e*x)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{C(bx)}{x^7} dx &= -\frac{C(bx)}{6x^6} + \frac{1}{6}b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} - \frac{C(bx)}{6x^6} - \frac{1}{30}(b^3\pi) \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} - \frac{C(bx)}{6x^6} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3} - \frac{1}{90}(b^5\pi^2) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x} - \frac{C(bx)}{6x^6} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3} + \frac{1}{90}(b^7\pi^3) \int \sin\left(\frac{1}{2}b^2\pi x^2\right) dx \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x} - \frac{C(bx)}{6x^6} + \frac{1}{90}b^6\pi^3 S(bx) + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}
 \end{aligned}$$

Mathematica [A] time = 0.115146, size = 74, normalized size = 0.79

$$\frac{1}{90} \left(\pi^3 b^6 S(bx) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^3} + \frac{b(\pi^2 b^4 x^4 - 3) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^5} - \frac{15 \text{FresnelC}(bx)}{x^6} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^7,x]
```

```
[Out] ((b*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^5 - (15*FresnelC[b*x])/x^6 + b^6*Pi^3*FresnelS[b*x] + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/x^3)/90
```

Maple [A] time = 0.047, size = 87, normalized size = 0.9

$$b^6 \left(-\frac{\text{FresnelC}(bx)}{6b^6x^6} - \frac{1}{30b^5x^5} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{30} \left(-\frac{1}{3x^3b^3} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{3} \left(-\frac{1}{bx} \cos\left(\frac{b^2\pi x^2}{2}\right) - \pi \text{FresnelS}(bx) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x^7,x)

[Out] b^6*(-1/6*FresnelC(b*x)/b^6/x^6-1/30/b^5/x^5*cos(1/2*b^2*Pi*x^2)-1/30*Pi*(-1/3*sin(1/2*b^2*Pi*x^2)/b^3/x^3+1/3*Pi*(-1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*FresnelS(b*x))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^7,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)/x^7, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^7,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)/x^7, x)

Sympy [A] time = 1.52775, size = 56, normalized size = 0.6

$$\frac{b\Gamma\left(-\frac{5}{4}\right)\Gamma\left(\frac{1}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{5}{4}, \frac{1}{4} \\ -\frac{1}{4}, \frac{1}{2}, \frac{5}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16x^5\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x**7,x)

[Out] b*gamma(-5/4)*gamma(1/4)*hyper((-5/4, 1/4), (-1/4, 1/2, 5/4), -pi**2*b**4*x**4/16)/(16*x**5*gamma(-1/4)*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^7,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)/x^7, x)

3.125 $\int \frac{\text{FresnelC}(bx)}{x^8} dx$

Optimal. Leaf size=102

$$\frac{1}{672}\pi^3 b^7 \text{Si}\left(\frac{1}{2}b^2\pi x^2\right) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{168x^4} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{336x^2} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{42x^6} - \frac{\text{FresnelC}(bx)}{7x^7}$$

[Out] $-(b*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(42*x^6) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(336*x^2) - \text{FresnelC}[b*x]/(7*x^7) + (b^3*\text{Pi}*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(168*x^4) + (b^7*\text{Pi}^3*\text{SinIntegral}[(b^2*\text{Pi}*x^2)/2])/672$

Rubi [A] time = 0.118397, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3380, 3297, 3299}

$$\frac{1}{672}\pi^3 b^7 \text{Si}\left(\frac{1}{2}b^2\pi x^2\right) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{168x^4} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{336x^2} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{42x^6} - \frac{\text{FresnelC}(bx)}{7x^7}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]/x^8,x]

[Out] $-(b*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(42*x^6) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(336*x^2) - \text{FresnelC}[b*x]/(7*x^7) + (b^3*\text{Pi}*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(168*x^4) + (b^7*\text{Pi}^3*\text{SinIntegral}[(b^2*\text{Pi}*x^2)/2])/672$

Rule 6427

Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)}{x^8} dx &= -\frac{C(bx)}{7x^7} + \frac{1}{7}b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx \\
&= -\frac{C(bx)}{7x^7} + \frac{1}{14}b \operatorname{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^4} dx, x, x^2\right) \\
&= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} - \frac{C(bx)}{7x^7} - \frac{1}{84}(b^3\pi) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3} dx, x, x^2\right) \\
&= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} - \frac{C(bx)}{7x^7} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{1}{336}(b^5\pi^2) \operatorname{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2} dx, x, x^2\right) \\
&= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{C(bx)}{7x^7} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} + \frac{1}{672}(b^7\pi^3) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x} dx, x, x^2\right) \\
&= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{C(bx)}{7x^7} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} + \frac{1}{672}b^7\pi^3 \operatorname{Si}\left(\frac{1}{2}b^2\pi x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.126535, size = 84, normalized size = 0.82

$$\frac{1}{672} \left(\pi^3 b^7 \operatorname{Si}\left(\frac{1}{2}b^2\pi x^2\right) + \frac{4\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^4} + \frac{2b(\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^6} - \frac{96 \operatorname{FresnelC}(bx)}{x^7} \right)$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x]/x^8,x]

[Out] $\frac{(2*b*(-8 + b^4*\pi^2*x^4)*\cos[(b^2*\pi*x^2)/2])/x^6 - (96*\text{FresnelC}[b*x])/x^7 + (4*b^3*\pi*\sin[(b^2*\pi*x^2)/2])/x^4 + b^7*\pi^3*\text{SinIntegral}[(b^2*\pi*x^2)/2])/672$

Maple [A] time = 0.05, size = 93, normalized size = 0.9

$$b^7 \left(-\frac{\text{FresnelC}(bx)}{7b^7x^7} - \frac{1}{42b^6x^6} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{42} \left(-\frac{1}{4x^4b^4} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{4} \left(-\frac{1}{2b^2x^2} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{4} \text{Si}\left(\frac{b^2\pi x^2}{2}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x^8,x)

[Out] $b^7*(-1/7*\text{FresnelC}(b*x)/b^7/x^7-1/42/b^6/x^6*\cos(1/2*b^2*\pi*x^2)-1/42*\pi*(-1/4*\sin(1/2*b^2*\pi*x^2)/b^4/x^4+1/4*\pi*(-1/2/b^2/x^2*\cos(1/2*b^2*\pi*x^2)-1/4*\pi*\text{Si}(1/2*b^2*\pi*x^2)))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^8,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)/x^8, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^8, x)
```

Sympy [A] time = 2.12267, size = 44, normalized size = 0.43

$$-\frac{b\Gamma\left(\frac{1}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2}, \frac{1}{2}, \frac{5}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{24x^6\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**8,x)
```

```
[Out] -b*gamma(1/4)*hyper((-3/2, 1/4), (-1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(24*x**6*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^8, x)
```

3.126 $\int \frac{\text{FresnelC}(bx)}{x^9} dx$

Optimal. Leaf size=119

$$\frac{1}{840}\pi^4 b^8 \text{FresnelC}(bx) - \frac{\pi^3 b^7 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{840x} + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{280x^5} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{840x^3} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{56x^7} - \frac{\text{FresnelC}(bx)}{8x^8}$$

[Out] $-(b*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(56*x^7) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(840*x^3) + (b^8*\text{Pi}^4*\text{FresnelC}[b*x])/840 - \text{FresnelC}[b*x]/(8*x^8) + (b^3*\text{Pi}*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(280*x^5) - (b^7*\text{Pi}^3*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(840*x)$

Rubi [A] time = 0.0784687, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3388, 3387, 3352}

$$\frac{1}{840}\pi^4 b^8 \text{FresnelC}(bx) - \frac{\pi^3 b^7 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{840x} + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{280x^5} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{840x^3} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{56x^7} - \frac{\text{FresnelC}(bx)}{8x^8}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]/x^9,x]

[Out] $-(b*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(56*x^7) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2])/(840*x^3) + (b^8*\text{Pi}^4*\text{FresnelC}[b*x])/840 - \text{FresnelC}[b*x]/(8*x^8) + (b^3*\text{Pi}*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(280*x^5) - (b^7*\text{Pi}^3*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(840*x)$

Rule 6427

Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*FresnelC[b*x])/(d*(m+1)), x] - Dist[b/(d*(m+1)), Int[(d*x)^(m+1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3388

Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] := Simp[((e*x)^(m+1)*Cos[c + d*x^n])/(e*(m+1)), x] + Dist[(d*n)/(e^n*(m+1)), Int[(e*x)^(m+n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3387

```
Int[((e._)*(x._))^(m_)*Sin[(c._) + (d._)*(x._)^(n_)], x_Symbol] := Simp[((e*x)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 3352

```
Int[Cos[(d._)*((e._) + (f._)*(x._))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{C(bx)}{x^9} dx &= -\frac{C(bx)}{8x^8} + \frac{1}{8}b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} - \frac{C(bx)}{8x^8} - \frac{1}{56}(b^3\pi) \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} - \frac{C(bx)}{8x^8} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{1}{280}(b^5\pi^2) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} - \frac{C(bx)}{8x^8} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} + \frac{1}{840}(b^7\pi^3) \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} - \frac{C(bx)}{8x^8} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{b^7\pi^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x} + \frac{1}{840}(b^9\pi^4) \int dx \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} + \frac{1}{840}b^8\pi^4 C(bx) - \frac{C(bx)}{8x^8} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{b^7\pi^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x}
 \end{aligned}$$

Mathematica [A] time = 0.0577638, size = 85, normalized size = 0.71

$$\frac{(\pi^4 b^8 x^8 - 105) \text{FresnelC}(bx) + \pi b^3 x^3 (3 - \pi^2 b^4 x^4) \sin\left(\frac{1}{2}\pi b^2 x^2\right) + bx (\pi^2 b^4 x^4 - 15) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{840x^8}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x]/x^9, x]

[Out] (b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*FresnelC[b*x] + b^3*Pi*x^3*(3 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)

Maple [A] time = 0.048, size = 108, normalized size = 0.9

$$b^8 \left(-\frac{\text{FresnelC}(bx)}{8b^8x^8} - \frac{1}{56b^7x^7} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{56} \left(-\frac{1}{5b^5x^5} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{5} \left(-\frac{1}{3x^3b^3} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{3} \left(-\frac{1}{bx} \sin\left(\frac{b^2\pi x^2}{2}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x^9,x)

[Out] b^8*(-1/8*FresnelC(b*x)/b^8/x^8-1/56/b^7/x^7*cos(1/2*b^2*Pi*x^2)-1/56*Pi*(-1/5*sin(1/2*b^2*Pi*x^2)/b^5/x^5+1/5*Pi*(-1/3/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/3*Pi*(-sin(1/2*b^2*Pi*x^2)/b/x+Pi*FresnelC(b*x))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^9,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)/x^9, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^9,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)/x^9, x)

Sympy [A] time = 3.70601, size = 185, normalized size = 1.55

$$\frac{\pi^4 b^8 C(bx) \Gamma\left(-\frac{7}{4}\right)}{2560 \Gamma\left(\frac{5}{4}\right)} - \frac{\pi^3 b^7 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{7}{4}\right)}{2560 x \Gamma\left(\frac{5}{4}\right)} + \frac{\pi^2 b^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{7}{4}\right)}{2560 x^3 \Gamma\left(\frac{5}{4}\right)} + \frac{3 \pi b^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{7}{4}\right)}{2560 x^5 \Gamma\left(\frac{5}{4}\right)} - \frac{3 b \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(-\frac{7}{4}\right)}{512 x^7 \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x**9, x)

[Out] pi**4*b**8*fresnelc(b*x)*gamma(-7/4)/(2560*gamma(5/4)) - pi**3*b**7*sin(pi*b**2*x**2/2)*gamma(-7/4)/(2560*x*gamma(5/4)) + pi**2*b**5*cos(pi*b**2*x**2/2)*gamma(-7/4)/(2560*x**3*gamma(5/4)) + 3*pi*b**3*sin(pi*b**2*x**2/2)*gamma(-7/4)/(2560*x**5*gamma(5/4)) - 3*b*cos(pi*b**2*x**2/2)*gamma(-7/4)/(512*x**7*gamma(5/4)) - 21*fresnelc(b*x)*gamma(-7/4)/(512*x**8*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^9, x, algorithm="giac")

[Out] integrate(fresnelc(b*x)/x^9, x)

3.127 $\int \frac{\text{FresnelC}(bx)}{x^{10}} dx$

Optimal. Leaf size=127

$$\frac{\pi^4 b^9 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right)}{6912} - \frac{\pi^3 b^7 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3456 x^2} + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{432 x^6} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{1728 x^4} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{72 x^8} - \frac{\text{FresnelC}(bx)}{9 x^9}$$

[Out] $-(b \cos[(b^2 \pi x^2)/2])/(72 x^8) + (b^5 \pi^2 \cos[(b^2 \pi x^2)/2])/(1728 x^4) + (b^9 \pi^4 \text{CosIntegral}[(b^2 \pi x^2)/2])/6912 - \text{FresnelC}[b x]/(9 x^9) + (b^3 \pi \sin[(b^2 \pi x^2)/2])/(432 x^6) - (b^7 \pi^3 \sin[(b^2 \pi x^2)/2])/(3456 x^2)$

Rubi [A] time = 0.143552, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6427, 3380, 3297, 3302}

$$\frac{\pi^4 b^9 \text{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right)}{6912} - \frac{\pi^3 b^7 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3456 x^2} + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{432 x^6} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{1728 x^4} - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{72 x^8} - \frac{\text{FresnelC}(bx)}{9 x^9}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]/x^10, x]

[Out] $-(b \cos[(b^2 \pi x^2)/2])/(72 x^8) + (b^5 \pi^2 \cos[(b^2 \pi x^2)/2])/(1728 x^4) + (b^9 \pi^4 \text{CosIntegral}[(b^2 \pi x^2)/2])/6912 - \text{FresnelC}[b x]/(9 x^9) + (b^3 \pi \sin[(b^2 \pi x^2)/2])/(432 x^6) - (b^7 \pi^3 \sin[(b^2 \pi x^2)/2])/(3456 x^2)$

Rule 6427

Int[FresnelC[(b_.)*(x_.)]*((d_.)*(x_.))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*FresnelC[b*x])/(d*(m+1)), x] - Dist[b/(d*(m+1)), Int[(d*x)^(m+1)*Cos[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]] && (EqQ[p, 1] || EqQ[m, n-1] || (IntegerQ[p] && GtQ[Simplify[(

$m + 1)/n], 0])$

Rule 3297

$\text{Int}[\left((c_{.}) + (d_{.}) \cdot (x_{.})\right)^{(m_{.})} \sin[(e_{.}) + (f_{.}) \cdot (x_{.})], x_{\text{Symbol}}] \rightarrow \text{Simp}[\left((c + d \cdot x)^{(m + 1)} \cdot \text{Sin}[e + f \cdot x]\right) / (d \cdot (m + 1)), x] - \text{Dist}[f / (d \cdot (m + 1)), \text{Int}[(c + d \cdot x)^{(m + 1)} \cdot \text{Cos}[e + f \cdot x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{LtQ}[m, -1]$

Rule 3302

$\text{Int}[\sin[(e_{.}) + (f_{.}) \cdot (x_{.})] / \left((c_{.}) + (d_{.}) \cdot (x_{.})\right), x_{\text{Symbol}}] \rightarrow \text{Simp}[\text{CosIntegral}[e - \text{Pi}/2 + f \cdot x] / d, x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{EqQ}[d \cdot (e - \text{Pi}/2) - c \cdot f, 0]$

Rubi steps

$$\begin{aligned}
 \int \frac{C(bx)}{x^{10}} dx &= -\frac{C(bx)}{9x^9} + \frac{1}{9}b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx \\
 &= -\frac{C(bx)}{9x^9} + \frac{1}{18}b \text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^5} dx, x, x^2\right) \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} - \frac{C(bx)}{9x^9} - \frac{1}{144}(b^3\pi) \text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^4} dx, x, x^2\right) \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} - \frac{C(bx)}{9x^9} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{1}{864}(b^5\pi^2) \text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3} dx, x, x^2\right) \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} - \frac{C(bx)}{9x^9} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} + \frac{(b^7\pi^3) \text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2} dx, x, x^2\right)}{3456} \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} - \frac{C(bx)}{9x^9} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{b^7\pi^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2} + \frac{(b^9\pi^4) \text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x} dx, x, x^2\right)}{6912} \\
 &= -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{b^9\pi^4 \text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)}{6912} - \frac{C(bx)}{9x^9} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{b^7\pi^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2}
 \end{aligned}$$

Mathematica [A] time = 0.0805378, size = 96, normalized size = 0.76

$$\frac{\pi^4 b^9 \operatorname{CosIntegral}\left(\frac{1}{2}\pi b^2 x^2\right) - \frac{2\pi b^3(\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^6} + \frac{4b(\pi^2 b^4 x^4 - 24) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^8} - \frac{768 \operatorname{FresnelC}(bx)}{x^9}}{6912}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x]/x^10,x]

[Out] ((4*b*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^8 + b^9*Pi^4*CosIntegral[(b^2*Pi*x^2)/2] - (768*FresnelC[b*x])/x^9 - (2*b^3*Pi*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^6)/6912

Maple [A] time = 0.047, size = 115, normalized size = 0.9

$$b^9 \left(-\frac{\operatorname{FresnelC}(bx)}{9b^9x^9} - \frac{1}{72b^8x^8} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{72} \left(-\frac{1}{6b^6x^6} \sin\left(\frac{b^2\pi x^2}{2}\right) + \frac{\pi}{6} \left(-\frac{1}{4x^4b^4} \cos\left(\frac{b^2\pi x^2}{2}\right) - \frac{\pi}{4} \left(-\frac{1}{2b^2x^2} \sin\left(\frac{b^2\pi x^2}{2}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x^10,x)

[Out] b^9*(-1/9*FresnelC(b*x)/b^9/x^9-1/72/b^8/x^8*cos(1/2*b^2*Pi*x^2)-1/72*Pi*(-1/6*sin(1/2*b^2*Pi*x^2)/b^6/x^6+1/6*Pi*(-1/4/b^4/x^4*cos(1/2*b^2*Pi*x^2)-1/4*Pi*(-1/2*sin(1/2*b^2*Pi*x^2)/b^2/x^2+1/4*Pi*Ci(1/2*b^2*Pi*x^2))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{fresnelc}(bx)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^10,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)/x^10, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^10,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)/x^10, x)

Sympy [A] time = 6.86518, size = 76, normalized size = 0.6

$$-\frac{\pi^6 b^{13} x^4 \Gamma\left(\frac{13}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{13}{4} \\ 2, \frac{7}{2}, 4, \frac{17}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{737280 \Gamma\left(\frac{17}{4}\right)} + \frac{\pi^4 b^9 \log(b^4 x^4)}{13824} + \frac{\pi^2 b^5}{160 x^4} - \frac{b}{8 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x**10,x)

[Out] -pi**6*b**13*x**4*gamma(13/4)*hyper((1, 1, 13/4), (2, 7/2, 4, 17/4), -pi**2*b**4*x**4/16)/(737280*gamma(17/4)) + pi**4*b**9*log(b**4*x**4)/13824 + pi**2*b**5/(160*x**4) - b/(8*x**8)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^10,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)/x^10, x)

3.128 $\int (c + dx)^3 \text{FresnelC}(a + bx) dx$

Optimal. Leaf size=298

$$-\frac{d^2(a+bx)^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4} - \frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2 b^4} - \frac{(bc-ad)^4 \text{FresnelC}(a+bx)}{4b^4 d} + \frac{3d(bc-ad)^2}{2\pi b}$$

[Out] $(-2*d^2*(b*c - a*d)*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}^2) - (3*d^3*(a + b*x)*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(4*b^4*\text{Pi}^2) - ((b*c - a*d)^4*\text{FresnelC}[a + b*x])/(4*b^4*d) + (3*d^3*\text{FresnelC}[a + b*x])/(4*b^4*\text{Pi}^2) + ((c + d*x)^4*\text{FresnelC}[a + b*x])/(4*d) + (3*d*(b*c - a*d)^2*\text{FresnelS}[a + b*x])/(2*b^4*\text{Pi}) - ((b*c - a*d)^3*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}) - (3*d*(b*c - a*d)^2*(a + b*x)*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(2*b^4*\text{Pi}) - (d^2*(b*c - a*d)*(a + b*x)^2*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}) - (d^3*(a + b*x)^3*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(4*b^4*\text{Pi})$

Rubi [A] time = 0.37306, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638, 3385}

$$-\frac{d^2(a+bx)^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4} - \frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2 b^4} - \frac{(bc-ad)^4 \text{FresnelC}(a+bx)}{4b^4 d} + \frac{3d(bc-ad)^2}{2\pi b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{FresnelC}[a + b*x], x]$

[Out] $(-2*d^2*(b*c - a*d)*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}^2) - (3*d^3*(a + b*x)*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/(4*b^4*\text{Pi}^2) - ((b*c - a*d)^4*\text{FresnelC}[a + b*x])/(4*b^4*d) + (3*d^3*\text{FresnelC}[a + b*x])/(4*b^4*\text{Pi}^2) + ((c + d*x)^4*\text{FresnelC}[a + b*x])/(4*d) + (3*d*(b*c - a*d)^2*\text{FresnelS}[a + b*x])/(2*b^4*\text{Pi}) - ((b*c - a*d)^3*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}) - (3*d*(b*c - a*d)^2*(a + b*x)*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(2*b^4*\text{Pi}) - (d^2*(b*c - a*d)*(a + b*x)^2*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(b^4*\text{Pi}) - (d^3*(a + b*x)^3*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(4*b^4*\text{Pi})$

Rule 6429

$\text{Int}[\text{FresnelC}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}*\text{FresnelC}[a + b*x]/(d*(m + 1)), x] - \text{Dist}[b/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2], x], x] /; \text{FreeQ}\{a, b,$

c, d}, x] && IGtQ[m, 0]

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*(e_.) + (f_.)*(x_)^(n_)])*(b_.)^(p_.)*((g_.) + (h_.)*(x_)^(m_.), x_Symbol] := Module[{k = If[FractionQ[n], Denominator[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_)^2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_)^2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_)^(m_.))*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

$e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3385

$\text{Int}[(e_.)*(x_.))^{(m_.)*\text{Sin}[(c_.) + (d_.)*(x_.)^{(n_.)}], x_Symbol] := -\text{Simp}[(e^{(n-1)}*(e*x)^{(m-n+1)}*\text{Cos}[c + d*x^n])/(d*n), x] + \text{Dist}[(e^n*(m-n+1))/(d*n), \text{Int}[(e*x)^{(m-n)}*\text{Cos}[c + d*x^n], x], x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[n, m+1]$

Rubi steps

$$\begin{aligned} \int (c + dx)^3 C(a + bx) dx &= \frac{(c + dx)^4 C(a + bx)}{4d} - \frac{b \int (c + dx)^4 \cos\left(\frac{1}{2}\pi(a + bx)^2\right) dx}{4d} \\ &= \frac{(c + dx)^4 C(a + bx)}{4d} - \frac{\text{Subst}\left(\int \left(b^4 c^4 \left(1 + \frac{ad(-4b^3 c^3 + 6ab^2 c^2 d - 4a^2 bcd^2 + a^3 d^3)}{b^4 c^4}\right) \cos\left(\frac{\pi x^2}{2}\right) + 4b^3 c^3 d\right) dx}{b^4 c^4}, x, a + bx\right)}{b^4 c^4} \\ &= \frac{(c + dx)^4 C(a + bx)}{4d} - \frac{d^3 \text{Subst}\left(\int x^4 \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{4b^4} - \frac{(d^2(bc - ad) \text{Subst}\left(\int x^3 \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right))}{b^4} \\ &= -\frac{(bc - ad)^4 C(a + bx)}{4b^4 d} + \frac{(c + dx)^4 C(a + bx)}{4d} - \frac{3d(bc - ad)^2(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{2b^4 \pi} - \frac{d^3(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{4b^4 \pi^2} \\ &= -\frac{3d^3(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{4b^4 \pi^2} - \frac{(bc - ad)^4 C(a + bx)}{4b^4 d} + \frac{(c + dx)^4 C(a + bx)}{4d} + \frac{3d(bc - ad)^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{2b^4 \pi} \\ &= -\frac{2d^2(bc - ad) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^4 \pi^2} - \frac{3d^3(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{4b^4 \pi^2} - \frac{(bc - ad)^4 C(a + bx)}{4b^4 d} + \frac{3d(bc - ad)^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{2b^4 \pi} \end{aligned}$$

Mathematica [A] time = 0.816556, size = 424, normalized size = 1.42

$\text{FresnelC}(a + bx) (6\pi^2 b^2 c^2 d (b^2 x^2 - a^2) + 4\pi^2 bcd^2 (a^3 + b^3 x^3) + d^3 (-\pi^2 a^4 + \pi^2 b^4 x^4 + 3) + 4\pi^2 b^3 c^3 (a + bx)) - 4\pi a^2 bcd$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*FresnelC[a + b*x],x]

```
[Out] (-8*b*c*d^2*cos[(Pi*(a + b*x)^2]/2] + 5*a*d^3*cos[(Pi*(a + b*x)^2]/2] - 3*b
*d^3*x*cos[(Pi*(a + b*x)^2]/2] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*Pi
^2*(-a^2 + b^2*x^2) + 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2 +
b^4*Pi^2*x^4))*FresnelC[a + b*x] + 6*d*(b*c - a*d)^2*Pi*FresnelS[a + b*x] -
4*b^3*c^3*Pi*Sin[(Pi*(a + b*x)^2]/2] + 6*a*b^2*c^2*d*Pi*Sin[(Pi*(a + b*x)^
2)/2] - 4*a^2*b*c*d^2*Pi*Sin[(Pi*(a + b*x)^2]/2] + a^3*d^3*Pi*Sin[(Pi*(a +
b*x)^2)/2] - 6*b^3*c^2*d*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + 4*a*b^2*c*d^2*Pi*x*
Sin[(Pi*(a + b*x)^2)/2] - a^2*b*d^3*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - 4*b^3*c*
d^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2] + a*b^2*d^3*Pi*x^2*Sin[(Pi*(a + b*x)^2)/
2] - b^3*d^3*Pi*x^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Maple [A] time = 0.056, size = 397, normalized size = 1.3

$$\frac{1}{b} \left(\frac{\text{FresnelC}(bx+a)(d(bx+a)-ad+bc)^4}{4db^3} - \frac{1}{4db^3} \left(\frac{d^4(bx+a)^3}{\pi} \sin\left(\frac{\pi(bx+a)^2}{2}\right) - 3 \frac{d^4}{\pi} \left(-\frac{(bx+a)\cos\left(\frac{1}{2}\pi(bx+a)^2\right)}{\pi} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*FresnelC(b*x+a), x)
```

```
[Out] 1/b*(1/4*FresnelC(b*x+a)*(d*(b*x+a)-a*d+b*c)^4/b^3/d-1/4/b^3/d*(d^4/Pi*(b*x
+a)^3*sin(1/2*Pi*(b*x+a)^2)-3*d^4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+1
/Pi*FresnelC(b*x+a))+(-4*a*d^4+4*b*c*d^3)/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2
)+2*(-4*a*d^4+4*b*c*d^3)/Pi^2*cos(1/2*Pi*(b*x+a)^2)+(6*a^2*d^4-12*a*b*c*d^3
+6*b^2*c^2*d^2)/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-(6*a^2*d^4-12*a*b*c*d^3+6*
b^2*c^2*d^2)/Pi*FresnelS(b*x+a)+(-4*a^3*d^4+12*a^2*b*c*d^3-12*a*b^2*c^2*d^2
+4*b^3*c^3*d)/Pi*sin(1/2*Pi*(b*x+a)^2)+a^4*d^4*FresnelC(b*x+a)-4*a^3*b*c*d^
3*FresnelC(b*x+a)+6*a^2*b^2*c^2*d^2*FresnelC(b*x+a)-4*a*b^3*c^3*d*FresnelC(
b*x+a)+b^4*c^4*FresnelC(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnelc(b*x+a), x, algorithm="maxima")
```

[Out] integrate((d*x + c)^3*fresnelc(b*x + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3\right)\text{fresnelc}(bx + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*fresnelc(b*x+a),x, algorithm="fricas")

[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*fresnelc(b*x + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 C(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*fresnelc(b*x+a),x)

[Out] Integral((c + d*x)**3*fresnelc(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*fresnelc(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^3*fresnelc(b*x + a), x)

3.129 $\int (c + dx)^2 \mathbf{FresnelC}(a + bx) dx$

Optimal. Leaf size=194

$$-\frac{(bc - ad)^3 \mathbf{FresnelC}(a + bx)}{3b^3 d} + \frac{d(bc - ad)S(a + bx)}{\pi b^3} - \frac{(bc - ad)^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} - \frac{d(a + bx)(bc - ad) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3}$$

[Out] $(-2*d^2*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/ (3*b^3*\text{Pi}^2) - ((b*c - a*d)^3*\text{FresnelC}[a + b*x])/ (3*b^3*d) + ((c + d*x)^3*\text{FresnelC}[a + b*x])/ (3*d) + (d*(b*c - a*d)*\text{FresnelS}[a + b*x])/ (b^3*\text{Pi}) - ((b*c - a*d)^2*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/ (b^3*\text{Pi}) - (d*(b*c - a*d)*(a + b*x)*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/ (b^3*\text{Pi}) - (d^2*(a + b*x)^2*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/ (3*b^3*\text{Pi})$

Rubi [A] time = 0.204614, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638}

$$-\frac{(bc - ad)^3 \mathbf{FresnelC}(a + bx)}{3b^3 d} + \frac{d(bc - ad)S(a + bx)}{\pi b^3} - \frac{(bc - ad)^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} - \frac{d(a + bx)(bc - ad) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{FresnelC}[a + b*x], x]$

[Out] $(-2*d^2*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2])/ (3*b^3*\text{Pi}^2) - ((b*c - a*d)^3*\text{FresnelC}[a + b*x])/ (3*b^3*d) + ((c + d*x)^3*\text{FresnelC}[a + b*x])/ (3*d) + (d*(b*c - a*d)*\text{FresnelS}[a + b*x])/ (b^3*\text{Pi}) - ((b*c - a*d)^2*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/ (b^3*\text{Pi}) - (d*(b*c - a*d)*(a + b*x)*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/ (b^3*\text{Pi}) - (d^2*(a + b*x)^2*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/ (3*b^3*\text{Pi})$

Rule 6429

$\text{Int}[\text{FresnelC}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[((c + d*x)^{(m + 1)}*\text{FresnelC}[a + b*x])/(d*(m + 1)), x] - \text{Dist}[b/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2], x], x] /;$ FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]

Rule 3434

$\text{Int}[(a_.) + \text{Cos}[(c_.) + (d_.)*((e_.) + (f_.)*(x_.))^{(n_.)}]*(b_.))^{(p_.)}*((g_.) + (h_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Module}[k = \text{If}[\text{FractionQ}[n], \text{Denominator}[\text{FractionQ}[n]], 1], \text{Int}[(a + \text{Cos}[c + d*(e + f*x)^n]*(b))^{p*(k)}*(g + h*x)^m, x]]$

or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3386

Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n]/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 C(a + bx) dx &= \frac{(c + dx)^3 C(a + bx)}{3d} - \frac{b \int (c + dx)^3 \cos\left(\frac{1}{2}\pi(a + bx)^2\right) dx}{3d} \\
&= \frac{(c + dx)^3 C(a + bx)}{3d} - \frac{\text{Subst}\left(\int \left(b^3 c^3 \left(1 - \frac{ad(3b^2 c^2 - 3abcd + a^2 d^2)}{b^3 c^3}\right) \cos\left(\frac{\pi x^2}{2}\right) + 3b^2 c^2 d \left(1 + \frac{ad(-2b^2 c^2 + a^2 d^2)}{b^2 c^3}\right)\right) dx, x, a + bx\right)}{3d} \\
&= \frac{(c + dx)^3 C(a + bx)}{3d} - \frac{d^2 \text{Subst}\left(\int x^3 \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{3b^3} - \frac{(d(bc - ad)) \text{Subst}\left(\int x^2 \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^3} \\
&= -\frac{(bc - ad)^3 C(a + bx)}{3b^3 d} + \frac{(c + dx)^3 C(a + bx)}{3d} - \frac{d(bc - ad)(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^3 \pi} - \frac{d^2 \text{Subst}\left(\int x \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^3} \\
&= -\frac{(bc - ad)^3 C(a + bx)}{3b^3 d} + \frac{(c + dx)^3 C(a + bx)}{3d} + \frac{d(bc - ad)S(a + bx)}{b^3 \pi} - \frac{(bc - ad)^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^3 \pi} \\
&= -\frac{2d^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{3b^3 \pi^2} - \frac{(bc - ad)^3 C(a + bx)}{3b^3 d} + \frac{(c + dx)^3 C(a + bx)}{3d} + \frac{d(bc - ad)S(a + bx)}{b^3 \pi}
\end{aligned}$$

Mathematica [A] time = 0.46643, size = 237, normalized size = 1.22

$$\pi^2 \text{FresnelC}(a + bx) \left(-3a^2 bcd + a^3 d^2 + 3ab^2 c^2 + b^3 x(3c^2 + 3cdx + d^2 x^2)\right) - \pi a^2 d^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right) - 3\pi b^2 c^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*FresnelC[a + b*x],x]

[Out] (-2*d^2*Cos[(Pi*(a + b*x)^2)/2] + Pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelC[a + b*x] + 3*d*(b*c - a*d)*Pi*FresnelS[a + b*x] - 3*b^2*c^2*Pi*Sin[(Pi*(a + b*x)^2)/2] + 3*a*b*c*d*Pi*Sin[(Pi*(a + b*x)^2)/2] - a^2*d^2*Pi*Sin[(Pi*(a + b*x)^2)/2] - 3*b^2*c*d*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + a*b*d^2*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - b^2*d^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)

Maple [A] time = 0.054, size = 249, normalized size = 1.3

$$\frac{1}{b} \left(\frac{\text{FresnelC}(bx + a) (d(bx + a) - ad + bc)^3}{3db^2} - \frac{1}{3db^2} \left(\frac{d^3(bx + a)^2}{\pi} \sin\left(\frac{\pi(bx + a)^2}{2}\right) + 2 \frac{d^3 \cos\left(\frac{1}{2}\pi(bx + a)^2\right)}{\pi^2} + \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^2*FresnelC(b*x+a),x)`

[Out] $\frac{1}{b} \left(\frac{1}{3} \text{FresnelC}(b*x+a) * (d*(b*x+a) - a*d + b*c)^3 / b^2 / d - \frac{1}{3} / b^2 / d * (d^3 / \text{Pi} * (b*x+a)^2 * \sin(1/2 * \text{Pi} * (b*x+a)^2) + 2*d^3 / \text{Pi}^2 * \cos(1/2 * \text{Pi} * (b*x+a)^2) + (-3*a*d^3 + 3*b*c*d^2) / \text{Pi} * (b*x+a) * \sin(1/2 * \text{Pi} * (b*x+a)^2) - (-3*a*d^3 + 3*b*c*d^2) / \text{Pi} * \text{FresnelS}(b*x+a) + (3*a^2*d^3 - 6*a*b*c*d^2 + 3*b^2*c^2*d) / \text{Pi} * \sin(1/2 * \text{Pi} * (b*x+a)^2) - a^3*d^3 * \text{FresnelC}(b*x+a) + 3*a^2*b*c*d^2 * \text{FresnelC}(b*x+a) - 3*a*b^2*c^2*d * \text{FresnelC}(b*x+a) + b^3*c^3 * \text{FresnelC}(b*x+a) \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*fresnelc(b*x+a),x, algorithm="maxima")`

[Out] `integrate((d*x + c)^2*fresnelc(b*x + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((d^2 x^2 + 2cdx + c^2) \text{fresnelc}(bx + a), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*fresnelc(b*x+a),x, algorithm="fricas")`

[Out] `integral((d^2*x^2 + 2*c*d*x + c^2)*fresnelc(b*x + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 C(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnelc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*fresnelc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*fresnelc(b*x + a), x)
```

3.130 $\int (c + dx) \mathbf{FresnelC}(a + bx) dx$

Optimal. Leaf size=122

$$-\frac{(bc - ad)^2 \mathbf{FresnelC}(a + bx)}{2b^2 d} - \frac{(bc - ad) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^2} + \frac{dS(a + bx)}{2\pi b^2} - \frac{d(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^2} + \frac{(c + dx)^2 \mathbf{FresnelC}(a + bx)}{2d}$$

[Out] $-\left(\frac{(b*c - a*d)^2*\mathbf{FresnelC}[a + b*x]}{(2*b^2*d)} + \left(\frac{(c + d*x)^2*\mathbf{FresnelC}[a + b*x]}{(2*d)} + \frac{d*\mathbf{FresnelS}[a + b*x]}{(2*b^2*\pi)} - \left(\frac{(b*c - a*d)*\sin[(\pi*(a + b*x)^2]/2)}{(b^2*\pi)} - \frac{d*(a + b*x)*\sin[(\pi*(a + b*x)^2]/2)}{(2*b^2*\pi)}\right)\right)$

Rubi [A] time = 0.112805, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351}

$$-\frac{(bc - ad)^2 \mathbf{FresnelC}(a + bx)}{2b^2 d} - \frac{(bc - ad) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^2} + \frac{dS(a + bx)}{2\pi b^2} - \frac{d(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^2} + \frac{(c + dx)^2 \mathbf{FresnelC}(a + bx)}{2d}$$

Antiderivative was successfully verified.

[In] `Int[(c + d*x)*FresnelC[a + b*x],x]`

[Out] $-\left(\frac{(b*c - a*d)^2*\mathbf{FresnelC}[a + b*x]}{(2*b^2*d)} + \left(\frac{(c + d*x)^2*\mathbf{FresnelC}[a + b*x]}{(2*d)} + \frac{d*\mathbf{FresnelS}[a + b*x]}{(2*b^2*\pi)} - \left(\frac{(b*c - a*d)*\sin[(\pi*(a + b*x)^2]/2)}{(b^2*\pi)} - \frac{d*(a + b*x)*\sin[(\pi*(a + b*x)^2]/2)}{(2*b^2*\pi)}\right)\right)$

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] :> Simp[(((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2]/2], x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((g_.) + (h_.)*(x_.))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominator[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x], (e + f*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)ⁿ])*(b_.))^p*(x_)^m, x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, xⁿ], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3386

Int[Cos[(c_.) + (d_.)*(x_)ⁿ]*(e_.)*(x_)^m, x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*xⁿ]/(d*n), x] - Dist[(eⁿ*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*xⁿ], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)C(a + bx) dx &= \frac{(c + dx)^2 C(a + bx)}{2d} - \frac{b \int (c + dx)^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right) dx}{2d} \\
&= \frac{(c + dx)^2 C(a + bx)}{2d} - \frac{\text{Subst}\left(\int \left(b^2 c^2 \left(1 + \frac{ad(-2bc+ad)}{b^2 c^2}\right) \cos\left(\frac{\pi x^2}{2}\right) + 2bcd\left(1 - \frac{ad}{bc}\right) x \cos\left(\frac{\pi x^2}{2}\right) + \dots\right) dx}{2b^2 d}}{2d} \\
&= \frac{(c + dx)^2 C(a + bx)}{2d} - \frac{d \text{Subst}\left(\int x^2 \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{2b^2} - \frac{(bc - ad) \text{Subst}\left(\int x \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^2} \\
&= -\frac{(bc - ad)^2 C(a + bx)}{2b^2 d} + \frac{(c + dx)^2 C(a + bx)}{2d} - \frac{d(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{2b^2 \pi} - \frac{(bc - ad) \text{Subst}\left(\int x \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^2} \\
&= -\frac{(bc - ad)^2 C(a + bx)}{2b^2 d} + \frac{(c + dx)^2 C(a + bx)}{2d} + \frac{dS(a + bx)}{2b^2 \pi} - \frac{(bc - ad) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^2 \pi} - \frac{d(a + bx) \cos\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.235525, size = 74, normalized size = 0.61

$$\frac{-\pi(a + bx)\text{FresnelC}(a + bx)(ad - b(2c + dx)) + \sin\left(\frac{1}{2}\pi(a + bx)^2\right)(ad - 2bc - bdx) + dS(a + bx)}{2\pi b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*FresnelC[a + b*x],x]

[Out] $(-\pi(a + b*x)*(a*d - b*(2*c + d*x))*\text{FresnelC}[a + b*x]) + d*\text{FresnelS}[a + b*x] + (-2*b*c + a*d - b*d*x)*\text{Sin}[(\pi*(a + b*x)^2)/2])/(2*b^2*\pi)$

Maple [A] time = 0.053, size = 107, normalized size = 0.9

$$\frac{1}{b} \left(\frac{\text{FresnelC}(bx + a)}{b} \left(\frac{d(bx + a)^2}{2} - ad(bx + a) + bc(bx + a) \right) - \frac{1}{2b} \left(\frac{d(bx + a)}{\pi} \sin\left(\frac{\pi(bx + a)^2}{2}\right) - \frac{d\text{FresnelS}(bx + a)}{\pi} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*FresnelC(b*x+a),x)

[Out] $1/b*(\text{FresnelC}(b*x+a)/b*(1/2*d*(b*x+a)^2-a*d*(b*x+a)+b*c*(b*x+a))-1/2/b*(d/Pi*(b*x+a)*\sin(1/2*Pi*(b*x+a)^2)-d/Pi*\text{FresnelS}(b*x+a)+(-2*a*d+2*b*c)/Pi*\sin(\dots))$

$1/2*\text{Pi}*(b*x+a)^2))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*fresnelc(b*x+a),x, algorithm="maxima")

[Out] integrate((d*x + c)*fresnelc(b*x + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)\text{fresnelc}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*fresnelc(b*x+a),x, algorithm="fricas")

[Out] integral((d*x + c)*fresnelc(b*x + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)C(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*fresnelc(b*x+a),x)

[Out] Integral((c + d*x)*fresnelc(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnelc(b*x + a), x)
```

3.131 $\int \text{FresnelC}(a + bx) dx$

Optimal. Leaf size=37

$$\frac{(a + bx)\text{FresnelC}(a + bx)}{b} - \frac{\sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b}$$

[Out] $((a + b*x)*\text{FresnelC}[a + b*x])/b - \text{Sin}[(\text{Pi}*(a + b*x)^2)/2]/(b*\text{Pi})$

Rubi [A] time = 0.0062556, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6419}

$$\frac{(a + bx)\text{FresnelC}(a + bx)}{b} - \frac{\sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{FresnelC}[a + b*x], x]$

[Out] $((a + b*x)*\text{FresnelC}[a + b*x])/b - \text{Sin}[(\text{Pi}*(a + b*x)^2)/2]/(b*\text{Pi})$

Rule 6419

$\text{Int}[\text{FresnelC}[(a_.) + (b_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[\frac{(a + b*x)*\text{FresnelC}[a + b*x]}{b}, x] - \text{Simp}[\frac{\text{Sin}[(\text{Pi}*(a + b*x)^2)/2]}{(b*\text{Pi})}, x] \text{ /; } \text{FreeQ}\{a, b\}, x]$

Rubi steps

$$\int C(a + bx) dx = \frac{(a + bx)C(a + bx)}{b} - \frac{\sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{b\pi}$$

Mathematica [B] time = 0.0292309, size = 90, normalized size = 2.43

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx + \frac{1}{2}\pi b^2 x^2\right)}{\pi b} - \frac{\cos\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx + \frac{1}{2}\pi b^2 x^2\right)}{\pi b} + x\text{FresnelC}(a + bx) + \frac{a\text{FresnelC}(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[a + b*x],x]

[Out] (a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^2)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi)

Maple [A] time = 0.049, size = 34, normalized size = 0.9

$$\frac{1}{b} \left(\text{FresnelC}(bx + a)(bx + a) - \frac{1}{\pi} \sin\left(\frac{\pi (bx + a)^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x+a),x)

[Out] 1/b*(FresnelC(b*x+a)*(b*x+a)-1/Pi*sin(1/2*Pi*(b*x+a)^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a),x, algorithm="maxima")

[Out] integrate(fresnelc(b*x + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{fresnelc}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a),x, algorithm="fricas")

[Out] `integral(fresnelc(b*x + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int C(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x+a),x)`

[Out] `Integral(fresnelc(a + b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x+a),x, algorithm="giac")`

[Out] `integrate(fresnelc(b*x + a), x)`

$$3.132 \quad \int \frac{\text{FresnelC}(a+bx)}{c+dx} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable[FresnelC[a + b*x]/(c + d*x), x]

Rubi [A] time = 0.014454, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[a + b*x]/(c + d*x), x]

[Out] Defer[Int][FresnelC[a + b*x]/(c + d*x), x]

Rubi steps

$$\int \frac{C(a+bx)}{c+dx} dx = \int \frac{C(a+bx)}{c+dx} dx$$

Mathematica [A] time = 0.025616, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[a + b*x]/(c + d*x), x]

[Out] Integrate[FresnelC[a + b*x]/(c + d*x), x]

Maple [A] time = 0.365, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x+a)/(d*x+c), x)

[Out] int(FresnelC(b*x+a)/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)/(d*x+c), x, algorithm="maxima")

[Out] integrate(fresnelc(b*x + a)/(d*x + c), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)/(d*x+c), x, algorithm="fricas")

[Out] integral(fresnelc(b*x + a)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(fresnelc(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a)/(d*x + c), x)
```


$$3.133 \quad \int \frac{\mathbf{FresnelC}(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\mathbf{FresnelC}(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[FresnelC[a + b*x]/(c + d*x)^2, x]

Rubi [A] time = 0.0142711, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\mathbf{FresnelC}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[a + b*x]/(c + d*x)^2, x]

[Out] Defer[Int][FresnelC[a + b*x]/(c + d*x)^2, x]

Rubi steps

$$\int \frac{C(a+bx)}{(c+dx)^2} dx = \int \frac{C(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 2.28707, size = 0, normalized size = 0.

$$\int \frac{\mathbf{FresnelC}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[a + b*x]/(c + d*x)^2, x]

[Out] Integrate[FresnelC[a + b*x]/(c + d*x)^2, x]

Maple [A] time = 0.373, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x+a)/(d*x+c)^2,x)

[Out] int(FresnelC(b*x+a)/(d*x+c)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x + a)/(d*x + c)^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx + a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(fresnelc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x+a)/(d*x+c)**2,x)`

[Out] `Integral(fresnelc(a + b*x)/(c + d*x)**2, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x+a)/(d*x+c)^2,x, algorithm="giac")`

[Out] `integrate(fresnelc(b*x + a)/(d*x + c)^2, x)`

3.134 $\int x^3 \text{FresnelC}(a + bx) dx$

Optimal. Leaf size=227

$$-\frac{a^4 \text{FresnelC}(a + bx)}{4b^4} + \frac{3a^2 S(a + bx)}{2\pi b^4} + \frac{a^3 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^4} - \frac{3a^2(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^4} + \frac{3 \text{FresnelC}(a + bx)}{4\pi^2 b^4} + \dots$$

```
[Out] (2*a*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2) - (a^4*FresnelC[a + b*x])/(4*b^4) + (3*FresnelC[a + b*x])/(4*b^4*Pi^2) + (x^4*FresnelC[a + b*x])/4 + (3*a^2*FresnelS[a + b*x])/(2*b^4*Pi) + (a^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*a^2*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) + (a*(a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - ((a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi)
```

Rubi [A] time = 0.18712, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638, 3385}

$$-\frac{a^4 \text{FresnelC}(a + bx)}{4b^4} + \frac{3a^2 S(a + bx)}{2\pi b^4} + \frac{a^3 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^4} - \frac{3a^2(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^4} + \frac{3 \text{FresnelC}(a + bx)}{4\pi^2 b^4} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[a + b*x], x]
```

```
[Out] (2*a*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2) - (a^4*FresnelC[a + b*x])/(4*b^4) + (3*FresnelC[a + b*x])/(4*b^4*Pi^2) + (x^4*FresnelC[a + b*x])/4 + (3*a^2*FresnelS[a + b*x])/(2*b^4*Pi) + (a^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*a^2*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) + (a*(a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - ((a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi)
```

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*(e_.) + (f_.)*(x_)^(n_)])*(b_.)^(p_.)*((g_.) + (h_.)*(x_)^(m_.), x_Symbol] := Module[{k = If[FractionQ[n], Denominator[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^(k*n)])]^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3352

```
Int[Cos[(d_.)*(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3351

```
Int[Sin[(d_.)*(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_)^(m_.))*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
 \int x^3 C(a + bx) dx &= \frac{1}{4} x^4 C(a + bx) - \frac{1}{4} b \int x^4 \cos\left(\frac{1}{2} \pi (a + bx)^2\right) dx \\
 &= \frac{1}{4} x^4 C(a + bx) - \frac{\text{Subst}\left(\int \left(a^4 \cos\left(\frac{\pi x^2}{2}\right) - 4a^3 x \cos\left(\frac{\pi x^2}{2}\right) + 6a^2 x^2 \cos\left(\frac{\pi x^2}{2}\right) - 4ax^3 \cos\left(\frac{\pi x^2}{2}\right) + x^4 \cos\left(\frac{\pi x^2}{2}\right)\right) dx, x, a + bx\right)}{4b^4} \\
 &= \frac{1}{4} x^4 C(a + bx) - \frac{\text{Subst}\left(\int x^4 \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{4b^4} + \frac{a \text{Subst}\left(\int x^3 \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^4} - \frac{3a^2 \text{Subst}\left(\int x^2 \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^4} + \frac{3a^3 \text{Subst}\left(\int x \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^4} - \frac{3a^4 \text{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right) dx, x, a + bx\right)}{b^4} \\
 &= -\frac{a^4 C(a + bx)}{4b^4} + \frac{1}{4} x^4 C(a + bx) - \frac{3a^2 (a + bx) \sin\left(\frac{1}{2} \pi (a + bx)^2\right)}{2b^4 \pi} - \frac{(a + bx)^3 \sin\left(\frac{1}{2} \pi (a + bx)^2\right)}{4b^4 \pi} + \frac{a^3 \sin\left(\frac{1}{2} \pi (a + bx)^2\right)}{b^4 \pi} \\
 &= -\frac{3(a + bx) \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{4b^4 \pi^2} - \frac{a^4 C(a + bx)}{4b^4} + \frac{1}{4} x^4 C(a + bx) + \frac{3a^2 S(a + bx)}{2b^4 \pi} + \frac{a^3 \sin\left(\frac{1}{2} \pi (a + bx)^2\right)}{b^4 \pi} \\
 &= \frac{2a \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{b^4 \pi^2} - \frac{3(a + bx) \cos\left(\frac{1}{2} \pi (a + bx)^2\right)}{4b^4 \pi^2} - \frac{a^4 C(a + bx)}{4b^4} + \frac{3C(a + bx)}{4b^4 \pi^2} + \frac{1}{4} x^4 C(a + bx) + \frac{3a^2 S(a + bx)}{2b^4 \pi} + \frac{a^3 \sin\left(\frac{1}{2} \pi (a + bx)^2\right)}{b^4 \pi}
 \end{aligned}$$

Mathematica [A] time = 0.294206, size = 166, normalized size = 0.73

$$\frac{(-\pi^2 a^4 + \pi^2 b^4 x^4 + 3) \text{FresnelC}(a + bx) + 6\pi a^2 S(a + bx) + \pi a^3 \sin\left(\frac{1}{2} \pi (a + bx)^2\right) - \pi a^2 b x \sin\left(\frac{1}{2} \pi (a + bx)^2\right) + \pi a b^2 x^2 \sin\left(\frac{1}{2} \pi (a + bx)^2\right)}{4\pi^2 b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[a + b*x], x]
```

```
[Out] (5*a*cos[(Pi*(a + b*x)^2]/2] - 3*b*x*cos[(Pi*(a + b*x)^2]/2] + (3 - a^4*Pi^
2 + b^4*Pi^2*x^4)*FresnelC[a + b*x] + 6*a^2*Pi*FresnelS[a + b*x] + a^3*Pi*S
in[(Pi*(a + b*x)^2]/2] - a^2*b*Pi*x*Sin[(Pi*(a + b*x)^2]/2] + a*b^2*Pi*x^2*
```

$$\frac{\sin\left(\frac{\pi(a + bx)^2}{2}\right) - b^3\pi x^3 \sin\left(\frac{\pi(a + bx)^2}{2}\right)}{4b^4\pi^2}$$

Maple [A] time = 0.05, size = 187, normalized size = 0.8

$$\frac{1}{b^4} \left(\frac{\text{FresnelC}(bx + a) b^4 x^4}{4} - \frac{(bx + a)^3}{4\pi} \sin\left(\frac{\pi(bx + a)^2}{2}\right) + \frac{3}{4\pi} \left(-\frac{bx + a}{\pi} \cos\left(\frac{\pi(bx + a)^2}{2}\right) + \frac{\text{FresnelC}(bx + a)}{\pi} \right) \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*FresnelC(b*x+a), x)

[Out] 1/b^4*(1/4*FresnelC(b*x+a)*b^4*x^4-1/4/Pi*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)+3/4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+1/Pi*FresnelC(b*x+a))+a/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*cos(1/2*Pi*(b*x+a)^2)-3/2*a^2/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)+3/2*a^2/Pi*FresnelS(b*x+a)+a^3/Pi*sin(1/2*Pi*(b*x+a)^2)-1/4*a^4*FresnelC(b*x+a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*fresnelc(b*x+a), x, algorithm="maxima")

[Out] integrate(x^3*fresnelc(b*x + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \text{fresnelc}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*fresnelc(b*x+a), x, algorithm="fricas")

[Out] integral(x^3*fresnelc(b*x + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 C(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*fresnelc(b*x+a),x)

[Out] Integral(x**3*fresnelc(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*fresnelc(b*x+a),x, algorithm="giac")

[Out] integrate(x^3*fresnelc(b*x + a), x)

3.135 $\int x^2 \text{FresnelC}(a + bx) dx$

Optimal. Leaf size=148

$$\frac{a^3 \text{FresnelC}(a + bx)}{3b^3} - \frac{a^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} - \frac{aS(a + bx)}{\pi b^3} + \frac{a(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} - \frac{(a + bx)^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{3\pi b^3}$$

```
[Out] (-2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) + (a^3*FresnelC[a + b*x])/(3*b^3)
+ (x^3*FresnelC[a + b*x])/3 - (a*FresnelS[a + b*x])/(b^3*Pi) - (a^2*Sin[(P
i*(a + b*x)^2)/2])/(b^3*Pi) + (a*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi
) - ((a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rubi [A] time = 0.122512, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638}

$$\frac{a^3 \text{FresnelC}(a + bx)}{3b^3} - \frac{a^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} - \frac{aS(a + bx)}{\pi b^3} + \frac{a(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^3} - \frac{(a + bx)^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{3\pi b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[a + b*x],x]
```

```
[Out] (-2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) + (a^3*FresnelC[a + b*x])/(3*b^3)
+ (x^3*FresnelC[a + b*x])/3 - (a*FresnelS[a + b*x])/(b^3*Pi) - (a^2*Sin[(P
i*(a + b*x)^2)/2])/(b^3*Pi) + (a*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi
) - ((a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])*(b_.)^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] := Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])]^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
```

] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]/(f*Rt[d, 2])), x] /; FreeQ[{d, e, f}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)ⁿ])*(b_.))^p*(x_)^m, x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, xⁿ], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3386

Int[Cos[(c_.) + (d_.)*(x_)ⁿ]*((e_.)*(x_)^m), x_Symbol] := Simp[(e^{n - 1}*(e*x)^{m - n + 1}*Sin[c + d*xⁿ]/(d*n), x] - Dist[(eⁿ*(m - n + 1))/(d*n), Int[(e*x)^{m - n}*Sin[c + d*xⁿ], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_)²), x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]/(f*Rt[d, 2])), x] /; FreeQ[{d, e, f}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_)^m)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^{m - 1}*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x^2 C(a+bx) dx &= \frac{1}{3} x^3 C(a+bx) - \frac{1}{3} b \int x^3 \cos\left(\frac{1}{2} \pi(a+bx)^2\right) dx \\
&= \frac{1}{3} x^3 C(a+bx) - \frac{\text{Subst}\left(\int\left(-a^3 \cos\left(\frac{\pi x^2}{2}\right) + 3a^2 x \cos\left(\frac{\pi x^2}{2}\right) - 3ax^2 \cos\left(\frac{\pi x^2}{2}\right) + x^3 \cos\left(\frac{\pi x^2}{2}\right)\right) dx, x, a+bx\right)}{3b^3} \\
&= \frac{1}{3} x^3 C(a+bx) - \frac{\text{Subst}\left(\int x^3 \cos\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{3b^3} + \frac{a \text{Subst}\left(\int x^2 \cos\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{b^3} - \frac{a^3 \text{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{b^3} \\
&= \frac{a^3 C(a+bx)}{3b^3} + \frac{1}{3} x^3 C(a+bx) + \frac{a(a+bx) \sin\left(\frac{1}{2} \pi(a+bx)^2\right)}{b^3 \pi} - \frac{\text{Subst}\left(\int x \cos\left(\frac{\pi x}{2}\right) dx, x, (a+bx)^2\right)}{6b^3} \\
&= \frac{a^3 C(a+bx)}{3b^3} + \frac{1}{3} x^3 C(a+bx) - \frac{aS(a+bx)}{b^3 \pi} - \frac{a^2 \sin\left(\frac{1}{2} \pi(a+bx)^2\right)}{b^3 \pi} + \frac{a(a+bx) \sin\left(\frac{1}{2} \pi(a+bx)^2\right)}{b^3 \pi} \\
&= -\frac{2 \cos\left(\frac{1}{2} \pi(a+bx)^2\right)}{3b^3 \pi^2} + \frac{a^3 C(a+bx)}{3b^3} + \frac{1}{3} x^3 C(a+bx) - \frac{aS(a+bx)}{b^3 \pi} - \frac{a^2 \sin\left(\frac{1}{2} \pi(a+bx)^2\right)}{b^3 \pi} + \frac{a(a+bx) \sin\left(\frac{1}{2} \pi(a+bx)^2\right)}{b^3 \pi}
\end{aligned}$$

Mathematica [A] time = 0.29704, size = 116, normalized size = 0.78

$$\frac{-\pi^2 (a^3 + b^3 x^3) \text{FresnelC}(a+bx) + \pi a^2 \sin\left(\frac{1}{2} \pi(a+bx)^2\right) + \pi b^2 x^2 \sin\left(\frac{1}{2} \pi(a+bx)^2\right) + 3\pi a S(a+bx) - \pi abx \sin\left(\frac{1}{2} \pi(a+bx)^2\right)}{3\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*FresnelC[a + b*x],x]

[Out] $-(2*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2] - \text{Pi}^2*(a^3 + b^3*x^3)*\text{FresnelC}[a + b*x] + 3*a*\text{Pi}*\text{FresnelS}[a + b*x] + a^2*\text{Pi}*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2] - a*b*\text{Pi}*x*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2] + b^2*\text{Pi}*x^2*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(3*b^3*\text{Pi}^2)$

Maple [A] time = 0.052, size = 122, normalized size = 0.8

$$\frac{1}{b^3} \left(\frac{b^3 x^3 \text{FresnelC}(bx+a)}{3} - \frac{(bx+a)^2}{3\pi} \sin\left(\frac{\pi (bx+a)^2}{2}\right) - \frac{2}{3\pi^2} \cos\left(\frac{\pi (bx+a)^2}{2}\right) + \frac{a(bx+a)}{\pi} \sin\left(\frac{\pi (bx+a)^2}{2}\right) - \frac{a^3}{3\pi} \cos\left(\frac{\pi (bx+a)^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*FresnelC(b*x+a),x)

[Out] $\frac{1}{b^3} \left(\frac{1}{3} b^3 x^3 \text{FresnelC}(b*x+a) - \frac{1}{3} \text{Pi} (b*x+a)^2 \sin\left(\frac{1}{2} \text{Pi} (b*x+a)^2\right) - \frac{2}{3} \text{Pi}^2 \cos\left(\frac{1}{2} \text{Pi} (b*x+a)^2\right) + \frac{a}{\text{Pi}} (b*x+a) \sin\left(\frac{1}{2} \text{Pi} (b*x+a)^2\right) - \frac{a}{\text{Pi}} \text{FresnelS}(b*x+a) - \frac{a^2}{\text{Pi}} \sin\left(\frac{1}{2} \text{Pi} (b*x+a)^2\right) + \frac{1}{3} a^3 \text{FresnelC}(b*x+a) \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnelc(b*x+a),x, algorithm="maxima")`

[Out] `integrate(x^2*fresnelc(b*x + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \text{fresnelc}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnelc(b*x+a),x, algorithm="fricas")`

[Out] `integral(x^2*fresnelc(b*x + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 C(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*fresnelc(b*x+a),x)`

[Out] `Integral(x**2*fresnelc(a + b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc(b*x + a), x)
```

3.136 $\int x \text{FresnelC}(a + bx) dx$

Optimal. Leaf size=95

$$-\frac{a^2 \text{FresnelC}(a + bx)}{2b^2} + \frac{S(a + bx)}{2\pi b^2} + \frac{a \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^2} - \frac{(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^2} + \frac{1}{2}x^2 \text{FresnelC}(a + bx)$$

[Out] $-(a^2 \text{FresnelC}[a + b*x])/(2*b^2) + (x^2 \text{FresnelC}[a + b*x])/2 + \text{FresnelS}[a + b*x]/(2*b^2*\text{Pi}) + (a*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(b^2*\text{Pi}) - ((a + b*x)*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(2*b^2*\text{Pi})$

Rubi [A] time = 0.0707863, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351}

$$-\frac{a^2 \text{FresnelC}(a + bx)}{2b^2} + \frac{S(a + bx)}{2\pi b^2} + \frac{a \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b^2} - \frac{(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{2\pi b^2} + \frac{1}{2}x^2 \text{FresnelC}(a + bx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{FresnelC}[a + b*x], x]$

[Out] $-(a^2 \text{FresnelC}[a + b*x])/(2*b^2) + (x^2 \text{FresnelC}[a + b*x])/2 + \text{FresnelS}[a + b*x]/(2*b^2*\text{Pi}) + (a*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(b^2*\text{Pi}) - ((a + b*x)*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(2*b^2*\text{Pi})$

Rule 6429

$\text{Int}[\text{FresnelC}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] :> \text{Simp}[\frac{(c + d*x)^{(m + 1)}*\text{FresnelC}[a + b*x]}{(d*(m + 1))}, x] - \text{Dist}[b/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2], x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 3434

$\text{Int}[\frac{(a_.) + \text{Cos}[(c_.) + (d_.)*((e_.) + (f_.)*(x_.))^{(n_.)}]*(b_.))^{(p_.)}*((g_.) + (h_.)*(x_.))^{(m_.)}, x_Symbol] :> \text{Module}[\{k = \text{If}[\text{FractionQ}[n], \text{Denominator}[n], 1]\}, \text{Dist}[k/f^{(m + 1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(a + b*\text{Cos}[c + d*x^{(k*n)}]]^{(p)}, x^{(k - 1)}*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^{(1/k)}], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)ⁿ])*(b_.))^p*(x_)^m, x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*cos[c + d*x])^p, x], x, xⁿ], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3386

Int[Cos[(c_.) + (d_.)*(x_)ⁿ]*(e_.)*(x_)^m, x_Symbol] := Simp[(e^{n - 1}*(e*x)^{m - n + 1}*Sin[c + d*xⁿ]/(d*n), x] - Dist[(eⁿ*(m - n + 1))/(d*n), Int[(e*x)^{m - n}*Sin[c + d*xⁿ], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int xC(a+bx) dx &= \frac{1}{2}x^2C(a+bx) - \frac{1}{2}b \int x^2 \cos\left(\frac{1}{2}\pi(a+bx)^2\right) dx \\
&= \frac{1}{2}x^2C(a+bx) - \frac{\text{Subst}\left(\int\left(a^2\cos\left(\frac{\pi x^2}{2}\right) - 2ax\cos\left(\frac{\pi x^2}{2}\right) + x^2\cos\left(\frac{\pi x^2}{2}\right)\right) dx, x, a+bx\right)}{2b^2} \\
&= \frac{1}{2}x^2C(a+bx) - \frac{\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{2b^2} + \frac{a\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{b^2} - \frac{a^2\text{Subst}\left(\int\cos\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{2b^2} \\
&= -\frac{a^2C(a+bx)}{2b^2} + \frac{1}{2}x^2C(a+bx) - \frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} + \frac{a\text{Subst}\left(\int\cos\left(\frac{\pi x}{2}\right) dx, x, (a+bx)^2\right)}{2b^2} \\
&= -\frac{a^2C(a+bx)}{2b^2} + \frac{1}{2}x^2C(a+bx) + \frac{S(a+bx)}{2b^2\pi} + \frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} - \frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}
\end{aligned}$$

Mathematica [A] time = 0.148444, size = 59, normalized size = 0.62

$$\frac{(\pi b^2 x^2 - \pi a^2) \text{FresnelC}(a+bx) + S(a+bx) + (a-bx) \sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*FresnelC[a + b*x], x]

[Out] ((-(a^2*Pi) + b^2*Pi*x^2)*FresnelC[a + b*x] + FresnelS[a + b*x] + (a - b*x)*Sin[(Pi*(a + b*x)^2]/2))/(2*b^2*Pi)

Maple [A] time = 0.052, size = 79, normalized size = 0.8

$$\frac{1}{b^2} \left(\text{FresnelC}(bx+a) \left(\frac{(bx+a)^2}{2} - a(bx+a) \right) - \frac{bx+a}{2\pi} \sin\left(\frac{\pi(bx+a)^2}{2}\right) + \frac{\text{FresnelS}(bx+a)}{2\pi} + \frac{a}{\pi} \sin\left(\frac{\pi(bx+a)^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*FresnelC(b*x+a), x)

[Out] 1/b^2*(FresnelC(b*x+a)*(1/2*(b*x+a)^2-a*(b*x+a))-1/2/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)+1/2/Pi*FresnelS(b*x+a)+a/Pi*sin(1/2*Pi*(b*x+a)^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*fresnelc(b*x+a),x, algorithm="maxima")`

[Out] `integrate(x*fresnelc(b*x + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x \operatorname{fresnelc}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*fresnelc(b*x+a),x, algorithm="fricas")`

[Out] `integral(x*fresnelc(b*x + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x C(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*fresnelc(b*x+a),x)`

[Out] `Integral(x*fresnelc(a + b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*fresnelc(b*x + a), x)
```

3.137 $\int \text{FresnelC}(a + bx) dx$

Optimal. Leaf size=37

$$\frac{(a + bx)\text{FresnelC}(a + bx)}{b} - \frac{\sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b}$$

[Out] $((a + b*x)*\text{FresnelC}[a + b*x])/b - \text{Sin}[(\text{Pi}*(a + b*x)^2)/2]/(b*\text{Pi})$

Rubi [A] time = 0.0067759, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6419}

$$\frac{(a + bx)\text{FresnelC}(a + bx)}{b} - \frac{\sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{FresnelC}[a + b*x], x]$

[Out] $((a + b*x)*\text{FresnelC}[a + b*x])/b - \text{Sin}[(\text{Pi}*(a + b*x)^2)/2]/(b*\text{Pi})$

Rule 6419

$\text{Int}[\text{FresnelC}[(a_.) + (b_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[\frac{(a + b*x)*\text{FresnelC}[a + b*x]}{b}, x] - \text{Simp}[\frac{\text{Sin}[(\text{Pi}*(a + b*x)^2)/2]}{(b*\text{Pi})}, x] \text{ /; } \text{FreeQ}\{a, b\}, x]$

Rubi steps

$$\int C(a + bx) dx = \frac{(a + bx)C(a + bx)}{b} - \frac{\sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{b\pi}$$

Mathematica [B] time = 0.0290167, size = 90, normalized size = 2.43

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx + \frac{1}{2}\pi b^2 x^2\right)}{\pi b} - \frac{\cos\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx + \frac{1}{2}\pi b^2 x^2\right)}{\pi b} + x\text{FresnelC}(a + bx) + \frac{a\text{FresnelC}(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[a + b*x],x]

[Out] (a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^2)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi)

Maple [A] time = 0.049, size = 34, normalized size = 0.9

$$\frac{1}{b} \left(\text{FresnelC}(bx + a)(bx + a) - \frac{1}{\pi} \sin\left(\frac{\pi (bx + a)^2}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x+a),x)

[Out] 1/b*(FresnelC(b*x+a)*(b*x+a)-1/Pi*sin(1/2*Pi*(b*x+a)^2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a),x, algorithm="maxima")

[Out] integrate(fresnelc(b*x + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{fresnelc}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a),x, algorithm="fricas")

[Out] `integral(fresnelc(b*x + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int C(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x+a),x)`

[Out] `Integral(fresnelc(a + b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x+a),x, algorithm="giac")`

[Out] `integrate(fresnelc(b*x + a), x)`

$$3.138 \quad \int \frac{\text{FresnelC}(a+bx)}{x} dx$$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(a+bx)}{x}, x\right)$$

[Out] Unintegrable[FresnelC[a + b*x]/x, x]

Rubi [A] time = 0.0103739, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[a + b*x]/x,x]

[Out] Defer[Int][FresnelC[a + b*x]/x, x]

Rubi steps

$$\int \frac{C(a+bx)}{x} dx = \int \frac{C(a+bx)}{x} dx$$

Mathematica [A] time = 0.0175946, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[a + b*x]/x,x]

[Out] Integrate[FresnelC[a + b*x]/x, x]

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x+a)/x,x)

[Out] int(FresnelC(b*x+a)/x,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)/x,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x + a)/x, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)/x,x, algorithm="fricas")

[Out] integral(fresnelc(b*x + a)/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x,x)
```

```
[Out] Integral(fresnelc(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a)/x, x)
```


$$3.139 \quad \int \frac{\mathbf{FresnelC}(a+bx)}{x^2} dx$$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\mathbf{FresnelC}(a+bx)}{x^2}, x\right)$$

[Out] Unintegrable[FresnelC[a + b*x]/x^2, x]

Rubi [A] time = 0.0109038, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\mathbf{FresnelC}(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[a + b*x]/x^2,x]

[Out] Defer[Int][FresnelC[a + b*x]/x^2, x]

Rubi steps

$$\int \frac{C(a+bx)}{x^2} dx = \int \frac{C(a+bx)}{x^2} dx$$

Mathematica [A] time = 1.46409, size = 0, normalized size = 0.

$$\int \frac{\mathbf{FresnelC}(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[a + b*x]/x^2,x]

[Out] Integrate[FresnelC[a + b*x]/x^2, x]

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x+a)/x^2,x)

[Out] int(FresnelC(b*x+a)/x^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)/x^2,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x + a)/x^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)/x^2,x, algorithm="fricas")

[Out] integral(fresnelc(b*x + a)/x^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C(a + bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a)/x^2, x)
```

3.140 $\int x^7 \text{FresnelC}(bx)^2 dx$

Optimal. Leaf size=253

$$-\frac{x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} + \frac{35x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^3 b^5} - \frac{7x^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{105x \text{FresnelC}(bx)^2}{4\pi b^5}$$

[Out] $(-105*x^2)/(16*b^6*\text{Pi}^4) + (7*x^6)/(48*b^2*\text{Pi}^2) + (55*x^2*\text{Cos}[b^2*\text{Pi}*x^2])/(16*b^6*\text{Pi}^4) - (x^6*\text{Cos}[b^2*\text{Pi}*x^2])/(16*b^2*\text{Pi}^2) + (105*x*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(4*b^7*\text{Pi}^4) - (7*x^5*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(4*b^3*\text{Pi}^2) - (105*\text{FresnelC}[b*x]^2)/(8*b^8*\text{Pi}^4) + (x^8*\text{FresnelC}[b*x]^2)/8 + (35*x^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(4*b^5*\text{Pi}^3) - (x^7*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(4*b*\text{Pi}) - (10*\text{Sin}[b^2*\text{Pi}*x^2])/(b^8*\text{Pi}^5) + (5*x^4*\text{Sin}[b^2*\text{Pi}*x^2])/(8*b^4*\text{Pi}^3)$

Rubi [A] time = 0.422278, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.1$, Rules used = {6431, 6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637, 3309}

$$-\frac{x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} + \frac{35x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^3 b^5} - \frac{7x^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{105x \text{FresnelC}(bx)^2}{4\pi b^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^7*\text{FresnelC}[b*x]^2, x]$

[Out] $(-105*x^2)/(16*b^6*\text{Pi}^4) + (7*x^6)/(48*b^2*\text{Pi}^2) + (55*x^2*\text{Cos}[b^2*\text{Pi}*x^2])/(16*b^6*\text{Pi}^4) - (x^6*\text{Cos}[b^2*\text{Pi}*x^2])/(16*b^2*\text{Pi}^2) + (105*x*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(4*b^7*\text{Pi}^4) - (7*x^5*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(4*b^3*\text{Pi}^2) - (105*\text{FresnelC}[b*x]^2)/(8*b^8*\text{Pi}^4) + (x^8*\text{FresnelC}[b*x]^2)/8 + (35*x^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(4*b^5*\text{Pi}^3) - (x^7*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(4*b*\text{Pi}) - (10*\text{Sin}[b^2*\text{Pi}*x^2])/(b^8*\text{Pi}^5) + (5*x^4*\text{Sin}[b^2*\text{Pi}*x^2])/(8*b^4*\text{Pi}^3)$

Rule 6431

$\text{Int}[\text{FresnelC}[(b_)*(x_)]^2*(x_)^{(m_)}, x_Symbol] := \text{Simp}[(x^{(m+1)}*\text{FresnelC}[b*x]^2)/(m+1), x] - \text{Dist}[(2*b)/(m+1), \text{Int}[x^{(m+1)}*\text{Cos}[(\text{Pi}*b^2*x^2)/2]*\text{FresnelC}[b*x], x], x] /; \text{FreeQ}[b, x] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{NeQ}[m, -1]$

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)^(m_) * Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x^(
(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.)], x_Symbol] := Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] := Simp[x/2, x] - Simp[Sin[2*c
+ d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

$m + 1)/n], 0))$

Rule 3296

$\text{Int}[(c + d \cdot x)^m \sin(e + f \cdot x), x_Symbol] \rightarrow -\text{Simp}[(c + d \cdot x)^m \cos(e + f \cdot x)/f, x] + \text{Dist}[(d \cdot m)/f, \text{Int}[(c + d \cdot x)^{m-1} \cos(e + f \cdot x), x], x] /;$ $\text{FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{GtQ}[m, 0]$

Rule 2637

$\text{Int}[\sin(\pi/2 + (c + d \cdot x)), x_Symbol] \rightarrow \text{Simp}[\sin(c + d \cdot x)/d, x] /;$ $\text{FreeQ}\{c, d, x\}$

Rule 3309

$\text{Int}[(c + d \cdot x)^m \sin(e + (f \cdot x)/2)^2, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[(c + d \cdot x)^m, x], x] - \text{Dist}[1/2, \text{Int}[(c + d \cdot x)^m \cos[2e + f \cdot x], x], x] /;$ $\text{FreeQ}\{c, d, e, f, m, x\}$

Rubi steps

$$\begin{aligned} \int x^7 C(bx)^2 dx &= \frac{1}{8} x^8 C(bx)^2 - \frac{1}{4} b \int x^8 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx) dx \\ &= \frac{1}{8} x^8 C(bx)^2 - \frac{x^7 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{4b\pi} + \frac{\int x^7 \sin(b^2 \pi x^2) dx}{8\pi} + \frac{7 \int x^6 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{4b\pi} \\ &= -\frac{7x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{4b^3 \pi^2} + \frac{1}{8} x^8 C(bx)^2 - \frac{x^7 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{4b\pi} + \frac{35 \int x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx) dx}{4b^3 \pi^2} + \dots \\ &= -\frac{x^6 \cos(b^2 \pi x^2)}{16b^2 \pi^2} - \frac{7x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{4b^3 \pi^2} + \frac{1}{8} x^8 C(bx)^2 + \frac{35x^3 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{4b^5 \pi^3} - \frac{x^7 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{4b\pi} \\ &= -\frac{x^6 \cos(b^2 \pi x^2)}{16b^2 \pi^2} + \frac{105x \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{4b^7 \pi^4} - \frac{7x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{4b^3 \pi^2} + \frac{1}{8} x^8 C(bx)^2 + \frac{35x^3 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{4b^5 \pi^3} \\ &= \frac{7x^6}{48b^2 \pi^2} + \frac{41x^2 \cos(b^2 \pi x^2)}{16b^6 \pi^4} - \frac{x^6 \cos(b^2 \pi x^2)}{16b^2 \pi^2} + \frac{105x \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{4b^7 \pi^4} - \frac{7x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{4b^3 \pi^2} + \dots \\ &= -\frac{105x^2}{16b^6 \pi^4} + \frac{7x^6}{48b^2 \pi^2} + \frac{55x^2 \cos(b^2 \pi x^2)}{16b^6 \pi^4} - \frac{x^6 \cos(b^2 \pi x^2)}{16b^2 \pi^2} + \frac{105x \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{4b^7 \pi^4} - \frac{7x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{4b^3 \pi^2} \\ &= -\frac{105x^2}{16b^6 \pi^4} + \frac{7x^6}{48b^2 \pi^2} + \frac{55x^2 \cos(b^2 \pi x^2)}{16b^6 \pi^4} - \frac{x^6 \cos(b^2 \pi x^2)}{16b^2 \pi^2} + \frac{105x \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{4b^7 \pi^4} - \frac{7x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{4b^3 \pi^2} \end{aligned}$$

Mathematica [A] time = 0.0172749, size = 253, normalized size = 1.

$$\frac{x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} + \frac{35x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^3 b^5} - \frac{7x^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{105x \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*FresnelC[b*x]^2,x]

[Out]
$$\begin{aligned} & (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) + (55*x^2*\text{Cos}[b^2*Pi*x^2]) \\ & / (16*b^6*Pi^4) - (x^6*\text{Cos}[b^2*Pi*x^2]) / (16*b^2*Pi^2) + (105*x*\text{Cos}[(b^2*Pi*x \\ & ^2)/2]*\text{FresnelC}[b*x]) / (4*b^7*Pi^4) - (7*x^5*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b* \\ & x]) / (4*b^3*Pi^2) - (105*\text{FresnelC}[b*x]^2) / (8*b^8*Pi^4) + (x^8*\text{FresnelC}[b*x]^ \\ & 2) / 8 + (35*x^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2]) / (4*b^5*Pi^3) - (x^7*\text{Fresn \\ & elC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2]) / (4*b*Pi) - (10*\text{Sin}[b^2*Pi*x^2]) / (b^8*Pi^5) + \\ & (5*x^4*\text{Sin}[b^2*Pi*x^2]) / (8*b^4*Pi^3) \end{aligned}$$

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int x^7 (\text{FresnelC}(bx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*FresnelC(b*x)^2,x)

[Out] int(x^7*FresnelC(b*x)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnelc(b*x)^2,x, algorithm="maxima")

[Out] integrate(x^7*fresnelc(b*x)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^7 \text{fresnelc}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnelc(b*x)^2,x, algorithm="fricas")

[Out] integral(x^7*fresnelc(b*x)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 C^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*fresnelc(b*x)**2,x)

[Out] Integral(x**7*fresnelc(b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*fresnelc(b*x)^2,x, algorithm="giac")

[Out] integrate(x^7*fresnelc(b*x)^2, x)

3.141 $\int x^6 \text{FresnelC}(bx)^2 dx$

Optimal. Leaf size=239

$$-\frac{2x^6 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi b} + \frac{48x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^3 b^5} - \frac{12x^4 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^2 b^3} + \frac{96 \text{FresnelC}(bx)^2}{7\pi^2 b^3}$$

[Out] $(-48*x)/(7*b^6*\text{Pi}^4) + (6*x^5)/(35*b^2*\text{Pi}^2) + (21*x*\text{Cos}[b^2*\text{Pi}*x^2])/(8*b^6*\text{Pi}^4) - (x^5*\text{Cos}[b^2*\text{Pi}*x^2])/(14*b^2*\text{Pi}^2) + (96*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(7*b^7*\text{Pi}^4) - (12*x^4*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(7*b^3*\text{Pi}^2) + (x^7*\text{FresnelC}[b*x]^2)/7 - (531*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(56*\text{Sqrt}[2]*b^7*\text{Pi}^4) + (48*x^2*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(7*b^5*\text{Pi}^3) - (2*x^6*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(7*b*\text{Pi}) + (17*x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(28*b^4*\text{Pi}^3)$

Rubi [A] time = 0.298269, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6431, 6455, 6463, 6461, 3358, 3352, 3385, 3392, 30, 3386}

$$-\frac{2x^6 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi b} + \frac{48x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^3 b^5} - \frac{12x^4 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^2 b^3} + \frac{96 \text{FresnelC}(bx)^2}{7\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^6*\text{FresnelC}[b*x]^2, x]$

[Out] $(-48*x)/(7*b^6*\text{Pi}^4) + (6*x^5)/(35*b^2*\text{Pi}^2) + (21*x*\text{Cos}[b^2*\text{Pi}*x^2])/(8*b^6*\text{Pi}^4) - (x^5*\text{Cos}[b^2*\text{Pi}*x^2])/(14*b^2*\text{Pi}^2) + (96*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(7*b^7*\text{Pi}^4) - (12*x^4*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(7*b^3*\text{Pi}^2) + (x^7*\text{FresnelC}[b*x]^2)/7 - (531*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(56*\text{Sqrt}[2]*b^7*\text{Pi}^4) + (48*x^2*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(7*b^5*\text{Pi}^3) - (2*x^6*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(7*b*\text{Pi}) + (17*x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(28*b^4*\text{Pi}^3)$

Rule 6431

$\text{Int}[\text{FresnelC}[(b_.)*(x_.)]^2*(x_.)^{(m_.)}, x_Symbol] \text{ :> } \text{Simp}[(x^{(m+1)}*\text{FresnelC}[b*x]^2)/(m+1), x] - \text{Dist}[(2*b)/(m+1), \text{Int}[x^{(m+1)}*\text{Cos}[(\text{Pi}*b^2*x^2)/2]*\text{FresnelC}[b*x], x], x] /; \text{FreeQ}[b, x] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{NeQ}[m, -1]$

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)*(x_)^(m_)], x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)*(x_)^(m_)]*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)*(x_)^2]*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])*(b_.)^(p_), x_Symbol] := Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_)]*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_), x_Symbol] := Dist[1/2, Int[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
```

$m, n\}, x]$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \text{ :> Simp}[x^{(m+1)}/(m+1), x] \text{ /; FreeQ}[m, x] \ \&\& \ \text{N eQ}[m, -1]$

Rule 3386

$\text{Int}[\text{Cos}[(c_.) + (d_.)*(x_)^{(n_.)}]*((e_.)*(x_))^{(m_.)}, x_Symbol] \text{ :> Simp}[(e^{(n-1)}*(e*x)^{(m-n+1)}*\text{Sin}[c+d*x^n])/(d*n), x] - \text{Dist}[(e^n*(m-n+1))/(d*n), \text{Int}[(e*x)^{(m-n)}*\text{Sin}[c+d*x^n], x], x] \text{ /; FreeQ}\{c, d, e\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[n, m+1]$

Rubi steps

$$\begin{aligned}
 \int x^6 C(bx)^2 dx &= \frac{1}{7} x^7 C(bx)^2 - \frac{1}{7} (2b) \int x^7 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx) dx \\
 &= \frac{1}{7} x^7 C(bx)^2 - \frac{2x^6 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} + \frac{\int x^6 \sin(b^2 \pi x^2) dx}{7\pi} + \frac{12 \int x^5 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{7b\pi} \\
 &= -\frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} - \frac{12x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{7b^3 \pi^2} + \frac{1}{7} x^7 C(bx)^2 - \frac{2x^6 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} + \frac{48 \int x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{7b\pi} \\
 &= -\frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} - \frac{12x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{7b^3 \pi^2} + \frac{1}{7} x^7 C(bx)^2 + \frac{48x^2 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^5 \pi^3} - \frac{2x^6 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} \\
 &= \frac{6x^5}{35b^2 \pi^2} + \frac{111x \cos(b^2 \pi x^2)}{56b^6 \pi^4} - \frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} + \frac{96 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{7b^7 \pi^4} - \frac{12x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{7b^3 \pi^2} + \frac{48x^2 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^5 \pi^3} - \frac{2x^6 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} \\
 &= \frac{6x^5}{35b^2 \pi^2} + \frac{21x \cos(b^2 \pi x^2)}{8b^6 \pi^4} - \frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} + \frac{96 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{7b^7 \pi^4} - \frac{12x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{7b^3 \pi^2} + \frac{48x^2 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^5 \pi^3} - \frac{2x^6 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} \\
 &= -\frac{48x}{7b^6 \pi^4} + \frac{6x^5}{35b^2 \pi^2} + \frac{21x \cos(b^2 \pi x^2)}{8b^6 \pi^4} - \frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} + \frac{96 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{7b^7 \pi^4} - \frac{12x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{7b^3 \pi^2} + \frac{48x^2 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^5 \pi^3} - \frac{2x^6 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi} \\
 &= -\frac{48x}{7b^6 \pi^4} + \frac{6x^5}{35b^2 \pi^2} + \frac{21x \cos(b^2 \pi x^2)}{8b^6 \pi^4} - \frac{x^5 \cos(b^2 \pi x^2)}{14b^2 \pi^2} + \frac{96 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{7b^7 \pi^4} - \frac{12x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{7b^3 \pi^2} + \frac{48x^2 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b^5 \pi^3} - \frac{2x^6 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{7b\pi}
 \end{aligned}$$

Mathematica [A] time = 0.240461, size = 170, normalized size = 0.71

$$\frac{80\pi^4 b^7 x^7 \text{FresnelC}(bx)^2 - 160 \text{FresnelC}(bx) \left(\pi b^2 x^2 (\pi^2 b^4 x^4 - 24) \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 6 (\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2} \pi b^2 x^2\right) \right) + 2bx}{560\pi^4 b^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^6*FresnelC[b*x]^2,x]

[Out] (80*b^7*Pi^4*x^7*FresnelC[b*x]^2 - 2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 160*FresnelC[b*x]*(6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] + 2*(-960 + 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))) / (560*b^7*Pi^4)

Maple [A] time = 0.077, size = 324, normalized size = 1.4

$$\frac{1}{b^7} \left(\frac{b^7 x^7 (\text{FresnelC}(bx))^2}{7} - 2 \text{FresnelC}(bx) \left(\frac{1}{7} \frac{b^6 x^6 \sin(1/2 b^2 \pi x^2)}{\pi} - \frac{6}{7} \frac{1}{\pi} \left(-\frac{x^4 b^4 \cos(1/2 b^2 \pi x^2)}{\pi} + \frac{1}{\pi} \left(\frac{b^2 x^2 \sin}{\pi} \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*FresnelC(b*x)^2,x)

[Out] 1/b^7*(1/7*b^7*x^7*FresnelC(b*x)^2-2*FresnelC(b*x)*(1/7/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/7/Pi*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*Pi^2*b^5*x^5-8*b*x)+6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))+1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnelc(b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^6 \text{fresnelc}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^6*fresnelc(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 C^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**6*fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnelc(b*x)^2, x)
```

3.142 $\int x^5 \text{FresnelC}(bx)^2 dx$

Optimal. Leaf size=265

$$\frac{5ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right)}{8\pi^3 b^4} + \frac{5ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)}{8\pi^3 b^4} - \frac{5 \text{FresnelC}(bx)^2}{3\pi b}$$

[Out] $(5x^4)/(24b^2\pi^2) + (11\cos[b^2\pi x^2])/(6b^6\pi^4) - (x^4\cos[b^2\pi x^2])/(12b^2\pi^2) - (5x^3\cos[(b^2\pi x^2)/2]*\text{FresnelC}[bx])/(3b^3\pi^2) + (x^6*\text{FresnelC}[bx]^2)/6 - (5*\text{FresnelC}[bx]*\text{FresnelS}[bx])/(2b^6\pi^3) - (((5*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2*\pi*x^2])/(b^4*\pi^3) + (((5*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2*\pi*x^2])/(b^4*\pi^3) + (5*x*\text{FresnelC}[bx]*\text{Sin}[(b^2*\pi*x^2)/2])/(b^5*\pi^3) - (x^5*\text{FresnelC}[bx]*\text{Sin}[(b^2*\pi*x^2)/2])/(3*b*\pi) + (7*x^2*\text{Sin}[b^2*\pi*x^2])/(12*b^4*\pi^3)$

Rubi [A] time = 0.288327, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6431, 6455, 6463, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$\frac{5ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3 b^4} + \frac{5ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi^3 b^4} - \frac{5 \text{FresnelC}(bx)S(bx)}{2\pi^3 b^6} - \frac{x^5 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi b}$$

Antiderivative was successfully verified.

[In] Int[x^5*FresnelC[b*x]^2,x]

[Out] $(5x^4)/(24b^2\pi^2) + (11\cos[b^2\pi x^2])/(6b^6\pi^4) - (x^4\cos[b^2\pi x^2])/(12b^2\pi^2) - (5x^3\cos[(b^2\pi x^2)/2]*\text{FresnelC}[bx])/(3b^3\pi^2) + (x^6*\text{FresnelC}[bx]^2)/6 - (5*\text{FresnelC}[bx]*\text{FresnelS}[bx])/(2b^6\pi^3) - (((5*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2*\pi*x^2])/(b^4*\pi^3) + (((5*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2*\pi*x^2])/(b^4*\pi^3) + (5*x*\text{FresnelC}[bx]*\text{Sin}[(b^2*\pi*x^2)/2])/(b^5*\pi^3) - (x^5*\text{FresnelC}[bx]*\text{Sin}[(b^2*\pi*x^2)/2])/(3*b*\pi) + (7*x^2*\text{Sin}[b^2*\pi*x^2])/(12*b^4*\pi^3)$

Rule 6431

Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*FresnelC[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/

2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6455

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 6463

Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 6447

Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(b*Pi*FresnelC[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(I*d*x^2)])]/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, I*d*x^2)])]/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

$m + 1)/n], 0]))$

Rule 3309

$\text{Int}[\left((c_{.}) + (d_{.}) \cdot (x_{.})\right)^{(m_{.})} \cdot \sin\left[\left(e_{.}\right) + \left(\left(f_{.}\right) \cdot (x_{.})\right)/2\right]^2, x_{\text{Symbol}}] \text{ :> } \text{Dist}\left[1/2, \text{Int}\left[(c + d \cdot x)^m, x\right], x\right] - \text{Dist}\left[1/2, \text{Int}\left[(c + d \cdot x)^m \cdot \cos[2 \cdot e + f \cdot x], x\right], x\right] \text{ /; FreeQ}\left[\{c, d, e, f, m\}, x\right]$

Rule 30

$\text{Int}\left[(x_{.})^{(m_{.})}, x_{\text{Symbol}}\right] \text{ :> } \text{Simp}\left[x^{(m + 1)} / (m + 1), x\right] \text{ /; FreeQ}\left[m, x\right] \ \&\& \ \text{NeQ}\left[m, -1\right]$

Rule 3296

$\text{Int}\left[\left((c_{.}) + (d_{.}) \cdot (x_{.})\right)^{(m_{.})} \cdot \sin\left[\left(e_{.}\right) + \left(f_{.}\right) \cdot (x_{.})\right], x_{\text{Symbol}}\right] \text{ :> } -\text{Simp}\left[\left((c + d \cdot x)^m \cdot \cos[e + f \cdot x]\right) / f, x\right] + \text{Dist}\left[(d \cdot m) / f, \text{Int}\left[(c + d \cdot x)^{(m - 1)} \cdot \cos[e + f \cdot x], x\right], x\right] \text{ /; FreeQ}\left[\{c, d, e, f\}, x\right] \ \&\& \ \text{GtQ}\left[m, 0\right]$

Rubi steps

$$\begin{aligned} \int x^5 C(bx)^2 dx &= \frac{1}{6} x^6 C(bx)^2 - \frac{1}{3} b \int x^6 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx) dx \\ &= \frac{1}{6} x^6 C(bx)^2 - \frac{x^5 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi} + \frac{\int x^5 \sin(b^2 \pi x^2) dx}{6\pi} + \frac{5 \int x^4 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{3b\pi} \\ &= -\frac{5x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{3b^3 \pi^2} + \frac{1}{6} x^6 C(bx)^2 - \frac{x^5 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi} + \frac{5 \int x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx) dx}{b^3 \pi^2} + \frac{5 \int x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right) C(bx) dx}{b^3 \pi^2} \\ &= -\frac{x^4 \cos(b^2 \pi x^2)}{12b^2 \pi^2} - \frac{5x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{3b^3 \pi^2} + \frac{1}{6} x^6 C(bx)^2 + \frac{5x C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{b^5 \pi^3} - \frac{x^5 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi} \\ &= -\frac{x^4 \cos(b^2 \pi x^2)}{12b^2 \pi^2} - \frac{5x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{3b^3 \pi^2} + \frac{1}{6} x^6 C(bx)^2 - \frac{5C(bx)S(bx)}{2b^6 \pi^3} - \frac{5ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2} ib^2 \pi x^2\right)}{8b^4 \pi^3} \\ &= \frac{5x^4}{24b^2 \pi^2} + \frac{17 \cos(b^2 \pi x^2)}{12b^6 \pi^4} - \frac{x^4 \cos(b^2 \pi x^2)}{12b^2 \pi^2} - \frac{5x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{3b^3 \pi^2} + \frac{1}{6} x^6 C(bx)^2 - \frac{5C(bx)S(bx)}{2b^6 \pi^3} - \frac{5ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2} ib^2 \pi x^2\right)}{8b^4 \pi^3} \\ &= \frac{5x^4}{24b^2 \pi^2} + \frac{11 \cos(b^2 \pi x^2)}{6b^6 \pi^4} - \frac{x^4 \cos(b^2 \pi x^2)}{12b^2 \pi^2} - \frac{5x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{3b^3 \pi^2} + \frac{1}{6} x^6 C(bx)^2 - \frac{5C(bx)S(bx)}{2b^6 \pi^3} - \frac{5ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2} ib^2 \pi x^2\right)}{8b^4 \pi^3} \end{aligned}$$

Mathematica [F] time = 0.197811, size = 0, normalized size = 0.

$$\int x^5 \text{FresnelC}(bx)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x^5*FresnelC[b*x]^2,x]

[Out] Integrate[x^5*FresnelC[b*x]^2, x]

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int x^5 (\text{FresnelC}(bx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*FresnelC(b*x)^2,x)

[Out] int(x^5*FresnelC(b*x)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*fresnelc(b*x)^2,x, algorithm="maxima")

[Out] integrate(x^5*fresnelc(b*x)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^5 \text{fresnelc}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnelc(b*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 C^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**5*fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnelc(b*x)^2, x)
```

3.143 $\int x^4 \text{FresnelC}(bx)^2 dx$

Optimal. Leaf size=177

$$-\frac{2x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi b} + \frac{16 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5} - \frac{8x^2 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3} - \frac{43S(\sqrt{2}bx)}{20\sqrt{2}\pi^3 b^5} +$$

[Out] $(4*x^3)/(15*b^2*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (8*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x]^2)/5 - (43*FresnelS[Sqrt[2]*b*x])/(20*Sqrt[2]*b^5*Pi^3) + (16*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) - (2*x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi) + (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)$

Rubi [A] time = 0.187988, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$, Rules used = {6431, 6455, 6463, 6453, 3351, 3392, 30, 3386, 3385}

$$-\frac{2x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi b} + \frac{16 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5} - \frac{8x^2 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3} - \frac{43S(\sqrt{2}bx)}{20\sqrt{2}\pi^3 b^5} +$$

Antiderivative was successfully verified.

[In] Int[x^4*FresnelC[b*x]^2,x]

[Out] $(4*x^3)/(15*b^2*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (8*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x]^2)/5 - (43*FresnelS[Sqrt[2]*b*x])/(20*Sqrt[2]*b^5*Pi^3) + (16*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) - (2*x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi) + (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)$

Rule 6431

Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*FresnelC[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6455

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(m - 1)*FresnelC[b*x]^2, x], x] /; FreeQ[b, x] && IntegerQ[m] && m > 1)

$m - 2) * \sin[d * x^2] * \text{FresnelC}[b * x], x], x] - \text{Dist}[b / (4 * d), \text{Int}[x^{(m - 1)} * \sin[2 * d * x^2], x], x]) /;$ FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2 * b^4) / 4] && IGtQ[m, 1]

Rule 6463

$\text{Int}[\text{FresnelC}[(b \cdot x)] * (x)^{(m)} * \sin[(d \cdot x)^2], x_Symbol] :> -\text{Simp}[(x^{(m - 1)} * \cos[d * x^2] * \text{FresnelC}[b * x]) / (2 * d), x] + (\text{Dist}[(m - 1) / (2 * d), \text{Int}[x^{(m - 2)} * \cos[d * x^2] * \text{FresnelC}[b * x], x], x] + \text{Dist}[b / (2 * d), \text{Int}[x^{(m - 1)} * \cos[d * x^2]^2, x], x]) /;$ FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2 * b^4) / 4] && IGtQ[m, 1]

Rule 6453

$\text{Int}[\cos[(d \cdot x)^2] * \text{FresnelC}[(b \cdot x)] * (x), x_Symbol] :> \text{Simp}[(\sin[d * x^2] * \text{FresnelC}[b * x]) / (2 * d), x] - \text{Dist}[b / (4 * d), \text{Int}[\sin[2 * d * x^2], x], x] /;$ FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2 * b^4) / 4]

Rule 3351

$\text{Int}[\sin[(d \cdot x) * ((e \cdot x) + (f \cdot x))^2], x_Symbol] :> \text{Simp}[(\text{Sqrt}[Pi / 2] * \text{FresnelS}[\text{Sqrt}[2 / Pi] * \text{Rt}[d, 2] * (e + f * x)]) / (f * \text{Rt}[d, 2]), x] /;$ FreeQ[{d, e, f}, x]

Rule 3392

$\text{Int}[\cos[(a \cdot x) + ((b \cdot x)^{(n)}) / 2]^2 * (x)^{(m)}, x_Symbol] :> \text{Dist}[1 / 2, \text{Int}[x^m, x], x] + \text{Dist}[1 / 2, \text{Int}[x^m * \cos[2 * a + b * x^n], x], x] /;$ FreeQ[{a, b, m, n}, x]

Rule 30

$\text{Int}[(x)^{(m)}, x_Symbol] :> \text{Simp}[x^{(m + 1)} / (m + 1), x] /;$ FreeQ[m, x] && NeQ[m, -1]

Rule 3386

$\text{Int}[\cos[(c \cdot x) + (d \cdot x)^{(n)}] * ((e \cdot x)^{(m)}), x_Symbol] :> \text{Simp}[(e^{(n - 1)} * (e * x)^{(m - n + 1)} * \sin[c + d * x^n]) / (d * n), x] - \text{Dist}[(e^n * (m - n + 1)) / (d * n), \text{Int}[(e * x)^{(m - n)} * \sin[c + d * x^n], x], x] /;$ FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3385

$\text{Int}[(e \cdot x)^{(m)} * \sin[(c \cdot x) + (d \cdot x)^{(n)}], x_Symbol] :> -\text{Simp}[(e^{(n - 1)} * (e * x)^{(m - n + 1)} * \cos[c + d * x^n]) / (d * n), x] + \text{Dist}[(e^n * (m - n + 1))$

)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]

Rubi steps

$$\begin{aligned}
 \int x^4 C(bx)^2 dx &= \frac{1}{5} x^5 C(bx)^2 - \frac{1}{5} (2b) \int x^5 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx) dx \\
 &= \frac{1}{5} x^5 C(bx)^2 - \frac{2x^4 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b\pi} + \frac{\int x^4 \sin(b^2 \pi x^2) dx}{5\pi} + \frac{8 \int x^3 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{5b\pi} \\
 &= -\frac{x^3 \cos(b^2 \pi x^2)}{10b^2 \pi^2} - \frac{8x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{5b^3 \pi^2} + \frac{1}{5} x^5 C(bx)^2 - \frac{2x^4 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b\pi} + \frac{16 \int x \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{5b^3} \\
 &= -\frac{x^3 \cos(b^2 \pi x^2)}{10b^2 \pi^2} - \frac{8x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{5b^3 \pi^2} + \frac{1}{5} x^5 C(bx)^2 + \frac{16C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b^5 \pi^3} - \frac{2x^4 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b\pi} \\
 &= \frac{4x^3}{15b^2 \pi^2} - \frac{x^3 \cos(b^2 \pi x^2)}{10b^2 \pi^2} - \frac{8x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{5b^3 \pi^2} + \frac{1}{5} x^5 C(bx)^2 - \frac{3S(\sqrt{2}bx)}{20\sqrt{2}b^5 \pi^3} - \frac{4\sqrt{2}S(\sqrt{2}bx)}{5b^5 \pi^3} + \frac{16C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b\pi} \\
 &= \frac{4x^3}{15b^2 \pi^2} - \frac{x^3 \cos(b^2 \pi x^2)}{10b^2 \pi^2} - \frac{8x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{5b^3 \pi^2} + \frac{1}{5} x^5 C(bx)^2 - \frac{3S(\sqrt{2}bx)}{20\sqrt{2}b^5 \pi^3} - \frac{\sqrt{2}S(\sqrt{2}bx)}{b^5 \pi^3} + \frac{16C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{5b\pi}
 \end{aligned}$$

Mathematica [A] time = 0.133759, size = 137, normalized size = 0.77

$$\frac{24\pi^3 b^5 x^5 \text{FresnelC}(bx)^2 - 48 \text{FresnelC}(bx) \left((\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 4\pi b^2 x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \right) + 32\pi b^3 x^3 + 66bx \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{120\pi^3 b^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*FresnelC[b*x]^2,x]

[Out] (32*b^3*Pi*x^3 - 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelC[b*x]^2 - 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 48*FresnelC[b*x]*(4*b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 66*b*x*SIN[b^2*Pi*x^2])/(120*b^5*Pi^3)

Maple [A] time = 0.076, size = 209, normalized size = 1.2

$$\frac{1}{b^5} \left(\frac{b^5 x^5 (\text{FresnelC}(bx))^2}{5} - 2 \text{FresnelC}(bx) \left(\frac{1}{5} \frac{x^4 b^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} - \frac{4}{5} \frac{1}{\pi} \left(-\frac{b^2 x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 2 \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*fresnelC(b*x)^2,x)`

[Out] $\frac{1}{b^5} \left(\frac{1}{5} b^5 x^5 \operatorname{FresnelC}(bx)^2 - 2 \operatorname{FresnelC}(bx) \left(\frac{1}{5} \pi b^4 x^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{4}{5} \pi \left(-\frac{1}{\pi} b^2 x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) + \frac{2}{\pi^2} \sin\left(\frac{1}{2} b^2 \pi x^2\right) \right) \right) + \frac{4}{15} \pi^2 b^3 x^3 + \frac{4}{5} \pi^2 \left(\frac{1}{2} \pi b x \sin(b^2 \pi x^2) - \frac{1}{4} \pi^2 \left(\frac{1}{2} \right) \operatorname{FresnelS}(bx^2)^{\frac{1}{2}} \right) + \frac{1}{5} \pi^3 \left(-\frac{1}{2} \pi b^3 x^3 \cos(b^2 \pi x^2) + \frac{3}{2} \pi \left(\frac{1}{2} \pi b x \sin(b^2 \pi x^2) - \frac{1}{4} \pi^2 \left(\frac{1}{2} \right) \operatorname{FresnelS}(bx^2)^{\frac{1}{2}} \right) \right) - 4 \cdot 2^{\frac{1}{2}} \operatorname{FresnelS}(bx^2)^{\frac{1}{2}} \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*fresnelc(b*x)^2,x, algorithm="maxima")`

[Out] `integrate(x^4*fresnelc(b*x)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^4 \operatorname{fresnelc}(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*fresnelc(b*x)^2,x, algorithm="fricas")`

[Out] `integral(x^4*fresnelc(b*x)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 C^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**4*fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnelc(b*x)^2, x)
```

3.144 $\int x^3 \text{FresnelC}(bx)^2 dx$

Optimal. Leaf size=140

$$-\frac{x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b} - \frac{3x \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi^2 b^3} + \frac{3 \text{FresnelC}(bx)^2}{4\pi^2 b^4} + \frac{3x^2}{8\pi^2 b^2} + \frac{\sin(\pi b^2 x^2)}{2\pi^3 b^4} - \frac{x^2 \cos(\pi b^2 x^2)}{8\pi^2 b^4}$$

[Out] $(3*x^2)/(8*b^2*Pi^2) - (x^2*\text{Cos}[b^2*Pi*x^2])/(8*b^2*Pi^2) - (3*x*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(2*b^3*Pi^2) + (3*\text{FresnelC}[b*x]^2)/(4*b^4*Pi^2) + (x^4*\text{FresnelC}[b*x]^2)/4 - (x^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(2*b*Pi) + \text{Sin}[b^2*Pi*x^2]/(2*b^4*Pi^3)$

Rubi [A] time = 0.155816, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6431, 6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637}

$$-\frac{x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b} - \frac{3x \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi^2 b^3} + \frac{3 \text{FresnelC}(bx)^2}{4\pi^2 b^4} + \frac{3x^2}{8\pi^2 b^2} + \frac{\sin(\pi b^2 x^2)}{2\pi^3 b^4} - \frac{x^2 \cos(\pi b^2 x^2)}{8\pi^2 b^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*\text{FresnelC}[b*x]^2, x]$

[Out] $(3*x^2)/(8*b^2*Pi^2) - (x^2*\text{Cos}[b^2*Pi*x^2])/(8*b^2*Pi^2) - (3*x*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(2*b^3*Pi^2) + (3*\text{FresnelC}[b*x]^2)/(4*b^4*Pi^2) + (x^4*\text{FresnelC}[b*x]^2)/4 - (x^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(2*b*Pi) + \text{Sin}[b^2*Pi*x^2]/(2*b^4*Pi^3)$

Rule 6431

$\text{Int}[\text{FresnelC}[(b_.)*(x_.)]^2*(x_.)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m+1)}*\text{FresnelC}[b*x]^2)/(m+1), x] - \text{Dist}[(2*b)/(m+1), \text{Int}[x^{(m+1)}*\text{Cos}[(Pi*b^2*x^2)/2]*\text{FresnelC}[b*x], x], x] /;$ FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6455

$\text{Int}[\text{Cos}[(d_.)*(x_.)^2]*\text{FresnelC}[(b_.)*(x_.)]*(x_.)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m-1)}*\text{Sin}[d*x^2]*\text{FresnelC}[b*x])/(2*d), x] + (-\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\text{Sin}[d*x^2]*\text{FresnelC}[b*x], x], x] - \text{Dist}[b/(4*d), \text{Int}[x^{(m-1)}*\text{Sin}[2*d*x^2], x], x]) /;$ FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

]

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.), x_Symbol] := Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eqQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] := Simp[x/2, x] - Simp[Sin[2*c
+ d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*SIN[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 C(bx)^2 dx &= \frac{1}{4} x^4 C(bx)^2 - \frac{1}{2} b \int x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx) dx \\
&= \frac{1}{4} x^4 C(bx)^2 - \frac{x^3 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{2b\pi} + \frac{\int x^3 \sin(b^2 \pi x^2) dx}{4\pi} + \frac{3 \int x^2 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{2b\pi} \\
&= -\frac{3x \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{2b^3 \pi^2} + \frac{1}{4} x^4 C(bx)^2 - \frac{x^3 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{2b\pi} + \frac{3 \int \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx) dx}{2b^3 \pi^2} + \frac{3 \int x \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx) dx}{2b^3 \pi^2} \\
&= -\frac{x^2 \cos(b^2 \pi x^2)}{8b^2 \pi^2} - \frac{3x \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{2b^3 \pi^2} + \frac{1}{4} x^4 C(bx)^2 - \frac{x^3 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{2b\pi} + \frac{3 \text{Subst}\left(\int x dx, x, \frac{1}{2} b^2 \pi x^2\right)}{2b^4 \pi^2} \\
&= \frac{3x^2}{8b^2 \pi^2} - \frac{x^2 \cos(b^2 \pi x^2)}{8b^2 \pi^2} - \frac{3x \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{2b^3 \pi^2} + \frac{3C(bx)^2}{4b^4 \pi^2} + \frac{1}{4} x^4 C(bx)^2 - \frac{x^3 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{2b\pi} + \frac{3 \text{Subst}\left(\int x dx, x, \frac{1}{2} b^2 \pi x^2\right)}{2b^4 \pi^2}
\end{aligned}$$

Mathematica [A] time = 0.0087466, size = 140, normalized size = 1.

$$-\frac{x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{2\pi b} - \frac{3x \text{FresnelC}(bx) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{2\pi^2 b^3} + \frac{3 \text{FresnelC}(bx)^2}{4\pi^2 b^4} + \frac{3x^2}{8\pi^2 b^2} + \frac{\sin(\pi b^2 x^2)}{2\pi^3 b^4} - \frac{x^2 \cos(\pi b^2 x^2)}{8\pi^2 b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) - (3*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(2*b^3*Pi^2) + (3*FresnelC[b*x]^2)/(4*b^4*Pi^2) + (x^4*FresnelC[b*x]^2)/4 - (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b*Pi) + Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int x^3 (\text{FresnelC}(bx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*FresnelC(b*x)^2,x)

[Out] int(x^3*FresnelC(b*x)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*fresnelc(b*x)^2,x, algorithm="maxima")

[Out] integrate(x^3*fresnelc(b*x)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \text{fresnelc}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*fresnelc(b*x)^2,x, algorithm="fricas")

[Out] integral(x^3*fresnelc(b*x)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 C^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**3*fresnelc(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnelc(b*x)^2, x)
```

3.145 $\int x^2 \text{FresnelC}(bx)^2 dx$

Optimal. Leaf size=124

$$-\frac{2x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi b} - \frac{4 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi^2 b^3} + \frac{5 \text{FresnelC}(\sqrt{2}bx)}{6\sqrt{2}\pi^2 b^3} - \frac{x \cos(\pi b^2 x^2)}{6\pi^2 b^2} + \frac{2x}{3\pi^2 b^2} + \frac{1}{3}$$

```
[Out] (2*x)/(3*b^2*Pi^2) - (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x]^2)/3 + (5*FresnelC[Sqrt[2]*b*x])/(6*Sqrt[2]*b^3*Pi^2) - (2*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b*Pi)
```

Rubi [A] time = 0.105312, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6431, 6455, 6461, 3358, 3352, 3385}

$$-\frac{2x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi b} - \frac{4 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi^2 b^3} + \frac{5 \text{FresnelC}(\sqrt{2}bx)}{6\sqrt{2}\pi^2 b^3} - \frac{x \cos(\pi b^2 x^2)}{6\pi^2 b^2} + \frac{2x}{3\pi^2 b^2} + \frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[b*x]^2,x]
```

```
[Out] (2*x)/(3*b^2*Pi^2) - (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x]^2)/3 + (5*FresnelC[Sqrt[2]*b*x])/(6*Sqrt[2]*b^3*Pi^2) - (2*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b*Pi)
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*FresnelC[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

]

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])*(b_.))^(p_), x_Sy
mbol] := Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^2 C(bx)^2 dx &= \frac{1}{3} x^3 C(bx)^2 - \frac{1}{3} (2b) \int x^3 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx) dx \\
&= \frac{1}{3} x^3 C(bx)^2 - \frac{2x^2 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi} + \frac{\int x^2 \sin(b^2 \pi x^2) dx}{3\pi} + \frac{4 \int x C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{3b\pi} \\
&= -\frac{x \cos(b^2 \pi x^2)}{6b^2 \pi^2} - \frac{4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{3b^3 \pi^2} + \frac{1}{3} x^3 C(bx)^2 - \frac{2x^2 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi} + \frac{\int \cos(b^2 \pi x^2) dx}{6b^2 \pi^2} \\
&= -\frac{x \cos(b^2 \pi x^2)}{6b^2 \pi^2} - \frac{4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{3b^3 \pi^2} + \frac{1}{3} x^3 C(bx)^2 + \frac{C(\sqrt{2}bx)}{6\sqrt{2}b^3 \pi^2} - \frac{2x^2 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi} + \frac{4 \int \cos(b^2 \pi x^2) dx}{6b^2 \pi^2} \\
&= \frac{2x}{3b^2 \pi^2} - \frac{x \cos(b^2 \pi x^2)}{6b^2 \pi^2} - \frac{4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{3b^3 \pi^2} + \frac{1}{3} x^3 C(bx)^2 + \frac{C(\sqrt{2}bx)}{6\sqrt{2}b^3 \pi^2} - \frac{2x^2 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi} \\
&= \frac{2x}{3b^2 \pi^2} - \frac{x \cos(b^2 \pi x^2)}{6b^2 \pi^2} - \frac{4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{3b^3 \pi^2} + \frac{1}{3} x^3 C(bx)^2 + \frac{C(\sqrt{2}bx)}{6\sqrt{2}b^3 \pi^2} + \frac{\sqrt{2}C(\sqrt{2}bx)}{3b^3 \pi^2} - \frac{2x^2 C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{3b\pi}
\end{aligned}$$

Mathematica [A] time = 0.0882343, size = 100, normalized size = 0.81

$$\frac{4\pi^2 b^3 x^3 \text{FresnelC}(bx)^2 - 8\text{FresnelC}(bx) \left(\pi b^2 x^2 \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \right) - 2bx \left(\cos(\pi b^2 x^2) - 4 \right) + 5\sqrt{2} \text{FresnelC}(bx)}{12\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*FresnelC[b*x]^2,x]

[Out] $(-2*b*x*(-4 + \text{Cos}[b^2*Pi*x^2]) + 4*b^3*Pi^2*x^3*\text{FresnelC}[b*x]^2 + 5*\text{Sqrt}[2]*\text{FresnelC}[\text{Sqrt}[2]*b*x] - 8*\text{FresnelC}[b*x]*(2*\text{Cos}[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*\text{Sin}[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)$

Maple [A] time = 0.075, size = 122, normalized size = 1.

$$\frac{1}{b^3} \left(\frac{b^3 x^3 (\text{FresnelC}(bx))^2}{3} - 2 \text{FresnelC}(bx) \left(\frac{1}{3} \frac{b^2 x^2 \sin(1/2 b^2 \pi x^2)}{\pi} + \frac{2}{3} \frac{\cos(1/2 b^2 \pi x^2)}{\pi^2} \right) + \frac{2bx}{3\pi^2} + \frac{\sqrt{2} \text{FresnelC}(bx)}{3\pi^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*FresnelC(b*x)^2,x)

[Out] $1/b^3*(1/3*b^3*x^3*\text{FresnelC}(b*x)^2-2*\text{FresnelC}(b*x)*(1/3/\text{Pi}*b^2*x^2*\sin(1/2*b^2*\text{Pi}*x^2)+2/3/\text{Pi}^2*\cos(1/2*b^2*\text{Pi}*x^2))+2/3/\text{Pi}^2*b*x+1/3/\text{Pi}^2*2^{(1/2)}*\text{FresnelC}(b*x*2^{(1/2)})+1/3/\text{Pi}*(-1/2/\text{Pi}*b*x*\cos(b^2*\text{Pi}*x^2)+1/4/\text{Pi}*2^{(1/2)}*\text{FresnelC}(b*x*2^{(1/2)})))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnelc(b*x)^2,x, algorithm="maxima")`

[Out] `integrate(x^2*fresnelc(b*x)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \text{fresnelc}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnelc(b*x)^2,x, algorithm="fricas")`

[Out] `integral(x^2*fresnelc(b*x)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 C^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*fresnelc(b*x)**2,x)`

[Out] `Integral(x**2*fresnelc(b*x)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc(b*x)^2, x)
```

3.146 $\int x \text{FresnelC}(bx)^2 dx$

Optimal. Leaf size=144

$$\frac{ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right)}{8\pi} - \frac{ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)}{8\pi} + \frac{\text{FresnelC}(bx)^2}{2\pi b}$$

[Out] $-\text{Cos}[b^2 \pi x^2]/(4b^2 \pi^2) + (x^2 \text{FresnelC}[bx]^2)/2 + (\text{FresnelC}[bx] * \text{FresnelS}[bx])/(2b^2 \pi) + ((I/8) * x^2 * \text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2) * b^2 \pi x^2])/ \pi - ((I/8) * x^2 * \text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2) * b^2 \pi x^2])/ \pi - (x * \text{FresnelC}[bx] * \text{Sin}[(b^2 \pi x^2)/2])/(b * \pi)$

Rubi [A] time = 0.0839067, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6431, 6455, 6447, 3379, 2638}

$$\frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2 \pi x^2\right)}{8\pi} - \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2 \pi x^2\right)}{8\pi} + \frac{\text{FresnelC}(bx) \text{S}(bx)}{2\pi b^2} - \frac{x \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b} - \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x * \text{FresnelC}[bx]^2, x]$

[Out] $-\text{Cos}[b^2 \pi x^2]/(4b^2 \pi^2) + (x^2 \text{FresnelC}[bx]^2)/2 + (\text{FresnelC}[bx] * \text{FresnelS}[bx])/(2b^2 \pi) + ((I/8) * x^2 * \text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2) * b^2 \pi x^2])/ \pi - ((I/8) * x^2 * \text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2) * b^2 \pi x^2])/ \pi - (x * \text{FresnelC}[bx] * \text{Sin}[(b^2 \pi x^2)/2])/(b * \pi)$

Rule 6431

$\text{Int}[\text{FresnelC}[(b_)] * (x_)]^2 * (x_)^{(m_)}, x_ \text{Symbol}] \rightarrow \text{Simp}[(x^{(m+1)} * \text{FresnelC}[bx]^2)/(m+1), x] - \text{Dist}[(2*b)/(m+1), \text{Int}[x^{(m+1)} * \text{Cos}[(\pi * b^2 * x^2)/2] * \text{FresnelC}[bx], x], x] /;$ FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6455

$\text{Int}[\text{Cos}[(d_)] * (x_)]^2 * \text{FresnelC}[(b_)] * (x_)] * (x_)^{(m_)}, x_ \text{Symbol}] \rightarrow \text{Simp}[(x^{(m-1)} * \text{Sin}[d * x^2] * \text{FresnelC}[bx])/(2*d), x] + (-\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)} * \text{Sin}[d * x^2] * \text{FresnelC}[bx], x], x] - \text{Dist}[b/(4*d), \text{Int}[x^{(m-1)} * \text{Sin}[2 * d * x^2], x], x]) /;$ FreeQ[{b, d}, x] && EqQ[d^2, (\pi^2 * b^4)/4] && IGtQ[m, 1]

]

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(b*Pi*FresnelC[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(I*d*x^2)])]/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, I*d*x^2)])]/8, x) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int xC(bx)^2 dx &= \frac{1}{2}x^2C(bx)^2 - b \int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx \\
&= \frac{1}{2}x^2C(bx)^2 - \frac{xC(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + \frac{\int x \sin(b^2\pi x^2) dx}{2\pi} + \frac{\int C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= \frac{1}{2}x^2C(bx)^2 + \frac{C(bx)S(bx)}{2b^2\pi} + \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{x C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} \\
&= -\frac{\cos(b^2\pi x^2)}{4b^2\pi^2} + \frac{1}{2}x^2C(bx)^2 + \frac{C(bx)S(bx)}{2b^2\pi} + \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi}
\end{aligned}$$

Mathematica [F] time = 0.174331, size = 0, normalized size = 0.

$$\int x \text{FresnelC}(bx)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x*FresnelC[b*x]^2,x]

[Out] Integrate[x*FresnelC[b*x]^2, x]

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int x (\text{FresnelC}(bx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*FresnelC(b*x)^2,x)

[Out] int(x*FresnelC(b*x)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int xfresnelc(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(b*x)^2,x, algorithm="maxima")

[Out] integrate(x*fresnelc(b*x)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(xfresnelc(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(b*x)^2,x, algorithm="fricas")

[Out] integral(x*fresnelc(b*x)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x C^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(b*x)**2,x)

[Out] Integral(x*fresnelc(b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(b*x)^2,x, algorithm="giac")

[Out] integrate(x*fresnelc(b*x)^2, x)

3.147 $\int \text{FresnelC}(bx)^2 dx$

Optimal. Leaf size=54

$$-\frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b} + x\text{FresnelC}(bx)^2 + \frac{S(\sqrt{2}bx)}{\sqrt{2}\pi b}$$

[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b*Pi)

Rubi [A] time = 0.0360543, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6421, 12, 6453, 3351}

$$-\frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b} + x\text{FresnelC}(bx)^2 + \frac{S(\sqrt{2}bx)}{\sqrt{2}\pi b}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]^2,x]

[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b*Pi)

Rule 6421

Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] := Simp[((a + b*x)*FresnelC[a + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Cos[(Pi*(a + b*x)^2]/2]*FresnelC[a + b*x], x], x] /; FreeQ[{a, b}, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 6453

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] := Simp[(Sin[d*x^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 3351

`Int[Sin[(d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]`

Rubi steps

$$\begin{aligned}
 \int C(bx)^2 dx &= xC(bx)^2 - 2 \int bx \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx \\
 &= xC(bx)^2 - (2b) \int x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx \\
 &= xC(bx)^2 - \frac{2C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + \frac{\int \sin(b^2\pi x^2) dx}{\pi} \\
 &= xC(bx)^2 + \frac{S(\sqrt{2}bx)}{\sqrt{2}b\pi} - \frac{2C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}
 \end{aligned}$$

Mathematica [A] time = 0.0100313, size = 54, normalized size = 1.

$$-\frac{2\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b} + x\text{FresnelC}(bx)^2 + \frac{S(\sqrt{2}bx)}{\sqrt{2}\pi b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x]^2,x]

[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b*Pi)

Maple [A] time = 0.053, size = 49, normalized size = 0.9

$$\frac{1}{b} \left(bx(\text{FresnelC}(bx))^2 - 2 \frac{\text{FresnelC}(bx) \sin(1/2 b^2 \pi x^2)}{\pi} + \frac{\sqrt{2}\text{FresnelS}(bx\sqrt{2})}{2\pi} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)^2,x)

[Out] $1/b*(b*x*\text{FresnelC}(b*x)^2-2*\text{FresnelC}(b*x)/\text{Pi}*\sin(1/2*b^2*\text{Pi}*x^2)+1/2/\text{Pi}*2^(1/2)*\text{FresnelS}(b*x*2^(1/2)))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)^2,x, algorithm="maxima")`

[Out] `integrate(fresnelc(b*x)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{fresnelc}(bx)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)^2,x, algorithm="fricas")`

[Out] `integral(fresnelc(b*x)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int C^2(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)**2,x)`

[Out] `Integral(fresnelc(b*x)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2, x)
```

$$3.148 \quad \int \frac{\text{FresnelC}(bx)^2}{x} dx$$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)^2}{x}, x\right)$$

[Out] Unintegrable[FresnelC[b*x]^2/x, x]

Rubi [A] time = 0.0161198, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[b*x]^2/x, x]

[Out] Defer[Int][FresnelC[b*x]^2/x, x]

Rubi steps

$$\int \frac{C(bx)^2}{x} dx = \int \frac{C(bx)^2}{x} dx$$

Mathematica [A] time = 0.0188291, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]^2/x, x]

[Out] Integrate[FresnelC[b*x]^2/x, x]

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)^2/x,x)

[Out] int(FresnelC(b*x)^2/x,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)^2/x, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)^2/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x, x)
```

$$3.149 \quad \int \frac{\text{FresnelC}(bx)^2}{x^2} dx$$

Optimal. Leaf size=37

$$2b\text{Unintegrable}\left(\frac{\text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) - \frac{\text{FresnelC}(bx)^2}{x}$$

[Out] -(FresnelC[b*x]^2/x) + 2*b*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]

Rubi [A] time = 0.0373831, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[b*x]^2/x^2, x]

[Out] -(FresnelC[b*x]^2/x) + 2*b*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]

Rubi steps

$$\int \frac{C(bx)^2}{x^2} dx = -\frac{C(bx)^2}{x} + (2b) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x} dx$$

Mathematica [A] time = 0.0259263, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]^2/x^2, x]

[Out] Integrate[FresnelC[b*x]^2/x^2, x]

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)^2/x^2,x)

[Out] int(FresnelC(b*x)^2/x^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^2,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)^2/x^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^2,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)^2/x^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)**2/x**2,x)

[Out] Integral(fresnelc(b*x)**2/x**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^2,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)^2/x^2, x)

$$3.150 \quad \int \frac{\text{FresnelC}(bx)^2}{x^3} dx$$

Optimal. Leaf size=38

$$b\text{Unintegrable}\left(\frac{\text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right) - \frac{\text{FresnelC}(bx)^2}{2x^2}$$

[Out] -FresnelC[b*x]^2/(2*x^2) + b*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]

Rubi [A] time = 0.0371539, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[b*x]^2/x^3, x]

[Out] -FresnelC[b*x]^2/(2*x^2) + b*Defer[Int] [(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]

Rubi steps

$$\int \frac{C(bx)^2}{x^3} dx = -\frac{C(bx)^2}{2x^2} + b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2} dx$$

Mathematica [A] time = 0.0198106, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]^2/x^3, x]

[Out] Integrate[FresnelC[b*x]^2/x^3, x]

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)^2/x^3,x)

[Out] int(FresnelC(b*x)^2/x^3,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^3,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)^2/x^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^3,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)^2/x^3, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)**2/x**3,x)

[Out] Integral(fresnelc(b*x)**2/x**3, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^3,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)^2/x^3, x)

$$3.151 \quad \int \frac{\text{FresnelC}(bx)^2}{x^4} dx$$

Optimal. Leaf size=119

$$-\frac{1}{3}\pi b^3 \text{Unintegrable} \left(\frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x \right) - \frac{b \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^2} - \frac{\pi b^3 S(\sqrt{2}bx)}{3\sqrt{2}} - \frac{b^2 \cos(\pi b^2 x^2)}{6x}$$

[Out] $-b^2/(6*x) - (b^2*\text{Cos}[b^2*Pi*x^2])/(6*x) - (b*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(3*x^2) - \text{FresnelC}[b*x]^2/(3*x^3) - (b^3*Pi*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(3*\text{Sqrt}[2]) - (b^3*Pi*\text{Unintegrable}[(\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x, x])/3$

Rubi [A] time = 0.0828138, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[b*x]^2/x^4, x]

[Out] $-b^2/(6*x) - (b^2*\text{Cos}[b^2*Pi*x^2])/(6*x) - (b*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(3*x^2) - \text{FresnelC}[b*x]^2/(3*x^3) - (b^3*Pi*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(3*\text{Sqrt}[2]) - (b^3*Pi*\text{Defer}[\text{Int}][(\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x, x])/3$

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^4} dx &= -\frac{C(bx)^2}{3x^3} + \frac{1}{3}(2b) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^3} dx \\
&= -\frac{b^2}{6x} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{3x^2} - \frac{C(bx)^2}{3x^3} + \frac{1}{6}b^2 \int \frac{\cos(b^2\pi x^2)}{x^2} dx - \frac{1}{3}(b^3\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx \\
&= -\frac{b^2}{6x} - \frac{b^2 \cos(b^2\pi x^2)}{6x} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{3x^2} - \frac{C(bx)^2}{3x^3} - \frac{1}{3}(b^3\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx - \frac{1}{3}(b^4\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx \\
&= -\frac{b^2}{6x} - \frac{b^2 \cos(b^2\pi x^2)}{6x} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{3x^2} - \frac{C(bx)^2}{3x^3} - \frac{b^3\pi S(\sqrt{2}bx)}{3\sqrt{2}} - \frac{1}{3}(b^3\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx
\end{aligned}$$

Mathematica [A] time = 0.0262054, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]^2/x^4, x]

[Out] Integrate[FresnelC[b*x]^2/x^4, x]

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)^2/x^4, x)

[Out] int(FresnelC(b*x)^2/x^4, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^4,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)^2/x^4, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^4,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)^2/x^4, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)**2/x**4,x)

[Out] Integral(fresnelc(b*x)**2/x**4, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^4,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)^2/x^4, x)

3.152 $\int \frac{\text{FresnelC}(bx)^2}{x^5} dx$

Optimal. Leaf size=127

$$\frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x} - \frac{b \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^3} - \frac{1}{12}\pi^2 b^4 \text{FresnelC}(bx)^2 - \frac{1}{12}\pi b^4 \text{Si}\left(b^2 \pi x^2\right) - \frac{b^2}{24x^2} - \frac{b^4 \pi^2}{24x^2}$$

[Out] $-b^2/(24*x^2) - (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(24*x^2) - (b*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(6*x^3) - (b^4*\text{Pi}^2*\text{FresnelC}[b*x]^2)/12 - \text{FresnelC}[b*x]^2/(4*x^4) + (b^3*\text{Pi}*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(6*x) - (b^4*\text{Pi}*\text{SinIntegral}[b^2*\text{Pi}*x^2])/12$

Rubi [A] time = 0.137878, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$, Rules used = {6431, 6457, 6465, 6441, 30, 3375, 3380, 3297, 3299}

$$\frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x} - \frac{b \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^3} - \frac{1}{12}\pi^2 b^4 \text{FresnelC}(bx)^2 - \frac{1}{12}\pi b^4 \text{Si}\left(b^2 \pi x^2\right) - \frac{b^2}{24x^2} - \frac{b^4 \pi^2}{24x^2}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]^2/x^5,x]

[Out] $-b^2/(24*x^2) - (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(24*x^2) - (b*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(6*x^3) - (b^4*\text{Pi}^2*\text{FresnelC}[b*x]^2)/12 - \text{FresnelC}[b*x]^2/(4*x^4) + (b^3*\text{Pi}*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(6*x) - (b^4*\text{Pi}*\text{SinIntegral}[b^2*\text{Pi}*x^2])/12$

Rule 6431

Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*FresnelC[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6457

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre

$eQ\{b, d\}, x \ \&\& \ EqQ[d^2, (Pi^2*b^4)/4] \ \&\& \ ILtQ[m, -2]$

Rule 6465

$Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] \ :> \ Simp[(x^(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1)*Sin[2*d*x^2], x], x]) /;$ $FreeQ\{b, d\}, x \ \&\& \ EqQ[d^2, (Pi^2*b^4)/4] \ \&\& \ ILtQ[m, -1]$

Rule 6441

$Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.)], x_Symbol] \ :> \ Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /;$ $FreeQ\{b, d, n\}, x \ \&\& \ EqQ[d^2, (Pi^2*b^4)/4]$

Rule 30

$Int[(x_)^(m_.), x_Symbol] \ :> \ Simp[x^(m + 1)/(m + 1), x] /;$ $FreeQ[m, x] \ \&\& \ NeQ[m, -1]$

Rule 3375

$Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] \ :> \ Simp[SinIntegral[d*x^n]/n, x] /;$ $FreeQ\{d, n\}, x]$

Rule 3380

$Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] \ :> \ Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /;$ $FreeQ\{a, b, c, d, m, n, p\}, x \ \&\& \ IntegerQ[Simplify[(m + 1)/n]] \ \&\& \ (EqQ[p, 1] \ || \ EqQ[m, n - 1] \ || \ (IntegerQ[p] \ \&\& \ GtQ[Simplify[(m + 1)/n], 0]))$

Rule 3297

$Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] \ :> \ Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /;$ $FreeQ\{c, d, e, f\}, x \ \&\& \ LtQ[m, -1]$

Rule 3299

$Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] \ :> \ Simp[SinIntegral[e + f*x]/d, x] /;$ $FreeQ\{c, d, e, f\}, x \ \&\& \ EqQ[d*e - c*f, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^5} dx &= -\frac{C(bx)^2}{4x^4} + \frac{1}{2}b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^4} dx \\
&= -\frac{b^2}{24x^2} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{1}{12}b^2 \int \frac{\cos(b^2\pi x^2)}{x^3} dx - \frac{1}{6}(b^3\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
&= -\frac{b^2}{24x^2} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} + \frac{1}{24}b^2 \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^2} dx, x, \right. \\
&= -\frac{b^2}{24x^2} - \frac{b^2 \cos(b^2\pi x^2)}{24x^2} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} - \frac{1}{24}b^4\pi \text{Si}(b^2\pi x^2) \\
&= -\frac{b^2}{24x^2} - \frac{b^2 \cos(b^2\pi x^2)}{24x^2} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{6x^3} - \frac{1}{12}b^4\pi^2 C(bx)^2 - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x}
\end{aligned}$$

Mathematica [A] time = 0.0069876, size = 127, normalized size = 1.

$$\frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x} - \frac{b \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^3} - \frac{1}{12}\pi^2 b^4 \text{FresnelC}(bx)^2 - \frac{1}{12}\pi b^4 \text{Si}(b^2\pi x^2) - \frac{b^2}{24x^2} - \frac{C(bx)^2}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x]^2/x^5, x]

[Out] $-\frac{b^2}{24x^2} - \frac{(b^2 \cos[b^2\pi x^2])}{(24x^2)} - \frac{(b \cos[(b^2\pi x^2)/2]) \text{FresnelC}[b*x]}{(6x^3)} - \frac{(b^4 \pi^2 \text{FresnelC}[b*x]^2)}{12} - \frac{\text{FresnelC}[b*x]^2}{(4x^4)} + \frac{(b^3 \pi \text{FresnelC}[b*x] \sin[(b^2\pi x^2)/2])}{(6x)} - \frac{(b^4 \pi \text{SinIntegral}[b^2\pi x^2])}{12}$

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(FresnelC(b*x)^2/x^5,x)`

[Out] `int(FresnelC(b*x)^2/x^5,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)^2/x^5,x, algorithm="maxima")`

[Out] `integrate(fresnelc(b*x)^2/x^5, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)^2/x^5,x, algorithm="fricas")`

[Out] `integral(fresnelc(b*x)^2/x^5, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)**2/x**5,x)`

[Out] `Integral(fresnelc(b*x)**2/x**5, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^5, x)
```

$$3.153 \quad \int \frac{\text{FresnelC}(bx)^2}{x^6} dx$$

Optimal. Leaf size=170

$$-\frac{1}{20}\pi^2 b^5 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) + \frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{20x^2} - \frac{b \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{10x^4}$$

[Out] $-b^2/(60*x^3) - (b^2*\text{Cos}[b^2*Pi*x^2])/(60*x^3) - (b*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(10*x^4) - \text{FresnelC}[b*x]^2/(5*x^5) - (7*b^5*Pi^2*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(60*\text{Sqrt}[2]) + (b^3*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(20*x^2) + (7*b^4*Pi*\text{Sin}[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*\text{Unintegrable}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x, x])/20$

Rubi [A] time = 0.145268, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^6} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[b*x]^2/x^6,x]

[Out] $-b^2/(60*x^3) - (b^2*\text{Cos}[b^2*Pi*x^2])/(60*x^3) - (b*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(10*x^4) - \text{FresnelC}[b*x]^2/(5*x^5) - (7*b^5*Pi^2*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(60*\text{Sqrt}[2]) + (b^3*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(20*x^2) + (7*b^4*Pi*\text{Sin}[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*\text{Defer}[\text{Int}][(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x, x])/20$

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^6} dx &= -\frac{C(bx)^2}{5x^5} + \frac{1}{5}(2b) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^5} dx \\
&= -\frac{b^2}{60x^3} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} + \frac{1}{20}b^2 \int \frac{\cos(b^2\pi x^2)}{x^4} dx - \frac{1}{10}(b^3\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx \\
&= -\frac{b^2}{60x^3} - \frac{b^2 \cos(b^2\pi x^2)}{60x^3} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2} - \frac{1}{40}(b^4\pi) \int \frac{\sin(b^2\pi x^2)}{x^2} dx \\
&= -\frac{b^2}{60x^3} - \frac{b^2 \cos(b^2\pi x^2)}{60x^3} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2} + \frac{7b^4\pi \sin(b^2\pi x^2)}{120x} \\
&= -\frac{b^2}{60x^3} - \frac{b^2 \cos(b^2\pi x^2)}{60x^3} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} - \frac{7b^5\pi^2 C(\sqrt{2}bx)}{60\sqrt{2}} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2}
\end{aligned}$$

Mathematica [A] time = 0.0265742, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^6} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]^2/x^6, x]

[Out] Integrate[FresnelC[b*x]^2/x^6, x]

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)^2/x^6, x)

[Out] int(FresnelC(b*x)^2/x^6, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^6,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)^2/x^6, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^6,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)^2/x^6, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)**2/x**6,x)

[Out] Integral(fresnelc(b*x)**2/x**6, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^6, x)
```

$$3.154 \quad \int \frac{\text{FresnelC}(bx)^2}{x^7} dx$$

Optimal. Leaf size=165

$$-\frac{1}{45}\pi^2b^5\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)-\frac{1}{72}\pi^2b^6\text{CosIntegral}(\pi b^2x^2)+\frac{\pi b^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{45x^3}$$

[Out] $-b^2/(120*x^4) - (b^2*\text{Cos}[b^2*Pi*x^2])/(120*x^4) - (b^6*Pi^2*\text{CosIntegral}[b^2*Pi*x^2])/72 - (b*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(15*x^5) - \text{FresnelC}[b*x]^2/(6*x^6) + (b^3*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(45*x^3) + (b^4*Pi*\text{Sin}[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*\text{Unintegrable}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x^2, x])/45$

Rubi [A] time = 0.220258, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^7} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[\text{FresnelC}[b*x]^2/x^7, x]$

[Out] $-b^2/(120*x^4) - (b^2*\text{Cos}[b^2*Pi*x^2])/(120*x^4) - (b^6*Pi^2*\text{CosIntegral}[b^2*Pi*x^2])/72 - (b*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(15*x^5) - \text{FresnelC}[b*x]^2/(6*x^6) + (b^3*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(45*x^3) + (b^4*Pi*\text{Sin}[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*\text{Defer}[\text{Int}][(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x^2, x])/45$

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^7} dx &= -\frac{C(bx)^2}{6x^6} + \frac{1}{3}b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^6} dx \\
&= -\frac{b^2}{120x^4} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{15x^5} - \frac{C(bx)^2}{6x^6} + \frac{1}{30}b^2 \int \frac{\cos(b^2\pi x^2)}{x^5} dx - \frac{1}{15}(b^3\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx \\
&= -\frac{b^2}{120x^4} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{15x^5} - \frac{C(bx)^2}{6x^6} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3} + \frac{1}{60}b^2 \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^3} dx, x\right) \\
&= -\frac{b^2}{120x^4} - \frac{b^2 \cos(b^2\pi x^2)}{120x^4} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{15x^5} - \frac{C(bx)^2}{6x^6} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3} - \frac{1}{180}(b^4\pi) \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^3} dx, x\right) \\
&= -\frac{b^2}{120x^4} - \frac{b^2 \cos(b^2\pi x^2)}{120x^4} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{15x^5} - \frac{C(bx)^2}{6x^6} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3} + \frac{b^4\pi \sin(b^2\pi x^2)}{72x^2} \\
&= -\frac{b^2}{120x^4} - \frac{b^2 \cos(b^2\pi x^2)}{120x^4} - \frac{1}{72}b^6\pi^2 \text{Ci}(b^2\pi x^2) - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{15x^5} - \frac{C(bx)^2}{6x^6} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3}
\end{aligned}$$

Mathematica [A] time = 0.0201974, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^7} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]^2/x^7, x]

[Out] Integrate[FresnelC[b*x]^2/x^7, x]

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)^2/x^7, x)

[Out] `int(FresnelC(b*x)^2/x^7,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)^2/x^7,x, algorithm="maxima")`

[Out] `integrate(fresnelc(b*x)^2/x^7, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)^2/x^7,x, algorithm="fricas")`

[Out] `integral(fresnelc(b*x)^2/x^7, x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)**2/x**7,x)`

[Out] `Integral(fresnelc(b*x)**2/x**7, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^7, x)
```

3.155 $\int \frac{\text{FresnelC}(bx)^2}{x^8} dx$

Optimal. Leaf size=258

$$\frac{1}{168}\pi^3b^7\text{Unintegrable}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}, x\right) + \frac{\pi b^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{84x^4} + \frac{\pi^2b^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{168x^2}$$

[Out] $-b^2/(210*x^5) + (b^6*\text{Pi}^2)/(336*x) - (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(210*x^5) + (67*b^6*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(5040*x) - (b*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(21*x^6) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(168*x^2) - \text{FresnelC}[b*x]^2/(7*x^7) + (b^7*\text{Pi}^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(72*\text{Sqrt}[2]) + (2*\text{Sqrt}[2]*b^7*\text{Pi}^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/315 + (b^3*\text{Pi}*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(84*x^4) + (13*b^4*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(2520*x^3) + (b^7*\text{Pi}^3*\text{Unintegrable}[(\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/x, x])/168$

Rubi [A] time = 0.239743, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^8} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[b*x]^2/x^8, x]

[Out] $-b^2/(210*x^5) + (b^6*\text{Pi}^2)/(336*x) - (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(210*x^5) + (67*b^6*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(5040*x) - (b*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(21*x^6) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(168*x^2) - \text{FresnelC}[b*x]^2/(7*x^7) + (b^7*\text{Pi}^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(72*\text{Sqrt}[2]) + (2*\text{Sqrt}[2]*b^7*\text{Pi}^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/315 + (b^3*\text{Pi}*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(84*x^4) + (13*b^4*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(2520*x^3) + (b^7*\text{Pi}^3*\text{Defier[Int]}[(\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/x, x])/168$

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^8} dx &= -\frac{C(bx)^2}{7x^7} + \frac{1}{7}(2b) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^7} dx \\
&= -\frac{b^2}{210x^5} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{21x^6} - \frac{C(bx)^2}{7x^7} + \frac{1}{42}b^2 \int \frac{\cos(b^2\pi x^2)}{x^6} dx - \frac{1}{21}(b^3\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx \\
&= -\frac{b^2}{210x^5} - \frac{b^2 \cos(b^2\pi x^2)}{210x^5} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{21x^6} - \frac{C(bx)^2}{7x^7} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{84x^4} - \frac{1}{168}(b^4\pi) \int \frac{\sin(b^2\pi x^2)}{x^4} dx \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2 \cos(b^2\pi x^2)}{210x^5} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{21x^6} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{168x^2} - \frac{C(bx)^2}{7x^7} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{84x^4} \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2 \cos(b^2\pi x^2)}{210x^5} + \frac{67b^6\pi^2 \cos(b^2\pi x^2)}{5040x} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{21x^6} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{168x^2} - \frac{C(bx)^2}{7x^7} \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2 \cos(b^2\pi x^2)}{210x^5} + \frac{67b^6\pi^2 \cos(b^2\pi x^2)}{5040x} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{21x^6} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{168x^2} - \frac{C(bx)^2}{7x^7}
\end{aligned}$$

Mathematica [A] time = 0.0270006, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^8} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]^2/x^8, x]

[Out] Integrate[FresnelC[b*x]^2/x^8, x]

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)^2/x^8, x)

[Out] int(FresnelC(b*x)^2/x^8, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^8,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)^2/x^8, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^8,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)^2/x^8, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)**2/x**8,x)

[Out] Integral(fresnelc(b*x)**2/x**8, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^8, x)
```

3.156 $\int \frac{\text{FresnelC}(bx)^2}{x^9} dx$

Optimal. Leaf size=242

$$\frac{\pi^3 b^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{420x} + \frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{140x^5} + \frac{\pi^2 b^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{420x^3} - \frac{b \text{FresnelC}(bx)^2}{x^8}$$

[Out] $-b^2/(336*x^6) + (b^6*\text{Pi}^2)/(1680*x^2) - (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(336*x^6) + (b^6*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(336*x^2) - (b*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(28*x^7) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(420*x^3) + (b^8*\text{Pi}^4*\text{FresnelC}[b*x]^2)/840 - \text{FresnelC}[b*x]^2/(8*x^8) + (b^3*\text{Pi}*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(140*x^5) - (b^7*\text{Pi}^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(420*x) + (b^4*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(420*x^4) + (b^8*\text{Pi}^3*\text{SinIntegral}[b^2*\text{Pi}*x^2])/280$

Rubi [A] time = 0.388084, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6431, 6457, 6465, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi^3 b^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{420x} + \frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{140x^5} + \frac{\pi^2 b^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{420x^3} - \frac{b \text{FresnelC}(bx)^2}{x^8}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]^2/x^9,x]

[Out] $-b^2/(336*x^6) + (b^6*\text{Pi}^2)/(1680*x^2) - (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(336*x^6) + (b^6*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(336*x^2) - (b*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(28*x^7) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(420*x^3) + (b^8*\text{Pi}^4*\text{FresnelC}[b*x]^2)/840 - \text{FresnelC}[b*x]^2/(8*x^8) + (b^3*\text{Pi}*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(140*x^5) - (b^7*\text{Pi}^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(420*x) + (b^4*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(420*x^4) + (b^8*\text{Pi}^3*\text{SinIntegral}[b^2*\text{Pi}*x^2])/280$

Rule 6431

Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*FresnelC[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)^(m_)]*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(x^(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_)], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_)^(m_))*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```


]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned}
 \int \frac{C(bx)^2}{x^9} dx &= -\frac{C(bx)^2}{8x^8} + \frac{1}{4}b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8} dx \\
 &= -\frac{b^2}{336x^6} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} - \frac{C(bx)^2}{8x^8} + \frac{1}{56}b^2 \int \frac{\cos(b^2\pi x^2)}{x^7} dx - \frac{1}{28}(b^3\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx \\
 &= -\frac{b^2}{336x^6} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} - \frac{C(bx)^2}{8x^8} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5} + \frac{1}{112}b^2 \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^4} dx\right) \\
 &= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2 \cos(b^2\pi x^2)}{336x^6} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{420x^3} - \frac{C(bx)^2}{8x^8} + \dots \\
 &= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2 \cos(b^2\pi x^2)}{336x^6} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{420x^3} - \frac{C(bx)^2}{8x^8} + \dots \\
 &= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2 \cos(b^2\pi x^2)}{336x^6} + \frac{b^6\pi^2 \cos(b^2\pi x^2)}{336x^2} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{420x^3} \\
 &= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2 \cos(b^2\pi x^2)}{336x^6} + \frac{b^6\pi^2 \cos(b^2\pi x^2)}{336x^2} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{420x^3}
 \end{aligned}$$

Mathematica [A] time = 0.0126167, size = 242, normalized size = 1.

$$-\frac{\pi^3 b^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{420x} + \frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{140x^5} + \frac{\pi^2 b^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{420x^3} - \frac{b \text{FresnelC}(bx)}{x^9}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x]^2/x^9,x]

[Out] $-\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos[b^2\pi x^2]}{336x^6} + \frac{b^6\pi^2\cos[b^2\pi x^2]}{336x^2} - \frac{b\cos\left[\frac{b^2\pi x^2}{2}\right]\text{FresnelC}[b*x]}{28x^7} + \frac{b^5\pi^2\cos\left[\frac{b^2\pi x^2}{2}\right]\text{FresnelC}[b*x]}{420x^3} + \frac{b^8\pi^4\text{FresnelC}[b*x]^2}{840} - \frac{\text{FresnelC}[b*x]^2}{8x^8} + \frac{b^3\pi\text{FresnelC}[b*x]\sin\left[\frac{b^2\pi x^2}{2}\right]}{140x^5} - \frac{b^7\pi^3\text{FresnelC}[b*x]\sin\left[\frac{b^2\pi x^2}{2}\right]}{420x} + \frac{b^4\pi\sin[b^2\pi x^2]}{420x^4} + \frac{b^8\pi^3\text{SinIntegral}[b^2\pi x^2]}{280}$

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)^2/x^9,x)

[Out] int(FresnelC(b*x)^2/x^9,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^9,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)^2/x^9, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^9,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)^2/x^9, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)**2/x**9,x)

[Out] Integral(fresnelc(b*x)**2/x**9, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^9,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)^2/x^9, x)

$$3.157 \quad \int \frac{\text{FresnelC}(bx)^2}{x^{10}} dx$$

Optimal. Leaf size=285

$$\frac{\pi^4 b^9 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right)}{1728} - \frac{\pi^3 b^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{1728 x^2} + \frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{216 x^6} + \dots$$

[Out] $-b^2/(504*x^7) + (b^6*\text{Pi}^2)/(5184*x^3) - (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(504*x^7) + (187*b^6*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(181440*x^3) - (b*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(36*x^8) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(864*x^4) - \text{FresnelC}[b*x]^2/(9*x^9) + (853*b^9*\text{Pi}^4*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(181440*\text{Sqrt}[2]) + (b^3*\text{Pi}*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(216*x^6) - (b^7*\text{Pi}^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(1728*x^2) + (19*b^4*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(15120*x^5) - (853*b^8*\text{Pi}^3*\text{Sin}[b^2*\text{Pi}*x^2])/(362880*x) + (b^9*\text{Pi}^4*\text{Unintegrable}[(\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/x, x])/1728$

Rubi [A] time = 0.342115, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[b*x]^2/x^10,x]

[Out] $-b^2/(504*x^7) + (b^6*\text{Pi}^2)/(5184*x^3) - (b^2*\text{Cos}[b^2*\text{Pi}*x^2])/(504*x^7) + (187*b^6*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(181440*x^3) - (b*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(36*x^8) + (b^5*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(864*x^4) - \text{FresnelC}[b*x]^2/(9*x^9) + (853*b^9*\text{Pi}^4*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(181440*\text{Sqrt}[2]) + (b^3*\text{Pi}*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(216*x^6) - (b^7*\text{Pi}^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(1728*x^2) + (19*b^4*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(15120*x^5) - (853*b^8*\text{Pi}^3*\text{Sin}[b^2*\text{Pi}*x^2])/(362880*x) + (b^9*\text{Pi}^4*\text{Defer[Int]}[(\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/x, x])/1728$

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^{10}} dx &= -\frac{C(bx)^2}{9x^9} + \frac{1}{9}(2b) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^9} dx \\
&= -\frac{b^2}{504x^7} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{36x^8} - \frac{C(bx)^2}{9x^9} + \frac{1}{72}b^2 \int \frac{\cos(b^2\pi x^2)}{x^8} dx - \frac{1}{36}(b^3\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx \\
&= -\frac{b^2}{504x^7} - \frac{b^2 \cos(b^2\pi x^2)}{504x^7} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{36x^8} - \frac{C(bx)^2}{9x^9} + \frac{b^3\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{216x^6} - \frac{1}{432}(b^4\pi) \int \frac{C(bx) \cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx \\
&= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2 \cos(b^2\pi x^2)}{504x^7} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{36x^8} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{864x^4} - \frac{C(bx)^2}{9x^9} + \frac{b^4\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^5} \\
&= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2 \cos(b^2\pi x^2)}{504x^7} + \frac{187b^6\pi^2 \cos(b^2\pi x^2)}{181440x^3} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{36x^8} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{864x^4} - \frac{C(bx)^2}{9x^9} + \frac{b^4\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^5} \\
&= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2 \cos(b^2\pi x^2)}{504x^7} + \frac{187b^6\pi^2 \cos(b^2\pi x^2)}{181440x^3} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{36x^8} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{864x^4} - \frac{C(bx)^2}{9x^9} + \frac{b^4\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^5} \\
&= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2 \cos(b^2\pi x^2)}{504x^7} + \frac{187b^6\pi^2 \cos(b^2\pi x^2)}{181440x^3} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{36x^8} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{864x^4} - \frac{C(bx)^2}{9x^9} + \frac{b^4\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^5}
\end{aligned}$$

Mathematica [A] time = 0.0265919, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]^2/x^10, x]

[Out] Integrate[FresnelC[b*x]^2/x^10, x]

Maple [A] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx))^2}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^10,x)
```

```
[Out] int(FresnelC(b*x)^2/x^10,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^10, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx)^2}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^10, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(bx)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**10,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**10, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx)^2}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^2/x^10,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)^2/x^10, x)

3.158 $\int (c + dx)^2 \mathbf{FresnelC}(a + bx)^2 dx$

Optimal. Leaf size=495

$$\frac{id(a + bx)^2(bc - ad)\text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi(a + bx)^2\right)}{4\pi b^3} - \frac{id(a + bx)^2(bc - ad)\text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi(a + bx)^2\right)}{4\pi b^3}$$

[Out] $(2*d^2*x)/(3*b^2*Pi^2) - (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) - (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (4*d^2*Cos[(Pi*(a + b*x)^2]/2)*FresnelC[a + b*x])/(3*b^3*Pi^2) + ((b*c - a*d)^2*(a + b*x)*FresnelC[a + b*x]^2)/b^3 + (d*(b*c - a*d)*(a + b*x)^2*FresnelC[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*FresnelC[a + b*x]^2)/(3*b^3) + (5*d^2*FresnelC[Sqrt[2]*(a + b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (d*(b*c - a*d)*FresnelC[a + b*x]*FresnelS[a + b*x])/(b^3*Pi) + ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*b^3*Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - (2*(b*c - a*d)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2]/2))/(b^3*Pi) - (2*d*(b*c - a*d)*(a + b*x)*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2]/2))/(b^3*Pi) - (2*d^2*(a + b*x)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2]/2))/(3*b^3*Pi)$

Rubi [A] time = 0.396293, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {6433, 6421, 6453, 3351, 6431, 6455, 6447, 3379, 2638, 6461, 3358, 3352, 3385}

$$\frac{id(a + bx)^2(bc - ad) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}i\pi(a + bx)^2\right)}{4\pi b^3} - \frac{id(a + bx)^2(bc - ad) {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}i\pi(a + bx)^2\right)}{4\pi b^3} + \frac{d(bc - ad)\text{FresnelC}[a + bx]^2}{4\pi b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*FresnelC[a + b*x]^2,x]

[Out] $(2*d^2*x)/(3*b^2*Pi^2) - (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) - (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (4*d^2*Cos[(Pi*(a + b*x)^2]/2)*FresnelC[a + b*x])/(3*b^3*Pi^2) + ((b*c - a*d)^2*(a + b*x)*FresnelC[a + b*x]^2)/b^3 + (d*(b*c - a*d)*(a + b*x)^2*FresnelC[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*FresnelC[a + b*x]^2)/(3*b^3) + (5*d^2*FresnelC[Sqrt[2]*(a + b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (d*(b*c - a*d)*FresnelC[a + b*x]*FresnelS[a + b*x])/(b^3*Pi) + ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*b^3*Pi)$

$\text{Pi} + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*\text{Pi}*(a + b*x)^2])/(b^3*\text{Pi}) - ((I/4)*d*(b*c - a*d)*(a + b*x)^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*\text{Pi}*(a + b*x)^2])/(b^3*\text{Pi}) - (2*(b*c - a*d)^2*\text{FresnelC}[a + b*x]*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(b^3*\text{Pi}) - (2*d*(b*c - a*d)*(a + b*x)*\text{FresnelC}[a + b*x]*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(b^3*\text{Pi}) - (2*d^2*(a + b*x)^2*\text{FresnelC}[a + b*x]*\text{Sin}[(\text{Pi}*(a + b*x)^2)/2])/(3*b^3*\text{Pi})$

Rule 6433

$\text{Int}[\text{FresnelC}[(a_) + (b_)*(x_)]^2*((c_) + (d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Dist}[1/b^{(m+1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[\text{FresnelC}[x]^2, (b*c - a*d + d*x)^m, x], x], x, a + b*x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{IGtQ}[m, 0]$

Rule 6421

$\text{Int}[\text{FresnelC}[(a_) + (b_)*(x_)]^2, x_Symbol] \rightarrow \text{Simp}[(a + b*x)*\text{FresnelC}[a + b*x]^2/b, x] - \text{Dist}[2, \text{Int}[(a + b*x)*\text{Cos}[(\text{Pi}*(a + b*x)^2)/2]*\text{FresnelC}[a + b*x], x], x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 6453

$\text{Int}[\text{Cos}[(d_)*(x_)^2]*\text{FresnelC}[(b_)*(x_)]*(x_), x_Symbol] \rightarrow \text{Simp}[(\text{Sin}[d*x^2]*\text{FresnelC}[b*x])/(2*d), x] - \text{Dist}[b/(4*d), \text{Int}[\text{Sin}[2*d*x^2], x], x] /; \text{FreeQ}[\{b, d\}, x] \&\& \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4]$

Rule 3351

$\text{Int}[\text{Sin}[(d_)*((e_) + (f_)*(x_))^{(m_)}], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}[\{d, e, f\}, x]$

Rule 6431

$\text{Int}[\text{FresnelC}[(b_)*(x_)]^2*(x_)^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m+1)}*\text{FresnelC}[b*x]^2)/(m+1), x] - \text{Dist}[(2*b)/(m+1), \text{Int}[x^{(m+1)}*\text{Cos}[(\text{Pi}*b^2*x^2)/2]*\text{FresnelC}[b*x], x], x] /; \text{FreeQ}[b, x] \&\& \text{IntegerQ}[m] \&\& \text{NeQ}[m, -1]$

Rule 6455

$\text{Int}[\text{Cos}[(d_)*(x_)^2]*\text{FresnelC}[(b_)*(x_)]*(x_)^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m-1)}*\text{Sin}[d*x^2]*\text{FresnelC}[b*x])/(2*d), x] + (-\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\text{Sin}[d*x^2]*\text{FresnelC}[b*x], x], x] - \text{Dist}[b/(4*d), \text{Int}[x^{(m-1)}*\text{Sin}[2*d*x^2], x], x]) /; \text{FreeQ}[\{b, d\}, x] \&\& \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4] \&\& \text{IGtQ}[m, 1]$

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2)]/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_)^(n_))]*(b_.))^(p_), x_Sy
mbol] := Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_)^2)], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_)^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 C(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \left(b^2 c^2 \left(1 + \frac{ad(-2bc+ad)}{b^2 c^2}\right) C(x)^2 + 2bcd \left(1 - \frac{ad}{bc}\right) x C(x)^2 + d^2 x^2 C(x)^2\right) dx, x, a + bx\right)}{b^3} \\
&= \frac{d^2 \text{Subst}\left(\int x^2 C(x)^2 dx, x, a + bx\right)}{b^3} + \frac{(2d(bc - ad)) \text{Subst}\left(\int x C(x)^2 dx, x, a + bx\right)}{b^3} + \frac{(bc - ad)^2 \text{Subst}\left(\int C(x)^2 dx, x, a + bx\right)}{b^3} \\
&= \frac{(bc - ad)^2 (a + bx) C(a + bx)^2}{b^3} + \frac{d(bc - ad)(a + bx)^2 C(a + bx)^2}{b^3} + \frac{d^2 (a + bx)^3 C(a + bx)^2}{3b^3} - \frac{d^2 (a + bx)^3 C(a + bx)^2}{3b^3} \\
&= \frac{(bc - ad)^2 (a + bx) C(a + bx)^2}{b^3} + \frac{d(bc - ad)(a + bx)^2 C(a + bx)^2}{b^3} + \frac{d^2 (a + bx)^3 C(a + bx)^2}{3b^3} - \frac{d^2 (a + bx)^3 C(a + bx)^2}{3b^3} \\
&= -\frac{d^2 (a + bx) \cos(\pi(a + bx)^2)}{6b^3 \pi^2} - \frac{4d^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right) C(a + bx)}{3b^3 \pi^2} + \frac{(bc - ad)^2 (a + bx) C(a + bx)^2}{b^3} \\
&= -\frac{d(bc - ad) \cos(\pi(a + bx)^2)}{2b^3 \pi^2} - \frac{d^2 (a + bx) \cos(\pi(a + bx)^2)}{6b^3 \pi^2} - \frac{4d^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right) C(a + bx)}{3b^3 \pi^2} \\
&= \frac{2d^2 x}{3b^2 \pi^2} - \frac{d(bc - ad) \cos(\pi(a + bx)^2)}{2b^3 \pi^2} - \frac{d^2 (a + bx) \cos(\pi(a + bx)^2)}{6b^3 \pi^2} - \frac{4d^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right) C(a + bx)}{3b^3 \pi^2} \\
&= \frac{2d^2 x}{3b^2 \pi^2} - \frac{d(bc - ad) \cos(\pi(a + bx)^2)}{2b^3 \pi^2} - \frac{d^2 (a + bx) \cos(\pi(a + bx)^2)}{6b^3 \pi^2} - \frac{4d^2 \cos\left(\frac{1}{2}\pi(a + bx)^2\right) C(a + bx)}{3b^3 \pi^2}
\end{aligned}$$

Mathematica [F] time = 0.6492, size = 0, normalized size = 0.

$$\int (c + dx)^2 \text{FresnelC}(a + bx)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^2*FresnelC[a + b*x]^2,x]

[Out] Integrate[(c + d*x)^2*FresnelC[a + b*x]^2, x]

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int (dx + c)^2 (\text{FresnelC}(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^2*FresnelC(b*x+a)^2,x)`

[Out] `int((d*x+c)^2*FresnelC(b*x+a)^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \text{fresnelc}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*fresnelc(b*x+a)^2,x, algorithm="maxima")`

[Out] `integrate((d*x + c)^2*fresnelc(b*x + a)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^2 + 2cdx + c^2\right)\text{fresnelc}(bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*fresnelc(b*x+a)^2,x, algorithm="fricas")`

[Out] `integral((d^2*x^2 + 2*c*d*x + c^2)*fresnelc(b*x + a)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 C^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**2*fresnelc(b*x+a)**2,x)`

[Out] `Integral((c + d*x)**2*fresnelc(a + b*x)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \text{fresnelc}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*fresnelc(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*fresnelc(b*x + a)^2, x)

3.159 $\int (c + dx)\mathbf{FresnelC}(a + bx)^2 dx$

Optimal. Leaf size=279

$$\frac{id(a + bx)^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi(a + bx)^2\right)}{8\pi b^2} - \frac{id(a + bx)^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi(a + bx)^2\right)}{8\pi b^2}$$

```
[Out] -(d*cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + ((b*c - a*d)*(a + b*x)*FresnelC[a +
b*x]^2)/b^2 + (d*(a + b*x)^2*FresnelC[a + b*x]^2)/(2*b^2) + (d*FresnelC[a
+ b*x]*FresnelS[a + b*x])/(2*b^2*Pi) + ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b
*x)])/(Sqrt[2]*b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - (2*(b*c - a*d)*Fr
esnelC[a + b*x]*Sin[(Pi*(a + b*x)^2]/2])/(b^2*Pi) - (d*(a + b*x)*FresnelC[a
+ b*x]*Sin[(Pi*(a + b*x)^2]/2])/(b^2*Pi)
```

Rubi [A] time = 0.185127, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6433, 6421, 6453, 3351, 6431, 6455, 6447, 3379, 2638}

$$\frac{id(a + bx)^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}i\pi(a + bx)^2\right)}{8\pi b^2} - \frac{id(a + bx)^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}i\pi(a + bx)^2\right)}{8\pi b^2} + \frac{(a + bx)(bc - ad)\mathbf{FresnelC}(a + bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelC[a + b*x]^2, x]
```

```
[Out] -(d*cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + ((b*c - a*d)*(a + b*x)*FresnelC[a +
b*x]^2)/b^2 + (d*(a + b*x)^2*FresnelC[a + b*x]^2)/(2*b^2) + (d*FresnelC[a
+ b*x]*FresnelS[a + b*x])/(2*b^2*Pi) + ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b
*x)])/(Sqrt[2]*b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - (2*(b*c - a*d)*Fr
esnelC[a + b*x]*Sin[(Pi*(a + b*x)^2]/2])/(b^2*Pi) - (d*(a + b*x)*FresnelC[a
+ b*x]*Sin[(Pi*(a + b*x)^2]/2])/(b^2*Pi)
```

Rule 6433

```
Int[FresnelC[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelC[x]^2, (b*c - a*d + d*x
```

$^m, x], x], x, a + b*x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 6421

$\text{Int}[\text{FresnelC}[(a_.) + (b_.)*(x_.)]^2, x_Symbol] \rightarrow \text{Simp}[\frac{(a + b*x)*\text{FresnelC}[a + b*x]^2}{b}, x] - \text{Dist}[2, \text{Int}[(a + b*x)*\text{Cos}[(\text{Pi}*(a + b*x)^2]/2)*\text{FresnelC}[a + b*x], x], x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 6453

$\text{Int}[\text{Cos}[(d_.)*(x_.)^2]*\text{FresnelC}[(b_.)*(x_.)]*(x_.), x_Symbol] \rightarrow \text{Simp}[\frac{\text{Sin}[d*x^2]*\text{FresnelC}[b*x]}{2*d}, x] - \text{Dist}[b/(4*d), \text{Int}[\text{Sin}[2*d*x^2], x], x] /; \text{FreeQ}[\{b, d\}, x] \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \rightarrow \text{Simp}[\frac{\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)]}{(f*\text{Rt}[d, 2])}, x] /; \text{FreeQ}[\{d, e, f\}, x]$

Rule 6431

$\text{Int}[\text{FresnelC}[(b_.)*(x_.)]^2*(x_.)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[\frac{(x^{(m+1)}*\text{FresnelC}[b*x]^2)}{(m+1)}, x] - \text{Dist}[\frac{2*b}{(m+1)}, \text{Int}[x^{(m+1)}*\text{Cos}[(\text{Pi}*b^2*x^2)/2]*\text{FresnelC}[b*x], x], x] /; \text{FreeQ}[b, x] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{NeQ}[m, -1]$

Rule 6455

$\text{Int}[\text{Cos}[(d_.)*(x_.)^2]*\text{FresnelC}[(b_.)*(x_.)]*(x_.)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[\frac{(x^{(m-1)}*\text{Sin}[d*x^2]*\text{FresnelC}[b*x])}{2*d}, x] + (-\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\text{Sin}[d*x^2]*\text{FresnelC}[b*x], x], x] - \text{Dist}[b/(4*d), \text{Int}[x^{(m-1)}*\text{Sin}[2*d*x^2], x], x]) /; \text{FreeQ}[\{b, d\}, x] \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

Rule 6447

$\text{Int}[\text{FresnelC}[(b_.)*(x_.)]*\text{Sin}[(d_.)*(x_.)^2], x_Symbol] \rightarrow \text{Simp}[\frac{(b*\text{Pi}*\text{FresnelC}[b*x]*\text{FresnelS}[b*x])}{4*d}, x] + (\text{Simp}[\frac{(1*I*b*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, -(I*d*x^2)])}{8}, x] - \text{Simp}[\frac{(1*I*b*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, I*d*x^2])}{8}, x]) /; \text{FreeQ}[\{b, d\}, x] \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4]$

Rule 3379

$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*\text{Sin}[(c_.) + (d_.)*(x_.)^{(n_.)])}^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Sin}[c + d*x])^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[($

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int (c + dx)C(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \left(bc\left(1 - \frac{ad}{bc}\right)C(x)^2 + dxC(x)^2\right) dx, x, a + bx\right)}{b^2} \\
 &= \frac{d \text{Subst}\left(\int xC(x)^2 dx, x, a + bx\right)}{b^2} + \frac{(bc - ad) \text{Subst}\left(\int C(x)^2 dx, x, a + bx\right)}{b^2} \\
 &= \frac{(bc - ad)(a + bx)C(a + bx)^2}{b^2} + \frac{d(a + bx)^2C(a + bx)^2}{2b^2} - \frac{d \text{Subst}\left(\int x^2 \cos\left(\frac{\pi x^2}{2}\right)C(x) dx, x, a + bx\right)}{b^2} \\
 &= \frac{(bc - ad)(a + bx)C(a + bx)^2}{b^2} + \frac{d(a + bx)^2C(a + bx)^2}{2b^2} - \frac{2(bc - ad)C(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{b^2\pi} \\
 &= \frac{(bc - ad)(a + bx)C(a + bx)^2}{b^2} + \frac{d(a + bx)^2C(a + bx)^2}{2b^2} + \frac{dC(a + bx)S(a + bx)}{2b^2\pi} + \frac{(bc - ad)S\left(\sqrt{2}b^2(a + bx)^2\right)}{\sqrt{2}b^2\pi} \\
 &= -\frac{d \cos\left(\pi(a + bx)^2\right)}{4b^2\pi^2} + \frac{(bc - ad)(a + bx)C(a + bx)^2}{b^2} + \frac{d(a + bx)^2C(a + bx)^2}{2b^2} + \frac{dC(a + bx)S(a + bx)}{2b^2\pi}
 \end{aligned}$$

Mathematica [F] time = 0.548492, size = 0, normalized size = 0.

$$\int (c + dx)\text{FresnelC}(a + bx)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)*FresnelC[a + b*x]^2, x]
```

```
[Out] Integrate[(c + d*x)*FresnelC[a + b*x]^2, x]
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int (dx + c) (\text{FresnelC}(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*FresnelC(b*x+a)^2,x)

[Out] int((d*x+c)*FresnelC(b*x+a)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \text{fresnelc}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*fresnelc(b*x+a)^2,x, algorithm="maxima")

[Out] integrate((d*x + c)*fresnelc(b*x + a)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c) \text{fresnelc}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*fresnelc(b*x+a)^2,x, algorithm="fricas")

[Out] integral((d*x + c)*fresnelc(b*x + a)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) C^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*fresnelc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\text{fresnelc}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnelc(b*x + a)^2, x)
```

3.160 $\int \text{FresnelC}(a + bx)^2 dx$

Optimal. Leaf size=69

$$\frac{(a + bx)\text{FresnelC}(a + bx)^2}{b} - \frac{2\text{FresnelC}(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b} + \frac{S\left(\sqrt{2}(a + bx)\right)}{\sqrt{2}\pi b}$$

[Out] ((a + b*x)*FresnelC[a + b*x]^2)/b + FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*Pi) - (2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2]/2)]/(b*Pi)

Rubi [A] time = 0.150686, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6421, 6453, 3351}

$$\frac{(a + bx)\text{FresnelC}(a + bx)^2}{b} - \frac{2\text{FresnelC}(a + bx) \sin\left(\frac{1}{2}\pi(a + bx)^2\right)}{\pi b} + \frac{S\left(\sqrt{2}(a + bx)\right)}{\sqrt{2}\pi b}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[a + b*x]^2,x]

[Out] ((a + b*x)*FresnelC[a + b*x]^2)/b + FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*Pi) - (2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2]/2)]/(b*Pi)

Rule 6421

Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] := Simp[((a + b*x)*FresnelC[a + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Cos[(Pi*(a + b*x)^2]/2]*FresnelC[a + b*x], x], x] /; FreeQ[{a, b}, x]

Rule 6453

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] := Simp[(Sin[d*x^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int C(a+bx)^2 dx &= \frac{(a+bx)C(a+bx)^2}{b} - 2 \int (a+bx) \cos\left(\frac{1}{2}\pi(a+bx)^2\right) C(a+bx) dx \\
&= \frac{(a+bx)C(a+bx)^2}{b} - \frac{2 \operatorname{Subst}\left(\int x \cos\left(\frac{\pi x^2}{2}\right) C(x) dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)C(a+bx)^2}{b} - \frac{2C(a+bx) \sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi} + \frac{\operatorname{Subst}\left(\int \sin(\pi x^2) dx, x, a+bx\right)}{b\pi} \\
&= \frac{(a+bx)C(a+bx)^2}{b} + \frac{S\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b\pi} - \frac{2C(a+bx) \sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}
\end{aligned}$$

Mathematica [A] time = 0.0099422, size = 66, normalized size = 0.96

$$\frac{2\pi(a+bx)\operatorname{FresnelC}(a+bx)^2 - 4\operatorname{FresnelC}(a+bx) \sin\left(\frac{1}{2}\pi(a+bx)^2\right) + \sqrt{2}S\left(\sqrt{2}(a+bx)\right)}{2\pi b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[a + b*x]^2, x]

[Out] (2*Pi*(a + b*x)*FresnelC[a + b*x]^2 + Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)] - 4*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2]/2))/(2*b*Pi)

Maple [A] time = 0.054, size = 60, normalized size = 0.9

$$\frac{1}{b} \left((bx+a) (\operatorname{FresnelC}(bx+a))^2 - 2 \frac{\operatorname{FresnelC}(bx+a) \sin\left(\frac{1}{2}\pi(bx+a)^2\right)}{\pi} + \frac{\sqrt{2}\operatorname{FresnelS}\left((bx+a)\sqrt{2}\right)}{2\pi} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x+a)^2, x)

[Out] 1/b*((b*x+a)*FresnelC(b*x+a)^2-2*FresnelC(b*x+a)/Pi*sin(1/2*Pi*(b*x+a)^2)+1/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{fresnelc}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{fresnelc}(bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int C^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{fresnelc}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a)^2, x)
```

$$3.161 \quad \int \frac{\mathbf{FresnelC}(a+bx)^2}{c+dx} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\mathbf{FresnelC}(a+bx)^2}{c+dx}, x\right)$$

[Out] Unintegrable[FresnelC[a + b*x]^2/(c + d*x), x]

Rubi [A] time = 0.023169, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\mathbf{FresnelC}(a+bx)^2}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[a + b*x]^2/(c + d*x), x]

[Out] Defer[Int][FresnelC[a + b*x]^2/(c + d*x), x]

Rubi steps

$$\int \frac{C(a+bx)^2}{c+dx} dx = \int \frac{C(a+bx)^2}{c+dx} dx$$

Mathematica [A] time = 0.0351198, size = 0, normalized size = 0.

$$\int \frac{\mathbf{FresnelC}(a+bx)^2}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[a + b*x]^2/(c + d*x), x]

[Out] Integrate[FresnelC[a + b*x]^2/(c + d*x), x]

Maple [A] time = 0.357, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx + a))^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x+a)^2/(d*x+c), x)

[Out] int(FresnelC(b*x+a)^2/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)^2/(d*x+c), x, algorithm="maxima")

[Out] integrate(fresnelc(b*x + a)^2/(d*x + c), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx + a)^2}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)^2/(d*x+c), x, algorithm="fricas")

[Out] integral(fresnelc(b*x + a)^2/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)**2/(d*x+c),x)

[Out] Integral(fresnelc(a + b*x)**2/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)^2/(d*x+c),x, algorithm="giac")

[Out] integrate(fresnelc(b*x + a)^2/(d*x + c), x)

$$3.162 \quad \int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[FresnelC[a + b*x]^2/(c + d*x)^2, x]

Rubi [A] time = 0.0228539, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[a + b*x]^2/(c + d*x)^2, x]

[Out] Defer[Int][FresnelC[a + b*x]^2/(c + d*x)^2, x]

Rubi steps

$$\int \frac{C(a+bx)^2}{(c+dx)^2} dx = \int \frac{C(a+bx)^2}{(c+dx)^2} dx$$

Mathematica [A] time = 0.0795137, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[a + b*x]^2/(c + d*x)^2, x]

[Out] Integrate[FresnelC[a + b*x]^2/(c + d*x)^2, x]

Maple [A] time = 0.347, size = 0, normalized size = 0.

$$\int \frac{(\text{FresnelC}(bx + a))^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x+a)^2/(d*x+c)^2,x)

[Out] int(FresnelC(b*x+a)^2/(d*x+c)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x + a)^2/(d*x + c)^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx + a)^2}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(fresnelc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C^2(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(fresnelc(a + b*x)**2/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx + a)^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(fresnelc(b*x + a)^2/(d*x + c)^2, x)

3.163 $\int x^2 \text{FresnelC}(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=231

$$\left(\frac{1}{12} + \frac{i}{12}\right) x^3 (cx^n)^{-3/n} e^{-\frac{3a}{bn} + \frac{9i}{2\pi b^2 d^2 n^2}} \text{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi a b d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{3}{n}\right)}{\sqrt{\pi} b d}\right) - \left(\frac{1}{12} + \frac{i}{12}\right) x^3 (cx^n)^{-3/n} e^{-\frac{3a}{bn} - \frac{9i}{2\pi b^2 d^2 n^2}}$$

[Out] $((1/12 + I/12)*E^{((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))}*x^3*\text{Erf}[(1/2 + I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*\text{Log}[c*x^n])]/(b*d*\text{Sqrt}[Pi])]/(c*x^n)^{(3/n)} - ((1/12 + I/12)*E^{((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))}*x^3*\text{Erfi}[(1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*\text{Log}[c*x^n])]/(b*d*\text{Sqrt}[Pi])]/(c*x^n)^{(3/n)} + (x^3*\text{FresnelC}[d*(a + b*\text{Log}[c*x^n])])/3$

Rubi [A] time = 0.522269, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{12} + \frac{i}{12}\right) x^3 (cx^n)^{-3/n} e^{-\frac{3a}{bn} + \frac{9i}{2\pi b^2 d^2 n^2}} \text{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi a b d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{3}{n}\right)}{\sqrt{\pi} b d}\right) - \left(\frac{1}{12} + \frac{i}{12}\right) x^3 (cx^n)^{-3/n} e^{-\frac{3a}{bn} - \frac{9i}{2\pi b^2 d^2 n^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*\text{FresnelC}[d*(a + b*\text{Log}[c*x^n])], x]$

[Out] $((1/12 + I/12)*E^{((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))}*x^3*\text{Erf}[(1/2 + I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*\text{Log}[c*x^n])]/(b*d*\text{Sqrt}[Pi])]/(c*x^n)^{(3/n)} - ((1/12 + I/12)*E^{((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))}*x^3*\text{Erfi}[(1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*\text{Log}[c*x^n])]/(b*d*\text{Sqrt}[Pi])]/(c*x^n)^{(3/n)} + (x^3*\text{FresnelC}[d*(a + b*\text{Log}[c*x^n])])/3$

Rule 6472

$\text{Int}[\text{FresnelC}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*(d_.)]*((e_.)*(x_.))^{(m_.)}, x_Symbol] :> \text{Simp}[(e*x)^{(m+1)}*\text{FresnelC}[d*(a + b*\text{Log}[c*x^n])]/(e*(m+1)), x] - \text{Dist}[(b*d*n)/(m+1), \text{Int}[(e*x)^m*\text{Cos}[(Pi*(d*(a + b*\text{Log}[c*x^n]))^2)/2], x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x\} \&\& \text{NeQ}[m, -1]$

Rule 4618

Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_)^(m_.),
 x_Symbol] := Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
 ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c,
 , d, e, m, n}, x]

Rule 2278

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_)^(m_.),
 x_Symbol] := Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
 ^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2274

Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] := Int[u*F^(a*v)*
 z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^F
 racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]

Rule 2276

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_)^(m_.),
 x_Symbol] := Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^(m + 1)/n), Subst[Int[E^(a
 *d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
 Q[{F, a, b, c, d, e, m, n}, x]

Rule 2234

Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[F^(a - b^2/
 (4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqr
 t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
 F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqr
 t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]]/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int x^2 C(d(a + b \log(cx^n))) dx &= \frac{1}{3} x^3 C(d(a + b \log(cx^n))) - \frac{1}{3} (bdn) \int x^2 \cos\left(\frac{1}{2} d^2 \pi (a + b \log(cx^n))^2\right) dx \\
 &= \frac{1}{3} x^3 C(d(a + b \log(cx^n))) - \frac{1}{6} (bdn) \int e^{-\frac{1}{2} i d^2 \pi (a + b \log(cx^n))^2} x^2 dx - \frac{1}{6} (bdn) \int e^{\frac{1}{2} i d^2 \pi (a + b \log(cx^n))^2} x^2 dx \\
 &= \frac{1}{3} x^3 C(d(a + b \log(cx^n))) - \frac{1}{6} (bdn) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - i a b d^2 \pi \log(cx^n) - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) x^2 dx \\
 &= \frac{1}{3} x^3 C(d(a + b \log(cx^n))) - \frac{1}{6} (bdn) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) x^2 (cx^n)^{-n} dx \\
 &= \frac{1}{3} x^3 C(d(a + b \log(cx^n))) - \frac{1}{6} \left(b d n x^{i a b d^2 n \pi} (cx^n)^{-i a b d^2 \pi}\right) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) x^2 (cx^n)^{-n} dx \\
 &= \frac{1}{3} x^3 C(d(a + b \log(cx^n))) - \frac{1}{6} \left(b d x^3 (cx^n)^{-i a b d^2 \pi - \frac{3 - i a b d^2 n \pi}{n}}\right) \text{Subst}\left(\int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(u)\right) u^2 du\right) \\
 &= \frac{1}{3} x^3 C(d(a + b \log(cx^n))) - \frac{1}{6} \left(b d e^{-\frac{3a}{bn} - \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (cx^n)^{-i a b d^2 \pi - \frac{3 - i a b d^2 n \pi}{n}}\right) \text{Subst}\left(\int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(u)\right) u^2 du\right) \\
 &= \left(\frac{1}{12} + \frac{i}{12}\right) e^{-\frac{3a}{bn} + \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (cx^n)^{-3/n} \text{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{3}{n} + i a b d^2 \pi + i b^2 d^2 \pi \log(cx^n)\right)}{b d \sqrt{\pi}}\right) - \left(\frac{1}{12} - \frac{i}{12}\right) e^{-\frac{3a}{bn} - \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (cx^n)^{-3/n} \text{erf}\left(\frac{\left(\frac{1}{2} - \frac{i}{2}\right) \left(\frac{3}{n} + i a b d^2 \pi + i b^2 d^2 \pi \log(cx^n)\right)}{b d \sqrt{\pi}}\right)
 \end{aligned}$$

Mathematica [A] time = 6.7753, size = 318, normalized size = 1.38

$$\frac{1}{12} x^3 \left(4 \text{FresnelC}(d(a + b \log(cx^n))) + \sqrt[4]{-1} \sqrt{2} (cx^n)^{-3/n} \left(i e^{\frac{9i}{\pi b^2 d^2 n^2}} \text{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) - 3i)}{\sqrt{\pi} b d n}\right) \right) + \text{Erfi}\left(\frac{\left(\frac{1}{2} - \frac{i}{2}\right) (\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) + 3i)}{\sqrt{\pi} b d n}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*FresnelC[d*(a + b*Log[c*x^n])], x]

[Out] (x^3*(4*FresnelC[d*(a + b*Log[c*x^n])]) + ((-1)^(1/4)*Sqrt[2]*E^(((6*a)/(b*n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(I*E^((9*I)/(b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n])]/(b*d*n*Sqrt[Pi]))))

$$\frac{\text{Pi} \cdot \text{Log}[c \cdot x^n]}{(b \cdot d \cdot n \cdot \text{Sqrt}[2 \cdot \text{Pi}])} \cdot (\text{Cos}[(d^2 \cdot \text{Pi} \cdot (a - b \cdot n \cdot \text{Log}[x] + b \cdot \text{Log}[c \cdot x^n])^2) / 2] + \text{I} \cdot \text{Sin}[(d^2 \cdot \text{Pi} \cdot (a - b \cdot n \cdot \text{Log}[x] + b \cdot \text{Log}[c \cdot x^n])^2) / 2]) / (c \cdot x^n)^{(3/n)}) / 12$$

Maple [F] time = 0.523, size = 0, normalized size = 0.

$$\int x^2 \text{FresnelC}(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)

[Out] int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")

[Out] integrate(x^2*fresnelc((b*log(c*x^n) + a)*d), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \text{fresnelc}(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] integral(x^2*fresnelc(b*d*log(c*x^n) + a*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 C(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*fresnelc(d*(a+b*ln(c*x**n))),x)

[Out] Integral(x**2*fresnelc(a*d + b*d*log(c*x**n)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")

[Out] integrate(x^2*fresnelc((b*log(c*x^n) + a)*d), x)

3.164 $\int x \text{FresnelC}(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=227

$$\left(\frac{1}{8} + \frac{i}{8}\right) x^2 (cx^n)^{-2/n} e^{-\frac{2\pi ab d^2 n + 2i}{\pi b^2 d^2 n^2}} \text{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n}\right)}{\sqrt{\pi} b d}\right) - \left(\frac{1}{8} + \frac{i}{8}\right) x^2 (cx^n)^{-2/n} e^{-\frac{2(\pi ab d^2 n + i)}{\pi b^2 d^2 n^2}} \text{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n}\right)}{\sqrt{\pi} b d}\right)$$

[Out] $((1/8 + I/8)*E^{((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[(((1/2 + I/2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi]))]/(c*x^n)^{(2/n)} - ((1/8 + I/8)*x^2*Erfi[(((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi]))]/(E^{((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*Pi))}*(c*x^n)^{(2/n))} + (x^2*FresnelC[d*(a + b*Log[c*x^n]))]/2$

Rubi [A] time = 0.407877, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{8} + \frac{i}{8}\right) x^2 (cx^n)^{-2/n} e^{-\frac{2\pi ab d^2 n + 2i}{\pi b^2 d^2 n^2}} \text{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n}\right)}{\sqrt{\pi} b d}\right) - \left(\frac{1}{8} + \frac{i}{8}\right) x^2 (cx^n)^{-2/n} e^{-\frac{2(\pi ab d^2 n + i)}{\pi b^2 d^2 n^2}} \text{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n}\right)}{\sqrt{\pi} b d}\right)$$

Antiderivative was successfully verified.

[In] Int[x*FresnelC[d*(a + b*Log[c*x^n]),x]

[Out] $((1/8 + I/8)*E^{((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[(((1/2 + I/2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi]))]/(c*x^n)^{(2/n)} - ((1/8 + I/8)*x^2*Erfi[(((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi]))]/(E^{((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*Pi))}*(c*x^n)^{(2/n))} + (x^2*FresnelC[d*(a + b*Log[c*x^n]))]/2$

Rule 6472

Int[FresnelC[(a_.) + Log[(c_.)*(x_.)^(n_.)]*(b_.)]*(d_.)]*((e_.)*(x_.))^(m_.), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])]/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]

Rule 4618

Int[Cos[(a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.)]^2*(d_.)]*((e_.)*(x_)^(m_.),
 x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + Dist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]

Rule 2278

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.)]^2*(d_.)]*((e_.)*(x_)^(m_.),
 x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2274

Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2276

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.)]*(d_.)]*((e_.)*(x_)^(m_.),
 x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2234

Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int x C(d(a + b \log(cx^n))) dx &= \frac{1}{2} x^2 C(d(a + b \log(cx^n))) - \frac{1}{2} (bdn) \int x \cos\left(\frac{1}{2} d^2 \pi (a + b \log(cx^n))^2\right) dx \\
 &= \frac{1}{2} x^2 C(d(a + b \log(cx^n))) - \frac{1}{4} (bdn) \int e^{-\frac{1}{2} i d^2 \pi (a + b \log(cx^n))^2} x dx - \frac{1}{4} (bdn) \int e^{\frac{1}{2} i d^2 \pi (a + b \log(cx^n))^2} x dx \\
 &= \frac{1}{2} x^2 C(d(a + b \log(cx^n))) - \frac{1}{4} (bdn) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - i a b d^2 \pi \log(cx^n) - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) x dx \\
 &= \frac{1}{2} x^2 C(d(a + b \log(cx^n))) - \frac{1}{4} (bdn) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) x (cx^n)^{-i a b d^2 \pi} dx \\
 &= \frac{1}{2} x^2 C(d(a + b \log(cx^n))) - \frac{1}{4} \left(b d n x^{i a b d^2 n \pi} (cx^n)^{-i a b d^2 \pi}\right) \int \exp\left(-\frac{1}{2} i a^2 d^2 \pi - \frac{1}{2} i b^2 d^2 \pi \log^2(cx^n)\right) dx \\
 &= \frac{1}{2} x^2 C(d(a + b \log(cx^n))) - \frac{1}{4} \left(b d x^2 (cx^n)^{-i a b d^2 \pi - \frac{2 - i a b d^2 n \pi}{n}}\right) \text{Subst}\left(\int \exp\left(-\frac{1}{2} i a^2 d^2 \pi + \frac{2 - i a b d^2 n \pi}{n} \log^2(u)\right) du\right) \\
 &= \frac{1}{2} x^2 C(d(a + b \log(cx^n))) - \frac{1}{4} \left(b d e^{-\frac{2(i + a b d^2 n \pi)}{b^2 d^2 n^2 \pi}} x^2 (cx^n)^{-i a b d^2 \pi - \frac{2 - i a b d^2 n \pi}{n}}\right) \text{Subst}\left(\int \exp\left(\frac{i(2 - i a b d^2 n \pi)}{b^2 d^2 n^2 \pi} \log^2(u)\right) du\right) \\
 &= \left(\frac{1}{8} + \frac{i}{8}\right) e^{\frac{2i - 2 a b d^2 n \pi}{b^2 d^2 n^2 \pi}} x^2 (cx^n)^{-2/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} + i a b d^2 \pi + i b^2 d^2 \pi \log(cx^n)\right)}{b d \sqrt{\pi}}\right) - \left(\frac{1}{8} + \frac{i}{8}\right) e^{-\frac{2i - 2 a b d^2 n \pi}{b^2 d^2 n^2 \pi}} x^2 (cx^n)^{-2/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} - \frac{i}{2}\right) \left(\frac{2}{n} + i a b d^2 \pi + i b^2 d^2 \pi \log(cx^n)\right)}{b d \sqrt{\pi}}\right)
 \end{aligned}$$

Mathematica [A] time = 6.64992, size = 318, normalized size = 1.4

$$\frac{1}{8} x^2 \left(4 \operatorname{FresnelC}(d(a + b \log(cx^n))) + \sqrt[4]{-1} \sqrt{2} (cx^n)^{-2/n} \left(i e^{\frac{4i}{\pi b^2 d^2 n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) - 2i)}{\sqrt{\pi} b d n}\right) \right) + \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} - \frac{i}{2}\right) (\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) + 2i)}{\sqrt{\pi} b d n}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x*FresnelC[d*(a + b*Log[c*x^n])],x]

[Out] (x^2*(4*FresnelC[d*(a + b*Log[c*x^n])]) + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*n)) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(I*E^((4*I)/(b^2*d^2*n^2*Pi)))*Erfi[((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])])

$$\frac{\text{Pi} \cdot \text{Log}[c \cdot x^n]}{(b \cdot d \cdot n \cdot \text{Sqrt}[2 \cdot \text{Pi}])}] \cdot (\text{Cos}[(d^2 \cdot \text{Pi} \cdot (a - b \cdot n \cdot \text{Log}[x] + b \cdot \text{Log}[c \cdot x^n])^2) / 2] + \text{I} \cdot \text{Sin}[(d^2 \cdot \text{Pi} \cdot (a - b \cdot n \cdot \text{Log}[x] + b \cdot \text{Log}[c \cdot x^n])^2) / 2]) / (c \cdot x^n)^{(2/n)}) / 8$$

Maple [F] time = 0.539, size = 0, normalized size = 0.

$$\int x \text{FresnelC}(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*FresnelC(d*(a+b*ln(c*x^n))),x)

[Out] int(x*FresnelC(d*(a+b*ln(c*x^n))),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \text{fresnelc}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")

[Out] integrate(x*fresnelc((b*log(c*x^n) + a)*d), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \text{fresnelc}(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] integral(x*fresnelc(b*d*log(c*x^n) + a*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x C(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(d*(a+b*ln(c*x**n))),x)

[Out] Integral(x*fresnelc(a*d + b*d*log(c*x**n)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \text{fresnelc}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")

[Out] integrate(x*fresnelc((b*log(c*x^n) + a)*d), x)

3.165 $\int \text{FresnelC}(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=214

$$\left(\frac{1}{4} + \frac{i}{4}\right) x (cx^n)^{-1/n} e^{-\frac{2abn - \frac{i}{\pi d^2}}{2b^2 n^2}} \text{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n}\right)}{\sqrt{\pi b d}}\right) - \left(\frac{1}{4} + \frac{i}{4}\right) x (cx^n)^{-1/n} e^{-\frac{2abn + \frac{i}{\pi d^2}}{2b^2 n^2}} \text{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n}\right)}{\sqrt{\pi b d}}\right)$$

[Out] $((1/4 + I/4)*x*\text{Erf}(((1/2 + I/2)*(n^{-1}) + I*a*b*d^2*\text{Pi} + I*b^2*d^2*\text{Pi}*Log[c*x^n]))/(b*d*\text{Sqrt}[\text{Pi}]])/(\text{E}^{((2*a*b*n - I/(d^2*\text{Pi}))/ (2*b^2*n^2))*(c*x^n)^{n^{-1}}}) - ((1/4 + I/4)*x*\text{Erfi}(((1/2 + I/2)*(n^{-1}) - I*a*b*d^2*\text{Pi} - I*b^2*d^2*\text{Pi}*Log[c*x^n]))/(b*d*\text{Sqrt}[\text{Pi}]])/(\text{E}^{((2*a*b*n + I/(d^2*\text{Pi}))/ (2*b^2*n^2))*(c*x^n)^{n^{-1}}}) + x*\text{FresnelC}[d*(a + b*Log[c*x^n])]$

Rubi [A] time = 0.263422, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {6469, 4616, 2277, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{4} + \frac{i}{4}\right) x (cx^n)^{-1/n} e^{-\frac{2abn - \frac{i}{\pi d^2}}{2b^2 n^2}} \text{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n}\right)}{\sqrt{\pi b d}}\right) - \left(\frac{1}{4} + \frac{i}{4}\right) x (cx^n)^{-1/n} e^{-\frac{2abn + \frac{i}{\pi d^2}}{2b^2 n^2}} \text{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(i\pi ab d^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n}\right)}{\sqrt{\pi b d}}\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{FresnelC}[d*(a + b*Log[c*x^n]), x]$

[Out] $((1/4 + I/4)*x*\text{Erf}(((1/2 + I/2)*(n^{-1}) + I*a*b*d^2*\text{Pi} + I*b^2*d^2*\text{Pi}*Log[c*x^n]))/(b*d*\text{Sqrt}[\text{Pi}]])/(\text{E}^{((2*a*b*n - I/(d^2*\text{Pi}))/ (2*b^2*n^2))*(c*x^n)^{n^{-1}}}) - ((1/4 + I/4)*x*\text{Erfi}(((1/2 + I/2)*(n^{-1}) - I*a*b*d^2*\text{Pi} - I*b^2*d^2*\text{Pi}*Log[c*x^n]))/(b*d*\text{Sqrt}[\text{Pi}]])/(\text{E}^{((2*a*b*n + I/(d^2*\text{Pi}))/ (2*b^2*n^2))*(c*x^n)^{n^{-1}}}) + x*\text{FresnelC}[d*(a + b*Log[c*x^n])]$

Rule 6469

$\text{Int}[\text{FresnelC}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*(d_.)], x_Symbol] \rightarrow \text{Simp}[x*\text{FresnelC}[d*(a + b*Log[c*x^n]), x] - \text{Dist}[b*d*n, \text{Int}[\text{Cos}[(\text{Pi}*(d*(a + b*Log[c*x^n]))^2)/2], x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x]$

Rule 4616

$\text{Int}[\text{Cos}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^2*(d_.)], x_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[\text{E}^{-(I*d*(a + b*Log[c*x^n])^2)}, x], x] + \text{Dist}[1/2, \text{Int}[\text{E}^{(I*d*(a + b*Log[c*x^n])^2)}, x], x]$

$b \cdot \log[c \cdot x^n]^2$), x], x] /; FreeQ[{a, b, c, d, n}, x]

Rule 2277

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)), x_Symbol] := Int[F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b, c, d, n}, x]

Rule 2274

Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] := Int[u*F^(a*v)*z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2276

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.), x_Symbol] := Dist[(e*x)^(m+1)/(e*n*(c*x^n)^((m+1)/n)), Subst[Int[E^(a*d*Log[F] + ((m+1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2234

Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
\int C(d(a + b \log(cx^n))) dx &= xC(d(a + b \log(cx^n))) - (bdn) \int \cos\left(\frac{1}{2}d^2\pi(a + b \log(cx^n))^2\right) dx \\
&= xC(d(a + b \log(cx^n))) - \frac{1}{2}(bdn) \int e^{-\frac{1}{2}id^2\pi(a+b \log(cx^n))^2} dx - \frac{1}{2}(bdn) \int e^{\frac{1}{2}id^2\pi(a+b \log(cx^n))^2} dx \\
&= xC(d(a + b \log(cx^n))) - \frac{1}{2}(bdn) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi \log(cx^n) - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) dx \\
&= xC(d(a + b \log(cx^n))) - \frac{1}{2}(bdn) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) (cx^n)^{-iabd^2\pi} dx \\
&= xC(d(a + b \log(cx^n))) - \frac{1}{2}\left(bdnx^{iabd^2\pi} (cx^n)^{-iabd^2\pi}\right) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2\right) dx \\
&= xC(d(a + b \log(cx^n))) - \frac{1}{2}\left(bdx (cx^n)^{-iabd^2\pi - \frac{1-iabd^2\pi}{n}}\right) \text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(1-iabd^2\pi)}{n} \log^2\right) dx\right) \\
&= xC(d(a + b \log(cx^n))) - \frac{1}{2}\left(bde^{-\frac{2abn + \frac{i}{d^2\pi}}{2b^2n^2}} x (cx^n)^{-iabd^2\pi - \frac{1-iabd^2\pi}{n}}\right) \text{Subst}\left(\int \exp\left(\frac{i(1-iabd^2\pi)}{n} \log^2\right) dx\right) \\
&= \left(\frac{1}{4} + \frac{i}{4}\right) e^{-\frac{2abn - \frac{i}{d^2\pi}}{2b^2n^2}} x (cx^n)^{-1/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{1}{n} + iabd^2\pi + ib^2d^2\pi \log(cx^n)\right)}{bd\sqrt{\pi}}\right) - \left(\frac{1}{4} + \frac{i}{4}\right) e^{\frac{2abn - \frac{i}{d^2\pi}}{2b^2n^2}} x (cx^n)^{-1/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{1}{n} + iabd^2\pi + ib^2d^2\pi \log(cx^n)\right)}{bd\sqrt{\pi}}\right)
\end{aligned}$$

Mathematica [A] time = 6.63245, size = 315, normalized size = 1.47

$$x\operatorname{FresnelC}(d(a + b \log(cx^n))) + \frac{\sqrt[4]{-1}x (cx^n)^{-1/n} \left(ie^{\frac{i}{\pi b^2 d^2 n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) - i)}{\sqrt{\pi} b d n}\right)\right) + \operatorname{Erfi}\left(\frac{(-1)^{3/4}(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) - i)}{\sqrt{2\pi} b d n}\right)}{\sqrt{2\pi} b d n}$$

Warning: Unable to verify antiderivative.

[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])], x]

[Out] x*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((2*a)/(b*n) - I/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(I*E^(I/(b^2*d^2*n^2*Pi))*Erfi[(1/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n])]/(b*d*n*Sqrt[Pi])] + Erfi[(-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n])]/(b*d*n*Sqrt[2*Pi])]*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-1))

Maple [F] time = 0.523, size = 0, normalized size = 0.

$$\int \text{FresnelC}(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(FresnelC(d*(a+b*ln(c*x^n))),x)`

[Out] `int(FresnelC(d*(a+b*ln(c*x^n))),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `integrate(fresnelc((b*log(c*x^n) + a)*d), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{fresnelc}(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral(fresnelc(b*d*log(c*x^n) + a*d), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int C(d(a + b \log(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(fresnelc(d*(a + b*log(c*x**n))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d), x)
```

$$3.166 \quad \int \frac{\mathbf{FresnelC}(d(a+b \log(cx^n)))}{x} dx$$

Optimal. Leaf size=66

$$\frac{(a + b \log(cx^n)) \mathbf{FresnelC}(d(a + b \log(cx^n)))}{bn} - \frac{\sin\left(\frac{1}{2}\pi d^2 (a + b \log(cx^n))^2\right)}{\pi bdn}$$

[Out] (FresnelC[d*(a + b*Log[c*x^n])]*(a + b*Log[c*x^n]))/(b*n) - Sin[(d^2*Pi*(a + b*Log[c*x^n])^2)/2]/(b*d*n*Pi)

Rubi [A] time = 0.0399113, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6419}

$$\frac{(a + b \log(cx^n)) \mathbf{FresnelC}(d(a + b \log(cx^n)))}{bn} - \frac{\sin\left(\frac{1}{2}\pi d^2 (a + b \log(cx^n))^2\right)}{\pi bdn}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x,x]

[Out] (FresnelC[d*(a + b*Log[c*x^n])]*(a + b*Log[c*x^n]))/(b*n) - Sin[(d^2*Pi*(a + b*Log[c*x^n])^2)/2]/(b*d*n*Pi)

Rule 6419

Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelC[a + b*x])/b, x] - Simp[Sin[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{C(d(a + b \log(cx^n)))}{x} dx &= \frac{\text{Subst}\left(\int C(d(a + bx)) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\text{Subst}\left(\int C(x) dx, x, ad + bd \log(cx^n)\right)}{bdn} \\ &= \frac{C(ad + bd \log(cx^n))(a + b \log(cx^n))}{bn} - \frac{\sin\left(\frac{1}{2}\pi (ad + bd \log(cx^n))^2\right)}{bdn\pi} \end{aligned}$$

Mathematica [B] time = 0.0946098, size = 165, normalized size = 2.5

$$\frac{\sin\left(\frac{1}{2}\pi a^2 d^2\right) \cos\left(\pi a b d^2 \log(cx^n) + \frac{1}{2}\pi b^2 d^2 \log^2(cx^n)\right)}{\pi b d n} - \frac{\cos\left(\frac{1}{2}\pi a^2 d^2\right) \sin\left(\pi a b d^2 \log(cx^n) + \frac{1}{2}\pi b^2 d^2 \log^2(cx^n)\right)}{\pi b d n} +$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x,x]

[Out] (a*FresnelC[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelC[d*(a + b*Log[c*x^n])]*Log[c*x^n])/n - (Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2]*Sin[(a^2*d^2*Pi)/2])/(b*d*n*Pi) - (Cos[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)

Maple [A] time = 0.069, size = 81, normalized size = 1.2

$$\frac{\ln(cx^n) \operatorname{FresnelC}(ad + bd \ln(cx^n))}{n} + \frac{\operatorname{FresnelC}(ad + bd \ln(cx^n)) a}{bn} - \frac{1}{bdn\pi} \sin\left(\frac{\pi(ad + bd \ln(cx^n))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x,x)

[Out] 1/n*ln(c*x^n)*FresnelC(a*d+b*d*ln(c*x^n))+1/n/b*FresnelC(a*d+b*d*ln(c*x^n))*a-1/n/b/d/Pi*sin(1/2*Pi*(a*d+b*d*ln(c*x^n))^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{fresnelc}((b \log(cx^n) + a)d)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")

[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bd \log(cx^n) + ad)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")

[Out] integral(fresnelc(b*d*log(c*x^n) + a*d)/x, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C(ad + bd \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x,x)

[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}((b \log(cx^n) + a)d)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")

[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x, x)

$$3.167 \quad \int \frac{\text{FresnelC}(d(a+b \log(cx^n)))}{x^2} dx$$

Optimal. Leaf size=217

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (cx^n)^{\frac{1}{n}} e^{\frac{2abn + \frac{i}{\pi d^2}}{2b^2 n^2}} \text{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (-i\pi abd^2 - i\pi b^2 d^2 \log(cx^n) + \frac{1}{n})}{\sqrt{\pi} bd}}{x} - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (cx^n)^{\frac{1}{n}} e^{\frac{2abn - \frac{i}{\pi d^2}}{2b^2 n^2}} \text{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (i\pi abd^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n})}{\sqrt{\pi} bd}}{x}$$

[Out] $((1/4 + I/4)*E^{((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^{(-1)}*Erf[((1/2 + I/2)*(n^{(-1)} - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/x - ((1/4 + I/4)*E^{((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^{(-1)}*Erfi[((1/2 + I/2)*(n^{(-1)} + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/x - \text{FresnelC}[d*(a + b*Log[c*x^n])]/x$

Rubi [A] time = 0.49256, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (cx^n)^{\frac{1}{n}} e^{\frac{2abn + \frac{i}{\pi d^2}}{2b^2 n^2}} \text{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (-i\pi abd^2 - i\pi b^2 d^2 \log(cx^n) + \frac{1}{n})}{\sqrt{\pi} bd}}{x} - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (cx^n)^{\frac{1}{n}} e^{\frac{2abn - \frac{i}{\pi d^2}}{2b^2 n^2}} \text{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (i\pi abd^2 + i\pi b^2 d^2 \log(cx^n) + \frac{1}{n})}{\sqrt{\pi} bd}}{x}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]

[Out] $((1/4 + I/4)*E^{((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^{(-1)}*Erf[((1/2 + I/2)*(n^{(-1)} - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/x - ((1/4 + I/4)*E^{((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^{(-1)}*Erfi[((1/2 + I/2)*(n^{(-1)} + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/x - \text{FresnelC}[d*(a + b*Log[c*x^n])]/x$

Rule 6472

Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_)^(m_.)), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]

Rule 4618

Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_)^(m_.),
x_Symbol] := Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + Dist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]

Rule 2278

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_)^(m_.),
x_Symbol] := Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2274

Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] := Int[u*F^(a*v)*z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2276

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_)^(m_.),
x_Symbol] := Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2234

Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int \frac{C(d(a + b \log(cx^n)))}{x^2} dx &= -\frac{C(d(a + b \log(cx^n)))}{x} + (bdn) \int \frac{\cos\left(\frac{1}{2}d^2\pi(a + b \log(cx^n))^2\right)}{x^2} dx \\
 &= -\frac{C(d(a + b \log(cx^n)))}{x} + \frac{1}{2}(bdn) \int \frac{e^{-\frac{1}{2}id^2\pi(a+b \log(cx^n))^2}}{x^2} dx + \frac{1}{2}(bdn) \int \frac{e^{\frac{1}{2}id^2\pi(a+b \log(cx^n))^2}}{x^2} dx \\
 &= -\frac{C(d(a + b \log(cx^n)))}{x} + \frac{1}{2}(bdn) \int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi \log(cx^n) - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right)}{x^2} dx \\
 &= -\frac{C(d(a + b \log(cx^n)))}{x} + \frac{1}{2}(bdn) \int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right)(cx^n)^{-iabd^2\pi}}{x^2} dx \\
 &= -\frac{C(d(a + b \log(cx^n)))}{x} + \frac{1}{2}\left(bdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2\left(\frac{cx^n}{bd}\right)\right) \frac{dx}{x} \\
 &= -\frac{C(d(a + b \log(cx^n)))}{x} + \frac{\left(bd(cx^n)^{-iabd^2\pi - \frac{-1-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(-1-iabd^2n\pi)}{n} \log^2\left(\frac{x}{bd}\right)\right) dx\right)}{2x} \\
 &= -\frac{C(d(a + b \log(cx^n)))}{x} + \frac{\left(bde^{\frac{2abn - \frac{i}{2}d^2\pi}{2b^2n^2}}(cx^n)^{-iabd^2\pi - \frac{-1-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(\frac{i\left(\frac{-1-iabd^2n\pi}{n} - \frac{1}{2}d^2\pi\right) \log^2\left(\frac{x}{bd}\right)\right) dx\right)}{2x} \\
 &= \frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{\frac{2abn - \frac{i}{2}d^2\pi}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} - iabd^2\pi - ib^2d^2\pi \log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{\frac{2abn - \frac{i}{2}d^2\pi}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} - iabd^2\pi - ib^2d^2\pi \log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x}
 \end{aligned}$$

Mathematica [A] time = 3.92977, size = 194, normalized size = 0.89

$$\frac{4\operatorname{FresnelC}\left(d(a + b \log(cx^n))\right) + \sqrt[4]{-1}\sqrt{2}(cx^n)^{\frac{1}{n}} e^{\frac{2abn - \frac{i}{2}d^2\pi}{2b^2n^2}} \left(ie^{\frac{i}{\pi b^2 d^2 n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) + i\right)}{\sqrt{\pi} b d n}\right)\right) + \operatorname{Erfi}\left(\frac{(-1)^{3/4} \left(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) + i\right)}{\sqrt{\pi} b d n}\right)}{4x}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]

[Out] -((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*(Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqr

$$t[2\pi]) + I \cdot E^{(I/(b^2 d^2 n^2 \pi))} \operatorname{Erfi}\left(\frac{(1/2 + I/2)(I + a b d^2 n \pi + b^2 d^2 n \pi \operatorname{Log}[c x^n])}{b d n \sqrt{\pi}}\right) + 4 \operatorname{FresnelC}\left[\frac{d(a + b \operatorname{Log}[c x^n])}{4x}\right]$$

Maple [F] time = 0.523, size = 0, normalized size = 0.

$$\int \frac{\operatorname{FresnelC}(d(a + b \ln(cx^n)))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)

[Out] int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{fresnelc}((b \log(cx^n) + a)d)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")

[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bd \log(cx^n) + ad)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")

[Out] integral(fresnelc(b*d*log(c*x^n) + a*d)/x^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C(ad + bd \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**2,x)

[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}((b \log(cx^n) + a)d)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")

[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^2, x)

$$3.168 \quad \int \frac{\text{FresnelC}(d(a+b \log(cx^n)))}{x^3} dx$$

Optimal. Leaf size=228

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (cx^n)^{2/n} e^{\frac{2\pi abd^2 n + 2i}{\pi b^2 d^2 n^2} \text{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (-i\pi abd^2 - i\pi b^2 d^2 \log(cx^n) + \frac{2}{n})}{\sqrt{\pi} bd}\right)}}{x^2} - \frac{\left(\frac{1}{8} + \frac{i}{8}\right) (cx^n)^{2/n} e^{-\frac{2(-\pi abd^2 n + i)}{\pi b^2 d^2 n^2} \text{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (i\pi abd^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n})}{\sqrt{\pi} bd}\right)}}{x^2}$$

[Out] $((1/8 + I/8)*E^{\left(\frac{2*I + 2*a*b*d^2*n*Pi}{b^2*d^2*n^2*Pi}\right)}*(c*x^n)^{(2/n)}*Erf\left[\frac{\left(\frac{1}{2} + I/2\right)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n])}{b*d*Sqrt[Pi]}\right])/x^2 - ((1/8 + I/8)*(c*x^n)^{(2/n)}*Erfi\left[\frac{\left(\frac{1}{2} + I/2\right)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n])}{b*d*Sqrt[Pi]}\right])/(E^{\left(\frac{2*(I - a*b*d^2*n*Pi)}{b^2*d^2*n^2*Pi}\right)}*x^2) - \text{FresnelC}[d*(a + b*Log[c*x^n])]/(2*x^2)$

Rubi [A] time = 0.493126, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (cx^n)^{2/n} e^{\frac{2\pi abd^2 n + 2i}{\pi b^2 d^2 n^2} \text{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (-i\pi abd^2 - i\pi b^2 d^2 \log(cx^n) + \frac{2}{n})}{\sqrt{\pi} bd}\right)}}{x^2} - \frac{\left(\frac{1}{8} + \frac{i}{8}\right) (cx^n)^{2/n} e^{-\frac{2(-\pi abd^2 n + i)}{\pi b^2 d^2 n^2} \text{Erfi}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (i\pi abd^2 + i\pi b^2 d^2 \log(cx^n) + \frac{2}{n})}{\sqrt{\pi} bd}\right)}}{x^2}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]

[Out] $((1/8 + I/8)*E^{\left(\frac{2*I + 2*a*b*d^2*n*Pi}{b^2*d^2*n^2*Pi}\right)}*(c*x^n)^{(2/n)}*Erf\left[\frac{\left(\frac{1}{2} + I/2\right)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n])}{b*d*Sqrt[Pi]}\right])/x^2 - ((1/8 + I/8)*(c*x^n)^{(2/n)}*Erfi\left[\frac{\left(\frac{1}{2} + I/2\right)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n])}{b*d*Sqrt[Pi]}\right])/(E^{\left(\frac{2*(I - a*b*d^2*n*Pi)}{b^2*d^2*n^2*Pi}\right)}*x^2) - \text{FresnelC}[d*(a + b*Log[c*x^n])]/(2*x^2)$

Rule 6472

Int[FresnelC[(a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.)]*(d_.)]*((e_.)*(x_)^(m_.), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])]/(e*(m + 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n])^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]

Rule 4618

Int[Cos[(a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.)]^2*(d_.)]*((e_.)*(x_)^(m_.),
 x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + Dist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]

Rule 2278

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.)]^2*(d_.)]*((e_.)*(x_)^(m_.),
 x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2274

Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2276

Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.)]*(d_.)]*((e_.)*(x_)^(m_.),
 x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

Rule 2234

Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int \frac{C(d(a+b \log(cx^n)))}{x^3} dx &= -\frac{C(d(a+b \log(cx^n)))}{2x^2} + \frac{1}{2}(bdn) \int \frac{\cos\left(\frac{1}{2}d^2\pi(a+b \log(cx^n))^2\right)}{x^3} dx \\
 &= -\frac{C(d(a+b \log(cx^n)))}{2x^2} + \frac{1}{4}(bdn) \int \frac{e^{-\frac{1}{2}id^2\pi(a+b \log(cx^n))^2}}{x^3} dx + \frac{1}{4}(bdn) \int \frac{e^{\frac{1}{2}id^2\pi(a+b \log(cx^n))^2}}{x^3} dx \\
 &= -\frac{C(d(a+b \log(cx^n)))}{2x^2} + \frac{1}{4}(bdn) \int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi \log(cx^n) - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right)}{x^3} dx \\
 &= -\frac{C(d(a+b \log(cx^n)))}{2x^2} + \frac{1}{4}(bdn) \int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right)(cx^n)^{-iabd^2\pi}}{x^3} dx \\
 &= -\frac{C(d(a+b \log(cx^n)))}{2x^2} + \frac{1}{4}\left(bdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2\left(\frac{cx^n}{bd}\right)\right) dx \\
 &= -\frac{C(d(a+b \log(cx^n)))}{2x^2} + \frac{\left(bd(cx^n)^{-iabd^2\pi - \frac{-2-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(-2-iabd^2n\pi)}{n} \log\left(\frac{cx^n}{bd}\right)\right) dx}{4x^2}}{4x^2} \\
 &= -\frac{C(d(a+b \log(cx^n)))}{2x^2} + \frac{\left(bde^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}}(cx^n)^{-iabd^2\pi - \frac{-2-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(\frac{i\left(\frac{-2-iabd^2n\pi}{n} - \frac{1}{2}ia^2d^2\pi\right) \log\left(\frac{cx^n}{bd}\right)\right) dx}{2b^2d^2n^2\pi}}{4x^2}}{4x^2} \\
 &= \frac{\left(\frac{1}{8} + \frac{i}{8}\right) e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}} (cx^n)^{2/n} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{2}{n} - iabd^2\pi - ib^2d^2\pi \log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} - \frac{\left(\frac{1}{8} + \frac{i}{8}\right) e^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}} (cx^n)^{2/n}}{x^2}
 \end{aligned}$$

Mathematica [A] time = 4.00641, size = 199, normalized size = 0.87

$$\frac{\operatorname{FresnelC}(d(a+b \log(cx^n)))}{2x^2} - \frac{\sqrt[4]{-1} \left(i e^{\frac{4i}{\pi b^2 d^2 n^2}} \operatorname{Erfi}\left(\frac{\sqrt[4]{-1}(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) + 2i)}{\sqrt{2\pi} b d n}\right) + \operatorname{Erfi}\left(\frac{(-1)^{3/4}(\pi a b d^2 n + \pi b^2 d^2 n \log(cx^n) - 2i)}{\sqrt{2\pi} b d n}\right) \right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]

[Out] -((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*x^n])))/n^2)*(Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))]/

$$(b*d*n*\text{Sqrt}[2*\text{Pi}]] + I*E^{((4*I)/(b^2*d^2*n^2*\text{Pi}))}*Erfi[(((-1)^{(1/4)}*(2*I + a*b*d^2*n*\text{Pi} + b^2*d^2*n*\text{Pi}*Log[c*x^n]))/(b*d*n*\text{Sqrt}[2*\text{Pi}]])]/(4*\text{Sqrt}[2]) - \text{FresnelC}[d*(a + b*Log[c*x^n])]/(2*x^2)$$

Maple [F] time = 0.527, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(d(a + b \ln(cx^n)))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)

[Out] int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}((b \log(cx^n) + a)d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")

[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bd \log(cx^n) + ad)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")

[Out] integral(fresnelc(b*d*log(c*x^n) + a*d)/x^3, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C(ad + bd \log(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**3,x)

[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}((b \log(cx^n) + a)d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")

[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^3, x)

3.169 $\int (ex)^m \mathbf{FresnelC}(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=280

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) x (ex)^m (cx^n)^{-\frac{m+1}{n}} \exp\left(\frac{i(m+1)(2i\pi abd^2 n + m+1)}{2\pi b^2 d^2 n^2}\right) \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(i\pi abd^2 n + i\pi b^2 d^2 n \log(cx^n) + m+1)}{\sqrt{\pi} b d n}\right)}{m+1} - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) x (ex)^m (cx^n)^{-\frac{m+1}{n}} e^{i\pi b d \log(cx^n)}}{m+1}$$

[Out] $((1/4 + I/4)*E^{((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))}/(b^2*d^2*n^2*Pi)) * x * (e*x)^m * \operatorname{Erf}(((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*\operatorname{Log}[c*x^n]))/(b*d*n*\operatorname{Sqrt}[Pi])))/((1 + m)*(c*x^n)^{((1 + m)/n)}) - ((1/4 + I/4)*x * (e*x)^m * \operatorname{Erfi}(((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*\operatorname{Log}[c*x^n]))/(b*d*n*\operatorname{Sqrt}[Pi])))/((1 + m)*(c*x^n)^{((1 + m)/n)}) + ((e*x)^{(1 + m)} * \operatorname{FresnelC}[d*(a + b*\operatorname{Log}[c*x^n])])/(e*(1 + m))$

Rubi [A] time = 0.607006, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {6472, 4618, 2278, 2274, 15, 20, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) x (ex)^m (cx^n)^{-\frac{m+1}{n}} \exp\left(\frac{i(m+1)(2i\pi abd^2 n + m+1)}{2\pi b^2 d^2 n^2}\right) \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(i\pi abd^2 n + i\pi b^2 d^2 n \log(cx^n) + m+1)}{\sqrt{\pi} b d n}\right)}{m+1} - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) x (ex)^m (cx^n)^{-\frac{m+1}{n}} e^{i\pi b d \log(cx^n)}}{m+1}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(e*x)^m * \operatorname{FresnelC}[d*(a + b*\operatorname{Log}[c*x^n])], x]$

[Out] $((1/4 + I/4)*E^{((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))}/(b^2*d^2*n^2*Pi)) * x * (e*x)^m * \operatorname{Erf}(((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*\operatorname{Log}[c*x^n]))/(b*d*n*\operatorname{Sqrt}[Pi])))/((1 + m)*(c*x^n)^{((1 + m)/n)}) - ((1/4 + I/4)*x * (e*x)^m * \operatorname{Erfi}(((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*\operatorname{Log}[c*x^n]))/(b*d*n*\operatorname{Sqrt}[Pi])))/((1 + m)*(c*x^n)^{((1 + m)/n)}) + ((e*x)^{(1 + m)} * \operatorname{FresnelC}[d*(a + b*\operatorname{Log}[c*x^n])])/(e*(1 + m))$

Rule 6472

$\operatorname{Int}[\operatorname{FresnelC}((a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.))*(d_.)]*(e_.)*(x_.)^{(m_.)}, x_Symbol] :> \operatorname{Simp}(((e*x)^{(m+1)} * \operatorname{FresnelC}[d*(a + b*\operatorname{Log}[c*x^n])])/(e*(m+1))), x] - \operatorname{Dist}[(b*d*n)/(m+1), \operatorname{Int}[(e*x)^m * \operatorname{Cos}[\operatorname{Pi}*(d*(a + b*\operatorname{Log}[c*x^n])])]$

$\wedge 2)/2], x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, n\}, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 4618

$\text{Int}[\text{Cos}[(a_.) + \text{Log}[(c_.)*(x_)^(n_.)]*(b_.)]^2*(d_.)]*((e_.)*(x_))^(m_.), x_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[(e*x)^m/E^{(I*d*(a + b*\text{Log}[c*x^n])^2)}, x], x] + \text{Dist}[1/2, \text{Int}[(e*x)^m*E^{(I*d*(a + b*\text{Log}[c*x^n])^2)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, n\}, x]$

Rule 2278

$\text{Int}[(F_)^(((a_.) + \text{Log}[(c_.)*(x_)^(n_.)]*(b_.))^{2*(d_.)}*((e_.)*(x_))^{(m_.)}), x_Symbol] \rightarrow \text{Int}[(e*x)^m*F^{(a^2*d + 2*a*b*d*\text{Log}[c*x^n] + b^2*d*\text{Log}[c*x^n]^2)}, x] /; \text{FreeQ}[\{F, a, b, c, d, e, m, n\}, x]$

Rule 2274

$\text{Int}[(u_.)*(F_)^{((a_.)*(\text{Log}[z_]*(b_.) + (v_.)))}, x_Symbol] \rightarrow \text{Int}[u*F^{(a*v)*z^{(a*b*\text{Log}[F])}}, x] /; \text{FreeQ}[\{F, a, b\}, x]$

Rule 15

$\text{Int}[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[m]}*(a*x^n)^{\text{FracPart}[m]})/x^{(n*\text{FracPart}[m])}, \text{Int}[u*x^{(m*n)}, x], x] /; \text{FreeQ}[\{a, m, n\}, x] \ \&\& \ !\text{IntegerQ}[m]$

Rule 20

$\text{Int}[(u_.)*((a_.)*(v_))^{(m_)}*((b_.)*(v_))^{(n_)}], x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ !\text{IntegerQ}[m+n]$

Rule 2276

$\text{Int}[(F_)^(((a_.) + \text{Log}[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.)]*((e_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \text{Subst}[\text{Int}[E^{(a*d*\text{Log}[F] + ((m+1)*x)/n + b*d*\text{Log}[F]*x^2)}, x], x, \text{Log}[c*x^n]], x] /; \text{FreeQ}[\{F, a, b, c, d, e, m, n\}, x]$

Rule 2234

$\text{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)}, x_Symbol] \rightarrow \text{Dist}[F^{(a - b^2/(4*c))}, \text{Int}[F^{((b + 2*c*x)^2/(4*c))}, x], x] /; \text{FreeQ}[\{F, a, b, c\}, x]$

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int (ex)^m C(d(a + b \log(cx^n))) dx &= \frac{(ex)^{1+m} C(d(a + b \log(cx^n)))}{e(1+m)} - \frac{(bdn) \int (ex)^m \cos\left(\frac{1}{2}d^2\pi(a + b \log(cx^n))^2\right) dx}{1+m} \\
 &= \frac{(ex)^{1+m} C(d(a + b \log(cx^n)))}{e(1+m)} - \frac{(bdn) \int e^{-\frac{1}{2}id^2\pi(a+b \log(cx^n))^2} (ex)^m dx}{2(1+m)} - \frac{(bdn) \int e^{\frac{1}{2}id^2\pi} (ex)^m dx}{2(1+m)} \\
 &= \frac{(ex)^{1+m} C(d(a + b \log(cx^n)))}{e(1+m)} - \frac{(bdn) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi \log(cx^n) - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) (ex)^m dx}{2(1+m)} \\
 &= \frac{(ex)^{1+m} C(d(a + b \log(cx^n)))}{e(1+m)} - \frac{(bdn) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) (ex)^m dx}{2(1+m)} \\
 &= \frac{(ex)^{1+m} C(d(a + b \log(cx^n)))}{e(1+m)} - \frac{\left(bdn x^{iabd^2n\pi} (cx^n)^{-iabd^2\pi}\right) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) (ex)^m dx}{2(1+m)} \\
 &= \frac{(ex)^{1+m} C(d(a + b \log(cx^n)))}{e(1+m)} - \frac{\left(bdn x^{-m+iabd^2n\pi} (ex)^m (cx^n)^{-iabd^2\pi}\right) \int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2(cx^n)\right) dx}{2(1+m)} \\
 &= \frac{(ex)^{1+m} C(d(a + b \log(cx^n)))}{e(1+m)} - \frac{\left(bdx (ex)^m (cx^n)^{-iabd^2\pi - \frac{1+m-iabd^2n\pi}{n}}\right) \text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi \log^2\left(\frac{x}{cx^n}\right)\right) dx\right)}{2(1+m)} \\
 &= \frac{(ex)^{1+m} C(d(a + b \log(cx^n)))}{e(1+m)} - \frac{\left(bd \exp\left(-\frac{i(1+m)(1+m-2iabd^2n\pi)}{2b^2d^2n^2\pi}\right) x (ex)^m (cx^n)^{-iabd^2\pi - \frac{1+m-iabd^2n\pi}{n}}\right)}{2(1+m)} \\
 &= \frac{\left(\frac{1}{4} + \frac{i}{4}\right) \exp\left(\frac{i(1+m)(1+m+2iabd^2n\pi)}{2b^2d^2n^2\pi}\right) x (ex)^m (cx^n)^{-\frac{1+m}{n}} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(1+m+iabd^2n\pi+ib^2d^2n\pi \log(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m}
 \end{aligned}$$

Mathematica [A] time = 5.35204, size = 244, normalized size = 0.87

$$(ex)^m \left(4x \operatorname{FresnelC}(d(a + b \log(cx^n))) + (-1)^{3/4} \sqrt{2} x^{-m} \exp\left(-\frac{(m+1)(2\pi ab d^2 n + 2\pi b^2 d^2 n (\log(cx^n) - n \log(x)) + im + i)}{2\pi b^2 d^2 n^2}\right) \right) \operatorname{Erf}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)(\pi ab d^2 n)}{2\pi b^2 d^2 n^2}\right)$$

$$4(m+1)$$

Antiderivative was successfully verified.

[In] Integrate[(e*x)^m*FresnelC[d*(a + b*Log[c*x^n])],x]

[Out] ((e*x)^m*(((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])) - E^(((I*(1 + m)^2)/(b^2*d^2*n^2*Pi))*Erfi[(((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi]))])/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2*d^2*n*Pi*(-n*Log[x] + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m) + 4*x*FresnelC[d*(a + b*Log[c*x^n])])/(4*(1 + m))

Maple [F] time = 0.205, size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{FresnelC}(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)

[Out] int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{fresnelc}((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")

[Out] integrate((e*x)^m*fresnelc((b*log(c*x^n) + a)*d), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((ex)^m \text{fresnelc} (bd \log (cx^n) + ad), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral((e*x)^m*fresnelc(b*d*log(c*x^n) + a*d), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m C(ad + bd \log (cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*fresnelc(d*(a+b*ln(c*x**n))),x)`

[Out] `Integral((e*x)**m*fresnelc(a*d + b*d*log(c*x**n)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \text{fresnelc} ((b \log (cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] `integrate((e*x)^m*fresnelc((b*log(c*x^n) + a)*d), x)`

3.170 $\int e^{c+\frac{1}{2}ib^2\pi x^2} \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=64

$$\frac{1}{4}be^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)-\frac{ie^c\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

[Out] $((-I/8)*E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*HypergeometricPFQ[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2*Pi*x^2])/4$

Rubi [A] time = 0.0693593, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6437, 6376, 6375, 30}

$$\frac{1}{4}be^cx^2{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

Antiderivative was successfully verified.

[In] Int[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]

[Out] $((-I/8)*E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*HypergeometricPFQ[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2*Pi*x^2])/4$

Rule 6437

Int[E^((c_.) + (d_.)*(x_)^2)*FresnelC[(b_.)*(x_)], x_Symbol] := Dist[(1 - I)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 + I)*b*x)/2], x], x] + Dist[(1 + I)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 - I)*b*x)/2], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -(Pi^2*b^4)/4]

Rule 6376

Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] := Simp[(b*E^c*x^2*HypergeometricPFQ[\{1, 1\}, \{3/2, 2\}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^2]

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)^(n_.), x_Symbol] := Dist[(E^c
*sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int e^{c+\frac{1}{2}ib^2\pi x^2} C(bx) dx &= \left(\frac{1}{4} - \frac{i}{4}\right) \int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right) dx + \left(\frac{1}{4} - \frac{i}{4}\right) \int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right) dx \\ &= \frac{1}{4}be^cx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) - \frac{(ie^c) \operatorname{Subst}\left(\int x dx, x, \operatorname{erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right)\right)}{4b} \\ &= -\frac{ie^c \operatorname{erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b} + \frac{1}{4}be^cx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) \end{aligned}$$

Mathematica [F] time = 0.0291736, size = 0, normalized size = 0.

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelC}(bx) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

```
[Out] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int e^{c+\frac{i}{2}b^2\pi x^2} \operatorname{FresnelC}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x), x)
```

[Out] `int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\frac{1}{2}i\pi b^2 x^2 + c\right)} \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")`

[Out] `integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{\left(\frac{1}{2}i\pi b^2 x^2 + c\right)} \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")`

[Out] `integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^c \int e^{\frac{inb^2x^2}{2}} C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)`

[Out] `exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnelc(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\frac{1}{2}i\pi b^2 x^2 + c\right)} \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnelc(b*x), x, algorithm="giac")
```

```
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

3.171 $\int e^{c-\frac{1}{2}ib^2\pi x^2} \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=64

$$\frac{1}{4}be^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

[Out] $((-I/8)*E^c*\text{Erf}[(1/2 + I/2)*b*\text{Sqrt}[Pi]*x]^2)/b + (b*E^c*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2*Pi*x^2])/4$

Rubi [A] time = 0.0662754, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6437, 6373, 30, 6378}

$$\frac{1}{4}be^cx^2 {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(c - (I/2)*b^2*Pi*x^2)}*\text{FresnelC}[b*x], x]$

[Out] $((-I/8)*E^c*\text{Erf}[(1/2 + I/2)*b*\text{Sqrt}[Pi]*x]^2)/b + (b*E^c*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2*Pi*x^2])/4$

Rule 6437

$\text{Int}[E^{((c_.) + (d_.)*(x_)^2)}*\text{FresnelC}[(b_.)*(x_)], x_Symbol] \rightarrow \text{Dist}[(1 - I)/4, \text{Int}[E^{(c + d*x^2)}*\text{Erf}[(\text{Sqrt}[Pi]*(1 + I)*b*x)/2], x], x] + \text{Dist}[(1 + I)/4, \text{Int}[E^{(c + d*x^2)}*\text{Erf}[(\text{Sqrt}[Pi]*(1 - I)*b*x)/2], x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{EqQ}[d^2, -((Pi^2*b^4)/4)]$

Rule 6373

$\text{Int}[E^{((c_.) + (d_.)*(x_)^2)}*\text{Erf}[(b_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(E^c*\text{Sqrt}[Pi])/(2*b), \text{Subst}[\text{Int}[x^n, x], x, \text{Erf}[b*x]], x] /; \text{FreeQ}[\{b, c, d, n\}, x] \ \&\& \ \text{EqQ}[d, -b^2]$

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 6378

Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] := Simp[(b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b, c, d}, x] && EqQ[d, -b^2]

Rubi steps

$$\begin{aligned} \int e^{c-\frac{1}{2}ib^2\pi x^2} C(bx) dx &= \left(\frac{1}{4} - \frac{i}{4}\right) \int e^{c-\frac{1}{2}ib^2\pi x^2} \operatorname{erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi x}\right) dx + \left(\frac{1}{4} - \frac{i}{4}\right) \int e^{c-\frac{1}{2}ib^2\pi x^2} \operatorname{erfi}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi x}\right) dx \\ &= \frac{1}{4}be^cx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) - \frac{(ie^c) \operatorname{Subst}\left(\int x dx, x, \operatorname{erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi x}\right)\right)}{4b} \\ &= -\frac{ie^c \operatorname{erf}\left(\left(\frac{1}{2} + \frac{i}{2}\right)b\sqrt{\pi x}\right)^2}{8b} + \frac{1}{4}be^cx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) \end{aligned}$$

Mathematica [F] time = 0.0304312, size = 0, normalized size = 0.

$$\int e^{c-\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelC}(bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]

[Out] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int e^{c-\frac{i}{2}b^2\pi x^2} \operatorname{FresnelC}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x), x)

[Out] `int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(-\frac{1}{2}i\pi b^2 x^2 + c\right)} \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")`

[Out] `integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{\left(-\frac{1}{2}i\pi b^2 x^2 + c\right)} \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")`

[Out] `integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^c \int e^{-\frac{i\pi b^2 x^2}{2}} C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)`

[Out] `exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnelc(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)} \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

3.172 $\int \mathbf{FresnelC}(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=101

$$\frac{1}{8}ibx^2 \cos(c) \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right) - \frac{1}{8}ibx^2 \cos(c) \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)$$

[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*Cos[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] - (I/8)*b*x^2*Cos[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] + (FresnelC[b*x]^2*Sin[c])/(2*b)

Rubi [A] time = 0.050629, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {6449, 6441, 30, 6447}

$$\frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) + \frac{\cos(c)\text{FresnelC}(bx)\text{S}(bx)}{2b} + \frac{\sin(c)\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]

[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*Cos[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] - (I/8)*b*x^2*Cos[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] + (FresnelC[b*x]^2*Sin[c])/(2*b)

Rule 6449

Int[FresnelC[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[Sin[c], Int[Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[Cos[c], Int[Sin[d*x^2]*FresnelC[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 6441

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.)], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 6447

Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(b*Pi*FresnelC[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(I*d*x^2)])]/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, I*d*x^2)])]/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rubi steps

$$\begin{aligned} \int C(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) dx &= \cos(c) \int C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx + \sin(c) \int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx \\ &= \frac{\cos(c)C(bx)S(bx)}{2b} + \frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) \\ &= \frac{\cos(c)C(bx)S(bx)}{2b} + \frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{8}ibx^2 \cos(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) \end{aligned}$$

Mathematica [F] time = 0.0440036, size = 0, normalized size = 0.

$$\int \text{FresnelC}(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]

[Out] Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \text{FresnelC}(bx) \sin\left(c + \frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2), x)

[Out] `int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")`

[Out] `integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2 + c), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2 + c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2 + c), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{\pi b^2 x^2}{2} + c\right) C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(c+1/2*b**2*pi*x**2),x)`

[Out] `Integral(sin(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

3.173 $\int \cos\left(c + \frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=101

$$-\frac{1}{8}ibx^2 \sin(c) \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right) + \frac{1}{8}ibx^2 \sin(c) \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}\right)$$

```
[Out] (Cos[c]*FresnelC[b*x]^2)/(2*b) - (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
- (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]*Sin[c]
+ (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin[c]
```

Rubi [A] time = 0.0478725, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {6443, 6441, 30, 6447}

$$-\frac{1}{8}ibx^2 \sin(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2 \sin(c) {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) - \frac{\sin(c)\mathbf{FresnelC}(bx)\mathbf{S}(bx)}{2b} + \frac{\cos(c)\mathbf{FresnelC}(bx)\mathbf{C}(bx)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

```
[Out] (Cos[c]*FresnelC[b*x]^2)/(2*b) - (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
- (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]*Sin[c]
+ (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin[c]
```

Rule 6443

```
Int[Cos[(c_) + (d_.)*(x_)^2]*FresnelC[(b_.)*(x_)], x_Symbol] := Dist[Cos[c],
Int[Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[Sin[c], Int[Sin[d*x^2]*FresnelC[b*x], x], x]
/; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.)], x_Symbol] := Dist[(Pi*b)/(2*d),
Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

`Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 6447

`Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(b*Pi*FresnelC[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(I*d*x^2)])]/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, I*d*x^2)])]/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]`

Rubi steps

$$\begin{aligned} \int \cos\left(c + \frac{1}{2}b^2\pi x^2\right) C(bx) dx &= \cos(c) \int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx - \sin(c) \int C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx \\ &= -\frac{C(bx)S(bx) \sin(c)}{2b} - \frac{1}{8}ibx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) \sin(c) + \frac{1}{8}ibx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) \sin(c) \\ &= \frac{\cos(c)C(bx)^2}{2b} - \frac{C(bx)S(bx) \sin(c)}{2b} - \frac{1}{8}ibx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right) \sin(c) + \frac{1}{8}ibx^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right) \sin(c) \end{aligned}$$

Mathematica [F] time = 0.0389271, size = 0, normalized size = 0.

$$\int \cos\left(c + \frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$$

Verification is Not applicable to the result.

[In] `Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x], x]`

[Out] `Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x], x]`

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \cos\left(c + \frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x), x)`

[Out] `int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2} \pi b^2 x^2 + c\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(c+1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")`

[Out] `integrate(cos(1/2*pi*b^2*x^2 + c)*fresnelc(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos\left(\frac{1}{2} \pi b^2 x^2 + c\right) \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(c+1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2 + c)*fresnelc(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{\pi b^2 x^2}{2} + c\right) C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(c+1/2*b**2*pi*x**2)*fresnelc(b*x),x)`

[Out] `Integral(cos(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2 x^2 + c\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

$$3.174 \quad \int \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx)^2 dx$$

Optimal. Leaf size=13

$$\frac{\mathbf{FresnelC}(bx)^3}{3b}$$

[Out] FresnelC[b*x]^3/(3*b)

Rubi [A] time = 0.0148672, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6441, 30}

$$\frac{\mathbf{FresnelC}(bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]

[Out] FresnelC[b*x]^3/(3*b)

Rule 6441

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)^2 dx &= \frac{\text{Subst}\left(\int x^2 dx, x, C(bx)\right)}{b} \\ &= \frac{C(bx)^3}{3b} \end{aligned}$$

Mathematica [A] time = 0.0087997, size = 13, normalized size = 1.

$$\frac{\text{FresnelC}(bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]

[Out] FresnelC[b*x]^3/(3*b)

Maple [A] time = 0.044, size = 12, normalized size = 0.9

$$\frac{(\text{FresnelC}(bx))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^2,x)

[Out] 1/3*FresnelC(b*x)^3/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^2,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)^2, x)
```

Sympy [A] time = 1.41009, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^3(bx)}{3b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**2,x)
```

```
[Out] Piecewise((fresnelc(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)^2, x)
```


$$3.175 \quad \int \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx) dx$$

Optimal. Leaf size=13

$$\frac{\mathbf{FresnelC}(bx)^2}{2b}$$

[Out] FresnelC[b*x]^2/(2*b)

Rubi [A] time = 0.0116175, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6441, 30}

$$\frac{\mathbf{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] FresnelC[b*x]^2/(2*b)

Rule 6441

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.), x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx &= \frac{\text{Subst}\left(\int x dx, x, C(bx)\right)}{b} \\ &= \frac{C(bx)^2}{2b} \end{aligned}$$

Mathematica [A] time = 0.0058943, size = 13, normalized size = 1.

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] FresnelC[b*x]^2/(2*b)

Maple [A] time = 0.044, size = 12, normalized size = 0.9

$$\frac{(\text{FresnelC}(bx))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)

[Out] 1/2*FresnelC(b*x)^2/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [A] time = 0.44121, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

$$3.176 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\mathbf{FresnelC}(bx)} dx$$

Optimal. Leaf size=9

$$\frac{\log(\mathbf{FresnelC}(bx))}{b}$$

[Out] Log[FresnelC[b*x]]/b

Rubi [A] time = 0.0155068, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6441, 29}

$$\frac{\log(\mathbf{FresnelC}(bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x],x]

[Out] Log[FresnelC[b*x]]/b

Rule 6441

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.), x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, C(bx)\right)}{b} \\ &= \frac{\log(C(bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.0109443, size = 9, normalized size = 1.

$$\frac{\log(\text{FresnelC}(bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x], x]

[Out] Log[FresnelC[b*x]]/b

Maple [A] time = 0.122, size = 10, normalized size = 1.1

$$\frac{\ln(\text{FresnelC}(bx))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x), x)

[Out] ln(FresnelC(b*x))/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{\text{fresnelc}(bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x), x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{\text{fresnelc}(bx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x),x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)/fresnelc(b*x), x)

Sympy [A] time = 0.242884, size = 10, normalized size = 1.11

$$\begin{cases} \frac{\log(C(bx))}{b} & \text{for } b \neq 0 \\ \infty x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x),x)

[Out] Piecewise((log(fresnelc(b*x))/b, Ne(b, 0)), (zoo*x, True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}(bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x),x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x), x)

$$3.177 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\mathbf{FresnelC}(bx)^2} dx$$

Optimal. Leaf size=11

$$-\frac{1}{b\mathbf{FresnelC}(bx)}$$

[Out] -(1/(b*FresnelC[b*x]))

Rubi [A] time = 0.0149059, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6441, 30}

$$-\frac{1}{b\mathbf{FresnelC}(bx)}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]

[Out] -(1/(b*FresnelC[b*x]))

Rule 6441

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.), x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2} dx, x, C(bx)\right)}{b} \\ &= -\frac{1}{bC(bx)} \end{aligned}$$

Mathematica [A] time = 0.0055501, size = 11, normalized size = 1.

$$-\frac{1}{b\text{FresnelC}(bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]

[Out] -(1/(b*FresnelC[b*x]))

Maple [A] time = 0.044, size = 12, normalized size = 1.1

$$-\frac{1}{b\text{FresnelC}(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^2,x)

[Out] -1/b/FresnelC(b*x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}(bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^2,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}(bx)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^2, x)
```

Sympy [A] time = 0.797724, size = 12, normalized size = 1.09

$$\begin{cases} -\frac{1}{bC(bx)} & \text{for } b \neq 0 \\ \infty x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**2,x)
```

```
[Out] Piecewise((-1/(b*fresnelc(b*x)), Ne(b, 0)), (zoo*x, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}(bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^2, x)
```

$$3.178 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\mathbf{FresnelC}(bx)^3} dx$$

Optimal. Leaf size=13

$$-\frac{1}{2b\mathbf{FresnelC}(bx)^2}$$

[Out] -1/(2*b*FresnelC[b*x]^2)

Rubi [A] time = 0.0156124, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6441, 30}

$$-\frac{1}{2b\mathbf{FresnelC}(bx)^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]

[Out] -1/(2*b*FresnelC[b*x]^2)

Rule 6441

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.), x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3} dx, x, C(bx)\right)}{b} \\ &= -\frac{1}{2bC(bx)^2} \end{aligned}$$

Mathematica [A] time = 0.0059558, size = 13, normalized size = 1.

$$-\frac{1}{2b\text{FresnelC}(bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]

[Out] -1/(2*b*FresnelC[b*x]^2)

Maple [A] time = 0.046, size = 12, normalized size = 0.9

$$-\frac{1}{2b(\text{FresnelC}(bx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^3,x)

[Out] -1/2/b/FresnelC(b*x)^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}(bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^3,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}(bx)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^3,x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^3, x)`

Sympy [A] time = 1.97152, size = 15, normalized size = 1.15

$$\begin{cases} -\frac{1}{2bC^2(bx)} & \text{for } b \neq 0 \\ \infty x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**3,x)`

[Out] `Piecewise((-1/(2*b*fresnelc(b*x)**2), Ne(b, 0)), (zoo*x, True))`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnelc}(bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^3,x, algorithm="giac")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^3, x)`

$$3.179 \quad \int \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx)^n dx$$

Optimal. Leaf size=17

$$\frac{\mathbf{FresnelC}(bx)^{n+1}}{b(n+1)}$$

[Out] FresnelC[b*x]^(1 + n)/(b*(1 + n))

Rubi [A] time = 0.0187874, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6441, 30}

$$\frac{\mathbf{FresnelC}(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]

[Out] FresnelC[b*x]^(1 + n)/(b*(1 + n))

Rule 6441

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.), x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)^n dx &= \frac{\text{Subst}\left(\int x^n dx, x, C(bx)\right)}{b} \\ &= \frac{C(bx)^{1+n}}{b(1+n)} \end{aligned}$$

Mathematica [A] time = 0.0102103, size = 17, normalized size = 1.

$$\frac{\text{FresnelC}(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]

[Out] FresnelC[b*x]^(1+n)/(b*(1+n))

Maple [A] time = 0.047, size = 18, normalized size = 1.1

$$\frac{(\text{FresnelC}(bx))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^n,x)

[Out] FresnelC(b*x)^(1+n)/b/(1+n)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx)^n \cos\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^n,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)^n*cos(1/2*pi*b^2*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnelc}(bx)^n \cos\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^n,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)^n*cos(1/2*pi*b^2*x^2), x)

Sympy [A] time = 4.59086, size = 34, normalized size = 2.

$$\begin{cases} \infty x & \text{for } b = 0 \wedge n = -1 \\ 0^n x & \text{for } b = 0 \\ \frac{\log(C(bx))}{b} & \text{for } n = -1 \\ \frac{C(bx)^b C^n(bx)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**n,x)

[Out] Piecewise((zoo*x, Eq(b, 0) & Eq(n, -1)), (0**n*x, Eq(b, 0)), (log(fresnelc(b*x))/b, Eq(n, -1)), (fresnelc(b*x)*fresnelc(b*x)**n/(b*n + b), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx)^n \cos\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^n,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)^n*cos(1/2*pi*b^2*x^2), x)

3.180 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=231

$$\frac{x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{35x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{7x^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{105x \text{FresnelC}(bx)}{\pi^4 b^2}$$

[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)

Rubi [A] time = 0.382908, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637, 3309}

$$\frac{x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{35x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{7x^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{105x \text{FresnelC}(bx)}{\pi^4 b^2}$$

Antiderivative was successfully verified.

[In] Int[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)

Rule 6455

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(m-1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m-1)/(2*d), Int[x^(m-2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m-1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.), x_Symbol] := Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eqQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] := Simp[x/2, x] - Simp[Sin[2*c
+ d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

$e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$
 $\text{FreeQ}\{c, d\}, x]$

Rule 3309

$\text{Int}[\{(c_.) + (d_.)*(x_.)\}^{(m_.)}*\sin\{(e_.) + \{(f_.)*(x_.)\}/2\}^2, x_Symbol] \rightarrow$
 $\text{Dist}[1/2, \text{Int}[(c + d*x)^m, x], x] - \text{Dist}[1/2, \text{Int}[(c + d*x)^m*\text{Cos}[2*e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned} \int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx &= \frac{x^7 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7 \int x^6 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^7 \sin(b^2\pi x^2) dx}{2b\pi} \\ &= \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{x^7 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{35 \int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx}{b^4\pi^2} - \frac{7 \int x^5 \sin(b^2\pi x^2) dx}{2b\pi} \\ &= \frac{x^6 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} - \frac{35x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\ &= \frac{x^6 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{105x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^8\pi^4} + \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} - \frac{35x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\ &= -\frac{7x^6}{12b^3\pi^2} - \frac{41x^2 \cos(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{105x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^8\pi^4} + \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} \\ &= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2 \cos(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{105x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^8\pi^4} + \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} \\ &= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2 \cos(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{105x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^8\pi^4} + \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} \end{aligned}$$

Mathematica [A] time = 0.0117987, size = 231, normalized size = 1.

$$\frac{x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{35x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{7x^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{105x \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*cos[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int x^8 \cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelC}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)

[Out] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")

[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^8 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^8*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.181 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=215

$$\frac{x^6 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{24x^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{6x^4 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{48 \mathbf{FresnelC}(bx)}{\pi^4}$$

[Out] $(24*x)/(b^7*\text{Pi}^4) - (3*x^5)/(5*b^3*\text{Pi}^2) - (147*x*\text{Cos}[b^2*\text{Pi}*x^2])/(16*b^7*\text{Pi}^4) + (x^5*\text{Cos}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2) - (48*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(b^8*\text{Pi}^4) + (6*x^4*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(b^4*\text{Pi}^2) + (531*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(16*\text{Sqrt}[2]*b^8*\text{Pi}^4) - (24*x^2*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^6*\text{Pi}^3) + (x^6*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi}) - (17*x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(8*b^5*\text{Pi}^3)$

Rubi [A] time = 0.259335, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6455, 6463, 6461, 3358, 3352, 3385, 3392, 30, 3386}

$$\frac{x^6 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{24x^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{6x^4 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{48 \mathbf{FresnelC}(bx)}{\pi^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^7*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x], x]$

[Out] $(24*x)/(b^7*\text{Pi}^4) - (3*x^5)/(5*b^3*\text{Pi}^2) - (147*x*\text{Cos}[b^2*\text{Pi}*x^2])/(16*b^7*\text{Pi}^4) + (x^5*\text{Cos}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2) - (48*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(b^8*\text{Pi}^4) + (6*x^4*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(b^4*\text{Pi}^2) + (531*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(16*\text{Sqrt}[2]*b^8*\text{Pi}^4) - (24*x^2*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^6*\text{Pi}^3) + (x^6*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^2*\text{Pi}) - (17*x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(8*b^5*\text{Pi}^3)$

Rule 6455

$\text{Int}[\text{Cos}[(d_.)*(x_)^2]*\text{FresnelC}[(b_.)*(x_)]*(x_)^(m_), x_Symbol] \text{ :> } \text{Simp}[(x^(m-1)*\text{Sin}[d*x^2]*\text{FresnelC}[b*x])/(2*d), x] + (-\text{Dist}[(m-1)/(2*d), \text{Int}[x^(m-2)*\text{Sin}[d*x^2]*\text{FresnelC}[b*x], x], x] - \text{Dist}[b/(4*d), \text{Int}[x^(m-1)*\text{Sin}[2*d*x^2], x], x]) /; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])*(b_.))^(p_), x_Sy
mbol] := Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_), x_Symbol] := Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
m, n}, x]
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
 \int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx &= \frac{x^6 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{6 \int x^5 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^6 \sin(b^2\pi x^2) dx}{2b\pi} \\
 &= \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{x^6 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{24 \int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} \\
 &= \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} - \frac{24x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^6 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
 &= -\frac{3x^5}{5b^3\pi^2} - \frac{111x \cos(b^2\pi x^2)}{16b^7\pi^4} + \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{48 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^8\pi^4} + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} \\
 &= -\frac{3x^5}{5b^3\pi^2} - \frac{147x \cos(b^2\pi x^2)}{16b^7\pi^4} + \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{48 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^8\pi^4} + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} \\
 &= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x \cos(b^2\pi x^2)}{16b^7\pi^4} + \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{48 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^8\pi^4} + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} \\
 &= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x \cos(b^2\pi x^2)}{16b^7\pi^4} + \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{48 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^8\pi^4} + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2}
 \end{aligned}$$

Mathematica [A] time = 0.234203, size = 154, normalized size = 0.72

$$\frac{160 \operatorname{FresnelC}(bx) \left(\pi b^2 x^2 (\pi^2 b^4 x^4 - 24) \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 6 (\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2} \pi b^2 x^2\right) \right) + 2bx (5 (4\pi^2 b^4 x^4 - 147) \cos(\pi b^2 x^2) - 24)}{160\pi^4 b^8}$$

Antiderivative was successfully verified.

[In] Integrate[x^7 * Cos[(b^2 * Pi * x^2) / 2] * FresnelC[b * x], x]

[Out] (2655 * Sqrt[2] * FresnelC[Sqrt[2] * b * x] + 160 * FresnelC[b * x] * (6 * (-8 + b^4 * Pi^2 * x^4) * Cos[(b^2 * Pi * x^2) / 2] + b^2 * Pi * x^2 * (-24 + b^4 * Pi^2 * x^4) * Sin[(b^2 * Pi * x^2) / 2]) + 2 * b * x * (5 * (-147 + 4 * b^4 * Pi^2 * x^4) * Cos[b^2 * Pi * x^2] - 2 * (-960 + 24 * b^4 * Pi^2 * x^4) * Sin[b^2 * Pi * x^2])) / (160 * Pi^4 * b^8)

$$i^2 x^4 + 85 b^2 \pi x^2 \sin[b^2 \pi x^2])) / (160 b^8 \pi^4)$$

Maple [A] time = 0.085, size = 317, normalized size = 1.5

$$\frac{1}{b} \left(\frac{\text{FresnelC}(bx)}{b^7} \left(\frac{b^6 x^6}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) - 6 \frac{1}{\pi} \left(-\frac{x^4 b^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 4 \frac{1}{\pi} \left(\frac{b^2 x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 2 \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)

[Out] (FresnelC(b*x)/b^7*(1/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/Pi*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*Pi^2*b^5*x^5-8*b*x)+3/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))+1/2/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")

[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^7 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")

[Out] integral(x^7*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")

[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)

3.182 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=247

$$\frac{15ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right)}{8\pi^3 b^5} - \frac{15ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)}{8\pi^3 b^5} + \frac{15 \text{FresnelC}(bx) \text{S}(bx)}{\pi b^2}$$

```
[Out] (-5*x^4)/(8*b^3*Pi^2) - (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (15*FresnelC[b*x]*FresnelS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (15*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```

Rubi [A] time = 0.25316, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6455, 6463, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$\frac{15ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3 b^5} - \frac{15ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi^3 b^5} + \frac{15 \text{FresnelC}(bx) \text{S}(bx)}{2\pi^3 b^7} + \frac{x^5 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

```
[Out] (-5*x^4)/(8*b^3*Pi^2) - (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (15*FresnelC[b*x]*FresnelS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (15*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

]

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])]/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx &= \frac{x^5 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{5 \int x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^5 \sin(b^2\pi x^2) dx}{2b\pi} \\
 &= \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{x^5 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{15 \int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx}{b^4\pi^2} - \frac{5 \int x^5 \sin(b^2\pi x^2) dx}{2b\pi} \\
 &= \frac{x^4 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} - \frac{15xC(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
 &= \frac{x^4 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{15C(bx)S(bx)}{2b^7\pi^3} + \frac{15ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} \\
 &= -\frac{5x^4}{8b^3\pi^2} - \frac{17 \cos(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^4 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{15C(bx)S(bx)}{2b^7\pi^3} \\
 &= -\frac{5x^4}{8b^3\pi^2} - \frac{11 \cos(b^2\pi x^2)}{2b^7\pi^4} + \frac{x^4 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{15C(bx)S(bx)}{2b^7\pi^3}
 \end{aligned}$$

Mathematica [F] time = 0.381465, size = 0, normalized size = 0.

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]

[Out] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int x^6 \cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelC}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)`

[Out] `int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")`

[Out] `integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^6 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")`

[Out] `integral(x^6*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")`

[Out] `integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

3.183 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$

Optimal. Leaf size=157

$$\frac{x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{8 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{4x^2 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} + \frac{43S(\sqrt{2}bx)}{8\sqrt{2}\pi^3 b^6} - \frac{2x}{3\pi^2}$$

[Out] $(-2*x^3)/(3*b^3*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (4*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) - (8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)$

Rubi [A] time = 0.151098, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6455, 6463, 6453, 3351, 3392, 30, 3386, 3385}

$$\frac{x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{8 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{4x^2 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} + \frac{43S(\sqrt{2}bx)}{8\sqrt{2}\pi^3 b^6} - \frac{2x}{3\pi^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^5*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x], x]$

[Out] $(-2*x^3)/(3*b^3*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (4*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) - (8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)$

Rule 6455

$\text{Int}[\text{Cos}[(d_.)*(x_)^2]*\text{FresnelC}[(b_.)*(x_)]*(x_)^(m_), x_Symbol] \rightarrow \text{Simp}[(x^(m-1)*\text{Sin}[d*x^2]*\text{FresnelC}[b*x])/(2*d), x] + (-\text{Dist}[(m-1)/(2*d), \text{Int}[x^(m-2)*\text{Sin}[d*x^2]*\text{FresnelC}[b*x], x], x] - \text{Dist}[b/(4*d), \text{Int}[x^(m-1)*\text{Sin}[2*d*x^2], x], x]) /; \text{FreeQ}\{b, d\}, x \&\& \text{EqQ}[d^2, (Pi^2*b^4)/4] \&\& \text{IGtQ}[m, 1]$

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] := Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] := Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] := Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx &= \frac{x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{4 \int x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^4 \sin(b^2\pi x^2) dx}{2b\pi} \\
&= \frac{x^3 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{8 \int x \cos\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} \\
&= \frac{x^3 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} - \frac{8C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&= -\frac{2x^3}{3b^3\pi^2} + \frac{x^3 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{3S(\sqrt{2}bx)}{8\sqrt{2}b^6\pi^3} + \frac{2\sqrt{2}S(\sqrt{2}bx)}{b^6\pi^3} - \frac{8}{b^6\pi^3} \\
&= -\frac{2x^3}{3b^3\pi^2} + \frac{x^3 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{11S(\sqrt{2}bx)}{8\sqrt{2}b^6\pi^3} + \frac{2\sqrt{2}S(\sqrt{2}bx)}{b^6\pi^3} - \frac{8}{b^6\pi^3}
\end{aligned}$$

Mathematica [A] time = 0.131994, size = 120, normalized size = 0.76

$$\frac{48\text{FresnelC}(bx) \left((\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2}\pi b^2 x^2\right) + 4\pi b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \right) - 32\pi b^3 x^3 - 66bx \sin(\pi b^2 x^2) + 12\pi b^3 x^3 \cos(\pi b^2 x^2)}{48\pi^3 b^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]

[Out] (-32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 48*FresnelC[b*x]*(4*b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(48*b^6*Pi^3)

Maple [A] time = 0.079, size = 202, normalized size = 1.3

$$\frac{1}{b} \left(\frac{\text{FresnelC}(bx)}{b^5} \left(\frac{x^4 b^4}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) - 4 \frac{1}{\pi} \left(-\frac{b^2 x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 2 \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right) \right) - \frac{1}{b^5} \left(\frac{2x^3 b^3}{3\pi^2} + 2 \frac{1}{\pi^2} \left(\frac{1}{2} b^2 \pi x^2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x), x)

[Out] $(\text{FresnelC}(b*x)/b^5*(1/\text{Pi}*b^4*x^4*\sin(1/2*b^2*\text{Pi}*x^2)-4/\text{Pi}*(-1/\text{Pi}*b^2*x^2*\cos(1/2*b^2*\text{Pi}*x^2)+2/\text{Pi}^2*\sin(1/2*b^2*\text{Pi}*x^2)))-1/b^5*(2/3/\text{Pi}^2*b^3*x^3+2/\text{Pi}^2*(1/2/\text{Pi}*b*x*\sin(b^2*\text{Pi}*x^2)-1/4/\text{Pi}^2*(1/2)*\text{FresnelS}(b*x*2^(1/2)))+1/2/\text{Pi}^3*(-1/2*\text{Pi}*b^3*x^3*\cos(b^2*\text{Pi}*x^2)+3/2*\text{Pi}*(1/2/\text{Pi}*b*x*\sin(b^2*\text{Pi}*x^2)-1/4/\text{Pi}^2*(1/2)*\text{FresnelS}(b*x*2^(1/2)))-4*2^(1/2)*\text{FresnelS}(b*x*2^(1/2))))/b$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")`

[Out] `integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^5 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")`

[Out] `integral(x^5*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)`

```
[Out] Integral(x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.184 $\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=120

$$\frac{x^3 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{3x \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{3 \mathbf{FresnelC}(bx)^2}{2\pi^2 b^5} - \frac{3x^2}{4\pi^2 b^3} - \frac{\sin(\pi b^2 x^2)}{\pi^3 b^5} + \frac{x^2 \cos(\pi b^2 x^2)}{4\pi^2 b^3}$$

[Out] $(-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3)$

Rubi [A] time = 0.118108, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637}

$$\frac{x^3 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{3x \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{3 \mathbf{FresnelC}(bx)^2}{2\pi^2 b^5} - \frac{3x^2}{4\pi^2 b^3} - \frac{\sin(\pi b^2 x^2)}{\pi^3 b^5} + \frac{x^2 \cos(\pi b^2 x^2)}{4\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x], x]$

[Out] $(-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3)$

Rule 6455

$\text{Int}[\text{Cos}[(d_.)*(x_)^2]*\text{FresnelC}[(b_.)*(x_)]*(x_)^(m_), x_Symbol] \rightarrow \text{Simp}[(x^(m-1)*\text{Sin}[d*x^2]*\text{FresnelC}[b*x])/(2*d), x] + (-\text{Dist}[(m-1)/(2*d), \text{Int}[x^(m-2)*\text{Sin}[d*x^2]*\text{FresnelC}[b*x], x], x] - \text{Dist}[b/(4*d), \text{Int}[x^(m-1)*\text{Sin}[2*d*x^2], x], x]) /; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (Pi^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

Rule 6463

$\text{Int}[\text{FresnelC}[(b_.)*(x_)]*(x_)^(m_)*\text{Sin}[(d_.)*(x_)^2], x_Symbol] \rightarrow -\text{Simp}[(x^(m-1)*\text{Cos}[d*x^2]*\text{FresnelC}[b*x])/(2*d), x] + (\text{Dist}[(m-1)/(2*d), \text{Int}[x^(m-2)*\text{Cos}[d*x^2]*\text{FresnelC}[b*x], x], x] + \text{Dist}[b/(2*d), \text{Int}[x^(m-1)*\text{Cos}[d$

$*x^2]^2, x], x]) /; \text{FreeQ}\{b, d\}, x\} \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$
 $]$

Rule 6441

$\text{Int}[\text{Cos}[(d_)*(x_)^2]*\text{FresnelC}[(b_)*(x_)]^{(n_)}, x_Symbol] \ :> \ \text{Dist}[(\text{Pi}*b)/$
 $(2*d), \text{Subst}[\text{Int}[x^n, x], x, \text{FresnelC}[b*x]], x] /; \text{FreeQ}\{b, d, n\}, x\} \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4]$

Rule 30

$\text{Int}[(x_)^{(m_)}, x_Symbol] \ :> \ \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 3380

$\text{Int}[(a_ + \text{Cos}[c_ + (d_)*(x_)^{(n_)}]*b_)]^{(p_)}*(x_)^{(m_)}, x_Symbol]$
 $] \ :> \ \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Cos}[c + d*x])^p}$
 $, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x\} \ \&\& \ \text{IntegerQ}[\text{Simplify}[($
 $m+1)/n]] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{EqQ}[m, n-1] \ || \ (\text{IntegerQ}[p] \ \&\& \ \text{GtQ}[\text{Simplify}[($
 $m+1)/n], 0]))$

Rule 2634

$\text{Int}[\sin[(c_ + ((d_)*(x_))/2)^2], x_Symbol] \ :> \ \text{Simp}[x/2, x] - \text{Simp}[\sin[2*c$
 $+ d*x]/(2*d), x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3379

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*\sin[(c_ + (d_)*(x_)^{(n_)}))]^{(p_)}, x_Symbol]$
 $] \ :> \ \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\sin[c + d*x])^p}$
 $, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x\} \ \&\& \ \text{IntegerQ}[\text{Simplify}[($
 $m+1)/n]] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{EqQ}[m, n-1] \ || \ (\text{IntegerQ}[p] \ \&\& \ \text{GtQ}[\text{Simplify}[($
 $m+1)/n], 0]))$

Rule 3296

$\text{Int}[(c_ + (d_)*(x_))^{(m_)}*\sin[(e_ + (f_)*(x_))], x_Symbol] \ :> \ -\text{Simp}[($
 $(c + d*x)^m*\text{Cos}[e + f*x])/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[$
 $e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[m, 0]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_ + (d_)*(x_))], x_Symbol] \ :> \ \text{Simp}[\sin[c + d*x]/d, x] /;$
 $\text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx &= \frac{x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3 \int x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^3 \sin(b^2\pi x^2) dx}{2b\pi} \\
&= \frac{3x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3 \int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx}{b^4\pi^2} - \frac{3 \int x \cos(b^2\pi x^2) dx}{b^4\pi^2} \\
&= \frac{x^2 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{3x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3 \text{Subst}\left(\int x dx, x, C(bx)\right)}{b^5\pi^2} \\
&= -\frac{3x^2}{4b^3\pi^2} + \frac{x^2 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{3x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} - \frac{3C(bx)^2}{2b^5\pi^2} + \frac{x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}
\end{aligned}$$

Mathematica [A] time = 0.006659, size = 120, normalized size = 1.

$$\frac{x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{3x \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{3 \text{FresnelC}(bx)^2}{2\pi^2 b^5} - \frac{3x^2}{4\pi^2 b^3} - \frac{\sin(\pi b^2 x^2)}{\pi^3 b^5} + \frac{x^2 \cos(\pi b^2 x^2)}{4\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] (-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3)

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)

[Out] $\int x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) \text{FresnelC}(b x), x$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(b x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")`

[Out] `integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(b x), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")`

[Out] `integral(x^4*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

Sympy [A] time = 26.6684, size = 151, normalized size = 1.26

$$\begin{cases} \frac{x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} - \frac{x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^2 b^3} - \frac{x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{3x \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi^2 b^4} - \frac{2 \sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^3 b^5} - \frac{3C^2(bx)}{2\pi^2 b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)`

[Out] `Piecewise((x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) - x**2*sin(pi*b**2*x**2/2)**2/(pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(2*pi**2*b**3) + 3*x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi**2*b**4) - 2*sin(pi*b**2*x**2/2)*`

```
cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnelc(b*x)**2/(2*pi**2*b**5), Ne(b,
0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```


3.185 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=104

$$\frac{x^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{2 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{5 \mathbf{FresnelC}(\sqrt{2}bx)}{4\sqrt{2}\pi^2 b^4} + \frac{x \cos(\pi b^2 x^2)}{4\pi^2 b^3} - \frac{x}{\pi^2 b^3}$$

[Out] $-(x/(b^3\pi^2)) + (x\cos[b^2\pi x^2])/(4b^3\pi^2) + (2\cos[(b^2\pi x^2)/2] * \mathbf{FresnelC}[b*x])/(b^4\pi^2) - (5*\mathbf{FresnelC}[\text{Sqrt}[2]*b*x])/(4*\text{Sqrt}[2]*b^4\pi^2) + (x^2*\mathbf{FresnelC}[b*x]*\text{Sin}[(b^2\pi x^2)/2])/(b^2\pi)$

Rubi [A] time = 0.0782079, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6455, 6461, 3358, 3352, 3385}

$$\frac{x^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{2 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{5 \mathbf{FresnelC}(\sqrt{2}bx)}{4\sqrt{2}\pi^2 b^4} + \frac{x \cos(\pi b^2 x^2)}{4\pi^2 b^3} - \frac{x}{\pi^2 b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\mathbf{FresnelC}[b*x], x]$

[Out] $-(x/(b^3\pi^2)) + (x\cos[b^2\pi x^2])/(4b^3\pi^2) + (2\cos[(b^2\pi x^2)/2] * \mathbf{FresnelC}[b*x])/(b^4\pi^2) - (5*\mathbf{FresnelC}[\text{Sqrt}[2]*b*x])/(4*\text{Sqrt}[2]*b^4\pi^2) + (x^2*\mathbf{FresnelC}[b*x]*\text{Sin}[(b^2\pi x^2)/2])/(b^2\pi)$

Rule 6455

$\text{Int}[\text{Cos}[(d_*)*(x_)^2]*\mathbf{FresnelC}[(b_*)*(x_)]*(x_)^{(m)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m-1)}*\text{Sin}[d*x^2]*\mathbf{FresnelC}[b*x])/(2*d), x] + (-\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\text{Sin}[d*x^2]*\mathbf{FresnelC}[b*x], x], x] - \text{Dist}[b/(4*d), \text{Int}[x^{(m-1)}*\text{Sin}[2*d*x^2], x], x]) /; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

Rule 6461

$\text{Int}[\mathbf{FresnelC}[(b_*)*(x_)]*(x_)*\text{Sin}[(d_*)*(x_)^2], x_Symbol] \rightarrow -\text{Simp}[(\text{Cos}[d*x^2]*\mathbf{FresnelC}[b*x])/(2*d), x] + \text{Dist}[b/(2*d), \text{Int}[\text{Cos}[d*x^2]^2, x], x] /; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4]$

Rule 3358

Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] :> Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3385

Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rubi steps

$$\begin{aligned}
 \int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx &= \frac{x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{2 \int x C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x^2 \sin(b^2\pi x^2) dx}{2b\pi} \\
 &= \frac{x \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} + \frac{x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \cos(b^2\pi x^2) dx}{4b^3\pi^2} - \frac{2 \int \left(\frac{1}{2} + \dots\right)}{2b\pi} \\
 &= \frac{x \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} - \frac{C(\sqrt{2}bx)}{4\sqrt{2}b^4\pi^2} + \frac{x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{2 \int \left(\frac{1}{2} + \dots\right)}{2b\pi} \\
 &= -\frac{x}{b^3\pi^2} + \frac{x \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} - \frac{C(\sqrt{2}bx)}{4\sqrt{2}b^4\pi^2} + \frac{x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{2 \int \left(\frac{1}{2} + \dots\right)}{2b\pi} \\
 &= -\frac{x}{b^3\pi^2} + \frac{x \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^4\pi^2} - \frac{5C(\sqrt{2}bx)}{4\sqrt{2}b^4\pi^2} + \frac{x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}
 \end{aligned}$$

Mathematica [A] time = 0.084731, size = 83, normalized size = 0.8

$$\frac{8\text{FresnelC}(bx) \left(\pi b^2 x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right) + 2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \right) + 2bx \left(\cos(\pi b^2 x^2) - 4 \right) - 5\sqrt{2}\text{FresnelC}(\sqrt{2}bx)}{8\pi^2 b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]

[Out] (2*b*x*(-4 + Cos[b^2*Pi*x^2]) - 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 8*FresnelC[b*x]*(2*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]))/(8*b^4*Pi^2)

Maple [A] time = 0.076, size = 114, normalized size = 1.1

$$\frac{1}{b} \left(\frac{\text{FresnelC}(bx)}{b^3} \left(\frac{b^2 x^2}{\pi} \sin\left(\frac{b^2 \pi x^2}{2}\right) + 2 \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) - \frac{1}{b^3} \left(\frac{bx}{\pi^2} + \frac{\sqrt{2} \text{FresnelC}(bx\sqrt{2})}{2\pi^2} + \frac{1}{2\pi} \left(-\frac{bx \cos(b^2 \pi x^2)}{2\pi} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x), x)

[Out] (FresnelC(b*x)/b^3*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2))-1/b^3*(1/Pi^2*b*x+1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))+1/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnelc(b*x), x, algorithm="maxima")

[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**3*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

3.186 $\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=136

$$\frac{ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right)}{8\pi b} + \frac{ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)}{8\pi b} - \frac{\text{FresnelC}(bx)}{2}$$

```
[Out] Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - (
(I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b*Pi) +
((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi) +
(x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rubi [A] time = 0.0595254, antiderivative size = 136, normalized size of antiderivative = 1, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {6455, 6447, 3379, 2638}

$$\frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi b} + \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi b} - \frac{\text{FresnelC}(bx)S(bx)}{2\pi b^3} + \frac{x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

```
[Out] Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - (
(I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b*Pi) +
((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi) +
(x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)^2]*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}

```

, {3/2, 2}, -(I*d*x^2))/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, I*d*x^2])/8, x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 3379

Int[(x_)^(m_)*((a_) + (b_)*Sin[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2638

Int[sin[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx &= \frac{x C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^2\pi} - \frac{\int x \sin(b^2\pi x^2) dx}{2b\pi} \\ &= -\frac{C(bx)S(bx)}{2b^3\pi} - \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{x C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\ &= \frac{\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{C(bx)S(bx)}{2b^3\pi} - \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8b\pi} \end{aligned}$$

Mathematica [F] time = 0.208928, size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]

[Out] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelC}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)`

[Out] `int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")`

[Out] `integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")`

[Out] `integral(x^2*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)`

[Out] `Integral(x**2*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")`

[Out] `integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

3.187 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=48

$$\frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{S(\sqrt{2}bx)}{2\sqrt{2}\pi b^2}$$

[Out] -FresnelS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi) + (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)

Rubi [A] time = 0.0208259, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6453, 3351}

$$\frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{S(\sqrt{2}bx)}{2\sqrt{2}\pi b^2}$$

Antiderivative was successfully verified.

[In] Int[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]

[Out] -FresnelS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi) + (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)

Rule 6453

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)*(x_)], x_Symbol] :> Simp[(Sin[d*x^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx = \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \sin(b^2\pi x^2) dx}{2b\pi}$$

$$= -\frac{S(\sqrt{2}bx)}{2\sqrt{2}b^2\pi} + \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

Mathematica [A] time = 0.0273675, size = 44, normalized size = 0.92

$$\frac{\sqrt{2}S(\sqrt{2}bx) - 4\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] -(Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^2*Pi)

Maple [A] time = 0.054, size = 45, normalized size = 0.9

$$\frac{1}{b} \left(\frac{\text{FresnelC}(bx)}{b\pi} \sin\left(\frac{b^2\pi x^2}{2}\right) - \frac{\sqrt{2}\text{FresnelS}(bx\sqrt{2})}{4b\pi} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)

[Out] (FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi-1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^(1/2))/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")`

[Out] `integrate(x*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")`

[Out] `integral(x*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)`

[Out] `Integral(x*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")`

[Out] `integrate(x*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)`

3.188 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx) dx$

Optimal. Leaf size=13

$$\frac{\mathbf{FresnelC}(bx)^2}{2b}$$

[Out] FresnelC[b*x]^2/(2*b)

Rubi [A] time = 0.0114821, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6441, 30}

$$\frac{\mathbf{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] FresnelC[b*x]^2/(2*b)

Rule 6441

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx &= \frac{\text{Subst}\left(\int x dx, x, C(bx)\right)}{b} \\ &= \frac{C(bx)^2}{2b} \end{aligned}$$

Mathematica [A] time = 0.0024072, size = 13, normalized size = 1.

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] FresnelC[b*x]^2/(2*b)

Maple [A] time = 0.046, size = 12, normalized size = 0.9

$$\frac{(\text{FresnelC}(bx))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)

[Out] 1/2*FresnelC(b*x)^2/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

Sympy [A] time = 0.458888, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

$$3.189 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}, x\right)$$

[Out] Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]

Rubi [A] time = 0.0168393, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]

[Out] Defer[Int] [(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x} dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x} dx$$

Mathematica [A] time = 0.029494, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]

Maple [A] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x,x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x, x)

$$3.190 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^2} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}, x\right)$$

[Out] Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]

Rubi [A] time = 0.0187246, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]

[Out] Defer[Int] [(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2} dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2} dx$$

Mathematica [A] time = 0.030572, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^2} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^2,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^2,x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**2,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^2,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^2, x)

$$3.191 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx)}{x^3} dx$$

Optimal. Leaf size=101

$$-\frac{1}{2}\pi b^2 \text{Unintegrable} \left(\frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x \right) - \frac{\mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2x^2} - \frac{\pi b^2 S(\sqrt{2}bx)}{2\sqrt{2}} - \frac{b \cos(\pi b^2 x^2)}{4x}$$

[Out] $-b/(4*x) - (b*\text{Cos}[b^2*\text{Pi}*x^2])/(4*x) - (\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(2*x^2) - (b^2*\text{Pi}*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(2*\text{Sqrt}[2]) - (b^2*\text{Pi}*\text{Unintegrable}[(\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/x, x])/2$

Rubi [A] time = 0.0564858, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx)}{x^3} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/x^3, x]$

[Out] $-b/(4*x) - (b*\text{Cos}[b^2*\text{Pi}*x^2])/(4*x) - (\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(2*x^2) - (b^2*\text{Pi}*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(2*\text{Sqrt}[2]) - (b^2*\text{Pi}*\text{Defer}[\text{Int}[(\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/x, x])/2$

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^3} dx &= -\frac{b}{4x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{2x^2} + \frac{1}{4}b \int \frac{\cos(b^2\pi x^2)}{x^2} dx - \frac{1}{2}(b^2\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx \\ &= -\frac{b}{4x} - \frac{b \cos(b^2\pi x^2)}{4x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{2x^2} - \frac{1}{2}(b^2\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx - \frac{1}{2}(b^3\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx \\ &= -\frac{b}{4x} - \frac{b \cos(b^2\pi x^2)}{4x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{2x^2} - \frac{b^2\pi S(\sqrt{2}bx)}{2\sqrt{2}} - \frac{1}{2}(b^2\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx \end{aligned}$$

Mathematica [A] time = 0.0296671, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3,x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3, x]

Maple [A] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^3} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^3,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^3,x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^3, x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**3,x)`

[Out] `Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right) \text{fresnelc}(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^3,x, algorithm="giac")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^3, x)`

$$3.192 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^4} dx$$

Optimal. Leaf size=109

$$\frac{\pi b^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3x} - \frac{\mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^3} - \frac{1}{6}\pi^2 b^3 \mathbf{FresnelC}(bx)^2 - \frac{1}{6}\pi b^3 \mathbf{Si}(b^2 \pi x^2) - \frac{b \cos(\pi b^2 x^2)}{12x^2}$$

[Out] -b/(12*x^2) - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6

Rubi [A] time = 0.115158, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6457, 6465, 6441, 30, 3375, 3380, 3297, 3299}

$$\frac{\pi b^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3x} - \frac{\mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^3} - \frac{1}{6}\pi^2 b^3 \mathbf{FresnelC}(bx)^2 - \frac{1}{6}\pi b^3 \mathbf{Si}(b^2 \pi x^2) - \frac{b \cos(\pi b^2 x^2)}{12x^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]

[Out] -b/(12*x^2) - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6

Rule 6457

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)*(x_)^(m_)], x_Symbol] :> Simp[(x^(m+1)*Cos[d*x^2]*FresnelC[b*x])/(m+1), x] + (Dist[(2*d)/(m+1), Int[x^(m+2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m+1)), Int[x^(m+1)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m+2))/(2*(m+1)*(m+2)), x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]

Rule 6465

Int[FresnelC[(b_.)*(x_)*(x_)^(m_) * Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^(m+1)*Sin[d*x^2]*FresnelC[b*x])/(m+1), x] + (-Dist[(2*d)/(m+1), Int[x^(m+2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m+1)), Int[x^(m+1)*Sin[d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]

1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]

Rule 6441

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.), x_Symbol] := Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]

Rule 3375

Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n/n, x] /
; FreeQ[{d, n}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^4} dx &= -\frac{b}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{3x^3} + \frac{1}{6}b \int \frac{\cos(b^2\pi x^2)}{x^3} dx - \frac{1}{3}(b^2\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx \\
&= -\frac{b}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{3x^3} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x} + \frac{1}{12}b \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^2} dx, x, \right. \\
&= -\frac{b}{12x^2} - \frac{b \cos(b^2\pi x^2)}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{3x^3} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x} - \frac{1}{12}b^3\pi \text{Si}(b^2\pi x^2) \\
&= -\frac{b}{12x^2} - \frac{b \cos(b^2\pi x^2)}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{3x^3} - \frac{1}{6}b^3\pi^2 C(bx)^2 + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x} - \frac{1}{12}b^3\pi \text{Si}(b^2\pi x^2)
\end{aligned}$$

Mathematica [A] time = 0.0069185, size = 109, normalized size = 1.

$$\frac{\pi b^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3x} - \frac{\text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^3} - \frac{1}{6}\pi^2 b^3 \text{FresnelC}(bx)^2 - \frac{1}{6}\pi b^3 \text{Si}(b^2\pi x^2) - \frac{b \cos(\pi b^2 x^2)}{12x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]

[Out] -b/(12*x^2) - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^4} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^4,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^4,x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^4, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**4,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \operatorname{fresnelc}(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^4, x)
```

$$3.193 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^5} dx$$

Optimal. Leaf size=152

$$-\frac{1}{8}\pi^2 b^4 \text{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) + \frac{\pi b^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^2} - \frac{\mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4x^4}$$

[Out] $-b/(24*x^3) - (b*\text{Cos}[b^2*Pi*x^2])/(24*x^3) - (\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(4*x^4) - (7*b^4*Pi^2*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(24*\text{Sqrt}[2]) + (b^2*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(8*x^2) + (7*b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(48*x) - (b^4*Pi^2*\text{Unintegrable}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x, x])/8$

Rubi [A] time = 0.115349, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^5} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x^5, x]$

[Out] $-b/(24*x^3) - (b*\text{Cos}[b^2*Pi*x^2])/(24*x^3) - (\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(4*x^4) - (7*b^4*Pi^2*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(24*\text{Sqrt}[2]) + (b^2*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(8*x^2) + (7*b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(48*x) - (b^4*Pi^2*\text{Defer}[\text{Int}][(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x, x])/8$

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^5} dx &= -\frac{b}{24x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{4x^4} + \frac{1}{8}b \int \frac{\cos(b^2\pi x^2)}{x^4} dx - \frac{1}{4}(b^2\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx \\
&= -\frac{b}{24x^3} - \frac{b \cos(b^2\pi x^2)}{24x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{4x^4} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2} - \frac{1}{16}(b^3\pi) \int \frac{\sin(b^2\pi x^2)}{x^2} dx \\
&= -\frac{b}{24x^3} - \frac{b \cos(b^2\pi x^2)}{24x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{4x^4} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2} + \frac{7b^3\pi \sin(b^2\pi x^2)}{48x} \\
&= -\frac{b}{24x^3} - \frac{b \cos(b^2\pi x^2)}{24x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{4x^4} - \frac{7b^4\pi^2 C(\sqrt{2}bx)}{24\sqrt{2}} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}
\end{aligned}$$

Mathematica [A] time = 0.0300349, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^5} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5,x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5, x]

Maple [A] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^5} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^5,x, algorithm="maxima")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^5, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^5,x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^5, x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**5,x)`

[Out] `Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^5, x)
```


$$3.194 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6} dx$$

Optimal. Leaf size=147

$$-\frac{1}{15}\pi^2 b^4 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right) - \frac{1}{24}\pi^2 b^5 \text{CosIntegral}(\pi b^2 x^2) + \frac{\pi b^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3}$$

[Out] $-b/(40*x^4) - (b*\text{Cos}[b^2*Pi*x^2])/(40*x^4) - (b^5*Pi^2*\text{CosIntegral}[b^2*Pi*x^2])/24 - (\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(5*x^5) + (b^2*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(15*x^3) + (b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(24*x^2) - (b^4*Pi^2*\text{Unintegrable}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x^2, x])/15$

Rubi [A] time = 0.192316, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x^6, x]$

[Out] $-b/(40*x^4) - (b*\text{Cos}[b^2*Pi*x^2])/(40*x^4) - (b^5*Pi^2*\text{CosIntegral}[b^2*Pi*x^2])/24 - (\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(5*x^5) + (b^2*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(15*x^3) + (b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(24*x^2) - (b^4*Pi^2*\text{Defer[Int]}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x^2, x])/15$

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^6} dx &= -\frac{b}{40x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{5x^5} + \frac{1}{10}b \int \frac{\cos(b^2\pi x^2)}{x^5} dx - \frac{1}{5}(b^2\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx \\
&= -\frac{b}{40x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{5x^5} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} + \frac{1}{20}b \operatorname{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^3} dx, x, \right. \\
&= -\frac{b}{40x^4} - \frac{b \cos(b^2\pi x^2)}{40x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{5x^5} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} - \frac{1}{60}(b^3\pi) \operatorname{Subst} \\
&= -\frac{b}{40x^4} - \frac{b \cos(b^2\pi x^2)}{40x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{5x^5} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} + \frac{b^3\pi \sin(b^2\pi x^2)}{24x^2} \\
&= -\frac{b}{40x^4} - \frac{b \cos(b^2\pi x^2)}{40x^4} - \frac{1}{24}b^5\pi^2 \operatorname{Ci}(b^2\pi x^2) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{5x^5} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}
\end{aligned}$$

Mathematica [A] time = 0.0296004, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{x^6} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6,x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6, x]

Maple [A] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{\operatorname{FresnelC}(bx)}{x^6} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^6,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^6, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^6,x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^6, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**6,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \operatorname{fresnelc}(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^6, x)
```

$$3.195 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^7} dx$$

Optimal. Leaf size=240

$$\frac{1}{48}\pi^3b^6\mathbf{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}, x\right) + \frac{\pi b^2\mathbf{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{24x^4} + \frac{\pi^2b^4\mathbf{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{48x^2}$$

[Out] $-b/(60*x^5) + (b^5*Pi^2)/(96*x) - (b*\text{Cos}[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*\text{Cos}[b^2*Pi*x^2])/(1440*x) - (\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(6*x^6) + (b^4*Pi^2*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(48*x^2) + (7*b^6*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(144*\text{Sqrt}[2]) + (\text{Sqrt}[2]*b^6*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/45 + (b^2*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(24*x^4) + (13*b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(720*x^3) + (b^6*Pi^3*\mathbf{Unintegrable}[(\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x, x])/48$

Rubi [A] time = 0.20372, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^7} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x^7, x]$

[Out] $-b/(60*x^5) + (b^5*Pi^2)/(96*x) - (b*\text{Cos}[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*\text{Cos}[b^2*Pi*x^2])/(1440*x) - (\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(6*x^6) + (b^4*Pi^2*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(48*x^2) + (7*b^6*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(144*\text{Sqrt}[2]) + (\text{Sqrt}[2]*b^6*Pi^3*\text{FresnelS}[\text{Sqrt}[2]*b*x])/45 + (b^2*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(24*x^4) + (13*b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(720*x^3) + (b^6*Pi^3*\text{Defer}[\text{Int}[(\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/x, x])/48$

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^7} dx &= -\frac{b}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{6x^6} + \frac{1}{12}b \int \frac{\cos(b^2\pi x^2)}{x^6} dx - \frac{1}{6}(b^2\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx \\
&= -\frac{b}{60x^5} - \frac{b \cos(b^2\pi x^2)}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{6x^6} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} - \frac{1}{48}(b^3\pi) \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx \\
&= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b \cos(b^2\pi x^2)}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{6x^6} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{48x^2} + \frac{b^2\pi C(bx)}{48x} \\
&= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b \cos(b^2\pi x^2)}{60x^5} + \frac{67b^5\pi^2 \cos(b^2\pi x^2)}{1440x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{6x^6} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{48x^2} + \frac{b^2\pi C(bx)}{48x} \\
&= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b \cos(b^2\pi x^2)}{60x^5} + \frac{67b^5\pi^2 \cos(b^2\pi x^2)}{1440x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{6x^6} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{48x^2} + \frac{b^2\pi C(bx)}{48x}
\end{aligned}$$

Mathematica [A] time = 0.0290908, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^7} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7, x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7, x]

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^7} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7, x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^7,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^7, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^7,x, algorithm="fricas")

[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^7, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**7,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \operatorname{fresnelc}(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^7, x)
```


$$3.196 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^8} dx$$

Optimal. Leaf size=224

$$-\frac{\pi^3 b^6 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{105x} + \frac{\pi b^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{35x^5} + \frac{\pi^2 b^4 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{105x^3} - \frac{\mathbf{FresnelC}(bx)}{7x^7}$$

[Out] $-b/(84*x^6) + (b^5*Pi^2)/(420*x^2) - (b*\text{Cos}[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*\text{Cos}[b^2*Pi*x^2])/(84*x^2) - (\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(7*x^7) + (b^4*Pi^2*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(105*x^3) + (b^7*Pi^4*\text{FresnelC}[b*x]^2)/210 + (b^2*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6*Pi^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(105*x^4) + (b^7*Pi^3*\text{SinIntegral}[b^2*Pi*x^2])/70$

Rubi [A] time = 0.351595, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6457, 6465, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$-\frac{\pi^3 b^6 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{105x} + \frac{\pi b^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{35x^5} + \frac{\pi^2 b^4 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{105x^3} - \frac{\mathbf{FresnelC}(bx)}{7x^7}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/x^8, x]$

[Out] $-b/(84*x^6) + (b^5*Pi^2)/(420*x^2) - (b*\text{Cos}[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*\text{Cos}[b^2*Pi*x^2])/(84*x^2) - (\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(7*x^7) + (b^4*Pi^2*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(105*x^3) + (b^7*Pi^4*\text{FresnelC}[b*x]^2)/210 + (b^2*Pi*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6*Pi^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*\text{Sin}[b^2*Pi*x^2])/(105*x^4) + (b^7*Pi^3*\text{SinIntegral}[b^2*Pi*x^2])/70$

Rule 6457

$\text{Int}[\text{Cos}[(d_.)*(x_)^2]*\text{FresnelC}[(b_.)*(x_)]*(x_)^(m_), x_Symbol] \text{ :> } \text{Simp}[(x^(m+1)*\text{Cos}[d*x^2]*\text{FresnelC}[b*x])/(m+1), x] + (\text{Dist}[(2*d)/(m+1), \text{Int}[x^(m+2)*\text{Sin}[d*x^2]*\text{FresnelC}[b*x], x], x] - \text{Dist}[b/(2*(m+1)), \text{Int}[x^(m+1)*\text{Cos}[2*d*x^2], x], x] - \text{Simp}[(b*x^(m+2))/(2*(m+1)*(m+2)), x]) /;$ FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(x^(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && LtQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.)], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_)*((a_) + (b_)*Sin[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8} dx &= -\frac{b}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{1}{14}b \int \frac{\cos(b^2\pi x^2)}{x^7} dx - \frac{1}{7}(b^2\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx \\
&= -\frac{b}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} + \frac{1}{28}b \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^4} dx, x, x^2\right) \\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b \cos(b^2\pi x^2)}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{105x^3} + \frac{b^2\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{105x^3} \\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b \cos(b^2\pi x^2)}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{105x^3} + \frac{b^2\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{105x^3} \\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b \cos(b^2\pi x^2)}{84x^6} + \frac{b^5\pi^2 \cos(b^2\pi x^2)}{84x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{105x^3} \\
&= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b \cos(b^2\pi x^2)}{84x^6} + \frac{b^5\pi^2 \cos(b^2\pi x^2)}{84x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{105x^3}
\end{aligned}$$

Mathematica [A] time = 0.0135264, size = 224, normalized size = 1.

$$\frac{\pi^3 b^6 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{105x} + \frac{\pi b^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{35x^5} + \frac{\pi^2 b^4 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{105x^3} - \frac{\text{FresnelC}(bx)}{105x}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^8,x]

[Out] -b/(84*x^6) + (b^5*Pi^2)/(420*x^2) - (b*Cos[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Cos[b^2*Pi*x^2])/(84*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*x^7) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(105*x^3) + (b^7*Pi^4*FresnelC[b*x]^2)/210 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*Sin[b^2*Pi*x^2])

$/(105*x^4) + (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70$

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^8} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)`

[Out] `int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^8,x, algorithm="maxima")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^8, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^8,x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^8, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**8,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^8,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^8, x)

$$3.197 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^9} dx$$

Optimal. Leaf size=267

$$\frac{1}{384}\pi^4 b^8 \text{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) - \frac{\pi^3 b^6 \mathbf{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{384x^2} + \frac{\pi b^2 \mathbf{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{48x^6}$$

[Out] $-b/(112*x^7) + (b^5*\pi^2)/(1152*x^3) - (b*\text{Cos}[b^2*\pi*x^2])/(112*x^7) + (187*b^5*\pi^2*\text{Cos}[b^2*\pi*x^2])/(40320*x^3) - (\text{Cos}[(b^2*\pi*x^2)/2]*\mathbf{FresnelC}[b*x])/(8*x^8) + (b^4*\pi^2*\text{Cos}[(b^2*\pi*x^2)/2]*\mathbf{FresnelC}[b*x])/(192*x^4) + (853*b^8*\pi^4*\mathbf{FresnelC}[\text{Sqrt}[2]*b*x])/(40320*\text{Sqrt}[2]) + (b^2*\pi*\mathbf{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(48*x^6) - (b^6*\pi^3*\mathbf{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(384*x^2) + (19*b^3*\pi*\text{Sin}[b^2*\pi*x^2])/(3360*x^5) - (853*b^7*\pi^3*\text{Sin}[b^2*\pi*x^2])/(80640*x) + (b^8*\pi^4*\text{Unintegrable}[(\text{Cos}[(b^2*\pi*x^2)/2]*\mathbf{FresnelC}[b*x])/x, x])/384$

Rubi [A] time = 0.303801, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^9} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{Cos}[(b^2*\pi*x^2)/2]*\mathbf{FresnelC}[b*x])/x^9, x]$

[Out] $-b/(112*x^7) + (b^5*\pi^2)/(1152*x^3) - (b*\text{Cos}[b^2*\pi*x^2])/(112*x^7) + (187*b^5*\pi^2*\text{Cos}[b^2*\pi*x^2])/(40320*x^3) - (\text{Cos}[(b^2*\pi*x^2)/2]*\mathbf{FresnelC}[b*x])/(8*x^8) + (b^4*\pi^2*\text{Cos}[(b^2*\pi*x^2)/2]*\mathbf{FresnelC}[b*x])/(192*x^4) + (853*b^8*\pi^4*\mathbf{FresnelC}[\text{Sqrt}[2]*b*x])/(40320*\text{Sqrt}[2]) + (b^2*\pi*\mathbf{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(48*x^6) - (b^6*\pi^3*\mathbf{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(384*x^2) + (19*b^3*\pi*\text{Sin}[b^2*\pi*x^2])/(3360*x^5) - (853*b^7*\pi^3*\text{Sin}[b^2*\pi*x^2])/(80640*x) + (b^8*\pi^4*\text{Defer}[\text{Int}[(\text{Cos}[(b^2*\pi*x^2)/2]*\mathbf{FresnelC}[b*x])/x, x])/384$

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^9} dx &= -\frac{b}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{1}{16}b \int \frac{\cos(b^2\pi x^2)}{x^8} dx - \frac{1}{8}(b^2\pi) \int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx \\
&= -\frac{b}{112x^7} - \frac{b \cos(b^2\pi x^2)}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{1}{96}(b^3\pi) \int \frac{C(bx)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx \\
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b \cos(b^2\pi x^2)}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{192x^4} + \frac{b^3\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{96x^2} \\
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b \cos(b^2\pi x^2)}{112x^7} + \frac{187b^5\pi^2 \cos(b^2\pi x^2)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{192x^4} + \frac{b^3\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{96x^2} \\
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b \cos(b^2\pi x^2)}{112x^7} + \frac{187b^5\pi^2 \cos(b^2\pi x^2)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{192x^4} + \frac{b^3\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{96x^2} \\
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b \cos(b^2\pi x^2)}{112x^7} + \frac{187b^5\pi^2 \cos(b^2\pi x^2)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{192x^4} + \frac{b^3\pi^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{96x^2}
\end{aligned}$$

Mathematica [A] time = 0.0311179, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9,x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9, x]

Maple [A] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^9} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)

[Out] `int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^9,x, algorithm="maxima")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^9, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^9,x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^9, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**9,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \operatorname{fresnelc}(bx)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^9,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^9, x)
```

$$3.198 \quad \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}} dx$$

Optimal. Leaf size=262

$$\frac{1}{945}\pi^4 b^8 \text{Unintegrable}\left(\frac{\text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right) + \frac{5\pi^4 b^9 \text{CosIntegral}(\pi b^2 x^2)}{2016} - \frac{\pi^3 b^6 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{945 x^3}$$

[Out] $-b/(144*x^8) + (b^5*\text{Pi}^2)/(2520*x^4) - (b*\text{Cos}[b^2*\text{Pi}*x^2])/(144*x^8) + (67*b^5*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(30240*x^4) + (5*b^9*\text{Pi}^4*\text{CosIntegral}[b^2*\text{Pi}*x^2])/2016 - (\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(9*x^9) + (b^4*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(315*x^5) + (b^2*\text{Pi}*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(63*x^7) - (b^6*\text{Pi}^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(945*x^3) + (11*b^3*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(3024*x^6) - (5*b^7*\text{Pi}^3*\text{Sin}[b^2*\text{Pi}*x^2])/(2016*x^2) + (b^8*\text{Pi}^4*\text{Unintegrable}[(\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/x^2, x])/945$

Rubi [A] time = 0.495136, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/x^{10}, x]$

[Out] $-b/(144*x^8) + (b^5*\text{Pi}^2)/(2520*x^4) - (b*\text{Cos}[b^2*\text{Pi}*x^2])/(144*x^8) + (67*b^5*\text{Pi}^2*\text{Cos}[b^2*\text{Pi}*x^2])/(30240*x^4) + (5*b^9*\text{Pi}^4*\text{CosIntegral}[b^2*\text{Pi}*x^2])/2016 - (\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(9*x^9) + (b^4*\text{Pi}^2*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(315*x^5) + (b^2*\text{Pi}*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(63*x^7) - (b^6*\text{Pi}^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(945*x^3) + (11*b^3*\text{Pi}*\text{Sin}[b^2*\text{Pi}*x^2])/(3024*x^6) - (5*b^7*\text{Pi}^3*\text{Sin}[b^2*\text{Pi}*x^2])/(2016*x^2) + (b^8*\text{Pi}^4*\text{Defer[Int]}[(\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/x^2, x])/945$

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^{10}} dx &= -\frac{b}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{9x^9} + \frac{1}{18}b \int \frac{\cos(b^2\pi x^2)}{x^9} dx - \frac{1}{9}(b^2\pi) \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx \\
&= -\frac{b}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{9x^9} + \frac{b^2\pi C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} + \frac{1}{36}b \text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^5} dx, bx\right) \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b \cos(b^2\pi x^2)}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{9x^9} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{315x^5} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{315x^5} \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b \cos(b^2\pi x^2)}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{9x^9} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{315x^5} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{315x^5} \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b \cos(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2 \cos(b^2\pi x^2)}{30240x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{9x^9} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{315x^5} \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b \cos(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2 \cos(b^2\pi x^2)}{30240x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{9x^9} + \frac{b^4\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{315x^5} \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b \cos(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2 \cos(b^2\pi x^2)}{30240x^4} + \frac{5b^9\pi^4 \text{Ci}(b^2\pi x^2)}{2016} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{9x^9}
\end{aligned}$$

Mathematica [A] time = 0.0328045, size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^{10}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10, x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10, x]

Maple [A] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^{10}} \cos\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)`

[Out] `int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^10,x, algorithm="maxima")`

[Out] `integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^10, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^10,x, algorithm="fricas")`

[Out] `integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^10, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**10,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnelc}(bx)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^10,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^10, x)

3.199 $\int \mathbf{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\sin\left(\frac{1}{2}\pi b^2 x^2\right)\mathbf{FresnelC}(bx)^n, x\right)$$

[Out] Unintegrable[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]

Rubi [A] time = 0.0129616, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \mathbf{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

[In] Int[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]

[Out] Defer[Int][FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]

Rubi steps

$$\int C(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int C(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Mathematica [A] time = 0.0690703, size = 0, normalized size = 0.

$$\int \mathbf{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]

[Out] Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int (\text{FresnelC}(bx))^n \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2), x)

[Out] int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx)^n \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^n*sin(1/2*b^2*pi*x^2), x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)^n*sin(1/2*pi*b^2*x^2), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnelc}(bx)^n \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^n*sin(1/2*b^2*pi*x^2), x, algorithm="fricas")

[Out] integral(fresnelc(b*x)^n*sin(1/2*pi*b^2*x^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{\pi b^2 x^2}{2}\right) C^n(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)**n*sin(1/2*b**2*pi*x**2),x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)**n, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx)^n \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")

[Out] integrate(fresnelc(b*x)^n*sin(1/2*pi*b^2*x^2), x)

3.200 $\int x^8 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=308

$$\frac{105ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right)}{8\pi^4 b^7} - \frac{105ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)}{8\pi^4 b^7} + 10$$

[Out] $(-35x^4)/(8b^5\pi^3) + x^8/(16b\pi) - (40\cos[b^2\pi x^2])/(b^9\pi^5) + (5x^4\cos[b^2\pi x^2])/(2b^5\pi^3) + (35x^3\cos[(b^2\pi x^2)/2]*\text{FresnelC}[bx])/(b^6\pi^3) - (x^7\cos[(b^2\pi x^2)/2]*\text{FresnelC}[bx])/(b^2\pi) + (105*\text{FresnelC}[bx]*\text{FresnelS}[bx])/(2b^9\pi^4) + (((105*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2*\pi x^2])/(b^7\pi^4) - (((105*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2*\pi x^2])/(b^7\pi^4) - (105*x*\text{FresnelC}[bx]*\text{Sin}[(b^2*\pi x^2)/2])/(b^8\pi^4) + (7*x^5*\text{FresnelC}[bx]*\text{Sin}[(b^2*\pi x^2)/2])/(b^4\pi^2) - (55*x^2*\text{Sin}[b^2*\pi x^2])/(4*b^7*\pi^4) + (x^6*\text{Sin}[b^2*\pi x^2])/(4*b^3*\pi^2)$

Rubi [A] time = 0.43063, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6463, 6455, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$\frac{105ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4 b^7} - \frac{105ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi^4 b^7} + \frac{105\text{FresnelC}(bx)\text{S}(bx)}{2\pi^4 b^9} + \frac{7x^5\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{\pi^2 b^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^8*\text{FresnelC}[bx]*\text{Sin}[(b^2*\pi x^2)/2], x]$

[Out] $(-35x^4)/(8b^5\pi^3) + x^8/(16b\pi) - (40\cos[b^2\pi x^2])/(b^9\pi^5) + (5x^4\cos[b^2\pi x^2])/(2b^5\pi^3) + (35x^3\cos[(b^2\pi x^2)/2]*\text{FresnelC}[bx])/(b^6\pi^3) - (x^7\cos[(b^2\pi x^2)/2]*\text{FresnelC}[bx])/(b^2\pi) + (105*\text{FresnelC}[bx]*\text{FresnelS}[bx])/(2b^9\pi^4) + (((105*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (-I/2)*b^2*\pi x^2])/(b^7\pi^4) - (((105*I)/8)*x^2*\text{HypergeometricPFQ}[\{1, 1\}, \{3/2, 2\}, (I/2)*b^2*\pi x^2])/(b^7\pi^4) - (105*x*\text{FresnelC}[bx]*\text{Sin}[(b^2*\pi x^2)/2])/(b^8\pi^4) + (7*x^5*\text{FresnelC}[bx]*\text{Sin}[(b^2*\pi x^2)/2])/(b^4\pi^2) - (55*x^2*\text{Sin}[b^2*\pi x^2])/(4*b^7*\pi^4) + (x^6*\text{Sin}[b^2*\pi x^2])/(4*b^3*\pi^2)$

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] := Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :=
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^8 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{7 \int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx}{b^2\pi} + \frac{\int x^7 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= -\frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{7x^5 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{35 \int x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} - \frac{7 \int x^7 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{7x^5 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{105 \int x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} \\
&= \frac{x^8}{16b\pi} + \frac{7x^4 \cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} - \frac{105x^4 \int x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} \\
&= \frac{x^8}{16b\pi} + \frac{5x^4 \cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{105C(bx) \int x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{2b^4\pi^2} \\
&= -\frac{35x^4}{8b^5\pi^3} + \frac{x^8}{16b\pi} - \frac{119 \cos\left(b^2\pi x^2\right)}{4b^9\pi^5} + \frac{5x^4 \cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} \\
&= -\frac{35x^4}{8b^5\pi^3} + \frac{x^8}{16b\pi} - \frac{40 \cos\left(b^2\pi x^2\right)}{b^9\pi^5} + \frac{5x^4 \cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi}
\end{aligned}$$

Mathematica [F] time = 0.0454054, size = 0, normalized size = 0.

$$\int x^8 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int x^8 \text{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)

[Out] int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(x^8*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^8 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")

[Out] `integral(x^8*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*fresnelc(b*x)*sin(1/2*b**2*pi*x**2), x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \operatorname{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*fresnelc(b*x)*sin(1/2*b^2*pi*x^2), x, algorithm="giac")`

[Out] `integrate(x^8*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

3.201 $\int x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=218

$$\frac{6x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{48 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^8} - \frac{x^6 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{24x^2 \text{FresnelC}(bx)}{\pi^3 b^6}$$

```
[Out] (-4*x^3)/(b^5*Pi^3) + x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
+ (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^6*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*
b^8*Pi^4) - (48*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*Fres
nelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7
*Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.27088, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6463, 6455, 6453, 3351, 3392, 30, 3386, 3385}

$$\frac{6x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{48 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^8} - \frac{x^6 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{24x^2 \text{FresnelC}(bx)}{\pi^3 b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*x^3)/(b^5*Pi^3) + x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
+ (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^6*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*
b^8*Pi^4) - (48*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*Fres
nelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7
*Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)*(x_)^(m_), x_Symbol] := Simp[(x^(
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)*(x_), x_Symbol] := Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] := Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.), x_Symbol] := Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_)^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^7 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{6 \int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx}{b^2\pi} + \frac{\int x^6 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= -\frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{6x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{24 \int x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} - \frac{3 \int x^6 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= \frac{x^7}{14b\pi} + \frac{3x^3 \cos(b^2\pi x^2)}{2b^5\pi^3} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{6x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&= \frac{x^7}{14b\pi} + \frac{17x^3 \cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} - \frac{48C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&= -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3 \cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} \\
&= -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3 \cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi}
\end{aligned}$$

Mathematica [A] time = 0.173901, size = 163, normalized size = 0.75

$$\frac{-224\text{FresnelC}(bx) \left(\pi b^2 x^2 (\pi^2 b^4 x^4 - 24) \cos\left(\frac{1}{2}\pi b^2 x^2\right) - 6 (\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2}\pi b^2 x^2\right) \right) + 16\pi^3 b^7 x^7 - 896\pi b^3 x^3 + 56\pi^2 b^5 x^5}{224\pi^4 b^8}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] (-896*b^3*Pi*x^3 + 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717*
Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 224*FresnelC[b*x]*(b^2*Pi*x^2*(-24 + b^4*Pi
^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) -
2058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi^4)

Maple [A] time = 0.083, size = 322, normalized size = 1.5

$$\frac{1}{b} \left(\frac{\text{FresnelC}(bx)}{b^7} \left(-\frac{b^6 x^6}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + 6 \frac{1}{\pi} \left(\frac{x^4 b^4 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} - 4 \frac{1}{\pi} \left(-\frac{b^2 x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 2 \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)`

[Out]
$$\frac{\begin{aligned} & \text{FresnelC}(b*x)/b^7*(-1/\text{Pi}*b^6*x^6*\cos(1/2*b^2*Pi*x^2)+6/\text{Pi}*(1/\text{Pi}*b^4*x^4*\sin(1/2*b^2*Pi*x^2)-4/\text{Pi}*(-1/\text{Pi}*b^2*x^2*\cos(1/2*b^2*Pi*x^2)+2/\text{Pi}^2*\sin(1/2*b^2*Pi*x^2))))-1/b^7*(-1/2/\text{Pi}^3*(1/7*Pi^2*b^7*x^7-8*x^3*b^3)+3/\text{Pi}^4*(-1/2*Pi*b^3*x^3*\cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*\sin(b^2*Pi*x^2)-1/4/Pi*2^{(1/2)}*FresnelS(b*x*2^{(1/2)}))-4*2^{(1/2)}*FresnelS(b*x*2^{(1/2)}))-1/2/\text{Pi}^3*(1/2*Pi*b^5*x^5*\sin(b^2*Pi*x^2)-5/2*Pi*(-1/2/\text{Pi}*b^3*x^3*\cos(b^2*Pi*x^2)+3/2/\text{Pi}*(1/2/Pi*b*x*\sin(b^2*Pi*x^2)-1/4/Pi*2^{(1/2)}*FresnelS(b*x*2^{(1/2)}))))-12/\text{Pi}*b*x*\sin(b^2*Pi*x^2)+6/\text{Pi}*2^{(1/2)}*FresnelS(b*x*2^{(1/2)})))/b \end{aligned}}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")`

[Out] `integrate(x^7*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^7 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(x^7*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \operatorname{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")`

[Out] `integrate(x^7*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

3.202 $\int x^6 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=185

$$\frac{5x^3 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^5 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{15x \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} - \frac{15 \mathbf{FresnelC}(bx)}{2\pi^3 b^7}$$

[Out] $(-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*\text{Cos}[b^2*Pi*x^2])/(4*b^5*Pi^3) + (15*x*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(b^6*Pi^3) - (x^5*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(b^2*Pi) - (15*\text{FresnelC}[b*x]^2)/(2*b^7*Pi^3) + (5*x^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*\text{Sin}[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*\text{Sin}[b^2*Pi*x^2])/(4*b^3*Pi^2)$

Rubi [A] time = 0.254223, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6463, 6455, 6441, 30, 3380, 2634, 3379, 3296, 2637, 3309}

$$\frac{5x^3 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^5 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{15x \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} - \frac{15 \mathbf{FresnelC}(bx)}{2\pi^3 b^7}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^6*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2], x]$

[Out] $(-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*\text{Cos}[b^2*Pi*x^2])/(4*b^5*Pi^3) + (15*x*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(b^6*Pi^3) - (x^5*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(b^2*Pi) - (15*\text{FresnelC}[b*x]^2)/(2*b^7*Pi^3) + (5*x^3*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*\text{Sin}[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*\text{Sin}[b^2*Pi*x^2])/(4*b^3*Pi^2)$

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :- Simp[(x^(m-1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m-1)/(2*d), Int[x^(m-2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m-1)*Cos[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x) + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x) - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.)], x_Symbol] := Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] := Simp[x/2, x] - Simp[Sin[2*c
+ d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :=
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int x^6 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{5 \int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx}{b^2\pi} + \frac{\int x^5 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
&= -\frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{5x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{15 \int x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} - 5 \\
&= \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{5x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{15 \int \cos}{b^4\pi^2} \\
&= \frac{x^6}{12b\pi} + \frac{5x^2 \cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{5x^3 C(bx)}{b^4\pi^2} \\
&= -\frac{15x^2}{4b^5\pi^3} + \frac{x^6}{12b\pi} + \frac{7x^2 \cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} \\
&= -\frac{15x^2}{4b^5\pi^3} + \frac{x^6}{12b\pi} + \frac{7x^2 \cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi}
\end{aligned}$$

Mathematica [A] time = 0.0092096, size = 185, normalized size = 1.

$$\frac{5x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{15x \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} - \frac{15 \text{FresnelC}(bx)}{2\pi^3 b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

```
[Out] (-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^5*Cos[(b^2*Pi
```

$x^2)/2] * \text{FresnelC}[b*x]) / (b^2 * \text{Pi}) - (15 * \text{FresnelC}[b*x]^2) / (2 * b^7 * \text{Pi}^3) + (5 * x^3 * \text{FresnelC}[b*x] * \text{Sin}[(b^2 * \text{Pi} * x^2) / 2]) / (b^4 * \text{Pi}^2) - (11 * \text{Sin}[b^2 * \text{Pi} * x^2]) / (2 * b^7 * \text{Pi}^4) + (x^4 * \text{Sin}[b^2 * \text{Pi} * x^2]) / (4 * b^3 * \text{Pi}^2)$

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int x^6 \text{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)`

[Out] `int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")`

[Out] `integrate(x^6*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^6 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(x^6*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [A] time = 123.898, size = 264, normalized size = 1.43

$$\left\{ \begin{array}{l} \frac{x^6 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} + \frac{x^6 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} - \frac{x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} + \frac{x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{5x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi^2 b^4} - \frac{11x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^3 b^5} - \frac{2x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^3 b^5} \\ 0 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*fresnelc(b*x)*sin(1/2*b**2*pi*x**2), x)

[Out] Piecewise((x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) + x**6*cos(pi*b**2*x**2/2)**2/(12*pi*b) - x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + x**4*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x)/(pi**2*b**4) - 11*x**2*sin(pi*b**2*x**2/2)**2/(2*pi**3*b**5) - 2*x**2*cos(pi*b**2*x**2/2)**2/(pi**3*b**5) + 15*x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi**4*b**7) - 15*fresnelc(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*fresnelc(b*x)*sin(1/2*b^2*pi*x^2), x, algorithm="giac")

[Out] integrate(x^6*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)

3.203 $\int x^5 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=167

$$\frac{4x^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^4 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{8 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} - \frac{43 \mathbf{FresnelC}(\sqrt{2}bx)}{8\sqrt{2}\pi^3 b^6}$$

[Out] $(-4*x)/(b^5*\text{Pi}^3) + x^5/(10*b*\text{Pi}) + (11*x*\text{Cos}[b^2*\text{Pi}*x^2])/(8*b^5*\text{Pi}^3) + (8*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(b^6*\text{Pi}^3) - (x^4*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(b^2*\text{Pi}) - (43*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(8*\text{Sqrt}[2]*b^6*\text{Pi}^3) + (4*x^2*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^4*\text{Pi}^2) + (x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2)$

Rubi [A] time = 0.174375, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6463, 6455, 6461, 3358, 3352, 3385, 3392, 30, 3386}

$$\frac{4x^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^4 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{8 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} - \frac{43 \mathbf{FresnelC}(\sqrt{2}bx)}{8\sqrt{2}\pi^3 b^6}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^5*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2], x]$

[Out] $(-4*x)/(b^5*\text{Pi}^3) + x^5/(10*b*\text{Pi}) + (11*x*\text{Cos}[b^2*\text{Pi}*x^2])/(8*b^5*\text{Pi}^3) + (8*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(b^6*\text{Pi}^3) - (x^4*\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(b^2*\text{Pi}) - (43*\text{FresnelC}[\text{Sqrt}[2]*b*x])/(8*\text{Sqrt}[2]*b^6*\text{Pi}^3) + (4*x^2*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2])/(b^4*\text{Pi}^2) + (x^3*\text{Sin}[b^2*\text{Pi}*x^2])/(4*b^3*\text{Pi}^2)$

Rule 6463

$\text{Int}[\text{FresnelC}[(b_)*(x_)]*(x_)^{(m_)}*\text{Sin}[(d_)*(x_)^2], x_Symbol] \rightarrow -\text{Simp}[(x^{(m-1)}*\text{Cos}[d*x^2]*\text{FresnelC}[b*x])/(2*d), x] + (\text{Dist}[(m-1)/(2*d), \text{Int}[x^{(m-2)}*\text{Cos}[d*x^2]*\text{FresnelC}[b*x], x], x] + \text{Dist}[b/(2*d), \text{Int}[x^{(m-1)}*\text{Cos}[d*x^2]^2, x], x]) /; \text{FreeQ}\{b, d\}, x \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4] \ \&\& \ \text{IGtQ}[m, 1]$

Rule 6455


```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)^(n_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] := -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] := Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] := Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
m, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eq[m, -1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] := Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
```

/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]

Rubi steps

$$\begin{aligned}
 \int x^5 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{4 \int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx}{b^2\pi} + \frac{\int x^4 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
 &= -\frac{x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{4x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{8 \int x C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} - \frac{2 \int x^4 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\
 &= \frac{x^5}{10b\pi} + \frac{x \cos(b^2\pi x^2)}{b^5\pi^3} + \frac{8 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{4x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
 &= \frac{x^5}{10b\pi} + \frac{11x \cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{8 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} - \frac{C(\sqrt{2}bx)}{\sqrt{2}b^6\pi^3} + \frac{11x^2 \cos(b^2\pi x^2)}{8b^5\pi^3} \\
 &= -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x \cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{8 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} - \frac{11x^2 \cos(b^2\pi x^2)}{8b^5\pi^3} \\
 &= -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x \cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{8 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^6\pi^3} - \frac{x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} - \frac{11x^2 \cos(b^2\pi x^2)}{8b^5\pi^3}
 \end{aligned}$$

Mathematica [A] time = 0.152275, size = 126, normalized size = 0.75

$$\frac{-80 \operatorname{FresnelC}(bx) \left((\pi^2 b^4 x^4 - 8) \cos\left(\frac{1}{2}\pi b^2 x^2\right) - 4\pi b^2 x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right) \right) + 2bx (4\pi^2 b^4 x^4 + 10\pi b^2 x^2 \sin(\pi b^2 x^2) + 55 \cos(\pi b^2 x^2))}{80\pi^3 b^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]

[Out] (-215*sqrt[2]*FresnelC[sqrt[2]*b*x] - 80*FresnelC[b*x]*((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(-160 + 4*b^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*b^6*Pi^3)

Maple [A] time = 0.079, size = 212, normalized size = 1.3

$$\frac{1}{b} \left(\frac{\text{FresnelC}(bx)}{b^5} \left(-\frac{x^4 b^4}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + 4 \frac{1}{\pi} \left(\frac{b^2 x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi} + 2 \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) \right) \right) - \frac{1}{b^5} \left(-\frac{1}{2 \pi^3} \left(\frac{\pi^2 b^5 x^5}{5} - 8 b \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)`

[Out] $(\text{FresnelC}(b*x)/b^5*(-1/\text{Pi}*b^4*x^4*\cos(1/2*b^2*\text{Pi}*x^2)+4/\text{Pi}*(1/\text{Pi}*b^2*x^2*\sin(1/2*b^2*\text{Pi}*x^2)+2/\text{Pi}^2*\cos(1/2*b^2*\text{Pi}*x^2)))-1/b^5*(-1/2/\text{Pi}^3*(1/5*\text{Pi}^2*b^5*x^5-8*b*x)+2/\text{Pi}^2*(-1/2/\text{Pi}*b*x*\cos(b^2*\text{Pi}*x^2)+1/4/\text{Pi}^2*(1/2)*\text{FresnelC}(b*x*x^2^(1/2)))-1/2/\text{Pi}^3*(1/2*\text{Pi}*b^3*x^3*\sin(b^2*\text{Pi}*x^2)-3/2*\text{Pi}*(-1/2/\text{Pi}*b*x*\cos(b^2*\text{Pi}*x^2)+1/4/\text{Pi}^2*(1/2)*\text{FresnelC}(b*x*x^2^(1/2)))-4*x^2^(1/2)*\text{FresnelC}(b*x*x^2^(1/2))))/b$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")`

[Out] `integrate(x^5*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^5 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(x^5*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)

[Out] Integral(x**5*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")

[Out] integrate(x^5*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)

3.204 $\int x^4 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=196

$$\frac{3ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, -\frac{1}{2}i\pi b^2 x^2\right)}{8\pi^2 b^3} + \frac{3ix^2 \text{HypergeometricPFQ}\left(\{1, 1\}, \left\{\frac{3}{2}, 2\right\}, \frac{1}{2}i\pi b^2 x^2\right)}{8\pi^2 b^3} - \frac{3 \text{FresnelC}(bx)}{\pi^2 b^4}$$

```
[Out] x^4/(8*b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) - (x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) - (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^3*Pi^2) + (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^3*Pi^2) + (3*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^2*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rubi [A] time = 0.15681, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6463, 6455, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$\frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2 b^3} + \frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8\pi^2 b^3} - \frac{3 \text{FresnelC}(bx) S(bx)}{2\pi^2 b^5} + \frac{3x \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

```
[Out] x^4/(8*b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) - (x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) - (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^3*Pi^2) + (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^3*Pi^2) + (3*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^2*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :-> -Simp[(x^(m-1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m-1)/(2*d), Int[x^(m-2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m-1)*Cos[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] := Simp[(b*Pi*FresnelC[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(I*d*x^2)])]/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, I*d*x^2)])]/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_)^(m_.))*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] := Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
```

eQ[m, -1]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int x^4 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{3 \int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx}{b^2\pi} + \frac{\int x^3 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\ &= -\frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{3xC(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3 \int C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} - \frac{3 \int x^3 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\ &= -\frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} - \frac{3C(bx)S(bx)}{2b^5\pi^2} - \frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} \\ &= \frac{x^4}{8b\pi} + \frac{3 \cos(b^2\pi x^2)}{4b^5\pi^3} - \frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} - \frac{3C(bx)S(bx)}{2b^5\pi^2} - \frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} \\ &= \frac{x^4}{8b\pi} + \frac{\cos(b^2\pi x^2)}{b^5\pi^3} - \frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} - \frac{3C(bx)S(bx)}{2b^5\pi^2} - \frac{3ix^2 {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} \end{aligned}$$

Mathematica [F] time = 0.0371357, size = 0, normalized size = 0.

$$\int x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]

[Out] Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int x^4 \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)`

[Out] `int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")`

[Out] `integrate(x^4*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(x^4*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)`

[Out] `Integral(x**4*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")`

[Out] `integrate(x^4*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

3.205 $\int x^3 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=109

$$\frac{2\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^2 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{5S(\sqrt{2}bx)}{4\sqrt{2}\pi^2 b^4} + \frac{x \sin(\pi b^2 x^2)}{4\pi^2 b^3} + \frac{x^3}{6\pi b}$$

[Out] $x^3/(6*b*Pi) - (x^2*\text{Cos}[(b^2*Pi*x^2)/2]*\mathbf{FresnelC}[b*x])/(b^2*Pi) - (5*\mathbf{FresnelS}[\text{Sqrt}[2]*b*x])/(4*\text{Sqrt}[2]*b^4*Pi^2) + (2*\mathbf{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x*\text{Sin}[b^2*Pi*x^2])/(4*b^3*Pi^2)$

Rubi [A] time = 0.0893288, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {6463, 6453, 3351, 3392, 30, 3386}

$$\frac{2\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^2 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{5S(\sqrt{2}bx)}{4\sqrt{2}\pi^2 b^4} + \frac{x \sin(\pi b^2 x^2)}{4\pi^2 b^3} + \frac{x^3}{6\pi b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*\mathbf{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2], x]$

[Out] $x^3/(6*b*Pi) - (x^2*\text{Cos}[(b^2*Pi*x^2)/2]*\mathbf{FresnelC}[b*x])/(b^2*Pi) - (5*\mathbf{FresnelS}[\text{Sqrt}[2]*b*x])/(4*\text{Sqrt}[2]*b^4*Pi^2) + (2*\mathbf{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x*\text{Sin}[b^2*Pi*x^2])/(4*b^3*Pi^2)$

Rule 6463

$\text{Int}[\mathbf{FresnelC}[(b_.)*(x_)]*(x_)^(m_)*\text{Sin}[(d_.)*(x_)^2], x_Symbol] \rightarrow -\text{Simp}[(x)^(m-1)*\text{Cos}[d*x^2]*\mathbf{FresnelC}[b*x]/(2*d), x] + (\text{Dist}[(m-1)/(2*d), \text{Int}[x^(m-2)*\text{Cos}[d*x^2]*\mathbf{FresnelC}[b*x], x], x] + \text{Dist}[b/(2*d), \text{Int}[x^(m-1)*\text{Cos}[d*x^2]^2, x], x]) /; \text{FreeQ}\{b, d\}, x \&\& \text{EqQ}[d^2, (Pi^2*b^4)/4] \&\& \text{IGtQ}[m, 1]$

Rule 6453

$\text{Int}[\text{Cos}[(d_.)*(x_)^2]*\mathbf{FresnelC}[(b_.)*(x_)]*(x_), x_Symbol] \rightarrow \text{Simp}[(\text{Sin}[d*x^2]*\mathbf{FresnelC}[b*x])/(2*d), x] - \text{Dist}[b/(4*d), \text{Int}[\text{Sin}[2*d*x^2], x], x] /; \text{FreeQ}\{b, d\}, x \&\& \text{EqQ}[d^2, (Pi^2*b^4)/4]$

Rule 3351

Int[Sin[(d_.)*(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3392

Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] := Dist[1/2, Int[x^m, x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b, m, n}, x]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3386

Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rubi steps

$$\begin{aligned} \int x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{2 \int x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx}{b^2\pi} + \frac{\int x^2 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\ &= -\frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{2C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{\int \sin(b^2\pi x^2) dx}{b^3\pi^2} + \frac{\int x^2 dx}{2b\pi} + \frac{\int x^2 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\ &= \frac{x^3}{6b\pi} - \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} - \frac{S(\sqrt{2}bx)}{\sqrt{2}b^4\pi^2} + \frac{2C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x \sin(b^2\pi x^2)}{4b^3\pi^2} - \frac{\int \sin(b^2\pi x^2) dx}{b^3\pi^2} \\ &= \frac{x^3}{6b\pi} - \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} - \frac{5S(\sqrt{2}bx)}{4\sqrt{2}b^4\pi^2} + \frac{2C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x \sin(b^2\pi x^2)}{4b^3\pi^2} \end{aligned}$$

Mathematica [A] time = 0.0841006, size = 90, normalized size = 0.83

$$\frac{-24\text{FresnelC}(bx) \left(\pi b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) - 2 \sin\left(\frac{1}{2}\pi b^2 x^2\right) \right) + 4\pi b^3 x^3 + 6bx \sin(\pi b^2 x^2) - 15\sqrt{2}S(\sqrt{2}bx)}{24\pi^2 b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (4*b^3*Pi*x^3 - 15*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 24*FresnelC[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]) + 6*b*x*Sin[b^2*Pi*x^2])/(24*b^4*Pi^2)
```

Maple [A] time = 0.08, size = 120, normalized size = 1.1

$$\frac{1}{b} \left(\frac{\text{FresnelC}(bx)}{b^3} \left(-\frac{b^2 x^2}{\pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + 2 \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{\pi^2} \right) - \frac{1}{b^3} \left(\frac{\sqrt{2} \text{FresnelS}(bx\sqrt{2})}{2\pi^2} - \frac{x^3 b^3}{6\pi} - \frac{1}{2\pi} \left(\frac{bx \sin(b^2 \pi x^2)}{2\pi} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelC(b*x)/b^3*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))-1/6/Pi*b^3*x^3-1/2/Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(x^3*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)`

[Out] `Integral(x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")`

[Out] `integrate(x^3*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

3.206 $\int x^2 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=74

$$-\frac{x \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\mathbf{FresnelC}(bx)^2}{2\pi b^3} + \frac{\sin(\pi b^2 x^2)}{4\pi^2 b^3} + \frac{x^2}{4\pi b}$$

[Out] $x^2/(4*b*Pi) - (x*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(b^2*Pi) + \text{FresnelC}[b*x]^2/(2*b^3*Pi) + \text{Sin}[b^2*Pi*x^2]/(4*b^3*Pi^2)$

Rubi [A] time = 0.0570508, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6463, 6441, 30, 3380, 2634}

$$-\frac{x \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\mathbf{FresnelC}(bx)^2}{2\pi b^3} + \frac{\sin(\pi b^2 x^2)}{4\pi^2 b^3} + \frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*\text{FresnelC}[b*x]*\text{Sin}[(b^2*Pi*x^2)/2], x]$

[Out] $x^2/(4*b*Pi) - (x*\text{Cos}[(b^2*Pi*x^2)/2]*\text{FresnelC}[b*x])/(b^2*Pi) + \text{FresnelC}[b*x]^2/(2*b^3*Pi) + \text{Sin}[b^2*Pi*x^2]/(4*b^3*Pi^2)$

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.)], x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 2634

Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] := Simp[x/2, x] - Simp[Sin[2*c + d*x]/(2*d), x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= -\frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx}{b^2\pi} + \frac{\int x \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b\pi} \\ &= -\frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{\text{Subst}\left(\int x dx, x, C(bx)\right)}{b^3\pi} + \frac{\text{Subst}\left(\int \cos^2\left(\frac{1}{2}b^2\pi x\right) dx, x, x^2\right)}{2b\pi} \\ &= \frac{x^2}{4b\pi} - \frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{b^2\pi} + \frac{C(bx)^2}{2b^3\pi} + \frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} \end{aligned}$$

Mathematica [A] time = 0.007526, size = 74, normalized size = 1.

$$-\frac{x \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\text{FresnelC}(bx)^2}{2\pi b^3} + \frac{\sin\left(\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]

[Out] x^2/(4*b*Pi) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[b*x]^2/(2*b^3*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x^2 \text{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)`

[Out] `int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")`

[Out] `integrate(x^2*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(x^2*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [A] time = 4.37083, size = 114, normalized size = 1.54

$$\begin{cases} \frac{x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{4\pi b} + \frac{x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{4\pi b} - \frac{x \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} + \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{C^2(bx)}{2\pi b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) + x**2*cos(pi*b**2*x**2/2)*
**2/(4*pi*b) - x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + sin(pi*b**2*x
**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + fresnelc(b*x)**2/(2*pi*b**3), N
e(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.207 $\int x \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=60

$$-\frac{\mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\mathbf{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}\pi b^2} + \frac{x}{2\pi b}$$

[Out] $x/(2*b*\text{Pi}) - (\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(b^2*\text{Pi}) + \text{FresnelC}[\text{Sqrt}[2]*b*x]/(2*\text{Sqrt}[2]*b^2*\text{Pi})$

Rubi [A] time = 0.0317032, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6461, 3358, 3352}

$$-\frac{\mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\mathbf{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}\pi b^2} + \frac{x}{2\pi b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\text{Pi}*x^2)/2], x]$

[Out] $x/(2*b*\text{Pi}) - (\text{Cos}[(b^2*\text{Pi}*x^2)/2]*\text{FresnelC}[b*x])/(b^2*\text{Pi}) + \text{FresnelC}[\text{Sqrt}[2]*b*x]/(2*\text{Sqrt}[2]*b^2*\text{Pi})$

Rule 6461

$\text{Int}[\text{FresnelC}[(b_)*(x_)]*(x_)*\text{Sin}[(d_)*(x_)^2], x_Symbol] \rightarrow -\text{Simp}[(\text{Cos}[d*x^2]*\text{FresnelC}[b*x])/(2*d), x] + \text{Dist}[b/(2*d), \text{Int}[\text{Cos}[d*x^2]^2, x], x] /; \text{FreeQ}\{b, d, x\} \ \&\& \ \text{EqQ}[d^2, (\text{Pi}^2*b^4)/4]$

Rule 3358

$\text{Int}[(a_ + \text{Cos}[c_ + (d_)*((e_ + (f_)*(x_))^n])*(b_)]^{(p_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(a + b*\text{Cos}[c + d*(e + f*x)^n])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{IGtQ}[p, 1] \ \&\& \ \text{IGtQ}[n, 1]$

Rule 3352

$\text{Int}[\text{Cos}[(d_)*((e_ + (f_)*(x_))^2)], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f, x\}$

Rubi steps

$$\begin{aligned}
\int x C(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx &= -\frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{b^2 \pi} + \frac{\int \cos^2\left(\frac{1}{2} b^2 \pi x^2\right) dx}{b \pi} \\
&= -\frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{b^2 \pi} + \frac{\int\left(\frac{1}{2} + \frac{1}{2} \cos\left(b^2 \pi x^2\right)\right) dx}{b \pi} \\
&= \frac{x}{2 b \pi} - \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{b^2 \pi} + \frac{\int \cos\left(b^2 \pi x^2\right) dx}{2 b \pi} \\
&= \frac{x}{2 b \pi} - \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right) C(bx)}{b^2 \pi} + \frac{C\left(\sqrt{2} b x\right)}{2 \sqrt{2} b^2 \pi}
\end{aligned}$$

Mathematica [A] time = 0.0263026, size = 48, normalized size = 0.8

$$\frac{-4 \operatorname{FresnelC}(b x) \cos\left(\frac{1}{2} \pi b^2 x^2\right) + \sqrt{2} \operatorname{FresnelC}\left(\sqrt{2} b x\right) + 2 b x}{4 \pi b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] (2*b*x - 4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x] + Sqrt[2]*FresnelC[Sqrt[2]*b*x])/ (4*b^2*Pi)

Maple [A] time = 0.061, size = 52, normalized size = 0.9

$$\frac{1}{b} \left(-\frac{\operatorname{FresnelC}(b x)}{b \pi} \cos\left(\frac{b^2 \pi x^2}{2}\right) + \frac{1}{b \pi} \left(\frac{b x}{2} + \frac{\sqrt{2} \operatorname{FresnelC}(b x \sqrt{2})}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)

[Out] (-FresnelC(b*x)/b/Pi*cos(1/2*b^2*Pi*x^2)+1/b/Pi*(1/2*b*x+1/4*2^(1/2)*FresnelC(b*x*2^(1/2))))/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(x*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x \operatorname{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")

[Out] integral(x*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)

[Out] Integral(x*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.208 $\int \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=80

$$\frac{1}{8}ibx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{2}i\pi b^2x^2\right)-\frac{1}{8}ibx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},\frac{1}{2}i\pi b^2x^2\right)+\frac{\text{FresnelC}(bx)S(bx)}{2b}$$

[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]

Rubi [A] time = 0.0157251, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6447}

$$\frac{1}{8}ibx^2 {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2 {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)S(bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]

Rule 6447

Int[FresnelC[(b_.)*(x_.)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*FresnelC[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rubi steps

$$\int C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{C(bx)S(bx)}{2b} + \frac{1}{8}ibx^2 {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{8}ibx^2 {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.0153241, size = 0, normalized size = 0.

$$\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]

[Out] Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2), x)

[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2), x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")`

[Out] `integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)`

[Out] `Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")`

[Out] `integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)`

$$3.209 \quad \int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right)$$

[Out] Unintegrable[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

Rubi [A] time = 0.0172122, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

[Out] Defer[Int] [(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

Rubi steps

$$\int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx = \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx$$

Mathematica [A] time = 0.0320878, size = 0, normalized size = 0.

$$\int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]

Maple [A] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)

[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x,x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x, x)

$$3.210 \quad \int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx$$

Optimal. Leaf size=48

$$-\frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x} + \frac{1}{4}b\mathbf{Si}(b^2\pi x^2) + \frac{1}{2}\pi b\mathbf{FresnelC}(bx)^2$$

[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIntegral[b^2*Pi*x^2])/4

Rubi [A] time = 0.0427923, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {6465, 6441, 30, 3375}

$$-\frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x} + \frac{1}{4}b\mathbf{Si}(b^2\pi x^2) + \frac{1}{2}\pi b\mathbf{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]

[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIntegral[b^2*Pi*x^2])/4

Rule 6465

Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IntegerQ[m, -1]

Rule 6441

Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.)], x_Symbol] :> Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3375

Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx &= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} + \frac{1}{2}b \int \frac{\sin(b^2\pi x^2)}{x} dx + (b^2\pi) \int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx) dx \\ &= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} + \frac{1}{4}b\text{Si}(b^2\pi x^2) + (b\pi) \text{Subst}\left(\int x dx, x, C(bx)\right) \\ &= \frac{1}{2}b\pi C(bx)^2 - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} + \frac{1}{4}b\text{Si}(b^2\pi x^2) \end{aligned}$$

Mathematica [A] time = 0.0050088, size = 48, normalized size = 1.

$$-\frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x} + \frac{1}{4}b\text{Si}(b^2\pi x^2) + \frac{1}{2}\pi b\text{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]

[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIntegral[b^2*Pi*x^2])/4

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^2} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)`

[Out] `int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")`

[Out] `integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")`

[Out] `integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)`

[Out] `Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")`

[Out] `integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)`

$$3.211 \quad \int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx$$

Optimal. Leaf size=93

$$\frac{1}{2}\pi b^2 \mathbf{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) - \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2x^2} + \frac{\pi b^2 \mathbf{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}} - \frac{b \sin(\pi b^2 x^2)}{4x}$$

[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) + (b^2*Pi*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/2

Rubi [A] time = 0.0563796, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]

[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) + (b^2*Pi*Defer[Int] [(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/2

Rubi steps

$$\begin{aligned} \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx &= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} + \frac{1}{4}b \int \frac{\sin(b^2\pi x^2)}{x^2} dx + \frac{1}{2}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x} dx \\ &= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} - \frac{b \sin(b^2\pi x^2)}{4x} + \frac{1}{2}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x} dx + \frac{1}{2}(b^3\pi) \int \cos \\ &= \frac{b^2\pi C(\sqrt{2}bx)}{2\sqrt{2}} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} - \frac{b \sin(b^2\pi x^2)}{4x} + \frac{1}{2}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x} dx \end{aligned}$$

Mathematica [A] time = 0.0330993, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]

[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)

[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)

$$3.212 \quad \int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx$$

Optimal. Leaf size=88

$$\frac{1}{3}\pi b^2 \text{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right) + \frac{1}{12}\pi b^3 \text{CosIntegral}(\pi b^2 x^2) - \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^3} - \dots$$

[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) + (b^2*Pi*Unintegrable[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/3

Rubi [A] time = 0.0893682, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4, x]

[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) + (b^2*Pi*Defer[Int] [(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/3

Rubi steps

$$\begin{aligned}
\int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx &= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{6}b \int \frac{\sin(b^2\pi x^2)}{x^3} dx + \frac{1}{3}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^2} dx \\
&= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{12}b \operatorname{Subst}\left(\int \frac{\sin(b^2\pi x)}{x^2} dx, x, x^2\right) + \frac{1}{3}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^2} dx \\
&= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} - \frac{b \sin(b^2\pi x^2)}{12x^2} + \frac{1}{3}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^2} dx + \frac{1}{12}(b^3\pi) \operatorname{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^2} dx, x, x^2\right) \\
&= \frac{1}{12}b^3\pi \operatorname{Ci}(b^2\pi x^2) - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} - \frac{b \sin(b^2\pi x^2)}{12x^2} + \frac{1}{3}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^2} dx
\end{aligned}$$

Mathematica [A] time = 0.0318848, size = 0, normalized size = 0.

$$\int \frac{\operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]

[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4, x]

Maple [A] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{\operatorname{FresnelC}(bx)}{x^4} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)

[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

$$3.213 \quad \int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx$$

Optimal. Leaf size=155

$$-\frac{1}{8}\pi^2 b^4 \text{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) - \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4x^4} - \frac{\pi b^2 \mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^2}$$

[Out] $-(b^3\pi)/(16*x) - (7*b^3\pi*\text{Cos}[b^2*\pi*x^2])/(48*x) - (b^2*\pi*\text{Cos}[(b^2*\pi*x^2)/2]*\text{FresnelC}[b*x])/(8*x^2) - (7*b^4*\pi^2*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(24*\text{Sqrt}[2]) - (\text{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(4*x^4) - (b*\text{Sin}[b^2*\pi*x^2])/(24*x^3) - (b^4*\pi^2*\text{Unintegrable}[(\text{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/x, x])/8$

Rubi [A] time = 0.119052, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/x^5, x]$

[Out] $-(b^3\pi)/(16*x) - (7*b^3\pi*\text{Cos}[b^2*\pi*x^2])/(48*x) - (b^2*\pi*\text{Cos}[(b^2*\pi*x^2)/2]*\text{FresnelC}[b*x])/(8*x^2) - (7*b^4*\pi^2*\text{FresnelS}[\text{Sqrt}[2]*b*x])/(24*\text{Sqrt}[2]) - (\text{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(4*x^4) - (b*\text{Sin}[b^2*\pi*x^2])/(24*x^3) - (b^4*\pi^2*\text{Defer}[\text{Int}[(\text{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/x, x])/8$

Rubi steps

$$\begin{aligned}
\int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx &= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} + \frac{1}{8}b \int \frac{\sin(b^2\pi x^2)}{x^4} dx + \frac{1}{4}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^3} dx \\
&= \frac{b^3\pi}{16x} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{8x^2} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} - \frac{b \sin(b^2\pi x^2)}{24x^3} + \frac{1}{16}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx \\
&= \frac{b^3\pi}{16x} - \frac{7b^3\pi \cos(b^2\pi x^2)}{48x} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{8x^2} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} - \frac{b \sin(b^2\pi x^2)}{24x^3} \\
&= \frac{b^3\pi}{16x} - \frac{7b^3\pi \cos(b^2\pi x^2)}{48x} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{8x^2} - \frac{7b^4\pi^2 S(\sqrt{2}bx)}{24\sqrt{2}} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.0324145, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]

[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)

[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")`

[Out] `integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")`

[Out] `integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)`

[Out] `Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

$$3.214 \quad \int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx$$

Optimal. Leaf size=163

$$\frac{\pi^2 b^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x} - \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5} - \frac{\pi b^2 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3} - \frac{1}{30} \pi^3 b^5 \text{FresnelC}$$

```
[Out] -(b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x) - (b*SIN[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2*Pi*x^2])/120
```

Rubi [A] time = 0.218457, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6465, 6457, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi^2 b^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x} - \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5} - \frac{\pi b^2 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3} - \frac{1}{30} \pi^3 b^5 \text{FresnelC}$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -(b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x) - (b*SIN[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2*Pi*x^2])/120
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^(m+1)*Sin[d*x^2]*FresnelC[b*x])/(m+1), x] + (-Dist[(2*d)/(m+1), Int[x^(m+2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m+1)), Int[x^(m+1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IntegerQ[m, -1]
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(m_)], x_Symbol] := Simp[(x^(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)^(n_.)], x_Symbol] := Dist[(Pi*b)/(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_)*((a_) + (b_)*Sin[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx &= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{10}b \int \frac{\sin(b^2\pi x^2)}{x^5} dx + \frac{1}{5}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^4} dx \\
&= -\frac{b^3\pi}{60x^2} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{15x^3} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{20}b \operatorname{Subst}\left(\int \frac{\sin(b^2\pi x)}{x^3} dx, x, x^2\right) \\
&= -\frac{b^3\pi}{60x^2} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{15x^3} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x} - \frac{b^5\pi^3 C(bx)^2}{30} \\
&= -\frac{b^3\pi}{60x^2} - \frac{b^3\pi \cos(b^2\pi x^2)}{24x^2} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{15x^3} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x} - \frac{b^5\pi^3 C(bx)^2}{30} \\
&= -\frac{b^3\pi}{60x^2} - \frac{b^3\pi \cos(b^2\pi x^2)}{24x^2} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{15x^3} - \frac{1}{30}b^5\pi^3 C(bx)^2 - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}
\end{aligned}$$

Mathematica [A] time = 0.0105432, size = 163, normalized size = 1.

$$\frac{\pi^2 b^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x} - \frac{\operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5} - \frac{\pi b^2 \operatorname{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3} - \frac{1}{30} \pi^3 b^5 \operatorname{FresnelC}(bx)^2 - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]

[Out] $-(b^3\pi)/(60x^2) - (b^3\pi\cos[b^2\pi x^2])/(24x^2) - (b^2\pi\cos[(b^2\pi x^2)/2]*\operatorname{FresnelC}[b*x])/(15x^3) - (b^5\pi^3\operatorname{FresnelC}[b*x]^2)/30 - (\operatorname{FresnelC}[b*x]*\sin[(b^2\pi x^2)/2])/(5x^5) + (b^4\pi^2\operatorname{FresnelC}[b*x]*\sin[(b^2\pi x^2)/2])/(15x) - (b*\sin[b^2\pi x^2])/(40x^4) - (7*b^5\pi^2*\operatorname{SinIntegral}[b^2\pi x^2])/120$

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^6} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)

[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)`

[Out] `Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")`

[Out] `integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)`

$$3.215 \quad \int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx$$

Optimal. Leaf size=230

$$-\frac{1}{48}\pi^3 b^6 \text{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) + \frac{\pi^2 b^4 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{48x^2} - \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^6}$$

[Out] $-(b^3\pi)/(144*x^3) - (13*b^3*\pi*\cos[b^2*\pi*x^2])/(720*x^3) - (b^2*\pi*\cos[(b^2*\pi*x^2)/2]*\mathbf{FresnelC}[b*x])/(24*x^4) - (7*b^6*\pi^3*\mathbf{FresnelC}[\text{Sqrt}[2]*b*x])/(144*\text{Sqrt}[2]) - (\text{Sqrt}[2]*b^6*\pi^3*\mathbf{FresnelC}[\text{Sqrt}[2]*b*x])/45 - (\mathbf{FresnelC}[b*x]*\sin[(b^2*\pi*x^2)/2])/(6*x^6) + (b^4*\pi^2*\mathbf{FresnelC}[b*x]*\sin[(b^2*\pi*x^2)/2])/(48*x^2) - (b*\sin[b^2*\pi*x^2])/(60*x^5) + (67*b^5*\pi^2*\sin[b^2*\pi*x^2])/(1440*x) - (b^6*\pi^3*\text{Unintegrable}[(\cos[(b^2*\pi*x^2)/2]*\mathbf{FresnelC}[b*x])/x, x])/48$

Rubi [A] time = 0.199967, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx$$

Verification is Not applicable to the result.

[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]

[Out] $-(b^3\pi)/(144*x^3) - (13*b^3*\pi*\cos[b^2*\pi*x^2])/(720*x^3) - (b^2*\pi*\cos[(b^2*\pi*x^2)/2]*\mathbf{FresnelC}[b*x])/(24*x^4) - (7*b^6*\pi^3*\mathbf{FresnelC}[\text{Sqrt}[2]*b*x])/(144*\text{Sqrt}[2]) - (\text{Sqrt}[2]*b^6*\pi^3*\mathbf{FresnelC}[\text{Sqrt}[2]*b*x])/45 - (\mathbf{FresnelC}[b*x]*\sin[(b^2*\pi*x^2)/2])/(6*x^6) + (b^4*\pi^2*\mathbf{FresnelC}[b*x]*\sin[(b^2*\pi*x^2)/2])/(48*x^2) - (b*\sin[b^2*\pi*x^2])/(60*x^5) + (67*b^5*\pi^2*\sin[b^2*\pi*x^2])/(1440*x) - (b^6*\pi^3*\text{Defer[Int]}[(\cos[(b^2*\pi*x^2)/2]*\mathbf{FresnelC}[b*x])/x, x])/48$

Rubi steps

$$\begin{aligned}
\int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx &= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{1}{12}b \int \frac{\sin(b^2\pi x^2)}{x^6} dx + \frac{1}{6}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^5} dx \\
&= -\frac{b^3\pi}{144x^3} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{24x^4} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} - \frac{b \sin(b^2\pi x^2)}{60x^5} + \frac{1}{48}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^3} dx \\
&= -\frac{b^3\pi}{144x^3} - \frac{13b^3\pi \cos(b^2\pi x^2)}{720x^3} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{24x^4} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{b^4\pi^2 C(bx)}{48x^3} \\
&= -\frac{b^3\pi}{144x^3} - \frac{13b^3\pi \cos(b^2\pi x^2)}{720x^3} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{24x^4} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{b^4\pi^2 C(bx)}{48x^3} \\
&= -\frac{b^3\pi}{144x^3} - \frac{13b^3\pi \cos(b^2\pi x^2)}{720x^3} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{24x^4} - \frac{7b^6\pi^3 C(\sqrt{2}bx)}{144\sqrt{2}} - \frac{1}{45}\sqrt{2}b^6\pi^3 C(\sqrt{2}bx)
\end{aligned}$$

Mathematica [A] time = 0.0313279, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]

[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^7} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)

[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

$$3.216 \quad \int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx$$

Optimal. Leaf size=201

$$-\frac{1}{105}\pi^3 b^6 \text{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2}, x\right) - \frac{1}{84}\pi^3 b^7 \text{CosIntegral}(\pi b^2 x^2) + \frac{\pi^2 b^4 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{105 x^3}$$

[Out] $-(b^3 \pi)/(280 x^4) - (b^3 \pi \cos[b^2 \pi x^2])/(105 x^4) - (b^7 \pi^3 \text{CosIntegral}[b^2 \pi x^2])/84 - (b^2 \pi \cos[(b^2 \pi x^2)/2] \text{FresnelC}[b x])/(35 x^5) - (\text{FresnelC}[b x] \sin[(b^2 \pi x^2)/2])/(7 x^7) + (b^4 \pi^2 \text{FresnelC}[b x] \sin[(b^2 \pi x^2)/2])/(105 x^3) - (b \sin[b^2 \pi x^2])/(84 x^6) + (b^5 \pi^2 \sin[b^2 \pi x^2])/(84 x^2) - (b^6 \pi^3 \text{Unintegrable}[(\cos[(b^2 \pi x^2)/2] \text{FresnelC}[b x])/x^2, x])/105$

Rubi [A] time = 0.314121, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{FresnelC}[b x] \sin[(b^2 \pi x^2)/2])/x^8, x]$

[Out] $-(b^3 \pi)/(280 x^4) - (b^3 \pi \cos[b^2 \pi x^2])/(105 x^4) - (b^7 \pi^3 \text{CosIntegral}[b^2 \pi x^2])/84 - (b^2 \pi \cos[(b^2 \pi x^2)/2] \text{FresnelC}[b x])/(35 x^5) - (\text{FresnelC}[b x] \sin[(b^2 \pi x^2)/2])/(7 x^7) + (b^4 \pi^2 \text{FresnelC}[b x] \sin[(b^2 \pi x^2)/2])/(105 x^3) - (b \sin[b^2 \pi x^2])/(84 x^6) + (b^5 \pi^2 \sin[b^2 \pi x^2])/(84 x^2) - (b^6 \pi^3 \text{Defer}[\text{Int}[(\cos[(b^2 \pi x^2)/2] \text{FresnelC}[b x])/x^2, x])/105$

Rubi steps

$$\begin{aligned}
\int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx &= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{1}{14}b \int \frac{\sin(b^2\pi x^2)}{x^7} dx + \frac{1}{7}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^6} dx \\
&= -\frac{b^3\pi}{280x^4} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{35x^5} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{1}{28}b \text{Subst}\left(\int \frac{\sin(b^2\pi x)}{x^4} dx, \sqrt{2}bx\right) \\
&= -\frac{b^3\pi}{280x^4} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{35x^5} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3} - \frac{b^4\pi^2 C(bx)}{105x^3} \\
&= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi \cos(b^2\pi x^2)}{105x^4} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{35x^5} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2 C(bx)}{105x^3} \\
&= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi \cos(b^2\pi x^2)}{105x^4} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{35x^5} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2 C(bx)}{105x^3} \\
&= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi \cos(b^2\pi x^2)}{105x^4} - \frac{1}{84}b^7\pi^3 \text{Ci}(b^2\pi x^2) - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{35x^5} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}
\end{aligned}$$

Mathematica [A] time = 0.0328026, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]

[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8, x]

Maple [A] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)

[Out] `int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")`

[Out] `integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")`

[Out] `integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

$$3.217 \quad \int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx$$

Optimal. Leaf size=270

$$\frac{1}{384}\pi^4 b^8 \text{Unintegrable}\left(\frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) + \frac{\pi^2 b^4 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{192x^4} - \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8}$$

[Out] $-(b^3\pi)/(480x^5) + (b^7\pi^3)/(768x) - (19b^3\pi\cos[b^2\pi x^2])/(3360x^5) + (853b^7\pi^3\cos[b^2\pi x^2])/(80640x) - (b^2\pi\cos[(b^2\pi x^2)/2]*\mathbf{FresnelC}[b*x])/(48x^6) + (b^6\pi^3\cos[(b^2\pi x^2)/2]*\mathbf{FresnelC}[b*x])/(384x^2) + (853b^8\pi^4*\mathbf{FresnelS}[\text{Sqrt}[2]*b*x])/(40320*\text{Sqrt}[2]) - (\mathbf{FresnelC}[b*x]*\sin[(b^2\pi x^2)/2])/(8x^8) + (b^4\pi^2*\mathbf{FresnelC}[b*x]*\sin[(b^2\pi x^2)/2])/(192x^4) - (b*\sin[b^2\pi x^2])/(112x^7) + (187b^5\pi^2*\sin[b^2\pi x^2])/(40320x^3) + (b^8\pi^4*\text{Unintegrable}[(\mathbf{FresnelC}[b*x]*\sin[(b^2\pi x^2)/2])/x, x])/384$

Rubi [A] time = 0.306279, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\mathbf{FresnelC}[b*x]*\sin[(b^2\pi x^2)/2])/x^9, x]$

[Out] $-(b^3\pi)/(480x^5) + (b^7\pi^3)/(768x) - (19b^3\pi\cos[b^2\pi x^2])/(3360x^5) + (853b^7\pi^3\cos[b^2\pi x^2])/(80640x) - (b^2\pi\cos[(b^2\pi x^2)/2]*\mathbf{FresnelC}[b*x])/(48x^6) + (b^6\pi^3\cos[(b^2\pi x^2)/2]*\mathbf{FresnelC}[b*x])/(384x^2) + (853b^8\pi^4*\mathbf{FresnelS}[\text{Sqrt}[2]*b*x])/(40320*\text{Sqrt}[2]) - (\mathbf{FresnelC}[b*x]*\sin[(b^2\pi x^2)/2])/(8x^8) + (b^4\pi^2*\mathbf{FresnelC}[b*x]*\sin[(b^2\pi x^2)/2])/(192x^4) - (b*\sin[b^2\pi x^2])/(112x^7) + (187b^5\pi^2*\sin[b^2\pi x^2])/(40320x^3) + (b^8\pi^4*\text{Defer}[\text{Int}[(\mathbf{FresnelC}[b*x]*\sin[(b^2\pi x^2)/2])/x, x])/384$

Rubi steps

$$\begin{aligned}
\int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx &= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{1}{16}b \int \frac{\sin(b^2\pi x^2)}{x^8} dx + \frac{1}{8}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^7} dx \\
&= -\frac{b^3\pi}{480x^5} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{48x^6} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} - \frac{b \sin(b^2\pi x^2)}{112x^7} + \frac{1}{96}(b^3\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^6} dx \\
&= -\frac{b^3\pi}{480x^5} - \frac{19b^3\pi \cos(b^2\pi x^2)}{3360x^5} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{48x^6} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{b^4\pi^2 C(bx)}{96x^6} \\
&= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi \cos(b^2\pi x^2)}{3360x^5} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{48x^6} + \frac{b^6\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{384x^2} \\
&= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi \cos(b^2\pi x^2)}{3360x^5} + \frac{853b^7\pi^3 \cos(b^2\pi x^2)}{80640x} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{48x^6} \\
&= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi \cos(b^2\pi x^2)}{3360x^5} + \frac{853b^7\pi^3 \cos(b^2\pi x^2)}{80640x} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{48x^6}
\end{aligned}$$

Mathematica [A] time = 0.0332526, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9} dx$$

Verification is Not applicable to the result.

[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]

[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^9} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)

[Out] `int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")`

[Out] `integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")`

[Out] `integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

$$3.218 \quad \int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}} dx$$

Optimal. Leaf size=278

$$-\frac{\pi^4 b^8 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{945x} + \frac{\pi^2 b^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{315x^5} - \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9} + \frac{\pi^3 b^6 \text{FresnelC}(bx)}{945x^9}$$

[Out] $-(b^3\pi)/(756*x^6) + (b^7\pi^3)/(3780*x^2) - (11*b^3*\pi*\text{Cos}[b^2*\pi*x^2])/(3024*x^6) + (5*b^7*\pi^3*\text{Cos}[b^2*\pi*x^2])/(2016*x^2) - (b^2*\pi*\text{Cos}[(b^2*\pi*x^2)/2]*\text{FresnelC}[b*x])/(63*x^7) + (b^6*\pi^3*\text{Cos}[(b^2*\pi*x^2)/2]*\text{FresnelC}[b*x])/(945*x^3) + (b^9*\pi^5*\text{FresnelC}[b*x]^2)/1890 - (\text{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(9*x^9) + (b^4*\pi^2*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(315*x^5) - (b^8*\pi^4*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(945*x) - (b*\text{Sin}[b^2*\pi*x^2])/(144*x^8) + (67*b^5*\pi^2*\text{Sin}[b^2*\pi*x^2])/(30240*x^4) + (83*b^9*\pi^4*\text{SinIntegral}[b^2*\pi*x^2])/30240$

Rubi [A] time = 0.520338, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {6465, 6457, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$-\frac{\pi^4 b^8 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{945x} + \frac{\pi^2 b^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{315x^5} - \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9} + \frac{\pi^3 b^6 \text{FresnelC}(bx)}{945x^9}$$

Antiderivative was successfully verified.

[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]

[Out] $-(b^3\pi)/(756*x^6) + (b^7\pi^3)/(3780*x^2) - (11*b^3*\pi*\text{Cos}[b^2*\pi*x^2])/(3024*x^6) + (5*b^7*\pi^3*\text{Cos}[b^2*\pi*x^2])/(2016*x^2) - (b^2*\pi*\text{Cos}[(b^2*\pi*x^2)/2]*\text{FresnelC}[b*x])/(63*x^7) + (b^6*\pi^3*\text{Cos}[(b^2*\pi*x^2)/2]*\text{FresnelC}[b*x])/(945*x^3) + (b^9*\pi^5*\text{FresnelC}[b*x]^2)/1890 - (\text{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(9*x^9) + (b^4*\pi^2*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(315*x^5) - (b^8*\pi^4*\text{FresnelC}[b*x]*\text{Sin}[(b^2*\pi*x^2)/2])/(945*x) - (b*\text{Sin}[b^2*\pi*x^2])/(144*x^8) + (67*b^5*\pi^2*\text{Sin}[b^2*\pi*x^2])/(30240*x^4) + (83*b^9*\pi^4*\text{SinIntegral}[b^2*\pi*x^2])/30240$

Rule 6465

Int[FresnelC[(b_.)*(x_)]*(x_)^(m)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^(m+1)*Sin[d*x^2]*FresnelC[b*x])/(m+1), x] + (-Dist[(2*d)/(m+1), Int[x

$$\int x^{m+2} \cos[d x^2] \operatorname{FresnelC}[b x], x] - \operatorname{Dist}[b/(2(m+1)), \int x^{m+1} \sin[2 d x^2], x] /; \operatorname{FreeQ}[\{b, d\}, x] \ \&\& \operatorname{EqQ}[d^2, (\pi^2 b^4)/4] \ \&\& \operatorname{LtQ}[m, -1]$$

Rule 6457

$$\operatorname{Int}[\operatorname{Cos}[(d \cdot) \cdot (x \cdot)^2] \operatorname{FresnelC}[(b \cdot) \cdot (x \cdot)] \cdot (x \cdot)^{(m \cdot)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m+1)} \operatorname{Cos}[d x^2] \operatorname{FresnelC}[b x]) / (m+1), x] + (\operatorname{Dist}[(2 d) / (m+1), \int x^{(m+2)} \sin[d x^2] \operatorname{FresnelC}[b x], x] - \operatorname{Dist}[b / (2(m+1)), \int x^{(m+1)} \operatorname{Cos}[2 d x^2], x] - \operatorname{Simp}[(b x^{(m+2)}) / (2(m+1)(m+2)), x]) /; \operatorname{FreeQ}[\{b, d\}, x] \ \&\& \operatorname{EqQ}[d^2, (\pi^2 b^4) / 4] \ \&\& \operatorname{LtQ}[m, -2]$$

Rule 6441

$$\operatorname{Int}[\operatorname{Cos}[(d \cdot) \cdot (x \cdot)^2] \operatorname{FresnelC}[(b \cdot) \cdot (x \cdot)]^{(n \cdot)}, x_Symbol] \rightarrow \operatorname{Dist}[(\pi \cdot b) / (2 d), \operatorname{Subst}[\int x^n, x, \operatorname{FresnelC}[b x]], x] /; \operatorname{FreeQ}[\{b, d, n\}, x] \ \&\& \operatorname{EqQ}[d^2, (\pi^2 b^4) / 4]$$

Rule 30

$$\operatorname{Int}[(x \cdot)^{(m \cdot)}, x_Symbol] \rightarrow \operatorname{Simp}[x^{(m+1)} / (m+1), x] /; \operatorname{FreeQ}[m, x] \ \&\& \operatorname{NeQ}[m, -1]$$

Rule 3375

$$\operatorname{Int}[\operatorname{Sin}[(d \cdot) \cdot (x \cdot)^{(n \cdot)}] / (x \cdot), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{SinIntegral}[d x^n] / n, x] /; \operatorname{FreeQ}[\{d, n\}, x]$$

Rule 3380

$$\operatorname{Int}[(a \cdot + \operatorname{Cos}[c \cdot] + (d \cdot) \cdot (x \cdot)^{(n \cdot)}) \cdot (b \cdot)^{(p \cdot)} \cdot (x \cdot)^{(m \cdot)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\int x^{(\operatorname{Simplify}[(m+1)/n] - 1) \cdot (a + b \operatorname{Cos}[c + d x])^p}, x], x, x^n], x] /; \operatorname{FreeQ}[\{a, b, c, d, m, n, p\}, x] \ \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]] \ \&\& (\operatorname{EqQ}[p, 1] \ \|\ \operatorname{EqQ}[m, n-1] \ \|\ (\operatorname{IntegerQ}[p] \ \&\& \operatorname{GtQ}[\operatorname{Simplify}[(m+1)/n], 0]))$$

Rule 3297

$$\operatorname{Int}[(c \cdot + (d \cdot) \cdot (x \cdot))^{(m \cdot)} \cdot \sin[(e \cdot) + (f \cdot) \cdot (x \cdot)], x_Symbol] \rightarrow \operatorname{Simp}[(c + d x)^{(m+1)} \operatorname{Sin}[e + f x] / (d(m+1)), x] - \operatorname{Dist}[f / (d(m+1)), \int (c + d x)^{(m+1)} \operatorname{Cos}[e + f x], x] /; \operatorname{FreeQ}[\{c, d, e, f\}, x] \ \&\& \operatorname{LtQ}[m, -1]$$

Rule 3299

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[SinIntegrals[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned}
 \int \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}} dx &= -\frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{1}{18}b \int \frac{\sin(b^2\pi x^2)}{x^9} dx + \frac{1}{9}(b^2\pi) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{x^8} dx \\
 &= -\frac{b^3\pi}{756x^6} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{63x^7} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{1}{36}b \operatorname{Subst}\left(\int \frac{\sin(b^2\pi x)}{x^5} dx, x, x^2\right) \\
 &= -\frac{b^3\pi}{756x^6} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{63x^7} - \frac{C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{b^4\pi^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5} - \frac{b \sin(b^2\pi x^2)}{315x^5} \\
 &= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi \cos(b^2\pi x^2)}{3024x^6} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{63x^7} + \frac{b^6\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{945x^3} \\
 &= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi \cos(b^2\pi x^2)}{3024x^6} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{63x^7} + \frac{b^6\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{945x^3} \\
 &= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi \cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3 \cos(b^2\pi x^2)}{2016x^2} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{63x^7} + \frac{b^6\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{945x^3} \\
 &= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi \cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3 \cos(b^2\pi x^2)}{2016x^2} - \frac{b^2\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{63x^7} + \frac{b^6\pi^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)}{945x^3}
 \end{aligned}$$

Mathematica [A] time = 0.0174078, size = 278, normalized size = 1.

$$-\frac{\pi^4 b^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{945x} + \frac{\pi^2 b^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{315x^5} - \frac{\operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9} + \frac{\pi^3 b^6 \operatorname{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{945x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]

[Out] $-(b^3\pi)/(756x^6) + (b^7\pi^3)/(3780x^2) - (11b^3\pi\cos[b^2\pi x^2])/(3024x^6) + (5b^7\pi^3\cos[b^2\pi x^2])/(2016x^2) - (b^2\pi\cos[(b^2\pi x^2)/2]*\text{FresnelC}[b*x])/(63x^7) + (b^6\pi^3\cos[(b^2\pi x^2)/2]*\text{FresnelC}[b*x])/(945x^3) + (b^9\pi^5*\text{FresnelC}[b*x]^2)/1890 - (\text{FresnelC}[b*x]*\text{Sin}[(b^2\pi x^2)/2])/(9x^9) + (b^4\pi^2*\text{FresnelC}[b*x]*\text{Sin}[(b^2\pi x^2)/2])/(315x^5) - (b^8\pi^4*\text{FresnelC}[b*x]*\text{Sin}[(b^2\pi x^2)/2])/(945x) - (b*\text{Sin}[b^2\pi x^2])/(144x^8) + (67b^5\pi^2*\text{Sin}[b^2\pi x^2])/(30240x^4) + (83b^9\pi^4*\text{SinIntegral}[b^2\pi x^2])/30240$

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{\text{FresnelC}(bx)}{x^{10}} \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)

[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")

[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{fresnelc}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65 else #result contains complex but optimal is not
66     if debug then
67         print("result contains complex but optimal is not");
68     fi;
69     return "C";
70 end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do not
as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+') or type(expn,'*') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```

```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                   asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                   asinh,acosh,atanh,acoth,asech,acsch
25                   ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                   fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                   gamma,loggamma,digamma,zeta,polylog,LambertW,
31                   elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                   ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```



```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```

```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```